

# Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

0-Independent-test-suites/5-Hearn-Problems

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# CHAPTER 1

## INTRODUCTION

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This report gives the result of running the computer algebra independent integration test. The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [ 284 ]. This is test number [ 5 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System      | % solved       | % Failed     |
|-------------|----------------|--------------|
| Mathematica | 100.00 ( 284 ) | 0.00 ( 0 )   |
| Maple       | 99.30 ( 282 )  | 0.70 ( 2 )   |
| Fricas      | 98.94 ( 281 )  | 1.06 ( 3 )   |
| Rubi        | 98.24 ( 279 )  | 1.76 ( 5 )   |
| Mupad       | 95.07 ( 270 )  | 4.93 ( 14 )  |
| Giac        | 94.72 ( 269 )  | 5.28 ( 15 )  |
| Sympy       | 89.44 ( 254 )  | 10.56 ( 30 ) |
| Maxima      | 88.38 ( 251 )  | 11.62 ( 33 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|-------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| A     | Integral was solved and antiderivative is optimal in quality and leaf size.                                                                                                                                                                                                                                                                                                                                                                                               |
| B     | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.                                                                                                                                                                                                                                                                                                                                    |
| C     | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol> |
| F     | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.                                                                                                                                                                                                                                                                                                        |

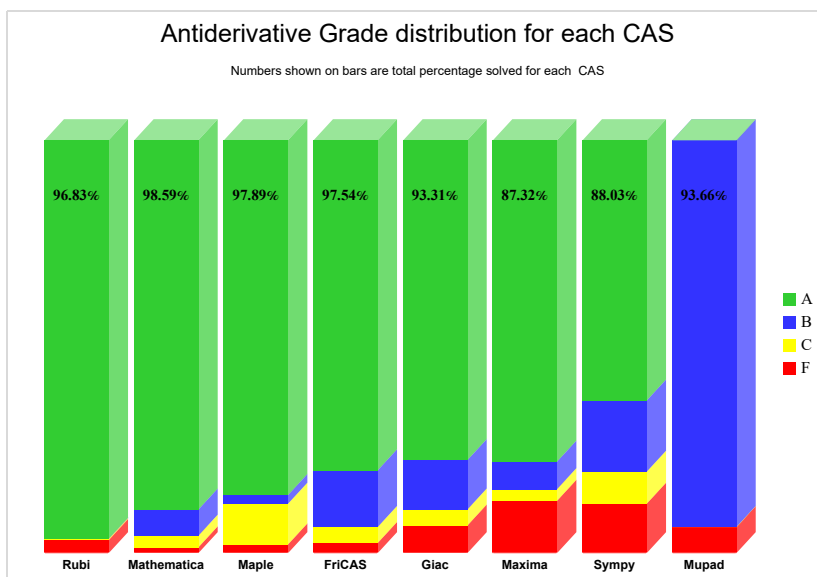
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

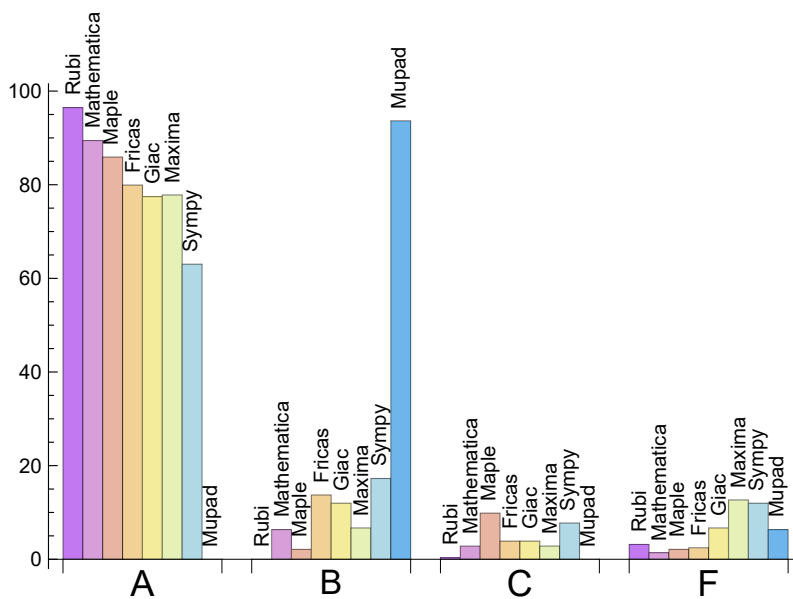
| System      | % A grade | % B grade | % C grade | % F grade |
|-------------|-----------|-----------|-----------|-----------|
| Rubi        | 96.479    | 0.000     | 0.352     | 3.169     |
| Mathematica | 89.437    | 6.338     | 2.817     | 1.408     |
| Maple       | 85.915    | 2.113     | 9.859     | 2.113     |
| Fricas      | 79.930    | 13.732    | 3.873     | 2.465     |
| Maxima      | 77.817    | 6.690     | 2.817     | 12.676    |
| Giac        | 77.465    | 11.972    | 3.873     | 6.690     |
| Sympy       | 63.028    | 17.254    | 7.746     | 11.972    |
| Mupad       | 0.000     | 93.662    | 0.000     | 6.338     |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima

and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

| System      | Number failed | Percentage normal failure | Percentage time-out failure | Percentage exception failure |
|-------------|---------------|---------------------------|-----------------------------|------------------------------|
| Mathematica | 0             | 0.00                      | 0.00                        | 0.00                         |
| Maple       | 2             | 50.00                     | 50.00                       | 0.00                         |
| Fricas      | 3             | 100.00                    | 0.00                        | 0.00                         |
| Rubi        | 5             | 100.00                    | 0.00                        | 0.00                         |
| Mupad       | 14            | 0.00                      | 100.00                      | 0.00                         |
| Giac        | 15            | 100.00                    | 0.00                        | 0.00                         |
| Sympy       | 30            | 86.67                     | 10.00                       | 3.33                         |
| Maxima      | 33            | 66.67                     | 0.00                        | 33.33                        |

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.



| System      | Mean time (sec) |
|-------------|-----------------|
| Mupad       | 0.17            |
| Maple       | 0.18            |
| Rubi        | 0.20            |
| Maxima      | 0.22            |
| Fricas      | 0.25            |
| Giac        | 0.29            |
| Mathematica | 0.34            |
| Sympy       | 2.34            |

Table 1.5: Time performance for each CAS

| System      | Mean size | Normalized mean | Median size | Normalized median |
|-------------|-----------|-----------------|-------------|-------------------|
| Mupad       | 35.80     | 1.05            | 18.00       | 0.86              |
| Maxima      | 35.94     | 1.30            | 19.00       | 0.88              |
| Rubi        | 41.24     | 1.03            | 22.00       | 1.00              |
| Fricas      | 44.57     | 1.36            | 22.00       | 1.00              |
| Maple       | 51.18     | 0.99            | 20.00       | 0.87              |
| Mathematica | 65.69     | 1.31            | 23.00       | 1.00              |
| Sympy       | 88.22     | 1.99            | 22.00       | 1.00              |
| Giac        | 105.03    | 1.78            | 23.00       | 1.00              |

Table 1.6: Leaf size performance for each CAS

## 1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the  $y$  axis is the percentage solved which Rubi itself needed the number of rules given the  $x$  axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

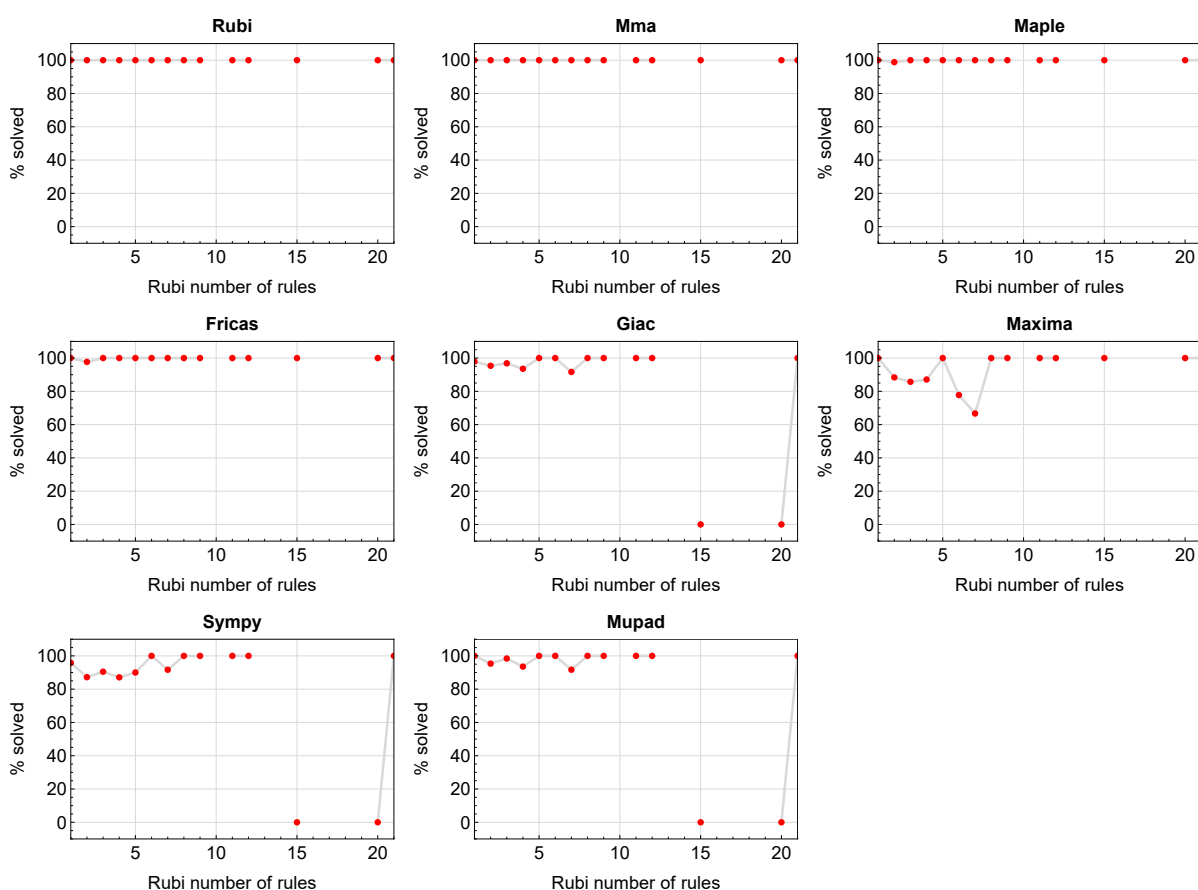


Figure 1.1: Solving statistics per number of Rubi rules used

## 1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

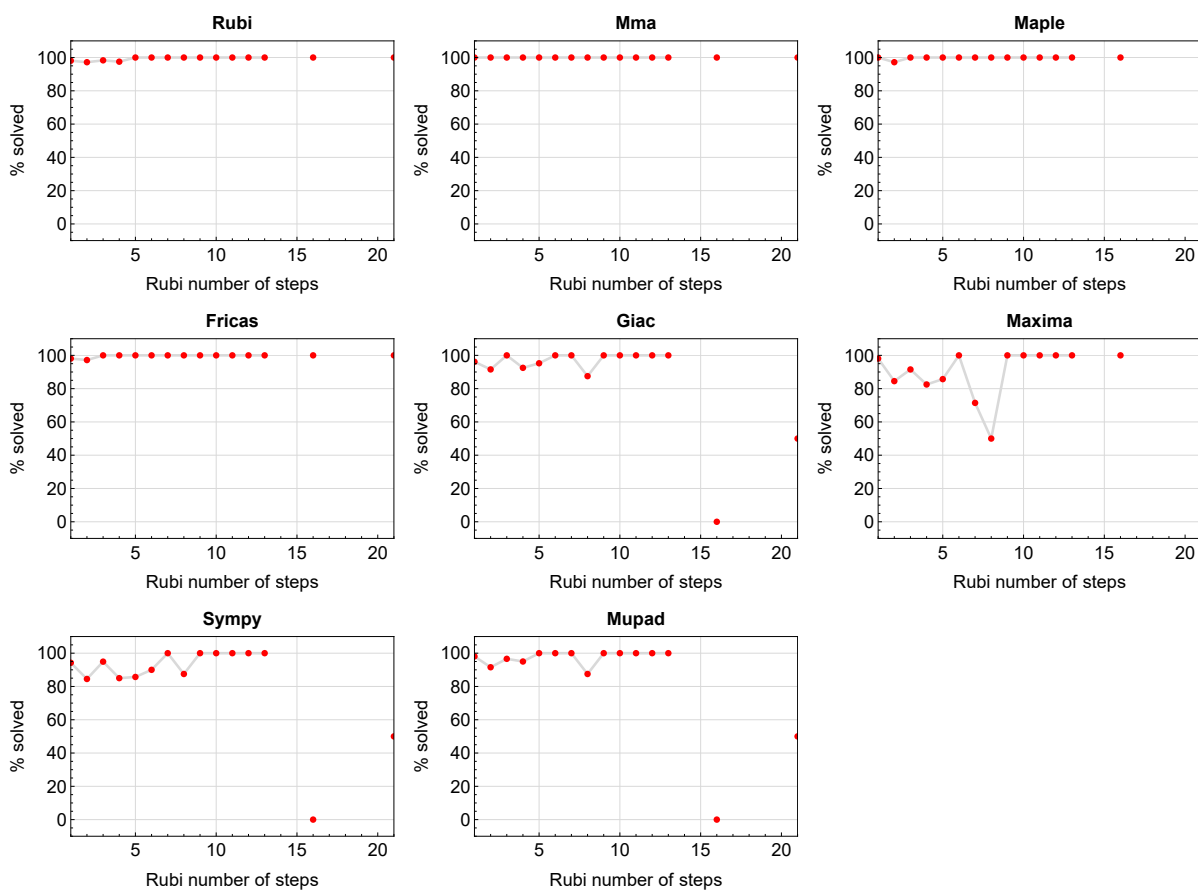


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the percentage of solved integrals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

## 1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

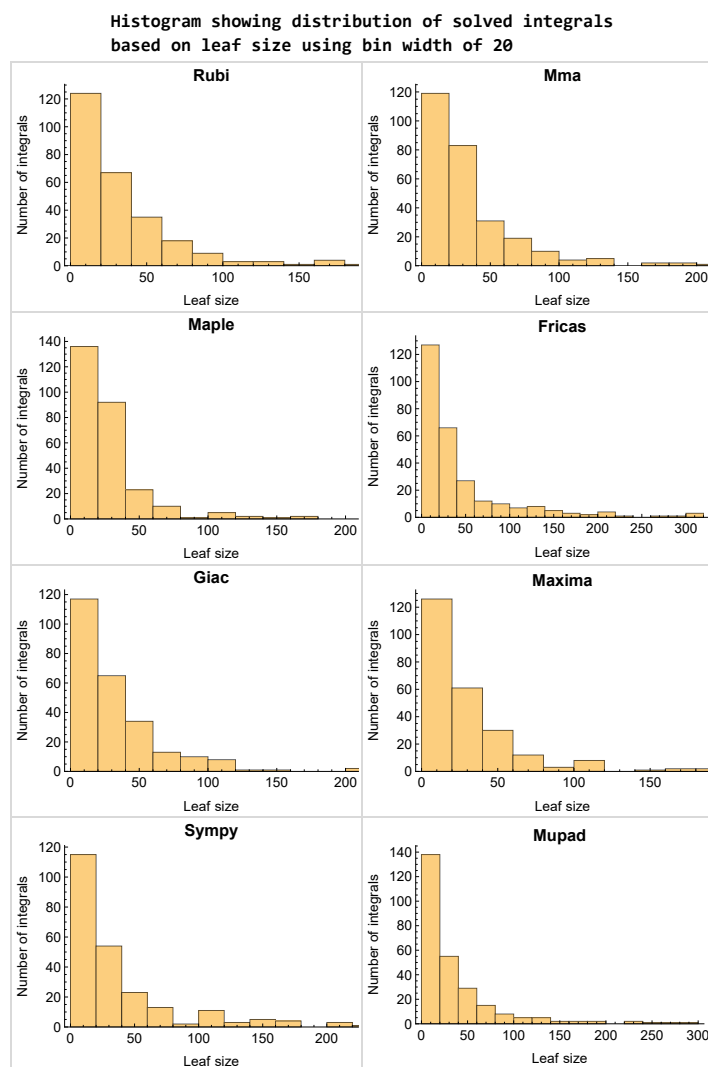


Figure 1.3: Solved integrals based on leaf size distribution

## 1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

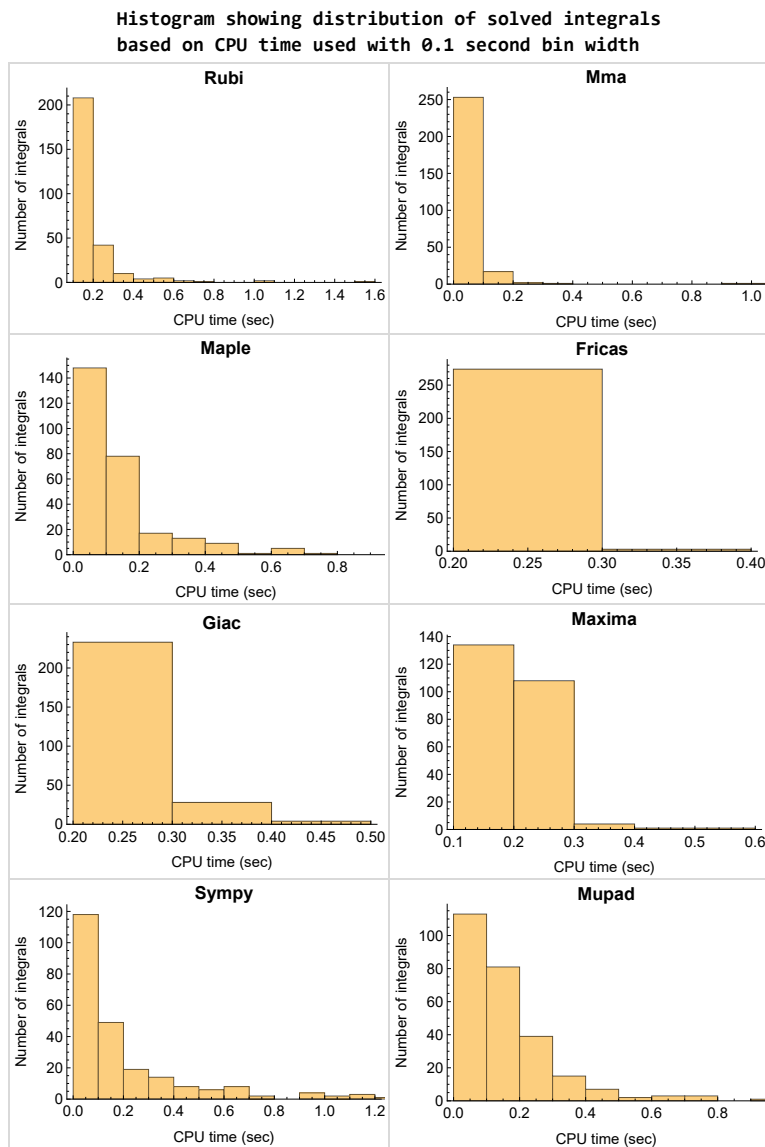


Figure 1.4: Solved integrals histogram based on CPU time used

## 1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

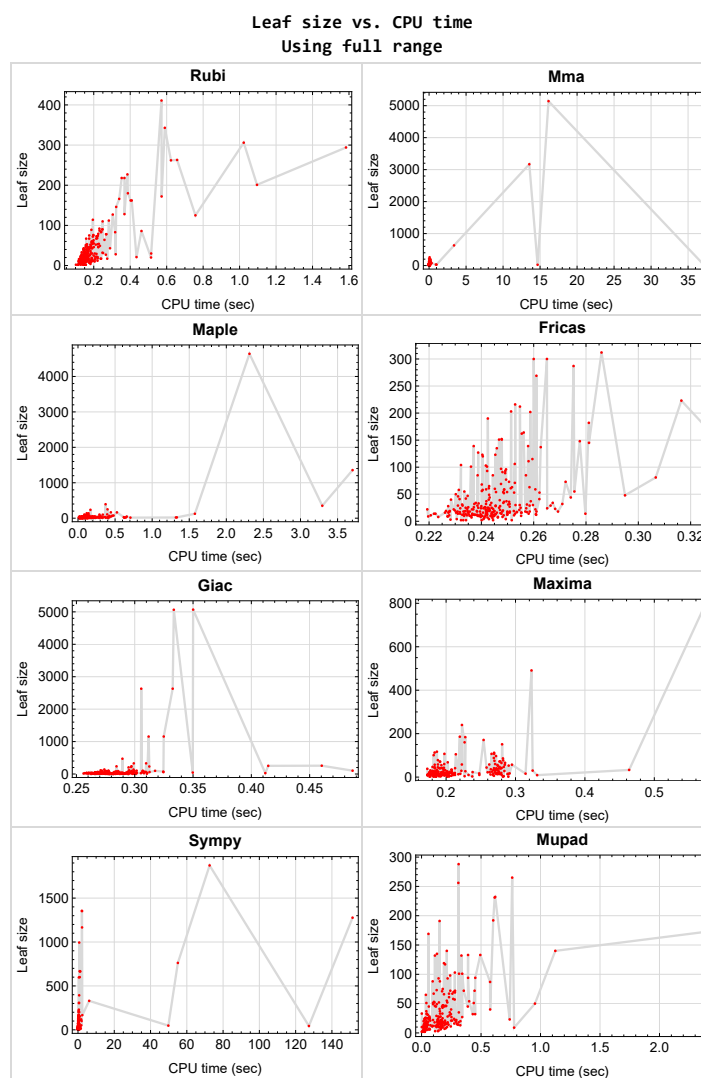


Figure 1.5: Leaf size vs. CPU time. Full range

## 1.9 list of integrals with no known antiderivative

{75, 145, 170, 273}

## 1.10 List of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {145}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

**Rubi** {240}

**Mathematica** {278, 279, 281}

**Maple** {281, 282}

**Maxima** Verification phase not currently implemented.

**Fricas** Verification phase not currently implemented.

**Sympy** Verification phase not currently implemented.

**Giac** Verification phase not currently implemented.

**Mupad** Verification phase not currently implemented.

## 1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.13 Verification

A verification phase was applied on the result of integration for **Rubi** and **Mathematica**.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.14 Important notes about some of the results

### 1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.



The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'  
'display2d : false'  
'domain : complex'  
'keepfloat : true'  
'load(to_poly_solve)'  
'load(simplify_sum)'  
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib  
maxima_lib.set('extra_definite_integration_methods', '[]')  
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

### 1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

### 1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at [https://ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

### 1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

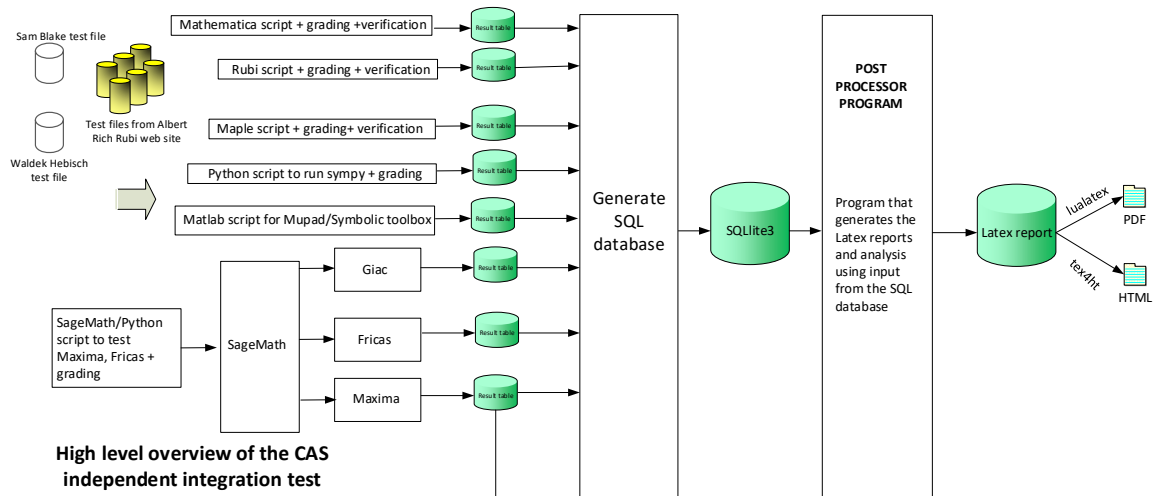
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives  $\sin(x)^2/2$

# 1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



**High level overview of the CAS independent integration test build system**

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in Rubi Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,...}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntax

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Design v0.01

# CHAPTER 2

## DETAILED SUMMARY TABLES OF RESULTS

|     |                                                                           |    |
|-----|---------------------------------------------------------------------------|----|
| 2.1 | List of integrals sorted by grade for each CAS . . . . .                  | 21 |
| 2.2 | Detailed conclusion table per each integral for all CAS systems . . . . . | 26 |
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## 2.1 List of integrals sorted by grade for each CAS

|       |                  |    |
|-------|------------------|----|
| 2.1.1 | Rubi . . . . .   | 21 |
| 2.1.2 | Mma . . . . .    | 22 |
| 2.1.3 | Maple . . . . .  | 22 |
| 2.1.4 | Fricas . . . . . | 23 |
| 2.1.5 | Maxima . . . . . | 23 |
| 2.1.6 | Giac . . . . .   | 24 |
| 2.1.7 | Mupad . . . . .  | 24 |
| 2.1.8 | Sympy . . . . .  | 25 |

### 2.1.1 Rubi

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 276, 277, 280, 282, 283 }

**B grade** { }

**C grade** { 225 }

**F normal fail** { 169, 278, 279, 281, 284 }

**F(-1) timeout fail** { }

**F(-2) exception fail** { }

### 2.1.2 Mma

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 112, 113, 114, 115, 116, 117, 118, 119, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 246, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 270, 271, 272, 274, 275, 276, 277, 280, 282, 283, 284 }

**B grade** { 52, 79, 82, 108, 110, 111, 120, 121, 190, 199, 202, 235, 236, 244, 247, 256, 260, 269 }

**C grade** { 20, 44, 45, 51, 245, 278, 279, 281 }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

### 2.1.3 Maple

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 41, 46, 52, 53, 54, 55, 56, 57, 58, 59, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 276, 277, 281, 283, 284 }

**B grade** { 60, 61, 110, 237, 257, 280 }

**C grade** { 18, 19, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 47, 48, 49, 50, 51, 95, 138, 139, 140, 141, 203, 259, 279, 282 }

**F normal fail** { 86 }

**F(-1) timeout fail** { 278 }

**F(-2) exception fail** { }

### 2.1.4 Fricas

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 46, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 83, 84, 85, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 196, 198, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 248, 249, 250, 251, 252, 253, 255, 258, 259, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 274, 275, 276, 277, 279, 282, 283 }

**B grade** { 39, 40, 41, 42, 43, 78, 79, 80, 81, 82, 90, 110, 111, 112, 127, 128, 133, 194, 195, 199, 204, 220, 222, 227, 228, 235, 236, 244, 245, 246, 247, 254, 256, 260, 268, 269, 278, 280, 284 }

**C grade** { 18, 19, 36, 37, 38, 44, 45, 49, 50, 51, 197 }

**F normal fail** { 86, 257, 281 }

**F(-1) timeout fail** { }

**F(-2) exception fail** { }

### 2.1.5 Maxima

**A grade** { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 41, 46, 47, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 106, 107, 108, 109, 110, 113, 114, 115, 116, 117, 118, 119, 120, 121, 124, 125, 126, 127, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 162, 164, 165, 167, 169, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 198, 199, 200, 201, 202, 205, 206, 207, 208, 209, 210, 215, 216, 217, 218, 219, 220, 221, 223, 224, 227, 228, 229, 230, 231, 232, 233, 234, 237, 238, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 276, 277, 283 }

**B grade** { 32, 81, 82, 111, 112, 128, 129, 130, 146, 191, 194, 195, 204, 222, 225, 226, 241, 256, 280 }



**C grade** { 102, 103, 104, 105, 145, 166, 168, 197 }

**F normal fail** { 39, 40, 42, 43, 44, 45, 49, 51, 86, 163, 196, 203, 235, 236, 239, 251, 257, 278, 279, 281, 282, 284 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { 8, 122, 123, 147, 160, 161, 176, 211, 212, 213, 214 }

### 2.1.6 Giac

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 80, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 165, 167, 168, 169, 171, 172, 173, 174, 175, 177, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 192, 193, 194, 196, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 244, 248, 249, 250, 251, 252, 253, 254, 255, 258, 259, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 274, 275, 276, 277, 282, 283, 284 }

**B grade** { 11, 23, 32, 67, 79, 81, 82, 110, 111, 112, 118, 127, 164, 176, 178, 179, 190, 195, 198, 199, 202, 220, 222, 228, 235, 236, 245, 246, 247, 256, 260, 268, 269, 280 }

**C grade** { 134, 135, 136, 137, 138, 139, 140, 141, 160, 161, 166 }

**F normal fail** { 56, 63, 86, 128, 129, 162, 163, 197, 200, 201, 203, 257, 278, 279, 281 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

### 2.1.7 Mupad

**A grade** { }

**B grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216,

217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 276, 277, 280, 282, 283, 284 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { 102, 103, 104, 105, 128, 129, 163, 169, 203, 249, 257, 278, 279, 281 }

**F(-2) exception fail** { }

### 2.1.8 Sympy

**A grade** { 1, 2, 3, 4, 5, 6, 10, 11, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 38, 40, 42, 43, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 74, 76, 77, 78, 79, 83, 84, 85, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 111, 114, 116, 118, 119, 120, 121, 124, 125, 126, 127, 130, 132, 144, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 177, 181, 182, 185, 186, 187, 190, 192, 193, 195, 199, 200, 201, 202, 206, 209, 210, 215, 216, 217, 218, 219, 221, 223, 225, 226, 227, 230, 231, 232, 240, 241, 243, 244, 245, 248, 250, 252, 253, 255, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 274, 275, 276, 277, 282, 283, 284 }

**B grade** { 7, 8, 13, 22, 23, 32, 39, 41, 44, 56, 80, 81, 82, 90, 103, 104, 110, 112, 113, 115, 117, 122, 123, 131, 133, 142, 143, 161, 178, 179, 180, 183, 184, 188, 189, 204, 212, 222, 228, 229, 233, 234, 237, 239, 246, 247, 254, 256, 272 }

**C grade** { 9, 14, 31, 50, 70, 72, 73, 134, 135, 136, 137, 138, 139, 140, 141, 175, 191, 194, 203, 205, 220, 224 }

**F normal fail** { 86, 128, 129, 146, 162, 163, 176, 196, 197, 198, 207, 208, 211, 213, 214, 235, 236, 238, 242, 249, 251, 257, 265, 278, 279, 281 }

**F(-1) timedout fail** { 12, 147, 280 }

**F(-2) exception fail** { 160 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column **N.S.** means **normalized size** and is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$ . To make the table fit the page, the name **Mathematica** was abbreviated to **MMA**.

| Problem 1  | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 16      | 16    | 16    | 13    | 12     | 12     | 10    | 12    | 13    |
| N.S.       | 1       | 1.00  | 1.00  | 0.81  | 0.75   | 0.75   | 0.62  | 0.75  | 0.81  |
| time (sec) | N/A     | 0.119 | 0.000 | 0.014 | 0.188  | 0.238  | 0.015 | 0.276 | 0.021 |

| Problem 2  | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 22      | 22    | 22    | 16    | 16     | 16     | 17    | 16    | 15    |
| N.S.       | 1       | 1.00  | 1.00  | 0.73  | 0.73   | 0.73   | 0.77  | 0.73  | 0.68  |
| time (sec) | N/A     | 0.132 | 0.001 | 0.046 | 0.197  | 0.227  | 0.015 | 0.268 | 0.038 |

| Problem 3  | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 22      | 22    | 22    | 16    | 16     | 16     | 15    | 16    | 15    |
| N.S.       | 1       | 1.00  | 1.00  | 0.73  | 0.73   | 0.73   | 0.68  | 0.73  | 0.68  |
| time (sec) | N/A     | 0.134 | 0.001 | 0.047 | 0.186  | 0.230  | 0.015 | 0.278 | 0.030 |

| Problem 4  | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 2       | 2     | 2     | 3     | 2      | 2      | 2     | 3     | 2     |
| N.S.       | 1       | 1.00  | 1.00  | 1.50  | 1.00   | 1.00   | 1.00  | 1.50  | 1.00  |
| time (sec) | N/A     | 0.103 | 0.000 | 0.013 | 0.183  | 0.232  | 0.044 | 0.277 | 0.009 |

| Problem 5  | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 36      | 36    | 24    | 22    | 32     | 46     | 29    | 23    | 22    |
| N.S.       | 1       | 1.00  | 0.67  | 0.61  | 0.89   | 1.28   | 0.81  | 0.64  | 0.61  |
| time (sec) | N/A     | 0.147 | 0.015 | 0.086 | 0.199  | 0.231  | 0.051 | 0.266 | 0.133 |

| Problem 6  | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 32      | 32    | 28    | 25    | 24     | 33     | 24    | 34    | 26    |
| N.S.       | 1       | 1.00  | 0.88  | 0.78  | 0.75   | 1.03   | 0.75  | 1.06  | 0.81  |
| time (sec) | N/A     | 0.150 | 0.011 | 0.079 | 0.189  | 0.239  | 0.065 | 0.282 | 0.034 |

| Problem 7  | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | B     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 40      | 40    | 34    | 41    | 40     | 34     | 144   | 42    | 40    |
| N.S.       | 1       | 1.00  | 0.85  | 1.02  | 1.00   | 0.85   | 3.60  | 1.05  | 1.00  |
| time (sec) | N/A     | 0.172 | 0.016 | 0.117 | 0.184  | 0.238  | 0.542 | 0.271 | 0.248 |

| Problem 8  | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | F(-2)  | A      | B     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 34      | 34    | 38    | 35    | 0      | 120    | 124   | 34    | 46    |
| N.S.       | 1       | 1.00  | 1.12  | 1.03  | 0.00   | 3.53   | 3.65  | 1.00  | 1.35  |
| time (sec) | N/A     | 0.154 | 0.011 | 0.365 | 0.000  | 0.241  | 0.105 | 0.299 | 0.196 |

| Problem 9  | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | C     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 16      | 16    | 16    | 15    | 14     | 14     | 26    | 14    | 14    |
| N.S.       | 1       | 1.00  | 1.00  | 0.94  | 0.88   | 0.88   | 1.62  | 0.88  | 0.88  |
| time (sec) | N/A     | 0.132 | 0.006 | 0.072 | 0.273  | 0.253  | 0.084 | 0.291 | 0.037 |

| Problem 10 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 19      | 21    | 16    | 15    | 14     | 14     | 22    | 14    | 14    |
| N.S.       | 1       | 1.11  | 0.84  | 0.79  | 0.74   | 0.74   | 1.16  | 0.74  | 0.74  |
| time (sec) | N/A     | 0.137 | 0.004 | 0.624 | 0.277  | 0.237  | 0.044 | 0.270 | 0.029 |

| Problem 11 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | B     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 49      | 52    | 35    | 36    | 39     | 71     | 41    | 80    | 49    |
| N.S.       | 1       | 1.06  | 0.71  | 0.73  | 0.80   | 1.45   | 0.84  | 1.63  | 1.00  |
| time (sec) | N/A     | 0.183 | 0.023 | 0.112 | 0.272  | 0.253  | 0.082 | 0.278 | 0.128 |

| Problem 12 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | F(-1) | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 68      | 68    | 62    | 69    | 78     | 81     | 0     | 81    | 87    |
| N.S.       | 1       | 1.00  | 0.91  | 1.01  | 1.15   | 1.19   | 0.00  | 1.19  | 1.28  |
| time (sec) | N/A     | 0.209 | 0.026 | 0.156 | 0.188  | 0.307  | 0.000 | 0.277 | 0.576 |

| Problem 13 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | B     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 47      | 46    | 34    | 33    | 43     | 32     | 121   | 43    | 256   |
| N.S.       | 1       | 0.98  | 0.72  | 0.70  | 0.91   | 0.68   | 2.57  | 0.91  | 5.45  |
| time (sec) | N/A     | 0.151 | 0.010 | 0.145 | 0.197  | 0.242  | 0.350 | 0.266 | 0.309 |

| Problem 14 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | C     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 40      | 40    | 30    | 41    | 40     | 30     | 393   | 40    | 191   |
| N.S.       | 1       | 1.00  | 0.75  | 1.02  | 1.00   | 0.75   | 9.82  | 1.00  | 4.78  |
| time (sec) | N/A     | 0.155 | 0.014 | 0.195 | 0.272  | 0.253  | 0.658 | 0.270 | 0.151 |

| Problem 15 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 27      | 28    | 27    | 20    | 19     | 19     | 19    | 20    | 25    |
| N.S.       | 1       | 1.04  | 1.00  | 0.74  | 0.70   | 0.70   | 0.70  | 0.74  | 0.93  |
| time (sec) | N/A     | 0.144 | 0.004 | 0.095 | 0.280  | 0.234  | 0.057 | 0.274 | 0.045 |

| Problem 16 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 41      | 45    | 40    | 33    | 34     | 34     | 41    | 35    | 46    |
| N.S.       | 1       | 1.10  | 0.98  | 0.80  | 0.83   | 0.83   | 1.00  | 0.85  | 1.12  |
| time (sec) | N/A     | 0.180 | 0.009 | 0.073 | 0.273  | 0.267  | 0.056 | 0.279 | 0.204 |

| Problem 17 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 43      | 43    | 34    | 32    | 31     | 40     | 31    | 36    | 33    |
| N.S.       | 1       | 1.00  | 0.79  | 0.74  | 0.72   | 0.93   | 0.72  | 0.84  | 0.77  |
| time (sec) | N/A     | 0.280 | 0.025 | 0.089 | 0.274  | 0.228  | 0.057 | 0.302 | 0.043 |

| Problem 18 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | C     | A      | C      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 85      | 90    | 64    | 22    | 72     | 61     | 73    | 72    | 33    |
| N.S.       | 1       | 1.06  | 0.75  | 0.26  | 0.85   | 0.72   | 0.86  | 0.85  | 0.39  |
| time (sec) | N/A     | 0.243 | 0.013 | 0.061 | 0.277  | 0.251  | 0.094 | 0.278 | 0.002 |

| Problem 19 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | C     | A      | C      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 85      | 90    | 64    | 22    | 72     | 61     | 73    | 72    | 33    |
| N.S.       | 1       | 1.06  | 0.75  | 0.26  | 0.85   | 0.72   | 0.86  | 0.85  | 0.39  |
| time (sec) | N/A     | 0.244 | 0.011 | 0.076 | 0.278  | 0.243  | 0.067 | 0.276 | 0.175 |

| Problem 20 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | C     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 67      | 71    | 73    | 54    | 53     | 53     | 70    | 53    | 47    |
| N.S.       | 1       | 1.06  | 1.09  | 0.81  | 0.79   | 0.79   | 1.04  | 0.79  | 0.70  |
| time (sec) | N/A     | 0.211 | 0.041 | 0.050 | 0.290  | 0.253  | 0.094 | 0.284 | 0.182 |

| Problem 21 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 18      | 18    | 18    | 19    | 18     | 20     | 20    | 18    | 18    |
| N.S.       | 1       | 1.00  | 1.00  | 1.06  | 1.00   | 1.11   | 1.11  | 1.00  | 1.00  |
| time (sec) | N/A     | 0.127 | 0.005 | 0.076 | 0.203  | 0.242  | 0.017 | 0.277 | 0.220 |

| Problem 22 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | B     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 39      | 39    | 33    | 36    | 42     | 53     | 201   | 76    | 94    |
| N.S.       | 1       | 1.00  | 0.85  | 0.92  | 1.08   | 1.36   | 5.15  | 1.95  | 2.41  |
| time (sec) | N/A     | 0.159 | 0.083 | 0.076 | 0.194  | 0.262  | 0.299 | 0.276 | 0.391 |

| Problem 23 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | B     | B     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 60      | 60    | 57    | 73    | 68     | 96     | 597   | 140   | 192   |
| N.S.       | 1       | 1.00  | 0.95  | 1.22  | 1.13   | 1.60   | 9.95  | 2.33  | 3.20  |
| time (sec) | N/A     | 0.177 | 0.056 | 0.086 | 0.202  | 0.249  | 0.479 | 0.274 | 0.603 |



| Problem 24 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 10      | 10    | 10    | 11    | 10     | 10     | 7     | 11    | 10    |
| N.S.       | 1       | 1.00  | 1.00  | 1.10  | 1.00   | 1.00   | 0.70  | 1.10  | 1.00  |
| time (sec) | N/A     | 0.119 | 0.000 | 0.069 | 0.193  | 0.244  | 0.017 | 0.285 | 0.027 |

| Problem 25 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 12      | 12    | 12    | 13    | 12     | 13     | 10    | 12    | 12    |
| N.S.       | 1       | 1.00  | 1.00  | 1.08  | 1.00   | 1.08   | 0.83  | 1.00  | 1.00  |
| time (sec) | N/A     | 0.126 | 0.002 | 0.064 | 0.186  | 0.242  | 0.056 | 0.271 | 0.116 |

| Problem 26 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 18      | 18    | 18    | 19    | 18     | 17     | 14    | 19    | 18    |
| N.S.       | 1       | 1.00  | 1.00  | 1.06  | 1.00   | 0.94   | 0.78  | 1.06  | 1.00  |
| time (sec) | N/A     | 0.159 | 0.004 | 0.070 | 0.183  | 0.231  | 0.044 | 0.273 | 0.036 |

| Problem 27 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 31      | 31    | 31    | 30    | 29     | 29     | 26    | 30    | 29    |
| N.S.       | 1       | 1.00  | 1.00  | 0.97  | 0.94   | 0.94   | 0.84  | 0.97  | 0.94  |
| time (sec) | N/A     | 0.159 | 0.003 | 0.068 | 0.222  | 0.231  | 0.048 | 0.262 | 0.125 |

| Problem 28 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 18      | 18    | 18    | 16    | 18     | 16     | 10    | 20    | 15    |
| N.S.       | 1       | 1.00  | 1.00  | 0.89  | 1.00   | 0.89   | 0.56  | 1.11  | 0.83  |
| time (sec) | N/A     | 0.131 | 0.004 | 0.074 | 0.194  | 0.255  | 0.094 | 0.268 | 0.134 |

| Problem 29 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 28      | 28    | 28    | 26    | 28     | 26     | 19    | 30    | 25    |
| N.S.       | 1       | 1.00  | 1.00  | 0.93  | 1.00   | 0.93   | 0.68  | 1.07  | 0.89  |
| time (sec) | N/A     | 0.153 | 0.004 | 0.074 | 0.181  | 0.241  | 0.079 | 0.275 | 0.053 |

| Problem 30 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 42      | 42    | 35    | 43    | 45     | 63     | 37    | 52    | 45    |
| N.S.       | 1       | 1.00  | 0.83  | 1.02  | 1.07   | 1.50   | 0.88  | 1.24  | 1.07  |
| time (sec) | N/A     | 0.167 | 0.030 | 0.080 | 0.184  | 0.252  | 0.122 | 0.283 | 0.188 |

| Problem 31 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | C     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 10      | 10    | 10    | 11    | 10     | 10     | 20    | 10    | 10    |
| N.S.       | 1       | 1.00  | 1.00  | 1.10  | 1.00   | 1.00   | 2.00  | 1.00  | 1.00  |
| time (sec) | N/A     | 0.125 | 0.002 | 0.112 | 0.267  | 0.245  | 0.045 | 0.274 | 0.041 |

| Problem 32 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | B      | A      | B     | B     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 10      | 10    | 10    | 19    | 21     | 18     | 15    | 23    | 10    |
| N.S.       | 1       | 1.00  | 1.00  | 1.90  | 2.10   | 1.80   | 1.50  | 2.30  | 1.00  |
| time (sec) | N/A     | 0.121 | 0.002 | 0.076 | 0.184  | 0.238  | 0.053 | 0.295 | 0.155 |

| Problem 33 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | C     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 78      | 87    | 66    | 24    | 66     | 63     | 78    | 57    | 72    |
| N.S.       | 1       | 1.12  | 0.85  | 0.31  | 0.85   | 0.81   | 1.00  | 0.73  | 0.92  |
| time (sec) | N/A     | 0.229 | 0.015 | 0.076 | 0.285  | 0.258  | 0.177 | 0.283 | 0.282 |

| Problem 34 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | C     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 74      | 73    | 65    | 22    | 56     | 68     | 71    | 57    | 72    |
| N.S.       | 1       | 0.99  | 0.88  | 0.30  | 0.76   | 0.92   | 0.96  | 0.77  | 0.97  |
| time (sec) | N/A     | 0.216 | 0.013 | 0.073 | 0.274  | 0.240  | 0.140 | 0.298 | 0.355 |

| Problem 35 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | C     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 115     | 110   | 89    | 29    | 97     | 300    | 20    | 104   | 101   |
| N.S.       | 1       | 0.96  | 0.77  | 0.25  | 0.84   | 2.61   | 0.17  | 0.90  | 0.88  |
| time (sec) | N/A     | 0.241 | 0.021 | 0.080 | 0.272  | 0.265  | 0.068 | 0.272 | 0.315 |

| Problem 36 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | C     | A      | C      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 35      | 35    | 43    | 22    | 34     | 59     | 46    | 39    | 20    |
| N.S.       | 1       | 1.00  | 1.23  | 0.63  | 0.97   | 1.69   | 1.31  | 1.11  | 0.57  |
| time (sec) | N/A     | 0.140 | 0.013 | 0.076 | 0.270  | 0.261  | 0.183 | 0.288 | 0.162 |

| Problem 37 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | C     | A      | C      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 35      | 35    | 43    | 24    | 41     | 59     | 48    | 39    | 18    |
| N.S.       | 1       | 1.00  | 1.23  | 0.69  | 1.17   | 1.69   | 1.37  | 1.11  | 0.51  |
| time (sec) | N/A     | 0.144 | 0.011 | 0.082 | 0.269  | 0.242  | 0.153 | 0.301 | 0.174 |

| Problem 38 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | C     | A      | C      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 171     | 166   | 120   | 24    | 151    | 85     | 151   | 95    | 45    |
| N.S.       | 1       | 0.97  | 0.70  | 0.14  | 0.88   | 0.50   | 0.88  | 0.56  | 0.26  |
| time (sec) | N/A     | 0.332 | 0.036 | 0.075 | 0.281  | 0.257  | 0.234 | 0.300 | 0.121 |

| Problem 39 | Optimal | Rubi  | MMA   | Maple | Maxima   | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|----------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | C     | <b>F</b> | B      | B     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD      | TBD    | TBD   | TBD   | TBD   |
| size       | 73      | 75    | 68    | 35    | 0        | 151    | 146   | 74    | 93    |
| N.S.       | 1       | 1.03  | 0.93  | 0.48  | 0.00     | 2.07   | 2.00  | 1.01  | 1.27  |
| time (sec) | N/A     | 0.197 | 0.038 | 0.079 | 0.000    | 0.247  | 0.258 | 0.297 | 0.219 |

| Problem 40 | Optimal | Rubi  | MMA   | Maple | Maxima   | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|----------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | C     | <b>F</b> | B      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD      | TBD    | TBD   | TBD   | TBD   |
| size       | 73      | 75    | 68    | 35    | 0        | 151    | 24    | 74    | 93    |
| N.S.       | 1       | 1.03  | 0.93  | 0.48  | 0.00     | 2.07   | 0.33  | 1.01  | 1.27  |
| time (sec) | N/A     | 0.170 | 0.020 | 0.070 | 0.000    | 0.248  | 0.214 | 0.324 | 0.143 |

| Problem 41 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | B      | B     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 72      | 72    | 83    | 54    | 75     | 91     | 158   | 81    | 67    |
| N.S.       | 1       | 1.00  | 1.15  | 0.75  | 1.04   | 1.26   | 2.19  | 1.12  | 0.93  |
| time (sec) | N/A     | 0.186 | 0.024 | 0.062 | 0.270  | 0.248  | 0.184 | 0.285 | 0.102 |

| Problem 42 | Optimal | Rubi  | MMA   | Maple | Maxima   | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|----------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | C     | <b>F</b> | B      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD      | TBD    | TBD   | TBD   | TBD   |
| size       | 67      | 67    | 67    | 33    | 0        | 123    | 24    | 101   | 98    |
| N.S.       | 1       | 1.00  | 1.00  | 0.49  | 0.00     | 1.84   | 0.36  | 1.51  | 1.46  |
| time (sec) | N/A     | 0.171 | 0.028 | 0.077 | 0.000    | 0.240  | 0.194 | 0.307 | 0.240 |

| Problem 43 | Optimal | Rubi  | MMA   | Maple | Maxima   | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|----------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | C     | <b>F</b> | B      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD      | TBD    | TBD   | TBD   | TBD   |
| size       | 67      | 67    | 67    | 33    | 0        | 123    | 92    | 51    | 117   |
| N.S.       | 1       | 1.00  | 1.00  | 0.49  | 0.00     | 1.84   | 1.37  | 0.76  | 1.75  |
| time (sec) | N/A     | 0.173 | 0.015 | 0.072 | 0.000    | 0.245  | 0.137 | 0.297 | 0.198 |

| Problem 44 | Optimal | Rubi  | MMA   | Maple | Maxima   | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|----------|--------|-------|-------|-------|
| grade      | N/A     | A     | C     | C     | <b>F</b> | C      | B     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD      | TBD    | TBD   | TBD   | TBD   |
| size       | 196     | 218   | 91    | 31    | 0        | 139    | 994   | 252   | 61    |
| N.S.       | 1       | 1.11  | 0.46  | 0.16  | 0.00     | 0.71   | 5.07  | 1.29  | 0.31  |
| time (sec) | N/A     | 0.361 | 0.041 | 0.139 | 0.000    | 0.258  | 0.681 | 0.414 | 0.231 |

| Problem 45 | Optimal | Rubi  | MMA   | Maple | Maxima   | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|----------|--------|-------|-------|-------|
| grade      | N/A     | A     | C     | C     | <b>F</b> | C      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD      | TBD    | TBD   | TBD   | TBD   |
| size       | 196     | 218   | 91    | 35    | 0        | 139    | 24    | 256   | 132   |
| N.S.       | 1       | 1.11  | 0.46  | 0.18  | 0.00     | 0.71   | 0.12  | 1.31  | 0.67  |
| time (sec) | N/A     | 0.352 | 0.053 | 0.131 | 0.000    | 0.237  | 0.343 | 0.460 | 0.112 |

| Problem 46 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 73      | 79    | 75    | 66    | 65     | 65     | 83    | 67    | 88    |
| N.S.       | 1       | 1.08  | 1.03  | 0.90  | 0.89   | 0.89   | 1.14  | 0.92  | 1.21  |
| time (sec) | N/A     | 0.220 | 0.011 | 0.106 | 0.280  | 0.243  | 0.132 | 0.309 | 0.093 |

| Problem 47 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | C     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 138     | 127   | 122   | 22    | 112    | 127    | 14    | 114   | 140   |
| N.S.       | 1       | 0.92  | 0.88  | 0.16  | 0.81   | 0.92   | 0.10  | 0.83  | 1.01  |
| time (sec) | N/A     | 0.301 | 0.026 | 0.178 | 0.269  | 0.239  | 0.318 | 0.295 | 0.212 |

| Problem 48 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | C     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 138     | 146   | 115   | 22    | 107    | 164    | 14    | 107   | 135   |
| N.S.       | 1       | 1.06  | 0.83  | 0.16  | 0.78   | 1.19   | 0.10  | 0.78  | 0.98  |
| time (sec) | N/A     | 0.325 | 0.019 | 0.218 | 0.266  | 0.256  | 0.138 | 0.277 | 0.130 |

| Problem 49 | Optimal | Rubi  | MMA   | Maple | Maxima   | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|----------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | C     | <b>F</b> | C      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD      | TBD    | TBD   | TBD   | TBD   |
| size       | 339     | 343   | 209   | 22    | 0        | 135    | 14    | 239   | 288   |
| N.S.       | 1       | 1.01  | 0.62  | 0.06  | 0.00     | 0.40   | 0.04  | 0.71  | 0.85  |
| time (sec) | N/A     | 0.579 | 0.007 | 0.085 | 0.000    | 0.246  | 1.340 | 0.297 | 0.311 |

| Problem 50 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy   | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|---------|-------|-------|
| grade      | N/A     | A     | A     | C     | A      | C      | C       | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD     | TBD   | TBD   |
| size       | 97      | 112   | 98    | 37    | 88     | 77     | 44      | 90    | 45    |
| N.S.       | 1       | 1.15  | 1.01  | 0.38  | 0.91   | 0.79   | 0.45    | 0.93  | 0.46  |
| time (sec) | N/A     | 0.279 | 0.025 | 0.125 | 0.269  | 0.249  | 127.316 | 0.286 | 0.173 |

| Problem 51 | Optimal | Rubi  | MMA   | Maple | Maxima   | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|----------|--------|-------|-------|-------|
| grade      | N/A     | A     | C     | C     | <b>F</b> | C      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD      | TBD    | TBD   | TBD   | TBD   |
| size       | 275     | 411   | 42    | 30    | 0        | 101    | 165   | 205   | 53    |
| N.S.       | 1       | 1.49  | 0.15  | 0.11  | 0.00     | 0.37   | 0.60  | 0.75  | 0.19  |
| time (sec) | N/A     | 0.561 | 0.010 | 0.073 | 0.000    | 0.242  | 0.179 | 0.295 | 0.104 |

| Problem 52 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | B     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 49      | 55    | 260   | 41    | 40     | 40     | 46    | 40    | 52    |
| N.S.       | 1       | 1.12  | 5.31  | 0.84  | 0.82   | 0.82   | 0.94  | 0.82  | 1.06  |
| time (sec) | N/A     | 0.192 | 0.071 | 0.116 | 0.276  | 0.243  | 0.109 | 0.289 | 0.217 |

| Problem 53 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 8       | 8     | 8     | 9     | 8      | 8      | 5     | 8     | 6     |
| N.S.       | 1       | 1.00  | 1.00  | 1.12  | 1.00   | 1.00   | 0.62  | 1.00  | 0.75  |
| time (sec) | N/A     | 0.131 | 0.002 | 0.017 | 0.177  | 0.240  | 0.046 | 0.301 | 0.023 |

| Problem 54 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 17      | 17    | 17    | 14    | 13     | 13     | 12    | 13    | 9     |
| N.S.       | 1       | 1.00  | 1.00  | 0.82  | 0.76   | 0.76   | 0.71  | 0.76  | 0.53  |
| time (sec) | N/A     | 0.135 | 0.000 | 0.017 | 0.179  | 0.237  | 0.061 | 0.285 | 0.033 |

| Problem 55 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 17      | 17    | 17    | 14    | 13     | 13     | 12    | 13    | 9     |
| N.S.       | 1       | 1.00  | 1.00  | 0.82  | 0.76   | 0.76   | 0.71  | 0.76  | 0.53  |
| time (sec) | N/A     | 0.136 | 0.002 | 0.024 | 0.184  | 0.250  | 0.055 | 0.283 | 0.032 |



| Problem 56 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac     | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|----------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | B     | <b>F</b> | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD      | TBD   |
| size       | 26      | 26    | 19    | 19    | 26     | 25     | 56    | 0        | 32    |
| N.S.       | 1       | 1.00  | 0.73  | 0.73  | 1.00   | 0.96   | 2.15  | 0.00     | 1.23  |
| time (sec) | N/A     | 0.148 | 0.006 | 0.045 | 0.183  | 0.255  | 0.391 | 0.000    | 0.247 |

| Problem 57 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 15      | 17    | 15    | 16    | 12     | 15     | 15    | 15    | 12    |
| N.S.       | 1       | 1.13  | 1.00  | 1.07  | 0.80   | 1.00   | 1.00  | 1.00  | 0.80  |
| time (sec) | N/A     | 0.150 | 0.000 | 0.017 | 0.189  | 0.242  | 0.038 | 0.289 | 0.032 |

| Problem 58 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 127     | 172   | 127   | 104   | 71     | 103    | 133   | 103   | 71    |
| N.S.       | 1       | 1.35  | 1.00  | 0.82  | 0.56   | 0.81   | 1.05  | 0.81  | 0.56  |
| time (sec) | N/A     | 0.556 | 0.004 | 0.086 | 0.191  | 0.242  | 0.166 | 0.302 | 0.173 |

| Problem 59 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 8       | 8     | 8     | 7     | 6      | 6      | 5     | 6     | 6     |
| N.S.       | 1       | 1.00  | 1.00  | 0.88  | 0.75   | 0.75   | 0.62  | 0.75  | 0.75  |
| time (sec) | N/A     | 0.135 | 0.003 | 0.027 | 0.191  | 0.244  | 0.048 | 0.298 | 0.057 |

| Problem 60 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | B     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 2       | 2     | 2     | 9     | 3      | 2      | 2     | 3     | 2     |
| N.S.       | 1       | 1.00  | 1.00  | 4.50  | 1.50   | 1.00   | 1.00  | 1.50  | 1.00  |
| time (sec) | N/A     | 0.120 | 0.004 | 0.019 | 0.204  | 0.234  | 0.204 | 0.283 | 0.004 |

| Problem 61 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | B     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 4       | 4     | 4     | 11    | 5      | 4      | 3     | 5     | 4     |
| N.S.       | 1       | 1.00  | 1.00  | 2.75  | 1.25   | 1.00   | 0.75  | 1.25  | 1.00  |
| time (sec) | N/A     | 0.139 | 0.004 | 0.029 | 0.205  | 0.241  | 0.207 | 0.281 | 0.014 |

| Problem 62 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 3       | 3     | 3     | 4     | 3      | 3      | 3     | 4     | 3     |
| N.S.       | 1       | 1.00  | 1.00  | 1.33  | 1.00   | 1.00   | 1.00  | 1.33  | 1.00  |
| time (sec) | N/A     | 0.133 | 0.001 | 0.021 | 0.182  | 0.228  | 0.040 | 0.278 | 0.114 |

| Problem 63 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac     | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|----------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | <b>F</b> | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD      | TBD   |
| size       | 17      | 17    | 17    | 15    | 6      | 19     | 14    | 0        | 17    |
| N.S.       | 1       | 1.00  | 1.00  | 0.88  | 0.35   | 1.12   | 0.82  | 0.00     | 1.00  |
| time (sec) | N/A     | 0.193 | 0.023 | 0.033 | 0.207  | 0.238  | 0.255 | 0.000    | 0.031 |

| Problem 64 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 12      | 12    | 12    | 13    | 12     | 12     | 15    | 12    | 22    |
| N.S.       | 1       | 1.00  | 1.00  | 1.08  | 1.00   | 1.00   | 1.25  | 1.00  | 1.83  |
| time (sec) | N/A     | 0.143 | 0.003 | 0.053 | 0.177  | 0.261  | 0.420 | 0.279 | 0.161 |

| Problem 65 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 28      | 46    | 28    | 25    | 25     | 25     | 22    | 24    | 21    |
| N.S.       | 1       | 1.64  | 1.00  | 0.89  | 0.89   | 0.89   | 0.79  | 0.86  | 0.75  |
| time (sec) | N/A     | 0.174 | 0.003 | 0.027 | 0.186  | 0.234  | 0.052 | 0.276 | 0.143 |

| Problem 66 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 54      | 59    | 53    | 47    | 47     | 47     | 44    | 47    | 47    |
| N.S.       | 1       | 1.09  | 0.98  | 0.87  | 0.87   | 0.87   | 0.81  | 0.87  | 0.87  |
| time (sec) | N/A     | 0.190 | 0.009 | 0.074 | 0.180  | 0.235  | 0.060 | 0.285 | 0.178 |

| Problem 67 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | B     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 29      | 29    | 27    | 30    | 38     | 34     | 24    | 138   | 35    |
| N.S.       | 1       | 1.00  | 0.93  | 1.03  | 1.31   | 1.17   | 0.83  | 4.76  | 1.21  |
| time (sec) | N/A     | 0.154 | 0.010 | 0.096 | 0.173  | 0.248  | 0.133 | 0.299 | 0.237 |

| Problem 68 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 46      | 50    | 46    | 39    | 44     | 39     | 42    | 58    | 66    |
| N.S.       | 1       | 1.09  | 1.00  | 0.85  | 0.96   | 0.85   | 0.91  | 1.26  | 1.43  |
| time (sec) | N/A     | 0.181 | 0.012 | 0.050 | 0.183  | 0.247  | 0.115 | 0.299 | 0.188 |

| Problem 69 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 59      | 63    | 59    | 50    | 57     | 49     | 54    | 94    | 75    |
| N.S.       | 1       | 1.07  | 1.00  | 0.85  | 0.97   | 0.83   | 0.92  | 1.59  | 1.27  |
| time (sec) | N/A     | 0.196 | 0.015 | 0.059 | 0.184  | 0.243  | 0.101 | 0.275 | 0.178 |

| Problem 70 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | C     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 23      | 24    | 23    | 24    | 23     | 23     | 36    | 23    | 23    |
| N.S.       | 1       | 1.04  | 1.00  | 1.04  | 1.00   | 1.00   | 1.57  | 1.00  | 1.00  |
| time (sec) | N/A     | 0.148 | 0.003 | 0.079 | 0.263  | 0.243  | 0.073 | 0.282 | 0.062 |

| Problem 71 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 27      | 31    | 26    | 29    | 28     | 23     | 31    | 28    | 51    |
| N.S.       | 1       | 1.15  | 0.96  | 1.07  | 1.04   | 0.85   | 1.15  | 1.04  | 1.89  |
| time (sec) | N/A     | 0.176 | 0.003 | 0.040 | 0.180  | 0.235  | 0.076 | 0.412 | 0.041 |

| Problem 72 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | C     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 44      | 44    | 44    | 37    | 36     | 36     | 53    | 36    | 65    |
| N.S.       | 1       | 1.00  | 1.00  | 0.84  | 0.82   | 0.82   | 1.20  | 0.82  | 1.48  |
| time (sec) | N/A     | 0.187 | 0.003 | 0.102 | 0.264  | 0.251  | 0.102 | 0.311 | 0.036 |

| Problem 73 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | C     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 54      | 54    | 54    | 45    | 44     | 44     | 63    | 44    | 73    |
| N.S.       | 1       | 1.00  | 1.00  | 0.83  | 0.81   | 0.81   | 1.17  | 0.81  | 1.35  |
| time (sec) | N/A     | 0.192 | 0.005 | 0.153 | 0.258  | 0.243  | 0.119 | 0.350 | 0.127 |

| Problem 74 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 25      | 27    | 25    | 32    | 31     | 31     | 29    | 33    | 25    |
| N.S.       | 1       | 1.08  | 1.00  | 1.28  | 1.24   | 1.24   | 1.16  | 1.32  | 1.00  |
| time (sec) | N/A     | 0.156 | 0.003 | 0.043 | 0.177  | 0.250  | 0.079 | 0.270 | 0.069 |

| Problem 75 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | N/A   | N/A   | N/A   | N/A    | N/A    | N/A   | N/A   | N/A   |
| verified   | N/A     | N/A   | N/A   | N/A   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 5       | 5     | 7     | 5     | 28     | 7      | 27    | 7     | 7     |
| N.S.       | 1       | 1.00  | 1.40  | 1.00  | 5.60   | 1.40   | 5.40  | 1.40  | 1.40  |
| time (sec) | N/A     | 0.155 | 0.039 | 0.050 | 0.263  | 0.252  | 3.797 | 0.275 | 0.584 |

| Problem 76 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 4       | 4     | 4     | 5     | 4      | 4      | 3     | 4     | 4     |
| N.S.       | 1       | 1.00  | 1.00  | 1.25  | 1.00   | 1.00   | 0.75  | 1.00  | 1.00  |
| time (sec) | N/A     | 0.138 | 0.001 | 0.056 | 0.190  | 0.243  | 0.036 | 0.279 | 0.020 |

| Problem 77 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 2       | 2     | 2     | 3     | 2      | 2      | 2     | 2     | 2     |
| N.S.       | 1       | 1.00  | 1.00  | 1.50  | 1.00   | 1.00   | 1.00  | 1.00  | 1.00  |
| time (sec) | N/A     | 0.134 | 0.004 | 0.043 | 0.185  | 0.252  | 0.043 | 0.278 | 0.027 |

| Problem 78 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | B      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 5       | 5     | 5     | 6     | 3      | 11     | 5     | 6     | 5     |
| N.S.       | 1       | 1.00  | 1.00  | 1.20  | 0.60   | 2.20   | 1.00  | 1.20  | 1.00  |
| time (sec) | N/A     | 0.138 | 0.003 | 0.030 | 0.177  | 0.249  | 0.036 | 0.265 | 0.029 |

| Problem 79 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | B     | A     | A      | B      | A     | B     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 3       | 3     | 7     | 4     | 3      | 16     | 3     | 17    | 13    |
| N.S.       | 1       | 1.00  | 2.33  | 1.33  | 1.00   | 5.33   | 1.00  | 5.67  | 4.33  |
| time (sec) | N/A     | 0.142 | 0.000 | 0.036 | 0.185  | 0.251  | 0.058 | 0.276 | 0.192 |

| Problem 80 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | B      | B     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 21      | 21    | 21    | 26    | 25     | 37     | 75    | 26    | 27    |
| N.S.       | 1       | 1.00  | 1.00  | 1.24  | 1.19   | 1.76   | 3.57  | 1.24  | 1.29  |
| time (sec) | N/A     | 0.218 | 0.073 | 0.079 | 0.266  | 0.259  | 0.170 | 0.296 | 0.251 |

| Problem 81 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | B      | B      | B     | B     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 3       | 3     | 3     | 7     | 15     | 17     | 15    | 17    | 11    |
| N.S.       | 1       | 1.00  | 1.00  | 2.33  | 5.00   | 5.67   | 5.00  | 5.67  | 3.67  |
| time (sec) | N/A     | 0.140 | 0.002 | 0.116 | 0.175  | 0.254  | 0.066 | 0.278 | 0.129 |

| Problem 82 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | B     | A     | B      | B      | B     | B     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 5       | 5     | 17    | 6     | 15     | 19     | 15    | 17    | 5     |
| N.S.       | 1       | 1.00  | 3.40  | 1.20  | 3.00   | 3.80   | 3.00  | 3.40  | 1.00  |
| time (sec) | N/A     | 0.142 | 0.006 | 0.063 | 0.178  | 0.252  | 0.061 | 0.273 | 0.044 |

| Problem 83 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 14      | 14    | 14    | 11    | 10     | 10     | 10    | 10    | 10    |
| N.S.       | 1       | 1.00  | 1.00  | 0.79  | 0.71   | 0.71   | 0.71  | 0.71  | 0.71  |
| time (sec) | N/A     | 0.145 | 0.001 | 0.078 | 0.204  | 0.239  | 0.019 | 0.276 | 0.029 |

| Problem 84 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 20      | 18    | 20    | 17    | 16     | 16     | 15    | 16    | 16    |
| N.S.       | 1       | 0.90  | 1.00  | 0.85  | 0.80   | 0.80   | 0.75  | 0.80  | 0.80  |
| time (sec) | N/A     | 0.210 | 0.003 | 0.327 | 0.194  | 0.246  | 0.184 | 0.283 | 0.164 |

| Problem 85 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 13      | 13    | 15    | 11    | 11     | 11     | 8     | 11    | 10    |
| N.S.       | 1       | 1.00  | 1.15  | 0.85  | 0.85   | 0.85   | 0.62  | 0.85  | 0.77  |
| time (sec) | N/A     | 0.162 | 0.001 | 0.151 | 0.173  | 0.238  | 0.020 | 0.271 | 0.034 |

| Problem 86 | Optimal | Rubi  | MMA   | Maple    | Maxima   | Fricas   | Sympy    | Giac     | Mupad |
|------------|---------|-------|-------|----------|----------|----------|----------|----------|-------|
| grade      | N/A     | A     | A     | <b>F</b> | <b>F</b> | <b>F</b> | <b>F</b> | <b>F</b> | B     |
| verified   | N/A     | Yes   | Yes   | N/A      | TBD      | TBD      | TBD      | TBD      | TBD   |
| size       | 44      | 44    | 44    | 0        | 0        | 0        | 0        | 0        | 35    |
| N.S.       | 1       | 1.00  | 1.00  | 0.00     | 0.00     | 0.00     | 0.00     | 0.00     | 0.80  |
| time (sec) | N/A     | 0.168 | 0.018 | 0.000    | 0.000    | 0.000    | 0.000    | 0.000    | 0.313 |

| Problem 87 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 19      | 19    | 19    | 16    | 15     | 18     | 17    | 15    | 15    |
| N.S.       | 1       | 1.00  | 1.00  | 0.84  | 0.79   | 0.95   | 0.89  | 0.79  | 0.79  |
| time (sec) | N/A     | 0.183 | 0.004 | 0.154 | 0.179  | 0.258  | 0.204 | 0.280 | 0.046 |



| Problem 88 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 14      | 14    | 14    | 11    | 10     | 10     | 10    | 10    | 10    |
| N.S.       | 1       | 1.00  | 1.00  | 0.79  | 0.71   | 0.71   | 0.71  | 0.71  | 0.71  |
| time (sec) | N/A     | 0.155 | 0.001 | 0.079 | 0.173  | 0.242  | 0.018 | 0.294 | 0.026 |

| Problem 89 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 11      | 11    | 11    | 11    | 9      | 10     | 8     | 9     | 9     |
| N.S.       | 1       | 1.00  | 1.00  | 1.00  | 0.82   | 0.91   | 0.73  | 0.82  | 0.82  |
| time (sec) | N/A     | 0.168 | 0.000 | 0.143 | 0.190  | 0.244  | 0.022 | 0.289 | 0.030 |

| Problem 90 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | B      | B     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 2       | 2     | 2     | 3     | 2      | 7      | 5     | 2     | 2     |
| N.S.       | 1       | 1.00  | 1.00  | 1.50  | 1.00   | 3.50   | 2.50  | 1.00  | 1.00  |
| time (sec) | N/A     | 0.155 | 0.001 | 0.132 | 0.176  | 0.242  | 0.038 | 0.282 | 0.023 |

| Problem 91 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 15      | 15    | 15    | 12    | 11     | 10     | 20    | 6     | 6     |
| N.S.       | 1       | 1.00  | 1.00  | 0.80  | 0.73   | 0.67   | 1.33  | 0.40  | 0.40  |
| time (sec) | N/A     | 0.160 | 0.018 | 0.167 | 0.182  | 0.244  | 0.149 | 0.300 | 0.028 |

| Problem 92 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 8       | 8     | 8     | 9     | 8      | 8      | 7     | 8     | 8     |
| N.S.       | 1       | 1.00  | 1.00  | 1.12  | 1.00   | 1.00   | 0.88  | 1.00  | 1.00  |
| time (sec) | N/A     | 0.179 | 0.002 | 0.050 | 0.173  | 0.246  | 0.068 | 0.300 | 0.021 |

| Problem 93 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 17      | 17    | 15    | 17    | 15     | 15     | 17    | 15    | 15    |
| N.S.       | 1       | 1.00  | 0.88  | 1.00  | 0.88   | 0.88   | 1.00  | 0.88  | 0.88  |
| time (sec) | N/A     | 0.242 | 0.005 | 0.086 | 0.184  | 0.247  | 0.098 | 0.279 | 0.027 |

| Problem 94 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 25      | 25    | 25    | 20    | 19     | 19     | 36    | 19    | 19    |
| N.S.       | 1       | 1.00  | 1.00  | 0.80  | 0.76   | 0.76   | 1.44  | 0.76  | 0.76  |
| time (sec) | N/A     | 0.164 | 0.002 | 0.100 | 0.208  | 0.243  | 0.097 | 0.283 | 0.056 |

| Problem 95 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | C     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 41      | 46    | 29    | 19    | 26     | 29     | 56    | 26    | 28    |
| N.S.       | 1       | 1.12  | 0.71  | 0.46  | 0.63   | 0.71   | 1.37  | 0.63  | 0.68  |
| time (sec) | N/A     | 0.219 | 0.017 | 0.181 | 0.184  | 0.258  | 0.132 | 0.264 | 0.160 |

| Problem 96 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 33      | 32    | 31    | 23    | 23     | 23     | 39    | 23    | 25    |
| N.S.       | 1       | 0.97  | 0.94  | 0.70  | 0.70   | 0.70   | 1.18  | 0.70  | 0.76  |
| time (sec) | N/A     | 0.243 | 0.003 | 0.223 | 0.182  | 0.268  | 0.145 | 0.266 | 0.068 |

| Problem 97 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 7       | 7     | 7     | 8     | 7      | 7      | 7     | 7     | 7     |
| N.S.       | 1       | 1.00  | 1.00  | 1.14  | 1.00   | 1.00   | 1.00  | 1.00  | 1.00  |
| time (sec) | N/A     | 0.176 | 0.007 | 0.078 | 0.193  | 0.246  | 0.064 | 0.298 | 0.020 |

| Problem 98 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 16      | 17    | 14    | 15    | 14     | 14     | 17    | 14    | 14    |
| N.S.       | 1       | 1.06  | 0.88  | 0.94  | 0.88   | 0.88   | 1.06  | 0.88  | 0.88  |
| time (sec) | N/A     | 0.233 | 0.011 | 0.097 | 0.178  | 0.248  | 0.124 | 0.288 | 0.025 |

| Problem 99 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade      | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified   | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size       | 25      | 25    | 25    | 20    | 19     | 19     | 36    | 19    | 19    |
| N.S.       | 1       | 1.00  | 1.00  | 0.80  | 0.76   | 0.76   | 1.44  | 0.76  | 0.76  |
| time (sec) | N/A     | 0.159 | 0.010 | 0.104 | 0.213  | 0.245  | 0.090 | 0.268 | 0.145 |

| Problem 100 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 41      | 46    | 29    | 27    | 26     | 29     | 56    | 26    | 28    |
| N.S.        | 1       | 1.12  | 0.71  | 0.66  | 0.63   | 0.71   | 1.37  | 0.63  | 0.68  |
| time (sec)  | N/A     | 0.219 | 0.027 | 0.184 | 0.202  | 0.245  | 0.146 | 0.266 | 0.066 |

| Problem 101 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 33      | 31    | 31    | 23    | 23     | 25     | 39    | 23    | 25    |
| N.S.        | 1       | 0.94  | 0.94  | 0.70  | 0.70   | 0.76   | 1.18  | 0.70  | 0.76  |
| time (sec)  | N/A     | 0.245 | 0.007 | 0.231 | 0.183  | 0.242  | 0.133 | 0.265 | 0.177 |

| Problem 102 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad        |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------------|
| grade       | N/A     | A     | A     | A     | C      | A      | A     | A     | <b>F(-1)</b> |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD          |
| size        | 2       | 2     | 2     | 3     | 13     | 2      | 2     | 2     | 0            |
| N.S.        | 1       | 1.00  | 1.00  | 1.50  | 6.50   | 1.00   | 1.00  | 1.00  | 0.00         |
| time (sec)  | N/A     | 0.149 | 0.003 | 0.060 | 0.212  | 0.244  | 0.392 | 0.294 | 0.000        |

| Problem 103 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad        |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|--------------|
| grade       | N/A     | A     | A     | A     | C      | A      | B     | A     | <b>F(-1)</b> |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD          |
| size        | 2       | 2     | 2     | 3     | 13     | 2      | 12    | 2     | 0            |
| N.S.        | 1       | 1.00  | 1.00  | 1.50  | 6.50   | 1.00   | 6.00  | 1.00  | 0.00         |
| time (sec)  | N/A     | 0.144 | 0.004 | 0.130 | 0.218  | 0.240  | 0.632 | 0.277 | 0.000        |

| Problem 104 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | C      | A      | B     | A     | F(-1) |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 10      | 10    | 10    | 11    | 15     | 13     | 17    | 13    | 0     |
| N.S.        | 1       | 1.00  | 1.00  | 1.10  | 1.50   | 1.30   | 1.70  | 1.30  | 0.00  |
| time (sec)  | N/A     | 0.193 | 0.003 | 0.142 | 0.220  | 0.242  | 0.905 | 0.290 | 0.000 |

| Problem 105 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | C      | A      | A     | A     | F(-1) |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 15      | 15    | 15    | 12    | 17     | 11     | 10    | 11    | 0     |
| N.S.        | 1       | 1.00  | 1.00  | 0.80  | 1.13   | 0.73   | 0.67  | 0.73  | 0.00  |
| time (sec)  | N/A     | 0.177 | 0.011 | 0.174 | 0.248  | 0.240  | 0.663 | 0.281 | 0.000 |

| Problem 106 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 12      | 12    | 12    | 17    | 20     | 18     | 12    | 16    | 16    |
| N.S.        | 1       | 1.00  | 1.00  | 1.42  | 1.67   | 1.50   | 1.00  | 1.33  | 1.33  |
| time (sec)  | N/A     | 0.177 | 0.006 | 0.049 | 0.201  | 0.261  | 0.047 | 0.298 | 0.022 |

| Problem 107 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 11      | 11    | 22    | 12    | 11     | 11     | 14    | 11    | 11    |
| N.S.        | 1       | 1.00  | 2.00  | 1.09  | 1.00   | 1.00   | 1.27  | 1.00  | 1.00  |
| time (sec)  | N/A     | 0.136 | 0.009 | 0.084 | 0.184  | 0.234  | 0.055 | 0.284 | 0.022 |

| Problem 108 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | B     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 10      | 10    | 21    | 11    | 10     | 10     | 12    | 10    | 10    |
| N.S.        | 1       | 1.00  | 2.10  | 1.10  | 1.00   | 1.00   | 1.20  | 1.00  | 1.00  |
| time (sec)  | N/A     | 0.141 | 0.009 | 0.073 | 0.184  | 0.235  | 0.073 | 0.266 | 0.022 |

| Problem 109 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 12      | 12    | 12    | 17    | 11     | 18     | 19    | 13    | 16    |
| N.S.        | 1       | 1.00  | 1.00  | 1.42  | 0.92   | 1.50   | 1.58  | 1.08  | 1.33  |
| time (sec)  | N/A     | 0.143 | 0.009 | 0.036 | 0.193  | 0.251  | 0.058 | 0.277 | 0.197 |

| Problem 110 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | B     | B     | A      | B      | B     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 11      | 13    | 23    | 26    | 11     | 27     | 29    | 56    | 28    |
| N.S.        | 1       | 1.18  | 2.09  | 2.36  | 1.00   | 2.45   | 2.64  | 5.09  | 2.55  |
| time (sec)  | N/A     | 0.150 | 0.007 | 0.041 | 0.188  | 0.251  | 0.069 | 0.277 | 0.141 |

| Problem 111 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | B     | A     | B      | B      | A     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 12      | 12    | 38    | 15    | 26     | 30     | 17    | 51    | 12    |
| N.S.        | 1       | 1.00  | 3.17  | 1.25  | 2.17   | 2.50   | 1.42  | 4.25  | 1.00  |
| time (sec)  | N/A     | 0.145 | 0.016 | 0.119 | 0.187  | 0.255  | 0.294 | 0.271 | 0.095 |

| Problem 112 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | B      | B      | B     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 11      | 11    | 11    | 19    | 26     | 28     | 34    | 28    | 11    |
| N.S.        | 1       | 1.00  | 1.00  | 1.73  | 2.36   | 2.55   | 3.09  | 2.55  | 1.00  |
| time (sec)  | N/A     | 0.157 | 0.002 | 0.164 | 0.201  | 0.245  | 0.334 | 0.291 | 0.023 |

| Problem 113 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 25      | 25    | 23    | 19    | 24     | 23     | 46    | 18    | 18    |
| N.S.        | 1       | 1.00  | 0.92  | 0.76  | 0.96   | 0.92   | 1.84  | 0.72  | 0.72  |
| time (sec)  | N/A     | 0.159 | 0.022 | 0.141 | 0.187  | 0.244  | 0.091 | 0.269 | 0.185 |

| Problem 114 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 27      | 24    | 29    | 22    | 22     | 22     | 37    | 25    | 24    |
| N.S.        | 1       | 0.89  | 1.07  | 0.81  | 0.81   | 0.81   | 1.37  | 0.93  | 0.89  |
| time (sec)  | N/A     | 0.168 | 0.011 | 0.204 | 0.176  | 0.242  | 0.132 | 0.284 | 0.139 |

| Problem 115 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 25      | 25    | 23    | 19    | 22     | 22     | 46    | 18    | 18    |
| N.S.        | 1       | 1.00  | 0.92  | 0.76  | 0.88   | 0.88   | 1.84  | 0.72  | 0.72  |
| time (sec)  | N/A     | 0.160 | 0.016 | 0.143 | 0.185  | 0.261  | 0.091 | 0.295 | 0.159 |

| Problem 116 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 26      | 26    | 26    | 22    | 22     | 21     | 36    | 22    | 24    |
| N.S.        | 1       | 1.00  | 1.00  | 0.85  | 0.85   | 0.81   | 1.38  | 0.85  | 0.92  |
| time (sec)  | N/A     | 0.166 | 0.006 | 0.200 | 0.181  | 0.247  | 0.119 | 0.276 | 0.025 |

| Problem 117 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 10      | 10    | 10    | 11    | 10     | 18     | 58    | 10    | 10    |
| N.S.        | 1       | 1.00  | 1.00  | 1.10  | 1.00   | 1.80   | 5.80  | 1.00  | 1.00  |
| time (sec)  | N/A     | 0.160 | 0.003 | 0.177 | 0.210  | 0.248  | 0.620 | 0.285 | 0.126 |

| Problem 118 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 9       | 9     | 6     | 5     | 9      | 9      | 3     | 30    | 4     |
| N.S.        | 1       | 1.00  | 0.67  | 0.56  | 1.00   | 1.00   | 0.33  | 3.33  | 0.44  |
| time (sec)  | N/A     | 0.146 | 0.004 | 0.056 | 0.183  | 0.244  | 0.114 | 0.284 | 0.143 |

| Problem 119 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 12      | 12    | 8     | 9     | 10     | 10     | 7     | 8     | 6     |
| N.S.        | 1       | 1.00  | 0.67  | 0.75  | 0.83   | 0.83   | 0.58  | 0.67  | 0.50  |
| time (sec)  | N/A     | 0.151 | 0.035 | 0.068 | 0.181  | 0.245  | 0.178 | 0.294 | 0.160 |



| Problem 120 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | B     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 10      | 10    | 23    | 11    | 15     | 18     | 8     | 10    | 10    |
| N.S.        | 1       | 1.00  | 2.30  | 1.10  | 1.50   | 1.80   | 0.80  | 1.00  | 1.00  |
| time (sec)  | N/A     | 0.144 | 0.010 | 0.073 | 0.185  | 0.242  | 0.201 | 0.282 | 0.023 |

| Problem 121 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | B     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 11      | 11    | 25    | 11    | 15     | 17     | 8     | 10    | 10    |
| N.S.        | 1       | 1.00  | 2.27  | 1.00  | 1.36   | 1.55   | 0.73  | 0.91  | 0.91  |
| time (sec)  | N/A     | 0.171 | 0.013 | 0.102 | 0.178  | 0.244  | 0.212 | 0.284 | 0.023 |

| Problem 122 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | F(-2)  | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 40      | 46    | 40    | 39    | 0      | 148    | 100   | 48    | 45    |
| N.S.        | 1       | 1.15  | 1.00  | 0.98  | 0.00   | 3.70   | 2.50  | 1.20  | 1.12  |
| time (sec)  | N/A     | 0.191 | 0.033 | 0.152 | 0.000  | 0.278  | 1.976 | 0.269 | 0.389 |

| Problem 123 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy  | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|--------|-------|-------|
| grade       | N/A     | A     | A     | A     | F(-2)  | A      | B      | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD    | TBD   | TBD   |
| size        | 47      | 52    | 44    | 43    | 0      | 287    | 1872   | 60    | 58    |
| N.S.        | 1       | 1.11  | 0.94  | 0.91  | 0.00   | 6.11   | 39.83  | 1.28  | 1.23  |
| time (sec)  | N/A     | 0.207 | 0.073 | 0.300 | 0.000  | 0.275  | 72.566 | 0.282 | 0.266 |

| Problem 124 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 73      | 78    | 47    | 46    | 117    | 54     | 105   | 45    | 52    |
| N.S.        | 1       | 1.07  | 0.64  | 0.63  | 1.60   | 0.74   | 1.44  | 0.62  | 0.71  |
| time (sec)  | N/A     | 0.264 | 0.106 | 0.237 | 0.187  | 0.250  | 0.359 | 0.267 | 0.146 |

| Problem 125 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 15      | 15    | 15    | 12    | 11     | 12     | 20    | 11    | 9     |
| N.S.        | 1       | 1.00  | 1.00  | 0.80  | 0.73   | 0.80   | 1.33  | 0.73  | 0.60  |
| time (sec)  | N/A     | 0.159 | 0.003 | 0.180 | 0.183  | 0.250  | 0.142 | 0.291 | 0.027 |

| Problem 126 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 73      | 78    | 47    | 46    | 113    | 54     | 105   | 45    | 52    |
| N.S.        | 1       | 1.07  | 0.64  | 0.63  | 1.55   | 0.74   | 1.44  | 0.62  | 0.71  |
| time (sec)  | N/A     | 0.264 | 0.085 | 0.276 | 0.183  | 0.246  | 0.246 | 0.270 | 0.109 |

| Problem 127 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | B      | A     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 14      | 14    | 19    | 22    | 14     | 31     | 14    | 29    | 21    |
| N.S.        | 1       | 1.00  | 1.36  | 1.57  | 1.00   | 2.21   | 1.00  | 2.07  | 1.50  |
| time (sec)  | N/A     | 0.188 | 0.006 | 0.051 | 0.185  | 0.257  | 0.039 | 0.281 | 0.172 |

| Problem 128 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | B      | B      | F     | F     | F(-1) |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 104     | 125   | 101   | 138   | 491    | 182    | 0     | 0     | 0     |
| N.S.        | 1       | 1.20  | 0.97  | 1.33  | 4.72   | 1.75   | 0.00  | 0.00  | 0.00  |
| time (sec)  | N/A     | 0.758 | 0.110 | 0.142 | 0.323  | 0.281  | 0.000 | 0.000 | 0.000 |

| Problem 129 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | B      | A      | F     | F     | F(-1) |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 153     | 294   | 133   | 237   | 777    | 212    | 0     | 0     | 0     |
| N.S.        | 1       | 1.92  | 0.87  | 1.55  | 5.08   | 1.39   | 0.00  | 0.00  | 0.00  |
| time (sec)  | N/A     | 1.573 | 0.228 | 0.168 | 0.571  | 0.255  | 0.000 | 0.000 | 0.000 |

| Problem 130 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | B      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 15      | 15    | 15    | 20    | 107    | 21     | 19    | 23    | 13    |
| N.S.        | 1       | 1.00  | 1.00  | 1.33  | 7.13   | 1.40   | 1.27  | 1.53  | 0.87  |
| time (sec)  | N/A     | 0.194 | 0.011 | 0.053 | 0.269  | 0.255  | 0.079 | 0.275 | 0.022 |

| Problem 131 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 15      | 15    | 15    | 12    | 11     | 13     | 26    | 11    | 13    |
| N.S.        | 1       | 1.00  | 1.00  | 0.80  | 0.73   | 0.87   | 1.73  | 0.73  | 0.87  |
| time (sec)  | N/A     | 0.161 | 0.008 | 0.204 | 0.181  | 0.245  | 0.140 | 0.278 | 0.044 |

| Problem 132 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 24      | 29    | 14    | 11    | 10     | 19     | 14    | 10    | 18    |
| N.S.        | 1       | 1.21  | 0.58  | 0.46  | 0.42   | 0.79   | 0.58  | 0.42  | 0.75  |
| time (sec)  | N/A     | 0.206 | 0.018 | 0.112 | 0.186  | 0.245  | 0.023 | 0.291 | 0.044 |

| Problem 133 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | B      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 7       | 7     | 6     | 11    | 9      | 18     | 12    | 9     | 6     |
| N.S.        | 1       | 1.00  | 0.86  | 1.57  | 1.29   | 2.57   | 1.71  | 1.29  | 0.86  |
| time (sec)  | N/A     | 0.174 | 0.023 | 0.208 | 0.197  | 0.234  | 0.025 | 0.290 | 0.064 |

| Problem 134 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | C     | C     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 32      | 32    | 22    | 23    | 25     | 22     | 104   | 328   | 22    |
| N.S.        | 1       | 1.00  | 0.69  | 0.72  | 0.78   | 0.69   | 3.25  | 10.25 | 0.69  |
| time (sec)  | N/A     | 0.156 | 0.016 | 0.130 | 0.196  | 0.243  | 0.307 | 0.299 | 0.025 |

| Problem 135 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | C     | C     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 31      | 31    | 20    | 21    | 24     | 20     | 107   | 329   | 20    |
| N.S.        | 1       | 1.00  | 0.65  | 0.68  | 0.77   | 0.65   | 3.45  | 10.61 | 0.65  |
| time (sec)  | N/A     | 0.156 | 0.014 | 0.141 | 0.186  | 0.248  | 0.290 | 0.310 | 0.017 |

| Problem 136 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | C     | C     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 84      | 84    | 50    | 56    | 60     | 60     | 308   | 1156  | 57    |
| N.S.        | 1       | 1.00  | 0.60  | 0.67  | 0.71   | 0.71   | 3.67  | 13.76 | 0.68  |
| time (sec)  | N/A     | 0.234 | 0.037 | 0.191 | 0.197  | 0.245  | 0.555 | 0.325 | 0.278 |

| Problem 137 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | C     | C     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 83      | 83    | 49    | 54    | 58     | 58     | 304   | 1155  | 55    |
| N.S.        | 1       | 1.00  | 0.59  | 0.65  | 0.70   | 0.70   | 3.66  | 13.92 | 0.66  |
| time (sec)  | N/A     | 0.234 | 0.037 | 0.217 | 0.188  | 0.249  | 0.546 | 0.312 | 0.212 |

| Problem 138 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | C     | A      | A      | C     | C     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 162     | 162   | 94    | 102   | 107    | 115    | 665   | 2631  | 133   |
| N.S.        | 1       | 1.00  | 0.58  | 0.63  | 0.66   | 0.71   | 4.10  | 16.24 | 0.82  |
| time (sec)  | N/A     | 0.395 | 0.060 | 0.289 | 0.198  | 0.260  | 1.284 | 0.333 | 0.391 |

| Problem 139 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | C     | A      | A      | C     | C     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 161     | 162   | 93    | 100   | 105    | 111    | 668   | 2631  | 132   |
| N.S.        | 1       | 1.01  | 0.58  | 0.62  | 0.65   | 0.69   | 4.15  | 16.34 | 0.82  |
| time (sec)  | N/A     | 0.385 | 0.052 | 0.335 | 0.214  | 0.258  | 1.011 | 0.306 | 0.335 |

| Problem 140 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | C     | A      | A      | C     | C     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 261     | 263   | 169   | 166   | 186    | 203    | 1355  | 5069  | 231   |
| N.S.        | 1       | 1.01  | 0.65  | 0.64  | 0.71   | 0.78   | 5.19  | 19.42 | 0.89  |
| time (sec)  | N/A     | 0.624 | 0.101 | 0.443 | 0.220  | 0.251  | 2.140 | 0.350 | 0.614 |

| Problem 141 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | C     | A      | A      | C     | C     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 260     | 262   | 168   | 164   | 184    | 202    | 1352  | 5065  | 232   |
| N.S.        | 1       | 1.01  | 0.65  | 0.63  | 0.71   | 0.78   | 5.20  | 19.48 | 0.89  |
| time (sec)  | N/A     | 0.617 | 0.083 | 0.525 | 0.228  | 0.259  | 2.215 | 0.334 | 0.619 |

| Problem 142 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 25      | 25    | 25    | 20    | 19     | 17     | 114   | 13    | 14    |
| N.S.        | 1       | 1.00  | 1.00  | 0.80  | 0.76   | 0.68   | 4.56  | 0.52  | 0.56  |
| time (sec)  | N/A     | 0.214 | 0.051 | 1.320 | 0.196  | 0.257  | 1.136 | 0.288 | 0.166 |

| Problem 143 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 30      | 30    | 30    | 23    | 22     | 25     | 114   | 22    | 22    |
| N.S.        | 1       | 1.00  | 1.00  | 0.77  | 0.73   | 0.83   | 3.80  | 0.73  | 0.73  |
| time (sec)  | N/A     | 0.197 | 0.034 | 1.333 | 0.193  | 0.247  | 1.142 | 0.279 | 0.280 |

| Problem 144 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 85      | 97    | 55    | 61    | 55     | 59     | 100   | 60    | 67    |
| N.S.        | 1       | 1.14  | 0.65  | 0.72  | 0.65   | 0.69   | 1.18  | 0.71  | 0.79  |
| time (sec)  | N/A     | 0.473 | 0.054 | 0.305 | 0.193  | 0.249  | 0.340 | 0.313 | 0.231 |

| Problem 145 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy  | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|--------|-------|-------|
| grade       | N/A     | N/A   | N/A   | N/A   | C      | N/A    | N/A    | N/A   | N/A   |
| verified    | N/A     | N/A   | N/A   | N/A   | TBD    | TBD    | TBD    | TBD   | TBD   |
| size        | 11      | 11    | 13    | 15    | 240    | 22     | 17     | 17    | 17    |
| N.S.        | 1       | 1.00  | 1.18  | 1.36  | 21.82  | 2.00   | 1.55   | 1.55  | 1.55  |
| time (sec)  | N/A     | 0.466 | 0.590 | 0.230 | 0.223  | 0.264  | 53.385 | 0.301 | 0.384 |

| Problem 146 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | B      | A      | F     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 12      | 18    | 12    | 11    | 41     | 16     | 0     | 18    | 10    |
| N.S.        | 1       | 1.50  | 1.00  | 0.92  | 3.42   | 1.33   | 0.00  | 1.50  | 0.83  |
| time (sec)  | N/A     | 0.188 | 0.017 | 0.375 | 0.213  | 0.250  | 0.000 | 0.299 | 0.177 |

| Problem 147 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | F(-2)  | A      | F(-1) | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 77      | 83    | 76    | 107   | 0      | 312    | 0     | 98    | 133   |
| N.S.        | 1       | 1.08  | 0.99  | 1.39  | 0.00   | 4.05   | 0.00  | 1.27  | 1.73  |
| time (sec)  | N/A     | 0.312 | 0.349 | 0.394 | 0.000  | 0.286  | 0.000 | 0.317 | 0.494 |

| Problem 148 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 17      | 17    | 17    | 13    | 12     | 13     | 15    | 13    | 13    |
| N.S.        | 1       | 1.00  | 1.00  | 0.76  | 0.71   | 0.76   | 0.88  | 0.76  | 0.76  |
| time (sec)  | N/A     | 0.142 | 0.005 | 0.079 | 0.189  | 0.245  | 0.137 | 0.284 | 0.129 |

| Problem 149 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 17      | 17    | 17    | 11    | 10     | 13     | 15    | 13    | 13    |
| N.S.        | 1       | 1.00  | 1.00  | 0.65  | 0.59   | 0.76   | 0.88  | 0.76  | 0.76  |
| time (sec)  | N/A     | 0.146 | 0.005 | 0.078 | 0.180  | 0.261  | 0.128 | 0.266 | 0.162 |

| Problem 150 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 3       | 3     | 3     | 3     | 2      | 2      | 2     | 2     | 2     |
| N.S.        | 1       | 1.00  | 1.00  | 1.00  | 0.67   | 0.67   | 0.67  | 0.67  | 0.67  |
| time (sec)  | N/A     | 0.122 | 0.000 | 0.023 | 0.201  | 0.233  | 0.030 | 0.281 | 0.008 |

| Problem 151 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 8       | 8     | 8     | 9     | 8      | 8      | 8     | 8     | 8     |
| N.S.        | 1       | 1.00  | 1.00  | 1.12  | 1.00   | 1.00   | 1.00  | 1.00  | 1.00  |
| time (sec)  | N/A     | 0.130 | 0.000 | 0.036 | 0.186  | 0.241  | 0.044 | 0.310 | 0.164 |



| Problem 152 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 9       | 9     | 9     | 9     | 8      | 8      | 7     | 8     | 8     |
| N.S.        | 1       | 1.00  | 1.00  | 1.00  | 0.89   | 0.89   | 0.78  | 0.89  | 0.89  |
| time (sec)  | N/A     | 0.129 | 0.006 | 0.029 | 0.183  | 0.224  | 0.041 | 0.292 | 0.027 |

| Problem 153 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 4       | 4     | 4     | 9     | 4      | 4      | 3     | 4     | 4     |
| N.S.        | 1       | 1.00  | 1.00  | 2.25  | 1.00   | 1.00   | 0.75  | 1.00  | 1.00  |
| time (sec)  | N/A     | 0.135 | 0.010 | 0.053 | 0.227  | 0.231  | 0.470 | 0.287 | 0.015 |

| Problem 154 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 24      | 30    | 38    | 23    | 23     | 22     | 15    | 26    | 22    |
| N.S.        | 1       | 1.25  | 1.58  | 0.96  | 0.96   | 0.92   | 0.62  | 1.08  | 0.92  |
| time (sec)  | N/A     | 0.157 | 0.017 | 0.040 | 0.238  | 0.237  | 0.050 | 0.308 | 0.096 |

| Problem 155 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 12      | 12    | 12    | 11    | 10     | 10     | 8     | 10    | 10    |
| N.S.        | 1       | 1.00  | 1.00  | 0.92  | 0.83   | 0.83   | 0.67  | 0.83  | 0.83  |
| time (sec)  | N/A     | 0.159 | 0.015 | 0.038 | 0.173  | 0.239  | 0.035 | 0.290 | 0.046 |

| Problem 156 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 13      | 13    | 13    | 13    | 14     | 12     | 14    | 14    | 14    |
| N.S.        | 1       | 1.00  | 1.00  | 1.00  | 1.08   | 0.92   | 1.08  | 1.08  | 1.08  |
| time (sec)  | N/A     | 0.137 | 0.009 | 0.035 | 0.192  | 0.237  | 0.058 | 0.276 | 0.055 |

| Problem 157 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 31      | 31    | 31    | 22    | 23     | 85     | 24    | 21    | 21    |
| N.S.        | 1       | 1.00  | 1.00  | 0.71  | 0.74   | 2.74   | 0.77  | 0.68  | 0.68  |
| time (sec)  | N/A     | 0.165 | 0.027 | 0.066 | 0.270  | 0.244  | 0.097 | 0.281 | 0.249 |

| Problem 158 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 21      | 21    | 14    | 14    | 13     | 13     | 19    | 13    | 13    |
| N.S.        | 1       | 1.00  | 0.67  | 0.67  | 0.62   | 0.62   | 0.90  | 0.62  | 0.62  |
| time (sec)  | N/A     | 0.158 | 0.010 | 0.039 | 0.187  | 0.240  | 0.039 | 0.276 | 0.027 |

| Problem 159 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 163     | 201   | 102   | 102   | 101    | 101    | 102   | 101   | 101   |
| N.S.        | 1       | 1.23  | 0.63  | 0.63  | 0.62   | 0.62   | 0.63  | 0.62  | 0.62  |
| time (sec)  | N/A     | 1.068 | 0.074 | 0.126 | 0.182  | 0.236  | 0.060 | 0.295 | 0.342 |

| Problem 160 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | F(-2)  | A      | F(-2) | C     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 18      | 18    | 18    | 19    | 0      | 18     | 0     | 216   | 18    |
| N.S.        | 1       | 1.00  | 1.00  | 1.06  | 0.00   | 1.00   | 0.00  | 12.00 | 1.00  |
| time (sec)  | N/A     | 0.168 | 0.016 | 0.078 | 0.000  | 0.257  | 0.000 | 0.297 | 0.200 |

| Problem 161 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | F(-2)  | A      | B     | C     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 14      | 14    | 14    | 15    | 0      | 14     | 31    | 237   | 14    |
| N.S.        | 1       | 1.00  | 1.00  | 1.07  | 0.00   | 1.00   | 2.21  | 16.93 | 1.00  |
| time (sec)  | N/A     | 0.157 | 0.013 | 0.076 | 0.000  | 0.244  | 0.304 | 0.284 | 0.174 |

| Problem 162 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | F     | F     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 17      | 17    | 17    | 21    | 10     | 19     | 0     | 0     | 19    |
| N.S.        | 1       | 1.00  | 1.00  | 1.24  | 0.59   | 1.12   | 0.00  | 0.00  | 1.12  |
| time (sec)  | N/A     | 0.165 | 0.014 | 0.076 | 0.232  | 0.228  | 0.000 | 0.000 | 0.132 |

| Problem 163 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | F      | A      | F     | F     | F(-1) |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 64      | 64    | 43    | 79    | 0      | 54     | 0     | 0     | 0     |
| N.S.        | 1       | 1.00  | 0.67  | 1.23  | 0.00   | 0.84   | 0.00  | 0.00  | 0.00  |
| time (sec)  | N/A     | 0.253 | 0.212 | 0.146 | 0.000  | 0.248  | 0.000 | 0.000 | 0.000 |

| Problem 164 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 16      | 16    | 16    | 16    | 16     | 16     | 12    | 45    | 15    |
| N.S.        | 1       | 1.00  | 1.00  | 1.00  | 1.00   | 1.00   | 0.75  | 2.81  | 0.94  |
| time (sec)  | N/A     | 0.167 | 0.106 | 0.086 | 0.182  | 0.241  | 0.049 | 0.306 | 0.205 |

| Problem 165 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 13      | 13    | 13    | 12    | 11     | 11     | 15    | 11    | 11    |
| N.S.        | 1       | 1.00  | 1.00  | 0.92  | 0.85   | 0.85   | 1.15  | 0.85  | 0.85  |
| time (sec)  | N/A     | 0.140 | 0.011 | 0.044 | 0.197  | 0.245  | 0.039 | 0.283 | 0.027 |

| Problem 166 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | C      | A      | A     | C     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 11      | 11    | 11    | 8     | 9      | 7      | 8     | 9     | 7     |
| N.S.        | 1       | 1.00  | 1.00  | 0.73  | 0.82   | 0.64   | 0.73  | 0.82  | 0.64  |
| time (sec)  | N/A     | 0.133 | 0.012 | 0.022 | 0.174  | 0.236  | 0.083 | 0.284 | 0.018 |

| Problem 167 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 9       | 9     | 9     | 7     | 6      | 6      | 5     | 6     | 6     |
| N.S.        | 1       | 1.00  | 1.00  | 0.78  | 0.67   | 0.67   | 0.56  | 0.67  | 0.67  |
| time (sec)  | N/A     | 0.134 | 0.009 | 0.033 | 0.185  | 0.241  | 0.034 | 0.279 | 0.019 |

| Problem 168 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | C      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 27      | 27    | 17    | 18    | 17     | 17     | 14    | 24    | 17    |
| N.S.        | 1       | 1.00  | 0.63  | 0.67  | 0.63   | 0.63   | 0.52  | 0.89  | 0.63  |
| time (sec)  | N/A     | 0.264 | 0.009 | 0.056 | 0.211  | 0.235  | 0.048 | 0.277 | 0.160 |

| Problem 169 | Optimal | Rubi     | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad        |
|-------------|---------|----------|-------|-------|--------|--------|-------|-------|--------------|
| grade       | N/A     | <b>F</b> | A     | A     | A      | A      | A     | A     | <b>F(-1)</b> |
| verified    | N/A     | N/A      | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD          |
| size        | 25      | 0        | 25    | 23    | 22     | 36     | 31    | 36    | 0            |
| N.S.        | 1       | 0.00     | 1.00  | 0.92  | 0.88   | 1.44   | 1.24  | 1.44  | 0.00         |
| time (sec)  | N/A     | 0.000    | 0.101 | 0.137 | 0.260  | 0.249  | 0.145 | 0.284 | 0.000        |

| Problem 170 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | N/A   | N/A   | N/A   | N/A    | N/A    | N/A   | N/A   | N/A   |
| verified    | N/A     | N/A   | N/A   | N/A   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 9       | 9     | 11    | 5     | 7      | 7      | 8     | 7     | 7     |
| N.S.        | 1       | 1.00  | 1.22  | 0.56  | 0.78   | 0.78   | 0.89  | 0.78  | 0.78  |
| time (sec)  | N/A     | 0.211 | 0.015 | 0.019 | 0.227  | 0.234  | 0.522 | 0.284 | 0.164 |

| Problem 171 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 11      | 11    | 11    | 12    | 10     | 10     | 8     | 10    | 10    |
| N.S.        | 1       | 1.00  | 1.00  | 1.09  | 0.91   | 0.91   | 0.73  | 0.91  | 0.91  |
| time (sec)  | N/A     | 0.162 | 0.006 | 0.049 | 0.213  | 0.233  | 1.134 | 0.277 | 0.028 |

| Problem 172 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 22      | 22    | 17    | 21    | 15     | 15     | 17    | 15    | 28    |
| N.S.        | 1       | 1.00  | 0.77  | 0.95  | 0.68   | 0.68   | 0.77  | 0.68  | 1.27  |
| time (sec)  | N/A     | 0.222 | 0.010 | 0.065 | 0.197  | 0.242  | 1.555 | 0.274 | 0.219 |

| Problem 173 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 23      | 23    | 17    | 17    | 11     | 11     | 10    | 11    | 11    |
| N.S.        | 1       | 1.00  | 0.74  | 0.74  | 0.48   | 0.48   | 0.43  | 0.48  | 0.48  |
| time (sec)  | N/A     | 0.167 | 0.005 | 0.036 | 0.187  | 0.228  | 0.043 | 0.274 | 0.044 |

| Problem 174 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 16      | 16    | 16    | 13    | 12     | 12     | 12    | 12    | 12    |
| N.S.        | 1       | 1.00  | 1.00  | 0.81  | 0.75   | 0.75   | 0.75  | 0.75  | 0.75  |
| time (sec)  | N/A     | 0.132 | 0.000 | 0.046 | 0.264  | 0.242  | 0.016 | 0.283 | 0.032 |

| Problem 175 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | C     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 57      | 57    | 43    | 43    | 58     | 89     | 1166  | 48    | 49    |
| N.S.        | 1       | 1.00  | 0.75  | 0.75  | 1.02   | 1.56   | 20.46 | 0.84  | 0.86  |
| time (sec)  | N/A     | 0.185 | 0.018 | 0.106 | 0.262  | 0.249  | 2.242 | 0.287 | 0.097 |

| Problem 176 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | F(-2)  | A      | F     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 116     | 114   | 95    | 140   | 0      | 300    | 0     | 232   | 88    |
| N.S.        | 1       | 0.98  | 0.82  | 1.21  | 0.00   | 2.59   | 0.00  | 2.00  | 0.76  |
| time (sec)  | N/A     | 0.195 | 0.175 | 0.118 | 0.000  | 0.260  | 0.000 | 0.312 | 0.154 |

| Problem 177 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 16      | 16    | 16    | 13    | 12     | 12     | 12    | 12    | 12    |
| N.S.        | 1       | 1.00  | 1.00  | 0.81  | 0.75   | 0.75   | 0.75  | 0.75  | 0.75  |
| time (sec)  | N/A     | 0.122 | 0.002 | 0.090 | 0.196  | 0.233  | 0.016 | 0.284 | 0.026 |

| Problem 178 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | B     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 34      | 34    | 34    | 21    | 26     | 30     | 202   | 66    | 25    |
| N.S.        | 1       | 1.00  | 1.00  | 0.62  | 0.76   | 0.88   | 5.94  | 1.94  | 0.74  |
| time (sec)  | N/A     | 0.145 | 0.015 | 0.127 | 0.181  | 0.243  | 0.667 | 0.268 | 0.031 |

| Problem 179 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | B     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 53      | 53    | 35    | 32    | 41     | 42     | 666   | 93    | 37    |
| N.S.        | 1       | 1.00  | 0.66  | 0.60  | 0.77   | 0.79   | 12.57 | 1.75  | 0.70  |
| time (sec)  | N/A     | 0.162 | 0.019 | 0.133 | 0.190  | 0.234  | 0.998 | 0.274 | 0.154 |

| Problem 180 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 35      | 35    | 35    | 28    | 42     | 73     | 68    | 32    | 27    |
| N.S.        | 1       | 1.00  | 1.00  | 0.80  | 1.20   | 2.09   | 1.94  | 0.91  | 0.77  |
| time (sec)  | N/A     | 0.145 | 0.022 | 0.101 | 0.280  | 0.272  | 0.792 | 0.271 | 0.149 |

| Problem 181 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 39      | 39    | 39    | 32    | 47     | 93     | 44    | 41    | 31    |
| N.S.        | 1       | 1.00  | 1.00  | 0.82  | 1.21   | 2.38   | 1.13  | 1.05  | 0.79  |
| time (sec)  | N/A     | 0.142 | 0.051 | 0.125 | 0.277  | 0.249  | 0.967 | 0.294 | 0.053 |

| Problem 182 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 14      | 14    | 14    | 13    | 12     | 12     | 10    | 12    | 12    |
| N.S.        | 1       | 1.00  | 1.00  | 0.93  | 0.86   | 0.86   | 0.71  | 0.86  | 0.86  |
| time (sec)  | N/A     | 0.120 | 0.001 | 0.089 | 0.182  | 0.237  | 0.016 | 0.290 | 0.017 |

| Problem 183 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 32      | 32    | 23    | 21    | 26     | 19     | 162   | 23    | 25    |
| N.S.        | 1       | 1.00  | 0.72  | 0.66  | 0.81   | 0.59   | 5.06  | 0.72  | 0.78  |
| time (sec)  | N/A     | 0.145 | 0.013 | 0.090 | 0.188  | 0.235  | 0.613 | 0.281 | 0.029 |



| Problem 184 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 51      | 51    | 35    | 32    | 41     | 31     | 600   | 37    | 37    |
| N.S.        | 1       | 1.00  | 0.69  | 0.63  | 0.80   | 0.61   | 11.76 | 0.73  | 0.73  |
| time (sec)  | N/A     | 0.151 | 0.019 | 0.099 | 0.192  | 0.233  | 0.987 | 0.276 | 0.040 |

| Problem 185 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 23      | 23    | 23    | 18    | 32     | 56     | 24    | 21    | 17    |
| N.S.        | 1       | 1.00  | 1.00  | 0.78  | 1.39   | 2.43   | 1.04  | 0.91  | 0.74  |
| time (sec)  | N/A     | 0.127 | 0.016 | 0.093 | 0.268  | 0.240  | 0.523 | 0.288 | 0.140 |

| Problem 186 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 41      | 41    | 41    | 34    | 60     | 93     | 44    | 47    | 33    |
| N.S.        | 1       | 1.00  | 1.00  | 0.83  | 1.46   | 2.27   | 1.07  | 1.15  | 0.80  |
| time (sec)  | N/A     | 0.141 | 0.046 | 0.122 | 0.271  | 0.242  | 1.339 | 0.297 | 0.156 |

| Problem 187 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 23      | 23    | 26    | 25    | 21     | 25     | 24    | 21    | 21    |
| N.S.        | 1       | 1.00  | 1.13  | 1.09  | 0.91   | 1.09   | 1.04  | 0.91  | 0.91  |
| time (sec)  | N/A     | 0.124 | 0.004 | 0.085 | 0.185  | 0.243  | 0.019 | 0.290 | 0.257 |

| Problem 188 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 48      | 48    | 38    | 43    | 45     | 58     | 216   | 86    | 94    |
| N.S.        | 1       | 1.00  | 0.79  | 0.90  | 0.94   | 1.21   | 4.50  | 1.79  | 1.96  |
| time (sec)  | N/A     | 0.162 | 0.035 | 0.083 | 0.188  | 0.231  | 0.320 | 0.307 | 0.452 |

| Problem 189 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 55      | 54    | 48    | 37    | 52     | 37     | 230   | 52    | 43    |
| N.S.        | 1       | 0.98  | 0.87  | 0.67  | 0.95   | 0.67   | 4.18  | 0.95  | 0.78  |
| time (sec)  | N/A     | 0.228 | 0.026 | 0.153 | 0.275  | 0.250  | 0.480 | 0.278 | 0.176 |

| Problem 190 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | B     | A     | A      | A      | A     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 12      | 12    | 38    | 11    | 14     | 14     | 10    | 26    | 10    |
| N.S.        | 1       | 1.00  | 3.17  | 0.92  | 1.17   | 1.17   | 0.83  | 2.17  | 0.83  |
| time (sec)  | N/A     | 0.125 | 0.003 | 0.135 | 0.190  | 0.244  | 0.065 | 0.257 | 0.172 |

| Problem 191 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | B      | A      | C     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 43      | 43    | 39    | 34    | 71     | 34     | 119   | 39    | 30    |
| N.S.        | 1       | 1.00  | 0.91  | 0.79  | 1.65   | 0.79   | 2.77  | 0.91  | 0.70  |
| time (sec)  | N/A     | 0.143 | 0.041 | 0.096 | 0.191  | 0.244  | 1.782 | 0.276 | 0.235 |

| Problem 192 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 22      | 22    | 22    | 17    | 16     | 16     | 20    | 16    | 16    |
| N.S.        | 1       | 1.00  | 1.00  | 0.77  | 0.73   | 0.73   | 0.91  | 0.73  | 0.73  |
| time (sec)  | N/A     | 0.211 | 0.012 | 0.064 | 0.200  | 0.235  | 0.102 | 0.272 | 0.217 |

| Problem 193 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 13      | 13    | 13    | 12    | 11     | 18     | 8     | 11    | 11    |
| N.S.        | 1       | 1.00  | 1.00  | 0.92  | 0.85   | 1.38   | 0.62  | 0.85  | 0.85  |
| time (sec)  | N/A     | 0.123 | 0.002 | 0.146 | 0.200  | 0.231  | 0.261 | 0.277 | 0.246 |

| Problem 194 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | B      | B      | C     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 8       | 8     | 8     | 7     | 16     | 18     | 19    | 6     | 16    |
| N.S.        | 1       | 1.00  | 1.00  | 0.88  | 2.00   | 2.25   | 2.38  | 0.75  | 2.00  |
| time (sec)  | N/A     | 0.127 | 0.092 | 0.287 | 0.314  | 0.236  | 0.500 | 0.288 | 0.275 |

| Problem 195 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | B      | B      | A     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 14      | 14    | 14    | 11    | 25     | 25     | 8     | 25    | 10    |
| N.S.        | 1       | 1.00  | 1.00  | 0.79  | 1.79   | 1.79   | 0.57  | 1.79  | 0.71  |
| time (sec)  | N/A     | 0.140 | 0.017 | 0.230 | 0.208  | 0.239  | 0.485 | 0.305 | 0.162 |

| Problem 196 | Optimal | Rubi  | MMA   | Maple | Maxima   | Fricas | Sympy    | Giac  | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|-------|
| grade       | N/A     | A     | A     | A     | <b>F</b> | A      | <b>F</b> | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD      | TBD    | TBD      | TBD   | TBD   |
| size        | 18      | 18    | 26    | 14    | 0        | 22     | 0        | 22    | 18    |
| N.S.        | 1       | 1.00  | 1.44  | 0.78  | 0.00     | 1.22   | 0.00     | 1.22  | 1.00  |
| time (sec)  | N/A     | 0.153 | 0.064 | 0.206 | 0.000    | 0.229  | 0.000    | 0.266 | 0.311 |

| Problem 197 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy    | Giac     | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|----------|-------|
| grade       | N/A     | A     | A     | A     | C      | C      | <b>F</b> | <b>F</b> | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD      | TBD      | TBD   |
| size        | 30      | 30    | 29    | 23    | 17     | 55     | 0        | 0        | 32    |
| N.S.        | 1       | 1.00  | 0.97  | 0.77  | 0.57   | 1.83   | 0.00     | 0.00     | 1.07  |
| time (sec)  | N/A     | 0.160 | 0.102 | 0.648 | 0.292  | 0.233  | 0.000    | 0.000    | 0.431 |

| Problem 198 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy    | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | <b>F</b> | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD      | TBD   | TBD   |
| size        | 17      | 17    | 17    | 15    | 14     | 17     | 0        | 35    | 14    |
| N.S.        | 1       | 1.00  | 1.00  | 0.88  | 0.82   | 1.00   | 0.00     | 2.06  | 0.82  |
| time (sec)  | N/A     | 0.131 | 0.117 | 0.115 | 0.275  | 0.232  | 0.000    | 0.290 | 0.143 |

| Problem 199 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | B     | A     | A      | B      | A     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 2       | 2     | 16    | 3     | 2      | 14     | 2     | 25    | 2     |
| N.S.        | 1       | 1.00  | 8.00  | 1.50  | 1.00   | 7.00   | 1.00  | 12.50 | 1.00  |
| time (sec)  | N/A     | 0.112 | 0.012 | 0.118 | 0.282  | 0.222  | 0.061 | 0.278 | 0.026 |

| Problem 200 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac     | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|----------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | <b>F</b> | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD      | TBD   |
| size        | 20      | 20    | 30    | 15    | 14     | 14     | 17    | 0        | 14    |
| N.S.        | 1       | 1.00  | 1.50  | 0.75  | 0.70   | 0.70   | 0.85  | 0.00     | 0.70  |
| time (sec)  | N/A     | 0.507 | 0.011 | 0.131 | 0.186  | 0.222  | 0.227 | 0.000    | 0.306 |

| Problem 201 | Optimal | Rubi  | MMA    | Maple | Maxima | Fricas | Sympy | Giac     | Mupad |
|-------------|---------|-------|--------|-------|--------|--------|-------|----------|-------|
| grade       | N/A     | A     | A      | A     | A      | A      | A     | <b>F</b> | B     |
| verified    | N/A     | Yes   | Yes    | Yes   | TBD    | TBD    | TBD   | TBD      | TBD   |
| size        | 24      | 30    | 24     | 21    | 30     | 20     | 19    | 0        | 20    |
| N.S.        | 1       | 1.25  | 1.00   | 0.88  | 1.25   | 0.83   | 0.79  | 0.00     | 0.83  |
| time (sec)  | N/A     | 0.498 | 14.666 | 0.195 | 0.325  | 0.230  | 0.280 | 0.000    | 0.327 |

| Problem 202 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | B     | A     | A      | A      | A     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 27      | 27    | 71    | 24    | 31     | 29     | 24    | 51    | 23    |
| N.S.        | 1       | 1.00  | 2.63  | 0.89  | 1.15   | 1.07   | 0.89  | 1.89  | 0.85  |
| time (sec)  | N/A     | 0.150 | 0.005 | 0.138 | 0.197  | 0.233  | 0.081 | 0.285 | 0.741 |

| Problem 203 | Optimal | Rubi  | MMA   | Maple | Maxima   | Fricas | Sympy | Giac     | Mupad        |
|-------------|---------|-------|-------|-------|----------|--------|-------|----------|--------------|
| grade       | N/A     | A     | A     | C     | <b>F</b> | A      | C     | <b>F</b> | <b>F(-1)</b> |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD      | TBD    | TBD   | TBD      | TBD          |
| size        | 82      | 92    | 82    | 25    | 0        | 216    | 51    | 0        | 0            |
| N.S.        | 1       | 1.12  | 1.00  | 0.30  | 0.00     | 2.63   | 0.62  | 0.00     | 0.00         |
| time (sec)  | N/A     | 0.226 | 0.070 | 0.072 | 0.000    | 0.253  | 1.829 | 0.000    | 0.000        |

| Problem 204 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy  | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|--------|-------|-------|
| grade       | N/A     | A     | A     | A     | B      | B      | B      | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD    | TBD   | TBD   |
| size        | 12      | 12    | 12    | 20    | 40     | 29     | 48     | 10    | 169   |
| N.S.        | 1       | 1.00  | 1.00  | 1.67  | 3.33   | 2.42   | 4.00   | 0.83  | 14.08 |
| time (sec)  | N/A     | 0.238 | 0.011 | 0.326 | 0.277  | 0.238  | 49.819 | 0.270 | 0.058 |

| Problem 205 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | C     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 40      | 40    | 40    | 33    | 36     | 85     | 66    | 55    | 32    |
| N.S.        | 1       | 1.00  | 1.00  | 0.82  | 0.90   | 2.12   | 1.65  | 1.38  | 0.80  |
| time (sec)  | N/A     | 0.144 | 0.008 | 0.142 | 0.196  | 0.239  | 0.556 | 0.266 | 0.450 |

| Problem 206 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 46      | 46    | 46    | 41    | 57     | 41     | 42    | 40    | 40    |
| N.S.        | 1       | 1.00  | 1.00  | 0.89  | 1.24   | 0.89   | 0.91  | 0.87  | 0.87  |
| time (sec)  | N/A     | 0.172 | 0.028 | 0.431 | 0.295  | 0.232  | 0.617 | 0.261 | 0.577 |

| Problem 207 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy    | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | <b>F</b> | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD      | TBD   | TBD   |
| size        | 37      | 37    | 45    | 52    | 40     | 52     | 0        | 40    | 51    |
| N.S.        | 1       | 1.00  | 1.22  | 1.41  | 1.08   | 1.41   | 0.00     | 1.08  | 1.38  |
| time (sec)  | N/A     | 0.158 | 0.075 | 0.451 | 0.279  | 0.249  | 0.000    | 0.294 | 0.096 |

| Problem 208 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy    | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | <b>F</b> | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD      | TBD   | TBD   |
| size        | 61      | 61    | 66    | 74    | 77     | 97     | 0        | 57    | 72    |
| N.S.        | 1       | 1.00  | 1.08  | 1.21  | 1.26   | 1.59   | 0.00     | 0.93  | 1.18  |
| time (sec)  | N/A     | 0.174 | 0.101 | 0.480 | 0.282  | 0.248  | 0.000    | 0.325 | 0.242 |

| Problem 209 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 23      | 23    | 23    | 20    | 19     | 19     | 29    | 19    | 19    |
| N.S.        | 1       | 1.00  | 1.00  | 0.87  | 0.83   | 0.83   | 1.26  | 0.83  | 0.83  |
| time (sec)  | N/A     | 0.131 | 0.003 | 0.112 | 0.194  | 0.242  | 0.155 | 0.281 | 0.289 |

| Problem 210 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 28      | 28    | 28    | 25    | 24     | 24     | 36    | 24    | 24    |
| N.S.        | 1       | 1.00  | 1.00  | 0.89  | 0.86   | 0.86   | 1.29  | 0.86  | 0.86  |
| time (sec)  | N/A     | 0.133 | 0.003 | 0.137 | 0.205  | 0.240  | 0.157 | 0.262 | 0.273 |

| Problem 211 | Optimal | Rubi  | MMA   | Maple | Maxima       | Fricas | Sympy    | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------------|--------|----------|-------|-------|
| grade       | N/A     | A     | A     | A     | <b>F(-2)</b> | A      | <b>F</b> | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD          | TBD    | TBD      | TBD   | TBD   |
| size        | 56      | 56    | 63    | 46    | 0            | 152    | 0        | 58    | 50    |
| N.S.        | 1       | 1.00  | 1.12  | 0.82  | 0.00         | 2.71   | 0.00     | 1.04  | 0.89  |
| time (sec)  | N/A     | 0.174 | 0.111 | 0.657 | 0.000        | 0.248  | 0.000    | 0.286 | 0.953 |

| Problem 212 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | F(-2)  | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 81      | 81    | 89    | 70    | 0      | 190    | 172   | 75    | 67    |
| N.S.        | 1       | 1.00  | 1.10  | 0.86  | 0.00   | 2.35   | 2.12  | 0.93  | 0.83  |
| time (sec)  | N/A     | 0.188 | 0.188 | 0.463 | 0.000  | 0.242  | 0.415 | 0.308 | 0.259 |

| Problem 213 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | F(-2)  | A      | F     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 44      | 44    | 53    | 56    | 0      | 58     | 0     | 45    | 54    |
| N.S.        | 1       | 1.00  | 1.20  | 1.27  | 0.00   | 1.32   | 0.00  | 1.02  | 1.23  |
| time (sec)  | N/A     | 0.179 | 0.080 | 0.134 | 0.000  | 0.241  | 0.000 | 0.271 | 0.400 |

| Problem 214 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | F(-2)  | A      | F     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 68      | 68    | 72    | 78    | 0      | 106    | 0     | 62    | 72    |
| N.S.        | 1       | 1.00  | 1.06  | 1.15  | 0.00   | 1.56   | 0.00  | 0.91  | 1.06  |
| time (sec)  | N/A     | 0.197 | 0.107 | 0.132 | 0.000  | 0.253  | 0.000 | 0.294 | 0.446 |

| Problem 215 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 13      | 13    | 13    | 12    | 11     | 22     | 10    | 11    | 11    |
| N.S.        | 1       | 1.00  | 1.00  | 0.92  | 0.85   | 1.69   | 0.77  | 0.85  | 0.85  |
| time (sec)  | N/A     | 0.172 | 0.005 | 0.125 | 0.197  | 0.236  | 0.092 | 0.276 | 0.090 |



| Problem 216 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 19      | 19    | 19    | 12    | 15     | 11     | 17    | 15    | 11    |
| N.S.        | 1       | 1.00  | 1.00  | 0.63  | 0.79   | 0.58   | 0.89  | 0.79  | 0.58  |
| time (sec)  | N/A     | 0.142 | 0.002 | 0.029 | 0.200  | 0.221  | 0.040 | 0.263 | 0.168 |

| Problem 217 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 12      | 12    | 12    | 13    | 20     | 12     | 10    | 12    | 12    |
| N.S.        | 1       | 1.00  | 1.00  | 1.08  | 1.67   | 1.00   | 0.83  | 1.00  | 1.00  |
| time (sec)  | N/A     | 0.189 | 0.020 | 0.138 | 0.200  | 0.238  | 0.068 | 0.279 | 0.086 |

| Problem 218 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 11      | 11    | 11    | 11    | 10     | 10     | 7     | 10    | 10    |
| N.S.        | 1       | 1.00  | 1.00  | 1.00  | 0.91   | 0.91   | 0.64  | 0.91  | 0.91  |
| time (sec)  | N/A     | 0.157 | 0.007 | 0.043 | 0.248  | 0.240  | 0.066 | 0.265 | 0.214 |

| Problem 219 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 27      | 26    | 27    | 23    | 23     | 22     | 20    | 24    | 22    |
| N.S.        | 1       | 0.96  | 1.00  | 0.85  | 0.85   | 0.81   | 0.74  | 0.89  | 0.81  |
| time (sec)  | N/A     | 0.153 | 0.004 | 0.095 | 0.194  | 0.219  | 0.063 | 0.292 | 0.168 |

| Problem 220 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | B      | C     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 33      | 33    | 21    | 16    | 20     | 50     | 167   | 66    | 15    |
| N.S.        | 1       | 1.00  | 0.64  | 0.48  | 0.61   | 1.52   | 5.06  | 2.00  | 0.45  |
| time (sec)  | N/A     | 0.128 | 0.046 | 0.089 | 0.205  | 0.230  | 2.378 | 0.299 | 0.297 |

| Problem 221 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 9       | 9     | 9     | 10    | 9      | 9      | 7     | 11    | 8     |
| N.S.        | 1       | 1.00  | 1.00  | 1.11  | 1.00   | 1.00   | 0.78  | 1.22  | 0.89  |
| time (sec)  | N/A     | 0.120 | 0.002 | 0.089 | 0.191  | 0.227  | 0.035 | 0.270 | 0.203 |

| Problem 222 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | B      | B      | B     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 19      | 19    | 19    | 14    | 28     | 28     | 39    | 32    | 13    |
| N.S.        | 1       | 1.00  | 1.00  | 0.74  | 1.47   | 1.47   | 2.05  | 1.68  | 0.68  |
| time (sec)  | N/A     | 0.129 | 0.021 | 0.151 | 0.284  | 0.227  | 0.111 | 0.259 | 0.074 |

| Problem 223 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 19      | 19    | 16    | 12    | 11     | 14     | 15    | 11    | 12    |
| N.S.        | 1       | 1.00  | 0.84  | 0.63  | 0.58   | 0.74   | 0.79  | 0.58  | 0.63  |
| time (sec)  | N/A     | 0.128 | 0.008 | 0.039 | 0.188  | 0.232  | 0.308 | 0.267 | 0.023 |

| Problem 224 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | C     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 88      | 89    | 112   | 89    | 86     | 104    | 153   | 103   | 119   |
| N.S.        | 1       | 1.01  | 1.27  | 1.01  | 0.98   | 1.18   | 1.74  | 1.17  | 1.35  |
| time (sec)  | N/A     | 0.182 | 0.065 | 0.103 | 0.282  | 0.232  | 1.096 | 0.487 | 0.188 |

| Problem 225 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | C     | A     | A     | B      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 9       | 18    | 9     | 10    | 34     | 9      | 7     | 17    | 9     |
| N.S.        | 1       | 2.00  | 1.00  | 1.11  | 3.78   | 1.00   | 0.78  | 1.89  | 1.00  |
| time (sec)  | N/A     | 0.189 | 0.007 | 0.085 | 0.208  | 0.220  | 0.077 | 0.275 | 0.021 |

| Problem 226 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | B      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 9       | 9     | 9     | 10    | 34     | 9      | 7     | 17    | 9     |
| N.S.        | 1       | 1.00  | 1.00  | 1.11  | 3.78   | 1.00   | 0.78  | 1.89  | 1.00  |
| time (sec)  | N/A     | 0.185 | 0.006 | 0.077 | 0.208  | 0.227  | 0.067 | 0.267 | 0.023 |

| Problem 227 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | B      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 9       | 9     | 9     | 8     | 7      | 26     | 7     | 13    | 7     |
| N.S.        | 1       | 1.00  | 1.00  | 0.89  | 0.78   | 2.89   | 0.78  | 1.44  | 0.78  |
| time (sec)  | N/A     | 0.145 | 0.005 | 0.063 | 0.190  | 0.235  | 0.055 | 0.286 | 0.193 |

| Problem 228 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | B      | B     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 12      | 12    | 12    | 16    | 13     | 26     | 22    | 23    | 13    |
| N.S.        | 1       | 1.00  | 1.00  | 1.33  | 1.08   | 2.17   | 1.83  | 1.92  | 1.08  |
| time (sec)  | N/A     | 0.182 | 0.096 | 0.707 | 0.193  | 0.241  | 0.175 | 0.290 | 0.326 |

| Problem 229 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 28      | 28    | 28    | 23    | 22     | 32     | 75    | 22    | 22    |
| N.S.        | 1       | 1.00  | 1.00  | 0.82  | 0.79   | 1.14   | 2.68  | 0.79  | 0.79  |
| time (sec)  | N/A     | 0.183 | 0.015 | 0.395 | 0.196  | 0.237  | 0.324 | 0.264 | 0.292 |

| Problem 230 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 21      | 20    | 16    | 16    | 15     | 13     | 15    | 15    | 14    |
| N.S.        | 1       | 0.95  | 0.76  | 0.76  | 0.71   | 0.62   | 0.71  | 0.71  | 0.67  |
| time (sec)  | N/A     | 0.157 | 0.001 | 0.024 | 0.274  | 0.233  | 0.085 | 0.270 | 0.019 |

| Problem 231 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 21      | 20    | 16    | 16    | 15     | 13     | 15    | 19    | 15    |
| N.S.        | 1       | 0.95  | 0.76  | 0.76  | 0.71   | 0.62   | 0.71  | 0.90  | 0.71  |
| time (sec)  | N/A     | 0.156 | 0.003 | 0.078 | 0.290  | 0.235  | 0.085 | 0.263 | 0.048 |

| Problem 232 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 23      | 25    | 22    | 23    | 22     | 19     | 26    | 22    | 41    |
| N.S.        | 1       | 1.09  | 0.96  | 1.00  | 0.96   | 0.83   | 1.13  | 0.96  | 1.78  |
| time (sec)  | N/A     | 0.179 | 0.003 | 0.044 | 0.194  | 0.226  | 0.065 | 0.258 | 0.151 |

| Problem 233 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 18      | 18    | 18    | 15    | 14     | 28     | 32    | 14    | 14    |
| N.S.        | 1       | 1.00  | 1.00  | 0.83  | 0.78   | 1.56   | 1.78  | 0.78  | 0.78  |
| time (sec)  | N/A     | 0.169 | 0.015 | 0.286 | 0.209  | 0.247  | 0.128 | 0.260 | 0.026 |

| Problem 234 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 18      | 18    | 18    | 15    | 14     | 28     | 32    | 14    | 14    |
| N.S.        | 1       | 1.00  | 1.00  | 0.83  | 0.78   | 1.56   | 1.78  | 0.78  | 0.78  |
| time (sec)  | N/A     | 0.162 | 0.008 | 0.251 | 0.197  | 0.240  | 0.130 | 0.301 | 0.026 |

| Problem 235 | Optimal | Rubi  | MMA   | Maple | Maxima   | Fricas | Sympy    | Giac  | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|-------|
| grade       | N/A     | A     | B     | A     | <b>F</b> | B      | <b>F</b> | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD      | TBD    | TBD      | TBD   | TBD   |
| size        | 12      | 12    | 40    | 17    | 0        | 24     | 0        | 22    | 16    |
| N.S.        | 1       | 1.00  | 3.33  | 1.42  | 0.00     | 2.00   | 0.00     | 1.83  | 1.33  |
| time (sec)  | N/A     | 0.150 | 0.008 | 0.309 | 0.000    | 0.232  | 0.000    | 0.273 | 0.164 |

| Problem 236 | Optimal | Rubi  | MMA   | Maple | Maxima   | Fricas | Sympy    | Giac  | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|-------|
| grade       | N/A     | A     | B     | A     | <b>F</b> | B      | <b>F</b> | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD      | TBD    | TBD      | TBD   | TBD   |
| size        | 14      | 14    | 42    | 23    | 0        | 26     | 0        | 35    | 18    |
| N.S.        | 1       | 1.00  | 3.00  | 1.64  | 0.00     | 1.86   | 0.00     | 2.50  | 1.29  |
| time (sec)  | N/A     | 0.157 | 0.008 | 0.408 | 0.000    | 0.237  | 0.000    | 0.289 | 0.136 |

| Problem 237 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | B     | A      | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 12      | 12    | 16    | 22    | 9      | 10     | 36    | 14    | 10    |
| N.S.        | 1       | 1.00  | 1.33  | 1.83  | 0.75   | 0.83   | 3.00  | 1.17  | 0.83  |
| time (sec)  | N/A     | 0.151 | 0.024 | 0.263 | 0.331  | 0.236  | 0.390 | 0.279 | 0.154 |

| Problem 238 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy    | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | <b>F</b> | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD      | TBD   | TBD   |
| size        | 14      | 14    | 18    | 22    | 20     | 18     | 0        | 23    | 12    |
| N.S.        | 1       | 1.00  | 1.29  | 1.57  | 1.43   | 1.29   | 0.00     | 1.64  | 0.86  |
| time (sec)  | N/A     | 0.158 | 0.026 | 0.342 | 0.276  | 0.233  | 0.000    | 0.274 | 0.032 |

| Problem 239 | Optimal | Rubi  | MMA   | Maple | Maxima   | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | <b>F</b> | A      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD      | TBD    | TBD   | TBD   | TBD   |
| size        | 21      | 21    | 17    | 14    | 0        | 13     | 63    | 13    | 21    |
| N.S.        | 1       | 1.00  | 0.81  | 0.67  | 0.00     | 0.62   | 3.00  | 0.62  | 1.00  |
| time (sec)  | N/A     | 0.143 | 0.064 | 0.083 | 0.000    | 0.234  | 0.204 | 0.276 | 0.197 |

| Problem 240 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | No    | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 24      | 13    | 22    | 19    | 18     | 18     | 20    | 19    | 18    |
| N.S.        | 1       | 0.54  | 0.92  | 0.79  | 0.75   | 0.75   | 0.83  | 0.79  | 0.75  |
| time (sec)  | N/A     | 0.151 | 0.010 | 0.073 | 0.194  | 0.228  | 0.055 | 0.278 | 0.110 |

| Problem 241 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | B      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 12      | 12    | 18    | 9     | 33     | 16     | 7     | 16    | 8     |
| N.S.        | 1       | 1.00  | 1.50  | 0.75  | 2.75   | 1.33   | 0.58  | 1.33  | 0.67  |
| time (sec)  | N/A     | 0.129 | 0.119 | 0.185 | 0.187  | 0.244  | 0.492 | 0.256 | 0.037 |

| Problem 242 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | F     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 32      | 36    | 33    | 25    | 24     | 24     | 0     | 24    | 24    |
| N.S.        | 1       | 1.12  | 1.03  | 0.78  | 0.75   | 0.75   | 0.00  | 0.75  | 0.75  |
| time (sec)  | N/A     | 0.161 | 0.020 | 0.089 | 0.178  | 0.241  | 0.000 | 0.260 | 0.032 |

| Problem 243 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 33      | 40    | 33    | 27    | 26     | 32     | 31    | 26    | 26    |
| N.S.        | 1       | 1.21  | 1.00  | 0.82  | 0.79   | 0.97   | 0.94  | 0.79  | 0.79  |
| time (sec)  | N/A     | 0.155 | 0.010 | 0.053 | 0.276  | 0.251  | 0.055 | 0.284 | 0.060 |

| Problem 244 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | B     | A     | A      | B      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 3       | 3     | 7     | 4     | 3      | 11     | 3     | 4     | 3     |
| N.S.        | 1       | 1.00  | 2.33  | 1.33  | 1.00   | 3.67   | 1.00  | 1.33  | 1.00  |
| time (sec)  | N/A     | 0.143 | 0.000 | 0.050 | 0.192  | 0.252  | 0.038 | 0.276 | 0.024 |

| Problem 245 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | C     | A     | A      | B      | A     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 12      | 12    | 22    | 11    | 16     | 48     | 19    | 34    | 10    |
| N.S.        | 1       | 1.00  | 1.83  | 0.92  | 1.33   | 4.00   | 1.58  | 2.83  | 0.83  |
| time (sec)  | N/A     | 0.195 | 0.001 | 0.065 | 0.268  | 0.295  | 0.026 | 0.264 | 0.024 |

| Problem 246 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | B      | B     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 3       | 3     | 3     | 4     | 3      | 18     | 7     | 11    | 3     |
| N.S.        | 1       | 1.00  | 1.00  | 1.33  | 1.00   | 6.00   | 2.33  | 3.67  | 1.00  |
| time (sec)  | N/A     | 0.154 | 0.004 | 0.055 | 0.186  | 0.269  | 0.062 | 0.275 | 0.012 |

| Problem 247 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | B     | A     | A      | B      | B     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 3       | 3     | 7     | 4     | 3      | 18     | 12    | 12    | 3     |
| N.S.        | 1       | 1.00  | 2.33  | 1.33  | 1.00   | 6.00   | 4.00  | 4.00  | 1.00  |
| time (sec)  | N/A     | 0.145 | 0.005 | 0.103 | 0.238  | 0.250  | 0.161 | 0.267 | 0.138 |



| Problem 248 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 8       | 8     | 8     | 9     | 8      | 8      | 8     | 8     | 8     |
| N.S.        | 1       | 1.00  | 1.00  | 1.12  | 1.00   | 1.00   | 1.00  | 1.00  | 1.00  |
| time (sec)  | N/A     | 0.128 | 0.000 | 0.033 | 0.194  | 0.248  | 0.034 | 0.270 | 0.165 |

| Problem 249 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy    | Giac  | Mupad        |
|-------------|---------|-------|-------|-------|--------|--------|----------|-------|--------------|
| grade       | N/A     | A     | A     | A     | A      | A      | <b>F</b> | A     | <b>F(-1)</b> |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD      | TBD   | TBD          |
| size        | 49      | 36    | 29    | 32    | 10     | 10     | 0        | 11    | 0            |
| N.S.        | 1       | 0.73  | 0.59  | 0.65  | 0.20   | 0.20   | 0.00     | 0.22  | 0.00         |
| time (sec)  | N/A     | 0.183 | 1.008 | 0.064 | 0.280  | 0.239  | 0.000    | 0.275 | 0.000        |

| Problem 250 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 16      | 16    | 17    | 11    | 12     | 12     | 12    | 13    | 10    |
| N.S.        | 1       | 1.00  | 1.06  | 0.69  | 0.75   | 0.75   | 0.75  | 0.81  | 0.62  |
| time (sec)  | N/A     | 0.139 | 0.003 | 0.088 | 0.201  | 0.237  | 0.028 | 0.281 | 0.145 |

| Problem 251 | Optimal | Rubi  | MMA   | Maple | Maxima   | Fricas | Sympy    | Giac  | Mupad |
|-------------|---------|-------|-------|-------|----------|--------|----------|-------|-------|
| grade       | N/A     | A     | A     | A     | <b>F</b> | A      | <b>F</b> | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD      | TBD    | TBD      | TBD   | TBD   |
| size        | 40      | 45    | 36    | 53    | 0        | 30     | 0        | 40    | 32    |
| N.S.        | 1       | 1.12  | 0.90  | 1.32  | 0.00     | 0.75   | 0.00     | 1.00  | 0.80  |
| time (sec)  | N/A     | 0.170 | 0.008 | 0.024 | 0.000    | 0.260  | 0.000    | 0.276 | 0.043 |

| Problem 252 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 128     | 128   | 67    | 63    | 58     | 55     | 109   | 57    | 69    |
| N.S.        | 1       | 1.00  | 0.52  | 0.49  | 0.45   | 0.43   | 0.85  | 0.45  | 0.54  |
| time (sec)  | N/A     | 0.367 | 0.108 | 0.422 | 0.223  | 0.276  | 0.577 | 0.273 | 0.280 |

| Problem 253 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 30      | 30    | 18    | 16    | 17     | 17     | 27    | 15    | 17    |
| N.S.        | 1       | 1.00  | 0.60  | 0.53  | 0.57   | 0.57   | 0.90  | 0.50  | 0.57  |
| time (sec)  | N/A     | 0.192 | 0.020 | 0.127 | 0.195  | 0.245  | 0.168 | 0.264 | 0.187 |

| Problem 254 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | B      | B     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 11      | 11    | 9     | 8     | 7      | 17     | 17    | 7     | 7     |
| N.S.        | 1       | 1.00  | 0.82  | 0.73  | 0.64   | 1.55   | 1.55  | 0.64  | 0.64  |
| time (sec)  | N/A     | 0.121 | 0.001 | 0.087 | 0.198  | 0.227  | 0.036 | 0.274 | 0.065 |

| Problem 255 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 40      | 45    | 40    | 31    | 32     | 32     | 41    | 33    | 46    |
| N.S.        | 1       | 1.12  | 1.00  | 0.78  | 0.80   | 0.80   | 1.02  | 0.82  | 1.15  |
| time (sec)  | N/A     | 0.181 | 0.006 | 0.106 | 0.270  | 0.271  | 0.053 | 0.259 | 0.112 |

| Problem 256 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | B     | A     | B      | B      | B     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 8       | 8     | 23    | 7     | 17     | 17     | 15    | 18    | 6     |
| N.S.        | 1       | 1.00  | 2.88  | 0.88  | 2.12   | 2.12   | 1.88  | 2.25  | 0.75  |
| time (sec)  | N/A     | 0.124 | 0.003 | 0.095 | 0.198  | 0.254  | 0.038 | 0.271 | 0.070 |

| Problem 257 | Optimal | Rubi  | MMA   | Maple | Maxima   | Fricas   | Sympy    | Giac     | Mupad        |
|-------------|---------|-------|-------|-------|----------|----------|----------|----------|--------------|
| grade       | N/A     | A     | A     | B     | <b>F</b> | <b>F</b> | <b>F</b> | <b>F</b> | <b>F(-1)</b> |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD      | TBD      | TBD      | TBD      | TBD          |
| size        | 227     | 227   | 194   | 394   | 0        | 0        | 0        | 0        | 0            |
| N.S.        | 1       | 1.00  | 0.85  | 1.74  | 0.00     | 0.00     | 0.00     | 0.00     | 0.00         |
| time (sec)  | N/A     | 0.379 | 0.188 | 0.372 | 0.000    | 0.000    | 0.000    | 0.000    | 0.000        |

| Problem 258 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 24      | 24    | 24    | 26    | 25     | 24     | 37    | 25    | 51    |
| N.S.        | 1       | 1.00  | 1.00  | 1.08  | 1.04   | 1.00   | 1.54  | 1.04  | 2.12  |
| time (sec)  | N/A     | 0.148 | 0.005 | 0.106 | 0.187  | 0.265  | 0.704 | 0.282 | 0.439 |

| Problem 259 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | C     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 78      | 74    | 71    | 22    | 57     | 69     | 73    | 58    | 70    |
| N.S.        | 1       | 0.95  | 0.91  | 0.28  | 0.73   | 0.88   | 0.94  | 0.74  | 0.90  |
| time (sec)  | N/A     | 0.223 | 0.016 | 0.093 | 0.285  | 0.248  | 0.147 | 0.277 | 0.269 |

| Problem 260 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | B     | A     | A      | B      | A     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 2       | 2     | 16    | 3     | 2      | 14     | 2     | 25    | 2     |
| N.S.        | 1       | 1.00  | 8.00  | 1.50  | 1.00   | 7.00   | 1.00  | 12.50 | 1.00  |
| time (sec)  | N/A     | 0.115 | 0.000 | 0.098 | 0.290  | 0.231  | 0.056 | 0.277 | 0.002 |

| Problem 261 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 27      | 27    | 33    | 21    | 20     | 25     | 24    | 25    | 20    |
| N.S.        | 1       | 1.00  | 1.22  | 0.78  | 0.74   | 0.93   | 0.89  | 0.93  | 0.74  |
| time (sec)  | N/A     | 0.131 | 0.020 | 0.186 | 0.273  | 0.244  | 0.079 | 0.269 | 0.042 |

| Problem 262 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 10      | 10    | 10    | 11    | 10     | 16     | 8     | 11    | 10    |
| N.S.        | 1       | 1.00  | 1.00  | 1.10  | 1.00   | 1.60   | 0.80  | 1.10  | 1.00  |
| time (sec)  | N/A     | 0.132 | 0.002 | 0.077 | 0.210  | 0.226  | 0.030 | 0.280 | 0.033 |

| Problem 263 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 16      | 16    | 16    | 15    | 14     | 14     | 12    | 14    | 14    |
| N.S.        | 1       | 1.00  | 1.00  | 0.94  | 0.88   | 0.88   | 0.75  | 0.88  | 0.88  |
| time (sec)  | N/A     | 0.147 | 0.002 | 0.013 | 0.288  | 0.280  | 0.051 | 0.292 | 0.227 |

| Problem 264 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 40      | 43    | 29    | 34    | 33     | 24     | 32    | 38    | 24    |
| N.S.        | 1       | 1.08  | 0.72  | 0.85  | 0.82   | 0.60   | 0.80  | 0.95  | 0.60  |
| time (sec)  | N/A     | 0.181 | 0.010 | 0.014 | 0.464  | 0.257  | 0.094 | 0.293 | 0.030 |

| Problem 265 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | F     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 21      | 17    | 29    | 14    | 13     | 29     | 0     | 15    | 9     |
| N.S.        | 1       | 0.81  | 1.38  | 0.67  | 0.62   | 1.38   | 0.00  | 0.71  | 0.43  |
| time (sec)  | N/A     | 0.273 | 0.081 | 0.401 | 0.200  | 0.266  | 0.000 | 0.282 | 0.778 |

| Problem 266 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 14      | 14    | 14    | 11    | 10     | 10     | 10    | 16    | 10    |
| N.S.        | 1       | 1.00  | 1.00  | 0.79  | 0.71   | 0.71   | 0.71  | 1.14  | 0.71  |
| time (sec)  | N/A     | 0.154 | 0.001 | 0.062 | 0.185  | 0.255  | 0.020 | 0.283 | 0.002 |

| Problem 267 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 18      | 18    | 18    | 17    | 16     | 18     | 14    | 18    | 16    |
| N.S.        | 1       | 1.00  | 1.00  | 0.94  | 0.89   | 1.00   | 0.78  | 1.00  | 0.89  |
| time (sec)  | N/A     | 0.155 | 0.003 | 0.119 | 0.189  | 0.239  | 0.056 | 0.266 | 0.153 |

| Problem 268 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | B      | A     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 10      | 10    | 20    | 7     | 6      | 16     | 7     | 29    | 6     |
| N.S.        | 1       | 1.00  | 2.00  | 0.70  | 0.60   | 1.60   | 0.70  | 2.90  | 0.60  |
| time (sec)  | N/A     | 0.115 | 0.012 | 0.155 | 0.269  | 0.250  | 0.076 | 0.260 | 0.041 |

| Problem 269 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | B     | A     | A      | B      | A     | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 6       | 6     | 16    | 5     | 4      | 14     | 3     | 25    | 4     |
| N.S.        | 1       | 1.00  | 2.67  | 0.83  | 0.67   | 2.33   | 0.50  | 4.17  | 0.67  |
| time (sec)  | N/A     | 0.115 | 0.012 | 0.126 | 0.283  | 0.258  | 0.071 | 0.278 | 0.030 |

| Problem 270 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 21      | 24    | 21    | 17    | 16     | 16     | 22    | 16    | 16    |
| N.S.        | 1       | 1.14  | 1.00  | 0.81  | 0.76   | 0.76   | 1.05  | 0.76  | 0.76  |
| time (sec)  | N/A     | 0.137 | 0.008 | 0.632 | 0.272  | 0.255  | 0.049 | 0.269 | 0.143 |

| Problem 271 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 53      | 53    | 51    | 42    | 47     | 73     | 46    | 46    | 45    |
| N.S.        | 1       | 1.00  | 0.96  | 0.79  | 0.89   | 1.38   | 0.87  | 0.87  | 0.85  |
| time (sec)  | N/A     | 0.218 | 0.014 | 0.055 | 0.283  | 0.251  | 0.077 | 0.267 | 0.061 |

| Problem 272 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy  | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|--------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | B      | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD    | TBD   | TBD   |
| size        | 49      | 49    | 49    | 46    | 41     | 41     | 762    | 44    | 46    |
| N.S.        | 1       | 1.00  | 1.00  | 0.94  | 0.84   | 0.84   | 15.55  | 0.90  | 0.94  |
| time (sec)  | N/A     | 0.232 | 0.018 | 0.123 | 0.191  | 0.262  | 55.063 | 0.268 | 0.207 |

| Problem 273 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | N/A   | N/A   | N/A   | N/A    | N/A    | N/A   | N/A   | N/A   |
| verified    | N/A     | N/A   | N/A   | N/A   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 12      | 12    | 14    | 12    | 207    | 54     | 270   | 14    | 14    |
| N.S.        | 1       | 1.00  | 1.17  | 1.00  | 17.25  | 4.50   | 22.50 | 1.17  | 1.17  |
| time (sec)  | N/A     | 0.144 | 2.289 | 0.102 | 0.306  | 0.252  | 7.443 | 0.259 | 0.286 |

| Problem 274 | Optimal | Rubi  | MMA    | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|--------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A      | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes    | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 28      | 28    | 28     | 28    | 27     | 44     | 26    | 27    | 27    |
| N.S.        | 1       | 1.00  | 1.00   | 1.00  | 0.96   | 1.57   | 0.93  | 0.96  | 0.96  |
| time (sec)  | N/A     | 0.320 | 37.104 | 0.617 | 0.228  | 0.274  | 0.166 | 0.278 | 0.342 |

| Problem 275 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy   | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|---------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A       | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD     | TBD   | TBD   |
| size        | 199     | 180   | 136   | 125   | 160    | 145    | 1277    | 114   | 140   |
| N.S.        | 1       | 0.90  | 0.68  | 0.63  | 0.80   | 0.73   | 6.42    | 0.57  | 0.70  |
| time (sec)  | N/A     | 0.384 | 0.148 | 1.576 | 0.227  | 0.281  | 151.384 | 0.271 | 1.124 |

| Problem 276 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 18      | 18    | 18    | 16    | 15     | 20     | 15    | 15    | 15    |
| N.S.        | 1       | 1.00  | 1.00  | 0.89  | 0.83   | 1.11   | 0.83  | 0.83  | 0.83  |
| time (sec)  | N/A     | 0.153 | 0.007 | 0.049 | 0.193  | 0.243  | 0.154 | 0.287 | 0.165 |

| Problem 277 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 24      | 24    | 24    | 22    | 21     | 37     | 36    | 37    | 21    |
| N.S.        | 1       | 1.00  | 1.00  | 0.92  | 0.88   | 1.54   | 1.50  | 1.54  | 0.88  |
| time (sec)  | N/A     | 0.163 | 0.006 | 0.065 | 0.205  | 0.242  | 0.213 | 0.263 | 0.073 |

| Problem 278 | Optimal | Rubi     | MMA       | Maple        | Maxima   | Fricas   | Sympy    | Giac     | Mupad        |
|-------------|---------|----------|-----------|--------------|----------|----------|----------|----------|--------------|
| grade       | N/A     | <b>F</b> | <b>C</b>  | <b>F(-1)</b> | <b>F</b> | <b>B</b> | <b>F</b> | <b>F</b> | <b>F(-1)</b> |
| verified    | N/A     | N/A      | <b>No</b> | N/A          | TBD      | TBD      | TBD      | TBD      | TBD          |
| size        | 94      | 0        | 5141      | 0            | 0        | 179      | 0        | 0        | 0            |
| N.S.        | 1       | 0.00     | 54.69     | 0.00         | 0.00     | 1.90     | 0.00     | 0.00     | 0.00         |
| time (sec)  | N/A     | 0.000    | 16.142    | 180.000      | 0.000    | 0.325    | 0.000    | 0.000    | 0.000        |

| Problem 279 | Optimal | Rubi     | MMA       | Maple    | Maxima   | Fricas   | Sympy    | Giac     | Mupad        |
|-------------|---------|----------|-----------|----------|----------|----------|----------|----------|--------------|
| grade       | N/A     | <b>F</b> | <b>C</b>  | <b>C</b> | <b>F</b> | <b>A</b> | <b>F</b> | <b>F</b> | <b>F(-1)</b> |
| verified    | N/A     | N/A      | <b>No</b> | Yes      | TBD      | TBD      | TBD      | TBD      | TBD          |
| size        | 142     | 0        | 630       | 352      | 0        | 223      | 0        | 0        | 0            |
| N.S.        | 1       | 0.00     | 4.44      | 2.48     | 0.00     | 1.57     | 0.00     | 0.00     | 0.00         |
| time (sec)  | N/A     | 0.000    | 3.375     | 3.294    | 0.000    | 0.316    | 0.000    | 0.000    | 0.000        |



| Problem 280 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | B     | B      | B      | F(-1) | B     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 21      | 21    | 31    | 250   | 171    | 162    | 0     | 76    | 172   |
| N.S.        | 1       | 1.00  | 1.48  | 11.90 | 8.14   | 7.71   | 0.00  | 3.62  | 8.19  |
| time (sec)  | N/A     | 0.431 | 0.923 | 0.398 | 0.254  | 0.255  | 0.000 | 0.284 | 2.370 |

| Problem 281 | Optimal | Rubi  | MMA    | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|--------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | F     | C      | A     | F      | F      | F     | F     | F(-1) |
| verified    | N/A     | N/A   | No     | No    | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 4030    | 0     | 3168   | 4640  | 0      | 0      | 0     | 0     | 0     |
| N.S.        | 1       | 0.00  | 0.79   | 1.15  | 0.00   | 0.00   | 0.00  | 0.00  | 0.00  |
| time (sec)  | N/A     | 0.000 | 13.546 | 2.312 | 0.000  | 0.000  | 0.000 | 0.000 | 0.000 |

| Problem 282 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | C     | F      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | No    | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 330     | 306   | 181   | 1356  | 0      | 269    | 330   | 472   | 265   |
| N.S.        | 1       | 0.93  | 0.55  | 4.11  | 0.00   | 0.82   | 1.00  | 1.43  | 0.80  |
| time (sec)  | N/A     | 1.035 | 0.157 | 3.702 | 0.000  | 0.261  | 6.170 | 0.289 | 0.762 |

| Problem 283 | Optimal | Rubi  | MMA   | Maple | Maxima | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|-------|-------|-------|--------|--------|-------|-------|-------|
| grade       | N/A     | A     | A     | A     | A      | A      | A     | A     | B     |
| verified    | N/A     | Yes   | Yes   | Yes   | TBD    | TBD    | TBD   | TBD   | TBD   |
| size        | 4       | 4     | 4     | 5     | 4      | 4      | 5     | 4     | 4     |
| N.S.        | 1       | 1.00  | 1.00  | 1.25  | 1.00   | 1.00   | 1.25  | 1.00  | 1.00  |
| time (sec)  | N/A     | 0.185 | 0.002 | 0.411 | 0.202  | 0.249  | 0.273 | 0.262 | 0.021 |

| Problem 284 | Optimal | Rubi     | MMA   | Maple | Maxima   | Fricas | Sympy | Giac  | Mupad |
|-------------|---------|----------|-------|-------|----------|--------|-------|-------|-------|
| grade       | N/A     | <b>F</b> | A     | A     | <b>F</b> | B      | A     | A     | B     |
| verified    | N/A     | N/A      | Yes   | Yes   | TBD      | TBD    | TBD   | TBD   | TBD   |
| size        | 71      | 0        | 71    | 61    | 0        | 137    | 76    | 94    | 103   |
| N.S.        | 1       | 0.00     | 1.00  | 0.86  | 0.00     | 1.93   | 1.07  | 1.32  | 1.45  |
| time (sec)  | N/A     | 0.000    | 0.066 | 0.094 | 0.000    | 0.263  | 0.106 | 0.297 | 0.280 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. The column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [159] had the largest ratio of [3]

Table 2.1: Rubi specific breakdown of results for each integral

| #  | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|----|-------|----------------------|------------------------|-------------------------------------|---------------------|-------------------------------------------------------------|
| 1  | A     | 1                    | 1                      | 1.00                                | 6                   | 0.167                                                       |
| 2  | A     | 3                    | 3                      | 1.00                                | 13                  | 0.231                                                       |
| 3  | A     | 3                    | 3                      | 1.00                                | 10                  | 0.300                                                       |
| 4  | A     | 1                    | 1                      | 1.00                                | 3                   | 0.333                                                       |
| 5  | A     | 2                    | 2                      | 1.00                                | 11                  | 0.182                                                       |
| 6  | A     | 2                    | 2                      | 1.00                                | 14                  | 0.143                                                       |
| 7  | A     | 2                    | 2                      | 1.00                                | 20                  | 0.100                                                       |
| 8  | A     | 3                    | 2                      | 1.00                                | 12                  | 0.167                                                       |
| 9  | A     | 3                    | 3                      | 1.00                                | 13                  | 0.231                                                       |
| 10 | A     | 3                    | 2                      | 1.11                                | 10                  | 0.200                                                       |
| 11 | A     | 4                    | 4                      | 1.06                                | 13                  | 0.308                                                       |
| 12 | A     | 2                    | 2                      | 1.00                                | 23                  | 0.087                                                       |
| 13 | A     | 4                    | 3                      | 0.98                                | 20                  | 0.150                                                       |
| 14 | A     | 2                    | 2                      | 1.00                                | 22                  | 0.091                                                       |
| 15 | A     | 5                    | 5                      | 1.04                                | 14                  | 0.357                                                       |
| 16 | A     | 8                    | 7                      | 1.10                                | 9                   | 0.778                                                       |
| 17 | A     | 2                    | 2                      | 1.00                                | 16                  | 0.125                                                       |
| 18 | A     | 9                    | 8                      | 1.06                                | 7                   | 1.143                                                       |
| 19 | A     | 9                    | 8                      | 1.06                                | 11                  | 0.727                                                       |
| 20 | A     | 7                    | 6                      | 1.06                                | 10                  | 0.600                                                       |
| 21 | A     | 1                    | 1                      | 1.00                                | 7                   | 0.143                                                       |
| 22 | A     | 2                    | 2                      | 1.00                                | 9                   | 0.222                                                       |

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Table 2.1 – continued from previous page

| #  | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|----|-------|----------------------|------------------------|-------------------------------------|---------------------|-------------------------------------------------------------|
| 23 | A     | 2                    | 2                      | 1.00                                | 11                  | 0.182                                                       |
| 24 | A     | 1                    | 1                      | 1.00                                | 7                   | 0.143                                                       |
| 25 | A     | 1                    | 1                      | 1.00                                | 7                   | 0.143                                                       |
| 26 | A     | 2                    | 2                      | 1.00                                | 9                   | 0.222                                                       |
| 27 | A     | 2                    | 2                      | 1.00                                | 11                  | 0.182                                                       |
| 28 | A     | 3                    | 3                      | 1.00                                | 11                  | 0.273                                                       |
| 29 | A     | 2                    | 2                      | 1.00                                | 11                  | 0.182                                                       |
| 30 | A     | 2                    | 2                      | 1.00                                | 11                  | 0.182                                                       |
| 31 | A     | 1                    | 1                      | 1.00                                | 9                   | 0.111                                                       |
| 32 | A     | 1                    | 1                      | 1.00                                | 11                  | 0.091                                                       |
| 33 | A     | 10                   | 9                      | 1.12                                | 9                   | 1.000                                                       |
| 34 | A     | 9                    | 8                      | 0.99                                | 7                   | 1.143                                                       |
| 35 | A     | 9                    | 8                      | 0.96                                | 11                  | 0.727                                                       |
| 36 | A     | 3                    | 3                      | 1.00                                | 7                   | 0.429                                                       |
| 37 | A     | 3                    | 3                      | 1.00                                | 9                   | 0.333                                                       |
| 38 | A     | 8                    | 7                      | 0.97                                | 9                   | 0.778                                                       |
| 39 | A     | 3                    | 3                      | 1.03                                | 12                  | 0.250                                                       |
| 40 | A     | 3                    | 3                      | 1.03                                | 12                  | 0.250                                                       |
| 41 | A     | 2                    | 2                      | 1.00                                | 12                  | 0.167                                                       |
| 42 | A     | 2                    | 2                      | 1.00                                | 12                  | 0.167                                                       |
| 43 | A     | 2                    | 2                      | 1.00                                | 12                  | 0.167                                                       |
| 44 | A     | 7                    | 6                      | 1.11                                | 10                  | 0.600                                                       |
| 45 | A     | 7                    | 6                      | 1.11                                | 12                  | 0.500                                                       |
| 46 | A     | 9                    | 8                      | 1.08                                | 7                   | 1.143                                                       |
| 47 | A     | 10                   | 9                      | 0.92                                | 7                   | 1.286                                                       |
| 48 | A     | 10                   | 9                      | 1.06                                | 7                   | 1.286                                                       |
| 49 | A     | 8                    | 7                      | 1.01                                | 7                   | 1.000                                                       |
| 50 | A     | 13                   | 12                     | 1.15                                | 7                   | 1.714                                                       |
| 51 | A     | 8                    | 7                      | 1.49                                | 12                  | 0.583                                                       |
| 52 | A     | 9                    | 8                      | 1.12                                | 11                  | 0.727                                                       |
| 53 | A     | 1                    | 1                      | 1.00                                | 2                   | 0.500                                                       |
| 54 | A     | 1                    | 1                      | 1.00                                | 4                   | 0.250                                                       |
| 55 | A     | 1                    | 1                      | 1.00                                | 6                   | 0.167                                                       |
| 56 | A     | 1                    | 1                      | 1.00                                | 6                   | 0.167                                                       |

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Table 2.1 – continued from previous page

| #  | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|----|-------|----------------------|------------------------|-------------------------------------|---------------------|-------------------------------------------------------------|
| 57 | A     | 2                    | 2                      | 1.13                                | 4                   | 0.500                                                       |
| 58 | A     | 11                   | 11                     | 1.35                                | 8                   | 1.375                                                       |
| 59 | A     | 3                    | 2                      | 1.00                                | 8                   | 0.250                                                       |
| 60 | A     | 1                    | 1                      | 1.00                                | 4                   | 0.250                                                       |
| 61 | A     | 3                    | 2                      | 1.00                                | 6                   | 0.333                                                       |
| 62 | A     | 3                    | 2                      | 1.00                                | 8                   | 0.250                                                       |
| 63 | A     | 4                    | 3                      | 1.00                                | 8                   | 0.375                                                       |
| 64 | A     | 3                    | 2                      | 1.00                                | 8                   | 0.250                                                       |
| 65 | A     | 4                    | 4                      | 1.64                                | 8                   | 0.500                                                       |
| 66 | A     | 4                    | 4                      | 1.09                                | 10                  | 0.400                                                       |
| 67 | A     | 2                    | 2                      | 1.00                                | 10                  | 0.200                                                       |
| 68 | A     | 3                    | 3                      | 1.09                                | 8                   | 0.375                                                       |
| 69 | A     | 3                    | 3                      | 1.07                                | 10                  | 0.300                                                       |
| 70 | A     | 3                    | 3                      | 1.04                                | 8                   | 0.375                                                       |
| 71 | A     | 4                    | 3                      | 1.15                                | 10                  | 0.300                                                       |
| 72 | A     | 3                    | 3                      | 1.00                                | 12                  | 0.250                                                       |
| 73 | A     | 3                    | 3                      | 1.00                                | 12                  | 0.250                                                       |
| 74 | A     | 4                    | 4                      | 1.08                                | 10                  | 0.400                                                       |
| 75 | N/A   | 1                    | 0                      | 1.00                                | 5                   | 0.000                                                       |
| 76 | A     | 2                    | 2                      | 1.00                                | 2                   | 1.000                                                       |
| 77 | A     | 2                    | 2                      | 1.00                                | 2                   | 1.000                                                       |
| 78 | A     | 2                    | 2                      | 1.00                                | 2                   | 1.000                                                       |
| 79 | A     | 3                    | 3                      | 1.00                                | 2                   | 1.500                                                       |
| 80 | A     | 4                    | 4                      | 1.00                                | 6                   | 0.667                                                       |
| 81 | A     | 2                    | 2                      | 1.00                                | 2                   | 1.000                                                       |
| 82 | A     | 2                    | 2                      | 1.00                                | 2                   | 1.000                                                       |
| 83 | A     | 3                    | 3                      | 1.00                                | 4                   | 0.750                                                       |
| 84 | A     | 6                    | 5                      | 0.90                                | 8                   | 0.625                                                       |
| 85 | A     | 4                    | 3                      | 1.00                                | 4                   | 0.750                                                       |
| 86 | A     | 2                    | 2                      | 1.00                                | 4                   | 0.500                                                       |
| 87 | A     | 5                    | 4                      | 1.00                                | 11                  | 0.364                                                       |
| 88 | A     | 3                    | 3                      | 1.00                                | 4                   | 0.750                                                       |
| 89 | A     | 4                    | 3                      | 1.00                                | 4                   | 0.750                                                       |

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Table 2.1 – continued from previous page

| #   | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|-------------------------------------------------------------|
| 90  | A     | 4                    | 3                      | 1.00                                | 4                   | 0.750                                                       |
| 91  | A     | 2                    | 2                      | 1.00                                | 7                   | 0.286                                                       |
| 92  | A     | 4                    | 4                      | 1.00                                | 4                   | 1.000                                                       |
| 93  | A     | 7                    | 7                      | 1.00                                | 6                   | 1.167                                                       |
| 94  | A     | 3                    | 3                      | 1.00                                | 6                   | 0.500                                                       |
| 95  | A     | 6                    | 6                      | 1.12                                | 8                   | 0.750                                                       |
| 96  | A     | 6                    | 6                      | 0.97                                | 6                   | 1.000                                                       |
| 97  | A     | 5                    | 5                      | 1.00                                | 4                   | 1.250                                                       |
| 98  | A     | 7                    | 7                      | 1.06                                | 6                   | 1.167                                                       |
| 99  | A     | 3                    | 3                      | 1.00                                | 6                   | 0.500                                                       |
| 100 | A     | 6                    | 6                      | 1.12                                | 8                   | 0.750                                                       |
| 101 | A     | 7                    | 7                      | 0.94                                | 6                   | 1.167                                                       |
| 102 | A     | 2                    | 2                      | 1.00                                | 6                   | 0.333                                                       |
| 103 | A     | 2                    | 2                      | 1.00                                | 6                   | 0.333                                                       |
| 104 | A     | 4                    | 4                      | 1.00                                | 6                   | 0.667                                                       |
| 105 | A     | 3                    | 3                      | 1.00                                | 8                   | 0.375                                                       |
| 106 | A     | 4                    | 4                      | 1.00                                | 4                   | 1.000                                                       |
| 107 | A     | 2                    | 2                      | 1.00                                | 6                   | 0.333                                                       |
| 108 | A     | 2                    | 2                      | 1.00                                | 6                   | 0.333                                                       |
| 109 | A     | 2                    | 2                      | 1.00                                | 6                   | 0.333                                                       |
| 110 | A     | 3                    | 3                      | 1.18                                | 6                   | 0.500                                                       |
| 111 | A     | 2                    | 2                      | 1.00                                | 6                   | 0.333                                                       |
| 112 | A     | 2                    | 2                      | 1.00                                | 6                   | 0.333                                                       |
| 113 | A     | 3                    | 3                      | 1.00                                | 8                   | 0.375                                                       |
| 114 | A     | 4                    | 3                      | 0.89                                | 8                   | 0.375                                                       |
| 115 | A     | 3                    | 3                      | 1.00                                | 8                   | 0.375                                                       |
| 116 | A     | 4                    | 3                      | 1.00                                | 8                   | 0.375                                                       |
| 117 | A     | 4                    | 3                      | 1.00                                | 8                   | 0.375                                                       |
| 118 | A     | 2                    | 2                      | 1.00                                | 6                   | 0.333                                                       |
| 119 | A     | 2                    | 2                      | 1.00                                | 8                   | 0.250                                                       |
| 120 | A     | 2                    | 2                      | 1.00                                | 6                   | 0.333                                                       |
| 121 | A     | 2                    | 2                      | 1.00                                | 8                   | 0.250                                                       |
| 122 | A     | 5                    | 4                      | 1.15                                | 8                   | 0.500                                                       |
| 123 | A     | 5                    | 4                      | 1.11                                | 10                  | 0.400                                                       |

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Table 2.1 – continued from previous page

| #   | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|-------------------------------------------------------------|
| 124 | A     | 6                    | 6                      | 1.07                                | 12                  | 0.500                                                       |
| 125 | A     | 2                    | 2                      | 1.00                                | 7                   | 0.286                                                       |
| 126 | A     | 6                    | 6                      | 1.07                                | 12                  | 0.500                                                       |
| 127 | A     | 7                    | 7                      | 1.00                                | 4                   | 1.750                                                       |
| 128 | A     | 16                   | 15                     | 1.20                                | 8                   | 1.875                                                       |
| 129 | A     | 21                   | 20                     | 1.92                                | 8                   | 2.500                                                       |
| 130 | A     | 5                    | 5                      | 1.00                                | 6                   | 0.833                                                       |
| 131 | A     | 2                    | 2                      | 1.00                                | 9                   | 0.222                                                       |
| 132 | A     | 5                    | 5                      | 1.21                                | 9                   | 0.556                                                       |
| 133 | A     | 5                    | 4                      | 1.00                                | 9                   | 0.444                                                       |
| 134 | A     | 1                    | 1                      | 1.00                                | 6                   | 0.167                                                       |
| 135 | A     | 1                    | 1                      | 1.00                                | 6                   | 0.167                                                       |
| 136 | A     | 2                    | 2                      | 1.00                                | 7                   | 0.286                                                       |
| 137 | A     | 2                    | 2                      | 1.00                                | 7                   | 0.286                                                       |
| 138 | A     | 4                    | 4                      | 1.00                                | 9                   | 0.444                                                       |
| 139 | A     | 3                    | 3                      | 1.01                                | 9                   | 0.333                                                       |
| 140 | A     | 4                    | 4                      | 1.01                                | 9                   | 0.444                                                       |
| 141 | A     | 3                    | 3                      | 1.01                                | 9                   | 0.333                                                       |
| 142 | A     | 3                    | 3                      | 1.00                                | 11                  | 0.273                                                       |
| 143 | A     | 3                    | 3                      | 1.00                                | 11                  | 0.273                                                       |
| 144 | A     | 12                   | 11                     | 1.14                                | 10                  | 1.100                                                       |
| 145 | N/A   | 1                    | 0                      | 1.00                                | 11                  | 0.000                                                       |
| 146 | A     | 6                    | 5                      | 1.50                                | 9                   | 0.556                                                       |
| 147 | A     | 8                    | 7                      | 1.08                                | 15                  | 0.467                                                       |
| 148 | A     | 1                    | 1                      | 1.00                                | 3                   | 0.333                                                       |
| 149 | A     | 1                    | 1                      | 1.00                                | 3                   | 0.333                                                       |
| 150 | A     | 1                    | 1                      | 1.00                                | 3                   | 0.333                                                       |
| 151 | A     | 1                    | 1                      | 1.00                                | 3                   | 0.333                                                       |
| 152 | A     | 1                    | 1                      | 1.00                                | 5                   | 0.200                                                       |
| 153 | A     | 1                    | 1                      | 1.00                                | 9                   | 0.111                                                       |
| 154 | A     | 5                    | 4                      | 1.25                                | 11                  | 0.364                                                       |
| 155 | A     | 4                    | 3                      | 1.00                                | 13                  | 0.231                                                       |
| 156 | A     | 1                    | 1                      | 1.00                                | 9                   | 0.111                                                       |

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Table 2.1 – continued from previous page

| #   | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|-------------------------------------------------------------|
| 157 | A     | 3                    | 2                      | 1.00                                | 18                  | 0.111                                                       |
| 158 | A     | 2                    | 2                      | 1.00                                | 7                   | 0.286                                                       |
| 159 | A     | 21                   | 21                     | 1.23                                | 7                   | 3.000                                                       |
| 160 | A     | 2                    | 2                      | 1.00                                | 9                   | 0.222                                                       |
| 161 | A     | 2                    | 2                      | 1.00                                | 7                   | 0.286                                                       |
| 162 | A     | 2                    | 2                      | 1.00                                | 7                   | 0.286                                                       |
| 163 | A     | 2                    | 2                      | 1.00                                | 12                  | 0.167                                                       |
| 164 | A     | 1                    | 1                      | 1.00                                | 14                  | 0.071                                                       |
| 165 | A     | 1                    | 1                      | 1.00                                | 7                   | 0.143                                                       |
| 166 | A     | 1                    | 1                      | 1.00                                | 5                   | 0.200                                                       |
| 167 | A     | 1                    | 1                      | 1.00                                | 7                   | 0.143                                                       |
| 168 | A     | 2                    | 2                      | 1.00                                | 12                  | 0.167                                                       |
| 169 | F     | 0                    | 0                      | N/A                                 | 0.000               | N/A                                                         |
| 170 | N/A   | 3                    | 0                      | 1.00                                | 9                   | 0.000                                                       |
| 171 | A     | 2                    | 2                      | 1.00                                | 6                   | 0.333                                                       |
| 172 | A     | 3                    | 3                      | 1.00                                | 7                   | 0.429                                                       |
| 173 | A     | 3                    | 3                      | 1.00                                | 10                  | 0.300                                                       |
| 174 | A     | 1                    | 1                      | 1.00                                | 13                  | 0.077                                                       |
| 175 | A     | 5                    | 4                      | 1.00                                | 12                  | 0.333                                                       |
| 176 | A     | 5                    | 4                      | 0.98                                | 19                  | 0.211                                                       |
| 177 | A     | 1                    | 1                      | 1.00                                | 9                   | 0.111                                                       |
| 178 | A     | 2                    | 2                      | 1.00                                | 11                  | 0.182                                                       |
| 179 | A     | 2                    | 2                      | 1.00                                | 13                  | 0.154                                                       |
| 180 | A     | 4                    | 3                      | 1.00                                | 13                  | 0.231                                                       |
| 181 | A     | 4                    | 3                      | 1.00                                | 13                  | 0.231                                                       |
| 182 | A     | 1                    | 1                      | 1.00                                | 9                   | 0.111                                                       |
| 183 | A     | 2                    | 2                      | 1.00                                | 11                  | 0.182                                                       |
| 184 | A     | 2                    | 2                      | 1.00                                | 13                  | 0.154                                                       |
| 185 | A     | 3                    | 2                      | 1.00                                | 13                  | 0.154                                                       |
| 186 | A     | 4                    | 3                      | 1.00                                | 13                  | 0.231                                                       |
| 187 | A     | 1                    | 1                      | 1.00                                | 11                  | 0.091                                                       |
| 188 | A     | 2                    | 2                      | 1.00                                | 13                  | 0.154                                                       |
| 189 | A     | 9                    | 8                      | 0.98                                | 18                  | 0.444                                                       |

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Table 2.1 – continued from previous page

| #   | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|-------------------------------------------------------------|
| 190 | A     | 3                    | 2                      | 1.00                                | 9                   | 0.222                                                       |
| 191 | A     | 5                    | 4                      | 1.00                                | 13                  | 0.308                                                       |
| 192 | A     | 6                    | 5                      | 1.00                                | 6                   | 0.833                                                       |
| 193 | A     | 1                    | 1                      | 1.00                                | 13                  | 0.077                                                       |
| 194 | A     | 3                    | 2                      | 1.00                                | 13                  | 0.154                                                       |
| 195 | A     | 4                    | 3                      | 1.00                                | 13                  | 0.231                                                       |
| 196 | A     | 4                    | 3                      | 1.00                                | 14                  | 0.214                                                       |
| 197 | A     | 4                    | 3                      | 1.00                                | 18                  | 0.167                                                       |
| 198 | A     | 1                    | 1                      | 1.00                                | 20                  | 0.050                                                       |
| 199 | A     | 1                    | 1                      | 1.00                                | 9                   | 0.111                                                       |
| 200 | A     | 5                    | 4                      | 1.00                                | 65                  | 0.062                                                       |
| 201 | A     | 4                    | 4                      | 1.25                                | 68                  | 0.059                                                       |
| 202 | A     | 1                    | 1                      | 1.00                                | 21                  | 0.048                                                       |
| 203 | A     | 8                    | 7                      | 1.12                                | 21                  | 0.333                                                       |
| 204 | A     | 5                    | 4                      | 1.00                                | 23                  | 0.174                                                       |
| 205 | A     | 3                    | 2                      | 1.00                                | 16                  | 0.125                                                       |
| 206 | A     | 4                    | 3                      | 1.00                                | 25                  | 0.120                                                       |
| 207 | A     | 3                    | 2                      | 1.00                                | 24                  | 0.083                                                       |
| 208 | A     | 3                    | 2                      | 1.00                                | 29                  | 0.069                                                       |
| 209 | A     | 1                    | 1                      | 1.00                                | 18                  | 0.056                                                       |
| 210 | A     | 1                    | 1                      | 1.00                                | 23                  | 0.043                                                       |
| 211 | A     | 4                    | 3                      | 1.00                                | 24                  | 0.125                                                       |
| 212 | A     | 4                    | 3                      | 1.00                                | 22                  | 0.136                                                       |
| 213 | A     | 4                    | 3                      | 1.00                                | 26                  | 0.115                                                       |
| 214 | A     | 4                    | 3                      | 1.00                                | 31                  | 0.097                                                       |
| 215 | A     | 5                    | 4                      | 1.00                                | 16                  | 0.250                                                       |
| 216 | A     | 1                    | 1                      | 1.00                                | 8                   | 0.125                                                       |
| 217 | A     | 4                    | 4                      | 1.00                                | 6                   | 0.667                                                       |
| 218 | A     | 1                    | 1                      | 1.00                                | 20                  | 0.050                                                       |
| 219 | A     | 4                    | 3                      | 0.96                                | 13                  | 0.231                                                       |
| 220 | A     | 2                    | 2                      | 1.00                                | 13                  | 0.154                                                       |
| 221 | A     | 3                    | 3                      | 1.00                                | 9                   | 0.333                                                       |
| 222 | A     | 3                    | 2                      | 1.00                                | 13                  | 0.154                                                       |
| 223 | A     | 2                    | 2                      | 1.00                                | 11                  | 0.182                                                       |

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Table 2.1 – continued from previous page

| #   | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|-------------------------------------------------------------|
| 224 | A     | 6                    | 5                      | 1.01                                | 13                  | 0.385                                                       |
| 225 | C     | 5                    | 5                      | 2.00                                | 4                   | 1.250                                                       |
| 226 | A     | 6                    | 6                      | 1.00                                | 4                   | 1.500                                                       |
| 227 | A     | 3                    | 3                      | 1.00                                | 4                   | 0.750                                                       |
| 228 | A     | 1                    | 1                      | 1.00                                | 28                  | 0.036                                                       |
| 229 | A     | 2                    | 2                      | 1.00                                | 11                  | 0.182                                                       |
| 230 | A     | 3                    | 3                      | 0.95                                | 4                   | 0.750                                                       |
| 231 | A     | 3                    | 3                      | 0.95                                | 4                   | 0.750                                                       |
| 232 | A     | 4                    | 3                      | 1.09                                | 8                   | 0.375                                                       |
| 233 | A     | 2                    | 2                      | 1.00                                | 7                   | 0.286                                                       |
| 234 | A     | 2                    | 2                      | 1.00                                | 7                   | 0.286                                                       |
| 235 | A     | 2                    | 2                      | 1.00                                | 8                   | 0.250                                                       |
| 236 | A     | 2                    | 2                      | 1.00                                | 10                  | 0.200                                                       |
| 237 | A     | 2                    | 2                      | 1.00                                | 8                   | 0.250                                                       |
| 238 | A     | 2                    | 2                      | 1.00                                | 10                  | 0.200                                                       |
| 239 | A     | 3                    | 3                      | 1.00                                | 17                  | 0.176                                                       |
| 240 | A     | 5                    | 4                      | 0.54                                | 13                  | 0.308                                                       |
| 241 | A     | 3                    | 2                      | 1.00                                | 11                  | 0.182                                                       |
| 242 | A     | 5                    | 4                      | 1.12                                | 13                  | 0.308                                                       |
| 243 | A     | 3                    | 3                      | 1.21                                | 8                   | 0.375                                                       |
| 244 | A     | 3                    | 3                      | 1.00                                | 2                   | 1.500                                                       |
| 245 | A     | 5                    | 5                      | 1.00                                | 4                   | 1.250                                                       |
| 246 | A     | 3                    | 3                      | 1.00                                | 2                   | 1.500                                                       |
| 247 | A     | 3                    | 3                      | 1.00                                | 2                   | 1.500                                                       |
| 248 | A     | 1                    | 1                      | 1.00                                | 3                   | 0.333                                                       |
| 249 | A     | 4                    | 4                      | 0.73                                | 12                  | 0.333                                                       |
| 250 | A     | 2                    | 2                      | 1.00                                | 13                  | 0.154                                                       |
| 251 | A     | 3                    | 3                      | 1.12                                | 14                  | 0.214                                                       |
| 252 | A     | 2                    | 2                      | 1.00                                | 12                  | 0.167                                                       |
| 253 | A     | 2                    | 2                      | 1.00                                | 7                   | 0.286                                                       |
| 254 | A     | 1                    | 1                      | 1.00                                | 5                   | 0.200                                                       |
| 255 | A     | 8                    | 7                      | 1.12                                | 9                   | 0.778                                                       |
| 256 | A     | 3                    | 2                      | 1.00                                | 9                   | 0.222                                                       |
| 257 | A     | 2                    | 2                      | 1.00                                | 15                  | 0.133                                                       |

Continued on next page

Table 2.1 – continued from previous page

| #   | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|-----|-------|----------------------|------------------------|-------------------------------------|---------------------|-------------------------------------------------------------|
| 258 | A     | 1                    | 1                      | 1.00                                | 11                  | 0.091                                                       |
| 259 | A     | 8                    | 7                      | 0.95                                | 7                   | 1.000                                                       |
| 260 | A     | 1                    | 1                      | 1.00                                | 9                   | 0.111                                                       |
| 261 | A     | 2                    | 2                      | 1.00                                | 9                   | 0.222                                                       |
| 262 | A     | 2                    | 2                      | 1.00                                | 7                   | 0.286                                                       |
| 263 | A     | 2                    | 2                      | 1.00                                | 2                   | 1.000                                                       |
| 264 | A     | 5                    | 4                      | 1.08                                | 6                   | 0.667                                                       |
| 265 | A     | 5                    | 4                      | 0.81                                | 17                  | 0.235                                                       |
| 266 | A     | 3                    | 3                      | 1.00                                | 4                   | 0.750                                                       |
| 267 | A     | 2                    | 2                      | 1.00                                | 19                  | 0.105                                                       |
| 268 | A     | 1                    | 1                      | 1.00                                | 11                  | 0.091                                                       |
| 269 | A     | 1                    | 1                      | 1.00                                | 9                   | 0.111                                                       |
| 270 | A     | 3                    | 2                      | 1.14                                | 12                  | 0.167                                                       |
| 271 | A     | 3                    | 3                      | 1.00                                | 24                  | 0.125                                                       |
| 272 | A     | 2                    | 2                      | 1.00                                | 29                  | 0.069                                                       |
| 273 | N/A   | 1                    | 0                      | 1.00                                | 12                  | 0.000                                                       |
| 274 | A     | 1                    | 1                      | 1.00                                | 54                  | 0.019                                                       |
| 275 | A     | 4                    | 4                      | 0.90                                | 21                  | 0.190                                                       |
| 276 | A     | 1                    | 1                      | 1.00                                | 2                   | 0.500                                                       |
| 277 | A     | 1                    | 1                      | 1.00                                | 4                   | 0.250                                                       |
| 278 | F     | 0                    | 0                      | N/A                                 | 0.000               | N/A                                                         |
| 279 | F     | 0                    | 0                      | N/A                                 | 0.000               | N/A                                                         |
| 280 | A     | 1                    | 1                      | 1.00                                | 85                  | 0.012                                                       |
| 281 | F     | 0                    | 0                      | N/A                                 | 0.000               | N/A                                                         |
| 282 | A     | 4                    | 4                      | 0.93                                | 107                 | 0.037                                                       |
| 283 | A     | 4                    | 4                      | 1.00                                | 7                   | 0.571                                                       |
| 284 | F     | 0                    | 0                      | N/A                                 | 0.000               | N/A                                                         |

# CHAPTER 3

## LISTING OF INTEGRALS

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| 3.227 | $\int \tanh(2x) dx$                                                               | 1229 |
| 3.228 | $\int \frac{-1+i\epsilon\text{ps} \sinh(x)}{ia-x+i\epsilon\text{ps} \cosh(x)} dx$ | 1233 |
| 3.229 | $\int \cos^2(x) \sin(3+2x) dx$                                                    | 1237 |
| 3.230 | $\int x \arctan(x) dx$                                                            | 1242 |
| 3.231 | $\int x \cot^{-1}(x) dx$                                                          | 1247 |
| 3.232 | $\int x \log(a+x^2) dx$                                                           | 1252 |
| 3.233 | $\int \cos(x) \sin(a+x) dx$                                                       | 1257 |
| 3.234 | $\int \cos(a+x) \sin(x) dx$                                                       | 1261 |
| 3.235 | $\int \sqrt{1+\sin(x)} dx$                                                        | 1265 |
| 3.236 | $\int \sqrt{1-\sin(x)} dx$                                                        | 1269 |
| 3.237 | $\int \sqrt{1+\cos(x)} dx$                                                        | 1273 |
| 3.238 | $\int \sqrt{1-\cos(x)} dx$                                                        | 1277 |
| 3.239 | $\int \frac{1}{-\sqrt{-1+x+\sqrt{x}}} dx$                                         | 1281 |
| 3.240 | $\int \frac{1}{1-\sqrt{1+x}} dx$                                                  | 1285 |
| 3.241 | $\int \frac{x}{\sqrt{36+x^4}} dx$                                                 | 1290 |
| 3.242 | $\int \frac{1}{\sqrt[3]{x+\sqrt{x}}} dx$                                          | 1295 |
| 3.243 | $\int \log(2+3x^2) dx$                                                            | 1300 |
| 3.244 | $\int \cot(x) dx$                                                                 | 1305 |
| 3.245 | $\int \cot^4(x) dx$                                                               | 1310 |
| 3.246 | $\int \tanh(x) dx$                                                                | 1315 |
| 3.247 | $\int \coth(x) dx$                                                                | 1319 |
| 3.248 | $\int b^x dx$                                                                     | 1323 |
| 3.249 | $\int \sqrt{2+\frac{1}{x^4}+x^4} dx$                                              | 1327 |
| 3.250 | $\int \frac{1+2x}{2+3x} dx$                                                       | 1332 |
| 3.251 | $\int x \log(x+\sqrt{1+x^2}) dx$                                                  | 1336 |
| 3.252 | $\int x(1+e^x \sin(x))^2 dx$                                                      | 1341 |
| 3.253 | $\int e^x x \cos(x) dx$                                                           | 1346 |
| 3.254 | $\int \frac{1}{(-3+x)^4} dx$                                                      | 1350 |
| 3.255 | $\int \frac{x}{-1+x^3} dx$                                                        | 1354 |
| 3.256 | $\int \frac{x}{-1+x^4} dx$                                                        | 1360 |
| 3.257 | $\int \frac{(1+x^3) \log(x)}{2+x^4} dx$                                           | 1365 |
| 3.258 | $\int (\log(x) + \log(1+x) + \log(2+x)) dx$                                       | 1370 |
| 3.259 | $\int \frac{1}{5+x^3} dx$                                                         | 1374 |

|       |                                                                                                                                                                         |      |
|-------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| 3.260 | $\int \frac{1}{\sqrt{1+x^2}} dx$                                                                                                                                        | 1380 |
| 3.261 | $\int \sqrt{3+x^2} dx$                                                                                                                                                  | 1384 |
| 3.262 | $\int \frac{x}{(1+x)^2} dx$                                                                                                                                             | 1389 |
| 3.263 | $\int \arcsin(x) dx$                                                                                                                                                    | 1393 |
| 3.264 | $\int x^2 \arcsin(x) dx$                                                                                                                                                | 1397 |
| 3.265 | $\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)} dx$                                                                                                                        | 1402 |
| 3.266 | $\int \cos^2(x) dx$                                                                                                                                                     | 1407 |
| 3.267 | $\int \frac{-2-3x+5x^2}{(-2+x)x^2} dx$                                                                                                                                  | 1411 |
| 3.268 | $\int \frac{1}{\sqrt{9+4x^2}} dx$                                                                                                                                       | 1415 |
| 3.269 | $\int \frac{1}{\sqrt{4+x^2}} dx$                                                                                                                                        | 1419 |
| 3.270 | $\int \frac{1}{10-12x+9x^2} dx$                                                                                                                                         | 1423 |
| 3.271 | $\int \frac{1}{x^4-2x^5+2x^6-2x^7+x^8} dx$                                                                                                                              | 1427 |
| 3.272 | $\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)} dx$                                                                                                                           | 1432 |
| 3.273 | $\int \frac{1}{(2-\log(1+x^2))^5} dx$                                                                                                                                   | 1438 |
| 3.274 | $\int \left( \frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2+\log(x)}{(x+\log^2(x))^2} + \frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)} \right) dx$             | 1443 |
| 3.275 | $\int e^{\frac{x}{2}+xz} x^4 \sin^4(\pi z) dz$                                                                                                                          | 1448 |
| 3.276 | $\int \operatorname{erf}(x) dx$                                                                                                                                         | 1454 |
| 3.277 | $\int \operatorname{erf}(a+x) dx$                                                                                                                                       | 1458 |
| 3.278 | $\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}} dx$                                                                                       | 1462 |
| 3.279 | $\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}} dy$                                                                                                      | 1467 |
| 3.280 | $\int \frac{x(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2})}{(4-5x^2+x^4)(1+\sqrt{-4+x^2}+\sqrt{-1+x^2})} dx$                                        | 1472 |
| 3.281 | $\int \left( \sqrt{9-4\sqrt{2}x} - \sqrt{2}\sqrt{1+4x+2x^2+x^4} \right) dx$                                                                                             | 1478 |
| 3.282 | $\int \frac{e^{-\frac{x}{y}} \left( \pi^2(-3mc^8+4mc^9+24mc^6x-48mc^7x-144mc^5x^2-24mc^2x^3+176mc^3x^3+3x^4+12mcx^4)+12mc^3\pi^2(3mc-12mc^2-8mc^3) \right)}{384x^2} dx$ |      |
| 3.283 | $\int \sec(x) \sin(2x) dx$                                                                                                                                              | 1492 |
| 3.284 | $\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}} dx$                                                                            | 1497 |

### 3.1 $\int (1 + x + x^2) dx$

|       |                                                     |     |
|-------|-----------------------------------------------------|-----|
| 3.1.1 | Optimal result . . . . .                            | 115 |
| 3.1.2 | Mathematica [A] (verified) . . . . .                | 115 |
| 3.1.3 | Rubi [A] (verified) . . . . .                       | 116 |
| 3.1.4 | Maple [A] (verified) . . . . .                      | 116 |
| 3.1.5 | Fricas [A] (verification not implemented) . . . . . | 117 |
| 3.1.6 | Sympy [A] (verification not implemented) . . . . .  | 117 |
| 3.1.7 | Maxima [A] (verification not implemented) . . . . . | 117 |
| 3.1.8 | Giac [A] (verification not implemented) . . . . .   | 118 |
| 3.1.9 | Mupad [B] (verification not implemented) . . . . .  | 118 |

#### 3.1.1 Optimal result

Integrand size = 6, antiderivative size = 16

$$\int (1 + x + x^2) dx = x + \frac{x^2}{2} + \frac{x^3}{3}$$

output `x+1/2*x^2+1/3*x^3`

#### 3.1.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (1 + x + x^2) dx = x + \frac{x^2}{2} + \frac{x^3}{3}$$

input `Integrate[1 + x + x^2,x]`

output `x + x^2/2 + x^3/3`

### 3.1.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^2 + x + 1) dx$$

$$\downarrow \text{2009}$$

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

input `Int[1 + x + x^2,x]`

output `x + x^2/2 + x^3/3`

#### 3.1.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

### 3.1.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

| method       | result                                | size |
|--------------|---------------------------------------|------|
| gospers      | $x + \frac{1}{2}x^2 + \frac{1}{3}x^3$ | 13   |
| default      | $x + \frac{1}{2}x^2 + \frac{1}{3}x^3$ | 13   |
| norman       | $x + \frac{1}{2}x^2 + \frac{1}{3}x^3$ | 13   |
| risch        | $x + \frac{1}{2}x^2 + \frac{1}{3}x^3$ | 13   |
| parallelrisc | $x + \frac{1}{2}x^2 + \frac{1}{3}x^3$ | 13   |
| parts        | $x + \frac{1}{2}x^2 + \frac{1}{3}x^3$ | 13   |

input `int(x^2+x+1,x,method=_RETURNVERBOSE)`

output `x+1/2*x^2+1/3*x^3`

### 3.1.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (1 + x + x^2) dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

input `integrate(x^2+x+1,x, algorithm="fricas")`

output `1/3*x^3 + 1/2*x^2 + x`

### 3.1.6 Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int (1 + x + x^2) dx = \frac{x^3}{3} + \frac{x^2}{2} + x$$

input `integrate(x**2+x+1,x)`

output `x**3/3 + x**2/2 + x`

### 3.1.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (1 + x + x^2) dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

input `integrate(x^2+x+1,x, algorithm="maxima")`

output `1/3*x^3 + 1/2*x^2 + x`

### 3.1.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (1 + x + x^2) dx = \frac{1}{3} x^3 + \frac{1}{2} x^2 + x$$

input `integrate(x^2+x+1,x, algorithm="giac")`

output `1/3*x^3 + 1/2*x^2 + x`

### 3.1.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int (1 + x + x^2) dx = \frac{x(2x^2 + 3x + 6)}{6}$$

input `int(x + x^2 + 1,x)`

output `(x*(3*x + 2*x^2 + 6))/6`

## 3.2 $\int x^2(x + 2x^2)^2 dx$

|       |                                                     |     |
|-------|-----------------------------------------------------|-----|
| 3.2.1 | Optimal result . . . . .                            | 119 |
| 3.2.2 | Mathematica [A] (verified) . . . . .                | 119 |
| 3.2.3 | Rubi [A] (verified) . . . . .                       | 120 |
| 3.2.4 | Maple [A] (verified) . . . . .                      | 121 |
| 3.2.5 | Fricas [A] (verification not implemented) . . . . . | 121 |
| 3.2.6 | Sympy [A] (verification not implemented) . . . . .  | 121 |
| 3.2.7 | Maxima [A] (verification not implemented) . . . . . | 122 |
| 3.2.8 | Giac [A] (verification not implemented) . . . . .   | 122 |
| 3.2.9 | Mupad [B] (verification not implemented) . . . . .  | 122 |

### 3.2.1 Optimal result

Integrand size = 13, antiderivative size = 22

$$\int x^2(x + 2x^2)^2 dx = \frac{x^5}{5} + \frac{2x^6}{3} + \frac{4x^7}{7}$$

output `1/5*x^5+2/3*x^6+4/7*x^7`

### 3.2.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2(x + 2x^2)^2 dx = \frac{x^5}{5} + \frac{2x^6}{3} + \frac{4x^7}{7}$$

input `Integrate[x^2*(x + 2*x^2)^2,x]`

output `x^5/5 + (2*x^6)/3 + (4*x^7)/7`



### 3.2.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int x^2(2x^2 + x)^2 dx \\ & \quad \downarrow 9 \\ & \int x^4(2x + 1)^2 dx \\ & \quad \downarrow 49 \\ & \int (4x^6 + 4x^5 + x^4) dx \\ & \quad \downarrow 2009 \\ & \frac{4x^7}{7} + \frac{2x^6}{3} + \frac{x^5}{5} \end{aligned}$$

input `Int[x^2*(x + 2*x^2)^2,x]`

output `x^5/5 + (2*x^6)/3 + (4*x^7)/7`

#### 3.2.3.1 Defintions of rubi rules used

rule 9 `Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] := With[{r = Expon[Px, x, Min]}, Simp[1/e^(p*r) Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p, x], x] /; IGtQ[r, 0] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] && !MonomialQ[Px, x]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

---

3.2.  $\int x^2(x + 2x^2)^2 dx$

### 3.2.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

| method        | result                                             | size |
|---------------|----------------------------------------------------|------|
| gospers       | $\frac{x^5(60x^2+70x+21)}{105}$                    | 16   |
| default       | $\frac{1}{5}x^5 + \frac{2}{3}x^6 + \frac{4}{7}x^7$ | 17   |
| norman        | $\frac{1}{5}x^5 + \frac{2}{3}x^6 + \frac{4}{7}x^7$ | 17   |
| risch         | $\frac{1}{5}x^5 + \frac{2}{3}x^6 + \frac{4}{7}x^7$ | 17   |
| parallelrisch | $\frac{1}{5}x^5 + \frac{2}{3}x^6 + \frac{4}{7}x^7$ | 17   |

input `int(x^2*(2*x^2+x)^2,x,method=_RETURNVERBOSE)`

output `1/105*x^5*(60*x^2+70*x+21)`

### 3.2.5 Fracas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x^2(x + 2x^2)^2 dx = \frac{4}{7}x^7 + \frac{2}{3}x^6 + \frac{1}{5}x^5$$

input `integrate(x^2*(2*x^2+x)^2,x, algorithm="fracas")`

output `4/7*x^7 + 2/3*x^6 + 1/5*x^5`

### 3.2.6 Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int x^2(x + 2x^2)^2 dx = \frac{4x^7}{7} + \frac{2x^6}{3} + \frac{x^5}{5}$$

input `integrate(x**2*(2*x**2+x)**2,x)`

output `4*x**7/7 + 2*x**6/3 + x**5/5`

**3.2.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x^2(x + 2x^2)^2 dx = \frac{4}{7}x^7 + \frac{2}{3}x^6 + \frac{1}{5}x^5$$

input `integrate(x^2*(2*x^2+x)^2,x, algorithm="maxima")`output `4/7*x^7 + 2/3*x^6 + 1/5*x^5`**3.2.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x^2(x + 2x^2)^2 dx = \frac{4}{7}x^7 + \frac{2}{3}x^6 + \frac{1}{5}x^5$$

input `integrate(x^2*(2*x^2+x)^2,x, algorithm="giac")`output `4/7*x^7 + 2/3*x^6 + 1/5*x^5`**3.2.9 Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int x^2(x + 2x^2)^2 dx = \frac{x^5(60x^2 + 70x + 21)}{105}$$

input `int(x^2*(x + 2*x^2)^2,x)`output `(x^5*(70*x + 60*x^2 + 21))/105`

### 3.3 $\int x(1 + 2x + x^2) dx$

|       |                                                     |     |
|-------|-----------------------------------------------------|-----|
| 3.3.1 | Optimal result . . . . .                            | 123 |
| 3.3.2 | Mathematica [A] (verified) . . . . .                | 123 |
| 3.3.3 | Rubi [A] (verified) . . . . .                       | 124 |
| 3.3.4 | Maple [A] (verified) . . . . .                      | 125 |
| 3.3.5 | Fricas [A] (verification not implemented) . . . . . | 125 |
| 3.3.6 | Sympy [A] (verification not implemented) . . . . .  | 125 |
| 3.3.7 | Maxima [A] (verification not implemented) . . . . . | 126 |
| 3.3.8 | Giac [A] (verification not implemented) . . . . .   | 126 |
| 3.3.9 | Mupad [B] (verification not implemented) . . . . .  | 126 |

#### 3.3.1 Optimal result

Integrand size = 10, antiderivative size = 22

$$\int x(1 + 2x + x^2) dx = \frac{x^2}{2} + \frac{2x^3}{3} + \frac{x^4}{4}$$

output `1/2*x^2+2/3*x^3+1/4*x^4`

#### 3.3.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(1 + 2x + x^2) dx = \frac{x^2}{2} + \frac{2x^3}{3} + \frac{x^4}{4}$$

input `Integrate[x*(1 + 2*x + x^2),x]`

output `x^2/2 + (2*x^3)/3 + x^4/4`

### 3.3.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {1098, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int x(x^2 + 2x + 1) dx \\ & \quad \downarrow \text{1098} \\ & \int x(x + 1)^2 dx \\ & \quad \downarrow \text{49} \\ & \int (x^3 + 2x^2 + x) dx \\ & \quad \downarrow \text{2009} \\ & \frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2} \end{aligned}$$

input `Int[x*(1 + 2*x + x^2),x]`

output `x^2/2 + (2*x^3)/3 + x^4/4`

#### 3.3.3.1 Defintions of rubi rules used

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 1098 `Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/c^p Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

### 3.3.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

| method        | result                                             | size |
|---------------|----------------------------------------------------|------|
| gospers       | $\frac{x^2(3x^2+8x+6)}{12}$                        | 16   |
| default       | $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{1}{4}x^4$ | 17   |
| norman        | $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{1}{4}x^4$ | 17   |
| risch         | $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{1}{4}x^4$ | 17   |
| parallelrisch | $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{1}{4}x^4$ | 17   |

input `int(x*(x^2+2*x+1),x,method=_RETURNVERBOSE)`

output `1/12*x^2*(3*x^2+8*x+6)`

### 3.3.5 Fracas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x(1 + 2x + x^2) dx = \frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2$$

input `integrate(x*(x^2+2*x+1),x, algorithm="fricas")`

output `1/4*x^4 + 2/3*x^3 + 1/2*x^2`

### 3.3.6 Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int x(1 + 2x + x^2) dx = \frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2}$$

input `integrate(x*(x**2+2*x+1),x)`

output `x**4/4 + 2*x**3/3 + x**2/2`

**3.3.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x(1 + 2x + x^2) dx = \frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2$$

input `integrate(x*(x^2+2*x+1),x, algorithm="maxima")`output `1/4*x^4 + 2/3*x^3 + 1/2*x^2`**3.3.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x(1 + 2x + x^2) dx = \frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2$$

input `integrate(x*(x^2+2*x+1),x, algorithm="giac")`output `1/4*x^4 + 2/3*x^3 + 1/2*x^2`**3.3.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int x(1 + 2x + x^2) dx = \frac{x^2(3x^2 + 8x + 6)}{12}$$

input `int(x*(2*x + x^2 + 1),x)`output `(x^2*(8*x + 3*x^2 + 6))/12`

## 3.4 $\int \frac{1}{x} dx$

|       |                                                     |     |
|-------|-----------------------------------------------------|-----|
| 3.4.1 | Optimal result . . . . .                            | 127 |
| 3.4.2 | Mathematica [A] (verified) . . . . .                | 127 |
| 3.4.3 | Rubi [A] (verified) . . . . .                       | 128 |
| 3.4.4 | Maple [A] (verified) . . . . .                      | 128 |
| 3.4.5 | Fricas [A] (verification not implemented) . . . . . | 129 |
| 3.4.6 | Sympy [A] (verification not implemented) . . . . .  | 129 |
| 3.4.7 | Maxima [A] (verification not implemented) . . . . . | 129 |
| 3.4.8 | Giac [A] (verification not implemented) . . . . .   | 130 |
| 3.4.9 | Mupad [B] (verification not implemented) . . . . .  | 130 |

### 3.4.1 Optimal result

Integrand size = 3, antiderivative size = 2

$$\int \frac{1}{x} dx = \log(x)$$

output

`ln(x)`

### 3.4.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} dx = \log(x)$$

input `Integrate[x^(-1), x]`

output `Log[x]`



### 3.4.3 Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x} dx$$

↓ 14

$$\log(x)$$

input `Int [x-1 , x]`

output `Log [x]`

#### 3.4.3.1 Defintions of rubi rules used

rule 14 `Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]`

### 3.4.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

| method        | result   | size |
|---------------|----------|------|
| default       | $\ln(x)$ | 3    |
| norman        | $\ln(x)$ | 3    |
| risch         | $\ln(x)$ | 3    |
| parallelrisch | $\ln(x)$ | 3    |

input `int(1/x,x,method=_RETURNVERBOSE)`

output `ln(x)`

### 3.4.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} dx = \log(x)$$

input `integrate(1/x,x, algorithm="fricas")`

output `log(x)`

### 3.4.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} dx = \log(x)$$

input `integrate(1/x,x)`

output `log(x)`

### 3.4.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} dx = \log(x)$$

input `integrate(1/x,x, algorithm="maxima")`

output `log(x)`

### 3.4.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{x} dx = \log(|x|)$$

input `integrate(1/x,x, algorithm="giac")`

output `log(abs(x))`

### 3.4.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} dx = \ln(x)$$

input `int(1/x,x)`

output `log(x)`

## 3.5 $\int \frac{(1+x)^3}{(-1+x)^4} dx$

|       |                                                     |     |
|-------|-----------------------------------------------------|-----|
| 3.5.1 | Optimal result . . . . .                            | 131 |
| 3.5.2 | Mathematica [A] (verified) . . . . .                | 131 |
| 3.5.3 | Rubi [A] (verified) . . . . .                       | 132 |
| 3.5.4 | Maple [A] (verified) . . . . .                      | 133 |
| 3.5.5 | Fricas [A] (verification not implemented) . . . . . | 133 |
| 3.5.6 | Sympy [A] (verification not implemented) . . . . .  | 134 |
| 3.5.7 | Maxima [A] (verification not implemented) . . . . . | 134 |
| 3.5.8 | Giac [A] (verification not implemented) . . . . .   | 134 |
| 3.5.9 | Mupad [B] (verification not implemented) . . . . .  | 135 |

### 3.5.1 Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{(1+x)^3}{(-1+x)^4} dx = \frac{8}{3(1-x)^3} - \frac{6}{(1-x)^2} + \frac{6}{1-x} + \log(1-x)$$

output `8/3/(1-x)^3-6/(1-x)^2+6/(1-x)+ln(1-x)`

### 3.5.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(1+x)^3}{(-1+x)^4} dx = -\frac{2(4-9x+9x^2)}{3(-1+x)^3} + \log(-1+x)$$

input `Integrate[(1 + x)^3/(-1 + x)^4,x]`

output `(-2*(4 - 9*x + 9*x^2))/(3*(-1 + x)^3) + Log[-1 + x]`

### 3.5.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^3}{(x-1)^4} dx$$

↓ 49

$$\int \left( \frac{1}{x-1} + \frac{6}{(x-1)^2} + \frac{12}{(x-1)^3} + \frac{8}{(x-1)^4} \right) dx$$

↓ 2009

$$\frac{6}{1-x} - \frac{6}{(1-x)^2} + \frac{8}{3(1-x)^3} + \log(1-x)$$

input `Int[(1 + x)^3/(-1 + x)^4,x]`

output `8/(3*(1 - x)^3) - 6/(1 - x)^2 + 6/(1 - x) + Log[1 - x]`

#### 3.5.3.1 Defintions of rubi rules used

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int  
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]  
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

### 3.5.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

| method       | result                                                                                                                          | size |
|--------------|---------------------------------------------------------------------------------------------------------------------------------|------|
| norman       | $\frac{-6x^2+6x-\frac{8}{3}}{(-1+x)^3} + \ln(-1+x)$                                                                             | 22   |
| risch        | $\frac{-6x^2+6x-\frac{8}{3}}{(-1+x)^3} + \ln(-1+x)$                                                                             | 22   |
| default      | $\ln(-1+x) - \frac{8}{3(-1+x)^3} - \frac{6}{-1+x} - \frac{6}{(-1+x)^2}$                                                         | 27   |
| parallelrisc | $\frac{3 \ln(-1+x)x^3 - 8 - 9 \ln(-1+x)x^2 + 9 \ln(-1+x)x - 18x^2 - 3 \ln(-1+x) + 18x}{3(-1+x)^3}$                              | 49   |
| meijerg      | $\frac{x(x^2-3x+3)}{3(1-x)^3} + \frac{x^2(3-x)}{2(1-x)^3} + \frac{x^3}{(1-x)^3} + \frac{x(22x^2-30x+12)}{12(1-x)^3} + \ln(1-x)$ | 74   |

input `int((1+x)^3/(-1+x)^4,x,method=_RETURNVERBOSE)`

output `(-6*x^2+6*x-8/3)/(-1+x)^3+ln(-1+x)`

### 3.5.5 Fracas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \frac{(1+x)^3}{(-1+x)^4} dx = -\frac{18x^2 - 3(x^3 - 3x^2 + 3x - 1)\log(x-1) - 18x + 8}{3(x^3 - 3x^2 + 3x - 1)}$$

input `integrate((1+x)^3/(-1+x)^4,x, algorithm="fricas")`

output `-1/3*(18*x^2 - 3*(x^3 - 3*x^2 + 3*x - 1)*log(x - 1) - 18*x + 8)/(x^3 - 3*x^2 + 3*x - 1)`

**3.5.6 Sympy [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{(1+x)^3}{(-1+x)^4} dx = \frac{-18x^2 + 18x - 8}{3x^3 - 9x^2 + 9x - 3} + \log(x-1)$$

input `integrate((1+x)**3/(-1+x)**4,x)`output `(-18*x**2 + 18*x - 8)/(3*x**3 - 9*x**2 + 9*x - 3) + log(x - 1)`**3.5.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{(1+x)^3}{(-1+x)^4} dx = -\frac{2(9x^2 - 9x + 4)}{3(x^3 - 3x^2 + 3x - 1)} + \log(x-1)$$

input `integrate((1+x)^3/(-1+x)^4,x, algorithm="maxima")`output `-2/3*(9*x^2 - 9*x + 4)/(x^3 - 3*x^2 + 3*x - 1) + log(x - 1)`**3.5.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{(1+x)^3}{(-1+x)^4} dx = -\frac{2(9x^2 - 9x + 4)}{3(x-1)^3} + \log(|x-1|)$$

input `integrate((1+x)^3/(-1+x)^4,x, algorithm="giac")`output `-2/3*(9*x^2 - 9*x + 4)/(x - 1)^3 + log(abs(x - 1))`

**3.5.9 Mupad [B] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{(1+x)^3}{(-1+x)^4} dx = \ln(x-1) - \frac{6x^2 - 6x + \frac{8}{3}}{(x-1)^3}$$

input `int((x + 1)^3/(x - 1)^4,x)`

output `log(x - 1) - (6*x^2 - 6*x + 8/3)/(x - 1)^3`



### 3.6 $\int \frac{1}{(-1+x)x(1+x)^2} dx$

|       |                                                     |     |
|-------|-----------------------------------------------------|-----|
| 3.6.1 | Optimal result . . . . .                            | 136 |
| 3.6.2 | Mathematica [A] (verified) . . . . .                | 136 |
| 3.6.3 | Rubi [A] (verified) . . . . .                       | 137 |
| 3.6.4 | Maple [A] (verified) . . . . .                      | 138 |
| 3.6.5 | Fricas [A] (verification not implemented) . . . . . | 138 |
| 3.6.6 | Sympy [A] (verification not implemented) . . . . .  | 138 |
| 3.6.7 | Maxima [A] (verification not implemented) . . . . . | 139 |
| 3.6.8 | Giac [A] (verification not implemented) . . . . .   | 139 |
| 3.6.9 | Mupad [B] (verification not implemented) . . . . .  | 139 |

#### 3.6.1 Optimal result

Integrand size = 14, antiderivative size = 32

$$\int \frac{1}{(-1+x)x(1+x)^2} dx = -\frac{1}{2(1+x)} + \frac{1}{4} \log(1-x) - \log(x) + \frac{3}{4} \log(1+x)$$

output `-1/2/(1+x)+1/4*ln(1-x)-ln(x)+3/4*ln(1+x)`

#### 3.6.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1}{(-1+x)x(1+x)^2} dx = \frac{1}{4} \left( -\frac{2}{1+x} + \log(1-x) - 4 \log(x) + 3 \log(1+x) \right)$$

input `Integrate[1/((-1 + x)*x*(1 + x)^2),x]`

output `(-2/(1 + x) + Log[1 - x] - 4*Log[x] + 3*Log[1 + x])/4`

### 3.6.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-1)x(x+1)^2} dx$$

↓ 93

$$\int \left( -\frac{1}{x} + \frac{3}{4(x+1)} + \frac{1}{2(x+1)^2} + \frac{1}{4(x-1)} \right) dx$$

↓ 2009

$$-\frac{1}{2(x+1)} + \frac{1}{4} \log(1-x) - \log(x) + \frac{3}{4} \log(x+1)$$

input `Int[1/((-1 + x)*x*(1 + x)^2),x]`

output `-1/2*1/(1 + x) + Log[1 - x]/4 - Log[x] + (3*Log[1 + x])/4`

#### 3.6.3.1 Defintions of rubi rules used

rule 93 `Int[((e_.) + (f_.)*(x_)^(p_))/((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)), x_] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

### 3.6.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

| method       | result                                                                                      | size |
|--------------|---------------------------------------------------------------------------------------------|------|
| default      | $\frac{\ln(-1+x)}{4} - \ln(x) - \frac{1}{2(1+x)} + \frac{3\ln(1+x)}{4}$                     | 25   |
| norman       | $\frac{\ln(-1+x)}{4} - \ln(x) - \frac{1}{2(1+x)} + \frac{3\ln(1+x)}{4}$                     | 25   |
| risch        | $\frac{\ln(-1+x)}{4} - \ln(x) - \frac{1}{2(1+x)} + \frac{3\ln(1+x)}{4}$                     | 25   |
| parallelrisc | $-\frac{4x \ln(x) - \ln(-1+x)x - 3\ln(1+x)x + 2 + 4\ln(x) - \ln(-1+x) - 3\ln(1+x)}{4(1+x)}$ | 45   |

input `int(1/(-1+x)/x/(1+x)^2,x,method=_RETURNVERBOSE)`

output `1/4*ln(-1+x)-ln(x)-1/2/(1+x)+3/4*ln(1+x)`

### 3.6.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{1}{(-1+x)x(1+x)^2} dx = \frac{3(x+1)\log(x+1) + (x+1)\log(x-1) - 4(x+1)\log(x) - 2}{4(x+1)}$$

input `integrate(1/(-1+x)/x/(1+x)^2,x, algorithm="fricas")`

output `1/4*(3*(x + 1)*log(x + 1) + (x + 1)*log(x - 1) - 4*(x + 1)*log(x) - 2)/(x + 1)`

### 3.6.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-1+x)x(1+x)^2} dx = -\log(x) + \frac{\log(x-1)}{4} + \frac{3\log(x+1)}{4} - \frac{1}{2x+2}$$

input `integrate(1/(-1+x)/x/(1+x)**2,x)`

output `-log(x) + log(x - 1)/4 + 3*log(x + 1)/4 - 1/(2*x + 2)`

---

3.6.  $\int \frac{1}{(-1+x)x(1+x)^2} dx$

**3.6.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-1+x)x(1+x)^2} dx = -\frac{1}{2(x+1)} + \frac{3}{4} \log(x+1) + \frac{1}{4} \log(x-1) - \log(x)$$

input `integrate(1/(-1+x)/x/(1+x)^2,x, algorithm="maxima")`output `-1/2/(x + 1) + 3/4*log(x + 1) + 1/4*log(x - 1) - log(x)`**3.6.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{1}{(-1+x)x(1+x)^2} dx = -\frac{1}{2(x+1)} - \log\left(\left|-\frac{1}{x+1} + 1\right|\right) + \frac{1}{4} \log\left(\left|-\frac{2}{x+1} + 1\right|\right)$$

input `integrate(1/(-1+x)/x/(1+x)^2,x, algorithm="giac")`output `-1/2/(x + 1) - log(abs(-1/(x + 1) + 1)) + 1/4*log(abs(-2/(x + 1) + 1))`**3.6.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-1+x)x(1+x)^2} dx = \frac{\ln(x-1)}{4} + \frac{3 \ln(x+1)}{4} - \ln(x) - \frac{1}{2(x+1)}$$

input `int(1/(x*(x - 1)*(x + 1)^2),x)`output `log(x - 1)/4 + (3*log(x + 1))/4 - log(x) - 1/(2*(x + 1))`

### 3.7 $\int \frac{b+ax}{(-p+x)(-q+x)} dx$

|       |                                                     |     |
|-------|-----------------------------------------------------|-----|
| 3.7.1 | Optimal result . . . . .                            | 140 |
| 3.7.2 | Mathematica [A] (verified) . . . . .                | 140 |
| 3.7.3 | Rubi [A] (verified) . . . . .                       | 141 |
| 3.7.4 | Maple [A] (verified) . . . . .                      | 142 |
| 3.7.5 | Fricas [A] (verification not implemented) . . . . . | 142 |
| 3.7.6 | Sympy [B] (verification not implemented) . . . . .  | 142 |
| 3.7.7 | Maxima [A] (verification not implemented) . . . . . | 143 |
| 3.7.8 | Giac [A] (verification not implemented) . . . . .   | 143 |
| 3.7.9 | Mupad [B] (verification not implemented) . . . . .  | 144 |

#### 3.7.1 Optimal result

Integrand size = 20, antiderivative size = 40

$$\int \frac{b+ax}{(-p+x)(-q+x)} dx = \frac{(b+ap)\log(p-x)}{p-q} - \frac{(b+aq)\log(q-x)}{p-q}$$

output  $(a*p+b)*\ln(p-x)/(p-q)-(a*q+b)*\ln(q-x)/(p-q)$

#### 3.7.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{b+ax}{(-p+x)(-q+x)} dx = \frac{(b+ap)\log(-p+x) - (b+aq)\log(-q+x)}{p-q}$$

input `Integrate[(b + a*x)/((-p + x)*(-q + x)),x]`

output  $((b + a*p)*\text{Log}[-p + x] - (b + a*q)*\text{Log}[-q + x])/(p - q)$

### 3.7.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax + b}{(x - p)(x - q)} dx$$

↓ 86

$$\int \left( \frac{-ap - b}{(p - q)(p - x)} + \frac{aq + b}{(p - q)(q - x)} \right) dx$$

↓ 2009

$$\frac{(ap + b) \log(p - x)}{p - q} - \frac{(aq + b) \log(q - x)}{p - q}$$

input `Int[(b + a*x)/((-p + x)*(-q + x)),x]`

output `((b + a*p)*Log[p - x])/(p - q) - ((b + a*q)*Log[q - x])/(p - q)`

#### 3.7.3.1 Defintions of rubi rules used

rule 86 `Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0]) || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

### 3.7.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

| method        | result                                                                                               | size |
|---------------|------------------------------------------------------------------------------------------------------|------|
| norman        | $\frac{(ap+b)\ln(p-x)}{p-q} - \frac{(aq+b)\ln(q-x)}{p-q}$                                            | 41   |
| default       | $\frac{(-aq-b)\ln(q-x)}{p-q} + \frac{(ap+b)\ln(p-x)}{p-q}$                                           | 43   |
| parallelrisch | $\frac{\ln(-p+x)ap - \ln(-q+x)aq + \ln(-p+x)b - \ln(-q+x)b}{p-q}$                                    | 46   |
| risch         | $-\frac{\ln(-q+x)aq}{p-q} - \frac{\ln(-q+x)b}{p-q} + \frac{\ln(p-x)ap}{p-q} + \frac{\ln(p-x)b}{p-q}$ | 66   |

input `int((a*x+b)/(-p+x)/(-q+x),x,method=_RETURNVERBOSE)`

output `(a*p+b)*ln(p-x)/(p-q)-(a*q+b)*ln(q-x)/(p-q)`

### 3.7.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{b+ax}{(-p+x)(-q+x)} dx = \frac{(ap+b)\log(-p+x) - (aq+b)\log(-q+x)}{p-q}$$

input `integrate((a*x+b)/(-p+x)/(-q+x),x, algorithm="fracas")`

output `((a*p + b)*log(-p + x) - (a*q + b)*log(-q + x))/(p - q)`

### 3.7.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(26) = 52.

Time = 0.54 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.60

$$\int \frac{b+ax}{(-p+x)(-q+x)} dx = \frac{(ap+b)\log\left(x + \frac{-2apq-bp-bq - \frac{p^2(ap+b)}{p-q} + \frac{2pq(ap+b)}{p-q} - \frac{q^2(ap+b)}{p-q}}{ap+aq+2b}\right)}{p-q} - \frac{(aq+b)\log\left(x + \frac{-2apq-bp-bq + \frac{p^2(aq+b)}{p-q} - \frac{2pq(aq+b)}{p-q} + \frac{q^2(aq+b)}{p-q}}{ap+aq+2b}\right)}{p-q}$$

---

3.7.  $\int \frac{b+ax}{(-p+x)(-q+x)} dx$

input `integrate((a*x+b)/(-p+x)/(-q+x),x)`

output  $(a*p + b)*\log(x + (-2*a*p*q - b*p - b*q - p**2*(a*p + b)/(p - q) + 2*p*q*(a*p + b)/(p - q) - q**2*(a*p + b)/(p - q))/(a*p + a*q + 2*b)/(p - q) - (a*q + b)*\log(x + (-2*a*p*q - b*p - b*q + p**2*(a*q + b)/(p - q) - 2*p*q*(a*q + b)/(p - q) + q**2*(a*q + b)/(p - q))/(a*p + a*q + 2*b)/(p - q)$

### 3.7.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{b + ax}{(-p + x)(-q + x)} dx = \frac{(ap + b) \log(-p + x)}{p - q} - \frac{(aq + b) \log(-q + x)}{p - q}$$

input `integrate((a*x+b)/(-p+x)/(-q+x),x, algorithm="maxima")`

output  $(a*p + b)*\log(-p + x)/(p - q) - (a*q + b)*\log(-q + x)/(p - q)$

### 3.7.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{b + ax}{(-p + x)(-q + x)} dx = \frac{(ap + b) \log(|-p + x|)}{p - q} - \frac{(aq + b) \log(|-q + x|)}{p - q}$$

input `integrate((a*x+b)/(-p+x)/(-q+x),x, algorithm="giac")`

output  $(a*p + b)*\log(\text{abs}(-p + x))/(p - q) - (a*q + b)*\log(\text{abs}(-q + x))/(p - q)$



**3.7.9 Mupad [B] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{b + ax}{(-p + x)(-q + x)} dx = \frac{\ln(x - p)(b + ap)}{p - q} - \frac{\ln(x - q)(b + aq)}{p - q}$$

input `int((b + a*x)/((p - x)*(q - x)),x)`

output `(log(x - p)*(b + a*p))/(p - q) - (log(x - q)*(b + a*q))/(p - q)`

### 3.8 $\int \frac{1}{c+bx+ax^2} dx$

|       |                                                     |     |
|-------|-----------------------------------------------------|-----|
| 3.8.1 | Optimal result . . . . .                            | 145 |
| 3.8.2 | Mathematica [A] (verified) . . . . .                | 145 |
| 3.8.3 | Rubi [A] (verified) . . . . .                       | 146 |
| 3.8.4 | Maple [A] (verified) . . . . .                      | 147 |
| 3.8.5 | Fricas [A] (verification not implemented) . . . . . | 147 |
| 3.8.6 | Sympy [B] (verification not implemented) . . . . .  | 148 |
| 3.8.7 | Maxima [F(-2)] . . . . .                            | 148 |
| 3.8.8 | Giac [A] (verification not implemented) . . . . .   | 149 |
| 3.8.9 | Mupad [B] (verification not implemented) . . . . .  | 149 |

#### 3.8.1 Optimal result

Integrand size = 12, antiderivative size = 34

$$\int \frac{1}{c + bx + ax^2} dx = -\frac{2\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output `-2*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)`

#### 3.8.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{1}{c + bx + ax^2} dx = \frac{2 \arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

input `Integrate[(c + b*x + a*x^2)^(-1),x]`

output `(2*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]`

### 3.8.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^2 + bx + c} dx$$

$$\downarrow \text{1083}$$

$$-2 \int \frac{1}{b^2 - (b + 2ax)^2 - 4ac} d(b + 2ax)$$

$$\downarrow \text{219}$$

$$-\frac{2 \operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input `Int[(c + b*x + a*x^2)^(-1),x]`

output `(-2*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]`

#### 3.8.3.1 Defintions of rubi rules used

rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

rule 1083 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Simp[-2 Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x]`

### 3.8.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

| method  | result                                                                                                      | size |
|---------|-------------------------------------------------------------------------------------------------------------|------|
| default | $\frac{2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$                                 | 35   |
| risch   | $-\frac{\ln(2ax+\sqrt{-4ac+b^2}+b)}{\sqrt{-4ac+b^2}} + \frac{\ln(-2ax+\sqrt{-4ac+b^2}-b)}{\sqrt{-4ac+b^2}}$ | 61   |

input `int(1/(a*x^2+b*x+c),x,method=_RETURNVERBOSE)`

output `2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))`

### 3.8.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.53

$$\int \frac{1}{c+bx+ax^2} dx = \left[ \frac{\log\left(\frac{2a^2x^2+2abx+b^2-2ac-\sqrt{b^2-4ac}(2ax+b)}{ax^2+bx+c}\right)}{\sqrt{b^2-4ac}}, \right. \\ \left. -\frac{2\sqrt{-b^2+4ac} \arctan\left(-\frac{\sqrt{-b^2+4ac}(2ax+b)}{b^2-4ac}\right)}{b^2-4ac} \right]$$

input `integrate(1/(a*x^2+b*x+c),x, algorithm="fracas")`

output `[log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]`

### 3.8.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs.  $2(34) = 68$ .

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.65

$$\int \frac{1}{c + bx + ax^2} dx = -\sqrt{-\frac{1}{4ac - b^2}} \log \left( x + \frac{-4ac\sqrt{-\frac{1}{4ac - b^2}} + b^2\sqrt{-\frac{1}{4ac - b^2}} + b}{2a} \right) \\ + \sqrt{-\frac{1}{4ac - b^2}} \log \left( x + \frac{4ac\sqrt{-\frac{1}{4ac - b^2}} - b^2\sqrt{-\frac{1}{4ac - b^2}} + b}{2a} \right)$$

input `integrate(1/(a*x**2+b*x+c),x)`

output `-sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*a)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*a))`

### 3.8.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{c + bx + ax^2} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(a*x^2+b*x+c),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more deta`

**3.8.8 Giac [A] (verification not implemented)**

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{c + bx + ax^2} dx = \frac{2 \arctan\left(\frac{2ax+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

input `integrate(1/(a*x^2+b*x+c),x, algorithm="giac")`output `2*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)`**3.8.9 Mupad [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{1}{c + bx + ax^2} dx = \frac{2 \operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2ax}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

input `int(1/(c + b*x + a*x^2),x)`output `(2*atan(b/(4*a*c - b^2)^(1/2) + (2*a*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)^(1/2)`

### 3.9 $\int \frac{b+ax}{1+x^2} dx$

|       |                                                     |     |
|-------|-----------------------------------------------------|-----|
| 3.9.1 | Optimal result . . . . .                            | 150 |
| 3.9.2 | Mathematica [A] (verified) . . . . .                | 150 |
| 3.9.3 | Rubi [A] (verified) . . . . .                       | 151 |
| 3.9.4 | Maple [A] (verified) . . . . .                      | 152 |
| 3.9.5 | Fricas [A] (verification not implemented) . . . . . | 152 |
| 3.9.6 | Sympy [C] (verification not implemented) . . . . .  | 152 |
| 3.9.7 | Maxima [A] (verification not implemented) . . . . . | 153 |
| 3.9.8 | Giac [A] (verification not implemented) . . . . .   | 153 |
| 3.9.9 | Mupad [B] (verification not implemented) . . . . .  | 153 |

#### 3.9.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{b+ax}{1+x^2} dx = b \arctan(x) + \frac{1}{2}a \log(1+x^2)$$

output `b*arctan(x)+1/2*a*ln(x^2+1)`

#### 3.9.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{b+ax}{1+x^2} dx = b \arctan(x) + \frac{1}{2}a \log(1+x^2)$$

input `Integrate[(b + a*x)/(1 + x^2), x]`

output `b*ArcTan[x] + (a*Log[1 + x^2])/2`

### 3.9.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{ax+b}{x^2+1} dx \\ & \quad \downarrow \text{452} \\ & a \int \frac{x}{x^2+1} dx + b \int \frac{1}{x^2+1} dx \\ & \quad \downarrow \text{216} \\ & a \int \frac{x}{x^2+1} dx + b \arctan(x) \\ & \quad \downarrow \text{240} \\ & \frac{1}{2} a \log(x^2+1) + b \arctan(x) \end{aligned}$$

input `Int[(b + a*x)/(1 + x^2),x]`

output `b*ArcTan[x] + (a*Log[1 + x^2])/2`

#### 3.9.3.1 Defintions of rubi rules used

rule 216 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

rule 240 `Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]`

rule 452 `Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] := Simp[c Int[1/(a + b*x^2), x], x] + Simp[d Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c^2 + a*d^2, 0]`



### 3.9.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

| method       | result                                                                                      | size |
|--------------|---------------------------------------------------------------------------------------------|------|
| default      | $b \arctan(x) + \frac{a \ln(x^2+1)}{2}$                                                     | 15   |
| meijerg      | $b \arctan(x) + \frac{a \ln(x^2+1)}{2}$                                                     | 15   |
| risch        | $b \arctan(x) + \frac{a \ln(x^2+1)}{2}$                                                     | 15   |
| parallelrisc | $\frac{\ln(x-i)a}{2} - \frac{i \ln(x-i)b}{2} + \frac{\ln(x+i)a}{2} + \frac{i \ln(x+i)b}{2}$ | 36   |

input `int((a*x+b)/(x^2+1),x,method=_RETURNVERBOSE)`

output `b*arctan(x)+1/2*a*ln(x^2+1)`

### 3.9.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+ax}{1+x^2} dx = b \arctan(x) + \frac{1}{2} a \log(x^2+1)$$

input `integrate((a*x+b)/(x^2+1),x, algorithm="fricas")`

output `b*arctan(x) + 1/2*a*log(x^2 + 1)`

### 3.9.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{b+ax}{1+x^2} dx = \left(\frac{a}{2} - \frac{ib}{2}\right) \log(x-i) + \left(\frac{a}{2} + \frac{ib}{2}\right) \log(x+i)$$

input `integrate((a*x+b)/(x**2+1),x)`

output `(a/2 - I*b/2)*log(x - I) + (a/2 + I*b/2)*log(x + I)`

**3.9.7 Maxima [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b + ax}{1 + x^2} dx = b \arctan(x) + \frac{1}{2} a \log(x^2 + 1)$$

input `integrate((a*x+b)/(x^2+1),x, algorithm="maxima")`

output `b*arctan(x) + 1/2*a*log(x^2 + 1)`

**3.9.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b + ax}{1 + x^2} dx = b \arctan(x) + \frac{1}{2} a \log(x^2 + 1)$$

input `integrate((a*x+b)/(x^2+1),x, algorithm="giac")`

output `b*arctan(x) + 1/2*a*log(x^2 + 1)`

**3.9.9 Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b + ax}{1 + x^2} dx = \frac{a \ln(x^2 + 1)}{2} + b \operatorname{atan}(x)$$

input `int((b + a*x)/(x^2 + 1),x)`

output `(a*log(x^2 + 1))/2 + b*atan(x)`

### 3.10 $\int \frac{1}{3-2x+x^2} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.10.1 | Optimal result . . . . .                            | 154 |
| 3.10.2 | Mathematica [A] (verified) . . . . .                | 154 |
| 3.10.3 | Rubi [A] (verified) . . . . .                       | 155 |
| 3.10.4 | Maple [A] (verified) . . . . .                      | 156 |
| 3.10.5 | Fricas [A] (verification not implemented) . . . . . | 156 |
| 3.10.6 | Sympy [A] (verification not implemented) . . . . .  | 156 |
| 3.10.7 | Maxima [A] (verification not implemented) . . . . . | 157 |
| 3.10.8 | Giac [A] (verification not implemented) . . . . .   | 157 |
| 3.10.9 | Mupad [B] (verification not implemented) . . . . .  | 157 |

#### 3.10.1 Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \frac{1}{3-2x+x^2} dx = -\frac{\arctan\left(\frac{1-x}{\sqrt{2}}\right)}{\sqrt{2}}$$

output `-1/2*arctan(1/2*(1-x)*2^(1/2))*2^(1/2)`

#### 3.10.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{3-2x+x^2} dx = \frac{\arctan\left(\frac{-1+x}{\sqrt{2}}\right)}{\sqrt{2}}$$

input `Integrate[(3 - 2*x + x^2)^(-1),x]`

output `ArcTan[(-1 + x)/Sqrt[2]]/Sqrt[2]`

### 3.10.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 - 2x + 3} dx$$

$$\downarrow \text{1083}$$

$$-2 \int \frac{1}{-(2x - 2)^2 - 8} d(2x - 2)$$

$$\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{2x-2}{2\sqrt{2}}\right)}{\sqrt{2}}$$

input `Int[(3 - 2*x + x^2)^(-1),x]`

output `ArcTan[(-2 + 2*x)/(2*Sqrt[2])]/Sqrt[2]`

#### 3.10.3.1 Defintions of rubi rules used

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])`

rule 1083 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Simp[-2 Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x]`

**3.10.4 Maple [A] (verified)**

Time = 0.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

| method  | result                                                             | size |
|---------|--------------------------------------------------------------------|------|
| risch   | $\frac{\sqrt{2} \arctan\left(\frac{(-1+x)\sqrt{2}}{2}\right)}{2}$  | 15   |
| default | $\frac{\sqrt{2} \arctan\left(\frac{(-2+2x)\sqrt{2}}{4}\right)}{2}$ | 17   |

input `int(1/(x^2-2*x+3),x,method=_RETURNVERBOSE)`

output `1/2*2^(1/2)*arctan(1/2*(-1+x)*2^(1/2))`

**3.10.5 Fricas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{1}{3-2x+x^2} dx = \frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(x-1)\right)$$

input `integrate(1/(x^2-2*x+3),x, algorithm="fricas")`

output `1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))`

**3.10.6 Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{3-2x+x^2} dx = \frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2} - \frac{\sqrt{2}}{2}\right)}{2}$$

input `integrate(1/(x**2-2*x+3),x)`

output `sqrt(2)*atan(sqrt(2)*x/2 - sqrt(2)/2)/2`

**3.10.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{1}{3-2x+x^2} dx = \frac{1}{2} \sqrt{2} \arctan \left( \frac{1}{2} \sqrt{2}(x-1) \right)$$

input `integrate(1/(x^2-2*x+3),x, algorithm="maxima")`output `1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))`**3.10.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{1}{3-2x+x^2} dx = \frac{1}{2} \sqrt{2} \arctan \left( \frac{1}{2} \sqrt{2}(x-1) \right)$$

input `integrate(1/(x^2-2*x+3),x, algorithm="giac")`output `1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))`**3.10.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{1}{3-2x+x^2} dx = \frac{\sqrt{2} \operatorname{atan} \left( \frac{\sqrt{2}(x-1)}{2} \right)}{2}$$

input `int(1/(x^2 - 2*x + 3),x)`output `(2^(1/2)*atan((2^(1/2)*(x - 1))/2))/2`

**3.11**      $\int \frac{1}{(-1+x)^2(1+x^2)^2} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.11.1 | Optimal result . . . . .                            | 158 |
| 3.11.2 | Mathematica [A] (verified) . . . . .                | 158 |
| 3.11.3 | Rubi [A] (verified) . . . . .                       | 159 |
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| 3.11.6 | Sympy [A] (verification not implemented) . . . . .  | 161 |
| 3.11.7 | Maxima [A] (verification not implemented) . . . . . | 161 |
| 3.11.8 | Giac [B] (verification not implemented) . . . . .   | 162 |
| 3.11.9 | Mupad [B] (verification not implemented) . . . . .  | 162 |

**3.11.1 Optimal result**

Integrand size = 13, antiderivative size = 49

$$\int \frac{1}{(-1+x)^2(1+x^2)^2} dx = \frac{1}{4(1-x)} - \frac{1}{4(1+x^2)} + \frac{\arctan(x)}{4} - \frac{1}{2} \log(1-x) + \frac{1}{4} \log(1+x^2)$$

output `1/4/(1-x)-1/4/(x^2+1)+1/4*arctan(x)-1/2*ln(1-x)+1/4*ln(x^2+1)`

**3.11.2 Mathematica [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-1+x)^2(1+x^2)^2} dx = \frac{1}{4} \left( \frac{1}{1-x} - \frac{1}{1+x^2} + \arctan(x) - 2 \log(-1+x) + \log(1+x^2) \right)$$

input `Integrate[1/((-1 + x)^2*(1 + x^2)^2),x]`

output `((1 - x)^(-1) - (1 + x^2)^(-1) + ArcTan[x] - 2*Log[-1 + x] + Log[1 + x^2]) /4`

### 3.11.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {496, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{(x-1)^2(x^2+1)^2} dx \\
 & \quad \downarrow 496 \\
 & -\frac{1}{4} \int -\frac{2(2-x)}{(1-x)^2(x^2+1)} dx - \frac{1}{4(x^2+1)} \\
 & \quad \downarrow 27 \\
 & \frac{1}{2} \int \frac{2-x}{(1-x)^2(x^2+1)} dx - \frac{1}{4(x^2+1)} \\
 & \quad \downarrow 657 \\
 & \frac{1}{2} \int \left( \frac{2x+1}{2(x^2+1)} + \frac{1}{1-x} + \frac{1}{2(x-1)^2} \right) dx - \frac{1}{4(x^2+1)} \\
 & \quad \downarrow 2009 \\
 & \frac{1}{2} \left( \frac{\arctan(x)}{2} + \frac{1}{2} \log(x^2+1) + \frac{1}{2(1-x)} - \log(1-x) \right) - \frac{1}{4(x^2+1)}
 \end{aligned}$$

input `Int[1/((-1 + x)^2*(1 + x^2)^2),x]`

output `-1/4*1/(1 + x^2) + (1/(2*(1 - x))) + ArcTan[x]/2 - Log[1 - x] + Log[1 + x^2]/2)/2`

#### 3.11.3.1 Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`



rule 496 `Int[((c_) + (d_)*(x_)^(n_))*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[(-a*d + b*c*x)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2)) Int[(c + d*x)^n*(a + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]`

rule 657 `Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)^(n_))/((a_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^2))], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

### 3.11.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

| method        | result                                                                                                                                      |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------|
| default       | $-\frac{1}{4(-1+x)} - \frac{\ln(-1+x)}{2} - \frac{1}{4(x^2+1)} + \frac{\ln(x^2+1)}{4} + \frac{\arctan(x)}{4}$                               |
| risch         | $\frac{-\frac{1}{4}x^2 - \frac{1}{4}x}{(x^2+1)(-1+x)} - \frac{\ln(-1+x)}{2} + \frac{\ln(x^2+1)}{4} + \frac{\arctan(x)}{4}$                  |
| parallelrisch | $-\frac{2x+2x^2+2\ln(x-i)+2\ln(x+i)-i\ln(x-i)x^2+i\ln(x-i)x^3+i\ln(x+i)-i\ln(x+i)x^3-i\ln(x+i)x+4\ln(-1+x)x^3-i\ln(x-i)-4}{4(x^3-x^2+x-1)}$ |

input `int(1/(-1+x)^2/(x^2+1)^2,x,method=_RETURNVERBOSE)`

output `-1/4/(-1+x)-1/2*ln(-1+x)-1/4/(x^2+1)+1/4*ln(x^2+1)+1/4*arctan(x)`

### 3.11.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

$$\int \frac{1}{(-1+x)^2(1+x^2)^2} dx = \frac{x^2 - (x^3 - x^2 + x - 1) \arctan(x) - (x^3 - x^2 + x - 1) \log(x^2 + 1) + 2(x^3 - x^2 + x - 1) \log(x - 1) + \dots}{4(x^3 - x^2 + x - 1)}$$

---

3.11.  $\int \frac{1}{(-1+x)^2(1+x^2)^2} dx$

input `integrate(1/(-1+x)^2/(x^2+1)^2,x, algorithm="fricas")`

output `-1/4*(x^2 - (x^3 - x^2 + x - 1)*arctan(x) - (x^3 - x^2 + x - 1)*log(x^2 + 1) + 2*(x^3 - x^2 + x - 1)*log(x - 1) + x)/(x^3 - x^2 + x - 1)`

### 3.11.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1}{(-1+x)^2(1+x^2)^2} dx = \frac{-x^2 - x}{4x^3 - 4x^2 + 4x - 4} - \frac{\log(x-1)}{2} + \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{4}$$

input `integrate(1/(-1+x)**2/(x**2+1)**2,x)`

output `(-x**2 - x)/(4*x**3 - 4*x**2 + 4*x - 4) - log(x - 1)/2 + log(x**2 + 1)/4 + atan(x)/4`

### 3.11.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{1}{(-1+x)^2(1+x^2)^2} dx = -\frac{x^2+x}{4(x^3-x^2+x-1)} + \frac{1}{4} \arctan(x) + \frac{1}{4} \log(x^2+1) - \frac{1}{2} \log(x-1)$$

input `integrate(1/(-1+x)^2/(x^2+1)^2,x, algorithm="maxima")`

output `-1/4*(x^2 + x)/(x^3 - x^2 + x - 1) + 1/4*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x - 1)`

**3.11.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 80 vs.  $2(37) = 74$ .

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.63

$$\int \frac{1}{(-1+x)^2(1+x^2)^2} dx = \frac{1}{16} \pi - \frac{1}{4} \pi \left[ \frac{\pi + 4 \arctan(x)}{4 \pi} + \frac{1}{2} \right] + \frac{\frac{2}{x-1} + 1}{8 \left( \frac{2}{x-1} + \frac{2}{(x-1)^2} + 1 \right)} - \frac{1}{4(x-1)} + \frac{1}{4} \arctan(x) + \frac{1}{4} \log \left( \frac{2}{x-1} + \frac{2}{(x-1)^2} + 1 \right)$$

input `integrate(1/(-1+x)^2/(x^2+1)^2,x, algorithm="giac")`

output `1/16*pi - 1/4*pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/8*(2/(x - 1) + 1)/(2/(x - 1) + 2/(x - 1)^2 + 1) - 1/4/(x - 1) + 1/4*arctan(x) + 1/4*log(2/(x - 1) + 2/(x - 1)^2 + 1)`

**3.11.9 Mupad [B] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-1+x)^2(1+x^2)^2} dx = -\frac{\ln(x-1)}{2} - \frac{\frac{x^2}{4} + \frac{x}{4}}{x^3 - x^2 + x - 1} + \ln(x-i) \left( \frac{1}{4} - \frac{1}{8}i \right) + \ln(x+1i) \left( \frac{1}{4} + \frac{1}{8}i \right)$$

input `int(1/((x^2 + 1)^2*(x - 1)^2),x)`

output `log(x - 1i)*(1/4 - 1i/8) - log(x - 1)/2 + log(x + 1i)*(1/4 + 1i/8) - (x/4 + x^2/4)/(x - x^2 + x^3 - 1)`

### 3.12 $\int \frac{x}{(-a+x)(-b+x)(-c+x)} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.12.1 | Optimal result . . . . .                            | 163 |
| 3.12.2 | Mathematica [A] (verified) . . . . .                | 163 |
| 3.12.3 | Rubi [A] (verified) . . . . .                       | 164 |
| 3.12.4 | Maple [A] (verified) . . . . .                      | 165 |
| 3.12.5 | Fricas [A] (verification not implemented) . . . . . | 165 |
| 3.12.6 | Sympy [F(-1)] . . . . .                             | 165 |
| 3.12.7 | Maxima [A] (verification not implemented) . . . . . | 166 |
| 3.12.8 | Giac [A] (verification not implemented) . . . . .   | 166 |
| 3.12.9 | Mupad [B] (verification not implemented) . . . . .  | 166 |

#### 3.12.1 Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \frac{x}{(-a+x)(-b+x)(-c+x)} dx = \frac{a \log(a-x)}{(a-b)(a-c)} - \frac{b \log(b-x)}{(a-b)(b-c)} + \frac{c \log(c-x)}{(a-c)(b-c)}$$

```
output a*ln(a-x)/(a-b)/(a-c)-b*ln(b-x)/(a-b)/(b-c)+c*ln(c-x)/(a-c)/(b-c)
```

#### 3.12.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{x}{(-a+x)(-b+x)(-c+x)} dx = \frac{a(b-c) \log(-a+x) + b(-a+c) \log(-b+x) + (a-b)c \log(-c+x)}{(a-b)(a-c)(b-c)}$$

```
input Integrate[x/((-a + x)*(-b + x)*(-c + x)),x]
```

```
output (a*(b - c)*Log[-a + x] + b*(-a + c)*Log[-b + x] + (a - b)*c*Log[-c + x])/((a - b)*(a - c)*(b - c))
```

### 3.12.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x-a)(x-b)(x-c)} dx$$

↓ 165

$$\int \left( -\frac{a}{(a-b)(a-c)(a-x)} + \frac{b}{(a-b)(b-c)(b-x)} + \frac{c}{(a-c)(c-b)(c-x)} \right) dx$$

↓ 2009

$$\frac{a \log(a-x)}{(a-b)(a-c)} - \frac{b \log(b-x)}{(a-b)(b-c)} + \frac{c \log(c-x)}{(a-c)(b-c)}$$

input `Int[x/((-a + x)*(-b + x)*(-c + x)),x]`

output `(a*Log[a - x])/((a - b)*(a - c)) - (b*Log[b - x])/((a - b)*(b - c)) + (c*Log[c - x])/((a - c)*(b - c))`

#### 3.12.3.1 Defintions of rubi rules used

rule 165 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

### 3.12.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

| method        | result                                                                                                          | size |
|---------------|-----------------------------------------------------------------------------------------------------------------|------|
| default       | $\frac{a \ln(a-x)}{(a-b)(a-c)} - \frac{b \ln(b-x)}{(a-b)(b-c)} + \frac{c \ln(c-x)}{(a-c)(b-c)}$                 | 69   |
| norman        | $\frac{c \ln(c-x)}{ab-ac-bc+c^2} + \frac{a \ln(a-x)}{(a-b)(a-c)} - \frac{b \ln(b-x)}{(a-b)(b-c)}$               | 72   |
| risch         | $-\frac{b \ln(-b+x)}{ab-ac-b^2+bc} + \frac{a \ln(-a+x)}{a^2-ab-ac+bc} + \frac{c \ln(c-x)}{ab-ac-bc+c^2}$        | 79   |
| parallelrisch | $\frac{\ln(-a+x)ab - \ln(-a+x)ac - \ln(-b+x)ab + \ln(-b+x)bc + \ln(-c+x)ac - \ln(-c+x)bc}{(ab-ac-bc+c^2)(a-b)}$ | 84   |

input `int(x/(-a+x)/(-b+x)/(-c+x),x,method=_RETURNVERBOSE)`

output `a*ln(a-x)/(a-b)/(a-c)-b*ln(b-x)/(a-b)/(b-c)+c*ln(c-x)/(a-c)/(b-c)`

### 3.12.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int \frac{x}{(-a+x)(-b+x)(-c+x)} dx$$

$$= \frac{(a-b)c \log(-c+x) + (ab-ac) \log(-a+x) - (ab-bc) \log(-b+x)}{a^2b - ab^2 + (a-b)c^2 - (a^2 - b^2)c}$$

input `integrate(x/(-a+x)/(-b+x)/(-c+x),x, algorithm="fricas")`

output `((a - b)*c*log(-c + x) + (a*b - a*c)*log(-a + x) - (a*b - b*c)*log(-b + x) )/(a^2*b - a*b^2 + (a - b)*c^2 - (a^2 - b^2)*c)`

### 3.12.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(-a+x)(-b+x)(-c+x)} dx = \text{Timed out}$$

input `integrate(x/(-a+x)/(-b+x)/(-c+x),x)`

---

3.12.  $\int \frac{x}{(-a+x)(-b+x)(-c+x)} dx$

output Timed out

### 3.12.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.15

$$\int \frac{x}{(-a+x)(-b+x)(-c+x)} dx = \frac{a \log(-a+x)}{a^2 - ab - (a-b)c} - \frac{b \log(-b+x)}{ab - b^2 - (a-b)c} + \frac{c \log(-c+x)}{ab - (a+b)c + c^2}$$

input `integrate(x/(-a+x)/(-b+x)/(-c+x),x, algorithm="maxima")`

output `a*log(-a + x)/(a^2 - a*b - (a - b)*c) - b*log(-b + x)/(a*b - b^2 - (a - b)*c) + c*log(-c + x)/(a*b - (a + b)*c + c^2)`

### 3.12.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int \frac{x}{(-a+x)(-b+x)(-c+x)} dx = \frac{a \log(|-a+x|)}{a^2 - ab - ac + bc} - \frac{b \log(|-b+x|)}{ab - b^2 - ac + bc} + \frac{c \log(|-c+x|)}{ab - ac - bc + c^2}$$

input `integrate(x/(-a+x)/(-b+x)/(-c+x),x, algorithm="giac")`

output `a*log(abs(-a + x))/(a^2 - a*b - a*c + b*c) - b*log(abs(-b + x))/(a*b - b^2 - a*c + b*c) + c*log(abs(-c + x))/(a*b - a*c - b*c + c^2)`

### 3.12.9 Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.28

$$\int \frac{x}{(-a+x)(-b+x)(-c+x)} dx = \ln(x-a) \left( \frac{b}{(a-b)(b-c)} - \frac{c}{(a-c)(b-c)} \right) - \frac{b \ln(x-b)}{(a-b)(b-c)} + \frac{c \ln(x-c)}{(a-c)(b-c)}$$

input `int(-x/((a - x)*(b - x)*(c - x)),x)`

output `log(x - a)*(b/((a - b)*(b - c)) - c/((a - c)*(b - c))) - (b*log(x - b))/((a - b)*(b - c)) + (c*log(x - c))/((a - c)*(b - c))`



### 3.13 $\int \frac{x}{(a^2+x^2)(b^2+x^2)} dx$

|        |                                           |     |
|--------|-------------------------------------------|-----|
| 3.13.1 | Optimal result                            | 168 |
| 3.13.2 | Mathematica [A] (verified)                | 168 |
| 3.13.3 | Rubi [A] (verified)                       | 169 |
| 3.13.4 | Maple [A] (verified)                      | 170 |
| 3.13.5 | Fricas [A] (verification not implemented) | 170 |
| 3.13.6 | Sympy [B] (verification not implemented)  | 171 |
| 3.13.7 | Maxima [A] (verification not implemented) | 171 |
| 3.13.8 | Giac [A] (verification not implemented)   | 171 |
| 3.13.9 | Mupad [B] (verification not implemented)  | 172 |

#### 3.13.1 Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{x}{(a^2+x^2)(b^2+x^2)} dx = -\frac{\log(a^2+x^2)}{2(a^2-b^2)} + \frac{\log(b^2+x^2)}{2(a^2-b^2)}$$

output `-1/2*ln(a^2+x^2)/(a^2-b^2)+1/2*ln(b^2+x^2)/(a^2-b^2)`

#### 3.13.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \frac{x}{(a^2+x^2)(b^2+x^2)} dx = \frac{-\log(a^2+x^2) + \log(b^2+x^2)}{2(a^2-b^2)}$$

input `Integrate[x/((a^2 + x^2)*(b^2 + x^2)),x]`

output `(-Log[a^2 + x^2] + Log[b^2 + x^2])/(2*(a^2 - b^2))`

### 3.13.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {353, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{x}{(a^2 + x^2)(b^2 + x^2)} dx \\ & \quad \downarrow \text{353} \\ & \frac{1}{2} \int \frac{1}{(a^2 + x^2)(b^2 + x^2)} dx^2 \\ & \quad \downarrow \text{47} \\ & \frac{1}{2} \left( \int \frac{1}{b^2 + x^2} dx^2 - \int \frac{1}{a^2 + x^2} dx^2 \right) \\ & \quad \downarrow \text{16} \\ & \frac{1}{2} \left( \frac{\log(b^2 + x^2)}{a^2 - b^2} - \frac{\log(a^2 + x^2)}{a^2 - b^2} \right) \end{aligned}$$

input `Int[x/((a^2 + x^2)*(b^2 + x^2)),x]`

output `(-(Log[a^2 + x^2]/(a^2 - b^2)) + Log[b^2 + x^2]/(a^2 - b^2))/2`

#### 3.13.3.1 Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 47 `Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Simp[b/(b*c - a*d) Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d) Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]`

```
rule 353 Int[(x_)*((a_) + (b_)*(x_)^2)^(p_)*((c_) + (d_)*(x_)^2)^(q_), x_Symbol]
  :> Simp[1/2 Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
  {a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

### 3.13.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

| method        | result                                                               | size |
|---------------|----------------------------------------------------------------------|------|
| parallelrisch | $-\frac{\ln(a^2+x^2)-\ln(b^2+x^2)}{2(a^2-b^2)}$                      | 33   |
| default       | $-\frac{\ln(a^2+x^2)}{2(a^2-b^2)} + \frac{\ln(b^2+x^2)}{2a^2-2b^2}$  | 44   |
| norman        | $-\frac{\ln(a^2+x^2)}{2(a^2-b^2)} + \frac{\ln(b^2+x^2)}{2a^2-2b^2}$  | 44   |
| risch         | $-\frac{\ln(-a^2-x^2)}{2(a^2-b^2)} + \frac{\ln(b^2+x^2)}{2a^2-2b^2}$ | 48   |

```
input int(x/(a^2+x^2)/(b^2+x^2),x,method=_RETURNVERBOSE)
```

```
output -1/2*(ln(a^2+x^2)-ln(b^2+x^2))/(a^2-b^2)
```

### 3.13.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68

$$\int \frac{x}{(a^2+x^2)(b^2+x^2)} dx = -\frac{\log(a^2+x^2) - \log(b^2+x^2)}{2(a^2-b^2)}$$

```
input integrate(x/(a^2+x^2)/(b^2+x^2),x, algorithm="fracas")
```

```
output -1/2*(log(a^2 + x^2) - log(b^2 + x^2))/(a^2 - b^2)
```

**3.13.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 121 vs.  $2(36) = 72$ .

Time = 0.35 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.57

$$\int \frac{x}{(a^2 + x^2)(b^2 + x^2)} dx = \frac{\log\left(-\frac{a^4}{2(a-b)(a+b)} + \frac{a^2b^2}{(a-b)(a+b)} + \frac{a^2}{2} - \frac{b^4}{2(a-b)(a+b)} + \frac{b^2}{2} + x^2\right)}{2(a-b)(a+b)} - \frac{\log\left(\frac{a^4}{2(a-b)(a+b)} - \frac{a^2b^2}{(a-b)(a+b)} + \frac{a^2}{2} + \frac{b^4}{2(a-b)(a+b)} + \frac{b^2}{2} + x^2\right)}{2(a-b)(a+b)}$$

input `integrate(x/(a**2+x**2)/(b**2+x**2),x)`

output `log(-a**4/(2*(a - b)*(a + b)) + a**2*b**2/((a - b)*(a + b)) + a**2/2 - b**4/(2*(a - b)*(a + b)) + b**2/2 + x**2)/(2*(a - b)*(a + b)) - log(a**4/(2*(a - b)*(a + b)) - a**2*b**2/((a - b)*(a + b)) + a**2/2 + b**4/(2*(a - b)*(a + b)) + b**2/2 + x**2)/(2*(a - b)*(a + b))`

**3.13.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{x}{(a^2 + x^2)(b^2 + x^2)} dx = -\frac{\log(a^2 + x^2)}{2(a^2 - b^2)} + \frac{\log(b^2 + x^2)}{2(a^2 - b^2)}$$

input `integrate(x/(a^2+x^2)/(b^2+x^2),x, algorithm="maxima")`

output `-1/2*log(a^2 + x^2)/(a^2 - b^2) + 1/2*log(b^2 + x^2)/(a^2 - b^2)`

**3.13.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{x}{(a^2 + x^2)(b^2 + x^2)} dx = -\frac{\log(a^2 + x^2)}{2(a^2 - b^2)} + \frac{\log(b^2 + x^2)}{2(a^2 - b^2)}$$

input `integrate(x/(a^2+x^2)/(b^2+x^2),x, algorithm="giac")`

output `-1/2*log(a^2 + x^2)/(a^2 - b^2) + 1/2*log(b^2 + x^2)/(a^2 - b^2)`

**3.13.9 Mupad [B] (verification not implemented)**

Time = 0.31 (sec) , antiderivative size = 256, normalized size of antiderivative = 5.45

$$\int \frac{x}{(a^2 + x^2)(b^2 + x^2)} dx = \frac{\operatorname{atan}\left(\frac{\left(\frac{x^2(8a^2+8b^2)+16a^2b^2}{2(a^2-b^2)}-4x^2\right)1i - \left(\frac{x^2(8a^2+8b^2)+16a^2b^2}{2(a^2-b^2)}+4x^2\right)1i}{\frac{x^2(8a^2+8b^2)+16a^2b^2}{2(a^2-b^2)}-4x^2 + \frac{x^2(8a^2+8b^2)+16a^2b^2}{2(a^2-b^2)}+4x^2}\right)1i}{a^2 - b^2}$$

input `int(x/((a^2 + x^2)*(b^2 + x^2)),x)`output `(atan((((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 - b^2)) - 4*x^2)*1i)/(2*(a^2 - b^2)) - (((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 - b^2)) + 4*x^2)*1i)/(2*(a^2 - b^2)))/(((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 - b^2)) - 4*x^2)/(2*(a^2 - b^2)) + ((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 - b^2)) + 4*x^2)/(2*(a^2 - b^2))))*1i)/(a^2 - b^2)`

### 3.14 $\int \frac{x^2}{(a^2+x^2)(b^2+x^2)} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.14.1 | Optimal result . . . . .                            | 173 |
| 3.14.2 | Mathematica [A] (verified) . . . . .                | 173 |
| 3.14.3 | Rubi [A] (verified) . . . . .                       | 174 |
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| 3.14.8 | Giac [A] (verification not implemented) . . . . .   | 177 |
| 3.14.9 | Mupad [B] (verification not implemented) . . . . .  | 177 |

#### 3.14.1 Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \frac{x^2}{(a^2 + x^2)(b^2 + x^2)} dx = \frac{a \arctan\left(\frac{x}{a}\right)}{a^2 - b^2} - \frac{b \arctan\left(\frac{x}{b}\right)}{a^2 - b^2}$$

output `a*arctan(x/a)/(a^2-b^2)-b*arctan(x/b)/(a^2-b^2)`

#### 3.14.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{(a^2 + x^2)(b^2 + x^2)} dx = \frac{a \arctan\left(\frac{x}{a}\right) - b \arctan\left(\frac{x}{b}\right)}{a^2 - b^2}$$

input `Integrate[x^2/((a^2 + x^2)*(b^2 + x^2)),x]`

output `(a*ArcTan[x/a] - b*ArcTan[x/b])/(a^2 - b^2)`

### 3.14.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {383, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^2 + x^2)(b^2 + x^2)} dx$$

$$\downarrow \text{383}$$

$$\frac{a^2 \int \frac{1}{a^2+x^2} dx}{a^2 - b^2} - \frac{b^2 \int \frac{1}{b^2+x^2} dx}{a^2 - b^2}$$

$$\downarrow \text{216}$$

$$\frac{a \arctan\left(\frac{x}{a}\right)}{a^2 - b^2} - \frac{b \arctan\left(\frac{x}{b}\right)}{a^2 - b^2}$$

input `Int[x^2/((a^2 + x^2)*(b^2 + x^2)),x]`

output `(a*ArcTan[x/a])/(a^2 - b^2) - (b*ArcTan[x/b])/(a^2 - b^2)`

#### 3.14.3.1 Defintions of rubi rules used

rule 216 `Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

rule 383 `Int[((e_)*(x_)^(m_))/(((a_) + (b_)*(x_)^2)*((c_) + (d_)*(x_)^2)), x_Symbol] := Simp[(-a)*(e^2/(b*c - a*d)) Int[(e*x)^(m - 2)/(a + b*x^2), x], x] + Simp[c*(e^2/(b*c - a*d)) Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]`

**3.14.4 Maple [A] (verified)**

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

| method        | result                                                                                                  | size |
|---------------|---------------------------------------------------------------------------------------------------------|------|
| default       | $\frac{a \arctan\left(\frac{x}{a}\right)}{a^2-b^2} - \frac{b \arctan\left(\frac{x}{b}\right)}{a^2-b^2}$ | 41   |
| risch         | $\frac{a \arctan\left(\frac{x}{a}\right)}{a^2-b^2} - \frac{b \arctan\left(\frac{x}{b}\right)}{a^2-b^2}$ | 41   |
| parallelrisch | $-\frac{ia \ln(-ia+x) - ib \ln(-ib+x) - ia \ln(ia+x) + ib \ln(ib+x)}{2(a^2-b^2)}$                       | 59   |

input `int(x^2/(a^2+x^2)/(b^2+x^2),x,method=_RETURNVERBOSE)`output `a*arctan(x/a)/(a^2-b^2)-b*arctan(x/b)/(a^2-b^2)`**3.14.5 Fricas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{(a^2+x^2)(b^2+x^2)} dx = \frac{a \arctan\left(\frac{x}{a}\right) - b \arctan\left(\frac{x}{b}\right)}{a^2 - b^2}$$

input `integrate(x^2/(a^2+x^2)/(b^2+x^2),x, algorithm="fricas")`output `(a*arctan(x/a) - b*arctan(x/b))/(a^2 - b^2)`



### 3.14.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 393, normalized size of antiderivative = 9.82

$$\int \frac{x^2}{(a^2 + x^2)(b^2 + x^2)} dx$$

$$= -\frac{ia \log\left(-\frac{2ia^7}{(a-b)^3(a+b)^3} + \frac{4ia^5b^2}{(a-b)^3(a+b)^3} - \frac{2ia^3b^4}{(a-b)^3(a+b)^3} + \frac{ia^3}{(a-b)(a+b)} + \frac{iab^2}{(a-b)(a+b)} + x\right)}{2(a-b)(a+b)}$$

$$+ \frac{ia \log\left(\frac{2ia^7}{(a-b)^3(a+b)^3} - \frac{4ia^5b^2}{(a-b)^3(a+b)^3} + \frac{2ia^3b^4}{(a-b)^3(a+b)^3} - \frac{ia^3}{(a-b)(a+b)} - \frac{iab^2}{(a-b)(a+b)} + x\right)}{2(a-b)(a+b)}$$

$$- \frac{ib \log\left(-\frac{2ia^4b^3}{(a-b)^3(a+b)^3} + \frac{4ia^2b^5}{(a-b)^3(a+b)^3} + \frac{ia^2b}{(a-b)(a+b)} - \frac{2ib^7}{(a-b)^3(a+b)^3} + \frac{ib^3}{(a-b)(a+b)} + x\right)}{2(a-b)(a+b)}$$

$$+ \frac{ib \log\left(\frac{2ia^4b^3}{(a-b)^3(a+b)^3} - \frac{4ia^2b^5}{(a-b)^3(a+b)^3} - \frac{ia^2b}{(a-b)(a+b)} + \frac{2ib^7}{(a-b)^3(a+b)^3} - \frac{ib^3}{(a-b)(a+b)} + x\right)}{2(a-b)(a+b)}$$

input `integrate(x**2/(a**2+x**2)/(b**2+x**2), x)`

output `-I*a*log(-2*I*a**7/((a - b)**3*(a + b)**3) + 4*I*a**5*b**2/((a - b)**3*(a + b)**3) - 2*I*a**3*b**4/((a - b)**3*(a + b)**3) + I*a**3/((a - b)*(a + b)) + I*a*b**2/((a - b)*(a + b)) + x)/(2*(a - b)*(a + b)) + I*a*log(2*I*a**7/((a - b)**3*(a + b)**3) - 4*I*a**5*b**2/((a - b)**3*(a + b)**3) + 2*I*a**3*b**4/((a - b)**3*(a + b)**3) - I*a**3/((a - b)*(a + b)) - I*a*b**2/((a - b)*(a + b)) + x)/(2*(a - b)*(a + b)) - I*b*log(-2*I*a**4*b**3/((a - b)**3*(a + b)**3) + 4*I*a**2*b**5/((a - b)**3*(a + b)**3) + I*a**2*b/((a - b)*(a + b)) - 2*I*b**7/((a - b)**3*(a + b)**3) + I*b**3/((a - b)*(a + b)) + x)/(2*(a - b)*(a + b)) + I*b*log(2*I*a**4*b**3/((a - b)**3*(a + b)**3) - 4*I*a**2*b**5/((a - b)**3*(a + b)**3) - I*a**2*b/((a - b)*(a + b)) + 2*I*b**7/((a - b)**3*(a + b)**3) - I*b**3/((a - b)*(a + b)) + x)/(2*(a - b)*(a + b))`

**3.14.7 Maxima [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a^2 + x^2)(b^2 + x^2)} dx = \frac{a \arctan\left(\frac{x}{a}\right)}{a^2 - b^2} - \frac{b \arctan\left(\frac{x}{b}\right)}{a^2 - b^2}$$

input `integrate(x^2/(a^2+x^2)/(b^2+x^2),x, algorithm="maxima")`output `a*arctan(x/a)/(a^2 - b^2) - b*arctan(x/b)/(a^2 - b^2)`**3.14.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a^2 + x^2)(b^2 + x^2)} dx = \frac{a \arctan\left(\frac{x}{a}\right)}{a^2 - b^2} - \frac{b \arctan\left(\frac{x}{b}\right)}{a^2 - b^2}$$

input `integrate(x^2/(a^2+x^2)/(b^2+x^2),x, algorithm="giac")`output `a*arctan(x/a)/(a^2 - b^2) - b*arctan(x/b)/(a^2 - b^2)`**3.14.9 Mupad [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 4.78

$$\int \frac{x^2}{(a^2 + x^2)(b^2 + x^2)} dx = - \frac{a \operatorname{atan}\left(\frac{x(2a^4+2b^4) - \frac{a^2 x(8a^6-8a^4b^2-8a^2b^4+8b^6)}{(2a^2-2b^2)^2}}{ab^2(2a^2-2b^2)}\right)}{a^2 - b^2} - \frac{b \operatorname{atan}\left(\frac{x(2a^4+2b^4) - \frac{b^2 x(8a^6-8a^4b^2-8a^2b^4+8b^6)}{(2a^2-2b^2)^2}}{a^2 b(2a^2-2b^2)}\right)}{a^2 - b^2}$$

input `int(x^2/((a^2 + x^2)*(b^2 + x^2)),x)`

output 
$$- \frac{(a \operatorname{atan}\left(\frac{x(2a^4 + 2b^4) - (a^2 x(8a^6 + 8b^6 - 8a^2 b^4 - 8a^4 b^2))}{2a^2 - 2b^2}\right) - (a^2 x(8a^6 + 8b^6 - 8a^2 b^4 - 8a^4 b^2))}{(2a^2 - 2b^2)^2} - \frac{(b \operatorname{atan}\left(\frac{x(2a^4 + 2b^4) - (b^2 x(8a^6 + 8b^6 - 8a^2 b^4 - 8a^4 b^2))}{2a^2 - 2b^2}\right) - (b^2 x(8a^6 + 8b^6 - 8a^2 b^4 - 8a^4 b^2))}{(2a^2 - 2b^2)^2}}{(a^2 - b^2)^2}$$

### 3.15 $\int \frac{x}{(-1+x)(1+x^2)} dx$

|        |                                           |     |
|--------|-------------------------------------------|-----|
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| 3.15.2 | Mathematica [A] (verified)                | 179 |
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| 3.15.5 | Fricas [A] (verification not implemented) | 182 |
| 3.15.6 | Sympy [A] (verification not implemented)  | 182 |
| 3.15.7 | Maxima [A] (verification not implemented) | 182 |
| 3.15.8 | Giac [A] (verification not implemented)   | 183 |
| 3.15.9 | Mupad [B] (verification not implemented)  | 183 |

#### 3.15.1 Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{x}{(-1+x)(1+x^2)} dx = \frac{\arctan(x)}{2} + \frac{1}{2} \log(1-x) - \frac{1}{4} \log(1+x^2)$$

output `1/2*arctan(x)+1/2*ln(1-x)-1/4*ln(x^2+1)`

#### 3.15.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x}{(-1+x)(1+x^2)} dx = \frac{\arctan(x)}{2} + \frac{1}{2} \log(1-x) - \frac{1}{4} \log(1+x^2)$$

input `Integrate[x/((-1 + x)*(1 + x^2)),x]`

output `ArcTan[x]/2 + Log[1 - x]/2 - Log[1 + x^2]/4`

**3.15.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {587, 16, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x}{(x-1)(x^2+1)} dx \\
 & \quad \downarrow \text{587} \\
 & \frac{1}{2} \int \frac{1-x}{x^2+1} dx + \frac{1}{2} \int \frac{1}{x-1} dx \\
 & \quad \downarrow \text{16} \\
 & \frac{1}{2} \int \frac{1-x}{x^2+1} dx + \frac{1}{2} \log(1-x) \\
 & \quad \downarrow \text{452} \\
 & \frac{1}{2} \left( \int \frac{1}{x^2+1} dx - \int \frac{x}{x^2+1} dx \right) + \frac{1}{2} \log(1-x) \\
 & \quad \downarrow \text{216} \\
 & \frac{1}{2} \left( \arctan(x) - \int \frac{x}{x^2+1} dx \right) + \frac{1}{2} \log(1-x) \\
 & \quad \downarrow \text{240} \\
 & \frac{1}{2} \left( \arctan(x) - \frac{1}{2} \log(x^2+1) \right) + \frac{1}{2} \log(1-x)
 \end{aligned}$$

input `Int[x/((-1 + x)*(1 + x^2)),x]`

output `Log[1 - x]/2 + (ArcTan[x] - Log[1 + x^2])/2`

## 3.15.3.1 Defintions of rubi rules used

- rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`
- rule 216 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`
- rule 240 `Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]`
- rule 452 `Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] := Simp[c Int[1/(a + b*x^2), x], x] + Simp[d Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c^2 + a*d^2, 0]`
- rule 587 `Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] := Simp[(-c)*(d/(b*c^2 + a*d^2)) Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2) Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c^2 + a*d^2, 0]`

## 3.15.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

| method        | result                                                                                                        | size |
|---------------|---------------------------------------------------------------------------------------------------------------|------|
| default       | $\frac{\ln(-1+x)}{2} - \frac{\ln(x^2+1)}{4} + \frac{\arctan(x)}{2}$                                           | 20   |
| risch         | $\frac{\ln(-1+x)}{2} - \frac{\ln(x^2+1)}{4} + \frac{\arctan(x)}{2}$                                           | 20   |
| parallelrisch | $\frac{\ln(-1+x)}{2} - \frac{\ln(x-i)}{4} - \frac{i \ln(x-i)}{4} - \frac{\ln(x+i)}{4} + \frac{i \ln(x+i)}{4}$ | 38   |

input `int(x/(-1+x)/(x^2+1),x,method=_RETURNVERBOSE)`

output `1/2*ln(-1+x)-1/4*ln(x^2+1)+1/2*arctan(x)`

**3.15.5 Fracas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x}{(-1+x)(1+x^2)} dx = \frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x - 1)$$

input `integrate(x/(-1+x)/(x^2+1),x, algorithm="fracas")`output `1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x - 1)`**3.15.6 Sympy [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x}{(-1+x)(1+x^2)} dx = \frac{\log(x-1)}{2} - \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

input `integrate(x/(-1+x)/(x**2+1),x)`output `log(x - 1)/2 - log(x**2 + 1)/4 + atan(x)/2`**3.15.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x}{(-1+x)(1+x^2)} dx = \frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x - 1)$$

input `integrate(x/(-1+x)/(x^2+1),x, algorithm="maxima")`output `1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x - 1)`

**3.15.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x}{(-1+x)(1+x^2)} dx = \frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(|x - 1|)$$

input `integrate(x/(-1+x)/(x^2+1),x, algorithm="giac")`output `1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x - 1))`**3.15.9 Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x}{(-1+x)(1+x^2)} dx = \frac{\ln(x-1)}{2} + \ln(x-i) \left(-\frac{1}{4} - \frac{1}{4}i\right) + \ln(x+1i) \left(-\frac{1}{4} + \frac{1}{4}i\right)$$

input `int(x/((x^2 + 1)*(x - 1)),x)`output `log(x - 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)`



### 3.16 $\int \frac{x}{1+x^3} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
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#### 3.16.1 Optimal result

Integrand size = 9, antiderivative size = 41

$$\int \frac{x}{1+x^3} dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3} \log(1+x) + \frac{1}{6} \log(1-x+x^2)$$

output `-1/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)`

#### 3.16.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{x}{1+x^3} dx = \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3} \log(1+x) + \frac{1}{6} \log(1-x+x^2)$$

input `Integrate[x/(1 + x^3),x]`

output `ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6`

**3.16.3 Rubi [A] (verified)**

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$ , Rules used = {821, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x}{x^3+1} dx \\
 & \quad \downarrow \text{821} \\
 & \frac{1}{3} \int \frac{x+1}{x^2-x+1} dx - \frac{1}{3} \int \frac{1}{x+1} dx \\
 & \quad \downarrow \text{16} \\
 & \frac{1}{3} \int \frac{x+1}{x^2-x+1} dx - \frac{1}{3} \log(x+1) \\
 & \quad \downarrow \text{1142} \\
 & \frac{1}{3} \left( \frac{3}{2} \int \frac{1}{x^2-x+1} dx + \frac{1}{2} \int -\frac{1-2x}{x^2-x+1} dx \right) - \frac{1}{3} \log(x+1) \\
 & \quad \downarrow \text{25} \\
 & \frac{1}{3} \left( \frac{3}{2} \int \frac{1}{x^2-x+1} dx - \frac{1}{2} \int \frac{1-2x}{x^2-x+1} dx \right) - \frac{1}{3} \log(x+1) \\
 & \quad \downarrow \text{1083} \\
 & \frac{1}{3} \left( -\frac{1}{2} \int \frac{1-2x}{x^2-x+1} dx - 3 \int \frac{1}{-(2x-1)^2-3} d(2x-1) \right) - \frac{1}{3} \log(x+1) \\
 & \quad \downarrow \text{217} \\
 & \frac{1}{3} \left( \sqrt{3} \arctan \left( \frac{2x-1}{\sqrt{3}} \right) - \frac{1}{2} \int \frac{1-2x}{x^2-x+1} dx \right) - \frac{1}{3} \log(x+1) \\
 & \quad \downarrow \text{1103} \\
 & \frac{1}{3} \left( \sqrt{3} \arctan \left( \frac{2x-1}{\sqrt{3}} \right) + \frac{1}{2} \log(x^2-x+1) \right) - \frac{1}{3} \log(x+1)
 \end{aligned}$$

input `Int[x/(1 + x^3), x]`

output 
$$-1/3 \cdot \text{Log}[1 + x] + (\text{Sqrt}[3] \cdot \text{ArcTan}[(-1 + 2x)/\text{Sqrt}[3]] + \text{Log}[1 - x + x^2/2])/3$$

### 3.16.3.1 Defintions of rubi rules used

rule 16 
$$\text{Int}[(c\_)/((a\_)+(b\_)(x\_)), x\_Symbol] \rightarrow \text{Simp}[c \cdot (\text{Log}[\text{RemoveContent}[a + b \cdot x, x]]/b), x] \text{ ; FreeQ}\{a, b, c\}, x]$$

rule 25 
$$\text{Int}[-(Fx\_), x\_Symbol] \rightarrow \text{Simp}[\text{Identity}[-1] \text{ Int}[Fx, x], x]$$

rule 217 
$$\text{Int}[(a\_)+(b\_)(x\_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2])^{-1}) \cdot \text{ArcTan}[\text{Rt}[-b, 2] \cdot (x/\text{Rt}[-a, 2])], x] \text{ ; FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])]$$

rule 821 
$$\text{Int}[(x\_)/((a\_)+(b\_)(x\_)^3), x\_Symbol] \rightarrow \text{Simp}[-(3 \cdot \text{Rt}[a, 3] \cdot \text{Rt}[b, 3])^{-1} \text{ Int}[1/(\text{Rt}[a, 3] + \text{Rt}[b, 3] \cdot x), x], x] + \text{Simp}[1/(3 \cdot \text{Rt}[a, 3] \cdot \text{Rt}[b, 3]) \text{ Int}[(\text{Rt}[a, 3] + \text{Rt}[b, 3] \cdot x)/(\text{Rt}[a, 3]^2 - \text{Rt}[a, 3] \cdot \text{Rt}[b, 3] \cdot x + \text{Rt}[b, 3]^2 \cdot x^2), x], x] \text{ ; FreeQ}\{a, b\}, x]$$

rule 1083 
$$\text{Int}[(a\_)+(b\_)(x\_)+(c\_)(x\_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[-2 \text{ Subst}[\text{Int}[1/\text{Simp}[b^2 - 4 \cdot a \cdot c - x^2, x], x], x, b + 2 \cdot c \cdot x], x] \text{ ; FreeQ}\{a, b, c\}, x]$$

rule 1103 
$$\text{Int}[(d\_)+(e\_)(x\_)/((a\_)+(b\_)(x\_)+(c\_)(x\_)^2), x\_Symbol] \rightarrow \text{Simp}[d \cdot (\text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]/b), x] \text{ ; FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$$

rule 1142 
$$\text{Int}[(d\_)+(e\_)(x\_)/((a\_)+(b\_)(x\_)+(c\_)(x\_)^2), x\_Symbol] \rightarrow \text{Simp}[(2 \cdot c \cdot d - b \cdot e)/(2 \cdot c) \text{ Int}[1/(a + b \cdot x + c \cdot x^2), x], x] + \text{Simp}[e/(2 \cdot c) \text{ Int}[(b + 2 \cdot c \cdot x)/(a + b \cdot x + c \cdot x^2), x], x] \text{ ; FreeQ}\{a, b, c, d, e\}, x]$$

### 3.16.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

| method  | result                                                                                                                                                                                                                                                                                           | size |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch   | $-\frac{\ln(1+x)}{3} + \frac{\ln(x^2-x+1)}{6} + \frac{\sqrt{3} \arctan\left(\frac{2(x-\frac{1}{2})\sqrt{3}}{3}\right)}{3}$                                                                                                                                                                       | 33   |
| default | $\frac{\ln(x^2-x+1)}{6} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3} - \frac{\ln(1+x)}{3}$                                                                                                                                                                                  | 35   |
| meijerg | $-\frac{x^2 \ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}} + \frac{x^2 \ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}} + \frac{x^2 \sqrt{3} \arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{2}{3}}}$ | 80   |

input `int(x/(x^3+1),x,method=_RETURNVERBOSE)`

output `-1/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))`

### 3.16.5 Fracas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+x^3} dx = \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) + \frac{1}{6} \log(x^2-x+1) - \frac{1}{3} \log(x+1)$$

input `integrate(x/(x^3+1),x, algorithm="fricas")`

output `1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(x + 1)`

### 3.16.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x^3} dx = -\frac{\log(x+1)}{3} + \frac{\log(x^2-x+1)}{6} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

input `integrate(x/(x**3+1),x)`

output `-log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3`

### 3.16.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+x^3} dx = \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) + \frac{1}{6} \log(x^2-x+1) - \frac{1}{3} \log(x+1)$$

input `integrate(x/(x^3+1),x, algorithm="maxima")`

output `1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(x + 1)`

### 3.16.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x}{1+x^3} dx = \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) + \frac{1}{6} \log(x^2-x+1) - \frac{1}{3} \log(|x+1|)$$

input `integrate(x/(x^3+1),x, algorithm="giac")`

output `1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(abs(x + 1))`

**3.16.9 Mupad [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{x}{1+x^3} dx = -\frac{\ln(x+1)}{3} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}1i}{2}\right) \left(-\frac{1}{6} + \frac{\sqrt{3}1i}{6}\right) \\ + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}1i}{2}\right) \left(\frac{1}{6} + \frac{\sqrt{3}1i}{6}\right)$$

input `int(x/(x^3 + 1),x)`output `log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6) - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x + 1)/3`

### 3.17 $\int \frac{x^3}{(-1+x)^2(1+x^3)} dx$

|        |                                           |     |
|--------|-------------------------------------------|-----|
| 3.17.1 | Optimal result                            | 190 |
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| 3.17.5 | Fricas [A] (verification not implemented) | 192 |
| 3.17.6 | Sympy [A] (verification not implemented)  | 193 |
| 3.17.7 | Maxima [A] (verification not implemented) | 193 |
| 3.17.8 | Giac [A] (verification not implemented)   | 193 |
| 3.17.9 | Mupad [B] (verification not implemented)  | 194 |

#### 3.17.1 Optimal result

Integrand size = 16, antiderivative size = 43

$$\int \frac{x^3}{(-1+x)^2(1+x^3)} dx = \frac{1}{2(1-x)} + \frac{3}{4} \log(1-x) - \frac{1}{12} \log(1+x) - \frac{1}{3} \log(1-x+x^2)$$

output `1/2/(1-x)+3/4*ln(1-x)-1/12*ln(1+x)-1/3*ln(x^2-x+1)`

#### 3.17.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{(-1+x)^2(1+x^3)} dx = \frac{1}{12} \left( -\frac{6}{-1+x} + 9 \log(-1+x) - \log(1+x) - 4 \log((-1+x)^2+x) \right)$$

input `Integrate[x^3/((-1 + x)^2*(1 + x^3)),x]`

output `(-6/(-1 + x) + 9*Log[-1 + x] - Log[1 + x] - 4*Log[(-1 + x)^2 + x])/12`

### 3.17.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(x-1)^2(x^3+1)} dx$$

↓ 7276

$$\int \left( \frac{1-2x}{3(x^2-x+1)} + \frac{3}{4(x-1)} - \frac{1}{12(x+1)} + \frac{1}{2(x-1)^2} \right) dx$$

↓ 2009

$$-\frac{1}{3} \log(x^2-x+1) + \frac{1}{2(1-x)} + \frac{3}{4} \log(1-x) - \frac{1}{12} \log(x+1)$$

input `Int[x^3/((-1+x)^2*(1+x^3)),x]`

output `1/(2*(1-x)) + (3*Log[1-x])/4 - Log[1+x]/12 - Log[1-x+x^2]/3`

#### 3.17.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 7276 `Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE  
x  
pand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ  
[n, 0]`



**3.17.4 Maple [A] (verified)**

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

| method        | result                                                                                                 | size |
|---------------|--------------------------------------------------------------------------------------------------------|------|
| default       | $-\frac{1}{2(-1+x)} + \frac{3\ln(-1+x)}{4} - \frac{\ln(x^2-x+1)}{3} - \frac{\ln(1+x)}{12}$             | 32   |
| norman        | $-\frac{1}{2(-1+x)} + \frac{3\ln(-1+x)}{4} - \frac{\ln(x^2-x+1)}{3} - \frac{\ln(1+x)}{12}$             | 32   |
| risch         | $-\frac{1}{2(-1+x)} + \frac{3\ln(-1+x)}{4} - \frac{\ln(x^2-x+1)}{3} - \frac{\ln(1+x)}{12}$             | 32   |
| parallelrisch | $\frac{9\ln(-1+x)x - \ln(1+x)x - 4\ln(x^2-x+1)x - 6 - 9\ln(-1+x) + \ln(1+x) + 4\ln(x^2-x+1)}{-12+12x}$ | 57   |

input `int(x^3/(-1+x)^2/(x^3+1),x,method=_RETURNVERBOSE)`output `-1/2/(-1+x)+3/4*ln(-1+x)-1/3*ln(x^2-x+1)-1/12*ln(1+x)`**3.17.5 Fracas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{(-1+x)^2(1+x^3)} dx$$

$$= -\frac{4(x-1)\log(x^2-x+1) + (x-1)\log(x+1) - 9(x-1)\log(x-1) + 6}{12(x-1)}$$

input `integrate(x^3/(-1+x)^2/(x^3+1),x, algorithm="fricas")`output `-1/12*(4*(x - 1)*log(x^2 - x + 1) + (x - 1)*log(x + 1) - 9*(x - 1)*log(x - 1) + 6)/(x - 1)`

**3.17.6 Sympy [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{(-1+x)^2(1+x^3)} dx = \frac{3 \log(x-1)}{4} - \frac{\log(x+1)}{12} - \frac{\log(x^2-x+1)}{3} - \frac{1}{2x-2}$$

input `integrate(x**3/(-1+x)**2/(x**3+1),x)`output `3*log(x - 1)/4 - log(x + 1)/12 - log(x**2 - x + 1)/3 - 1/(2*x - 2)`**3.17.7 Maxima [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{(-1+x)^2(1+x^3)} dx = -\frac{1}{2(x-1)} - \frac{1}{3} \log(x^2-x+1) - \frac{1}{12} \log(x+1) + \frac{3}{4} \log(x-1)$$

input `integrate(x^3/(-1+x)^2/(x^3+1),x, algorithm="maxima")`output `-1/2/(x - 1) - 1/3*log(x^2 - x + 1) - 1/12*log(x + 1) + 3/4*log(x - 1)`**3.17.8 Giac [A] (verification not implemented)**

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{(-1+x)^2(1+x^3)} dx = -\frac{1}{2(x-1)} - \frac{1}{3} \log\left(\frac{1}{x-1} + \frac{1}{(x-1)^2} + 1\right) - \frac{1}{12} \log\left(\left|-\frac{2}{x-1} - 1\right|\right)$$

input `integrate(x^3/(-1+x)^2/(x^3+1),x, algorithm="giac")`output `-1/2/(x - 1) - 1/3*log(1/(x - 1) + 1/(x - 1)^2 + 1) - 1/12*log(abs(-2/(x - 1) - 1))`

**3.17.9 Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{(-1+x)^2(1+x^3)} dx = \frac{3 \ln(x-1)}{4} - \frac{\ln(x+1)}{12} - \frac{\ln(x^2-x+1)}{3} - \frac{1}{2(x-1)}$$

input `int(x^3/((x^3 + 1)*(x - 1)^2),x)`

output `(3*log(x - 1))/4 - log(x + 1)/12 - log(x^2 - x + 1)/3 - 1/(2*(x - 1))`

### 3.18 $\int \frac{1}{1+x^4} dx$

|        |                                                     |     |
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| 3.18.1 | Optimal result . . . . .                            | 195 |
| 3.18.2 | Mathematica [A] (verified) . . . . .                | 195 |
| 3.18.3 | Rubi [A] (verified) . . . . .                       | 196 |
| 3.18.4 | Maple [C] (verified) . . . . .                      | 198 |
| 3.18.5 | Fricas [C] (verification not implemented) . . . . . | 199 |
| 3.18.6 | Sympy [A] (verification not implemented) . . . . .  | 199 |
| 3.18.7 | Maxima [A] (verification not implemented) . . . . . | 200 |
| 3.18.8 | Giac [A] (verification not implemented) . . . . .   | 200 |
| 3.18.9 | Mupad [B] (verification not implemented) . . . . .  | 200 |

#### 3.18.1 Optimal result

Integrand size = 7, antiderivative size = 85

$$\int \frac{1}{1+x^4} dx = -\frac{\arctan(1-\sqrt{2}x)}{2\sqrt{2}} + \frac{\arctan(1+\sqrt{2}x)}{2\sqrt{2}} - \frac{\log(1-\sqrt{2}x+x^2)}{4\sqrt{2}} + \frac{\log(1+\sqrt{2}x+x^2)}{4\sqrt{2}}$$

output `1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)-1/8*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)`

#### 3.18.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+x^4} dx = \frac{-2 \arctan(1-\sqrt{2}x) + 2 \arctan(1+\sqrt{2}x) - \log(1-\sqrt{2}x+x^2) + \log(1+\sqrt{2}x+x^2)}{4\sqrt{2}}$$

input `Integrate[(1 + x^4)^(-1),x]`

output `(-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqrt[2]*x + x^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])`

**3.18.3 Rubi [A] (verified)**

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$ , Rules used = {755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{x^4 + 1} dx \\
 & \quad \downarrow \text{755} \\
 & \frac{1}{2} \int \frac{1 - x^2}{x^4 + 1} dx + \frac{1}{2} \int \frac{x^2 + 1}{x^4 + 1} dx \\
 & \quad \downarrow \text{1476} \\
 & \frac{1}{2} \left( \int \frac{1}{x^2 - \sqrt{2}x + 1} dx + \int \frac{1}{x^2 + \sqrt{2}x + 1} dx \right) + \frac{1}{2} \int \frac{1 - x^2}{x^4 + 1} dx \\
 & \quad \downarrow \text{1082} \\
 & \frac{1}{2} \int \frac{1 - x^2}{x^4 + 1} dx + \frac{1}{2} \left( \frac{\int \frac{1}{-(1 - \sqrt{2}x)^2 - 1} d(1 - \sqrt{2}x)}{\sqrt{2}} - \frac{\int \frac{1}{-(\sqrt{2}x + 1)^2 - 1} d(\sqrt{2}x + 1)}{\sqrt{2}} \right) \\
 & \quad \downarrow \text{217} \\
 & \frac{1}{2} \int \frac{1 - x^2}{x^4 + 1} dx + \frac{1}{2} \left( \frac{\arctan(\sqrt{2}x + 1)}{\sqrt{2}} - \frac{\arctan(1 - \sqrt{2}x)}{\sqrt{2}} \right) \\
 & \quad \downarrow \text{1479} \\
 & \frac{1}{2} \left( -\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1} dx}{2\sqrt{2}} - \frac{\int -\frac{\sqrt{2}(\sqrt{2}x+1)}{x^2+\sqrt{2}x+1} dx}{2\sqrt{2}} \right) + \frac{1}{2} \left( \frac{\arctan(\sqrt{2}x + 1)}{\sqrt{2}} - \frac{\arctan(1 - \sqrt{2}x)}{\sqrt{2}} \right) \\
 & \quad \downarrow \text{25} \\
 & \frac{1}{2} \left( \frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1} dx}{2\sqrt{2}} + \frac{\int \frac{\sqrt{2}(\sqrt{2}x+1)}{x^2+\sqrt{2}x+1} dx}{2\sqrt{2}} \right) + \frac{1}{2} \left( \frac{\arctan(\sqrt{2}x + 1)}{\sqrt{2}} - \frac{\arctan(1 - \sqrt{2}x)}{\sqrt{2}} \right) \\
 & \quad \downarrow \text{27}
 \end{aligned}$$

$$\frac{1}{2} \left( \int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1} dx + \frac{1}{2} \int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1} dx \right) + \frac{1}{2} \left( \frac{\arctan(\sqrt{2}x+1)}{\sqrt{2}} - \frac{\arctan(1-\sqrt{2}x)}{\sqrt{2}} \right)$$

↓ 1103

$$\frac{1}{2} \left( \frac{\arctan(\sqrt{2}x+1)}{\sqrt{2}} - \frac{\arctan(1-\sqrt{2}x)}{\sqrt{2}} \right) + \frac{1}{2} \left( \frac{\log(x^2+\sqrt{2}x+1)}{2\sqrt{2}} - \frac{\log(x^2-\sqrt{2}x+1)}{2\sqrt{2}} \right)$$

input `Int[(1 + x^4)^(-1), x]`

output `(-(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*x + x^2]/Sqrt[2] + Log[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2]))/2`

### 3.18.3.1 Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

rule 755 `Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r) Int[(r - s*x^2)/(a + b*x^4), x], x] + Simp[1/(2*r) Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))`

rule 1082 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Simp[-2/b Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x]`

rule 1103 `Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

rule 1476 `Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Simp[e/(2*c) Int[1/Simp[d/e + q*x + x^2, x], x] + Simp[e/(2*c) Int[1/Simp[d/e - q*x + x^2, x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]`

rule 1479 `Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Simp[e/(2*c*q) Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x] + Simp[e/(2*c*q) Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]`

### 3.18.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.26

| method  | result                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch   | $\frac{\left( \sum_{-R=\text{RootOf}(-Z^4+1)} \frac{\ln(x-R)}{-R^3} \right)}{4}$                                                                                                                                                                                                                                                                                                                                                                                   |
| default | $\frac{\sqrt{2} \left( \ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right) + 2 \arctan(1+x\sqrt{2}) + 2 \arctan(-1+x\sqrt{2}) \right)}{8}$                                                                                                                                                                                                                                                                                                                     |
| meijerg | $-\frac{x\sqrt{2} \ln\left(1-\sqrt{2}(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{1}{4}}} + \frac{x\sqrt{2} \arctan\left(\frac{\sqrt{2}(x^4)^{\frac{1}{4}}}{2-\sqrt{2}(x^4)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{1}{4}}} + \frac{x\sqrt{2} \ln\left(1+\sqrt{2}(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{1}{4}}} + \frac{x\sqrt{2} \arctan\left(\frac{\sqrt{2}(x^4)^{\frac{1}{4}}}{2+\sqrt{2}(x^4)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{1}{4}}}$ |

input `int(1/(x^4+1),x,method=_RETURNVERBOSE)`

output `1/4*sum(1/_R^3*ln(x-_R),_R=RootOf(-Z^4+1))`

**3.18.5 Fracas [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

$$\int \frac{1}{1+x^4} dx = \left(\frac{1}{8}i + \frac{1}{8}\right) \sqrt{2} \log(2x + (i+1)\sqrt{2}) - \left(\frac{1}{8}i - \frac{1}{8}\right) \sqrt{2} \log(2x - (i-1)\sqrt{2}) \\ + \left(\frac{1}{8}i - \frac{1}{8}\right) \sqrt{2} \log(2x + (i-1)\sqrt{2}) - \left(\frac{1}{8}i + \frac{1}{8}\right) \sqrt{2} \log(2x - (i+1)\sqrt{2})$$

input `integrate(1/(x^4+1),x, algorithm="fricas")`

output `(1/8*I + 1/8)*sqrt(2)*log(2*x + (I + 1)*sqrt(2)) - (1/8*I - 1/8)*sqrt(2)*log(2*x - (I - 1)*sqrt(2)) + (1/8*I - 1/8)*sqrt(2)*log(2*x + (I - 1)*sqrt(2)) - (1/8*I + 1/8)*sqrt(2)*log(2*x - (I + 1)*sqrt(2))`

**3.18.6 Sympy [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{1}{1+x^4} dx = -\frac{\sqrt{2} \log(x^2 - \sqrt{2}x + 1)}{8} + \frac{\sqrt{2} \log(x^2 + \sqrt{2}x + 1)}{8} \\ + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}x - 1)}{4} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}x + 1)}{4}$$

input `integrate(1/(x**4+1),x)`

output `-sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8 + sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4`



**3.18.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+x^4} dx = \frac{1}{4} \sqrt{2} \arctan \left( \frac{1}{2} \sqrt{2} (2x + \sqrt{2}) \right) + \frac{1}{4} \sqrt{2} \arctan \left( \frac{1}{2} \sqrt{2} (2x - \sqrt{2}) \right) \\ + \frac{1}{8} \sqrt{2} \log (x^2 + \sqrt{2}x + 1) - \frac{1}{8} \sqrt{2} \log (x^2 - \sqrt{2}x + 1)$$

input `integrate(1/(x^4+1),x, algorithm="maxima")`output `1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1)`**3.18.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+x^4} dx = \frac{1}{4} \sqrt{2} \arctan \left( \frac{1}{2} \sqrt{2} (2x + \sqrt{2}) \right) + \frac{1}{4} \sqrt{2} \arctan \left( \frac{1}{2} \sqrt{2} (2x - \sqrt{2}) \right) \\ + \frac{1}{8} \sqrt{2} \log (x^2 + \sqrt{2}x + 1) - \frac{1}{8} \sqrt{2} \log (x^2 - \sqrt{2}x + 1)$$

input `integrate(1/(x^4+1),x, algorithm="giac")`output `1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1)`**3.18.9 Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.39

$$\int \frac{1}{1+x^4} dx = \sqrt{2} \operatorname{atan} \left( \sqrt{2}x \left( \frac{1}{2} - \frac{1}{2}i \right) \right) \left( \frac{1}{4} + \frac{1}{4}i \right) + \sqrt{2} \operatorname{atan} \left( \sqrt{2}x \left( \frac{1}{2} + \frac{1}{2}i \right) \right) \left( \frac{1}{4} - \frac{1}{4}i \right)$$

input `int(1/(x^4 + 1),x)`

output `2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 + 1i/4) + 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/4 - 1i/4)`

### 3.19 $\int \frac{x^2}{1+x^4} dx$

|        |                                                     |     |
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#### 3.19.1 Optimal result

Integrand size = 11, antiderivative size = 85

$$\int \frac{x^2}{1+x^4} dx = -\frac{\arctan(1-\sqrt{2}x)}{2\sqrt{2}} + \frac{\arctan(1+\sqrt{2}x)}{2\sqrt{2}} + \frac{\log(1-\sqrt{2}x+x^2)}{4\sqrt{2}} - \frac{\log(1+\sqrt{2}x+x^2)}{4\sqrt{2}}$$

output `1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)`

#### 3.19.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{1+x^4} dx = \frac{-2 \arctan(1-\sqrt{2}x) + 2 \arctan(1+\sqrt{2}x) + \log(1-\sqrt{2}x+x^2) - \log(1+\sqrt{2}x+x^2)}{4\sqrt{2}}$$

input `Integrate[x^2/(1 + x^4),x]`

output `(-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] + Log[1 - Sqrt[2]*x + x^2] - Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])`

**3.19.3 Rubi [A] (verified)**

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$ , Rules used = {826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^2}{x^4 + 1} dx \\
 & \quad \downarrow \text{826} \\
 & \frac{1}{2} \int \frac{x^2 + 1}{x^4 + 1} dx - \frac{1}{2} \int \frac{1 - x^2}{x^4 + 1} dx \\
 & \quad \downarrow \text{1476} \\
 & \frac{1}{2} \left( \frac{1}{2} \int \frac{1}{x^2 - \sqrt{2}x + 1} dx + \frac{1}{2} \int \frac{1}{x^2 + \sqrt{2}x + 1} dx \right) - \frac{1}{2} \int \frac{1 - x^2}{x^4 + 1} dx \\
 & \quad \downarrow \text{1082} \\
 & \frac{1}{2} \left( \frac{\int \frac{1}{-(1-\sqrt{2}x)^2 - 1} d(1 - \sqrt{2}x)}{\sqrt{2}} - \frac{\int \frac{1}{-(\sqrt{2}x+1)^2 - 1} d(\sqrt{2}x + 1)}{\sqrt{2}} \right) - \frac{1}{2} \int \frac{1 - x^2}{x^4 + 1} dx \\
 & \quad \downarrow \text{217} \\
 & \frac{1}{2} \left( \frac{\arctan(\sqrt{2}x + 1)}{\sqrt{2}} - \frac{\arctan(1 - \sqrt{2}x)}{\sqrt{2}} \right) - \frac{1}{2} \int \frac{1 - x^2}{x^4 + 1} dx \\
 & \quad \downarrow \text{1479} \\
 & \frac{1}{2} \left( \frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1} dx}{2\sqrt{2}} + \frac{\int -\frac{\sqrt{2}(\sqrt{2}x+1)}{x^2+\sqrt{2}x+1} dx}{2\sqrt{2}} \right) + \frac{1}{2} \left( \frac{\arctan(\sqrt{2}x + 1)}{\sqrt{2}} - \frac{\arctan(1 - \sqrt{2}x)}{\sqrt{2}} \right) \\
 & \quad \downarrow \text{25} \\
 & \frac{1}{2} \left( -\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1} dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{2}(\sqrt{2}x+1)}{x^2+\sqrt{2}x+1} dx}{2\sqrt{2}} \right) + \frac{1}{2} \left( \frac{\arctan(\sqrt{2}x + 1)}{\sqrt{2}} - \frac{\arctan(1 - \sqrt{2}x)}{\sqrt{2}} \right) \\
 & \quad \downarrow \text{27}
 \end{aligned}$$

$$\frac{1}{2} \left( -\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1} dx}{2\sqrt{2}} - \frac{1}{2} \int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1} dx \right) + \frac{1}{2} \left( \frac{\arctan(\sqrt{2}x+1)}{\sqrt{2}} - \frac{\arctan(1-\sqrt{2}x)}{\sqrt{2}} \right)$$

↓ 1103

$$\frac{1}{2} \left( \frac{\arctan(\sqrt{2}x+1)}{\sqrt{2}} - \frac{\arctan(1-\sqrt{2}x)}{\sqrt{2}} \right) + \frac{1}{2} \left( \frac{\log(x^2-\sqrt{2}x+1)}{2\sqrt{2}} - \frac{\log(x^2+\sqrt{2}x+1)}{2\sqrt{2}} \right)$$

input `Int[x^2/(1 + x^4), x]`

output `(-ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*x + x^2]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2]))/2`

### 3.19.3.1 Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

rule 826 `Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s) Int[(r + s*x^2)/(a + b*x^4), x], x] - Simp[1/(2*s) Int[(r - s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))`

rule 1082 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Simp[-2/b Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x]`

rule 1103 `Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

rule 1476 `Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Simp[e/(2*c) Int[1/Simp[d/e + q*x + x^2, x], x] + Simp[e/(2*c) Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]`

rule 1479 `Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Simp[e/(2*c*q) Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x] + Simp[e/(2*c*q) Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]`

### 3.19.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.26

| method  | result                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|---------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch   | $\frac{\left( \sum_{-R=\text{RootOf}(-Z^4+1)} \frac{\ln(x-R)}{-R} \right)}{4}$                                                                                                                                                                                                                                                                                                                                                                                            |
| default | $\frac{\sqrt{2} \left( \ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right) + 2 \arctan(1+x\sqrt{2}) + 2 \arctan(-1+x\sqrt{2}) \right)}{8}$                                                                                                                                                                                                                                                                                                                            |
| meijerg | $\frac{x^3\sqrt{2} \ln\left(1-\sqrt{2}(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}} + \frac{x^3\sqrt{2} \arctan\left(\frac{\sqrt{2}(x^4)^{\frac{1}{4}}}{2-\sqrt{2}(x^4)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{3}{4}}} - \frac{x^3\sqrt{2} \ln\left(1+\sqrt{2}(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}} + \frac{x^3\sqrt{2} \arctan\left(\frac{\sqrt{2}(x^4)^{\frac{1}{4}}}{2+\sqrt{2}(x^4)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{3}{4}}}$ |

input `int(x^2/(x^4+1), x, method=_RETURNVERBOSE)`

output `1/4*sum(1/_R*ln(x-_R), _R=RootOf(-Z^4+1))`

**3.19.5 Fracas [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{1+x^4} dx = \left(\frac{1}{8}i - \frac{1}{8}\right) \sqrt{2} \log(2x + (i+1)\sqrt{2}) - \left(\frac{1}{8}i + \frac{1}{8}\right) \sqrt{2} \log(2x - (i-1)\sqrt{2}) \\ + \left(\frac{1}{8}i + \frac{1}{8}\right) \sqrt{2} \log(2x + (i-1)\sqrt{2}) - \left(\frac{1}{8}i - \frac{1}{8}\right) \sqrt{2} \log(2x - (i+1)\sqrt{2})$$

input `integrate(x^2/(x^4+1),x, algorithm="fricas")`

output `(1/8*I - 1/8)*sqrt(2)*log(2*x + (I + 1)*sqrt(2)) - (1/8*I + 1/8)*sqrt(2)*log(2*x - (I - 1)*sqrt(2)) + (1/8*I + 1/8)*sqrt(2)*log(2*x + (I - 1)*sqrt(2)) - (1/8*I - 1/8)*sqrt(2)*log(2*x - (I + 1)*sqrt(2))`

**3.19.6 Sympy [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{1+x^4} dx = \frac{\sqrt{2} \log(x^2 - \sqrt{2}x + 1)}{8} - \frac{\sqrt{2} \log(x^2 + \sqrt{2}x + 1)}{8} \\ + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}x - 1)}{4} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}x + 1)}{4}$$

input `integrate(x**2/(x**4+1),x)`

output `sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 - sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8 + sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4`

**3.19.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{1+x^4} dx = \frac{1}{4} \sqrt{2} \arctan \left( \frac{1}{2} \sqrt{2} (2x + \sqrt{2}) \right) + \frac{1}{4} \sqrt{2} \arctan \left( \frac{1}{2} \sqrt{2} (2x - \sqrt{2}) \right) - \frac{1}{8} \sqrt{2} \log (x^2 + \sqrt{2}x + 1) + \frac{1}{8} \sqrt{2} \log (x^2 - \sqrt{2}x + 1)$$

input `integrate(x^2/(x^4+1),x, algorithm="maxima")`output `1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1)`**3.19.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{1+x^4} dx = \frac{1}{4} \sqrt{2} \arctan \left( \frac{1}{2} \sqrt{2} (2x + \sqrt{2}) \right) + \frac{1}{4} \sqrt{2} \arctan \left( \frac{1}{2} \sqrt{2} (2x - \sqrt{2}) \right) - \frac{1}{8} \sqrt{2} \log (x^2 + \sqrt{2}x + 1) + \frac{1}{8} \sqrt{2} \log (x^2 - \sqrt{2}x + 1)$$

input `integrate(x^2/(x^4+1),x, algorithm="giac")`output `1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1)`**3.19.9 Mupad [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{1+x^4} dx = \sqrt{2} \operatorname{atan} \left( \sqrt{2}x \left( \frac{1}{2} - \frac{1}{2}i \right) \right) \left( \frac{1}{4} - \frac{1}{4}i \right) + \sqrt{2} \operatorname{atan} \left( \sqrt{2}x \left( \frac{1}{2} + \frac{1}{2}i \right) \right) \left( \frac{1}{4} + \frac{1}{4}i \right)$$



input `int(x^2/(x^4 + 1),x)`

output `2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 - 1i/4) + 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/4 + 1i/4)`

### 3.20 $\int \frac{1}{1+x^2+x^4} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.20.1 | Optimal result . . . . .                            | 209 |
| 3.20.2 | Mathematica [C] (verified) . . . . .                | 209 |
| 3.20.3 | Rubi [A] (verified) . . . . .                       | 210 |
| 3.20.4 | Maple [A] (verified) . . . . .                      | 212 |
| 3.20.5 | Fricas [A] (verification not implemented) . . . . . | 212 |
| 3.20.6 | Sympy [A] (verification not implemented) . . . . .  | 212 |
| 3.20.7 | Maxima [A] (verification not implemented) . . . . . | 213 |
| 3.20.8 | Giac [A] (verification not implemented) . . . . .   | 213 |
| 3.20.9 | Mupad [B] (verification not implemented) . . . . .  | 214 |

#### 3.20.1 Optimal result

Integrand size = 10, antiderivative size = 67

$$\int \frac{1}{1+x^2+x^4} dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4} \log(1-x+x^2) + \frac{1}{4} \log(1+x+x^2)$$

output `-1/4*ln(x^2-x+1)+1/4*ln(x^2+x+1)-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)`

#### 3.20.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \frac{1}{1+x^2+x^4} dx = \frac{i\left(\sqrt{1-i\sqrt{3}} \arctan\left(\frac{1}{2}(-i+\sqrt{3})x\right) - \sqrt{1+i\sqrt{3}} \arctan\left(\frac{1}{2}(i+\sqrt{3})x\right)\right)}{\sqrt{6}}$$

input `Integrate[(1 + x^2 + x^4)^(-1),x]`

output `(I*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x)/2] - Sqrt[1 + I*Sqrt[3]]*ArcTan[((I + Sqrt[3])*x)/2]))/Sqrt[6]`

### 3.20.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{x^4 + x^2 + 1} dx \\
 & \quad \downarrow \text{1407} \\
 & \frac{1}{2} \int \frac{1-x}{x^2-x+1} dx + \frac{1}{2} \int \frac{x+1}{x^2+x+1} dx \\
 & \quad \downarrow \text{1142} \\
 & \frac{1}{2} \left( \frac{1}{2} \int \frac{1}{x^2-x+1} dx - \frac{1}{2} \int \frac{1-2x}{x^2-x+1} dx \right) + \frac{1}{2} \left( \frac{1}{2} \int \frac{1}{x^2+x+1} dx + \frac{1}{2} \int \frac{2x+1}{x^2+x+1} dx \right) \\
 & \quad \downarrow \text{25} \\
 & \frac{1}{2} \left( \frac{1}{2} \int \frac{1}{x^2-x+1} dx + \frac{1}{2} \int \frac{1-2x}{x^2-x+1} dx \right) + \frac{1}{2} \left( \frac{1}{2} \int \frac{1}{x^2+x+1} dx + \frac{1}{2} \int \frac{2x+1}{x^2+x+1} dx \right) \\
 & \quad \downarrow \text{1083} \\
 & \frac{1}{2} \left( \frac{1}{2} \int \frac{1-2x}{x^2-x+1} dx - \int \frac{1}{-(2x-1)^2-3} d(2x-1) \right) + \\
 & \quad \frac{1}{2} \left( \frac{1}{2} \int \frac{2x+1}{x^2+x+1} dx - \int \frac{1}{-(2x+1)^2-3} d(2x+1) \right) \\
 & \quad \downarrow \text{217} \\
 & \frac{1}{2} \left( \frac{1}{2} \int \frac{1-2x}{x^2-x+1} dx + \frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}} \right) + \frac{1}{2} \left( \frac{1}{2} \int \frac{2x+1}{x^2+x+1} dx + \frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} \right) \\
 & \quad \downarrow \text{1103} \\
 & \frac{1}{2} \left( \frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{2} \log(x^2-x+1) \right) + \frac{1}{2} \left( \frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2} \log(x^2+x+1) \right)
 \end{aligned}$$

input `Int[(1 + x^2 + x^4)^(-1), x]`

output  $(\text{ArcTan}[-1 + 2x]/\sqrt{3})/\sqrt{3} - \text{Log}[1 - x + x^2]/2 + (\text{ArcTan}[(1 + 2x)/\sqrt{3}]/\sqrt{3} + \text{Log}[1 + x + x^2]/2)/2$

### 3.20.3.1 Defintions of rubi rules used

rule 25  $\text{Int}[-(F_x), x\_Symbol] \rightarrow \text{Simp}[\text{Identity}[-1] \quad \text{Int}[F_x, x], x]$

rule 217  $\text{Int}[(a + (b \cdot x)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2])^{-1} \cdot \text{ArcTan}[\text{Rt}[-b, 2] \cdot (x/\text{Rt}[-a, 2])], x] \text{ ; FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

rule 1083  $\text{Int}[(a + (b \cdot x) + (c \cdot x)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[-2 \quad \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4ac - x^2, x], x], x, b + 2cx], x] \text{ ; FreeQ}\{a, b, c, x\}$

rule 1103  $\text{Int}[(d + (e \cdot x))/(a + (b \cdot x) + (c \cdot x)^2), x\_Symbol] \rightarrow \text{Simp}[d \cdot (\text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]/b), x] \text{ ; FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[2cd - be, 0]$

rule 1142  $\text{Int}[(d + (e \cdot x))/(a + (b \cdot x) + (c \cdot x)^2), x\_Symbol] \rightarrow \text{Simp}[(2cd - be)/(2c) \quad \text{Int}[1/(a + bx + cx^2), x], x] + \text{Simp}[e/(2c) \quad \text{Int}[(b + 2cx)/(a + bx + cx^2), x], x] \text{ ; FreeQ}\{a, b, c, d, e, x\}$

rule 1407  $\text{Int}[(a + (b \cdot x)^2 + (c \cdot x)^4)^{-1}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[a/c, 2]\}, \text{With}\{r = \text{Rt}[2q - b/c, 2]\}, \text{Simp}[1/(2cq)r \quad \text{Int}[(r - x)/(q - rx + x^2), x], x] + \text{Simp}[1/(2cq)r \quad \text{Int}[(r + x)/(q + rx + x^2), x], x]] \text{ ; FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NegQ}[b^2 - 4ac]$

### 3.20.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

| method  | result                                                                                                                                                                                   | size |
|---------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default | $-\frac{\ln(x^2-x+1)}{4} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\ln(x^2+x+1)}{4} + \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}$    | 54   |
| risch   | $\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6} + \frac{\ln(4x^2+4x+4)}{4} - \frac{\ln(4x^2-4x+4)}{4} + \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$ | 60   |

input `int(1/(x^4+x^2+1),x,method=_RETURNVERBOSE)`

output `-1/4*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/4*ln(x^2+x+1)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)`

### 3.20.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^4} dx = \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) + \frac{1}{4} \log(x^2+x+1) - \frac{1}{4} \log(x^2-x+1)$$

input `integrate(1/(x^4+x^2+1),x, algorithm="fricas")`

output `1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)`

### 3.20.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{1}{1+x^2+x^4} dx = -\frac{\log(x^2-x+1)}{4} + \frac{\log(x^2+x+1)}{4} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{6}$$

input `integrate(1/(x**4+x**2+1),x)`

output `-log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6`

### 3.20.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^4} dx = \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) + \frac{1}{4} \log(x^2+x+1) - \frac{1}{4} \log(x^2-x+1)$$

input `integrate(1/(x^4+x^2+1),x, algorithm="maxima")`

output `1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)`

### 3.20.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^4} dx = \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) + \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) + \frac{1}{4} \log(x^2+x+1) - \frac{1}{4} \log(x^2-x+1)$$

input `integrate(1/(x^4+x^2+1),x, algorithm="giac")`

output `1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)`

**3.20.9 Mupad [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{1}{1+x^2+x^4} dx = \operatorname{atanh}\left(\frac{2x}{-1+\sqrt{3}1i}\right) \left(-\frac{1}{2} + \frac{\sqrt{3}1i}{6}\right) + \operatorname{atanh}\left(\frac{2x}{1+\sqrt{3}1i}\right) \left(\frac{1}{2} + \frac{\sqrt{3}1i}{6}\right)$$

input `int(1/(x^2 + x^4 + 1),x)`

output `atanh((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) + atanh((2*x)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/6 + 1/2)`

## 3.21 $\int (a + bx)^p dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.21.1 | Optimal result . . . . .                            | 215 |
| 3.21.2 | Mathematica [A] (verified) . . . . .                | 215 |
| 3.21.3 | Rubi [A] (verified) . . . . .                       | 216 |
| 3.21.4 | Maple [A] (verified) . . . . .                      | 216 |
| 3.21.5 | Fricas [A] (verification not implemented) . . . . . | 217 |
| 3.21.6 | Sympy [A] (verification not implemented) . . . . .  | 217 |
| 3.21.7 | Maxima [A] (verification not implemented) . . . . . | 217 |
| 3.21.8 | Giac [A] (verification not implemented) . . . . .   | 218 |
| 3.21.9 | Mupad [B] (verification not implemented) . . . . .  | 218 |

### 3.21.1 Optimal result

Integrand size = 7, antiderivative size = 18

$$\int (a + bx)^p dx = \frac{(a + bx)^{1+p}}{b(1+p)}$$

output  $(b*x+a)^{(p+1)}/b/(p+1)$

### 3.21.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a + bx)^p dx = \frac{(a + bx)^{1+p}}{b(1+p)}$$

input `Integrate[(a + b*x)^p,x]`

output  $(a + b*x)^{(1 + p)}/(b*(1 + p))$



### 3.21.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + bx)^p dx$$

$$\downarrow 17$$

$$\frac{(a + bx)^{p+1}}{b(p + 1)}$$

input `Int[(a + b*x)^p,x]`

output `(a + b*x)^(1 + p)/(b*(1 + p))`

#### 3.21.3.1 Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

### 3.21.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

| method        | result                                                             | size |
|---------------|--------------------------------------------------------------------|------|
| gospers       | $\frac{(bx+a)^{1+p}}{b(1+p)}$                                      | 19   |
| default       | $\frac{(bx+a)^{1+p}}{b(1+p)}$                                      | 19   |
| risch         | $\frac{(bx+a)(bx+a)^p}{b(1+p)}$                                    | 22   |
| parallelrisch | $\frac{x(bx+a)^p ab + (bx+a)^p a^2}{(1+p)ab}$                      | 36   |
| norman        | $\frac{x e^{p \ln(bx+a)}}{1+p} + \frac{a e^{p \ln(bx+a)}}{b(1+p)}$ | 37   |

input `int((b*x+a)^p,x,method=_RETURNVERBOSE)`

output  $(b*x+a)^{(1+p)}/b/(1+p)$

### 3.21.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (a + bx)^p dx = \frac{(bx + a)(bx + a)^p}{bp + b}$$

input `integrate((b*x+a)^p,x, algorithm="fricas")`

output  $(b*x + a)*(b*x + a)^p/(b*p + b)$

### 3.21.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (a + bx)^p dx = \frac{\begin{cases} \frac{(a+bx)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(a + bx) & \text{otherwise} \end{cases}}{b}$$

input `integrate((b*x+a)**p,x)`

output `Piecewise(((a + b*x)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x), True))/b`

### 3.21.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a + bx)^p dx = \frac{(bx + a)^{p+1}}{b(p + 1)}$$

input `integrate((b*x+a)^p,x, algorithm="maxima")`

output  $(b*x + a)^{(p + 1)}/(b*(p + 1))$

**3.21.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a + bx)^p dx = \frac{(bx + a)^{p+1}}{b(p+1)}$$

input `integrate((b*x+a)^p,x, algorithm="giac")`

output `(b*x + a)^(p + 1)/(b*(p + 1))`

**3.21.9 Mupad [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a + bx)^p dx = \frac{(a + bx)^{p+1}}{b(p+1)}$$

input `int((a + b*x)^p,x)`

output `(a + b*x)^(p + 1)/(b*(p + 1))`

## 3.22 $\int x(a + bx)^p dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.22.1 | Optimal result . . . . .                            | 219 |
| 3.22.2 | Mathematica [A] (verified) . . . . .                | 219 |
| 3.22.3 | Rubi [A] (verified) . . . . .                       | 220 |
| 3.22.4 | Maple [A] (verified) . . . . .                      | 221 |
| 3.22.5 | Fricas [A] (verification not implemented) . . . . . | 221 |
| 3.22.6 | Sympy [B] (verification not implemented) . . . . .  | 221 |
| 3.22.7 | Maxima [A] (verification not implemented) . . . . . | 222 |
| 3.22.8 | Giac [A] (verification not implemented) . . . . .   | 222 |
| 3.22.9 | Mupad [B] (verification not implemented) . . . . .  | 223 |

### 3.22.1 Optimal result

Integrand size = 9, antiderivative size = 39

$$\int x(a + bx)^p dx = -\frac{a(a + bx)^{1+p}}{b^2(1 + p)} + \frac{(a + bx)^{2+p}}{b^2(2 + p)}$$

output `-a*(b*x+a)^(p+1)/b^2/(p+1)+(b*x+a)^(2+p)/b^2/(2+p)`

### 3.22.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int x(a + bx)^p dx = \frac{(a + bx)^{1+p}(-a + b(1 + p)x)}{b^2(1 + p)(2 + p)}$$

input `Integrate[x*(a + b*x)^p,x]`

output `((a + b*x)^(1 + p)*(-a + b*(1 + p)*x))/(b^2*(1 + p)*(2 + p))`

### 3.22.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^p dx$$

$$\downarrow \text{53}$$

$$\int \left( \frac{(a+bx)^{p+1}}{b} - \frac{a(a+bx)^p}{b} \right) dx$$

$$\downarrow \text{2009}$$

$$\frac{(a+bx)^{p+2}}{b^2(p+2)} - \frac{a(a+bx)^{p+1}}{b^2(p+1)}$$

input `Int[x*(a + b*x)^p,x]`

output `-((a*(a + b*x)^(1 + p))/(b^2*(1 + p))) + (a + b*x)^(2 + p)/(b^2*(2 + p))`

#### 3.22.3.1 Defintions of rubi rules used

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int [ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

### 3.22.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

| method        | result                                                                                                                | size |
|---------------|-----------------------------------------------------------------------------------------------------------------------|------|
| gospers       | $-\frac{(bx+a)^{1+p}(-xpb-bx+a)}{b^2(p^2+3p+2)}$                                                                      | 36   |
| risch         | $-\frac{(-x^2b^2p-xapb-x^2b^2+a^2)(bx+a)^p}{b^2(2+p)(1+p)}$                                                           | 50   |
| parallelrisch | $\frac{x^2(bx+a)^pb^2p+x^2(bx+a)^pb^2+x(bx+a)^pabp-(bx+a)^pa^2}{b^2(p^2+3p+2)}$                                       | 69   |
| norman        | $\frac{x^2e^{p \ln(bx+a)}}{2+p} + \frac{pax e^{p \ln(bx+a)}}{b(p^2+3p+2)} - \frac{a^2e^{p \ln(bx+a)}}{b^2(p^2+3p+2)}$ | 73   |

input `int(x*(b*x+a)^p,x,method=_RETURNVERBOSE)`

output `-1/b^2*(b*x+a)^(1+p)/(p^2+3*p+2)*(-b*p*x-b*x+a)`

### 3.22.5 Fracas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36

$$\int x(a+bx)^p dx = \frac{(abpx + (b^2p + b^2)x^2 - a^2)(bx+a)^p}{b^2p^2 + 3b^2p + 2b^2}$$

input `integrate(x*(b*x+a)^p,x, algorithm="fracas")`

output `(a*b*p*x + (b^2*p + b^2)*x^2 - a^2)*(b*x + a)^p/(b^2*p^2 + 3*b^2*p + 2*b^2)`

### 3.22.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(31) = 62.

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 5.15

$$\int x(a+bx)^p dx = \begin{cases} \frac{a^p x^2}{2} & \text{for } b = 0 \\ \frac{a \log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} + \frac{a}{ab^2+b^3x} + \frac{bx \log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for } p = -2 \\ -\frac{a \log\left(\frac{a}{b}+x\right)}{b^2} + \frac{x}{b} & \text{for } p = -1 \\ -\frac{a^2(a+bx)^p}{b^2p^2+3b^2p+2b^2} + \frac{abpx(a+bx)^p}{b^2p^2+3b^2p+2b^2} + \frac{b^2px^2(a+bx)^p}{b^2p^2+3b^2p+2b^2} + \frac{b^2x^2(a+bx)^p}{b^2p^2+3b^2p+2b^2} & \text{otherwise} \end{cases}$$

input `integrate(x*(b*x+a)**p,x)`

output `Piecewise((a**p*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) + a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(p, -2)), (-a*log(a/b + x)/b**2 + x/b, Eq(p, -1)), (-a**2*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2) + a*b*p*x*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2) + b**2*p*x**2*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2) + b**2*x**2*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2), True))`

### 3.22.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int x(a + bx)^p dx = \frac{(b^2(p + 1)x^2 + abpx - a^2)(bx + a)^p}{(p^2 + 3p + 2)b^2}$$

input `integrate(x*(b*x+a)^p,x, algorithm="maxima")`

output `(b^2*(p + 1)*x^2 + a*b*p*x - a^2)*(b*x + a)^p/((p^2 + 3*p + 2)*b^2)`

### 3.22.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.95

$$\int x(a + bx)^p dx = \frac{(bx + a)^p b^2 p x^2 + (bx + a)^p abpx + (bx + a)^p b^2 x^2 - (bx + a)^p a^2}{b^2 p^2 + 3 b^2 p + 2 b^2}$$

input `integrate(x*(b*x+a)^p,x, algorithm="giac")`

output `((b*x + a)^p*b^2*p*x^2 + (b*x + a)^p*a*b*p*x + (b*x + a)^p*b^2*x^2 - (b*x + a)^p*a^2)/(b^2*p^2 + 3*b^2*p + 2*b^2)`

**3.22.9 Mupad [B] (verification not implemented)**

Time = 0.39 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.41

$$\int x(a+bx)^p dx = \begin{cases} -\frac{a \ln(a+bx) - bx}{b^2} & \text{if } p = -1 \\ \frac{\ln(a+bx) + \frac{a}{a+bx}}{b^2} & \text{if } p = -2 \\ \frac{2 \left( \frac{(a+bx)^{p+2}}{2p+4} - \frac{a(a+bx)^{p+1}}{2p+2} \right)}{b^2} & \text{if } p \neq -1 \wedge p \neq -2 \end{cases}$$

input `int(x*(a + b*x)^p,x)`output `piecewise(p == -1, -(a*log(a + b*x) - b*x)/b^2, p == -2, (log(a + b*x) + a/(a + b*x))/b^2, p ~= -1 & p ~= -2, (2*((a + b*x)^(p + 2)/(2*p + 4) - (a*(a + b*x)^(p + 1))/(2*p + 2)))/b^2)`



### 3.23 $\int x^2(a + bx)^p dx$

|        |                                                     |     |
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| 3.23.1 | Optimal result . . . . .                            | 224 |
| 3.23.2 | Mathematica [A] (verified) . . . . .                | 224 |
| 3.23.3 | Rubi [A] (verified) . . . . .                       | 225 |
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| 3.23.5 | Fricas [A] (verification not implemented) . . . . . | 226 |
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| 3.23.9 | Mupad [B] (verification not implemented) . . . . .  | 228 |

#### 3.23.1 Optimal result

Integrand size = 11, antiderivative size = 60

$$\int x^2(a + bx)^p dx = \frac{a^2(a + bx)^{1+p}}{b^3(1 + p)} - \frac{2a(a + bx)^{2+p}}{b^3(2 + p)} + \frac{(a + bx)^{3+p}}{b^3(3 + p)}$$

output  $a^2*(b*x+a)^{(p+1)}/b^3/(p+1)-2*a*(b*x+a)^{(2+p)}/b^3/(2+p)+(b*x+a)^{(3+p)}/b^3/(3+p)$

#### 3.23.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int x^2(a + bx)^p dx = \frac{(a + bx)^{1+p} (2a^2 - 2ab(1 + p)x + b^2(2 + 3p + p^2)x^2)}{b^3(1 + p)(2 + p)(3 + p)}$$

input `Integrate[x^2*(a + b*x)^p,x]`

output  $((a + b*x)^{(1 + p)}*(2*a^2 - 2*a*b*(1 + p)*x + b^2*(2 + 3*p + p^2)*x^2))/(b^3*(1 + p)*(2 + p)*(3 + p))$

### 3.23.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^p dx$$

$$\downarrow \text{53}$$

$$\int \left( \frac{a^2(a+bx)^p}{b^2} - \frac{2a(a+bx)^{p+1}}{b^2} + \frac{(a+bx)^{p+2}}{b^2} \right) dx$$

$$\downarrow \text{2009}$$

$$\frac{a^2(a+bx)^{p+1}}{b^3(p+1)} - \frac{2a(a+bx)^{p+2}}{b^3(p+2)} + \frac{(a+bx)^{p+3}}{b^3(p+3)}$$

input `Int[x^2*(a + b*x)^p,x]`

output `(a^2*(a + b*x)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x)^(2 + p))/(b^3*(2 + p)) + (a + b*x)^(3 + p)/(b^3*(3 + p))`

#### 3.23.3.1 Defintions of rubi rules used

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]`

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

### 3.23.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

| method        | result                                                                                                                                                                            |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| gospers       | $\frac{(bx+a)^{1+p}(b^2p^2x^2+3x^2b^2p-2xapb+2x^2b^2-2axb+2a^2)}{b^3(p^3+6p^2+11p+6)}$                                                                                            |
| risch         | $\frac{(b^3p^2x^3+ab^2p^2x^2+3b^3px^3+x^2apb^2+2x^3b^3-2a^2pxb+2a^3)(bx+a)^p}{(2+p)(3+p)(1+p)b^3}$                                                                                |
| norman        | $\frac{x^3e^{p \ln(bx+a)}}{3+p} + \frac{apx^2e^{p \ln(bx+a)}}{b(p^2+5p+6)} + \frac{2a^3e^{p \ln(bx+a)}}{b^3(p^3+6p^2+11p+6)} - \frac{2pa^2xe^{p \ln(bx+a)}}{b^2(p^3+6p^2+11p+6)}$ |
| parallelrisch | $\frac{x^3(bx+a)^pa^3b^3p^2+3x^3(bx+a)^pa^3b^3p+x^2(bx+a)^pa^2b^2p^2+2x^3(bx+a)^pa^3b^3+x^2(bx+a)^pa^2b^2p-2x(bx+a)^pa^3bp+2a^4(bx+a)^p}{ab^3(p^2+5p+6)(1+p)}$                    |

input `int(x^2*(b*x+a)^p,x,method=_RETURNVERBOSE)`

output 
$$\frac{1}{b^3} \frac{(bx+a)^{1+p}}{(p^3+6p^2+11p+6)} \frac{(b^2p^2x^2+3b^2p^2x^2+3b^2p^2x^2-2a^2b^2p^2x^2-2a^2b^2p^2x^2+2a^2b^2p^2x^2)}{(p^3+6p^2+11p+6)}$$

### 3.23.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.60

$$\int x^2(a+bx)^p dx = -\frac{(2a^2bpx - (b^3p^2 + 3b^3p + 2b^3)x^3 - 2a^3 - (ab^2p^2 + ab^2p)x^2)(bx+a)^p}{b^3p^3 + 6b^3p^2 + 11b^3p + 6b^3}$$

input `integrate(x^2*(b*x+a)^p,x, algorithm="fricas")`

output 
$$-(2a^2b^2p^2x - (b^3p^2 + 3b^3p + 2b^3)x^3 - 2a^3 - (ab^2p^2 + ab^2p)x^2) \frac{(bx+a)^p}{(b^3p^3 + 6b^3p^2 + 11b^3p + 6b^3)}$$

### 3.23.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs.  $2(51) = 102$ .

Time = 0.48 (sec) , antiderivative size = 597, normalized size of antiderivative = 9.95

$$\int x^2(a+bx)^p dx$$

$$= \begin{cases} \frac{a^p x^3}{3} \\ \frac{2a^2 \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{3a^2}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{2b^2x^2 \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} \\ - \frac{2a^2 \log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} - \frac{2a^2}{ab^3+b^4x} - \frac{2abx \log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} + \frac{b^2x^2}{ab^3+b^4x} \\ \frac{a^2 \log\left(\frac{a}{b}+x\right)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b} \\ \frac{2a^3(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} - \frac{2a^2bpx(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{ab^2p^2x^2(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{ab^2px^2(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{b^3p^2x^3(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} \end{cases}$$

input `integrate(x**2*(b*x+a)**p,x)`

output `Piecewise((a**p*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(p, -3)), (-2*a**2*log(a/b + x)/(a*b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 + b**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(p, -2)), (a**2*log(a/b + x)/b**3 - a*x/b**2 + x**2/(2*b), Eq(p, -1)), (2*a**3*(a + b*x)**p/(b**3*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) - 2*a**2*b*p*x*(a + b*x)**p/(b**3*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) + a*b**2*p**2*x**2*(a + b*x)**p/(b**3*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) + a*b**2*p*x**2*(a + b*x)**p/(b**3*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) + b**3*p**2*x**3*(a + b*x)**p/(b**3*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) + 3*b**3*p*x**3*(a + b*x)**p/(b**3*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3) + 2*b**3*x**3*(a + b*x)**p/(b**3*p**3 + 6*b**3*p**2 + 11*b**3*p + 6*b**3), True))`

### 3.23.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int x^2(a+bx)^p dx = \frac{((p^2 + 3p + 2)b^3x^3 + (p^2 + p)ab^2x^2 - 2a^2bpx + 2a^3)(bx + a)^p}{(p^3 + 6p^2 + 11p + 6)b^3}$$

input `integrate(x^2*(b*x+a)^p,x, algorithm="maxima")`

output  $((p^2 + 3p + 2)*b^3*x^3 + (p^2 + p)*a*b^2*x^2 - 2*a^2*b*p*x + 2*a^3)*(b*x + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)$

### 3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs.  $2(60) = 120$ .

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.33

$$\int x^2(a+bx)^p dx = \frac{(bx+a)^p b^3 p^2 x^3 + (bx+a)^p a b^2 p^2 x^2 + 3(bx+a)^p b^3 p x^3 + (bx+a)^p a b^2 p x^2 + 2(bx+a)^p b^3 x^3 - 2(bx+a)^p}{b^3 p^3 + 6 b^3 p^2 + 11 b^3 p + 6 b^3}$$

input `integrate(x^2*(b*x+a)^p,x, algorithm="giac")`

output  $((b*x + a)^p*b^3*p^2*x^3 + (b*x + a)^p*a*b^2*p^2*x^2 + 3*(b*x + a)^p*b^3*p*x^3 + (b*x + a)^p*a*b^2*p*x^2 + 2*(b*x + a)^p*b^3*x^3 - 2*(b*x + a)^p*a^2*b*p*x + 2*(b*x + a)^p*a^3)/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)$

### 3.23.9 Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.20

$$\int x^2(a+bx)^p dx = \begin{cases} \frac{2a^2 \ln(a+bx) + b^2 x^2 - 2abx}{2b^3} & \text{if } p = -1 \\ \frac{x}{b^2} - \frac{a^2}{b^3(a+bx)} - \frac{2a \ln(a+bx)}{b^3} & \text{if } p = -2 \\ \frac{\ln(a+bx) + \frac{2a}{a+bx} - \frac{a^2}{2(a+bx)^2}}{b^3} & \text{if } p = -3 \\ \frac{2(a+bx)^{p+1} (8a^2 - 8abpx - 8abx + 4b^2 p^2 x^2 + 12b^2 px^2 + 8b^2 x^2)}{b^3 (8p^3 + 48p^2 + 88p + 48)} & \text{if } p \neq -1 \wedge p \neq -2 \wedge p \neq -3 \end{cases}$$

input `int(x^2*(a + b*x)^p,x)`

```
output piecewise(p == -1, (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3), p ==
-2, x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*log(a + b*x))/b^3, p == -3, (log(a
+ b*x) + (2*a)/(a + b*x) - a^2/(2*(a + b*x)^2))/b^3, p ~= -1 & p ~= -2 & p
~= -3, (2*(a + b*x)^(p + 1)*(8*a^2 + 8*b^2*x^2 + 12*b^2*p*x^2 - 8*a*b*x +
4*b^2*p^2*x^2 - 8*a*b*p*x))/(b^3*(88*p + 48*p^2 + 8*p^3 + 48)))
```

## 3.24 $\int \frac{1}{a+bx} dx$

|        |                                                     |     |
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| 3.24.1 | Optimal result . . . . .                            | 230 |
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| 3.24.3 | Rubi [A] (verified) . . . . .                       | 231 |
| 3.24.4 | Maple [A] (verified) . . . . .                      | 231 |
| 3.24.5 | Fricas [A] (verification not implemented) . . . . . | 232 |
| 3.24.6 | Sympy [A] (verification not implemented) . . . . .  | 232 |
| 3.24.7 | Maxima [A] (verification not implemented) . . . . . | 232 |
| 3.24.8 | Giac [A] (verification not implemented) . . . . .   | 233 |
| 3.24.9 | Mupad [B] (verification not implemented) . . . . .  | 233 |

### 3.24.1 Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{a+bx} dx = \frac{\log(a+bx)}{b}$$

output `ln(b*x+a)/b`

### 3.24.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx} dx = \frac{\log(a+bx)}{b}$$

input `Integrate[(a + b*x)^(-1),x]`

output `Log[a + b*x]/b`

### 3.24.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx} dx$$

↓ 16

$$\frac{\log(a+bx)}{b}$$

input `Int[(a + b*x)^(-1),x]`

output `Log[a + b*x]/b`

#### 3.24.3.1 Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

### 3.24.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

| method        | result                | size |
|---------------|-----------------------|------|
| default       | $\frac{\ln(bx+a)}{b}$ | 11   |
| norman        | $\frac{\ln(bx+a)}{b}$ | 11   |
| risch         | $\frac{\ln(bx+a)}{b}$ | 11   |
| parallelrisch | $\frac{\ln(bx+a)}{b}$ | 11   |

input `int(1/(b*x+a),x,method=_RETURNVERBOSE)`

output `ln(b*x+a)/b`



**3.24.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx} dx = \frac{\log(bx+a)}{b}$$

input `integrate(1/(b*x+a),x, algorithm="fricas")`

output `log(b*x + a)/b`

**3.24.6 Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{a+bx} dx = \frac{\log(a+bx)}{b}$$

input `integrate(1/(b*x+a),x)`

output `log(a + b*x)/b`

**3.24.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx} dx = \frac{\log(bx+a)}{b}$$

input `integrate(1/(b*x+a),x, algorithm="maxima")`

output `log(b*x + a)/b`

**3.24.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+bx} dx = \frac{\log(|bx+a|)}{b}$$

input `integrate(1/(b*x+a),x, algorithm="giac")`

output `log(abs(b*x + a))/b`

**3.24.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx} dx = \frac{\ln(a+bx)}{b}$$

input `int(1/(a + b*x),x)`

output `log(a + b*x)/b`

## 3.25 $\int \frac{1}{(a+bx)^2} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.25.1 | Optimal result . . . . .                            | 234 |
| 3.25.2 | Mathematica [A] (verified) . . . . .                | 234 |
| 3.25.3 | Rubi [A] (verified) . . . . .                       | 235 |
| 3.25.4 | Maple [A] (verified) . . . . .                      | 235 |
| 3.25.5 | Fricas [A] (verification not implemented) . . . . . | 236 |
| 3.25.6 | Sympy [A] (verification not implemented) . . . . .  | 236 |
| 3.25.7 | Maxima [A] (verification not implemented) . . . . . | 236 |
| 3.25.8 | Giac [A] (verification not implemented) . . . . .   | 237 |
| 3.25.9 | Mupad [B] (verification not implemented) . . . . .  | 237 |

### 3.25.1 Optimal result

Integrand size = 7, antiderivative size = 12

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b(a+bx)}$$

output `-1/b/(b*x+a)`

### 3.25.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b(a+bx)}$$

input `Integrate[(a + b*x)^(-2),x]`

output `-(1/(b*(a + b*x)))`

### 3.25.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^2} dx$$

↓ 17

$$-\frac{1}{b(a+bx)}$$

input `Int[(a + b*x)^(-2), x]`

output `-(1/(b*(a + b*x)))`

#### 3.25.3.1 Defintions of rubi rules used

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

### 3.25.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

| method       | result               | size |
|--------------|----------------------|------|
| gospers      | $-\frac{1}{b(bx+a)}$ | 13   |
| default      | $-\frac{1}{b(bx+a)}$ | 13   |
| norman       | $\frac{x}{a(bx+a)}$  | 13   |
| risch        | $-\frac{1}{b(bx+a)}$ | 13   |
| parallelrisc | $-\frac{1}{b(bx+a)}$ | 13   |

input `int(1/(b*x+a)^2, x, method=_RETURNVERBOSE)`

output `-1/b/(b*x+a)`

### 3.25.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b^2x+ab}$$

input `integrate(1/(b*x+a)^2,x, algorithm="fricas")`

output `-1/(b^2*x + a*b)`

### 3.25.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{ab+b^2x}$$

input `integrate(1/(b*x+a)**2,x)`

output `-1/(a*b + b**2*x)`

### 3.25.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{(bx+a)b}$$

input `integrate(1/(b*x+a)^2,x, algorithm="maxima")`

output `-1/((b*x + a)*b)`

**3.25.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a + bx)^2} dx = -\frac{1}{(bx + a)b}$$

input `integrate(1/(b*x+a)^2,x, algorithm="giac")`

output `-1/((b*x + a)*b)`

**3.25.9 Mupad [B] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a + bx)^2} dx = -\frac{1}{b(a + bx)}$$

input `int(1/(a + b*x)^2,x)`

output `-1/(b*(a + b*x))`

## 3.26 $\int \frac{x}{a+bx} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.26.1 | Optimal result . . . . .                            | 238 |
| 3.26.2 | Mathematica [A] (verified) . . . . .                | 238 |
| 3.26.3 | Rubi [A] (verified) . . . . .                       | 239 |
| 3.26.4 | Maple [A] (verified) . . . . .                      | 240 |
| 3.26.5 | Fricas [A] (verification not implemented) . . . . . | 240 |
| 3.26.6 | Sympy [A] (verification not implemented) . . . . .  | 240 |
| 3.26.7 | Maxima [A] (verification not implemented) . . . . . | 241 |
| 3.26.8 | Giac [A] (verification not implemented) . . . . .   | 241 |
| 3.26.9 | Mupad [B] (verification not implemented) . . . . .  | 241 |

### 3.26.1 Optimal result

Integrand size = 9, antiderivative size = 18

$$\int \frac{x}{a+bx} dx = \frac{x}{b} - \frac{a \log(a+bx)}{b^2}$$

output `x/b-a*ln(b*x+a)/b^2`

### 3.26.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+bx} dx = \frac{x}{b} - \frac{a \log(a+bx)}{b^2}$$

input `Integrate[x/(a + b*x),x]`

output `x/b - (a*Log[a + b*x])/b^2`

### 3.26.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+bx} dx$$

↓ 49

$$\int \left( \frac{1}{b} - \frac{a}{b(a+bx)} \right) dx$$

↓ 2009

$$\frac{x}{b} - \frac{a \log(a+bx)}{b^2}$$

input `Int[x/(a + b*x), x]`

output `x/b - (a*Log[a + b*x])/b^2`

#### 3.26.3.1 Defintions of rubi rules used

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`



**3.26.4 Maple [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

| method        | result                                  | size |
|---------------|-----------------------------------------|------|
| default       | $\frac{x}{b} - \frac{a \ln(bx+a)}{b^2}$ | 19   |
| norman        | $\frac{x}{b} - \frac{a \ln(bx+a)}{b^2}$ | 19   |
| risch         | $\frac{x}{b} - \frac{a \ln(bx+a)}{b^2}$ | 19   |
| parallelrisch | $-\frac{a \ln(bx+a)-bx}{b^2}$           | 19   |

input `int(x/(b*x+a),x,method=_RETURNVERBOSE)`output `x/b-a*ln(b*x+a)/b^2`**3.26.5 Fracas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x}{a+bx} dx = \frac{bx - a \log(bx+a)}{b^2}$$

input `integrate(x/(b*x+a),x, algorithm="fricas")`output `(b*x - a*log(b*x + a))/b^2`**3.26.6 Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{a+bx} dx = -\frac{a \log(a+bx)}{b^2} + \frac{x}{b}$$

input `integrate(x/(b*x+a),x)`output `-a*log(a + b*x)/b**2 + x/b`

**3.26.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+bx} dx = \frac{x}{b} - \frac{a \log(bx+a)}{b^2}$$

input `integrate(x/(b*x+a),x, algorithm="maxima")`output `x/b - a*log(b*x + a)/b^2`**3.26.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{x}{a+bx} dx = \frac{x}{b} - \frac{a \log(|bx+a|)}{b^2}$$

input `integrate(x/(b*x+a),x, algorithm="giac")`output `x/b - a*log(abs(b*x + a))/b^2`**3.26.9 Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+bx} dx = -\frac{a \ln(a+bx) - bx}{b^2}$$

input `int(x/(a + b*x),x)`output `-(a*log(a + b*x) - b*x)/b^2`

## 3.27 $\int \frac{x^2}{a+bx} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.27.1 | Optimal result . . . . .                            | 242 |
| 3.27.2 | Mathematica [A] (verified) . . . . .                | 242 |
| 3.27.3 | Rubi [A] (verified) . . . . .                       | 243 |
| 3.27.4 | Maple [A] (verified) . . . . .                      | 244 |
| 3.27.5 | Fricas [A] (verification not implemented) . . . . . | 244 |
| 3.27.6 | Sympy [A] (verification not implemented) . . . . .  | 244 |
| 3.27.7 | Maxima [A] (verification not implemented) . . . . . | 245 |
| 3.27.8 | Giac [A] (verification not implemented) . . . . .   | 245 |
| 3.27.9 | Mupad [B] (verification not implemented) . . . . .  | 245 |

### 3.27.1 Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{x^2}{a+bx} dx = -\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \log(a+bx)}{b^3}$$

output `-a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3`

### 3.27.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+bx} dx = -\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \log(a+bx)}{b^3}$$

input `Integrate[x^2/(a + b*x),x]`

output `-((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3`

### 3.27.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+bx} dx$$

↓ 49

$$\int \left( \frac{a^2}{b^2(a+bx)} - \frac{a}{b^2} + \frac{x}{b} \right) dx$$

↓ 2009

$$\frac{a^2 \log(a+bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

input `Int[x^2/(a + b*x), x]`

output `-((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3`

#### 3.27.3.1 Defintions of rubi rules used

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int  
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]  
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

**3.27.4 Maple [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

| method        | result                                                         | size |
|---------------|----------------------------------------------------------------|------|
| default       | $-\frac{\frac{1}{2}x^2b+ax}{b^2} + \frac{a^2 \ln(bx+a)}{b^3}$  | 30   |
| norman        | $-\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \ln(bx+a)}{b^3}$ | 30   |
| risch         | $-\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \ln(bx+a)}{b^3}$ | 30   |
| parallelrisch | $\frac{x^2b^2+2a^2 \ln(bx+a)-2axb}{2b^3}$                      | 30   |

input `int(x^2/(b*x+a),x,method=_RETURNVERBOSE)`output `-1/b^2*(-1/2*x^2*b+a*x)+a^2*ln(b*x+a)/b^3`**3.27.5 Fricas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a+bx} dx = \frac{b^2x^2 - 2abx + 2a^2 \log(bx+a)}{2b^3}$$

input `integrate(x^2/(b*x+a),x, algorithm="fricas")`output `1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3`**3.27.6 Sympy [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{a+bx} dx = \frac{a^2 \log(a+bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

input `integrate(x**2/(b*x+a),x)`output `a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)`

**3.27.7 Maxima [A] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a+bx} dx = \frac{a^2 \log(bx+a)}{b^3} + \frac{bx^2 - 2ax}{2b^2}$$

input `integrate(x^2/(b*x+a),x, algorithm="maxima")`output `a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2`**3.27.8 Giac [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{a+bx} dx = \frac{a^2 \log(|bx+a|)}{b^3} + \frac{bx^2 - 2ax}{2b^2}$$

input `integrate(x^2/(b*x+a),x, algorithm="giac")`output `a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2`**3.27.9 Mupad [B] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a+bx} dx = \frac{2a^2 \ln(a+bx) + b^2 x^2 - 2abx}{2b^3}$$

input `int(x^2/(a + b*x),x)`output `(2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3)`

### 3.28 $\int \frac{1}{x(a+bx)} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.28.1 | Optimal result . . . . .                            | 246 |
| 3.28.2 | Mathematica [A] (verified) . . . . .                | 246 |
| 3.28.3 | Rubi [A] (verified) . . . . .                       | 247 |
| 3.28.4 | Maple [A] (verified) . . . . .                      | 248 |
| 3.28.5 | Fricas [A] (verification not implemented) . . . . . | 248 |
| 3.28.6 | Sympy [A] (verification not implemented) . . . . .  | 248 |
| 3.28.7 | Maxima [A] (verification not implemented) . . . . . | 249 |
| 3.28.8 | Giac [A] (verification not implemented) . . . . .   | 249 |
| 3.28.9 | Mupad [B] (verification not implemented) . . . . .  | 249 |

#### 3.28.1 Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{x(a+bx)} dx = \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

output `ln(x)/a-ln(b*x+a)/a`

#### 3.28.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+bx)} dx = \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

input `Integrate[1/(x*(a + b*x)),x]`

output `Log[x]/a - Log[a + b*x]/a`

### 3.28.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{x(a+bx)} dx \\ & \quad \downarrow 47 \\ & \frac{\int \frac{1}{x} dx}{a} - \frac{b \int \frac{1}{a+bx} dx}{a} \\ & \quad \downarrow 14 \\ & \frac{\log(x)}{a} - \frac{b \int \frac{1}{a+bx} dx}{a} \\ & \quad \downarrow 16 \\ & \frac{\log(x)}{a} - \frac{\log(a+bx)}{a} \end{aligned}$$

input `Int[1/(x*(a + b*x)),x]`

output `Log[x]/a - Log[a + b*x]/a`

#### 3.28.3.1 Defintions of rubi rules used

rule 14 `Int[(a_.)/(x_), x_Symbol] := Simp[a*Log[x], x] /; FreeQ[a, x]`

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 47 `Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Simp[b/(b*c - a*d) Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d) Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]`



**3.28.4 Maple [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

| method       | result                                     | size |
|--------------|--------------------------------------------|------|
| parallelrisc | $\frac{\ln(x) - \ln(bx+a)}{a}$             | 16   |
| default      | $\frac{\ln(x)}{a} - \frac{\ln(bx+a)}{a}$   | 19   |
| norman       | $\frac{\ln(x)}{a} - \frac{\ln(bx+a)}{a}$   | 19   |
| risc         | $-\frac{\ln(bx+a)}{a} + \frac{\ln(-x)}{a}$ | 21   |

input `int(1/x/(b*x+a),x,method=_RETURNVERBOSE)`output `(ln(x)-ln(b*x+a))/a`**3.28.5 Fracas [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(a+bx)} dx = -\frac{\log(bx+a) - \log(x)}{a}$$

input `integrate(1/x/(b*x+a),x, algorithm="fricas")`output `-(log(b*x + a) - log(x))/a`**3.28.6 Sympy [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{1}{x(a+bx)} dx = \frac{\log(x) - \log(\frac{a}{b} + x)}{a}$$

input `integrate(1/x/(b*x+a),x)`output `(log(x) - log(a/b + x))/a`

**3.28.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+bx)} dx = -\frac{\log(bx+a)}{a} + \frac{\log(x)}{a}$$

input `integrate(1/x/(b*x+a),x, algorithm="maxima")`output `-log(b*x + a)/a + log(x)/a`**3.28.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(a+bx)} dx = -\frac{\log(|bx+a|)}{a} + \frac{\log(|x|)}{a}$$

input `integrate(1/x/(b*x+a),x, algorithm="giac")`output `-log(abs(b*x + a))/a + log(abs(x))/a`**3.28.9 Mupad [B] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(a+bx)} dx = -\frac{2 \operatorname{atanh}\left(\frac{2bx}{a} + 1\right)}{a}$$

input `int(1/(x*(a + b*x)),x)`output `-(2*atanh((2*b*x)/a + 1))/a`

### 3.29 $\int \frac{1}{x^2(a+bx)} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.29.1 | Optimal result . . . . .                            | 250 |
| 3.29.2 | Mathematica [A] (verified) . . . . .                | 250 |
| 3.29.3 | Rubi [A] (verified) . . . . .                       | 251 |
| 3.29.4 | Maple [A] (verified) . . . . .                      | 252 |
| 3.29.5 | Fricas [A] (verification not implemented) . . . . . | 252 |
| 3.29.6 | Sympy [A] (verification not implemented) . . . . .  | 252 |
| 3.29.7 | Maxima [A] (verification not implemented) . . . . . | 253 |
| 3.29.8 | Giac [A] (verification not implemented) . . . . .   | 253 |
| 3.29.9 | Mupad [B] (verification not implemented) . . . . .  | 253 |

#### 3.29.1 Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \frac{1}{x^2(a+bx)} dx = -\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{b \log(a+bx)}{a^2}$$

output `-1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2`

#### 3.29.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+bx)} dx = -\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{b \log(a+bx)}{a^2}$$

input `Integrate[1/(x^2*(a + b*x)),x]`

output `-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2`

### 3.29.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+bx)} dx$$

↓ 54

$$\int \left( \frac{b^2}{a^2(a+bx)} - \frac{b}{a^2x} + \frac{1}{ax^2} \right) dx$$

↓ 2009

$$-\frac{b \log(x)}{a^2} + \frac{b \log(a+bx)}{a^2} - \frac{1}{ax}$$

input `Int[1/(x^2*(a + b*x)),x]`

output `-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2`

#### 3.29.3.1 Defintions of rubi rules used

rule 54 `Int[((a_) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

**3.29.4 Maple [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

| method      | result                                                            | size |
|-------------|-------------------------------------------------------------------|------|
| parallelsch | $-\frac{bx \ln(x) - b \ln(bx+a)x + a}{a^2 x}$                     | 26   |
| default     | $-\frac{1}{ax} - \frac{b \ln(x)}{a^2} + \frac{b \ln(bx+a)}{a^2}$  | 29   |
| norman      | $-\frac{1}{ax} - \frac{b \ln(x)}{a^2} + \frac{b \ln(bx+a)}{a^2}$  | 29   |
| risch       | $-\frac{1}{ax} + \frac{b \ln(-bx-a)}{a^2} - \frac{b \ln(x)}{a^2}$ | 32   |

input `int(1/x^2/(b*x+a),x,method=_RETURNVERBOSE)`output `-(b*x*ln(x)-b*ln(b*x+a)*x+a)/a^2/x`**3.29.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2(a+bx)} dx = \frac{bx \log(bx+a) - bx \log(x) - a}{a^2 x}$$

input `integrate(1/x^2/(b*x+a),x, algorithm="fricas")`output `(b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)`**3.29.6 Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^2(a+bx)} dx = -\frac{1}{ax} + \frac{b(-\log(x) + \log(\frac{a}{b} + x))}{a^2}$$

input `integrate(1/x**2/(b*x+a),x)`output `-1/(a*x) + b*(-log(x) + log(a/b + x))/a**2`

**3.29.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+bx)} dx = \frac{b \log(bx+a)}{a^2} - \frac{b \log(x)}{a^2} - \frac{1}{ax}$$

input `integrate(1/x^2/(b*x+a),x, algorithm="maxima")`output `b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)`**3.29.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(a+bx)} dx = \frac{b \log(|bx+a|)}{a^2} - \frac{b \log(|x|)}{a^2} - \frac{1}{ax}$$

input `integrate(1/x^2/(b*x+a),x, algorithm="giac")`output `b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)`**3.29.9 Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2(a+bx)} dx = \frac{2b \operatorname{atanh}\left(\frac{2bx}{a} + 1\right)}{a^2} - \frac{1}{ax}$$

input `int(1/(x^2*(a + b*x)),x)`output `(2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)`

### 3.30 $\int \frac{1}{x^2(a+bx)^2} dx$

|                                                            |     |
|------------------------------------------------------------|-----|
| 3.30.1 Optimal result . . . . .                            | 254 |
| 3.30.2 Mathematica [A] (verified) . . . . .                | 254 |
| 3.30.3 Rubi [A] (verified) . . . . .                       | 255 |
| 3.30.4 Maple [A] (verified) . . . . .                      | 256 |
| 3.30.5 Fricas [A] (verification not implemented) . . . . . | 256 |
| 3.30.6 Sympy [A] (verification not implemented) . . . . .  | 256 |
| 3.30.7 Maxima [A] (verification not implemented) . . . . . | 257 |
| 3.30.8 Giac [A] (verification not implemented) . . . . .   | 257 |
| 3.30.9 Mupad [B] (verification not implemented) . . . . .  | 257 |

#### 3.30.1 Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \frac{1}{x^2(a+bx)^2} dx = -\frac{1}{a^2x} - \frac{b}{a^2(a+bx)} - \frac{2b \log(x)}{a^3} + \frac{2b \log(a+bx)}{a^3}$$

output  $-1/a^2/x - b/a^2/(b*x+a) - 2*b*\ln(x)/a^3 + 2*b*\ln(b*x+a)/a^3$

#### 3.30.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2(a+bx)^2} dx = -\frac{a\left(\frac{1}{x} + \frac{b}{a+bx}\right) + 2b \log(x) - 2b \log(a+bx)}{a^3}$$

input `Integrate[1/(x^2*(a + b*x)^2),x]`

output  $-((a*(x^{-1}) + b/(a + b*x)) + 2*b*\text{Log}[x] - 2*b*\text{Log}[a + b*x])/a^3$

### 3.30.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+bx)^2} dx$$

↓ 54

$$\int \left( \frac{2b^2}{a^3(a+bx)} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{1}{a^2x^2} \right) dx$$

↓ 2009

$$-\frac{2b \log(x)}{a^3} + \frac{2b \log(a+bx)}{a^3} - \frac{b}{a^2(a+bx)} - \frac{1}{a^2x}$$

input `Int[1/(x^2*(a + b*x)^2),x]`

output `-(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^3`

#### 3.30.3.1 Defintions of rubi rules used

rule 54 `Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`



**3.30.4 Maple [A] (verified)**

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

| method        | result                                                                                                  | size |
|---------------|---------------------------------------------------------------------------------------------------------|------|
| default       | $-\frac{1}{a^2x} - \frac{b}{a^2(bx+a)} - \frac{2b \ln(x)}{a^3} + \frac{2b \ln(bx+a)}{a^3}$              | 43   |
| risch         | $\frac{-\frac{2bx}{a^2} - \frac{1}{a}}{x(bx+a)} + \frac{2b \ln(-bx-a)}{a^3} - \frac{2b \ln(x)}{a^3}$    | 49   |
| norman        | $\frac{\frac{2b^2x^2}{a^3} - \frac{1}{a}}{x(bx+a)} - \frac{2b \ln(x)}{a^3} + \frac{2b \ln(bx+a)}{a^3}$  | 50   |
| parallelrisch | $-\frac{2 \ln(x)x^2b^2 - 2 \ln(bx+a)x^2b^2 + 2 \ln(x)xab - 2 \ln(bx+a)xab - 2x^2b^2 + a^2}{a^3x(bx+a)}$ | 70   |

input `int(1/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)`output `-1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3`**3.30.5 Fracas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.50

$$\int \frac{1}{x^2(a+bx)^2} dx = -\frac{2abx + a^2 - 2(b^2x^2 + abx) \log(bx+a) + 2(b^2x^2 + abx) \log(x)}{a^3bx^2 + a^4x}$$

input `integrate(1/x^2/(b*x+a)^2,x, algorithm="fricas")`output `-(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*log(x))/(a^3*b*x^2 + a^4*x)`**3.30.6 Sympy [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2(a+bx)^2} dx = \frac{-a - 2bx}{a^3x + a^2bx^2} + \frac{2b(-\log(x) + \log(\frac{a}{b} + x))}{a^3}$$

input `integrate(1/x**2/(b*x+a)**2,x)`output `(-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3`

**3.30.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(a+bx)^2} dx = -\frac{2bx+a}{a^2bx^2+a^3x} + \frac{2b\log(bx+a)}{a^3} - \frac{2b\log(x)}{a^3}$$

input `integrate(1/x^2/(b*x+a)^2,x, algorithm="maxima")`output `-(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3`**3.30.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^2(a+bx)^2} dx = -\frac{2b\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^3} - \frac{b}{(bx+a)a^2} + \frac{b}{a^3\left(\frac{a}{bx+a}-1\right)}$$

input `integrate(1/x^2/(b*x+a)^2,x, algorithm="giac")`output `-2*b*log(abs(-a/(b*x + a) + 1))/a^3 - b/((b*x + a)*a^2) + b/(a^3*(a/(b*x + a) - 1))`**3.30.9 Mupad [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(a+bx)^2} dx = \frac{2b\ln\left(\frac{a+bx}{x}\right)}{a^3} - \frac{1}{ax(a+bx)} - \frac{2b}{a^2(a+bx)}$$

input `int(1/(x^2*(a + b*x)^2),x)`output `(2*b*log((a + b*x)/x))/a^3 - 1/(a*x*(a + b*x)) - (2*b)/(a^2*(a + b*x))`

### 3.31 $\int \frac{1}{c^2+x^2} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.31.1 | Optimal result . . . . .                            | 258 |
| 3.31.2 | Mathematica [A] (verified) . . . . .                | 258 |
| 3.31.3 | Rubi [A] (verified) . . . . .                       | 259 |
| 3.31.4 | Maple [A] (verified) . . . . .                      | 259 |
| 3.31.5 | Fricas [A] (verification not implemented) . . . . . | 260 |
| 3.31.6 | Sympy [C] (verification not implemented) . . . . .  | 260 |
| 3.31.7 | Maxima [A] (verification not implemented) . . . . . | 260 |
| 3.31.8 | Giac [A] (verification not implemented) . . . . .   | 261 |
| 3.31.9 | Mupad [B] (verification not implemented) . . . . .  | 261 |

#### 3.31.1 Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{1}{c^2 + x^2} dx = \frac{\arctan\left(\frac{x}{c}\right)}{c}$$

output `arctan(x/c)/c`

#### 3.31.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{c^2 + x^2} dx = \frac{\arctan\left(\frac{x}{c}\right)}{c}$$

input `Integrate[(c^2 + x^2)^(-1),x]`

output `ArcTan[x/c]/c`

### 3.31.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{c^2 + x^2} dx$$

↓ 216

$$\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

input `Int[(c^2 + x^2)^(-1),x]`

output `ArcTan[x/c]/c`

#### 3.31.3.1 Defintions of rubi rules used

rule 216 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

### 3.31.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

| method       | result                                      | size |
|--------------|---------------------------------------------|------|
| default      | $\frac{\arctan\left(\frac{x}{c}\right)}{c}$ | 11   |
| risch        | $\frac{\arctan\left(\frac{x}{c}\right)}{c}$ | 11   |
| parallelrisc | $-\frac{i \ln(-ic+x) - i \ln(ic+x)}{2c}$    | 27   |

input `int(1/(c^2+x^2),x,method=_RETURNVERBOSE)`

output `arctan(x/c)/c`

**3.31.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{c^2 + x^2} dx = \frac{\arctan\left(\frac{x}{c}\right)}{c}$$

input `integrate(1/(c^2+x^2),x, algorithm="fricas")`

output `arctan(x/c)/c`

**3.31.6 Sympy [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{1}{c^2 + x^2} dx = \frac{-\frac{i \log(-ic+x)}{2} + \frac{i \log(ic+x)}{2}}{c}$$

input `integrate(1/(c**2+x**2),x)`

output `(-I*log(-I*c + x)/2 + I*log(I*c + x)/2)/c`

**3.31.7 Maxima [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{c^2 + x^2} dx = \frac{\arctan\left(\frac{x}{c}\right)}{c}$$

input `integrate(1/(c^2+x^2),x, algorithm="maxima")`

output `arctan(x/c)/c`

**3.31.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{c^2 + x^2} dx = \frac{\arctan\left(\frac{x}{c}\right)}{c}$$

input `integrate(1/(c^2+x^2),x, algorithm="giac")`

output `arctan(x/c)/c`

**3.31.9 Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{c^2 + x^2} dx = \frac{\operatorname{atan}\left(\frac{x}{c}\right)}{c}$$

input `int(1/(c^2 + x^2),x)`

output `atan(x/c)/c`

### 3.32 $\int \frac{1}{c^2-x^2} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.32.1 | Optimal result . . . . .                            | 262 |
| 3.32.2 | Mathematica [A] (verified) . . . . .                | 262 |
| 3.32.3 | Rubi [A] (verified) . . . . .                       | 263 |
| 3.32.4 | Maple [A] (verified) . . . . .                      | 263 |
| 3.32.5 | Fricas [A] (verification not implemented) . . . . . | 264 |
| 3.32.6 | Sympy [B] (verification not implemented) . . . . .  | 264 |
| 3.32.7 | Maxima [B] (verification not implemented) . . . . . | 264 |
| 3.32.8 | Giac [B] (verification not implemented) . . . . .   | 265 |
| 3.32.9 | Mupad [B] (verification not implemented) . . . . .  | 265 |

#### 3.32.1 Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{1}{c^2-x^2} dx = \frac{\operatorname{arctanh}\left(\frac{x}{c}\right)}{c}$$

output `arctanh(x/c)/c`

#### 3.32.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{c^2-x^2} dx = \frac{\operatorname{arctanh}\left(\frac{x}{c}\right)}{c}$$

input `Integrate[(c^2 - x^2)^(-1),x]`

output `ArcTanh[x/c]/c`

### 3.32.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{c^2 - x^2} dx$$

↓ 219

$$\frac{\operatorname{arctanh}\left(\frac{x}{c}\right)}{c}$$

input `Int[(c^2 - x^2)^(-1),x]`

output `ArcTanh[x/c]/c`

#### 3.32.3.1 Defintions of rubi rules used

rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

### 3.32.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

| method        | result                                       | size |
|---------------|----------------------------------------------|------|
| parallelrisch | $-\frac{\ln(-c+x)-\ln(c+x)}{2c}$             | 19   |
| default       | $-\frac{\ln(c-x)}{2c} + \frac{\ln(c+x)}{2c}$ | 22   |
| norman        | $-\frac{\ln(c-x)}{2c} + \frac{\ln(c+x)}{2c}$ | 22   |
| risch         | $\frac{\ln(c+x)}{2c} - \frac{\ln(-c+x)}{2c}$ | 22   |

input `int(1/(c^2-x^2),x,method=_RETURNVERBOSE)`



output  $-1/2*(\ln(-c+x)-\ln(c+x))/c$

### 3.32.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{1}{c^2 - x^2} dx = \frac{\log(c+x) - \log(-c+x)}{2c}$$

input `integrate(1/(c^2-x^2),x, algorithm="fricas")`

output  $1/2*(\log(c+x) - \log(-c+x))/c$

### 3.32.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs.  $2(5) = 10$ .

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{1}{c^2 - x^2} dx = -\frac{\frac{\log(-c+x)}{2} - \frac{\log(c+x)}{2}}{c}$$

input `integrate(1/(c**2-x**2),x)`

output  $-(\log(-c+x)/2 - \log(c+x)/2)/c$

### 3.32.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs.  $2(10) = 20$ .

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \frac{1}{c^2 - x^2} dx = \frac{\log(c+x)}{2c} - \frac{\log(-c+x)}{2c}$$

input `integrate(1/(c^2-x^2),x, algorithm="maxima")`

output  $1/2*\log(c+x)/c - 1/2*\log(-c+x)/c$

**3.32.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 23 vs.  $2(10) = 20$ .

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{1}{c^2 - x^2} dx = \frac{\log(|c + x|)}{2c} - \frac{\log(|-c + x|)}{2c}$$

input `integrate(1/(c^2-x^2),x, algorithm="giac")`

output `1/2*log(abs(c + x))/c - 1/2*log(abs(-c + x))/c`

**3.32.9 Mupad [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{c^2 - x^2} dx = \frac{\operatorname{atanh}\left(\frac{x}{c}\right)}{c}$$

input `int(1/(c^2 - x^2),x)`

output `atanh(x/c)/c`

### 3.33 $\int \frac{1}{-1+2x^3} dx$

|        |                                                     |     |
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#### 3.33.1 Optimal result

Integrand size = 9, antiderivative size = 78

$$\int \frac{1}{-1+2x^3} dx = -\frac{\arctan\left(\frac{1+2\sqrt[3]{2}x}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}} - \frac{\log\left(1+\sqrt[3]{2}x+2^{2/3}x^2\right)}{6\sqrt[3]{2}}$$

output `1/6*ln(1-2^(1/3)*x)*2^(2/3)-1/12*ln(1+2^(1/3)*x+2^(2/3)*x^2)*2^(2/3)-1/6*arctan(1/3*(1+2*2^(1/3)*x)*3^(1/2))*2^(2/3)*3^(1/2)`

#### 3.33.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{1}{-1+2x^3} dx = -\frac{2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{2}x}{\sqrt{3}}\right) - 2\log\left(1-\sqrt[3]{2}x\right) + \log\left(1+\sqrt[3]{2}x+2^{2/3}x^2\right)}{6\sqrt[3]{2}}$$

input `Integrate[(-1 + 2*x^3)^(-1), x]`

output `-1/6*(2*Sqrt[3]*ArcTan[(1 + 2*2^(1/3)*x)/Sqrt[3]] - 2*Log[1 - 2^(1/3)*x] + Log[1 + 2^(1/3)*x + 2^(2/3)*x^2])/2^(1/3)`

**3.33.3 Rubi [A] (verified)**

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {750, 16, 25, 27, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{2x^3 - 1} dx \\
 & \quad \downarrow \text{750} \\
 & \frac{1}{3} \int -\frac{\sqrt[3]{2}x + 2}{2^{2/3}x^2 + \sqrt[3]{2}x + 1} dx + \frac{1}{3} \int \frac{1}{\sqrt[3]{2}x - 1} dx \\
 & \quad \downarrow \text{16} \\
 & \frac{1}{3} \int -\frac{\sqrt[3]{2}x + 2}{2^{2/3}x^2 + \sqrt[3]{2}x + 1} dx + \frac{\log(1 - \sqrt[3]{2}x)}{3\sqrt[3]{2}} \\
 & \quad \downarrow \text{25} \\
 & \frac{\log(1 - \sqrt[3]{2}x)}{3\sqrt[3]{2}} - \frac{1}{3} \int \frac{\sqrt[3]{2}(x + 2^{2/3})}{2^{2/3}x^2 + \sqrt[3]{2}x + 1} dx \\
 & \quad \downarrow \text{27} \\
 & \frac{\log(1 - \sqrt[3]{2}x)}{3\sqrt[3]{2}} - \frac{1}{3} \sqrt[3]{2} \int \frac{x + 2^{2/3}}{2^{2/3}x^2 + \sqrt[3]{2}x + 1} dx \\
 & \quad \downarrow \text{1142} \\
 & \frac{\log(1 - \sqrt[3]{2}x)}{3\sqrt[3]{2}} - \frac{1}{3} \sqrt[3]{2} \left( \frac{3 \int \frac{1}{2^{2/3}x^2 + \sqrt[3]{2}x + 1} dx}{2\sqrt[3]{2}} + \frac{\int \frac{\sqrt[3]{2}(2\sqrt[3]{2}x + 1)}{2^{2/3}x^2 + \sqrt[3]{2}x + 1} dx}{2 \cdot 2^{2/3}} \right) \\
 & \quad \downarrow \text{27} \\
 & \frac{\log(1 - \sqrt[3]{2}x)}{3\sqrt[3]{2}} - \frac{1}{3} \sqrt[3]{2} \left( \frac{3 \int \frac{1}{2^{2/3}x^2 + \sqrt[3]{2}x + 1} dx}{2\sqrt[3]{2}} + \frac{\int \frac{2\sqrt[3]{2}x + 1}{2^{2/3}x^2 + \sqrt[3]{2}x + 1} dx}{2\sqrt[3]{2}} \right) \\
 & \quad \downarrow \text{1082}
 \end{aligned}$$

$$\frac{\log(1 - \sqrt[3]{2x})}{3\sqrt[3]{2}} - \frac{1}{3}\sqrt[3]{2} \left( \frac{\int \frac{2\sqrt[3]{2x+1}}{2^{2/3}x^2 + \sqrt[3]{2x+1}} dx}{2\sqrt[3]{2}} - \frac{3 \int \frac{1}{-(2\sqrt[3]{2x+1})^2 - 3} d(2\sqrt[3]{2x+1})}{2^{2/3}} \right)$$

↓ 217

$$\frac{\log(1 - \sqrt[3]{2x})}{3\sqrt[3]{2}} - \frac{1}{3}\sqrt[3]{2} \left( \frac{\int \frac{2\sqrt[3]{2x+1}}{2^{2/3}x^2 + \sqrt[3]{2x+1}} dx}{2\sqrt[3]{2}} + \frac{\sqrt{3} \arctan\left(\frac{2\sqrt[3]{2x+1}}{\sqrt{3}}\right)}{2^{2/3}} \right)$$

↓ 1103

$$\frac{\log(1 - \sqrt[3]{2x})}{3\sqrt[3]{2}} - \frac{1}{3}\sqrt[3]{2} \left( \frac{\sqrt{3} \arctan\left(\frac{2\sqrt[3]{2x+1}}{\sqrt{3}}\right)}{2^{2/3}} + \frac{\log(2^{2/3}x^2 + \sqrt[3]{2x+1})}{2 \cdot 2^{2/3}} \right)$$

input `Int[(-1 + 2*x^3)^(-1), x]`

output `Log[1 - 2^(1/3)*x]/(3*2^(1/3)) - (2^(1/3)*((Sqrt[3]*ArcTan[(1 + 2*2^(1/3)*x)/Sqrt[3]])/2^(2/3) + Log[1 + 2^(1/3)*x + 2^(2/3)*x^2]/(2*2^(2/3))))/3`

### 3.33.3.1 Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_.)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_.)*(Gx_)] /; FreeQ[b, x]`

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

rule 750 `Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Simp[1/(3*Rt[a, 3]^2) Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2) Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;`  
`FreeQ[{a, b}, x]`

rule 1082 `Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Simp[-2/b Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /;`  
`RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /;`  
`FreeQ[{a, b, c}, x]`

rule 1103 `Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /;`  
`FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

rule 1142 `Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(2*c*d - b*e)/(2*c) Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c) Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /;`  
`FreeQ[{a, b, c, d, e}, x]`

### 3.33.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.31

| method  | result                                                                                                                                                                                                                                                                                                                                                  | size |
|---------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch   | $\frac{\sum_{-R=\text{RootOf}(2-Z^3-1)} \frac{\ln(x-R)}{-R^2}}{6}$                                                                                                                                                                                                                                                                                      | 24   |
| default | $\frac{2^{\frac{2}{3}} \ln\left(x - \frac{2^{\frac{2}{3}}}{2}\right)}{6} - \frac{2^{\frac{2}{3}} \ln\left(x^2 + \frac{2^{\frac{2}{3}}x}{2} + \frac{1}{2}\right)}{12} - \frac{\arctan\left(\frac{(1+2^{\frac{1}{3}}x)\sqrt{3}}{3}\right)}{6} 2^{\frac{2}{3}}\sqrt{3}$                                                                                    | 58   |
| meijerg | $\frac{2^{\frac{2}{3}}x \left( \ln\left(1 - 2^{\frac{1}{3}}(x^3)^{\frac{1}{3}}\right) - \frac{\ln\left(1 + 2^{\frac{1}{3}}(x^3)^{\frac{1}{3}} + 2^{\frac{2}{3}}(x^3)^{\frac{2}{3}}\right)}{2} - \sqrt{3} \arctan\left(\frac{\sqrt{3} 2^{\frac{1}{3}}(x^3)^{\frac{1}{3}}}{2 + 2^{\frac{1}{3}}(x^3)^{\frac{1}{3}}}\right) \right)}{6(x^3)^{\frac{1}{3}}}$ | 82   |

input `int(1/(2*x^3-1),x,method=_RETURNVERBOSE)`

output `1/6*sum(1/_R^2*ln(x-_R),_R=RootOf(2*_Z^3-1))`

### 3.33.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

$$\int \frac{1}{-1+2x^3} dx = -\frac{1}{6} \sqrt{6} 2^{\frac{1}{6}} \arctan \left( \frac{1}{6} \sqrt{6} 2^{\frac{1}{6}} \left( 2 \cdot 2^{\frac{2}{3}} x + 2^{\frac{1}{3}} \right) \right) - \frac{1}{12} \cdot 2^{\frac{2}{3}} \log \left( 2x^2 + 2^{\frac{2}{3}} x + 2^{\frac{1}{3}} \right) + \frac{1}{6} \cdot 2^{\frac{2}{3}} \log \left( 2x - 2^{\frac{2}{3}} \right)$$

input `integrate(1/(2*x^3-1),x, algorithm="fricas")`

output `-1/6*sqrt(6)*2^(1/6)*arctan(1/6*sqrt(6)*2^(1/6)*(2*2^(2/3)*x + 2^(1/3))) - 1/12*2^(2/3)*log(2*x^2 + 2^(2/3)*x + 2^(1/3)) + 1/6*2^(2/3)*log(2*x - 2^(2/3))`

### 3.33.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{1}{-1+2x^3} dx = \frac{2^{\frac{2}{3}} \log \left( x - \frac{2^{\frac{2}{3}}}{2} \right)}{6} - \frac{2^{\frac{2}{3}} \log \left( x^2 + \frac{2^{\frac{2}{3}} x}{2} + \frac{\sqrt[3]{2}}{2} \right)}{12} - \frac{2^{\frac{2}{3}} \sqrt{3} \operatorname{atan} \left( \frac{2 \cdot \sqrt[3]{2} \sqrt{3} x + \sqrt{3}}{3} \right)}{6}$$

input `integrate(1/(2*x**3-1),x)`

output `2**(2/3)*log(x - 2**(2/3)/2)/6 - 2**(2/3)*log(x**2 + 2**(2/3)*x/2 + 2**(1/3)/2)/12 - 2**(2/3)*sqrt(3)*atan(2*2**(1/3)*sqrt(3)*x/3 + sqrt(3)/3)/6`

**3.33.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{1}{-1+2x^3} dx = -\frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} \arctan \left( \frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} \left( 2 \cdot 2^{\frac{2}{3}} x + 2^{\frac{1}{3}} \right) \right) - \frac{1}{12} \cdot 2^{\frac{2}{3}} \log \left( 2^{\frac{2}{3}} x^2 + 2^{\frac{1}{3}} x + 1 \right) + \frac{1}{6} \cdot 2^{\frac{2}{3}} \log \left( \frac{1}{2} \cdot 2^{\frac{2}{3}} \left( 2^{\frac{1}{3}} x - 1 \right) \right)$$

input `integrate(1/(2*x^3-1),x, algorithm="maxima")`output `-1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2*2^(2/3)*x + 2^(1/3))) - 1/12*2^(2/3)*log(2^(2/3)*x^2 + 2^(1/3)*x + 1) + 1/6*2^(2/3)*log(1/2*2^(2/3)*(2^(1/3)*x - 1))`**3.33.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int \frac{1}{-1+2x^3} dx = -\frac{1}{3} \sqrt{3} \left( \frac{1}{2} \right)^{\frac{1}{3}} \arctan \left( \frac{2}{3} \sqrt{3} \left( \frac{1}{2} \right)^{\frac{2}{3}} \left( 2x + \left( \frac{1}{2} \right)^{\frac{1}{3}} \right) \right) - \frac{1}{12} \cdot 4^{\frac{1}{3}} \log \left( x^2 + \left( \frac{1}{2} \right)^{\frac{1}{3}} x + \left( \frac{1}{2} \right)^{\frac{2}{3}} \right) + \frac{1}{3} \left( \frac{1}{2} \right)^{\frac{1}{3}} \log \left( \left| x - \left( \frac{1}{2} \right)^{\frac{1}{3}} \right| \right)$$

input `integrate(1/(2*x^3-1),x, algorithm="giac")`output `-1/3*sqrt(3)*(1/2)^(1/3)*arctan(2/3*sqrt(3)*(1/2)^(2/3)*(2*x + (1/2)^(1/3))) - 1/12*4^(1/3)*log(x^2 + (1/2)^(1/3)*x + (1/2)^(2/3)) + 1/3*(1/2)^(1/3)*log(abs(x - (1/2)^(1/3)))`



**3.33.9 Mupad [B] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int \frac{1}{-1+2x^3} dx = \frac{2^{2/3} \ln\left(x - \frac{2^{2/3}}{2}\right)}{6} + \frac{2^{2/3} \ln\left(x - \frac{2^{2/3}(-1+\sqrt{3}i)}{4}\right) (-1 + \sqrt{3}i)}{12} - \frac{2^{2/3} \ln\left(x + \frac{2^{2/3}(1+\sqrt{3}i)}{4}\right) (1 + \sqrt{3}i)}{12}$$

input `int(1/(2*x^3 - 1),x)`output `(2^(2/3)*log(x - 2^(2/3)/2))/6 + (2^(2/3)*log(x - (2^(2/3)*(3^(1/2)*1i - 1))/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i + 1))/4)*(3^(1/2)*1i + 1))/12`

### 3.34 $\int \frac{1}{-2+x^3} dx$

|        |                                                     |     |
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| 3.34.8 | Giac [A] (verification not implemented) . . . . .   | 278 |
| 3.34.9 | Mupad [B] (verification not implemented) . . . . .  | 278 |

#### 3.34.1 Optimal result

Integrand size = 7, antiderivative size = 74

$$\int \frac{1}{-2+x^3} dx = -\frac{\arctan\left(\frac{1+2^{2/3}x}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(\sqrt[3]{2}-x\right)}{3 \cdot 2^{2/3}} - \frac{\log\left(2^{2/3} + \sqrt[3]{2}x + x^2\right)}{6 \cdot 2^{2/3}}$$

output `1/6*ln(2^(1/3)-x)*2^(1/3)-1/12*ln(2^(2/3)+2^(1/3)*x+x^2)*2^(1/3)-1/6*arctan(1/3*(1+2^(2/3)*x)*3^(1/2))*2^(1/3)*3^(1/2)`

#### 3.34.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{1}{-2+x^3} dx = -\frac{2\sqrt{3} \arctan\left(\frac{1+2^{2/3}x}{\sqrt{3}}\right) - 2 \log(2 - 2^{2/3}x) + \log(2 + 2^{2/3}x + \sqrt[3]{2}x^2)}{6 \cdot 2^{2/3}}$$

input `Integrate[(-2 + x^3)^(-1),x]`

output `-1/6*(2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*x)/Sqrt[3]] - 2*Log[2 - 2^(2/3)*x] + Log[2 + 2^(2/3)*x + 2^(1/3)*x^2])/2^(2/3)`

**3.34.3 Rubi [A] (verified)**

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$ , Rules used = {750, 16, 25, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{x^3 - 2} dx \\
 & \quad \downarrow \text{750} \\
 & \frac{\int -\frac{x+2\sqrt[3]{2}}{x^2+\sqrt[3]{2}x+2^{2/3}} dx}{3 \cdot 2^{2/3}} + \frac{\int \frac{1}{x-\sqrt[3]{2}} dx}{3 \cdot 2^{2/3}} \\
 & \quad \downarrow \text{16} \\
 & \frac{\int -\frac{x+2\sqrt[3]{2}}{x^2+\sqrt[3]{2}x+2^{2/3}} dx}{3 \cdot 2^{2/3}} + \frac{\log(\sqrt[3]{2}-x)}{3 \cdot 2^{2/3}} \\
 & \quad \downarrow \text{25} \\
 & \frac{\log(\sqrt[3]{2}-x)}{3 \cdot 2^{2/3}} - \frac{\int \frac{x+2\sqrt[3]{2}}{x^2+\sqrt[3]{2}x+2^{2/3}} dx}{3 \cdot 2^{2/3}} \\
 & \quad \downarrow \text{1142} \\
 & \frac{\log(\sqrt[3]{2}-x)}{3 \cdot 2^{2/3}} - \frac{3 \int \frac{1}{x^2+\sqrt[3]{2}x+2^{2/3}} dx}{2^{2/3}} + \frac{1}{2} \int \frac{\sqrt[3]{2}(2^{2/3}x+1)}{x^2+\sqrt[3]{2}x+2^{2/3}} dx}{3 \cdot 2^{2/3}} \\
 & \quad \downarrow \text{27} \\
 & \frac{\log(\sqrt[3]{2}-x)}{3 \cdot 2^{2/3}} - \frac{3 \int \frac{1}{x^2+\sqrt[3]{2}x+2^{2/3}} dx}{2^{2/3}} + \frac{\int \frac{2^{2/3}x+1}{x^2+\sqrt[3]{2}x+2^{2/3}} dx}{3 \cdot 2^{2/3}} \\
 & \quad \downarrow \text{1082} \\
 & \frac{\log(\sqrt[3]{2}-x)}{3 \cdot 2^{2/3}} - \frac{\int \frac{2^{2/3}x+1}{x^2+\sqrt[3]{2}x+2^{2/3}} dx}{2^{2/3}} - 3 \int \frac{1}{-(2^{2/3}x+1)^2-3} d(2^{2/3}x+1)}{3 \cdot 2^{2/3}} \\
 & \quad \downarrow \text{217}
 \end{aligned}$$

$$\frac{\log\left(\sqrt[3]{2}-x\right)}{3 \cdot 2^{2/3}} - \frac{\int \frac{2^{2/3}x+1}{x^2+\sqrt[3]{2}x+2^{2/3}} dx}{2^{2/3}} + \sqrt{3} \arctan\left(\frac{2^{2/3}x+1}{\sqrt{3}}\right)}{3 \cdot 2^{2/3}}$$

↓ 1103

$$\frac{\log\left(\sqrt[3]{2}-x\right)}{3 \cdot 2^{2/3}} - \frac{\sqrt{3} \arctan\left(\frac{2^{2/3}x+1}{\sqrt{3}}\right) + \frac{1}{2} \log\left(x^2 + \sqrt[3]{2}x + 2^{2/3}\right)}{3 \cdot 2^{2/3}}$$

input `Int[(-2 + x^3)^(-1), x]`

output `Log[2^(1/3) - x]/(3*2^(2/3)) - (Sqrt[3]*ArcTan[(1 + 2^(2/3)*x)/Sqrt[3]] + Log[2^(2/3) + 2^(1/3)*x + x^2]/2)/(3*2^(2/3))`

### 3.34.3.1 Defintions of rubi rules used

rule 16 `Int[(c_)/((a_) + (b_)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 217 `Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

rule 750 `Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Simp[1/(3*Rt[a, 3]^2) Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2) Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]`

rule 1082 `Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Simp[-2/b Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x]`

rule 1103 `Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

rule 1142 `Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(2*c*d - b*e)/(2*c) Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c) Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]`

### 3.34.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.30

| method  | result                                                                                                                                                                                                                                                                                                                                                                             | size |
|---------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch   | $\frac{\left( \sum_{-R=\text{RootOf}(\_Z^3-2)} \frac{\ln(x-R)}{-R^2} \right)}{3}$                                                                                                                                                                                                                                                                                                  | 22   |
| default | $\frac{2^{\frac{1}{3}} \ln(x-2^{\frac{1}{3}})}{6} - \frac{\ln(2^{\frac{2}{3}}+2^{\frac{1}{3}}x+x^2)2^{\frac{1}{3}}}{12} - \frac{\arctan\left(\frac{(1+2^{\frac{2}{3}}x)\sqrt{3}}{3}\right)2^{\frac{1}{3}}\sqrt{3}}{6}$                                                                                                                                                             | 54   |
| meijerg | $\frac{2^{\frac{1}{3}}x \left( \ln\left(1 - \frac{2^{\frac{2}{3}}(x^3)^{\frac{1}{3}}}{2}\right) - \frac{\ln\left(1 + \frac{2^{\frac{2}{3}}(x^3)^{\frac{1}{3}}}{2} + \frac{2^{\frac{1}{3}}(x^3)^{\frac{2}{3}}}{2}\right)}{2} - \sqrt{3} \arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(x^3)^{\frac{1}{3}}}{4+2^{\frac{2}{3}}(x^3)^{\frac{1}{3}}}\right) \right)}{6(x^3)^{\frac{1}{3}}}$ | 84   |

input `int(1/(x^3-2),x,method=_RETURNVERBOSE)`

output `1/3*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3-2))`

**3.34.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{1}{-2+x^3} dx = -\frac{1}{6} \cdot 4^{\frac{1}{6}} \sqrt{3} \arctan \left( \frac{1}{6} \cdot 4^{\frac{1}{6}} \left( 4^{\frac{2}{3}} \sqrt{3} x + 4^{\frac{1}{3}} \sqrt{3} \right) \right) - \frac{1}{24} \cdot 4^{\frac{2}{3}} \log \left( 2x^2 + 4^{\frac{2}{3}} x + 2 \cdot 4^{\frac{1}{3}} \right) + \frac{1}{12} \cdot 4^{\frac{2}{3}} \log \left( 2x - 4^{\frac{2}{3}} \right)$$

input `integrate(1/(x^3-2),x, algorithm="fricas")`output `-1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*x + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*log(2*x^2 + 4^(2/3)*x + 2*4^(1/3)) + 1/12*4^(2/3)*log(2*x - 4^(2/3))`**3.34.6 Sympy [A] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{1}{-2+x^3} dx = \frac{\sqrt[3]{2} \log(x - \sqrt[3]{2})}{6} - \frac{\sqrt[3]{2} \log(x^2 + \sqrt[3]{2}x + 2^{\frac{2}{3}})}{12} - \frac{\sqrt[3]{2} \sqrt{3} \operatorname{atan}\left(\frac{2^{\frac{2}{3}} \sqrt{3} x}{3} + \frac{\sqrt{3}}{3}\right)}{6}$$

input `integrate(1/(x**3-2),x)`output `2**(1/3)*log(x - 2**(1/3))/6 - 2**(1/3)*log(x**2 + 2**(1/3)*x + 2**(2/3))/12 - 2**(1/3)*sqrt(3)*atan(2**(2/3)*sqrt(3)*x/3 + sqrt(3)/3)/6`**3.34.7 Maxima [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int \frac{1}{-2+x^3} dx = -\frac{1}{6} \sqrt{3} 2^{\frac{1}{3}} \arctan \left( \frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} \left( 2x + 2^{\frac{1}{3}} \right) \right) - \frac{1}{12} \cdot 2^{\frac{1}{3}} \log \left( x^2 + 2^{\frac{1}{3}} x + 2^{\frac{2}{3}} \right) + \frac{1}{6} \cdot 2^{\frac{1}{3}} \log \left( x - 2^{\frac{1}{3}} \right)$$

input `integrate(1/(x^3-2),x, algorithm="maxima")`

output `-1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2*x + 2^(1/3))) - 1/12*2^(1/3)*log(x^2 + 2^(1/3)*x + 2^(2/3)) + 1/6*2^(1/3)*log(x - 2^(1/3))`

### 3.34.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{1}{-2+x^3} dx = -\frac{1}{6} \sqrt{3} 2^{\frac{1}{3}} \arctan\left(\frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} (2x + 2^{\frac{1}{3}})\right) - \frac{1}{12} \cdot 2^{\frac{1}{3}} \log\left(x^2 + 2^{\frac{1}{3}}x + 2^{\frac{2}{3}}\right) + \frac{1}{6} \cdot 2^{\frac{1}{3}} \log\left|x - 2^{\frac{1}{3}}\right|$$

input `integrate(1/(x^3-2),x, algorithm="giac")`

output `-1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2*x + 2^(1/3))) - 1/12*2^(1/3)*log(x^2 + 2^(1/3)*x + 2^(2/3)) + 1/6*2^(1/3)*log(abs(x - 2^(1/3)))`

### 3.34.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{1}{-2+x^3} dx = \frac{2^{1/3} \ln(x - 2^{1/3})}{6} + \frac{2^{1/3} \ln\left(x - \frac{2^{1/3}(-1+\sqrt{3}i)}{2}\right) (-1 + \sqrt{3}i)}{12} - \frac{2^{1/3} \ln\left(x + \frac{2^{1/3}(1+\sqrt{3}i)}{2}\right) (1 + \sqrt{3}i)}{12}$$

input `int(1/(x^3 - 2),x)`

output `(2^(1/3)*log(x - 2^(1/3)))/6 + (2^(1/3)*log(x - (2^(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 - (2^(1/3)*log(x + (2^(1/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/12`

### 3.35 $\int \frac{1}{-b+ax^3} dx$

|        |                                                     |     |
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#### 3.35.1 Optimal result

Integrand size = 11, antiderivative size = 115

$$\int \frac{1}{-b+ax^3} dx = -\frac{\arctan\left(\frac{\sqrt[3]{b+2\sqrt[3]{ax}}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{ab^{2/3}}} + \frac{\log\left(\sqrt[3]{b}-\sqrt[3]{ax}\right)}{3\sqrt[3]{ab^{2/3}}} - \frac{\log\left(b^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+a^{2/3}x^2\right)}{6\sqrt[3]{ab^{2/3}}}$$

```
output 1/3*ln(b^(1/3)-a^(1/3)*x)/a^(1/3)/b^(2/3)-1/6*ln(b^(2/3)+a^(1/3)*b^(1/3)*x
+a^(2/3)*x^2)/a^(1/3)/b^(2/3)-1/3*arctan(1/3*(b^(1/3)+2*a^(1/3)*x)/b^(1/3)
*3^(1/2))/a^(1/3)/b^(2/3)*3^(1/2)
```

#### 3.35.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{1}{-b+ax^3} dx = \frac{2\sqrt{3} \arctan\left(\frac{1+\frac{2\sqrt[3]{ax}}{\sqrt[3]{b}}}{\sqrt{3}}\right) - 2 \log\left(\sqrt[3]{b}-\sqrt[3]{ax}\right) + \log\left(b^{2/3}+\sqrt[3]{a}\sqrt[3]{bx}+a^{2/3}x^2\right)}{6\sqrt[3]{ab^{2/3}}}$$

```
input Integrate[(-b + a*x^3)^(-1),x]
```



output  $-1/6*(2*\text{Sqrt}[3]*\text{ArcTan}[(1 + (2*a^{(1/3)}*x)/b^{(1/3)})/\text{Sqrt}[3]] - 2*\text{Log}[b^{(1/3)} - a^{(1/3)}*x] + \text{Log}[b^{(2/3)} + a^{(1/3)}*b^{(1/3)}*x + a^{(2/3)}*x^2])/(a^{(1/3)}*b^{(2/3)})$

### 3.35.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$ , Rules used = {750, 16, 25, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{ax^3 - b} dx \\
 & \quad \downarrow 750 \\
 & \frac{\int -\frac{\sqrt[3]{ax+2}\sqrt[3]{b}}{a^{2/3}x^2 + \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}} dx}{3b^{2/3}} + \frac{\int \frac{1}{\sqrt[3]{ax} - \sqrt[3]{b}} dx}{3b^{2/3}} \\
 & \quad \downarrow 16 \\
 & \frac{\int -\frac{\sqrt[3]{ax+2}\sqrt[3]{b}}{a^{2/3}x^2 + \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}} dx}{3b^{2/3}} + \frac{\log(\sqrt[3]{b} - \sqrt[3]{ax})}{3\sqrt[3]{ab^{2/3}}} \\
 & \quad \downarrow 25 \\
 & \frac{\log(\sqrt[3]{b} - \sqrt[3]{ax})}{3\sqrt[3]{ab^{2/3}}} - \frac{\int \frac{\sqrt[3]{ax+2}\sqrt[3]{b}}{a^{2/3}x^2 + \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}} dx}{3b^{2/3}} \\
 & \quad \downarrow 1142 \\
 & \frac{\log(\sqrt[3]{b} - \sqrt[3]{ax})}{3\sqrt[3]{ab^{2/3}}} - \frac{\frac{3}{2}\sqrt[3]{b} \int \frac{1}{a^{2/3}x^2 + \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}} dx + \frac{\int \frac{\sqrt[3]{a}(2\sqrt[3]{ax} + \sqrt[3]{b})}{a^{2/3}x^2 + \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}} dx}{2\sqrt[3]{a}}}{3b^{2/3}} \\
 & \quad \downarrow 27 \\
 & \frac{\log(\sqrt[3]{b} - \sqrt[3]{ax})}{3\sqrt[3]{ab^{2/3}}} - \frac{\frac{3}{2}\sqrt[3]{b} \int \frac{1}{a^{2/3}x^2 + \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}} dx + \frac{1}{2} \int \frac{2\sqrt[3]{ax} + \sqrt[3]{b}}{a^{2/3}x^2 + \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}} dx}{3b^{2/3}}
 \end{aligned}$$

$$\begin{aligned}
 & \downarrow 1082 \\
 & \frac{\log\left(\sqrt[3]{b} - \sqrt[3]{ax}\right)}{3\sqrt[3]{ab^{2/3}}} - \frac{\frac{1}{2} \int \frac{2\sqrt[3]{ax} + \sqrt[3]{b}}{a^{2/3}x^2 + \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}} dx}{3b^{2/3}} - \frac{3 \int \frac{1}{\left(\frac{2\sqrt[3]{ax}}{\sqrt[3]{b}} + 1\right)^2} d\left(\frac{2\sqrt[3]{ax}}{\sqrt[3]{b}} + 1\right)}{\sqrt[3]{a}} \\
 & \downarrow 217 \\
 & \frac{\log\left(\sqrt[3]{b} - \sqrt[3]{ax}\right)}{3\sqrt[3]{ab^{2/3}}} - \frac{\frac{1}{2} \int \frac{2\sqrt[3]{ax} + \sqrt[3]{b}}{a^{2/3}x^2 + \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}} dx}{3b^{2/3}} + \frac{\sqrt{3} \arctan\left(\frac{\frac{2\sqrt[3]{ax}}{\sqrt[3]{b}} + 1}{\sqrt{3}}\right)}{\sqrt[3]{a}} \\
 & \downarrow 1103 \\
 & \frac{\log\left(\sqrt[3]{b} - \sqrt[3]{ax}\right)}{3\sqrt[3]{ab^{2/3}}} - \frac{\frac{\log\left(a^{2/3}x^2 + \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}\right)}{2\sqrt[3]{a}}}{3b^{2/3}} + \frac{\sqrt{3} \arctan\left(\frac{\frac{2\sqrt[3]{ax}}{\sqrt[3]{b}} + 1}{\sqrt{3}}\right)}{\sqrt[3]{a}}
 \end{aligned}$$

input `Int[(-b + a*x^3)^(-1),x]`

output `Log[b^(1/3) - a^(1/3)*x]/(3*a^(1/3)*b^(2/3)) - ((Sqrt[3]*ArcTan[(1 + (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]])/a^(1/3) + Log[b^(2/3) + a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/(2*a^(1/3)))/(3*b^(2/3))`

### 3.35.3.1 Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])`

rule 750 `Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Simp[1/(3*Rt[a, 3]^2) Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2) Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]`

rule 1082 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Simp[-2/b Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x]`

rule 1103 `Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x])/b], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

rule 1142 `Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(2*c*d - b*e)/(2*c) Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c) Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]`

### 3.35.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.25

| method  | result                                                                                                                                                                                                                                                                                                                                                                                                                           | size |
|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch   | $\frac{\sum_{-R=\text{RootOf}(a-Z^3-b)} \frac{\ln(x-\frac{R}{a})}{-R^2}}{3a}$                                                                                                                                                                                                                                                                                                                                                    | 29   |
| default | $\frac{\ln\left(x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{\ln\left(x^2+\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}+1\right)}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$ | 92   |

3.35.  $\int \frac{1}{-b+ax^3} dx$

input `int(1/(a*x^3-b),x,method=_RETURNVERBOSE)`

output `1/3/a*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*a-b))`

### 3.35.5 Fracas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.61

$$\int \frac{1}{-b + ax^3} dx$$

$$= \frac{3 \sqrt{\frac{1}{3}} ab \sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}} \log \left( \frac{2 abx^3 - 3 (ab^2)^{\frac{1}{3}} bx + b^2 - 3 \sqrt{\frac{1}{3}} \left( 2 abx^2 - (ab^2)^{\frac{2}{3}} x - (ab^2)^{\frac{1}{3}} b \right) \sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}}{ax^3 - b} \right) - (ab^2)^{\frac{2}{3}} \log \left( abx^2 + (ab^2)^{\frac{2}{3}} x + (ab^2)^{\frac{1}{3}} b \right)}{6 ab^2}$$

$$+ \frac{6 \sqrt{\frac{1}{3}} ab \sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}} \arctan \left( \frac{\sqrt{\frac{1}{3}} \left( 2 (ab^2)^{\frac{2}{3}} x + (ab^2)^{\frac{1}{3}} b \right) \sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}}{b^2} \right) + (ab^2)^{\frac{2}{3}} \log \left( abx^2 + (ab^2)^{\frac{2}{3}} x + (ab^2)^{\frac{1}{3}} b \right) - 2 (ab^2)^{\frac{2}{3}} \log \left( abx^2 + (ab^2)^{\frac{2}{3}} x + (ab^2)^{\frac{1}{3}} b \right)}{6 ab^2}$$

input `integrate(1/(a*x^3-b),x, algorithm="fricas")`

output `[1/6*(3*sqrt(1/3)*a*b*sqrt(-(a*b^2)^(1/3)/a)*log((2*a*b*x^3 - 3*(a*b^2)^(1/3)*b*x + b^2 - 3*sqrt(1/3)*(2*a*b*x^2 - (a*b^2)^(2/3)*x - (a*b^2)^(1/3)*b)*sqrt(-(a*b^2)^(1/3)/a))/(a*x^3 - b)) - (a*b^2)^(2/3)*log(a*b*x^2 + (a*b^2)^(2/3)*x + (a*b^2)^(1/3)*b) + 2*(a*b^2)^(2/3)*log(a*b*x - (a*b^2)^(2/3))/(a*b^2), -1/6*(6*sqrt(1/3)*a*b*sqrt((a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*(a*b^2)^(2/3)*x + (a*b^2)^(1/3)*b)*sqrt((a*b^2)^(1/3)/a)/b^2) + (a*b^2)^(2/3)*log(a*b*x^2 + (a*b^2)^(2/3)*x + (a*b^2)^(1/3)*b) - 2*(a*b^2)^(2/3)*log(a*b*x - (a*b^2)^(2/3))/(a*b^2)]`

**3.35.6 Sympy [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.17

$$\int \frac{1}{-b + ax^3} dx = \text{RootSum} (27t^3 ab^2 - 1, (t \mapsto t \log(-3tb + x)))$$

input `integrate(1/(a*x**3-b),x)`output `RootSum(27*_t**3*a*b**2 - 1, Lambda(_t, _t*log(-3*_t*b + x)))`**3.35.7 Maxima [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

$$\int \frac{1}{-b + ax^3} dx$$

$$= -\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2 + x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{\log\left(x - \left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

input `integrate(1/(a*x^3-b),x, algorithm="maxima")`output `-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (b/a)^(1/3))/(b/a)^(1/3))/(a*(b/a)^(2/3)) - 1/6*log(x^2 + x*(b/a)^(1/3) + (b/a)^(2/3))/(a*(b/a)^(2/3)) + 1/3*log(x - (b/a)^(1/3))/(a*(b/a)^(2/3))`**3.35.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{1}{-b + ax^3} dx = \frac{\left(\frac{b}{a}\right)^{\frac{1}{3}} \log\left(\left|x - \left(\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3b} - \frac{\sqrt{3}(a^2b)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3ab}$$

$$- \frac{(a^2b)^{\frac{1}{3}} \log\left(x^2 + x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6ab}$$

input `integrate(1/(a*x^3-b),x, algorithm="giac")`

output  $\frac{1}{3} \cdot \left(\frac{b}{a}\right)^{1/3} \cdot \log\left(\frac{\text{abs}\left(x - \left(\frac{b}{a}\right)^{1/3}\right)}{b} - \frac{1}{3} \cdot \sqrt{3} \cdot \left(a^2 b\right)^{1/3} \cdot \arctan\left(\frac{1}{3} \cdot \sqrt{3} \cdot \left(2x + \left(\frac{b}{a}\right)^{1/3}\right) / \left(\frac{b}{a}\right)^{1/3}\right) / (a \cdot b) - \frac{1}{6} \cdot \left(a^2 b\right)^{1/3} \cdot \log\left(x^2 + x \cdot \left(\frac{b}{a}\right)^{1/3} + \left(\frac{b}{a}\right)^{2/3}\right) / (a \cdot b)}\right)$

### 3.35.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.88

$$\int \frac{1}{-b + ax^3} dx = \frac{\ln\left(a^{1/3}x - b^{1/3}\right)}{3a^{1/3}b^{2/3}} + \frac{\ln\left(3a^2x - \frac{3a^{5/3}b^{1/3}(-1+\sqrt{3}i)}{2}\right)(-1+\sqrt{3}i)}{6a^{1/3}b^{2/3}} - \frac{\ln\left(3a^2x + \frac{3a^{5/3}b^{1/3}(1+\sqrt{3}i)}{2}\right)(1+\sqrt{3}i)}{6a^{1/3}b^{2/3}}$$

input `int(-1/(b - a*x^3),x)`

output  $\log\left(\frac{a^{1/3}x - b^{1/3}}{3a^{1/3}b^{2/3}}\right) + \frac{\log\left(3a^2x - \frac{3a^{5/3}b^{1/3}(3^{1/2}i - 1)}{2}\right) \cdot (3^{1/2}i - 1)}{6a^{1/3}b^{2/3}} - \frac{\log\left(3a^2x + \frac{3a^{5/3}b^{1/3}(3^{1/2}i + 1)}{2}\right) \cdot (3^{1/2}i + 1)}{6a^{1/3}b^{2/3}}$

### 3.36 $\int \frac{1}{-2+x^4} dx$

|        |                                                     |     |
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| 3.36.9 | Mupad [B] (verification not implemented) . . . . .  | 290 |

#### 3.36.1 Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \frac{1}{-2+x^4} dx = -\frac{\arctan\left(\frac{x}{\sqrt[4]{2}}\right)}{2 \cdot 2^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}}\right)}{2 \cdot 2^{3/4}}$$

output `-1/4*arctan(1/2*x*2^(3/4))*2^(1/4)-1/4*arctanh(1/2*x*2^(3/4))*2^(1/4)`

#### 3.36.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{-2+x^4} dx = -\frac{2 \arctan\left(\frac{x}{\sqrt[4]{2}}\right) - \log(2 - 2^{3/4}x) + \log(2 + 2^{3/4}x)}{4 \cdot 2^{3/4}}$$

input `Integrate[(-2 + x^4)^(-1),x]`

output `-1/4*(2*ArcTan[x/2^(1/4)] - Log[2 - 2^(3/4)*x] + Log[2 + 2^(3/4)*x])/2^(3/4)`

### 3.36.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{x^4 - 2} dx \\
 & \quad \downarrow \text{756} \\
 & -\frac{\int \frac{1}{\sqrt{2}-x^2} dx}{2\sqrt{2}} - \frac{\int \frac{1}{x^2+\sqrt{2}} dx}{2\sqrt{2}} \\
 & \quad \downarrow \text{216} \\
 & -\frac{\int \frac{1}{\sqrt{2}-x^2} dx}{2\sqrt{2}} - \frac{\arctan\left(\frac{x}{\sqrt[4]{2}}\right)}{2 \cdot 2^{3/4}} \\
 & \quad \downarrow \text{219} \\
 & -\frac{\arctan\left(\frac{x}{\sqrt[4]{2}}\right)}{2 \cdot 2^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}}\right)}{2 \cdot 2^{3/4}}
 \end{aligned}$$

input `Int[(-2 + x^4)^(-1), x]`

output `-1/2*ArcTan[x/2^(1/4)]/2^(3/4) - ArcTanh[x/2^(1/4)]/(2*2^(3/4))`

#### 3.36.3.1 Defintions of rubi rules used

rule 216 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`



```
rule 756 Int[((a_) + (b_)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a) Int[1/(r - s*x^2), x], x]
+ Simp[r/(2*a) Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a
/b, 0]
```

### 3.36.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

| method  | result                                                                                                                                                                                                                                                                   | size |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch   | $\frac{\sum_{R=\text{RootOf}(\_Z^4-2)} \frac{\ln(x-R)}{-R^3}}{4}$                                                                                                                                                                                                        | 22   |
| default | $-\frac{2^{\frac{1}{4}} \left( \ln\left(\frac{x+2^{\frac{1}{4}}}{x-2^{\frac{1}{4}}}\right) + 2 \arctan\left(\frac{x 2^{\frac{3}{4}}}{2}\right) \right)}{8}$                                                                                                              | 32   |
| meijerg | $\frac{2^{\frac{1}{4}} x \left( \ln\left(1 - \frac{2^{\frac{3}{4}}(x^4)^{\frac{1}{4}}}{2}\right) - \ln\left(1 + \frac{2^{\frac{3}{4}}(x^4)^{\frac{1}{4}}}{2}\right) - 2 \arctan\left(\frac{2^{\frac{3}{4}}(x^4)^{\frac{1}{4}}}{2}\right) \right)}{8(x^4)^{\frac{1}{4}}}$ | 54   |

```
input int(1/(x^4-2),x,method=_RETURNVERBOSE)
```

```
output 1/4*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4-2))
```

### 3.36.5 Fracas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{1}{-2+x^4} dx = -\frac{1}{32} \cdot 8^{\frac{3}{4}} \log\left(4x + 8^{\frac{3}{4}}\right) - \frac{1}{32} i \cdot 8^{\frac{3}{4}} \log\left(4x + i \cdot 8^{\frac{3}{4}}\right) \\ + \frac{1}{32} i \cdot 8^{\frac{3}{4}} \log\left(4x - i \cdot 8^{\frac{3}{4}}\right) + \frac{1}{32} \cdot 8^{\frac{3}{4}} \log\left(4x - 8^{\frac{3}{4}}\right)$$

```
input integrate(1/(x^4-2),x, algorithm="fricas")
```

output  $-1/32*8^{(3/4)}*\log(4*x + 8^{(3/4)}) - 1/32*I*8^{(3/4)}*\log(4*x + I*8^{(3/4)}) + 1/32*I*8^{(3/4)}*\log(4*x - I*8^{(3/4)}) + 1/32*8^{(3/4)}*\log(4*x - 8^{(3/4)})$

### 3.36.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \frac{1}{-2+x^4} dx = \frac{\sqrt[4]{2} \log(x - \sqrt[4]{2})}{8} - \frac{\sqrt[4]{2} \log(x + \sqrt[4]{2})}{8} - \frac{\sqrt[4]{2} \operatorname{atan}\left(\frac{2^{\frac{3}{4}}x}{2}\right)}{4}$$

input `integrate(1/(x**4-2),x)`

output  $2^{(1/4)}*\log(x - 2^{(1/4)})/8 - 2^{(1/4)}*\log(x + 2^{(1/4)})/8 - 2^{(1/4)}*\operatorname{atan}(2^{(3/4)}*x/2)/4$

### 3.36.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{1}{-2+x^4} dx = -\frac{1}{4} \cdot 2^{\frac{1}{4}} \arctan\left(\frac{1}{2} \cdot 2^{\frac{3}{4}}x\right) + \frac{1}{8} \cdot 2^{\frac{1}{4}} \log\left(\frac{x - 2^{\frac{1}{4}}}{x + 2^{\frac{1}{4}}}\right)$$

input `integrate(1/(x^4-2),x, algorithm="maxima")`

output  $-1/4*2^{(1/4)}*\arctan(1/2*2^{(3/4)}*x) + 1/8*2^{(1/4)}*\log((x - 2^{(1/4)})/(x + 2^{(1/4)}))$

### 3.36.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{1}{-2+x^4} dx = -\frac{1}{4} \cdot 2^{\frac{1}{4}} \arctan\left(\frac{1}{2} \cdot 2^{\frac{3}{4}}x\right) - \frac{1}{8} \cdot 2^{\frac{1}{4}} \log\left(\left|x + 2^{\frac{1}{4}}\right|\right) + \frac{1}{8} \cdot 2^{\frac{1}{4}} \log\left(\left|x - 2^{\frac{1}{4}}\right|\right)$$

input `integrate(1/(x^4-2),x, algorithm="giac")`

output 
$$-1/4*2^{1/4}*\arctan(1/2*2^{3/4}*x) - 1/8*2^{1/4}*\log(\text{abs}(x + 2^{1/4})) + 1/8*2^{1/4}*\log(\text{abs}(x - 2^{1/4}))$$

### 3.36.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.57

$$\int \frac{1}{-2 + x^4} dx = -\frac{2^{1/4} \left( \operatorname{atan}\left(\frac{2^{3/4}x}{2}\right) + \operatorname{atanh}\left(\frac{2^{3/4}x}{2}\right) \right)}{4}$$

input `int(1/(x^4 - 2),x)`

output 
$$-(2^{1/4}*(\operatorname{atan}((2^{3/4}*x)/2) + \operatorname{atanh}((2^{3/4}*x)/2)))/4$$

### 3.37 $\int \frac{1}{-1+5x^4} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.37.1 | Optimal result . . . . .                            | 291 |
| 3.37.2 | Mathematica [A] (verified) . . . . .                | 291 |
| 3.37.3 | Rubi [A] (verified) . . . . .                       | 292 |
| 3.37.4 | Maple [C] (verified) . . . . .                      | 293 |
| 3.37.5 | Fricas [C] (verification not implemented) . . . . . | 293 |
| 3.37.6 | Sympy [A] (verification not implemented) . . . . .  | 294 |
| 3.37.7 | Maxima [A] (verification not implemented) . . . . . | 294 |
| 3.37.8 | Giac [A] (verification not implemented) . . . . .   | 294 |
| 3.37.9 | Mupad [B] (verification not implemented) . . . . .  | 295 |

#### 3.37.1 Optimal result

Integrand size = 9, antiderivative size = 35

$$\int \frac{1}{-1+5x^4} dx = -\frac{\arctan\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}} - \frac{\operatorname{arctanh}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}$$

output `-1/10*arctan(5^(1/4)*x)*5^(3/4)-1/10*arctanh(5^(1/4)*x)*5^(3/4)`

#### 3.37.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{-1+5x^4} dx = -\frac{2 \arctan\left(\sqrt[4]{5}x\right) - \log\left(1 - \sqrt[4]{5}x\right) + \log\left(1 + \sqrt[4]{5}x\right)}{4\sqrt[4]{5}}$$

input `Integrate[(-1 + 5*x^4)^(-1), x]`

output `-1/4*(2*ArcTan[5^(1/4)*x] - Log[1 - 5^(1/4)*x] + Log[1 + 5^(1/4)*x])/5^(1/4)`

### 3.37.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{5x^4 - 1} dx \\
 & \quad \downarrow \text{756} \\
 & -\frac{1}{2} \int \frac{1}{1 - \sqrt{5}x^2} dx - \frac{1}{2} \int \frac{1}{\sqrt{5}x^2 + 1} dx \\
 & \quad \downarrow \text{216} \\
 & -\frac{1}{2} \int \frac{1}{1 - \sqrt{5}x^2} dx - \frac{\arctan\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}} \\
 & \quad \downarrow \text{219} \\
 & -\frac{\arctan\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}} - \frac{\operatorname{arctanh}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}
 \end{aligned}$$

input `Int[(-1 + 5*x^4)^(-1), x]`

output `-1/2*ArcTan[5^(1/4)*x]/5^(1/4) - ArcTanh[5^(1/4)*x]/(2*5^(1/4))`

#### 3.37.3.1 Defintions of rubi rules used

rule 216 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

```
rule 756 Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a) Int[1/(r - s*x^2), x], x]
+ Simp[r/(2*a) Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a
/b, 0]
```

### 3.37.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

| method  | result                                                                                                                                                                                                                                 | size |
|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch   | $\frac{\sum_{-R=\text{RootOf}(5Z^4-1)} \frac{\ln(x-R)}{-R^3}}{20}$                                                                                                                                                                     | 24   |
| default | $-\frac{5^{\frac{3}{4}} \left( \ln\left(\frac{x+\frac{5^{\frac{3}{4}}}{5}}{x-\frac{5^{\frac{3}{4}}}{5}}\right) + 2 \arctan\left(5^{\frac{1}{4}}x\right) \right)}{20}$                                                                  | 33   |
| meijerg | $\frac{5^{\frac{3}{4}}x \left( \ln\left(1-5^{\frac{1}{4}}(x^4)^{\frac{1}{4}}\right) - \ln\left(1+5^{\frac{1}{4}}(x^4)^{\frac{1}{4}}\right) - 2 \arctan\left(5^{\frac{1}{4}}(x^4)^{\frac{1}{4}}\right) \right)}{20(x^4)^{\frac{1}{4}}}$ | 52   |

```
input int(1/(5*x^4-1),x,method=_RETURNVERBOSE)
```

```
output 1/20*sum(1/_R^3*ln(x-_R),_R=RootOf(5*_Z^4-1))
```

### 3.37.5 Fracas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{1}{-1+5x^4} dx = -\frac{1}{20} \cdot 5^{\frac{3}{4}} \log\left(5x+5^{\frac{3}{4}}\right) - \frac{1}{20}i \cdot 5^{\frac{3}{4}} \log\left(5x+i \cdot 5^{\frac{3}{4}}\right) \\ + \frac{1}{20}i \cdot 5^{\frac{3}{4}} \log\left(5x-i \cdot 5^{\frac{3}{4}}\right) + \frac{1}{20} \cdot 5^{\frac{3}{4}} \log\left(5x-5^{\frac{3}{4}}\right)$$

```
input integrate(1/(5*x^4-1),x, algorithm="fracas")
```

output  $-1/20*5^{(3/4)}*\log(5*x + 5^{(3/4)}) - 1/20*I*5^{(3/4)}*\log(5*x + I*5^{(3/4)}) + 1/20*I*5^{(3/4)}*\log(5*x - I*5^{(3/4)}) + 1/20*5^{(3/4)}*\log(5*x - 5^{(3/4)})$

### 3.37.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{1}{-1 + 5x^4} dx = \frac{5^{\frac{3}{4}} \log\left(x - \frac{5^{\frac{3}{4}}}{5}\right)}{20} - \frac{5^{\frac{3}{4}} \log\left(x + \frac{5^{\frac{3}{4}}}{5}\right)}{20} - \frac{5^{\frac{3}{4}} \operatorname{atan}\left(\sqrt[4]{5}x\right)}{10}$$

input `integrate(1/(5*x**4-1),x)`

output  $5^{(3/4)}*\log(x - 5^{(3/4)}/5)/20 - 5^{(3/4)}*\log(x + 5^{(3/4)}/5)/20 - 5^{(3/4)}*\operatorname{atan}(5^{(1/4)}*x)/10$

### 3.37.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{1}{-1 + 5x^4} dx = -\frac{1}{10} \cdot 5^{\frac{3}{4}} \arctan\left(5^{\frac{1}{4}}x\right) + \frac{1}{20} \cdot 5^{\frac{3}{4}} \log\left(\frac{\sqrt{5}x - 5^{\frac{1}{4}}}{\sqrt{5}x + 5^{\frac{1}{4}}}\right)$$

input `integrate(1/(5*x^4-1),x, algorithm="maxima")`

output  $-1/10*5^{(3/4)}*\arctan(5^{(1/4)}*x) + 1/20*5^{(3/4)}*\log((\operatorname{sqrt}(5)*x - 5^{(1/4)})/(\operatorname{sqrt}(5)*x + 5^{(1/4)}))$

### 3.37.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{1}{-1 + 5x^4} dx = -\frac{1}{10} \cdot 5^{\frac{3}{4}} \arctan\left(5 \left(\frac{1}{5}\right)^{\frac{3}{4}} x\right) - \frac{1}{20} \cdot 5^{\frac{3}{4}} \log\left(\left|x + \left(\frac{1}{5}\right)^{\frac{1}{4}}\right|\right) + \frac{1}{20} \cdot 5^{\frac{3}{4}} \log\left(\left|x - \left(\frac{1}{5}\right)^{\frac{1}{4}}\right|\right)$$

input `integrate(1/(5*x^4-1),x, algorithm="giac")`

output `-1/10*5^(3/4)*arctan(5*(1/5)^(3/4)*x) - 1/20*5^(3/4)*log(abs(x + (1/5)^(1/4))) + 1/20*5^(3/4)*log(abs(x - (1/5)^(1/4)))`

### 3.37.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

$$\int \frac{1}{-1+5x^4} dx = -\frac{5^{3/4} (\operatorname{atan}(5^{1/4}x) + \operatorname{atanh}(5^{1/4}x))}{10}$$

input `int(1/(5*x^4 - 1),x)`

output `-(5^(3/4)*(atan(5^(1/4)*x) + atanh(5^(1/4)*x)))/10`



### 3.38 $\int \frac{1}{7+3x^4} dx$

|        |                                                     |     |
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#### 3.38.1 Optimal result

Integrand size = 9, antiderivative size = 171

$$\int \frac{1}{7+3x^4} dx = -\frac{\arctan\left(1 - \sqrt[4]{\frac{3}{7}}\sqrt{2x}\right)}{2\sqrt{2}\sqrt[4]{3}7^{3/4}} + \frac{\arctan\left(1 + \sqrt[4]{\frac{3}{7}}\sqrt{2x}\right)}{2\sqrt{2}\sqrt[4]{3}7^{3/4}} - \frac{\log\left(\sqrt{21} - \sqrt{2}3^{3/4}\sqrt[4]{7x+3x^2}\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}} + \frac{\log\left(\sqrt{21} + \sqrt{2}3^{3/4}\sqrt[4]{7x+3x^2}\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}}$$

```
output 1/84*arctan(-1+1/7*3^(1/4)*7^(3/4)*x*2^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)+1/84
*arctan(1+1/7*3^(1/4)*7^(3/4)*x*2^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)-1/168*ln(
3*x^2-3^(3/4)*7^(1/4)*x*2^(1/2)+21^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)+1/168*ln
(3*x^2+3^(3/4)*7^(1/4)*x*2^(1/2)+21^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)
```

#### 3.38.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.70

$$\int \frac{1}{7+3x^4} dx = \frac{-2 \arctan\left(1 - \sqrt[4]{\frac{3}{7}}\sqrt{2x}\right) + 2 \arctan\left(1 + \sqrt[4]{\frac{3}{7}}\sqrt{2x}\right) - \log\left(7 - \sqrt{2}\sqrt[4]{3}7^{3/4}x + \sqrt{21}x^2\right) + \log\left(7 + \sqrt{2}\sqrt[4]{3}7^{3/4}x + \sqrt{21}x^2\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}}$$

input `Integrate[(7 + 3*x^4)^(-1),x]`

output `(-2*ArcTan[1 - (3/7)^(1/4)*Sqrt[2]*x] + 2*ArcTan[1 + (3/7)^(1/4)*Sqrt[2]*x] - Log[7 - Sqrt[2]*3^(1/4)*7^(3/4)*x + Sqrt[21]*x^2] + Log[7 + Sqrt[2]*3^(1/4)*7^(3/4)*x + Sqrt[21]*x^2])/(4*Sqrt[2]*3^(1/4)*7^(3/4))`

### 3.38.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$ , Rules used = {755, 1476, 1082, 217, 1479, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{3x^4 + 7} dx \\
 & \quad \downarrow 755 \\
 & \frac{\int \frac{\sqrt{7}-\sqrt{3}x^2}{3x^4+7} dx}{2\sqrt{7}} + \frac{\int \frac{\sqrt{3}x^2+\sqrt{7}}{3x^4+7} dx}{2\sqrt{7}} \\
 & \quad \downarrow 1476 \\
 & \frac{\int \frac{1}{x^2-\sqrt{2}\sqrt[4]{\frac{7}{3}}x+\sqrt{\frac{7}{3}}} dx}{2\sqrt{3}} + \frac{\int \frac{1}{x^2+\sqrt{2}\sqrt[4]{\frac{7}{3}}x+\sqrt{\frac{7}{3}}} dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{7}-\sqrt{3}x^2}{3x^4+7} dx}{2\sqrt{7}} \\
 & \quad \downarrow 1082 \\
 & \frac{\int \frac{\sqrt{7}-\sqrt{3}x^2}{3x^4+7} dx}{2\sqrt{7}} + \frac{\int \frac{1}{\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)^2 - \left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)^{-1}} dx}{\sqrt{2}\sqrt[4]{21}} - \frac{\int \frac{1}{\left(\sqrt[4]{\frac{3}{7}}\sqrt{2}x+1\right)^2 - \left(\sqrt[4]{\frac{3}{7}}\sqrt{2}x+1\right)^{-1}} dx}{\sqrt{2}\sqrt[4]{21}} \\
 & \quad \downarrow 217 \\
 & \frac{\int \frac{\sqrt{7}-\sqrt{3}x^2}{3x^4+7} dx}{2\sqrt{7}} + \frac{\arctan\left(\sqrt[4]{\frac{3}{7}}\sqrt{2}x+1\right)}{\sqrt{2}\sqrt[4]{21}} - \frac{\arctan\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{\sqrt{2}\sqrt[4]{21}}
 \end{aligned}$$

---

3.38.  $\int \frac{1}{7+3x^4} dx$

$$\begin{aligned}
& \downarrow 1479 \\
& \frac{\int -\frac{\sqrt{23}^{3/4} \sqrt[4]{7} - 6x}{3x^2 - \sqrt{23}^{3/4} \sqrt[4]{7} x + \sqrt{21}} dx}{2\sqrt{2} \sqrt[4]{21}} - \frac{\int -\frac{6x + \sqrt{23}^{3/4} \sqrt[4]{7}}{3x^2 + \sqrt{23}^{3/4} \sqrt[4]{7} x + \sqrt{21}} dx}{2\sqrt{2} \sqrt[4]{21}} + \frac{\arctan\left(\sqrt[4]{\frac{3}{7}} \sqrt{2x+1}\right)}{\sqrt{2} \sqrt[4]{21}} - \frac{\arctan\left(1 - \sqrt[4]{\frac{3}{7}} \sqrt{2x}\right)}{\sqrt{2} \sqrt[4]{21}} \\
& \downarrow 25 \\
& \frac{\int -\frac{\sqrt{23}^{3/4} \sqrt[4]{7} - 6x}{3x^2 - \sqrt{23}^{3/4} \sqrt[4]{7} x + \sqrt{21}} dx}{2\sqrt{2} \sqrt[4]{21}} + \frac{\int -\frac{6x + \sqrt{23}^{3/4} \sqrt[4]{7}}{3x^2 + \sqrt{23}^{3/4} \sqrt[4]{7} x + \sqrt{21}} dx}{2\sqrt{2} \sqrt[4]{21}} + \frac{\arctan\left(\sqrt[4]{\frac{3}{7}} \sqrt{2x+1}\right)}{\sqrt{2} \sqrt[4]{21}} - \frac{\arctan\left(1 - \sqrt[4]{\frac{3}{7}} \sqrt{2x}\right)}{\sqrt{2} \sqrt[4]{21}} \\
& \downarrow 1103 \\
& \frac{\arctan\left(\sqrt[4]{\frac{3}{7}} \sqrt{2x+1}\right)}{\sqrt{2} \sqrt[4]{21}} - \frac{\arctan\left(1 - \sqrt[4]{\frac{3}{7}} \sqrt{2x}\right)}{\sqrt{2} \sqrt[4]{21}} + \frac{\log\left(3x^2 + \sqrt{23}^{3/4} \sqrt[4]{7} x + \sqrt{21}\right)}{2\sqrt{2} \sqrt[4]{21}} - \frac{\log\left(3x^2 - \sqrt{23}^{3/4} \sqrt[4]{7} x + \sqrt{21}\right)}{2\sqrt{2} \sqrt[4]{21}}
\end{aligned}$$

input `Int[(7 + 3*x^4)^(-1), x]`

output `(-(ArcTan[1 - (3/7)^(1/4)*Sqrt[2]*x]/(Sqrt[2]*21^(1/4))) + ArcTan[1 + (3/7)^(1/4)*Sqrt[2]*x]/(Sqrt[2]*21^(1/4)))/(2*Sqrt[7]) + (-1/2*Log[Sqrt[21] - Sqrt[2]*3^(3/4)*7^(1/4)*x + 3*x^2]/(Sqrt[2]*21^(1/4)) + Log[Sqrt[21] + Sqrt[2]*3^(3/4)*7^(1/4)*x + 3*x^2]/(2*Sqrt[2]*21^(1/4)))/(2*Sqrt[7])`

### 3.38.3.1 Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

```
rule 755 Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r) Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r) Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

```
rule 1082 Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

```
rule 1103 Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

```
rule 1476 Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c) Int[1/Simp[d/e + q*x + x^2, x], x] + Simp[
e/(2*c) Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
&& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

```
rule 1479 Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q) Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Simp[e/(2*c*q) Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### 3.38.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.14

| method  | result                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch   | $\frac{\sum_{-R=\text{RootOf}(3\_Z^4+7)} \frac{\ln(x-R)}{-R^3}}{12}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| default | $\frac{\sqrt{3} 21^{\frac{1}{4}} \sqrt{2} \left( \ln \left( \frac{x^2 + \sqrt{3} 21^{\frac{1}{4}} x \sqrt{2} + \sqrt{21}}{x^2 - \sqrt{3} 21^{\frac{1}{4}} x \sqrt{2} + \sqrt{21}} \right) + 2 \arctan \left( \frac{\sqrt{2} \sqrt{3} 21^{\frac{3}{4}} x + 1}{21} \right) + 2 \arctan \left( \frac{\sqrt{2} \sqrt{3} 21^{\frac{3}{4}} x - 1}{21} \right) \right)}{168}$                                                                                                                                                                                                                                                   |
| meijerg | $\frac{1029^{\frac{3}{4}} \left( -\frac{x \sqrt{2} \ln \left( 1 - \frac{\sqrt{2} 3^{\frac{1}{4}} 7^{\frac{3}{4}} (x^4)^{\frac{1}{4}}}{7} + \frac{\sqrt{3} \sqrt{7} \sqrt{x^4}}{7} \right)}{2(x^4)^{\frac{1}{4}}} + \frac{x \sqrt{2} \arctan \left( \frac{\sqrt{2} 3^{\frac{1}{4}} 7^{\frac{3}{4}} (x^4)^{\frac{1}{4}}}{14 - \sqrt{2} 3^{\frac{1}{4}} 7^{\frac{3}{4}} (x^4)^{\frac{1}{4}}} \right)}{(x^4)^{\frac{1}{4}}} + \frac{x \sqrt{2} \ln \left( 1 + \frac{\sqrt{2} 3^{\frac{1}{4}} 7^{\frac{3}{4}} (x^4)^{\frac{1}{4}}}{7} + \frac{\sqrt{3} \sqrt{7} \sqrt{x^4}}{7} \right)}{2(x^4)^{\frac{1}{4}}} \right)}{4116}$ |

input `int(1/(3*x^4+7),x,method=_RETURNVERBOSE)`

output `1/12*sum(1/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+7))`

### 3.38.5 Fracas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.50

$$\begin{aligned} \int \frac{1}{7+3x^4} dx &= \left( \frac{1}{8232}i + \frac{1}{8232} \right) \cdot 1029^{\frac{3}{4}} \sqrt{2} \log \left( (i+1) \cdot 1029^{\frac{3}{4}} \sqrt{2} + 294x \right) \\ &\quad - \left( \frac{1}{8232}i - \frac{1}{8232} \right) \cdot 1029^{\frac{3}{4}} \sqrt{2} \log \left( -(i-1) \cdot 1029^{\frac{3}{4}} \sqrt{2} + 294x \right) \\ &\quad + \left( \frac{1}{8232}i - \frac{1}{8232} \right) \cdot 1029^{\frac{3}{4}} \sqrt{2} \log \left( (i-1) \cdot 1029^{\frac{3}{4}} \sqrt{2} + 294x \right) \\ &\quad - \left( \frac{1}{8232}i + \frac{1}{8232} \right) \cdot 1029^{\frac{3}{4}} \sqrt{2} \log \left( -(i+1) \cdot 1029^{\frac{3}{4}} \sqrt{2} + 294x \right) \end{aligned}$$

input `integrate(1/(3*x^4+7),x, algorithm="fricas")`

output `(1/8232*I + 1/8232)*1029^(3/4)*sqrt(2)*log((I + 1)*1029^(3/4)*sqrt(2) + 294*x) - (1/8232*I - 1/8232)*1029^(3/4)*sqrt(2)*log(-(I - 1)*1029^(3/4)*sqrt(2) + 294*x) + (1/8232*I - 1/8232)*1029^(3/4)*sqrt(2)*log((I - 1)*1029^(3/4)*sqrt(2) + 294*x) - (1/8232*I + 1/8232)*1029^(3/4)*sqrt(2)*log(-(I + 1)*1029^(3/4)*sqrt(2) + 294*x)`

**3.38.6 Sympy [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int \frac{1}{7+3x^4} dx = -\frac{\sqrt[4]{189}\sqrt{2} \log\left(x^2 - \frac{\sqrt[4]{189}\sqrt{2}x}{3} + \frac{\sqrt{21}}{3}\right)}{168}$$

$$+ \frac{\sqrt[4]{189}\sqrt{2} \log\left(x^2 + \frac{\sqrt[4]{189}\sqrt{2}x}{3} + \frac{\sqrt{21}}{3}\right)}{168}$$

$$+ \frac{\sqrt{2} \cdot 3^{\frac{3}{4}} \cdot \sqrt[4]{7} \operatorname{atan}\left(\frac{\sqrt{2} \cdot \sqrt[4]{3} \cdot 7^{\frac{3}{4}}x}{7} - 1\right)}{84}$$

$$+ \frac{\sqrt{2} \cdot 3^{\frac{3}{4}} \cdot \sqrt[4]{7} \operatorname{atan}\left(\frac{\sqrt{2} \cdot \sqrt[4]{3} \cdot 7^{\frac{3}{4}}x}{7} + 1\right)}{84}$$

input `integrate(1/(3*x**4+7),x)`output `-189**(1/4)*sqrt(2)*log(x**2 - 189**(1/4)*sqrt(2)*x/3 + sqrt(21)/3)/168 + 189**(1/4)*sqrt(2)*log(x**2 + 189**(1/4)*sqrt(2)*x/3 + sqrt(21)/3)/168 + sqrt(2)*3**(3/4)*7**(1/4)*atan(sqrt(2)*3**(1/4)*7**(3/4)*x/7 - 1)/84 + sqrt(2)*3**(3/4)*7**(1/4)*atan(sqrt(2)*3**(1/4)*7**(3/4)*x/7 + 1)/84`**3.38.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int \frac{1}{7+3x^4} dx = \frac{1}{84} \cdot 7^{\frac{1}{4}} 3^{\frac{3}{4}} \sqrt{2} \operatorname{arctan}\left(\frac{1}{42} \cdot 7^{\frac{3}{4}} 3^{\frac{3}{4}} \sqrt{2} (2\sqrt{3}x + 7^{\frac{1}{4}} 3^{\frac{1}{4}} \sqrt{2})\right)$$

$$+ \frac{1}{84} \cdot 7^{\frac{1}{4}} 3^{\frac{3}{4}} \sqrt{2} \operatorname{arctan}\left(\frac{1}{42} \cdot 7^{\frac{3}{4}} 3^{\frac{3}{4}} \sqrt{2} (2\sqrt{3}x - 7^{\frac{1}{4}} 3^{\frac{1}{4}} \sqrt{2})\right)$$

$$+ \frac{1}{168} \cdot 7^{\frac{1}{4}} 3^{\frac{3}{4}} \sqrt{2} \log\left(\sqrt{3}x^2 + 7^{\frac{1}{4}} 3^{\frac{1}{4}} \sqrt{2}x + \sqrt{7}\right)$$

$$- \frac{1}{168} \cdot 7^{\frac{1}{4}} 3^{\frac{3}{4}} \sqrt{2} \log\left(\sqrt{3}x^2 - 7^{\frac{1}{4}} 3^{\frac{1}{4}} \sqrt{2}x + \sqrt{7}\right)$$

input `integrate(1/(3*x^4+7),x, algorithm="maxima")`

output  $1/84*7^{(1/4)}*3^{(3/4)}*\text{sqrt}(2)*\text{arctan}(1/42*7^{(3/4)}*3^{(3/4)}*\text{sqrt}(2)*(2*\text{sqrt}(3)*x + 7^{(1/4)}*3^{(1/4)}*\text{sqrt}(2))) + 1/84*7^{(1/4)}*3^{(3/4)}*\text{sqrt}(2)*\text{arctan}(1/42*7^{(3/4)}*3^{(3/4)}*\text{sqrt}(2)*(2*\text{sqrt}(3)*x - 7^{(1/4)}*3^{(1/4)}*\text{sqrt}(2))) + 1/168*7^{(1/4)}*3^{(3/4)}*\text{sqrt}(2)*\log(\text{sqrt}(3)*x^2 + 7^{(1/4)}*3^{(1/4)}*\text{sqrt}(2)*x + \text{sqrt}(7)) - 1/168*7^{(1/4)}*3^{(3/4)}*\text{sqrt}(2)*\log(\text{sqrt}(3)*x^2 - 7^{(1/4)}*3^{(1/4)}*\text{sqrt}(2)*x + \text{sqrt}(7))$

### 3.38.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.56

$$\begin{aligned} \int \frac{1}{7+3x^4} dx &= \frac{1}{84} \cdot 756^{\frac{1}{4}} \arctan \left( \frac{3}{14} \left( \frac{7}{3} \right)^{\frac{3}{4}} \sqrt{2} \left( 2x + \left( \frac{7}{3} \right)^{\frac{1}{4}} \sqrt{2} \right) \right) \\ &+ \frac{1}{84} \cdot 756^{\frac{1}{4}} \arctan \left( \frac{3}{14} \left( \frac{7}{3} \right)^{\frac{3}{4}} \sqrt{2} \left( 2x - \left( \frac{7}{3} \right)^{\frac{1}{4}} \sqrt{2} \right) \right) \\ &+ \frac{1}{168} \cdot 756^{\frac{1}{4}} \log \left( x^2 + \left( \frac{7}{3} \right)^{\frac{1}{4}} \sqrt{2}x + \sqrt{\frac{7}{3}} \right) \\ &- \frac{1}{168} \cdot 756^{\frac{1}{4}} \log \left( x^2 - \left( \frac{7}{3} \right)^{\frac{1}{4}} \sqrt{2}x + \sqrt{\frac{7}{3}} \right) \end{aligned}$$

input `integrate(1/(3*x^4+7),x, algorithm="giac")`

output  $1/84*756^{(1/4)}*\text{arctan}(3/14*(7/3)^{(3/4)}*\text{sqrt}(2)*(2*x + (7/3)^{(1/4)}*\text{sqrt}(2))) + 1/84*756^{(1/4)}*\text{arctan}(3/14*(7/3)^{(3/4)}*\text{sqrt}(2)*(2*x - (7/3)^{(1/4)}*\text{sqrt}(2))) + 1/168*756^{(1/4)}*\log(x^2 + (7/3)^{(1/4)}*\text{sqrt}(2)*x + \text{sqrt}(7/3)) - 1/168*756^{(1/4)}*\log(x^2 - (7/3)^{(1/4)}*\text{sqrt}(2)*x + \text{sqrt}(7/3))$

### 3.38.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.26

$$\begin{aligned} \int \frac{1}{7+3x^4} dx &= \sqrt{2} 189^{1/4} \text{atan} \left( \sqrt{2} 189^{3/4} x \left( \frac{1}{126} - \frac{1}{126}i \right) \right) \left( \frac{1}{84} + \frac{1}{84}i \right) \\ &+ \sqrt{2} 189^{1/4} \text{atan} \left( \sqrt{2} 189^{3/4} x \left( \frac{1}{126} + \frac{1}{126}i \right) \right) \left( \frac{1}{84} - \frac{1}{84}i \right) \end{aligned}$$

input `int(1/(3*x^4 + 7),x)`

output `2^(1/2)*189^(1/4)*atan(2^(1/2)*189^(3/4)*x*(1/126 - 1i/126))*(1/84 + 1i/84) + 2^(1/2)*189^(1/4)*atan(2^(1/2)*189^(3/4)*x*(1/126 + 1i/126))*(1/84 - 1i/84)`



### 3.39 $\int \frac{1}{-1+3x^2+x^4} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.39.1 | Optimal result . . . . .                            | 304 |
| 3.39.2 | Mathematica [A] (verified) . . . . .                | 304 |
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| 3.39.5 | Fricas [B] (verification not implemented) . . . . . | 306 |
| 3.39.6 | Sympy [B] (verification not implemented) . . . . .  | 307 |
| 3.39.7 | Maxima [F] . . . . .                                | 308 |
| 3.39.8 | Giac [A] (verification not implemented) . . . . .   | 308 |
| 3.39.9 | Mupad [B] (verification not implemented) . . . . .  | 308 |

#### 3.39.1 Optimal result

Integrand size = 12, antiderivative size = 73

$$\int \frac{1}{-1+3x^2+x^4} dx = -\sqrt{\frac{2}{13(3+\sqrt{13})}} \arctan\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right) - \sqrt{\frac{1}{26}(3+\sqrt{13})} \operatorname{arctanh}\left(\sqrt{\frac{2}{-3+\sqrt{13}}}x\right)$$

output `-1/13*arctan(x*2^(1/2)/(3+13^(1/2))^(1/2))*26^(1/2)/(3+13^(1/2))^(1/2)-1/26*arctanh(x*2^(1/2)/(-3+13^(1/2))^(1/2))*(78+26*13^(1/2))^(1/2)`

#### 3.39.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{1}{-1+3x^2+x^4} dx = -\frac{\sqrt{-3+\sqrt{13}} \arctan\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right) + \sqrt{3+\sqrt{13}} \operatorname{arctanh}\left(\sqrt{\frac{2}{-3+\sqrt{13}}}x\right)}{\sqrt{26}}$$

input `Integrate[(-1 + 3*x^2 + x^4)^(-1),x]`

output `-((Sqrt[-3 + Sqrt[13]]*ArcTan[Sqrt[2/(3 + Sqrt[13])]]*x) + Sqrt[3 + Sqrt[13]]*ArcTanh[Sqrt[2/(-3 + Sqrt[13])]]*x)/Sqrt[26])`

**3.39.3 Rubi [A] (verified)**

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {1406, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{x^4 + 3x^2 - 1} dx \\
 & \quad \downarrow \text{1406} \\
 & \frac{\int \frac{1}{x^2 + \frac{1}{2}(3 - \sqrt{13})} dx}{\sqrt{13}} - \frac{\int \frac{1}{x^2 + \frac{1}{2}(3 + \sqrt{13})} dx}{\sqrt{13}} \\
 & \quad \downarrow \text{216} \\
 & \frac{\int \frac{1}{x^2 + \frac{1}{2}(3 - \sqrt{13})} dx}{\sqrt{13}} - \sqrt{\frac{2}{13(3 + \sqrt{13})}} \arctan\left(\sqrt{\frac{2}{3 + \sqrt{13}}} x\right) \\
 & \quad \downarrow \text{220} \\
 & -\sqrt{\frac{2}{13(3 + \sqrt{13})}} \arctan\left(\sqrt{\frac{2}{3 + \sqrt{13}}} x\right) - \sqrt{\frac{2}{13(\sqrt{13} - 3)}} \operatorname{arctanh}\left(\sqrt{\frac{2}{\sqrt{13} - 3}} x\right)
 \end{aligned}$$

input `Int[(-1 + 3*x^2 + x^4)^(-1), x]`

output `-(Sqrt[2/(13*(3 + Sqrt[13]))]*ArcTan[Sqrt[2/(3 + Sqrt[13])]*x]) - Sqrt[2/(13*(-3 + Sqrt[13]))]*ArcTanh[Sqrt[2/(-3 + Sqrt[13])]*x]`

**3.39.3.1 Defintions of rubi rules used**

rule 216 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

rule 220 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

```
rule 1406 Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q Int[1/(b/2 + q/2 + c*x^2), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

### 3.39.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.48

| method  | result                                                                                                                                                                                                        | size |
|---------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch   | $\frac{\left( \sum_{R=\text{RootOf}(\_Z^4+3\_Z^2-1)} \frac{\ln(x-\_R)}{2\_R^3+3\_R} \right)}{2}$                                                                                                              | 35   |
| default | $-\frac{2\sqrt{13} \arctan\left(\frac{2x}{\sqrt{6+2\sqrt{13}}}\right)}{13\sqrt{6+2\sqrt{13}}} - \frac{2\sqrt{13} \operatorname{arctanh}\left(\frac{2x}{\sqrt{-6+2\sqrt{13}}}\right)}{13\sqrt{-6+2\sqrt{13}}}$ | 56   |

```
input int(1/(x^4+3*x^2-1),x,method=_RETURNVERBOSE)
```

```
output 1/2*sum(1/(2*_R^3+3*_R)*ln(x-_R),_R=RootOf(_Z^4+3*_Z^2-1))
```

### 3.39.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(50) = 100.

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.07

$$\int \frac{1}{-1+3x^2+x^4} dx = \frac{1}{52} \sqrt{26} \sqrt{\sqrt{13}+3} \log \left( \sqrt{26} (3\sqrt{13}-13) \sqrt{\sqrt{13}+3+52x} \right) \\ - \frac{1}{52} \sqrt{26} \sqrt{\sqrt{13}+3} \log \left( -\sqrt{26} (3\sqrt{13}-13) \sqrt{\sqrt{13}+3+52x} \right) \\ - \frac{1}{52} \sqrt{26} \sqrt{-\sqrt{13}+3} \log \left( \sqrt{26} (3\sqrt{13}+13) \sqrt{-\sqrt{13}+3+52x} \right) \\ + \frac{1}{52} \sqrt{26} \sqrt{-\sqrt{13}+3} \log \left( -\sqrt{26} (3\sqrt{13}+13) \sqrt{-\sqrt{13}+3} \right. \\ \left. + 52x \right)$$

```
input integrate(1/(x^4+3*x^2-1),x, algorithm="fricas")
```

```
output 1/52*sqrt(26)*sqrt(sqrt(13) + 3)*log(sqrt(26)*(3*sqrt(13) - 13)*sqrt(sqrt(13) + 3) + 52*x) - 1/52*sqrt(26)*sqrt(sqrt(13) + 3)*log(-sqrt(26)*(3*sqrt(13) - 13)*sqrt(sqrt(13) + 3) + 52*x) - 1/52*sqrt(26)*sqrt(-sqrt(13) + 3)*log(sqrt(26)*(3*sqrt(13) + 13)*sqrt(-sqrt(13) + 3) + 52*x) + 1/52*sqrt(26)*sqrt(-sqrt(13) + 3)*log(-sqrt(26)*(3*sqrt(13) + 13)*sqrt(-sqrt(13) + 3) + 52*x)
```

### 3.39.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(68) = 136.

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.00

$$\int \frac{1}{-1 + 3x^2 + x^4} dx = \sqrt{\frac{3}{104} + \frac{\sqrt{13}}{104}} \log \left( x - 22\sqrt{\frac{3}{104} + \frac{\sqrt{13}}{104}} + 312 \left( \frac{3}{104} + \frac{\sqrt{13}}{104} \right)^{\frac{3}{2}} \right) - \sqrt{\frac{3}{104} + \frac{\sqrt{13}}{104}} \log \left( x - 312 \left( \frac{3}{104} + \frac{\sqrt{13}}{104} \right)^{\frac{3}{2}} + 22\sqrt{\frac{3}{104} + \frac{\sqrt{13}}{104}} \right) - 2\sqrt{-\frac{3}{104} + \frac{\sqrt{13}}{104}} \operatorname{atan} \left( \frac{2\sqrt{2}x}{3\sqrt{-3 + \sqrt{13}} + \sqrt{13}\sqrt{-3 + \sqrt{13}}} \right)$$

```
input integrate(1/(x**4+3*x**2-1),x)
```

```
output sqrt(3/104 + sqrt(13)/104)*log(x - 22*sqrt(3/104 + sqrt(13)/104) + 312*(3/104 + sqrt(13)/104)**(3/2)) - sqrt(3/104 + sqrt(13)/104)*log(x - 312*(3/104 + sqrt(13)/104)**(3/2) + 22*sqrt(3/104 + sqrt(13)/104)) - 2*sqrt(-3/104 + sqrt(13)/104)*atan(2*sqrt(2)*x/(3*sqrt(-3 + sqrt(13)) + sqrt(13)*sqrt(-3 + sqrt(13))))
```

**3.39.7 Maxima [F]**

$$\int \frac{1}{-1 + 3x^2 + x^4} dx = \int \frac{1}{x^4 + 3x^2 - 1} dx$$

input `integrate(1/(x^4+3*x^2-1),x, algorithm="maxima")`

output `integrate(1/(x^4 + 3*x^2 - 1), x)`

**3.39.8 Giac [A] (verification not implemented)**

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\begin{aligned} \int \frac{1}{-1 + 3x^2 + x^4} dx = & -\frac{1}{26} \sqrt{26\sqrt{13} - 78} \arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{13} + \frac{3}{2}}}\right) \\ & - \frac{1}{52} \sqrt{26\sqrt{13} + 78} \log\left(x + \sqrt{\frac{1}{2}\sqrt{13} - \frac{3}{2}}\right) \\ & + \frac{1}{52} \sqrt{26\sqrt{13} + 78} \log\left(x - \sqrt{\frac{1}{2}\sqrt{13} - \frac{3}{2}}\right) \end{aligned}$$

input `integrate(1/(x^4+3*x^2-1),x, algorithm="giac")`

output `-1/26*sqrt(26*sqrt(13) - 78)*arctan(x/sqrt(1/2*sqrt(13) + 3/2)) - 1/52*sqrt(26*sqrt(13) + 78)*log(abs(x + sqrt(1/2*sqrt(13) - 3/2))) + 1/52*sqrt(26*sqrt(13) + 78)*log(abs(x - sqrt(1/2*sqrt(13) - 3/2)))`

**3.39.9 Mupad [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\begin{aligned} \int \frac{1}{-1 + 3x^2 + x^4} dx = & -\frac{\sqrt{26} \operatorname{atanh}\left(\frac{\sqrt{26}x}{2\sqrt{\sqrt{13}+3}} + \frac{3\sqrt{13}\sqrt{26}x}{26\sqrt{\sqrt{13}+3}}\right) \sqrt{\sqrt{13}+3}}{26} \\ & - \frac{\sqrt{26} \operatorname{atanh}\left(\frac{\sqrt{26}x}{2\sqrt{3-\sqrt{13}}} - \frac{3\sqrt{13}\sqrt{26}x}{26\sqrt{3-\sqrt{13}}}\right) \sqrt{3-\sqrt{13}}}{26} \end{aligned}$$

input `int(1/(3*x^2 + x^4 - 1),x)`

output 
$$- \frac{26^{1/2} \operatorname{atanh}\left(\frac{26^{1/2} x}{2(13^{1/2} + 3)^{1/2}}\right) + (3 \cdot 13^{1/2} \cdot 26^{1/2} x) / (26(13^{1/2} + 3)^{1/2})}{26} - \frac{26^{1/2} \operatorname{atanh}\left(\frac{26^{1/2} x}{2(3 - 13^{1/2})^{1/2}}\right) - (3 \cdot 13^{1/2} \cdot 26^{1/2} x) / (26(3 - 13^{1/2})^{1/2})}{26}$$

### 3.40 $\int \frac{1}{-1-3x^2+x^4} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.40.1 | Optimal result . . . . .                            | 310 |
| 3.40.2 | Mathematica [A] (verified) . . . . .                | 310 |
| 3.40.3 | Rubi [A] (verified) . . . . .                       | 311 |
| 3.40.4 | Maple [C] (verified) . . . . .                      | 312 |
| 3.40.5 | Fricas [B] (verification not implemented) . . . . . | 312 |
| 3.40.6 | Sympy [A] (verification not implemented) . . . . .  | 313 |
| 3.40.7 | Maxima [F] . . . . .                                | 313 |
| 3.40.8 | Giac [A] (verification not implemented) . . . . .   | 314 |
| 3.40.9 | Mupad [B] (verification not implemented) . . . . .  | 314 |

#### 3.40.1 Optimal result

Integrand size = 12, antiderivative size = 73

$$\int \frac{1}{-1-3x^2+x^4} dx = -\sqrt{\frac{1}{26} (3 + \sqrt{13})} \arctan \left( \sqrt{\frac{2}{-3 + \sqrt{13}}} x \right) - \sqrt{\frac{2}{13 (3 + \sqrt{13})}} \operatorname{arctanh} \left( \sqrt{\frac{2}{3 + \sqrt{13}}} x \right)$$

output `-1/13*arctanh(x*2^(1/2)/(3+13^(1/2))^(1/2))*26^(1/2)/(3+13^(1/2))^(1/2)-1/26*arctan(x*2^(1/2)/(-3+13^(1/2))^(1/2))*(78+26*13^(1/2))^(1/2)`

#### 3.40.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{1}{-1-3x^2+x^4} dx = -\frac{\sqrt{3 + \sqrt{13}} \arctan \left( \sqrt{\frac{2}{-3 + \sqrt{13}}} x \right) + \sqrt{-3 + \sqrt{13}} \operatorname{arctanh} \left( \sqrt{\frac{2}{3 + \sqrt{13}}} x \right)}{\sqrt{26}}$$

input `Integrate[(-1 - 3*x^2 + x^4)^(-1), x]`

output `-((Sqrt[3 + Sqrt[13]]*ArcTan[Sqrt[2/(-3 + Sqrt[13])]]*x) + Sqrt[-3 + Sqrt[13]]*ArcTanh[Sqrt[2/(3 + Sqrt[13])]]*x)/Sqrt[26])`

### 3.40.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {1406, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{x^4 - 3x^2 - 1} dx \\
 & \quad \downarrow \text{1406} \\
 & \frac{\int \frac{1}{x^2 + \frac{1}{2}(-3 - \sqrt{13})} dx}{\sqrt{13}} - \frac{\int \frac{1}{x^2 + \frac{1}{2}(-3 + \sqrt{13})} dx}{\sqrt{13}} \\
 & \quad \downarrow \text{216} \\
 & \frac{\int \frac{1}{x^2 + \frac{1}{2}(-3 - \sqrt{13})} dx}{\sqrt{13}} - \sqrt{\frac{2}{13(\sqrt{13} - 3)}} \arctan\left(\sqrt{\frac{2}{\sqrt{13} - 3}} x\right) \\
 & \quad \downarrow \text{220} \\
 & -\sqrt{\frac{2}{13(\sqrt{13} - 3)}} \arctan\left(\sqrt{\frac{2}{\sqrt{13} - 3}} x\right) - \sqrt{\frac{2}{13(3 + \sqrt{13})}} \operatorname{arctanh}\left(\sqrt{\frac{2}{3 + \sqrt{13}}} x\right)
 \end{aligned}$$

input `Int[(-1 - 3*x^2 + x^4)^(-1), x]`

output `-(Sqrt[2/(13*(-3 + Sqrt[13]))])*ArcTan[Sqrt[2/(-3 + Sqrt[13])]*x]) - Sqrt[2/(13*(3 + Sqrt[13]))]*ArcTanh[Sqrt[2/(3 + Sqrt[13])]*x]`

#### 3.40.3.1 Defintions of rubi rules used

rule 216 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

rule 220 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

---

3.40.  $\int \frac{1}{-1-3x^2+x^4} dx$



```
rule 1406 Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q Int[1/(b/2 + q/2 + c*x^2), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

### 3.40.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.48

| method  | result                                                                                                                                                                                                                      | size |
|---------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch   | $\frac{\left( \sum_{R=\text{RootOf}(\_Z^4-3\_Z^2-1)} \frac{\ln(x-R)}{2R^3-3R} \right)}{2}$                                                                                                                                  | 35   |
| default | $-\frac{2\sqrt{13} \operatorname{arctanh}\left(\frac{2x}{\sqrt{6+2\sqrt{13}}}\right)}{13\sqrt{6+2\sqrt{13}}} - \frac{2\sqrt{13} \operatorname{arctan}\left(\frac{2x}{\sqrt{-6+2\sqrt{13}}}\right)}{13\sqrt{-6+2\sqrt{13}}}$ | 56   |

```
input int(1/(x^4-3*x^2-1),x,method=_RETURNVERBOSE)
```

```
output 1/2*sum(1/(2*_R^3-3*_R)*ln(x-_R),_R=RootOf(_Z^4-3*_Z^2-1))
```

### 3.40.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(50) = 100.

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.07

$$\int \frac{1}{-1-3x^2+x^4} dx = -\frac{1}{52} \sqrt{26} \sqrt{\sqrt{13}-3} \log\left(\sqrt{26}(3\sqrt{13}+13)\sqrt{\sqrt{13}-3+52x}\right) + \frac{1}{52} \sqrt{26} \sqrt{\sqrt{13}-3} \log\left(-\sqrt{26}(3\sqrt{13}+13)\sqrt{\sqrt{13}-3+52x}\right) + \frac{1}{52} \sqrt{26} \sqrt{-\sqrt{13}-3} \log\left(\sqrt{26}(3\sqrt{13}-13)\sqrt{-\sqrt{13}-3+52x}\right) - \frac{1}{52} \sqrt{26} \sqrt{-\sqrt{13}-3} \log\left(-\sqrt{26}(3\sqrt{13}-13)\sqrt{-\sqrt{13}-3+52x}\right)$$

```
input integrate(1/(x^4-3*x^2-1),x, algorithm="fricas")
```

```
output -1/52*sqrt(26)*sqrt(sqrt(13) - 3)*log(sqrt(26)*(3*sqrt(13) + 13)*sqrt(sqrt(13) - 3) + 52*x) + 1/52*sqrt(26)*sqrt(sqrt(13) - 3)*log(-sqrt(26)*(3*sqrt(13) + 13)*sqrt(sqrt(13) - 3) + 52*x) + 1/52*sqrt(26)*sqrt(-sqrt(13) - 3)*log(sqrt(26)*(3*sqrt(13) - 13)*sqrt(-sqrt(13) - 3) + 52*x) - 1/52*sqrt(26)*sqrt(-sqrt(13) - 3)*log(-sqrt(26)*(3*sqrt(13) - 13)*sqrt(-sqrt(13) - 3) + 52*x)
```

### 3.40.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.33

$$\int \frac{1}{-1 - 3x^2 + x^4} dx = \text{RootSum}(2704t^4 + 156t^2 - 1, (t \mapsto t \log(-312t^3 - 22t + x)))$$

```
input integrate(1/(x**4-3*x**2-1),x)
```

```
output RootSum(2704*_t**4 + 156*_t**2 - 1, Lambda(_t, _t*log(-312*_t**3 - 22*_t + x)))
```

### 3.40.7 Maxima [F]

$$\int \frac{1}{-1 - 3x^2 + x^4} dx = \int \frac{1}{x^4 - 3x^2 - 1} dx$$

```
input integrate(1/(x^4-3*x^2-1),x, algorithm="maxima")
```

```
output integrate(1/(x^4 - 3*x^2 - 1), x)
```

**3.40.8 Giac [A] (verification not implemented)**

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{1}{-1 - 3x^2 + x^4} dx = -\frac{1}{26} \sqrt{26\sqrt{13} + 78} \arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{13} - \frac{3}{2}}}\right) - \frac{1}{52} \sqrt{26\sqrt{13} - 78} \log\left(\left|x + \sqrt{\frac{1}{2}\sqrt{13} + \frac{3}{2}}\right|\right) + \frac{1}{52} \sqrt{26\sqrt{13} - 78} \log\left(\left|x - \sqrt{\frac{1}{2}\sqrt{13} + \frac{3}{2}}\right|\right)$$

input `integrate(1/(x^4-3*x^2-1),x, algorithm="giac")`output `-1/26*sqrt(26*sqrt(13) + 78)*arctan(x/sqrt(1/2*sqrt(13) - 3/2)) - 1/52*sqrt(26*sqrt(13) - 78)*log(abs(x + sqrt(1/2*sqrt(13) + 3/2))) + 1/52*sqrt(26*sqrt(13) - 78)*log(abs(x - sqrt(1/2*sqrt(13) + 3/2)))`**3.40.9 Mupad [B] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int \frac{1}{-1 - 3x^2 + x^4} dx = -\frac{\sqrt{26} \operatorname{atanh}\left(\frac{\sqrt{26}x}{2\sqrt{\sqrt{13}-3}} - \frac{3\sqrt{13}\sqrt{26}x}{26\sqrt{\sqrt{13}-3}}\right) \sqrt{\sqrt{13}-3}}{26} - \frac{\sqrt{26} \operatorname{atanh}\left(\frac{\sqrt{26}x}{2\sqrt{-\sqrt{13}-3}} + \frac{3\sqrt{13}\sqrt{26}x}{26\sqrt{-\sqrt{13}-3}}\right) \sqrt{-\sqrt{13}-3}}{26}$$

input `int(-1/(3*x^2 - x^4 + 1),x)`output `-(26^(1/2)*atanh((26^(1/2)*x)/(2*(13^(1/2) - 3)^(1/2)) - (3*13^(1/2)*26^(1/2)*x)/(26*(13^(1/2) - 3)^(1/2)))*(13^(1/2) - 3)^(1/2))/26 - (26^(1/2)*atanh((26^(1/2)*x)/(2*(-13^(1/2) - 3)^(1/2)) + (3*13^(1/2)*26^(1/2)*x)/(26*(-13^(1/2) - 3)^(1/2)))*(-13^(1/2) - 3)^(1/2))/26`

### 3.41 $\int \frac{1}{1-3x^2+x^4} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.41.1 | Optimal result . . . . .                            | 315 |
| 3.41.2 | Mathematica [A] (verified) . . . . .                | 315 |
| 3.41.3 | Rubi [A] (verified) . . . . .                       | 316 |
| 3.41.4 | Maple [A] (verified) . . . . .                      | 317 |
| 3.41.5 | Fricas [B] (verification not implemented) . . . . . | 317 |
| 3.41.6 | Sympy [B] (verification not implemented) . . . . .  | 318 |
| 3.41.7 | Maxima [A] (verification not implemented) . . . . . | 318 |
| 3.41.8 | Giac [A] (verification not implemented) . . . . .   | 319 |
| 3.41.9 | Mupad [B] (verification not implemented) . . . . .  | 319 |

#### 3.41.1 Optimal result

Integrand size = 12, antiderivative size = 72

$$\int \frac{1}{1-3x^2+x^4} dx = -\sqrt{\frac{2}{5(3+\sqrt{5})}} \operatorname{arctanh}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right) + \sqrt{\frac{1}{10(3+\sqrt{5})}} \operatorname{arctanh}\left(\sqrt{\frac{1}{2}(3+\sqrt{5})}x\right)$$

output `-1/5*arctanh(x*2^(1/2)/(3+5^(1/2))^(1/2))*10^(1/2)/(3+5^(1/2))^(1/2)+arctanh(x*(1/2+1/2*5^(1/2)))*(1/2+1/10*5^(1/2))`

#### 3.41.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int \frac{1}{1-3x^2+x^4} dx = \frac{1}{20} \left( -\left( (5+\sqrt{5}) \log(-1+\sqrt{5}-2x) \right) - \left( -5+\sqrt{5} \right) \log(1+\sqrt{5}-2x) \right) + \left( 5+\sqrt{5} \right) \log(-1+\sqrt{5}+2x) + \left( -5+\sqrt{5} \right) \log(1+\sqrt{5}+2x) \right)$$

input `Integrate[(1 - 3*x^2 + x^4)^(-1), x]`

output `(-((5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x]) - (-5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*x] + (5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x] + (-5 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/20`

### 3.41.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 - 3x^2 + 1} dx$$

$$\downarrow 1406$$

$$\frac{\int \frac{1}{x^2 + \frac{1}{2}(-3 - \sqrt{5})} dx}{\sqrt{5}} - \frac{\int \frac{1}{x^2 + \frac{1}{2}(-3 + \sqrt{5})} dx}{\sqrt{5}}$$

$$\downarrow 220$$

$$\sqrt{\frac{1}{10}(3 + \sqrt{5})} \operatorname{arctanh}\left(\sqrt{\frac{1}{2}(3 + \sqrt{5})}x\right) - \sqrt{\frac{2}{5(3 + \sqrt{5})}} \operatorname{arctanh}\left(\sqrt{\frac{2}{3 + \sqrt{5}}}x\right)$$

input `Int[(1 - 3*x^2 + x^4)^(-1),x]`

output `-(Sqrt[2/(5*(3 + Sqrt[5]))]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x]) + Sqrt[(3 + Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x]`

#### 3.41.3.1 Defintions of rubi rules used

rule 220 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

rule 1406 `Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]`

**3.41.4 Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

| method  | result                                                                                                                                                                                                                     |
|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| default | $\frac{\ln(x^2-x-1)}{4} + \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)}{10} - \frac{\ln(x^2+x-1)}{4} + \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)}{10}$      |
| risch   | $\frac{\ln(2x-1+\sqrt{5})}{4} + \frac{\ln(2x-1+\sqrt{5})\sqrt{5}}{20} + \frac{\ln(2x-1-\sqrt{5})}{4} - \frac{\ln(2x-1-\sqrt{5})\sqrt{5}}{20} - \frac{\ln(2x+\sqrt{5}+1)}{4} + \frac{\ln(2x+\sqrt{5}+1)\sqrt{5}}{20} - \ln$ |

input `int(1/(x^4-3*x^2+1),x,method=_RETURNVERBOSE)`

output `1/4*ln(x^2-x-1)+1/10*5^(1/2)*arctanh(1/5*(2*x-1)*5^(1/2))-1/4*ln(x^2+x-1)+1/10*5^(1/2)*arctanh(1/5*(1+2*x)*5^(1/2))`

**3.41.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 91 vs.  $2(42) = 84$ .

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \frac{1}{1-3x^2+x^4} dx = \frac{1}{20} \sqrt{5} \log \left( \frac{2x^2 + \sqrt{5}(2x+1) + 2x+3}{x^2+x-1} \right) + \frac{1}{20} \sqrt{5} \log \left( \frac{2x^2 + \sqrt{5}(2x-1) - 2x+3}{x^2-x-1} \right) - \frac{1}{4} \log(x^2+x-1) + \frac{1}{4} \log(x^2-x-1)$$

input `integrate(1/(x^4-3*x^2+1),x, algorithm="fracas")`

output `1/20*sqrt(5)*log((2*x^2 + sqrt(5)*(2*x + 1) + 2*x + 3)/(x^2 + x - 1)) + 1/20*sqrt(5)*log((2*x^2 + sqrt(5)*(2*x - 1) - 2*x + 3)/(x^2 - x - 1)) - 1/4*log(x^2 + x - 1) + 1/4*log(x^2 - x - 1)`

**3.41.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(58) = 116.

Time = 0.18 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.19

$$\begin{aligned} \int \frac{1}{1-3x^2+x^4} dx = & \left( \frac{\sqrt{5}}{20} + \frac{1}{4} \right) \log \left( x - \frac{7}{2} - \frac{7\sqrt{5}}{10} + 120 \left( \frac{\sqrt{5}}{20} + \frac{1}{4} \right)^3 \right) \\ & + \left( \frac{1}{4} - \frac{\sqrt{5}}{20} \right) \log \left( x - \frac{7}{2} + 120 \left( \frac{1}{4} - \frac{\sqrt{5}}{20} \right)^3 + \frac{7\sqrt{5}}{10} \right) \\ & + \left( -\frac{1}{4} + \frac{\sqrt{5}}{20} \right) \log \left( x - \frac{7\sqrt{5}}{10} + 120 \left( -\frac{1}{4} + \frac{\sqrt{5}}{20} \right)^3 + \frac{7}{2} \right) \\ & + \left( -\frac{1}{4} - \frac{\sqrt{5}}{20} \right) \log \left( x + 120 \left( -\frac{1}{4} - \frac{\sqrt{5}}{20} \right)^3 + \frac{7\sqrt{5}}{10} + \frac{7}{2} \right) \end{aligned}$$

input `integrate(1/(x**4-3*x**2+1),x)`

output `(sqrt(5)/20 + 1/4)*log(x - 7/2 - 7*sqrt(5)/10 + 120*(sqrt(5)/20 + 1/4)**3) + (1/4 - sqrt(5)/20)*log(x - 7/2 + 120*(1/4 - sqrt(5)/20)**3 + 7*sqrt(5)/10) + (-1/4 + sqrt(5)/20)*log(x - 7*sqrt(5)/10 + 120*(-1/4 + sqrt(5)/20)**3 + 7/2) + (-1/4 - sqrt(5)/20)*log(x + 120*(-1/4 - sqrt(5)/20)**3 + 7*sqrt(5)/10 + 7/2)`

**3.41.7 Maxima [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\begin{aligned} \int \frac{1}{1-3x^2+x^4} dx = & -\frac{1}{20} \sqrt{5} \log \left( \frac{2x - \sqrt{5} + 1}{2x + \sqrt{5} + 1} \right) - \frac{1}{20} \sqrt{5} \log \left( \frac{2x - \sqrt{5} - 1}{2x + \sqrt{5} - 1} \right) \\ & - \frac{1}{4} \log(x^2 + x - 1) + \frac{1}{4} \log(x^2 - x - 1) \end{aligned}$$

input `integrate(1/(x^4-3*x^2+1),x, algorithm="maxima")`

output `-1/20*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/20*sqrt(5)*log((2*x - sqrt(5) - 1)/(2*x + sqrt(5) - 1)) - 1/4*log(x^2 + x - 1) + 1/4*log(x^2 - x - 1)`

**3.41.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12

$$\int \frac{1}{1-3x^2+x^4} dx = -\frac{1}{20} \sqrt{5} \log \left( \frac{|2x - \sqrt{5} + 1|}{|2x + \sqrt{5} + 1|} \right) - \frac{1}{20} \sqrt{5} \log \left( \frac{|2x - \sqrt{5} - 1|}{|2x + \sqrt{5} - 1|} \right) \\ - \frac{1}{4} \log (|x^2 + x - 1|) + \frac{1}{4} \log (|x^2 - x - 1|)$$

input `integrate(1/(x^4-3*x^2+1),x, algorithm="giac")`output `-1/20*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) - 1/20*sqrt(5)*log(abs(2*x - sqrt(5) - 1)/abs(2*x + sqrt(5) - 1)) - 1/4*log(abs(x^2 + x - 1)) + 1/4*log(abs(x^2 - x - 1))`**3.41.9 Mupad [B] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \frac{1}{1-3x^2+x^4} dx = \operatorname{atanh} \left( \frac{4x}{\sqrt{5}-3} - \frac{2\sqrt{5}x}{\sqrt{5}-3} \right) \left( \frac{\sqrt{5}}{10} - \frac{1}{2} \right) \\ + \operatorname{atanh} \left( \frac{4x}{\sqrt{5}+3} + \frac{2\sqrt{5}x}{\sqrt{5}+3} \right) \left( \frac{\sqrt{5}}{10} + \frac{1}{2} \right)$$

input `int(1/(x^4 - 3*x^2 + 1),x)`output `atanh((4*x)/(5^(1/2) - 3) - (2*5^(1/2)*x)/(5^(1/2) - 3))*(5^(1/2)/10 - 1/2) + atanh((4*x)/(5^(1/2) + 3) + (2*5^(1/2)*x)/(5^(1/2) + 3))*(5^(1/2)/10 + 1/2)`



### 3.42 $\int \frac{1}{1-4x^2+x^4} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.42.1 | Optimal result . . . . .                            | 320 |
| 3.42.2 | Mathematica [A] (verified) . . . . .                | 320 |
| 3.42.3 | Rubi [A] (verified) . . . . .                       | 321 |
| 3.42.4 | Maple [C] (verified) . . . . .                      | 322 |
| 3.42.5 | Fricas [B] (verification not implemented) . . . . . | 322 |
| 3.42.6 | Sympy [A] (verification not implemented) . . . . .  | 323 |
| 3.42.7 | Maxima [F] . . . . .                                | 323 |
| 3.42.8 | Giac [A] (verification not implemented) . . . . .   | 323 |
| 3.42.9 | Mupad [B] (verification not implemented) . . . . .  | 324 |

#### 3.42.1 Optimal result

Integrand size = 12, antiderivative size = 67

$$\int \frac{1}{1-4x^2+x^4} dx = \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}(2-\sqrt{3})} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}(2+\sqrt{3})}$$

output `1/2*arctanh(x/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/2*arctanh(x/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))`

#### 3.42.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-4x^2+x^4} dx = \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}(2-\sqrt{3})} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}(2+\sqrt{3})}$$

input `Integrate[(1 - 4*x^2 + x^4)^(-1), x]`

output `ArcTanh[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTanh[x/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])`

### 3.42.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 - 4x^2 + 1} dx$$

$$\downarrow 1406$$

$$\frac{\int \frac{1}{x^2 - \sqrt{3} - 2} dx}{2\sqrt{3}} - \frac{\int \frac{1}{x^2 + \sqrt{3} - 2} dx}{2\sqrt{3}}$$

$$\downarrow 220$$

$$\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}(2-\sqrt{3})} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}(2+\sqrt{3})}$$

input `Int[(1 - 4*x^2 + x^4)^(-1),x]`

output `ArcTanh[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTanh[x/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])`

#### 3.42.3.1 Defintions of rubi rules used

rule 220 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

rule 1406 `Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]`

### 3.42.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.49

| method  | result                                                                                                                                                                                                          | size |
|---------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch   | $\frac{\left( \sum_{R=\text{RootOf}(\_Z^4-4\_Z^2+1)} \frac{\ln(x-\_R)}{\_R^3-\_2\_R} \right)}{4}$                                                                                                               | 33   |
| default | $-\frac{\sqrt{3} \operatorname{arctanh}\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3(\sqrt{6}+\sqrt{2})} + \frac{\sqrt{3} \operatorname{arctanh}\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}}$ | 60   |

input `int(1/(x^4-4*x^2+1),x,method=_RETURNVERBOSE)`

output `1/4*sum(1/(_R^3-2*_R)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^2+1))`

### 3.42.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(55) = 110.

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.84

$$\begin{aligned} \int \frac{1}{1-4x^2+x^4} dx = & -\frac{1}{12} \sqrt{3} \sqrt{\sqrt{3}+2} \log \left( \sqrt{\sqrt{3}+2} (\sqrt{3}-2) + x \right) \\ & + \frac{1}{12} \sqrt{3} \sqrt{\sqrt{3}+2} \log \left( -\sqrt{\sqrt{3}+2} (\sqrt{3}-2) + x \right) \\ & - \frac{1}{12} \sqrt{3} \sqrt{-\sqrt{3}+2} \log \left( (\sqrt{3}+2) \sqrt{-\sqrt{3}+2} + x \right) \\ & + \frac{1}{12} \sqrt{3} \sqrt{-\sqrt{3}+2} \log \left( -(\sqrt{3}+2) \sqrt{-\sqrt{3}+2} + x \right) \end{aligned}$$

input `integrate(1/(x^4-4*x^2+1),x, algorithm="fracas")`

output `-1/12*sqrt(3)*sqrt(sqrt(3)+2)*log(sqrt(sqrt(3)+2)*(sqrt(3)-2)+x) +  
1/12*sqrt(3)*sqrt(sqrt(3)+2)*log(-sqrt(sqrt(3)+2)*(sqrt(3)-2)+x)  
- 1/12*sqrt(3)*sqrt(-sqrt(3)+2)*log((sqrt(3)+2)*sqrt(-sqrt(3)+2)+x)  
) + 1/12*sqrt(3)*sqrt(-sqrt(3)+2)*log(-(sqrt(3)+2)*sqrt(-sqrt(3)+2)  
+ x)`

**3.42.6 Sympy [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.36

$$\int \frac{1}{1-4x^2+x^4} dx = \text{RootSum}(2304t^4 - 192t^2 + 1, (t \mapsto t \log(384t^3 - 28t + x)))$$

input `integrate(1/(x**4-4*x**2+1),x)`output `RootSum(2304*_t**4 - 192*_t**2 + 1, Lambda(_t, _t*log(384*_t**3 - 28*_t + x)))`**3.42.7 Maxima [F]**

$$\int \frac{1}{1-4x^2+x^4} dx = \int \frac{1}{x^4-4x^2+1} dx$$

input `integrate(1/(x^4-4*x^2+1),x, algorithm="maxima")`output `integrate(1/(x^4 - 4*x^2 + 1), x)`**3.42.8 Giac [A] (verification not implemented)**

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.51

$$\begin{aligned} \int \frac{1}{1-4x^2+x^4} dx = & \frac{1}{24} (\sqrt{6} - 3\sqrt{2}) \log \left( \left| x + \frac{1}{2}\sqrt{6} + \frac{1}{2}\sqrt{2} \right| \right) \\ & + \frac{1}{24} (\sqrt{6} + 3\sqrt{2}) \log \left( \left| x + \frac{1}{2}\sqrt{6} - \frac{1}{2}\sqrt{2} \right| \right) \\ & - \frac{1}{24} (\sqrt{6} + 3\sqrt{2}) \log \left( \left| x - \frac{1}{2}\sqrt{6} + \frac{1}{2}\sqrt{2} \right| \right) \\ & - \frac{1}{24} (\sqrt{6} - 3\sqrt{2}) \log \left( \left| x - \frac{1}{2}\sqrt{6} - \frac{1}{2}\sqrt{2} \right| \right) \end{aligned}$$

input `integrate(1/(x^4-4*x^2+1),x, algorithm="giac")`

output  $1/24*(\sqrt{6} - 3*\sqrt{2})*\log(\text{abs}(x + 1/2*\sqrt{6} + 1/2*\sqrt{2})) + 1/24*(\sqrt{6} + 3*\sqrt{2})*\log(\text{abs}(x + 1/2*\sqrt{6} - 1/2*\sqrt{2})) - 1/24*(\sqrt{6} + 3*\sqrt{2})*\log(\text{abs}(x - 1/2*\sqrt{6} + 1/2*\sqrt{2})) - 1/24*(\sqrt{6} - 3*\sqrt{2})*\log(\text{abs}(x - 1/2*\sqrt{6} - 1/2*\sqrt{2}))$

### 3.42.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.46

$$\int \frac{1}{1-4x^2+x^4} dx = \operatorname{atanh}\left(\frac{5\sqrt{2}x}{\sqrt{2}\sqrt{6+4}} + \frac{3\sqrt{6}x}{\sqrt{2}\sqrt{6+4}}\right) \left(\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{12}\right) - \operatorname{atanh}\left(\frac{5\sqrt{2}x}{\sqrt{2}\sqrt{6-4}} - \frac{3\sqrt{6}x}{\sqrt{2}\sqrt{6-4}}\right) \left(\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{12}\right)$$

input `int(1/(x^4 - 4*x^2 + 1),x)`

output  $\operatorname{atanh}((5*2^{(1/2)}*x)/(2^{(1/2)}*6^{(1/2)} + 4) + (3*6^{(1/2)}*x)/(2^{(1/2)}*6^{(1/2)} + 4))*(2^{(1/2)}/4 + 6^{(1/2)}/12) - \operatorname{atanh}((5*2^{(1/2)}*x)/(2^{(1/2)}*6^{(1/2)} - 4) - (3*6^{(1/2)}*x)/(2^{(1/2)}*6^{(1/2)} - 4))*(2^{(1/2)}/4 - 6^{(1/2)}/12)$

### 3.43 $\int \frac{1}{1+4x^2+x^4} dx$

|                                                            |     |
|------------------------------------------------------------|-----|
| 3.43.1 Optimal result . . . . .                            | 325 |
| 3.43.2 Mathematica [A] (verified) . . . . .                | 325 |
| 3.43.3 Rubi [A] (verified) . . . . .                       | 326 |
| 3.43.4 Maple [C] (verified) . . . . .                      | 327 |
| 3.43.5 Fricas [B] (verification not implemented) . . . . . | 327 |
| 3.43.6 Sympy [A] (verification not implemented) . . . . .  | 328 |
| 3.43.7 Maxima [F] . . . . .                                | 328 |
| 3.43.8 Giac [A] (verification not implemented) . . . . .   | 328 |
| 3.43.9 Mupad [B] (verification not implemented) . . . . .  | 329 |

#### 3.43.1 Optimal result

Integrand size = 12, antiderivative size = 67

$$\int \frac{1}{1 + 4x^2 + x^4} dx = \frac{\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}(2-\sqrt{3})} - \frac{\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}(2+\sqrt{3})}$$

output `1/2*arctan(x/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/2*arctan(x/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))`

#### 3.43.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{1}{1 + 4x^2 + x^4} dx = \frac{\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}(2-\sqrt{3})} - \frac{\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}(2+\sqrt{3})}$$

input `Integrate[(1 + 4*x^2 + x^4)^(-1), x]`

output `ArcTan[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[x/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])`

### 3.43.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1406, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 + 4x^2 + 1} dx$$

↓ 1406

$$\frac{\int \frac{1}{x^2 - \sqrt{3} + 2} dx}{2\sqrt{3}} - \frac{\int \frac{1}{x^2 + \sqrt{3} + 2} dx}{2\sqrt{3}}$$

↓ 216

$$\frac{\arctan\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3}(2-\sqrt{3})} - \frac{\arctan\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}(2+\sqrt{3})}$$

input `Int[(1 + 4*x^2 + x^4)^(-1),x]`

output `ArcTan[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[x/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])`

#### 3.43.3.1 Defintions of rubi rules used

rule 216 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

rule 1406 `Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]`

### 3.43.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.49

| method  | result                                                                                                                                                                           | size |
|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch   | $\frac{\left( \sum_{R=\text{RootOf}(-Z^4+4Z^2+1)} \frac{\ln(x-R)}{R^3+2R} \right)}{4}$                                                                                           | 33   |
| default | $\frac{\sqrt{3} \arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}} - \frac{\sqrt{3} \arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3(\sqrt{6}+\sqrt{2})}$ | 60   |

input `int(1/(x^4+4*x^2+1),x,method=_RETURNVERBOSE)`

output `1/4*sum(1/(_R^3+2*_R)*ln(x-_R),_R=RootOf(_Z^4+4*_Z^2+1))`

### 3.43.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(55) = 110.

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.84

$$\begin{aligned} \int \frac{1}{1+4x^2+x^4} dx = & -\frac{1}{12} \sqrt{3} \sqrt{\sqrt{3}-2} \log\left(\left(\sqrt{3}+2\right) \sqrt{\sqrt{3}-2+x}\right) \\ & + \frac{1}{12} \sqrt{3} \sqrt{\sqrt{3}-2} \log\left(-\left(\sqrt{3}+2\right) \sqrt{\sqrt{3}-2+x}\right) \\ & - \frac{1}{12} \sqrt{3} \sqrt{-\sqrt{3}-2} \log\left(\left(\sqrt{3}-2\right) \sqrt{-\sqrt{3}-2+x}\right) \\ & + \frac{1}{12} \sqrt{3} \sqrt{-\sqrt{3}-2} \log\left(-\left(\sqrt{3}-2\right) \sqrt{-\sqrt{3}-2+x}\right) \end{aligned}$$

input `integrate(1/(x^4+4*x^2+1),x, algorithm="fracas")`

output `-1/12*sqrt(3)*sqrt(sqrt(3) - 2)*log((sqrt(3) + 2)*sqrt(sqrt(3) - 2) + x) +  
1/12*sqrt(3)*sqrt(sqrt(3) - 2)*log(-(sqrt(3) + 2)*sqrt(sqrt(3) - 2) + x)  
- 1/12*sqrt(3)*sqrt(-sqrt(3) - 2)*log((sqrt(3) - 2)*sqrt(-sqrt(3) - 2) + x)  
) + 1/12*sqrt(3)*sqrt(-sqrt(3) - 2)*log(-(sqrt(3) - 2)*sqrt(-sqrt(3) - 2)  
+ x)`



**3.43.6 Sympy [A] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37

$$\int \frac{1}{1+4x^2+x^4} dx = -2\sqrt{\frac{1}{24} - \frac{\sqrt{3}}{48}} \operatorname{atan}\left(\frac{x}{\sqrt{3}\sqrt{2-\sqrt{3}}+2\sqrt{2-\sqrt{3}}}\right) - 2\sqrt{\frac{\sqrt{3}}{48} + \frac{1}{24}} \operatorname{atan}\left(\frac{x}{-2\sqrt{\sqrt{3}+2}+\sqrt{3}\sqrt{\sqrt{3}+2}}\right)$$

input `integrate(1/(x**4+4*x**2+1),x)`output `-2*sqrt(1/24 - sqrt(3)/48)*atan(x/(sqrt(3)*sqrt(2 - sqrt(3)) + 2*sqrt(2 - sqrt(3)))) - 2*sqrt(sqrt(3)/48 + 1/24)*atan(x/(-2*sqrt(sqrt(3) + 2) + sqrt(3)*sqrt(sqrt(3) + 2)))`**3.43.7 Maxima [F]**

$$\int \frac{1}{1+4x^2+x^4} dx = \int \frac{1}{x^4+4x^2+1} dx$$

input `integrate(1/(x^4+4*x^2+1),x, algorithm="maxima")`output `integrate(1/(x^4 + 4*x^2 + 1), x)`**3.43.8 Giac [A] (verification not implemented)**

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+4x^2+x^4} dx = \frac{1}{12} (\sqrt{6} - 3\sqrt{2}) \arctan\left(\frac{2x}{\sqrt{6} + \sqrt{2}}\right) + \frac{1}{12} (\sqrt{6} + 3\sqrt{2}) \arctan\left(\frac{2x}{\sqrt{6} - \sqrt{2}}\right)$$

input `integrate(1/(x^4+4*x^2+1),x, algorithm="giac")`output `1/12*(sqrt(6) - 3*sqrt(2))*arctan(2*x/(sqrt(6) + sqrt(2))) + 1/12*(sqrt(6) + 3*sqrt(2))*arctan(2*x/(sqrt(6) - sqrt(2)))`

**3.43.9 Mupad [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.75

$$\int \frac{1}{1+4x^2+x^4} dx = 2 \operatorname{atanh} \left( \frac{24x \sqrt{\frac{\sqrt{3}}{48} - \frac{1}{24}}}{2\sqrt{3}-4} - \frac{16\sqrt{3}x \sqrt{\frac{\sqrt{3}}{48} - \frac{1}{24}}}{2\sqrt{3}-4} \right) \sqrt{\frac{\sqrt{3}}{48} - \frac{1}{24}} \\ - 2 \operatorname{atanh} \left( \frac{24x \sqrt{-\frac{\sqrt{3}}{48} - \frac{1}{24}}}{2\sqrt{3}+4} + \frac{16\sqrt{3}x \sqrt{-\frac{\sqrt{3}}{48} - \frac{1}{24}}}{2\sqrt{3}+4} \right) \sqrt{-\frac{\sqrt{3}}{48} - \frac{1}{24}}$$

input `int(1/(4*x^2 + x^4 + 1),x)`output `2*atanh((24*x*(3^(1/2)/48 - 1/24)^(1/2))/(2*3^(1/2) - 4) - (16*3^(1/2)*x*(3^(1/2)/48 - 1/24)^(1/2))/(2*3^(1/2) - 4))*(3^(1/2)/48 - 1/24)^(1/2) - 2*atanh((24*x*(- 3^(1/2)/48 - 1/24)^(1/2))/(2*3^(1/2) + 4) + (16*3^(1/2)*x*(- 3^(1/2)/48 - 1/24)^(1/2))/(2*3^(1/2) + 4))*(- 3^(1/2)/48 - 1/24)^(1/2)`

### 3.44 $\int \frac{1}{2+x^2+x^4} dx$

|        |                                           |     |
|--------|-------------------------------------------|-----|
| 3.44.1 | Optimal result                            | 330 |
| 3.44.2 | Mathematica [C] (verified)                | 330 |
| 3.44.3 | Rubi [A] (verified)                       | 331 |
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| 3.44.8 | Giac [A] (verification not implemented)   | 337 |
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#### 3.44.1 Optimal result

Integrand size = 10, antiderivative size = 196

$$\int \frac{1}{2+x^2+x^4} dx = -\frac{1}{2} \sqrt{\frac{1}{14}(-1+2\sqrt{2})} \arctan\left(\frac{\sqrt{-1+2\sqrt{2}-2x}}{\sqrt{1+2\sqrt{2}}}\right) + \frac{1}{2} \sqrt{\frac{1}{14}(-1+2\sqrt{2})} \arctan\left(\frac{\sqrt{-1+2\sqrt{2}+2x}}{\sqrt{1+2\sqrt{2}}}\right) - \frac{\log(\sqrt{2}-\sqrt{-1+2\sqrt{2}x+x^2})}{4\sqrt{2}(-1+2\sqrt{2})} + \frac{\log(\sqrt{2}+\sqrt{-1+2\sqrt{2}x+x^2})}{4\sqrt{2}(-1+2\sqrt{2})}$$

output 
$$-1/28*\arctan((-2*x+(-1+2*2^{(1/2)})^{(1/2)})/(1+2*2^{(1/2)})^{(1/2)})*(-14+28*2^{(1/2)})^{(1/2)}+1/28*\arctan((2*x+(-1+2*2^{(1/2)})^{(1/2)})/(1+2*2^{(1/2)})^{(1/2)})*(-14+28*2^{(1/2)})^{(1/2)}-1/4*\ln(x^2+2^{(1/2)}-x*(-1+2*2^{(1/2)})^{(1/2)})/(-2+4*2^{(1/2)})^{(1/2)}+1/4*\ln(x^2+2^{(1/2)}+x*(-1+2*2^{(1/2)})^{(1/2)})/(-2+4*2^{(1/2)})^{(1/2)}$$

#### 3.44.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.46

$$\int \frac{1}{2+x^2+x^4} dx = -\frac{i \arctan\left(\frac{x}{\sqrt{\frac{1}{2}(1-i\sqrt{7})}}\right)}{\sqrt{\frac{7}{2}(1-i\sqrt{7})}} + \frac{i \arctan\left(\frac{x}{\sqrt{\frac{1}{2}(1+i\sqrt{7})}}\right)}{\sqrt{\frac{7}{2}(1+i\sqrt{7})}}$$

input `Integrate[(2 + x^2 + x^4)^(-1),x]`

output `((-I)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/Sqrt[(7*(1 - I*Sqrt[7]))/2] + (I*ArcTan[x/Sqrt[(1 + I*Sqrt[7])/2]])/Sqrt[(7*(1 + I*Sqrt[7]))/2]`

### 3.44.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{x^4 + x^2 + 2} dx \\
 & \quad \downarrow \text{1407} \\
 & \int \frac{\frac{\sqrt{-1+2\sqrt{2}}-x}{x^2-\sqrt{-1+2\sqrt{2}x+\sqrt{2}}}}{2\sqrt{2}(2\sqrt{2}-1)} dx + \int \frac{\frac{x+\sqrt{-1+2\sqrt{2}}}{x^2+\sqrt{-1+2\sqrt{2}x+\sqrt{2}}}}{2\sqrt{2}(2\sqrt{2}-1)} dx \\
 & \quad \downarrow \text{1142} \\
 & \frac{\frac{1}{2}\sqrt{2\sqrt{2}-1} \int \frac{1}{x^2-\sqrt{-1+2\sqrt{2}x+\sqrt{2}}} dx - \frac{1}{2} \int \frac{\sqrt{-1+2\sqrt{2}}-2x}{x^2-\sqrt{-1+2\sqrt{2}x+\sqrt{2}}} dx}{2\sqrt{2}(2\sqrt{2}-1)} + \\
 & \frac{\frac{1}{2}\sqrt{2\sqrt{2}-1} \int \frac{1}{x^2+\sqrt{-1+2\sqrt{2}x+\sqrt{2}}} dx + \frac{1}{2} \int \frac{2x+\sqrt{-1+2\sqrt{2}}}{x^2+\sqrt{-1+2\sqrt{2}x+\sqrt{2}}} dx}{2\sqrt{2}(2\sqrt{2}-1)} \\
 & \quad \downarrow \text{25} \\
 & \frac{\frac{1}{2}\sqrt{2\sqrt{2}-1} \int \frac{1}{x^2-\sqrt{-1+2\sqrt{2}x+\sqrt{2}}} dx + \frac{1}{2} \int \frac{\sqrt{-1+2\sqrt{2}}-2x}{x^2-\sqrt{-1+2\sqrt{2}x+\sqrt{2}}} dx}{2\sqrt{2}(2\sqrt{2}-1)} + \\
 & \frac{\frac{1}{2}\sqrt{2\sqrt{2}-1} \int \frac{1}{x^2+\sqrt{-1+2\sqrt{2}x+\sqrt{2}}} dx + \frac{1}{2} \int \frac{2x+\sqrt{-1+2\sqrt{2}}}{x^2+\sqrt{-1+2\sqrt{2}x+\sqrt{2}}} dx}{2\sqrt{2}(2\sqrt{2}-1)} \\
 & \quad \downarrow \text{1083}
 \end{aligned}$$

$$\begin{aligned}
& \frac{\frac{1}{2} \int \frac{\sqrt{-1+2\sqrt{2}}-2x}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}} dx - \sqrt{2\sqrt{2}-1} \int \frac{1}{-(2x-\sqrt{-1+2\sqrt{2}})^2-2\sqrt{2}-1} d(2x-\sqrt{-1+2\sqrt{2}})}{2\sqrt{2}(2\sqrt{2}-1)} + \\
& \frac{\frac{1}{2} \int \frac{2x+\sqrt{-1+2\sqrt{2}}}{x^2+\sqrt{-1+2\sqrt{2}}x+\sqrt{2}} dx - \sqrt{2\sqrt{2}-1} \int \frac{1}{-(2x+\sqrt{-1+2\sqrt{2}})^2-2\sqrt{2}-1} d(2x+\sqrt{-1+2\sqrt{2}})}{2\sqrt{2}(2\sqrt{2}-1)} \\
& \quad \downarrow \text{217} \\
& \frac{\frac{1}{2} \int \frac{\sqrt{-1+2\sqrt{2}}-2x}{x^2-\sqrt{-1+2\sqrt{2}}x+\sqrt{2}} dx + \sqrt{\frac{2\sqrt{2}-1}{1+2\sqrt{2}}} \arctan\left(\frac{2x-\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2}(2\sqrt{2}-1)} + \\
& \frac{\frac{1}{2} \int \frac{2x+\sqrt{-1+2\sqrt{2}}}{x^2+\sqrt{-1+2\sqrt{2}}x+\sqrt{2}} dx + \sqrt{\frac{2\sqrt{2}-1}{1+2\sqrt{2}}} \arctan\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2}(2\sqrt{2}-1)} \\
& \quad \downarrow \text{1103} \\
& \frac{\sqrt{\frac{2\sqrt{2}-1}{1+2\sqrt{2}}} \arctan\left(\frac{2x-\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right) - \frac{1}{2} \log\left(x^2 - \sqrt{2\sqrt{2}-1}x + \sqrt{2}\right)}{2\sqrt{2}(2\sqrt{2}-1)} + \\
& \frac{\sqrt{\frac{2\sqrt{2}-1}{1+2\sqrt{2}}} \arctan\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right) + \frac{1}{2} \log\left(x^2 + \sqrt{2\sqrt{2}-1}x + \sqrt{2}\right)}{2\sqrt{2}(2\sqrt{2}-1)}
\end{aligned}$$

input `Int[(2 + x^2 + x^4)^(-1),x]`

output `(Sqrt[(-1 + 2*Sqrt[2])/(1 + 2*Sqrt[2])]*ArcTan[(-Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]]] - Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2/2]/(2*Sqrt[2*(-1 + 2*Sqrt[2])])) + (Sqrt[(-1 + 2*Sqrt[2])/(1 + 2*Sqrt[2])]*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]]] + Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2/2]/(2*Sqrt[2*(-1 + 2*Sqrt[2])]))`

## 3.44.3.1 Defintions of rubi rules used

- rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`
- rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])`
- rule 1083 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Simp[-2 Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x]`
- rule 1103 `Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`
- rule 1142 `Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(2*c*d - b*e)/(2*c) Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c) Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]`
- rule 1407 `Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r) Int[(r - x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r) Int[(r + x)/(q + r*x + x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]`

### 3.44.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.16

| method  | result                                                                                                                                                                                                                                                                                                             |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch   | $\frac{\sum_{-R=\text{RootOf}(-Z^4+Z^2+2)} \frac{\ln(x-R)}{2R^3+R}}{2}$                                                                                                                                                                                                                                            |
| default | $\frac{(-\sqrt{-1+2\sqrt{2}}\sqrt{2}-4\sqrt{-1+2\sqrt{2}})\ln(x^2+\sqrt{2}-x\sqrt{-1+2\sqrt{2}})}{56} + \frac{\left(7\sqrt{2}+\frac{(-\sqrt{-1+2\sqrt{2}}\sqrt{2}-4\sqrt{-1+2\sqrt{2}})\sqrt{-1+2\sqrt{2}}}{2}\right)\arctan\left(\frac{2x-\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right)}{14\sqrt{1+2\sqrt{2}}}$ |

input `int(1/(x^4+x^2+2),x,method=_RETURNVERBOSE)`

output `1/2*sum(1/(2*_R^3+_R)*ln(x-_R),_R=RootOf(-Z^4+Z^2+2))`

### 3.44.5 Fracas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.71

$$\begin{aligned} \int \frac{1}{2+x^2+x^4} dx &= \frac{1}{28} \sqrt{7} \sqrt{i\sqrt{7}+1} \log\left(\left(\sqrt{7}+i\right)\sqrt{i\sqrt{7}+1+4x}\right) \\ &\quad - \frac{1}{28} \sqrt{7} \sqrt{i\sqrt{7}+1} \log\left(-\left(\sqrt{7}+i\right)\sqrt{i\sqrt{7}+1+4x}\right) \\ &\quad + \frac{1}{28} \sqrt{7} \sqrt{-i\sqrt{7}+1} \log\left(\left(\sqrt{7}-i\right)\sqrt{-i\sqrt{7}+1+4x}\right) \\ &\quad - \frac{1}{28} \sqrt{7} \sqrt{-i\sqrt{7}+1} \log\left(-\left(\sqrt{7}-i\right)\sqrt{-i\sqrt{7}+1+4x}\right) \end{aligned}$$

input `integrate(1/(x^4+x^2+2),x, algorithm="fracas")`

output `1/28*sqrt(7)*sqrt(I*sqrt(7)+1)*log((sqrt(7)+I)*sqrt(I*sqrt(7)+1)+4*x) - 1/28*sqrt(7)*sqrt(I*sqrt(7)+1)*log(-(sqrt(7)+I)*sqrt(I*sqrt(7)+1)+4*x) + 1/28*sqrt(7)*sqrt(-I*sqrt(7)+1)*log((sqrt(7)-I)*sqrt(-I*sqrt(7)+1)+4*x) - 1/28*sqrt(7)*sqrt(-I*sqrt(7)+1)*log(-(sqrt(7)-I)*sqrt(-I*sqrt(7)+1)+4*x)`

**3.44.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 994 vs.  $2(158) = 316$ .

Time = 0.68 (sec) , antiderivative size = 994, normalized size of antiderivative = 5.07

$$\int \frac{1}{2+x^2+x^4} dx$$

$$= \sqrt{\frac{1}{224} + \frac{\sqrt{2}}{112}} \log \left( x^2 + x \left( -\frac{4\sqrt{7}\sqrt{1+2\sqrt{2}}}{7} + \frac{5\sqrt{14}\sqrt{1+2\sqrt{2}}}{28} + \frac{3\sqrt{14}\sqrt{1+2\sqrt{2}}\sqrt{4\sqrt{2}+9}}{28} \right) - \frac{33\sqrt{2}}{224} \right)$$

$$- \sqrt{\frac{1}{224} + \frac{\sqrt{2}}{112}} \log \left( x^2 + x \left( -\frac{3\sqrt{14}\sqrt{1+2\sqrt{2}}\sqrt{4\sqrt{2}+9}}{28} - \frac{5\sqrt{14}\sqrt{1+2\sqrt{2}}}{28} + \frac{4\sqrt{7}\sqrt{1+2\sqrt{2}}}{7} \right) - \frac{33\sqrt{2}}{224} \right)$$

$$+ 2\sqrt{-\frac{\sqrt{4\sqrt{2}+9}}{112} + \frac{1}{224} + \frac{3\sqrt{2}}{112}} \operatorname{atan} \left( \frac{4\sqrt{14}x}{\sqrt{4\sqrt{2}+9}\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}+7\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}} \right)$$

$$+ 2\sqrt{-\frac{\sqrt{4\sqrt{2}+9}}{112} + \frac{1}{224} + \frac{3\sqrt{2}}{112}} \operatorname{atan} \left( \frac{4\sqrt{14}x}{\sqrt{4\sqrt{2}+9}\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}+7\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}} \right)$$

input `integrate(1/(x**4+x**2+2),x)`



output `sqrt(1/224 + sqrt(2)/112)*log(x**2 + x*(-4*sqrt(7)*sqrt(1 + 2*sqrt(2)))/7 + 5*sqrt(14)*sqrt(1 + 2*sqrt(2)))/28 + 3*sqrt(14)*sqrt(1 + 2*sqrt(2))*sqrt(4*sqrt(2) + 9)/28 - 33*sqrt(4*sqrt(2) + 9)/28 - 11/28 + 11*sqrt(2)*sqrt(4*sqrt(2) + 9)/28 + 83*sqrt(2)/28 - sqrt(1/224 + sqrt(2)/112)*log(x**2 + x*(-3*sqrt(14)*sqrt(1 + 2*sqrt(2)))*sqrt(4*sqrt(2) + 9)/28 - 5*sqrt(14)*sqrt(1 + 2*sqrt(2)))/28 + 4*sqrt(7)*sqrt(1 + 2*sqrt(2)))/7 - 33*sqrt(4*sqrt(2) + 9)/28 - 11/28 + 11*sqrt(2)*sqrt(4*sqrt(2) + 9)/28 + 83*sqrt(2)/28) + 2*sqrt(-sqrt(4*sqrt(2) + 9)/112 + 1/224 + 3*sqrt(2)/112)*atan(4*sqrt(14)*x/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)))) - 8*sqrt(2)*sqrt(1 + 2*sqrt(2))/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2))) + 5*sqrt(1 + 2*sqrt(2))/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2))) + 3*sqrt(1 + 2*sqrt(2))*sqrt(4*sqrt(2) + 9)/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)))) + 2*sqrt(-sqrt(4*sqrt(2) + 9)/112 + 1/224 + 3*sqrt(2)/112)*atan(4*sqrt(14)*x/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)))) - 3*sqrt(1 + 2*sqrt(2))*sqrt(4*sqrt(2) + 9)/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)))`

### 3.44.7 Maxima [F]

$$\int \frac{1}{2 + x^2 + x^4} dx = \int \frac{1}{x^4 + x^2 + 2} dx$$

input `integrate(1/(x^4+x^2+2),x, algorithm="maxima")`

output `integrate(1/(x^4 + x^2 + 2), x)`

**3.44.8 Giac [A] (verification not implemented)**

Time = 0.41 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.29

$$\begin{aligned}
& \int \frac{1}{2+x^2+x^4} dx \\
&= \frac{1}{112} \sqrt{7} \left( \sqrt{7} 2^{\frac{1}{4}} \sqrt{2\sqrt{2}+8} - 2^{\frac{1}{4}} \sqrt{-2\sqrt{2}+8} \right) \arctan \left( \frac{2 \cdot 2^{\frac{3}{4}} \sqrt{\frac{1}{2}} \left( x + 2^{\frac{1}{4}} \sqrt{-\frac{1}{8}\sqrt{2} + \frac{1}{2}} \right)}{\sqrt{\sqrt{2}+4}} \right) \\
&+ \frac{1}{112} \sqrt{7} \left( \sqrt{7} 2^{\frac{1}{4}} \sqrt{2\sqrt{2}+8} - 2^{\frac{1}{4}} \sqrt{-2\sqrt{2}+8} \right) \arctan \left( \frac{2 \cdot 2^{\frac{3}{4}} \sqrt{\frac{1}{2}} \left( x - 2^{\frac{1}{4}} \sqrt{-\frac{1}{8}\sqrt{2} + \frac{1}{2}} \right)}{\sqrt{\sqrt{2}+4}} \right) \\
&+ \frac{1}{224} \sqrt{7} \left( \sqrt{7} 2^{\frac{1}{4}} \sqrt{-2\sqrt{2}+8} + 2^{\frac{1}{4}} \sqrt{2\sqrt{2}+8} \right) \log \left( x^2 + 2 \cdot 2^{\frac{1}{4}} x \sqrt{-\frac{1}{8}\sqrt{2} + \frac{1}{2}} + \sqrt{2} \right) \\
&- \frac{1}{224} \sqrt{7} \left( \sqrt{7} 2^{\frac{1}{4}} \sqrt{-2\sqrt{2}+8} + 2^{\frac{1}{4}} \sqrt{2\sqrt{2}+8} \right) \log \left( x^2 - 2 \cdot 2^{\frac{1}{4}} x \sqrt{-\frac{1}{8}\sqrt{2} + \frac{1}{2}} + \sqrt{2} \right)
\end{aligned}$$

input `integrate(1/(x^4+x^2+2),x, algorithm="giac")`

```

output 1/112*sqrt(7)*(sqrt(7)*2^(1/4)*sqrt(2*sqrt(2) + 8) - 2^(1/4)*sqrt(-2*sqrt(
2) + 8))*arctan(2*2^(3/4)*sqrt(1/2)*(x + 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))
/sqrt(sqrt(2) + 4)) + 1/112*sqrt(7)*(sqrt(7)*2^(1/4)*sqrt(2*sqrt(2) + 8) -
2^(1/4)*sqrt(-2*sqrt(2) + 8))*arctan(2*2^(3/4)*sqrt(1/2)*(x - 2^(1/4)*sqr
t(-1/8*sqrt(2) + 1/2))/sqrt(sqrt(2) + 4)) + 1/224*sqrt(7)*(sqrt(7)*2^(1/4)
*sqrt(-2*sqrt(2) + 8) + 2^(1/4)*sqrt(2*sqrt(2) + 8))*log(x^2 + 2*2^(1/4)*x
*sqrt(-1/8*sqrt(2) + 1/2) + sqrt(2)) - 1/224*sqrt(7)*(sqrt(7)*2^(1/4)*sqrt
(-2*sqrt(2) + 8) + 2^(1/4)*sqrt(2*sqrt(2) + 8))*log(x^2 - 2*2^(1/4)*x*sqrt
(-1/8*sqrt(2) + 1/2) + sqrt(2))

```

**3.44.9 Mupad [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.31

$$\int \frac{1}{2 + x^2 + x^4} dx = \frac{\operatorname{atan}\left(\frac{\sqrt{7}x\sqrt{7-\sqrt{7}7i}}{14}\right) \sqrt{7-\sqrt{7}7i} \operatorname{li} - \sqrt{7} \operatorname{atan}\left(\frac{x\sqrt{1+\sqrt{7}7i}}{2}\right) \sqrt{1+\sqrt{7}7i} \operatorname{li}}{14}$$

input `int(1/(x^2 + x^4 + 2),x)`

output `(atan((7^(1/2)*x*(7 - 7^(1/2)*7i)^(1/2))/14)*(7 - 7^(1/2)*7i)^(1/2)*1i)/14  
- (7^(1/2)*atan((x*(7^(1/2)*1i + 1)^(1/2))/2)*(7^(1/2)*1i + 1)^(1/2)*1i)/  
14`

### 3.45 $\int \frac{1}{2-x^2+x^4} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.45.1 | Optimal result . . . . .                            | 339 |
| 3.45.2 | Mathematica [C] (verified) . . . . .                | 339 |
| 3.45.3 | Rubi [A] (verified) . . . . .                       | 340 |
| 3.45.4 | Maple [C] (verified) . . . . .                      | 343 |
| 3.45.5 | Fricas [C] (verification not implemented) . . . . . | 343 |
| 3.45.6 | Sympy [A] (verification not implemented) . . . . .  | 344 |
| 3.45.7 | Maxima [F] . . . . .                                | 344 |
| 3.45.8 | Giac [A] (verification not implemented) . . . . .   | 344 |
| 3.45.9 | Mupad [B] (verification not implemented) . . . . .  | 345 |

#### 3.45.1 Optimal result

Integrand size = 12, antiderivative size = 196

$$\int \frac{1}{2-x^2+x^4} dx = -\frac{1}{2}\sqrt{\frac{1}{14}}(1+2\sqrt{2}) \arctan\left(\frac{\sqrt{1+2\sqrt{2}}-2x}{\sqrt{-1+2\sqrt{2}}}\right) + \frac{1}{2}\sqrt{\frac{1}{14}}(1+2\sqrt{2}) \arctan\left(\frac{\sqrt{1+2\sqrt{2}}+2x}{\sqrt{-1+2\sqrt{2}}}\right) - \frac{\log(\sqrt{2}-\sqrt{1+2\sqrt{2}}x+x^2)}{4\sqrt{2}(1+2\sqrt{2})} + \frac{\log(\sqrt{2}+\sqrt{1+2\sqrt{2}}x+x^2)}{4\sqrt{2}(1+2\sqrt{2})}$$

output `-1/28*arctan((-2*x+(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*(14+28*2^(1/2))^(1/2)+1/28*arctan((2*x+(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*(14+28*2^(1/2))^(1/2)-1/4*ln(x^2+2^(1/2)-x*(1+2*2^(1/2))^(1/2))/(2+4*2^(1/2))^(1/2)+1/4*ln(x^2+2^(1/2)+x*(1+2*2^(1/2))^(1/2))/(2+4*2^(1/2))^(1/2)`

#### 3.45.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.46

$$\int \frac{1}{2-x^2+x^4} dx = -\frac{i \arctan\left(\frac{x}{\sqrt{\frac{1}{2}}(-1-i\sqrt{7})}\right)}{\sqrt{\frac{7}{2}}(-1-i\sqrt{7})} + \frac{i \arctan\left(\frac{x}{\sqrt{\frac{1}{2}}(-1+i\sqrt{7})}\right)}{\sqrt{\frac{7}{2}}(-1+i\sqrt{7})}$$

input `Integrate[(2 - x^2 + x^4)^(-1),x]`

output `((-I)*ArcTan[x/Sqrt[(-1 - I*Sqrt[7])/2]])/Sqrt[(7*(-1 - I*Sqrt[7]))/2] + (I*ArcTan[x/Sqrt[(-1 + I*Sqrt[7])/2]])/Sqrt[(7*(-1 + I*Sqrt[7]))/2]`

### 3.45.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{x^4 - x^2 + 2} dx \\
 & \quad \downarrow \text{1407} \\
 & \frac{\int \frac{\sqrt{1+2\sqrt{2}}-x}{x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}} dx}{2\sqrt{2}(1+2\sqrt{2})} + \frac{\int \frac{x+\sqrt{1+2\sqrt{2}}}{x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}} dx}{2\sqrt{2}(1+2\sqrt{2})} \\
 & \quad \downarrow \text{1142} \\
 & \frac{\frac{1}{2}\sqrt{1+2\sqrt{2}} \int \frac{1}{x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}} dx - \frac{1}{2} \int \frac{\sqrt{1+2\sqrt{2}}-2x}{x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}} dx}{2\sqrt{2}(1+2\sqrt{2})} + \\
 & \frac{\frac{1}{2}\sqrt{1+2\sqrt{2}} \int \frac{1}{x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}} dx + \frac{1}{2} \int \frac{2x+\sqrt{1+2\sqrt{2}}}{x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}} dx}{2\sqrt{2}(1+2\sqrt{2})} \\
 & \quad \downarrow \text{25} \\
 & \frac{\frac{1}{2}\sqrt{1+2\sqrt{2}} \int \frac{1}{x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}} dx + \frac{1}{2} \int \frac{\sqrt{1+2\sqrt{2}}-2x}{x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}} dx}{2\sqrt{2}(1+2\sqrt{2})} + \\
 & \frac{\frac{1}{2}\sqrt{1+2\sqrt{2}} \int \frac{1}{x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}} dx + \frac{1}{2} \int \frac{2x+\sqrt{1+2\sqrt{2}}}{x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}} dx}{2\sqrt{2}(1+2\sqrt{2})} \\
 & \quad \downarrow \text{1083}
 \end{aligned}$$

$$\begin{aligned}
& \frac{\frac{1}{2} \int \frac{\sqrt{1+2\sqrt{2}}-2x}{x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}} dx - \sqrt{1+2\sqrt{2}} \int \frac{1}{-(2x-\sqrt{1+2\sqrt{2}})^2-2\sqrt{2}+1} d(2x-\sqrt{1+2\sqrt{2}})}{2\sqrt{2}(1+2\sqrt{2})} + \\
& \frac{\frac{1}{2} \int \frac{2x+\sqrt{1+2\sqrt{2}}}{x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}} dx - \sqrt{1+2\sqrt{2}} \int \frac{1}{-(2x+\sqrt{1+2\sqrt{2}})^2-2\sqrt{2}+1} d(2x+\sqrt{1+2\sqrt{2}})}{2\sqrt{2}(1+2\sqrt{2})} \\
& \quad \downarrow \text{217} \\
& \frac{\frac{1}{2} \int \frac{\sqrt{1+2\sqrt{2}}-2x}{x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}} dx + \sqrt{\frac{1+2\sqrt{2}}{2\sqrt{2}-1}} \arctan\left(\frac{2x-\sqrt{1+2\sqrt{2}}}{\sqrt{2\sqrt{2}-1}}\right)}{2\sqrt{2}(1+2\sqrt{2})} + \\
& \frac{\frac{1}{2} \int \frac{2x+\sqrt{1+2\sqrt{2}}}{x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}} dx + \sqrt{\frac{1+2\sqrt{2}}{2\sqrt{2}-1}} \arctan\left(\frac{2x+\sqrt{1+2\sqrt{2}}}{\sqrt{2\sqrt{2}-1}}\right)}{2\sqrt{2}(1+2\sqrt{2})} \\
& \quad \downarrow \text{1103} \\
& \frac{\sqrt{\frac{1+2\sqrt{2}}{2\sqrt{2}-1}} \arctan\left(\frac{2x-\sqrt{1+2\sqrt{2}}}{\sqrt{2\sqrt{2}-1}}\right) - \frac{1}{2} \log\left(x^2 - \sqrt{1+2\sqrt{2}}x + \sqrt{2}\right)}{2\sqrt{2}(1+2\sqrt{2})} + \\
& \frac{\sqrt{\frac{1+2\sqrt{2}}{2\sqrt{2}-1}} \arctan\left(\frac{2x+\sqrt{1+2\sqrt{2}}}{\sqrt{2\sqrt{2}-1}}\right) + \frac{1}{2} \log\left(x^2 + \sqrt{1+2\sqrt{2}}x + \sqrt{2}\right)}{2\sqrt{2}(1+2\sqrt{2})}
\end{aligned}$$

input `Int[(2 - x^2 + x^4)^(-1),x]`

output `(Sqrt[(1 + 2*Sqrt[2])/(-1 + 2*Sqrt[2])]*ArcTan[(-Sqrt[1 + 2*Sqrt[2]] + 2*x)/Sqrt[-1 + 2*Sqrt[2]]] - Log[Sqrt[2] - Sqrt[1 + 2*Sqrt[2]]*x + x^2]/(2*Sqrt[2*(1 + 2*Sqrt[2])])) + (Sqrt[(1 + 2*Sqrt[2])/(-1 + 2*Sqrt[2])]*ArcTan[(Sqrt[1 + 2*Sqrt[2]] + 2*x)/Sqrt[-1 + 2*Sqrt[2]]] + Log[Sqrt[2] + Sqrt[1 + 2*Sqrt[2]]*x + x^2]/(2*Sqrt[2*(1 + 2*Sqrt[2])]))`

## 3.45.3.1 Defintions of rubi rules used

- rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`
- rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])`
- rule 1083 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Simp[-2 Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x]`
- rule 1103 `Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`
- rule 1142 `Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(2*c*d - b*e)/(2*c) Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c) Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]`
- rule 1407 `Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r) Int[(r - x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r) Int[(r + x)/(q + r*x + x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]`

### 3.45.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.18

| method  | result                                                                                                                                                                                                                                                                                                              |
|---------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch   | $\frac{\left( \sum_{R=\text{RootOf}(\_Z^4-\_Z^2+2)} \frac{\ln(x-R)}{2R^3-R} \right)}{2}$                                                                                                                                                                                                                            |
| default | $\frac{(\sqrt{1+2\sqrt{2}}\sqrt{2}-4\sqrt{1+2\sqrt{2}})\ln(x^2+\sqrt{2}-x\sqrt{1+2\sqrt{2}})}{56} + \frac{\left(7\sqrt{2}+\frac{(\sqrt{1+2\sqrt{2}}\sqrt{2}-4\sqrt{1+2\sqrt{2}})\sqrt{1+2\sqrt{2}}}{2}\right)\arctan\left(\frac{2x-\sqrt{1+2\sqrt{2}}}{\sqrt{-1+2\sqrt{2}}}\right)}{14\sqrt{-1+2\sqrt{2}}} + \dots$ |

input `int(1/(x^4-x^2+2),x,method=_RETURNVERBOSE)`

output `1/2*sum(1/(2*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^4-_Z^2+2))`

### 3.45.5 Fracas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.71

$$\begin{aligned} \int \frac{1}{2-x^2+x^4} dx &= \frac{1}{28} \sqrt{7} \sqrt{i\sqrt{7}-1} \log\left(\left(\sqrt{7}-i\right)\sqrt{i\sqrt{7}-1+4x}\right) \\ &\quad - \frac{1}{28} \sqrt{7} \sqrt{i\sqrt{7}-1} \log\left(-\left(\sqrt{7}-i\right)\sqrt{i\sqrt{7}-1+4x}\right) \\ &\quad + \frac{1}{28} \sqrt{7} \sqrt{-i\sqrt{7}-1} \log\left(\left(\sqrt{7}+i\right)\sqrt{-i\sqrt{7}-1+4x}\right) \\ &\quad - \frac{1}{28} \sqrt{7} \sqrt{-i\sqrt{7}-1} \log\left(-\left(\sqrt{7}+i\right)\sqrt{-i\sqrt{7}-1+4x}\right) \end{aligned}$$

input `integrate(1/(x^4-x^2+2),x, algorithm="fracas")`

output `1/28*sqrt(7)*sqrt(I*sqrt(7) - 1)*log((sqrt(7) - I)*sqrt(I*sqrt(7) - 1) + 4*x) - 1/28*sqrt(7)*sqrt(I*sqrt(7) - 1)*log(-(sqrt(7) - I)*sqrt(I*sqrt(7) - 1) + 4*x) + 1/28*sqrt(7)*sqrt(-I*sqrt(7) - 1)*log((sqrt(7) + I)*sqrt(-I*sqrt(7) - 1) + 4*x) - 1/28*sqrt(7)*sqrt(-I*sqrt(7) - 1)*log(-(sqrt(7) + I)*sqrt(-I*sqrt(7) - 1) + 4*x)`



**3.45.6 Sympy [A] (verification not implemented)**

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.12

$$\int \frac{1}{2 - x^2 + x^4} dx = \text{RootSum}(1568t^4 + 28t^2 + 1, (t \mapsto t \log(-112t^3 + 6t + x)))$$

input `integrate(1/(x**4-x**2+2),x)`output `RootSum(1568*_t**4 + 28*_t**2 + 1, Lambda(_t, _t*log(-112*_t**3 + 6*_t + x)))`**3.45.7 Maxima [F]**

$$\int \frac{1}{2 - x^2 + x^4} dx = \int \frac{1}{x^4 - x^2 + 2} dx$$

input `integrate(1/(x^4-x^2+2),x, algorithm="maxima")`output `integrate(1/(x^4 - x^2 + 2), x)`**3.45.8 Giac [A] (verification not implemented)**

Time = 0.46 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.31

$$\begin{aligned} & \int \frac{1}{2 - x^2 + x^4} dx \\ &= \frac{1}{112} \sqrt{7} \left( \sqrt{7} 2^{\frac{1}{4}} \sqrt{-2\sqrt{2} + 8} + 2^{\frac{1}{4}} \sqrt{2\sqrt{2} + 8} \right) \arctan \left( \frac{2^{\frac{3}{4}} \left( 2^{\frac{1}{4}} \sqrt{\frac{1}{2}} \sqrt{\sqrt{2} + 4} + 2x \right)}{4 \sqrt{-\frac{1}{8}\sqrt{2} + \frac{1}{2}}} \right) \\ &+ \frac{1}{112} \sqrt{7} \left( \sqrt{7} 2^{\frac{1}{4}} \sqrt{-2\sqrt{2} + 8} + 2^{\frac{1}{4}} \sqrt{2\sqrt{2} + 8} \right) \arctan \left( -\frac{2^{\frac{3}{4}} \left( 2^{\frac{1}{4}} \sqrt{\frac{1}{2}} \sqrt{\sqrt{2} + 4} - 2x \right)}{4 \sqrt{-\frac{1}{8}\sqrt{2} + \frac{1}{2}}} \right) \\ &+ \frac{1}{224} \sqrt{7} \left( \sqrt{7} 2^{\frac{1}{4}} \sqrt{2\sqrt{2} + 8} - 2^{\frac{1}{4}} \sqrt{-2\sqrt{2} + 8} \right) \log \left( 2^{\frac{1}{4}} \sqrt{\frac{1}{2}} x \sqrt{\sqrt{2} + 4} + x^2 + \sqrt{2} \right) \\ &- \frac{1}{224} \sqrt{7} \left( \sqrt{7} 2^{\frac{1}{4}} \sqrt{2\sqrt{2} + 8} - 2^{\frac{1}{4}} \sqrt{-2\sqrt{2} + 8} \right) \log \left( -2^{\frac{1}{4}} \sqrt{\frac{1}{2}} x \sqrt{\sqrt{2} + 4} + x^2 + \sqrt{2} \right) \end{aligned}$$

input `integrate(1/(x^4-x^2+2),x, algorithm="giac")`

output 
$$\begin{aligned} & 1/112*\sqrt{7}*(\sqrt{7}*2^{(1/4)}*\sqrt{-2*\sqrt{2} + 8} + 2^{(1/4)}*\sqrt{2*\sqrt{2} + 8})*\arctan(1/4*2^{(3/4)}*(2^{(1/4)}*\sqrt{1/2}*\sqrt{\sqrt{2} + 4} + 2*x)/\sqrt{-1/8*\sqrt{2} + 1/2}) + 1/112*\sqrt{7}*(\sqrt{7}*2^{(1/4)}*\sqrt{-2*\sqrt{2} + 8} + 2^{(1/4)}*\sqrt{2*\sqrt{2} + 8})*\arctan(-1/4*2^{(3/4)}*(2^{(1/4)}*\sqrt{1/2}*\sqrt{\sqrt{2} + 4} - 2*x)/\sqrt{-1/8*\sqrt{2} + 1/2}) + 1/224*\sqrt{7}*(\sqrt{7})*2^{(1/4)}*\sqrt{2*\sqrt{2} + 8} - 2^{(1/4)}*\sqrt{-2*\sqrt{2} + 8})*\log(2^{(1/4)}*\sqrt{1/2}*x*\sqrt{\sqrt{2} + 4} + x^2 + \sqrt{2}) - 1/224*\sqrt{7}*(\sqrt{7})*2^{(1/4)}*\sqrt{2*\sqrt{2} + 8} - 2^{(1/4)}*\sqrt{-2*\sqrt{2} + 8})*\log(-2^{(1/4)}*\sqrt{1/2}*x*\sqrt{\sqrt{2} + 4} + x^2 + \sqrt{2}) \end{aligned}$$

### 3.45.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.67

$$\int \frac{1}{2-x^2+x^4} dx = \frac{\operatorname{atan}\left(\frac{x\sqrt{-7-\sqrt{7}7i}i}{4\left(\frac{1}{2}+\frac{\sqrt{7}i}{2}\right)} + \frac{\sqrt{7}x\sqrt{-7-\sqrt{7}7i}}{28\left(\frac{1}{2}+\frac{\sqrt{7}i}{2}\right)}\right) \sqrt{-7-\sqrt{7}7i}i}{14} + \frac{\sqrt{7}\operatorname{atan}\left(\frac{x\sqrt{-1+\sqrt{7}i}i}{4\left(-\frac{1}{2}+\frac{\sqrt{7}i}{2}\right)} - \frac{\sqrt{7}x\sqrt{-1+\sqrt{7}i}i}{4\left(-\frac{1}{2}+\frac{\sqrt{7}i}{2}\right)}\right) \sqrt{-1+\sqrt{7}i}i}{14}$$

input `int(1/(x^4 - x^2 + 2),x)`

output 
$$\begin{aligned} & (\operatorname{atan}((x*(-7^{(1/2)}*7i - 7)^{(1/2)}*1i)/(4*((7^{(1/2)}*1i)/2 + 1/2))) + (7^{(1/2)})*x*(-7^{(1/2)}*7i - 7)^{(1/2)})/(28*((7^{(1/2)}*1i)/2 + 1/2)))*(-7^{(1/2)}*7i - 7)^{(1/2)}*1i/14 + (7^{(1/2)}*\operatorname{atan}((x*(7^{(1/2)}*1i - 1)^{(1/2)})/(4*((7^{(1/2)}*1i)/2 - 1/2))) - (7^{(1/2)})*x*(7^{(1/2)}*1i - 1)^{(1/2)}*1i)/(4*((7^{(1/2)}*1i)/2 - 1/2)))*(7^{(1/2)}*1i - 1)^{(1/2)}*1i/14 \end{aligned}$$

### 3.46 $\int \frac{1}{-1+x^6} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.46.1 | Optimal result . . . . .                            | 346 |
| 3.46.2 | Mathematica [A] (verified) . . . . .                | 346 |
| 3.46.3 | Rubi [A] (verified) . . . . .                       | 347 |
| 3.46.4 | Maple [A] (verified) . . . . .                      | 349 |
| 3.46.5 | Fricas [A] (verification not implemented) . . . . . | 350 |
| 3.46.6 | Sympy [A] (verification not implemented) . . . . .  | 350 |
| 3.46.7 | Maxima [A] (verification not implemented) . . . . . | 350 |
| 3.46.8 | Giac [A] (verification not implemented) . . . . .   | 351 |
| 3.46.9 | Mupad [B] (verification not implemented) . . . . .  | 351 |

#### 3.46.1 Optimal result

Integrand size = 7, antiderivative size = 73

$$\int \frac{1}{-1+x^6} dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\operatorname{arctanh}(x)}{3} \\ + \frac{1}{12} \log(1-x+x^2) - \frac{1}{12} \log(1+x+x^2)$$

output `-1/3*arctanh(x)+1/12*ln(x^2-x+1)-1/12*ln(x^2+x+1)+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)`

#### 3.46.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{1}{-1+x^6} dx = \frac{1}{12} \left( -2\sqrt{3} \arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\sqrt{3} \arctan\left(\frac{1+2x}{\sqrt{3}}\right) + 2\log(1-x) \right. \\ \left. - 2\log(1+x) + \log(1-x+x^2) - \log(1+x+x^2) \right)$$

input `Integrate[(-1 + x^6)^(-1),x]`

output `(-2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])/12`

**3.46.3 Rubi [A] (verified)**

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$ , Rules used = {754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{x^6 - 1} dx \\
 & \quad \downarrow \text{754} \\
 & -\frac{1}{3} \int \frac{1}{1 - x^2} dx - \frac{1}{3} \int \frac{2 - x}{2(x^2 - x + 1)} dx - \frac{1}{3} \int \frac{x + 2}{2(x^2 + x + 1)} dx \\
 & \quad \downarrow \text{27} \\
 & -\frac{1}{3} \int \frac{1}{1 - x^2} dx - \frac{1}{6} \int \frac{2 - x}{x^2 - x + 1} dx - \frac{1}{6} \int \frac{x + 2}{x^2 + x + 1} dx \\
 & \quad \downarrow \text{219} \\
 & -\frac{1}{6} \int \frac{2 - x}{x^2 - x + 1} dx - \frac{1}{6} \int \frac{x + 2}{x^2 + x + 1} dx - \frac{\operatorname{arctanh}(x)}{3} \\
 & \quad \downarrow \text{1142} \\
 & \frac{1}{6} \left( \frac{1}{2} \int -\frac{1 - 2x}{x^2 - x + 1} dx - \frac{3}{2} \int \frac{1}{x^2 - x + 1} dx \right) + \frac{1}{6} \left( -\frac{3}{2} \int \frac{1}{x^2 + x + 1} dx - \frac{1}{2} \int \frac{2x + 1}{x^2 + x + 1} dx \right) - \frac{\operatorname{arctanh}(x)}{3} \\
 & \quad \downarrow \text{25} \\
 & \frac{1}{6} \left( -\frac{3}{2} \int \frac{1}{x^2 - x + 1} dx - \frac{1}{2} \int \frac{1 - 2x}{x^2 - x + 1} dx \right) + \frac{1}{6} \left( -\frac{3}{2} \int \frac{1}{x^2 + x + 1} dx - \frac{1}{2} \int \frac{2x + 1}{x^2 + x + 1} dx \right) - \frac{\operatorname{arctanh}(x)}{3} \\
 & \quad \downarrow \text{1083} \\
 & \frac{1}{6} \left( 3 \int \frac{1}{-(2x - 1)^2 - 3} d(2x - 1) - \frac{1}{2} \int \frac{1 - 2x}{x^2 - x + 1} dx \right) + \\
 & \frac{1}{6} \left( 3 \int \frac{1}{-(2x + 1)^2 - 3} d(2x + 1) - \frac{1}{2} \int \frac{2x + 1}{x^2 + x + 1} dx \right) - \frac{\operatorname{arctanh}(x)}{3} \\
 & \quad \downarrow \text{217}
 \end{aligned}$$

$$\frac{1}{6} \left( -\frac{1}{2} \int \frac{1-2x}{x^2-x+1} dx - \sqrt{3} \arctan \left( \frac{2x-1}{\sqrt{3}} \right) \right) +$$

$$\frac{1}{6} \left( -\frac{1}{2} \int \frac{2x+1}{x^2+x+1} dx - \sqrt{3} \arctan \left( \frac{2x+1}{\sqrt{3}} \right) \right) - \frac{\operatorname{arctanh}(x)}{3}$$

↓ 1103

$$\frac{1}{6} \left( \frac{1}{2} \log(x^2-x+1) - \sqrt{3} \arctan \left( \frac{2x-1}{\sqrt{3}} \right) \right) +$$

$$\frac{1}{6} \left( -\sqrt{3} \arctan \left( \frac{2x+1}{\sqrt{3}} \right) - \frac{1}{2} \log(x^2+x+1) \right) - \frac{\operatorname{arctanh}(x)}{3}$$

input `Int[(-1 + x^6)^(-1), x]`

output `-1/3*ArcTanh[x] + (-Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]]) + Log[1 - x + x^2  
]/2)/6 + (-Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]) - Log[1 + x + x^2]/2)/6`

### 3.46.3.1 Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma  
tchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(  
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &  
& (LtQ[a, 0] || LtQ[b, 0])`

rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*  
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt  
Q[a, 0] || LtQ[b, 0])`

rule 754 `Int[((a_) + (b_.)*(x_)^(n_))^(1), x_Symbol] := Module[{r = Numerator[Rt[-a  
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*  
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2  
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))  
Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n)) Sum[u, {k, 1, (n - 2)/4}], x] /  
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]`

rule 1083 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Simp[-2 Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x]`

rule 1103 `Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

rule 1142 `Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(2*c*d - b*e)/(2*c) Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c) Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]`

### 3.46.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

| method  | result                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
|---------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| default | $\frac{\ln(-1+x)}{6} - \frac{\ln(x^2+x+1)}{12} - \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6} + \frac{\ln(x^2-x+1)}{12} - \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} - \frac{\ln(1+x)}{6}$                                                                                                                                                                                                                               |
| risch   | $\frac{\ln(4x^2-4x+4)}{12} - \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} - \frac{\ln(1+x)}{6} - \frac{\ln(x^2+x+1)}{12} - \frac{\sqrt{3} \arctan\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{6} + \frac{\ln(-1+x)}{6}$                                                                                                                                                                                                       |
| meijerg | $x \left( \frac{\ln\left(1-(x^6)^{\frac{1}{6}}\right) - \ln\left(1+(x^6)^{\frac{1}{6}}\right) + \frac{\ln\left(1-(x^6)^{\frac{1}{6}} + (x^6)^{\frac{1}{3}}\right)}{2}}{6(x^6)^{\frac{1}{6}}} - \sqrt{3} \arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{2-(x^6)^{\frac{1}{6}}}\right) - \frac{\ln\left(1+(x^6)^{\frac{1}{6}} + (x^6)^{\frac{1}{3}}\right)}{2} - \sqrt{3} \arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{2+(x^6)^{\frac{1}{6}}}\right) \right)$ |

input `int(1/(x^6-1),x,method=_RETURNVERBOSE)`

output `1/6*ln(-1+x)-1/12*ln(x^2+x+1)-1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/6*ln(1+x)`

**3.46.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{1}{-1+x^6} dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) \\ - \frac{1}{12}\log(x^2+x+1) + \frac{1}{12}\log(x^2-x+1) - \frac{1}{6}\log(x+1) + \frac{1}{6}\log(x-1)$$

input `integrate(1/(x^6-1),x, algorithm="fricas")`output `-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1) + 1/6*log(x - 1)`**3.46.6 Sympy [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{1}{-1+x^6} dx = \frac{\log(x-1)}{6} - \frac{\log(x+1)}{6} + \frac{\log(x^2-x+1)}{12} - \frac{\log(x^2+x+1)}{12} \\ - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{6}$$

input `integrate(1/(x**6-1),x)`output `log(x - 1)/6 - log(x + 1)/6 + log(x**2 - x + 1)/12 - log(x**2 + x + 1)/12 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6`**3.46.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{1}{-1+x^6} dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) \\ - \frac{1}{12}\log(x^2+x+1) + \frac{1}{12}\log(x^2-x+1) - \frac{1}{6}\log(x+1) + \frac{1}{6}\log(x-1)$$

input `integrate(1/(x^6-1),x, algorithm="maxima")`

output `-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1) + 1/6*log(x - 1)`

### 3.46.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \frac{1}{-1+x^6} dx = -\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) - \frac{1}{12} \log(x^2+x+1) + \frac{1}{12} \log(x^2-x+1) - \frac{1}{6} \log(|x+1|) + \frac{1}{6} \log(|x-1|)$$

input `integrate(1/(x^6-1),x, algorithm="giac")`

output `-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x + 1)) + 1/6*log(abs(x - 1))`

### 3.46.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.21

$$\int \frac{1}{-1+x^6} dx = -\frac{\operatorname{atanh}(x)}{3} - \operatorname{atan}\left(\frac{x \operatorname{li}}{1+\sqrt{3} \operatorname{li}} + \frac{\sqrt{3} x}{1+\sqrt{3} \operatorname{li}}\right) \left(\frac{\sqrt{3}}{6} + \frac{1}{6} i\right) - \operatorname{atan}\left(\frac{x \operatorname{li}}{-1+\sqrt{3} \operatorname{li}} - \frac{\sqrt{3} x}{-1+\sqrt{3} \operatorname{li}}\right) \left(\frac{\sqrt{3}}{6} - \frac{1}{6} i\right)$$

input `int(1/(x^6 - 1),x)`

output `- atanh(x)/3 - atan((x*1i)/(3^(1/2)*1i + 1) + (3^(1/2)*x)/(3^(1/2)*1i + 1))*(3^(1/2)/6 + 1i/6) - atan((x*1i)/(3^(1/2)*1i - 1) - (3^(1/2)*x)/(3^(1/2)*1i - 1))*(3^(1/2)/6 - 1i/6)`



### 3.47 $\int \frac{1}{-2+x^6} dx$

|        |                                                     |     |
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#### 3.47.1 Optimal result

Integrand size = 7, antiderivative size = 138

$$\int \frac{1}{-2+x^6} dx = \frac{\arctan\left(\frac{1}{\sqrt{3}} - \frac{2^{5/6}x}{\sqrt{3}}\right)}{2 \cdot 2^{5/6} \sqrt{3}} - \frac{\arctan\left(\frac{1}{\sqrt{3}} + \frac{2^{5/6}x}{\sqrt{3}}\right)}{2 \cdot 2^{5/6} \sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} + \frac{\log\left(\sqrt[3]{2} - \sqrt[6]{2}x + x^2\right)}{12 \cdot 2^{5/6}} - \frac{\log\left(\sqrt[3]{2} + \sqrt[6]{2}x + x^2\right)}{12 \cdot 2^{5/6}}$$

```
output -1/6*arctanh(1/2*x*2^(5/6))*2^(1/6)+1/24*ln(2^(1/3)-2^(1/6)*x+x^2)*2^(1/6)
-1/24*ln(2^(1/3)+2^(1/6)*x+x^2)*2^(1/6)-1/12*arctan(-1/3*3^(1/2)+1/3*2^(5/6)*x*3^(1/2))*2^(1/6)*3^(1/2)-1/12*arctan(1/3*3^(1/2)+1/3*2^(5/6)*x*3^(1/2))*2^(1/6)*3^(1/2)
```

#### 3.47.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.88

$$\int \frac{1}{-2+x^6} dx = \frac{2\sqrt{3} \arctan\left(\frac{-1+2^{5/6}x}{\sqrt{3}}\right) + 2\sqrt{3} \arctan\left(\frac{1+2^{5/6}x}{\sqrt{3}}\right) - 2 \log(2 - 2^{5/6}x) + 2 \log(2 + 2^{5/6}x) - \log(2 - 2^{5/6}x)}{12 \cdot 2^{5/6}}$$

```
input Integrate[(-2 + x^6)^(-1),x]
```

output  $-1/12*(2*\text{Sqrt}[3]*\text{ArcTan}[(-1 + 2^{(5/6)*x})/\text{Sqrt}[3]] + 2*\text{Sqrt}[3]*\text{ArcTan}[(1 + 2^{(5/6)*x})/\text{Sqrt}[3]] - 2*\text{Log}[2 - 2^{(5/6)*x}] + 2*\text{Log}[2 + 2^{(5/6)*x}] - \text{Log}[2 - 2^{(5/6)*x} + 2^{(2/3)*x^2}] + \text{Log}[2 + 2^{(5/6)*x} + 2^{(2/3)*x^2}])/2^{(5/6)}$

### 3.47.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.286$ , Rules used = {754, 27, 219, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{x^6 - 2} dx \\
 & \quad \downarrow 754 \\
 & -\frac{\int \frac{1}{\sqrt[3]{2-x^2}} dx}{3 \cdot 2^{2/3}} - \frac{\int \frac{2\sqrt[6]{2-x}}{2(x^2 - \sqrt[6]{2x} + \sqrt[3]{2})} dx}{3 \cdot 2^{5/6}} - \frac{\int \frac{x+2\sqrt[6]{2}}{2(x^2 + \sqrt[6]{2x} + \sqrt[3]{2})} dx}{3 \cdot 2^{5/6}} \\
 & \quad \downarrow 27 \\
 & -\frac{\int \frac{1}{\sqrt[3]{2-x^2}} dx}{3 \cdot 2^{2/3}} - \frac{\int \frac{2\sqrt[6]{2-x}}{x^2 - \sqrt[6]{2x} + \sqrt[3]{2}} dx}{6 \cdot 2^{5/6}} - \frac{\int \frac{x+2\sqrt[6]{2}}{x^2 + \sqrt[6]{2x} + \sqrt[3]{2}} dx}{6 \cdot 2^{5/6}} \\
 & \quad \downarrow 219 \\
 & -\frac{\int \frac{2\sqrt[6]{2-x}}{x^2 - \sqrt[6]{2x} + \sqrt[3]{2}} dx}{6 \cdot 2^{5/6}} - \frac{\int \frac{x+2\sqrt[6]{2}}{x^2 + \sqrt[6]{2x} + \sqrt[3]{2}} dx}{6 \cdot 2^{5/6}} - \frac{\text{arctanh}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} \\
 & \quad \downarrow 1142 \\
 & -\frac{3 \int \frac{1}{x^2 - \sqrt[6]{2x} + \sqrt[3]{2}} dx}{6 \cdot 2^{5/6}} - \frac{1}{2} \int -\frac{\sqrt[6]{2}(1-2^{5/6}x)}{x^2 - \sqrt[6]{2x} + \sqrt[3]{2}} dx - \frac{3 \int \frac{1}{x^2 + \sqrt[6]{2x} + \sqrt[3]{2}} dx}{6 \cdot 2^{5/6}} + \frac{1}{2} \int \frac{\sqrt[6]{2}(2^{5/6}x+1)}{x^2 + \sqrt[6]{2x} + \sqrt[3]{2}} dx \\
 & \quad \downarrow 25 \\
 & \frac{\text{arctanh}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}}
 \end{aligned}$$

---

3.47.  $\int \frac{1}{-2+x^6} dx$

$$\begin{aligned}
& \frac{3 \int \frac{1}{x^2 - \sqrt[6]{2}x + \sqrt[3]{2}} dx}{6 \cdot 2^{5/6}} + \frac{1}{2} \int \frac{\sqrt[6]{2}(1-2^{5/6}x)}{x^2 - \sqrt[6]{2}x + \sqrt[3]{2}} dx - \frac{3 \int \frac{1}{x^2 + \sqrt[6]{2}x + \sqrt[3]{2}} dx}{6 \cdot 2^{5/6}} + \frac{1}{2} \int \frac{\sqrt[6]{2}(2^{5/6}x+1)}{x^2 + \sqrt[6]{2}x + \sqrt[3]{2}} dx \\
& \quad - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} \\
& \quad \downarrow 27 \\
& \frac{3 \int \frac{1}{x^2 - \sqrt[6]{2}x + \sqrt[3]{2}} dx}{6 \cdot 2^{5/6}} + \frac{\int \frac{1-2^{5/6}x}{x^2 - \sqrt[6]{2}x + \sqrt[3]{2}} dx}{2^{5/6}} - \frac{3 \int \frac{1}{x^2 + \sqrt[6]{2}x + \sqrt[3]{2}} dx}{6 \cdot 2^{5/6}} + \frac{\int \frac{2^{5/6}x+1}{x^2 + \sqrt[6]{2}x + \sqrt[3]{2}} dx}{2^{5/6}} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} \\
& \quad \downarrow 1082 \\
& \frac{\int \frac{1-2^{5/6}x}{x^2 - \sqrt[6]{2}x + \sqrt[3]{2}} dx}{2^{5/6}} + 3 \int \frac{1}{-(1-2^{5/6}x)^2-3} d(1-2^{5/6}x) \\
& \quad - \frac{\int \frac{2^{5/6}x+1}{x^2 + \sqrt[6]{2}x + \sqrt[3]{2}} dx}{2^{5/6}} - 3 \int \frac{1}{-(2^{5/6}x+1)^2-3} d(2^{5/6}x+1) - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} \\
& \quad \downarrow 217 \\
& \frac{\int \frac{1-2^{5/6}x}{x^2 - \sqrt[6]{2}x + \sqrt[3]{2}} dx}{2^{5/6}} - \sqrt{3} \operatorname{arctan}\left(\frac{1-2^{5/6}x}{\sqrt{3}}\right) - \frac{\int \frac{2^{5/6}x+1}{x^2 + \sqrt[6]{2}x + \sqrt[3]{2}} dx}{2^{5/6}} + \sqrt{3} \operatorname{arctan}\left(\frac{2^{5/6}x+1}{\sqrt{3}}\right) \\
& \quad - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} \\
& \quad \downarrow 1103 \\
& \frac{-\sqrt{3} \operatorname{arctan}\left(\frac{1-2^{5/6}x}{\sqrt{3}}\right) - \frac{1}{2} \log\left(x^2 - \sqrt[6]{2}x + \sqrt[3]{2}\right)}{6 \cdot 2^{5/6}} \\
& \quad - \frac{\sqrt{3} \operatorname{arctan}\left(\frac{2^{5/6}x+1}{\sqrt{3}}\right) + \frac{1}{2} \log\left(x^2 + \sqrt[6]{2}x + \sqrt[3]{2}\right) - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}}}{6 \cdot 2^{5/6}}
\end{aligned}$$

input `Int[(-2 + x^6)^(-1), x]`

output `-1/3*ArcTanh[x/2^(1/6)]/2^(5/6) - (- (Sqrt[3]*ArcTan[(1 - 2^(5/6)*x)/Sqrt[3]]) - Log[2^(1/3) - 2^(1/6)*x + x^2]/2)/(6*2^(5/6)) - (Sqrt[3]*ArcTan[(1 + 2^(5/6)*x)/Sqrt[3]] + Log[2^(1/3) + 2^(1/6)*x + x^2]/2)/(6*2^(5/6))`

## 3.47.3.1 Defintions of rubi rules used

- rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`
- rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`
- rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`
- rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`
- rule 754 `Int[((a_) + (b_.)*(x_)^(n_))(-1), x_Symbol] := Module[{r = Numerator[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n)) Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n)) Sum[u, {k, 1, (n - 2)/4}], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]`
- rule 1082 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Simp[-2/b Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x]`
- rule 1103 `Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`
- rule 1142 `Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(2*c*d - b*e)/(2*c) Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c) Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]`

**3.47.4 Maple [C] (verified)**

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.16

| method  | result                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|---------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch   | $\frac{\sum_{-R=\text{RootOf}(-Z^6-2)} \frac{\ln(x-R)}{-R^5}}{6}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| default | $\frac{\ln(2^{\frac{1}{3}}-2^{\frac{1}{6}}x+x^2)2^{\frac{1}{6}}}{24} - \frac{\arctan\left(-\frac{\sqrt{3}}{3}+\frac{2^{\frac{5}{6}}x\sqrt{3}}{3}\right)2^{\frac{1}{6}}\sqrt{3}}{12} + \frac{2^{\frac{1}{6}}\ln(x-2^{\frac{1}{6}})}{12} - \frac{\ln(2^{\frac{1}{3}}+2^{\frac{1}{6}}x+x^2)2^{\frac{1}{6}}}{24} - \frac{\arctan\left(\frac{\sqrt{3}}{3}+\frac{2^{\frac{5}{6}}x\sqrt{3}}{3}\right)2^{\frac{1}{6}}\sqrt{3}}{12}$                                                                                                              |
| meijerg | $2^{\frac{1}{6}}x \left( \ln\left(1-\frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2}\right) - \ln\left(1+\frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2}\right) + \frac{\ln\left(1-\frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2}+\frac{2^{\frac{2}{3}}(x^6)^{\frac{1}{3}}}{2}\right)}{2} - \sqrt{3} \arctan\left(\frac{\sqrt{3}2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{4-2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}\right) - \frac{\ln\left(1+\frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2}+\frac{2^{\frac{2}{3}}(x^6)^{\frac{1}{3}}}{2}\right)}{2} \right)$ |

input `int(1/(x^6-2),x,method=_RETURNVERBOSE)`

output `1/6*sum(1/_R^5*ln(x-_R),_R=RootOf(_Z^6-2))`

**3.47.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

$$\begin{aligned} \int \frac{1}{-2+x^6} dx &= -\frac{1}{384} \cdot 32^{\frac{5}{6}}(\sqrt{-3}+1) \log\left(32^{\frac{5}{6}}(\sqrt{-3}+1)+32x\right) + \frac{1}{384} \\ &\quad \cdot 32^{\frac{5}{6}}(\sqrt{-3}+1) \log\left(-32^{\frac{5}{6}}(\sqrt{-3}+1)+32x\right) - \frac{1}{384} \\ &\quad \cdot 32^{\frac{5}{6}}(\sqrt{-3}-1) \log\left(32^{\frac{5}{6}}(\sqrt{-3}-1)+32x\right) + \frac{1}{384} \\ &\quad \cdot 32^{\frac{5}{6}}(\sqrt{-3}-1) \log\left(-32^{\frac{5}{6}}(\sqrt{-3}-1)+32x\right) - \frac{1}{192} \\ &\quad \cdot 32^{\frac{5}{6}} \log\left(16x+32^{\frac{5}{6}}\right) + \frac{1}{192} \cdot 32^{\frac{5}{6}} \log\left(16x-32^{\frac{5}{6}}\right) \end{aligned}$$

input `integrate(1/(x^6-2),x, algorithm="fracas")`

output `-1/384*32^(5/6)*(sqrt(-3) + 1)*log(32^(5/6)*(sqrt(-3) + 1) + 32*x) + 1/384*32^(5/6)*(sqrt(-3) + 1)*log(-32^(5/6)*(sqrt(-3) + 1) + 32*x) - 1/384*32^(5/6)*(sqrt(-3) - 1)*log(32^(5/6)*(sqrt(-3) - 1) + 32*x) + 1/384*32^(5/6)*(sqrt(-3) - 1)*log(-32^(5/6)*(sqrt(-3) - 1) + 32*x) - 1/192*32^(5/6)*log(16*x + 32^(5/6)) + 1/192*32^(5/6)*log(16*x - 32^(5/6))`

### 3.47.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.10

$$\int \frac{1}{-2+x^6} dx = \text{RootSum}(1492992t^6 - 1, (t \mapsto t \log(-12t + x)))$$

input `integrate(1/(x**6-2),x)`

output `RootSum(1492992*_t**6 - 1, Lambda(_t, _t*log(-12*_t + x)))`

### 3.47.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.81

$$\begin{aligned} \int \frac{1}{-2+x^6} dx = & -\frac{1}{12} \sqrt{32^{\frac{1}{6}}} \arctan\left(\frac{1}{6} \sqrt{32^{\frac{5}{6}}} (2x + 2^{\frac{1}{6}})\right) \\ & - \frac{1}{12} \sqrt{32^{\frac{1}{6}}} \arctan\left(\frac{1}{6} \sqrt{32^{\frac{5}{6}}} (2x - 2^{\frac{1}{6}})\right) - \frac{1}{24} \cdot 2^{\frac{1}{6}} \log\left(x^2 + 2^{\frac{1}{6}}x + 2^{\frac{1}{3}}\right) \\ & + \frac{1}{24} \cdot 2^{\frac{1}{6}} \log\left(x^2 - 2^{\frac{1}{6}}x + 2^{\frac{1}{3}}\right) - \frac{1}{12} \cdot 2^{\frac{1}{6}} \log\left(x + 2^{\frac{1}{6}}\right) + \frac{1}{12} \cdot 2^{\frac{1}{6}} \log\left(x - 2^{\frac{1}{6}}\right) \end{aligned}$$

input `integrate(1/(x^6-2),x, algorithm="maxima")`

output `-1/12*sqrt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x + 2^(1/6))) - 1/12*sqrt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x - 2^(1/6))) - 1/24*2^(1/6)*log(x^2 + 2^(1/6)*x + 2^(1/3)) + 1/24*2^(1/6)*log(x^2 - 2^(1/6)*x + 2^(1/3)) - 1/12*2^(1/6)*log(x + 2^(1/6)) + 1/12*2^(1/6)*log(x - 2^(1/6))`

**3.47.8 Giac [A] (verification not implemented)**

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \frac{1}{-2+x^6} dx = -\frac{1}{12} \sqrt{3} 2^{\frac{1}{6}} \arctan\left(\frac{1}{6} \sqrt{3} 2^{\frac{5}{6}} (2x + 2^{\frac{1}{6}})\right) - \frac{1}{12} \sqrt{3} 2^{\frac{1}{6}} \arctan\left(\frac{1}{6} \sqrt{3} 2^{\frac{5}{6}} (2x - 2^{\frac{1}{6}})\right) - \frac{1}{24} \cdot 2^{\frac{1}{6}} \log\left(x^2 + 2^{\frac{1}{6}}x + 2^{\frac{1}{3}}\right) + \frac{1}{24} \cdot 2^{\frac{1}{6}} \log\left(x^2 - 2^{\frac{1}{6}}x + 2^{\frac{1}{3}}\right) - \frac{1}{12} \cdot 2^{\frac{1}{6}} \log\left(|x + 2^{\frac{1}{6}}|\right) + \frac{1}{12} \cdot 2^{\frac{1}{6}} \log\left(|x - 2^{\frac{1}{6}}|\right)$$

input `integrate(1/(x^6-2),x, algorithm="giac")`

```
output -1/12*sqrt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x + 2^(1/6))) - 1/12*sqrt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x - 2^(1/6))) - 1/24*2^(1/6)*log(x^2 + 2^(1/6)*x + 2^(1/3)) + 1/24*2^(1/6)*log(x^2 - 2^(1/6)*x + 2^(1/3)) - 1/12*2^(1/6)*log(abs(x + 2^(1/6))) + 1/12*2^(1/6)*log(abs(x - 2^(1/6)))
```

**3.47.9 Mupad [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01

$$\int \frac{1}{-2+x^6} dx = -\frac{2^{1/6} \operatorname{atanh}\left(\frac{2^{5/6}x}{2}\right)}{6} + \frac{2^{1/6} \operatorname{atan}\left(\frac{2^{1/6}x \operatorname{li}}{2\left(-\frac{2^{1/3}}{2} + \frac{2^{1/3}\sqrt{3}\operatorname{li}}{2}\right)} - \frac{2^{1/6}\sqrt{3}x}{2\left(-\frac{2^{1/3}}{2} + \frac{2^{1/3}\sqrt{3}\operatorname{li}}{2}\right)}\right) (1 + \sqrt{3}\operatorname{li}) \operatorname{li}}{12} + \frac{2^{1/6} \operatorname{atan}\left(\frac{2^{1/6}x \operatorname{li}}{2\left(\frac{2^{1/3}}{2} + \frac{2^{1/3}\sqrt{3}\operatorname{li}}{2}\right)} + \frac{2^{1/6}\sqrt{3}x}{2\left(\frac{2^{1/3}}{2} + \frac{2^{1/3}\sqrt{3}\operatorname{li}}{2}\right)}\right) (-1 + \sqrt{3}\operatorname{li}) \operatorname{li}}{12}$$

input `int(1/(x^6 - 2),x)`

```
output (2^(1/6)*atan((2^(1/6)*x*li)/(2*((2^(1/3)*3^(1/2)*li)/2 - 2^(1/3)/2)) - (2^(1/6)*3^(1/2)*x)/(2*((2^(1/3)*3^(1/2)*li)/2 - 2^(1/3)/2)))*(3^(1/2)*li + 1)*li)/12 - (2^(1/6)*atanh((2^(5/6)*x)/2))/6 + (2^(1/6)*atan((2^(1/6)*x*li)/(2*((2^(1/3)*3^(1/2)*li)/2 + 2^(1/3)/2)) + (2^(1/6)*3^(1/2)*x)/(2*((2^(1/3)*3^(1/2)*li)/2 + 2^(1/3)/2)))*(3^(1/2)*li - 1)*li)/12
```

### 3.48 $\int \frac{1}{2+x^6} dx$

|        |                                                     |     |
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#### 3.48.1 Optimal result

Integrand size = 7, antiderivative size = 138

$$\int \frac{1}{2+x^6} dx = \frac{\arctan\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} - \frac{\arctan(\sqrt{3} - 2^{5/6}x)}{6 \cdot 2^{5/6}} + \frac{\arctan(\sqrt{3} + 2^{5/6}x)}{6 \cdot 2^{5/6}} - \frac{\log\left(\sqrt[3]{2} - \sqrt[6]{2}\sqrt{3}x + x^2\right)}{4 \cdot 2^{5/6}\sqrt{3}} + \frac{\log\left(\sqrt[3]{2} + \sqrt[6]{2}\sqrt{3}x + x^2\right)}{4 \cdot 2^{5/6}\sqrt{3}}$$

```
output 1/6*arctan(1/2*x*2^(5/6))*2^(1/6)+1/12*arctan(x*2^(5/6)-3^(1/2))*2^(1/6)+1/12*arctan(x*2^(5/6)+3^(1/2))*2^(1/6)-1/24*ln(2^(1/3)+x^2-2^(1/6)*x*3^(1/2))*2^(1/6)*3^(1/2)+1/24*ln(2^(1/3)+x^2+2^(1/6)*x*3^(1/2))*2^(1/6)*3^(1/2)
```

#### 3.48.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

$$\int \frac{1}{2+x^6} dx = \frac{4 \arctan\left(\frac{x}{\sqrt[6]{2}}\right) - 2 \arctan(\sqrt{3} - 2^{5/6}x) + 2 \arctan(\sqrt{3} + 2^{5/6}x) - \sqrt{3} \log(2 - 2^{5/6}\sqrt{3}x + 2^{2/3}x^2) + \sqrt{3} \log(2 + 2^{5/6}\sqrt{3}x + 2^{2/3}x^2)}{12 \cdot 2^{5/6}}$$

```
input Integrate[(2 + x^6)^(-1),x]
```



output  $(4*\text{ArcTan}[x/2^{(1/6)}] - 2*\text{ArcTan}[\text{Sqrt}[3] - 2^{(5/6)}*x] + 2*\text{ArcTan}[\text{Sqrt}[3] + 2^{(5/6)}*x] - \text{Sqrt}[3]*\text{Log}[2 - 2^{(5/6)}*\text{Sqrt}[3]*x + 2^{(2/3)}*x^2] + \text{Sqrt}[3]*\text{Log}[2 + 2^{(5/6)}*\text{Sqrt}[3]*x + 2^{(2/3)}*x^2])/(12*2^{(5/6)})$

### 3.48.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.286$ , Rules used = {753, 27, 216, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{x^6 + 2} dx \\
 & \quad \downarrow 753 \\
 & \int \frac{1}{x^2 + \sqrt[3]{2}} dx + \int \frac{2\sqrt[6]{2} - \sqrt{3}x}{2(x^2 - \sqrt[6]{2}\sqrt{3}x + \sqrt[3]{2})} dx + \int \frac{\sqrt{3}x + 2\sqrt[6]{2}}{2(x^2 + \sqrt[6]{2}\sqrt{3}x + \sqrt[3]{2})} dx \\
 & \quad \downarrow 27 \\
 & \int \frac{1}{x^2 + \sqrt[3]{2}} dx + \int \frac{2\sqrt[6]{2} - \sqrt{3}x}{6 \cdot 2^{5/6}} dx + \int \frac{\sqrt{3}x + 2\sqrt[6]{2}}{6 \cdot 2^{5/6}} dx \\
 & \quad \downarrow 216 \\
 & \int \frac{2\sqrt[6]{2} - \sqrt{3}x}{6 \cdot 2^{5/6}} dx + \int \frac{\sqrt{3}x + 2\sqrt[6]{2}}{6 \cdot 2^{5/6}} dx + \frac{\arctan\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} \\
 & \quad \downarrow 1142 \\
 & \frac{\int \frac{1}{x^2 - \sqrt[6]{2}\sqrt{3}x + \sqrt[3]{2}} dx}{2^{5/6}} - \frac{1}{2}\sqrt{3} \int \frac{\sqrt[6]{2}(\sqrt{3} - 2^{5/6}x)}{x^2 - \sqrt[6]{2}\sqrt{3}x + \sqrt[3]{2}} dx}{6 \cdot 2^{5/6}} + \\
 & \frac{\int \frac{1}{x^2 + \sqrt[6]{2}\sqrt{3}x + \sqrt[3]{2}} dx}{2^{5/6}} + \frac{1}{2}\sqrt{3} \int \frac{\sqrt[6]{2}(2^{5/6}x + \sqrt{3})}{x^2 + \sqrt[6]{2}\sqrt{3}x + \sqrt[3]{2}} dx}{6 \cdot 2^{5/6}} + \frac{\arctan\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} \\
 & \quad \downarrow 25
 \end{aligned}$$

$$\begin{aligned}
& \frac{\int \frac{1}{x^2 - \sqrt[6]{2}\sqrt[3]{x} + \sqrt[3]{2}} dx}{6 \cdot 2^{5/6}} + \frac{1}{2}\sqrt{3} \int \frac{\sqrt[6]{2}(\sqrt{3} - 2^{5/6}x)}{x^2 - \sqrt[6]{2}\sqrt[3]{x} + \sqrt[3]{2}} dx + \frac{\int \frac{1}{x^2 + \sqrt[6]{2}\sqrt[3]{x} + \sqrt[3]{2}} dx}{6 \cdot 2^{5/6}} + \frac{1}{2}\sqrt{3} \int \frac{\sqrt[6]{2}(2^{5/6}x + \sqrt{3})}{x^2 + \sqrt[6]{2}\sqrt[3]{x} + \sqrt[3]{2}} dx + \\
& \frac{\arctan\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} \\
& \quad \downarrow \text{27} \\
& \frac{\int \frac{1}{x^2 - \sqrt[6]{2}\sqrt[3]{x} + \sqrt[3]{2}} dx}{6 \cdot 2^{5/6}} + \frac{\sqrt{3} \int \frac{\sqrt{3} - 2^{5/6}x}{x^2 - \sqrt[6]{2}\sqrt[3]{x} + \sqrt[3]{2}} dx}{2^{5/6}} + \frac{\int \frac{1}{x^2 + \sqrt[6]{2}\sqrt[3]{x} + \sqrt[3]{2}} dx}{6 \cdot 2^{5/6}} + \frac{\sqrt{3} \int \frac{2^{5/6}x + \sqrt{3}}{x^2 + \sqrt[6]{2}\sqrt[3]{x} + \sqrt[3]{2}} dx}{2^{5/6}} + \\
& \frac{\arctan\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} \\
& \quad \downarrow \text{1082} \\
& \frac{\sqrt{3} \int \frac{\sqrt{3} - 2^{5/6}x}{x^2 - \sqrt[6]{2}\sqrt[3]{x} + \sqrt[3]{2}} dx}{2^{5/6}} + \frac{\int \frac{1}{-(1 - \frac{2^{5/6}x}{\sqrt{3}})^2 - \frac{1}{3}} d\left(1 - \frac{2^{5/6}x}{\sqrt{3}}\right)}{\sqrt{3}} + \\
& \frac{\sqrt{3} \int \frac{2^{5/6}x + \sqrt{3}}{x^2 + \sqrt[6]{2}\sqrt[3]{x} + \sqrt[3]{2}} dx}{2^{5/6}} - \frac{\int \frac{1}{-(\frac{2^{5/6}x}{\sqrt{3}} + 1)^2 - \frac{1}{3}} d\left(\frac{2^{5/6}x}{\sqrt{3}} + 1\right)}{\sqrt{3}} + \frac{\arctan\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} \\
& \quad \downarrow \text{217} \\
& \frac{\sqrt{3} \int \frac{\sqrt{3} - 2^{5/6}x}{x^2 - \sqrt[6]{2}\sqrt[3]{x} + \sqrt[3]{2}} dx}{2^{5/6}} - \frac{\arctan\left(\sqrt{3}\left(1 - \frac{2^{5/6}x}{\sqrt{3}}\right)\right)}{6 \cdot 2^{5/6}} + \\
& \frac{\sqrt{3} \int \frac{2^{5/6}x + \sqrt{3}}{x^2 + \sqrt[6]{2}\sqrt[3]{x} + \sqrt[3]{2}} dx}{2^{5/6}} + \frac{\arctan\left(\sqrt{3}\left(\frac{2^{5/6}x}{\sqrt{3}} + 1\right)\right)}{6 \cdot 2^{5/6}} + \frac{\arctan\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}} \\
& \quad \downarrow \text{1103} \\
& \frac{-\arctan\left(\sqrt{3}\left(1 - \frac{2^{5/6}x}{\sqrt{3}}\right)\right) - \frac{1}{2}\sqrt{3} \log\left(x^2 - \sqrt[6]{2}\sqrt[3]{x} + \sqrt[3]{2}\right)}{6 \cdot 2^{5/6}} + \\
& \frac{\arctan\left(\sqrt{3}\left(\frac{2^{5/6}x}{\sqrt{3}} + 1\right)\right) + \frac{1}{2}\sqrt{3} \log\left(x^2 + \sqrt[6]{2}\sqrt[3]{x} + \sqrt[3]{2}\right)}{6 \cdot 2^{5/6}} + \frac{\arctan\left(\frac{x}{\sqrt[6]{2}}\right)}{3 \cdot 2^{5/6}}
\end{aligned}$$

input `Int[(2 + x^6)^(-1), x]`

```
output ArcTan[x/2^(1/6)]/(3*2^(5/6)) + (-ArcTan[Sqrt[3]*(1 - (2^(5/6)*x)/Sqrt[3])
] - (Sqrt[3]*Log[2^(1/3) - 2^(1/6)*Sqrt[3]*x + x^2])/2)/(6*2^(5/6)) + (Arc
Tan[Sqrt[3]*(1 + (2^(5/6)*x)/Sqrt[3])] + (Sqrt[3]*Log[2^(1/3) + 2^(1/6)*Sq
rt[3]*x + x^2])/2)/(6*2^(5/6))
```

### 3.48.3.1 Defintions of rubi rules used

```
rule 25 Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

```
rule 27 Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

```
rule 216 Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

```
rule 217 Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(
-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

```
rule 753 Int[((a_) + (b_.)*(x_)^(n_))(-1), x_Symbol] := Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
- 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n)) Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n)) Sum[u,
{k, 1, (n - 2)/4}], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

```
rule 1082 Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

```
rule 1103 Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

```
rule 1142 Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(2*c*d - b*e)/(2*c) Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

### 3.48.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.16

| method  | result                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
|---------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch   | $\frac{\sum_{-R=\text{RootOf}(-Z^6+2)} \frac{\ln(x-R)}{-R^5}}{6}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| default | $\frac{\arctan\left(\frac{x2^{\frac{5}{6}}}{2}\right)2^{\frac{1}{6}}}{6} + \frac{\arctan(x2^{\frac{5}{6}}-\sqrt{3})2^{\frac{1}{6}}}{12} + \frac{\arctan(x2^{\frac{5}{6}}+\sqrt{3})2^{\frac{1}{6}}}{12} - \frac{\ln(2^{\frac{1}{3}}+x^2-2^{\frac{1}{6}}x\sqrt{3})2^{\frac{1}{6}}\sqrt{3}}{24} + \frac{\ln(2^{\frac{1}{3}}+x^2+2^{\frac{1}{6}}x\sqrt{3})2^{\frac{1}{6}}}{24}$                                                                                                                                                                                                                                     |
| meijerg | $2^{\frac{1}{6}} \left( -\frac{x\sqrt{3} \ln\left(1 - \frac{\sqrt{3}2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2} + \frac{2^{\frac{2}{3}}(x^6)^{\frac{1}{3}}}{2}\right)}{2(x^6)^{\frac{1}{6}}} + \frac{x \arctan\left(\frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{4-\sqrt{3}2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}\right)}{(x^6)^{\frac{1}{6}}} + \frac{2x \arctan\left(\frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2}\right)}{(x^6)^{\frac{1}{6}}} + \frac{x\sqrt{3} \ln\left(1 + \frac{\sqrt{3}2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2} + \frac{2^{\frac{2}{3}}(x^6)^{\frac{1}{3}}}{2}\right)}{2(x^6)^{\frac{1}{6}}} \right)$ |

```
input int(1/(x^6+2), x, method=_RETURNVERBOSE)
```

```
output 1/6*sum(1/_R^5*ln(x-_R), _R=RootOf(-_Z^6+2))
```

**3.48.5 Fracas [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.19

$$\int \frac{1}{2+x^6} dx = \frac{1}{384} \cdot 32^{\frac{5}{6}}(-1)^{\frac{1}{6}}(\sqrt{-3}+1) \log\left(32^{\frac{5}{6}}(-1)^{\frac{1}{6}}(\sqrt{-3}+1)+32x\right) - \frac{1}{384} \cdot 32^{\frac{5}{6}}(-1)^{\frac{1}{6}}(\sqrt{-3}+1) \log\left(-32^{\frac{5}{6}}(-1)^{\frac{1}{6}}(\sqrt{-3}+1)+32x\right) + \frac{1}{384} \cdot 32^{\frac{5}{6}}(-1)^{\frac{1}{6}}(\sqrt{-3}-1) \log\left(32^{\frac{5}{6}}(-1)^{\frac{1}{6}}(\sqrt{-3}-1)+32x\right) - \frac{1}{384} \cdot 32^{\frac{5}{6}}(-1)^{\frac{1}{6}}(\sqrt{-3}-1) \log\left(-32^{\frac{5}{6}}(-1)^{\frac{1}{6}}(\sqrt{-3}-1)+32x\right) + \frac{1}{192} \cdot 32^{\frac{5}{6}}(-1)^{\frac{1}{6}} \log\left(32^{\frac{5}{6}}(-1)^{\frac{1}{6}}+16x\right) - \frac{1}{192} \cdot 32^{\frac{5}{6}}(-1)^{\frac{1}{6}} \log\left(-32^{\frac{5}{6}}(-1)^{\frac{1}{6}}+16x\right)$$

input `integrate(1/(x^6+2),x, algorithm="fricas")`output `1/384*32^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1)*log(32^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1) + 32*x) - 1/384*32^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1)*log(-32^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1) + 32*x) + 1/384*32^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*log(32^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1) + 32*x) - 1/384*32^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*log(-32^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1) + 32*x) + 1/192*32^(5/6)*(-1)^(1/6)*log(32^(5/6)*(-1)^(1/6) + 16*x) - 1/192*32^(5/6)*(-1)^(1/6)*log(-32^(5/6)*(-1)^(1/6) + 16*x)`**3.48.6 Sympy [A] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.10

$$\int \frac{1}{2+x^6} dx = \text{RootSum}(1492992t^6 + 1, (t \mapsto t \log(12t + x)))$$

input `integrate(1/(x**6+2),x)`output `RootSum(1492992*_t**6 + 1, Lambda(_t, _t*log(12*_t + x)))`

**3.48.7 Maxima [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78

$$\int \frac{1}{2+x^6} dx = \frac{1}{24} \sqrt{32^{\frac{1}{6}}} \log \left( x^2 + \sqrt{32^{\frac{1}{6}}} x + 2^{\frac{1}{3}} \right) - \frac{1}{24} \sqrt{32^{\frac{1}{6}}} \log \left( x^2 - \sqrt{32^{\frac{1}{6}}} x + 2^{\frac{1}{3}} \right) \\ + \frac{1}{12} \cdot 2^{\frac{1}{6}} \arctan \left( \frac{1}{2} \cdot 2^{\frac{5}{6}} \left( 2x + \sqrt{32^{\frac{1}{6}}} \right) \right) + \frac{1}{12} \\ \cdot 2^{\frac{1}{6}} \arctan \left( \frac{1}{2} \cdot 2^{\frac{5}{6}} \left( 2x - \sqrt{32^{\frac{1}{6}}} \right) \right) + \frac{1}{6} \cdot 2^{\frac{1}{6}} \arctan \left( \frac{1}{2} \cdot 2^{\frac{5}{6}} x \right)$$

input `integrate(1/(x^6+2),x, algorithm="maxima")`output `1/24*sqrt(3)*2^(1/6)*log(x^2 + sqrt(3)*2^(1/6)*x + 2^(1/3)) - 1/24*sqrt(3)*2^(1/6)*log(x^2 - sqrt(3)*2^(1/6)*x + 2^(1/3)) + 1/12*2^(1/6)*arctan(1/2*2^(5/6)*(2*x + sqrt(3)*2^(1/6))) + 1/12*2^(1/6)*arctan(1/2*2^(5/6)*(2*x - sqrt(3)*2^(1/6))) + 1/6*2^(1/6)*arctan(1/2*2^(5/6)*x)`**3.48.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78

$$\int \frac{1}{2+x^6} dx = \frac{1}{24} \sqrt{32^{\frac{1}{6}}} \log \left( x^2 + \sqrt{32^{\frac{1}{6}}} x + 2^{\frac{1}{3}} \right) - \frac{1}{24} \sqrt{32^{\frac{1}{6}}} \log \left( x^2 - \sqrt{32^{\frac{1}{6}}} x + 2^{\frac{1}{3}} \right) \\ + \frac{1}{12} \cdot 2^{\frac{1}{6}} \arctan \left( \frac{1}{2} \cdot 2^{\frac{5}{6}} \left( 2x + \sqrt{32^{\frac{1}{6}}} \right) \right) + \frac{1}{12} \\ \cdot 2^{\frac{1}{6}} \arctan \left( \frac{1}{2} \cdot 2^{\frac{5}{6}} \left( 2x - \sqrt{32^{\frac{1}{6}}} \right) \right) + \frac{1}{6} \cdot 2^{\frac{1}{6}} \arctan \left( \frac{1}{2} \cdot 2^{\frac{5}{6}} x \right)$$

input `integrate(1/(x^6+2),x, algorithm="giac")`output `1/24*sqrt(3)*2^(1/6)*log(x^2 + sqrt(3)*2^(1/6)*x + 2^(1/3)) - 1/24*sqrt(3)*2^(1/6)*log(x^2 - sqrt(3)*2^(1/6)*x + 2^(1/3)) + 1/12*2^(1/6)*arctan(1/2*2^(5/6)*(2*x + sqrt(3)*2^(1/6))) + 1/12*2^(1/6)*arctan(1/2*2^(5/6)*(2*x - sqrt(3)*2^(1/6))) + 1/6*2^(1/6)*arctan(1/2*2^(5/6)*x)`

**3.48.9 Mupad [B] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.98

$$\int \frac{1}{2+x^6} dx = \frac{2^{1/6} \operatorname{atan}\left(\frac{2^{5/6}x}{2}\right)}{6} + \frac{2^{1/6} \operatorname{atan}\left(\frac{2^{1/6}x}{2\left(-\frac{2^{1/3}}{2} + \frac{2^{1/3}\sqrt{3}1i}{2}\right)} + \frac{2^{1/6}\sqrt{3}x1i}{2\left(-\frac{2^{1/3}}{2} + \frac{2^{1/3}\sqrt{3}1i}{2}\right)}\right) (\sqrt{3}-i) 1i}{12} + \frac{2^{1/6} \operatorname{atan}\left(\frac{2^{1/6}x}{2\left(\frac{2^{1/3}}{2} + \frac{2^{1/3}\sqrt{3}1i}{2}\right)} - \frac{2^{1/6}\sqrt{3}x1i}{2\left(\frac{2^{1/3}}{2} + \frac{2^{1/3}\sqrt{3}1i}{2}\right)}\right) (\sqrt{3}+1i) 1i}{12}$$

input `int(1/(x^6 + 2),x)`

output `(2^(1/6)*atan((2^(5/6)*x)/2))/6 + (2^(1/6)*atan((2^(1/6)*x)/(2*((2^(1/3)*3^(1/2)*1i)/2 - 2^(1/3)/2)) + (2^(1/6)*3^(1/2)*x*1i)/(2*((2^(1/3)*3^(1/2)*1i)/2 - 2^(1/3)/2)))*(3^(1/2) - 1i)*1i)/12 + (2^(1/6)*atan((2^(1/6)*x)/(2*((2^(1/3)*3^(1/2)*1i)/2 + 2^(1/3)/2)) - (2^(1/6)*3^(1/2)*x*1i)/(2*((2^(1/3)*3^(1/2)*1i)/2 + 2^(1/3)/2)))*(3^(1/2) + 1i)*1i)/12`

### 3.49 $\int \frac{1}{1+x^8} dx$

|        |                                                     |     |
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#### 3.49.1 Optimal result

Integrand size = 7, antiderivative size = 339

$$\int \frac{1}{1+x^8} dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}-2x}}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2}(2-\sqrt{2})} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}-2x}}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2}(2+\sqrt{2})} + \frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}+2x}}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2}(2-\sqrt{2})}$$

$$+ \frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}+2x}}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2}(2+\sqrt{2})} - \frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right)$$

$$+ \frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right)$$

$$- \frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(1-\sqrt{2+\sqrt{2}}x+x^2\right)$$

$$+ \frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(1+\sqrt{2+\sqrt{2}}x+x^2\right)$$

output

```
-1/16*ln(1+x^2-x*(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/16*ln(1+x^2+x*(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/4*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/4*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/16*ln(1+x^2-x*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/16*ln(1+x^2+x*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)-1/4*arctan((-2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)+1/4*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```



### 3.49.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.62

$$\begin{aligned} \int \frac{1}{1+x^8} dx = & \frac{1}{4} \arctan \left( \sec \left( \frac{\pi}{8} \right) \left( x - \sin \left( \frac{\pi}{8} \right) \right) \right) \cos \left( \frac{\pi}{8} \right) \\ & + \frac{1}{4} \arctan \left( \sec \left( \frac{\pi}{8} \right) \left( x + \sin \left( \frac{\pi}{8} \right) \right) \right) \cos \left( \frac{\pi}{8} \right) \\ & - \frac{1}{8} \cos \left( \frac{\pi}{8} \right) \log \left( 1 + x^2 - 2x \cos \left( \frac{\pi}{8} \right) \right) + \frac{1}{8} \cos \left( \frac{\pi}{8} \right) \log \left( 1 + x^2 + 2x \cos \left( \frac{\pi}{8} \right) \right) \\ & + \frac{1}{4} \arctan \left( \left( x - \cos \left( \frac{\pi}{8} \right) \right) \csc \left( \frac{\pi}{8} \right) \right) \sin \left( \frac{\pi}{8} \right) \\ & + \frac{1}{4} \arctan \left( \left( x + \cos \left( \frac{\pi}{8} \right) \right) \csc \left( \frac{\pi}{8} \right) \right) \sin \left( \frac{\pi}{8} \right) \\ & - \frac{1}{8} \log \left( 1 + x^2 - 2x \sin \left( \frac{\pi}{8} \right) \right) \sin \left( \frac{\pi}{8} \right) + \frac{1}{8} \log \left( 1 + x^2 + 2x \sin \left( \frac{\pi}{8} \right) \right) \sin \left( \frac{\pi}{8} \right) \end{aligned}$$

input `Integrate[(1 + x^8)^(-1),x]`

output `(ArcTan[Sec[Pi/8]*(x - Sin[Pi/8])*Cos[Pi/8])/4 + (ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])*Cos[Pi/8])/4 - (Cos[Pi/8]*Log[1 + x^2 - 2*x*Cos[Pi/8]])/8 + (Cos[Pi/8]*Log[1 + x^2 + 2*x*Cos[Pi/8]])/8 + (ArcTan[(x - Cos[Pi/8])*Csc[Pi/8])*Sin[Pi/8])/4 + (ArcTan[(x + Cos[Pi/8])*Csc[Pi/8])*Sin[Pi/8])/4 - (Log[1 + x^2 - 2*x*Sin[Pi/8])*Sin[Pi/8])/8 + (Log[1 + x^2 + 2*x*Sin[Pi/8])*Sin[Pi/8])/8`

### 3.49.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {757, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{x^8 + 1} dx \\ & \quad \downarrow \text{757} \\ & \frac{\int \frac{\sqrt{2}-x^2}{x^4-\sqrt{2}x^2+1} dx}{2\sqrt{2}} + \frac{\int \frac{x^2+\sqrt{2}}{x^4+\sqrt{2}x^2+1} dx}{2\sqrt{2}} \end{aligned}$$

$$\begin{array}{c}
 \int \frac{\frac{(1-\sqrt{2})x + \sqrt{2(2-\sqrt{2})}}{x^2 - \sqrt{2-\sqrt{2}}x + 1} dx + \int \frac{\sqrt{2(2-\sqrt{2})} - (1-\sqrt{2})x}{x^2 + \sqrt{2-\sqrt{2}}x + 1} dx}{2\sqrt{2-\sqrt{2}}} + \int \frac{\frac{\sqrt{2(2+\sqrt{2})} - (1+\sqrt{2})x}{x^2 - \sqrt{2+\sqrt{2}}x + 1} dx + \int \frac{(1+\sqrt{2})x + \sqrt{2(2+\sqrt{2})}}{x^2 + \sqrt{2+\sqrt{2}}x + 1} dx}{2\sqrt{2+\sqrt{2}}} \\
 \downarrow \text{1483} \\
 \frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{2}} \int \frac{1}{x^2 - \sqrt{2-\sqrt{2}}x + 1} dx + \frac{1}{2}(1-\sqrt{2}) \int -\frac{\sqrt{2-\sqrt{2}}-2x}{x^2 - \sqrt{2-\sqrt{2}}x + 1} dx}{2\sqrt{2-\sqrt{2}}} + \frac{\frac{1}{2}\sqrt{2+\sqrt{2}} \int \frac{1}{x^2 + \sqrt{2-\sqrt{2}}x + 1} dx - \frac{1}{2}(1-\sqrt{2}) \int \frac{2x + \sqrt{2-\sqrt{2}}}{x^2 + \sqrt{2-\sqrt{2}}x + 1} dx}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}} + \\
 \frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{2}} \int \frac{1}{x^2 - \sqrt{2+\sqrt{2}}x + 1} dx - \frac{1}{2}(1+\sqrt{2}) \int -\frac{\sqrt{2+\sqrt{2}}-2x}{x^2 - \sqrt{2+\sqrt{2}}x + 1} dx}{2\sqrt{2+\sqrt{2}}} + \frac{\frac{1}{2}\sqrt{2-\sqrt{2}} \int \frac{1}{x^2 + \sqrt{2+\sqrt{2}}x + 1} dx + \frac{1}{2}(1+\sqrt{2}) \int \frac{2x + \sqrt{2+\sqrt{2}}}{x^2 + \sqrt{2+\sqrt{2}}x + 1} dx}{2\sqrt{2+\sqrt{2}}}}{2\sqrt{2}} \\
 \downarrow \text{1142} \\
 \frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{2}} \int \frac{1}{x^2 - \sqrt{2-\sqrt{2}}x + 1} dx - \frac{1}{2}(1-\sqrt{2}) \int \frac{\sqrt{2-\sqrt{2}}-2x}{x^2 - \sqrt{2-\sqrt{2}}x + 1} dx}{2\sqrt{2-\sqrt{2}}} + \frac{\frac{1}{2}\sqrt{2+\sqrt{2}} \int \frac{1}{x^2 + \sqrt{2-\sqrt{2}}x + 1} dx - \frac{1}{2}(1-\sqrt{2}) \int \frac{2x + \sqrt{2-\sqrt{2}}}{x^2 + \sqrt{2-\sqrt{2}}x + 1} dx}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}} + \\
 \frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{2}} \int \frac{1}{x^2 - \sqrt{2+\sqrt{2}}x + 1} dx + \frac{1}{2}(1+\sqrt{2}) \int -\frac{\sqrt{2+\sqrt{2}}-2x}{x^2 - \sqrt{2+\sqrt{2}}x + 1} dx}{2\sqrt{2+\sqrt{2}}} + \frac{\frac{1}{2}\sqrt{2-\sqrt{2}} \int \frac{1}{x^2 + \sqrt{2+\sqrt{2}}x + 1} dx + \frac{1}{2}(1+\sqrt{2}) \int \frac{2x + \sqrt{2+\sqrt{2}}}{x^2 + \sqrt{2+\sqrt{2}}x + 1} dx}{2\sqrt{2+\sqrt{2}}}}{2\sqrt{2}} \\
 \downarrow \text{25} \\
 \frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{2}} \int \frac{1}{x^2 - \sqrt{2-\sqrt{2}}x + 1} dx - \frac{1}{2}(1-\sqrt{2}) \int \frac{\sqrt{2-\sqrt{2}}-2x}{x^2 - \sqrt{2-\sqrt{2}}x + 1} dx}{2\sqrt{2-\sqrt{2}}} + \frac{\frac{1}{2}\sqrt{2+\sqrt{2}} \int \frac{1}{x^2 + \sqrt{2-\sqrt{2}}x + 1} dx - \frac{1}{2}(1-\sqrt{2}) \int \frac{2x + \sqrt{2-\sqrt{2}}}{x^2 + \sqrt{2-\sqrt{2}}x + 1} dx}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}} + \\
 \frac{\frac{\frac{1}{2}\sqrt{2-\sqrt{2}} \int \frac{1}{x^2 - \sqrt{2+\sqrt{2}}x + 1} dx + \frac{1}{2}(1+\sqrt{2}) \int -\frac{\sqrt{2+\sqrt{2}}-2x}{x^2 - \sqrt{2+\sqrt{2}}x + 1} dx}{2\sqrt{2+\sqrt{2}}} + \frac{\frac{1}{2}\sqrt{2-\sqrt{2}} \int \frac{1}{x^2 + \sqrt{2+\sqrt{2}}x + 1} dx + \frac{1}{2}(1+\sqrt{2}) \int \frac{2x + \sqrt{2+\sqrt{2}}}{x^2 + \sqrt{2+\sqrt{2}}x + 1} dx}{2\sqrt{2+\sqrt{2}}}}{2\sqrt{2}} \\
 \downarrow \text{1083} \\
 \frac{-\frac{1}{2}(1-\sqrt{2}) \int \frac{\sqrt{2-\sqrt{2}}-2x}{x^2 - \sqrt{2-\sqrt{2}}x + 1} dx - \sqrt{2+\sqrt{2}} \int \frac{1}{-(2x - \sqrt{2-\sqrt{2}})^2 - \sqrt{2}-2} d(2x - \sqrt{2-\sqrt{2}})}{2\sqrt{2-\sqrt{2}}} + \frac{-\frac{1}{2}(1-\sqrt{2}) \int \frac{2x + \sqrt{2-\sqrt{2}}}{x^2 + \sqrt{2-\sqrt{2}}x + 1} dx - \sqrt{2+\sqrt{2}} \int \frac{1}{-(2x + \sqrt{2-\sqrt{2}})^2 - \sqrt{2}-2} d(2x + \sqrt{2-\sqrt{2}})}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}} + \\
 \frac{\frac{1}{2}(1+\sqrt{2}) \int \frac{\sqrt{2+\sqrt{2}}-2x}{x^2 - \sqrt{2+\sqrt{2}}x + 1} dx - \sqrt{2-\sqrt{2}} \int \frac{1}{-(2x - \sqrt{2+\sqrt{2}})^2 + \sqrt{2}-2} d(2x - \sqrt{2+\sqrt{2}})}{2\sqrt{2+\sqrt{2}}} + \frac{\frac{1}{2}(1+\sqrt{2}) \int \frac{2x + \sqrt{2+\sqrt{2}}}{x^2 + \sqrt{2+\sqrt{2}}x + 1} dx - \sqrt{2-\sqrt{2}} \int \frac{1}{-(2x + \sqrt{2+\sqrt{2}})^2 + \sqrt{2}-2} d(2x + \sqrt{2+\sqrt{2}})}{2\sqrt{2+\sqrt{2}}}}{2\sqrt{2}} \\
 \downarrow \text{217} \\
 \frac{\arctan\left(\frac{2x - \sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{2}(1-\sqrt{2}) \int \frac{\sqrt{2-\sqrt{2}}-2x}{x^2 - \sqrt{2-\sqrt{2}}x + 1} dx}{2\sqrt{2-\sqrt{2}}} + \frac{\arctan\left(\frac{2x + \sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{2}(1-\sqrt{2}) \int \frac{2x + \sqrt{2-\sqrt{2}}}{x^2 + \sqrt{2-\sqrt{2}}x + 1} dx}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}} + \\
 \frac{\frac{1}{2}(1+\sqrt{2}) \int \frac{\sqrt{2+\sqrt{2}}-2x}{x^2 - \sqrt{2+\sqrt{2}}x + 1} dx + \arctan\left(\frac{2x - \sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}} + \frac{\frac{1}{2}(1+\sqrt{2}) \int \frac{2x + \sqrt{2+\sqrt{2}}}{x^2 + \sqrt{2+\sqrt{2}}x + 1} dx + \arctan\left(\frac{2x + \sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}}{2\sqrt{2}} \\
 \downarrow \text{1103}
 \end{array}$$

$$\frac{\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right) + \frac{1}{2}(1-\sqrt{2}) \log(x^2 - \sqrt{2-\sqrt{2}}x + 1)}{2\sqrt{2-\sqrt{2}}} + \frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{2}(1-\sqrt{2}) \log(x^2 + \sqrt{2-\sqrt{2}}x + 1)}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}} +$$

$$\frac{\arctan\left(\frac{2x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{2}(1+\sqrt{2}) \log(x^2 - \sqrt{2+\sqrt{2}}x + 1)}{2\sqrt{2+\sqrt{2}}} + \frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{2}(1+\sqrt{2}) \log(x^2 + \sqrt{2+\sqrt{2}}x + 1)}{2\sqrt{2+\sqrt{2}}}}{2\sqrt{2}}$$

input `Int[(1 + x^8)^(-1), x]`

output `((ArcTan[(-Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]] + ((1 - Sqrt[2])*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[2]]) + (ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]] - ((1 - Sqrt[2])*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[2]]))/(2*Sqrt[2]) + ((ArcTan[(-Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]] - ((1 + Sqrt[2])*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[2]]) + (ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]] + ((1 + Sqrt[2])*Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[2]]))/(2*Sqrt[2])`

### 3.49.3.1 Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

rule 757 `Int[((a_) + (b_.)*(x_)^(n_))(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Simp[r/(2*Sqrt[2]*a) Int[(Sqrt[2]*r - s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Simp[r/(2*Sqrt[2]*a) Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]`

rule 1083 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Simp[-2 Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x]`

rule 1103 `Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

rule 1142 `Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(2*c*d - b*e)/(2*c) Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c) Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]`

rule 1483 `Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r) Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r) Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]`

### 3.49.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.06

| method  | result                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
|---------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| default | $\frac{\left( \sum_{-R=\text{RootOf}(-Z^8+1)} \frac{\ln(x-R)}{-R^7} \right)}{8}$                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| risch   | $\frac{\left( \sum_{-R=\text{RootOf}(-Z^8+1)} \frac{\ln(x-R)}{-R^7} \right)}{8}$                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| meijerg | $-\frac{x \cos\left(\frac{\pi}{8}\right) \ln\left(1-2 \cos\left(\frac{\pi}{8}\right) (x^8)^{\frac{1}{8}}+(x^8)^{\frac{1}{4}}\right)}{8(x^8)^{\frac{1}{8}}} + \frac{x \sin\left(\frac{\pi}{8}\right) \arctan\left(\frac{\sin\left(\frac{\pi}{8}\right) (x^8)^{\frac{1}{8}}}{1-\cos\left(\frac{\pi}{8}\right) (x^8)^{\frac{1}{8}}}\right)}{4(x^8)^{\frac{1}{8}}} - \frac{x \cos\left(\frac{3\pi}{8}\right) \ln\left(1-2 \cos\left(\frac{3\pi}{8}\right) (x^8)^{\frac{1}{8}}+(x^8)^{\frac{1}{4}}\right)}{8(x^8)^{\frac{1}{8}}}$ |

input `int(1/(x^8+1),x,method=_RETURNVERBOSE)`

output `1/8*sum(1/_R^7*ln(x-_R),_R=RootOf(-_Z^8+1))`

**3.49.5 Fracas [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.40

$$\begin{aligned} \int \frac{1}{1+x^8} dx = & \left( \frac{1}{16}i + \frac{1}{16} \right) \sqrt{2}(-1)^{\frac{1}{8}} \log \left( 2x + (i+1) \sqrt{2}(-1)^{\frac{1}{8}} \right) \\ & - \left( \frac{1}{16}i - \frac{1}{16} \right) \sqrt{2}(-1)^{\frac{1}{8}} \log \left( 2x - (i-1) \sqrt{2}(-1)^{\frac{1}{8}} \right) \\ & + \left( \frac{1}{16}i - \frac{1}{16} \right) \sqrt{2}(-1)^{\frac{1}{8}} \log \left( 2x + (i-1) \sqrt{2}(-1)^{\frac{1}{8}} \right) \\ & - \left( \frac{1}{16}i + \frac{1}{16} \right) \sqrt{2}(-1)^{\frac{1}{8}} \log \left( 2x - (i+1) \sqrt{2}(-1)^{\frac{1}{8}} \right) \\ & + \frac{1}{8} (-1)^{\frac{1}{8}} \log \left( x + (-1)^{\frac{1}{8}} \right) + \frac{1}{8} i (-1)^{\frac{1}{8}} \log \left( x + i(-1)^{\frac{1}{8}} \right) \\ & - \frac{1}{8} i (-1)^{\frac{1}{8}} \log \left( x - i(-1)^{\frac{1}{8}} \right) - \frac{1}{8} (-1)^{\frac{1}{8}} \log \left( x - (-1)^{\frac{1}{8}} \right) \end{aligned}$$

input `integrate(1/(x^8+1),x, algorithm="fricas")`

output `(1/16*I + 1/16)*sqrt(2)*(-1)^(1/8)*log(2*x + (I + 1)*sqrt(2)*(-1)^(1/8)) - (1/16*I - 1/16)*sqrt(2)*(-1)^(1/8)*log(2*x - (I - 1)*sqrt(2)*(-1)^(1/8)) + (1/16*I - 1/16)*sqrt(2)*(-1)^(1/8)*log(2*x + (I - 1)*sqrt(2)*(-1)^(1/8)) - (1/16*I + 1/16)*sqrt(2)*(-1)^(1/8)*log(2*x - (I + 1)*sqrt(2)*(-1)^(1/8)) + 1/8*(-1)^(1/8)*log(x + (-1)^(1/8)) + 1/8*I*(-1)^(1/8)*log(x + I*(-1)^(1/8)) - 1/8*I*(-1)^(1/8)*log(x - I*(-1)^(1/8)) - 1/8*(-1)^(1/8)*log(x - (-1)^(1/8))`

**3.49.6 Sympy [A] (verification not implemented)**

Time = 1.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.04

$$\int \frac{1}{1+x^8} dx = \text{RootSum} (16777216t^8 + 1, (t \mapsto t \log(8t + x)))$$

input `integrate(1/(x**8+1),x)`

output `RootSum(16777216*_t**8 + 1, Lambda(_t, _t*log(8*_t + x)))`

**3.49.7 Maxima [F]**

$$\int \frac{1}{1+x^8} dx = \int \frac{1}{x^8+1} dx$$

input `integrate(1/(x^8+1),x, algorithm="maxima")`

output `integrate(1/(x^8 + 1), x)`

**3.49.8 Giac [A] (verification not implemented)**

Time = 0.30 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.71

$$\begin{aligned} \int \frac{1}{1+x^8} dx &= \frac{1}{8} \sqrt{\sqrt{2}+2} \arctan \left( \frac{2x + \sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}} \right) \\ &+ \frac{1}{8} \sqrt{\sqrt{2}+2} \arctan \left( \frac{2x - \sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}} \right) \\ &+ \frac{1}{8} \sqrt{-\sqrt{2}+2} \arctan \left( \frac{2x + \sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}} \right) \\ &+ \frac{1}{8} \sqrt{-\sqrt{2}+2} \arctan \left( \frac{2x - \sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}} \right) \\ &+ \frac{1}{16} \sqrt{\sqrt{2}+2} \log \left( x^2 + x\sqrt{\sqrt{2}+2} + 1 \right) \\ &- \frac{1}{16} \sqrt{\sqrt{2}+2} \log \left( x^2 - x\sqrt{\sqrt{2}+2} + 1 \right) \\ &+ \frac{1}{16} \sqrt{-\sqrt{2}+2} \log \left( x^2 + x\sqrt{-\sqrt{2}+2} + 1 \right) \\ &- \frac{1}{16} \sqrt{-\sqrt{2}+2} \log \left( x^2 - x\sqrt{-\sqrt{2}+2} + 1 \right) \end{aligned}$$

input `integrate(1/(x^8+1),x, algorithm="giac")`

```
output 1/8*sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))
+ 1/8*sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) +
2)) + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2)
+ 2)) + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sq
rt(2) + 2)) + 1/16*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) -
1/16*sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/16*sqrt(-sq
rt(2) + 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(-sqrt(2) + 2)*lo
g(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

### 3.49.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+x^8} dx = \operatorname{atan}\left(\frac{x\sqrt{-\sqrt{2}-2}1i}{\sqrt{2}-\sqrt{2}\sqrt{-\sqrt{2}-2}+\sqrt{2}} - \frac{x\sqrt{2-\sqrt{2}}1i}{\sqrt{2}-\sqrt{2}\sqrt{-\sqrt{2}-2}+\sqrt{2}}\right) \left(\frac{\sqrt{-\sqrt{2}-2}1i}{8} - \frac{\sqrt{2-\sqrt{2}}1i}{8}\right) - \operatorname{atan}\left(\frac{x\sqrt{\sqrt{2}-2}1i}{\sqrt{2}+\sqrt{\sqrt{2}-2}\sqrt{\sqrt{2}+2}} + \frac{x\sqrt{\sqrt{2}+2}1i}{\sqrt{2}+\sqrt{\sqrt{2}-2}\sqrt{\sqrt{2}+2}}\right) \left(\frac{\sqrt{\sqrt{2}-2}1i}{8} + \frac{\sqrt{\sqrt{2}+2}1i}{8}\right) + \operatorname{atan}\left(-\frac{\sqrt{2}x\sqrt{\sqrt{2}+2}}{2} + x\sqrt{\sqrt{2}+2}\left(\frac{1}{2} + \frac{1}{2}i\right)\right) \left(\frac{\sqrt{2}1i}{16} - \frac{1}{16} - \frac{1}{16}i\right) \sqrt{\sqrt{2}+2}2i - \operatorname{atan}\left(x\sqrt{\sqrt{2}+2}\left(\frac{1}{2} - \frac{1}{2}i\right) + \frac{\sqrt{2}x\sqrt{\sqrt{2}+2}1i}{2}\right) \left(\frac{\sqrt{2}}{16} - \frac{1}{16} + \frac{1}{16}i\right) \sqrt{\sqrt{2}+2}2i$$

```
input int(1/(x^8 + 1),x)
```

output

```
atan((x*(- 2^(1/2) - 2)^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/2) - 2)^(1/2) + 2^(1/2)) - (x*(2 - 2^(1/2))^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/2) - 2)^(1/2) + 2^(1/2)))*((- 2^(1/2) - 2)^(1/2)*1i)/8 - ((2 - 2^(1/2))^(1/2)*1i)/8) - atan((x*(2^(1/2) - 2)^(1/2)*1i)/(2^(1/2) + (2^(1/2) - 2)^(1/2) * (2^(1/2) + 2)^(1/2)) + (x*(2^(1/2) + 2)^(1/2)*1i)/(2^(1/2) + (2^(1/2) - 2)^(1/2) * (2^(1/2) + 2)^(1/2)))*((2^(1/2) - 2)^(1/2)*1i)/8 + ((2^(1/2) + 2)^(1/2)*1i)/8) + atan(x*(2^(1/2) + 2)^(1/2)*(1/2 + 1i/2) - (2^(1/2)*x*(2^(1/2) + 2)^(1/2))/2)*((2^(1/2)*1i)/16 - (1/16 + 1i/16))*(2^(1/2) + 2)^(1/2)*2i - atan(x*(2^(1/2) + 2)^(1/2)*(1/2 - 1i/2) + (2^(1/2)*x*(2^(1/2) + 2)^(1/2)*1i)/2)*(2^(1/2)/16 - (1/16 - 1i/16))*(2^(1/2) + 2)^(1/2)*2i
```



### 3.50 $\int \frac{1}{-1+x^8} dx$

|        |                                                     |     |
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#### 3.50.1 Optimal result

Integrand size = 7, antiderivative size = 97

$$\int \frac{1}{-1+x^8} dx = -\frac{\arctan(x)}{4} + \frac{\arctan(1-\sqrt{2}x)}{4\sqrt{2}} - \frac{\arctan(1+\sqrt{2}x)}{4\sqrt{2}} - \frac{\operatorname{arctanh}(x)}{4} + \frac{\log(1-\sqrt{2}x+x^2)}{8\sqrt{2}} - \frac{\log(1+\sqrt{2}x+x^2)}{8\sqrt{2}}$$

output `-1/4*arctan(x)-1/4*arctanh(x)-1/8*arctan(-1+x*2^(1/2))*2^(1/2)-1/8*arctan(1+x*2^(1/2))*2^(1/2)+1/16*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/16*ln(1+x^2+x*2^(1/2))*2^(1/2)`

#### 3.50.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

$$\int \frac{1}{-1+x^8} dx = \frac{1}{16} \left( -4 \arctan(x) + 2\sqrt{2} \arctan(1-\sqrt{2}x) - 2\sqrt{2} \arctan(1+\sqrt{2}x) + 2 \log(1-x) - 2 \log(1+x) + \sqrt{2} \log(1-\sqrt{2}x+x^2) - \sqrt{2} \log(1+\sqrt{2}x+x^2) \right)$$

input `Integrate[(-1 + x^8)^(-1),x]`

output  $(-4*\text{ArcTan}[x] + 2*\text{Sqrt}[2]*\text{ArcTan}[1 - \text{Sqrt}[2]*x] - 2*\text{Sqrt}[2]*\text{ArcTan}[1 + \text{Sqrt}[2]*x] + 2*\text{Log}[1 - x] - 2*\text{Log}[1 + x] + \text{Sqrt}[2]*\text{Log}[1 - \text{Sqrt}[2]*x + x^2] - \text{Sqrt}[2]*\text{Log}[1 + \text{Sqrt}[2]*x + x^2])/16$

### 3.50.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.714$ , Rules used = {758, 755, 756, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{x^8 - 1} dx \\
 & \quad \downarrow 758 \\
 & -\frac{1}{2} \int \frac{1}{1 - x^4} dx - \frac{1}{2} \int \frac{1}{x^4 + 1} dx \\
 & \quad \downarrow 755 \\
 & \frac{1}{2} \left( -\frac{1}{2} \int \frac{1 - x^2}{x^4 + 1} dx - \frac{1}{2} \int \frac{x^2 + 1}{x^4 + 1} dx \right) - \frac{1}{2} \int \frac{1}{1 - x^4} dx \\
 & \quad \downarrow 756 \\
 & \frac{1}{2} \left( -\frac{1}{2} \int \frac{1}{1 - x^2} dx - \frac{1}{2} \int \frac{1}{x^2 + 1} dx \right) + \frac{1}{2} \left( -\frac{1}{2} \int \frac{1 - x^2}{x^4 + 1} dx - \frac{1}{2} \int \frac{x^2 + 1}{x^4 + 1} dx \right) \\
 & \quad \downarrow 216 \\
 & \frac{1}{2} \left( -\frac{1}{2} \int \frac{1}{1 - x^2} dx - \frac{\arctan(x)}{2} \right) + \frac{1}{2} \left( -\frac{1}{2} \int \frac{1 - x^2}{x^4 + 1} dx - \frac{1}{2} \int \frac{x^2 + 1}{x^4 + 1} dx \right) \\
 & \quad \downarrow 219 \\
 & \frac{1}{2} \left( -\frac{1}{2} \int \frac{1 - x^2}{x^4 + 1} dx - \frac{1}{2} \int \frac{x^2 + 1}{x^4 + 1} dx \right) + \frac{1}{2} \left( -\frac{\arctan(x)}{2} - \frac{\text{arctanh}(x)}{2} \right) \\
 & \quad \downarrow 1476 \\
 & \frac{1}{2} \left( \frac{1}{2} \left( -\frac{1}{2} \int \frac{1}{x^2 - \sqrt{2}x + 1} dx - \frac{1}{2} \int \frac{1}{x^2 + \sqrt{2}x + 1} dx \right) - \frac{1}{2} \int \frac{1 - x^2}{x^4 + 1} dx \right) + \\
 & \quad \frac{1}{2} \left( -\frac{\arctan(x)}{2} - \frac{\text{arctanh}(x)}{2} \right) \\
 & \quad \downarrow 1082
 \end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \left( \frac{1}{2} \left( \frac{\int \frac{1}{-(\sqrt{2}x+1)^2-1} d(\sqrt{2}x+1)}{\sqrt{2}} - \frac{\int \frac{1}{-(1-\sqrt{2}x)^2-1} d(1-\sqrt{2}x)}{\sqrt{2}} \right) - \frac{1}{2} \int \frac{1-x^2}{x^4+1} dx \right) + \\
& \qquad \qquad \qquad \frac{1}{2} \left( -\frac{\arctan(x)}{2} - \frac{\operatorname{arctanh}(x)}{2} \right) \\
& \qquad \qquad \qquad \downarrow \text{217} \\
& \frac{1}{2} \left( \frac{1}{2} \left( \frac{\arctan(1-\sqrt{2}x)}{\sqrt{2}} - \frac{\arctan(\sqrt{2}x+1)}{\sqrt{2}} \right) - \frac{1}{2} \int \frac{1-x^2}{x^4+1} dx \right) + \\
& \qquad \qquad \qquad \frac{1}{2} \left( -\frac{\arctan(x)}{2} - \frac{\operatorname{arctanh}(x)}{2} \right) \\
& \qquad \qquad \qquad \downarrow \text{1479} \\
& \frac{1}{2} \left( \frac{1}{2} \left( \frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1} dx}{2\sqrt{2}} + \frac{\int -\frac{\sqrt{2}(\sqrt{2}x+1)}{x^2+\sqrt{2}x+1} dx}{2\sqrt{2}} \right) + \frac{1}{2} \left( \frac{\arctan(1-\sqrt{2}x)}{\sqrt{2}} - \frac{\arctan(\sqrt{2}x+1)}{\sqrt{2}} \right) \right) + \\
& \qquad \qquad \qquad \frac{1}{2} \left( -\frac{\arctan(x)}{2} - \frac{\operatorname{arctanh}(x)}{2} \right) \\
& \qquad \qquad \qquad \downarrow \text{25} \\
& \frac{1}{2} \left( \frac{1}{2} \left( -\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1} dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{2}(\sqrt{2}x+1)}{x^2+\sqrt{2}x+1} dx}{2\sqrt{2}} \right) + \frac{1}{2} \left( \frac{\arctan(1-\sqrt{2}x)}{\sqrt{2}} - \frac{\arctan(\sqrt{2}x+1)}{\sqrt{2}} \right) \right) + \\
& \qquad \qquad \qquad \frac{1}{2} \left( -\frac{\arctan(x)}{2} - \frac{\operatorname{arctanh}(x)}{2} \right) \\
& \qquad \qquad \qquad \downarrow \text{27} \\
& \frac{1}{2} \left( \frac{1}{2} \left( -\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1} dx}{2\sqrt{2}} - \frac{1}{2} \int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1} dx \right) + \frac{1}{2} \left( \frac{\arctan(1-\sqrt{2}x)}{\sqrt{2}} - \frac{\arctan(\sqrt{2}x+1)}{\sqrt{2}} \right) \right) + \\
& \qquad \qquad \qquad \frac{1}{2} \left( -\frac{\arctan(x)}{2} - \frac{\operatorname{arctanh}(x)}{2} \right) \\
& \qquad \qquad \qquad \downarrow \text{1103} \\
& \frac{1}{2} \left( \frac{1}{2} \left( \frac{\arctan(1-\sqrt{2}x)}{\sqrt{2}} - \frac{\arctan(\sqrt{2}x+1)}{\sqrt{2}} \right) + \frac{1}{2} \left( \frac{\log(x^2-\sqrt{2}x+1)}{2\sqrt{2}} - \frac{\log(x^2+\sqrt{2}x+1)}{2\sqrt{2}} \right) \right) + \\
& \qquad \qquad \qquad \frac{1}{2} \left( -\frac{\arctan(x)}{2} - \frac{\operatorname{arctanh}(x)}{2} \right)
\end{aligned}$$

input `Int[(-1 + x^8)^(-1), x]`

---

3.50.  $\int \frac{1}{-1+x^8} dx$

output  $(-1/2 \cdot \text{ArcTan}[x] - \text{ArcTanh}[x]/2)/2 + ((\text{ArcTan}[1 - \text{Sqrt}[2] \cdot x]/\text{Sqrt}[2] - \text{ArcTan}[1 + \text{Sqrt}[2] \cdot x]/\text{Sqrt}[2])/2 + (\text{Log}[1 - \text{Sqrt}[2] \cdot x + x^2]/(2 \cdot \text{Sqrt}[2]) - \text{Log}[1 + \text{Sqrt}[2] \cdot x + x^2]/(2 \cdot \text{Sqrt}[2]))/2)/2$

### 3.50.3.1 Defintions of rubi rules used

rule 25  $\text{Int}[-(\text{Fx}_), x\_Symbol] \rightarrow \text{Simp}[\text{Identity}[-1] \quad \text{Int}[\text{Fx}, x], x]$

rule 27  $\text{Int}[(a_)*(\text{Fx}_), x\_Symbol] \rightarrow \text{Simp}[a \quad \text{Int}[\text{Fx}, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[\text{Fx}, (b_)*(\text{Gx}_) /; \text{FreeQ}[b, x]]$

rule 216  $\text{Int}[(a_) + (b_.) \cdot (x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2] \cdot \text{Rt}[b, 2])) \cdot \text{ArcTan}[\text{Rt}[b, 2] \cdot (x/\text{Rt}[a, 2])], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

rule 217  $\text{Int}[(a_) + (b_.) \cdot (x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(-\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2])^{-1} \cdot \text{ArcTan}[\text{Rt}[-b, 2] \cdot (x/\text{Rt}[-a, 2])], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

rule 219  $\text{Int}[(a_) + (b_.) \cdot (x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2])) \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot (x/\text{Rt}[a, 2])], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

rule 755  $\text{Int}[(a_) + (b_.) \cdot (x_)^4)^{-1}, x\_Symbol] \rightarrow \text{With}[\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Simp}[1/(2 \cdot r) \quad \text{Int}[(r - s \cdot x^2)/(a + b \cdot x^4), x], x] + \text{Simp}[1/(2 \cdot r) \quad \text{Int}[(r + s \cdot x^2)/(a + b \cdot x^4), x], x]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ (\text{GtQ}[a/b, 0] \ || \ (\text{PosQ}[a/b] \ \&\& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, a]] \ \&\& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, b]]))$

rule 756  $\text{Int}[(a_) + (b_.) \cdot (x_)^4)^{-1}, x\_Symbol] \rightarrow \text{With}[\{r = \text{Numerator}[\text{Rt}[-a/b, 2]], s = \text{Denominator}[\text{Rt}[-a/b, 2]]\}, \text{Simp}[r/(2 \cdot a) \quad \text{Int}[1/(r - s \cdot x^2), x], x] + \text{Simp}[r/(2 \cdot a) \quad \text{Int}[1/(r + s \cdot x^2), x], x]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ !\text{GtQ}[a/b, 0]$

- rule 758 `Int[((a_) + (b_)*(x_)^(n_))^(n_)-1, x_Symbol] := With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a) Int[1/(r - s*x^(n/2)), x], x] + Simp[r/(2*a) Int[1/(r + s*x^(n/2)), x], x] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && !GtQ[a/b, 0]`
- rule 1082 `Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(n_)-1, x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Simp[-2/b Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x]`
- rule 1103 `Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`
- rule 1476 `Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[2*(d/e), 2]}, Simp[e/(2*c) Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[e/(2*c) Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]`
- rule 1479 `Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[-2*(d/e), 2]}, Simp[e/(2*c*q) Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Simp[e/(2*c*q) Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]`

### 3.50.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.38

| method  | result                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|---------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch   | $\left( \frac{\sum_{-R=\text{RootOf}(-Z^4+1)} -R \ln(x-R)}{8} \right) - \frac{\arctan(x)}{4} - \frac{\ln(1+x)}{8} + \frac{\ln(-1+x)}{8}$                                                                                                                                                                                                                                                                                                 |
| default | $-\frac{\arctan(x)}{4} - \frac{\operatorname{arctanh}(x)}{4} - \frac{\sqrt{2} \left( \ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right) + 2 \arctan(1+x\sqrt{2}) + 2 \arctan(-1+x\sqrt{2}) \right)}{16}$                                                                                                                                                                                                                            |
| meijerg | $x \left( \ln\left(1-(x^8)^{\frac{1}{8}}\right) - \ln\left(1+(x^8)^{\frac{1}{8}}\right) + \frac{\sqrt{2} \ln\left(1-\sqrt{2}(x^8)^{\frac{1}{8}}+(x^8)^{\frac{1}{4}}\right)}{2} - \sqrt{2} \arctan\left(\frac{\sqrt{2}(x^8)^{\frac{1}{8}}}{2-\sqrt{2}(x^8)^{\frac{1}{8}}}\right) - 2 \arctan\left((x^8)^{\frac{1}{8}}\right) - \frac{\sqrt{2} \ln\left(1+\sqrt{2}(x^8)^{\frac{1}{8}}\right)}{2} \right) - \frac{1}{8(x^8)^{\frac{1}{8}}}$ |

input `int(1/(x^8-1),x,method=_RETURNVERBOSE)`

output `1/8*sum(_R*ln(x-_R),_R=RootOf(_Z^4+1))-1/4*arctan(x)-1/8*ln(1+x)+1/8*ln(-1+x)`

### 3.50.5 Fracas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\begin{aligned} \int \frac{1}{-1+x^8} dx = & -\left(\frac{1}{16}i + \frac{1}{16}\right) \sqrt{2} \log\left(2x + (i+1)\sqrt{2}\right) \\ & + \left(\frac{1}{16}i - \frac{1}{16}\right) \sqrt{2} \log\left(2x - (i-1)\sqrt{2}\right) \\ & - \left(\frac{1}{16}i - \frac{1}{16}\right) \sqrt{2} \log\left(2x + (i-1)\sqrt{2}\right) \\ & + \left(\frac{1}{16}i + \frac{1}{16}\right) \sqrt{2} \log\left(2x - (i+1)\sqrt{2}\right) \\ & - \frac{1}{4} \arctan(x) - \frac{1}{8} \log(x+1) + \frac{1}{8} \log(x-1) \end{aligned}$$

input `integrate(1/(x^8-1),x, algorithm="fricas")`

output `-(1/16*I + 1/16)*sqrt(2)*log(2*x + (I + 1)*sqrt(2)) + (1/16*I - 1/16)*sqrt(2)*log(2*x - (I - 1)*sqrt(2)) - (1/16*I - 1/16)*sqrt(2)*log(2*x + (I - 1)*sqrt(2)) + (1/16*I + 1/16)*sqrt(2)*log(2*x - (I + 1)*sqrt(2)) - 1/4*arctan(x) - 1/8*log(x + 1) + 1/8*log(x - 1)`

**3.50.6 Sympy [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 127.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.45

$$\int \frac{1}{-1+x^8} dx = \frac{\log(x-1)}{8} - \frac{\log(x+1)}{8} + \frac{i \log(x-i)}{8} - \frac{i \log(x+i)}{8} + \text{RootSum}(4096t^4 + 1, (t \mapsto t \log(-8t + x)))$$

input `integrate(1/(x**8-1),x)`

output `log(x - 1)/8 - log(x + 1)/8 + I*log(x - I)/8 - I*log(x + I)/8 + RootSum(4096*_t**4 + 1, Lambda(_t, _t*log(-8*_t + x)))`

**3.50.7 Maxima [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int \frac{1}{-1+x^8} dx = -\frac{1}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(2x + \sqrt{2})\right) - \frac{1}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(2x - \sqrt{2})\right) - \frac{1}{16} \sqrt{2} \log(x^2 + \sqrt{2}x + 1) + \frac{1}{16} \sqrt{2} \log(x^2 - \sqrt{2}x + 1) - \frac{1}{4} \arctan(x) - \frac{1}{8} \log(x + 1) + \frac{1}{8} \log(x - 1)$$

input `integrate(1/(x^8-1),x, algorithm="maxima")`

output `-1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x) - 1/8*log(x + 1) + 1/8*log(x - 1)`

**3.50.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \frac{1}{-1+x^8} dx = -\frac{1}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(2x + \sqrt{2})\right) - \frac{1}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(2x - \sqrt{2})\right) \\ - \frac{1}{16} \sqrt{2} \log(x^2 + \sqrt{2}x + 1) + \frac{1}{16} \sqrt{2} \log(x^2 - \sqrt{2}x + 1) \\ - \frac{1}{4} \arctan(x) - \frac{1}{8} \log(|x + 1|) + \frac{1}{8} \log(|x - 1|)$$

input `integrate(1/(x^8-1),x, algorithm="giac")`output `-1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x) - 1/8*log(abs(x + 1)) + 1/8*log(abs(x - 1))`**3.50.9 Mupad [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.46

$$\int \frac{1}{-1+x^8} dx = \frac{\operatorname{atan}(x \operatorname{li} \operatorname{li})}{4} - \frac{\operatorname{atan}(x)}{4} + \sqrt{2} \operatorname{atan}\left(\sqrt{2} x \left(\frac{1}{2} - \frac{1}{2}i\right)\right) \left(-\frac{1}{8} - \frac{1}{8}i\right) \\ + \sqrt{2} \operatorname{atan}\left(\sqrt{2} x \left(\frac{1}{2} + \frac{1}{2}i\right)\right) \left(-\frac{1}{8} + \frac{1}{8}i\right)$$

input `int(1/(x^8 - 1),x)`output `(atan(x*1i)*1i)/4 - atan(x)/4 - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/8 + 1i/8) - 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/8 - 1i/8)`



### 3.51 $\int \frac{1}{1-x^4+x^8} dx$

|        |                                                     |     |
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#### 3.51.1 Optimal result

Integrand size = 12, antiderivative size = 275

$$\int \frac{1}{1-x^4+x^8} dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}-2x}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}-2x}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

$$+ \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}+2x}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}+2x}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

$$- \frac{\log\left(1 - \sqrt{2 - \sqrt{3}}x + x^2\right)}{4\sqrt{6}} + \frac{\log\left(1 + \sqrt{2 - \sqrt{3}}x + x^2\right)}{4\sqrt{6}}$$

$$- \frac{\log\left(1 - \sqrt{2 + \sqrt{3}}x + x^2\right)}{4\sqrt{6}} + \frac{\log\left(1 + \sqrt{2 + \sqrt{3}}x + x^2\right)}{4\sqrt{6}}$$

output

```
-1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(
1/2)+1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*
6^(1/2)-1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2
)))*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(
1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(
1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/
2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

### 3.51.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.15

$$\int \frac{1}{1-x^4+x^8} dx = \frac{1}{4} \text{RootSum} \left[ 1 - \#1^4 + \#1^8 \&, \frac{\log(x - \#1)}{-\#1^3 + 2\#1^7} \& \right]$$

input `Integrate[(1 - x^4 + x^8)^(-1), x]`

output `RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4`

### 3.51.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.49, number of steps used = 8, number of rules used = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$ , Rules used = {1684, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{x^8 - x^4 + 1} dx \\ & \quad \downarrow \text{1684} \\ & \frac{\int \frac{\sqrt{3}-x^2}{x^4-\sqrt{3}x^2+1} dx}{2\sqrt{3}} + \frac{\int \frac{x^2+\sqrt{3}}{x^4+\sqrt{3}x^2+1} dx}{2\sqrt{3}} \\ & \quad \downarrow \text{1483} \\ & \frac{\int \frac{(1-\sqrt{3})x+\sqrt{3(2-\sqrt{3})}}{x^2-\sqrt{2-\sqrt{3}}x+1} dx}{2\sqrt{2-\sqrt{3}}} + \frac{\int \frac{\sqrt{3(2-\sqrt{3})}-(1-\sqrt{3})x}{x^2+\sqrt{2-\sqrt{3}}x+1} dx}{2\sqrt{2-\sqrt{3}}} + \frac{\int \frac{\sqrt{3(2+\sqrt{3})}-(1+\sqrt{3})x}{x^2-\sqrt{2+\sqrt{3}}x+1} dx}{2\sqrt{2+\sqrt{3}}} + \frac{\int \frac{(1+\sqrt{3})x+\sqrt{3(2+\sqrt{3})}}{x^2+\sqrt{2+\sqrt{3}}x+1} dx}{2\sqrt{2+\sqrt{3}}} \\ & \quad \downarrow \text{1142} \end{aligned}$$

$$\begin{aligned}
& \frac{\int \frac{1}{x^2 - \sqrt{2 - \sqrt{3}x + 1}} dx}{2\sqrt{2 - \sqrt{3}}} + \frac{\frac{1}{2}(1 - \sqrt{3}) \int -\frac{\sqrt{2 - \sqrt{3}} - 2x}{x^2 - \sqrt{2 - \sqrt{3}x + 1}} dx}{2\sqrt{2 - \sqrt{3}}} + \frac{\int \frac{1}{x^2 + \sqrt{2 + \sqrt{3}x + 1}} dx}{2\sqrt{2 + \sqrt{3}}} - \frac{\frac{1}{2}(1 - \sqrt{3}) \int \frac{2x + \sqrt{2 - \sqrt{3}}}{x^2 + \sqrt{2 + \sqrt{3}x + 1}} dx}{2\sqrt{2 - \sqrt{3}}} + \\
& \frac{\int \frac{1}{x^2 - \sqrt{2 + \sqrt{3}x + 1}} dx}{2\sqrt{2 + \sqrt{3}}} - \frac{\frac{1}{2}(1 + \sqrt{3}) \int -\frac{\sqrt{2 + \sqrt{3}} - 2x}{x^2 - \sqrt{2 + \sqrt{3}x + 1}} dx}{2\sqrt{2 + \sqrt{3}}} + \frac{\int \frac{1}{x^2 + \sqrt{2 + \sqrt{3}x + 1}} dx}{2\sqrt{2 + \sqrt{3}}} + \frac{\frac{1}{2}(1 + \sqrt{3}) \int \frac{2x + \sqrt{2 + \sqrt{3}}}{x^2 + \sqrt{2 + \sqrt{3}x + 1}} dx}{2\sqrt{2 + \sqrt{3}}} + \\
& \hspace{10em} \downarrow \text{25} \\
& \frac{\int \frac{1}{x^2 - \sqrt{2 - \sqrt{3}x + 1}} dx}{2\sqrt{2 - \sqrt{3}}} - \frac{\frac{1}{2}(1 - \sqrt{3}) \int \frac{\sqrt{2 - \sqrt{3}} - 2x}{x^2 - \sqrt{2 - \sqrt{3}x + 1}} dx}{2\sqrt{2 - \sqrt{3}}} + \frac{\int \frac{1}{x^2 + \sqrt{2 + \sqrt{3}x + 1}} dx}{2\sqrt{2 + \sqrt{3}}} - \frac{\frac{1}{2}(1 - \sqrt{3}) \int \frac{2x + \sqrt{2 - \sqrt{3}}}{x^2 + \sqrt{2 - \sqrt{3}x + 1}} dx}{2\sqrt{2 - \sqrt{3}}} + \\
& \frac{\int \frac{1}{x^2 - \sqrt{2 + \sqrt{3}x + 1}} dx}{2\sqrt{2 + \sqrt{3}}} + \frac{\frac{1}{2}(1 + \sqrt{3}) \int \frac{\sqrt{2 + \sqrt{3}} - 2x}{x^2 - \sqrt{2 + \sqrt{3}x + 1}} dx}{2\sqrt{2 + \sqrt{3}}} + \frac{\int \frac{1}{x^2 + \sqrt{2 + \sqrt{3}x + 1}} dx}{2\sqrt{2 + \sqrt{3}}} + \frac{\frac{1}{2}(1 + \sqrt{3}) \int \frac{2x + \sqrt{2 + \sqrt{3}}}{x^2 + \sqrt{2 + \sqrt{3}x + 1}} dx}{2\sqrt{2 + \sqrt{3}}} + \\
& \hspace{10em} \downarrow \text{1083} \\
& \frac{-\frac{1}{2}(1 - \sqrt{3}) \int \frac{\sqrt{2 - \sqrt{3}} - 2x}{x^2 - \sqrt{2 - \sqrt{3}x + 1}} dx - \sqrt{2} \int \frac{1}{-(2x - \sqrt{2 - \sqrt{3}})^2 - \sqrt{3} - 2} d(2x - \sqrt{2 - \sqrt{3}})}{2\sqrt{2 - \sqrt{3}}} + \frac{-\frac{1}{2}(1 - \sqrt{3}) \int \frac{2x + \sqrt{2 - \sqrt{3}}}{x^2 + \sqrt{2 - \sqrt{3}x + 1}} dx - \sqrt{2} \int \frac{1}{-(2x + \sqrt{2 - \sqrt{3}})^2 - \sqrt{3} - 2} d(2x + \sqrt{2 - \sqrt{3}})}{2\sqrt{2 - \sqrt{3}}} + \\
& \frac{\frac{1}{2}(1 + \sqrt{3}) \int \frac{\sqrt{2 + \sqrt{3}} - 2x}{x^2 - \sqrt{2 + \sqrt{3}x + 1}} dx - \sqrt{2} \int \frac{1}{-(2x - \sqrt{2 + \sqrt{3}})^2 + \sqrt{3} - 2} d(2x - \sqrt{2 + \sqrt{3}})}{2\sqrt{2 + \sqrt{3}}} + \frac{\frac{1}{2}(1 + \sqrt{3}) \int \frac{2x + \sqrt{2 + \sqrt{3}}}{x^2 + \sqrt{2 + \sqrt{3}x + 1}} dx - \sqrt{2} \int \frac{1}{-(2x + \sqrt{2 + \sqrt{3}})^2 + \sqrt{3} - 2} d(2x + \sqrt{2 + \sqrt{3}})}{2\sqrt{2 + \sqrt{3}}} + \\
& \hspace{10em} \downarrow \text{217} \\
& \frac{\sqrt{\frac{2}{2 + \sqrt{3}}} \arctan\left(\frac{2x - \sqrt{2 - \sqrt{3}}}{\sqrt{2 + \sqrt{3}}}\right) - \frac{1}{2}(1 - \sqrt{3}) \int \frac{\sqrt{2 - \sqrt{3}} - 2x}{x^2 - \sqrt{2 - \sqrt{3}x + 1}} dx}{2\sqrt{2 - \sqrt{3}}} + \frac{\sqrt{\frac{2}{2 + \sqrt{3}}} \arctan\left(\frac{2x + \sqrt{2 - \sqrt{3}}}{\sqrt{2 + \sqrt{3}}}\right) - \frac{1}{2}(1 - \sqrt{3}) \int \frac{2x + \sqrt{2 - \sqrt{3}}}{x^2 + \sqrt{2 - \sqrt{3}x + 1}} dx}{2\sqrt{2 - \sqrt{3}}} + \\
& \frac{\frac{1}{2}(1 + \sqrt{3}) \int \frac{\sqrt{2 + \sqrt{3}} - 2x}{x^2 - \sqrt{2 + \sqrt{3}x + 1}} dx + \sqrt{\frac{2}{2 - \sqrt{3}}} \arctan\left(\frac{2x - \sqrt{2 + \sqrt{3}}}{\sqrt{2 - \sqrt{3}}}\right)}{2\sqrt{2 + \sqrt{3}}} + \frac{\frac{1}{2}(1 + \sqrt{3}) \int \frac{2x + \sqrt{2 + \sqrt{3}}}{x^2 + \sqrt{2 + \sqrt{3}x + 1}} dx + \sqrt{\frac{2}{2 - \sqrt{3}}} \arctan\left(\frac{2x + \sqrt{2 + \sqrt{3}}}{\sqrt{2 - \sqrt{3}}}\right)}{2\sqrt{2 + \sqrt{3}}} + \\
& \hspace{10em} \downarrow \text{1103} \\
& \frac{\sqrt{\frac{2}{2 + \sqrt{3}}} \arctan\left(\frac{2x - \sqrt{2 - \sqrt{3}}}{\sqrt{2 + \sqrt{3}}}\right) + \frac{1}{2}(1 - \sqrt{3}) \log(x^2 - \sqrt{2 - \sqrt{3}x + 1})}{2\sqrt{2 - \sqrt{3}}} + \frac{\sqrt{\frac{2}{2 + \sqrt{3}}} \arctan\left(\frac{2x + \sqrt{2 - \sqrt{3}}}{\sqrt{2 + \sqrt{3}}}\right) - \frac{1}{2}(1 - \sqrt{3}) \log(x^2 + \sqrt{2 - \sqrt{3}x + 1})}{2\sqrt{2 - \sqrt{3}}} + \\
& \frac{\sqrt{\frac{2}{2 - \sqrt{3}}} \arctan\left(\frac{2x - \sqrt{2 + \sqrt{3}}}{\sqrt{2 - \sqrt{3}}}\right) - \frac{1}{2}(1 + \sqrt{3}) \log(x^2 - \sqrt{2 + \sqrt{3}x + 1})}{2\sqrt{2 + \sqrt{3}}} + \frac{\sqrt{\frac{2}{2 - \sqrt{3}}} \arctan\left(\frac{2x + \sqrt{2 + \sqrt{3}}}{\sqrt{2 - \sqrt{3}}}\right) + \frac{1}{2}(1 + \sqrt{3}) \log(x^2 + \sqrt{2 + \sqrt{3}x + 1})}{2\sqrt{2 + \sqrt{3}}} + \\
& \hspace{10em} \downarrow \text{25}
\end{aligned}$$

---

3.51.  $\int \frac{1}{1 - x^4 + x^8} dx$

input `Int[(1 - x^4 + x^8)^(-1),x]`

output `((Sqrt[2/(2 + Sqrt[3])]*ArcTan[(-Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]] + ((1 - Sqrt[3])*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]) + (Sqrt[2/(2 + Sqrt[3])]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]) - ((1 - Sqrt[3])*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 - Sqrt[3]]) + ((Sqrt[2/(2 - Sqrt[3])]*ArcTan[(-Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]) - ((1 + Sqrt[3])*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]]) + (Sqrt[2/(2 - Sqrt[3])]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]) + ((1 + Sqrt[3])*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/2)/(2*Sqrt[2 + Sqrt[3]])))/(2*Sqrt[3])`

### 3.51.3.1 Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

rule 1083 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Simp[-2 Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x]`

rule 1103 `Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

rule 1142 `Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(2*c*d - b*e)/(2*c) Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c) Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]`

```
rule 1483 Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r) Int[
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r) Int[(d*r
+ (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

```
rule 1684 Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(n_+1), x_Symbol] := With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r) Int[(r - x^(
n/2))/(q - r*x^(n/2) + x^n), x], x] + Simp[1/(2*c*q*r) Int[(r + x^(n/2))/
(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

### 3.51.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.11

| method  | result                                                                         | size |
|---------|--------------------------------------------------------------------------------|------|
| default | $\frac{\left( \sum_{R=\text{RootOf}(9Z^4+1)} -R \ln(3R^2+3Rx+x^2) \right)}{4}$ | 30   |
| risch   | $\frac{\left( \sum_{R=\text{RootOf}(9Z^4+1)} -R \ln(3R^2+3Rx+x^2) \right)}{4}$ | 30   |

```
input int(1/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
output 1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

**3.51.5 Fracas [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.37

$$\int \frac{1}{1-x^4+x^8} dx = \left(\frac{1}{24}i + \frac{1}{24}\right) \sqrt{3}\sqrt{2} \log\left((3i+3) \sqrt{3}\sqrt{2}x + 6x^2 + 6i\right) \\ - \left(\frac{1}{24}i - \frac{1}{24}\right) \sqrt{3}\sqrt{2} \log\left(-(3i-3) \sqrt{3}\sqrt{2}x + 6x^2 - 6i\right) \\ + \left(\frac{1}{24}i - \frac{1}{24}\right) \sqrt{3}\sqrt{2} \log\left((3i-3) \sqrt{3}\sqrt{2}x + 6x^2 - 6i\right) \\ - \left(\frac{1}{24}i + \frac{1}{24}\right) \sqrt{3}\sqrt{2} \log\left(-(3i+3) \sqrt{3}\sqrt{2}x + 6x^2 + 6i\right)$$

input `integrate(1/(x^8-x^4+1),x, algorithm="fricas")`

output `(1/24*I + 1/24)*sqrt(3)*sqrt(2)*log((3*I + 3)*sqrt(3)*sqrt(2)*x + 6*x^2 + 6*I) - (1/24*I - 1/24)*sqrt(3)*sqrt(2)*log(-(3*I - 3)*sqrt(3)*sqrt(2)*x + 6*x^2 - 6*I) + (1/24*I - 1/24)*sqrt(3)*sqrt(2)*log((3*I - 3)*sqrt(3)*sqrt(2)*x + 6*x^2 - 6*I) - (1/24*I + 1/24)*sqrt(3)*sqrt(2)*log(-(3*I + 3)*sqrt(3)*sqrt(2)*x + 6*x^2 + 6*I)`

**3.51.6 Sympy [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.60

$$\int \frac{1}{1-x^4+x^8} dx = \frac{\sqrt{6} \cdot \left(2 \operatorname{atan}\left(\frac{\sqrt{6}x}{3} - \frac{1}{3}\right) + 2 \operatorname{atan}\left(\sqrt{6}x^3 - 4x^2 + 2\sqrt{6}x - 3\right)\right)}{24} \\ + \frac{\sqrt{6} \cdot \left(2 \operatorname{atan}\left(\frac{\sqrt{6}x}{3} + \frac{1}{3}\right) + 2 \operatorname{atan}\left(\sqrt{6}x^3 + 4x^2 + 2\sqrt{6}x + 3\right)\right)}{24} \\ - \frac{\sqrt{6} \log\left(x^4 - \sqrt{6}x^3 + 3x^2 - \sqrt{6}x + 1\right)}{24} \\ + \frac{\sqrt{6} \log\left(x^4 + \sqrt{6}x^3 + 3x^2 + \sqrt{6}x + 1\right)}{24}$$

input `integrate(1/(x**8-x**4+1),x)`

output `sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x + 1)/24`

### 3.51.7 Maxima [F]

$$\int \frac{1}{1-x^4+x^8} dx = \int \frac{1}{x^8-x^4+1} dx$$

input `integrate(1/(x^8-x^4+1),x, algorithm="maxima")`

output `integrate(1/(x^8 - x^4 + 1), x)`

### 3.51.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.75

$$\begin{aligned} \int \frac{1}{1-x^4+x^8} dx &= \frac{1}{12} \sqrt{6} \arctan \left( \frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} \right) + \frac{1}{12} \sqrt{6} \arctan \left( \frac{4x - \sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}} \right) \\ &+ \frac{1}{12} \sqrt{6} \arctan \left( \frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}} \right) + \frac{1}{12} \sqrt{6} \arctan \left( \frac{4x - \sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}} \right) \\ &+ \frac{1}{24} \sqrt{6} \log \left( x^2 + \frac{1}{2} x (\sqrt{6} + \sqrt{2}) + 1 \right) \\ &- \frac{1}{24} \sqrt{6} \log \left( x^2 - \frac{1}{2} x (\sqrt{6} + \sqrt{2}) + 1 \right) \\ &+ \frac{1}{24} \sqrt{6} \log \left( x^2 + \frac{1}{2} x (\sqrt{6} - \sqrt{2}) + 1 \right) \\ &- \frac{1}{24} \sqrt{6} \log \left( x^2 - \frac{1}{2} x (\sqrt{6} - \sqrt{2}) + 1 \right) \end{aligned}$$

input `integrate(1/(x^8-x^4+1),x, algorithm="giac")`

```
output 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(
6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*ar
ctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2
+ 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6)
+ sqrt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) -
1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

### 3.51.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.19

$$\int \frac{1}{1-x^4+x^8} dx = \sqrt{6} \operatorname{atan} \left( \frac{\sqrt{6} x \left( \frac{1}{3} + \frac{1}{3}i \right)}{\frac{2x^2}{3} - \frac{2}{3}i} \right) \left( -\frac{1}{12} - \frac{1}{12}i \right) \\ + \sqrt{6} \operatorname{atan} \left( \frac{\sqrt{6} x \left( \frac{1}{3} - \frac{1}{3}i \right)}{\frac{2x^2}{3} + \frac{2}{3}i} \right) \left( -\frac{1}{12} + \frac{1}{12}i \right)$$

```
input int(1/(x^8 - x^4 + 1),x)
```

```
output - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i/12)
- 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1i/12
)
```



## 3.52 $\int \frac{x^7}{1+x^{12}} dx$

|        |                                                     |     |
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| 3.52.2 | Mathematica [B] (verified) . . . . .                | 392 |
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### 3.52.1 Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{x^7}{1+x^{12}} dx = -\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{12} \log(1+x^4) + \frac{1}{24} \log(1-x^4+x^8)$$

output `-1/12*ln(x^4+1)+1/24*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)`

### 3.52.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 260 vs.  $2(49) = 98$ .

Time = 0.07 (sec) , antiderivative size = 260, normalized size of antiderivative = 5.31

$$\begin{aligned} \int \frac{x^7}{1+x^{12}} dx = & \frac{1}{24} \left( 2\sqrt{3} \arctan\left(\frac{1+\sqrt{3}-2\sqrt{2}x}{1-\sqrt{3}}\right) - 2\sqrt{3} \arctan\left(\frac{1-\sqrt{3}+2\sqrt{2}x}{1+\sqrt{3}}\right) \right. \\ & + 2\sqrt{3} \arctan\left(\frac{-1+\sqrt{3}+2\sqrt{2}x}{1+\sqrt{3}}\right) - 2\sqrt{3} \arctan\left(\frac{1+\sqrt{3}+2\sqrt{2}x}{-1+\sqrt{3}}\right) \\ & - 2 \log(1-\sqrt{2}x+x^2) - 2 \log(1+\sqrt{2}x+x^2) \\ & + \log(2+\sqrt{2}x-\sqrt{6}x+2x^2) + \log(2+\sqrt{2}(-1+\sqrt{3})x+2x^2) \\ & \left. + \log(2-(\sqrt{2}+\sqrt{6})x+2x^2) + \log(2+(\sqrt{2}+\sqrt{6})x+2x^2) \right) \end{aligned}$$

input `Integrate[x^7/(1 + x^12),x]`

output `(2*Sqrt[3]*ArcTan[(1 + Sqrt[3] - 2*Sqrt[2]*x)/(1 - Sqrt[3])] - 2*Sqrt[3]*ArcTan[(1 - Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] + 2*Sqrt[3]*ArcTan[(-1 + Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] - 2*Sqrt[3]*ArcTan[(1 + Sqrt[3] + 2*Sqrt[2]*x)/(-1 + Sqrt[3])] - 2*Log[1 - Sqrt[2]*x + x^2] - 2*Log[1 + Sqrt[2]*x + x^2] + Log[2 + Sqrt[2]*x - Sqrt[6]*x + 2*x^2] + Log[2 + Sqrt[2]*(-1 + Sqrt[3])*x + 2*x^2] + Log[2 - (Sqrt[2] + Sqrt[6])*x + 2*x^2] + Log[2 + (Sqrt[2] + Sqrt[6])*x + 2*x^2])/24`

### 3.52.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$ , Rules used = {807, 821, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^7}{x^{12} + 1} dx \\
 & \quad \downarrow \text{807} \\
 & \frac{1}{4} \int \frac{x^4}{x^{12} + 1} dx^4 \\
 & \quad \downarrow \text{821} \\
 & \frac{1}{4} \left( \frac{1}{3} \int \frac{x^4 + 1}{x^8 - x^4 + 1} dx^4 - \frac{1}{3} \int \frac{1}{x^4 + 1} dx^4 \right) \\
 & \quad \downarrow \text{16} \\
 & \frac{1}{4} \left( \frac{1}{3} \int \frac{x^4 + 1}{x^8 - x^4 + 1} dx^4 - \frac{1}{3} \log(x^4 + 1) \right) \\
 & \quad \downarrow \text{1142} \\
 & \frac{1}{4} \left( \frac{1}{3} \left( \frac{3}{2} \int \frac{1}{x^8 - x^4 + 1} dx^4 + \frac{1}{2} \int -\frac{1 - 2x^4}{x^8 - x^4 + 1} dx^4 \right) - \frac{1}{3} \log(x^4 + 1) \right) \\
 & \quad \downarrow \text{25} \\
 & \frac{1}{4} \left( \frac{1}{3} \left( \frac{3}{2} \int \frac{1}{x^8 - x^4 + 1} dx^4 - \frac{1}{2} \int \frac{1 - 2x^4}{x^8 - x^4 + 1} dx^4 \right) - \frac{1}{3} \log(x^4 + 1) \right)
 \end{aligned}$$

$$\begin{aligned}
 & \downarrow 1083 \\
 & \frac{1}{4} \left( \frac{1}{3} \left( -3 \int \frac{1}{-x^8 - 3} d(2x^4 - 1) - \frac{1}{2} \int \frac{1 - 2x^4}{x^8 - x^4 + 1} dx^4 \right) - \frac{1}{3} \log(x^4 + 1) \right) \\
 & \downarrow 217 \\
 & \frac{1}{4} \left( \frac{1}{3} \left( \sqrt{3} \arctan \left( \frac{2x^4 - 1}{\sqrt{3}} \right) - \frac{1}{2} \int \frac{1 - 2x^4}{x^8 - x^4 + 1} dx^4 \right) - \frac{1}{3} \log(x^4 + 1) \right) \\
 & \downarrow 1103 \\
 & \frac{1}{4} \left( \frac{1}{3} \left( \sqrt{3} \arctan \left( \frac{2x^4 - 1}{\sqrt{3}} \right) + \frac{1}{2} \log(x^8 - x^4 + 1) \right) - \frac{1}{3} \log(x^4 + 1) \right)
 \end{aligned}$$

input `Int[x^7/(1 + x^12),x]`

output `(-1/3*Log[1 + x^4] + (Sqrt[3]*ArcTan[(-1 + 2*x^4)/Sqrt[3]] + Log[1 - x^4 + x^8]/2)/3)/4`

### 3.52.3.1 Defintions of rubi rules used

rule 16 `Int[(c_)/((a_) + (b_)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 217 `Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

rule 807 `Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Simp[1/k Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]`

rule 821 `Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-1) Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3]) Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]`

rule 1083 `Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Simp[-2 Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x]`

rule 1103 `Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

rule 1142 `Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(2*c*d - b*e)/(2*c) Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c) Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]`

### 3.52.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

| method  | result                                                                                                                                                                                                                                                                                                                              | size |
|---------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default | $\frac{\ln(x^8 - x^4 + 1)}{24} + \frac{\arctan\left(\frac{(2x^4 - 1)\sqrt{3}}{3}\right)\sqrt{3}}{12} - \frac{\ln(x^4 + 1)}{12}$                                                                                                                                                                                                     | 41   |
| risch   | $\frac{\ln(4x^8 - 4x^4 + 4)}{24} + \frac{\arctan\left(\frac{(2x^4 - 1)\sqrt{3}}{3}\right)\sqrt{3}}{12} - \frac{\ln(x^4 + 1)}{12}$                                                                                                                                                                                                   | 43   |
| meijerg | $-\frac{x^8 \ln\left(1 + (x^{12})^{\frac{1}{3}}\right)}{12(x^{12})^{\frac{2}{3}}} + \frac{x^8 \ln\left(1 - (x^{12})^{\frac{1}{3}} + (x^{12})^{\frac{2}{3}}\right)}{24(x^{12})^{\frac{2}{3}}} + \frac{x^8 \sqrt{3} \arctan\left(\frac{\sqrt{3}(x^{12})^{\frac{1}{3}}}{2 - (x^{12})^{\frac{1}{3}}}\right)}{12(x^{12})^{\frac{2}{3}}}$ | 80   |

input `int(x^7/(x^12+1),x,method=_RETURNVERBOSE)`

output `1/24*ln(x^8-x^4+1)+1/12*arctan(1/3*(2*x^4-1)*3^(1/2))*3^(1/2)-1/12*ln(x^4+1)`

**3.52.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+x^{12}} dx = \frac{1}{12} \sqrt{3} \arctan \left( \frac{1}{3} \sqrt{3} (2x^4 - 1) \right) + \frac{1}{24} \log(x^8 - x^4 + 1) - \frac{1}{12} \log(x^4 + 1)$$

input `integrate(x^7/(x^12+1),x, algorithm="fracas")`output `1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/12*log(x^4 + 1)`**3.52.6 Sympy [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{x^7}{1+x^{12}} dx = -\frac{\log(x^4 + 1)}{12} + \frac{\log(x^8 - x^4 + 1)}{24} + \frac{\sqrt{3} \operatorname{atan} \left( \frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3} \right)}{12}$$

input `integrate(x**7/(x**12+1),x)`output `-log(x**4 + 1)/12 + log(x**8 - x**4 + 1)/24 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12`**3.52.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+x^{12}} dx = \frac{1}{12} \sqrt{3} \arctan \left( \frac{1}{3} \sqrt{3} (2x^4 - 1) \right) + \frac{1}{24} \log(x^8 - x^4 + 1) - \frac{1}{12} \log(x^4 + 1)$$

input `integrate(x^7/(x^12+1),x, algorithm="maxima")`output `1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/12*log(x^4 + 1)`

**3.52.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+x^{12}} dx = \frac{1}{12} \sqrt{3} \arctan \left( \frac{1}{3} \sqrt{3} (2x^4 - 1) \right) + \frac{1}{24} \log(x^8 - x^4 + 1) - \frac{1}{12} \log(x^4 + 1)$$

input `integrate(x^7/(x^12+1),x, algorithm="giac")`output `1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/12*log(x^4 + 1)`**3.52.9 Mupad [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{x^7}{1+x^{12}} dx = -\frac{\ln(x^4 + 1)}{12} - \ln \left( x^4 - \frac{\sqrt{3} \text{li}}{2} - \frac{1}{2} \right) \left( -\frac{1}{24} + \frac{\sqrt{3} \text{li}}{24} \right) \\ + \ln \left( x^4 + \frac{\sqrt{3} \text{li}}{2} - \frac{1}{2} \right) \left( \frac{1}{24} + \frac{\sqrt{3} \text{li}}{24} \right)$$

input `int(x^7/(x^12 + 1),x)`output `log((3^(1/2)*li)/2 + x^4 - 1/2)*((3^(1/2)*li)/24 + 1/24) - log(x^4 - (3^(1/2)*li)/2 - 1/2)*((3^(1/2)*li)/24 - 1/24) - log(x^4 + 1)/12`

## 3.53 $\int \log(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.53.1 | Optimal result . . . . .                            | 398 |
| 3.53.2 | Mathematica [A] (verified) . . . . .                | 398 |
| 3.53.3 | Rubi [A] (verified) . . . . .                       | 399 |
| 3.53.4 | Maple [A] (verified) . . . . .                      | 399 |
| 3.53.5 | Fricas [A] (verification not implemented) . . . . . | 400 |
| 3.53.6 | Sympy [A] (verification not implemented) . . . . .  | 400 |
| 3.53.7 | Maxima [A] (verification not implemented) . . . . . | 400 |
| 3.53.8 | Giac [A] (verification not implemented) . . . . .   | 401 |
| 3.53.9 | Mupad [B] (verification not implemented) . . . . .  | 401 |

### 3.53.1 Optimal result

Integrand size = 2, antiderivative size = 8

$$\int \log(x) dx = -x + x \log(x)$$

output `-x+x*ln(x)`

### 3.53.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x) dx = -x + x \log(x)$$

input `Integrate[Log[x],x]`

output `-x + x*Log[x]`

### 3.53.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(x) dx$$

$$\downarrow \text{2732}$$

$$x \log(x) - x$$

input `Int [Log[x] , x]`

output `-x + x*Log[x]`

#### 3.53.3.1 Defintions of rubi rules used

rule 2732 `Int [Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]`

### 3.53.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

| method        | result          | size |
|---------------|-----------------|------|
| lookup        | $-x + x \ln(x)$ | 9    |
| default       | $-x + x \ln(x)$ | 9    |
| norman        | $-x + x \ln(x)$ | 9    |
| risch         | $-x + x \ln(x)$ | 9    |
| parallelrisch | $-x + x \ln(x)$ | 9    |
| parts         | $-x + x \ln(x)$ | 9    |

input `int(ln(x), x, method=_RETURNVERBOSE)`



output `-x+x*ln(x)`

### 3.53.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x) dx = x \log(x) - x$$

input `integrate(log(x),x, algorithm="fricas")`

output `x*log(x) - x`

### 3.53.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \log(x) dx = x \log(x) - x$$

input `integrate(ln(x),x)`

output `x*log(x) - x`

### 3.53.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x) dx = x \log(x) - x$$

input `integrate(log(x),x, algorithm="maxima")`

output `x*log(x) - x`

**3.53.8 Giac [A] (verification not implemented)**

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \log(x) dx = x \log(x) - x$$

input `integrate(log(x),x, algorithm="giac")`

output `x*log(x) - x`

**3.53.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \log(x) dx = x (\ln(x) - 1)$$

input `int(log(x),x)`

output `x*(log(x) - 1)`

## 3.54 $\int x \log(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.54.1 | Optimal result . . . . .                            | 402 |
| 3.54.2 | Mathematica [A] (verified) . . . . .                | 402 |
| 3.54.3 | Rubi [A] (verified) . . . . .                       | 403 |
| 3.54.4 | Maple [A] (verified) . . . . .                      | 403 |
| 3.54.5 | Fricas [A] (verification not implemented) . . . . . | 404 |
| 3.54.6 | Sympy [A] (verification not implemented) . . . . .  | 404 |
| 3.54.7 | Maxima [A] (verification not implemented) . . . . . | 404 |
| 3.54.8 | Giac [A] (verification not implemented) . . . . .   | 405 |
| 3.54.9 | Mupad [B] (verification not implemented) . . . . .  | 405 |

### 3.54.1 Optimal result

Integrand size = 4, antiderivative size = 17

$$\int x \log(x) dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

output `-1/4*x^2+1/2*x^2*ln(x)`

### 3.54.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x \log(x) dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

input `Integrate[x*Log[x],x]`

output `-1/4*x^2 + (x^2*Log[x])/2`

### 3.54.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log(x) dx$$

$$\downarrow 2741$$

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

input `Int[x*Log[x],x]`

output `-1/4*x^2 + (x^2*Log[x])/2`

#### 3.54.3.1 Defintions of rubi rules used

rule 2741 `Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]`

### 3.54.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

| method        | result                                  | size |
|---------------|-----------------------------------------|------|
| default       | $-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}$ | 14   |
| norman        | $-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}$ | 14   |
| risch         | $-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}$ | 14   |
| parallelrisch | $-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}$ | 14   |
| parts         | $-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}$ | 14   |

input `int(x*ln(x),x,method=_RETURNVERBOSE)`

output `-1/4*x^2+1/2*x^2*ln(x)`

### 3.54.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x \log(x) dx = \frac{1}{2} x^2 \log(x) - \frac{1}{4} x^2$$

input `integrate(x*log(x),x, algorithm="fricas")`

output `1/2*x^2*log(x) - 1/4*x^2`

### 3.54.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x \log(x) dx = \frac{x^2 \log(x)}{2} - \frac{x^2}{4}$$

input `integrate(x*ln(x),x)`

output `x**2*log(x)/2 - x**2/4`

### 3.54.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x \log(x) dx = \frac{1}{2} x^2 \log(x) - \frac{1}{4} x^2$$

input `integrate(x*log(x),x, algorithm="maxima")`

output `1/2*x^2*log(x) - 1/4*x^2`

**3.54.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x \log(x) dx = \frac{1}{2} x^2 \log(x) - \frac{1}{4} x^2$$

input `integrate(x*log(x),x, algorithm="giac")`

output `1/2*x^2*log(x) - 1/4*x^2`

**3.54.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int x \log(x) dx = \frac{x^2 (\ln(x) - \frac{1}{2})}{2}$$

input `int(x*log(x),x)`

output `(x^2*(log(x) - 1/2))/2`

## 3.55 $\int x^2 \log(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.55.1 | Optimal result . . . . .                            | 406 |
| 3.55.2 | Mathematica [A] (verified) . . . . .                | 406 |
| 3.55.3 | Rubi [A] (verified) . . . . .                       | 407 |
| 3.55.4 | Maple [A] (verified) . . . . .                      | 407 |
| 3.55.5 | Fricas [A] (verification not implemented) . . . . . | 408 |
| 3.55.6 | Sympy [A] (verification not implemented) . . . . .  | 408 |
| 3.55.7 | Maxima [A] (verification not implemented) . . . . . | 408 |
| 3.55.8 | Giac [A] (verification not implemented) . . . . .   | 409 |
| 3.55.9 | Mupad [B] (verification not implemented) . . . . .  | 409 |

### 3.55.1 Optimal result

Integrand size = 6, antiderivative size = 17

$$\int x^2 \log(x) dx = -\frac{x^3}{9} + \frac{1}{3}x^3 \log(x)$$

output `-1/9*x^3+1/3*x^3*ln(x)`

### 3.55.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2 \log(x) dx = -\frac{x^3}{9} + \frac{1}{3}x^3 \log(x)$$

input `Integrate[x^2*Log[x],x]`

output `-1/9*x^3 + (x^3*Log[x])/3`

### 3.55.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log(x) dx$$

$$\downarrow 2741$$

$$\frac{1}{3}x^3 \log(x) - \frac{x^3}{9}$$

input `Int[x^2*Log[x],x]`

output `-1/9*x^3 + (x^3*Log[x])/3`

#### 3.55.3.1 Defintions of rubi rules used

rule 2741 `Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol] :=  
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(  
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]`

### 3.55.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

| method        | result                                  | size |
|---------------|-----------------------------------------|------|
| default       | $-\frac{x^3}{9} + \frac{x^3 \ln(x)}{3}$ | 14   |
| norman        | $-\frac{x^3}{9} + \frac{x^3 \ln(x)}{3}$ | 14   |
| risch         | $-\frac{x^3}{9} + \frac{x^3 \ln(x)}{3}$ | 14   |
| parallelrisch | $-\frac{x^3}{9} + \frac{x^3 \ln(x)}{3}$ | 14   |
| parts         | $-\frac{x^3}{9} + \frac{x^3 \ln(x)}{3}$ | 14   |



input `int(x^2*ln(x),x,method=_RETURNVERBOSE)`

output `-1/9*x^3+1/3*x^3*ln(x)`

### 3.55.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2 \log(x) dx = \frac{1}{3} x^3 \log(x) - \frac{1}{9} x^3$$

input `integrate(x^2*log(x),x, algorithm="fricas")`

output `1/3*x^3*log(x) - 1/9*x^3`

### 3.55.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^2 \log(x) dx = \frac{x^3 \log(x)}{3} - \frac{x^3}{9}$$

input `integrate(x**2*ln(x),x)`

output `x**3*log(x)/3 - x**3/9`

### 3.55.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2 \log(x) dx = \frac{1}{3} x^3 \log(x) - \frac{1}{9} x^3$$

input `integrate(x^2*log(x),x, algorithm="maxima")`

output `1/3*x^3*log(x) - 1/9*x^3`

**3.55.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2 \log(x) dx = \frac{1}{3} x^3 \log(x) - \frac{1}{9} x^3$$

input `integrate(x^2*log(x),x, algorithm="giac")`

output `1/3*x^3*log(x) - 1/9*x^3`

**3.55.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int x^2 \log(x) dx = \frac{x^3 (\ln(x) - \frac{1}{3})}{3}$$

input `int(x^2*log(x),x)`

output `(x^3*(log(x) - 1/3))/3`

### 3.56 $\int x^p \log(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.56.1 | Optimal result . . . . .                            | 410 |
| 3.56.2 | Mathematica [A] (verified) . . . . .                | 410 |
| 3.56.3 | Rubi [A] (verified) . . . . .                       | 411 |
| 3.56.4 | Maple [A] (verified) . . . . .                      | 411 |
| 3.56.5 | Fricas [A] (verification not implemented) . . . . . | 412 |
| 3.56.6 | Sympy [B] (verification not implemented) . . . . .  | 412 |
| 3.56.7 | Maxima [A] (verification not implemented) . . . . . | 412 |
| 3.56.8 | Giac [F] . . . . .                                  | 413 |
| 3.56.9 | Mupad [B] (verification not implemented) . . . . .  | 413 |

#### 3.56.1 Optimal result

Integrand size = 6, antiderivative size = 26

$$\int x^p \log(x) dx = -\frac{x^{1+p}}{(1+p)^2} + \frac{x^{1+p} \log(x)}{1+p}$$

output `-x^(p+1)/(p+1)^2+x^(p+1)*ln(x)/(p+1)`

#### 3.56.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int x^p \log(x) dx = \frac{x^{1+p}(-1 + (1+p) \log(x))}{(1+p)^2}$$

input `Integrate[x^p*Log[x],x]`

output `(x^(1 + p)*(-1 + (1 + p)*Log[x]))/(1 + p)^2`

### 3.56.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^p \log(x) dx$$

$$\downarrow \text{2741}$$

$$\frac{x^{p+1} \log(x)}{p+1} - \frac{x^{p+1}}{(p+1)^2}$$

input `Int[x^p*Log[x],x]`

output `-(x^(1+p))/(1+p)^2 + (x^(1+p)*Log[x])/(1+p)`

#### 3.56.3.1 Defintions of rubi rules used

rule 2741 `Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m+1)*((a + b*Log[c*x^n])/(d*(m+1))), x] - Simp[b*n*((d*x)^(m+1)/(d*(m+1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]`

### 3.56.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

| method        | result                                                                  | size |
|---------------|-------------------------------------------------------------------------|------|
| risch         | $\frac{x(\ln(x)p + \ln(x) - 1)x^p}{(1+p)^2}$                            | 19   |
| norman        | $\frac{x \ln(x) e^{\ln(x)p}}{1+p} - \frac{x e^{\ln(x)p}}{p^2 + 2p + 1}$ | 34   |
| parallelrisch | $\frac{x x^p \ln(x)p + x^p \ln(x)x - x x^p}{p^2 + 2p + 1}$              | 34   |

input `int(x^p*ln(x),x,method=_RETURNVERBOSE)`

output `x*(ln(x)*p+ln(x)-1)/(1+p)^2*x^p`

### 3.56.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int x^p \log(x) dx = \frac{((p+1)x \log(x) - x)x^p}{p^2 + 2p + 1}$$

input `integrate(x^p*log(x),x, algorithm="fricas")`

output `((p + 1)*x*log(x) - x)*x^p/(p^2 + 2*p + 1)`

### 3.56.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(20) = 40.

Time = 0.39 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.15

$$\int x^p \log(x) dx = \begin{cases} \frac{pxx^p \log(x)}{p^2+2p+1} + \frac{xx^p \log(x)}{p^2+2p+1} - \frac{xx^p}{p^2+2p+1} & \text{for } p \neq -1 \\ \frac{\log(x)^2}{2} & \text{otherwise} \end{cases}$$

input `integrate(x**p*ln(x),x)`

output `Piecewise((p*x*x**p*log(x)/(p**2 + 2*p + 1) + x*x**p*log(x)/(p**2 + 2*p + 1) - x*x**p/(p**2 + 2*p + 1), Ne(p, -1)), (log(x)**2/2, True))`

### 3.56.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^p \log(x) dx = \frac{x^{p+1} \log(x)}{p+1} - \frac{x^{p+1}}{(p+1)^2}$$

input `integrate(x^p*log(x),x, algorithm="maxima")`

output `x^(p + 1)*log(x)/(p + 1) - x^(p + 1)/(p + 1)^2`

**3.56.8 Giac [F]**

$$\int x^p \log(x) dx = \int x^p \log(x) dx$$

input `integrate(x^p*log(x),x, algorithm="giac")`

output `integrate(x^p*log(x), x)`

**3.56.9 Mupad [B] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int x^p \log(x) dx = \begin{cases} \frac{\ln(x)^2}{2} & \text{if } p = -1 \\ \frac{x^{p+1} (\ln(x) (p+1) - 1)}{(p+1)^2} & \text{if } p \neq -1 \end{cases}$$

input `int(x^p*log(x),x)`

output `piecewise(p == -1, log(x)^2/2, p ~= -1, (x^(p + 1)*(log(x)*(p + 1) - 1))/(p + 1)^2)`

## 3.57 $\int \log^2(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.57.1 | Optimal result . . . . .                            | 414 |
| 3.57.2 | Mathematica [A] (verified) . . . . .                | 414 |
| 3.57.3 | Rubi [A] (verified) . . . . .                       | 415 |
| 3.57.4 | Maple [A] (verified) . . . . .                      | 416 |
| 3.57.5 | Fricas [A] (verification not implemented) . . . . . | 416 |
| 3.57.6 | Sympy [A] (verification not implemented) . . . . .  | 416 |
| 3.57.7 | Maxima [A] (verification not implemented) . . . . . | 417 |
| 3.57.8 | Giac [A] (verification not implemented) . . . . .   | 417 |
| 3.57.9 | Mupad [B] (verification not implemented) . . . . .  | 417 |

### 3.57.1 Optimal result

Integrand size = 4, antiderivative size = 15

$$\int \log^2(x) dx = 2x - 2x \log(x) + x \log^2(x)$$

output `2*x-2*x*ln(x)+x*ln(x)^2`

### 3.57.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x) dx = 2x - 2x \log(x) + x \log^2(x)$$

input `Integrate[Log[x]^2,x]`

output `2*x - 2*x*Log[x] + x*Log[x]^2`

### 3.57.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \log^2(x) dx \\ & \quad \downarrow \text{2733} \\ & x \log^2(x) - 2 \int \log(x) dx \\ & \quad \downarrow \text{2732} \\ & x \log^2(x) - 2(x \log(x) - x) \end{aligned}$$

input `Int [Log[x]^2,x]`

output `x*Log[x]^2 - 2*(-x + x*Log[x])`

#### 3.57.3.1 Defintions of rubi rules used

rule 2732 `Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]`

rule 2733 `Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b *Log[c*x^n])^p, x] - Simp[b*n*p Int[(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]`



**3.57.4 Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

| method       | result                        | size |
|--------------|-------------------------------|------|
| default      | $2x - 2x \ln(x) + x \ln(x)^2$ | 16   |
| norman       | $2x - 2x \ln(x) + x \ln(x)^2$ | 16   |
| risch        | $2x - 2x \ln(x) + x \ln(x)^2$ | 16   |
| parallelrisc | $2x - 2x \ln(x) + x \ln(x)^2$ | 16   |

input `int(ln(x)^2,x,method=_RETURNVERBOSE)`output `2*x-2*x*ln(x)+x*ln(x)^2`**3.57.5 Fricas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x) dx = x \log(x)^2 - 2x \log(x) + 2x$$

input `integrate(log(x)^2,x, algorithm="fricas")`output `x*log(x)^2 - 2*x*log(x) + 2*x`**3.57.6 Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x) dx = x \log(x)^2 - 2x \log(x) + 2x$$

input `integrate(ln(x)**2,x)`output `x*log(x)**2 - 2*x*log(x) + 2*x`

**3.57.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \log^2(x) dx = (\log(x)^2 - 2 \log(x) + 2)x$$

input `integrate(log(x)^2,x, algorithm="maxima")`

output `(log(x)^2 - 2*log(x) + 2)*x`

**3.57.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \log^2(x) dx = x \log(x)^2 - 2x \log(x) + 2x$$

input `integrate(log(x)^2,x, algorithm="giac")`

output `x*log(x)^2 - 2*x*log(x) + 2*x`

**3.57.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \log^2(x) dx = x (\ln(x)^2 - 2 \ln(x) + 2)$$

input `int(log(x)^2,x)`

output `x*(log(x)^2 - 2*log(x) + 2)`

### 3.58 $\int x^9 \log^{11}(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.58.1 | Optimal result . . . . .                            | 418 |
| 3.58.2 | Mathematica [A] (verified) . . . . .                | 418 |
| 3.58.3 | Rubi [A] (verified) . . . . .                       | 419 |
| 3.58.4 | Maple [A] (verified) . . . . .                      | 421 |
| 3.58.5 | Fricas [A] (verification not implemented) . . . . . | 422 |
| 3.58.6 | Sympy [A] (verification not implemented) . . . . .  | 422 |
| 3.58.7 | Maxima [A] (verification not implemented) . . . . . | 423 |
| 3.58.8 | Giac [A] (verification not implemented) . . . . .   | 423 |
| 3.58.9 | Mupad [B] (verification not implemented) . . . . .  | 424 |

#### 3.58.1 Optimal result

Integrand size = 8, antiderivative size = 127

$$\int x^9 \log^{11}(x) dx = -\frac{6237x^{10}}{156250000} + \frac{6237x^{10} \log(x)}{15625000} - \frac{6237x^{10} \log^2(x)}{3125000} + \frac{2079x^{10} \log^3(x)}{312500}$$

$$- \frac{2079x^{10} \log^4(x)}{125000} + \frac{2079x^{10} \log^5(x)}{62500} - \frac{693x^{10} \log^6(x)}{12500} + \frac{99x^{10} \log^7(x)}{1250}$$

$$- \frac{99x^{10} \log^8(x)}{1000} + \frac{11}{100}x^{10} \log^9(x) - \frac{11}{100}x^{10} \log^{10}(x) + \frac{1}{10}x^{10} \log^{11}(x)$$

output

```
-6237/156250000*x^10+6237/15625000*x^10*ln(x)-6237/3125000*x^10*ln(x)^2+2079/312500*x^10*ln(x)^3-2079/125000*x^10*ln(x)^4+2079/62500*x^10*ln(x)^5-693/12500*x^10*ln(x)^6+99/1250*x^10*ln(x)^7-99/1000*x^10*ln(x)^8+11/100*x^10*ln(x)^9-11/100*x^10*ln(x)^10+1/10*x^10*ln(x)^11
```

#### 3.58.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int x^9 \log^{11}(x) dx = -\frac{6237x^{10}}{156250000} + \frac{6237x^{10} \log(x)}{15625000} - \frac{6237x^{10} \log^2(x)}{3125000} + \frac{2079x^{10} \log^3(x)}{312500}$$

$$- \frac{2079x^{10} \log^4(x)}{125000} + \frac{2079x^{10} \log^5(x)}{62500} - \frac{693x^{10} \log^6(x)}{12500} + \frac{99x^{10} \log^7(x)}{1250}$$

$$- \frac{99x^{10} \log^8(x)}{1000} + \frac{11}{100}x^{10} \log^9(x) - \frac{11}{100}x^{10} \log^{10}(x) + \frac{1}{10}x^{10} \log^{11}(x)$$

input `Integrate[x^9*Log[x]^11,x]`

output  $(-6237*x^{10})/156250000 + (6237*x^{10}*Log[x])/15625000 - (6237*x^{10}*Log[x]^2)/3125000 + (2079*x^{10}*Log[x]^3)/312500 - (2079*x^{10}*Log[x]^4)/125000 + (2079*x^{10}*Log[x]^5)/62500 - (693*x^{10}*Log[x]^6)/12500 + (99*x^{10}*Log[x]^7)/1250 - (99*x^{10}*Log[x]^8)/1000 + (11*x^{10}*Log[x]^9)/100 - (11*x^{10}*Log[x]^10)/100 + (x^{10}*Log[x]^11)/10$

### 3.58.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.35, number of steps used = 11, number of rules used = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.375$ , Rules used = {2742, 2742, 2742, 2742, 2742, 2742, 2742, 2742, 2742, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^9 \log^{11}(x) dx \\
 & \quad \downarrow \text{2742} \\
 & \frac{1}{10} x^{10} \log^{11}(x) - \frac{11}{10} \int x^9 \log^{10}(x) dx \\
 & \quad \downarrow \text{2742} \\
 & \frac{1}{10} x^{10} \log^{11}(x) - \frac{11}{10} \left( \frac{1}{10} x^{10} \log^{10}(x) - \int x^9 \log^9(x) dx \right) \\
 & \quad \downarrow \text{2742} \\
 & \frac{1}{10} x^{10} \log^{11}(x) - \frac{11}{10} \left( \frac{9}{10} \int x^9 \log^8(x) dx + \frac{1}{10} x^{10} \log^{10}(x) - \frac{1}{10} x^{10} \log^9(x) \right) \\
 & \quad \downarrow \text{2742} \\
 & \frac{1}{10} x^{10} \log^{11}(x) - \frac{11}{10} \left( \frac{9}{10} \left( \frac{1}{10} x^{10} \log^8(x) - \frac{4}{5} \int x^9 \log^7(x) dx \right) + \frac{1}{10} x^{10} \log^{10}(x) - \frac{1}{10} x^{10} \log^9(x) \right) \\
 & \quad \downarrow \text{2742} \\
 & \frac{1}{10} x^{10} \log^{11}(x) - \frac{11}{10} \left( \frac{9}{10} \left( \frac{1}{10} x^{10} \log^8(x) - \frac{4}{5} \left( \frac{1}{10} x^{10} \log^7(x) - \frac{7}{10} \int x^9 \log^6(x) dx \right) \right) + \frac{1}{10} x^{10} \log^{10}(x) - \frac{1}{10} x^{10} \log^9(x) \right) \\
 & \quad \downarrow \text{2742}
 \end{aligned}$$

$$\begin{aligned}
& \frac{1}{10}x^{10}\log^{11}(x) - \\
& \frac{11}{10}\left(\frac{9}{10}\left(\frac{1}{10}x^{10}\log^8(x) - \frac{4}{5}\left(\frac{1}{10}x^{10}\log^7(x) - \frac{7}{10}\left(\frac{1}{10}x^{10}\log^6(x) - \frac{3}{5}\int x^9\log^5(x)dx\right)\right)\right)\right) + \frac{1}{10}x^{10}\log^{10}(x) - \frac{1}{10}x^{10}\log^9(x) \\
& \quad \downarrow \text{2742} \\
& \frac{1}{10}x^{10}\log^{11}(x) - \\
& \frac{11}{10}\left(\frac{9}{10}\left(\frac{1}{10}x^{10}\log^8(x) - \frac{4}{5}\left(\frac{1}{10}x^{10}\log^7(x) - \frac{7}{10}\left(\frac{1}{10}x^{10}\log^6(x) - \frac{3}{5}\left(\frac{1}{10}x^{10}\log^5(x) - \frac{1}{2}\int x^9\log^4(x)dx\right)\right)\right)\right)\right) + \frac{1}{10}x^{10}\log^{10}(x) - \frac{1}{10}x^{10}\log^9(x) \\
& \quad \downarrow \text{2742} \\
& \frac{1}{10}x^{10}\log^{11}(x) - \\
& \frac{11}{10}\left(\frac{9}{10}\left(\frac{1}{10}x^{10}\log^8(x) - \frac{4}{5}\left(\frac{1}{10}x^{10}\log^7(x) - \frac{7}{10}\left(\frac{1}{10}x^{10}\log^6(x) - \frac{3}{5}\left(\frac{1}{2}\left(\frac{2}{5}\int x^9\log^3(x)dx - \frac{1}{10}x^{10}\log^4(x)\right)\right)\right)\right)\right)\right) + \frac{1}{10}x^{10}\log^{10}(x) - \frac{1}{10}x^{10}\log^9(x) \\
& \quad \downarrow \text{2742} \\
& \frac{1}{10}x^{10}\log^{11}(x) - \\
& \frac{11}{10}\left(\frac{9}{10}\left(\frac{1}{10}x^{10}\log^8(x) - \frac{4}{5}\left(\frac{1}{10}x^{10}\log^7(x) - \frac{7}{10}\left(\frac{1}{10}x^{10}\log^6(x) - \frac{3}{5}\left(\frac{1}{2}\left(\frac{2}{5}\left(\frac{1}{10}x^{10}\log^3(x) - \frac{3}{10}\int x^9\log^2(x)dx\right)\right)\right)\right)\right)\right)\right) + \frac{1}{10}x^{10}\log^{10}(x) - \frac{1}{10}x^{10}\log^9(x) \\
& \quad \downarrow \text{2742} \\
& \frac{1}{10}x^{10}\log^{11}(x) - \\
& \frac{11}{10}\left(\frac{9}{10}\left(\frac{1}{10}x^{10}\log^8(x) - \frac{4}{5}\left(\frac{1}{10}x^{10}\log^7(x) - \frac{7}{10}\left(\frac{1}{10}x^{10}\log^6(x) - \frac{3}{5}\left(\frac{1}{2}\left(\frac{2}{5}\left(\frac{1}{10}x^{10}\log^3(x) - \frac{3}{10}\left(\frac{1}{10}x^{10}\log^2(x) - \frac{3}{10}\int x^9\log(x)dx\right)\right)\right)\right)\right)\right)\right)\right) + \frac{1}{10}x^{10}\log^{10}(x) - \frac{1}{10}x^{10}\log^9(x) \\
& \quad \downarrow \text{2741} \\
& \frac{1}{10}x^{10}\log^{11}(x) - \\
& \frac{11}{10}\left(\frac{1}{10}x^{10}\log^{10}(x) - \frac{1}{10}x^{10}\log^9(x) + \frac{9}{10}\left(\frac{1}{10}x^{10}\log^8(x) - \frac{4}{5}\left(\frac{1}{10}x^{10}\log^7(x) - \frac{7}{10}\left(\frac{1}{10}x^{10}\log^6(x) - \frac{3}{5}\left(\frac{1}{10}x^{10}\log^5(x) - \frac{1}{2}\int x^9\log^4(x)dx\right)\right)\right)\right)\right) + \frac{1}{10}x^{10}\log^{10}(x) - \frac{1}{10}x^{10}\log^9(x)
\end{aligned}$$

input `Int[x^9*Log[x]^11,x]`

output `(x^10*Log[x]^11)/10 - (11*(-1/10*(x^10*Log[x]^9) + (x^10*Log[x]^10)/10 + (9*((x^10*Log[x]^8)/10 - (4*((x^10*Log[x]^7)/10 - (7*((x^10*Log[x]^6)/10 - (3*((x^10*Log[x]^5)/10 + (-1/10*(x^10*Log[x]^4) + (2*((x^10*Log[x]^3)/10 - (3*((x^10*Log[x]^2)/10 + (x^10/100 - (x^10*Log[x])/10)/5))/10))/5)/2))/5)/10))/5)/10)/10`

## 3.58.3.1 Defintions of rubi rules used

rule 2741 `Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol] :=  
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(  
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]`

rule 2742 `Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbo  
l] := Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*  
(p/(m + 1)) Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b  
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]`

## 3.58.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.82

| method        | result                                                                                                                                                                                                                                                      |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| default       | $-\frac{6237x^{10}}{156250000} + \frac{6237x^{10} \ln(x)}{15625000} - \frac{6237x^{10} \ln(x)^2}{3125000} + \frac{2079x^{10} \ln(x)^3}{312500} - \frac{2079x^{10} \ln(x)^4}{125000} + \frac{2079x^{10} \ln(x)^5}{62500} - \frac{693x^{10} \ln(x)^6}{12500}$ |
| risch         | $-\frac{6237x^{10}}{156250000} + \frac{6237x^{10} \ln(x)}{15625000} - \frac{6237x^{10} \ln(x)^2}{3125000} + \frac{2079x^{10} \ln(x)^3}{312500} - \frac{2079x^{10} \ln(x)^4}{125000} + \frac{2079x^{10} \ln(x)^5}{62500} - \frac{693x^{10} \ln(x)^6}{12500}$ |
| parallelrisch | $-\frac{6237x^{10}}{156250000} + \frac{6237x^{10} \ln(x)}{15625000} - \frac{6237x^{10} \ln(x)^2}{3125000} + \frac{2079x^{10} \ln(x)^3}{312500} - \frac{2079x^{10} \ln(x)^4}{125000} + \frac{2079x^{10} \ln(x)^5}{62500} - \frac{693x^{10} \ln(x)^6}{12500}$ |
| parts         | $-\frac{6237x^{10}}{156250000} + \frac{6237x^{10} \ln(x)}{15625000} - \frac{6237x^{10} \ln(x)^2}{3125000} + \frac{2079x^{10} \ln(x)^3}{312500} - \frac{2079x^{10} \ln(x)^4}{125000} + \frac{2079x^{10} \ln(x)^5}{62500} - \frac{693x^{10} \ln(x)^6}{12500}$ |

input `int(x^9*ln(x)^11,x,method=_RETURNVERBOSE)`

output  $-6237/156250000*x^{10}+6237/15625000*x^{10}*\ln(x)-6237/3125000*x^{10}*\ln(x)^2+2079/312500*x^{10}*\ln(x)^3-2079/125000*x^{10}*\ln(x)^4+2079/62500*x^{10}*\ln(x)^5-693/12500*x^{10}*\ln(x)^6+99/1250*x^{10}*\ln(x)^7-99/1000*x^{10}*\ln(x)^8+11/100*x^{10}*\ln(x)^9-11/100*x^{10}*\ln(x)^{10}+1/10*x^{10}*\ln(x)^{11}$

**3.58.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.81

$$\int x^9 \log^{11}(x) dx = \frac{1}{10} x^{10} \log(x)^{11} - \frac{11}{100} x^{10} \log(x)^{10} + \frac{11}{100} x^{10} \log(x)^9 - \frac{99}{1000} x^{10} \log(x)^8 + \frac{99}{1250} x^{10} \log(x)^7 - \frac{693}{12500} x^{10} \log(x)^6 + \frac{2079}{62500} x^{10} \log(x)^5 - \frac{2079}{125000} x^{10} \log(x)^4 + \frac{2079}{312500} x^{10} \log(x)^3 - \frac{6237}{3125000} x^{10} \log(x)^2 + \frac{6237}{15625000} x^{10} \log(x) - \frac{6237}{156250000} x^{10}$$

input `integrate(x^9*log(x)^11,x, algorithm="fricas")`output `1/10*x^10*log(x)^11 - 11/100*x^10*log(x)^10 + 11/100*x^10*log(x)^9 - 99/1000*x^10*log(x)^8 + 99/1250*x^10*log(x)^7 - 693/12500*x^10*log(x)^6 + 2079/62500*x^10*log(x)^5 - 2079/125000*x^10*log(x)^4 + 2079/312500*x^10*log(x)^3 - 6237/3125000*x^10*log(x)^2 + 6237/15625000*x^10*log(x) - 6237/156250000*x^10`**3.58.6 Sympy [A] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05

$$\int x^9 \log^{11}(x) dx = \frac{x^{10} \log(x)^{11}}{10} - \frac{11x^{10} \log(x)^{10}}{100} + \frac{11x^{10} \log(x)^9}{100} - \frac{99x^{10} \log(x)^8}{1000} + \frac{99x^{10} \log(x)^7}{1250} - \frac{693x^{10} \log(x)^6}{12500} + \frac{2079x^{10} \log(x)^5}{62500} - \frac{2079x^{10} \log(x)^4}{125000} + \frac{2079x^{10} \log(x)^3}{312500} - \frac{6237x^{10} \log(x)^2}{3125000} + \frac{6237x^{10} \log(x)}{15625000} - \frac{6237x^{10}}{156250000}$$

input `integrate(x**9*ln(x)**11,x)`output `x**10*log(x)**11/10 - 11*x**10*log(x)**10/100 + 11*x**10*log(x)**9/100 - 99*x**10*log(x)**8/1000 + 99*x**10*log(x)**7/1250 - 693*x**10*log(x)**6/12500 + 2079*x**10*log(x)**5/62500 - 2079*x**10*log(x)**4/125000 + 2079*x**10*log(x)**3/312500 - 6237*x**10*log(x)**2/3125000 + 6237*x**10*log(x)/15625000 - 6237*x**10/156250000`

**3.58.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.56

$$\int x^9 \log^{11}(x) dx$$

$$= \frac{1}{156250000} (15625000 \log(x)^{11} - 17187500 \log(x)^{10} + 17187500 \log(x)^9 - 15468750 \log(x)^8 + 12375000 \log(x)^7 - 8662500 \log(x)^6 + 5197500 \log(x)^5 - 2598750 \log(x)^4 + 1039500 \log(x)^3 - 311850 \log(x)^2 + 62370 \log(x) - 6237) x^{10}$$

input `integrate(x^9*log(x)^11,x, algorithm="maxima")`

output `1/156250000*(15625000*log(x)^11 - 17187500*log(x)^10 + 17187500*log(x)^9 - 15468750*log(x)^8 + 12375000*log(x)^7 - 8662500*log(x)^6 + 5197500*log(x)^5 - 2598750*log(x)^4 + 1039500*log(x)^3 - 311850*log(x)^2 + 62370*log(x) - 6237)*x^10`

**3.58.8 Giac [A] (verification not implemented)**

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.81

$$\int x^9 \log^{11}(x) dx = \frac{1}{10} x^{10} \log(x)^{11} - \frac{11}{100} x^{10} \log(x)^{10} + \frac{11}{100} x^{10} \log(x)^9 - \frac{99}{1000} x^{10} \log(x)^8 + \frac{99}{1250} x^{10} \log(x)^7 - \frac{693}{12500} x^{10} \log(x)^6 + \frac{2079}{62500} x^{10} \log(x)^5 - \frac{2079}{125000} x^{10} \log(x)^4 + \frac{2079}{312500} x^{10} \log(x)^3 - \frac{6237}{3125000} x^{10} \log(x)^2 + \frac{6237}{15625000} x^{10} \log(x) - \frac{6237}{156250000} x^{10}$$

input `integrate(x^9*log(x)^11,x, algorithm="giac")`

output `1/10*x^10*log(x)^11 - 11/100*x^10*log(x)^10 + 11/100*x^10*log(x)^9 - 99/1000*x^10*log(x)^8 + 99/1250*x^10*log(x)^7 - 693/12500*x^10*log(x)^6 + 2079/62500*x^10*log(x)^5 - 2079/125000*x^10*log(x)^4 + 2079/312500*x^10*log(x)^3 - 6237/3125000*x^10*log(x)^2 + 6237/15625000*x^10*log(x) - 6237/156250000*x^10`



**3.58.9 Mupad [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.56

$$\int x^9 \log^{11}(x) dx$$

$$= \frac{6237 x^{10} \left( \frac{15625000 \ln(x)^{11}}{6237} - \frac{1562500 \ln(x)^{10}}{567} + \frac{1562500 \ln(x)^9}{567} - \frac{156250 \ln(x)^8}{63} + \frac{125000 \ln(x)^7}{63} - \frac{12500 \ln(x)^6}{9} + \frac{2500 \ln(x)^5}{3} \right)}{156250000}$$

input `int(x^9*log(x)^11,x)`

output `(6237*x^10*(10*log(x) - 50*log(x)^2 + (500*log(x)^3)/3 - (1250*log(x)^4)/3 + (2500*log(x)^5)/3 - (12500*log(x)^6)/9 + (125000*log(x)^7)/63 - (156250*log(x)^8)/63 + (1562500*log(x)^9)/567 - (1562500*log(x)^10)/567 + (15625000*log(x)^11)/6237 - 1))/156250000`

### 3.59 $\int \frac{\log^2(x)}{x} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.59.1 | Optimal result . . . . .                            | 425 |
| 3.59.2 | Mathematica [A] (verified) . . . . .                | 425 |
| 3.59.3 | Rubi [A] (verified) . . . . .                       | 426 |
| 3.59.4 | Maple [A] (verified) . . . . .                      | 427 |
| 3.59.5 | Fricas [A] (verification not implemented) . . . . . | 427 |
| 3.59.6 | Sympy [A] (verification not implemented) . . . . .  | 427 |
| 3.59.7 | Maxima [A] (verification not implemented) . . . . . | 428 |
| 3.59.8 | Giac [A] (verification not implemented) . . . . .   | 428 |
| 3.59.9 | Mupad [B] (verification not implemented) . . . . .  | 428 |

#### 3.59.1 Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{\log^2(x)}{x} dx = \frac{\log^3(x)}{3}$$

output `1/3*ln(x)^3`

#### 3.59.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(x)}{x} dx = \frac{\log^3(x)}{3}$$

input `Integrate[Log[x]^2/x,x]`

output `Log[x]^3/3`

### 3.59.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2(x)}{x} dx$$

↓ 2739

$$\int \log^2(x) d \log(x)$$

↓ 15

$$\frac{\log^3(x)}{3}$$

input `Int [Log[x]^2/x,x]`

output `Log[x]^3/3`

#### 3.59.3.1 Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 2739 `Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] := Simp[1/(b*n) Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]`

**3.59.4 Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

| method           | result               | size |
|------------------|----------------------|------|
| derivativdivides | $\frac{\ln(x)^3}{3}$ | 7    |
| default          | $\frac{\ln(x)^3}{3}$ | 7    |
| norman           | $\frac{\ln(x)^3}{3}$ | 7    |
| risch            | $\frac{\ln(x)^3}{3}$ | 7    |
| parts            | $\frac{\ln(x)^3}{3}$ | 7    |

input `int(ln(x)^2/x,x,method=_RETURNVERBOSE)`output `1/3*ln(x)^3`**3.59.5 Fricas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\log^2(x)}{x} dx = \frac{1}{3} \log(x)^3$$

input `integrate(log(x)^2/x,x, algorithm="fricas")`output `1/3*log(x)^3`**3.59.6 Sympy [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{\log^2(x)}{x} dx = \frac{\log(x)^3}{3}$$

input `integrate(ln(x)**2/x,x)`output `log(x)**3/3`

**3.59.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\log^2(x)}{x} dx = \frac{1}{3} \log(x)^3$$

input `integrate(log(x)^2/x,x, algorithm="maxima")`

output `1/3*log(x)^3`

**3.59.8 Giac [A] (verification not implemented)**

Time = 0.30 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\log^2(x)}{x} dx = \frac{1}{3} \log(x)^3$$

input `integrate(log(x)^2/x,x, algorithm="giac")`

output `1/3*log(x)^3`

**3.59.9 Mupad [B] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\log^2(x)}{x} dx = \frac{\ln(x)^3}{3}$$

input `int(log(x)^2/x,x)`

output `log(x)^3/3`

## 3.60 $\int \frac{1}{\log(x)} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.60.1 | Optimal result . . . . .                            | 429 |
| 3.60.2 | Mathematica [A] (verified) . . . . .                | 429 |
| 3.60.3 | Rubi [A] (verified) . . . . .                       | 430 |
| 3.60.4 | Maple [B] (verified) . . . . .                      | 430 |
| 3.60.5 | Fricas [A] (verification not implemented) . . . . . | 431 |
| 3.60.6 | Sympy [A] (verification not implemented) . . . . .  | 431 |
| 3.60.7 | Maxima [A] (verification not implemented) . . . . . | 431 |
| 3.60.8 | Giac [A] (verification not implemented) . . . . .   | 432 |
| 3.60.9 | Mupad [B] (verification not implemented) . . . . .  | 432 |

### 3.60.1 Optimal result

Integrand size = 4, antiderivative size = 2

$$\int \frac{1}{\log(x)} dx = \text{LogIntegral}(x)$$

output `Li(x)`

### 3.60.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(x)} dx = \text{LogIntegral}(x)$$

input `Integrate[Log[x]^(-1),x]`

output `LogIntegral[x]`

### 3.60.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2735}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log(x)} dx$$

↓ 2735

$$\text{LogIntegral}(x)$$

input `Int [Log [x] ^(-1) , x]`

output `LogIntegral [x]`

#### 3.60.3.1 Defintions of rubi rules used

rule 2735 `Int [Log [(c_.)*(x_)] ^(-1) , x_Symbol] :> Simp [LogIntegral [c*x]/c , x] /; FreeQ [c , x]`

### 3.60.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs.  $2(2) = 4$ .

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 4.50

| method  | result                  | size |
|---------|-------------------------|------|
| default | $-\text{Ei}_1(-\ln(x))$ | 9    |
| risch   | $-\text{Ei}_1(-\ln(x))$ | 9    |

input `int (1/ln(x) , x , method=_RETURNVERBOSE)`

output `-Ei (1, -ln(x))`

**3.60.5 Fracas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(x)} dx = \text{log\_integral}(x)$$

input `integrate(1/log(x),x, algorithm="fricas")`

output `log_integral(x)`

**3.60.6 Sympy [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(x)} dx = \text{li}(x)$$

input `integrate(1/ln(x),x)`

output `li(x)`

**3.60.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{\log(x)} dx = \text{Ei}(\log(x))$$

input `integrate(1/log(x),x, algorithm="maxima")`

output `Ei(log(x))`



**3.60.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{\log(x)} dx = \text{Ei}(\log(x))$$

input `integrate(1/log(x),x, algorithm="giac")`

output `Ei(log(x))`

**3.60.9 Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(x)} dx = \text{logint}(x)$$

input `int(1/log(x),x)`

output `logint(x)`

### 3.61 $\int \frac{1}{\log(1+x)} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.61.1 | Optimal result . . . . .                            | 433 |
| 3.61.2 | Mathematica [A] (verified) . . . . .                | 433 |
| 3.61.3 | Rubi [A] (verified) . . . . .                       | 434 |
| 3.61.4 | Maple [B] (verified) . . . . .                      | 435 |
| 3.61.5 | Fricas [A] (verification not implemented) . . . . . | 435 |
| 3.61.6 | Sympy [A] (verification not implemented) . . . . .  | 435 |
| 3.61.7 | Maxima [A] (verification not implemented) . . . . . | 436 |
| 3.61.8 | Giac [A] (verification not implemented) . . . . .   | 436 |
| 3.61.9 | Mupad [B] (verification not implemented) . . . . .  | 436 |

#### 3.61.1 Optimal result

Integrand size = 6, antiderivative size = 4

$$\int \frac{1}{\log(1+x)} dx = \text{LogIntegral}(1+x)$$

output `Li(1+x)`

#### 3.61.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(1+x)} dx = \text{LogIntegral}(1+x)$$

input `Integrate[Log[1 + x]^(-1),x]`

output `LogIntegral[1 + x]`

### 3.61.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2836, 2735}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \frac{1}{\log(x+1)} dx \\ \downarrow \text{2836} \\ \int \frac{1}{\log(x+1)} d(x+1) \\ \downarrow \text{2735} \\ \text{LogIntegral}(x+1) \end{array}$$

input `Int[Log[1 + x]^(-1),x]`

output `LogIntegral[1 + x]`

#### 3.61.3.1 Defintions of rubi rules used

rule 2735 `Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ[c, x]`

rule 2836 `Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[1/e Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]`

**3.61.4 Maple [B] (verified)**

Leaf count of result is larger than twice the leaf count of optimal. 10 vs.  $2(4) = 8$ .

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.75

| method            | result             | size |
|-------------------|--------------------|------|
| derivativedivides | $-Ei_1(-\ln(1+x))$ | 11   |
| default           | $-Ei_1(-\ln(1+x))$ | 11   |
| risch             | $-Ei_1(-\ln(1+x))$ | 11   |

input `int(1/ln(1+x),x,method=_RETURNVERBOSE)`

output `-Ei(1,-ln(1+x))`

**3.61.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(1+x)} dx = \log\_integral(x+1)$$

input `integrate(1/log(1+x),x, algorithm="fricas")`

output `log_integral(x + 1)`

**3.61.6 Sympy [A] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{1}{\log(1+x)} dx = \text{li}(x+1)$$

input `integrate(1/ln(1+x),x)`

output `li(x + 1)`

**3.61.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \frac{1}{\log(1+x)} dx = \text{Ei}(\log(x+1))$$

input `integrate(1/log(1+x),x, algorithm="maxima")`output `Ei(log(x + 1))`**3.61.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \frac{1}{\log(1+x)} dx = \text{Ei}(\log(x+1))$$

input `integrate(1/log(1+x),x, algorithm="giac")`output `Ei(log(x + 1))`**3.61.9 Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(1+x)} dx = \text{logint}(x+1)$$

input `int(1/log(x + 1),x)`output `logint(x + 1)`

### 3.62 $\int \frac{1}{x \log(x)} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.62.1 | Optimal result . . . . .                            | 437 |
| 3.62.2 | Mathematica [A] (verified) . . . . .                | 437 |
| 3.62.3 | Rubi [A] (verified) . . . . .                       | 438 |
| 3.62.4 | Maple [A] (verified) . . . . .                      | 439 |
| 3.62.5 | Fricas [A] (verification not implemented) . . . . . | 439 |
| 3.62.6 | Sympy [A] (verification not implemented) . . . . .  | 439 |
| 3.62.7 | Maxima [A] (verification not implemented) . . . . . | 440 |
| 3.62.8 | Giac [A] (verification not implemented) . . . . .   | 440 |
| 3.62.9 | Mupad [B] (verification not implemented) . . . . .  | 440 |

#### 3.62.1 Optimal result

Integrand size = 8, antiderivative size = 3

$$\int \frac{1}{x \log(x)} dx = \log(\log(x))$$

output `ln(ln(x))`

#### 3.62.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log(x)} dx = \log(\log(x))$$

input `Integrate[1/(x*Log[x]),x]`

output `Log[Log[x]]`

### 3.62.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \frac{1}{x \log(x)} dx \\ \downarrow \text{2739} \\ \int \frac{1}{\log(x)} d\log(x) \\ \downarrow \text{14} \\ \log(\log(x)) \end{array}$$

input `Int[1/(x*Log[x]),x]`

output `Log[Log[x]]`

#### 3.62.3.1 Defintions of rubi rules used

rule 14 `Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]`

rule 2739 `Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(b*n) Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]`

**3.62.4 Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

| method            | result        | size |
|-------------------|---------------|------|
| derivativedivides | $\ln(\ln(x))$ | 4    |
| default           | $\ln(\ln(x))$ | 4    |
| norman            | $\ln(\ln(x))$ | 4    |
| risch             | $\ln(\ln(x))$ | 4    |
| parallelrisch     | $\ln(\ln(x))$ | 4    |

input `int(1/x/ln(x),x,method=_RETURNVERBOSE)`

output `ln(ln(x))`

**3.62.5 Fricas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log(x)} dx = \log(\log(x))$$

input `integrate(1/x/log(x),x, algorithm="fricas")`

output `log(log(x))`

**3.62.6 Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log(x)} dx = \log(\log(x))$$

input `integrate(1/x/ln(x),x)`

output `log(log(x))`



**3.62.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log(x)} dx = \log(\log(x))$$

input `integrate(1/x/log(x),x, algorithm="maxima")`output `log(log(x))`**3.62.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int \frac{1}{x \log(x)} dx = \log(|\log(x)|)$$

input `integrate(1/x/log(x),x, algorithm="giac")`output `log(abs(log(x)))`**3.62.9 Mupad [B] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log(x)} dx = \ln(\ln(x))$$

input `int(1/(x*log(x)),x)`output `log(log(x))`

### 3.63 $\int \frac{1}{x^2 \log^2(x)} dx$

|        |                                           |     |
|--------|-------------------------------------------|-----|
| 3.63.1 | Optimal result                            | 441 |
| 3.63.2 | Mathematica [A] (verified)                | 441 |
| 3.63.3 | Rubi [A] (verified)                       | 442 |
| 3.63.4 | Maple [A] (verified)                      | 443 |
| 3.63.5 | Fricas [A] (verification not implemented) | 443 |
| 3.63.6 | Sympy [A] (verification not implemented)  | 443 |
| 3.63.7 | Maxima [A] (verification not implemented) | 444 |
| 3.63.8 | Giac [F]                                  | 444 |
| 3.63.9 | Mupad [B] (verification not implemented)  | 444 |

#### 3.63.1 Optimal result

Integrand size = 8, antiderivative size = 17

$$\int \frac{1}{x^2 \log^2(x)} dx = -\text{ExpIntegralEi}(-\log(x)) - \frac{1}{x \log(x)}$$

output `-Ei(-ln(x))-1/x/ln(x)`

#### 3.63.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \log^2(x)} dx = -\text{ExpIntegralEi}(-\log(x)) - \frac{1}{x \log(x)}$$

input `Integrate[1/(x^2*Log[x]^2),x]`

output `-ExpIntegralEi[-Log[x]] - 1/(x*Log[x])`

### 3.63.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {2743, 2746, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{x^2 \log^2(x)} dx \\ & \quad \downarrow \text{2743} \\ & - \int \frac{1}{x^2 \log(x)} dx - \frac{1}{x \log(x)} \\ & \quad \downarrow \text{2746} \\ & - \int \frac{1}{x \log(x)} d \log(x) - \frac{1}{x \log(x)} \\ & \quad \downarrow \text{2609} \\ & - \text{ExpIntegralEi}(-\log(x)) - \frac{1}{x \log(x)} \end{aligned}$$

input `Int[1/(x^2*Log[x]^2),x]`

output `-ExpIntegralEi[-Log[x]] - 1/(x*Log[x])`

#### 3.63.3.1 Defintions of rubi rules used

rule 2609 `Int[(F_)^((g_.)*(e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

rule 2743 `Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_)^(m_.), x_Symbol] := Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] - Simp[(m + 1)/(b*n*(p + 1)) Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]`

```
rule 2746 Int[((a_.) + Log[(c_.)*(x_.)]*(b_.))^(p_.)*(x_.)^(m_.), x_Symbol] := Simp[1/c^(
(m + 1) Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[m]
```

### 3.63.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

| method  | result                                      | size |
|---------|---------------------------------------------|------|
| default | $-\frac{1}{x \ln(x)} + \text{Ei}_1(\ln(x))$ | 15   |
| risch   | $-\frac{1}{x \ln(x)} + \text{Ei}_1(\ln(x))$ | 15   |

```
input int(1/x^2/ln(x)^2,x,method=_RETURNVERBOSE)
```

```
output -1/x/ln(x)+Ei(1,ln(x))
```

### 3.63.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2 \log^2(x)} dx = -\frac{x \log(x) \log\_integral\left(\frac{1}{x}\right) + 1}{x \log(x)}$$

```
input integrate(1/x^2/log(x)^2,x, algorithm="fricas")
```

```
output -(x*log(x)*log_integral(1/x) + 1)/(x*log(x))
```

### 3.63.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2 \log^2(x)} dx = -\text{Ei}(-\log(x)) - \frac{1}{x \log(x)}$$

input `integrate(1/x**2/ln(x)**2,x)`

output `-Ei(-log(x)) - 1/(x*log(x))`

### 3.63.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^2 \log^2(x)} dx = -\Gamma(-1, \log(x))$$

input `integrate(1/x^2/log(x)^2,x, algorithm="maxima")`

output `-gamma(-1, log(x))`

### 3.63.8 Giac [F]

$$\int \frac{1}{x^2 \log^2(x)} dx = \int \frac{1}{x^2 \log(x)^2} dx$$

input `integrate(1/x^2/log(x)^2,x, algorithm="giac")`

output `integrate(1/(x^2*log(x)^2), x)`

### 3.63.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \log^2(x)} dx = -\text{ei}(-\ln(x)) - \frac{1}{x \ln(x)}$$

input `int(1/(x^2*log(x)^2),x)`

output `- ei(-log(x)) - 1/(x*log(x))`

## 3.64 $\int \frac{\log^p(x)}{x} dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.64.1 | Optimal result . . . . .                            | 445 |
| 3.64.2 | Mathematica [A] (verified) . . . . .                | 445 |
| 3.64.3 | Rubi [A] (verified) . . . . .                       | 446 |
| 3.64.4 | Maple [A] (verified) . . . . .                      | 447 |
| 3.64.5 | Fricas [A] (verification not implemented) . . . . . | 447 |
| 3.64.6 | Sympy [A] (verification not implemented) . . . . .  | 447 |
| 3.64.7 | Maxima [A] (verification not implemented) . . . . . | 448 |
| 3.64.8 | Giac [A] (verification not implemented) . . . . .   | 448 |
| 3.64.9 | Mupad [B] (verification not implemented) . . . . .  | 448 |

### 3.64.1 Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \frac{\log^p(x)}{x} dx = \frac{\log^{1+p}(x)}{1+p}$$

output `ln(x)^(p+1)/(p+1)`

### 3.64.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^p(x)}{x} dx = \frac{\log^{1+p}(x)}{1+p}$$

input `Integrate[Log[x]^p/x,x]`

output `Log[x]^(1 + p)/(1 + p)`

### 3.64.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^p(x)}{x} dx$$

↓ 2739

$$\int \log^p(x) d \log(x)$$

↓ 15

$$\frac{\log^{p+1}(x)}{p+1}$$

input `Int [Log[x]^p/x,x]`

output `Log[x]^(1 + p)/(1 + p)`

#### 3.64.3.1 Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 2739 `Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] := Simp[1/(b*n) Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]`

**3.64.4 Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

| method            | result                               | size |
|-------------------|--------------------------------------|------|
| derivativedivides | $\frac{\ln(x)^{1+p}}{1+p}$           | 13   |
| default           | $\frac{\ln(x)^{1+p}}{1+p}$           | 13   |
| risch             | $\frac{\ln(x)\ln(x)^p}{1+p}$         | 13   |
| norman            | $\frac{\ln(x)e^{p\ln(\ln(x))}}{1+p}$ | 15   |

input `int(ln(x)^p/x,x,method=_RETURNVERBOSE)`output `ln(x)^(1+p)/(1+p)`**3.64.5 Fricas [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^p(x)}{x} dx = \frac{\log(x)^p \log(x)}{p+1}$$

input `integrate(log(x)^p/x,x, algorithm="fricas")`output `log(x)^p*log(x)/(p + 1)`**3.64.6 Sympy [A] (verification not implemented)**

Time = 0.42 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\log^p(x)}{x} dx = \begin{cases} \frac{\log(x)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(\log(x)) & \text{otherwise} \end{cases}$$

input `integrate(ln(x)**p/x,x)`output `Piecewise((log(x)**(p + 1)/(p + 1), Ne(p, -1)), (log(log(x)), True))`



**3.64.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^p(x)}{x} dx = \frac{\log(x)^{p+1}}{p+1}$$

input `integrate(log(x)^p/x,x, algorithm="maxima")`output `log(x)^(p + 1)/(p + 1)`**3.64.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log^p(x)}{x} dx = \frac{\log(x)^{p+1}}{p+1}$$

input `integrate(log(x)^p/x,x, algorithm="giac")`output `log(x)^(p + 1)/(p + 1)`**3.64.9 Mupad [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\log^p(x)}{x} dx = \begin{cases} \ln(\ln(x)) & \text{if } p = -1 \\ \frac{\ln(x)^{p+1}}{p+1} & \text{if } p \neq -1 \end{cases}$$

input `int(log(x)^p/x,x)`output `piecewise(p == -1, log(log(x)), p ~= -1, log(x)^(p + 1)/(p + 1))`

## 3.65 $\int (b + ax) \log(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.65.1 | Optimal result . . . . .                            | 449 |
| 3.65.2 | Mathematica [A] (verified) . . . . .                | 449 |
| 3.65.3 | Rubi [A] (verified) . . . . .                       | 450 |
| 3.65.4 | Maple [A] (verified) . . . . .                      | 451 |
| 3.65.5 | Fricas [A] (verification not implemented) . . . . . | 451 |
| 3.65.6 | Sympy [A] (verification not implemented) . . . . .  | 452 |
| 3.65.7 | Maxima [A] (verification not implemented) . . . . . | 452 |
| 3.65.8 | Giac [A] (verification not implemented) . . . . .   | 452 |
| 3.65.9 | Mupad [B] (verification not implemented) . . . . .  | 453 |

### 3.65.1 Optimal result

Integrand size = 8, antiderivative size = 28

$$\int (b + ax) \log(x) dx = -bx - \frac{ax^2}{4} + bx \log(x) + \frac{1}{2}ax^2 \log(x)$$

output `-b*x-1/4*a*x^2+b*x*ln(x)+1/2*a*x^2*ln(x)`

### 3.65.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (b + ax) \log(x) dx = -bx - \frac{ax^2}{4} + bx \log(x) + \frac{1}{2}ax^2 \log(x)$$

input `Integrate[(b + a*x)*Log[x],x]`

output `-(b*x) - (a*x^2)/4 + b*x*Log[x] + (a*x^2*Log[x])/2`

### 3.65.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {2750, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \log(x)(ax + b) dx \\
 & \quad \downarrow \text{2750} \\
 & \frac{\log(x)(ax + b)^2}{2a} - \int \frac{(b + ax)^2}{2ax} dx \\
 & \quad \downarrow \text{27} \\
 & \frac{\log(x)(ax + b)^2}{2a} - \frac{\int \frac{(b+ax)^2}{x} dx}{2a} \\
 & \quad \downarrow \text{49} \\
 & \frac{\log(x)(ax + b)^2}{2a} - \frac{\int \left( xa^2 + 2ba + \frac{b^2}{x} \right) dx}{2a} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\log(x)(ax + b)^2}{2a} - \frac{\frac{a^2 x^2}{2} + 2abx + b^2 \log(x)}{2a}
 \end{aligned}$$

input `Int[(b + a*x)*Log[x],x]`

output `((b + a*x)^2*Log[x])/(2*a) - (2*a*b*x + (a^2*x^2)/2 + b^2*Log[x])/(2*a)`

#### 3.65.3.1 Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2750 `Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.), x_Symbol] := With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[(a + b*Log[c*x^n]) u, x] - Simp[b*n Int[SimplifyIntegrand[u/x, x], x]] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0]`

### 3.65.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

| method        | result                                                                   | size |
|---------------|--------------------------------------------------------------------------|------|
| norman        | $-bx - \frac{ax^2}{4} + bx \ln(x) + \frac{ax^2 \ln(x)}{2}$               | 25   |
| risch         | $(\frac{1}{2}ax^2 + bx) \ln(x) - \frac{ax^2}{4} - bx$                    | 25   |
| parallelrisch | $-bx - \frac{ax^2}{4} + bx \ln(x) + \frac{ax^2 \ln(x)}{2}$               | 25   |
| parts         | $-bx - \frac{ax^2}{4} + bx \ln(x) + \frac{ax^2 \ln(x)}{2}$               | 25   |
| default       | $a\left(-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}\right) + b(-x + x \ln(x))$ | 27   |

input `int((a*x+b)*ln(x),x,method=_RETURNVERBOSE)`

output `-b*x-1/4*a*x^2+b*x*ln(x)+1/2*a*x^2*ln(x)`

### 3.65.5 Fracas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (b + ax) \log(x) dx = -\frac{1}{4}ax^2 - bx + \frac{1}{2}(ax^2 + 2bx) \log(x)$$

input `integrate((a*x+b)*log(x),x, algorithm="fricas")`

output `-1/4*a*x^2 - b*x + 1/2*(a*x^2 + 2*b*x)*log(x)`

**3.65.6 Sympy [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int (b + ax) \log(x) dx = -\frac{ax^2}{4} - bx + \left(\frac{ax^2}{2} + bx\right) \log(x)$$

input `integrate((a*x+b)*ln(x),x)`output `-a*x**2/4 - b*x + (a*x**2/2 + b*x)*log(x)`**3.65.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (b + ax) \log(x) dx = -\frac{1}{4} ax^2 - bx + \frac{1}{2} (ax^2 + 2bx) \log(x)$$

input `integrate((a*x+b)*log(x),x, algorithm="maxima")`output `-1/4*a*x^2 - b*x + 1/2*(a*x^2 + 2*b*x)*log(x)`**3.65.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (b + ax) \log(x) dx = \frac{1}{2} ax^2 \log(x) - \frac{1}{4} ax^2 + bx \log(x) - bx$$

input `integrate((a*x+b)*log(x),x, algorithm="giac")`output `1/2*a*x^2*log(x) - 1/4*a*x^2 + b*x*log(x) - b*x`

**3.65.9 Mupad [B] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int (b + ax) \log(x) dx = -\frac{x(4b + ax - 4b \ln(x) - 2ax \ln(x))}{4}$$

input `int(log(x)*(b + a*x),x)`

output `-(x*(4*b + a*x - 4*b*log(x) - 2*a*x*log(x)))/4`

### 3.66 $\int (b + ax)^2 \log(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.66.1 | Optimal result . . . . .                            | 454 |
| 3.66.2 | Mathematica [A] (verified) . . . . .                | 454 |
| 3.66.3 | Rubi [A] (verified) . . . . .                       | 455 |
| 3.66.4 | Maple [A] (verified) . . . . .                      | 456 |
| 3.66.5 | Fricas [A] (verification not implemented) . . . . . | 457 |
| 3.66.6 | Sympy [A] (verification not implemented) . . . . .  | 457 |
| 3.66.7 | Maxima [A] (verification not implemented) . . . . . | 457 |
| 3.66.8 | Giac [A] (verification not implemented) . . . . .   | 458 |
| 3.66.9 | Mupad [B] (verification not implemented) . . . . .  | 458 |

#### 3.66.1 Optimal result

Integrand size = 10, antiderivative size = 54

$$\int (b + ax)^2 \log(x) dx = -b^2x - \frac{1}{2}abx^2 - \frac{a^2x^3}{9} - \frac{b^3 \log(x)}{3a} + \frac{(b + ax)^3 \log(x)}{3a}$$

output `-b^2*x-1/2*a*b*x^2-1/9*a^2*x^3-1/3*b^3*ln(x)/a+1/3*(a*x+b)^3*ln(x)/a`

#### 3.66.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int (b + ax)^2 \log(x) dx = -b^2x - \frac{1}{2}abx^2 - \frac{a^2x^3}{9} + b^2x \log(x) + abx^2 \log(x) + \frac{1}{3}a^2x^3 \log(x)$$

input `Integrate[(b + a*x)^2*Log[x],x]`

output `-(b^2*x) - (a*b*x^2)/2 - (a^2*x^3)/9 + b^2*x*Log[x] + a*b*x^2*Log[x] + (a^2*x^3*Log[x])/3`

**3.66.3 Rubi [A] (verified)**

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {2750, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \log(x)(ax + b)^2 dx \\
 & \quad \downarrow \text{2750} \\
 & \frac{\log(x)(ax + b)^3}{3a} - \int \frac{(b + ax)^3}{3ax} dx \\
 & \quad \downarrow \text{27} \\
 & \frac{\log(x)(ax + b)^3}{3a} - \frac{\int \frac{(b+ax)^3}{x} dx}{3a} \\
 & \quad \downarrow \text{49} \\
 & \frac{\log(x)(ax + b)^3}{3a} - \frac{\int \left( x^2 a^3 + 3bxa^2 + 3b^2a + \frac{b^3}{x} \right) dx}{3a} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\log(x)(ax + b)^3}{3a} - \frac{\frac{a^3 x^3}{3} + \frac{3}{2} a^2 b x^2 + 3ab^2 x + b^3 \log(x)}{3a}
 \end{aligned}$$

input `Int[(b + a*x)^2*Log[x],x]`

output `((b + a*x)^3*Log[x])/(3*a) - (3*a*b^2*x + (3*a^2*b*x^2)/2 + (a^3*x^3)/3 + b^3*Log[x])/(3*a)`



## 3.66.3.1 Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2750 `Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.) + (e_.)*(x_)^(r_.))^(q_.), x_Symbol] := With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[(a + b*Log[c*x^n]) u, x] - Simp[b*n Int[SimplifyIntegrand[u/x, x], x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0]`

## 3.66.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

| method        | result                                                                                                                                     | size |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch         | $-b^2x - \frac{abx^2}{2} - \frac{a^2x^3}{9} - \frac{b^3 \ln(x)}{3a} + \frac{(ax+b)^3 \ln(x)}{3a}$                                          | 47   |
| default       | $a^2 \left( -\frac{x^3}{9} + \frac{x^3 \ln(x)}{3} \right) + 2ab \left( -\frac{x^2}{4} + \frac{x^2 \ln(x)}{2} \right) + b^2(-x + x \ln(x))$ | 48   |
| norman        | $b^2x \ln(x) + abx^2 \ln(x) - \frac{a^2x^3}{9} - b^2x - \frac{abx^2}{2} + \frac{a^2x^3 \ln(x)}{3}$                                         | 48   |
| parallelrisch | $b^2x \ln(x) + abx^2 \ln(x) - \frac{a^2x^3}{9} - b^2x - \frac{abx^2}{2} + \frac{a^2x^3 \ln(x)}{3}$                                         | 48   |
| parts         | $\frac{a^2x^3 \ln(x)}{3} + abx^2 \ln(x) + b^2x \ln(x) + \frac{b^3 \ln(x)}{3a} - \frac{a^3x^3 + 3a^2bx^2 + 3ab^2x + b^3 \ln(x)}{3a}$        | 73   |

input `int((a*x+b)^2*ln(x),x,method=_RETURNVERBOSE)`

output `-b^2*x-1/2*a*b*x^2-1/9*a^2*x^3-1/3*b^3*ln(x)/a+1/3*(a*x+b)^3*ln(x)/a`

**3.66.5 Fracas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int (b + ax)^2 \log(x) dx = -\frac{1}{9} a^2 x^3 - \frac{1}{2} abx^2 - b^2 x + \frac{1}{3} (a^2 x^3 + 3 abx^2 + 3 b^2 x) \log(x)$$

input `integrate((a*x+b)^2*log(x),x, algorithm="fricas")`output `-1/9*a^2*x^3 - 1/2*a*b*x^2 - b^2*x + 1/3*(a^2*x^3 + 3*a*b*x^2 + 3*b^2*x)*log(x)`**3.66.6 Sympy [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int (b + ax)^2 \log(x) dx = -\frac{a^2 x^3}{9} - \frac{abx^2}{2} - b^2 x + \left( \frac{a^2 x^3}{3} + abx^2 + b^2 x \right) \log(x)$$

input `integrate((a*x+b)**2*ln(x),x)`output `-a**2*x**3/9 - a*b*x**2/2 - b**2*x + (a**2*x**3/3 + a*b*x**2 + b**2*x)*log(x)`**3.66.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int (b + ax)^2 \log(x) dx = -\frac{1}{9} a^2 x^3 - \frac{1}{2} abx^2 - b^2 x + \frac{1}{3} (a^2 x^3 + 3 abx^2 + 3 b^2 x) \log(x)$$

input `integrate((a*x+b)^2*log(x),x, algorithm="maxima")`output `-1/9*a^2*x^3 - 1/2*a*b*x^2 - b^2*x + 1/3*(a^2*x^3 + 3*a*b*x^2 + 3*b^2*x)*log(x)`

**3.66.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int (b + ax)^2 \log(x) dx = \frac{1}{3} a^2 x^3 \log(x) - \frac{1}{9} a^2 x^3 + abx^2 \log(x) - \frac{1}{2} abx^2 + b^2 x \log(x) - b^2 x$$

input `integrate((a*x+b)^2*log(x),x, algorithm="giac")`output `1/3*a^2*x^3*log(x) - 1/9*a^2*x^3 + a*b*x^2*log(x) - 1/2*a*b*x^2 + b^2*x*log(x) - b^2*x`**3.66.9 Mupad [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int (b + ax)^2 \log(x) dx = b^2 x \ln(x) - \frac{a^2 x^3}{9} - b^2 x + \frac{a^2 x^3 \ln(x)}{3} - \frac{a b x^2}{2} + a b x^2 \ln(x)$$

input `int(log(x)*(b + a*x)^2,x)`output `b^2*x*log(x) - (a^2*x^3)/9 - b^2*x + (a^2*x^3*log(x))/3 - (a*b*x^2)/2 + a*b*x^2*log(x)`

### 3.67 $\int \frac{\log(x)}{(b+ax)^2} dx$

|        |                                           |     |
|--------|-------------------------------------------|-----|
| 3.67.1 | Optimal result                            | 459 |
| 3.67.2 | Mathematica [A] (verified)                | 459 |
| 3.67.3 | Rubi [A] (verified)                       | 460 |
| 3.67.4 | Maple [A] (verified)                      | 461 |
| 3.67.5 | Fricas [A] (verification not implemented) | 461 |
| 3.67.6 | Sympy [A] (verification not implemented)  | 461 |
| 3.67.7 | Maxima [A] (verification not implemented) | 462 |
| 3.67.8 | Giac [B] (verification not implemented)   | 462 |
| 3.67.9 | Mupad [B] (verification not implemented)  | 463 |

#### 3.67.1 Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{\log(x)}{(b+ax)^2} dx = \frac{x \log(x)}{b(b+ax)} - \frac{\log(b+ax)}{ab}$$

output `x*ln(x)/b/(a*x+b)-ln(a*x+b)/a/b`

#### 3.67.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\log(x)}{(b+ax)^2} dx = \frac{\frac{x \log(x)}{b+ax} - \frac{\log(b+ax)}{a}}{b}$$

input `Integrate[Log[x]/(b + a*x)^2,x]`

output `((x*Log[x])/(b + a*x) - Log[b + a*x]/a)/b`

### 3.67.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2751, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{(ax+b)^2} dx$$

$$\downarrow \text{2751}$$

$$\frac{x \log(x)}{b(ax+b)} - \frac{\int \frac{1}{b+ax} dx}{b}$$

$$\downarrow \text{16}$$

$$\frac{x \log(x)}{b(ax+b)} - \frac{\log(ax+b)}{ab}$$

input `Int[Log[x]/(b + a*x)^2,x]`

output `(x*Log[x])/(b*(b + a*x)) - Log[b + a*x]/(a*b)`

#### 3.67.3.1 Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_.)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 2751 `Int[((a_.) + Log[(c_.)*(x_.)^(n_.)]*(b_.))*((d_.) + (e_.)*(x_.)^(r_.))^(q_.), x_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*(n/d) Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x] && EqQ[r*(q + 1) + 1, 0]`

**3.67.4 Maple [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

| method        | result                                                                       | size |
|---------------|------------------------------------------------------------------------------|------|
| default       | $\frac{x \ln(x)}{b(ax+b)} - \frac{\ln(ax+b)}{ab}$                            | 30   |
| norman        | $\frac{x \ln(x)}{b(ax+b)} - \frac{\ln(ax+b)}{ab}$                            | 30   |
| parts         | $-\frac{\ln(x)}{a(ax+b)} + \frac{\frac{\ln(x)}{b} - \frac{\ln(ax+b)}{b}}{a}$ | 38   |
| parallelrisch | $\frac{-\ln(ax+b)xa + \ln(x)ax - \ln(ax+b)b}{ab(ax+b)}$                      | 40   |
| risch         | $-\frac{\ln(x)}{a(ax+b)} + \frac{\ln(-x)}{ba} - \frac{\ln(ax+b)}{ab}$        | 41   |

input `int(ln(x)/(a*x+b)^2,x,method=_RETURNVERBOSE)`output `x*ln(x)/b/(a*x+b)-ln(a*x+b)/a/b`**3.67.5 Fricas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{\log(x)}{(b+ax)^2} dx = \frac{ax \log(x) - (ax+b) \log(ax+b)}{a^2bx + ab^2}$$

input `integrate(log(x)/(a*x+b)^2,x, algorithm="fricas")`output `(a*x*log(x) - (a*x + b)*log(a*x + b))/(a^2*b*x + a*b^2)`**3.67.6 Sympy [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{\log(x)}{(b+ax)^2} dx = -\frac{\log(x)}{a^2x+ab} + \frac{\log(x) - \log(x+\frac{b}{a})}{ab}$$

input `integrate(ln(x)/(a*x+b)**2,x)`output `-log(x)/(a**2*x + a*b) + (log(x) - log(x + b/a))/(a*b)`

---

3.67.  $\int \frac{\log(x)}{(b+ax)^2} dx$

**3.67.7 Maxima [A] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int \frac{\log(x)}{(b+ax)^2} dx = -\frac{\frac{\log(ax+b)}{b} - \frac{\log(x)}{b}}{a} - \frac{\log(x)}{(ax+b)a}$$

input `integrate(log(x)/(a*x+b)^2,x, algorithm="maxima")`

output `-(log(a*x + b)/b - log(x)/b)/a - log(x)/((a*x + b)*a)`

**3.67.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(29) = 58.

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 4.76

$$\int \frac{\log(x)}{(b+ax)^2} dx = a^2 \left( \frac{\log\left(\frac{(ax+b)^2 |a| \left|\frac{b}{ax+b} - 1\right|}{a^2 |ax+b|}\right)}{a^3 b} + \frac{\log\left(-\frac{b + \frac{(ax+b)a\left(\frac{b}{ax+b} - 1\right) - ab}{a}}{a}\right)}{\left((ax+b)\left(\frac{b}{ax+b} - 1\right) - b\right)a^3} - \frac{\log\left(\left|-(ax+b)\left(\frac{b}{ax+b} - 1\right) + b\right|\right)}{a^3 b} \right)$$

input `integrate(log(x)/(a*x+b)^2,x, algorithm="giac")`

output `a^2*(log((a*x + b)^2*abs(a)*abs(b/(a*x + b) - 1)/(a^2*abs(a*x + b)))/(a^3*b) + log(-(b + ((a*x + b)*a*(b/(a*x + b) - 1) - a*b)/a)/a)/(((a*x + b)*(b/(a*x + b) - 1) - b)*a^3) - log(abs(-(a*x + b)*(b/(a*x + b) - 1) + b))/(a^3*b))`

**3.67.9 Mupad [B] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\log(x)}{(b+ax)^2} dx = \frac{x^2 \ln(x)}{b(ax^2+bx)} - \frac{\ln(b+ax)}{ab}$$

input `int(log(x)/(b + a*x)^2,x)`

output `(x^2*log(x))/(b*(b*x + a*x^2)) - log(b + a*x)/(a*b)`



### 3.68 $\int x \log(b + ax) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.68.1 | Optimal result . . . . .                            | 464 |
| 3.68.2 | Mathematica [A] (verified) . . . . .                | 464 |
| 3.68.3 | Rubi [A] (verified) . . . . .                       | 465 |
| 3.68.4 | Maple [A] (verified) . . . . .                      | 466 |
| 3.68.5 | Fricas [A] (verification not implemented) . . . . . | 466 |
| 3.68.6 | Sympy [A] (verification not implemented) . . . . .  | 467 |
| 3.68.7 | Maxima [A] (verification not implemented) . . . . . | 467 |
| 3.68.8 | Giac [A] (verification not implemented) . . . . .   | 467 |
| 3.68.9 | Mupad [B] (verification not implemented) . . . . .  | 468 |

#### 3.68.1 Optimal result

Integrand size = 8, antiderivative size = 46

$$\int x \log(b + ax) dx = \frac{bx}{2a} - \frac{x^2}{4} - \frac{b^2 \log(b + ax)}{2a^2} + \frac{1}{2}x^2 \log(b + ax)$$

output `1/2*b*x/a-1/4*x^2-1/2*b^2*ln(a*x+b)/a^2+1/2*x^2*ln(a*x+b)`

#### 3.68.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int x \log(b + ax) dx = \frac{bx}{2a} - \frac{x^2}{4} - \frac{b^2 \log(b + ax)}{2a^2} + \frac{1}{2}x^2 \log(b + ax)$$

input `Integrate[x*Log[b + a*x],x]`

output `(b*x)/(2*a) - x^2/4 - (b^2*Log[b + a*x])/(2*a^2) + (x^2*Log[b + a*x])/2`

### 3.68.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x \log(ax + b) dx \\
 & \quad \downarrow \text{2842} \\
 & \frac{1}{2}x^2 \log(ax + b) - \frac{1}{2}a \int \frac{x^2}{b + ax} dx \\
 & \quad \downarrow \text{49} \\
 & \frac{1}{2}x^2 \log(ax + b) - \frac{1}{2}a \int \left( \frac{b^2}{a^2(b + ax)} - \frac{b}{a^2} + \frac{x}{a} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & \frac{1}{2}x^2 \log(ax + b) - \frac{1}{2}a \left( \frac{b^2 \log(ax + b)}{a^3} - \frac{bx}{a^2} + \frac{x^2}{2a} \right)
 \end{aligned}$$

input `Int[x*Log[b + a*x],x]`

output `(x^2*Log[b + a*x])/2 - (a*(-((b*x)/a^2) + x^2/(2*a) + (b^2*Log[b + a*x])/a^3))/2`

#### 3.68.3.1 Defintions of rubi rules used

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

```
rule 2842 Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_)
)^(q_.), x_Symbol] := Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1))) Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

### 3.68.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

| method            | result                                                                                                               | size |
|-------------------|----------------------------------------------------------------------------------------------------------------------|------|
| norman            | $\frac{bx}{2a} - \frac{x^2}{4} - \frac{b^2 \ln(ax+b)}{2a^2} + \frac{x^2 \ln(ax+b)}{2}$                               | 39   |
| risch             | $\frac{bx}{2a} - \frac{x^2}{4} - \frac{b^2 \ln(ax+b)}{2a^2} + \frac{x^2 \ln(ax+b)}{2}$                               | 39   |
| parts             | $\frac{x^2 \ln(ax+b)}{2} - \frac{a \left( \frac{1}{2} \frac{ax^2 - bx}{a^2} + \frac{b^2 \ln(ax+b)}{a^3} \right)}{2}$ | 45   |
| parallelrisch     | $\frac{2x^2 \ln(ax+b)a^2 - a^2x^2 + 2axb - 2b^2 \ln(ax+b) - 2b^2}{4a^2}$                                             | 50   |
| derivativedivides | $\frac{-b(\ln(ax+b)(ax+b) - ax - b) + \frac{(ax+b)^2 \ln(ax+b)}{2} - \frac{(ax+b)^2}{4}}{a^2}$                       | 53   |
| default           | $\frac{-b(\ln(ax+b)(ax+b) - ax - b) + \frac{(ax+b)^2 \ln(ax+b)}{2} - \frac{(ax+b)^2}{4}}{a^2}$                       | 53   |

input `int(x*ln(a*x+b),x,method=_RETURNVERBOSE)`

output `1/2*b*x/a-1/4*x^2-1/2*b^2*ln(a*x+b)/a^2+1/2*x^2*ln(a*x+b)`

### 3.68.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int x \log(b + ax) dx = -\frac{a^2 x^2 - 2 abx - 2 (a^2 x^2 - b^2) \log(ax + b)}{4 a^2}$$

input `integrate(x*log(a*x+b),x, algorithm="fricas")`

output `-1/4*(a^2*x^2 - 2*a*b*x - 2*(a^2*x^2 - b^2)*log(a*x + b))/a^2`

**3.68.6 Sympy [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int x \log(b + ax) dx = -a \left( \frac{x^2}{4a} - \frac{bx}{2a^2} + \frac{b^2 \log(ax + b)}{2a^3} \right) + \frac{x^2 \log(ax + b)}{2}$$

input `integrate(x*ln(a*x+b),x)`output `-a*(x**2/(4*a) - b*x/(2*a**2) + b**2*log(a*x + b)/(2*a**3)) + x**2*log(a*x + b)/2`**3.68.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x \log(b + ax) dx = \frac{1}{2} x^2 \log(ax + b) - \frac{1}{4} a \left( \frac{2b^2 \log(ax + b)}{a^3} + \frac{ax^2 - 2bx}{a^2} \right)$$

input `integrate(x*log(a*x+b),x, algorithm="maxima")`output `1/2*x^2*log(a*x + b) - 1/4*a*(2*b^2*log(a*x + b)/a^3 + (a*x^2 - 2*b*x)/a^2)`**3.68.8 Giac [A] (verification not implemented)**

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int x \log(b + ax) dx = \frac{(ax + b)^2 \log(ax + b)}{2a^2} - \frac{(ax + b)b \log(ax + b)}{a^2} - \frac{(ax + b)^2}{4a^2} + \frac{(ax + b)b}{a^2}$$

input `integrate(x*log(a*x+b),x, algorithm="giac")`output `1/2*(a*x + b)^2*log(a*x + b)/a^2 - (a*x + b)*b*log(a*x + b)/a^2 - 1/4*(a*x + b)^2/a^2 + (a*x + b)*b/a^2`

**3.68.9 Mupad [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int x \log(b + ax) dx = \begin{cases} \frac{x^2 (\ln(ax) - \frac{1}{2})}{2} & \text{if } b = 0 \\ \frac{\ln(b+ax) \left(x^2 - \frac{b^2}{a^2}\right)}{2} - \frac{b^2 \left(\frac{a^2 x^2}{2b^2} - \frac{ax}{b}\right)}{2a^2} & \text{if } b \neq 0 \end{cases}$$

input `int(x*log(b + a*x),x)`output `piecewise(b == 0, (x^2*(log(a*x) - 1/2))/2, b ~= 0, (log(b + a*x)*(x^2 - b^2/a^2))/2 - (b^2*((a^2*x^2)/(2*b^2) - (a*x)/b))/(2*a^2))`

## 3.69 $\int x^2 \log(b + ax) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.69.1 | Optimal result . . . . .                            | 469 |
| 3.69.2 | Mathematica [A] (verified) . . . . .                | 469 |
| 3.69.3 | Rubi [A] (verified) . . . . .                       | 470 |
| 3.69.4 | Maple [A] (verified) . . . . .                      | 471 |
| 3.69.5 | Fricas [A] (verification not implemented) . . . . . | 471 |
| 3.69.6 | Sympy [A] (verification not implemented) . . . . .  | 472 |
| 3.69.7 | Maxima [A] (verification not implemented) . . . . . | 472 |
| 3.69.8 | Giac [A] (verification not implemented) . . . . .   | 472 |
| 3.69.9 | Mupad [B] (verification not implemented) . . . . .  | 473 |

### 3.69.1 Optimal result

Integrand size = 10, antiderivative size = 59

$$\int x^2 \log(b + ax) dx = -\frac{b^2 x}{3a^2} + \frac{bx^2}{6a} - \frac{x^3}{9} + \frac{b^3 \log(b + ax)}{3a^3} + \frac{1}{3}x^3 \log(b + ax)$$

output `-1/3*b^2*x/a^2+1/6*b*x^2/a-1/9*x^3+1/3*b^3*ln(a*x+b)/a^3+1/3*x^3*ln(a*x+b)`

### 3.69.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int x^2 \log(b + ax) dx = -\frac{b^2 x}{3a^2} + \frac{bx^2}{6a} - \frac{x^3}{9} + \frac{b^3 \log(b + ax)}{3a^3} + \frac{1}{3}x^3 \log(b + ax)$$

input `Integrate[x^2*Log[b + a*x],x]`

output `-1/3*(b^2*x)/a^2 + (b*x^2)/(6*a) - x^3/9 + (b^3*Log[b + a*x])/(3*a^3) + (x^3*Log[b + a*x])/3`

### 3.69.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log(ax + b) dx$$

$$\downarrow 2842$$

$$\frac{1}{3}x^3 \log(ax + b) - \frac{1}{3}a \int \frac{x^3}{b + ax} dx$$

$$\downarrow 49$$

$$\frac{1}{3}x^3 \log(ax + b) - \frac{1}{3}a \int \left( -\frac{b^3}{a^3(b + ax)} + \frac{b^2}{a^3} - \frac{xb}{a^2} + \frac{x^2}{a} \right) dx$$

$$\downarrow 2009$$

$$\frac{1}{3}x^3 \log(ax + b) - \frac{1}{3}a \left( -\frac{b^3 \log(ax + b)}{a^4} + \frac{b^2 x}{a^3} - \frac{bx^2}{2a^2} + \frac{x^3}{3a} \right)$$

input `Int[x^2*Log[b + a*x],x]`

output `(x^3*Log[b + a*x])/3 - (a*((b^2*x)/a^3 - (b*x^2)/(2*a^2) + x^3/(3*a) - (b^3*Log[b + a*x])/a^4))/3`

#### 3.69.3.1 Defintions of rubi rules used

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

```
rule 2842 Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_
))^((q_.), x_Symbol] := Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1))) Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

### 3.69.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

| method            | result                                                                                                                                                                | size |
|-------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| norman            | $-\frac{b^2x}{3a^2} + \frac{bx^2}{6a} - \frac{x^3}{9} + \frac{b^3 \ln(ax+b)}{3a^3} + \frac{x^3 \ln(ax+b)}{3}$                                                         | 50   |
| risch             | $-\frac{b^2x}{3a^2} + \frac{bx^2}{6a} - \frac{x^3}{9} + \frac{b^3 \ln(ax+b)}{3a^3} + \frac{x^3 \ln(ax+b)}{3}$                                                         | 50   |
| parts             | $\frac{x^3 \ln(ax+b)}{3} - \frac{a \left( \frac{1}{3}a^2x^3 - \frac{1}{2}abx^2 + b^2x - \frac{b^3 \ln(ax+b)}{a^4} \right)}{3}$                                        | 56   |
| parallelrisch     | $\frac{6x^3 \ln(ax+b)a^3 - 2a^3x^3 + 3a^2bx^2 - 6ab^2x + 6b^3 \ln(ax+b) + 6b^3}{18a^3}$                                                                               | 61   |
| derivativedivides | $\frac{b^2(\ln(ax+b)(ax+b) - ax - b) - 2b \left( \frac{(ax+b)^2 \ln(ax+b)}{2} - \frac{(ax+b)^2}{4} \right) + \frac{(ax+b)^3 \ln(ax+b)}{3} - \frac{(ax+b)^3}{9}}{a^3}$ | 82   |
| default           | $\frac{b^2(\ln(ax+b)(ax+b) - ax - b) - 2b \left( \frac{(ax+b)^2 \ln(ax+b)}{2} - \frac{(ax+b)^2}{4} \right) + \frac{(ax+b)^3 \ln(ax+b)}{3} - \frac{(ax+b)^3}{9}}{a^3}$ | 82   |

```
input int(x^2*ln(a*x+b),x,method=_RETURNVERBOSE)
```

```
output -1/3*b^2*x/a^2+1/6*b*x^2/a-1/9*x^3+1/3*b^3*ln(a*x+b)/a^3+1/3*x^3*ln(a*x+b)
```

### 3.69.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int x^2 \log(b + ax) dx = -\frac{2a^3x^3 - 3a^2bx^2 + 6ab^2x - 6(a^3x^3 + b^3) \log(ax + b)}{18a^3}$$

```
input integrate(x^2*log(a*x+b),x, algorithm="fricas")
```

```
output -1/18*(2*a^3*x^3 - 3*a^2*b*x^2 + 6*a*b^2*x - 6*(a^3*x^3 + b^3)*log(a*x + b
))/a^3
```



**3.69.6 Sympy [A] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^2 \log(b + ax) dx = -a \left( \frac{x^3}{9a} - \frac{bx^2}{6a^2} + \frac{b^2x}{3a^3} - \frac{b^3 \log(ax + b)}{3a^4} \right) + \frac{x^3 \log(ax + b)}{3}$$

input `integrate(x**2*ln(a*x+b),x)`output `-a*(x**3/(9*a) - b*x**2/(6*a**2) + b**2*x/(3*a**3) - b**3*log(a*x + b)/(3*a**4)) + x**3*log(a*x + b)/3`**3.69.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int x^2 \log(b + ax) dx = \frac{1}{3} x^3 \log(ax + b) + \frac{1}{18} a \left( \frac{6b^3 \log(ax + b)}{a^4} - \frac{2a^2x^3 - 3abx^2 + 6b^2x}{a^3} \right)$$

input `integrate(x^2*log(a*x+b),x, algorithm="maxima")`output `1/3*x^3*log(a*x + b) + 1/18*a*(6*b^3*log(a*x + b)/a^4 - (2*a^2*x^3 - 3*a*b*x^2 + 6*b^2*x)/a^3)`**3.69.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.59

$$\int x^2 \log(b + ax) dx = \frac{(ax + b)^3 \log(ax + b)}{3a^3} - \frac{(ax + b)^2 b \log(ax + b)}{a^3} + \frac{(ax + b)b^2 \log(ax + b)}{a^3} - \frac{(ax + b)^3}{9a^3} + \frac{(ax + b)^2 b}{2a^3} - \frac{(ax + b)b^2}{a^3}$$

input `integrate(x^2*log(a*x+b),x, algorithm="giac")`output `1/3*(a*x + b)^3*log(a*x + b)/a^3 - (a*x + b)^2*b*log(a*x + b)/a^3 + (a*x + b)*b^2*log(a*x + b)/a^3 - 1/9*(a*x + b)^3/a^3 + 1/2*(a*x + b)^2*b/a^3 - (a*x + b)*b^2/a^3`

**3.69.9 Mupad [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int x^2 \log(b + ax) dx = \begin{cases} \frac{x^3 (\ln(ax) - \frac{1}{3})}{3} & \text{if } b = 0 \\ \frac{\ln(b+ax) \left(x^3 + \frac{b^3}{a^3}\right)}{3} - \frac{b^3 \left(\frac{a^3 x^3}{3b^3} - \frac{a^2 x^2}{2b^2} + \frac{ax}{b}\right)}{3a^3} & \text{if } b \neq 0 \end{cases}$$

input `int(x^2*log(b + a*x),x)`output `piecewise(b == 0, (x^3*(log(a*x) - 1/3))/3, b ~= 0, (log(b + a*x)*(x^3 + b^3/a^3))/3 - (b^3*(-(a^2*x^2)/(2*b^2) + (a^3*x^3)/(3*b^3) + (a*x)/b))/(3*a^3))`

## 3.70 $\int \log(a^2 + x^2) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.70.1 | Optimal result . . . . .                            | 474 |
| 3.70.2 | Mathematica [A] (verified) . . . . .                | 474 |
| 3.70.3 | Rubi [A] (verified) . . . . .                       | 475 |
| 3.70.4 | Maple [A] (verified) . . . . .                      | 476 |
| 3.70.5 | Fricas [A] (verification not implemented) . . . . . | 476 |
| 3.70.6 | Sympy [C] (verification not implemented) . . . . .  | 477 |
| 3.70.7 | Maxima [A] (verification not implemented) . . . . . | 477 |
| 3.70.8 | Giac [A] (verification not implemented) . . . . .   | 477 |
| 3.70.9 | Mupad [B] (verification not implemented) . . . . .  | 478 |

### 3.70.1 Optimal result

Integrand size = 8, antiderivative size = 23

$$\int \log(a^2 + x^2) dx = -2x + 2a \arctan\left(\frac{x}{a}\right) + x \log(a^2 + x^2)$$

output `-2*x+2*a*arctan(x/a)+x*ln(a^2+x^2)`

### 3.70.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \log(a^2 + x^2) dx = -2x + 2a \arctan\left(\frac{x}{a}\right) + x \log(a^2 + x^2)$$

input `Integrate[Log[a^2 + x^2],x]`

output `-2*x + 2*a*ArcTan[x/a] + x*Log[a^2 + x^2]`

### 3.70.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {2898, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \log(a^2 + x^2) dx \\
 & \quad \downarrow \text{2898} \\
 & x \log(a^2 + x^2) - 2 \int \frac{x^2}{a^2 + x^2} dx \\
 & \quad \downarrow \text{262} \\
 & x \log(a^2 + x^2) - 2 \left( x - a^2 \int \frac{1}{a^2 + x^2} dx \right) \\
 & \quad \downarrow \text{216} \\
 & x \log(a^2 + x^2) - 2 \left( x - a \arctan\left(\frac{x}{a}\right) \right)
 \end{aligned}$$

input `Int[Log[a^2 + x^2],x]`

output `-2*(x - a*ArcTan[x/a]) + x*Log[a^2 + x^2]`

#### 3.70.3.1 Defintions of rubi rules used

rule 216 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

rule 262 `Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/(b*(m + 2*p + 1))) Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]`

rule 2898 `Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d + e*x^n)^p], x] - Simp[e*n*p Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]`

### 3.70.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

| method       | result                                                           | size |
|--------------|------------------------------------------------------------------|------|
| default      | $-2x + 2a \arctan\left(\frac{x}{a}\right) + x \ln(a^2 + x^2)$    | 24   |
| risch        | $-2x + 2a \arctan\left(\frac{x}{a}\right) + x \ln(a^2 + x^2)$    | 24   |
| parts        | $-2x + 2a \arctan\left(\frac{x}{a}\right) + x \ln(a^2 + x^2)$    | 24   |
| paralelrisch | $-2i \ln(-ia + x) a + ia \ln(a^2 + x^2) + x \ln(a^2 + x^2) - 2x$ | 38   |

input `int(ln(a^2+x^2),x,method=_RETURNVERBOSE)`

output `-2*x+2*a*arctan(x/a)+x*ln(a^2+x^2)`

### 3.70.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \log(a^2 + x^2) dx = 2a \arctan\left(\frac{x}{a}\right) + x \log(a^2 + x^2) - 2x$$

input `integrate(log(a^2+x^2),x, algorithm="fricas")`

output `2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x`

**3.70.6 Sympy [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \log(a^2 + x^2) dx = -2a \left( \frac{i \log(-ia + x)}{2} - \frac{i \log(ia + x)}{2} \right) + x \log(a^2 + x^2) - 2x$$

input `integrate(ln(a**2+x**2),x)`

output `-2*a*(I*log(-I*a + x)/2 - I*log(I*a + x)/2) + x*log(a**2 + x**2) - 2*x`

**3.70.7 Maxima [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \log(a^2 + x^2) dx = 2a \arctan\left(\frac{x}{a}\right) + x \log(a^2 + x^2) - 2x$$

input `integrate(log(a^2+x^2),x, algorithm="maxima")`

output `2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x`

**3.70.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \log(a^2 + x^2) dx = 2a \arctan\left(\frac{x}{a}\right) + x \log(a^2 + x^2) - 2x$$

input `integrate(log(a^2+x^2),x, algorithm="giac")`

output `2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x`

**3.70.9 Mupad [B] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \log(a^2 + x^2) dx = x \ln(a^2 + x^2) - 2x + 2a \operatorname{atan}\left(\frac{x}{a}\right)$$

input `int(log(a^2 + x^2),x)`

output `x*log(a^2 + x^2) - 2*x + 2*a*atan(x/a)`

## 3.71 $\int x \log(a^2 + x^2) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.71.1 | Optimal result . . . . .                            | 479 |
| 3.71.2 | Mathematica [A] (verified) . . . . .                | 479 |
| 3.71.3 | Rubi [A] (verified) . . . . .                       | 480 |
| 3.71.4 | Maple [A] (verified) . . . . .                      | 481 |
| 3.71.5 | Fricas [A] (verification not implemented) . . . . . | 481 |
| 3.71.6 | Sympy [A] (verification not implemented) . . . . .  | 482 |
| 3.71.7 | Maxima [A] (verification not implemented) . . . . . | 482 |
| 3.71.8 | Giac [A] (verification not implemented) . . . . .   | 482 |
| 3.71.9 | Mupad [B] (verification not implemented) . . . . .  | 483 |

### 3.71.1 Optimal result

Integrand size = 10, antiderivative size = 27

$$\int x \log(a^2 + x^2) dx = -\frac{x^2}{2} + \frac{1}{2}(a^2 + x^2) \log(a^2 + x^2)$$

output `-1/2*x^2+1/2*(a^2+x^2)*ln(a^2+x^2)`

### 3.71.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int x \log(a^2 + x^2) dx = \frac{1}{2}(-x^2 + (a^2 + x^2) \log(a^2 + x^2))$$

input `Integrate[x*Log[a^2 + x^2],x]`

output `(-x^2 + (a^2 + x^2)*Log[a^2 + x^2])/2`



### 3.71.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {2904, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int x \log (a^2 + x^2) dx \\ & \quad \downarrow \text{2904} \\ & \frac{1}{2} \int \log (a^2 + x^2) dx^2 \\ & \quad \downarrow \text{2836} \\ & \frac{1}{2} \int \log (a^2 + x^2) d(a^2 + x^2) \\ & \quad \downarrow \text{2732} \\ & \frac{1}{2} ((a^2 + x^2) \log (a^2 + x^2) - a^2 - x^2) \end{aligned}$$

input `Int[x*Log[a^2 + x^2],x]`

output `(-a^2 - x^2 + (a^2 + x^2)*Log[a^2 + x^2])/2`

#### 3.71.3.1 Defintions of rubi rules used

rule 2732 `Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]`

rule 2836 `Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.), x_Symbol] :> Simp[1/e Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]`

```
rule 2904 Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
& !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

### 3.71.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

| method           | result                                                                                  | size |
|------------------|-----------------------------------------------------------------------------------------|------|
| derivativdivides | $\frac{(a^2+x^2)\ln(a^2+x^2)}{2} - \frac{a^2}{2} - \frac{x^2}{2}$                       | 29   |
| default          | $\frac{(a^2+x^2)\ln(a^2+x^2)}{2} - \frac{a^2}{2} - \frac{x^2}{2}$                       | 29   |
| norman           | $-\frac{x^2}{2} + \frac{x^2\ln(a^2+x^2)}{2} + \frac{\ln(a^2+x^2)a^2}{2}$                | 33   |
| risch            | $-\frac{x^2}{2} + \frac{x^2\ln(a^2+x^2)}{2} + \frac{\ln(a^2+x^2)a^2}{2}$                | 33   |
| parts            | $-\frac{x^2}{2} + \frac{x^2\ln(a^2+x^2)}{2} + \frac{\ln(a^2+x^2)a^2}{2}$                | 33   |
| parallelrisch    | $\frac{x^2\ln(a^2+x^2)}{2} + \frac{\ln(a^2+x^2)a^2}{2} - \frac{x^2}{2} + \frac{a^2}{2}$ | 38   |

input `int(x*ln(a^2+x^2),x,method=_RETURNVERBOSE)`

output `1/2*(a^2+x^2)*ln(a^2+x^2)-1/2*a^2-1/2*x^2`

### 3.71.5 Fracas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x \log(a^2 + x^2) dx = -\frac{1}{2}x^2 + \frac{1}{2}(a^2 + x^2) \log(a^2 + x^2)$$

input `integrate(x*log(a^2+x^2),x, algorithm="fricas")`

output `-1/2*x^2 + 1/2*(a^2 + x^2)*log(a^2 + x^2)`

**3.71.6 Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int x \log(a^2 + x^2) dx = \frac{a^2 \log(a^2 + x^2)}{2} + \frac{x^2 \log(a^2 + x^2)}{2} - \frac{x^2}{2}$$

input `integrate(x*ln(a**2+x**2),x)`output `a**2*log(a**2 + x**2)/2 + x**2*log(a**2 + x**2)/2 - x**2/2`**3.71.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int x \log(a^2 + x^2) dx = -\frac{1}{2}a^2 - \frac{1}{2}x^2 + \frac{1}{2}(a^2 + x^2) \log(a^2 + x^2)$$

input `integrate(x*log(a^2+x^2),x, algorithm="maxima")`output `-1/2*a^2 - 1/2*x^2 + 1/2*(a^2 + x^2)*log(a^2 + x^2)`**3.71.8 Giac [A] (verification not implemented)**

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int x \log(a^2 + x^2) dx = -\frac{1}{2}a^2 - \frac{1}{2}x^2 + \frac{1}{2}(a^2 + x^2) \log(a^2 + x^2)$$

input `integrate(x*log(a^2+x^2),x, algorithm="giac")`output `-1/2*a^2 - 1/2*x^2 + 1/2*(a^2 + x^2)*log(a^2 + x^2)`

**3.71.9 Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.89

$$\int x \log(a^2 + x^2) dx = \frac{a^2 \ln(x - \sqrt{-a^2})}{2} + \frac{x^2 \ln(a^2 + x^2)}{2} - \frac{x^2}{2} + \frac{a^2 \ln(x + \sqrt{-a^2})}{2}$$

input `int(x*log(a^2 + x^2),x)`

output `(a^2*log(x - (-a^2)^(1/2)))/2 + (x^2*log(a^2 + x^2))/2 - x^2/2 + (a^2*log(x + (-a^2)^(1/2)))/2`

## 3.72 $\int x^2 \log(a^2 + x^2) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.72.1 | Optimal result . . . . .                            | 484 |
| 3.72.2 | Mathematica [A] (verified) . . . . .                | 484 |
| 3.72.3 | Rubi [A] (verified) . . . . .                       | 485 |
| 3.72.4 | Maple [A] (verified) . . . . .                      | 486 |
| 3.72.5 | Fricas [A] (verification not implemented) . . . . . | 486 |
| 3.72.6 | Sympy [C] (verification not implemented) . . . . .  | 486 |
| 3.72.7 | Maxima [A] (verification not implemented) . . . . . | 487 |
| 3.72.8 | Giac [A] (verification not implemented) . . . . .   | 487 |
| 3.72.9 | Mupad [B] (verification not implemented) . . . . .  | 487 |

### 3.72.1 Optimal result

Integrand size = 12, antiderivative size = 44

$$\int x^2 \log(a^2 + x^2) dx = \frac{2a^2x}{3} - \frac{2x^3}{9} - \frac{2}{3}a^3 \arctan\left(\frac{x}{a}\right) + \frac{1}{3}x^3 \log(a^2 + x^2)$$

output `2/3*a^2*x-2/9*x^3-2/3*a^3*arctan(x/a)+1/3*x^3*ln(a^2+x^2)`

### 3.72.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int x^2 \log(a^2 + x^2) dx = \frac{2a^2x}{3} - \frac{2x^3}{9} - \frac{2}{3}a^3 \arctan\left(\frac{x}{a}\right) + \frac{1}{3}x^3 \log(a^2 + x^2)$$

input `Integrate[x^2*Log[a^2 + x^2],x]`

output `(2*a^2*x)/3 - (2*x^3)/9 - (2*a^3*ArcTan[x/a])/3 + (x^3*Log[a^2 + x^2])/3`

### 3.72.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2905, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int x^2 \log(a^2 + x^2) dx \\ & \quad \downarrow \text{2905} \\ & \frac{1}{3}x^3 \log(a^2 + x^2) - \frac{2}{3} \int \frac{x^4}{a^2 + x^2} dx \\ & \quad \downarrow \text{254} \\ & \frac{1}{3}x^3 \log(a^2 + x^2) - \frac{2}{3} \int \left( \frac{a^4}{a^2 + x^2} - a^2 + x^2 \right) dx \\ & \quad \downarrow \text{2009} \\ & \frac{1}{3}x^3 \log(a^2 + x^2) - \frac{2}{3} \left( a^3 \arctan\left(\frac{x}{a}\right) - a^2x + \frac{x^3}{3} \right) \end{aligned}$$

input `Int[x^2*Log[a^2 + x^2],x]`

output `(-2*(-(a^2*x) + x^3/3 + a^3*ArcTan[x/a]))/3 + (x^3*Log[a^2 + x^2])/3`

#### 3.72.3.1 Defintions of rubi rules used

rule 254 `Int[(x_)^(m_)/((a_) + (b_)*(x_)^2), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2905 `Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_)^(m_.), x_Symbol] := Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m + 1))), x] - Simp[b*e*n*(p/(f*(m + 1))) Int[x^(n - 1)*((f*x)^(m + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]`

**3.72.4 Maple [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

| method       | result                                                                                                                     | size |
|--------------|----------------------------------------------------------------------------------------------------------------------------|------|
| default      | $\frac{2a^2x}{3} - \frac{2x^3}{9} - \frac{2a^3 \arctan\left(\frac{x}{a}\right)}{3} + \frac{x^3 \ln(a^2+x^2)}{3}$           | 37   |
| risch        | $\frac{2a^2x}{3} - \frac{2x^3}{9} - \frac{2a^3 \arctan\left(\frac{x}{a}\right)}{3} + \frac{x^3 \ln(a^2+x^2)}{3}$           | 37   |
| parts        | $\frac{2a^2x}{3} - \frac{2x^3}{9} - \frac{2a^3 \arctan\left(\frac{x}{a}\right)}{3} + \frac{x^3 \ln(a^2+x^2)}{3}$           | 37   |
| parallelrisc | $\frac{2i \ln(-ia+x)a^3}{3} - \frac{i \ln(a^2+x^2)a^3}{3} + \frac{x^3 \ln(a^2+x^2)}{3} - \frac{2x^3}{9} + \frac{2a^2x}{3}$ | 53   |

input `int(x^2*ln(a^2+x^2),x,method=_RETURNVERBOSE)`output `2/3*a^2*x-2/9*x^3-2/3*a^3*arctan(x/a)+1/3*x^3*ln(a^2+x^2)`**3.72.5 Fracas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int x^2 \log(a^2 + x^2) dx = -\frac{2}{3} a^3 \arctan\left(\frac{x}{a}\right) + \frac{1}{3} x^3 \log(a^2 + x^2) + \frac{2}{3} a^2 x - \frac{2}{9} x^3$$

input `integrate(x^2*log(a^2+x^2),x, algorithm="fracas")`output `-2/3*a^3*arctan(x/a) + 1/3*x^3*log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3`**3.72.6 Sympy [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int x^2 \log(a^2 + x^2) dx = -2a^3 \left( -\frac{i \log(-ia + x)}{6} + \frac{i \log(ia + x)}{6} \right) + \frac{2a^2x}{3} + \frac{x^3 \log(a^2 + x^2)}{3} - \frac{2x^3}{9}$$

input `integrate(x**2*ln(a**2+x**2),x)`

output `-2*a**3*(-I*log(-I*a + x)/6 + I*log(I*a + x)/6) + 2*a**2*x/3 + x**3*log(a**2 + x**2)/3 - 2*x**3/9`

### 3.72.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int x^2 \log(a^2 + x^2) dx = -\frac{2}{3} a^3 \arctan\left(\frac{x}{a}\right) + \frac{1}{3} x^3 \log(a^2 + x^2) + \frac{2}{3} a^2 x - \frac{2}{9} x^3$$

input `integrate(x^2*log(a^2+x^2),x, algorithm="maxima")`

output `-2/3*a^3*arctan(x/a) + 1/3*x^3*log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3`

### 3.72.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int x^2 \log(a^2 + x^2) dx = -\frac{2}{3} a^3 \arctan\left(\frac{x}{a}\right) + \frac{1}{3} x^3 \log(a^2 + x^2) + \frac{2}{3} a^2 x - \frac{2}{9} x^3$$

input `integrate(x^2*log(a^2+x^2),x, algorithm="giac")`

output `-2/3*a^3*arctan(x/a) + 1/3*x^3*log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3`

### 3.72.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.48

$$\int x^2 \log(a^2 + x^2) dx = \frac{2a^2x}{3} - \frac{\ln(x - \sqrt{-a^2}) (-a^2)^{3/2}}{3} + \frac{x^3 \ln(a^2 + x^2)}{3} + \frac{\ln(x + \sqrt{-a^2}) (-a^2)^{3/2}}{3} - \frac{2x^3}{9}$$



input `int(x^2*log(a^2 + x^2),x)`

output  $(2*a^2*x)/3 - (\log(x - (-a^2)^{(1/2)})*(-a^2)^{(3/2)})/3 + (x^3*\log(a^2 + x^2))/3 + (\log(x + (-a^2)^{(1/2)})*(-a^2)^{(3/2)})/3 - (2*x^3)/9$

### 3.73 $\int x^4 \log(a^2 + x^2) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.73.1 | Optimal result . . . . .                            | 489 |
| 3.73.2 | Mathematica [A] (verified) . . . . .                | 489 |
| 3.73.3 | Rubi [A] (verified) . . . . .                       | 490 |
| 3.73.4 | Maple [A] (verified) . . . . .                      | 491 |
| 3.73.5 | Fricas [A] (verification not implemented) . . . . . | 491 |
| 3.73.6 | Sympy [C] (verification not implemented) . . . . .  | 492 |
| 3.73.7 | Maxima [A] (verification not implemented) . . . . . | 492 |
| 3.73.8 | Giac [A] (verification not implemented) . . . . .   | 492 |
| 3.73.9 | Mupad [B] (verification not implemented) . . . . .  | 493 |

#### 3.73.1 Optimal result

Integrand size = 12, antiderivative size = 54

$$\int x^4 \log(a^2 + x^2) dx = -\frac{2a^4x}{5} + \frac{2a^2x^3}{15} - \frac{2x^5}{25} + \frac{2}{5}a^5 \arctan\left(\frac{x}{a}\right) + \frac{1}{5}x^5 \log(a^2 + x^2)$$

output `-2/5*a^4*x+2/15*a^2*x^3-2/25*x^5+2/5*a^5*arctan(x/a)+1/5*x^5*ln(a^2+x^2)`

#### 3.73.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x^4 \log(a^2 + x^2) dx = -\frac{2a^4x}{5} + \frac{2a^2x^3}{15} - \frac{2x^5}{25} + \frac{2}{5}a^5 \arctan\left(\frac{x}{a}\right) + \frac{1}{5}x^5 \log(a^2 + x^2)$$

input `Integrate[x^4*Log[a^2 + x^2],x]`

output `(-2*a^4*x)/5 + (2*a^2*x^3)/15 - (2*x^5)/25 + (2*a^5*ArcTan[x/a])/5 + (x^5*Log[a^2 + x^2])/5`

**3.73.3 Rubi [A] (verified)**

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2905, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int x^4 \log(a^2 + x^2) dx \\ & \quad \downarrow \text{2905} \\ & \frac{1}{5}x^5 \log(a^2 + x^2) - \frac{2}{5} \int \frac{x^6}{a^2 + x^2} dx \\ & \quad \downarrow \text{254} \\ & \frac{1}{5}x^5 \log(a^2 + x^2) - \frac{2}{5} \int \left( -\frac{a^6}{a^2 + x^2} + a^4 - x^2 a^2 + x^4 \right) dx \\ & \quad \downarrow \text{2009} \\ & \frac{1}{5}x^5 \log(a^2 + x^2) - \frac{2}{5} \left( a^5 \left( -\arctan\left(\frac{x}{a}\right) \right) + a^4 x - \frac{a^2 x^3}{3} + \frac{x^5}{5} \right) \end{aligned}$$

input `Int[x^4*Log[a^2 + x^2],x]`

output `(-2*(a^4*x - (a^2*x^3)/3 + x^5/5 - a^5*ArcTan[x/a]))/5 + (x^5*Log[a^2 + x^2])/5`

**3.73.3.1 Defintions of rubi rules used**

rule 254 `Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2905 `Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_)^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m + 1))), x] - Simp[b*e*n*(p/(f*(m + 1))) Int[x^(n - 1)*((f*x)^(m + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]`

### 3.73.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

| method        | result                                                                                                                                            | size |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default       | $-\frac{2a^4x}{5} + \frac{2a^2x^3}{15} - \frac{2x^5}{25} + \frac{2a^5 \arctan\left(\frac{x}{a}\right)}{5} + \frac{x^5 \ln(a^2+x^2)}{5}$           | 45   |
| risch         | $-\frac{2a^4x}{5} + \frac{2a^2x^3}{15} - \frac{2x^5}{25} + \frac{2a^5 \arctan\left(\frac{x}{a}\right)}{5} + \frac{x^5 \ln(a^2+x^2)}{5}$           | 45   |
| parts         | $-\frac{2a^4x}{5} + \frac{2a^2x^3}{15} - \frac{2x^5}{25} + \frac{2a^5 \arctan\left(\frac{x}{a}\right)}{5} + \frac{x^5 \ln(a^2+x^2)}{5}$           | 45   |
| parallelrisch | $-\frac{2i \ln(-ia+x)a^5}{5} + \frac{i \ln(a^2+x^2)a^5}{5} + \frac{x^5 \ln(a^2+x^2)}{5} - \frac{2x^5}{25} + \frac{2a^2x^3}{15} - \frac{2a^4x}{5}$ | 61   |

input `int(x^4*ln(a^2+x^2),x,method=_RETURNVERBOSE)`

output `-2/5*a^4*x+2/15*a^2*x^3-2/25*x^5+2/5*a^5*arctan(x/a)+1/5*x^5*ln(a^2+x^2)`

### 3.73.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^4 \log(a^2 + x^2) dx = \frac{2}{5} a^5 \arctan\left(\frac{x}{a}\right) + \frac{1}{5} x^5 \log(a^2 + x^2) - \frac{2}{5} a^4 x + \frac{2}{15} a^2 x^3 - \frac{2}{25} x^5$$

input `integrate(x^4*log(a^2+x^2),x, algorithm="fracas")`

output `2/5*a^5*arctan(x/a) + 1/5*x^5*log(a^2 + x^2) - 2/5*a^4*x + 2/15*a^2*x^3 - 2/25*x^5`

**3.73.6 Sympy [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int x^4 \log(a^2 + x^2) dx = -2a^5 \left( \frac{i \log(-ia + x)}{10} - \frac{i \log(ia + x)}{10} \right) - \frac{2a^4 x}{5} + \frac{2a^2 x^3}{15} + \frac{x^5 \log(a^2 + x^2)}{5} - \frac{2x^5}{25}$$

input `integrate(x**4*ln(a**2+x**2),x)`

output `-2*a**5*(I*log(-I*a + x)/10 - I*log(I*a + x)/10) - 2*a**4*x/5 + 2*a**2*x**3/15 + x**5*log(a**2 + x**2)/5 - 2*x**5/25`

**3.73.7 Maxima [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^4 \log(a^2 + x^2) dx = \frac{2}{5} a^5 \arctan\left(\frac{x}{a}\right) + \frac{1}{5} x^5 \log(a^2 + x^2) - \frac{2}{5} a^4 x + \frac{2}{15} a^2 x^3 - \frac{2}{25} x^5$$

input `integrate(x^4*log(a^2+x^2),x, algorithm="maxima")`

output `2/5*a^5*arctan(x/a) + 1/5*x^5*log(a^2 + x^2) - 2/5*a^4*x + 2/15*a^2*x^3 - 2/25*x^5`

**3.73.8 Giac [A] (verification not implemented)**

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^4 \log(a^2 + x^2) dx = \frac{2}{5} a^5 \arctan\left(\frac{x}{a}\right) + \frac{1}{5} x^5 \log(a^2 + x^2) - \frac{2}{5} a^4 x + \frac{2}{15} a^2 x^3 - \frac{2}{25} x^5$$

input `integrate(x^4*log(a^2+x^2),x, algorithm="giac")`

output `2/5*a^5*arctan(x/a) + 1/5*x^5*log(a^2 + x^2) - 2/5*a^4*x + 2/15*a^2*x^3 - 2/25*x^5`

**3.73.9 Mupad [B] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int x^4 \log(a^2 + x^2) dx = \frac{x^5 \ln(a^2 + x^2)}{5} - \frac{2a^4 x}{5} - \frac{\ln(x - \sqrt{-a^2}) (-a^2)^{5/2}}{5} \\ + \frac{\ln(x + \sqrt{-a^2}) (-a^2)^{5/2}}{5} - \frac{2x^5}{25} + \frac{2a^2 x^3}{15}$$

input `int(x^4*log(a^2 + x^2),x)`output `(x^5*log(a^2 + x^2))/5 - (2*a^4*x)/5 - (log(x - (-a^2)^(1/2))*(-a^2)^(5/2))/5 + (log(x + (-a^2)^(1/2))*(-a^2)^(5/2))/5 - (2*x^5)/25 + (2*a^2*x^3)/15`

## 3.74 $\int \log(-a^2 + x^2) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.74.1 | Optimal result . . . . .                            | 494 |
| 3.74.2 | Mathematica [A] (verified) . . . . .                | 494 |
| 3.74.3 | Rubi [A] (verified) . . . . .                       | 495 |
| 3.74.4 | Maple [A] (verified) . . . . .                      | 496 |
| 3.74.5 | Fricas [A] (verification not implemented) . . . . . | 496 |
| 3.74.6 | Sympy [A] (verification not implemented) . . . . .  | 497 |
| 3.74.7 | Maxima [A] (verification not implemented) . . . . . | 497 |
| 3.74.8 | Giac [A] (verification not implemented) . . . . .   | 497 |
| 3.74.9 | Mupad [B] (verification not implemented) . . . . .  | 498 |

### 3.74.1 Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \log(-a^2 + x^2) dx = -2x + 2a \operatorname{arctanh}\left(\frac{x}{a}\right) + x \log(-a^2 + x^2)$$

output `-2*x+2*a*arctanh(x/a)+x*ln(-a^2+x^2)`

### 3.74.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \log(-a^2 + x^2) dx = -2x + 2a \operatorname{arctanh}\left(\frac{x}{a}\right) + x \log(-a^2 + x^2)$$

input `Integrate[Log[-a^2 + x^2],x]`

output `-2*x + 2*a*ArcTanh[x/a] + x*Log[-a^2 + x^2]`

### 3.74.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {2898, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \log(x^2 - a^2) dx \\
 & \quad \downarrow \text{2898} \\
 & x \log(x^2 - a^2) - 2 \int -\frac{x^2}{a^2 - x^2} dx \\
 & \quad \downarrow \text{25} \\
 & 2 \int \frac{x^2}{a^2 - x^2} dx + x \log(x^2 - a^2) \\
 & \quad \downarrow \text{262} \\
 & 2 \left( a^2 \int \frac{1}{a^2 - x^2} dx - x \right) + x \log(x^2 - a^2) \\
 & \quad \downarrow \text{219} \\
 & x \log(x^2 - a^2) + 2 \left( a \operatorname{arctanh}\left(\frac{x}{a}\right) - x \right)
 \end{aligned}$$

input `Int[Log[-a^2 + x^2],x]`

output `2*(-x + a*ArcTanh[x/a]) + x*Log[-a^2 + x^2]`

#### 3.74.3.1 Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`



rule 262 `Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[c*(c*x)^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/(b*(m + 2*p + 1)) Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]`

rule 2898 `Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] := Simp[x*Log[c*(d + e*x^n)^p], x] - Simp[e*n*p Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]`

### 3.74.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

| method        | result                                                         | size |
|---------------|----------------------------------------------------------------|------|
| default       | $x \ln(-a^2 + x^2) - 2x - a \ln(a - x) + a \ln(a + x)$         | 32   |
| risch         | $x \ln(-a^2 + x^2) - 2x + a \ln(a + x) - a \ln(-a + x)$        | 32   |
| parts         | $x \ln(-a^2 + x^2) - 2x - a \ln(a - x) + a \ln(a + x)$         | 32   |
| parallelrisch | $-2a \ln(-a + x) + x \ln(-a^2 + x^2) + a \ln(-a^2 + x^2) - 2x$ | 38   |

input `int(ln(-a^2+x^2),x,method=_RETURNVERBOSE)`

output `x*ln(-a^2+x^2)-2*x-a*ln(a-x)+a*ln(a+x)`

### 3.74.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \log(-a^2 + x^2) dx = x \log(-a^2 + x^2) + a \log(a + x) - a \log(-a + x) - 2x$$

input `integrate(log(-a^2+x^2),x, algorithm="fricas")`

output `x*log(-a^2 + x^2) + a*log(a + x) - a*log(-a + x) - 2*x`

**3.74.6 Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \log(-a^2 + x^2) dx = -2a \left( \frac{\log(-a + x)}{2} - \frac{\log(a + x)}{2} \right) + x \log(-a^2 + x^2) - 2x$$

input `integrate(ln(-a**2+x**2),x)`output `-2*a*(log(-a + x)/2 - log(a + x)/2) + x*log(-a**2 + x**2) - 2*x`**3.74.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \log(-a^2 + x^2) dx = x \log(-a^2 + x^2) + a \log(a + x) - a \log(-a + x) - 2x$$

input `integrate(log(-a^2+x^2),x, algorithm="maxima")`output `x*log(-a^2 + x^2) + a*log(a + x) - a*log(-a + x) - 2*x`**3.74.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \log(-a^2 + x^2) dx = x \log(-a^2 + x^2) + a \log(|a + x|) - a \log(|-a + x|) - 2x$$

input `integrate(log(-a^2+x^2),x, algorithm="giac")`output `x*log(-a^2 + x^2) + a*log(abs(a + x)) - a*log(abs(-a + x)) - 2*x`

**3.74.9 Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \log(-a^2 + x^2) dx = x \ln(x^2 - a^2) - 2x + 2a \operatorname{atanh}\left(\frac{x}{a}\right)$$

input `int(log(x^2 - a^2),x)`

output `x*log(x^2 - a^2) - 2*x + 2*a*atanh(x/a)`

### 3.75 $\int \log(\log(\log(\log(x)))) dx$

|        |                             |     |
|--------|-----------------------------|-----|
| 3.75.1 | Optimal result . . . . .    | 499 |
| 3.75.2 | Mathematica [N/A] . . . . . | 499 |
| 3.75.3 | Rubi [N/A] . . . . .        | 500 |
| 3.75.4 | Maple [N/A] . . . . .       | 500 |
| 3.75.5 | Fricas [N/A] . . . . .      | 501 |
| 3.75.6 | Sympy [N/A] . . . . .       | 501 |
| 3.75.7 | Maxima [N/A] . . . . .      | 501 |
| 3.75.8 | Giac [N/A] . . . . .        | 502 |
| 3.75.9 | Mupad [N/A] . . . . .       | 502 |

#### 3.75.1 Optimal result

Integrand size = 5, antiderivative size = 5

$$\int \log(\log(\log(\log(x)))) dx = \text{Int}(\log(\log(\log(\log(x))))), x)$$

output `CannotIntegrate(ln(ln(ln(ln(x))))), x)`

#### 3.75.2 Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \log(\log(\log(\log(x)))) dx = \int \log(\log(\log(\log(x)))) dx$$

input `Integrate[Log[Log[Log[Log[x]]]], x]`

output `Integrate[Log[Log[Log[Log[x]]]], x]`

**3.75.3 Rubi [N/A]**

Not integrable

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log(\log(\log(\log(x)))) dx$$

↓ 7299

$$\int \log(\log(\log(\log(x)))) dx$$

input `Int [Log [Log [Log [Log [x]]]]] , x]`

output `$Aborted`

**3.75.3.1 Defintions of rubi rules used**

rule 7299 `Int [u_ , x_] :> CannotIntegrate [u , x]`

**3.75.4 Maple [N/A]**

Not integrable

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \ln (\ln (\ln (\ln (x)))) dx$$

input `int (ln (ln (ln (ln (x)))) , x)`

output `int (ln (ln (ln (ln (x)))) , x)`

**3.75.5 Fracas [N/A]**

Not integrable

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \log(\log(\log(\log(x)))) dx = \int \log(\log(\log(\log(x)))) dx$$

input `integrate(log(log(log(log(x)))) , x, algorithm="fricas")`output `integral(log(log(log(log(x)))) , x)`**3.75.6 Sympy [N/A]**

Not integrable

Time = 3.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 5.40

$$\int \log(\log(\log(\log(x)))) dx = x \log(\log(\log(\log(x)))) - \int \frac{1}{\log(x) \log(\log(x)) \log(\log(\log(x)))} dx$$

input `integrate(ln(ln(ln(ln(x)))) , x)`output `x*log(log(log(log(x)))) - Integral(1/(log(x)*log(log(x))*log(log(log(x)))) , x)`**3.75.7 Maxima [N/A]**

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 5.60

$$\int \log(\log(\log(\log(x)))) dx = \int \log(\log(\log(\log(x)))) dx$$

input `integrate(log(log(log(log(x)))) , x, algorithm="maxima")`output `x*log(log(log(log(x)))) - integrate(1/(log(x)*log(log(x))*log(log(log(x)))) , x)`

**3.75.8 Giac [N/A]**

Not integrable

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \log(\log(\log(\log(x)))) dx = \int \log(\log(\log(\log(x)))) dx$$

input `integrate(log(log(log(log(x)))) , x, algorithm="giac")`output `integrate(log(log(log(log(x)))) , x)`**3.75.9 Mupad [N/A]**

Not integrable

Time = 0.58 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \log(\log(\log(\log(x)))) dx = \int \ln(\ln(\ln(\ln(x)))) dx$$

input `int(log(log(log(log(x)))) , x)`output `int(log(log(log(log(x)))) , x)`

## 3.76 $\int \sin(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.76.1 | Optimal result . . . . .                            | 503 |
| 3.76.2 | Mathematica [A] (verified) . . . . .                | 503 |
| 3.76.3 | Rubi [A] (verified) . . . . .                       | 504 |
| 3.76.4 | Maple [A] (verified) . . . . .                      | 505 |
| 3.76.5 | Fricas [A] (verification not implemented) . . . . . | 505 |
| 3.76.6 | Sympy [A] (verification not implemented) . . . . .  | 505 |
| 3.76.7 | Maxima [A] (verification not implemented) . . . . . | 506 |
| 3.76.8 | Giac [A] (verification not implemented) . . . . .   | 506 |
| 3.76.9 | Mupad [B] (verification not implemented) . . . . .  | 506 |

### 3.76.1 Optimal result

Integrand size = 2, antiderivative size = 4

$$\int \sin(x) dx = -\cos(x)$$

output `-cos(x)`

### 3.76.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x) dx = -\cos(x)$$

input `Integrate[Sin[x],x]`

output `-Cos[x]`



### 3.76.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \sin(x) dx \\ \downarrow 3042 \\ \int \sin(x) dx \\ \downarrow 3118 \\ -\cos(x) \end{array}$$

input `Int[Sin[x],x]`

output `-Cos[x]`

#### 3.76.3.1 Defintions of rubi rules used

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3118 `Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

**3.76.4 Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

| method        | result                                                                        | size |
|---------------|-------------------------------------------------------------------------------|------|
| lookup        | $-\cos(x)$                                                                    | 5    |
| default       | $-\cos(x)$                                                                    | 5    |
| risch         | $-\cos(x)$                                                                    | 5    |
| parallelrisch | $-\cos(x) - 1$                                                                | 7    |
| norman        | $-\frac{2}{1+\tan^2(\frac{x}{2})}$                                            | 13   |
| meijerg       | $\sqrt{\pi} \left( \frac{1}{\sqrt{\pi}} - \frac{\cos(x)}{\sqrt{\pi}} \right)$ | 16   |

input `int(sin(x),x,method=_RETURNVERBOSE)`output `-cos(x)`**3.76.5 Fricas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x) dx = -\cos(x)$$

input `integrate(sin(x),x, algorithm="fricas")`output `-cos(x)`**3.76.6 Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \sin(x) dx = -\cos(x)$$

input `integrate(sin(x),x)`output `-cos(x)`

**3.76.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x) dx = -\cos(x)$$

input `integrate(sin(x),x, algorithm="maxima")`

output `-cos(x)`

**3.76.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x) dx = -\cos(x)$$

input `integrate(sin(x),x, algorithm="giac")`

output `-cos(x)`

**3.76.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(x) dx = -\cos(x)$$

input `int(sin(x),x)`

output `-cos(x)`

## 3.77 $\int \cos(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.77.1 | Optimal result . . . . .                            | 507 |
| 3.77.2 | Mathematica [A] (verified) . . . . .                | 507 |
| 3.77.3 | Rubi [A] (verified) . . . . .                       | 508 |
| 3.77.4 | Maple [A] (verified) . . . . .                      | 509 |
| 3.77.5 | Fricas [A] (verification not implemented) . . . . . | 509 |
| 3.77.6 | Sympy [A] (verification not implemented) . . . . .  | 509 |
| 3.77.7 | Maxima [A] (verification not implemented) . . . . . | 510 |
| 3.77.8 | Giac [A] (verification not implemented) . . . . .   | 510 |
| 3.77.9 | Mupad [B] (verification not implemented) . . . . .  | 510 |

### 3.77.1 Optimal result

Integrand size = 2, antiderivative size = 2

$$\int \cos(x) dx = \sin(x)$$

output `sin(x)`

### 3.77.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x) dx = \sin(x)$$

input `Integrate[Cos[x],x]`

output `Sin[x]`

**3.77.3 Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \cos(x) dx \\ \downarrow \text{3042} \\ \int \sin\left(x + \frac{\pi}{2}\right) dx \\ \downarrow \text{3117} \\ \sin(x) \end{array}$$

input `Int[Cos[x], x]`

output `Sin[x]`

**3.77.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3117 `Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

**3.77.4 Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

| method        | result                                                | size |
|---------------|-------------------------------------------------------|------|
| lookup        | $\sin(x)$                                             | 3    |
| default       | $\sin(x)$                                             | 3    |
| meijerg       | $\sin(x)$                                             | 3    |
| risch         | $\sin(x)$                                             | 3    |
| parallelrisch | $\sin(x)$                                             | 3    |
| norman        | $\frac{2 \tan(\frac{x}{2})}{1 + \tan^2(\frac{x}{2})}$ | 17   |

input `int(cos(x),x,method=_RETURNVERBOSE)`

output `sin(x)`

**3.77.5 Fricas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x) dx = \sin(x)$$

input `integrate(cos(x),x, algorithm="fricas")`

output `sin(x)`

**3.77.6 Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x) dx = \sin(x)$$

input `integrate(cos(x),x)`

output `sin(x)`

**3.77.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x) dx = \sin(x)$$

input `integrate(cos(x),x, algorithm="maxima")`

output `sin(x)`

**3.77.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x) dx = \sin(x)$$

input `integrate(cos(x),x, algorithm="giac")`

output `sin(x)`

**3.77.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x) dx = \sin(x)$$

input `int(cos(x),x)`

output `sin(x)`

## 3.78 $\int \tan(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.78.1 | Optimal result . . . . .                            | 511 |
| 3.78.2 | Mathematica [A] (verified) . . . . .                | 511 |
| 3.78.3 | Rubi [A] (verified) . . . . .                       | 512 |
| 3.78.4 | Maple [A] (verified) . . . . .                      | 513 |
| 3.78.5 | Fricas [B] (verification not implemented) . . . . . | 513 |
| 3.78.6 | Sympy [A] (verification not implemented) . . . . .  | 514 |
| 3.78.7 | Maxima [A] (verification not implemented) . . . . . | 514 |
| 3.78.8 | Giac [A] (verification not implemented) . . . . .   | 514 |
| 3.78.9 | Mupad [B] (verification not implemented) . . . . .  | 515 |

### 3.78.1 Optimal result

Integrand size = 2, antiderivative size = 5

$$\int \tan(x) dx = -\log(\cos(x))$$

output `-ln(cos(x))`

### 3.78.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \tan(x) dx = -\log(\cos(x))$$

input `Integrate[Tan[x],x]`

output `-Log[Cos[x]]`



### 3.78.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \tan(x) dx \\ \downarrow 3042 \\ \int \tan(x) dx \\ \downarrow 3956 \\ -\log(\cos(x)) \end{array}$$

input `Int [Tan [x] , x]`

output `-Log [Cos [x]]`

#### 3.78.3.1 Defintions of rubi rules used

rule 3042 `Int [u_ , x_Symbol] :> Int [DeactivateTrig [u , x] , x] /; FunctionOfTrigOfLinear Q [u , x]`

rule 3956 `Int [tan [(c_) + (d_)*(x_)] , x_Symbol] :> Simp [-Log [RemoveContent [Cos [c + d *x] , x]]/d , x] /; FreeQ [{c , d} , x]`

**3.78.4 Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

| method            | result                       | size |
|-------------------|------------------------------|------|
| lookup            | $-\ln(\cos(x))$              | 6    |
| default           | $-\ln(\cos(x))$              | 6    |
| derivativedivides | $\frac{\ln(1+\tan^2(x))}{2}$ | 10   |
| norman            | $\frac{\ln(1+\tan^2(x))}{2}$ | 10   |
| parallelrisc      | $\frac{\ln(1+\tan^2(x))}{2}$ | 10   |
| risc              | $ix - \ln(e^{2ix} + 1)$      | 16   |

input `int(tan(x),x,method=_RETURNVERBOSE)`

output `-ln(cos(x))`

**3.78.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 11 vs.  $2(5) = 10$ .

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int \tan(x) dx = -\frac{1}{2} \log\left(\frac{1}{\tan(x)^2 + 1}\right)$$

input `integrate(tan(x),x, algorithm="fracas")`

output `-1/2*log(1/(tan(x)^2 + 1))`

**3.78.6 Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \tan(x) dx = -\log(\cos(x))$$

input `integrate(tan(x),x)`

output `-log(cos(x))`

**3.78.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int \tan(x) dx = \log(\sec(x))$$

input `integrate(tan(x),x, algorithm="maxima")`

output `log(sec(x))`

**3.78.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

$$\int \tan(x) dx = -\log(|\cos(x)|)$$

input `integrate(tan(x),x, algorithm="giac")`

output `-log(abs(cos(x)))`

**3.78.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \tan(x) dx = -\ln(\cos(x))$$

input `int(tan(x),x)`

output `-log(cos(x))`

### 3.79 $\int \cot(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.79.1 | Optimal result . . . . .                            | 516 |
| 3.79.2 | Mathematica [B] (verified) . . . . .                | 516 |
| 3.79.3 | Rubi [A] (verified) . . . . .                       | 517 |
| 3.79.4 | Maple [A] (verified) . . . . .                      | 518 |
| 3.79.5 | Fricas [B] (verification not implemented) . . . . . | 518 |
| 3.79.6 | Sympy [A] (verification not implemented) . . . . .  | 519 |
| 3.79.7 | Maxima [A] (verification not implemented) . . . . . | 519 |
| 3.79.8 | Giac [B] (verification not implemented) . . . . .   | 519 |
| 3.79.9 | Mupad [B] (verification not implemented) . . . . .  | 520 |

#### 3.79.1 Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \cot(x) dx = \log(\sin(x))$$

output `ln(sin(x))`

#### 3.79.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \cot(x) dx = \log(\cos(x)) + \log(\tan(x))$$

input `Integrate[Cot[x], x]`

output `Log[Cos[x]] + Log[Tan[x]]`

**3.79.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$ , Rules used = {3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c}
 \int \cot(x) dx \\
 \downarrow \text{3042} \\
 \int -\tan\left(x + \frac{\pi}{2}\right) dx \\
 \downarrow \text{25} \\
 -\int \tan\left(x + \frac{\pi}{2}\right) dx \\
 \downarrow \text{3956} \\
 \log(\sin(x))
 \end{array}$$

input `Int[Cot[x], x]`

output `Log[Sin[x]]`

**3.79.3.1 Defintions of rubi rules used**

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 3956 `Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d *x], x]]/d, x] /; FreeQ[{c, d}, x]`

**3.79.4 Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

| method            | result                                       | size |
|-------------------|----------------------------------------------|------|
| lookup            | $\ln(\sin(x))$                               | 4    |
| default           | $\ln(\sin(x))$                               | 4    |
| derivativedivides | $-\frac{\ln(1+\tan^2(x))}{2} + \ln(\tan(x))$ | 14   |
| norman            | $-\frac{\ln(1+\tan^2(x))}{2} + \ln(\tan(x))$ | 14   |
| risch             | $-ix + \ln(e^{2ix} - 1)$                     | 14   |
| parallelrisc      | $-\frac{\ln(1+\tan^2(x))}{2} + \ln(\tan(x))$ | 14   |

input `int(1/tan(x),x,method=_RETURNVERBOSE)`

output `ln(sin(x))`

**3.79.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 16 vs.  $2(3) = 6$ .

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 5.33

$$\int \cot(x) dx = \frac{1}{2} \log\left(\frac{\tan(x)^2}{\tan(x)^2 + 1}\right)$$

input `integrate(1/tan(x),x, algorithm="fricas")`

output `1/2*log(tan(x)^2/(tan(x)^2 + 1))`

**3.79.6 Sympy [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x) dx = \log(\sin(x))$$

input `integrate(1/tan(x),x)`

output `log(sin(x))`

**3.79.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x) dx = \log(\sin(x))$$

input `integrate(1/tan(x),x, algorithm="maxima")`

output `log(sin(x))`

**3.79.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(3) = 6.

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \cot(x) dx = -\frac{1}{2} \log(\tan(x)^2 + 1) + \frac{1}{2} \log(\tan(x)^2)$$

input `integrate(1/tan(x),x, algorithm="giac")`

output `-1/2*log(tan(x)^2 + 1) + 1/2*log(tan(x)^2)`



**3.79.9 Mupad [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 4.33

$$\int \cot(x) dx = \ln(\tan(x)) - \frac{\ln(\tan(x)^2 + 1)}{2}$$

input `int(1/tan(x),x)`

output `log(tan(x)) - log(tan(x)^2 + 1)/2`

### 3.80 $\int \frac{1}{(1+\tan(x))^2} dx$

|                                                            |     |
|------------------------------------------------------------|-----|
| 3.80.1 Optimal result . . . . .                            | 521 |
| 3.80.2 Mathematica [A] (verified) . . . . .                | 521 |
| 3.80.3 Rubi [A] (verified) . . . . .                       | 522 |
| 3.80.4 Maple [A] (verified) . . . . .                      | 523 |
| 3.80.5 Fricas [B] (verification not implemented) . . . . . | 523 |
| 3.80.6 Sympy [B] (verification not implemented) . . . . .  | 524 |
| 3.80.7 Maxima [A] (verification not implemented) . . . . . | 524 |
| 3.80.8 Giac [A] (verification not implemented) . . . . .   | 524 |
| 3.80.9 Mupad [B] (verification not implemented) . . . . .  | 525 |

#### 3.80.1 Optimal result

Integrand size = 6, antiderivative size = 21

$$\int \frac{1}{(1 + \tan(x))^2} dx = \frac{1}{2} \log(\cos(x) + \sin(x)) - \frac{1}{2(1 + \tan(x))}$$

output `1/2*ln(cos(x)+sin(x))-1/2/(1+tan(x))`

#### 3.80.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1 + \tan(x))^2} dx = \frac{1}{2} \left( \log(\cos(x)) + \log(1 + \tan(x)) - \frac{1}{1 + \tan(x)} \right)$$

input `Integrate[(1 + Tan[x])^(-2), x]`

output `(Log[Cos[x]] + Log[1 + Tan[x]] - (1 + Tan[x])^(-1))/2`

### 3.80.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {3042, 3964, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{(\tan(x) + 1)^2} dx \\
 & \quad \downarrow \text{3042} \\
 & \int \frac{1}{(\tan(x) + 1)^2} dx \\
 & \quad \downarrow \text{3964} \\
 & \frac{1}{2} \int \frac{1 - \tan(x)}{\tan(x) + 1} dx - \frac{1}{2(\tan(x) + 1)} \\
 & \quad \downarrow \text{3042} \\
 & \frac{1}{2} \int \frac{1 - \tan(x)}{\tan(x) + 1} dx - \frac{1}{2(\tan(x) + 1)} \\
 & \quad \downarrow \text{4013} \\
 & \frac{1}{2} \log(\sin(x) + \cos(x)) - \frac{1}{2(\tan(x) + 1)}
 \end{aligned}$$

input `Int[(1 + Tan[x])^(-2), x]`

output `Log[Cos[x] + Sin[x]]/2 - 1/(2*(1 + Tan[x]))`

#### 3.80.3.1 Defintions of rubi rules used

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3964 `Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*((a + b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2) Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]`

```
rule 4013 Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)]) / ((a_) + (b_)*tan[(e_) + (f_)*
(x_)]), x_Symbol] := Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

### 3.80.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

| method            | result                                                                                                              | size |
|-------------------|---------------------------------------------------------------------------------------------------------------------|------|
| derivativedivides | $-\frac{1}{2(\tan(x)+1)} + \frac{\ln(\tan(x)+1)}{2} - \frac{\ln(1+\tan^2(x))}{4}$                                   | 26   |
| default           | $-\frac{1}{2(\tan(x)+1)} + \frac{\ln(\tan(x)+1)}{2} - \frac{\ln(1+\tan^2(x))}{4}$                                   | 26   |
| norman            | $-\frac{1}{2(\tan(x)+1)} + \frac{\ln(\tan(x)+1)}{2} - \frac{\ln(1+\tan^2(x))}{4}$                                   | 26   |
| risch             | $-\frac{ix}{2} - \frac{1}{2(e^{2ix}+i)} + \frac{\ln(e^{2ix}+i)}{2}$                                                 | 29   |
| parallelrisch     | $\frac{2 \ln(\tan(x)+1) \tan(x) - \ln(1+\tan^2(x)) \tan(x) - 2 + 2 \ln(\tan(x)+1) - \ln(1+\tan^2(x))}{4 \tan(x)+4}$ | 47   |

```
input int(1/(tan(x)+1)^2,x,method=_RETURNVERBOSE)
```

```
output -1/2/(tan(x)+1)+1/2*ln(tan(x)+1)-1/4*ln(1+tan(x)^2)
```

### 3.80.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs.  $2(17) = 34$ .

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{1}{(1+\tan(x))^2} dx = \frac{(\tan(x)+1) \log\left(\frac{\tan(x)^2+2\tan(x)+1}{\tan(x)^2+1}\right) + \tan(x) - 1}{4(\tan(x)+1)}$$

```
input integrate(1/(1+tan(x))^2,x, algorithm="fricas")
```

```
output 1/4*((tan(x) + 1)*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) + tan(x) -
1)/(tan(x) + 1)
```

**3.80.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 75 vs.  $2(17) = 34$ .

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.57

$$\int \frac{1}{(1 + \tan(x))^2} dx = \frac{2 \log(\tan(x) + 1) \tan(x)}{4 \tan(x) + 4} + \frac{2 \log(\tan(x) + 1)}{4 \tan(x) + 4} - \frac{\log(\tan^2(x) + 1) \tan(x)}{4 \tan(x) + 4} - \frac{\log(\tan^2(x) + 1)}{4 \tan(x) + 4} - \frac{2}{4 \tan(x) + 4}$$

input `integrate(1/(1+tan(x))**2,x)`

output `2*log(tan(x) + 1)*tan(x)/(4*tan(x) + 4) + 2*log(tan(x) + 1)/(4*tan(x) + 4) - log(tan(x)**2 + 1)*tan(x)/(4*tan(x) + 4) - log(tan(x)**2 + 1)/(4*tan(x) + 4) - 2/(4*tan(x) + 4)`

**3.80.7 Maxima [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{(1 + \tan(x))^2} dx = -\frac{1}{2(\tan(x) + 1)} - \frac{1}{4} \log(\tan(x)^2 + 1) + \frac{1}{2} \log(\tan(x) + 1)$$

input `integrate(1/(1+tan(x))^2,x, algorithm="maxima")`

output `-1/2/(tan(x) + 1) - 1/4*log(tan(x)^2 + 1) + 1/2*log(tan(x) + 1)`

**3.80.8 Giac [A] (verification not implemented)**

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{1}{(1 + \tan(x))^2} dx = -\frac{1}{2(\tan(x) + 1)} - \frac{1}{4} \log(\tan(x)^2 + 1) + \frac{1}{2} \log(|\tan(x) + 1|)$$

input `integrate(1/(1+tan(x))^2,x, algorithm="giac")`

output `-1/2/(tan(x) + 1) - 1/4*log(tan(x)^2 + 1) + 1/2*log(abs(tan(x) + 1))`

**3.80.9 Mupad [B] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{1}{(1 + \tan(x))^2} dx = \frac{\ln(\tan(x) + 1)}{2} - \frac{\ln(\tan(x)^2 + 1)}{4} - \frac{1}{2(\tan(x) + 1)}$$

input `int(1/(tan(x) + 1)^2,x)`

output `log(tan(x) + 1)/2 - log(tan(x)^2 + 1)/4 - 1/(2*(tan(x) + 1))`

## 3.81 $\int \sec(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.81.1 | Optimal result . . . . .                            | 526 |
| 3.81.2 | Mathematica [A] (verified) . . . . .                | 526 |
| 3.81.3 | Rubi [A] (verified) . . . . .                       | 527 |
| 3.81.4 | Maple [A] (verified) . . . . .                      | 528 |
| 3.81.5 | Fricas [B] (verification not implemented) . . . . . | 528 |
| 3.81.6 | Sympy [B] (verification not implemented) . . . . .  | 528 |
| 3.81.7 | Maxima [B] (verification not implemented) . . . . . | 529 |
| 3.81.8 | Giac [B] (verification not implemented) . . . . .   | 529 |
| 3.81.9 | Mupad [B] (verification not implemented) . . . . .  | 529 |

### 3.81.1 Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \sec(x) dx = \operatorname{arctanh}(\sin(x))$$

output `arctanh(sin(x))`

### 3.81.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \sec(x) dx = \operatorname{arctanh}(\sin(x))$$

input `Integrate[Sec[x],x]`

output `ArcTanh[Sin[x]]`

### 3.81.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \sec(x) dx \\ \downarrow 3042 \\ \int \csc\left(x + \frac{\pi}{2}\right) dx \\ \downarrow 4257 \\ \operatorname{arctanh}(\sin(x)) \end{array}$$

input `Int[Sec[x], x]`

output `ArcTanh[Sin[x]]`

#### 3.81.3.1 Defintions of rubi rules used

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4257 `Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`



**3.81.4 Maple [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

| method       | result                                                     | size |
|--------------|------------------------------------------------------------|------|
| default      | $\ln(\sec(x) + \tan(x))$                                   | 7    |
| norman       | $-\ln(\tan(\frac{x}{2}) - 1) + \ln(1 + \tan(\frac{x}{2}))$ | 18   |
| parallelrisc | $-\ln(\tan(\frac{x}{2}) - 1) + \ln(1 + \tan(\frac{x}{2}))$ | 18   |
| risc         | $\ln(i + e^{ix}) - \ln(e^{ix} - i)$                        | 22   |

input `int(1/cos(x),x,method=_RETURNVERBOSE)`

output `ln(sec(x)+tan(x))`

**3.81.5 Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 17 vs.  $2(3) = 6$ .

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \sec(x) dx = \frac{1}{2} \log(\sin(x) + 1) - \frac{1}{2} \log(-\sin(x) + 1)$$

input `integrate(1/cos(x),x, algorithm="fricas")`

output `1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)`

**3.81.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 15 vs.  $2(3) = 6$ .

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \sec(x) dx = -\frac{\log(\sin(x) - 1)}{2} + \frac{\log(\sin(x) + 1)}{2}$$

input `integrate(1/cos(x),x)`

output `-log(sin(x) - 1)/2 + log(sin(x) + 1)/2`

**3.81.7 Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 15 vs.  $2(3) = 6$ .

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \sec(x) dx = \frac{1}{2} \log(\sin(x) + 1) - \frac{1}{2} \log(\sin(x) - 1)$$

input `integrate(1/cos(x),x, algorithm="maxima")`

output `1/2*log(sin(x) + 1) - 1/2*log(sin(x) - 1)`

**3.81.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 17 vs.  $2(3) = 6$ .

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \sec(x) dx = \frac{1}{2} \log(\sin(x) + 1) - \frac{1}{2} \log(-\sin(x) + 1)$$

input `integrate(1/cos(x),x, algorithm="giac")`

output `1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)`

**3.81.9 Mupad [B] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \sec(x) dx = \ln\left(\frac{1}{\cos(x)}\right) + \ln(\sin(x) + 1)$$

input `int(1/cos(x),x)`

output `log(1/cos(x)) + log(sin(x) + 1)`

## 3.82 $\int \csc(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.82.1 | Optimal result . . . . .                            | 530 |
| 3.82.2 | Mathematica [B] (verified) . . . . .                | 530 |
| 3.82.3 | Rubi [A] (verified) . . . . .                       | 531 |
| 3.82.4 | Maple [A] (verified) . . . . .                      | 532 |
| 3.82.5 | Fricas [B] (verification not implemented) . . . . . | 532 |
| 3.82.6 | Sympy [B] (verification not implemented) . . . . .  | 532 |
| 3.82.7 | Maxima [B] (verification not implemented) . . . . . | 533 |
| 3.82.8 | Giac [B] (verification not implemented) . . . . .   | 533 |
| 3.82.9 | Mupad [B] (verification not implemented) . . . . .  | 533 |

### 3.82.1 Optimal result

Integrand size = 2, antiderivative size = 5

$$\int \csc(x) dx = -\operatorname{arctanh}(\cos(x))$$

output `-arctanh(cos(x))`

### 3.82.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 17 vs.  $2(5) = 10$ .

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 3.40

$$\int \csc(x) dx = -\log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)$$

input `Integrate[Csc[x], x]`

output `-Log[Cos[x/2]] + Log[Sin[x/2]]`

### 3.82.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \csc(x) dx \\ \downarrow 3042 \\ \int \csc(x) dx \\ \downarrow 4257 \\ -\operatorname{arctanh}(\cos(x)) \end{array}$$

input `Int [Csc [x] , x]`

output `-ArcTanh [Cos [x]]`

#### 3.82.3.1 Defintions of rubi rules used

rule 3042 `Int [u_ , x_Symbol] :> Int [DeactivateTrig [u , x] , x] /; FunctionOfTrigOfLinear Q [u , x]`

rule 4257 `Int [csc [(c_.) + (d_.)*(x_)], x_Symbol] :> Simp [-ArcTanh [Cos [c + d*x]]/d , x] /; FreeQ [{c , d} , x]`

**3.82.4 Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

| method        | result                                         | size |
|---------------|------------------------------------------------|------|
| norman        | $\ln\left(\tan\left(\frac{x}{2}\right)\right)$ | 6    |
| parallelrisch | $\ln\left(\tan\left(\frac{x}{2}\right)\right)$ | 6    |
| default       | $\ln(\csc(x) - \cot(x))$                       | 9    |
| risch         | $\ln(e^{ix} - 1) - \ln(e^{ix} + 1)$            | 20   |

input `int(1/sin(x),x,method=_RETURNVERBOSE)`

output `ln(tan(1/2*x))`

**3.82.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 19 vs.  $2(5) = 10$ .

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 3.80

$$\int \csc(x) dx = -\frac{1}{2} \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + \frac{1}{2} \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right)$$

input `integrate(1/sin(x),x, algorithm="fricas")`

output `-1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)`

**3.82.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 15 vs.  $2(5) = 10$ .

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.00

$$\int \csc(x) dx = \frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2}$$

input `integrate(1/sin(x),x)`

output `log(cos(x) - 1)/2 - log(cos(x) + 1)/2`

**3.82.7 Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 15 vs.  $2(5) = 10$ .

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.00

$$\int \csc(x) dx = -\frac{1}{2} \log(\cos(x) + 1) + \frac{1}{2} \log(\cos(x) - 1)$$

input `integrate(1/sin(x),x, algorithm="maxima")`

output `-1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)`

**3.82.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 17 vs.  $2(5) = 10$ .

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 3.40

$$\int \csc(x) dx = -\frac{1}{2} \log(\cos(x) + 1) + \frac{1}{2} \log(-\cos(x) + 1)$$

input `integrate(1/sin(x),x, algorithm="giac")`

output `-1/2*log(cos(x) + 1) + 1/2*log(-cos(x) + 1)`

**3.82.9 Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \csc(x) dx = \ln\left(\tan\left(\frac{x}{2}\right)\right)$$

input `int(1/sin(x),x)`

output `log(tan(x/2))`

### 3.83 $\int \sin^2(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.83.1 | Optimal result . . . . .                            | 534 |
| 3.83.2 | Mathematica [A] (verified) . . . . .                | 534 |
| 3.83.3 | Rubi [A] (verified) . . . . .                       | 535 |
| 3.83.4 | Maple [A] (verified) . . . . .                      | 536 |
| 3.83.5 | Fricas [A] (verification not implemented) . . . . . | 536 |
| 3.83.6 | Sympy [A] (verification not implemented) . . . . .  | 536 |
| 3.83.7 | Maxima [A] (verification not implemented) . . . . . | 537 |
| 3.83.8 | Giac [A] (verification not implemented) . . . . .   | 537 |
| 3.83.9 | Mupad [B] (verification not implemented) . . . . .  | 537 |

#### 3.83.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \sin^2(x) dx = \frac{x}{2} - \frac{1}{2} \cos(x) \sin(x)$$

output `1/2*x-1/2*cos(x)*sin(x)`

#### 3.83.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sin^2(x) dx = \frac{x}{2} - \frac{1}{4} \sin(2x)$$

input `Integrate[Sin[x]^2,x]`

output `x/2 - Sin[2*x]/4`

### 3.83.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$ , Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c}
 \int \sin^2(x) dx \\
 \downarrow \text{3042} \\
 \int \sin(x)^2 dx \\
 \downarrow \text{3115} \\
 \frac{\int 1 dx}{2} - \frac{1}{2} \sin(x) \cos(x) \\
 \downarrow \text{24} \\
 \frac{x}{2} - \frac{1}{2} \sin(x) \cos(x)
 \end{array}$$

input `Int[Sin[x]^2,x]`

output `x/2 - (Cos[x]*Sin[x])/2`

#### 3.83.3.1 Defintions of rubi rules used

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3115 `Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sine[c + d*x])^(n - 1)/(d*n), x] + Simp[b^2*((n - 1)/n) Int[(b*Sine[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`



**3.83.4 Maple [A] (verified)**

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

| method        | result                                                                                                                                                                                                                                                | size |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default       | $\frac{x}{2} - \frac{\cos(x)\sin(x)}{2}$                                                                                                                                                                                                              | 11   |
| risch         | $\frac{x}{2} - \frac{\sin(2x)}{4}$                                                                                                                                                                                                                    | 11   |
| parallelrisch | $\frac{x}{2} - \frac{\sin(2x)}{4}$                                                                                                                                                                                                                    | 11   |
| meijerg       | $\frac{\sqrt{\pi} \left( \frac{2x}{\sqrt{\pi}} - \frac{\sin(2x)}{\sqrt{\pi}} \right)}{4}$                                                                                                                                                             | 22   |
| norman        | $\frac{\tan^3\left(\frac{x}{2}\right) + x \left( \tan^2\left(\frac{x}{2}\right) + \frac{x}{2} + \frac{x \left( \tan^4\left(\frac{x}{2}\right) \right)}{2} - \tan\left(\frac{x}{2}\right) \right)}{\left(1 + \tan^2\left(\frac{x}{2}\right)\right)^2}$ | 45   |

input `int(sin(x)^2,x,method=_RETURNVERBOSE)`output `1/2*x-1/2*cos(x)*sin(x)`**3.83.5 Fricas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x) dx = -\frac{1}{2} \cos(x) \sin(x) + \frac{1}{2} x$$

input `integrate(sin(x)^2,x, algorithm="fricas")`output `-1/2*cos(x)*sin(x) + 1/2*x`**3.83.6 Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x) dx = \frac{x}{2} - \frac{\sin(x) \cos(x)}{2}$$

input `integrate(sin(x)**2,x)`output `x/2 - sin(x)*cos(x)/2`

**3.83.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x) dx = \frac{1}{2}x - \frac{1}{4}\sin(2x)$$

input `integrate(sin(x)^2,x, algorithm="maxima")`output `1/2*x - 1/4*sin(2*x)`**3.83.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x) dx = \frac{1}{2}x - \frac{1}{4}\sin(2x)$$

input `integrate(sin(x)^2,x, algorithm="giac")`output `1/2*x - 1/4*sin(2*x)`**3.83.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x) dx = \frac{x}{2} - \frac{\sin(2x)}{4}$$

input `int(sin(x)^2,x)`output `x/2 - sin(2*x)/4`

## 3.84 $\int x^3 \sin(x^2) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.84.1 | Optimal result . . . . .                            | 538 |
| 3.84.2 | Mathematica [A] (verified) . . . . .                | 538 |
| 3.84.3 | Rubi [A] (verified) . . . . .                       | 539 |
| 3.84.4 | Maple [A] (verified) . . . . .                      | 540 |
| 3.84.5 | Fricas [A] (verification not implemented) . . . . . | 541 |
| 3.84.6 | Sympy [A] (verification not implemented) . . . . .  | 541 |
| 3.84.7 | Maxima [A] (verification not implemented) . . . . . | 541 |
| 3.84.8 | Giac [A] (verification not implemented) . . . . .   | 542 |
| 3.84.9 | Mupad [B] (verification not implemented) . . . . .  | 542 |

### 3.84.1 Optimal result

Integrand size = 8, antiderivative size = 20

$$\int x^3 \sin(x^2) dx = -\frac{1}{2}x^2 \cos(x^2) + \frac{\sin(x^2)}{2}$$

output `-1/2*x^2*cos(x^2)+1/2*sin(x^2)`

### 3.84.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^3 \sin(x^2) dx = -\frac{1}{2}x^2 \cos(x^2) + \frac{\sin(x^2)}{2}$$

input `Integrate[x^3*Sin[x^2],x]`

output `-1/2*(x^2*Cos[x^2]) + Sin[x^2]/2`

**3.84.3 Rubi [A] (verified)**

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {3860, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^3 \sin(x^2) dx \\
 & \quad \downarrow \text{3860} \\
 & \frac{1}{2} \int x^2 \sin(x^2) dx^2 \\
 & \quad \downarrow \text{3042} \\
 & \frac{1}{2} \int x^2 \sin(x^2) dx^2 \\
 & \quad \downarrow \text{3777} \\
 & \frac{1}{2} \left( \int \cos(x^2) dx^2 - x^2 \cos(x^2) \right) \\
 & \quad \downarrow \text{3042} \\
 & \frac{1}{2} \left( \int \sin\left(x^2 + \frac{\pi}{2}\right) dx^2 - x^2 \cos(x^2) \right) \\
 & \quad \downarrow \text{3117} \\
 & \frac{1}{2} (\sin(x^2) - x^2 \cos(x^2))
 \end{aligned}$$

input `Int[x^3*Sin[x^2],x]`

output `(-x^2*Cos[x^2]) + Sin[x^2])/2`

## 3.84.3.1 Defintions of rubi rules used

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3117 `Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

rule 3777 `Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

rule 3860 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*SIN[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))`

## 3.84.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

| method           | result                                                                                                                                                                                                                                                               | size |
|------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| derivativdivides | $-\frac{x^2 \cos(x^2)}{2} + \frac{\sin(x^2)}{2}$                                                                                                                                                                                                                     | 17   |
| default          | $-\frac{x^2 \cos(x^2)}{2} + \frac{\sin(x^2)}{2}$                                                                                                                                                                                                                     | 17   |
| risch            | $-\frac{x^2 \cos(x^2)}{2} + \frac{\sin(x^2)}{2}$                                                                                                                                                                                                                     | 17   |
| parallelrisc     | $-\frac{x^2 \cos(x^2)}{2} + \frac{\sin(x^2)}{2}$                                                                                                                                                                                                                     | 17   |
| meijerg          | $\sqrt{\pi} \left( -\frac{x^2 \cos(x^2)}{2\sqrt{\pi}} + \frac{\sin(x^2)}{2\sqrt{\pi}} \right)$                                                                                                                                                                       | 27   |
| norman           | $\frac{-\frac{x^2}{2} + \frac{x^2 \left( \tan^2\left(\frac{x^2}{2}\right) \right)}{2} + \tan\left(\frac{x^2}{2}\right)}{1 + \tan^2\left(\frac{x^2}{2}\right)}$                                                                                                       | 39   |
| parts            | $\frac{\sqrt{2} \sqrt{\pi} S\left(\frac{\sqrt{2} x}{\sqrt{\pi}}\right) x^3}{2} - \frac{3\pi^2 \left( \frac{2 S\left(\frac{\sqrt{2} x}{\sqrt{\pi}}\right) \sqrt{2} x^3}{3\pi^{\frac{3}{2}}} + \frac{2x^2 \cos(x^2)}{3\pi^2} - \frac{2 \sin(x^2)}{3\pi^2} \right)}{4}$ | 69   |

input `int(x^3*sin(x^2),x,method=_RETURNVERBOSE)`

output `-1/2*x^2*cos(x^2)+1/2*sin(x^2)`

### 3.84.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin(x^2) dx = -\frac{1}{2} x^2 \cos(x^2) + \frac{1}{2} \sin(x^2)$$

input `integrate(x^3*sin(x^2),x, algorithm="fricas")`

output `-1/2*x^2*cos(x^2) + 1/2*sin(x^2)`

### 3.84.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int x^3 \sin(x^2) dx = -\frac{x^2 \cos(x^2)}{2} + \frac{\sin(x^2)}{2}$$

input `integrate(x**3*sin(x**2),x)`

output `-x**2*cos(x**2)/2 + sin(x**2)/2`

### 3.84.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin(x^2) dx = -\frac{1}{2} x^2 \cos(x^2) + \frac{1}{2} \sin(x^2)$$

input `integrate(x^3*sin(x^2),x, algorithm="maxima")`

output `-1/2*x^2*cos(x^2) + 1/2*sin(x^2)`

**3.84.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin(x^2) dx = -\frac{1}{2} x^2 \cos(x^2) + \frac{1}{2} \sin(x^2)$$

input `integrate(x^3*sin(x^2),x, algorithm="giac")`

output `-1/2*x^2*cos(x^2) + 1/2*sin(x^2)`

**3.84.9 Mupad [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin(x^2) dx = \frac{\sin(x^2)}{2} - \frac{x^2 \cos(x^2)}{2}$$

input `int(x^3*sin(x^2),x)`

output `sin(x^2)/2 - (x^2*cos(x^2))/2`

### 3.85 $\int \sin^3(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.85.1 | Optimal result . . . . .                            | 543 |
| 3.85.2 | Mathematica [A] (verified) . . . . .                | 543 |
| 3.85.3 | Rubi [A] (verified) . . . . .                       | 544 |
| 3.85.4 | Maple [A] (verified) . . . . .                      | 545 |
| 3.85.5 | Fricas [A] (verification not implemented) . . . . . | 545 |
| 3.85.6 | Sympy [A] (verification not implemented) . . . . .  | 545 |
| 3.85.7 | Maxima [A] (verification not implemented) . . . . . | 546 |
| 3.85.8 | Giac [A] (verification not implemented) . . . . .   | 546 |
| 3.85.9 | Mupad [B] (verification not implemented) . . . . .  | 546 |

#### 3.85.1 Optimal result

Integrand size = 4, antiderivative size = 13

$$\int \sin^3(x) dx = -\cos(x) + \frac{\cos^3(x)}{3}$$

output `-cos(x)+1/3*cos(x)^3`

#### 3.85.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sin^3(x) dx = -\frac{3 \cos(x)}{4} + \frac{1}{12} \cos(3x)$$

input `Integrate[Sin[x]^3,x]`

output `(-3*Cos[x])/4 + Cos[3*x]/12`



### 3.85.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$ , Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c}
 \int \sin^3(x) dx \\
 \downarrow \text{3042} \\
 \int \sin(x)^3 dx \\
 \downarrow \text{3113} \\
 - \int (1 - \cos^2(x)) d \cos(x) \\
 \downarrow \text{2009} \\
 \frac{\cos^3(x)}{3} - \cos(x)
 \end{array}$$

input `Int[Sin[x]^3,x]`

output `-Cos[x] + Cos[x]^3/3`

#### 3.85.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3113 `Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[-d^(-1) Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

**3.85.4 Maple [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

| method        | result                                                                    | size |
|---------------|---------------------------------------------------------------------------|------|
| default       | $-\frac{(2+\sin^2(x))\cos(x)}{3}$                                         | 11   |
| risch         | $-\frac{3\cos(x)}{4} + \frac{\cos(3x)}{12}$                               | 12   |
| parallelrisch | $-\frac{2}{3} - \frac{3\cos(x)}{4} + \frac{\cos(3x)}{12}$                 | 13   |
| norman        | $\frac{-4(\tan^2(\frac{x}{2})) - \frac{4}{3}}{(1+\tan^2(\frac{x}{2}))^3}$ | 22   |

input `int(sin(x)^3,x,method=_RETURNVERBOSE)`output `-1/3*(2+sin(x)^2)*cos(x)`**3.85.5 Fricas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x) dx = \frac{1}{3} \cos^3(x) - \cos(x)$$

input `integrate(sin(x)^3,x, algorithm="fricas")`output `1/3*cos(x)^3 - cos(x)`**3.85.6 Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \sin^3(x) dx = \frac{\cos^3(x)}{3} - \cos(x)$$

input `integrate(sin(x)**3,x)`output `cos(x)**3/3 - cos(x)`

**3.85.7 Maxima [A] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x) dx = \frac{1}{3} \cos(x)^3 - \cos(x)$$

input `integrate(sin(x)^3,x, algorithm="maxima")`output `1/3*cos(x)^3 - cos(x)`**3.85.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sin^3(x) dx = \frac{1}{3} \cos(x)^3 - \cos(x)$$

input `integrate(sin(x)^3,x, algorithm="giac")`output `1/3*cos(x)^3 - cos(x)`**3.85.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \sin^3(x) dx = \frac{\cos(x) (\cos(x)^2 - 3)}{3}$$

input `int(sin(x)^3,x)`output `(cos(x)*(cos(x)^2 - 3))/3`

### 3.86 $\int \sin^p(x) dx$

|        |                                          |     |
|--------|------------------------------------------|-----|
| 3.86.1 | Optimal result                           | 547 |
| 3.86.2 | Mathematica [A] (verified)               | 547 |
| 3.86.3 | Rubi [A] (verified)                      | 548 |
| 3.86.4 | Maple [F]                                | 549 |
| 3.86.5 | Fricas [F]                               | 549 |
| 3.86.6 | Sympy [F]                                | 549 |
| 3.86.7 | Maxima [F]                               | 550 |
| 3.86.8 | Giac [F]                                 | 550 |
| 3.86.9 | Mupad [B] (verification not implemented) | 550 |

#### 3.86.1 Optimal result

Integrand size = 4, antiderivative size = 44

$$\int \sin^p(x) dx = \frac{\cos(x) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \sin^2(x)\right) \sin^{1+p}(x)}{(1+p)\sqrt{\cos^2(x)}}$$

output `cos(x)*hypergeom([1/2, 1/2+1/2*p], [3/2+1/2*p], sin(x)^2)*sin(x)^(p+1)/(p+1) / (cos(x)^2)^(1/2)`

#### 3.86.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \sin^p(x) dx = \frac{\sqrt{\cos^2(x)} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \sin^2(x)\right) \sec(x) \sin^{1+p}(x)}{1+p}$$

input `Integrate[Sin[x]^p,x]`

output `(Sqrt[Cos[x]^2]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[x]^2]*Sec[x]*Sin[x]^(1 + p))/(1 + p)`

### 3.86.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^p(x) dx$$

↓ 3042

$$\int \sin(x)^p dx$$

↓ 3122

$$\frac{\cos(x) \sin^{p+1}(x) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{p+1}{2}, \frac{p+3}{2}, \sin^2(x)\right)}{(p+1)\sqrt{\cos^2(x)}}$$

input `Int[Sin[x]^p,x]`

output `(Cos[x]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[x]^2]*Sin[x]^(1 + p))/((1 + p)*Sqrt[Cos[x]^2])`

#### 3.86.3.1 Defintions of rubi rules used

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3122 `Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]`

**3.86.4 Maple [F]**

$$\int (\sin^p(x)) dx$$

input `int(sin(x)^p,x)`

output `int(sin(x)^p,x)`

**3.86.5 Fricas [F]**

$$\int \sin^p(x) dx = \int \sin(x)^p dx$$

input `integrate(sin(x)^p,x, algorithm="fricas")`

output `integral(sin(x)^p, x)`

**3.86.6 Sympy [F]**

$$\int \sin^p(x) dx = \int \sin^p(x) dx$$

input `integrate(sin(x)**p,x)`

output `Integral(sin(x)**p, x)`

**3.86.7 Maxima [F]**

$$\int \sin^p(x) dx = \int \sin(x)^p dx$$

input `integrate(sin(x)^p,x, algorithm="maxima")`

output `integrate(sin(x)^p, x)`

**3.86.8 Giac [F]**

$$\int \sin^p(x) dx = \int \sin(x)^p dx$$

input `integrate(sin(x)^p,x, algorithm="giac")`

output `integrate(sin(x)^p, x)`

**3.86.9 Mupad [B] (verification not implemented)**

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \sin^p(x) dx = -\frac{\cos(x) \sin(x)^{p+1} {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - \frac{p}{2}; \frac{3}{2}; \cos(x)^2\right)}{(\sin(x)^2)^{\frac{p}{2} + \frac{1}{2}}}$$

input `int(sin(x)^p,x)`

output `-(cos(x)*sin(x)^(p + 1)*hypergeom([1/2, 1/2 - p/2], 3/2, cos(x)^2))/(sin(x)^2)^(p/2 + 1/2)`

### 3.87 $\int \cos(x) (1 + \sin^2(x))^2 dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.87.1 | Optimal result . . . . .                            | 551 |
| 3.87.2 | Mathematica [A] (verified) . . . . .                | 551 |
| 3.87.3 | Rubi [A] (verified) . . . . .                       | 552 |
| 3.87.4 | Maple [A] (verified) . . . . .                      | 553 |
| 3.87.5 | Fricas [A] (verification not implemented) . . . . . | 553 |
| 3.87.6 | Sympy [A] (verification not implemented) . . . . .  | 554 |
| 3.87.7 | Maxima [A] (verification not implemented) . . . . . | 554 |
| 3.87.8 | Giac [A] (verification not implemented) . . . . .   | 554 |
| 3.87.9 | Mupad [B] (verification not implemented) . . . . .  | 555 |

#### 3.87.1 Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \cos(x) (1 + \sin^2(x))^2 dx = \sin(x) + \frac{2 \sin^3(x)}{3} + \frac{\sin^5(x)}{5}$$

output `sin(x)+2/3*sin(x)^3+1/5*sin(x)^5`

#### 3.87.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos(x) (1 + \sin^2(x))^2 dx = \sin(x) + \frac{2 \sin^3(x)}{3} + \frac{\sin^5(x)}{5}$$

input `Integrate[Cos[x]*(1 + Sin[x]^2)^2,x]`

output `Sin[x] + (2*Sin[x]^3)/3 + Sin[x]^5/5`



### 3.87.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {3042, 3669, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (\sin^2(x) + 1)^2 \cos(x) dx \\
 & \quad \downarrow \text{3042} \\
 & \int (\sin(x)^2 + 1)^2 \cos(x) dx \\
 & \quad \downarrow \text{3669} \\
 & \int (\sin^2(x) + 1)^2 d\sin(x) \\
 & \quad \downarrow \text{210} \\
 & \int (\sin^4(x) + 2\sin^2(x) + 1) d\sin(x) \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sin^5(x)}{5} + \frac{2\sin^3(x)}{3} + \sin(x)
 \end{aligned}$$

input `Int[Cos[x]*(1 + Sin[x]^2)^2,x]`

output `Sin[x] + (2*Sin[x]^3)/3 + Sin[x]^5/5`

#### 3.87.3.1 Defintions of rubi rules used

rule 210 `Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^(p, x), x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]`

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

```
rule 3042 Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

```
rule 3669 Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

### 3.87.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

| method            | result                                                                | size |
|-------------------|-----------------------------------------------------------------------|------|
| derivativedivides | $\sin(x) + \frac{2(\sin^3(x))}{3} + \frac{(\sin^5(x))}{5}$            | 16   |
| default           | $\sin(x) + \frac{2(\sin^3(x))}{3} + \frac{(\sin^5(x))}{5}$            | 16   |
| risch             | $\frac{13 \sin(x)}{8} + \frac{\sin(5x)}{80} - \frac{11 \sin(3x)}{48}$ | 18   |
| parallelrisc      | $\frac{13 \sin(x)}{8} + \frac{\sin(5x)}{80} - \frac{11 \sin(3x)}{48}$ | 18   |

```
input int(cos(x)*(1+sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
output sin(x)+2/3*sin(x)^3+1/5*sin(x)^5
```

### 3.87.5 Fracas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \cos(x) (1 + \sin^2(x))^2 dx = \frac{1}{15} (3 \cos(x)^4 - 16 \cos(x)^2 + 28) \sin(x)$$

```
input integrate(cos(x)*(1+sin(x)^2)^2,x, algorithm="fricas")
```

```
output 1/15*(3*cos(x)^4 - 16*cos(x)^2 + 28)*sin(x)
```

**3.87.6 Sympy [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \cos(x) (1 + \sin^2(x))^2 dx = \frac{\sin^5(x)}{5} + \frac{2 \sin^3(x)}{3} + \sin(x)$$

input `integrate(cos(x)*(1+sin(x)**2)**2,x)`output `sin(x)**5/5 + 2*sin(x)**3/3 + sin(x)`**3.87.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cos(x) (1 + \sin^2(x))^2 dx = \frac{1}{5} \sin(x)^5 + \frac{2}{3} \sin(x)^3 + \sin(x)$$

input `integrate(cos(x)*(1+sin(x)^2)^2,x, algorithm="maxima")`output `1/5*sin(x)^5 + 2/3*sin(x)^3 + sin(x)`**3.87.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cos(x) (1 + \sin^2(x))^2 dx = \frac{1}{5} \sin(x)^5 + \frac{2}{3} \sin(x)^3 + \sin(x)$$

input `integrate(cos(x)*(1+sin(x)^2)^2,x, algorithm="giac")`output `1/5*sin(x)^5 + 2/3*sin(x)^3 + sin(x)`

**3.87.9 Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \cos(x) (1 + \sin^2(x))^2 dx = \frac{\sin(x)^5}{5} + \frac{2 \sin(x)^3}{3} + \sin(x)$$

input `int(cos(x)*(sin(x)^2 + 1)^2,x)`

output `sin(x) + (2*sin(x)^3)/3 + sin(x)^5/5`

## 3.88 $\int \cos^2(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.88.1 | Optimal result . . . . .                            | 556 |
| 3.88.2 | Mathematica [A] (verified) . . . . .                | 556 |
| 3.88.3 | Rubi [A] (verified) . . . . .                       | 557 |
| 3.88.4 | Maple [A] (verified) . . . . .                      | 558 |
| 3.88.5 | Fricas [A] (verification not implemented) . . . . . | 558 |
| 3.88.6 | Sympy [A] (verification not implemented) . . . . .  | 558 |
| 3.88.7 | Maxima [A] (verification not implemented) . . . . . | 559 |
| 3.88.8 | Giac [A] (verification not implemented) . . . . .   | 559 |
| 3.88.9 | Mupad [B] (verification not implemented) . . . . .  | 559 |

### 3.88.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cos^2(x) dx = \frac{x}{2} + \frac{1}{2} \cos(x) \sin(x)$$

output `1/2*x+1/2*cos(x)*sin(x)`

### 3.88.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(x) dx = \frac{x}{2} + \frac{1}{4} \sin(2x)$$

input `Integrate[Cos[x]^2,x]`

output `x/2 + Sin[2*x]/4`

### 3.88.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$ , Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c}
 \int \cos^2(x) dx \\
 \downarrow \text{3042} \\
 \int \sin\left(x + \frac{\pi}{2}\right)^2 dx \\
 \downarrow \text{3115} \\
 \frac{\int 1 dx}{2} + \frac{1}{2} \sin(x) \cos(x) \\
 \downarrow \text{24} \\
 \frac{x}{2} + \frac{1}{2} \sin(x) \cos(x)
 \end{array}$$

input `Int[Cos[x]^2,x]`

output `x/2 + (Cos[x]*Sin[x])/2`

#### 3.88.3.1 Defintions of rubi rules used

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3115 `Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sine[c + d*x])^(n - 1)/(d*n), x] + Simp[b^2*((n - 1)/n) Int[(b*Sine[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

**3.88.4 Maple [A] (verified)**

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

| method        | result                                                                                                                                                  | size |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default       | $\frac{x}{2} + \frac{\cos(x) \sin(x)}{2}$                                                                                                               | 11   |
| risch         | $\frac{x}{2} + \frac{\sin(2x)}{4}$                                                                                                                      | 11   |
| parallelrisch | $\frac{x}{2} + \frac{\sin(2x)}{4}$                                                                                                                      | 11   |
| norman        | $\frac{x(\tan^2(\frac{x}{2})) + \frac{x}{2} - (\tan^3(\frac{x}{2})) + \frac{x(\tan^4(\frac{x}{2}))}{2} + \tan(\frac{x}{2})}{(1+\tan^2(\frac{x}{2}))^2}$ | 45   |

input `int(cos(x)^2,x,method=_RETURNVERBOSE)`output `1/2*x+1/2*cos(x)*sin(x)`**3.88.5 Fricas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x) dx = \frac{1}{2} \cos(x) \sin(x) + \frac{1}{2} x$$

input `integrate(cos(x)^2,x, algorithm="fricas")`output `1/2*cos(x)*sin(x) + 1/2*x`**3.88.6 Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x) dx = \frac{x}{2} + \frac{\sin(x) \cos(x)}{2}$$

input `integrate(cos(x)**2,x)`output `x/2 + sin(x)*cos(x)/2`

**3.88.7 Maxima [A] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x) dx = \frac{1}{2}x + \frac{1}{4}\sin(2x)$$

input `integrate(cos(x)^2,x, algorithm="maxima")`output `1/2*x + 1/4*sin(2*x)`**3.88.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x) dx = \frac{1}{2}x + \frac{1}{4}\sin(2x)$$

input `integrate(cos(x)^2,x, algorithm="giac")`output `1/2*x + 1/4*sin(2*x)`**3.88.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x) dx = \frac{x}{2} + \frac{\sin(2x)}{4}$$

input `int(cos(x)^2,x)`output `x/2 + sin(2*x)/4`



### 3.89 $\int \cos^3(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.89.1 | Optimal result . . . . .                            | 560 |
| 3.89.2 | Mathematica [A] (verified) . . . . .                | 560 |
| 3.89.3 | Rubi [A] (verified) . . . . .                       | 561 |
| 3.89.4 | Maple [A] (verified) . . . . .                      | 562 |
| 3.89.5 | Fricas [A] (verification not implemented) . . . . . | 562 |
| 3.89.6 | Sympy [A] (verification not implemented) . . . . .  | 562 |
| 3.89.7 | Maxima [A] (verification not implemented) . . . . . | 563 |
| 3.89.8 | Giac [A] (verification not implemented) . . . . .   | 563 |
| 3.89.9 | Mupad [B] (verification not implemented) . . . . .  | 563 |

#### 3.89.1 Optimal result

Integrand size = 4, antiderivative size = 11

$$\int \cos^3(x) dx = \sin(x) - \frac{\sin^3(x)}{3}$$

output `sin(x)-1/3*sin(x)^3`

#### 3.89.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos^3(x) dx = \sin(x) - \frac{\sin^3(x)}{3}$$

input `Integrate[Cos[x]^3,x]`

output `Sin[x] - Sin[x]^3/3`

**3.89.3 Rubi [A] (verified)**

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$ , Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \cos^3(x) dx \\ & \quad \downarrow \text{3042} \\ & \int \sin\left(x + \frac{\pi}{2}\right)^3 dx \\ & \quad \downarrow \text{3113} \\ & - \int (1 - \sin^2(x)) d(-\sin(x)) \\ & \quad \downarrow \text{2009} \\ & \sin(x) - \frac{\sin^3(x)}{3} \end{aligned}$$

input `Int[Cos[x]^3,x]`

output `Sin[x] - Sin[x]^3/3`

**3.89.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3113 `Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[-d^(-1) Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

**3.89.4 Maple [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

| method        | result                                      | size |
|---------------|---------------------------------------------|------|
| default       | $\frac{(2+\cos^2(x)) \sin(x)}{3}$           | 11   |
| risch         | $\frac{3 \sin(x)}{4} + \frac{\sin(3x)}{12}$ | 12   |
| parallelrisch | $\frac{3 \sin(x)}{4} + \frac{\sin(3x)}{12}$ | 12   |

input `int(cos(x)^3,x,method=_RETURNVERBOSE)`output `1/3*(2+cos(x)^2)*sin(x)`**3.89.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \cos^3(x) dx = \frac{1}{3} (\cos(x)^2 + 2) \sin(x)$$

input `integrate(cos(x)^3,x, algorithm="fricas")`output `1/3*(cos(x)^2 + 2)*sin(x)`**3.89.6 Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \cos^3(x) dx = -\frac{\sin^3(x)}{3} + \sin(x)$$

input `integrate(cos(x)**3,x)`output `-sin(x)**3/3 + sin(x)`

**3.89.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x) dx = -\frac{1}{3} \sin(x)^3 + \sin(x)$$

input `integrate(cos(x)^3,x, algorithm="maxima")`output `-1/3*sin(x)^3 + sin(x)`**3.89.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x) dx = -\frac{1}{3} \sin(x)^3 + \sin(x)$$

input `integrate(cos(x)^3,x, algorithm="giac")`output `-1/3*sin(x)^3 + sin(x)`**3.89.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos^3(x) dx = \sin(x) - \frac{\sin(x)^3}{3}$$

input `int(cos(x)^3,x)`output `sin(x) - sin(x)^3/3`

## 3.90 $\int \sec^2(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.90.1 | Optimal result . . . . .                            | 564 |
| 3.90.2 | Mathematica [A] (verified) . . . . .                | 564 |
| 3.90.3 | Rubi [A] (verified) . . . . .                       | 565 |
| 3.90.4 | Maple [A] (verified) . . . . .                      | 566 |
| 3.90.5 | Fricas [B] (verification not implemented) . . . . . | 566 |
| 3.90.6 | Sympy [B] (verification not implemented) . . . . .  | 566 |
| 3.90.7 | Maxima [A] (verification not implemented) . . . . . | 567 |
| 3.90.8 | Giac [A] (verification not implemented) . . . . .   | 567 |
| 3.90.9 | Mupad [B] (verification not implemented) . . . . .  | 567 |

### 3.90.1 Optimal result

Integrand size = 4, antiderivative size = 2

$$\int \sec^2(x) dx = \tan(x)$$

output `tan(x)`

### 3.90.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec^2(x) dx = \tan(x)$$

input `Integrate[Sec[x]^2,x]`

output `Tan[x]`

### 3.90.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$ , Rules used = {3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c}
 \int \sec^2(x) dx \\
 \downarrow 3042 \\
 \int \csc\left(x + \frac{\pi}{2}\right)^2 dx \\
 \downarrow 4254 \\
 - \int 1d(-\tan(x)) \\
 \downarrow 24 \\
 \tan(x)
 \end{array}$$

input `Int[Sec[x]^2,x]`

output `Tan[x]`

#### 3.90.3.1 Defintions of rubi rules used

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4254 `Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[-d^(-1) Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

**3.90.4 Maple [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

| method        | result                                               | size |
|---------------|------------------------------------------------------|------|
| default       | $\tan(x)$                                            | 3    |
| parallelrisch | $\tan(x)$                                            | 3    |
| risch         | $\frac{2i}{e^{2ix}+1}$                               | 13   |
| norman        | $-\frac{2 \tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1}$ | 17   |

input `int(1/cos(x)^2,x,method=_RETURNVERBOSE)`

output `tan(x)`

**3.90.5 Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 7 vs.  $2(2) = 4$ .

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 3.50

$$\int \sec^2(x) dx = \frac{\sin(x)}{\cos(x)}$$

input `integrate(1/cos(x)^2,x, algorithm="fricas")`

output `sin(x)/cos(x)`

**3.90.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 5 vs.  $2(2) = 4$ .

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 2.50

$$\int \sec^2(x) dx = \frac{\sin(x)}{\cos(x)}$$

input `integrate(1/cos(x)**2,x)`

output `sin(x)/cos(x)`

**3.90.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec^2(x) dx = \tan(x)$$

input `integrate(1/cos(x)^2,x, algorithm="maxima")`

output `tan(x)`

**3.90.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec^2(x) dx = \tan(x)$$

input `integrate(1/cos(x)^2,x, algorithm="giac")`

output `tan(x)`

**3.90.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \sec^2(x) dx = \tan(x)$$

input `int(1/cos(x)^2,x)`

output `tan(x)`



## 3.91 $\int \sin(x) \sin(2x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.91.1 | Optimal result . . . . .                            | 568 |
| 3.91.2 | Mathematica [A] (verified) . . . . .                | 568 |
| 3.91.3 | Rubi [A] (verified) . . . . .                       | 569 |
| 3.91.4 | Maple [A] (verified) . . . . .                      | 570 |
| 3.91.5 | Fricas [A] (verification not implemented) . . . . . | 570 |
| 3.91.6 | Sympy [A] (verification not implemented) . . . . .  | 570 |
| 3.91.7 | Maxima [A] (verification not implemented) . . . . . | 571 |
| 3.91.8 | Giac [A] (verification not implemented) . . . . .   | 571 |
| 3.91.9 | Mupad [B] (verification not implemented) . . . . .  | 571 |

### 3.91.1 Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \sin(x) \sin(2x) dx = \frac{\sin(x)}{2} - \frac{1}{6} \sin(3x)$$

output `1/2*sin(x)-1/6*sin(3*x)`

### 3.91.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sin(x) \sin(2x) dx = \frac{\sin(x)}{2} - \frac{1}{6} \sin(3x)$$

input `Integrate[Sin[x]*Sin[2*x],x]`

output `Sin[x]/2 - Sin[3*x]/6`

### 3.91.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3042, 4770}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \sin(x) \sin(2x) dx \\ \downarrow \text{3042} \\ \int \sin(x) \sin(2x) dx \\ \downarrow \text{4770} \\ \frac{\sin(x)}{2} - \frac{1}{6} \sin(3x) \end{array}$$

input `Int[Sin[x]*Sin[2*x],x]`

output `Sin[x]/2 - Sin[3*x]/6`

#### 3.91.3.1 Defintions of rubi rules used

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 4770 `Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]`

**3.91.4 Maple [A] (verified)**

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

| method        | result                                                                                                                                                                                                                                                                               | size |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default       | $\frac{\sin(x)}{2} - \frac{\sin(3x)}{6}$                                                                                                                                                                                                                                             | 12   |
| risch         | $\frac{\sin(x)}{2} - \frac{\sin(3x)}{6}$                                                                                                                                                                                                                                             | 12   |
| parallelrisch | $\frac{\sin(x)}{2} - \frac{\sin(3x)}{6}$                                                                                                                                                                                                                                             | 12   |
| norman        | $-\frac{2 \tan(x) \left( \tan^2\left(\frac{x}{2}\right) \right)}{3} + \frac{4 \left( \tan^2(x) \right) \tan\left(\frac{x}{2}\right)}{3} + \frac{2 \tan(x)}{3} - \frac{4 \tan\left(\frac{x}{2}\right)}{3}$<br>$\frac{\hspace{10em}}{(1+\tan^2\left(\frac{x}{2}\right))(1+\tan^2(x))}$ | 51   |

input `int(sin(x)*sin(2*x),x,method=_RETURNVERBOSE)`output `1/2*sin(x)-1/6*sin(3*x)`**3.91.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \sin(x) \sin(2x) dx = -\frac{2}{3} (\cos(x)^2 - 1) \sin(x)$$

input `integrate(sin(x)*sin(2*x),x, algorithm="fricas")`output `-2/3*(cos(x)^2 - 1)*sin(x)`**3.91.6 Sympy [A] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \sin(x) \sin(2x) dx = -\frac{2 \sin(x) \cos(2x)}{3} + \frac{\sin(2x) \cos(x)}{3}$$

input `integrate(sin(x)*sin(2*x),x)`output `-2*sin(x)*cos(2*x)/3 + sin(2*x)*cos(x)/3`

**3.91.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \sin(x) \sin(2x) dx = -\frac{1}{6} \sin(3x) + \frac{1}{2} \sin(x)$$

input `integrate(sin(x)*sin(2*x),x, algorithm="maxima")`output `-1/6*sin(3*x) + 1/2*sin(x)`**3.91.8 Giac [A] (verification not implemented)**

Time = 0.30 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \sin(x) \sin(2x) dx = \frac{2}{3} \sin(x)^3$$

input `integrate(sin(x)*sin(2*x),x, algorithm="giac")`output `2/3*sin(x)^3`**3.91.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \sin(x) \sin(2x) dx = \frac{2 \sin(x)^3}{3}$$

input `int(sin(2*x)*sin(x),x)`output `(2*sin(x)^3)/3`

## 3.92 $\int x \sin(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.92.1 | Optimal result . . . . .                            | 572 |
| 3.92.2 | Mathematica [A] (verified) . . . . .                | 572 |
| 3.92.3 | Rubi [A] (verified) . . . . .                       | 573 |
| 3.92.4 | Maple [A] (verified) . . . . .                      | 574 |
| 3.92.5 | Fricas [A] (verification not implemented) . . . . . | 574 |
| 3.92.6 | Sympy [A] (verification not implemented) . . . . .  | 575 |
| 3.92.7 | Maxima [A] (verification not implemented) . . . . . | 575 |
| 3.92.8 | Giac [A] (verification not implemented) . . . . .   | 575 |
| 3.92.9 | Mupad [B] (verification not implemented) . . . . .  | 576 |

### 3.92.1 Optimal result

Integrand size = 4, antiderivative size = 8

$$\int x \sin(x) dx = -x \cos(x) + \sin(x)$$

output `-x*cos(x)+sin(x)`

### 3.92.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x \sin(x) dx = -x \cos(x) + \sin(x)$$

input `Integrate[x*Sin[x],x]`

output `-(x*Cos[x]) + Sin[x]`

### 3.92.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c}
 \int x \sin(x) dx \\
 \downarrow \text{3042} \\
 \int x \sin(x) dx \\
 \downarrow \text{3777} \\
 \int \cos(x) dx - x \cos(x) \\
 \downarrow \text{3042} \\
 \int \sin\left(x + \frac{\pi}{2}\right) dx - x \cos(x) \\
 \downarrow \text{3117} \\
 \sin(x) - x \cos(x)
 \end{array}$$

input `Int[x*Sin[x],x]`

output `-(x*Cos[x]) + Sin[x]`

#### 3.92.3.1 Defintions of rubi rules used

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3117 `Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

```
rule 3777 Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(-
(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### 3.92.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

| method        | result                                                                                    | size |
|---------------|-------------------------------------------------------------------------------------------|------|
| default       | $-x \cos(x) + \sin(x)$                                                                    | 9    |
| risch         | $-x \cos(x) + \sin(x)$                                                                    | 9    |
| parallelrisch | $-x \cos(x) + \sin(x)$                                                                    | 9    |
| parts         | $-x \cos(x) + \sin(x)$                                                                    | 9    |
| meijerg       | $2\sqrt{\pi} \left( -\frac{x \cos(x)}{2\sqrt{\pi}} + \frac{\sin(x)}{2\sqrt{\pi}} \right)$ | 22   |
| norman        | $\frac{x(\tan^2(\frac{x}{2}) - x + 2 \tan(\frac{x}{2}))}{1 + \tan^2(\frac{x}{2})}$        | 30   |

```
input int(x*sin(x),x,method=_RETURNVERBOSE)
```

```
output -x*cos(x)+sin(x)
```

### 3.92.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x \sin(x) dx = -x \cos(x) + \sin(x)$$

```
input integrate(x*sin(x),x, algorithm="fricas")
```

```
output -x*cos(x) + sin(x)
```

**3.92.6 Sympy [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int x \sin(x) dx = -x \cos(x) + \sin(x)$$

input `integrate(x*sin(x),x)`

output `-x*cos(x) + sin(x)`

**3.92.7 Maxima [A] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x \sin(x) dx = -x \cos(x) + \sin(x)$$

input `integrate(x*sin(x),x, algorithm="maxima")`

output `-x*cos(x) + sin(x)`

**3.92.8 Giac [A] (verification not implemented)**

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x \sin(x) dx = -x \cos(x) + \sin(x)$$

input `integrate(x*sin(x),x, algorithm="giac")`

output `-x*cos(x) + sin(x)`



**3.92.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x \sin(x) dx = \sin(x) - x \cos(x)$$

input `int(x*sin(x),x)`

output `sin(x) - x*cos(x)`

### 3.93 $\int x^2 \sin(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.93.1 | Optimal result . . . . .                            | 577 |
| 3.93.2 | Mathematica [A] (verified) . . . . .                | 577 |
| 3.93.3 | Rubi [A] (verified) . . . . .                       | 578 |
| 3.93.4 | Maple [A] (verified) . . . . .                      | 579 |
| 3.93.5 | Fricas [A] (verification not implemented) . . . . . | 580 |
| 3.93.6 | Sympy [A] (verification not implemented) . . . . .  | 580 |
| 3.93.7 | Maxima [A] (verification not implemented) . . . . . | 580 |
| 3.93.8 | Giac [A] (verification not implemented) . . . . .   | 581 |
| 3.93.9 | Mupad [B] (verification not implemented) . . . . .  | 581 |

#### 3.93.1 Optimal result

Integrand size = 6, antiderivative size = 17

$$\int x^2 \sin(x) dx = 2 \cos(x) - x^2 \cos(x) + 2x \sin(x)$$

output `2*cos(x)-x^2*cos(x)+2*x*sin(x)`

#### 3.93.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x) dx = -((-2 + x^2) \cos(x)) + 2x \sin(x)$$

input `Integrate[x^2*Sin[x],x]`

output `-((-2 + x^2)*Cos[x]) + 2*x*Sin[x]`

**3.93.3 Rubi [A] (verified)**

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$ , Rules used = {3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^2 \sin(x) dx \\
 & \quad \downarrow \text{3042} \\
 & \int x^2 \sin(x) dx \\
 & \quad \downarrow \text{3777} \\
 & 2 \int x \cos(x) dx - x^2 \cos(x) \\
 & \quad \downarrow \text{3042} \\
 & 2 \int x \sin\left(x + \frac{\pi}{2}\right) dx - x^2 \cos(x) \\
 & \quad \downarrow \text{3777} \\
 & 2\left(\int -\sin(x) dx + x \sin(x)\right) - x^2 \cos(x) \\
 & \quad \downarrow \text{25} \\
 & 2(x \sin(x) - \int \sin(x) dx) - x^2 \cos(x) \\
 & \quad \downarrow \text{3042} \\
 & 2(x \sin(x) - \int \sin(x) dx) - x^2 \cos(x) \\
 & \quad \downarrow \text{3118} \\
 & 2(x \sin(x) + \cos(x)) - x^2 \cos(x)
 \end{aligned}$$

input `Int[x^2*Sin[x],x]`

output `-(x^2*Cos[x]) + 2*(Cos[x] + x*Sin[x])`

## 3.93.3.1 Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3118 `Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

rule 3777 `Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

## 3.93.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

| method       | result                                                                                                                                 | size |
|--------------|----------------------------------------------------------------------------------------------------------------------------------------|------|
| risch        | $(-x^2 + 2) \cos(x) + 2x \sin(x)$                                                                                                      | 17   |
| default      | $2 \cos(x) - x^2 \cos(x) + 2x \sin(x)$                                                                                                 | 18   |
| parts        | $2 \cos(x) - x^2 \cos(x) + 2x \sin(x)$                                                                                                 | 18   |
| parallelrisc | $-x^2 \cos(x) + 2x \sin(x) + 2 \cos(x) + 2$                                                                                            | 19   |
| meijerg      | $4\sqrt{\pi} \left( -\frac{1}{2\sqrt{\pi}} + \frac{(-\frac{x^2}{2} + 1) \cos(x)}{2\sqrt{\pi}} + \frac{x \sin(x)}{2\sqrt{\pi}} \right)$ | 34   |
| norman       | $\frac{x^2 (\tan^2(\frac{x}{2}) - x^2 + 4x \tan(\frac{x}{2}) + 4)}{1 + \tan^2(\frac{x}{2})}$                                           | 36   |

input `int(x^2*sin(x), x, method=_RETURNVERBOSE)`

output `(-x^2+2)*cos(x)+2*x*sin(x)`

**3.93.5 Fracas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x) dx = -(x^2 - 2) \cos(x) + 2x \sin(x)$$

input `integrate(x^2*sin(x),x, algorithm="fricas")`output `-(x^2 - 2)*cos(x) + 2*x*sin(x)`**3.93.6 Sympy [A] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2 \sin(x) dx = -x^2 \cos(x) + 2x \sin(x) + 2 \cos(x)$$

input `integrate(x**2*sin(x),x)`output `-x**2*cos(x) + 2*x*sin(x) + 2*cos(x)`**3.93.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x) dx = -(x^2 - 2) \cos(x) + 2x \sin(x)$$

input `integrate(x^2*sin(x),x, algorithm="maxima")`output `-(x^2 - 2)*cos(x) + 2*x*sin(x)`

**3.93.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x) dx = -(x^2 - 2) \cos(x) + 2x \sin(x)$$

input `integrate(x^2*sin(x),x, algorithm="giac")`

output `-(x^2 - 2)*cos(x) + 2*x*sin(x)`

**3.93.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \sin(x) dx = 2x \sin(x) - \cos(x) (x^2 - 2)$$

input `int(x^2*sin(x),x)`

output `2*x*sin(x) - cos(x)*(x^2 - 2)`

## 3.94 $\int x \sin^2(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.94.1 | Optimal result . . . . .                            | 582 |
| 3.94.2 | Mathematica [A] (verified) . . . . .                | 582 |
| 3.94.3 | Rubi [A] (verified) . . . . .                       | 583 |
| 3.94.4 | Maple [A] (verified) . . . . .                      | 584 |
| 3.94.5 | Fricas [A] (verification not implemented) . . . . . | 584 |
| 3.94.6 | Sympy [A] (verification not implemented) . . . . .  | 585 |
| 3.94.7 | Maxima [A] (verification not implemented) . . . . . | 585 |
| 3.94.8 | Giac [A] (verification not implemented) . . . . .   | 585 |
| 3.94.9 | Mupad [B] (verification not implemented) . . . . .  | 586 |

### 3.94.1 Optimal result

Integrand size = 6, antiderivative size = 25

$$\int x \sin^2(x) dx = \frac{x^2}{4} - \frac{1}{2}x \cos(x) \sin(x) + \frac{\sin^2(x)}{4}$$

output `1/4*x^2-1/2*x*cos(x)*sin(x)+1/4*sin(x)^2`

### 3.94.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x \sin^2(x) dx = \frac{x^2}{4} - \frac{1}{8} \cos(2x) - \frac{1}{4}x \sin(2x)$$

input `Integrate[x*Sin[x]^2,x]`

output `x^2/4 - Cos[2*x]/8 - (x*Sin[2*x])/4`

**3.94.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int x \sin^2(x) dx \\ & \quad \downarrow \text{3042} \\ & \int x \sin(x)^2 dx \\ & \quad \downarrow \text{3791} \\ & \frac{\int x dx}{2} + \frac{\sin^2(x)}{4} - \frac{1}{2} x \sin(x) \cos(x) \\ & \quad \downarrow \text{15} \\ & \frac{x^2}{4} + \frac{\sin^2(x)}{4} - \frac{1}{2} x \sin(x) \cos(x) \end{aligned}$$

input `Int[x*Sin[x]^2,x]`

output `x^2/4 - (x*Cos[x]*Sin[x])/2 + Sin[x]^2/4`

**3.94.3.1 Defintions of rubi rules used**

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`



```
rule 3791 Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
  Simp[d*((b*SIN[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x]
  ]*((b*SIN[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n) Int[(c + d*
  x)*(b*SIN[e + f*x])^(n - 2), x], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
  1]
```

### 3.94.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

| method  | result                                                                                                                                                                                                                                                                                                             | size |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch   | $\frac{x^2}{4} - \frac{\cos(2x)}{8} - \frac{x \sin(2x)}{4}$                                                                                                                                                                                                                                                        | 20   |
| default | $x \left( \frac{x}{2} - \frac{\cos(x) \sin(x)}{2} \right) - \frac{x^2}{4} + \frac{(\sin^2(x))}{4}$                                                                                                                                                                                                                 | 25   |
| meijerg | $\frac{\sqrt{\pi} \left( \frac{2x^2+1}{2\sqrt{\pi}} - \frac{\cos(2x)}{2\sqrt{\pi}} - \frac{x \sin(2x)}{\sqrt{\pi}} \right)}{4}$                                                                                                                                                                                    | 38   |
| norman  | $\frac{\tan^2\left(\frac{x}{2}\right) + \left(\tan^3\left(\frac{x}{2}\right)\right)x + \frac{x^2}{4} - x \tan\left(\frac{x}{2}\right) + \frac{x^2 \left(\tan^2\left(\frac{x}{2}\right)\right)}{2} + \frac{x^2 \left(\tan^4\left(\frac{x}{2}\right)\right)}{4}}{\left(1 + \tan^2\left(\frac{x}{2}\right)\right)^2}$ | 61   |

```
input int(x*sin(x)^2,x,method=_RETURNVERBOSE)
```

```
output 1/4*x^2-1/8*cos(2*x)-1/4*x*sin(2*x)
```

### 3.94.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x \sin^2(x) dx = -\frac{1}{2} x \cos(x) \sin(x) + \frac{1}{4} x^2 - \frac{1}{4} \cos(x)^2$$

```
input integrate(x*sin(x)^2,x, algorithm="fricas")
```

```
output -1/2*x*cos(x)*sin(x) + 1/4*x^2 - 1/4*cos(x)^2
```

**3.94.6 Sympy [A] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int x \sin^2(x) dx = \frac{x^2 \sin^2(x)}{4} + \frac{x^2 \cos^2(x)}{4} - \frac{x \sin(x) \cos(x)}{2} - \frac{\cos^2(x)}{4}$$

input `integrate(x*sin(x)**2,x)`output `x**2*sin(x)**2/4 + x**2*cos(x)**2/4 - x*sin(x)*cos(x)/2 - cos(x)**2/4`**3.94.7 Maxima [A] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x \sin^2(x) dx = \frac{1}{4} x^2 - \frac{1}{4} x \sin(2x) - \frac{1}{8} \cos(2x)$$

input `integrate(x*sin(x)^2,x, algorithm="maxima")`output `1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)`**3.94.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x \sin^2(x) dx = \frac{1}{4} x^2 - \frac{1}{4} x \sin(2x) - \frac{1}{8} \cos(2x)$$

input `integrate(x*sin(x)^2,x, algorithm="giac")`output `1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)`

**3.94.9 Mupad [B] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x \sin^2(x) dx = \frac{\sin(x)^2}{4} - \frac{x \sin(2x)}{4} + \frac{x^2}{4}$$

input `int(x*sin(x)^2,x)`

output `sin(x)^2/4 - (x*sin(2*x))/4 + x^2/4`

### 3.95 $\int x^2 \sin^2(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.95.1 | Optimal result . . . . .                            | 587 |
| 3.95.2 | Mathematica [A] (verified) . . . . .                | 587 |
| 3.95.3 | Rubi [A] (verified) . . . . .                       | 588 |
| 3.95.4 | Maple [C] (verified) . . . . .                      | 589 |
| 3.95.5 | Fricas [A] (verification not implemented) . . . . . | 590 |
| 3.95.6 | Sympy [A] (verification not implemented) . . . . .  | 590 |
| 3.95.7 | Maxima [A] (verification not implemented) . . . . . | 591 |
| 3.95.8 | Giac [A] (verification not implemented) . . . . .   | 591 |
| 3.95.9 | Mupad [B] (verification not implemented) . . . . .  | 591 |

#### 3.95.1 Optimal result

Integrand size = 8, antiderivative size = 41

$$\int x^2 \sin^2(x) dx = -\frac{x}{4} + \frac{x^3}{6} + \frac{1}{4} \cos(x) \sin(x) - \frac{1}{2} x^2 \cos(x) \sin(x) + \frac{1}{2} x \sin^2(x)$$

output `-1/4*x+1/6*x^3+1/4*cos(x)*sin(x)-1/2*x^2*cos(x)*sin(x)+1/2*x*sin(x)^2`

#### 3.95.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int x^2 \sin^2(x) dx = \frac{1}{24} (4x^3 - 6x \cos(2x) + (3 - 6x^2) \sin(2x))$$

input `Integrate[x^2*Sin[x]^2,x]`

output `(4*x^3 - 6*x*Cos[2*x] + (3 - 6*x^2)*Sin[2*x])/24`

**3.95.3 Rubi [A] (verified)**

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$ , Rules used = {3042, 3792, 15, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^2 \sin^2(x) dx \\
 & \quad \downarrow \text{3042} \\
 & \int x^2 \sin(x)^2 dx \\
 & \quad \downarrow \text{3792} \\
 & \frac{\int x^2 dx}{2} - \frac{1}{2} \int \sin^2(x) dx - \frac{1}{2} x^2 \sin(x) \cos(x) + \frac{1}{2} x \sin^2(x) \\
 & \quad \downarrow \text{15} \\
 & -\frac{1}{2} \int \sin^2(x) dx + \frac{x^3}{6} - \frac{1}{2} x^2 \sin(x) \cos(x) + \frac{1}{2} x \sin^2(x) \\
 & \quad \downarrow \text{3042} \\
 & -\frac{1}{2} \int \sin(x)^2 dx + \frac{x^3}{6} - \frac{1}{2} x^2 \sin(x) \cos(x) + \frac{1}{2} x \sin^2(x) \\
 & \quad \downarrow \text{3115} \\
 & \frac{1}{2} \left( \frac{1}{2} \sin(x) \cos(x) - \frac{\int 1 dx}{2} \right) + \frac{x^3}{6} - \frac{1}{2} x^2 \sin(x) \cos(x) + \frac{1}{2} x \sin^2(x) \\
 & \quad \downarrow \text{24} \\
 & \frac{x^3}{6} - \frac{1}{2} x^2 \sin(x) \cos(x) + \frac{1}{2} x \sin^2(x) + \frac{1}{2} \left( \frac{1}{2} \sin(x) \cos(x) - \frac{x}{2} \right)
 \end{aligned}$$

input `Int[x^2*Sin[x]^2,x]`

output `x^3/6 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin[x]^2)/2 + (-1/2*x + (Cos[x]*Sin[x])/2)/2`

## 3.95.3.1 Defintions of rubi rules used

- rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`
- rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`
- rule 3115 `Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n) Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`
- rule 3792 `Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n) Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2*m*((m - 1)/(f^2*n^2)) Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]`

## 3.95.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

| method  | result                                                                                                                                                                                                                                                                                                                                    | size |
|---------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| meijerg | $\frac{x^5 {}_2F_3(1, \frac{5}{2}; \frac{3}{2}, 2, \frac{7}{2}; -x^2)}{5}$                                                                                                                                                                                                                                                                | 19   |
| risch   | $\frac{x^3}{6} - \frac{x \cos(2x)}{4} - \frac{(2x^2 - 1) \sin(2x)}{8}$                                                                                                                                                                                                                                                                    | 27   |
| default | $x^2 \left( \frac{x}{2} - \frac{\cos(x) \sin(x)}{2} \right) - \frac{(\cos^2(x))x}{2} + \frac{\cos(x) \sin(x)}{4} + \frac{x}{4} - \frac{x^3}{3}$                                                                                                                                                                                           | 37   |
| norman  | $\frac{x^2 (\tan^3(\frac{x}{2}) - \frac{x}{4} + \frac{x^3}{6} - \frac{(\tan^3(\frac{x}{2}))}{2} + \frac{3x (\tan^2(\frac{x}{2}))}{2} - \frac{x (\tan^4(\frac{x}{2}))}{4} - x^2 \tan(\frac{x}{2}) + \frac{x^3 (\tan^2(\frac{x}{2}))}{3} + \frac{x^3 (\tan^4(\frac{x}{2}))}{6} + \frac{\tan(\frac{x}{2})}{2}}{(1 + \tan^2(\frac{x}{2}))^2}$ | 94   |

input `int(x^2*sin(x)^2,x,method=_RETURNVERBOSE)`

output `1/5*x^5*hypergeom([1,5/2],[3/2,2,7/2],-x^2)`

### 3.95.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int x^2 \sin^2(x) dx = \frac{1}{6} x^3 - \frac{1}{2} x \cos(x)^2 - \frac{1}{4} (2x^2 - 1) \cos(x) \sin(x) + \frac{1}{4} x$$

input `integrate(x^2*sin(x)^2,x, algorithm="fricas")`

output `1/6*x^3 - 1/2*x*cos(x)^2 - 1/4*(2*x^2 - 1)*cos(x)*sin(x) + 1/4*x`

### 3.95.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int x^2 \sin^2(x) dx = \frac{x^3 \sin^2(x)}{6} + \frac{x^3 \cos^2(x)}{6} - \frac{x^2 \sin(x) \cos(x)}{2} + \frac{x \sin^2(x)}{4} - \frac{x \cos^2(x)}{4} + \frac{\sin(x) \cos(x)}{4}$$

input `integrate(x**2*sin(x)**2,x)`

output `x**3*sin(x)**2/6 + x**3*cos(x)**2/6 - x**2*sin(x)*cos(x)/2 + x*sin(x)**2/4 - x*cos(x)**2/4 + sin(x)*cos(x)/4`

**3.95.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int x^2 \sin^2(x) dx = \frac{1}{6} x^3 - \frac{1}{4} x \cos(2x) - \frac{1}{8} (2x^2 - 1) \sin(2x)$$

input `integrate(x^2*sin(x)^2,x, algorithm="maxima")`output `1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)`**3.95.8 Giac [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int x^2 \sin^2(x) dx = \frac{1}{6} x^3 - \frac{1}{4} x \cos(2x) - \frac{1}{8} (2x^2 - 1) \sin(2x)$$

input `integrate(x^2*sin(x)^2,x, algorithm="giac")`output `1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)`**3.95.9 Mupad [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int x^2 \sin^2(x) dx = \frac{\sin(2x)}{8} - \frac{x \cos(2x)}{4} - \frac{x^2 \sin(2x)}{4} + \frac{x^3}{6}$$

input `int(x^2*sin(x)^2,x)`output `sin(2*x)/8 - (x*cos(2*x))/4 - (x^2*sin(2*x))/4 + x^3/6`



### 3.96 $\int x \sin^3(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.96.1 | Optimal result . . . . .                            | 592 |
| 3.96.2 | Mathematica [A] (verified) . . . . .                | 592 |
| 3.96.3 | Rubi [A] (verified) . . . . .                       | 593 |
| 3.96.4 | Maple [A] (verified) . . . . .                      | 594 |
| 3.96.5 | Fricas [A] (verification not implemented) . . . . . | 595 |
| 3.96.6 | Sympy [A] (verification not implemented) . . . . .  | 595 |
| 3.96.7 | Maxima [A] (verification not implemented) . . . . . | 595 |
| 3.96.8 | Giac [A] (verification not implemented) . . . . .   | 596 |
| 3.96.9 | Mupad [B] (verification not implemented) . . . . .  | 596 |

#### 3.96.1 Optimal result

Integrand size = 6, antiderivative size = 33

$$\int x \sin^3(x) dx = -\frac{2}{3}x \cos(x) + \frac{2 \sin(x)}{3} - \frac{1}{3}x \cos(x) \sin^2(x) + \frac{\sin^3(x)}{9}$$

output `-2/3*x*cos(x)+2/3*sin(x)-1/3*x*cos(x)*sin(x)^2+1/9*sin(x)^3`

#### 3.96.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int x \sin^3(x) dx = -\frac{3}{4}x \cos(x) + \frac{1}{12}x \cos(3x) + \frac{3 \sin(x)}{4} - \frac{1}{36} \sin(3x)$$

input `Integrate[x*Sin[x]^3,x]`

output `(-3*x*Cos[x])/4 + (x*Cos[3*x])/12 + (3*Sin[x])/4 - Sin[3*x]/36`

**3.96.3 Rubi [A] (verified)**

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {3042, 3791, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x \sin^3(x) dx \\
 & \quad \downarrow \text{3042} \\
 & \int x \sin(x)^3 dx \\
 & \quad \downarrow \text{3791} \\
 & \frac{2}{3} \int x \sin(x) dx + \frac{\sin^3(x)}{9} - \frac{1}{3} x \sin^2(x) \cos(x) \\
 & \quad \downarrow \text{3042} \\
 & \frac{2}{3} \int x \sin(x) dx + \frac{\sin^3(x)}{9} - \frac{1}{3} x \sin^2(x) \cos(x) \\
 & \quad \downarrow \text{3777} \\
 & \frac{2}{3} \left( \int \cos(x) dx - x \cos(x) \right) + \frac{\sin^3(x)}{9} - \frac{1}{3} x \sin^2(x) \cos(x) \\
 & \quad \downarrow \text{3042} \\
 & \frac{2}{3} \left( \int \sin \left( x + \frac{\pi}{2} \right) dx - x \cos(x) \right) + \frac{\sin^3(x)}{9} - \frac{1}{3} x \sin^2(x) \cos(x) \\
 & \quad \downarrow \text{3117} \\
 & \frac{\sin^3(x)}{9} - \frac{1}{3} x \sin^2(x) \cos(x) + \frac{2}{3} (\sin(x) - x \cos(x))
 \end{aligned}$$

input `Int [x*Sin[x]^3,x]`

output `-1/3*(x*Cos[x]*Sin[x]^2) + Sin[x]^3/9 + (2*(-(x*Cos[x]) + Sin[x]))/3`

## 3.96.3.1 Defintions of rubi rules used

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3117 `Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

rule 3777 `Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

rule 3791 `Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[d*((b*Sine[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x]*((b*Sine[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n) Int[(c + d*x)*(b*Sine[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]`

## 3.96.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

| method       | result                                                                                                                                                                                                                                          | size |
|--------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default      | $-\frac{x(2+\sin^2(x))\cos(x)}{3} + \frac{(\sin^3(x))}{9} + \frac{2\sin(x)}{3}$                                                                                                                                                                 | 23   |
| risch        | $-\frac{3x\cos(x)}{4} + \frac{3\sin(x)}{4} + \frac{x\cos(3x)}{12} - \frac{\sin(3x)}{36}$                                                                                                                                                        | 24   |
| parallerisch | $-\frac{3x\cos(x)}{4} + \frac{3\sin(x)}{4} + \frac{x\cos(3x)}{12} - \frac{\sin(3x)}{36}$                                                                                                                                                        | 24   |
| norman       | $-\frac{2x}{3} + \frac{32(\tan^3(\frac{x}{2}))}{9} + \frac{4(\tan^5(\frac{x}{2}))}{3} - \frac{2x(\tan^2(\frac{x}{2})) + 2x(\tan^4(\frac{x}{2})) + \frac{2x(\tan^6(\frac{x}{2}))}{3} + \frac{4\tan(\frac{x}{2})}{3}}{(1+\tan^2(\frac{x}{2}))^3}$ | 65   |

input `int(x*sin(x)^3,x,method=_RETURNVERBOSE)`

output `-1/3*x*(2+sin(x)^2)*cos(x)+1/9*sin(x)^3+2/3*sin(x)`

**3.96.5 Fracas [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int x \sin^3(x) dx = \frac{1}{3} x \cos(x)^3 - x \cos(x) - \frac{1}{9} (\cos(x)^2 - 7) \sin(x)$$

input `integrate(x*sin(x)^3,x, algorithm="fracas")`output `1/3*x*cos(x)^3 - x*cos(x) - 1/9*(cos(x)^2 - 7)*sin(x)`**3.96.6 Sympy [A] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int x \sin^3(x) dx = -x \sin^2(x) \cos(x) - \frac{2x \cos^3(x)}{3} + \frac{7 \sin^3(x)}{9} + \frac{2 \sin(x) \cos^2(x)}{3}$$

input `integrate(x*sin(x)**3,x)`output `-x*sin(x)**2*cos(x) - 2*x*cos(x)**3/3 + 7*sin(x)**3/9 + 2*sin(x)*cos(x)**2/3`**3.96.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int x \sin^3(x) dx = \frac{1}{12} x \cos(3x) - \frac{3}{4} x \cos(x) - \frac{1}{36} \sin(3x) + \frac{3}{4} \sin(x)$$

input `integrate(x*sin(x)^3,x, algorithm="maxima")`output `1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)`

**3.96.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int x \sin^3(x) dx = \frac{1}{12} x \cos(3x) - \frac{3}{4} x \cos(x) - \frac{1}{36} \sin(3x) + \frac{3}{4} \sin(x)$$

input `integrate(x*sin(x)^3,x, algorithm="giac")`output `1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)`**3.96.9 Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int x \sin^3(x) dx = \frac{x \cos(x)^3}{3} - \frac{\sin(x) \cos(x)^2}{9} - x \cos(x) + \frac{7 \sin(x)}{9}$$

input `int(x*sin(x)^3,x)`output `(7*sin(x))/9 + (x*cos(x)^3)/3 - (cos(x)^2*sin(x))/9 - x*cos(x)`

## 3.97 $\int x \cos(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.97.1 | Optimal result . . . . .                            | 597 |
| 3.97.2 | Mathematica [A] (verified) . . . . .                | 597 |
| 3.97.3 | Rubi [A] (verified) . . . . .                       | 598 |
| 3.97.4 | Maple [A] (verified) . . . . .                      | 599 |
| 3.97.5 | Fricas [A] (verification not implemented) . . . . . | 600 |
| 3.97.6 | Sympy [A] (verification not implemented) . . . . .  | 600 |
| 3.97.7 | Maxima [A] (verification not implemented) . . . . . | 600 |
| 3.97.8 | Giac [A] (verification not implemented) . . . . .   | 601 |
| 3.97.9 | Mupad [B] (verification not implemented) . . . . .  | 601 |

### 3.97.1 Optimal result

Integrand size = 4, antiderivative size = 7

$$\int x \cos(x) dx = \cos(x) + x \sin(x)$$

output `cos(x)+x*sin(x)`

### 3.97.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x \cos(x) dx = \cos(x) + x \sin(x)$$

input `Integrate[x*Cos[x],x]`

output `Cos[x] + x*Sin[x]`

**3.97.3 Rubi [A] (verified)**

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$ , Rules used = {3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x \cos(x) dx \\
 & \quad \downarrow \text{3042} \\
 & \int x \sin\left(x + \frac{\pi}{2}\right) dx \\
 & \quad \downarrow \text{3777} \\
 & \int -\sin(x) dx + x \sin(x) \\
 & \quad \downarrow \text{25} \\
 & x \sin(x) - \int \sin(x) dx \\
 & \quad \downarrow \text{3042} \\
 & x \sin(x) - \int \sin(x) dx \\
 & \quad \downarrow \text{3118} \\
 & x \sin(x) + \cos(x)
 \end{aligned}$$

input `Int [x*Cos [x] , x]`

output `Cos [x] + x*Sin [x]`

## 3.97.3.1 Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3118 `Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

rule 3777 `Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

## 3.97.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

| method       | result                                                                                                            | size |
|--------------|-------------------------------------------------------------------------------------------------------------------|------|
| default      | $\cos(x) + x \sin(x)$                                                                                             | 8    |
| risch        | $\cos(x) + x \sin(x)$                                                                                             | 8    |
| parts        | $\cos(x) + x \sin(x)$                                                                                             | 8    |
| parallelrisc | $x \sin(x) + \cos(x) + 1$                                                                                         | 9    |
| norman       | $\frac{2x \tan(\frac{x}{2}) + 2}{1 + \tan^2(\frac{x}{2})}$                                                        | 21   |
| meijerg      | $2\sqrt{\pi} \left( -\frac{1}{2\sqrt{\pi}} + \frac{\cos(x)}{2\sqrt{\pi}} + \frac{x \sin(x)}{2\sqrt{\pi}} \right)$ | 27   |

input `int(x*cos(x),x,method=_RETURNVERBOSE)`

output `cos(x)+x*sin(x)`



**3.97.5 Fracas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x \cos(x) dx = x \sin(x) + \cos(x)$$

input `integrate(x*cos(x),x, algorithm="fricas")`

output `x*sin(x) + cos(x)`

**3.97.6 Sympy [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x \cos(x) dx = x \sin(x) + \cos(x)$$

input `integrate(x*cos(x),x)`

output `x*sin(x) + cos(x)`

**3.97.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x \cos(x) dx = x \sin(x) + \cos(x)$$

input `integrate(x*cos(x),x, algorithm="maxima")`

output `x*sin(x) + cos(x)`

**3.97.8 Giac [A] (verification not implemented)**

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x \cos(x) dx = x \sin(x) + \cos(x)$$

input `integrate(x*cos(x),x, algorithm="giac")`

output `x*sin(x) + cos(x)`

**3.97.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x \cos(x) dx = \cos(x) + x \sin(x)$$

input `int(x*cos(x),x)`

output `cos(x) + x*sin(x)`

### 3.98 $\int x^2 \cos(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.98.1 | Optimal result . . . . .                            | 602 |
| 3.98.2 | Mathematica [A] (verified) . . . . .                | 602 |
| 3.98.3 | Rubi [A] (verified) . . . . .                       | 603 |
| 3.98.4 | Maple [A] (verified) . . . . .                      | 604 |
| 3.98.5 | Fricas [A] (verification not implemented) . . . . . | 605 |
| 3.98.6 | Sympy [A] (verification not implemented) . . . . .  | 605 |
| 3.98.7 | Maxima [A] (verification not implemented) . . . . . | 605 |
| 3.98.8 | Giac [A] (verification not implemented) . . . . .   | 606 |
| 3.98.9 | Mupad [B] (verification not implemented) . . . . .  | 606 |

#### 3.98.1 Optimal result

Integrand size = 6, antiderivative size = 16

$$\int x^2 \cos(x) dx = 2x \cos(x) - 2 \sin(x) + x^2 \sin(x)$$

output `2*x*cos(x)-2*sin(x)+x^2*sin(x)`

#### 3.98.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2 \cos(x) dx = 2x \cos(x) + (-2 + x^2) \sin(x)$$

input `Integrate[x^2*Cos[x],x]`

output `2*x*Cos[x] + (-2 + x^2)*Sin[x]`

**3.98.3 Rubi [A] (verified)**

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$ , Rules used = {3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^2 \cos(x) dx \\
 & \quad \downarrow \text{3042} \\
 & \int x^2 \sin\left(x + \frac{\pi}{2}\right) dx \\
 & \quad \downarrow \text{3777} \\
 & 2 \int -x \sin(x) dx + x^2 \sin(x) \\
 & \quad \downarrow \text{25} \\
 & x^2 \sin(x) - 2 \int x \sin(x) dx \\
 & \quad \downarrow \text{3042} \\
 & x^2 \sin(x) - 2 \int x \sin(x) dx \\
 & \quad \downarrow \text{3777} \\
 & x^2 \sin(x) - 2\left(\int \cos(x) dx - x \cos(x)\right) \\
 & \quad \downarrow \text{3042} \\
 & x^2 \sin(x) - 2\left(\int \sin\left(x + \frac{\pi}{2}\right) dx - x \cos(x)\right) \\
 & \quad \downarrow \text{3117} \\
 & x^2 \sin(x) - 2(\sin(x) - x \cos(x))
 \end{aligned}$$

input `Int[x^2*Cos[x],x]`

output `x^2*Sin[x] - 2*(-(x*Cos[x]) + Sin[x])`

## 3.98.3.1 Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3117 `Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

rule 3777 `Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

## 3.98.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

| method        | result                                                                                                         | size |
|---------------|----------------------------------------------------------------------------------------------------------------|------|
| risch         | $2x \cos(x) + (x^2 - 2) \sin(x)$                                                                               | 15   |
| default       | $2x \cos(x) - 2 \sin(x) + x^2 \sin(x)$                                                                         | 17   |
| parallelrisch | $2x \cos(x) - 2 \sin(x) + x^2 \sin(x)$                                                                         | 17   |
| parts         | $2x \cos(x) - 2 \sin(x) + x^2 \sin(x)$                                                                         | 17   |
| meijerg       | $4\sqrt{\pi} \left( \frac{x \cos(x)}{2\sqrt{\pi}} - \frac{(-\frac{3x^2}{2} + 3) \sin(x)}{6\sqrt{\pi}} \right)$ | 29   |
| norman        | $\frac{2x - 2x(\tan^2(\frac{x}{2})) + 2x^2 \tan(\frac{x}{2}) - 4 \tan(\frac{x}{2})}{1 + \tan^2(\frac{x}{2})}$  | 40   |

input `int(x^2*cos(x), x, method=_RETURNVERBOSE)`

output `2*x*cos(x)+(x^2-2)*sin(x)`

**3.98.5 Fracas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2 \cos(x) dx = 2x \cos(x) + (x^2 - 2) \sin(x)$$

input `integrate(x^2*cos(x),x, algorithm="fricas")`output `2*x*cos(x) + (x^2 - 2)*sin(x)`**3.98.6 Sympy [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int x^2 \cos(x) dx = x^2 \sin(x) + 2x \cos(x) - 2 \sin(x)$$

input `integrate(x**2*cos(x),x)`output `x**2*sin(x) + 2*x*cos(x) - 2*sin(x)`**3.98.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2 \cos(x) dx = 2x \cos(x) + (x^2 - 2) \sin(x)$$

input `integrate(x^2*cos(x),x, algorithm="maxima")`output `2*x*cos(x) + (x^2 - 2)*sin(x)`

**3.98.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2 \cos(x) dx = 2x \cos(x) + (x^2 - 2) \sin(x)$$

input `integrate(x^2*cos(x),x, algorithm="giac")`

output `2*x*cos(x) + (x^2 - 2)*sin(x)`

**3.98.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2 \cos(x) dx = \sin(x) (x^2 - 2) + 2x \cos(x)$$

input `int(x^2*cos(x),x)`

output `sin(x)*(x^2 - 2) + 2*x*cos(x)`

## 3.99 $\int x \cos^2(x) dx$

|        |                                                     |     |
|--------|-----------------------------------------------------|-----|
| 3.99.1 | Optimal result . . . . .                            | 607 |
| 3.99.2 | Mathematica [A] (verified) . . . . .                | 607 |
| 3.99.3 | Rubi [A] (verified) . . . . .                       | 608 |
| 3.99.4 | Maple [A] (verified) . . . . .                      | 609 |
| 3.99.5 | Fricas [A] (verification not implemented) . . . . . | 609 |
| 3.99.6 | Sympy [A] (verification not implemented) . . . . .  | 610 |
| 3.99.7 | Maxima [A] (verification not implemented) . . . . . | 610 |
| 3.99.8 | Giac [A] (verification not implemented) . . . . .   | 610 |
| 3.99.9 | Mupad [B] (verification not implemented) . . . . .  | 611 |

### 3.99.1 Optimal result

Integrand size = 6, antiderivative size = 25

$$\int x \cos^2(x) dx = \frac{x^2}{4} + \frac{\cos^2(x)}{4} + \frac{1}{2}x \cos(x) \sin(x)$$

output `1/4*x^2+1/4*cos(x)^2+1/2*x*cos(x)*sin(x)`

### 3.99.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x \cos^2(x) dx = \frac{x^2}{4} + \frac{1}{8} \cos(2x) + \frac{1}{4}x \sin(2x)$$

input `Integrate[x*Cos[x]^2,x]`

output `x^2/4 + Cos[2*x]/8 + (x*Sin[2*x])/4`



**3.99.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int x \cos^2(x) dx \\ & \quad \downarrow \text{3042} \\ & \int x \sin\left(x + \frac{\pi}{2}\right)^2 dx \\ & \quad \downarrow \text{3791} \\ & \frac{\int x dx}{2} + \frac{\cos^2(x)}{4} + \frac{1}{2} x \sin(x) \cos(x) \\ & \quad \downarrow \text{15} \\ & \frac{x^2}{4} + \frac{\cos^2(x)}{4} + \frac{1}{2} x \sin(x) \cos(x) \end{aligned}$$

input `Int[x*Cos[x]^2,x]`

output `x^2/4 + Cos[x]^2/4 + (x*Cos[x]*Sin[x])/2`

**3.99.3.1 Defintions of rubi rules used**

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

```
rule 3791 Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
  Simp[d*((b*Sine[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x]
  ]*((b*Sine[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n) Int[(c + d*
  x)*(b*Sine[e + f*x])^(n - 2), x], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
  1]
```

### 3.99.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

| method        | result                                                                                                                                                                                                 | size |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch         | $\frac{x^2}{4} + \frac{\cos(2x)}{8} + \frac{x \sin(2x)}{4}$                                                                                                                                            | 20   |
| parallelrisch | $\frac{x^2}{4} - \frac{1}{8} + \frac{x \sin(2x)}{4} + \frac{\cos(2x)}{8}$                                                                                                                              | 21   |
| default       | $x \left( \frac{x}{2} + \frac{\cos(x) \sin(x)}{2} \right) - \frac{x^2}{4} - \frac{(\sin^2(x))}{4}$                                                                                                     | 25   |
| norman        | $\frac{x \tan(\frac{x}{2}) - (\tan^2(\frac{x}{2})) + \frac{x^2}{4} + \frac{x^2 (\tan^2(\frac{x}{2}))}{2} + \frac{x^2 (\tan^4(\frac{x}{2}))}{4} - (\tan^3(\frac{x}{2}))x}{(1 + \tan^2(\frac{x}{2}))^2}$ | 63   |

input `int(cos(x)^2*x,x,method=_RETURNVERBOSE)`

output `1/4*x^2+1/8*cos(2*x)+1/4*x*sin(2*x)`

### 3.99.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x \cos^2(x) dx = \frac{1}{2} x \cos(x) \sin(x) + \frac{1}{4} x^2 + \frac{1}{4} \cos(x)^2$$

input `integrate(x*cos(x)^2,x, algorithm="fricas")`

output `1/2*x*cos(x)*sin(x) + 1/4*x^2 + 1/4*cos(x)^2`

**3.99.6 Sympy [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int x \cos^2(x) dx = \frac{x^2 \sin^2(x)}{4} + \frac{x^2 \cos^2(x)}{4} + \frac{x \sin(x) \cos(x)}{2} + \frac{\cos^2(x)}{4}$$

input `integrate(x*cos(x)**2,x)`output `x**2*sin(x)**2/4 + x**2*cos(x)**2/4 + x*sin(x)*cos(x)/2 + cos(x)**2/4`**3.99.7 Maxima [A] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x \cos^2(x) dx = \frac{1}{4} x^2 + \frac{1}{4} x \sin(2x) + \frac{1}{8} \cos(2x)$$

input `integrate(x*cos(x)^2,x, algorithm="maxima")`output `1/4*x^2 + 1/4*x*sin(2*x) + 1/8*cos(2*x)`**3.99.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x \cos^2(x) dx = \frac{1}{4} x^2 + \frac{1}{4} x \sin(2x) + \frac{1}{8} \cos(2x)$$

input `integrate(x*cos(x)^2,x, algorithm="giac")`output `1/4*x^2 + 1/4*x*sin(2*x) + 1/8*cos(2*x)`

**3.99.9 Mupad [B] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x \cos^2(x) dx = \frac{x \sin(2x)}{4} - \frac{\sin(x)^2}{4} + \frac{x^2}{4}$$

input `int(x*cos(x)^2,x)`

output `(x*sin(2*x))/4 - sin(x)^2/4 + x^2/4`

### 3.100 $\int x^2 \cos^2(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.100.1 Optimal result . . . . .                            | 612 |
| 3.100.2 Mathematica [A] (verified) . . . . .                | 612 |
| 3.100.3 Rubi [A] (verified) . . . . .                       | 613 |
| 3.100.4 Maple [A] (verified) . . . . .                      | 614 |
| 3.100.5 Fricas [A] (verification not implemented) . . . . . | 615 |
| 3.100.6 Sympy [A] (verification not implemented) . . . . .  | 615 |
| 3.100.7 Maxima [A] (verification not implemented) . . . . . | 615 |
| 3.100.8 Giac [A] (verification not implemented) . . . . .   | 616 |
| 3.100.9 Mupad [B] (verification not implemented) . . . . .  | 616 |

#### 3.100.1 Optimal result

Integrand size = 8, antiderivative size = 41

$$\int x^2 \cos^2(x) dx = -\frac{x}{4} + \frac{x^3}{6} + \frac{1}{2}x \cos^2(x) - \frac{1}{4} \cos(x) \sin(x) + \frac{1}{2}x^2 \cos(x) \sin(x)$$

output `-1/4*x+1/6*x^3+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)+1/2*x^2*cos(x)*sin(x)`

#### 3.100.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int x^2 \cos^2(x) dx = \frac{1}{24}(4x^3 + 6x \cos(2x) + (-3 + 6x^2) \sin(2x))$$

input `Integrate[x^2*Cos[x]^2,x]`

output `(4*x^3 + 6*x*Cos[2*x] + (-3 + 6*x^2)*Sin[2*x])/24`

**3.100.3 Rubi [A] (verified)**

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$ , Rules used = {3042, 3792, 15, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^2 \cos^2(x) dx \\
 & \quad \downarrow \text{3042} \\
 & \int x^2 \sin\left(x + \frac{\pi}{2}\right)^2 dx \\
 & \quad \downarrow \text{3792} \\
 & \frac{\int x^2 dx}{2} - \frac{1}{2} \int \cos^2(x) dx + \frac{1}{2} x^2 \sin(x) \cos(x) + \frac{1}{2} x \cos^2(x) \\
 & \quad \downarrow \text{15} \\
 & -\frac{1}{2} \int \cos^2(x) dx + \frac{x^3}{6} + \frac{1}{2} x^2 \sin(x) \cos(x) + \frac{1}{2} x \cos^2(x) \\
 & \quad \downarrow \text{3042} \\
 & -\frac{1}{2} \int \sin\left(x + \frac{\pi}{2}\right)^2 dx + \frac{x^3}{6} + \frac{1}{2} x^2 \sin(x) \cos(x) + \frac{1}{2} x \cos^2(x) \\
 & \quad \downarrow \text{3115} \\
 & \frac{1}{2} \left( -\frac{\int 1 dx}{2} - \frac{1}{2} \sin(x) \cos(x) \right) + \frac{x^3}{6} + \frac{1}{2} x^2 \sin(x) \cos(x) + \frac{1}{2} x \cos^2(x) \\
 & \quad \downarrow \text{24} \\
 & \frac{x^3}{6} + \frac{1}{2} x^2 \sin(x) \cos(x) + \frac{1}{2} x \cos^2(x) + \frac{1}{2} \left( -\frac{x}{2} - \frac{1}{2} \sin(x) \cos(x) \right)
 \end{aligned}$$

input `Int[x^2*Cos[x]^2,x]`

output `x^3/6 + (x*Cos[x]^2)/2 + (x^2*Cos[x]*Sin[x])/2 + (-1/2*x - (Cos[x]*Sin[x])/2)/2`

3.100.3.1 Defintions of rubi rules used

- rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`
  
- rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`
  
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`
  
- rule 3115 `Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n) Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`
  
- rule 3792 `Int[((c_.) + (d_.)*(x_)^(m_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n) Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2*m*((m - 1)/(f^2*n^2)) Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]`

3.100.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

| method        | result                                                                                                                                                                                                                                                                                                                              | size |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch         | $\frac{x^3}{6} + \frac{x \cos(2x)}{4} + \frac{(2x^2-1) \sin(2x)}{8}$                                                                                                                                                                                                                                                                | 27   |
| parallelrisch | $\frac{x^3}{6} + \frac{x \cos(2x)}{4} + \frac{x^2 \sin(2x)}{4} - \frac{\sin(2x)}{8}$                                                                                                                                                                                                                                                | 29   |
| default       | $x^2 \left( \frac{x}{2} + \frac{\cos(x) \sin(x)}{2} \right) + \frac{\cos^2(x)x}{2} - \frac{\cos(x) \sin(x)}{4} - \frac{x}{4} - \frac{x^3}{3}$                                                                                                                                                                                       | 37   |
| norman        | $\frac{x^2 \tan(\frac{x}{2}) + \frac{x}{4} + \frac{x^3}{6} + \frac{(\tan^3(\frac{x}{2}))}{2} - \frac{3x(\tan^2(\frac{x}{2}))}{2} + \frac{x(\tan^4(\frac{x}{2}))}{4} - x^2(\tan^3(\frac{x}{2})) + \frac{x^3(\tan^2(\frac{x}{2}))}{3} + \frac{x^3(\tan^4(\frac{x}{2}))}{6} - \frac{\tan(\frac{x}{2})}{2}}{(1+\tan^2(\frac{x}{2}))^2}$ | 94   |

input `int(x^2*cos(x)^2,x,method=_RETURNVERBOSE)`

output `1/6*x^3+1/4*x*cos(2*x)+1/8*(2*x^2-1)*sin(2*x)`

### 3.100.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int x^2 \cos^2(x) dx = \frac{1}{6} x^3 + \frac{1}{2} x \cos(x)^2 + \frac{1}{4} (2x^2 - 1) \cos(x) \sin(x) - \frac{1}{4} x$$

input `integrate(x^2*cos(x)^2,x, algorithm="fricas")`

output `1/6*x^3 + 1/2*x*cos(x)^2 + 1/4*(2*x^2 - 1)*cos(x)*sin(x) - 1/4*x`

### 3.100.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int x^2 \cos^2(x) dx = \frac{x^3 \sin^2(x)}{6} + \frac{x^3 \cos^2(x)}{6} + \frac{x^2 \sin(x) \cos(x)}{2} - \frac{x \sin^2(x)}{4} + \frac{x \cos^2(x)}{4} - \frac{\sin(x) \cos(x)}{4}$$

input `integrate(x**2*cos(x)**2,x)`

output `x**3*sin(x)**2/6 + x**3*cos(x)**2/6 + x**2*sin(x)*cos(x)/2 - x*sin(x)**2/4 + x*cos(x)**2/4 - sin(x)*cos(x)/4`

### 3.100.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int x^2 \cos^2(x) dx = \frac{1}{6} x^3 + \frac{1}{4} x \cos(2x) + \frac{1}{8} (2x^2 - 1) \sin(2x)$$

input `integrate(x^2*cos(x)^2,x, algorithm="maxima")`

output `1/6*x^3 + 1/4*x*cos(2*x) + 1/8*(2*x^2 - 1)*sin(2*x)`



**3.100.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int x^2 \cos^2(x) dx = \frac{1}{6} x^3 + \frac{1}{4} x \cos(2x) + \frac{1}{8} (2x^2 - 1) \sin(2x)$$

input `integrate(x^2*cos(x)^2,x, algorithm="giac")`output `1/6*x^3 + 1/4*x*cos(2*x) + 1/8*(2*x^2 - 1)*sin(2*x)`**3.100.9 Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int x^2 \cos^2(x) dx = \frac{x \cos(2x)}{4} - \frac{\sin(2x)}{8} + \frac{x^2 \sin(2x)}{4} + \frac{x^3}{6}$$

input `int(x^2*cos(x)^2,x)`output `(x*cos(2*x))/4 - sin(2*x)/8 + (x^2*sin(2*x))/4 + x^3/6`

### 3.101 $\int x \cos^3(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.101.1 Optimal result . . . . .                            | 617 |
| 3.101.2 Mathematica [A] (verified) . . . . .                | 617 |
| 3.101.3 Rubi [A] (verified) . . . . .                       | 618 |
| 3.101.4 Maple [A] (verified) . . . . .                      | 619 |
| 3.101.5 Fricas [A] (verification not implemented) . . . . . | 620 |
| 3.101.6 Sympy [A] (verification not implemented) . . . . .  | 620 |
| 3.101.7 Maxima [A] (verification not implemented) . . . . . | 620 |
| 3.101.8 Giac [A] (verification not implemented) . . . . .   | 621 |
| 3.101.9 Mupad [B] (verification not implemented) . . . . .  | 621 |

#### 3.101.1 Optimal result

Integrand size = 6, antiderivative size = 33

$$\int x \cos^3(x) dx = \frac{2 \cos(x)}{3} + \frac{\cos^3(x)}{9} + \frac{2}{3}x \sin(x) + \frac{1}{3}x \cos^2(x) \sin(x)$$

output `2/3*cos(x)+1/9*cos(x)^3+2/3*x*sin(x)+1/3*x*cos(x)^2*sin(x)`

#### 3.101.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int x \cos^3(x) dx = \frac{3 \cos(x)}{4} + \frac{1}{36} \cos(3x) + \frac{3}{4}x \sin(x) + \frac{1}{12}x \sin(3x)$$

input `Integrate[x*Cos[x]^3,x]`

output `(3*Cos[x])/4 + Cos[3*x]/36 + (3*x*Sin[x])/4 + (x*Sin[3*x])/12`

**3.101.3 Rubi [A] (verified)**

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$ , Rules used = {3042, 3791, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x \cos^3(x) dx \\
 & \quad \downarrow \text{3042} \\
 & \int x \sin\left(x + \frac{\pi}{2}\right)^3 dx \\
 & \quad \downarrow \text{3791} \\
 & \frac{2}{3} \int x \cos(x) dx + \frac{\cos^3(x)}{9} + \frac{1}{3} x \sin(x) \cos^2(x) \\
 & \quad \downarrow \text{3042} \\
 & \frac{2}{3} \int x \sin\left(x + \frac{\pi}{2}\right) dx + \frac{\cos^3(x)}{9} + \frac{1}{3} x \sin(x) \cos^2(x) \\
 & \quad \downarrow \text{3777} \\
 & \frac{2}{3} \left( \int -\sin(x) dx + x \sin(x) \right) + \frac{\cos^3(x)}{9} + \frac{1}{3} x \sin(x) \cos^2(x) \\
 & \quad \downarrow \text{25} \\
 & \frac{2}{3} (x \sin(x) - \int \sin(x) dx) + \frac{\cos^3(x)}{9} + \frac{1}{3} x \sin(x) \cos^2(x) \\
 & \quad \downarrow \text{3042} \\
 & \frac{2}{3} (x \sin(x) - \int \sin(x) dx) + \frac{\cos^3(x)}{9} + \frac{1}{3} x \sin(x) \cos^2(x) \\
 & \quad \downarrow \text{3118} \\
 & \frac{\cos^3(x)}{9} + \frac{1}{3} x \sin(x) \cos^2(x) + \frac{2}{3} (x \sin(x) + \cos(x))
 \end{aligned}$$

input `Int[x*Cos[x]^3,x]`

output `Cos[x]^3/9 + (x*Cos[x]^2*Sin[x])/3 + (2*(Cos[x] + x*Sin[x]))/3`

## 3.101.3.1 Defintions of rubi rules used

- rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`
- rule 3118 `Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`
- rule 3777 `Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`
- rule 3791 `Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1)/(f*n), x] + Simp[b^2*((n - 1)/n) Int[(c + d*x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]`

## 3.101.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

| method        | result                                                                                                                                                                                           | size |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default       | $\frac{x(2+\cos^2(x))\sin(x)}{3} + \frac{(\cos^3(x))}{9} + \frac{2\cos(x)}{3}$                                                                                                                   | 23   |
| risch         | $\frac{3\cos(x)}{4} + \frac{3x\sin(x)}{4} + \frac{\cos(3x)}{36} + \frac{x\sin(3x)}{12}$                                                                                                          | 24   |
| parallelrisch | $\frac{7}{9} + \frac{3\cos(x)}{4} + \frac{3x\sin(x)}{4} + \frac{\cos(3x)}{36} + \frac{x\sin(3x)}{12}$                                                                                            | 25   |
| norman        | $\frac{2(\tan^4(\frac{x}{2})) + \frac{8(\tan^2(\frac{x}{2}))}{3} + 2x\tan(\frac{x}{2}) + 2x(\tan^5(\frac{x}{2})) + \frac{4(\tan^3(\frac{x}{2}))x}{3} + \frac{14}{9}}{(1+\tan^2(\frac{x}{2}))^3}$ | 55   |

input `int(x*cos(x)^3,x,method=_RETURNVERBOSE)`

output `1/3*x*(2+cos(x)^2)*sin(x)+1/9*cos(x)^3+2/3*cos(x)`

**3.101.5 Fricas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int x \cos^3(x) dx = \frac{1}{9} \cos(x)^3 + \frac{1}{3} (x \cos(x)^2 + 2x) \sin(x) + \frac{2}{3} \cos(x)$$

input `integrate(x*cos(x)^3,x, algorithm="fricas")`output `1/9*cos(x)^3 + 1/3*(x*cos(x)^2 + 2*x)*sin(x) + 2/3*cos(x)`**3.101.6 Sympy [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int x \cos^3(x) dx = \frac{2x \sin^3(x)}{3} + x \sin(x) \cos^2(x) + \frac{2 \sin^2(x) \cos(x)}{3} + \frac{7 \cos^3(x)}{9}$$

input `integrate(x*cos(x)**3,x)`output `2*x*sin(x)**3/3 + x*sin(x)*cos(x)**2 + 2*sin(x)**2*cos(x)/3 + 7*cos(x)**3/9`**3.101.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int x \cos^3(x) dx = \frac{1}{12} x \sin(3x) + \frac{3}{4} x \sin(x) + \frac{1}{36} \cos(3x) + \frac{3}{4} \cos(x)$$

input `integrate(x*cos(x)^3,x, algorithm="maxima")`output `1/12*x*sin(3*x) + 3/4*x*sin(x) + 1/36*cos(3*x) + 3/4*cos(x)`

**3.101.8 Giac [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int x \cos^3(x) dx = \frac{1}{12} x \sin(3x) + \frac{3}{4} x \sin(x) + \frac{1}{36} \cos(3x) + \frac{3}{4} \cos(x)$$

input `integrate(x*cos(x)^3,x, algorithm="giac")`output `1/12*x*sin(3*x) + 3/4*x*sin(x) + 1/36*cos(3*x) + 3/4*cos(x)`**3.101.9 Mupad [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int x \cos^3(x) dx = \frac{\cos(x)^3}{9} + \frac{x \sin(x) \cos(x)^2}{3} + \frac{2 \cos(x)}{3} + \frac{2x \sin(x)}{3}$$

input `int(x*cos(x)^3,x)`output `(2*cos(x))/3 + cos(x)^3/9 + (2*x*sin(x))/3 + (x*cos(x)^2*sin(x))/3`

### 3.102 $\int \frac{\sin(x)}{x} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.102.1 Optimal result . . . . .                            | 622 |
| 3.102.2 Mathematica [A] (verified) . . . . .                | 622 |
| 3.102.3 Rubi [A] (verified) . . . . .                       | 623 |
| 3.102.4 Maple [A] (verified) . . . . .                      | 624 |
| 3.102.5 Fricas [A] (verification not implemented) . . . . . | 624 |
| 3.102.6 Sympy [A] (verification not implemented) . . . . .  | 624 |
| 3.102.7 Maxima [C] (verification not implemented) . . . . . | 625 |
| 3.102.8 Giac [A] (verification not implemented) . . . . .   | 625 |
| 3.102.9 Mupad [F(-1)] . . . . .                             | 625 |

#### 3.102.1 Optimal result

Integrand size = 6, antiderivative size = 2

$$\int \frac{\sin(x)}{x} dx = \text{Si}(x)$$

output

```
Si(x)
```

#### 3.102.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{x} dx = \text{Si}(x)$$

input `Integrate[Sin[x]/x,x]`

output `SinIntegral[x]`

**3.102.3 Rubi [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{x} dx$$

↓ 3042

$$\int \frac{\sin(x)}{x} dx$$

↓ 3780

$$\text{Si}(x)$$

input `Int[Sin[x]/x,x]`

output `SinIntegral[x]`

**3.102.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3780 `Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]`



**3.102.4 Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

| method  | result                                                 | size |
|---------|--------------------------------------------------------|------|
| default | $\text{Si}(x)$                                         | 3    |
| meijerg | $\text{Si}(x)$                                         | 3    |
| risch   | $-\frac{\pi \operatorname{csgn}(x)}{2} + \text{Si}(x)$ | 9    |

input `int(sin(x)/x,x,method=_RETURNVERBOSE)`

output `Si(x)`

**3.102.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{x} dx = \text{Si}(x)$$

input `integrate(sin(x)/x,x, algorithm="fricas")`

output `sin_integral(x)`

**3.102.6 Sympy [A] (verification not implemented)**

Time = 0.39 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{x} dx = \text{Si}(x)$$

input `integrate(sin(x)/x,x)`

output `Si(x)`

**3.102.7 Maxima [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{\sin(x)}{x} dx = -\frac{1}{2}i \operatorname{Ei}(ix) + \frac{1}{2}i \operatorname{Ei}(-ix)$$

input `integrate(sin(x)/x,x, algorithm="maxima")`

output `-1/2*I*Ei(I*x) + 1/2*I*Ei(-I*x)`

**3.102.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{x} dx = \operatorname{Si}(x)$$

input `integrate(sin(x)/x,x, algorithm="giac")`

output `sin_integral(x)`

**3.102.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{\sin(x)}{x} dx = \operatorname{sinint}(x)$$

input `int(sin(x)/x,x)`

output `sinint(x)`

### 3.103 $\int \frac{\cos(x)}{x} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.103.1 Optimal result . . . . .                            | 626 |
| 3.103.2 Mathematica [A] (verified) . . . . .                | 626 |
| 3.103.3 Rubi [A] (verified) . . . . .                       | 627 |
| 3.103.4 Maple [A] (verified) . . . . .                      | 628 |
| 3.103.5 Fricas [A] (verification not implemented) . . . . . | 628 |
| 3.103.6 Sympy [B] (verification not implemented) . . . . .  | 628 |
| 3.103.7 Maxima [C] (verification not implemented) . . . . . | 629 |
| 3.103.8 Giac [A] (verification not implemented) . . . . .   | 629 |
| 3.103.9 Mupad [F(-1)] . . . . .                             | 629 |

#### 3.103.1 Optimal result

Integrand size = 6, antiderivative size = 2

$$\int \frac{\cos(x)}{x} dx = \text{CosIntegral}(x)$$

output

```
Ci(x)
```

#### 3.103.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{x} dx = \text{CosIntegral}(x)$$

input `Integrate[Cos[x]/x,x]`

output `CosIntegral[x]`

**3.103.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \frac{\cos(x)}{x} dx \\ \downarrow \text{3042} \\ \int \frac{\sin(x + \frac{\pi}{2})}{x} dx \\ \downarrow \text{3783} \\ \text{CosIntegral}(x) \end{array}$$

input `Int[Cos[x]/x,x]`

output `CosIntegral[x]`

**3.103.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3783 `Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]`

**3.103.4 Maple [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

| method  | result                                                                                                                                                                                                                             | size |
|---------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default | $\text{Ci}(x)$                                                                                                                                                                                                                     | 3    |
| risch   | $\text{Ci}(x) - \frac{i\pi \operatorname{csgn}(ix) \operatorname{csgn}(x)}{2} + \frac{i\pi \operatorname{csgn}(ix)}{2}$                                                                                                            | 24   |
| meijerg | $\frac{\sqrt{\pi} \left( \frac{2\gamma + 2\ln(x)}{\sqrt{\pi}} - \frac{2\gamma}{\sqrt{\pi}} - \frac{2\ln(2)}{\sqrt{\pi}} - \frac{2\ln\left(\frac{x}{2}\right)}{\sqrt{\pi}} + \frac{2 \operatorname{Ci}(x)}{\sqrt{\pi}} \right)}{2}$ | 48   |

input `int(cos(x)/x,x,method=_RETURNVERBOSE)`output `Ci(x)`**3.103.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{x} dx = \text{Ci}(x)$$

input `integrate(cos(x)/x,x, algorithm="fricas")`output `cos_integral(x)`**3.103.6 Sympy [B] (verification not implemented)**Leaf count of result is larger than twice the leaf count of optimal. 12 vs.  $2(2) = 4$ .

Time = 0.63 (sec) , antiderivative size = 12, normalized size of antiderivative = 6.00

$$\int \frac{\cos(x)}{x} dx = -\log(x) + \frac{\log(x^2)}{2} + \text{Ci}(x)$$

input `integrate(cos(x)/x,x)`output `-log(x) + log(x**2)/2 + Ci(x)`

**3.103.7 Maxima [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{\cos(x)}{x} dx = \frac{1}{2} \operatorname{Ei}(ix) + \frac{1}{2} \operatorname{Ei}(-ix)$$

input `integrate(cos(x)/x,x, algorithm="maxima")`

output `1/2*Ei(I*x) + 1/2*Ei(-I*x)`

**3.103.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{x} dx = \operatorname{Ci}(x)$$

input `integrate(cos(x)/x,x, algorithm="giac")`

output `cos_integral(x)`

**3.103.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{\cos(x)}{x} dx = \operatorname{cosint}(x)$$

input `int(cos(x)/x,x)`

output `cosint(x)`

### 3.104 $\int \frac{\sin(x)}{x^2} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.104.1 Optimal result . . . . .                            | 630 |
| 3.104.2 Mathematica [A] (verified) . . . . .                | 630 |
| 3.104.3 Rubi [A] (verified) . . . . .                       | 631 |
| 3.104.4 Maple [A] (verified) . . . . .                      | 632 |
| 3.104.5 Fricas [A] (verification not implemented) . . . . . | 632 |
| 3.104.6 Sympy [B] (verification not implemented) . . . . .  | 633 |
| 3.104.7 Maxima [C] (verification not implemented) . . . . . | 633 |
| 3.104.8 Giac [A] (verification not implemented) . . . . .   | 633 |
| 3.104.9 Mupad [F(-1)] . . . . .                             | 634 |

#### 3.104.1 Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \frac{\sin(x)}{x^2} dx = \text{CosIntegral}(x) - \frac{\sin(x)}{x}$$

output `Ci(x)-sin(x)/x`

#### 3.104.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{x^2} dx = \text{CosIntegral}(x) - \frac{\sin(x)}{x}$$

input `Integrate[Sin[x]/x^2,x]`

output `CosIntegral[x] - Sin[x]/x`

**3.104.3 Rubi [A] (verified)**

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {3042, 3778, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c}
 \int \frac{\sin(x)}{x^2} dx \\
 \downarrow \text{3042} \\
 \int \frac{\sin(x)}{x^2} dx \\
 \downarrow \text{3778} \\
 \int \frac{\cos(x)}{x} dx - \frac{\sin(x)}{x} \\
 \downarrow \text{3042} \\
 \int \frac{\sin\left(x + \frac{\pi}{2}\right)}{x} dx - \frac{\sin(x)}{x} \\
 \downarrow \text{3783} \\
 \text{CosIntegral}(x) - \frac{\sin(x)}{x}
 \end{array}$$

input `Int[Sin[x]/x^2,x]`

output `CosIntegral[x] - Sin[x]/x`

**3.104.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 3778 `Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(c + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1)) Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]`



```
rule 3783 Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

### 3.104.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

| method  | result                                                                                                                                                                                                                                                                              | size |
|---------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default | $\text{Ci}(x) - \frac{\sin(x)}{x}$                                                                                                                                                                                                                                                  | 11   |
| risch   | $\text{Ci}(x) - \frac{i\pi \operatorname{csgn}(ix) \operatorname{csgn}(x)}{2} + \frac{i\pi \operatorname{csgn}(ix)}{2} - \frac{\sin(x)}{x}$                                                                                                                                         | 31   |
| meijerg | $\frac{\sqrt{\pi} \left( \frac{4\gamma - 4 + 4\ln(x)}{\sqrt{\pi}} + \frac{4}{\sqrt{\pi}} - \frac{4\gamma}{\sqrt{\pi}} - \frac{4\ln(2)}{\sqrt{\pi}} - \frac{4\ln\left(\frac{x}{2}\right)}{\sqrt{\pi}} - \frac{4\sin(x)}{\sqrt{\pi}x} + \frac{4\text{Ci}(x)}{\sqrt{\pi}} \right)}{4}$ | 65   |

```
input int(sin(x)/x^2,x,method=_RETURNVERBOSE)
```

```
output Ci(x)-sin(x)/x
```

### 3.104.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{\sin(x)}{x^2} dx = \frac{x \text{Ci}(x) - \sin(x)}{x}$$

```
input integrate(sin(x)/x^2,x, algorithm="fricas")
```

```
output (x*cos_integral(x) - sin(x))/x
```

**3.104.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 17 vs.  $2(7) = 14$ .

Time = 0.90 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{\sin(x)}{x^2} dx = -\log(x) + \frac{\log(x^2)}{2} + \text{Ci}(x) - \frac{\sin(x)}{x}$$

input `integrate(sin(x)/x**2,x)`

output `-log(x) + log(x**2)/2 + Ci(x) - sin(x)/x`

**3.104.7 Maxima [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{\sin(x)}{x^2} dx = \frac{1}{2} \Gamma(-1, ix) + \frac{1}{2} \Gamma(-1, -ix)$$

input `integrate(sin(x)/x^2,x, algorithm="maxima")`

output `1/2*gamma(-1, I*x) + 1/2*gamma(-1, -I*x)`

**3.104.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{\sin(x)}{x^2} dx = \frac{x \text{Ci}(x) - \sin(x)}{x}$$

input `integrate(sin(x)/x^2,x, algorithm="giac")`

output `(x*cos_integral(x) - sin(x))/x`

**3.104.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{\sin(x)}{x^2} dx = \operatorname{cosint}(x) - \frac{\sin(x)}{x}$$

input `int(sin(x)/x^2,x)`output `cosint(x) - sin(x)/x`

### 3.105 $\int \frac{\sin^2(x)}{x} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.105.1 Optimal result . . . . .                            | 635 |
| 3.105.2 Mathematica [A] (verified) . . . . .                | 635 |
| 3.105.3 Rubi [A] (verified) . . . . .                       | 636 |
| 3.105.4 Maple [A] (verified) . . . . .                      | 637 |
| 3.105.5 Fricas [A] (verification not implemented) . . . . . | 637 |
| 3.105.6 Sympy [A] (verification not implemented) . . . . .  | 637 |
| 3.105.7 Maxima [C] (verification not implemented) . . . . . | 638 |
| 3.105.8 Giac [A] (verification not implemented) . . . . .   | 638 |
| 3.105.9 Mupad [F(-1)] . . . . .                             | 638 |

#### 3.105.1 Optimal result

Integrand size = 8, antiderivative size = 15

$$\int \frac{\sin^2(x)}{x} dx = -\frac{\text{CosIntegral}(2x)}{2} + \frac{\log(x)}{2}$$

output `-1/2*Ci(2*x)+1/2*ln(x)`

#### 3.105.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(x)}{x} dx = -\frac{\text{CosIntegral}(2x)}{2} + \frac{\log(x)}{2}$$

input `Integrate[Sin[x]^2/x,x]`

output `-1/2*CosIntegral[2*x] + Log[x]/2`

**3.105.3 Rubi [A] (verified)**

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{\sin^2(x)}{x} dx \\ & \quad \downarrow \text{3042} \\ & \int \frac{\sin(x)^2}{x} dx \\ & \quad \downarrow \text{3793} \\ & \int \left( \frac{1}{2x} - \frac{\cos(2x)}{2x} \right) dx \\ & \quad \downarrow \text{2009} \\ & \frac{\log(x)}{2} - \frac{\text{CosIntegral}(2x)}{2} \end{aligned}$$

input `Int[Sin[x]^2/x,x]`

output `-1/2*CosIntegral[2*x] + Log[x]/2`

**3.105.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3793 `Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))`

**3.105.4 Maple [A] (verified)**

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

| method  | result                                                                                                                                                         | size |
|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default | $-\frac{\text{Ci}(2x)}{2} + \frac{\ln(x)}{2}$                                                                                                                  | 12   |
| risch   | $-\frac{\text{Ci}(2x)}{2} + \frac{i\pi \text{csgn}(ix) \text{csgn}(x)}{4} - \frac{i\pi \text{csgn}(ix)}{4} + \frac{\ln(x)}{2}$                                 | 32   |
| meijerg | $\frac{\sqrt{\pi} \left( \frac{2\gamma}{\sqrt{\pi}} + \frac{2\ln(2)}{\sqrt{\pi}} + \frac{2\ln(x)}{\sqrt{\pi}} - \frac{2\text{Ci}(2x)}{\sqrt{\pi}} \right)}{4}$ | 36   |

input `int(sin(x)^2/x,x,method=_RETURNVERBOSE)`output `-1/2*Ci(2*x)+1/2*ln(x)`**3.105.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\sin^2(x)}{x} dx = -\frac{1}{2} \text{Ci}(2x) + \frac{1}{2} \log(x)$$

input `integrate(sin(x)^2/x,x, algorithm="fricas")`output `-1/2*cos_integral(2*x) + 1/2*log(x)`**3.105.6 Sympy [A] (verification not implemented)**

Time = 0.66 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{\sin^2(x)}{x} dx = \frac{\log(x)}{2} - \frac{\text{Ci}(2x)}{2}$$

input `integrate(sin(x)**2/x,x)`output `log(x)/2 - Ci(2*x)/2`

**3.105.7 Maxima [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\sin^2(x)}{x} dx = -\frac{1}{4} \operatorname{Ei}(2i x) - \frac{1}{4} \operatorname{Ei}(-2i x) + \frac{1}{2} \log(x)$$

input `integrate(sin(x)^2/x,x, algorithm="maxima")`

output `-1/4*Ei(2*I*x) - 1/4*Ei(-2*I*x) + 1/2*log(x)`

**3.105.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\sin^2(x)}{x} dx = -\frac{1}{2} \operatorname{Ci}(2 x) + \frac{1}{2} \log(x)$$

input `integrate(sin(x)^2/x,x, algorithm="giac")`

output `-1/2*cos_integral(2*x) + 1/2*log(x)`

**3.105.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{\sin^2(x)}{x} dx = \frac{\ln(x)}{2} - \frac{\operatorname{cosint}(2 x)}{2}$$

input `int(sin(x)^2/x,x)`

output `log(x)/2 - cosint(2*x)/2`

## 3.106 $\int \tan^3(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.106.1 Optimal result . . . . .                            | 639 |
| 3.106.2 Mathematica [A] (verified) . . . . .                | 639 |
| 3.106.3 Rubi [A] (verified) . . . . .                       | 640 |
| 3.106.4 Maple [A] (verified) . . . . .                      | 641 |
| 3.106.5 Fricas [A] (verification not implemented) . . . . . | 641 |
| 3.106.6 Sympy [A] (verification not implemented) . . . . .  | 642 |
| 3.106.7 Maxima [A] (verification not implemented) . . . . . | 642 |
| 3.106.8 Giac [A] (verification not implemented) . . . . .   | 642 |
| 3.106.9 Mupad [B] (verification not implemented) . . . . .  | 643 |

### 3.106.1 Optimal result

Integrand size = 4, antiderivative size = 12

$$\int \tan^3(x) dx = \log(\cos(x)) + \frac{\tan^2(x)}{2}$$

output `ln(cos(x))+1/2*tan(x)^2`

### 3.106.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \tan^3(x) dx = \log(\cos(x)) + \frac{\tan^2(x)}{2}$$

input `Integrate[Tan[x]^3,x]`

output `Log[Cos[x]] + Tan[x]^2/2`



**3.106.3 Rubi [A] (verified)**

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c}
 \int \tan^3(x) dx \\
 \downarrow \text{3042} \\
 \int \tan(x)^3 dx \\
 \downarrow \text{3954} \\
 \frac{\tan^2(x)}{2} - \int \tan(x) dx \\
 \downarrow \text{3042} \\
 \frac{\tan^2(x)}{2} - \int \tan(x) dx \\
 \downarrow \text{3956} \\
 \frac{\tan^2(x)}{2} + \log(\cos(x))
 \end{array}$$

input `Int[Tan[x]^3,x]`

output `Log[Cos[x]] + Tan[x]^2/2`

**3.106.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3954 `Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2 Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]`

rule 3956 `Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

### 3.106.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

| method           | result                                                    | size |
|------------------|-----------------------------------------------------------|------|
| derivativdivides | $\frac{\tan^2(x)}{2} - \frac{\ln(1+\tan^2(x))}{2}$        | 17   |
| default          | $\frac{\tan^2(x)}{2} - \frac{\ln(1+\tan^2(x))}{2}$        | 17   |
| norman           | $\frac{\tan^2(x)}{2} - \frac{\ln(1+\tan^2(x))}{2}$        | 17   |
| parallelrisc     | $\frac{\tan^2(x)}{2} - \frac{\ln(1+\tan^2(x))}{2}$        | 17   |
| risc             | $-ix + \frac{2e^{2ix}}{(e^{2ix}+1)^2} + \ln(e^{2ix} + 1)$ | 30   |

input `int(tan(x)^3,x,method=_RETURNVERBOSE)`

output `1/2*tan(x)^2-1/2*ln(1+tan(x)^2)`

### 3.106.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \tan^3(x) dx = \frac{1}{2} \tan(x)^2 + \frac{1}{2} \log\left(\frac{1}{\tan(x)^2 + 1}\right)$$

input `integrate(tan(x)^3,x, algorithm="fricas")`

output `1/2*tan(x)^2 + 1/2*log(1/(tan(x)^2 + 1))`

**3.106.6 Sympy [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \tan^3(x) dx = \log(\cos(x)) + \frac{1}{2\cos^2(x)}$$

input `integrate(tan(x)**3,x)`output `log(cos(x)) + 1/(2*cos(x)**2)`**3.106.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \tan^3(x) dx = -\frac{1}{2(\sin(x)^2 - 1)} + \frac{1}{2} \log(\sin(x)^2 - 1)$$

input `integrate(tan(x)^3,x, algorithm="maxima")`output `-1/2/(sin(x)^2 - 1) + 1/2*log(sin(x)^2 - 1)`**3.106.8 Giac [A] (verification not implemented)**

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \tan^3(x) dx = \frac{1}{2} \tan(x)^2 - \frac{1}{2} \log(\tan(x)^2 + 1)$$

input `integrate(tan(x)^3,x, algorithm="giac")`output `1/2*tan(x)^2 - 1/2*log(tan(x)^2 + 1)`

**3.106.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \tan^3(x) dx = \ln(\cos(x)) - \frac{\cos(x)^2 - 1}{2\cos(x)^2}$$

input `int(tan(x)^3,x)`

output `log(cos(x)) - (cos(x)^2 - 1)/(2*cos(x)^2)`

### 3.107 $\int \sin(a + bx) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.107.1 Optimal result . . . . .                            | 644 |
| 3.107.2 Mathematica [A] (verified) . . . . .                | 644 |
| 3.107.3 Rubi [A] (verified) . . . . .                       | 645 |
| 3.107.4 Maple [A] (verified) . . . . .                      | 646 |
| 3.107.5 Fricas [A] (verification not implemented) . . . . . | 646 |
| 3.107.6 Sympy [A] (verification not implemented) . . . . .  | 647 |
| 3.107.7 Maxima [A] (verification not implemented) . . . . . | 647 |
| 3.107.8 Giac [A] (verification not implemented) . . . . .   | 647 |
| 3.107.9 Mupad [B] (verification not implemented) . . . . .  | 648 |

#### 3.107.1 Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \sin(a + bx) dx = -\frac{\cos(a + bx)}{b}$$

output `-cos(b*x+a)/b`

#### 3.107.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \sin(a + bx) dx = -\frac{\cos(a) \cos(bx)}{b} + \frac{\sin(a) \sin(bx)}{b}$$

input `Integrate[Sin[a + b*x],x]`

output `-((Cos[a]*Cos[b*x])/b) + (Sin[a]*Sin[b*x])/b`

**3.107.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \sin(a + bx) dx \\ \downarrow \text{3042} \\ \int \sin(a + bx) dx \\ \downarrow \text{3118} \\ -\frac{\cos(a + bx)}{b} \end{array}$$

input `Int[Sin[a + b*x],x]`

output `-(Cos[a + b*x]/b)`

**3.107.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 3118 `Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

**3.107.4 Maple [A] (verified)**

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

| method            | result                                                                                                                          | size |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------|------|
| derivativedivides | $-\frac{\cos(bx+a)}{b}$                                                                                                         | 12   |
| default           | $-\frac{\cos(bx+a)}{b}$                                                                                                         | 12   |
| risch             | $-\frac{\cos(bx+a)}{b}$                                                                                                         | 12   |
| parallelrisch     | $\frac{-\cos(bx+a)-1}{b}$                                                                                                       | 15   |
| norman            | $\frac{2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$ | 32   |
| meijerg           | $\frac{\sin(a)\sin(bx)}{b} + \frac{\cos(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(bx)}{\sqrt{\pi}}\right)}{b}$          | 34   |

input `int(sin(b*x+a),x,method=_RETURNVERBOSE)`output  `-cos(b*x+a)/b`**3.107.5 Fricas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a + bx) dx = -\frac{\cos(bx + a)}{b}$$

input `integrate(sin(b*x+a),x, algorithm="fricas")`output  `-cos(b*x + a)/b`

**3.107.6 Sympy [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \sin(a + bx) dx = \begin{cases} -\frac{\cos(a+bx)}{b} & \text{for } b \neq 0 \\ x \sin(a) & \text{otherwise} \end{cases}$$

input `integrate(sin(b*x+a),x)`output `Piecewise((-cos(a + b*x)/b, Ne(b, 0)), (x*sin(a), True))`**3.107.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a + bx) dx = -\frac{\cos(bx + a)}{b}$$

input `integrate(sin(b*x+a),x, algorithm="maxima")`output `-cos(b*x + a)/b`**3.107.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a + bx) dx = -\frac{\cos(bx + a)}{b}$$

input `integrate(sin(b*x+a),x, algorithm="giac")`output `-cos(b*x + a)/b`



**3.107.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a + bx) dx = -\frac{\cos(a + bx)}{b}$$

input `int(sin(a + b*x),x)`

output `-cos(a + b*x)/b`

## 3.108 $\int \cos(a + bx) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.108.1 Optimal result . . . . .                            | 649 |
| 3.108.2 Mathematica [B] (verified) . . . . .                | 649 |
| 3.108.3 Rubi [A] (verified) . . . . .                       | 650 |
| 3.108.4 Maple [A] (verified) . . . . .                      | 651 |
| 3.108.5 Fricas [A] (verification not implemented) . . . . . | 651 |
| 3.108.6 Sympy [A] (verification not implemented) . . . . .  | 652 |
| 3.108.7 Maxima [A] (verification not implemented) . . . . . | 652 |
| 3.108.8 Giac [A] (verification not implemented) . . . . .   | 652 |
| 3.108.9 Mupad [B] (verification not implemented) . . . . .  | 653 |

### 3.108.1 Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \cos(a + bx) dx = \frac{\sin(a + bx)}{b}$$

output `sin(b*x+a)/b`

### 3.108.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \cos(a + bx) dx = \frac{\cos(bx) \sin(a)}{b} + \frac{\cos(a) \sin(bx)}{b}$$

input `Integrate[Cos[a + b*x],x]`

output `(Cos[b*x]*Sin[a])/b + (Cos[a]*Sin[b*x])/b`

**3.108.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \cos(a + bx) dx \\ \downarrow \text{3042} \\ \int \sin\left(a + bx + \frac{\pi}{2}\right) dx \\ \downarrow \text{3117} \\ \frac{\sin(a + bx)}{b} \end{array}$$

input `Int[Cos[a + b*x],x]`

output `Sin[a + b*x]/b`

**3.108.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 3117 `Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

**3.108.4 Maple [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

| method            | result                                                                                                                    | size |
|-------------------|---------------------------------------------------------------------------------------------------------------------------|------|
| derivativedivides | $\frac{\sin(bx+a)}{b}$                                                                                                    | 11   |
| default           | $\frac{\sin(bx+a)}{b}$                                                                                                    | 11   |
| risch             | $\frac{\sin(bx+a)}{b}$                                                                                                    | 11   |
| parallelrisch     | $\frac{\sin(bx+a)}{b}$                                                                                                    | 11   |
| norman            | $\frac{2 \tan\left(\frac{bx+a}{2}\right)}{b\left(1+\tan^2\left(\frac{bx+a}{2}\right)\right)}$                             | 30   |
| meijerg           | $\frac{\cos(a) \sin(bx)}{b} - \frac{\sin(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}} - \frac{\cos(bx)}{\sqrt{\pi}}\right)}{b}$ | 35   |

input `int(cos(b*x+a),x,method=_RETURNVERBOSE)`output `sin(b*x+a)/b`**3.108.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(a + bx) dx = \frac{\sin(bx + a)}{b}$$

input `integrate(cos(b*x+a),x, algorithm="fricas")`output `sin(b*x + a)/b`

**3.108.6 Sympy [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \cos(a + bx) dx = \begin{cases} \frac{\sin(a+bx)}{b} & \text{for } b \neq 0 \\ x \cos(a) & \text{otherwise} \end{cases}$$

input `integrate(cos(b*x+a),x)`output `Piecewise((sin(a + b*x)/b, Ne(b, 0)), (x*cos(a), True))`**3.108.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(a + bx) dx = \frac{\sin(bx + a)}{b}$$

input `integrate(cos(b*x+a),x, algorithm="maxima")`output `sin(b*x + a)/b`**3.108.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(a + bx) dx = \frac{\sin(bx + a)}{b}$$

input `integrate(cos(b*x+a),x, algorithm="giac")`output `sin(b*x + a)/b`

**3.108.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(a + bx) dx = \frac{\sin(a + bx)}{b}$$

input `int(cos(a + b*x),x)`

output `sin(a + b*x)/b`

### 3.109 $\int \tan(a + bx) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.109.1 Optimal result . . . . .                            | 654 |
| 3.109.2 Mathematica [A] (verified) . . . . .                | 654 |
| 3.109.3 Rubi [A] (verified) . . . . .                       | 655 |
| 3.109.4 Maple [A] (verified) . . . . .                      | 656 |
| 3.109.5 Fricas [A] (verification not implemented) . . . . . | 656 |
| 3.109.6 Sympy [A] (verification not implemented) . . . . .  | 656 |
| 3.109.7 Maxima [A] (verification not implemented) . . . . . | 657 |
| 3.109.8 Giac [A] (verification not implemented) . . . . .   | 657 |
| 3.109.9 Mupad [B] (verification not implemented) . . . . .  | 657 |

#### 3.109.1 Optimal result

Integrand size = 6, antiderivative size = 12

$$\int \tan(a + bx) dx = -\frac{\log(\cos(a + bx))}{b}$$

output `-ln(cos(b*x+a))/b`

#### 3.109.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \tan(a + bx) dx = -\frac{\log(\cos(a + bx))}{b}$$

input `Integrate[Tan[a + b*x],x]`

output `-(Log[Cos[a + b*x]]/b)`

**3.109.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \tan(a + bx) dx \\ \downarrow \text{3042} \\ \int \tan(a + bx) dx \\ \downarrow \text{3956} \\ -\frac{\log(\cos(a + bx))}{b} \end{array}$$

input `Int[Tan[a + b*x],x]`

output `-(Log[Cos[a + b*x]]/b)`

**3.109.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 3956 `Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d *x], x]]/d, x] /; FreeQ[{c, d}, x]`



**3.109.4 Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

| method            | result                                               | size |
|-------------------|------------------------------------------------------|------|
| derivativedivides | $\frac{\ln(1+\tan^2(bx+a))}{2b}$                     | 17   |
| default           | $\frac{\ln(1+\tan^2(bx+a))}{2b}$                     | 17   |
| norman            | $\frac{\ln(1+\tan^2(bx+a))}{2b}$                     | 17   |
| parallelrisch     | $\frac{\ln(1+\tan^2(bx+a))}{2b}$                     | 17   |
| risch             | $ix + \frac{2ia}{b} - \frac{\ln(e^{2i(bx+a)}+1)}{b}$ | 30   |

input `int(tan(b*x+a),x,method=_RETURNVERBOSE)`output `1/2/b*ln(1+tan(b*x+a)^2)`**3.109.5 Fricas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \tan(a + bx) dx = -\frac{\log\left(\frac{1}{\tan(bx+a)^2+1}\right)}{2b}$$

input `integrate(tan(b*x+a),x, algorithm="fricas")`output `-1/2*log(1/(tan(b*x + a)^2 + 1))/b`**3.109.6 Sympy [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \tan(a + bx) dx = \begin{cases} \frac{\log(\tan^2(a+bx)+1)}{2b} & \text{for } b \neq 0 \\ x \tan(a) & \text{otherwise} \end{cases}$$

input `integrate(tan(b*x+a),x)`

output `Piecewise((log(tan(a + b*x)**2 + 1)/(2*b), Ne(b, 0)), (x*tan(a), True))`

### 3.109.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \tan(a + bx) dx = \frac{\log(\sec(bx + a))}{b}$$

input `integrate(tan(b*x+a),x, algorithm="maxima")`

output `log(sec(b*x + a))/b`

### 3.109.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \tan(a + bx) dx = -\frac{\log(|\cos(bx + a)|)}{b}$$

input `integrate(tan(b*x+a),x, algorithm="giac")`

output `-log(abs(cos(b*x + a)))/b`

### 3.109.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \tan(a + bx) dx = \frac{\ln(\tan(a + bx)^2 + 1)}{2b}$$

input `int(tan(a + b*x),x)`

output `log(tan(a + b*x)^2 + 1)/(2*b)`

### 3.110 $\int \cot(a + bx) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.110.1 Optimal result . . . . .                            | 658 |
| 3.110.2 Mathematica [B] (verified) . . . . .                | 658 |
| 3.110.3 Rubi [A] (verified) . . . . .                       | 659 |
| 3.110.4 Maple [B] (verified) . . . . .                      | 660 |
| 3.110.5 Fricas [B] (verification not implemented) . . . . . | 660 |
| 3.110.6 Sympy [B] (verification not implemented) . . . . .  | 661 |
| 3.110.7 Maxima [A] (verification not implemented) . . . . . | 661 |
| 3.110.8 Giac [B] (verification not implemented) . . . . .   | 661 |
| 3.110.9 Mupad [B] (verification not implemented) . . . . .  | 662 |

#### 3.110.1 Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \cot(a + bx) dx = \frac{\log(\sin(a + bx))}{b}$$

output `ln(sin(b*x+a))/b`

#### 3.110.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \cot(a + bx) dx = \frac{\log(\cos(a + bx))}{b} + \frac{\log(\tan(a + bx))}{b}$$

input `Integrate[Cot[a + b*x],x]`

output `Log[Cos[a + b*x]]/b + Log[Tan[a + b*x]]/b`

**3.110.3 Rubi [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \cot(a + bx) dx \\ & \quad \downarrow \text{3042} \\ & \int -\tan\left(a + bx + \frac{\pi}{2}\right) dx \\ & \quad \downarrow \text{25} \\ & -\int \tan\left(\frac{1}{2}(2a + \pi) + bx\right) dx \\ & \quad \downarrow \text{3956} \\ & \frac{\log(-\sin(a + bx))}{b} \end{aligned}$$

input `Int[Cot[a + b*x],x]`

output `Log[-Sin[a + b*x]]/b`

**3.110.3.1 Defintions of rubi rules used**

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 3956 `Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

**3.110.4 Maple [B] (verified)**

Leaf count of result is larger than twice the leaf count of optimal. 25 vs.  $2(11) = 22$ .

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

| method            | result                                                       | size |
|-------------------|--------------------------------------------------------------|------|
| derivativedivides | $\frac{\ln(\tan(bx+a)) - \frac{\ln(1+\tan^2(bx+a))}{2}}{b}$  | 26   |
| default           | $\frac{\ln(\tan(bx+a)) - \frac{\ln(1+\tan^2(bx+a))}{2}}{b}$  | 26   |
| norman            | $\frac{\ln(\tan(bx+a))}{b} - \frac{\ln(1+\tan^2(bx+a))}{2b}$ | 29   |
| risch             | $-ix - \frac{2ia}{b} + \frac{\ln(e^{2i(bx+a)} - 1)}{b}$      | 29   |
| parallelrisch     | $\frac{2\ln(\tan(bx+a)) - \ln(1+\tan^2(bx+a))}{2b}$          | 29   |

input `int(1/tan(b*x+a),x,method=_RETURNVERBOSE)`

output `1/b*(ln(tan(b*x+a))-1/2*ln(1+tan(b*x+a)^2))`

**3.110.5 Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 27 vs.  $2(11) = 22$ .

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int \cot(a + bx) dx = \frac{\log\left(\frac{\tan(bx+a)^2}{\tan(bx+a)^2+1}\right)}{2b}$$

input `integrate(1/tan(b*x+a),x, algorithm="fricas")`

output `1/2*log(tan(b*x + a)^2/(tan(b*x + a)^2 + 1))/b`

**3.110.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 29 vs.  $2(8) = 16$ .

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.64

$$\int \cot(a + bx) dx = \begin{cases} -\frac{\log(\tan^2(a+bx)+1)}{2b} + \frac{\log(\tan(a+bx))}{b} & \text{for } b \neq 0 \\ \frac{x}{\tan(a)} & \text{otherwise} \end{cases}$$

input `integrate(1/tan(b*x+a),x)`

output `Piecewise((-log(tan(a + b*x)**2 + 1)/(2*b) + log(tan(a + b*x))/b, Ne(b, 0)), (x/tan(a), True))`

**3.110.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot(a + bx) dx = \frac{\log(\sin(bx + a))}{b}$$

input `integrate(1/tan(b*x+a),x, algorithm="maxima")`

output `log(sin(b*x + a))/b`

**3.110.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 56 vs.  $2(11) = 22$ .

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 5.09

$$\int \cot(a + bx) dx = \frac{\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) - 2 \log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1} + 1\right|\right)}{2b}$$

input `integrate(1/tan(b*x+a),x, algorithm="giac")`

output `1/2*(log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 2*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b`

**3.110.9 Mupad [B] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \cot(a + bx) dx = \frac{\ln(\tan(a + bx))}{b} - \frac{\ln(\tan(a + bx)^2 + 1)}{2b}$$

input `int(1/tan(a + b*x),x)`

output `log(tan(a + b*x))/b - log(tan(a + b*x)^2 + 1)/(2*b)`

### 3.111 $\int \csc(a + bx) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.111.1 Optimal result . . . . .                            | 663 |
| 3.111.2 Mathematica [B] (verified) . . . . .                | 663 |
| 3.111.3 Rubi [A] (verified) . . . . .                       | 664 |
| 3.111.4 Maple [A] (verified) . . . . .                      | 665 |
| 3.111.5 Fricas [B] (verification not implemented) . . . . . | 665 |
| 3.111.6 Sympy [A] (verification not implemented) . . . . .  | 666 |
| 3.111.7 Maxima [B] (verification not implemented) . . . . . | 666 |
| 3.111.8 Giac [B] (verification not implemented) . . . . .   | 666 |
| 3.111.9 Mupad [B] (verification not implemented) . . . . .  | 667 |

#### 3.111.1 Optimal result

Integrand size = 6, antiderivative size = 12

$$\int \csc(a + bx) dx = -\frac{\operatorname{arctanh}(\cos(a + bx))}{b}$$

output `-arctanh(cos(b*x+a))/b`

#### 3.111.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 38 vs.  $2(12) = 24$ .

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.17

$$\int \csc(a + bx) dx = -\frac{\log\left(\cos\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b} + \frac{\log\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b}$$

input `Integrate[Csc[a + b*x],x]`

output `-(Log[Cos[a/2 + (b*x)/2]]/b) + Log[Sin[a/2 + (b*x)/2]]/b`



**3.111.3 Rubi [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \csc(a + bx) dx \\ \downarrow \text{3042} \\ \int \csc(a + bx) dx \\ \downarrow \text{4257} \\ -\frac{\operatorname{arctanh}(\cos(a + bx))}{b} \end{array}$$

input `Int[Csc[a + b*x],x]`

output `-(ArcTanh[Cos[a + b*x]]/b)`

**3.111.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 4257 `Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

**3.111.4 Maple [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

| method            | result                                                                  | size |
|-------------------|-------------------------------------------------------------------------|------|
| norman            | $\frac{\ln\left(\tan\left(\frac{bx}{2} + \frac{a}{2}\right)\right)}{b}$ | 15   |
| parallelrisch     | $\frac{\ln\left(\tan\left(\frac{bx}{2} + \frac{a}{2}\right)\right)}{b}$ | 15   |
| derivativedivides | $\frac{\ln(\csc(bx+a) - \cot(bx+a))}{b}$                                | 21   |
| default           | $\frac{\ln(\csc(bx+a) - \cot(bx+a))}{b}$                                | 21   |
| risch             | $-\frac{\ln(e^{i(bx+a)}+1)}{b} + \frac{\ln(e^{i(bx+a)}-1)}{b}$          | 35   |

input `int(1/sin(b*x+a),x,method=_RETURNVERBOSE)`

output `1/b*ln(tan(1/2*b*x+1/2*a))`

**3.111.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 30 vs.  $2(12) = 24$ .

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.50

$$\int \csc(a + bx) dx = -\frac{\log\left(\frac{1}{2} \cos(bx + a) + \frac{1}{2}\right) - \log\left(-\frac{1}{2} \cos(bx + a) + \frac{1}{2}\right)}{2b}$$

input `integrate(1/sin(b*x+a),x, algorithm="fracas")`

output `-1/2*(log(1/2*cos(b*x + a) + 1/2) - log(-1/2*cos(b*x + a) + 1/2))/b`

**3.111.6 Sympy [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \csc(a + bx) dx = \begin{cases} \frac{\log\left(\tan\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b} & \text{for } b \neq 0 \\ \frac{x}{\sin(a)} & \text{otherwise} \end{cases}$$

input `integrate(1/sin(b*x+a),x)`

output `Piecewise((log(tan(a/2 + b*x/2))/b, Ne(b, 0)), (x/sin(a), True))`

**3.111.7 Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \csc(a + bx) dx = -\frac{\log(\cos(bx + a) + 1) - \log(\cos(bx + a) - 1)}{2b}$$

input `integrate(1/sin(b*x+a),x, algorithm="maxima")`

output `-1/2*(log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b`

**3.111.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 4.25

$$\int \csc(a + bx) dx = \frac{\log\left(\left|-\frac{\cos(bx+a)}{b} + \frac{1}{|b|}\right|\right)}{2|b|} - \frac{\log\left(\left|-\frac{\cos(bx+a)}{b} - \frac{1}{|b|}\right|\right)}{2|b|}$$

input `integrate(1/sin(b*x+a),x, algorithm="giac")`

output `1/2*log(abs(-cos(b*x + a)/b + 1/abs(b)))/abs(b) - 1/2*log(abs(-cos(b*x + a)/b - 1/abs(b)))/abs(b)`

**3.111.9 Mupad [B] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \csc(a + bx) dx = -\frac{\operatorname{atanh}(\cos(a + bx))}{b}$$

input `int(1/sin(a + b*x),x)`

output `-atanh(cos(a + b*x))/b`

## 3.112 $\int \sec(a + bx) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.112.1 Optimal result . . . . .                            | 668 |
| 3.112.2 Mathematica [A] (verified) . . . . .                | 668 |
| 3.112.3 Rubi [A] (verified) . . . . .                       | 669 |
| 3.112.4 Maple [A] (verified) . . . . .                      | 670 |
| 3.112.5 Fricas [B] (verification not implemented) . . . . . | 670 |
| 3.112.6 Sympy [B] (verification not implemented) . . . . .  | 671 |
| 3.112.7 Maxima [B] (verification not implemented) . . . . . | 671 |
| 3.112.8 Giac [B] (verification not implemented) . . . . .   | 671 |
| 3.112.9 Mupad [B] (verification not implemented) . . . . .  | 672 |

### 3.112.1 Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \sec(a + bx) dx = \frac{\operatorname{arctanh}(\sin(a + bx))}{b}$$

output `arctanh(sin(b*x+a))/b`

### 3.112.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sec(a + bx) dx = \frac{\operatorname{arctanh}(\sin(a + bx))}{b}$$

input `Integrate[Sec[a + b*x],x]`

output `ArcTanh[Sin[a + b*x]]/b`

**3.112.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \sec(a + bx) dx \\ \downarrow \text{3042} \\ \int \csc\left(a + bx + \frac{\pi}{2}\right) dx \\ \downarrow \text{4257} \\ \frac{\operatorname{arctanh}(\sin(a + bx))}{b} \end{array}$$

input `Int[Sec[a + b*x],x]`

output `ArcTanh[Sin[a + b*x]]/b`

**3.112.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 4257 `Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

**3.112.4 Maple [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

| method            | result                                                                                                                                          | size |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|------|
| derivativedivides | $\frac{\ln(\sec(bx+a)+\tan(bx+a))}{b}$                                                                                                          | 19   |
| default           | $\frac{\ln(\sec(bx+a)+\tan(bx+a))}{b}$                                                                                                          | 19   |
| parallelrisc      | $\frac{-\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}$            | 32   |
| norman            | $\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b} - \frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}$ | 35   |
| risc              | $\frac{\ln(e^{i(bx+a)}+i)}{b} - \frac{\ln(e^{i(bx+a)}-i)}{b}$                                                                                   | 37   |

input `int(1/cos(b*x+a),x,method=_RETURNVERBOSE)`

output `1/b*ln(sec(b*x+a)+tan(b*x+a))`

**3.112.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \sec(a + bx) dx = \frac{\log(\sin(bx + a) + 1) - \log(-\sin(bx + a) + 1)}{2b}$$

input `integrate(1/cos(b*x+a),x, algorithm="fracas")`

output `1/2*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b`

**3.112.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 34 vs.  $2(8) = 16$ .

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.09

$$\int \sec(a + bx) dx = \begin{cases} -\frac{\log\left(\tan\left(\frac{a}{2} + \frac{bx}{2}\right) - 1\right)}{b} + \frac{\log\left(\tan\left(\frac{a}{2} + \frac{bx}{2}\right) + 1\right)}{b} & \text{for } b \neq 0 \\ \frac{x}{\cos(a)} & \text{otherwise} \end{cases}$$

input `integrate(1/cos(b*x+a),x)`

output `Piecewise((-log(tan(a/2 + b*x/2) - 1)/b + log(tan(a/2 + b*x/2) + 1)/b, Ne(b, 0)), (x/cos(a), True))`

**3.112.7 Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 26 vs.  $2(11) = 22$ .

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int \sec(a + bx) dx = \frac{\log(\sin(bx + a) + 1) - \log(\sin(bx + a) - 1)}{2b}$$

input `integrate(1/cos(b*x+a),x, algorithm="maxima")`

output `1/2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b`

**3.112.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 28 vs.  $2(11) = 22$ .

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \sec(a + bx) dx = \frac{\log(|\sin(bx + a) + 1|) - \log(|\sin(bx + a) - 1|)}{2b}$$

input `integrate(1/cos(b*x+a),x, algorithm="giac")`

output `1/2*(log(abs(sin(b*x + a) + 1)) - log(abs(sin(b*x + a) - 1)))/b`



**3.112.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sec(a + bx) dx = \frac{\operatorname{atanh}(\sin(a + bx))}{b}$$

input `int(1/cos(a + b*x),x)`

output `atanh(sin(a + b*x))/b`

### 3.113 $\int \sin^2(a + bx) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.113.1 Optimal result . . . . .                            | 673 |
| 3.113.2 Mathematica [A] (verified) . . . . .                | 673 |
| 3.113.3 Rubi [A] (verified) . . . . .                       | 674 |
| 3.113.4 Maple [A] (verified) . . . . .                      | 675 |
| 3.113.5 Fricas [A] (verification not implemented) . . . . . | 675 |
| 3.113.6 Sympy [B] (verification not implemented) . . . . .  | 675 |
| 3.113.7 Maxima [A] (verification not implemented) . . . . . | 676 |
| 3.113.8 Giac [A] (verification not implemented) . . . . .   | 676 |
| 3.113.9 Mupad [B] (verification not implemented) . . . . .  | 676 |

#### 3.113.1 Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \sin^2(a + bx) dx = \frac{x}{2} - \frac{\cos(a + bx) \sin(a + bx)}{2b}$$

output `1/2*x-1/2*cos(b*x+a)*sin(b*x+a)/b`

#### 3.113.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sin^2(a + bx) dx = -\frac{-2(a + bx) + \sin(2(a + bx))}{4b}$$

input `Integrate[Sin[a + b*x]^2,x]`

output `-1/4*(-2*(a + b*x) + Sin[2*(a + b*x)])/b`

**3.113.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \sin^2(a + bx) dx \\ & \quad \downarrow \text{3042} \\ & \int \sin(a + bx)^2 dx \\ & \quad \downarrow \text{3115} \\ & \frac{\int 1 dx}{2} - \frac{\sin(a + bx) \cos(a + bx)}{2b} \\ & \quad \downarrow \text{24} \\ & \frac{x}{2} - \frac{\sin(a + bx) \cos(a + bx)}{2b} \end{aligned}$$

input `Int[Sin[a + b*x]^2,x]`

output `x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)`

**3.113.3.1 Defintions of rubi rules used**

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3115 `Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sine[c + d*x])^(n - 1)/(d*n), x] + Simp[b^2*((n - 1)/n) Int[(b*Sine[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

**3.113.4 Maple [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

| method            | result                                                                                                                                                                                                                                                                                                                                      | size |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch             | $\frac{x}{2} - \frac{\sin(2bx+2a)}{4b}$                                                                                                                                                                                                                                                                                                     | 19   |
| parallelrisch     | $\frac{2bx - \sin(2bx+2a)}{4b}$                                                                                                                                                                                                                                                                                                             | 22   |
| derivativedivides | $-\frac{\cos(bx+a)\sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2}$<br>$b$                                                                                                                                                                                                                                                                       | 27   |
| default           | $-\frac{\cos(bx+a)\sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2}$<br>$b$                                                                                                                                                                                                                                                                       | 27   |
| norman            | $\frac{\tan^3\left(\frac{bx}{2} + \frac{a}{2}\right)}{b} + x\left(\tan^2\left(\frac{bx}{2} + \frac{a}{2}\right)\right) + \frac{x}{2} - \frac{\tan\left(\frac{bx}{2} + \frac{a}{2}\right)}{b} + \frac{x\left(\tan^4\left(\frac{bx}{2} + \frac{a}{2}\right)\right)}{2}$<br>$\left(1 + \tan^2\left(\frac{bx}{2} + \frac{a}{2}\right)\right)^2$ | 77   |

input `int(sin(b*x+a)^2,x,method=_RETURNVERBOSE)`output `1/2*x-1/4/b*sin(2*b*x+2*a)`**3.113.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sin^2(a + bx) dx = \frac{bx - \cos(bx + a)\sin(bx + a)}{2b}$$

input `integrate(sin(b*x+a)^2,x, algorithm="fracas")`output `1/2*(b*x - cos(b*x + a)*sin(b*x + a))/b`**3.113.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(19) = 38.

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \sin^2(a + bx) dx = \begin{cases} \frac{x \sin^2(a+bx)}{2} + \frac{x \cos^2(a+bx)}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x \sin^2(a) & \text{otherwise} \end{cases}$$

input `integrate(sin(b*x+a)**2,x)`

output `Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*sin(a)**2, True))`

### 3.113.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \sin^2(a + bx) dx = \frac{2bx + 2a - \sin(2bx + 2a)}{4b}$$

input `integrate(sin(b*x+a)^2,x, algorithm="maxima")`

output `1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))/b`

### 3.113.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \sin^2(a + bx) dx = \frac{1}{2}x - \frac{\sin(2bx + 2a)}{4b}$$

input `integrate(sin(b*x+a)^2,x, algorithm="giac")`

output `1/2*x - 1/4*sin(2*b*x + 2*a)/b`

### 3.113.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \sin^2(a + bx) dx = \frac{x}{2} - \frac{\sin(2a + 2bx)}{4b}$$

input `int(sin(a + b*x)^2,x)`

output `x/2 - sin(2*a + 2*b*x)/(4*b)`

### 3.114 $\int \sin^3(a + bx) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.114.1 Optimal result . . . . .                            | 677 |
| 3.114.2 Mathematica [A] (verified) . . . . .                | 677 |
| 3.114.3 Rubi [A] (verified) . . . . .                       | 678 |
| 3.114.4 Maple [A] (verified) . . . . .                      | 679 |
| 3.114.5 Fricas [A] (verification not implemented) . . . . . | 679 |
| 3.114.6 Sympy [A] (verification not implemented) . . . . .  | 679 |
| 3.114.7 Maxima [A] (verification not implemented) . . . . . | 680 |
| 3.114.8 Giac [A] (verification not implemented) . . . . .   | 680 |
| 3.114.9 Mupad [B] (verification not implemented) . . . . .  | 680 |

#### 3.114.1 Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \sin^3(a + bx) dx = -\frac{\cos(a + bx)}{b} + \frac{\cos^3(a + bx)}{3b}$$

output `-cos(b*x+a)/b+1/3*cos(b*x+a)^3/b`

#### 3.114.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \sin^3(a + bx) dx = -\frac{3 \cos(a + bx)}{4b} + \frac{\cos(3(a + bx))}{12b}$$

input `Integrate[Sin[a + b*x]^3,x]`

output `(-3*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b)`

**3.114.3 Rubi [A] (verified)**

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \sin^3(a + bx) dx \\ & \quad \downarrow \text{3042} \\ & \int \sin(a + bx)^3 dx \\ & \quad \downarrow \text{3113} \\ & -\frac{\int (1 - \cos^2(a + bx)) d \cos(a + bx)}{b} \\ & \quad \downarrow \text{2009} \\ & -\frac{\cos(a + bx) - \frac{1}{3} \cos^3(a + bx)}{b} \end{aligned}$$

input `Int[Sin[a + b*x]^3,x]`

output `-((Cos[a + b*x] - Cos[a + b*x]^3/3)/b)`

**3.114.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3113 `Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[-d^(-1) Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

**3.114.4 Maple [A] (verified)**

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

| method            | result                                                                                                                                                       | size |
|-------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| derivativedivides | $-\frac{(2+\sin^2(bx+a))\cos(bx+a)}{3b}$                                                                                                                     | 22   |
| default           | $-\frac{(2+\sin^2(bx+a))\cos(bx+a)}{3b}$                                                                                                                     | 22   |
| parallelrisc      | $-\frac{8-9\cos(bx+a)+\cos(3bx+3a)}{12b}$                                                                                                                    | 25   |
| risc              | $-\frac{3\cos(bx+a)}{4b} + \frac{\cos(3bx+3a)}{12b}$                                                                                                         | 27   |
| norman            | $-\frac{4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b} - \frac{4}{3b} \frac{1}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3}$ | 39   |

input `int(sin(b*x+a)^3,x,method=_RETURNVERBOSE)`output `-1/3/b*(2+sin(b*x+a)^2)*cos(b*x+a)`**3.114.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \sin^3(a+bx) dx = \frac{\cos(bx+a)^3 - 3\cos(bx+a)}{3b}$$

input `integrate(sin(b*x+a)^3,x, algorithm="fricas")`output `1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b`**3.114.6 Sympy [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \sin^3(a+bx) dx = \begin{cases} -\frac{\sin^2(a+bx)\cos(a+bx)}{b} - \frac{2\cos^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x \sin^3(a) & \text{otherwise} \end{cases}$$



input `integrate(sin(b*x+a)**3,x)`

output `Piecewise((-sin(a + b*x)**2*cos(a + b*x)/b - 2*cos(a + b*x)**3/(3*b), Ne(b, 0)), (x*sin(a)**3, True))`

### 3.114.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \sin^3(a + bx) dx = \frac{\cos(bx + a)^3 - 3 \cos(bx + a)}{3b}$$

input `integrate(sin(b*x+a)^3,x, algorithm="maxima")`

output `1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b`

### 3.114.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sin^3(a + bx) dx = \frac{\cos(bx + a)^3}{3b} - \frac{\cos(bx + a)}{b}$$

input `integrate(sin(b*x+a)^3,x, algorithm="giac")`

output `1/3*cos(b*x + a)^3/b - cos(b*x + a)/b`

### 3.114.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \sin^3(a + bx) dx = -\frac{3 \cos(a + bx) - \cos(a + bx)^3}{3b}$$

input `int(sin(a + b*x)^3,x)`

output `-(3*cos(a + b*x) - cos(a + b*x)^3)/(3*b)`

### 3.115 $\int \cos^2(a + bx) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.115.1 Optimal result . . . . .                            | 681 |
| 3.115.2 Mathematica [A] (verified) . . . . .                | 681 |
| 3.115.3 Rubi [A] (verified) . . . . .                       | 682 |
| 3.115.4 Maple [A] (verified) . . . . .                      | 683 |
| 3.115.5 Fricas [A] (verification not implemented) . . . . . | 683 |
| 3.115.6 Sympy [B] (verification not implemented) . . . . .  | 683 |
| 3.115.7 Maxima [A] (verification not implemented) . . . . . | 684 |
| 3.115.8 Giac [A] (verification not implemented) . . . . .   | 684 |
| 3.115.9 Mupad [B] (verification not implemented) . . . . .  | 684 |

#### 3.115.1 Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \cos^2(a + bx) dx = \frac{x}{2} + \frac{\cos(a + bx) \sin(a + bx)}{2b}$$

output `1/2*x+1/2*cos(b*x+a)*sin(b*x+a)/b`

#### 3.115.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \cos^2(a + bx) dx = \frac{2(a + bx) + \sin(2(a + bx))}{4b}$$

input `Integrate[Cos[a + b*x]^2,x]`

output `(2*(a + b*x) + Sin[2*(a + b*x)])/(4*b)`

**3.115.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \cos^2(a + bx) dx \\ & \quad \downarrow \text{3042} \\ & \int \sin\left(a + bx + \frac{\pi}{2}\right)^2 dx \\ & \quad \downarrow \text{3115} \\ & \frac{\int 1 dx}{2} + \frac{\sin(a + bx) \cos(a + bx)}{2b} \\ & \quad \downarrow \text{24} \\ & \frac{\sin(a + bx) \cos(a + bx)}{2b} + \frac{x}{2} \end{aligned}$$

input `Int[Cos[a + b*x]^2,x]`

output `x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)`

**3.115.3.1 Defintions of rubi rules used**

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3115 `Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sine[c + d*x])^(n - 1)/(d*n), x] + Simp[b^2*((n - 1)/n) Int[(b*Sine[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

**3.115.4 Maple [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

| method            | result                                                                                                                                                                                                                                                                                                                                    | size |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch             | $\frac{x}{2} + \frac{\sin(2bx+2a)}{4b}$                                                                                                                                                                                                                                                                                                   | 19   |
| parallelrisch     | $\frac{2bx+\sin(2bx+2a)}{4b}$                                                                                                                                                                                                                                                                                                             | 20   |
| derivativedivides | $\frac{\cos(bx+a)\sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2}$<br>$b$                                                                                                                                                                                                                                                                      | 27   |
| default           | $\frac{\cos(bx+a)\sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2}$<br>$b$                                                                                                                                                                                                                                                                      | 27   |
| norman            | $\frac{\tan\left(\frac{bx}{2} + \frac{a}{2}\right)}{b} + x\left(\tan^2\left(\frac{bx}{2} + \frac{a}{2}\right)\right) + \frac{x}{2} - \frac{\tan^3\left(\frac{bx}{2} + \frac{a}{2}\right)}{b} + \frac{x\left(\tan^4\left(\frac{bx}{2} + \frac{a}{2}\right)\right)}{2}$<br>$\left(1+\tan^2\left(\frac{bx}{2} + \frac{a}{2}\right)\right)^2$ | 77   |

input `int(cos(b*x+a)^2,x,method=_RETURNVERBOSE)`output `1/2*x+1/4/b*sin(2*b*x+2*a)`**3.115.5 Fracas [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \cos^2(a + bx) dx = \frac{bx + \cos(bx + a)\sin(bx + a)}{2b}$$

input `integrate(cos(b*x+a)^2,x, algorithm="fricas")`output `1/2*(b*x + cos(b*x + a)*sin(b*x + a))/b`**3.115.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(19) = 38.

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \cos^2(a + bx) dx = \begin{cases} \frac{x \sin^2(a+bx)}{2} + \frac{x \cos^2(a+bx)}{2} + \frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x \cos^2(a) & \text{otherwise} \end{cases}$$

input `integrate(cos(b*x+a)**2,x)`

output `Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 + sin(a + b*x)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*cos(a)**2, True))`

### 3.115.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \cos^2(a + bx) dx = \frac{2bx + 2a + \sin(2bx + 2a)}{4b}$$

input `integrate(cos(b*x+a)^2,x, algorithm="maxima")`

output `1/4*(2*b*x + 2*a + sin(2*b*x + 2*a))/b`

### 3.115.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \cos^2(a + bx) dx = \frac{1}{2}x + \frac{\sin(2bx + 2a)}{4b}$$

input `integrate(cos(b*x+a)^2,x, algorithm="giac")`

output `1/2*x + 1/4*sin(2*b*x + 2*a)/b`

### 3.115.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \cos^2(a + bx) dx = \frac{x}{2} + \frac{\sin(2a + 2bx)}{4b}$$

input `int(cos(a + b*x)^2,x)`

output `x/2 + sin(2*a + 2*b*x)/(4*b)`

## 3.116 $\int \cos^3(a + bx) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.116.1 Optimal result . . . . .                            | 685 |
| 3.116.2 Mathematica [A] (verified) . . . . .                | 685 |
| 3.116.3 Rubi [A] (verified) . . . . .                       | 686 |
| 3.116.4 Maple [A] (verified) . . . . .                      | 687 |
| 3.116.5 Fricas [A] (verification not implemented) . . . . . | 687 |
| 3.116.6 Sympy [A] (verification not implemented) . . . . .  | 687 |
| 3.116.7 Maxima [A] (verification not implemented) . . . . . | 688 |
| 3.116.8 Giac [A] (verification not implemented) . . . . .   | 688 |
| 3.116.9 Mupad [B] (verification not implemented) . . . . .  | 688 |

### 3.116.1 Optimal result

Integrand size = 8, antiderivative size = 26

$$\int \cos^3(a + bx) dx = \frac{\sin(a + bx)}{b} - \frac{\sin^3(a + bx)}{3b}$$

output `sin(b*x+a)/b-1/3*sin(b*x+a)^3/b`

### 3.116.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cos^3(a + bx) dx = \frac{\sin(a + bx)}{b} - \frac{\sin^3(a + bx)}{3b}$$

input `Integrate[Cos[a + b*x]^3,x]`

output `Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)`

**3.116.3 Rubi [A] (verified)**

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \cos^3(a + bx) dx \\ & \quad \downarrow \text{3042} \\ & \int \sin\left(a + bx + \frac{\pi}{2}\right)^3 dx \\ & \quad \downarrow \text{3113} \\ & -\frac{\int (1 - \sin^2(a + bx)) d(-\sin(a + bx))}{b} \\ & \quad \downarrow \text{2009} \\ & -\frac{\frac{1}{3} \sin^3(a + bx) - \sin(a + bx)}{b} \end{aligned}$$

input `Int[Cos[a + b*x]^3,x]`

output `-((-Sin[a + b*x] + Sin[a + b*x]^3/3)/b)`

**3.116.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3113 `Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[-d^(-1) Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

**3.116.4 Maple [A] (verified)**

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

| method            | result                                               | size |
|-------------------|------------------------------------------------------|------|
| derivativedivides | $\frac{(2+\cos^2(bx+a)) \sin(bx+a)}{3b}$             | 22   |
| default           | $\frac{(2+\cos^2(bx+a)) \sin(bx+a)}{3b}$             | 22   |
| parallelrisch     | $\frac{9 \sin(bx+a) + \sin(3bx+3a)}{12b}$            | 24   |
| risch             | $\frac{3 \sin(bx+a)}{4b} + \frac{\sin(3bx+3a)}{12b}$ | 27   |

input `int(cos(b*x+a)^3,x,method=_RETURNVERBOSE)`output `1/3/b*(2+cos(b*x+a)^2)*sin(b*x+a)`**3.116.5 Fracas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \cos^3(a + bx) dx = \frac{(\cos(bx + a))^2 + 2}{3b} \sin(bx + a)$$

input `integrate(cos(b*x+a)^3,x, algorithm="fricas")`output `1/3*(cos(b*x + a)^2 + 2)*sin(b*x + a)/b`**3.116.6 Sympy [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \cos^3(a + bx) dx = \begin{cases} \frac{2 \sin^3(a+bx)}{3b} + \frac{\sin(a+bx) \cos^2(a+bx)}{b} & \text{for } b \neq 0 \\ x \cos^3(a) & \text{otherwise} \end{cases}$$

input `integrate(cos(b*x+a)**3,x)`output `Piecewise((2*sin(a + b*x)**3/(3*b) + sin(a + b*x)*cos(a + b*x)**2/b, Ne(b, 0)), (x*cos(a)**3, True))`



**3.116.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \cos^3(a + bx) dx = -\frac{\sin(bx + a)^3 - 3 \sin(bx + a)}{3b}$$

input `integrate(cos(b*x+a)^3,x, algorithm="maxima")`output `-1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b`**3.116.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \cos^3(a + bx) dx = -\frac{\sin(bx + a)^3 - 3 \sin(bx + a)}{3b}$$

input `integrate(cos(b*x+a)^3,x, algorithm="giac")`output `-1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b`**3.116.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \cos^3(a + bx) dx = \frac{3 \sin(a + bx) - \sin(a + bx)^3}{3b}$$

input `int(cos(a + b*x)^3,x)`output `(3*sin(a + b*x) - sin(a + b*x)^3)/(3*b)`

### 3.117 $\int \sec^2(a + bx) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.117.1 Optimal result . . . . .                            | 689 |
| 3.117.2 Mathematica [A] (verified) . . . . .                | 689 |
| 3.117.3 Rubi [A] (verified) . . . . .                       | 690 |
| 3.117.4 Maple [A] (verified) . . . . .                      | 691 |
| 3.117.5 Fricas [A] (verification not implemented) . . . . . | 691 |
| 3.117.6 Sympy [B] (verification not implemented) . . . . .  | 691 |
| 3.117.7 Maxima [A] (verification not implemented) . . . . . | 692 |
| 3.117.8 Giac [A] (verification not implemented) . . . . .   | 692 |
| 3.117.9 Mupad [B] (verification not implemented) . . . . .  | 693 |

#### 3.117.1 Optimal result

Integrand size = 8, antiderivative size = 10

$$\int \sec^2(a + bx) dx = \frac{\tan(a + bx)}{b}$$

output `tan(b*x+a)/b`

#### 3.117.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sec^2(a + bx) dx = \frac{\tan(a + bx)}{b}$$

input `Integrate[Sec[a + b*x]^2,x]`

output `Tan[a + b*x]/b`

**3.117.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \sec^2(a + bx) dx \\ \downarrow 3042 \\ \int \csc\left(a + bx + \frac{\pi}{2}\right)^2 dx \\ \downarrow 4254 \\ -\frac{\int 1d(-\tan(a + bx))}{b} \\ \downarrow 24 \\ \frac{\tan(a + bx)}{b} \end{array}$$

input `Int[Sec[a + b*x]^2,x]`

output `Tan[a + b*x]/b`

**3.117.3.1 Defintions of rubi rules used**

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4254 `Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[-d^(-1) Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

**3.117.4 Maple [A] (verified)**

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

| method            | result                                                                                                                   | size |
|-------------------|--------------------------------------------------------------------------------------------------------------------------|------|
| derivativedivides | $\frac{\tan(bx+a)}{b}$                                                                                                   | 11   |
| default           | $\frac{\tan(bx+a)}{b}$                                                                                                   | 11   |
| risch             | $\frac{2i}{b(e^{2i(bx+a)}+1)}$                                                                                           | 20   |
| norman            | $-\frac{2 \tan\left(\frac{bx}{2} + \frac{a}{2}\right)}{b\left(\tan^2\left(\frac{bx}{2} + \frac{a}{2}\right) - 1\right)}$ | 30   |
| parallelrisc      | $-\frac{2 \tan\left(\frac{bx}{2} + \frac{a}{2}\right)}{b\left(\tan^2\left(\frac{bx}{2} + \frac{a}{2}\right) - 1\right)}$ | 30   |

input `int(1/cos(b*x+a)^2,x,method=_RETURNVERBOSE)`output `tan(b*x+a)/b`**3.117.5 Fricas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \sec^2(a + bx) dx = \frac{\sin(bx + a)}{b \cos(bx + a)}$$

input `integrate(1/cos(b*x+a)^2,x, algorithm="fricas")`output `sin(b*x + a)/(b*cos(b*x + a))`**3.117.6 Sympy [B] (verification not implemented)**Leaf count of result is larger than twice the leaf count of optimal. 58 vs.  $2(7) = 14$ .

Time = 0.62 (sec) , antiderivative size = 58, normalized size of antiderivative = 5.80

$$\int \sec^2(a + bx) dx = \begin{cases} \tilde{\infty}x & \text{for } (a = -\frac{\pi}{2} \vee a = -bx - \frac{\pi}{2}) \wedge (a = -bx - \frac{\pi}{2} \vee b = 0) \\ \frac{x}{\cos^2(a)} & \text{for } b = 0 \\ -\frac{2 \tan(\frac{a}{2} + \frac{bx}{2})}{b \tan^2(\frac{a}{2} + \frac{bx}{2}) - b} & \text{otherwise} \end{cases}$$

input `integrate(1/cos(b*x+a)**2,x)`

output `Piecewise((zoo*x, (Eq(b, 0) | Eq(a, -b*x - pi/2)) & (Eq(a, -pi/2) | Eq(a, -b*x - pi/2))), (x/cos(a)**2, Eq(b, 0)), (-2*tan(a/2 + b*x/2)/(b*tan(a/2 + b*x/2)**2 - b), True))`

### 3.117.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sec^2(a + bx) dx = \frac{\tan(bx + a)}{b}$$

input `integrate(1/cos(b*x+a)^2,x, algorithm="maxima")`

output `tan(b*x + a)/b`

### 3.117.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sec^2(a + bx) dx = \frac{\tan(bx + a)}{b}$$

input `integrate(1/cos(b*x+a)^2,x, algorithm="giac")`

output `tan(b*x + a)/b`

**3.117.9 Mupad [B] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sec^2(a + bx) dx = \frac{\tan(a + bx)}{b}$$

input `int(1/cos(a + b*x)^2,x)`

output `tan(a + b*x)/b`

### 3.118 $\int \frac{1}{1+\cos(x)} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.118.1 Optimal result . . . . .                            | 694 |
| 3.118.2 Mathematica [A] (verified) . . . . .                | 694 |
| 3.118.3 Rubi [A] (verified) . . . . .                       | 695 |
| 3.118.4 Maple [A] (verified) . . . . .                      | 696 |
| 3.118.5 Fricas [A] (verification not implemented) . . . . . | 696 |
| 3.118.6 Sympy [A] (verification not implemented) . . . . .  | 696 |
| 3.118.7 Maxima [A] (verification not implemented) . . . . . | 697 |
| 3.118.8 Giac [B] (verification not implemented) . . . . .   | 697 |
| 3.118.9 Mupad [B] (verification not implemented) . . . . .  | 697 |

#### 3.118.1 Optimal result

Integrand size = 6, antiderivative size = 9

$$\int \frac{1}{1+\cos(x)} dx = \frac{\sin(x)}{1+\cos(x)}$$

output `sin(x)/(1+cos(x))`

#### 3.118.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{1}{1+\cos(x)} dx = \tan\left(\frac{x}{2}\right)$$

input `Integrate[(1 + Cos[x])^(-1), x]`

output `Tan[x/2]`

**3.118.3 Rubi [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos(x) + 1} dx$$

↓ 3042

$$\int \frac{1}{\sin\left(x + \frac{\pi}{2}\right) + 1} dx$$

↓ 3127

$$\frac{\sin(x)}{\cos(x) + 1}$$

input `Int[(1 + Cos[x])^(-1),x]`

output `Sin[x]/(1 + Cos[x])`

**3.118.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 3127 `Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`



**3.118.4 Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

| method       | result                         | size |
|--------------|--------------------------------|------|
| default      | $\tan\left(\frac{x}{2}\right)$ | 5    |
| norman       | $\tan\left(\frac{x}{2}\right)$ | 5    |
| parallelrisc | $\tan\left(\frac{x}{2}\right)$ | 5    |
| risc         | $\frac{2i}{e^{ix}+1}$          | 13   |

input `int(1/(cos(x)+1),x,method=_RETURNVERBOSE)`

output `tan(1/2*x)`

**3.118.5 Fricas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{1 + \cos(x)} dx = \frac{\sin(x)}{\cos(x) + 1}$$

input `integrate(1/(1+cos(x)),x, algorithm="fricas")`

output `sin(x)/(cos(x) + 1)`

**3.118.6 Sympy [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.33

$$\int \frac{1}{1 + \cos(x)} dx = \tan\left(\frac{x}{2}\right)$$

input `integrate(1/(1+cos(x)),x)`

output `tan(x/2)`

**3.118.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{1 + \cos(x)} dx = \frac{\sin(x)}{\cos(x) + 1}$$

input `integrate(1/(1+cos(x)),x, algorithm="maxima")`

output `sin(x)/(cos(x) + 1)`

**3.118.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(9) = 18.

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 3.33

$$\int \frac{1}{1 + \cos(x)} dx = -\frac{2 \tan\left(\frac{1}{2}x\right)}{(x^2 + 1)\left(\frac{x^2 - 1}{x^2 + 1} - 1\right)}$$

input `integrate(1/(1+cos(x)),x, algorithm="giac")`

output `-2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))`

**3.118.9 Mupad [B] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \frac{1}{1 + \cos(x)} dx = \tan\left(\frac{x}{2}\right)$$

input `int(1/(cos(x) + 1),x)`

output `tan(x/2)`

### 3.119 $\int \frac{1}{1-\cos(x)} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.119.1 Optimal result . . . . .                            | 698 |
| 3.119.2 Mathematica [A] (verified) . . . . .                | 698 |
| 3.119.3 Rubi [A] (verified) . . . . .                       | 699 |
| 3.119.4 Maple [A] (verified) . . . . .                      | 700 |
| 3.119.5 Fricas [A] (verification not implemented) . . . . . | 700 |
| 3.119.6 Sympy [A] (verification not implemented) . . . . .  | 700 |
| 3.119.7 Maxima [A] (verification not implemented) . . . . . | 701 |
| 3.119.8 Giac [A] (verification not implemented) . . . . .   | 701 |
| 3.119.9 Mupad [B] (verification not implemented) . . . . .  | 701 |

#### 3.119.1 Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \frac{1}{1-\cos(x)} dx = -\frac{\sin(x)}{1-\cos(x)}$$

output `-sin(x)/(1-cos(x))`

#### 3.119.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)} dx = -\cot\left(\frac{x}{2}\right)$$

input `Integrate[(1 - Cos[x])^(-1),x]`

output `-Cot[x/2]`

**3.119.3 Rubi [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1 - \cos(x)} dx$$

↓ 3042

$$\int \frac{1}{1 - \sin\left(x + \frac{\pi}{2}\right)} dx$$

↓ 3127

$$-\frac{\sin(x)}{1 - \cos(x)}$$

input `Int[(1 - Cos[x])^(-1),x]`

output `-(Sin[x]/(1 - Cos[x]))`

**3.119.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 3127 `Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

**3.119.4 Maple [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

| method        | result                         | size |
|---------------|--------------------------------|------|
| default       | $-\frac{1}{\tan(\frac{x}{2})}$ | 9    |
| norman        | $-\frac{1}{\tan(\frac{x}{2})}$ | 9    |
| parallelrisch | $-\frac{1}{\tan(\frac{x}{2})}$ | 9    |
| risch         | $-\frac{2i}{e^{ix}-1}$         | 13   |

input `int(1/(1-cos(x)),x,method=_RETURNVERBOSE)`output `-1/tan(1/2*x)`**3.119.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cos(x)} dx = -\frac{\cos(x)+1}{\sin(x)}$$

input `integrate(1/(1-cos(x)),x, algorithm="fricas")`output `-(cos(x) + 1)/sin(x)`**3.119.6 Sympy [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1}{1-\cos(x)} dx = -\frac{1}{\tan(\frac{x}{2})}$$

input `integrate(1/(1-cos(x)),x)`output `-1/tan(x/2)`

**3.119.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1 - \cos(x)} dx = -\frac{\cos(x) + 1}{\sin(x)}$$

input `integrate(1/(1-cos(x)),x, algorithm="maxima")`output `-(cos(x) + 1)/sin(x)`**3.119.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1 - \cos(x)} dx = -\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

input `integrate(1/(1-cos(x)),x, algorithm="giac")`output `-1/tan(1/2*x)`**3.119.9 Mupad [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{1 - \cos(x)} dx = -\cot\left(\frac{x}{2}\right)$$

input `int(-1/(cos(x) - 1),x)`output `-cot(x/2)`

### 3.120 $\int \frac{1}{1+\sin(x)} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.120.1 Optimal result . . . . .                            | 702 |
| 3.120.2 Mathematica [B] (verified) . . . . .                | 702 |
| 3.120.3 Rubi [A] (verified) . . . . .                       | 703 |
| 3.120.4 Maple [A] (verified) . . . . .                      | 704 |
| 3.120.5 Fricas [A] (verification not implemented) . . . . . | 704 |
| 3.120.6 Sympy [A] (verification not implemented) . . . . .  | 704 |
| 3.120.7 Maxima [A] (verification not implemented) . . . . . | 705 |
| 3.120.8 Giac [A] (verification not implemented) . . . . .   | 705 |
| 3.120.9 Mupad [B] (verification not implemented) . . . . .  | 705 |

#### 3.120.1 Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \frac{1}{1+\sin(x)} dx = -\frac{\cos(x)}{1+\sin(x)}$$

output `-cos(x)/(1+sin(x))`

#### 3.120.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs.  $2(10) = 20$ .

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{1}{1+\sin(x)} dx = \frac{2 \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}$$

input `Integrate[(1 + Sin[x])^(-1),x]`

output `(2*Sin[x/2])/(Cos[x/2] + Sin[x/2])`

**3.120.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin(x) + 1} dx$$

↓ 3042

$$\int \frac{1}{\sin(x) + 1} dx$$

↓ 3127

$$-\frac{\cos(x)}{\sin(x) + 1}$$

input `Int[(1 + Sin[x])^(-1),x]`

output `-(Cos[x]/(1 + Sin[x]))`

**3.120.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3127 `Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`



**3.120.4 Maple [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

| method        | result                           | size |
|---------------|----------------------------------|------|
| default       | $-\frac{2}{1+\tan(\frac{x}{2})}$ | 11   |
| norman        | $-\frac{2}{1+\tan(\frac{x}{2})}$ | 11   |
| parallelrisch | $-\frac{2}{1+\tan(\frac{x}{2})}$ | 11   |
| risch         | $-\frac{2}{i+e^{ix}}$            | 13   |

input `int(1/(sin(x)+1),x,method=_RETURNVERBOSE)`output `-2/(1+tan(1/2*x))`**3.120.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{1}{1 + \sin(x)} dx = -\frac{\cos(x) - \sin(x) + 1}{\cos(x) + \sin(x) + 1}$$

input `integrate(1/(1+sin(x)),x, algorithm="fricas")`output `-(cos(x) - sin(x) + 1)/(cos(x) + sin(x) + 1)`**3.120.6 Sympy [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{1 + \sin(x)} dx = -\frac{2}{\tan\left(\frac{x}{2}\right) + 1}$$

input `integrate(1/(1+sin(x)),x)`output `-2/(tan(x/2) + 1)`

**3.120.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{1}{1 + \sin(x)} dx = -\frac{2}{\frac{\sin(x)}{\cos(x)+1} + 1}$$

input `integrate(1/(1+sin(x)),x, algorithm="maxima")`output `-2/(sin(x)/(cos(x) + 1) + 1)`**3.120.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1 + \sin(x)} dx = -\frac{2}{\tan\left(\frac{1}{2}x\right) + 1}$$

input `integrate(1/(1+sin(x)),x, algorithm="giac")`output `-2/(tan(1/2*x) + 1)`**3.120.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1 + \sin(x)} dx = -\frac{2}{\tan\left(\frac{x}{2}\right) + 1}$$

input `int(1/(sin(x) + 1),x)`output `-2/(tan(x/2) + 1)`

### 3.121 $\int \frac{1}{1-\sin(x)} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.121.1 Optimal result . . . . .                            | 706 |
| 3.121.2 Mathematica [B] (verified) . . . . .                | 706 |
| 3.121.3 Rubi [A] (verified) . . . . .                       | 707 |
| 3.121.4 Maple [A] (verified) . . . . .                      | 708 |
| 3.121.5 Fricas [A] (verification not implemented) . . . . . | 708 |
| 3.121.6 Sympy [A] (verification not implemented) . . . . .  | 708 |
| 3.121.7 Maxima [A] (verification not implemented) . . . . . | 709 |
| 3.121.8 Giac [A] (verification not implemented) . . . . .   | 709 |
| 3.121.9 Mupad [B] (verification not implemented) . . . . .  | 709 |

#### 3.121.1 Optimal result

Integrand size = 8, antiderivative size = 11

$$\int \frac{1}{1-\sin(x)} dx = \frac{\cos(x)}{1-\sin(x)}$$

output `cos(x)/(1-sin(x))`

#### 3.121.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{1}{1-\sin(x)} dx = \frac{2 \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}$$

input `Integrate[(1 - Sin[x])^(-1),x]`

output `(2*Sin[x/2])/(Cos[x/2] - Sin[x/2])`

**3.121.3 Rubi [A] (verified)**

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1 - \sin(x)} dx$$

↓ 3042

$$\int \frac{1}{1 - \sin(x)} dx$$

↓ 3127

$$\frac{\cos(x)}{1 - \sin(x)}$$

input `Int[(1 - Sin[x])^(-1),x]`

output `Cos[x]/(1 - Sin[x])`

**3.121.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3127 `Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

**3.121.4 Maple [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

| method        | result                           | size |
|---------------|----------------------------------|------|
| default       | $-\frac{2}{\tan(\frac{x}{2})-1}$ | 11   |
| norman        | $-\frac{2}{\tan(\frac{x}{2})-1}$ | 11   |
| parallelrisch | $-\frac{2}{\tan(\frac{x}{2})-1}$ | 11   |
| risch         | $\frac{2}{e^{ix}-i}$             | 13   |

input `int(1/(-sin(x)+1),x,method=_RETURNVERBOSE)`output `-2/(tan(1/2*x)-1)`**3.121.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1}{1-\sin(x)} dx = \frac{\cos(x) + \sin(x) + 1}{\cos(x) - \sin(x) + 1}$$

input `integrate(1/(1-sin(x)),x, algorithm="fricas")`output `(cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)`**3.121.6 Sympy [A] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{1-\sin(x)} dx = -\frac{2}{\tan(\frac{x}{2})-1}$$

input `integrate(1/(1-sin(x)),x)`output `-2/(tan(x/2) - 1)`

**3.121.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1}{1 - \sin(x)} dx = -\frac{2}{\frac{\sin(x)}{\cos(x)+1} - 1}$$

input `integrate(1/(1-sin(x)),x, algorithm="maxima")`output `-2/(sin(x)/(cos(x) + 1) - 1)`**3.121.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{1 - \sin(x)} dx = -\frac{2}{\tan\left(\frac{1}{2}x\right) - 1}$$

input `integrate(1/(1-sin(x)),x, algorithm="giac")`output `-2/(tan(1/2*x) - 1)`**3.121.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{1 - \sin(x)} dx = -\frac{2}{\tan\left(\frac{x}{2}\right) - 1}$$

input `int(-1/(sin(x) - 1),x)`output `-2/(tan(x/2) - 1)`

### 3.122 $\int \frac{1}{a+b \sin(x)} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.122.1 Optimal result . . . . .                            | 710 |
| 3.122.2 Mathematica [A] (verified) . . . . .                | 710 |
| 3.122.3 Rubi [A] (verified) . . . . .                       | 711 |
| 3.122.4 Maple [A] (verified) . . . . .                      | 712 |
| 3.122.5 Fricas [A] (verification not implemented) . . . . . | 713 |
| 3.122.6 Sympy [B] (verification not implemented) . . . . .  | 713 |
| 3.122.7 Maxima [F(-2)] . . . . .                            | 714 |
| 3.122.8 Giac [A] (verification not implemented) . . . . .   | 714 |
| 3.122.9 Mupad [B] (verification not implemented) . . . . .  | 715 |

#### 3.122.1 Optimal result

Integrand size = 8, antiderivative size = 40

$$\int \frac{1}{a + b \sin(x)} dx = \frac{2 \arctan\left(\frac{b+a \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

output `2*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)`

#### 3.122.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{a + b \sin(x)} dx = \frac{2 \arctan\left(\frac{b+a \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input `Integrate[(a + b*Sin[x])^(-1),x]`

output `(2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]`

**3.122.3 Rubi [A] (verified)**

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{a + b \sin(x)} dx \\
 & \quad \downarrow \text{3042} \\
 & \int \frac{1}{a + b \sin(x)} dx \\
 & \quad \downarrow \text{3139} \\
 & 2 \int \frac{1}{a \tan^2\left(\frac{x}{2}\right) + 2b \tan\left(\frac{x}{2}\right) + a} d \tan\left(\frac{x}{2}\right) \\
 & \quad \downarrow \text{1083} \\
 & -4 \int \frac{1}{-(2b + 2a \tan\left(\frac{x}{2}\right))^2 - 4(a^2 - b^2)} d\left(2b + 2a \tan\left(\frac{x}{2}\right)\right) \\
 & \quad \downarrow \text{217} \\
 & \frac{2 \arctan\left(\frac{2a \tan\left(\frac{x}{2}\right) + 2b}{2\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2}}
 \end{aligned}$$

input `Int[(a + b*Sin[x])^(-1),x]`

output `(2*ArcTan[(2*b + 2*a*Tan[x/2])/(2*Sqrt[a^2 - b^2])])/Sqrt[a^2 - b^2]`

**3.122.3.1 Defintions of rubi rules used**

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])`



```
rule 1083 Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Simp[-2 Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x]
```

```
rule 3042 Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]
```

```
rule 3139 Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d) Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

### 3.122.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

| method  | result                                                                                                                                                                                                                           | size |
|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default | $\frac{2 \arctan\left(\frac{2a \tan\left(\frac{x}{2}\right) + 2b}{2\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2}}$                                                                                                                  | 39   |
| risch   | $-\frac{\ln\left(e^{ix} + \frac{ia\sqrt{-a^2 + b^2} - a^2 + b^2}{b\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}} + \frac{\ln\left(e^{ix} + \frac{ia\sqrt{-a^2 + b^2} + a^2 - b^2}{b\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}}$ | 119  |

```
input int(1/(a+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
output 2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

**3.122.5 Fricas [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.70

$$\int \frac{1}{a + b \sin(x)} dx = \left[ -\frac{\sqrt{-a^2 + b^2} \log\left(\frac{(2a^2 - b^2) \cos(x)^2 - 2ab \sin(x) - a^2 - b^2 + 2(a \cos(x) \sin(x) + b \cos(x)) \sqrt{-a^2 + b^2}}{b^2 \cos(x)^2 - 2ab \sin(x) - a^2 - b^2}\right)}{2(a^2 - b^2)}, -\frac{\arctan\left(-\frac{a \sin(x) + b}{\sqrt{a^2 - b^2} \cos(x)}\right)}{\sqrt{a^2 - b^2}} \right]$$

input `integrate(1/(a+b*sin(x)),x, algorithm="fricas")`output `[-1/2*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2))/(a^2 - b^2), -arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x)))/sqrt(a^2 - b^2)]`**3.122.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(31) = 62.

Time = 1.98 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.50

$$\int \frac{1}{a + b \sin(x)} dx = \begin{cases} \infty \log\left(\tan\left(\frac{x}{2}\right)\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b} & \text{for } a = 0 \\ \frac{2}{b \tan\left(\frac{x}{2}\right) - b} & \text{for } a = -b \\ -\frac{2}{b \tan\left(\frac{x}{2}\right) + b} & \text{for } a = b \\ \frac{\log\left(\tan\left(\frac{x}{2}\right) + \frac{b}{a} - \frac{\sqrt{-a^2 + b^2}}{a}\right)}{\sqrt{-a^2 + b^2}} - \frac{\log\left(\tan\left(\frac{x}{2}\right) + \frac{b}{a} + \frac{\sqrt{-a^2 + b^2}}{a}\right)}{\sqrt{-a^2 + b^2}} & \text{otherwise} \end{cases}$$

input `integrate(1/(a+b*sin(x)),x)`

output `Piecewise((zoo*log(tan(x/2)), Eq(a, 0) & Eq(b, 0)), (log(tan(x/2))/b, Eq(a, 0)), (2/(b*tan(x/2) - b), Eq(a, -b)), (-2/(b*tan(x/2) + b), Eq(a, b)), (log(tan(x/2) + b/a - sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2) - log(tan(x/2) + b/a + sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2), True))`

### 3.122.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a + b \sin(x)} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(a+b*sin(x)),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f or more de`

### 3.122.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{1}{a + b \sin(x)} dx = \frac{2 \left( \pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(a) + \arctan \left( \frac{a \tan(\frac{1}{2}x) + b}{\sqrt{a^2 - b^2}} \right) \right)}{\sqrt{a^2 - b^2}}$$

input `integrate(1/(a+b*sin(x)),x, algorithm="giac")`

output `2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b^2)))/sqrt(a^2 - b^2)`

**3.122.9 Mupad [B] (verification not implemented)**

Time = 0.39 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{1}{a + b \sin(x)} dx = \frac{2 \operatorname{atan}\left(\frac{b}{\sqrt{a^2 - b^2}} + \frac{a \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2}}$$

input `int(1/(a + b*sin(x)),x)`

output `(2*atan(b/(a^2 - b^2)^(1/2) + (a*tan(x/2))/(a^2 - b^2)^(1/2)))/(a^2 - b^2)^(1/2)`

### 3.123 $\int \frac{1}{a+\cos(x)+b \sin(x)} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.123.1 Optimal result . . . . .                            | 716 |
| 3.123.2 Mathematica [A] (verified) . . . . .                | 716 |
| 3.123.3 Rubi [A] (verified) . . . . .                       | 717 |
| 3.123.4 Maple [A] (verified) . . . . .                      | 718 |
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#### 3.123.1 Optimal result

Integrand size = 10, antiderivative size = 47

$$\int \frac{1}{a + \cos(x) + b \sin(x)} dx = -\frac{2 \operatorname{arctanh}\left(\frac{b-(1-a) \tan\left(\frac{x}{2}\right)}{\sqrt{1-a^2+b^2}}\right)}{\sqrt{1-a^2+b^2}}$$

output `-2*arctanh((b-(1-a)*tan(1/2*x))/(-a^2+b^2+1)^(1/2))/(-a^2+b^2+1)^(1/2)`

#### 3.123.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{1}{a + \cos(x) + b \sin(x)} dx = \frac{2 \arctan\left(\frac{b+(-1+a) \tan\left(\frac{x}{2}\right)}{\sqrt{-1+a^2-b^2}}\right)}{\sqrt{-1+a^2-b^2}}$$

input `Integrate[(a + Cos[x] + b*Sin[x])^(-1),x]`

output `(2*ArcTan[(b + (-1 + a)*Tan[x/2])/Sqrt[-1 + a^2 - b^2]])/Sqrt[-1 + a^2 - b^2]`

**3.123.3 Rubi [A] (verified)**

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{a + b \sin(x) + \cos(x)} dx \\
 & \quad \downarrow \text{3042} \\
 & \int \frac{1}{a + b \sin(x) + \cos(x)} dx \\
 & \quad \downarrow \text{3603} \\
 & 2 \int \frac{1}{-((1-a)\tan^2(\frac{x}{2})) + 2b\tan(\frac{x}{2}) + a + 1} d\tan\left(\frac{x}{2}\right) \\
 & \quad \downarrow \text{1083} \\
 & -4 \int \frac{1}{4(-a^2 + b^2 + 1) - (2b - 2(1-a)\tan(\frac{x}{2}))^2} d\left(2b - 2(1-a)\tan\left(\frac{x}{2}\right)\right) \\
 & \quad \downarrow \text{219} \\
 & \frac{2\text{arctanh}\left(\frac{2b - 2(1-a)\tan(\frac{x}{2})}{2\sqrt{-a^2 + b^2 + 1}}\right)}{\sqrt{-a^2 + b^2 + 1}}
 \end{aligned}$$

input `Int[(a + Cos[x] + b*Sin[x])^(-1),x]`

output `(-2*ArcTanh[(2*b - 2*(1 - a)*Tan[x/2])/(2*Sqrt[1 - a^2 + b^2])])/Sqrt[1 - a^2 + b^2]`

3.123.3.1 Defintions of rubi rules used

- rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`
  
- rule 1083 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Simp[-2 Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x]`
  
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`
  
- rule 3603 `Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f/e) Subst[Int[1/(a + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]`

3.123.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

| method  | result                                                                                                                                                                                                                                                                                                   |
|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| default | $\frac{2 \arctan\left(\frac{2(a-1)\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2-1}}\right)}{\sqrt{a^2-b^2-1}}$                                                                                                                                                                                         |
| risch   | $-\frac{\ln\left(e^{ix} + \frac{iab\sqrt{-a^2+b^2+1}+ia^2-ib^2-a^2b+b^3+a\sqrt{-a^2+b^2+1}-ib}{(b^2+1)\sqrt{-a^2+b^2+1}}\right)}{\sqrt{-a^2+b^2+1}} + \frac{\ln\left(e^{ix} + \frac{iab\sqrt{-a^2+b^2+1}-ia^2+ib^2+a^2b-b^3+a\sqrt{-a^2+b^2+1}+ib}{(b^2+1)\sqrt{-a^2+b^2+1}}\right)}{\sqrt{-a^2+b^2+1}}$ |

input `int(1/(a+cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)`

output `2/(a^2-b^2-1)^(1/2)*arctan(1/2*(2*(a-1)*tan(1/2*x)+2*b)/(a^2-b^2-1)^(1/2))`

**3.123.5 Fricas [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 287, normalized size of antiderivative = 6.11

$$\int \frac{1}{a + \cos(x) + b \sin(x)} dx$$

$$= \left[ -\frac{\sqrt{-a^2 + b^2 + 1} \log \left( -\frac{b^4 + (a^2 + 3)b^2 - (2a^2b^2 - b^4 - 2a^2 + 1) \cos(x)^2 - a^2 + 2(ab^2 + a) \cos(x) + 2(ab^3 + ab - (b^3 - (2a^2 - 1)b) \cos(x)) \sin(x)}{(b^2 - 1) \cos(x)^2 - a^2 - b^2 - 2a \cos(x) - 2} \right)}{2(a^2 - b^2 - 1)} \right]$$

input `integrate(1/(a+cos(x)+b*sin(x)),x, algorithm="fricas")`

output `[-1/2*sqrt(-a^2 + b^2 + 1)*log(-(b^4 + (a^2 + 3)*b^2 - (2*a^2*b^2 - b^4 - 2*a^2 + 1)*cos(x)^2 - a^2 + 2*(a*b^2 + a)*cos(x) + 2*(a*b^3 + a*b - (b^3 - (2*a^2 - 1)*b)*cos(x))*sin(x) - 2*(2*a*b*cos(x)^2 - a*b + (b^3 + b)*cos(x) - (b^2 - (a*b^2 - a)*cos(x) + 1)*sin(x))*sqrt(-a^2 + b^2 + 1) + 2)/((b^2 - 1)*cos(x)^2 - a^2 - b^2 - 2*a*cos(x) - 2*(a*b + b*cos(x))*sin(x)))/(a^2 - b^2 - 1), arctan(-(a*b*sin(x) + b^2 + a*cos(x) + 1)*sqrt(a^2 - b^2 - 1)/((b^3 - (a^2 - 1)*b)*cos(x) + (a^2 - b^2 - 1)*sin(x)))/sqrt(a^2 - b^2 - 1)]`

**3.123.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 1872 vs. 2(37) = 74.

Time = 72.57 (sec) , antiderivative size = 1872, normalized size of antiderivative = 39.83

$$\int \frac{1}{a + \cos(x) + b \sin(x)} dx = \text{Too large to display}$$

input `integrate(1/(a+cos(x)+b*sin(x)),x)`



output `Piecewise((log(tan(x/2) + 1/b)/b, Eq(a, 1)), (2*b**4*sqrt(b**2 + 1)/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 10*b**4/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 24*b**2*sqrt(b**2 + 1)/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 40*b**2/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 32*sqrt(b**2 + 1)/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 32/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**...`

### 3.123.7 Maxima [**F(-2)**]

Exception generated.

$$\int \frac{1}{a + \cos(x) + b \sin(x)} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(a+cos(x)+b*sin(x)),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(b^2-a^2+1>0)', see `assume?` for more deta`

**3.123.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int \frac{1}{a + \cos(x) + b \sin(x)} dx = \frac{2 \left( \pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(2a - 2) + \arctan \left( \frac{a \tan(\frac{1}{2}x) + b - \tan(\frac{1}{2}x)}{\sqrt{a^2 - b^2 - 1}} \right) \right)}{\sqrt{a^2 - b^2 - 1}}$$

input `integrate(1/(a+cos(x)+b*sin(x)),x, algorithm="giac")`output `2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*a - 2) + arctan((a*tan(1/2*x) + b - tan(1/2*x))/sqrt(a^2 - b^2 - 1)))/sqrt(a^2 - b^2 - 1)`**3.123.9 Mupad [B] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \frac{1}{a + \cos(x) + b \sin(x)} dx = \begin{cases} \frac{\ln(b \tan(\frac{x}{2}) + 1)}{b} & \text{if } a = 1 \\ \frac{2 \operatorname{atan} \left( \frac{b + \tan(\frac{x}{2})(a-1)}{\sqrt{a^2 - b^2 - 1}} \right)}{\sqrt{a^2 - b^2 - 1}} & \text{if } a \neq 1 \end{cases}$$

input `int(1/(a + cos(x) + b*sin(x)),x)`output `piecewise(a == 1, log(b*tan(x/2) + 1)/b, a ~= 1, (2*atan((b + tan(x/2)*(a - 1))/sqrt(a^2 - b^2 - 1)))/sqrt(a^2 - b^2 - 1))`

## 3.124 $\int x^2 \sin^2(a + bx) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
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| 3.124.2 Mathematica [A] (verified) . . . . .                | 722 |
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| 3.124.4 Maple [A] (verified) . . . . .                      | 724 |
| 3.124.5 Fricas [A] (verification not implemented) . . . . . | 725 |
| 3.124.6 Sympy [A] (verification not implemented) . . . . .  | 725 |
| 3.124.7 Maxima [A] (verification not implemented) . . . . . | 726 |
| 3.124.8 Giac [A] (verification not implemented) . . . . .   | 726 |
| 3.124.9 Mupad [B] (verification not implemented) . . . . .  | 726 |

### 3.124.1 Optimal result

Integrand size = 12, antiderivative size = 73

$$\int x^2 \sin^2(a + bx) dx = -\frac{x}{4b^2} + \frac{x^3}{6} + \frac{\cos(a + bx) \sin(a + bx)}{4b^3} - \frac{x^2 \cos(a + bx) \sin(a + bx)}{2b} + \frac{x \sin^2(a + bx)}{2b^2}$$

output `-1/4*x/b^2+1/6*x^3+1/4*cos(b*x+a)*sin(b*x+a)/b^3-1/2*x^2*cos(b*x+a)*sin(b*x+a)/b+1/2*x*sin(b*x+a)^2/b^2`

### 3.124.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int x^2 \sin^2(a + bx) dx = \frac{4b^3x^3 - 6bx \cos(2(a + bx)) + (3 - 6b^2x^2) \sin(2(a + bx))}{24b^3}$$

input `Integrate[x^2*Sin[a + b*x]^2,x]`

output `(4*b^3*x^3 - 6*b*x*Cos[2*(a + b*x)] + (3 - 6*b^2*x^2)*Sin[2*(a + b*x)])/(24*b^3)`

**3.124.3 Rubi [A] (verified)**

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {3042, 3792, 15, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^2 \sin^2(a + bx) dx \\
 & \quad \downarrow \text{3042} \\
 & \int x^2 \sin(a + bx)^2 dx \\
 & \quad \downarrow \text{3792} \\
 & -\frac{\int \sin^2(a + bx) dx}{2b^2} + \frac{\int x^2 dx}{2} + \frac{x \sin^2(a + bx)}{2b^2} - \frac{x^2 \sin(a + bx) \cos(a + bx)}{2b} \\
 & \quad \downarrow \text{15} \\
 & -\frac{\int \sin^2(a + bx) dx}{2b^2} + \frac{x \sin^2(a + bx)}{2b^2} - \frac{x^2 \sin(a + bx) \cos(a + bx)}{2b} + \frac{x^3}{6} \\
 & \quad \downarrow \text{3042} \\
 & -\frac{\int \sin(a + bx)^2 dx}{2b^2} + \frac{x \sin^2(a + bx)}{2b^2} - \frac{x^2 \sin(a + bx) \cos(a + bx)}{2b} + \frac{x^3}{6} \\
 & \quad \downarrow \text{3115} \\
 & -\frac{\frac{\int 1 dx}{2} - \frac{\sin(a+bx) \cos(a+bx)}{2b}}{2b^2} + \frac{x \sin^2(a + bx)}{2b^2} - \frac{x^2 \sin(a + bx) \cos(a + bx)}{2b} + \frac{x^3}{6} \\
 & \quad \downarrow \text{24} \\
 & \frac{x \sin^2(a + bx)}{2b^2} - \frac{\frac{x}{2} - \frac{\sin(a+bx) \cos(a+bx)}{2b}}{2b^2} - \frac{x^2 \sin(a + bx) \cos(a + bx)}{2b} + \frac{x^3}{6}
 \end{aligned}$$

input `Int[x^2*Sin[a + b*x]^2,x]`

output `x^3/6 - (x^2*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (x*Sin[a + b*x]^2)/(2*b^2) - (x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b))/(2*b^2)`

3.124.3.1 Defintions of rubi rules used

- rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`
  
- rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`
  
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`
  
- rule 3115 `Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n) Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`
  
- rule 3792 `Int[((c_.) + (d_.)*(x_)^(m_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n) Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2*m*((m - 1)/(f^2*n^2)) Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]`

3.124.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

| method            | result                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|-------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch             | $\frac{x^3}{6} - \frac{x \cos(2bx+2a)}{4b^2} - \frac{(2x^2b^2-1) \sin(2bx+2a)}{8b^3}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| derivativedivides | $\frac{a^2 \left( -\frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - 2a \left( (bx+a) \left( -\frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - \frac{(bx+a)^2}{4} + \frac{\sin^2(bx+a)}{4} \right) + (bx+a)^2}{b^3}$                                                                                                                                                                                                                                                                                                                    |
| default           | $\frac{a^2 \left( -\frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - 2a \left( (bx+a) \left( -\frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - \frac{(bx+a)^2}{4} + \frac{\sin^2(bx+a)}{4} \right) + (bx+a)^2}{b^3}$                                                                                                                                                                                                                                                                                                                    |
| norman            | $\frac{\frac{x^2 \left( \tan^3 \left( \frac{bx}{2} + \frac{a}{2} \right) \right)}{b} + \frac{x^3}{6} + \frac{\tan \left( \frac{bx}{2} + \frac{a}{2} \right)}{2b^3} - \frac{\tan^3 \left( \frac{bx}{2} + \frac{a}{2} \right)}{2b^3} - \frac{x}{4b^2} + \frac{x^3 \left( \tan^2 \left( \frac{bx}{2} + \frac{a}{2} \right) \right)}{3} + \frac{x^3 \left( \tan^4 \left( \frac{bx}{2} + \frac{a}{2} \right) \right)}{6} + \frac{3x \left( \tan^2 \left( \frac{bx}{2} + \frac{a}{2} \right) \right)}{2b^2} - x}{\left( 1 + \tan^2 \left( \frac{bx}{2} + \frac{a}{2} \right) \right)^2}$ |

input `int(x^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)`

output `1/6*x^3-1/4/b^2*x*cos(2*b*x+2*a)-1/8*(2*b^2*x^2-1)/b^3*sin(2*b*x+2*a)`

### 3.124.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int x^2 \sin^2(a + bx) dx$$

$$= \frac{2b^3x^3 - 6bx \cos(bx + a)^2 - 3(2b^2x^2 - 1) \cos(bx + a) \sin(bx + a) + 3bx}{12b^3}$$

input `integrate(x^2*sin(b*x+a)^2,x, algorithm="fricas")`

output `1/12*(2*b^3*x^3 - 6*b*x*cos(b*x + a)^2 - 3*(2*b^2*x^2 - 1)*cos(b*x + a)*sin(b*x + a) + 3*b*x)/b^3`

### 3.124.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int x^2 \sin^2(a + bx) dx$$

$$= \begin{cases} \frac{x^3 \sin^2(a+bx)}{6} + \frac{x^3 \cos^2(a+bx)}{6} - \frac{x^2 \sin(a+bx) \cos(a+bx)}{2b} + \frac{x \sin^2(a+bx)}{4b^2} - \frac{x \cos^2(a+bx)}{4b^2} + \frac{\sin(a+bx) \cos(a+bx)}{4b^3} & \text{for } b \neq 0 \\ \frac{x^3 \sin^2(a)}{3} & \text{otherwise} \end{cases}$$

input `integrate(x**2*sin(b*x+a)**2,x)`

output `Piecewise((x**3*sin(a + b*x)**2/6 + x**3*cos(a + b*x)**2/6 - x**2*sin(a + b*x)*cos(a + b*x)/(2*b) + x*sin(a + b*x)**2/(4*b**2) - x*cos(a + b*x)**2/(4*b**2) + sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne(b, 0)), (x**3*sin(a)**2/3, True))`

**3.124.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\int x^2 \sin^2(a + bx) dx = \frac{4(bx + a)^3 + 6(2bx + 2a - \sin(2bx + 2a))a^2 - 6(2(bx + a)^2 - 2(bx + a)\sin(2bx + 2a) - \cos(2bx + 2a))}{24b^3}$$

input `integrate(x^2*sin(b*x+a)^2,x, algorithm="maxima")`output `1/24*(4*(b*x + a)^3 + 6*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^2 - 6*(2*(b*x + a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a - 6*(b*x + a)*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))/b^3`**3.124.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int x^2 \sin^2(a + bx) dx = \frac{1}{6}x^3 - \frac{x \cos(2bx + 2a)}{4b^2} - \frac{(2b^2x^2 - 1)\sin(2bx + 2a)}{8b^3}$$

input `integrate(x^2*sin(b*x+a)^2,x, algorithm="giac")`output `1/6*x^3 - 1/4*x*cos(2*b*x + 2*a)/b^2 - 1/8*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a)/b^3`**3.124.9 Mupad [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int x^2 \sin^2(a + bx) dx = \frac{x^3}{6} + \frac{\sin(2a + 2bx)}{8b^3} - \frac{x \cos(2a + 2bx)}{4b^2} - \frac{x^2 \sin(2a + 2bx)}{4b}$$

input `int(x^2*sin(a + b*x)^2,x)`output `x^3/6 + sin(2*a + 2*b*x)/(8*b^3) - (x*cos(2*a + 2*b*x))/(4*b^2) - (x^2*sin(2*a + 2*b*x))/(4*b)`

### 3.125 $\int \cos(x) \cos(2x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.125.1 Optimal result . . . . .                            | 727 |
| 3.125.2 Mathematica [A] (verified) . . . . .                | 727 |
| 3.125.3 Rubi [A] (verified) . . . . .                       | 728 |
| 3.125.4 Maple [A] (verified) . . . . .                      | 729 |
| 3.125.5 Fricas [A] (verification not implemented) . . . . . | 729 |
| 3.125.6 Sympy [A] (verification not implemented) . . . . .  | 729 |
| 3.125.7 Maxima [A] (verification not implemented) . . . . . | 730 |
| 3.125.8 Giac [A] (verification not implemented) . . . . .   | 730 |
| 3.125.9 Mupad [B] (verification not implemented) . . . . .  | 730 |

#### 3.125.1 Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cos(x) \cos(2x) dx = \frac{\sin(x)}{2} + \frac{1}{6} \sin(3x)$$

output `1/2*sin(x)+1/6*sin(3*x)`

#### 3.125.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(x) \cos(2x) dx = \frac{\sin(x)}{2} + \frac{1}{6} \sin(3x)$$

input `Integrate[Cos[x]*Cos[2*x],x]`

output `Sin[x]/2 + Sin[3*x]/6`



**3.125.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3042, 4771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x) \cos(2x) dx$$

$$\downarrow \text{3042}$$

$$\int \cos(x) \cos(2x) dx$$

$$\downarrow \text{4771}$$

$$\frac{\sin(x)}{2} + \frac{1}{6} \sin(3x)$$

input `Int[Cos[x]*Cos[2*x],x]`

output `Sin[x]/2 + Sin[3*x]/6`

**3.125.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 4771 `Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]`

**3.125.4 Maple [A] (verified)**

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

| method        | result                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | size |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default       | $\frac{\sin(x)}{2} + \frac{\sin(3x)}{6}$                                                                                                                                                                                                                                                                                                                                                                                                                                        | 12   |
| risch         | $\frac{\sin(x)}{2} + \frac{\sin(3x)}{6}$                                                                                                                                                                                                                                                                                                                                                                                                                                        | 12   |
| parallelrisch | $\frac{\sin(x)}{2} + \frac{\sin(3x)}{6}$                                                                                                                                                                                                                                                                                                                                                                                                                                        | 12   |
| norman        | $-\frac{4 \tan(x) \left(\tan^2\left(\frac{x}{2}\right)\right)}{3} + \frac{2 \left(\tan^2(x)\right) \tan\left(\frac{x}{2}\right)}{3} + \frac{4 \tan(x)}{3} - \frac{2 \tan\left(\frac{x}{2}\right)}{3}$<br>$\frac{\phantom{-\frac{4 \tan(x) \left(\tan^2\left(\frac{x}{2}\right)\right)}{3} + \frac{2 \left(\tan^2(x)\right) \tan\left(\frac{x}{2}\right)}{3} + \frac{4 \tan(x)}{3} - \frac{2 \tan\left(\frac{x}{2}\right)}{3}}{(1+\tan^2\left(\frac{x}{2}\right))(1+\tan^2(x))}$ | 51   |

input `int(cos(x)*cos(2*x),x,method=_RETURNVERBOSE)`output `1/2*sin(x)+1/6*sin(3*x)`**3.125.5 Fracas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \cos(x) \cos(2x) dx = \frac{1}{3} (2 \cos(x)^2 + 1) \sin(x)$$

input `integrate(cos(x)*cos(2*x),x, algorithm="fricas")`output `1/3*(2*cos(x)^2 + 1)*sin(x)`**3.125.6 Sympy [A] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(x) \cos(2x) dx = -\frac{\sin(x) \cos(2x)}{3} + \frac{2 \sin(2x) \cos(x)}{3}$$

input `integrate(cos(x)*cos(2*x),x)`output `-sin(x)*cos(2*x)/3 + 2*sin(2*x)*cos(x)/3`

**3.125.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(x) \cos(2x) dx = \frac{1}{6} \sin(3x) + \frac{1}{2} \sin(x)$$

input `integrate(cos(x)*cos(2*x),x, algorithm="maxima")`output `1/6*sin(3*x) + 1/2*sin(x)`**3.125.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(x) \cos(2x) dx = \frac{1}{6} \sin(3x) + \frac{1}{2} \sin(x)$$

input `integrate(cos(x)*cos(2*x),x, algorithm="giac")`output `1/6*sin(3*x) + 1/2*sin(x)`**3.125.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \cos(x) \cos(2x) dx = \sin(x) - \frac{2 \sin(x)^3}{3}$$

input `int(cos(2*x)*cos(x),x)`output `sin(x) - (2*sin(x)^3)/3`

### 3.126 $\int x^2 \cos^2(a + bx) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.126.1 Optimal result . . . . .                            | 731 |
| 3.126.2 Mathematica [A] (verified) . . . . .                | 731 |
| 3.126.3 Rubi [A] (verified) . . . . .                       | 732 |
| 3.126.4 Maple [A] (verified) . . . . .                      | 733 |
| 3.126.5 Fracas [A] (verification not implemented) . . . . . | 734 |
| 3.126.6 Sympy [A] (verification not implemented) . . . . .  | 734 |
| 3.126.7 Maxima [A] (verification not implemented) . . . . . | 735 |
| 3.126.8 Giac [A] (verification not implemented) . . . . .   | 735 |
| 3.126.9 Mupad [B] (verification not implemented) . . . . .  | 735 |

#### 3.126.1 Optimal result

Integrand size = 12, antiderivative size = 73

$$\int x^2 \cos^2(a + bx) dx = -\frac{x}{4b^2} + \frac{x^3}{6} + \frac{x \cos^2(a + bx)}{2b^2} - \frac{\cos(a + bx) \sin(a + bx)}{4b^3} + \frac{x^2 \cos(a + bx) \sin(a + bx)}{2b}$$

output `-1/4*x/b^2+1/6*x^3+1/2*x*cos(b*x+a)^2/b^2-1/4*cos(b*x+a)*sin(b*x+a)/b^3+1/2*x^2*cos(b*x+a)*sin(b*x+a)/b`

#### 3.126.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int x^2 \cos^2(a + bx) dx = \frac{4b^3 x^3 + 6bx \cos(2(a + bx)) + (-3 + 6b^2 x^2) \sin(2(a + bx))}{24b^3}$$

input `Integrate[x^2*Cos[a + b*x]^2,x]`

output `(4*b^3*x^3 + 6*b*x*Cos[2*(a + b*x)] + (-3 + 6*b^2*x^2)*Sin[2*(a + b*x)])/(24*b^3)`

**3.126.3 Rubi [A] (verified)**

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {3042, 3792, 15, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^2 \cos^2(a + bx) dx \\
 & \quad \downarrow \text{3042} \\
 & \int x^2 \sin\left(a + bx + \frac{\pi}{2}\right)^2 dx \\
 & \quad \downarrow \text{3792} \\
 & -\frac{\int \cos^2(a + bx) dx}{2b^2} + \frac{\int x^2 dx}{2} + \frac{x \cos^2(a + bx)}{2b^2} + \frac{x^2 \sin(a + bx) \cos(a + bx)}{2b} \\
 & \quad \downarrow \text{15} \\
 & -\frac{\int \cos^2(a + bx) dx}{2b^2} + \frac{x \cos^2(a + bx)}{2b^2} + \frac{x^2 \sin(a + bx) \cos(a + bx)}{2b} + \frac{x^3}{6} \\
 & \quad \downarrow \text{3042} \\
 & -\frac{\int \sin\left(a + bx + \frac{\pi}{2}\right)^2 dx}{2b^2} + \frac{x \cos^2(a + bx)}{2b^2} + \frac{x^2 \sin(a + bx) \cos(a + bx)}{2b} + \frac{x^3}{6} \\
 & \quad \downarrow \text{3115} \\
 & -\frac{\frac{\int 1 dx}{2} + \frac{\sin(a + bx) \cos(a + bx)}{2b}}{2b^2} + \frac{x \cos^2(a + bx)}{2b^2} + \frac{x^2 \sin(a + bx) \cos(a + bx)}{2b} + \frac{x^3}{6} \\
 & \quad \downarrow \text{24} \\
 & \frac{x \cos^2(a + bx)}{2b^2} - \frac{\sin(a + bx) \cos(a + bx)}{2b^2} + \frac{x}{2} + \frac{x^2 \sin(a + bx) \cos(a + bx)}{2b} + \frac{x^3}{6}
 \end{aligned}$$

input `Int[x^2*Cos[a + b*x]^2,x]`

output `x^3/6 + (x*Cos[a + b*x]^2)/(2*b^2) + (x^2*Cos[a + b*x]*Sin[a + b*x])/(2*b) - (x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b))/(2*b^2)`

3.126.3.1 Defintions of rubi rules used

- rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`
- rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`
- rule 3115 `Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n) Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`
- rule 3792 `Int[((c_.) + (d_.)*(x_)^(m_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n) Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2*m*((m - 1)/(f^2*n^2)) Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]`

3.126.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

| method            | result                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|-------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch             | $\frac{x^3}{6} + \frac{x \cos(2bx+2a)}{4b^2} + \frac{(2x^2b^2-1) \sin(2bx+2a)}{8b^3}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| parallelrisch     | $\frac{(6x^2b^2-3) \sin(2bx+2a)+4x^3b^3+6x \cos(2bx+2a)b}{24b^3}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| derivativedivides | $\frac{a^2 \left( \frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - 2a \left( (bx+a) \left( \frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - \frac{(bx+a)^2}{4} - \frac{(\sin^2(bx+a))}{4} \right) + (bx+a)^2 \left( \frac{\cos(bx+a)}{2} \right)}{b^3}$                                                                                                                                                                                                                                                                                                                  |
| default           | $\frac{a^2 \left( \frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - 2a \left( (bx+a) \left( \frac{\cos(bx+a) \sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right) - \frac{(bx+a)^2}{4} - \frac{(\sin^2(bx+a))}{4} \right) + (bx+a)^2 \left( \frac{\cos(bx+a)}{2} \right)}{b^3}$                                                                                                                                                                                                                                                                                                                  |
| norman            | $\frac{\frac{x^2 \tan\left(\frac{bx}{2} + \frac{a}{2}\right)}{b} + \frac{x^3}{6} - \frac{\tan\left(\frac{bx}{2} + \frac{a}{2}\right)}{2b^3} + \frac{\tan^3\left(\frac{bx}{2} + \frac{a}{2}\right)}{2b^3} + \frac{x}{4b^2} + \frac{x^3 \left( \tan^2\left(\frac{bx}{2} + \frac{a}{2}\right) \right)}{3} + \frac{x^3 \left( \tan^4\left(\frac{bx}{2} + \frac{a}{2}\right) \right)}{6} - \frac{3x \left( \tan^2\left(\frac{bx}{2} + \frac{a}{2}\right) \right)}{2b^2} + \frac{x \left( \tan^4\left(\frac{bx}{2} + \frac{a}{2}\right) \right)}{2b^2}}{\left(1 + \tan^2\left(\frac{bx}{2} + \frac{a}{2}\right)\right)^2}$ |

3.126.  $\int x^2 \cos^2(a + bx) dx$

input `int(x^2*cos(b*x+a)^2,x,method=_RETURNVERBOSE)`

output `1/6*x^3+1/4/b^2*x*cos(2*b*x+2*a)+1/8*(2*b^2*x^2-1)/b^3*sin(2*b*x+2*a)`

### 3.126.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int x^2 \cos^2(a + bx) dx$$

$$= \frac{2b^3x^3 + 6bx \cos(bx + a)^2 + 3(2b^2x^2 - 1) \cos(bx + a) \sin(bx + a) - 3bx}{12b^3}$$

input `integrate(x^2*cos(b*x+a)^2,x, algorithm="fricas")`

output `1/12*(2*b^3*x^3 + 6*b*x*cos(b*x + a)^2 + 3*(2*b^2*x^2 - 1)*cos(b*x + a)*sin(b*x + a) - 3*b*x)/b^3`

### 3.126.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int x^2 \cos^2(a + bx) dx$$

$$= \begin{cases} \frac{x^3 \sin^2(a+bx)}{6} + \frac{x^3 \cos^2(a+bx)}{6} + \frac{x^2 \sin(a+bx) \cos(a+bx)}{2b} - \frac{x \sin^2(a+bx)}{4b^2} + \frac{x \cos^2(a+bx)}{4b^2} - \frac{\sin(a+bx) \cos(a+bx)}{4b^3} & \text{for } b \neq 0 \\ \frac{x^3 \cos^2(a)}{3} & \text{otherwise} \end{cases}$$

input `integrate(x**2*cos(b*x+a)**2,x)`

output `Piecewise((x**3*sin(a + b*x)**2/6 + x**3*cos(a + b*x)**2/6 + x**2*sin(a + b*x)*cos(a + b*x)/(2*b) - x*sin(a + b*x)**2/(4*b**2) + x*cos(a + b*x)**2/(4*b**2) - sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne(b, 0)), (x**3*cos(a)**2/3, True))`

**3.126.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.55

$$\int x^2 \cos^2(a + bx) dx = \frac{4(bx + a)^3 + 6(2bx + 2a + \sin(2bx + 2a))a^2 - 6(2(bx + a)^2 + 2(bx + a)\sin(2bx + 2a) + \cos(2bx + 2a))}{24b^3}$$

input `integrate(x^2*cos(b*x+a)^2,x, algorithm="maxima")`output `1/24*(4*(b*x + a)^3 + 6*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2 - 6*(2*(b*x + a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))/b^3`**3.126.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int x^2 \cos^2(a + bx) dx = \frac{1}{6}x^3 + \frac{x \cos(2bx + 2a)}{4b^2} + \frac{(2b^2x^2 - 1)\sin(2bx + 2a)}{8b^3}$$

input `integrate(x^2*cos(b*x+a)^2,x, algorithm="giac")`output `1/6*x^3 + 1/4*x*cos(2*b*x + 2*a)/b^2 + 1/8*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a)/b^3`**3.126.9 Mupad [B] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int x^2 \cos^2(a + bx) dx = \frac{x^3}{6} - \frac{\sin(2a + 2bx)}{8b^3} + \frac{x \cos(2a + 2bx)}{4b^2} + \frac{x^2 \sin(2a + 2bx)}{4b}$$

input `int(x^2*cos(a + b*x)^2,x)`output `x^3/6 - sin(2*a + 2*b*x)/(8*b^3) + (x*cos(2*a + 2*b*x))/(4*b^2) + (x^2*sin(2*a + 2*b*x))/(4*b)`



## 3.127 $\int \cot^3(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.127.1 Optimal result . . . . .                            | 736 |
| 3.127.2 Mathematica [A] (verified) . . . . .                | 736 |
| 3.127.3 Rubi [A] (verified) . . . . .                       | 737 |
| 3.127.4 Maple [A] (verified) . . . . .                      | 738 |
| 3.127.5 Fricas [B] (verification not implemented) . . . . . | 739 |
| 3.127.6 Sympy [A] (verification not implemented) . . . . .  | 739 |
| 3.127.7 Maxima [A] (verification not implemented) . . . . . | 739 |
| 3.127.8 Giac [B] (verification not implemented) . . . . .   | 740 |
| 3.127.9 Mupad [B] (verification not implemented) . . . . .  | 740 |

### 3.127.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cot^3(x) dx = -\frac{1}{2} \cot^2(x) - \log(\sin(x))$$

output `-1/2*cot(x)^2-ln(sin(x))`

### 3.127.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \cot^3(x) dx = -\frac{1}{2} \cot^2(x) - \log(\cos(x)) - \log(\tan(x))$$

input `Integrate[Cot[x]^3,x]`

output `-1/2*Cot[x]^2 - Log[Cos[x]] - Log[Tan[x]]`

**3.127.3 Rubi [A] (verified)**

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.750$ , Rules used = {3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \cot^3(x) dx \\
 & \quad \downarrow \text{3042} \\
 & \int -\tan\left(x + \frac{\pi}{2}\right)^3 dx \\
 & \quad \downarrow \text{25} \\
 & -\int \tan\left(x + \frac{\pi}{2}\right)^3 dx \\
 & \quad \downarrow \text{3954} \\
 & \int -\cot(x) dx - \frac{\cot^2(x)}{2} \\
 & \quad \downarrow \text{25} \\
 & -\int \cot(x) dx - \frac{1}{2} \cot^2(x) \\
 & \quad \downarrow \text{3042} \\
 & -\int -\tan\left(x + \frac{\pi}{2}\right) dx - \frac{1}{2} \cot^2(x) \\
 & \quad \downarrow \text{25} \\
 & \int \tan\left(x + \frac{\pi}{2}\right) dx - \frac{\cot^2(x)}{2} \\
 & \quad \downarrow \text{3956} \\
 & -\frac{1}{2} \cot^2(x) - \log(\sin(x))
 \end{aligned}$$

input `Int[Cot[x]^3,x]`

output `-1/2*Cot[x]^2 - Log[Sin[x]]`

## 3.127.3.1 Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3954 `Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2 Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]`

rule 3956 `Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

## 3.127.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

| method            | result                                                                             | size |
|-------------------|------------------------------------------------------------------------------------|------|
| derivativedivides | $-\frac{1}{2 \tan(x)^2} - \ln(\tan(x)) + \frac{\ln(1+\tan^2(x))}{2}$               | 22   |
| default           | $-\frac{1}{2 \tan(x)^2} - \ln(\tan(x)) + \frac{\ln(1+\tan^2(x))}{2}$               | 22   |
| norman            | $-\frac{1}{2 \tan(x)^2} - \ln(\tan(x)) + \frac{\ln(1+\tan^2(x))}{2}$               | 22   |
| parallelrisc      | $-\frac{2 \ln(\tan(x))(\tan^2(x)) - \ln(1+\tan^2(x))(\tan^2(x)) + 1}{2 \tan(x)^2}$ | 31   |
| risc              | $ix + \frac{2e^{2ix}}{(e^{2ix}-1)^2} - \ln(e^{2ix} - 1)$                           | 32   |

input `int(1/tan(x)^3,x,method=_RETURNVERBOSE)`

output `-1/2/tan(x)^2-ln(tan(x))+1/2*ln(1+tan(x)^2)`

**3.127.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 31 vs.  $2(12) = 24$ .

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \cot^3(x) dx = -\frac{\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right) \tan(x)^2 + \tan(x)^2 + 1}{2 \tan(x)^2}$$

input `integrate(1/tan(x)^3,x, algorithm="fricas")`

output `-1/2*(log(tan(x)^2/(tan(x)^2 + 1))*tan(x)^2 + tan(x)^2 + 1)/tan(x)^2`

**3.127.6 Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cot^3(x) dx = -\log(\sin(x)) - \frac{1}{2 \sin^2(x)}$$

input `integrate(1/tan(x)**3,x)`

output `-log(sin(x)) - 1/(2*sin(x)**2)`

**3.127.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cot^3(x) dx = -\frac{1}{2 \sin(x)^2} - \frac{1}{2} \log(\sin(x)^2)$$

input `integrate(1/tan(x)^3,x, algorithm="maxima")`

output `-1/2/sin(x)^2 - 1/2*log(sin(x)^2)`

**3.127.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 29 vs.  $2(12) = 24$ .

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.07

$$\int \cot^3(x) dx = \frac{\tan(x)^2 - 1}{2 \tan(x)^2} + \frac{1}{2} \log(\tan(x)^2 + 1) - \frac{1}{2} \log(\tan(x)^2)$$

input `integrate(1/tan(x)^3,x, algorithm="giac")`

output `1/2*(tan(x)^2 - 1)/tan(x)^2 + 1/2*log(tan(x)^2 + 1) - 1/2*log(tan(x)^2)`

**3.127.9 Mupad [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \cot^3(x) dx = \frac{\ln(\tan(x)^2 + 1)}{2} - \ln(\tan(x)) - \frac{1}{2 \tan(x)^2}$$

input `int(1/tan(x)^3,x)`

output `log(tan(x)^2 + 1)/2 - log(tan(x)) - 1/(2*tan(x)^2)`

### 3.128 $\int x^3 \tan^4(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
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| 3.128.2 Mathematica [A] (verified) . . . . .                | 741 |
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#### 3.128.1 Optimal result

Integrand size = 8, antiderivative size = 104

$$\int x^3 \tan^4(x) dx = -\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2 \log(1 + e^{2ix}) + \log(\cos(x)) + 4ix \operatorname{PolyLog}(2, -e^{2ix}) - 2 \operatorname{PolyLog}(3, -e^{2ix}) + x \tan(x) - x^3 \tan(x) - \frac{1}{2}x^2 \tan^2(x) + \frac{1}{3}x^3 \tan^3(x)$$

```
output -1/2*x^2+4/3*I*x^3+1/4*x^4-4*x^2*ln(1+exp(2*I*x))+ln(cos(x))+4*I*x*polylog
(2,-exp(2*I*x))-2*polylog(3,-exp(2*I*x))+x*tan(x)-x^3*tan(x)-1/2*x^2*tan(x)
)^2+1/3*x^3*tan(x)^3
```

#### 3.128.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int x^3 \tan^4(x) dx = \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2 \log(1 + e^{2ix}) + \log(\cos(x)) + 4ix \operatorname{PolyLog}(2, -e^{2ix}) - 2 \operatorname{PolyLog}(3, -e^{2ix}) - \frac{1}{2}x^2 \sec^2(x) + x \tan(x) - \frac{4}{3}x^3 \tan(x) + \frac{1}{3}x^3 \sec^2(x) \tan(x)$$

```
input Integrate[x^3*Tan[x]^4,x]
```

output  $((4*I)/3)*x^3 + x^4/4 - 4*x^2*Log[1 + E^((2*I)*x)] + Log[Cos[x]] + (4*I)*x$   
 $*PolyLog[2, -E^((2*I)*x)] - 2*PolyLog[3, -E^((2*I)*x)] - (x^2*Sec[x]^2)/2$   
 $+ x*Tan[x] - (4*x^3*Tan[x])/3 + (x^3*Sec[x]^2*Tan[x])/3$

### 3.128.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.20, number of steps used = 16, number of rules used = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.875$ , Rules used = {3042, 4203, 3042, 4203, 15, 3042, 4202, 2620, 3011, 2720, 4203, 15, 3042, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int x^3 \tan^4(x) dx \\ & \quad \downarrow \text{3042} \\ & \int x^3 \tan(x)^4 dx \\ & \quad \downarrow \text{4203} \\ & - \int x^3 \tan^2(x) dx - \int x^2 \tan^3(x) dx + \frac{1}{3} x^3 \tan^3(x) \\ & \quad \downarrow \text{3042} \\ & - \int x^3 \tan(x)^2 dx - \int x^2 \tan(x)^3 dx + \frac{1}{3} x^3 \tan^3(x) \\ & \quad \downarrow \text{4203} \\ & \int x^3 dx + 4 \int x^2 \tan(x) dx + \int x \tan^2(x) dx + \frac{1}{3} x^3 \tan^3(x) - x^3 \tan(x) - \frac{1}{2} x^2 \tan^2(x) \\ & \quad \downarrow \text{15} \\ & 4 \int x^2 \tan(x) dx + \int x \tan^2(x) dx + \frac{x^4}{4} + \frac{1}{3} x^3 \tan^3(x) - x^3 \tan(x) - \frac{1}{2} x^2 \tan^2(x) \\ & \quad \downarrow \text{3042} \\ & 4 \int x^2 \tan(x) dx + \int x \tan(x)^2 dx + \frac{x^4}{4} + \frac{1}{3} x^3 \tan^3(x) - x^3 \tan(x) - \frac{1}{2} x^2 \tan^2(x) \\ & \quad \downarrow \text{4202} \\ & 4 \left( \frac{ix^3}{3} - 2i \int \frac{e^{2ix} x^2}{1 + e^{2ix}} dx \right) + \int x \tan(x)^2 dx + \frac{x^4}{4} + \frac{1}{3} x^3 \tan^3(x) - x^3 \tan(x) - \frac{1}{2} x^2 \tan^2(x) \end{aligned}$$

$$\begin{aligned}
& \downarrow 2620 \\
& 4\left(\frac{ix^3}{3} - 2i\left(i\int x \log(1 + e^{2ix}) dx - \frac{1}{2}ix^2 \log(1 + e^{2ix})\right)\right) + \int x \tan(x)^2 dx + \frac{x^4}{4} + \\
& \quad \frac{1}{3}x^3 \tan^3(x) - x^3 \tan(x) - \frac{1}{2}x^2 \tan^2(x) \\
& \downarrow 3011 \\
& 4\left(\frac{ix^3}{3} - 2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{2}i\int \operatorname{PolyLog}(2, -e^{2ix}) dx\right) - \frac{1}{2}ix^2 \log(1 + e^{2ix})\right)\right) + \\
& \quad \int x \tan(x)^2 dx + \frac{x^4}{4} + \frac{1}{3}x^3 \tan^3(x) - x^3 \tan(x) - \frac{1}{2}x^2 \tan^2(x) \\
& \downarrow 2720 \\
& 4\left(\frac{ix^3}{3} - 2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4}\int e^{-2ix} \operatorname{PolyLog}(2, -e^{2ix}) de^{2ix}\right) - \frac{1}{2}ix^2 \log(1 + e^{2ix})\right)\right) + \\
& \quad \int x \tan(x)^2 dx + \frac{x^4}{4} + \frac{1}{3}x^3 \tan^3(x) - x^3 \tan(x) - \frac{1}{2}x^2 \tan^2(x) \\
& \downarrow 4203 \\
& 4\left(\frac{ix^3}{3} - 2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4}\int e^{-2ix} \operatorname{PolyLog}(2, -e^{2ix}) de^{2ix}\right) - \frac{1}{2}ix^2 \log(1 + e^{2ix})\right)\right) - \\
& \quad \int x dx - \int \tan(x) dx + \frac{x^4}{4} + \frac{1}{3}x^3 \tan^3(x) - x^3 \tan(x) - \frac{1}{2}x^2 \tan^2(x) + x \tan(x) \\
& \downarrow 15 \\
& 4\left(\frac{ix^3}{3} - 2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4}\int e^{-2ix} \operatorname{PolyLog}(2, -e^{2ix}) de^{2ix}\right) - \frac{1}{2}ix^2 \log(1 + e^{2ix})\right)\right) - \\
& \quad \int \tan(x) dx + \frac{x^4}{4} + \frac{1}{3}x^3 \tan^3(x) - x^3 \tan(x) - \frac{x^2}{2} - \frac{1}{2}x^2 \tan^2(x) + x \tan(x) \\
& \downarrow 3042 \\
& 4\left(\frac{ix^3}{3} - 2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4}\int e^{-2ix} \operatorname{PolyLog}(2, -e^{2ix}) de^{2ix}\right) - \frac{1}{2}ix^2 \log(1 + e^{2ix})\right)\right) - \\
& \quad \int \tan(x) dx + \frac{x^4}{4} + \frac{1}{3}x^3 \tan^3(x) - x^3 \tan(x) - \frac{x^2}{2} - \frac{1}{2}x^2 \tan^2(x) + x \tan(x) \\
& \downarrow 3956
\end{aligned}$$



$$4\left(\frac{ix^3}{3} - 2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4} \int e^{-2ix} \operatorname{PolyLog}(2, -e^{2ix}) de^{2ix}\right) - \frac{1}{2}ix^2 \log(1 + e^{2ix})\right)\right) + \frac{x^4}{4} + \frac{1}{3}x^3 \tan^3(x) - x^3 \tan(x) - \frac{x^2}{2} - \frac{1}{2}x^2 \tan^2(x) + x \tan(x) + \log(\cos(x))$$

↓ 7143

$$4\left(\frac{ix^3}{3} - 2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4} \operatorname{PolyLog}(3, -e^{2ix})\right) - \frac{1}{2}ix^2 \log(1 + e^{2ix})\right)\right) + \frac{x^4}{4} + \frac{1}{3}x^3 \tan^3(x) - x^3 \tan(x) - \frac{x^2}{2} - \frac{1}{2}x^2 \tan^2(x) + x \tan(x) + \log(\cos(x))$$

input `Int[x^3*Tan[x]^4,x]`

output `-1/2*x^2 + x^4/4 + Log[Cos[x]] + 4*((I/3)*x^3 - (2*I)*((-1/2*I)*x^2*Log[1 + E^((2*I)*x)] + I*((I/2)*x*PolyLog[2, -E^((2*I)*x)] - PolyLog[3, -E^((2*I)*x)])/4)) + x*Tan[x] - x^3*Tan[x] - (x^2*Tan[x]^2)/2 + (x^3*Tan[x]^3)/3`

### 3.128.3.1 Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 2620 `Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x)*(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

```
rule 3011 Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

```
rule 3042 Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

```
rule 3956 Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

```
rule 4202 Int[((c_.) + (d_.)*(x_)^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m*(E^(2*I*(
e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

```
rule 4203 Int[((c_.) + (d_.)*(x_)^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] := Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1))) Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2 Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

```
rule 7143 Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.128.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.33

| method | result                                                                                                                                                                                              |
|--------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch  | $\frac{x^4}{4} - \frac{2ix(6x^2e^{4ix} + 6x^2e^{2ix} - 3e^{4ix} - 3ixe^{4ix} + 4x^2 - 6e^{2ix} - 3ixe^{2ix} - 3)}{3(e^{2ix} + 1)^3} + \ln(e^{2ix} + 1) - 2\ln(e^{ix}) + \frac{8ix^3}{3} - 4x^2 \ln$ |

```
input int(x^3*tan(x)^4,x,method=_RETURNVERBOSE)
```

3.128.  $\int x^3 \tan^4(x) dx$

output  $1/4*x^4-2/3*I*x*(6*x^2*\exp(4*I*x)+6*x^2*\exp(2*I*x)-3*\exp(4*I*x)-3*I*x*\exp(4*I*x)+4*x^2-6*\exp(2*I*x)-3*I*x*\exp(2*I*x)-3)/(\exp(2*I*x)+1)^3+\ln(\exp(2*I*x)+1)-2*\ln(\exp(I*x))+8/3*I*x^3-4*x^2*\ln(\exp(2*I*x)+1)+4*I*x*\text{polylog}(2,-\exp(2*I*x))-2*\text{polylog}(3,-\exp(2*I*x))$

### 3.128.5 Fracas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs.  $2(80) = 160$ .

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.75

$$\begin{aligned} \int x^3 \tan^4(x) dx &= \frac{1}{3} x^3 \tan(x)^3 + \frac{1}{4} x^4 - \frac{1}{2} x^2 \tan(x)^2 - \frac{1}{2} x^2 \\ &\quad - 2i x \text{Li}_2\left(\frac{2(i \tan(x) - 1)}{\tan(x)^2 + 1} + 1\right) + 2i x \text{Li}_2\left(\frac{2(-i \tan(x) - 1)}{\tan(x)^2 + 1} + 1\right) \\ &\quad - \frac{1}{2} (4x^2 - 1) \log\left(-\frac{2(i \tan(x) - 1)}{\tan(x)^2 + 1}\right) \\ &\quad - \frac{1}{2} (4x^2 - 1) \log\left(-\frac{2(-i \tan(x) - 1)}{\tan(x)^2 + 1}\right) \\ &\quad - (x^3 - x) \tan(x) - \text{polylog}\left(3, \frac{\tan(x)^2 + 2i \tan(x) - 1}{\tan(x)^2 + 1}\right) \\ &\quad - \text{polylog}\left(3, \frac{\tan(x)^2 - 2i \tan(x) - 1}{\tan(x)^2 + 1}\right) \end{aligned}$$

input `integrate(x^3*tan(x)^4,x, algorithm="fricas")`

output  $1/3*x^3*\tan(x)^3 + 1/4*x^4 - 1/2*x^2*\tan(x)^2 - 1/2*x^2 - 2*I*x*\text{dilog}(2*(I*\tan(x) - 1)/(\tan(x)^2 + 1) + 1) + 2*I*x*\text{dilog}(2*(-I*\tan(x) - 1)/(\tan(x)^2 + 1) + 1) - 1/2*(4*x^2 - 1)*\log(-2*(I*\tan(x) - 1)/(\tan(x)^2 + 1)) - 1/2*(4*x^2 - 1)*\log(-2*(-I*\tan(x) - 1)/(\tan(x)^2 + 1)) - (x^3 - x)*\tan(x) - \text{polylog}(3, (\tan(x)^2 + 2*I*\tan(x) - 1)/(\tan(x)^2 + 1)) - \text{polylog}(3, (\tan(x)^2 - 2*I*\tan(x) - 1)/(\tan(x)^2 + 1))$

**3.128.6 Sympy [F]**

$$\int x^3 \tan^4(x) dx = \int x^3 \tan^4(x) dx$$

input `integrate(x**3*tan(x)**4,x)`

output `Integral(x**3*tan(x)**4, x)`

**3.128.7 Maxima [B] (verification not implemented)**

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 491 vs.  $2(80) = 160$ .

Time = 0.32 (sec) , antiderivative size = 491, normalized size of antiderivative = 4.72

$$\int x^3 \tan^4(x) dx = \frac{3i x^4 + 12(4x^2 + (4x^2 - 1)\cos(6x) + 3(4x^2 - 1)\cos(4x) + 3(4x^2 - 1)\cos(2x) - (-4ix^2 + i)\sin(6x) - 3(-4ix^2 + i)\sin(4x) - 3(-4ix^2 + i)\sin(2x) - \arctan(\frac{\sin(2x)}{\cos(2x) + 1}) + (3ix^4 - 32x^3 + 24x)\cos(6x) - 3(-3ix^4 + 16x^3 + 8ix^2 - 16x)\cos(4x) - 3(-3ix^4 + 16x^3 + 8ix^2 - 8x)\cos(2x) - 48(x\cos(6x) + 3x\cos(4x) + 3x\cos(2x) + ix\sin(6x) + 3ix\sin(4x) + 3ix\sin(2x) + x)\operatorname{dilog}(-e^{2ix}) - 6(4ix^2 + (4ix^2 - i)\cos(6x) + 3(4ix^2 - i)\cos(4x) + 3(4ix^2 - i)\cos(2x) - (4x^2 - 1)\sin(6x) - 3(4x^2 - 1)\sin(4x) - 3(4x^2 - 1)\sin(2x) - i)\log(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1) - 24(i\cos(6x) + 3i\cos(4x) + 3i\cos(2x) - \sin(6x) - 3\sin(4x) - 3\sin(2x) + i)\operatorname{polylog}(3, -e^{2ix}) - (3x^4 + 32ix^3 - 24ix)\sin(6x) - 3(3x^4 + 16ix^3 - 8x^2 - 16ix)\sin(4x) - 3(3x^4 + 16ix^3 - 8x^2 - 8ix)\sin(2x))}{(-12i\cos(6x) - 36i\cos(4x) - 36i\cos(2x) + 12\sin(6x) + 36\sin(4x) + 36\sin(2x) - 12i)}$$

input `integrate(x^3*tan(x)^4,x, algorithm="maxima")`

output `(-3*I*x^4 + 12*(4*x^2 + (4*x^2 - 1)*cos(6*x) + 3*(4*x^2 - 1)*cos(4*x) + 3*(4*x^2 - 1)*cos(2*x) - (-4*I*x^2 + I)*sin(6*x) - 3*(-4*I*x^2 + I)*sin(4*x) - 3*(-4*I*x^2 + I)*sin(2*x) - 1)*arctan2(sin(2*x), cos(2*x) + 1) + (3*I*x^4 - 32*x^3 + 24*x)*cos(6*x) - 3*(-3*I*x^4 + 16*x^3 + 8*I*x^2 - 16*x)*cos(4*x) - 3*(-3*I*x^4 + 16*x^3 + 8*I*x^2 - 8*x)*cos(2*x) - 48*(x*cos(6*x) + 3*x*cos(4*x) + 3*x*cos(2*x) + I*x*sin(6*x) + 3*I*x*sin(4*x) + 3*I*x*sin(2*x) + x)*dilog(-e^(2*I*x)) - 6*(4*I*x^2 + (4*I*x^2 - I)*cos(6*x) + 3*(4*I*x^2 - I)*cos(4*x) + 3*(4*I*x^2 - I)*cos(2*x) - (4*x^2 - 1)*sin(6*x) - 3*(4*x^2 - 1)*sin(4*x) - 3*(4*x^2 - 1)*sin(2*x) - I)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - 24*(I*cos(6*x) + 3*I*cos(4*x) + 3*I*cos(2*x) - sin(6*x) - 3*sin(4*x) - 3*sin(2*x) + I)*polylog(3, -e^(2*I*x)) - (3*x^4 + 32*I*x^3 - 24*I*x)*sin(6*x) - 3*(3*x^4 + 16*I*x^3 - 8*x^2 - 16*I*x)*sin(4*x) - 3*(3*x^4 + 16*I*x^3 - 8*x^2 - 8*I*x)*sin(2*x))/(-12*I*cos(6*x) - 36*I*cos(4*x) - 36*I*cos(2*x) + 12*sin(6*x) + 36*sin(4*x) + 36*sin(2*x) - 12*I)`

**3.128.8 Giac [F]**

$$\int x^3 \tan^4(x) dx = \int x^3 \tan(x)^4 dx$$

input `integrate(x^3*tan(x)^4,x, algorithm="giac")`

output `integrate(x^3*tan(x)^4, x)`

**3.128.9 Mupad [F(-1)]**

Timed out.

$$\int x^3 \tan^4(x) dx = \int x^3 \tan(x)^4 dx$$

input `int(x^3*tan(x)^4,x)`

output `int(x^3*tan(x)^4, x)`

### 3.129 $\int x^3 \tan^6(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.129.1 Optimal result . . . . .                            | 749 |
| 3.129.2 Mathematica [A] (verified) . . . . .                | 749 |
| 3.129.3 Rubi [A] (verified) . . . . .                       | 750 |
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| 3.129.5 Fricas [A] (verification not implemented) . . . . . | 756 |
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#### 3.129.1 Optimal result

Integrand size = 8, antiderivative size = 153

$$\begin{aligned} \int x^3 \tan^6(x) dx = & \frac{19x^2}{20} - \frac{23ix^3}{15} - \frac{x^4}{4} + \frac{23}{5}x^2 \log(1 + e^{2ix}) - 2 \log(\cos(x)) \\ & - \frac{23}{5}ix \operatorname{PolyLog}(2, -e^{2ix}) + \frac{23}{10} \operatorname{PolyLog}(3, -e^{2ix}) \\ & - \frac{19}{10}x \tan(x) + x^3 \tan(x) - \frac{\tan^2(x)}{20} + \frac{4}{5}x^2 \tan^2(x) \\ & + \frac{1}{10}x \tan^3(x) - \frac{1}{3}x^3 \tan^3(x) - \frac{3}{20}x^2 \tan^4(x) + \frac{1}{5}x^3 \tan^5(x) \end{aligned}$$

```
output 19/20*x^2-23/15*I*x^3-1/4*x^4+23/5*x^2*ln(1+exp(2*I*x))-2*ln(cos(x))-23/5*
I*x*polylog(2,-exp(2*I*x))+23/10*polylog(3,-exp(2*I*x))-19/10*x*tan(x)+x^3
*tan(x)-1/20*tan(x)^2+4/5*x^2*tan(x)^2+1/10*x*tan(x)^3-1/3*x^3*tan(x)^3-3/
20*x^2*tan(x)^4+1/5*x^3*tan(x)^5
```

#### 3.129.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.87

$$\begin{aligned} \int x^3 \tan^6(x) dx = & \frac{1}{60}(-92ix^3 - 15x^4 + 276x^2 \log(1 + e^{2ix}) - 120 \log(\cos(x)) \\ & - 276ix \operatorname{PolyLog}(2, -e^{2ix}) + 138 \operatorname{PolyLog}(3, -e^{2ix}) - 3 \sec^2(x) \\ & + 66x^2 \sec^2(x) - 9x^2 \sec^4(x) - 120x \tan(x) + 92x^3 \tan(x) \\ & + 6x \sec^2(x) \tan(x) - 44x^3 \sec^2(x) \tan(x) + 12x^3 \sec^4(x) \tan(x)) \end{aligned}$$

input `Integrate[x^3*Tan[x]^6,x]`

output  $((-92I)*x^3 - 15*x^4 + 276*x^2*\text{Log}[1 + E^{((2*I)*x)}] - 120*\text{Log}[\text{Cos}[x]] - (276*I)*x*\text{PolyLog}[2, -E^{((2*I)*x)}] + 138*\text{PolyLog}[3, -E^{((2*I)*x)}] - 3*\text{Sec}[x]^2 + 66*x^2*\text{Sec}[x]^2 - 9*x^2*\text{Sec}[x]^4 - 120*x*\text{Tan}[x] + 92*x^3*\text{Tan}[x] + 6*x*\text{Sec}[x]^2*\text{Tan}[x] - 44*x^3*\text{Sec}[x]^2*\text{Tan}[x] + 12*x^3*\text{Sec}[x]^4*\text{Tan}[x])/60$

### 3.129.3 Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.92, number of steps used = 21, number of rules used = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 2.500$ , Rules used = {3042, 4203, 3042, 4203, 3042, 4203, 15, 3042, 3954, 3042, 3956, 4202, 2620, 3011, 2720, 4203, 15, 3042, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^3 \tan^6(x) dx \\
 & \quad \downarrow \text{3042} \\
 & \int x^3 \tan(x)^6 dx \\
 & \quad \downarrow \text{4203} \\
 & - \int x^3 \tan^4(x) dx - \frac{3}{5} \int x^2 \tan^5(x) dx + \frac{1}{5} x^3 \tan^5(x) \\
 & \quad \downarrow \text{3042} \\
 & - \int x^3 \tan(x)^4 dx - \frac{3}{5} \int x^2 \tan(x)^5 dx + \frac{1}{5} x^3 \tan^5(x) \\
 & \quad \downarrow \text{4203} \\
 & \int x^3 \tan^2(x) dx + \int x^2 \tan^3(x) dx - \frac{3}{5} \left( - \int x^2 \tan^3(x) dx - \frac{1}{2} \int x \tan^4(x) dx + \frac{1}{4} x^2 \tan^4(x) \right) + \\
 & \quad \frac{1}{5} x^3 \tan^5(x) - \frac{1}{3} x^3 \tan^3(x) \\
 & \quad \downarrow \text{3042} \\
 & \int x^3 \tan(x)^2 dx - \frac{3}{5} \left( - \int x^2 \tan(x)^3 dx - \frac{1}{2} \int x \tan(x)^4 dx + \frac{1}{4} x^2 \tan^4(x) \right) + \int x^2 \tan(x)^3 dx + \\
 & \quad \frac{1}{5} x^3 \tan^5(x) - \frac{1}{3} x^3 \tan^3(x)
 \end{aligned}$$

$$\begin{aligned} & \downarrow 4203 \\ & - \int x^3 dx - \\ & \frac{3}{5} \left( \int x^2 \tan(x) dx + \int x \tan^2(x) dx + \frac{1}{2} \left( \frac{1}{3} \int \tan^3(x) dx + \int x \tan^2(x) dx - \frac{1}{3} x \tan^3(x) \right) + \frac{1}{4} x^2 \tan^4(x) - \frac{1}{2} x^2 \tan^2(x) \right) \\ & 4 \int x^2 \tan(x) dx - \int x \tan^2(x) dx + \frac{1}{5} x^3 \tan^5(x) - \frac{1}{3} x^3 \tan^3(x) + x^3 \tan(x) + \frac{1}{2} x^2 \tan^2(x) \end{aligned}$$

$$\begin{aligned} & \downarrow 15 \\ & - \frac{3}{5} \left( \int x^2 \tan(x) dx + \int x \tan^2(x) dx + \frac{1}{2} \left( \frac{1}{3} \int \tan^3(x) dx + \int x \tan^2(x) dx - \frac{1}{3} x \tan^3(x) \right) + \frac{1}{4} x^2 \tan^4(x) - \frac{1}{2} x^2 \tan^2(x) \right) \\ & 4 \int x^2 \tan(x) dx - \int x \tan^2(x) dx - \frac{x^4}{4} + \frac{1}{5} x^3 \tan^5(x) - \frac{1}{3} x^3 \tan^3(x) + x^3 \tan(x) + \frac{1}{2} x^2 \tan^2(x) \end{aligned}$$

$$\begin{aligned} & \downarrow 3042 \\ & - \frac{3}{5} \left( \int x^2 \tan(x) dx + \frac{1}{2} \left( \int x \tan(x)^2 dx + \frac{1}{3} \int \tan(x)^3 dx - \frac{1}{3} x \tan^3(x) \right) + \int x \tan(x)^2 dx + \frac{1}{4} x^2 \tan^4(x) - \frac{1}{2} x^2 \tan^2(x) \right) \\ & 4 \int x^2 \tan(x) dx - \int x \tan(x)^2 dx - \frac{x^4}{4} + \frac{1}{5} x^3 \tan^5(x) - \frac{1}{3} x^3 \tan^3(x) + x^3 \tan(x) + \frac{1}{2} x^2 \tan^2(x) \end{aligned}$$

$$\begin{aligned} & \downarrow 3954 \\ & - \frac{3}{5} \left( \int x^2 \tan(x) dx + \frac{1}{2} \left( \frac{1}{3} \left( \frac{\tan^2(x)}{2} - \int \tan(x) dx \right) + \int x \tan(x)^2 dx - \frac{1}{3} x \tan^3(x) \right) + \int x \tan(x)^2 dx + \frac{1}{4} x^2 \tan^4(x) - \frac{1}{2} x^2 \tan^2(x) \right) \\ & 4 \int x^2 \tan(x) dx - \int x \tan(x)^2 dx - \frac{x^4}{4} + \frac{1}{5} x^3 \tan^5(x) - \frac{1}{3} x^3 \tan^3(x) + x^3 \tan(x) + \frac{1}{2} x^2 \tan^2(x) \end{aligned}$$

$$\begin{aligned} & \downarrow 3042 \\ & - \frac{3}{5} \left( \int x^2 \tan(x) dx + \frac{1}{2} \left( \frac{1}{3} \left( \frac{\tan^2(x)}{2} - \int \tan(x) dx \right) + \int x \tan(x)^2 dx - \frac{1}{3} x \tan^3(x) \right) + \int x \tan(x)^2 dx + \frac{1}{4} x^2 \tan^4(x) - \frac{1}{2} x^2 \tan^2(x) \right) \\ & 4 \int x^2 \tan(x) dx - \int x \tan(x)^2 dx - \frac{x^4}{4} + \frac{1}{5} x^3 \tan^5(x) - \frac{1}{3} x^3 \tan^3(x) + x^3 \tan(x) + \frac{1}{2} x^2 \tan^2(x) \end{aligned}$$

$$\begin{aligned} & \downarrow 3956 \\ & -4 \int x^2 \tan(x) dx - \\ & \frac{3}{5} \left( \int x^2 \tan(x) dx + \int x \tan(x)^2 dx + \frac{1}{2} \left( \int x \tan(x)^2 dx - \frac{1}{3} x \tan^3(x) + \frac{1}{3} \left( \frac{\tan^2(x)}{2} + \log(\cos(x)) \right) \right) + \frac{1}{4} x^2 \tan^4(x) - \frac{1}{2} x^2 \tan^2(x) \right) \\ & \int x \tan(x)^2 dx - \frac{x^4}{4} + \frac{1}{5} x^3 \tan^5(x) - \frac{1}{3} x^3 \tan^3(x) + x^3 \tan(x) + \frac{1}{2} x^2 \tan^2(x) \end{aligned}$$

$$\downarrow 4202$$



$$\begin{aligned}
& -4\left(\frac{ix^3}{3} - 2i \int \frac{e^{2ix}x^2}{1+e^{2ix}} dx\right) - \\
\frac{3}{5}\left(-2i \int \frac{e^{2ix}x^2}{1+e^{2ix}} dx + \int x \tan(x)^2 dx + \frac{1}{2}\left(\int x \tan(x)^2 dx - \frac{1}{3}x \tan^3(x) + \frac{1}{3}\left(\frac{\tan^2(x)}{2} + \log(\cos(x))\right)\right) + \frac{ix^3}{3} - \int x \tan(x)^2 dx - \frac{x^4}{4} + \frac{1}{5}x^3 \tan^5(x) - \frac{1}{3}x^3 \tan^3(x) + x^3 \tan(x) + \frac{1}{2}x^2 \tan^2(x)\right) \\
& \downarrow \text{2620} \\
& -4\left(\frac{ix^3}{3} - 2i\left(i \int x \log(1+e^{2ix}) dx - \frac{1}{2}ix^2 \log(1+e^{2ix})\right)\right) - \\
\frac{3}{5}\left(-2i\left(i \int x \log(1+e^{2ix}) dx - \frac{1}{2}ix^2 \log(1+e^{2ix})\right) + \int x \tan(x)^2 dx + \frac{1}{2}\left(\int x \tan(x)^2 dx - \frac{1}{3}x \tan^3(x) + \frac{1}{3}\left(\frac{\tan^2(x)}{2} + \log(\cos(x))\right)\right) + \frac{ix^3}{3} - \int x \tan(x)^2 dx - \frac{x^4}{4} + \frac{1}{5}x^3 \tan^5(x) - \frac{1}{3}x^3 \tan^3(x) + x^3 \tan(x) + \frac{1}{2}x^2 \tan^2(x)\right) \\
& \downarrow \text{3011} \\
& -4\left(\frac{ix^3}{3} - 2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{2}i \int \operatorname{PolyLog}(2, -e^{2ix}) dx\right) - \frac{1}{2}ix^2 \log(1+e^{2ix})\right)\right) - \\
\frac{3}{5}\left(-2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{2}i \int \operatorname{PolyLog}(2, -e^{2ix}) dx\right) - \frac{1}{2}ix^2 \log(1+e^{2ix})\right) + \int x \tan(x)^2 dx + \frac{1}{2}\left(\int x \tan(x)^2 dx - \frac{1}{3}x \tan^3(x) + \frac{1}{3}\left(\frac{\tan^2(x)}{2} + \log(\cos(x))\right)\right) + \frac{ix^3}{3} - \int x \tan(x)^2 dx - \frac{x^4}{4} + \frac{1}{5}x^3 \tan^5(x) - \frac{1}{3}x^3 \tan^3(x) + x^3 \tan(x) + \frac{1}{2}x^2 \tan^2(x)\right) \\
& \downarrow \text{2720} \\
& -4\left(\frac{ix^3}{3} - 2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4} \int e^{-2ix} \operatorname{PolyLog}(2, -e^{2ix}) de^{2ix}\right) - \frac{1}{2}ix^2 \log(1+e^{2ix})\right)\right) - \\
\frac{3}{5}\left(-2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4} \int e^{-2ix} \operatorname{PolyLog}(2, -e^{2ix}) de^{2ix}\right) - \frac{1}{2}ix^2 \log(1+e^{2ix})\right) + \int x \tan(x)^2 dx + \frac{1}{2}\left(\int x \tan(x)^2 dx - \frac{1}{3}x \tan^3(x) + \frac{1}{3}\left(\frac{\tan^2(x)}{2} + \log(\cos(x))\right)\right) + \frac{ix^3}{3} - \int x \tan(x)^2 dx - \frac{x^4}{4} + \frac{1}{5}x^3 \tan^5(x) - \frac{1}{3}x^3 \tan^3(x) + x^3 \tan(x) + \frac{1}{2}x^2 \tan^2(x)\right) \\
& \downarrow \text{4203} \\
& -4\left(\frac{ix^3}{3} - 2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4} \int e^{-2ix} \operatorname{PolyLog}(2, -e^{2ix}) de^{2ix}\right) - \frac{1}{2}ix^2 \log(1+e^{2ix})\right)\right) - \\
\frac{3}{5}\left(-2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4} \int e^{-2ix} \operatorname{PolyLog}(2, -e^{2ix}) de^{2ix}\right) - \frac{1}{2}ix^2 \log(1+e^{2ix})\right) - \int x dx - \int \tan(x) dx + \int x dx + \int \tan(x) dx - \frac{x^4}{4} + \frac{1}{5}x^3 \tan^5(x) - \frac{1}{3}x^3 \tan^3(x) + x^3 \tan(x) + \frac{1}{2}x^2 \tan^2(x) - x \tan(x)\right) \\
& \downarrow \text{15}
\end{aligned}$$

$$-4\left(\frac{ix^3}{3} - 2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4}\int e^{-2ix} \operatorname{PolyLog}(2, -e^{2ix}) de^{2ix}\right) - \frac{1}{2}ix^2 \log(1 + e^{2ix})\right)\right) - \frac{3}{5}\left(-2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4}\int e^{-2ix} \operatorname{PolyLog}(2, -e^{2ix}) de^{2ix}\right) - \frac{1}{2}ix^2 \log(1 + e^{2ix})\right) + \frac{1}{2}\left(-\int \tan(x) dx - \int \tan(x) dx - \frac{x^4}{4} + \frac{1}{5}x^3 \tan^5(x) - \frac{1}{3}x^3 \tan^3(x) + x^3 \tan(x) + \frac{x^2}{2} + \frac{1}{2}x^2 \tan^2(x) - x \tan(x)\right)\right)$$

↓ 3042

$$-4\left(\frac{ix^3}{3} - 2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4}\int e^{-2ix} \operatorname{PolyLog}(2, -e^{2ix}) de^{2ix}\right) - \frac{1}{2}ix^2 \log(1 + e^{2ix})\right)\right) - \frac{3}{5}\left(-2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4}\int e^{-2ix} \operatorname{PolyLog}(2, -e^{2ix}) de^{2ix}\right) - \frac{1}{2}ix^2 \log(1 + e^{2ix})\right) + \frac{1}{2}\left(-\int \tan(x) dx - \int \tan(x) dx - \frac{x^4}{4} + \frac{1}{5}x^3 \tan^5(x) - \frac{1}{3}x^3 \tan^3(x) + x^3 \tan(x) + \frac{x^2}{2} + \frac{1}{2}x^2 \tan^2(x) - x \tan(x)\right)\right)$$

↓ 3956

$$-4\left(\frac{ix^3}{3} - 2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4}\int e^{-2ix} \operatorname{PolyLog}(2, -e^{2ix}) de^{2ix}\right) - \frac{1}{2}ix^2 \log(1 + e^{2ix})\right)\right) - \frac{3}{5}\left(-2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4}\int e^{-2ix} \operatorname{PolyLog}(2, -e^{2ix}) de^{2ix}\right) - \frac{1}{2}ix^2 \log(1 + e^{2ix})\right) + \frac{ix^3}{3} - \frac{x^2}{2} + \frac{1}{4}\frac{x^4}{4} + \frac{1}{5}x^3 \tan^5(x) - \frac{1}{3}x^3 \tan^3(x) + x^3 \tan(x) + \frac{x^2}{2} + \frac{1}{2}x^2 \tan^2(x) - x \tan(x) - \log(\cos(x))\right)$$

↓ 7143

$$-4\left(\frac{ix^3}{3} - 2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4}\operatorname{PolyLog}(3, -e^{2ix})\right) - \frac{1}{2}ix^2 \log(1 + e^{2ix})\right)\right) - \frac{3}{5}\left(-2i\left(i\left(\frac{1}{2}ix \operatorname{PolyLog}(2, -e^{2ix}) - \frac{1}{4}\operatorname{PolyLog}(3, -e^{2ix})\right) - \frac{1}{2}ix^2 \log(1 + e^{2ix})\right) + \frac{ix^3}{3} - \frac{x^2}{2} + \frac{1}{4}x^2 \tan^4(x) - \frac{x^4}{4} + \frac{1}{5}x^3 \tan^5(x) - \frac{1}{3}x^3 \tan^3(x) + x^3 \tan(x) + \frac{x^2}{2} + \frac{1}{2}x^2 \tan^2(x) - x \tan(x) - \log(\cos(x))\right)$$

input `Int[x^3*Tan[x]^6,x]`

```
output x^2/2 - x^4/4 - Log[Cos[x]] - 4*((I/3)*x^3 - (2*I)*((-1/2*I)*x^2*Log[1 + E
^((2*I)*x)] + I*((I/2)*x*PolyLog[2, -E^((2*I)*x)] - PolyLog[3, -E^((2*I)*x
)]/4))) - x*Tan[x] + x^3*Tan[x] + (x^2*Tan[x]^2)/2 - (x^3*Tan[x]^3)/3 + (x
^3*Tan[x]^5)/5 - (3*(-1/2*x^2 + (I/3)*x^3 + Log[Cos[x]] - (2*I)*((-1/2*I)*
x^2*Log[1 + E^((2*I)*x)] + I*((I/2)*x*PolyLog[2, -E^((2*I)*x)] - PolyLog[3
, -E^((2*I)*x)]/4)) + x*Tan[x] - (x^2*Tan[x]^2)/2 + (x^2*Tan[x]^4)/4 + (-1
/2*x^2 + Log[Cos[x]] + x*Tan[x] - (x*Tan[x]^3)/3 + (Log[Cos[x]] + Tan[x]^2
/2)/3)/2)/5
```

### 3.129.3.1 Defintions of rubi rules used

```
rule 15 Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

```
rule 2620 Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_)^(m_.)))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
))))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

```
rule 2720 Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_)^(m_)) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

```
rule 3011 Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

```
rule 3042 Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954 `Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2 Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]`

rule 3956 `Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

rule 4202 `Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m*(E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 4203 `Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Simp[b*d*(m/(f*(n - 1))) Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Simp[b^2 Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]`

rule 7143 `Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]`

### 3.129.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.55

| method | result                                                                                                                                                                                                                    |
|--------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch  | $-\frac{x^4}{4} + \frac{i(9ie^{6ix} + 90x^3e^{8ix} - 162ix^2e^{4ix} - 66ix^2e^{2ix} + 180x^3e^{6ix} - 66xe^{8ix} + 3ie^{8ix} + 9ie^{4ix} + 280e^{4ix}x^3 - 246xe^{6ix} - 162ix^2e^{6ix} + 3ie^{2ix})}{15(e^{2ix} + 1)^5}$ |

input `int(x^3*tan(x)^6,x,method=_RETURNVERBOSE)`

```
output -1/4*x^4+1/15*I*(9*I*exp(6*I*x)+90*x^3*exp(8*I*x)-162*I*x^2*exp(4*I*x)-66*
I*x^2*exp(2*I*x)+180*x^3*exp(6*I*x)-66*x*exp(8*I*x)+3*I*exp(8*I*x)+9*I*exp
(4*I*x)+280*exp(4*I*x)*x^3-246*x*exp(6*I*x)-162*I*x^2*exp(6*I*x)+3*I*exp(2
*I*x)+140*exp(2*I*x)*x^3-354*x*exp(4*I*x)-66*I*x^2*exp(8*I*x)+46*x^3-234*x
*exp(2*I*x)-60*x)/(exp(2*I*x)+1)^5-2*ln(exp(2*I*x)+1)+4*ln(exp(I*x))-46/15
*I*x^3+23/5*x^2*ln(exp(2*I*x)+1)-23/5*I*x*polylog(2,-exp(2*I*x))+23/10*pol
ylog(3,-exp(2*I*x))
```

### 3.129.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.39

$$\begin{aligned} \int x^3 \tan^6(x) dx &= \frac{1}{5} x^3 \tan(x)^5 - \frac{3}{20} x^2 \tan(x)^4 - \frac{1}{4} x^4 - \frac{1}{30} (10x^3 - 3x) \tan(x)^3 \\ &+ \frac{1}{20} (16x^2 - 1) \tan(x)^2 + \frac{19}{20} x^2 + \frac{23}{10} i x \operatorname{Li}_2\left(\frac{2(i \tan(x) - 1)}{\tan(x)^2 + 1} + 1\right) \\ &- \frac{23}{10} i x \operatorname{Li}_2\left(\frac{2(-i \tan(x) - 1)}{\tan(x)^2 + 1} + 1\right) \\ &+ \frac{1}{10} (23x^2 - 10) \log\left(-\frac{2(i \tan(x) - 1)}{\tan(x)^2 + 1}\right) \\ &+ \frac{1}{10} (23x^2 - 10) \log\left(-\frac{2(-i \tan(x) - 1)}{\tan(x)^2 + 1}\right) \\ &+ \frac{1}{10} (10x^3 - 19x) \tan(x) + \frac{23}{20} \operatorname{polylog}\left(3, \frac{\tan(x)^2 + 2i \tan(x) - 1}{\tan(x)^2 + 1}\right) \\ &+ \frac{23}{20} \operatorname{polylog}\left(3, \frac{\tan(x)^2 - 2i \tan(x) - 1}{\tan(x)^2 + 1}\right) \end{aligned}$$

```
input integrate(x^3*tan(x)^6,x, algorithm="fracas")
```

```
output 1/5*x^3*tan(x)^5 - 3/20*x^2*tan(x)^4 - 1/4*x^4 - 1/30*(10*x^3 - 3*x)*tan(x)
)^3 + 1/20*(16*x^2 - 1)*tan(x)^2 + 19/20*x^2 + 23/10*I*x*dilog(2*(I*tan(x)
- 1)/(tan(x)^2 + 1) + 1) - 23/10*I*x*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 +
1) + 1) + 1/10*(23*x^2 - 10)*log(-2*(I*tan(x) - 1)/(tan(x)^2 + 1)) + 1/10*
(23*x^2 - 10)*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) + 1/10*(10*x^3 - 19*x
)*tan(x) + 23/20*polylog(3, (tan(x)^2 + 2*I*tan(x) - 1)/(tan(x)^2 + 1)) +
23/20*polylog(3, (tan(x)^2 - 2*I*tan(x) - 1)/(tan(x)^2 + 1))
```

**3.129.6 Sympy [F]**

$$\int x^3 \tan^6(x) dx = \int x^3 \tan^6(x) dx$$

input `integrate(x**3*tan(x)**6,x)`

output `Integral(x**3*tan(x)**6, x)`

**3.129.7 Maxima [B] (verification not implemented)**

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 777 vs.  $2(113) = 226$ .

Time = 0.57 (sec) , antiderivative size = 777, normalized size of antiderivative = 5.08

$$\int x^3 \tan^6(x) dx = \text{Too large to display}$$

input `integrate(x^3*tan(x)^6,x, algorithm="maxima")`

output `(15*I*x^4 + 12*(23*x^2 + (23*x^2 - 10)*cos(10*x) + 5*(23*x^2 - 10)*cos(8*x) + 10*(23*x^2 - 10)*cos(6*x) + 10*(23*x^2 - 10)*cos(4*x) + 5*(23*x^2 - 10)*cos(2*x) - (-23*I*x^2 + 10*I)*sin(10*x) - 5*(-23*I*x^2 + 10*I)*sin(8*x) - 10*(-23*I*x^2 + 10*I)*sin(6*x) - 10*(-23*I*x^2 + 10*I)*sin(4*x) - 5*(-23*I*x^2 + 10*I)*sin(2*x) - 10)*arctan2(sin(2*x), cos(2*x) + 1) + (15*I*x^4 - 184*x^3 + 240*x)*cos(10*x) + (75*I*x^4 - 560*x^3 - 264*I*x^2 + 936*x + 12*I)*cos(8*x) - 2*(-75*I*x^4 + 560*x^3 + 324*I*x^2 - 708*x - 18*I)*cos(6*x) - 6*(-25*I*x^4 + 120*x^3 + 108*I*x^2 - 164*x - 6*I)*cos(4*x) - 3*(-25*I*x^4 + 120*x^3 + 88*I*x^2 - 88*x - 4*I)*cos(2*x) - 276*(x*cos(10*x) + 5*x*cos(8*x) + 10*x*cos(6*x) + 10*x*cos(4*x) + 5*x*cos(2*x) + I*x*sin(10*x) + 5*I*x*sin(8*x) + 10*I*x*sin(6*x) + 10*I*x*sin(4*x) + 5*I*x*sin(2*x) + x)*dilog(-e^(2*I*x)) - 6*(23*I*x^2 + (23*I*x^2 - 10*I)*cos(10*x) + 5*(23*I*x^2 - 10*I)*cos(8*x) + 10*(23*I*x^2 - 10*I)*cos(6*x) + 10*(23*I*x^2 - 10*I)*cos(4*x) + 5*(23*I*x^2 - 10*I)*cos(2*x) - (23*x^2 - 10)*sin(10*x) - 5*(23*x^2 - 10)*sin(8*x) - 10*(23*x^2 - 10)*sin(6*x) - 10*(23*x^2 - 10)*sin(4*x) - 5*(23*x^2 - 10)*sin(2*x) - 10*I)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - 138*(I*cos(10*x) + 5*I*cos(8*x) + 10*I*cos(6*x) + 10*I*cos(4*x) + 5*I*cos(2*x) - sin(10*x) - 5*sin(8*x) - 10*sin(6*x) - 10*sin(4*x) - 5*sin(2*x) + I)*polylog(3, -e^(2*I*x)) - (15*x^4 + 184*I*x^3 - 240*I*x)*sin(10*x) - (75*x^4 + 560*I*x^3 - 264*x^2 - 936*I*x + 12)*sin(8*x) - 2*(75*x^4 ...`

**3.129.8 Giac [F]**

$$\int x^3 \tan^6(x) dx = \int x^3 \tan(x)^6 dx$$

input `integrate(x^3*tan(x)^6,x, algorithm="giac")`

output `integrate(x^3*tan(x)^6, x)`

**3.129.9 Mupad [F(-1)]**

Timed out.

$$\int x^3 \tan^6(x) dx = \int x^3 \tan(x)^6 dx$$

input `int(x^3*tan(x)^6,x)`

output `int(x^3*tan(x)^6, x)`

### 3.130 $\int x \tan^2(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.130.1 Optimal result . . . . .                            | 759 |
| 3.130.2 Mathematica [A] (verified) . . . . .                | 759 |
| 3.130.3 Rubi [A] (verified) . . . . .                       | 760 |
| 3.130.4 Maple [A] (verified) . . . . .                      | 761 |
| 3.130.5 Fricas [A] (verification not implemented) . . . . . | 762 |
| 3.130.6 Sympy [A] (verification not implemented) . . . . .  | 762 |
| 3.130.7 Maxima [B] (verification not implemented) . . . . . | 762 |
| 3.130.8 Giac [A] (verification not implemented) . . . . .   | 763 |
| 3.130.9 Mupad [B] (verification not implemented) . . . . .  | 763 |

#### 3.130.1 Optimal result

Integrand size = 6, antiderivative size = 15

$$\int x \tan^2(x) dx = -\frac{x^2}{2} + \log(\cos(x)) + x \tan(x)$$

output `-1/2*x^2+ln(cos(x))+x*tan(x)`

#### 3.130.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x \tan^2(x) dx = -\frac{x^2}{2} + \log(\cos(x)) + x \tan(x)$$

input `Integrate[x*Tan[x]^2,x]`

output `-1/2*x^2 + Log[Cos[x]] + x*Tan[x]`



**3.130.3 Rubi [A] (verified)**

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$ , Rules used = {3042, 4203, 15, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x \tan^2(x) dx \\
 & \quad \downarrow \text{3042} \\
 & \int x \tan(x)^2 dx \\
 & \quad \downarrow \text{4203} \\
 & - \int x dx - \int \tan(x) dx + x \tan(x) \\
 & \quad \downarrow \text{15} \\
 & - \int \tan(x) dx - \frac{x^2}{2} + x \tan(x) \\
 & \quad \downarrow \text{3042} \\
 & - \int \tan(x) dx - \frac{x^2}{2} + x \tan(x) \\
 & \quad \downarrow \text{3956} \\
 & -\frac{x^2}{2} + x \tan(x) + \log(\cos(x))
 \end{aligned}$$

input `Int [x*Tan[x]^2,x]`

output `-1/2*x^2 + Log[Cos[x]] + x*Tan[x]`

## 3.130.3.1 Defintions of rubi rules used

- rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`
- rule 3956 `Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`
- rule 4203 `Int[((c_.) + (d_.)*(x_)^(m_.))*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Simp[b*d*(m/(f*(n - 1))) Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Simp[b^2 Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]`

## 3.130.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

| method       | result                                                            | size |
|--------------|-------------------------------------------------------------------|------|
| norman       | $x \tan(x) - \frac{x^2}{2} - \frac{\ln(1+\tan^2(x))}{2}$          | 20   |
| parallelrisc | $x \tan(x) - \frac{x^2}{2} - \frac{\ln(1+\tan^2(x))}{2}$          | 20   |
| risc         | $-\frac{x^2}{2} - 2ix + \frac{2ix}{e^{2ix}+1} + \ln(e^{2ix} + 1)$ | 32   |

input `int(x*tan(x)^2,x,method=_RETURNVERBOSE)`

output `x*tan(x)-1/2*x^2-1/2*ln(1+tan(x)^2)`

**3.130.5 Fricas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int x \tan^2(x) dx = -\frac{1}{2} x^2 + x \tan(x) + \frac{1}{2} \log\left(\frac{1}{\tan(x)^2 + 1}\right)$$

input `integrate(x*tan(x)^2,x, algorithm="fricas")`output `-1/2*x^2 + x*tan(x) + 1/2*log(1/(tan(x)^2 + 1))`**3.130.6 Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int x \tan^2(x) dx = -\frac{x^2}{2} + x \tan(x) - \frac{\log(\tan^2(x) + 1)}{2}$$

input `integrate(x*tan(x)**2,x)`output `-x**2/2 + x*tan(x) - log(tan(x)**2 + 1)/2`**3.130.7 Maxima [B] (verification not implemented)**Leaf count of result is larger than twice the leaf count of optimal. 107 vs.  $2(13) = 26$ .

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 7.13

$$\int x \tan^2(x) dx = \frac{x^2 \cos(2x)^2 + x^2 \sin(2x)^2 + 2x^2 \cos(2x) + x^2 - (\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1) \log(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1)}{2(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1)}$$

input `integrate(x*tan(x)^2,x, algorithm="maxima")`output `-1/2*(x^2*cos(2*x)^2 + x^2*sin(2*x)^2 + 2*x^2*cos(2*x) + x^2 - (cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)`

**3.130.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int x \tan^2(x) dx = -\frac{1}{2} x^2 + x \tan(x) + \frac{1}{2} \log\left(\frac{4}{\tan(x)^2 + 1}\right)$$

input `integrate(x*tan(x)^2,x, algorithm="giac")`

output `-1/2*x^2 + x*tan(x) + 1/2*log(4/(tan(x)^2 + 1))`

**3.130.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x \tan^2(x) dx = \ln(\cos(x)) + x \tan(x) - \frac{x^2}{2}$$

input `int(x*tan(x)^2,x)`

output `log(cos(x)) + x*tan(x) - x^2/2`

### 3.131 $\int \cos(3x) \sin(2x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.131.1 Optimal result . . . . .                            | 764 |
| 3.131.2 Mathematica [A] (verified) . . . . .                | 764 |
| 3.131.3 Rubi [A] (verified) . . . . .                       | 765 |
| 3.131.4 Maple [A] (verified) . . . . .                      | 766 |
| 3.131.5 Fricas [A] (verification not implemented) . . . . . | 766 |
| 3.131.6 Sympy [B] (verification not implemented) . . . . .  | 766 |
| 3.131.7 Maxima [A] (verification not implemented) . . . . . | 767 |
| 3.131.8 Giac [A] (verification not implemented) . . . . .   | 767 |
| 3.131.9 Mupad [B] (verification not implemented) . . . . .  | 767 |

#### 3.131.1 Optimal result

Integrand size = 9, antiderivative size = 15

$$\int \cos(3x) \sin(2x) dx = \frac{\cos(x)}{2} - \frac{1}{10} \cos(5x)$$

output `1/2*cos(x)-1/10*cos(5*x)`

#### 3.131.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(3x) \sin(2x) dx = \frac{\cos(x)}{2} - \frac{1}{10} \cos(5x)$$

input `Integrate[Cos[3*x]*Sin[2*x],x]`

output `Cos[x]/2 - Cos[5*x]/10`

**3.131.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3042, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \sin(2x) \cos(3x) dx \\ \downarrow \text{3042} \\ \int \sin(2x) \cos(3x) dx \\ \downarrow \text{4772} \\ \frac{\cos(x)}{2} - \frac{1}{10} \cos(5x) \end{array}$$

input `Int[Cos[3*x]*Sin[2*x],x]`

output `Cos[x]/2 - Cos[5*x]/10`

**3.131.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 4772 `Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] := Simp[-Cos[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]`

**3.131.4 Maple [A] (verified)**

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

| method       | result                                                                                                                             | size |
|--------------|------------------------------------------------------------------------------------------------------------------------------------|------|
| default      | $\frac{\cos(x)}{2} - \frac{\cos(5x)}{10}$                                                                                          | 12   |
| risch        | $\frac{\cos(x)}{2} - \frac{\cos(5x)}{10}$                                                                                          | 12   |
| parallelrisc | $\frac{\cos(x)}{2} - \frac{\cos(5x)}{10} - \frac{2}{5}$                                                                            | 13   |
| norman       | $-\frac{4(\tan^2(x)) - 4(\tan^2(\frac{3x}{2}))}{5} + \frac{12 \tan(x) \tan(\frac{3x}{2})}{5(1+\tan^2(\frac{3x}{2}))(1+\tan^2(x))}$ | 43   |

input `int(cos(3*x)*sin(2*x),x,method=_RETURNVERBOSE)`output `1/2*cos(x)-1/10*cos(5*x)`**3.131.5 Fracas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(3x) \sin(2x) dx = -\frac{8}{5} \cos(x)^5 + 2 \cos(x)^3$$

input `integrate(cos(3*x)*sin(2*x),x, algorithm="fricas")`output `-8/5*cos(x)^5 + 2*cos(x)^3`**3.131.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \cos(3x) \sin(2x) dx = \frac{3 \sin(2x) \sin(3x)}{5} + \frac{2 \cos(2x) \cos(3x)}{5}$$

input `integrate(cos(3*x)*sin(2*x),x)`output `3*sin(2*x)*sin(3*x)/5 + 2*cos(2*x)*cos(3*x)/5`

**3.131.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x) \sin(2x) dx = -\frac{1}{10} \cos(5x) + \frac{1}{2} \cos(x)$$

input `integrate(cos(3*x)*sin(2*x),x, algorithm="maxima")`output `-1/10*cos(5*x) + 1/2*cos(x)`**3.131.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(3x) \sin(2x) dx = -\frac{1}{10} \cos(5x) + \frac{1}{2} \cos(x)$$

input `integrate(cos(3*x)*sin(2*x),x, algorithm="giac")`output `-1/10*cos(5*x) + 1/2*cos(x)`**3.131.9 Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(3x) \sin(2x) dx = 2 \cos(x)^3 - \frac{8 \cos(x)^5}{5}$$

input `int(cos(3*x)*sin(2*x),x)`output `2*cos(x)^3 - (8*cos(x)^5)/5`



### 3.132 $\int \cos^2(x) \sin^2(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.132.1 Optimal result . . . . .                            | 768 |
| 3.132.2 Mathematica [A] (verified) . . . . .                | 768 |
| 3.132.3 Rubi [A] (verified) . . . . .                       | 769 |
| 3.132.4 Maple [A] (verified) . . . . .                      | 770 |
| 3.132.5 Fricas [A] (verification not implemented) . . . . . | 771 |
| 3.132.6 Sympy [A] (verification not implemented) . . . . .  | 771 |
| 3.132.7 Maxima [A] (verification not implemented) . . . . . | 771 |
| 3.132.8 Giac [A] (verification not implemented) . . . . .   | 772 |
| 3.132.9 Mupad [B] (verification not implemented) . . . . .  | 772 |

#### 3.132.1 Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \cos^2(x) \sin^2(x) dx = \frac{x}{8} + \frac{1}{8} \cos(x) \sin(x) - \frac{1}{4} \cos^3(x) \sin(x)$$

output `1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)`

#### 3.132.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int \cos^2(x) \sin^2(x) dx = \frac{x}{8} - \frac{1}{32} \sin(4x)$$

input `Integrate[Cos[x]^2*Sin[x]^2,x]`

output `x/8 - Sin[4*x]/32`

**3.132.3 Rubi [A] (verified)**

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {3042, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \sin^2(x) \cos^2(x) dx \\
 & \quad \downarrow \text{3042} \\
 & \int \sin(x)^2 \cos(x)^2 dx \\
 & \quad \downarrow \text{3048} \\
 & \frac{1}{4} \int \cos^2(x) dx - \frac{1}{4} \sin(x) \cos^3(x) \\
 & \quad \downarrow \text{3042} \\
 & \frac{1}{4} \int \sin\left(x + \frac{\pi}{2}\right)^2 dx - \frac{1}{4} \sin(x) \cos^3(x) \\
 & \quad \downarrow \text{3115} \\
 & \frac{1}{4} \left( \int \frac{1 dx}{2} + \frac{1}{2} \sin(x) \cos(x) \right) - \frac{1}{4} \sin(x) \cos^3(x) \\
 & \quad \downarrow \text{24} \\
 & \frac{1}{4} \left( \frac{x}{2} + \frac{1}{2} \sin(x) \cos(x) \right) - \frac{1}{4} \sin(x) \cos^3(x)
 \end{aligned}$$

input `Int[Cos[x]^2*Sin[x]^2,x]`

output `-1/4*(Cos[x]^3*Sin[x]) + (x/2 + (Cos[x]*Sin[x])/2)/4`

## 3.132.3.1 Defintions of rubi rules used

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3048 `Int[(cos[(e_.) + (f_.)*(x_.)]*(b_.))^n_]*((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_ - 1), x_Symbol] := Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m - 1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n)) Int[(b*Cos[e + f*x])^n *(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]`

rule 3115 `Int[(b_.)*sin[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n) Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

## 3.132.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

| method        | result                                                                                                                                                                                                                                                                                                                            | size |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch         | $\frac{x}{8} - \frac{\sin(4x)}{32}$                                                                                                                                                                                                                                                                                               | 11   |
| parallelrisch | $\frac{x}{8} - \frac{\sin(4x)}{32}$                                                                                                                                                                                                                                                                                               | 11   |
| default       | $\frac{x}{8} + \frac{\cos(x)\sin(x)}{8} - \frac{(\cos^3(x))\sin(x)}{4}$                                                                                                                                                                                                                                                           | 19   |
| norman        | $\frac{x}{8} + \frac{7(\tan^3(\frac{x}{2}))}{4} - \frac{7(\tan^5(\frac{x}{2}))}{4} + \frac{(\tan^7(\frac{x}{2}))}{4} + \frac{x(\tan^2(\frac{x}{2}))}{2} + \frac{3x(\tan^4(\frac{x}{2}))}{4} + \frac{x(\tan^6(\frac{x}{2}))}{2} + \frac{x(\tan^8(\frac{x}{2}))}{8} - \frac{\tan(\frac{x}{2})}{4}$<br>$(1 + \tan^2(\frac{x}{2}))^4$ | 82   |

input `int(cos(x)^2*sin(x)^2,x,method=_RETURNVERBOSE)`

output `1/8*x-1/32*sin(4*x)`

**3.132.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \cos^2(x) \sin^2(x) dx = -\frac{1}{8} (2 \cos(x)^3 - \cos(x)) \sin(x) + \frac{1}{8} x$$

input `integrate(cos(x)^2*sin(x)^2,x, algorithm="fracas")`output `-1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x`**3.132.6 Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int \cos^2(x) \sin^2(x) dx = \frac{x}{8} - \frac{\sin(2x) \cos(2x)}{16}$$

input `integrate(cos(x)**2*sin(x)**2,x)`output `x/8 - sin(2*x)*cos(2*x)/16`**3.132.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.42

$$\int \cos^2(x) \sin^2(x) dx = \frac{1}{8} x - \frac{1}{32} \sin(4x)$$

input `integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")`output `1/8*x - 1/32*sin(4*x)`

**3.132.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.42

$$\int \cos^2(x) \sin^2(x) dx = \frac{1}{8} x - \frac{1}{32} \sin(4x)$$

input `integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")`output `1/8*x - 1/32*sin(4*x)`**3.132.9 Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \cos^2(x) \sin^2(x) dx = \frac{\cos(x) \sin(x)^3}{4} - \frac{\cos(x) \sin(x)}{8} + \frac{x}{8}$$

input `int(cos(x)^2*sin(x)^2,x)`output `x/8 - (cos(x)*sin(x))/8 + (cos(x)*sin(x)^3)/4`

### 3.133 $\int \csc^2(x) \sec^2(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.133.1 Optimal result . . . . .                            | 773 |
| 3.133.2 Mathematica [A] (verified) . . . . .                | 773 |
| 3.133.3 Rubi [A] (verified) . . . . .                       | 774 |
| 3.133.4 Maple [A] (verified) . . . . .                      | 775 |
| 3.133.5 Fricas [B] (verification not implemented) . . . . . | 775 |
| 3.133.6 Sympy [B] (verification not implemented) . . . . .  | 776 |
| 3.133.7 Maxima [A] (verification not implemented) . . . . . | 776 |
| 3.133.8 Giac [A] (verification not implemented) . . . . .   | 776 |
| 3.133.9 Mupad [B] (verification not implemented) . . . . .  | 777 |

#### 3.133.1 Optimal result

Integrand size = 9, antiderivative size = 7

$$\int \csc^2(x) \sec^2(x) dx = -\cot(x) + \tan(x)$$

output `-cot(x)+tan(x)`

#### 3.133.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \csc^2(x) \sec^2(x) dx = -2 \cot(2x)$$

input `Integrate[Csc[x]^2*Sec[x]^2,x]`

output `-2*Cot[2*x]`

**3.133.3 Rubi [A] (verified)**

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \csc^2(x) \sec^2(x) dx \\
 & \quad \downarrow \text{3042} \\
 & \int \csc(x)^2 \sec(x)^2 dx \\
 & \quad \downarrow \text{3100} \\
 & \int (\tan^2(x) + 1) \cot^2(x) d \tan(x) \\
 & \quad \downarrow \text{244} \\
 & \int (\cot^2(x) + 1) d \tan(x) \\
 & \quad \downarrow \text{2009} \\
 & \tan(x) - \cot(x)
 \end{aligned}$$

input `Int[Csc[x]^2*Sec[x]^2,x]`

output `-Cot[x] + Tan[x]`

**3.133.3.1 Defintions of rubi rules used**

rule 244 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[Expand Integrand[(c*x)^m*(a + b*x^2)^p, x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3100 `Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Simp[1/f Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]`

### 3.133.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

| method        | result                                                                                                                       | size |
|---------------|------------------------------------------------------------------------------------------------------------------------------|------|
| parallelrisch | $-2 \cot(x) + \sec(x) \csc(x)$                                                                                               | 11   |
| default       | $\frac{1}{\cos(x) \sin(x)} - 2 \cot(x)$                                                                                      | 15   |
| risch         | $-\frac{4i}{(e^{2ix}+1)(e^{2ix}-1)}$                                                                                         | 22   |
| norman        | $\frac{\frac{1}{2} - 3(\tan^2(\frac{x}{2})) + \frac{(\tan^4(\frac{x}{2}))}{2}}{(\tan^2(\frac{x}{2}) - 1) \tan(\frac{x}{2})}$ | 36   |

input `int(1/cos(x)^2/sin(x)^2,x,method=_RETURNVERBOSE)`

output `-2*cot(x)+sec(x)*csc(x)`

### 3.133.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs.  $2(7) = 14$ .

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.57

$$\int \csc^2(x) \sec^2(x) dx = -\frac{2 \cos(x)^2 - 1}{\cos(x) \sin(x)}$$

input `integrate(1/cos(x)^2/sin(x)^2,x, algorithm="fricas")`

output `-(2*cos(x)^2 - 1)/(cos(x)*sin(x))`



**3.133.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 12 vs.  $2(5) = 10$ .

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int \csc^2(x) \sec^2(x) dx = -\frac{2 \cos(2x)}{\sin(2x)}$$

input `integrate(1/cos(x)**2/sin(x)**2,x)`

output `-2*cos(2*x)/sin(2*x)`

**3.133.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc^2(x) \sec^2(x) dx = -\frac{1}{\tan(x)} + \tan(x)$$

input `integrate(1/cos(x)^2/sin(x)^2,x, algorithm="maxima")`

output `-1/tan(x) + tan(x)`

**3.133.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc^2(x) \sec^2(x) dx = -\frac{1}{\tan(x)} + \tan(x)$$

input `integrate(1/cos(x)^2/sin(x)^2,x, algorithm="giac")`

output `-1/tan(x) + tan(x)`

**3.133.9 Mupad [B] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \csc^2(x) \sec^2(x) dx = -2 \cot(2x)$$

input `int(1/(cos(x)^2*sin(x)^2),x)`

output `-2*cot(2*x)`

### 3.134 $\int d^x \sin(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.134.1 Optimal result . . . . .                            | 778 |
| 3.134.2 Mathematica [A] (verified) . . . . .                | 778 |
| 3.134.3 Rubi [A] (verified) . . . . .                       | 779 |
| 3.134.4 Maple [A] (verified) . . . . .                      | 779 |
| 3.134.5 Fricas [A] (verification not implemented) . . . . . | 780 |
| 3.134.6 Sympy [C] (verification not implemented) . . . . .  | 780 |
| 3.134.7 Maxima [A] (verification not implemented) . . . . . | 781 |
| 3.134.8 Giac [C] (verification not implemented) . . . . .   | 781 |
| 3.134.9 Mupad [B] (verification not implemented) . . . . .  | 782 |

#### 3.134.1 Optimal result

Integrand size = 6, antiderivative size = 32

$$\int d^x \sin(x) dx = -\frac{d^x \cos(x)}{1 + \log^2(d)} + \frac{d^x \log(d) \sin(x)}{1 + \log^2(d)}$$

output `-d^x*cos(x)/(1+ln(d)^2)+d^x*ln(d)*sin(x)/(1+ln(d)^2)`

#### 3.134.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int d^x \sin(x) dx = \frac{d^x(-\cos(x) + \log(d) \sin(x))}{1 + \log^2(d)}$$

input `Integrate[d^x*Sin[x],x]`

output `(d^x*(-Cos[x] + Log[d]*Sin[x]))/(1 + Log[d]^2)`

### 3.134.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int d^x \sin(x) dx$$

$$\downarrow 4932$$

$$\frac{d^x \log(d) \sin(x)}{\log^2(d) + 1} - \frac{d^x \cos(x)}{\log^2(d) + 1}$$

input `Int[d^x*Sin[x],x]`

output `-((d^x*Cos[x])/((1 + Log[d]^2)) + (d^x*Log[d]*Sin[x])/((1 + Log[d]^2))`

#### 3.134.3.1 Defintions of rubi rules used

rule 4932 `Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :=  
Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x  
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F  
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]`

### 3.134.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

| method        | result                                                                                                                                                                                                                             | size |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| parallelrisch | $\frac{d^x (\ln(d) \sin(x) - \cos(x))}{1 + \ln(d)^2}$                                                                                                                                                                              | 23   |
| risch         | $-\frac{d^x \cos(x)}{1 + \ln(d)^2} + \frac{d^x \ln(d) \sin(x)}{1 + \ln(d)^2}$                                                                                                                                                      | 33   |
| norman        | $\frac{\frac{e^x \ln(d) \left(\tan^2\left(\frac{x}{2}\right)\right)}{1 + \ln(d)^2} - \frac{e^x \ln(d)}{1 + \ln(d)^2} + \frac{2 \ln(d) e^x \ln(d) \tan\left(\frac{x}{2}\right)}{1 + \ln(d)^2}}{1 + \tan^2\left(\frac{x}{2}\right)}$ | 69   |

input `int(d^x*sin(x),x,method=_RETURNVERBOSE)`

output `dx*(ln(d)*sin(x)-cos(x))/(1+ln(d)2)`

### 3.134.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int d^x \sin(x) dx = \frac{(\log(d) \sin(x) - \cos(x))d^x}{\log(d)^2 + 1}$$

input `integrate(dx*sin(x),x, algorithm="fricas")`

output `(log(d)*sin(x) - cos(x))*dx/(log(d)2 + 1)`

### 3.134.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.25

$$\int d^x \sin(x) dx = \begin{cases} \frac{xe^{-ix} \sin(x)}{2} - \frac{ixe^{-ix} \cos(x)}{2} - \frac{e^{-ix} \cos(x)}{2} & \text{for } d = e^{-i} \\ \frac{xe^{ix} \sin(x)}{2} + \frac{ixe^{ix} \cos(x)}{2} - \frac{e^{ix} \cos(x)}{2} & \text{for } d = e^i \\ \frac{d^x \log(d) \sin(x)}{\log(d)^2 + 1} - \frac{d^x \cos(x)}{\log(d)^2 + 1} & \text{otherwise} \end{cases}$$

input `integrate(d**x*sin(x),x)`

output `Piecewise((x*exp(-I*x)*sin(x)/2 - I*x*exp(-I*x)*cos(x)/2 - exp(-I*x)*cos(x)/2, Eq(d, exp(-I))), (x*exp(I*x)*sin(x)/2 + I*x*exp(I*x)*cos(x)/2 - exp(I*x)*cos(x)/2, Eq(d, exp(I))), (d**x*log(d)*sin(x)/(log(d)**2 + 1) - d**x*cos(x)/(log(d)**2 + 1), True))`

**3.134.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int d^x \sin(x) dx = \frac{d^x \log(d) \sin(x) - d^x \cos(x)}{\log(d)^2 + 1}$$

input `integrate(d^x*sin(x),x, algorithm="maxima")`output `(d^x*log(d)*sin(x) - d^x*cos(x))/(log(d)^2 + 1)`**3.134.8 Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 10.25

$$\begin{aligned} & \int d^x \sin(x) dx \\ &= |d|^x \left( \frac{(\pi - \pi \operatorname{sgn}(d) - 2) \cos\left(\frac{1}{2} \pi x \operatorname{sgn}(d) - \frac{1}{2} \pi x + x\right)}{(\pi - \pi \operatorname{sgn}(d) - 2)^2 + 4 \log(|d|)^2} + \frac{2 \log(|d|) \sin\left(\frac{1}{2} \pi x \operatorname{sgn}(d) - \frac{1}{2} \pi x + x\right)}{(\pi - \pi \operatorname{sgn}(d) - 2)^2 + 4 \log(|d|)^2} \right) \\ & - |d|^x \left( \frac{(\pi - \pi \operatorname{sgn}(d) + 2) \cos\left(\frac{1}{2} \pi x \operatorname{sgn}(d) - \frac{1}{2} \pi x - x\right)}{(\pi - \pi \operatorname{sgn}(d) + 2)^2 + 4 \log(|d|)^2} + \frac{2 \log(|d|) \sin\left(\frac{1}{2} \pi x \operatorname{sgn}(d) - \frac{1}{2} \pi x - x\right)}{(\pi - \pi \operatorname{sgn}(d) + 2)^2 + 4 \log(|d|)^2} \right) \\ & - |d|^x \left( -\frac{i e^{\left(\frac{1}{2} i \pi x \operatorname{sgn}(d) - \frac{1}{2} i \pi x + i x\right)}}{-2i \pi + 2i \pi \operatorname{sgn}(d) + 4 \log(|d|) + 4i} - \frac{i e^{\left(-\frac{1}{2} i \pi x \operatorname{sgn}(d) + \frac{1}{2} i \pi x - i x\right)}}{2i \pi - 2i \pi \operatorname{sgn}(d) + 4 \log(|d|) - 4i} \right) \\ & - |d|^x \left( \frac{i e^{\left(\frac{1}{2} i \pi x \operatorname{sgn}(d) - \frac{1}{2} i \pi x - i x\right)}}{-2i \pi + 2i \pi \operatorname{sgn}(d) + 4 \log(|d|) - 4i} + \frac{i e^{\left(-\frac{1}{2} i \pi x \operatorname{sgn}(d) + \frac{1}{2} i \pi x + i x\right)}}{2i \pi - 2i \pi \operatorname{sgn}(d) + 4 \log(|d|) + 4i} \right) \end{aligned}$$

input `integrate(d^x*sin(x),x, algorithm="giac")`

output  $\text{abs}(d)^x \cdot ((\pi - \pi \cdot \text{sgn}(d) - 2) \cdot \cos(1/2 \cdot \pi \cdot x \cdot \text{sgn}(d) - 1/2 \cdot \pi \cdot x + x) / ((\pi - \pi \cdot \text{sgn}(d) - 2)^2 + 4 \cdot \log(\text{abs}(d))^2) + 2 \cdot \log(\text{abs}(d)) \cdot \sin(1/2 \cdot \pi \cdot x \cdot \text{sgn}(d) - 1/2 \cdot \pi \cdot x + x) / ((\pi - \pi \cdot \text{sgn}(d) - 2)^2 + 4 \cdot \log(\text{abs}(d))^2)) - \text{abs}(d)^x \cdot ((\pi - \pi \cdot \text{sgn}(d) + 2) \cdot \cos(1/2 \cdot \pi \cdot x \cdot \text{sgn}(d) - 1/2 \cdot \pi \cdot x - x) / ((\pi - \pi \cdot \text{sgn}(d) + 2)^2 + 4 \cdot \log(\text{abs}(d))^2) + 2 \cdot \log(\text{abs}(d)) \cdot \sin(1/2 \cdot \pi \cdot x \cdot \text{sgn}(d) - 1/2 \cdot \pi \cdot x - x) / ((\pi - \pi \cdot \text{sgn}(d) + 2)^2 + 4 \cdot \log(\text{abs}(d))^2)) - \text{abs}(d)^x \cdot (-I \cdot e^{(1/2 \cdot I \cdot \pi \cdot x \cdot \text{sgn}(d) - 1/2 \cdot I \cdot \pi \cdot x + I \cdot x)} / (-2 \cdot I \cdot \pi + 2 \cdot I \cdot \pi \cdot \text{sgn}(d) + 4 \cdot \log(\text{abs}(d)) + 4 \cdot I) - I \cdot e^{(-1/2 \cdot I \cdot \pi \cdot x \cdot \text{sgn}(d) + 1/2 \cdot I \cdot \pi \cdot x - I \cdot x)} / (2 \cdot I \cdot \pi - 2 \cdot I \cdot \pi \cdot \text{sgn}(d) + 4 \cdot \log(\text{abs}(d)) - 4 \cdot I)) - \text{abs}(d)^x \cdot (I \cdot e^{(1/2 \cdot I \cdot \pi \cdot x \cdot \text{sgn}(d) - 1/2 \cdot I \cdot \pi \cdot x - I \cdot x)} / (-2 \cdot I \cdot \pi + 2 \cdot I \cdot \pi \cdot \text{sgn}(d) + 4 \cdot \log(\text{abs}(d)) - 4 \cdot I) + I \cdot e^{(-1/2 \cdot I \cdot \pi \cdot x \cdot \text{sgn}(d) + 1/2 \cdot I \cdot \pi \cdot x + I \cdot x)} / (2 \cdot I \cdot \pi - 2 \cdot I \cdot \pi \cdot \text{sgn}(d) + 4 \cdot \log(\text{abs}(d)) + 4 \cdot I))$

### 3.134.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int d^x \sin(x) dx = -\frac{d^x (\cos(x) - \ln(d) \sin(x))}{\ln(d)^2 + 1}$$

input `int(d^x*sin(x),x)`

output  $-(d^x \cdot (\cos(x) - \log(d) \cdot \sin(x))) / (\log(d)^2 + 1)$

### 3.135 $\int d^x \cos(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
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| 3.135.2 Mathematica [A] (verified) . . . . .                | 783 |
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| 3.135.5 Fricas [A] (verification not implemented) . . . . . | 785 |
| 3.135.6 Sympy [C] (verification not implemented) . . . . .  | 785 |
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#### 3.135.1 Optimal result

Integrand size = 6, antiderivative size = 31

$$\int d^x \cos(x) dx = \frac{d^x \cos(x) \log(d)}{1 + \log^2(d)} + \frac{d^x \sin(x)}{1 + \log^2(d)}$$

output `d^x*cos(x)*ln(d)/(1+ln(d)^2)+d^x*sin(x)/(1+ln(d)^2)`

#### 3.135.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int d^x \cos(x) dx = \frac{d^x (\cos(x) \log(d) + \sin(x))}{1 + \log^2(d)}$$

input `Integrate[d^x*Cos[x],x]`

output `(d^x*(Cos[x]*Log[d] + Sin[x]))/(1 + Log[d]^2)`



### 3.135.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int d^x \cos(x) dx$$

↓ 4933

$$\frac{d^x \sin(x)}{\log^2(d) + 1} + \frac{d^x \log(d) \cos(x)}{\log^2(d) + 1}$$

input `Int [d^x*Cos [x] , x]`

output `(d^x*Cos [x]*Log [d])/(1 + Log [d]^2) + (d^x*Sin [x])/(1 + Log [d]^2)`

#### 3.135.3.1 Defintions of rubi rules used

rule 4933 `Int [Cos [(d_.) + (e_.)*(x_.)]*(F_)^((c_.)*((a_.) + (b_.)*(x_.))), x_Symbol] :=  
Simp [b*c*Log [F]*F^(c*(a + b*x))*(Cos [d + e*x]/(e^2 + b^2*c^2*Log [F]^2)), x  
] + Simp [e*F^(c*(a + b*x))*(Sin [d + e*x]/(e^2 + b^2*c^2*Log [F]^2)), x] /;  
FreeQ [{F, a, b, c, d, e}, x] && NeQ [e^2 + b^2*c^2*Log [F]^2, 0]`

### 3.135.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

| method       | result                                                                                                                                                                                     | size |
|--------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| parallelrisc | $\frac{d^x (\ln(d) \cos(x) + \sin(x))}{1 + \ln(d)^2}$                                                                                                                                      | 21   |
| risc         | $\frac{d^x \cos(x) \ln(d)}{1 + \ln(d)^2} + \frac{d^x \sin(x)}{1 + \ln(d)^2}$                                                                                                               | 32   |
| norman       | $\frac{\frac{\ln(d)e^x \ln(d)}{1 + \ln(d)^2} + \frac{2e^x \ln(d) \tan(\frac{x}{2})}{1 + \ln(d)^2} - \frac{\ln(d)e^x \ln(d) (\tan^2(\frac{x}{2}))}{1 + \ln(d)^2}}{1 + \tan^2(\frac{x}{2})}$ | 71   |

input `int (d^x*cos (x) , x, method=_RETURNVERBOSE)`

output  $d^x(\ln(d)\cos(x)+\sin(x))/(1+\ln(d)^2)$

### 3.135.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int d^x \cos(x) dx = \frac{(\cos(x) \log(d) + \sin(x))d^x}{\log(d)^2 + 1}$$

input `integrate(d^x*cos(x),x, algorithm="fricas")`

output  $(\cos(x)\log(d) + \sin(x))*d^x/(\log(d)^2 + 1)$

### 3.135.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.45

$$\int d^x \cos(x) dx = \begin{cases} \frac{ixe^{-ix} \sin(x)}{2} + \frac{xe^{-ix} \cos(x)}{2} + \frac{ie^{-ix} \cos(x)}{2} & \text{for } d = e^{-i} \\ -\frac{ixe^{ix} \sin(x)}{2} + \frac{xe^{ix} \cos(x)}{2} - \frac{ie^{ix} \cos(x)}{2} & \text{for } d = e^i \\ \frac{d^x \log(d) \cos(x)}{\log(d)^2+1} + \frac{d^x \sin(x)}{\log(d)^2+1} & \text{otherwise} \end{cases}$$

input `integrate(d**x*cos(x),x)`

output `Piecewise((I*x*exp(-I*x)*sin(x)/2 + x*exp(-I*x)*cos(x)/2 + I*exp(-I*x)*cos(x)/2, Eq(d, exp(-I))), (-I*x*exp(I*x)*sin(x)/2 + x*exp(I*x)*cos(x)/2 - I*exp(I*x)*cos(x)/2, Eq(d, exp(I))), (d**x*log(d)*cos(x)/(log(d)**2 + 1) + d**x*sin(x)/(log(d)**2 + 1), True))`

**3.135.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int d^x \cos(x) dx = \frac{d^x \cos(x) \log(d) + d^x \sin(x)}{\log(d)^2 + 1}$$

input `integrate(d^x*cos(x),x, algorithm="maxima")`output `(d^x*cos(x)*log(d) + d^x*sin(x))/(log(d)^2 + 1)`**3.135.8 Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 329, normalized size of antiderivative = 10.61

$$\begin{aligned} & \int d^x \cos(x) dx \\ &= |d|^x \left( \frac{2 \cos\left(\frac{1}{2} \pi x \operatorname{sgn}(d) - \frac{1}{2} \pi x + x\right) \log(|d|)}{(\pi - \pi \operatorname{sgn}(d) - 2)^2 + 4 \log(|d|)^2} - \frac{(\pi - \pi \operatorname{sgn}(d) - 2) \sin\left(\frac{1}{2} \pi x \operatorname{sgn}(d) - \frac{1}{2} \pi x + x\right)}{(\pi - \pi \operatorname{sgn}(d) - 2)^2 + 4 \log(|d|)^2} \right) \\ &+ |d|^x \left( \frac{2 \cos\left(\frac{1}{2} \pi x \operatorname{sgn}(d) - \frac{1}{2} \pi x - x\right) \log(|d|)}{(\pi - \pi \operatorname{sgn}(d) + 2)^2 + 4 \log(|d|)^2} - \frac{(\pi - \pi \operatorname{sgn}(d) + 2) \sin\left(\frac{1}{2} \pi x \operatorname{sgn}(d) - \frac{1}{2} \pi x - x\right)}{(\pi - \pi \operatorname{sgn}(d) + 2)^2 + 4 \log(|d|)^2} \right) \\ &+ i |d|^x \left( \frac{i e^{\left(\frac{1}{2} i \pi x \operatorname{sgn}(d) - \frac{1}{2} i \pi x + i x\right)}}{-2i \pi + 2i \pi \operatorname{sgn}(d) + 4 \log(|d|) + 4i} - \frac{i e^{\left(-\frac{1}{2} i \pi x \operatorname{sgn}(d) + \frac{1}{2} i \pi x - i x\right)}}{2i \pi - 2i \pi \operatorname{sgn}(d) + 4 \log(|d|) - 4i} \right) \\ &+ i |d|^x \left( \frac{i e^{\left(\frac{1}{2} i \pi x \operatorname{sgn}(d) - \frac{1}{2} i \pi x - i x\right)}}{-2i \pi + 2i \pi \operatorname{sgn}(d) + 4 \log(|d|) - 4i} - \frac{i e^{\left(-\frac{1}{2} i \pi x \operatorname{sgn}(d) + \frac{1}{2} i \pi x + i x\right)}}{2i \pi - 2i \pi \operatorname{sgn}(d) + 4 \log(|d|) + 4i} \right) \end{aligned}$$

input `integrate(d^x*cos(x),x, algorithm="giac")`

output  $\text{abs}(d)^x \cdot (2 \cos(1/2 \pi x \text{sgn}(d)) - 1/2 \pi x + x) \log(\text{abs}(d)) / ((\pi - \pi \text{sgn}(d) - 2)^2 + 4 \log(\text{abs}(d))^2) - (\pi - \pi \text{sgn}(d) - 2) \sin(1/2 \pi x \text{sgn}(d) - 1/2 \pi x + x) / ((\pi - \pi \text{sgn}(d) - 2)^2 + 4 \log(\text{abs}(d))^2) + \text{abs}(d)^x \cdot (2 \cos(1/2 \pi x \text{sgn}(d)) - 1/2 \pi x - x) \log(\text{abs}(d)) / ((\pi - \pi \text{sgn}(d) + 2)^2 + 4 \log(\text{abs}(d))^2) - (\pi - \pi \text{sgn}(d) + 2) \sin(1/2 \pi x \text{sgn}(d) - 1/2 \pi x - x) / ((\pi - \pi \text{sgn}(d) + 2)^2 + 4 \log(\text{abs}(d))^2) + I \text{abs}(d)^x \cdot (I e^{(1/2 I \pi x \text{sgn}(d) - 1/2 I \pi x + I x)} / (-2 I \pi + 2 I \pi \text{sgn}(d) + 4 \log(\text{abs}(d)) + 4 I) - I e^{-(1/2 I \pi x \text{sgn}(d) + 1/2 I \pi x - I x)} / (2 I \pi - 2 I \pi \text{sgn}(d) + 4 \log(\text{abs}(d)) - 4 I)) + I \text{abs}(d)^x \cdot (I e^{(1/2 I \pi x \text{sgn}(d) - 1/2 I \pi x - I x)} / (-2 I \pi + 2 I \pi \text{sgn}(d) + 4 \log(\text{abs}(d)) - 4 I) - I e^{-(1/2 I \pi x \text{sgn}(d) + 1/2 I \pi x + I x)} / (2 I \pi - 2 I \pi \text{sgn}(d) + 4 \log(\text{abs}(d)) + 4 I))$

### 3.135.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int d^x \cos(x) dx = \frac{d^x (\sin(x) + \ln(d) \cos(x))}{\ln(d)^2 + 1}$$

input `int(d^x*cos(x),x)`

output  $(d^x \cdot (\sin(x) + \log(d) \cos(x))) / (\log(d)^2 + 1)$

### 3.136 $\int d^x x \sin(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.136.1 Optimal result . . . . .                            | 788 |
| 3.136.2 Mathematica [A] (verified) . . . . .                | 788 |
| 3.136.3 Rubi [A] (verified) . . . . .                       | 789 |
| 3.136.4 Maple [A] (verified) . . . . .                      | 790 |
| 3.136.5 Fricas [A] (verification not implemented) . . . . . | 790 |
| 3.136.6 Sympy [C] (verification not implemented) . . . . .  | 791 |
| 3.136.7 Maxima [A] (verification not implemented) . . . . . | 791 |
| 3.136.8 Giac [C] (verification not implemented) . . . . .   | 792 |
| 3.136.9 Mupad [B] (verification not implemented) . . . . .  | 792 |

#### 3.136.1 Optimal result

Integrand size = 7, antiderivative size = 84

$$\int d^x x \sin(x) dx = \frac{2d^x \cos(x) \log(d)}{(1 + \log^2(d))^2} - \frac{d^x x \cos(x)}{1 + \log^2(d)} + \frac{d^x \sin(x)}{(1 + \log^2(d))^2} - \frac{d^x \log^2(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x \log(d) \sin(x)}{1 + \log^2(d)}$$

output `2*d^x*cos(x)*ln(d)/(1+ln(d)^2)^2-d^x*x*cos(x)/(1+ln(d)^2)+d^x*sin(x)/(1+ln(d)^2)^2-d^x*ln(d)^2*sin(x)/(1+ln(d)^2)^2+d^x*x*ln(d)*sin(x)/(1+ln(d)^2)`

#### 3.136.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.60

$$\int d^x x \sin(x) dx = \frac{d^x (-\cos(x) (x - 2 \log(d) + x \log^2(d)) + (1 + x \log(d) - \log^2(d) + x \log^3(d)) \sin(x))}{(1 + \log^2(d))^2}$$

input `Integrate[d^x*x*Sin[x],x]`

output `(d^x*(-(Cos[x]*(x - 2*Log[d] + x*Log[d]^2)) + (1 + x*Log[d] - Log[d]^2 + x*Log[d]^3)*Sin[x]))/(1 + Log[d]^2)^2`

### 3.136.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {4968, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x d^x \sin(x) dx$$

$$\downarrow 4968$$

$$-\int \left( \frac{d^x \log(d) \sin(x)}{\log^2(d) + 1} - \frac{d^x \cos(x)}{\log^2(d) + 1} \right) dx + \frac{x d^x \log(d) \sin(x)}{\log^2(d) + 1} - \frac{x d^x \cos(x)}{\log^2(d) + 1}$$

$$\downarrow 2009$$

$$\frac{x d^x \log(d) \sin(x)}{\log^2(d) + 1} - \frac{d^x \log^2(d) \sin(x)}{(\log^2(d) + 1)^2} + \frac{d^x \sin(x)}{(\log^2(d) + 1)^2} - \frac{x d^x \cos(x)}{\log^2(d) + 1} + \frac{2 d^x \log(d) \cos(x)}{(\log^2(d) + 1)^2}$$

input `Int[d^x*x*Sin[x],x]`

output  $(2*d^x*\text{Cos}[x]*\text{Log}[d])/((1 + \text{Log}[d]^2)^2) - (d^x*x*\text{Cos}[x])/((1 + \text{Log}[d]^2) + (d^x*\text{Sin}[x])/((1 + \text{Log}[d]^2)^2) - (d^x*\text{Log}[d]^2*\text{Sin}[x])/((1 + \text{Log}[d]^2)^2) + (d^x*x*\text{Log}[d]*\text{Sin}[x])/((1 + \text{Log}[d]^2)$

#### 3.136.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 4968 `Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*(x_)]^(n_.), x_Symbol] := Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^n, x]}, Simp[(f*x)^m u, x] - Simp[f*m Int[(f*x)^(m - 1)*u, x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]`

**3.136.4 Maple [A] (verified)**

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

| method       | result                                                                                                                                                                                                                                                                                                                                                                                                                                      |
|--------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| parallelrisc | $\frac{d^x (\ln(d)^3 x \sin(x) + (-x \cos(x) - \sin(x)) \ln(d)^2 + (x \sin(x) + 2 \cos(x)) \ln(d) - x \cos(x) + \sin(x))}{(1 + \ln(d)^2)^2}$                                                                                                                                                                                                                                                                                                |
| risc         | $-\frac{i(-1+x \ln(d)+ix)d^x e^{ix}}{2(\ln(d)+i)^2} + \frac{i(-1+x \ln(d)-ix)d^x e^{-ix}}{2(\ln(d)-i)^2}$                                                                                                                                                                                                                                                                                                                                   |
| norman       | $\frac{x e^{x \ln(d)} \left( \tan^2\left(\frac{x}{2}\right) \right)}{1 + \ln(d)^2} + \frac{2 \ln(d) e^{x \ln(d)}}{(1 + \ln(d)^2)^2} - \frac{x e^{x \ln(d)}}{1 + \ln(d)^2} - \frac{2 \ln(d) e^{x \ln(d)} \left( \tan^2\left(\frac{x}{2}\right) \right)}{(1 + \ln(d)^2)^2} - \frac{2 (\ln(d)^2 - 1) e^{x \ln(d)} \tan\left(\frac{x}{2}\right)}{(1 + \ln(d)^2)^2} + \frac{2 \ln(d) x e^{x \ln(d)} \tan\left(\frac{x}{2}\right)}{1 + \ln(d)^2}$ |

input `int(d^x*x*sin(x),x,method=_RETURNVERBOSE)`output `d^x*(ln(d)^3*x*sin(x)+(-x*cos(x)-sin(x))*ln(d)^2+(x*sin(x)+2*cos(x))*ln(d)-x*cos(x)+sin(x))/(1+ln(d)^2)^2`**3.136.5 Fracas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int d^x x \sin(x) dx = \frac{(x \cos(x) \log(d)^2 + x \cos(x) - 2 \cos(x) \log(d) - (x \log(d)^3 + x \log(d) - \log(d)^2 + 1) \sin(x)) d^x}{\log(d)^4 + 2 \log(d)^2 + 1}$$

input `integrate(d^x*x*sin(x),x, algorithm="fricas")`output `-(x*cos(x)*log(d)^2 + x*cos(x) - 2*cos(x)*log(d) - (x*log(d)^3 + x*log(d) - log(d)^2 + 1)*sin(x))*d^x/(log(d)^4 + 2*log(d)^2 + 1)`

**3.136.6 Sympy [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.67

$$\int d^x x \sin(x) dx$$

$$= \begin{cases} \frac{x^2 e^{-ix} \sin(x)}{4} - \frac{ix^2 e^{-ix} \cos(x)}{4} + \frac{ixe^{-ix} \sin(x)}{4} - \frac{xe^{-ix} \cos(x)}{4} + \frac{ie^{-ix} \cos(x)}{4} \\ \frac{x^2 e^{ix} \sin(x)}{4} + \frac{ix^2 e^{ix} \cos(x)}{4} - \frac{ixe^{ix} \sin(x)}{4} - \frac{xe^{ix} \cos(x)}{4} - \frac{ie^{ix} \cos(x)}{4} \\ \frac{d^x x \log(d)^3 \sin(x)}{\log(d)^4 + 2 \log(d)^2 + 1} - \frac{d^x x \log(d)^2 \cos(x)}{\log(d)^4 + 2 \log(d)^2 + 1} + \frac{d^x x \log(d) \sin(x)}{\log(d)^4 + 2 \log(d)^2 + 1} - \frac{d^x x \cos(x)}{\log(d)^4 + 2 \log(d)^2 + 1} - \frac{d^x \log(d)^2 \sin(x)}{\log(d)^4 + 2 \log(d)^2 + 1} + \frac{2d^x \log(d)}{\log(d)^4 + 2 \log(d)^2 + 1} \end{cases}$$

input `integrate(d**x*x*sin(x),x)`

output `Piecewise((x**2*exp(-I*x)*sin(x)/4 - I*x**2*exp(-I*x)*cos(x)/4 + I*x*exp(-I*x)*sin(x)/4 - x*exp(-I*x)*cos(x)/4 + I*exp(-I*x)*cos(x)/4, Eq(d, exp(-I))), (x**2*exp(I*x)*sin(x)/4 + I*x**2*exp(I*x)*cos(x)/4 - I*x*exp(I*x)*sin(x)/4 - x*exp(I*x)*cos(x)/4 - I*exp(I*x)*cos(x)/4, Eq(d, exp(I))), (d**x*x*log(d)**3*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*x*log(d)**2*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) + d**x*x*log(d)*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*x*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*log(d)**2*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) + 2*d**x*log(d)*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) + d**x*sin(x)/(log(d)**4 + 2*log(d)**2 + 1), True))`

**3.136.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int d^x x \sin(x) dx$$

$$= \frac{((\log(d)^2 + 1)x - 2 \log(d))d^x \cos(x) - ((\log(d)^3 + \log(d))x - \log(d)^2 + 1)d^x \sin(x)}{\log(d)^4 + 2 \log(d)^2 + 1}$$

input `integrate(d^x*x*sin(x),x, algorithm="maxima")`

output `-(((log(d)^2 + 1)*x - 2*log(d))*d^x*cos(x) - ((log(d)^3 + log(d))*x - log(d)^2 + 1)*d^x*sin(x))/(log(d)^4 + 2*log(d)^2 + 1)`



**3.136.8 Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1156, normalized size of antiderivative = 13.76

$$\int d^x x \sin(x) dx = \text{Too large to display}$$

input `integrate(d^x*x*sin(x),x, algorithm="giac")`

output

```

1/2*(((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)*(pi*
x*sgn(d) - pi*x + 2*x)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi
i*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)
))^2) - 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))*(x*log(
abs(d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) -
2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d))^2))*cos
(1/2*pi*x*sgn(d) - 1/2*pi*x + x) + 2*((pi*x*sgn(d) - pi*x + 2*x)*(pi*log(a
bs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))/((2*pi + pi^2*sgn(d) - pi^
2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*l
og(abs(d)) + 2*log(abs(d))^2) + (2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d)
)^2 - 2*pi*sgn(d) - 2)*(x*log(abs(d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2
*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(ab
s(d)) + 2*log(abs(d))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d)^x +
1/2*(((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)*(pi
*x*sgn(d) - pi*x - 2*x)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*
pi*sgn(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)
))^2) + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))*(x*log
(abs(d)) - 1)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d)
+ 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d))^2))*co
s(1/2*pi*x*sgn(d) - 1/2*pi*x - x) - 2*((pi*x*sgn(d) - pi*x - 2*x)*(pi*1...

```

**3.136.9 Mupad [B] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.68

$$\int d^x x \sin(x) dx = \frac{d^x (\sin(x) + 2 \ln(d) \cos(x) - \ln(d)^2 \sin(x) - x \cos(x) + x \ln(d) \sin(x) - x \ln(d)^2 \cos(x) + x \ln(d)^3 \sin(x))}{(\ln(d)^2 + 1)^2}$$

input `int(d^x*x*sin(x),x)`

output  $(d^x*(\sin(x) + 2*\log(d)*\cos(x) - \log(d)^2*\sin(x) - x*\cos(x) + x*\log(d)*\sin(x) - x*\log(d)^2*\cos(x) + x*\log(d)^3*\sin(x)))/(\log(d)^2 + 1)^2$

### 3.137 $\int d^x x \cos(x) dx$

|                                                             |     |
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#### 3.137.1 Optimal result

Integrand size = 7, antiderivative size = 83

$$\int d^x x \cos(x) dx = \frac{d^x \cos(x)}{(1 + \log^2(d))^2} - \frac{d^x \cos(x) \log^2(d)}{(1 + \log^2(d))^2} + \frac{d^x x \cos(x) \log(d)}{1 + \log^2(d)} - \frac{2d^x \log(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x \sin(x)}{1 + \log^2(d)}$$

output  $d^x \cos(x) / (1 + \ln(d)^2)^2 - d^x \cos(x) \ln(d)^2 / (1 + \ln(d)^2)^2 + d^x x \cos(x) \ln(d) / (1 + \ln(d)^2) - 2 d^x \ln(d) \sin(x) / (1 + \ln(d)^2)^2 + d^x x \sin(x) / (1 + \ln(d)^2)$

#### 3.137.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.59

$$\int d^x x \cos(x) dx = \frac{d^x (\cos(x) (1 + x \log(d) - \log^2(d) + x \log^3(d)) + (x - 2 \log(d) + x \log^2(d)) \sin(x))}{(1 + \log^2(d))^2}$$

input `Integrate[d^x*x*Cos[x],x]`

output  $(d^x * (\cos[x] * (1 + x * \log[d] - \log[d]^2 + x * \log[d]^3) + (x - 2 * \log[d] + x * \log[d]^2) * \sin[x])) / (1 + \log[d]^2)^2$

### 3.137.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {4969, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x d^x \cos(x) dx$$

$$\downarrow 4969$$

$$-\int \left( \frac{\sin(x) d^x}{\log^2(d) + 1} + \frac{\cos(x) \log(d) d^x}{\log^2(d) + 1} \right) dx + \frac{x d^x \sin(x)}{\log^2(d) + 1} + \frac{x d^x \log(d) \cos(x)}{\log^2(d) + 1}$$

$$\downarrow 2009$$

$$\frac{x d^x \sin(x)}{\log^2(d) + 1} - \frac{2 d^x \log(d) \sin(x)}{(\log^2(d) + 1)^2} + \frac{x d^x \log(d) \cos(x)}{\log^2(d) + 1} - \frac{d^x \log^2(d) \cos(x)}{(\log^2(d) + 1)^2} + \frac{d^x \cos(x)}{(\log^2(d) + 1)^2}$$

input `Int[d^x*x*Cos[x],x]`

output  $(d^x \cos(x)) / (1 + \log(d)^2)^2 - (d^x \cos(x) \log(d)) / (1 + \log(d)^2)^2 + (d^x x \cos(x) \log(d)) / (1 + \log(d)^2) - (2 d^x x \log(d) \sin(x)) / (1 + \log(d)^2)^2 + (d^x x \sin(x)) / (1 + \log(d)^2)$

#### 3.137.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 4969 `Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.), x_Symbol] := Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^n, x]}, Simp[(f*x)^m u, x] - Simp[f*m Int[(f*x)^(m - 1)*u, x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]`

**3.137.4 Maple [A] (verified)**

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.65

| method        | result                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| parallelrisch | $\frac{d^x \left( \ln(d)^3 \cos(x)x + (x \sin(x) - \cos(x)) \ln(d)^2 + (x \cos(x) - 2 \sin(x)) \ln(d) + x \sin(x) + \cos(x) \right)}{(1 + \ln(d)^2)^2}$                                                                                                                                                                                                                                                                                                                |
| risch         | $\frac{(-1 + x \ln(d) + ix)d^x e^{ix}}{2(\ln(d) + i)^2} + \frac{(-1 + x \ln(d) - ix)d^x e^{-ix}}{2(\ln(d) - i)^2}$                                                                                                                                                                                                                                                                                                                                                     |
| norman        | $\frac{\frac{(\ln(d)^2 - 1)e^{x \ln(d)} \left( \tan^2\left(\frac{x}{2}\right) \right) + \ln(d)x e^{x \ln(d)}}{(1 + \ln(d)^2)^2} - \frac{(\ln(d)^2 - 1)e^{x \ln(d)}}{(1 + \ln(d)^2)^2} - \frac{4 \ln(d)e^{x \ln(d)} \tan\left(\frac{x}{2}\right)}{(1 + \ln(d)^2)^2} + \frac{2x e^{x \ln(d)} \tan\left(\frac{x}{2}\right)}{1 + \ln(d)^2} - \frac{\ln(d)x e^{x \ln(d)} \left( \tan^2\left(\frac{x}{2}\right) \right)}{1 + \ln(d)^2}}{1 + \tan^2\left(\frac{x}{2}\right)}$ |

input `int(d^x*x*cos(x),x,method=_RETURNVERBOSE)`output `d^x*(ln(d)^3*cos(x)*x+(x*sin(x)-cos(x))*ln(d)^2+(x*cos(x)-2*sin(x))*ln(d)+x*sin(x)+cos(x))/(1+ln(d)^2)^2`**3.137.5 Fracas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int d^x x \cos(x) dx$$

$$= \frac{(x \cos(x) \log(d)^3 + x \cos(x) \log(d) - \cos(x) \log(d)^2 + (x \log(d)^2 + x - 2 \log(d)) \sin(x) + \cos(x)) d^x}{\log(d)^4 + 2 \log(d)^2 + 1}$$

input `integrate(d^x*x*cos(x),x, algorithm="fricas")`output `(x*cos(x)*log(d)^3 + x*cos(x)*log(d) - cos(x)*log(d)^2 + (x*log(d)^2 + x - 2*log(d))*sin(x) + cos(x))*d^x/(log(d)^4 + 2*log(d)^2 + 1)`

**3.137.6 Sympy [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.66

$$\int d^x x \cos(x) dx$$

$$= \begin{cases} \frac{ix^2 e^{-ix} \sin(x)}{4} + \frac{x^2 e^{-ix} \cos(x)}{4} + \frac{x e^{-ix} \sin(x)}{4} + \frac{ix e^{-ix} \cos(x)}{4} + \frac{e^{-ix} \cos(x)}{4} \\ -\frac{ix^2 e^{ix} \sin(x)}{4} + \frac{x^2 e^{ix} \cos(x)}{4} + \frac{x e^{ix} \sin(x)}{4} - \frac{ix e^{ix} \cos(x)}{4} + \frac{e^{ix} \cos(x)}{4} \\ \frac{d^x x \log(d)^3 \cos(x)}{\log(d)^4 + 2 \log(d)^2 + 1} + \frac{d^x x \log(d)^2 \sin(x)}{\log(d)^4 + 2 \log(d)^2 + 1} + \frac{d^x x \log(d) \cos(x)}{\log(d)^4 + 2 \log(d)^2 + 1} + \frac{d^x x \sin(x)}{\log(d)^4 + 2 \log(d)^2 + 1} - \frac{d^x \log(d)^2 \cos(x)}{\log(d)^4 + 2 \log(d)^2 + 1} - \frac{2 d^x \log(d)}{\log(d)^4 + 2 \log(d)^2 + 1} \end{cases}$$

input `integrate(d**x*x*cos(x),x)`

output `Piecewise((I*x**2*exp(-I*x)*sin(x)/4 + x**2*exp(-I*x)*cos(x)/4 + x*exp(-I*x)*sin(x)/4 + I*x*exp(-I*x)*cos(x)/4 + exp(-I*x)*cos(x)/4, Eq(d, exp(-I))), (-I*x**2*exp(I*x)*sin(x)/4 + x**2*exp(I*x)*cos(x)/4 + x*exp(I*x)*sin(x)/4 - I*x*exp(I*x)*cos(x)/4 + exp(I*x)*cos(x)/4, Eq(d, exp(I))), (d**x*x*log(d)**3*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) + d**x*x*log(d)**2*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) + d**x*x*log(d)*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) + d**x*x*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*log(d)**2*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) - 2*d**x*log(d)*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) + d**x*cos(x)/(log(d)**4 + 2*log(d)**2 + 1), True))`

**3.137.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int d^x x \cos(x) dx$$

$$= \frac{((\log(d)^3 + \log(d))x - \log(d)^2 + 1)d^x \cos(x) + ((\log(d)^2 + 1)x - 2 \log(d))d^x \sin(x)}{\log(d)^4 + 2 \log(d)^2 + 1}$$

input `integrate(d^x*x*cos(x),x, algorithm="maxima")`

output `((((log(d)^3 + log(d))*x - log(d)^2 + 1)*d^x*cos(x) + ((log(d)^2 + 1)*x - 2*log(d))*d^x*sin(x))/(log(d)^4 + 2*log(d)^2 + 1)`

**3.137.8 Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 1155, normalized size of antiderivative = 13.92

$$\int d^x x \cos(x) dx = \text{Too large to display}$$

input `integrate(d^x*x*cos(x),x, algorithm="giac")`

output

```

1/2*(2*((pi*x*sgn(d) - pi*x + 2*x)*(pi*log(abs(d))*sgn(d) - pi*log(abs(d))
+ 2*log(abs(d)))/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn
(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^2)
+ (2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)*(x*log(
abs(d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) -
2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^2))*cos
(1/2*pi*x*sgn(d) - 1/2*pi*x + x) - ((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs
(d))^2 - 2*pi*sgn(d) - 2)*(pi*x*sgn(d) - pi*x + 2*x)/((2*pi + pi^2*sgn(d)
- pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) -
pi*log(abs(d)) + 2*log(abs(d)))^2) - 4*(pi*log(abs(d))*sgn(d) - pi*log(ab
s(d)) + 2*log(abs(d)))*(x*log(abs(d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2
*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(ab
s(d)) + 2*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d)^x +
1/2*(2*((pi*x*sgn(d) - pi*x - 2*x)*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)
) - 2*log(abs(d)))/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sg
n(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2)
) - (2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)*(x*log
(abs(d)) - 1)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d)
+ 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2))*co
s(1/2*pi*x*sgn(d) - 1/2*pi*x - x) + ((2*pi - pi^2*sgn(d) + pi^2 - 2*log...

```

**3.137.9 Mupad [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

$$\int d^x x \cos(x) dx = \frac{d^x (\cos(x) - 2 \ln(d) \sin(x) - \ln(d)^2 \cos(x) + x \sin(x) + x \ln(d) \cos(x) + x \ln(d)^3 \cos(x) + x \ln(d)^2)}{(\ln(d)^2 + 1)^2}$$

input `int(d^x*x*cos(x),x)`

output `(d^x*(cos(x) - 2*log(d)*sin(x) - log(d)^2*cos(x) + x*sin(x) + x*log(d)*cos(x) + x*log(d)^3*cos(x) + x*log(d)^2*sin(x)))/(log(d)^2 + 1)^2`



### 3.138 $\int d^x x^2 \sin(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
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| 3.138.2 Mathematica [A] (verified) . . . . .                | 800 |
| 3.138.3 Rubi [A] (verified) . . . . .                       | 801 |
| 3.138.4 Maple [C] (verified) . . . . .                      | 802 |
| 3.138.5 Fricas [A] (verification not implemented) . . . . . | 803 |
| 3.138.6 Sympy [C] (verification not implemented) . . . . .  | 803 |
| 3.138.7 Maxima [A] (verification not implemented) . . . . . | 804 |
| 3.138.8 Giac [C] (verification not implemented) . . . . .   | 805 |
| 3.138.9 Mupad [B] (verification not implemented) . . . . .  | 805 |

#### 3.138.1 Optimal result

Integrand size = 9, antiderivative size = 162

$$\int d^x x^2 \sin(x) dx = \frac{2d^x \cos(x)}{(1 + \log^2(d))^3} - \frac{6d^x \cos(x) \log^2(d)}{(1 + \log^2(d))^3} + \frac{4d^x x \cos(x) \log(d)}{(1 + \log^2(d))^2} - \frac{d^x x^2 \cos(x)}{1 + \log^2(d)} - \frac{6d^x \log(d) \sin(x)}{(1 + \log^2(d))^3} + \frac{2d^x \log^3(d) \sin(x)}{(1 + \log^2(d))^3} + \frac{2d^x x \sin(x)}{(1 + \log^2(d))^2} - \frac{2d^x x \log^2(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x^2 \log(d) \sin(x)}{1 + \log^2(d)}$$

```
output 2*d^x*cos(x)/(1+ln(d)^2)^3-6*d^x*cos(x)*ln(d)^2/(1+ln(d)^2)^3+4*d^x*x*cos(x)*ln(d)/(1+ln(d)^2)^2-d^x*x^2*cos(x)/(1+ln(d)^2)-6*d^x*ln(d)*sin(x)/(1+ln(d)^2)^3+2*d^x*ln(d)^3*sin(x)/(1+ln(d)^2)^3+2*d^x*x*sin(x)/(1+ln(d)^2)^2-2*d^x*x*ln(d)^2*sin(x)/(1+ln(d)^2)^2+d^x*x^2*ln(d)*sin(x)/(1+ln(d)^2)
```

#### 3.138.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.58

$$\int d^x x^2 \sin(x) dx = \frac{d^x (-\cos(x) (-2 + x^2 - 4x \log(d) + 2(3 + x^2) \log^2(d) - 4x \log^3(d) + x^2 \log^4(d)) + (2x + (-6 + x^2) \log(d))}{(1 + \log^2(d))^3}$$

input `Integrate[d^x*x^2*Sin[x],x]`

output  $(d^x * (-(\cos[x] * (-2 + x^2 - 4*x*\log[d] + 2*(3 + x^2)*\log[d]^2 - 4*x*\log[d]^3 + x^2*\log[d]^4)) + (2*x + (-6 + x^2)*\log[d] + 2*(1 + x^2)*\log[d]^3 - 2*x*\log[d]^4 + x^2*\log[d]^5)*\sin[x])) / (1 + \log[d]^2)^3$

### 3.138.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {4968, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^2 d^x \sin(x) dx \\
 & \quad \downarrow 4968 \\
 & -2 \int -x \left( \frac{d^x \cos(x)}{\log^2(d) + 1} - \frac{d^x \log(d) \sin(x)}{\log^2(d) + 1} \right) dx + \frac{x^2 d^x \log(d) \sin(x)}{\log^2(d) + 1} - \frac{x^2 d^x \cos(x)}{\log^2(d) + 1} \\
 & \quad \downarrow 25 \\
 & 2 \int x \left( \frac{d^x \cos(x)}{\log^2(d) + 1} - \frac{d^x \log(d) \sin(x)}{\log^2(d) + 1} \right) dx + \frac{x^2 d^x \log(d) \sin(x)}{\log^2(d) + 1} - \frac{x^2 d^x \cos(x)}{\log^2(d) + 1} \\
 & \quad \downarrow 2010 \\
 & 2 \int \left( \frac{d^x x \cos(x)}{\log^2(d) + 1} - \frac{d^x x \log(d) \sin(x)}{\log^2(d) + 1} \right) dx + \frac{x^2 d^x \log(d) \sin(x)}{\log^2(d) + 1} - \frac{x^2 d^x \cos(x)}{\log^2(d) + 1} \\
 & \quad \downarrow 2009 \\
 & \frac{x^2 d^x \log(d) \sin(x)}{\log^2(d) + 1} - \frac{x^2 d^x \cos(x)}{\log^2(d) + 1} + \\
 & 2 \left( -\frac{x d^x \log^2(d) \sin(x)}{(\log^2(d) + 1)^2} + \frac{x d^x \sin(x)}{(\log^2(d) + 1)^2} - \frac{3 d^x \log(d) \sin(x)}{(\log^2(d) + 1)^3} + \frac{d^x \log^3(d) \sin(x)}{(\log^2(d) + 1)^3} + \frac{2 x d^x \log(d) \cos(x)}{(\log^2(d) + 1)^2} - \frac{3 d^x \log^2(d) \cos(x)}{(\log^2(d) + 1)^2} \right)
 \end{aligned}$$

input `Int[d^x*x^2*Sin[x],x]`

```
output -((d^x*x^2*Cos[x])/(1 + Log[d]^2)) + (d^x*x^2*Log[d]*Sin[x])/(1 + Log[d]^2)
) + 2*((d^x*Cos[x])/(1 + Log[d]^2)^3 - (3*d^x*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^3 + (2*d^x*x*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (3*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^3 + (d^x*Log[d]^3*Sin[x])/(1 + Log[d]^2)^3 + (d^x*x*Sin[x])/(1 + Log[d]^2)^2 - (d^x*x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^2)
```

### 3.138.3.1 Defintions of rubi rules used

```
rule 25 Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 2010 Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

```
rule 4968 Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*(x_)]^(n_.), x_Symbol] := Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^n, x]}, Simp[(f*x)^m u, x] - Simp[f*m Int[(f*x)^(m - 1)*u, x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

### 3.138.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.63

| method        | result                                                                                                                                                                                                                                                                                                                                                                                                     |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch         | $-\frac{i(2+\ln(d)^2x^2+2i\ln(d)x^2-x^2-2x\ln(d)-2ix)d^xe^{ix}}{2(\ln(d)+i)^3} + \frac{i(2-2x\ln(d)+2ix+\ln(d)^2x^2-2i\ln(d)x^2-x^2)d^xe^{-ix}}{2(\ln(d)-i)^3}$                                                                                                                                                                                                                                            |
| parallelrisch | $\frac{d^x(\ln(d)^5x^2\sin(x)+(-x^2\cos(x)-2x\sin(x))\ln(d)^4+(2x^2\sin(x)+4x\cos(x)+2\sin(x))\ln(d)^3+(-2x^2-6)\cos(x)\ln(d)^2+(x^2\sin(x)+2x\cos(x))\ln(d))}{(1+\ln(d)^2)^3}$                                                                                                                                                                                                                            |
| norman        | $\frac{x^2e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\ln(d)^2} - \frac{x^2e^{x\ln(d)}}{1+\ln(d)^2} - \frac{2(3\ln(d)^2-1)e^{x\ln(d)}}{(1+\ln(d)^2)^3} + \frac{4\ln(d)x e^{x\ln(d)}}{(1+\ln(d)^2)^2} + \frac{2(3\ln(d)^2-1)e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{(1+\ln(d)^2)^3} - \frac{4\ln(d)x e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{(1+\ln(d)^2)^2}$ |

input `int(d^x*x^2*sin(x),x,method=_RETURNVERBOSE)`

output 
$$-1/2*I*(2+\ln(d)^2*x^2+2*I*\ln(d)*x^2-x^2-2*x*\ln(d)-2*I*x)*d^x/(\ln(d)+I)^3*\exp(I*x)+1/2*I*(2-2*x*\ln(d)+2*I*x+\ln(d)^2*x^2-2*I*\ln(d)*x^2-x^2)*d^x/(\ln(d)-I)^3*\exp(-I*x)$$

### 3.138.5 Fracas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.71

$$\int d^x x^2 \sin(x) dx = \frac{(x^2 \cos(x) \log(d)^4 - 4x \cos(x) \log(d)^3 + 2(x^2 + 3) \cos(x) \log(d)^2 - 4x \cos(x) \log(d) + (x^2 - 2) \cos(x)) d^x}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1}$$

input `integrate(d^x*x^2*sin(x),x, algorithm="fracas")`

output 
$$-(x^2*\cos(x)*\log(d)^4 - 4*x*\cos(x)*\log(d)^3 + 2*(x^2 + 3)*\cos(x)*\log(d)^2 - 4*x*\cos(x)*\log(d) + (x^2 - 2)*\cos(x) - (x^2*\log(d)^5 - 2*x*\log(d)^4 + 2*(x^2 + 1)*\log(d)^3 + (x^2 - 6)*\log(d) + 2*x)*\sin(x))*d^x/(\log(d)^6 + 3*\log(d)^4 + 3*\log(d)^2 + 1)$$

### 3.138.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 665, normalized size of antiderivative = 4.10

$$\int d^x x^2 \sin(x) dx = \left\{ \begin{array}{l} \frac{x^3 e^{-ix} \sin(x)}{6} - \frac{ix^3 e^{-ix} \cos(x)}{6} + \frac{ix^2 e^{-ix} \sin(x)}{4} - \frac{x^2 e^{-ix} \cos(x)}{4} + \frac{xe^{-ix} \sin(x)}{4} + \frac{ixe^{-ix} \cos(x)}{4} + \frac{e^{-ix} \cos(x)}{4} \\ \frac{x^3 e^{ix} \sin(x)}{6} + \frac{ix^3 e^{ix} \cos(x)}{6} - \frac{ix^2 e^{ix} \sin(x)}{4} - \frac{x^2 e^{ix} \cos(x)}{4} + \frac{xe^{ix} \sin(x)}{4} - \frac{ixe^{ix} \cos(x)}{4} + \frac{e^{ix} \cos(x)}{4} \\ \frac{d^x x^2 \log(d)^5 \sin(x)}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1} - \frac{d^x x^2 \log(d)^4 \cos(x)}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1} + \frac{2d^x x^2 \log(d)^3 \sin(x)}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1} - \frac{2d^x x^2 \log(d)^2 \cos(x)}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1} \end{array} \right.$$

input `integrate(d**x*x**2*sin(x),x)`

output `Piecewise((x**3*exp(-I*x)*sin(x)/6 - I*x**3*exp(-I*x)*cos(x)/6 + I*x**2*exp(-I*x)*sin(x)/4 - x**2*exp(-I*x)*cos(x)/4 + x*exp(-I*x)*sin(x)/4 + I*x*exp(-I*x)*cos(x)/4 + exp(-I*x)*cos(x)/4, Eq(d, exp(-I))), (x**3*exp(I*x)*sin(x)/6 + I*x**3*exp(I*x)*cos(x)/6 - I*x**2*exp(I*x)*sin(x)/4 - x**2*exp(I*x)*cos(x)/4 + x*exp(I*x)*sin(x)/4 - I*x*exp(I*x)*cos(x)/4 + exp(I*x)*cos(x)/4, Eq(d, exp(I))), (d**x*x**2*log(d)**5*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - d**x*x**2*log(d)**4*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*x**2*log(d)**3*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 2*d**x*x**2*log(d)**2*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + d**x*x**2*log(d)*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - d**x*x**2*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 2*d**x*x*log(d)**4*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 4*d**x*x*log(d)**3*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 4*d**x*x*log(d)*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*x*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*log(d)**3*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)**2*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1), True))`

### 3.138.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66

$$\int d^x x^2 \sin(x) dx = \frac{((\log(d)^4 + 2 \log(d)^2 + 1)x^2 - 4(\log(d)^3 + \log(d))x + 6 \log(d)^2 - 2)d^x \cos(x) - ((\log(d)^5 + 2 \log(d)^3 + \log(d)^2 + 1)d^x \sin(x) - 6 \log(d)^4 x + 6 \log(d)^3 - 2 \log(d)^2)x}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1}$$

input `integrate(d^x*x^2*sin(x),x, algorithm="maxima")`

output `-(((log(d)^4 + 2*log(d)^2 + 1)*x^2 - 4*(log(d)^3 + log(d))*x + 6*log(d)^2 - 2)*d^x*cos(x) - ((log(d)^5 + 2*log(d)^3 + log(d))*x^2 + 2*log(d)^3 - 2*(log(d)^4 - 1)*x - 6*log(d))*d^x*sin(x))/(log(d)^6 + 3*log(d)^4 + 3*log(d)^2 + 1)`

**3.138.8 Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 2631, normalized size of antiderivative = 16.24

$$\int d^x x^2 \sin(x) dx = \text{Too large to display}$$

input `integrate(d^x*x^2*sin(x),x, algorithm="giac")`

output

```
-1/2*(((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(a
bs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)*(pi
^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d) + 2*pi*x^
2 - 2*x^2 - 4*x*log(abs(d)) + 4)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2
*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d
))^2 - 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)
) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(a
bs(d)))^2) - 2*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^2*log
(abs(d)) - pi*x*sgn(d) + pi*x - 2*x)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*l
og(abs(d)) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d))
- 6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 -
3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(
d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d)
)^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*cos(
1/2*pi*x*sgn(d) - 1/2*pi*x + x) - (2*(3*pi - pi^3*sgn(d) + 3*pi*log(abs(d)
))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(ab
s(d))^2 - 3*pi*sgn(d) - 2)*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d))
+ 2*x^2*log(abs(d)) - pi*x*sgn(d) + pi*x - 2*x)/((3*pi - pi^3*sgn(d) + 3*
pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi
^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) ...
```

**3.138.9 Mupad [B] (verification not implemented)**

Time = 0.39 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int d^x x^2 \sin(x) dx$$

$$= \frac{d^x (2 \cos(x) - x^2 \cos(x) + 2x \sin(x)) + d^x \ln(d)^3 (2 \sin(x) + 2x^2 \sin(x) + 4x \cos(x)) - d^x \ln(d)^2 (6 \cos(x) - 2x^2 \cos(x) + 4x \sin(x))}{d^x}$$

input `int(d^x*x^2*sin(x),x)`

output  $(d^x*(2*\cos(x) - x^2*\cos(x) + 2*x*\sin(x)) + d^x*\log(d)^3*(2*\sin(x) + 2*x^2*\sin(x) + 4*x*\cos(x)) - d^x*\log(d)^2*(6*\cos(x) + 2*x^2*\cos(x)) + d^x*\log(d)*(x^2*\sin(x) - 6*\sin(x) + 4*x*\cos(x)) - d^x*\log(d)^4*(x^2*\cos(x) + 2*x*\sin(x)) + d^x*x^2*\log(d)^5*\sin(x))/(\log(d)^2 + 1)^3$

### 3.139 $\int d^x x^2 \cos(x) dx$

|                                                             |     |
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#### 3.139.1 Optimal result

Integrand size = 9, antiderivative size = 161

$$\int d^x x^2 \cos(x) dx = -\frac{6d^x \cos(x) \log(d)}{(1 + \log^2(d))^3} + \frac{2d^x \cos(x) \log^3(d)}{(1 + \log^2(d))^3} + \frac{2d^x x \cos(x)}{(1 + \log^2(d))^2} - \frac{2d^x x \cos(x) \log^2(d)}{(1 + \log^2(d))^2} + \frac{d^x x^2 \cos(x) \log(d)}{1 + \log^2(d)} - \frac{2d^x \sin(x)}{(1 + \log^2(d))^3} + \frac{6d^x \log^2(d) \sin(x)}{(1 + \log^2(d))^3} - \frac{4d^x x \log(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x^2 \sin(x)}{1 + \log^2(d)}$$

```
output -6*d^x*cos(x)*ln(d)/(1+ln(d)^2)^3+2*d^x*cos(x)*ln(d)^3/(1+ln(d)^2)^3+2*d^x*x*cos(x)/(1+ln(d)^2)^2-2*d^x*x*cos(x)*ln(d)^2/(1+ln(d)^2)^2+d^x*x^2*cos(x)*ln(d)/(1+ln(d)^2)-2*d^x*sin(x)/(1+ln(d)^2)^3+6*d^x*ln(d)^2*sin(x)/(1+ln(d)^2)^3-4*d^x*x*ln(d)*sin(x)/(1+ln(d)^2)^2+d^x*x^2*sin(x)/(1+ln(d)^2)
```

#### 3.139.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.58

$$\int d^x x^2 \cos(x) dx = \frac{d^x (\cos(x) (2x + (-6 + x^2) \log(d) + 2(1 + x^2) \log^3(d) - 2x \log^4(d) + x^2 \log^5(d)) + (-2 + x^2 - 4x \log(d) - 2x \log^2(d) + x^2 \log^3(d)))}{(1 + \log^2(d))^3}$$



input `Integrate[d^x*x^2*Cos[x],x]`

output  $(d^x*(\cos(x)*(2x + (-6 + x^2)*\log[d] + 2*(1 + x^2)*\log[d]^3 - 2*x*\log[d]^4 + x^2*\log[d]^5) + (-2 + x^2 - 4*x*\log[d] + 2*(3 + x^2)*\log[d]^2 - 4*x*\log[d]^3 + x^2*\log[d]^4)*\sin(x)))/(1 + \log[d]^2)^3$

### 3.139.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {4969, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 d^x \cos(x) dx$$

$$\downarrow 4969$$

$$-2 \int x \left( \frac{\sin(x) d^x}{\log^2(d) + 1} + \frac{\cos(x) \log(d) d^x}{\log^2(d) + 1} \right) dx + \frac{x^2 d^x \sin(x)}{\log^2(d) + 1} + \frac{x^2 d^x \log(d) \cos(x)}{\log^2(d) + 1}$$

$$\downarrow 2010$$

$$-2 \int \left( \frac{x \sin(x) d^x}{\log^2(d) + 1} + \frac{x \cos(x) \log(d) d^x}{\log^2(d) + 1} \right) dx + \frac{x^2 d^x \sin(x)}{\log^2(d) + 1} + \frac{x^2 d^x \log(d) \cos(x)}{\log^2(d) + 1}$$

$$\downarrow 2009$$

$$\frac{x^2 d^x \sin(x)}{\log^2(d) + 1} + \frac{x^2 d^x \log(d) \cos(x)}{\log^2(d) + 1} -$$

$$2 \left( \frac{2x d^x \log(d) \sin(x)}{(\log^2(d) + 1)^2} - \frac{3d^x \log^2(d) \sin(x)}{(\log^2(d) + 1)^3} + \frac{d^x \sin(x)}{(\log^2(d) + 1)^3} + \frac{x d^x \log^2(d) \cos(x)}{(\log^2(d) + 1)^2} - \frac{x d^x \cos(x)}{(\log^2(d) + 1)^2} + \frac{3d^x \log(d) \cos(x)}{(\log^2(d) + 1)^2} \right)$$

input `Int[d^x*x^2*Cos[x],x]`

output  $(d^x*x^2*\cos[x]*\log[d])/(1 + \log[d]^2) + (d^x*x^2*\sin[x])/(1 + \log[d]^2) - 2*((3*d^x*\cos[x]*\log[d])/(1 + \log[d]^2)^3 - (d^x*\cos[x]*\log[d]^3)/(1 + \log[d]^2)^3 - (d^x*x*\cos[x])/(1 + \log[d]^2)^2 + (d^x*x*\cos[x]*\log[d]^2)/(1 + \log[d]^2)^2 + (d^x*\sin[x])/(1 + \log[d]^2)^3 - (3*d^x*\log[d]^2*\sin[x])/(1 + \log[d]^2)^3 + (2*d^x*x*\log[d]*\sin[x])/(1 + \log[d]^2)^2)$

### 3.139.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_)*(x_)^(m_)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]`

rule 4969 `Int[Cos[(d_.) + (e_)*(x_)]^(n_)*(F_)^((c_)*((a_.) + (b_)*(x_)))*((f_)*(x_))^(m_), x_Symbol] := Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^n, x]}, Simp[(f*x)^m u, x] - Simp[f*m Int[(f*x)^(m - 1)*u, x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]`

### 3.139.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.62

| method       | result                                                                                                                                                                                                                                                                                                                                                          |
|--------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch        | $\frac{(2+\ln(d)^2x^2+2i\ln(d)x^2-x^2-2x\ln(d)-2ix)d^xe^{ix}}{2(\ln(d)+i)^3} + \frac{(2-2x\ln(d)+2ix+\ln(d)^2x^2-2i\ln(d)x^2-x^2)d^xe^{-ix}}{2(\ln(d)-i)^3}$                                                                                                                                                                                                    |
| parallelrisc | $\frac{d^x(\ln(d)^5x^2\cos(x)+x(-2\cos(x)+x\sin(x))\ln(d)^4+(2x^2\cos(x)-4x\sin(x)+2\cos(x))\ln(d)^3+(2x^2+6)\sin(x)\ln(d)^2+(x^2\cos(x)-2x\sin(x))\ln(d))}{(1+\ln(d)^2)^3}$                                                                                                                                                                                    |
| norman       | $\frac{\ln(d)x^2e^{x\ln(d)}}{1+\ln(d)^2} + \frac{2x^2e^{x\ln(d)}\tan(\frac{x}{2})}{1+\ln(d)^2} - \frac{2(\ln(d)^2-1)x e^{x\ln(d)}}{(1+\ln(d)^2)^2} + \frac{4(3\ln(d)^2-1)e^{x\ln(d)}\tan(\frac{x}{2})}{(1+\ln(d)^2)^3} + \frac{2\ln(d)(\ln(d)^2-3)e^{x\ln(d)}}{(1+\ln(d)^2)^3} - \frac{8\ln(d)x e^{x\ln(d)}}{(1+\ln(d)^2)^3} - \frac{1}{1+\tan^2(\frac{x}{2})}$ |

input `int(d^x*x^2*cos(x), x, method=_RETURNVERBOSE)`

output `1/2*(2+ln(d)^2*x^2+2*I*ln(d)*x^2-x^2-2*x*ln(d)-2*I*x)*d^x/(ln(d)+I)^3*exp(I*x)+1/2*(2-2*x*ln(d)+2*I*x+ln(d)^2*x^2-2*I*ln(d)*x^2-x^2)*d^x/(ln(d)-I)^3*exp(-I*x)`

**3.139.5 Fracas [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.69

$$\int d^x x^2 \cos(x) dx$$

$$= \frac{(x^2 \cos(x) \log(d)^5 - 2x \cos(x) \log(d)^4 + 2(x^2 + 1) \cos(x) \log(d)^3 + (x^2 - 6) \cos(x) \log(d) + 2x \cos(x) \log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1)}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1}$$

input `integrate(d^x*x^2*cos(x),x, algorithm="fricas")`output `(x^2*cos(x)*log(d)^5 - 2*x*cos(x)*log(d)^4 + 2*(x^2 + 1)*cos(x)*log(d)^3 + (x^2 - 6)*cos(x)*log(d) + 2*x*cos(x) + (x^2*log(d)^4 - 4*x*log(d)^3 + 2*(x^2 + 3)*log(d)^2 + x^2 - 4*x*log(d) - 2)*sin(x))*d^x/(log(d)^6 + 3*log(d)^4 + 3*log(d)^2 + 1)`**3.139.6 Sympy [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 668, normalized size of antiderivative = 4.15

$$\int d^x x^2 \cos(x) dx$$

$$= \left\{ \begin{array}{l} \frac{ix^3 e^{-ix} \sin(x)}{6} + \frac{x^3 e^{-ix} \cos(x)}{6} + \frac{x^2 e^{-ix} \sin(x)}{4} + \frac{ix^2 e^{-ix} \cos(x)}{4} - \frac{ixe^{-ix} \sin(x)}{4} + \frac{xe^{-ix} \cos(x)}{4} - \frac{ie^{-ix} \cos(x)}{4} \\ - \frac{ix^3 e^{ix} \sin(x)}{6} + \frac{x^3 e^{ix} \cos(x)}{6} + \frac{x^2 e^{ix} \sin(x)}{4} - \frac{ix^2 e^{ix} \cos(x)}{4} + \frac{ixe^{ix} \sin(x)}{4} + \frac{xe^{ix} \cos(x)}{4} + \frac{ie^{ix} \cos(x)}{4} \\ \frac{d^x x^2 \log(d)^5 \cos(x)}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1} + \frac{d^x x^2 \log(d)^4 \sin(x)}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1} + \frac{2d^x x^2 \log(d)^3 \cos(x)}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1} + \frac{2d^x x^2 \log(d)^2 \sin(x)}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1} \end{array} \right.$$

input `integrate(d**x*x**2*cos(x),x)`

output `Piecewise((I*x**3*exp(-I*x)*sin(x)/6 + x**3*exp(-I*x)*cos(x)/6 + x**2*exp(-I*x)*sin(x)/4 + I*x**2*exp(-I*x)*cos(x)/4 - I*x*exp(-I*x)*sin(x)/4 + x*exp(-I*x)*cos(x)/4 - I*exp(-I*x)*cos(x)/4, Eq(d, exp(-I))), (-I*x**3*exp(I*x)*sin(x)/6 + x**3*exp(I*x)*cos(x)/6 + x**2*exp(I*x)*sin(x)/4 - I*x**2*exp(I*x)*cos(x)/4 + I*x*exp(I*x)*sin(x)/4 + x*exp(I*x)*cos(x)/4 + I*exp(I*x)*cos(x)/4, Eq(d, exp(I))), (d**x*x**2*log(d)**5*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + d**x*x**2*log(d)**4*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*x**2*log(d)**3*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*x**2*log(d)**2*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + d**x*x**2*log(d)*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + d**x*x**2*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 2*d**x*x*log(d)**4*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 4*d**x*x*log(d)**3*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 4*d**x*x*log(d)*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*x*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*log(d)**3*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 6*d**x*log(d)**2*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 2*d**x*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1), True))`

### 3.139.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.65

$$\int d^x x^2 \cos(x) dx$$

$$= \frac{((\log(d))^5 + 2 \log(d)^3 + \log(d))x^2 + 2 \log(d)^3 - 2(\log(d)^4 - 1)x - 6 \log(d)d^x \cos(x) + ((\log(d))^4 + 2 \log(d)^2 - 2)d^x \sin(x)}{\log(d)^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1}$$

input `integrate(d^x*x^2*cos(x),x, algorithm="maxima")`

output `((((log(d))^5 + 2*log(d)^3 + log(d))*x^2 + 2*log(d)^3 - 2*(log(d)^4 - 1)*x - 6*log(d)*d^x*cos(x) + ((log(d)^4 + 2*log(d)^2 + 1)*x^2 - 4*(log(d)^3 + 1*log(d))*x + 6*log(d)^2 - 2)*d^x*sin(x))/(log(d)^6 + 3*log(d)^4 + 3*log(d)^2 + 1)`

**3.139.8 Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 2631, normalized size of antiderivative = 16.34

$$\int d^x x^2 \cos(x) dx = \text{Too large to display}$$

input `integrate(d^x*x^2*cos(x),x, algorithm="giac")`

output

```
1/2*((2*(3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(
abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)*(p
i*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^2*log(abs(d)) - pi*x*s
gn(d) + pi*x - 2*x)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^
3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*s
gn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs
(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2) +
(pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d) + 2*pi
*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*lo
g(abs(d)) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) -
6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 -
3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d)
- 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))
^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*cos(1
/2*pi*x*sgn(d) - 1/2*pi*x + x) + ((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2
*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d)
))^2 - 3*pi*sgn(d) - 2)*(pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2
- 2*pi*x^2*sgn(d) + 2*pi*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)/((3*pi - pi^3*
sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sg
n(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(...
```

**3.139.9 Mupad [B] (verification not implemented)**

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82

$$\int d^x x^2 \cos(x) dx$$

$$= \frac{d^x (x^2 \sin(x) - 2 \sin(x) + 2x \cos(x)) + d^x \ln(d)^3 (2 \cos(x) + 2x^2 \cos(x) - 4x \sin(x)) + d^x \ln(d)^2 (6 \sin(x) - 2x \cos(x))}{d^x}$$

input `int(d^x*x^2*cos(x),x)`

output  $(d^x(x^2\sin(x) - 2\sin(x) + 2x\cos(x)) + d^x\log(d)^3(2\cos(x) + 2x^2\cos(x) - 4x\sin(x)) + d^x\log(d)^2(6\sin(x) + 2x^2\sin(x)) - d^x\log(d)(6\cos(x) - x^2\cos(x) + 4x\sin(x)) + d^x\log(d)^4(x^2\sin(x) - 2x\cos(x)) + d^x x^2 \log(d)^5 \cos(x)) / (\log(d)^2 + 1)^3$

### 3.140 $\int d^x x^3 \sin(x) dx$

|                                                             |     |
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#### 3.140.1 Optimal result

Integrand size = 9, antiderivative size = 261

$$\int d^x x^3 \sin(x) dx = -\frac{24d^x \cos(x) \log(d)}{(1 + \log^2(d))^4} + \frac{24d^x \cos(x) \log^3(d)}{(1 + \log^2(d))^4} + \frac{6d^x x \cos(x)}{(1 + \log^2(d))^3}$$

$$- \frac{18d^x x \cos(x) \log^2(d)}{(1 + \log^2(d))^3} + \frac{6d^x x^2 \cos(x) \log(d)}{(1 + \log^2(d))^2}$$

$$- \frac{d^x x^3 \cos(x)}{1 + \log^2(d)} - \frac{6d^x \sin(x)}{(1 + \log^2(d))^4} + \frac{36d^x \log^2(d) \sin(x)}{(1 + \log^2(d))^4}$$

$$- \frac{6d^x \log^4(d) \sin(x)}{(1 + \log^2(d))^4} - \frac{18d^x x \log(d) \sin(x)}{(1 + \log^2(d))^3} + \frac{6d^x x \log^3(d) \sin(x)}{(1 + \log^2(d))^3}$$

$$+ \frac{3d^x x^2 \sin(x)}{(1 + \log^2(d))^2} - \frac{3d^x x^2 \log^2(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x^3 \log(d) \sin(x)}{1 + \log^2(d)}$$

output

```
-24*d^x*cos(x)*ln(d)/(1+ln(d)^2)^4+24*d^x*cos(x)*ln(d)^3/(1+ln(d)^2)^4+6*d^x*x*cos(x)/(1+ln(d)^2)^3-18*d^x*x*cos(x)*ln(d)^2/(1+ln(d)^2)^3+6*d^x*x^2*cos(x)*ln(d)/(1+ln(d)^2)^2-d^x*x^3*cos(x)/(1+ln(d)^2)-6*d^x*sin(x)/(1+ln(d)^2)^4+36*d^x*ln(d)^2*sin(x)/(1+ln(d)^2)^4-6*d^x*ln(d)^4*sin(x)/(1+ln(d)^2)^4-18*d^x*x*ln(d)*sin(x)/(1+ln(d)^2)^3+6*d^x*x*ln(d)^3*sin(x)/(1+ln(d)^2)^3+3*d^x*x^2*sin(x)/(1+ln(d)^2)^2-3*d^x*x^2*ln(d)^2*sin(x)/(1+ln(d)^2)^2+d^x*x^3*ln(d)*sin(x)/(1+ln(d)^2)
```

**3.140.2 Mathematica [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.65

$$\int d^x x^3 \sin(x) dx$$

$$= \frac{d^x (-\cos(x) (x(-6 + x^2) - 6(-4 + x^2) \log(d) + 3x(4 + x^2) \log^2(d) - 12(2 + x^2) \log^3(d) + 3x(6 + x^2) \log^4(d) + 3x^2 \log^5(d) - 3x^3 \log^6(d) + x^4 \log^7(d) \sin(x))}{(1 + \log(d)^2)^4}$$

input `Integrate[d^x*x^3*Sin[x],x]`

output  $(d^x * (-(\text{Cos}[x] * (x * (-6 + x^2) - 6 * (-4 + x^2) * \text{Log}[d] + 3 * x * (4 + x^2) * \text{Log}[d]^2 - 12 * (2 + x^2) * \text{Log}[d]^3 + 3 * x * (6 + x^2) * \text{Log}[d]^4 - 6 * x^2 * \text{Log}[d]^5 + x^3 * \text{Log}[d]^6)) + (3 * (-2 + x^2) + x * (-18 + x^2) * \text{Log}[d] + 3 * (12 + x^2) * \text{Log}[d]^2 + 3 * x * (-4 + x^2) * \text{Log}[d]^3 - 3 * (2 + x^2) * \text{Log}[d]^4 + 3 * x * (2 + x^2) * \text{Log}[d]^5 - 3 * x^2 * \text{Log}[d]^6 + x^3 * \text{Log}[d]^7) * \text{Sin}[x])) / (1 + \text{Log}[d]^2)^4$

**3.140.3 Rubi [A] (verified)**Time = 0.62 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {4968, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 d^x \sin(x) dx$$

$$\downarrow 4968$$

$$-3 \int -x^2 \left( \frac{d^x \cos(x)}{\log^2(d) + 1} - \frac{d^x \log(d) \sin(x)}{\log^2(d) + 1} \right) dx + \frac{x^3 d^x \log(d) \sin(x)}{\log^2(d) + 1} - \frac{x^3 d^x \cos(x)}{\log^2(d) + 1}$$

$$\downarrow 25$$

$$3 \int x^2 \left( \frac{d^x \cos(x)}{\log^2(d) + 1} - \frac{d^x \log(d) \sin(x)}{\log^2(d) + 1} \right) dx + \frac{x^3 d^x \log(d) \sin(x)}{\log^2(d) + 1} - \frac{x^3 d^x \cos(x)}{\log^2(d) + 1}$$

$$\downarrow 2010$$

$$3 \int \left( \frac{d^x x^2 \cos(x)}{\log^2(d) + 1} - \frac{d^x x^2 \log(d) \sin(x)}{\log^2(d) + 1} \right) dx + \frac{x^3 d^x \log(d) \sin(x)}{\log^2(d) + 1} - \frac{x^3 d^x \cos(x)}{\log^2(d) + 1}$$



$$\begin{aligned}
 & \downarrow \text{2009} \\
 & \frac{x^3 d^x \log(d) \sin(x)}{\log^2(d) + 1} - \frac{x^3 d^x \cos(x)}{\log^2(d) + 1} + \\
 & 3 \left( \frac{x^2 d^x \sin(x)}{(\log^2(d) + 1)^2} - \frac{x^2 d^x \log^2(d) \sin(x)}{(\log^2(d) + 1)^2} + \frac{2x^2 d^x \log(d) \cos(x)}{(\log^2(d) + 1)^2} - \frac{6x d^x \log(d) \sin(x)}{(\log^2(d) + 1)^3} + \frac{12d^x \log^2(d) \sin(x)}{(\log^2(d) + 1)^4} - \frac{2d^x}{(\log^2(d) + 1)^4} \right)
 \end{aligned}$$

input `Int[d^x*x^3*Sin[x],x]`

output `-((d^x*x^3*Cos[x])/(1 + Log[d]^2)) + (d^x*x^3*Log[d]*Sin[x])/(1 + Log[d]^2)
 ) + 3*((-8*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^4 + (8*d^x*Cos[x]*Log[d]^3)/(
 1 + Log[d]^2)^4 + (2*d^x*x*Cos[x])/(1 + Log[d]^2)^3 - (6*d^x*x*Cos[x]*Log[
 d]^2)/(1 + Log[d]^2)^3 + (2*d^x*x^2*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (2*d
 ^x*Sin[x])/(1 + Log[d]^2)^4 + (12*d^x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^4 -
 (2*d^x*Log[d]^4*Sin[x])/(1 + Log[d]^2)^4 - (6*d^x*x*Log[d]*Sin[x])/(1 + Lo
 g[d]^2)^3 + (2*d^x*x*Log[d]^3*Sin[x])/(1 + Log[d]^2)^3 + (d^x*x^2*Sin[x])/
 (1 + Log[d]^2)^2 - (d^x*x^2*Log[d]^2*Sin[x])/(1 + Log[d]^2)^2)`

### 3.140.3.1 Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]`

rule 4968 `Int[(F_)^((c_.)*(a_.) + (b_.)*(x_))*((f_.)*(x_))^(m_)*Sin[(d_.) + (e_.)*
(x_)]^(n_), x_Symbol] := Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Simp[(f*x)^m u, x] - Simp[f*m Int[(f*x)^(m - 1)*u, x], x] /; F
reeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]`

### 3.140.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.64

| method        | result                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch         | $\frac{i(-6+\ln(d)^3x^3+3i\ln(d)^2x^3-3\ln(d)x^3-ix^3+6x\ln(d)+6ix-3\ln(d)^2x^2-6i\ln(d)x^2+3x^2)d^xe^{ix}}{2(\ln(d)+i)^4} + \frac{i(-6+6x\ln(d)-6ix-3\ln(d)^2x^2+3x^2)}{2(\ln(d)+i)^4}$                                                                                                                                                                                                                                                                            |
| parallelrisch | $d^x \left( \ln(d)^7 x^3 \sin(x) + (-x^3 \cos(x) - 3x^2 \sin(x)) \ln(d)^6 + 3((x^3 + 2x) \sin(x) + 2x^2 \cos(x)) \ln(d)^5 + 3(-x^3 \cos(x) - x^2 \sin(x) - 6x \cos(x)) \ln(d)^4 \right)$                                                                                                                                                                                                                                                                            |
| norman        | $\frac{x^3 e^{x \ln(d)} \left( \tan^2\left(\frac{x}{2}\right) \right)}{1 + \ln(d)^2} - \frac{x^3 e^{x \ln(d)}}{1 + \ln(d)^2} + \frac{6 \ln(d) x^2 e^{x \ln(d)}}{\ln(d)^4 + 2 \ln(d)^2 + 1} - \frac{6(\ln(d)^2 - 1) x^2 e^{x \ln(d)} \tan\left(\frac{x}{2}\right)}{\ln(d)^4 + 2 \ln(d)^2 + 1} - \frac{6(3 \ln(d)^2 - 1) x e^{x \ln(d)}}{(1 + \ln(d)^2)(\ln(d)^4 + 2 \ln(d)^2 + 1)} - \frac{12(\ln(d)^4 - 3 \ln(d)^2 + 1)}{(\ln(d)^6 + 3 \ln(d)^4 + 6 \ln(d)^2 + 1)}$ |

input `int(d^x*x^3*sin(x),x,method=_RETURNVERBOSE)`

output `-1/2*I*(-6+ln(d)^3*x^3+3*I*ln(d)^2*x^3-3*ln(d)*x^3-I*x^3+6*x*ln(d)+6*I*x-3*ln(d)^2*x^2-6*I*ln(d)*x^2+3*x^2)*d^x/(ln(d)+I)^4*exp(I*x)+1/2*I*(-6+6*x*ln(d)-6*I*x-3*ln(d)^2*x^2+6*I*ln(d)*x^2+3*x^2+ln(d)^3*x^3-3*I*ln(d)^2*x^3-3*ln(d)*x^3+I*x^3)*d^x/(ln(d)-I)^4*exp(-I*x)`

### 3.140.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.78

$$\int d^x x^3 \sin(x) dx = \frac{(x^3 \cos(x) \log(d)^6 - 6x^2 \cos(x) \log(d)^5 + 3(x^3 + 6x) \cos(x) \log(d)^4 - 12(x^2 + 2) \cos(x) \log(d)^3 + 3(x^3 - 6x) \cos(x) \log(d)^2 - 6(x^2 - 4) \cos(x) \log(d) + (x^3 - 6x) \cos(x) - (x^3 \log(d)^7 - 3x^2 \log(d)^6 + 3(x^3 + 2x) \log(d)^5 - 3(x^2 + 2) \log(d)^4 + 3(x^3 - 4x) \log(d)^3 + 3(x^2 + 12) \log(d)^2 + 3x^2 + (x^3 - 18x) \log(d) - 6) \sin(x)) d^x}{\log(d)^8 + 4 \log(d)^6 + 6 \log(d)^4 + 4 \log(d)^2 + 1}$$

input `integrate(d^x*x^3*sin(x),x, algorithm="fricas")`

output `-(x^3*cos(x)*log(d)^6 - 6*x^2*cos(x)*log(d)^5 + 3*(x^3 + 6*x)*cos(x)*log(d)^4 - 12*(x^2 + 2)*cos(x)*log(d)^3 + 3*(x^3 + 4*x)*cos(x)*log(d)^2 - 6*(x^2 - 4)*cos(x)*log(d) + (x^3 - 6*x)*cos(x) - (x^3*log(d)^7 - 3*x^2*log(d)^6 + 3*(x^3 + 2*x)*log(d)^5 - 3*(x^2 + 2)*log(d)^4 + 3*(x^3 - 4*x)*log(d)^3 + 3*(x^2 + 12)*log(d)^2 + 3*x^2 + (x^3 - 18*x)*log(d) - 6)*sin(x))*d^x/(log(d)^8 + 4*log(d)^6 + 6*log(d)^4 + 4*log(d)^2 + 1)`

**3.140.6 Sympy [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 1355, normalized size of antiderivative = 5.19

$$\int d^x x^3 \sin(x) dx = \text{Too large to display}$$

input `integrate(d**x*x**3*sin(x),x)`

output `Piecewise((x**4*exp(-I*x)*sin(x)/8 - I*x**4*exp(-I*x)*cos(x)/8 + I*x**3*exp(-I*x)*sin(x)/4 - x**3*exp(-I*x)*cos(x)/4 + 3*x**2*exp(-I*x)*sin(x)/8 + 3*I*x**2*exp(-I*x)*cos(x)/8 - 3*I*x*exp(-I*x)*sin(x)/8 + 3*x*exp(-I*x)*cos(x)/8 - 3*I*exp(-I*x)*cos(x)/8, Eq(d, exp(-I))), (x**4*exp(I*x)*sin(x)/8 + I*x**4*exp(I*x)*cos(x)/8 - I*x**3*exp(I*x)*sin(x)/4 - x**3*exp(I*x)*cos(x)/4 + 3*x**2*exp(I*x)*sin(x)/8 - 3*I*x**2*exp(I*x)*cos(x)/8 + 3*I*x*exp(I*x)*sin(x)/8 + 3*x*exp(I*x)*cos(x)/8 + 3*I*exp(I*x)*cos(x)/8, Eq(d, exp(I))), (d**x*x**3*log(d)**7*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - d**x*x**3*log(d)**6*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**5*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**3*log(d)**4*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**3*log(d)**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + d**x*x**3*log(d)*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - d**x*x**3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**6*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 6*d**x*x**2*log(d)**5*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**4*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) ...`

**3.140.7 Maxima [A] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.71

$$\int d^x x^3 \sin(x) dx = \frac{((\log(d))^6 + 3 \log(d)^4 + 3 \log(d)^2 + 1)x^3 - 6(\log(d))^5 + 2 \log(d)^3 + \log(d)x^2 - 24 \log(d)^3 + 6(3 \log(d)^2 + 1)x - 6 \log(d)^3}{(\log(d))^8 + 4 \log(d)^6 + 6 \log(d)^4 + 4 \log(d)^2 + 1}$$

input `integrate(d^x*x^3*sin(x),x, algorithm="maxima")`

output 
$$-\left(\left(\log(d)^6 + 3\log(d)^4 + 3\log(d)^2 + 1\right)x^3 - 6\left(\log(d)^5 + 2\log(d)^3 + \log(d)\right)x^2 - 24\log(d)^3 + 6\left(3\log(d)^4 + 2\log(d)^2 - 1\right)x + 24\log(d)\right)d^x\cos(x) - \left(\left(\log(d)^7 + 3\log(d)^5 + 3\log(d)^3 + \log(d)\right)x^3 - 6\log(d)^4 - 3\left(\log(d)^6 + \log(d)^4 - \log(d)^2 - 1\right)x^2 + 6\left(\log(d)^5 - 2\log(d)^3 - 3\log(d)\right)x + 36\log(d)^2 - 6\right)d^x\sin(x)\right)/\left(\log(d)^8 + 4\log(d)^6 + 6\log(d)^4 + 4\log(d)^2 + 1\right)$$

### 3.140.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 5069, normalized size of antiderivative = 19.42

$$\int d^x x^3 \sin(x) dx = \text{Too large to display}$$

input `integrate(d^x*x^3*sin(x),x, algorithm="giac")`

output 
$$\frac{1}{2}\left(\left(4\pi + \pi^4\operatorname{sgn}(d) - 6\pi^2\log(\operatorname{abs}(d))^2\operatorname{sgn}(d) - \pi^4 + 6\pi^2\log(\operatorname{abs}(d))^2 - 2\log(\operatorname{abs}(d))^4 - 4\pi^3\operatorname{sgn}(d) + 12\pi\log(\operatorname{abs}(d))^2\operatorname{sgn}(d) + 4\pi^3 - 12\pi\log(\operatorname{abs}(d))^2 + 6\pi^2\operatorname{sgn}(d) - 6\pi^2 + 12\log(\operatorname{abs}(d))^2 - 4\pi\operatorname{sgn}(d) - 2\right)\left(\pi^3x^3\operatorname{sgn}(d) - 3\pi x^3\log(\operatorname{abs}(d))^2\operatorname{sgn}(d) - \pi^3x^3 + 3\pi x^3\log(\operatorname{abs}(d))^2 - 3\pi^2x^3\operatorname{sgn}(d) + 3\pi^2x^3 - 6x^3\log(\operatorname{abs}(d))^2 + 3\pi x^3\operatorname{sgn}(d) + 6\pi x^2\log(\operatorname{abs}(d))\operatorname{sgn}(d) - 3\pi x^3 - 6\pi x^2\log(\operatorname{abs}(d)) + 2x^3 + 12x^2\log(\operatorname{abs}(d)) - 6\pi x\operatorname{sgn}(d) + 6\pi x - 12x\right)\right)/\left(\left(4\pi + \pi^4\operatorname{sgn}(d) - 6\pi^2\log(\operatorname{abs}(d))^2\operatorname{sgn}(d) - \pi^4 + 6\pi^2\log(\operatorname{abs}(d))^2 - 2\log(\operatorname{abs}(d))^4 - 4\pi^3\operatorname{sgn}(d) + 12\pi\log(\operatorname{abs}(d))^2\operatorname{sgn}(d) + 4\pi^3 - 12\pi\log(\operatorname{abs}(d))^2 + 6\pi^2\operatorname{sgn}(d) - 6\pi^2 + 12\log(\operatorname{abs}(d))^2 - 4\pi\operatorname{sgn}(d) - 2\right)^2 + 16\left(\pi^3\log(\operatorname{abs}(d))\operatorname{sgn}(d) - \pi\log(\operatorname{abs}(d))^3\operatorname{sgn}(d) - \pi^3\log(\operatorname{abs}(d)) + \pi\log(\operatorname{abs}(d))^3 - 3\pi^2\log(\operatorname{abs}(d))\operatorname{sgn}(d) + 3\pi^2\log(\operatorname{abs}(d)) - 2\log(\operatorname{abs}(d))^3 + 3\pi\log(\operatorname{abs}(d))\operatorname{sgn}(d) - 3\pi\log(\operatorname{abs}(d)) + 2\log(\operatorname{abs}(d))^2\right) + 4\left(3\pi^2x^3\log(\operatorname{abs}(d))\operatorname{sgn}(d) - 3\pi^2x^3\log(\operatorname{abs}(d)) + 2x^3\log(\operatorname{abs}(d))^3 - 6\pi x^3\log(\operatorname{abs}(d))\operatorname{sgn}(d) + 6\pi x^3\log(\operatorname{abs}(d)) - 3\pi^2x^2\operatorname{sgn}(d) + 3\pi^2x^2 - 6x^3\log(\operatorname{abs}(d)) - 6x^2\log(\operatorname{abs}(d))^2 + 6\pi x^2\operatorname{sgn}(d) - 6\pi x^2 + 6x^2 + 12x\log(\operatorname{abs}(d)) - 12\right)\left(\pi^3\log(\operatorname{abs}(d))\operatorname{sgn}(d) - \pi\log(\operatorname{abs}(d))^3\operatorname{sgn}(d) - \pi^3\log(\operatorname{abs}(d)) + \pi\log(\operatorname{abs}(d))^3 - 3\pi^2\log(\operatorname{abs}(d))\operatorname{sgn}(d) + 3\pi^2\log(\operatorname{abs}(d)) - 2\log(\operatorname{abs}(d))^3 + 3\pi\log(\operatorname{abs}(d))\operatorname{sgn}(d) - 3\pi\log(\operatorname{abs}(d)) + 2\log(\operatorname{abs}...\right)$$

**3.140.9 Mupad [B] (verification not implemented)**

Time = 0.61 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.89

$$\int d^x x^3 \sin(x) dx =$$

$$\frac{d^x (6 \sin(x) + x^3 \cos(x) - 3x^2 \sin(x) - 6x \cos(x)) - d^x \ln(d)^5 (6x^2 \cos(x) + 3x^3 \sin(x) + 6x \sin(x))}{(\log(d)^2 + 1)^4}$$

input `int(d^x*x^3*sin(x),x)`

output

$$\frac{-(d^x*(6*\sin(x) + x^3*\cos(x) - 3*x^2*\sin(x) - 6*x*\cos(x)) - d^x*\log(d)^5*(6*x^2*\cos(x) + 3*x^3*\sin(x) + 6*x*\sin(x)) + d^x*\log(d)^4*(6*\sin(x) + 3*x^3*\cos(x) + 3*x^2*\sin(x) + 18*x*\cos(x)) - d^x*\log(d)^3*(24*\cos(x) + 12*x^2*\cos(x) + 3*x^3*\sin(x) - 12*x*\sin(x)) - d^x*\log(d)^2*(36*\sin(x) - 3*x^3*\cos(x) + 3*x^2*\sin(x) - 12*x*\cos(x)) + d^x*\log(d)^6*(x^3*\cos(x) + 3*x^2*\sin(x)) + d^x*\log(d)*(24*\cos(x) - 6*x^2*\cos(x) - x^3*\sin(x) + 18*x*\sin(x)) - d^x*x^3*\log(d)^7*\sin(x))}{(\log(d)^2 + 1)^4}$$

### 3.141 $\int d^x x^3 \cos(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
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| 3.141.2 Mathematica [A] (verified) . . . . .                | 822 |
| 3.141.3 Rubi [A] (verified) . . . . .                       | 822 |
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| 3.141.5 Fricas [A] (verification not implemented) . . . . . | 824 |
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| 3.141.7 Maxima [A] (verification not implemented) . . . . . | 825 |
| 3.141.8 Giac [C] (verification not implemented) . . . . .   | 826 |
| 3.141.9 Mupad [B] (verification not implemented) . . . . .  | 827 |

#### 3.141.1 Optimal result

Integrand size = 9, antiderivative size = 260

$$\int d^x x^3 \cos(x) dx = -\frac{6d^x \cos(x)}{(1 + \log^2(d))^4} + \frac{36d^x \cos(x) \log^2(d)}{(1 + \log^2(d))^4} - \frac{6d^x \cos(x) \log^4(d)}{(1 + \log^2(d))^4}$$

$$- \frac{18d^x x \cos(x) \log(d)}{(1 + \log^2(d))^3} + \frac{6d^x x \cos(x) \log^3(d)}{(1 + \log^2(d))^3}$$

$$+ \frac{3d^x x^2 \cos(x)}{(1 + \log^2(d))^2} - \frac{3d^x x^2 \cos(x) \log^2(d)}{(1 + \log^2(d))^2} + \frac{d^x x^3 \cos(x) \log(d)}{1 + \log^2(d)}$$

$$+ \frac{24d^x \log(d) \sin(x)}{(1 + \log^2(d))^4} - \frac{24d^x \log^3(d) \sin(x)}{(1 + \log^2(d))^4} - \frac{6d^x x \sin(x)}{(1 + \log^2(d))^3}$$

$$+ \frac{18d^x x \log^2(d) \sin(x)}{(1 + \log^2(d))^3} - \frac{6d^x x^2 \log(d) \sin(x)}{(1 + \log^2(d))^2} + \frac{d^x x^3 \sin(x)}{1 + \log^2(d)}$$

```
output -6*d^x*cos(x)/(1+ln(d)^2)^4+36*d^x*cos(x)*ln(d)^2/(1+ln(d)^2)^4-6*d^x*cos(x)*ln(d)^4/(1+ln(d)^2)^4-18*d^x*x*cos(x)*ln(d)/(1+ln(d)^2)^3+6*d^x*x*cos(x)*ln(d)^3/(1+ln(d)^2)^3+3*d^x*x^2*cos(x)/(1+ln(d)^2)^2-3*d^x*x^2*cos(x)*ln(d)^2/(1+ln(d)^2)^2+d^x*x^3*cos(x)*ln(d)/(1+ln(d)^2)+24*d^x*ln(d)*sin(x)/(1+ln(d)^2)^4-24*d^x*ln(d)^3*sin(x)/(1+ln(d)^2)^4-6*d^x*x*sin(x)/(1+ln(d)^2)^3+18*d^x*x*ln(d)^2*sin(x)/(1+ln(d)^2)^3-6*d^x*x^2*ln(d)*sin(x)/(1+ln(d)^2)^2+d^x*x^3*sin(x)/(1+ln(d)^2)
```

**3.141.2 Mathematica [A] (verified)**

Time = 0.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.65

$$\int d^x x^3 \cos(x) dx$$

$$= \frac{d^x (\cos(x) (3(-2 + x^2) + x(-18 + x^2) \log(d) + 3(12 + x^2) \log^2(d) + 3x(-4 + x^2) \log^3(d) - 3(2 + x^2) \log^4(d))}{1 + \log(d)^2}$$

input `Integrate[d^x*x^3*Cos[x],x]`

output `(d^x*(Cos[x]*(3*(-2 + x^2) + x*(-18 + x^2)*Log[d] + 3*(12 + x^2)*Log[d]^2 + 3*x*(-4 + x^2)*Log[d]^3 - 3*(2 + x^2)*Log[d]^4 + 3*x*(2 + x^2)*Log[d]^5 - 3*x^2*Log[d]^6 + x^3*Log[d]^7) + (x*(-6 + x^2) - 6*(-4 + x^2)*Log[d] + 3*x*(4 + x^2)*Log[d]^2 - 12*(2 + x^2)*Log[d]^3 + 3*x*(6 + x^2)*Log[d]^4 - 6*x^2*Log[d]^5 + x^3*Log[d]^6)*Sin[x]))/(1 + Log[d]^2)^4`

**3.141.3 Rubi [A] (verified)**

Time = 0.62 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {4969, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 d^x \cos(x) dx$$

$$\downarrow 4969$$

$$-3 \int x^2 \left( \frac{\sin(x) d^x}{\log^2(d) + 1} + \frac{\cos(x) \log(d) d^x}{\log^2(d) + 1} \right) dx + \frac{x^3 d^x \sin(x)}{\log^2(d) + 1} + \frac{x^3 d^x \log(d) \cos(x)}{\log^2(d) + 1}$$

$$\downarrow 2010$$

$$-3 \int \left( \frac{x^2 \sin(x) d^x}{\log^2(d) + 1} + \frac{x^2 \cos(x) \log(d) d^x}{\log^2(d) + 1} \right) dx + \frac{x^3 d^x \sin(x)}{\log^2(d) + 1} + \frac{x^3 d^x \log(d) \cos(x)}{\log^2(d) + 1}$$

$$\downarrow 2009$$

$$3 \left( \frac{2x^2 d^x \log(d) \sin(x)}{(\log^2(d) + 1)^2} + \frac{x^2 d^x \log^2(d) \cos(x)}{(\log^2(d) + 1)^2} - \frac{x^2 d^x \cos(x)}{(\log^2(d) + 1)^2} - \frac{6x d^x \log^2(d) \sin(x)}{(\log^2(d) + 1)^3} + \frac{2x d^x \sin(x)}{(\log^2(d) + 1)^3} - \frac{8d^x \log(d)}{(\log^2(d) + 1)^3} \right) + \frac{x^3 d^x \sin(x)}{\log^2(d) + 1} + \frac{x^3 d^x \log(d) \cos(x)}{\log^2(d) + 1} -$$

input `Int[d^x*x^3*Cos[x],x]`

output `(d^x*x^3*Cos[x]*Log[d])/(1 + Log[d]^2) + (d^x*x^3*Sin[x])/(1 + Log[d]^2) - 3*((2*d^x*Cos[x])/(1 + Log[d]^2)^4 - (12*d^x*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^4 + (2*d^x*Cos[x]*Log[d]^4)/(1 + Log[d]^2)^4 + (6*d^x*x*Cos[x]*Log[d])/(1 + Log[d]^2)^3 - (2*d^x*x*Cos[x]*Log[d]^3)/(1 + Log[d]^2)^3 - (d^x*x^2*Cos[x])/(1 + Log[d]^2)^2 + (d^x*x^2*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^2 - (8*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^4 + (8*d^x*Log[d]^3*Sin[x])/(1 + Log[d]^2)^4 + (2*d^x*x*Sin[x])/(1 + Log[d]^2)^3 - (6*d^x*x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^3 + (2*d^x*x^2*Log[d]*Sin[x])/(1 + Log[d]^2)^2)`

### 3.141.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

rule 4969 `Int[Cos[(d_) + (e_)*(x_)]^(n_)*(F_)^((c_)*((a_) + (b_)*(x_)))*((f_)*(x_))^(m_), x_Symbol] := Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^n, x]}, Simp[(f*x)^m u, x] - Simp[f*m Int[(f*x)^(m - 1)*u, x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]`



### 3.141.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.63

| method        | result                                                                                                                                                                                                                                                                                                                                                                                                                |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch         | $\frac{(-6 + \ln(d)^3 x^3 + 3i \ln(d)^2 x^3 - 3 \ln(d) x^3 - i x^3 + 6x \ln(d) + 6ix - 3 \ln(d)^2 x^2 - 6i \ln(d) x^2 + 3x^2) d^x e^{ix}}{2(\ln(d) + i)^4} + \frac{(-6 + 6x \ln(d) - 6ix - 3 \ln(d)^2 x^2 + 3x^2)}{2(\ln(d) + i)^4}$                                                                                                                                                                                  |
| parallelrisch | $d^x \left( \ln(d)^7 x^3 \cos(x) + x^2 (x \sin(x) - 3 \cos(x)) \ln(d)^6 + 3((x^3 + 2x) \cos(x) - 2x^2 \sin(x)) \ln(d)^5 + 3(x^3 \sin(x) - x^2 \cos(x) + 6x \sin(x) - 3x^2 \cos(x)) \ln(d)^4 \right)$                                                                                                                                                                                                                  |
| norman        | $\frac{\ln(d) x^3 e^{x \ln(d)}}{1 + \ln(d)^2} + \frac{2x^3 e^{x \ln(d)} \tan\left(\frac{x}{2}\right)}{1 + \ln(d)^2} - \frac{3(\ln(d)^2 - 1) x^2 e^{x \ln(d)}}{\ln(d)^4 + 2 \ln(d)^2 + 1} - \frac{6(\ln(d)^4 - 6 \ln(d)^2 + 1) e^{x \ln(d)}}{(\ln(d)^6 + 3 \ln(d)^4 + 3 \ln(d)^2 + 1)(1 + \ln(d)^2)} + \frac{3(\ln(d)^2 - 1) x^2 e^{x \ln(d)} \left(\tan\left(\frac{x}{2}\right)\right)^2}{\ln(d)^4 + 2 \ln(d)^2 + 1}$ |

input `int(d^x*x^3*cos(x),x,method=_RETURNVERBOSE)`

output `1/2*(-6+ln(d)^3*x^3+3*I*ln(d)^2*x^3-3*ln(d)*x^3-I*x^3+6*x*ln(d)+6*I*x-3*ln(d)^2*x^2-6*I*ln(d)*x^2+3*x^2)*d^x/(ln(d)+I)^4*exp(I*x)+1/2*(-6+6*x*ln(d)-6*I*x-3*ln(d)^2*x^2+6*I*ln(d)*x^2+3*x^2+ln(d)^3*x^3-3*I*ln(d)^2*x^3-3*ln(d)*x^3+I*x^3)*d^x/(ln(d)-I)^4*exp(-I*x)`

### 3.141.5 Fracas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.78

$$\int d^x x^3 \cos(x) dx$$


---


$$= (x^3 \cos(x) \log(d)^7 - 3x^2 \cos(x) \log(d)^6 + 3(x^3 + 2x) \cos(x) \log(d)^5 - 3(x^2 + 2) \cos(x) \log(d)^4 + 3(x^3 \sin(x) \log(d)^6 - 6x^2 \sin(x) \log(d)^5 + 3(x^3 + 6x) \sin(x) \log(d)^4 - 12(x^2 + 2) \sin(x) \log(d)^3 + x^3 + 3(x^3 + 4x) \sin(x) \log(d)^2 - 6(x^2 - 4) \sin(x) \log(d) - 6x \sin(x)) d^x / (\log(d)^8 + 4 \log(d)^6 + 6 \log(d)^4 + 4 \log(d)^2 + 1)$$

input `integrate(d^x*x^3*cos(x),x, algorithm="fricas")`

output `(x^3*cos(x)*log(d)^7 - 3*x^2*cos(x)*log(d)^6 + 3*(x^3 + 2*x)*cos(x)*log(d)^5 - 3*(x^2 + 2)*cos(x)*log(d)^4 + 3*(x^3 - 4*x)*cos(x)*log(d)^3 + 3*(x^2 + 12)*cos(x)*log(d)^2 + (x^3 - 18*x)*cos(x)*log(d) + 3*(x^2 - 2)*cos(x) + (x^3*log(d)^6 - 6*x^2*log(d)^5 + 3*(x^3 + 6*x)*log(d)^4 - 12*(x^2 + 2)*log(d)^3 + x^3 + 3*(x^3 + 4*x)*log(d)^2 - 6*(x^2 - 4)*log(d) - 6*x)*sin(x))*d^x/(log(d)^8 + 4*log(d)^6 + 6*log(d)^4 + 4*log(d)^2 + 1)`

**3.141.6 Sympy [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 2.22 (sec) , antiderivative size = 1352, normalized size of antiderivative = 5.20

$$\int d^x x^3 \cos(x) dx = \text{Too large to display}$$

input `integrate(d**x*x**3*cos(x),x)`

output `Piecewise((I*x**4*exp(-I*x)*sin(x)/8 + x**4*exp(-I*x)*cos(x)/8 + x**3*exp(-I*x)*sin(x)/4 + I*x**3*exp(-I*x)*cos(x)/4 - 3*I*x**2*exp(-I*x)*sin(x)/8 + 3*x**2*exp(-I*x)*cos(x)/8 - 3*x*exp(-I*x)*sin(x)/8 - 3*I*x*exp(-I*x)*cos(x)/8 - 3*exp(-I*x)*cos(x)/8, Eq(d, exp(-I))), (-I*x**4*exp(I*x)*sin(x)/8 + x**4*exp(I*x)*cos(x)/8 + x**3*exp(I*x)*sin(x)/4 - I*x**3*exp(I*x)*cos(x)/4 + 3*I*x**2*exp(I*x)*sin(x)/8 + 3*x**2*exp(I*x)*cos(x)/8 - 3*x*exp(I*x)*sin(x)/8 + 3*I*x*exp(I*x)*cos(x)/8 - 3*exp(I*x)*cos(x)/8, Eq(d, exp(I))), (d**x*x**3*log(d)**7*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + d**x*x**3*log(d)**6*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**5*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**4*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + d**x*x**3*log(d)*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + d**x*x**3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**6*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*x**2*log(d)**5*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**4*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 1...`

**3.141.7 Maxima [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.71

$$\int d^x x^3 \cos(x) dx$$

$$= \frac{((\log(d))^7 + 3 \log(d)^5 + 3 \log(d)^3 + \log(d))x^3 - 6 \log(d)^4 - 3(\log(d)^6 + \log(d)^4 - \log(d)^2 - 1)x^2 + 6 \log(d)^5 - 6 \log(d)^3 - 3 \log(d)^2 - 3 \log(d) - 3}{(\log(d))^8 + 4 \log(d)^6 + 6 \log(d)^4 + 4 \log(d)^2 + 1}$$

```
input integrate(d^x*x^3*cos(x),x, algorithm="maxima")
```

```
output (((log(d)^7 + 3*log(d)^5 + 3*log(d)^3 + log(d))*x^3 - 6*log(d)^4 - 3*(log(d)^6 + log(d)^4 - log(d)^2 - 1)*x^2 + 6*(log(d)^5 - 2*log(d)^3 - 3*log(d))*x + 36*log(d)^2 - 6)*d^x*cos(x) + ((log(d)^6 + 3*log(d)^4 + 3*log(d)^2 + 1)*x^3 - 6*(log(d)^5 + 2*log(d)^3 + log(d))*x^2 - 24*log(d)^3 + 6*(3*log(d))^4 + 2*log(d)^2 - 1)*x + 24*log(d))*d^x*sin(x))/(log(d)^8 + 4*log(d)^6 + 6*log(d)^4 + 4*log(d)^2 + 1)
```

### 3.141.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 5065, normalized size of antiderivative = 19.48

$$\int d^x x^3 \cos(x) dx = \text{Too large to display}$$

```
input integrate(d^x*x^3*cos(x),x, algorithm="giac")
```

```
output -1/2*(((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)*(3*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi^2*x^3*log(abs(d)) + 2*x^3*log(abs(d))^3 - 6*pi*x^3*log(abs(d))*sgn(d) + 6*pi*x^3*log(abs(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*log(abs(d)) - 6*x^2*log(abs(d))^2 + 6*pi*x^2*sgn(d) - 6*pi*x^2 + 6*x^2 + 12*x*log(abs(d)) - 12)/((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d))^2) - 4*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(abs(d))^2*sgn(d) - pi^3*x^3 + 3*pi*x^3*log(abs(d))^2 - 3*pi^2*x^3*sgn(d) + 3*pi^2*x^3 - 6*x^3*log(abs(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*pi*x^3 - 6*pi*x^2*log(abs(d)) + 2*x^3 + 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x - 12*x)*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(ab...
```

**3.141.9 Mupad [B] (verification not implemented)**

Time = 0.62 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.89

$$\int d^x x^3 \cos(x) dx =$$

$$\frac{d^x (6 \cos(x) - 3x^2 \cos(x) - x^3 \sin(x) + 6x \sin(x)) - d^x \ln(d)^5 (3x^3 \cos(x) - 6x^2 \sin(x) + 6x \cos(x))}{(\log(d)^2 + 1)^4}$$

input `int(d^x*x^3*cos(x),x)`

output

```

-(d^x*(6*cos(x) - 3*x^2*cos(x) - x^3*sin(x) + 6*x*sin(x)) - d^x*log(d)^5*(
3*x^3*cos(x) - 6*x^2*sin(x) + 6*x*cos(x)) + d^x*log(d)^4*(6*cos(x) + 3*x^2
*cos(x) - 3*x^3*sin(x) - 18*x*sin(x)) + d^x*log(d)^3*(24*sin(x) - 3*x^3*co
s(x) + 12*x^2*sin(x) + 12*x*cos(x)) - d^x*log(d)^2*(36*cos(x) + 3*x^2*cos(
x) + 3*x^3*sin(x) + 12*x*sin(x)) + d^x*log(d)^6*(3*x^2*cos(x) - x^3*sin(x)
) - d^x*log(d)*(24*sin(x) + x^3*cos(x) - 6*x^2*sin(x) - 18*x*cos(x)) - d^x
*x^3*log(d)^7*cos(x))/(log(d)^2 + 1)^4

```

### 3.142 $\int \sin(x) \sin(2x) \sin(3x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.142.1 Optimal result . . . . .                            | 828 |
| 3.142.2 Mathematica [A] (verified) . . . . .                | 828 |
| 3.142.3 Rubi [A] (verified) . . . . .                       | 829 |
| 3.142.4 Maple [A] (verified) . . . . .                      | 830 |
| 3.142.5 Fricas [A] (verification not implemented) . . . . . | 830 |
| 3.142.6 Sympy [B] (verification not implemented) . . . . .  | 830 |
| 3.142.7 Maxima [A] (verification not implemented) . . . . . | 831 |
| 3.142.8 Giac [A] (verification not implemented) . . . . .   | 831 |
| 3.142.9 Mupad [B] (verification not implemented) . . . . .  | 831 |

#### 3.142.1 Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \sin(x) \sin(2x) \sin(3x) dx = -\frac{1}{8} \cos(2x) - \frac{1}{16} \cos(4x) + \frac{1}{24} \cos(6x)$$

output `-1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)`

#### 3.142.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sin(x) \sin(2x) \sin(3x) dx = -\frac{1}{8} \cos(2x) - \frac{1}{16} \cos(4x) + \frac{1}{24} \cos(6x)$$

input `Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]`

output `-1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24`

### 3.142.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {3042, 4855, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c}
 \int \sin(x) \sin(2x) \sin(3x) dx \\
 \downarrow \text{3042} \\
 \int \sin(x) \sin(2x) \sin(3x) dx \\
 \downarrow \text{4855} \\
 \int \left( \frac{1}{4} \sin(2x) + \frac{1}{4} \sin(4x) - \frac{1}{4} \sin(6x) \right) dx \\
 \downarrow \text{2009} \\
 -\frac{1}{8} \cos(2x) - \frac{1}{16} \cos(4x) + \frac{1}{24} \cos(6x)
 \end{array}$$

input `Int[Sin[x]*Sin[2*x]*Sin[3*x],x]`

output `-1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24`

#### 3.142.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4855 `Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.) + (f_.)*(x_)]^(r_.), x_Symbol] := Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H, sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]`

**3.142.4 Maple [A] (verified)**

Time = 1.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

| method        | result                                                                           | size |
|---------------|----------------------------------------------------------------------------------|------|
| default       | $-\frac{\cos(2x)}{8} - \frac{\cos(4x)}{16} + \frac{\cos(6x)}{24}$                | 20   |
| risch         | $-\frac{\cos(2x)}{8} - \frac{\cos(4x)}{16} + \frac{\cos(6x)}{24}$                | 20   |
| parallelrisch | $-\frac{3}{16} + \frac{\cos(6x)}{24} - \frac{\cos(4x)}{16} - \frac{\cos(2x)}{8}$ | 21   |

input `int(sin(x)*sin(2*x)*sin(3*x),x,method=_RETURNVERBOSE)`output `-1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)`**3.142.5 Fracas [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \sin(x) \sin(2x) \sin(3x) dx = \frac{4}{3} \cos(x)^6 - \frac{5}{2} \cos(x)^4 + \cos(x)^2$$

input `integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")`output `4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2`**3.142.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(19) = 38.

Time = 1.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.56

$$\int \sin(x) \sin(2x) \sin(3x) dx = \frac{x \sin(x) \sin(2x) \sin(3x)}{4} + \frac{x \sin(x) \cos(2x) \cos(3x)}{4} + \frac{x \sin(2x) \cos(x) \cos(3x)}{4} - \frac{x \sin(3x) \cos(x) \cos(2x)}{4} - \frac{5 \sin(x) \sin(2x) \cos(3x)}{24} - \frac{\sin(2x) \sin(3x) \cos(x)}{8} - \frac{\cos(x) \cos(2x) \cos(3x)}{6}$$

input `integrate(sin(x)*sin(2*x)*sin(3*x),x)`

output `x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*cos(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - 5*sin(x)*sin(2*x)*cos(3*x)/24 - sin(2*x)*sin(3*x)*cos(x)/8 - cos(x)*cos(2*x)*cos(3*x)/6`

### 3.142.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \sin(x) \sin(2x) \sin(3x) dx = \frac{1}{24} \cos(6x) - \frac{1}{16} \cos(4x) - \frac{1}{8} \cos(2x)$$

input `integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")`

output `1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)`

### 3.142.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \sin(x) \sin(2x) \sin(3x) dx = -\frac{4}{3} \sin(x)^6 + \frac{3}{2} \sin(x)^4$$

input `integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="giac")`

output `-4/3*sin(x)^6 + 3/2*sin(x)^4`

### 3.142.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sin(x) \sin(2x) \sin(3x) dx = -\frac{\sin(x)^4 (8 \sin(x)^2 - 9)}{6}$$

input `int(sin(2*x)*sin(3*x)*sin(x),x)`

output `-(sin(x)^4*(8*sin(x)^2 - 9))/6`



### 3.143 $\int \cos(x) \cos(2x) \cos(3x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.143.1 Optimal result . . . . .                            | 832 |
| 3.143.2 Mathematica [A] (verified) . . . . .                | 832 |
| 3.143.3 Rubi [A] (verified) . . . . .                       | 833 |
| 3.143.4 Maple [A] (verified) . . . . .                      | 834 |
| 3.143.5 Fricas [A] (verification not implemented) . . . . . | 834 |
| 3.143.6 Sympy [B] (verification not implemented) . . . . .  | 834 |
| 3.143.7 Maxima [A] (verification not implemented) . . . . . | 835 |
| 3.143.8 Giac [A] (verification not implemented) . . . . .   | 835 |
| 3.143.9 Mupad [B] (verification not implemented) . . . . .  | 835 |

#### 3.143.1 Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \cos(x) \cos(2x) \cos(3x) dx = \frac{x}{4} + \frac{1}{8} \sin(2x) + \frac{1}{16} \sin(4x) + \frac{1}{24} \sin(6x)$$

output `1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)`

#### 3.143.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \cos(x) \cos(2x) \cos(3x) dx = \frac{x}{4} + \frac{1}{8} \sin(2x) + \frac{1}{16} \sin(4x) + \frac{1}{24} \sin(6x)$$

input `Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]`

output `x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24`

**3.143.3 Rubi [A] (verified)**

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {3042, 4855, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x) \cos(2x) \cos(3x) dx$$

$$\downarrow \text{3042}$$

$$\int \cos(x) \cos(2x) \cos(3x) dx$$

$$\downarrow \text{4855}$$

$$\int \left( \frac{1}{4} \cos(2x) + \frac{1}{4} \cos(4x) + \frac{1}{4} \cos(6x) + \frac{1}{4} \right) dx$$

$$\downarrow \text{2009}$$

$$\frac{x}{4} + \frac{1}{8} \sin(2x) + \frac{1}{16} \sin(4x) + \frac{1}{24} \sin(6x)$$

input `Int[Cos[x]*Cos[2*x]*Cos[3*x],x]`

output `x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24`

**3.143.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4855 `Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.) + (f_.)*(x_)]^(r_.), x_Symbol] := Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H, sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]`

**3.143.4 Maple [A] (verified)**

Time = 1.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

| method        | result                                                                         | size |
|---------------|--------------------------------------------------------------------------------|------|
| default       | $\frac{x}{4} + \frac{\sin(2x)}{8} + \frac{\sin(4x)}{16} + \frac{\sin(6x)}{24}$ | 23   |
| risch         | $\frac{x}{4} + \frac{\sin(2x)}{8} + \frac{\sin(4x)}{16} + \frac{\sin(6x)}{24}$ | 23   |
| parallelrisch | $\frac{x}{4} + \frac{\sin(2x)}{8} + \frac{\sin(4x)}{16} + \frac{\sin(6x)}{24}$ | 23   |

input `int(cos(x)*cos(2*x)*cos(3*x),x,method=_RETURNVERBOSE)`output `1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)`**3.143.5 Fracas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \cos(x) \cos(2x) \cos(3x) dx = \frac{1}{12} (16 \cos(x)^5 - 10 \cos(x)^3 + 3 \cos(x)) \sin(x) + \frac{1}{4} x$$

input `integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="fricas")`output `1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x`**3.143.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(22) = 44.

Time = 1.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.80

$$\begin{aligned} \int \cos(x) \cos(2x) \cos(3x) dx = & -\frac{x \sin(x) \sin(2x) \cos(3x)}{4} + \frac{x \sin(x) \sin(3x) \cos(2x)}{4} \\ & + \frac{x \sin(2x) \sin(3x) \cos(x)}{4} + \frac{x \cos(x) \cos(2x) \cos(3x)}{4} \\ & - \frac{\sin(x) \cos(2x) \cos(3x)}{24} - \frac{\sin(2x) \cos(x) \cos(3x)}{6} \\ & + \frac{3 \sin(3x) \cos(x) \cos(2x)}{8} \end{aligned}$$

input `integrate(cos(x)*cos(2*x)*cos(3*x),x)`

output `-x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*sin(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 - sin(x)*cos(2*x)*cos(3*x)/24 - sin(2*x)*cos(x)*cos(3*x)/6 + 3*sin(3*x)*cos(x)*cos(2*x)/8`

### 3.143.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x) \cos(2x) \cos(3x) dx = \frac{1}{4}x + \frac{1}{24} \sin(6x) + \frac{1}{16} \sin(4x) + \frac{1}{8} \sin(2x)$$

input `integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="maxima")`

output `1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)`

### 3.143.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x) \cos(2x) \cos(3x) dx = \frac{1}{4}x + \frac{1}{24} \sin(6x) + \frac{1}{16} \sin(4x) + \frac{1}{8} \sin(2x)$$

input `integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="giac")`

output `1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)`

### 3.143.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x) \cos(2x) \cos(3x) dx = \frac{x}{4} + \frac{\sin(2x)}{8} + \frac{\sin(4x)}{16} + \frac{\sin(6x)}{24}$$

input `int(cos(2*x)*cos(3*x)*cos(x),x)`

output `x/4 + sin(2*x)/8 + sin(4*x)/16 + sin(6*x)/24`

### 3.144 $\int x^2 \sin^3(kx) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.144.1 Optimal result . . . . .                            | 836 |
| 3.144.2 Mathematica [A] (verified) . . . . .                | 836 |
| 3.144.3 Rubi [A] (verified) . . . . .                       | 837 |
| 3.144.4 Maple [A] (verified) . . . . .                      | 839 |
| 3.144.5 Fricas [A] (verification not implemented) . . . . . | 840 |
| 3.144.6 Sympy [A] (verification not implemented) . . . . .  | 840 |
| 3.144.7 Maxima [A] (verification not implemented) . . . . . | 841 |
| 3.144.8 Giac [A] (verification not implemented) . . . . .   | 841 |
| 3.144.9 Mupad [B] (verification not implemented) . . . . .  | 841 |

#### 3.144.1 Optimal result

Integrand size = 10, antiderivative size = 85

$$\int x^2 \sin^3(kx) dx = \frac{14 \cos(kx)}{9k^3} - \frac{2x^2 \cos(kx)}{3k} - \frac{2 \cos^3(kx)}{27k^3} + \frac{4x \sin(kx)}{3k^2} - \frac{x^2 \cos(kx) \sin^2(kx)}{3k} + \frac{2x \sin^3(kx)}{9k^2}$$

output  $14/9*\cos(k*x)/k^3-2/3*x^2*\cos(k*x)/k-2/27*\cos(k*x)^3/k^3+4/3*x*\sin(k*x)/k^2-1/3*x^2*\cos(k*x)*\sin(k*x)^2/k+2/9*x*\sin(k*x)^3/k^2$

#### 3.144.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

$$\int x^2 \sin^3(kx) dx = \frac{-81(-2 + k^2 x^2) \cos(kx) + (-2 + 9k^2 x^2) \cos(3kx) - 6kx(-27 \sin(kx) + \sin(3kx))}{108k^3}$$

input `Integrate[x^2*Sin[k*x]^3,x]`

output  $(-81*(-2 + k^2*x^2)*Cos[k*x] + (-2 + 9*k^2*x^2)*Cos[3*k*x] - 6*k*x*(-27*Sin[k*x] + Sin[3*k*x]))/(108*k^3)$

**3.144.3 Rubi [A] (verified)**

Time = 0.47 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$ , Rules used = {3042, 3792, 3042, 3113, 2009, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^2 \sin^3(kx) dx \\
 & \quad \downarrow \text{3042} \\
 & \int x^2 \sin(kx)^3 dx \\
 & \quad \downarrow \text{3792} \\
 & -\frac{2 \int \sin^3(kx) dx}{9k^2} + \frac{2}{3} \int x^2 \sin(kx) dx + \frac{2x \sin^3(kx)}{9k^2} - \frac{x^2 \sin^2(kx) \cos(kx)}{3k} \\
 & \quad \downarrow \text{3042} \\
 & -\frac{2 \int \sin(kx)^3 dx}{9k^2} + \frac{2}{3} \int x^2 \sin(kx) dx + \frac{2x \sin^3(kx)}{9k^2} - \frac{x^2 \sin^2(kx) \cos(kx)}{3k} \\
 & \quad \downarrow \text{3113} \\
 & \frac{2 \int (1 - \cos^2(kx)) d \cos(kx)}{9k^3} + \frac{2}{3} \int x^2 \sin(kx) dx + \frac{2x \sin^3(kx)}{9k^2} - \frac{x^2 \sin^2(kx) \cos(kx)}{3k} \\
 & \quad \downarrow \text{2009} \\
 & \frac{2}{3} \int x^2 \sin(kx) dx + \frac{2(\cos(kx) - \frac{1}{3} \cos^3(kx))}{9k^3} + \frac{2x \sin^3(kx)}{9k^2} - \frac{x^2 \sin^2(kx) \cos(kx)}{3k} \\
 & \quad \downarrow \text{3777} \\
 & \frac{2}{3} \left( \frac{2 \int x \cos(kx) dx}{k} - \frac{x^2 \cos(kx)}{k} \right) + \frac{2(\cos(kx) - \frac{1}{3} \cos^3(kx))}{9k^3} + \frac{2x \sin^3(kx)}{9k^2} - \frac{x^2 \sin^2(kx) \cos(kx)}{3k} \\
 & \quad \downarrow \text{3042} \\
 & \frac{2}{3} \left( \frac{2 \int x \sin(kx + \frac{\pi}{2}) dx}{k} - \frac{x^2 \cos(kx)}{k} \right) + \frac{2(\cos(kx) - \frac{1}{3} \cos^3(kx))}{9k^3} + \frac{2x \sin^3(kx)}{9k^2} - \\
 & \quad \frac{x^2 \sin^2(kx) \cos(kx)}{3k} \\
 & \quad \downarrow \text{3777}
 \end{aligned}$$

$$\begin{aligned}
& \frac{2}{3} \left( \frac{2 \left( \frac{\int -\sin(kx) dx}{k} + \frac{x \sin(kx)}{k} \right) - \frac{x^2 \cos(kx)}{k}}{\frac{x^2 \sin^2(kx) \cos(kx)}{3k}} + \frac{2(\cos(kx) - \frac{1}{3} \cos^3(kx))}{9k^3} + \frac{2x \sin^3(kx)}{9k^2} - \right. \\
& \qquad \qquad \qquad \downarrow \text{25} \\
& \frac{2}{3} \left( \frac{2 \left( \frac{x \sin(kx)}{k} - \frac{\int \sin(kx) dx}{k} \right) - \frac{x^2 \cos(kx)}{k}}{\frac{x^2 \sin^2(kx) \cos(kx)}{3k}} + \frac{2(\cos(kx) - \frac{1}{3} \cos^3(kx))}{9k^3} + \frac{2x \sin^3(kx)}{9k^2} - \right. \\
& \qquad \qquad \qquad \downarrow \text{3042} \\
& \frac{2}{3} \left( \frac{2 \left( \frac{x \sin(kx)}{k} - \frac{\int \sin(kx) dx}{k} \right) - \frac{x^2 \cos(kx)}{k}}{\frac{x^2 \sin^2(kx) \cos(kx)}{3k}} + \frac{2(\cos(kx) - \frac{1}{3} \cos^3(kx))}{9k^3} + \frac{2x \sin^3(kx)}{9k^2} - \right. \\
& \qquad \qquad \qquad \downarrow \text{3118} \\
& \frac{2(\cos(kx) - \frac{1}{3} \cos^3(kx))}{9k^3} + \frac{2}{3} \left( \frac{2 \left( \frac{\cos(kx)}{k^2} + \frac{x \sin(kx)}{k} \right) - \frac{x^2 \cos(kx)}{k}}{\frac{x^2 \sin^2(kx) \cos(kx)}{3k}} + \frac{2x \sin^3(kx)}{9k^2} - \right.
\end{aligned}$$

input `Int[x^2*Sin[k*x]^3,x]`

output `(2*(Cos[k*x] - Cos[k*x]^3/3))/(9*k^3) - (x^2*Cos[k*x]*Sin[k*x]^2)/(3*k) + (2*x*Sin[k*x]^3)/(9*k^2) + (2*(-((x^2*Cos[k*x])/k) + (2*(Cos[k*x]/k^2 + (x*Sin[k*x])/k))/k))/3`

### 3.144.3.1 Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 3113 `Int[sin[(c_.) + (d_.)*(x_)^(n_), x_Symbol] := Simp[-d^(-1) Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

rule 3118 `Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

rule 3777 `Int[((c_.) + (d_.)*(x_)^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[( - (c + d*x)^m*(Cos[e + f*x]/f), x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

rule 3792 `Int[((c_.) + (d_.)*(x_)^(m_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n) Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2*m*((m - 1)/(f^2*n^2)) Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]`

### 3.144.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

| method            | result                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch             | $-\frac{3(x^2k^2-2)\cos(kx)}{4k^3} + \frac{3x\sin(kx)}{2k^2} + \frac{(9x^2k^2-2)\cos(3kx)}{108k^3} - \frac{x\sin(3kx)}{18k^2}$                                                                                                                                                                                                                                                                                                                                                                                                |
| derivativedivides | $-\frac{k^2x^2(2+\sin^2(kx))\cos(kx)}{3} + \frac{4\cos(kx)}{3} + \frac{4kx\sin(kx)}{3} + \frac{2kx(\sin^3(kx))}{9} + \frac{2(2+\sin^2(kx))\cos(kx)}{27}$                                                                                                                                                                                                                                                                                                                                                                      |
| default           | $-\frac{k^2x^2(2+\sin^2(kx))\cos(kx)}{3} + \frac{4\cos(kx)}{3} + \frac{4kx\sin(kx)}{3} + \frac{2kx(\sin^3(kx))}{9} + \frac{2(2+\sin^2(kx))\cos(kx)}{27}$                                                                                                                                                                                                                                                                                                                                                                      |
| norman            | $\frac{-\frac{2x^2}{3k} + \frac{80}{27k^3} + \frac{8x\tan\left(\frac{kx}{2}\right)}{3k^2} + \frac{64x\left(\tan^3\left(\frac{kx}{2}\right)\right)}{9k^2} + \frac{8x\left(\tan^5\left(\frac{kx}{2}\right)\right)}{3k^2} - \frac{2x^2\left(\tan^2\left(\frac{kx}{2}\right)\right)}{k} + \frac{2x^2\left(\tan^4\left(\frac{kx}{2}\right)\right)}{k} + \frac{2x^2\left(\tan^6\left(\frac{kx}{2}\right)\right)}{3k} + \frac{8\left(\tan^8\left(\frac{kx}{2}\right)\right)}{3k}}{\left(1+\tan^2\left(\frac{kx}{2}\right)\right)^3}$ |

input `int(x^2*sin(k*x)^3,x,method=_RETURNVERBOSE)`



output `-3/4*(k^2*x^2-2)/k^3*cos(k*x)+3/2*x*sin(k*x)/k^2+1/108*(9*k^2*x^2-2)/k^3*cos(3*k*x)-1/18*x/k^2*sin(3*k*x)`

### 3.144.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

$$\int x^2 \sin^3(kx) dx = \frac{(9k^2x^2 - 2) \cos(kx)^3 - 3(9k^2x^2 - 14) \cos(kx) - 6(kx \cos(kx))^2 - 7kx \sin(kx)}{27k^3}$$

input `integrate(x^2*sin(k*x)^3,x, algorithm="fricas")`

output `1/27*((9*k^2*x^2 - 2)*cos(k*x)^3 - 3*(9*k^2*x^2 - 14)*cos(k*x) - 6*(k*x*cos(k*x))^2 - 7*k*x*sin(k*x))/k^3`

### 3.144.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

$$\int x^2 \sin^3(kx) dx = \begin{cases} -\frac{x^2 \sin^2(kx) \cos(kx)}{k} - \frac{2x^2 \cos^3(kx)}{3k} + \frac{14x \sin^3(kx)}{9k^2} + \frac{4x \sin(kx) \cos^2(kx)}{3k^2} + \frac{14 \sin^2(kx) \cos(kx)}{9k^3} + \frac{40 \cos^3(kx)}{27k^3} & \text{for } k \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input `integrate(x**2*sin(k*x)**3,x)`

output `Piecewise((-x**2*sin(k*x)**2*cos(k*x)/k - 2*x**2*cos(k*x)**3/(3*k) + 14*x*sin(k*x)**3/(9*k**2) + 4*x*sin(k*x)*cos(k*x)**2/(3*k**2) + 14*sin(k*x)**2*cos(k*x)/(9*k**3) + 40*cos(k*x)**3/(27*k**3), Ne(k, 0)), (0, True))`

**3.144.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

$$\int x^2 \sin^3(kx) dx = -\frac{6 kx \sin(3 kx) - 162 kx \sin(kx) - (9 k^2 x^2 - 2) \cos(3 kx) + 81 (k^2 x^2 - 2) \cos(kx)}{108 k^3}$$

input `integrate(x^2*sin(k*x)^3,x, algorithm="maxima")`output `-1/108*(6*k*x*sin(3*k*x) - 162*k*x*sin(k*x) - (9*k^2*x^2 - 2)*cos(3*k*x) + 81*(k^2*x^2 - 2)*cos(k*x))/k^3`**3.144.8 Giac [A] (verification not implemented)**

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int x^2 \sin^3(kx) dx = -\frac{x \sin(3 kx)}{18 k^2} + \frac{3 x \sin(kx)}{2 k^2} + \frac{(9 k^2 x^2 - 2) \cos(3 kx)}{108 k^3} - \frac{3 (k^2 x^2 - 2) \cos(kx)}{4 k^3}$$

input `integrate(x^2*sin(k*x)^3,x, algorithm="giac")`output `-1/18*x*sin(3*k*x)/k^2 + 3/2*x*sin(k*x)/k^2 + 1/108*(9*k^2*x^2 - 2)*cos(3*k*x)/k^3 - 3/4*(k^2*x^2 - 2)*cos(k*x)/k^3`**3.144.9 Mupad [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.79

$$\int x^2 \sin^3(kx) dx = \frac{\frac{14 \cos(kx)}{9} - \frac{2 \cos(kx)^3}{27} + k \left( \frac{14 x \sin(kx)}{9} - \frac{2 x \cos(kx)^2 \sin(kx)}{9} \right) + k^2 \left( \frac{x^2 \cos(kx)^3}{3} - x^2 \cos(kx) \right)}{k^3}$$

input `int(x^2*sin(k*x)^3,x)`output `((14*cos(k*x))/9 - (2*cos(k*x)^3)/27 + k*((14*x*sin(k*x))/9 - (2*x*cos(k*x))^2*sin(k*x))/9) + k^2*((x^2*cos(k*x)^3)/3 - x^2*cos(k*x))/k^3`

### 3.145 $\int x \cos(k \csc(x)) \cot(x) \csc(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.145.1 Optimal result . . . . .                            | 842 |
| 3.145.2 Mathematica [N/A] . . . . .                         | 842 |
| 3.145.3 Rubi [N/A] . . . . .                                | 843 |
| 3.145.4 Maple [N/A] . . . . .                               | 843 |
| 3.145.5 Fricas [N/A] . . . . .                              | 844 |
| 3.145.6 Sympy [N/A] . . . . .                               | 844 |
| 3.145.7 Maxima [C] (verification not implemented) . . . . . | 844 |
| 3.145.8 Giac [N/A] . . . . .                                | 845 |
| 3.145.9 Mupad [N/A] . . . . .                               | 845 |

#### 3.145.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) dx = \text{Int}(x \cos(k \csc(x)) \cot(x) \csc(x), x)$$

output `CannotIntegrate(x*cos(k*csc(x))*cot(x)*csc(x),x)`

#### 3.145.2 Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) dx = \int x \cos(k \csc(x)) \cot(x) \csc(x) dx$$

input `Integrate[x*Cos[k*Csc[x]]*Cot[x]*Csc[x],x]`

output `Integrate[x*Cos[k*Csc[x]]*Cot[x]*Csc[x],x]`

**3.145.3 Rubi [N/A]**

Not integrable

Time = 0.47 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cot(x) \csc(x) \cos(k \csc(x)) dx$$

↓ 7299

$$\int x \cot(x) \csc(x) \cos(k \csc(x)) dx$$

input `Int[x*Cos[k*Csc[x]]*Cot[x]*Csc[x],x]`

output `$Aborted`

**3.145.3.1 Defintions of rubi rules used**

rule 7299 `Int[u_, x_] :> CannotIntegrate[u, x]`

**3.145.4 Maple [N/A]**

Not integrable

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{x \cos(x) \cos\left(\frac{k}{\sin(x)}\right)}{\sin(x)^2} dx$$

input `int(x*cos(x)*cos(k/sin(x))/sin(x)^2,x)`

output `int(x*cos(x)*cos(k/sin(x))/sin(x)^2,x)`

**3.145.5 Fricas [N/A]**

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) dx = \int \frac{x \cos(x) \cos\left(\frac{k}{\sin(x)}\right)}{\sin(x)^2} dx$$

input `integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="fricas")`output `integral(-x*cos(x)*cos(k/sin(x))/(cos(x)^2 - 1), x)`**3.145.6 Sympy [N/A]**

Not integrable

Time = 53.39 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) dx = \int \frac{x \cos(x) \cos\left(\frac{k}{\sin(x)}\right)}{\sin^2(x)} dx$$

input `integrate(x*cos(x)*cos(k/sin(x))/sin(x)**2,x)`output `Integral(x*cos(x)*cos(k/sin(x))/sin(x)**2, x)`**3.145.7 Maxima [C] (verification not implemented)**

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.22 (sec) , antiderivative size = 240, normalized size of antiderivative = 21.82

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) dx = \frac{\left( x e^{\left( \frac{4k \cos(2x) \cos(x)}{\cos(2x)^2 + \sin(2x)^2 - 2 \cos(2x) + 1} + \frac{4k \sin(2x) \sin(x)}{\cos(2x)^2 + \sin(2x)^2 - 2 \cos(2x) + 1} \right)} + x e^{\left( \frac{4k \cos(x)}{\cos(2x)^2 + \sin(2x)^2 - 2 \cos(2x) + 1} \right)} \right) e^{\left( -\frac{2k \cos(2x) \cos(x)}{\cos(2x)^2 + \sin(2x)^2 - 2 \cos(2x) + 1} \right)}}{2k}$$

input `integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="maxima")`

output `-1/2*(x*e^(4*k*cos(2*x)*cos(x))/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) + 4*k*sin(2*x)*sin(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)) + x*e^(4*k*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1))*e^(-2*k*cos(2*x)*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) - 2*k*sin(2*x)*sin(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) - 2*k*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1))*sin(2*(k*cos(x)*sin(2*x) - k*cos(2*x)*sin(x) + k*sin(x)))/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1))/k`

### 3.145.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) dx = \int \frac{x \cos(x) \cos\left(\frac{k}{\sin(x)}\right)}{\sin(x)^2} dx$$

input `integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="giac")`

output `integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2, x)`

### 3.145.9 Mupad [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) dx = \int \frac{x \cos\left(\frac{k}{\sin(x)}\right) \cos(x)}{\sin(x)^2} dx$$

input `int((x*cos(k/sin(x))*cos(x))/sin(x)^2,x)`

output `int((x*cos(k/sin(x))*cos(x))/sin(x)^2, x)`

### 3.146 $\int \cot\left(\frac{x}{2}\right) \cot(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.146.1 Optimal result . . . . .                            | 846 |
| 3.146.2 Mathematica [A] (verified) . . . . .                | 846 |
| 3.146.3 Rubi [A] (verified) . . . . .                       | 847 |
| 3.146.4 Maple [A] (verified) . . . . .                      | 848 |
| 3.146.5 Fricas [A] (verification not implemented) . . . . . | 849 |
| 3.146.6 Sympy [F] . . . . .                                 | 849 |
| 3.146.7 Maxima [B] (verification not implemented) . . . . . | 849 |
| 3.146.8 Giac [A] (verification not implemented) . . . . .   | 850 |
| 3.146.9 Mupad [B] (verification not implemented) . . . . .  | 850 |

#### 3.146.1 Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \cot\left(\frac{x}{2}\right) \cot(x) dx = -x - \cot\left(\frac{x}{2}\right)$$

output `-x-cot(1/2*x)`

#### 3.146.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \cot\left(\frac{x}{2}\right) \cot(x) dx = -x - \cot\left(\frac{x}{2}\right)$$

input `Integrate[Cot[x/2]*Cot[x],x]`

output `-x - Cot[x/2]`

**3.146.3 Rubi [A] (verified)**

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50, number of steps used = 6, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {3042, 4889, 27, 359, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \cot\left(\frac{x}{2}\right) \cot(x) dx \\
 & \quad \downarrow \text{3042} \\
 & \int \cot\left(\frac{x}{2}\right) \cot(x) dx \\
 & \quad \downarrow \text{4889} \\
 & 2 \int \frac{\cot^2\left(\frac{x}{2}\right) (1 - \tan^2\left(\frac{x}{2}\right))}{2(\tan^2\left(\frac{x}{2}\right) + 1)} d \tan\left(\frac{x}{2}\right) \\
 & \quad \downarrow \text{27} \\
 & \int \frac{(1 - \tan^2\left(\frac{x}{2}\right)) \cot^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) + 1} d \tan\left(\frac{x}{2}\right) \\
 & \quad \downarrow \text{359} \\
 & -2 \int \frac{1}{\tan^2\left(\frac{x}{2}\right) + 1} d \tan\left(\frac{x}{2}\right) - \cot\left(\frac{x}{2}\right) \\
 & \quad \downarrow \text{216} \\
 & -2 \arctan\left(\tan\left(\frac{x}{2}\right)\right) - \cot\left(\frac{x}{2}\right)
 \end{aligned}$$

input `Int[Cot[x/2]*Cot[x],x]`

output `-2*ArcTan[Tan[x/2]] - Cot[x/2]`



## 3.146.3.1 Defintions of rubi rules used

- rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(G_x_) /; FreeQ[b, x]]`
- rule 216 `Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`
- rule 359 `Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^2)^(p_)*((c_) + (d_)*(x_)^2), x_Symbol] := Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1)) Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !ILtQ[p, -1]`
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`
- rule 4889 `Int[u_, x_Symbol] := With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors[Tan[v], x]}, Simp[d/Coefficient[v, x, 1] Subst[Int[SubstFor[1/(1 + d^2*x^2), Tan[v]/d, u, x], x], x, Tan[v]/d, x]]] /; !FalseQ[v] && FunctionOfQ[NonfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] && !MatchQ[u, (v_)*((c_)*tan[w_]^(n_)*tan[z_]^(n_))^(p_)] /; FreeQ[{c, p}, x] && IntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]`

## 3.146.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

| method       | result                                                                     | size |
|--------------|----------------------------------------------------------------------------|------|
| default      | $-x - \cot\left(\frac{x}{2}\right)$                                        | 11   |
| norman       | $\frac{-1 - x \tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)}$ | 17   |
| risch        | $-x - \frac{2i}{e^{ix} - 1}$                                               | 17   |
| parallelrisc | $\frac{-1 - x \tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)}$ | 17   |

input `int(cos(x)/sin(x)/tan(1/2*x),x,method=_RETURNVERBOSE)`

output `-x-cot(1/2*x)`

### 3.146.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \cot\left(\frac{x}{2}\right) \cot(x) dx = -\frac{x \tan\left(\frac{1}{2}x\right) + 1}{\tan\left(\frac{1}{2}x\right)}$$

input `integrate(cos(x)/sin(x)/tan(1/2*x),x, algorithm="fricas")`

output `-(x*tan(1/2*x) + 1)/tan(1/2*x)`

### 3.146.6 Sympy [F]

$$\int \cot\left(\frac{x}{2}\right) \cot(x) dx = \int \frac{\cos(x)}{\sin(x) \tan\left(\frac{x}{2}\right)} dx$$

input `integrate(cos(x)/sin(x)/tan(1/2*x),x)`

output `Integral(cos(x)/(sin(x)*tan(x/2)), x)`

### 3.146.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs.  $2(10) = 20$ .

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.42

$$\int \cot\left(\frac{x}{2}\right) \cot(x) dx = -\frac{x \cos(x)^2 + x \sin(x)^2 - 2x \cos(x) + x + 2 \sin(x)}{\cos(x)^2 + \sin(x)^2 - 2 \cos(x) + 1}$$

input `integrate(cos(x)/sin(x)/tan(1/2*x),x, algorithm="maxima")`

output  $-(x*\cos(x)^2 + x*\sin(x)^2 - 2*x*\cos(x) + x + 2*\sin(x))/(\cos(x)^2 + \sin(x)^2 - 2*\cos(x) + 1)$

### 3.146.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \cot\left(\frac{x}{2}\right) \cot(x) dx = -x - \frac{1}{2 \tan\left(\frac{1}{4}x\right)} + \frac{1}{2} \tan\left(\frac{1}{4}x\right)$$

input `integrate(cos(x)/sin(x)/tan(1/2*x),x, algorithm="giac")`

output  $-x - 1/2/\tan(1/4*x) + 1/2*\tan(1/4*x)$

### 3.146.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \cot\left(\frac{x}{2}\right) \cot(x) dx = -x - \cot\left(\frac{x}{2}\right)$$

input `int(cos(x)/(tan(x/2)*sin(x)),x)`

output  $-x - \cot(x/2)$

### 3.147 $\int \frac{\sin(ax)}{(b+c \sin(ax))^2} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.147.1 Optimal result . . . . .                            | 851 |
| 3.147.2 Mathematica [A] (verified) . . . . .                | 851 |
| 3.147.3 Rubi [A] (verified) . . . . .                       | 852 |
| 3.147.4 Maple [A] (verified) . . . . .                      | 854 |
| 3.147.5 Fricas [A] (verification not implemented) . . . . . | 854 |
| 3.147.6 Sympy [F(-1)] . . . . .                             | 855 |
| 3.147.7 Maxima [F(-2)] . . . . .                            | 855 |
| 3.147.8 Giac [A] (verification not implemented) . . . . .   | 855 |
| 3.147.9 Mupad [B] (verification not implemented) . . . . .  | 856 |

#### 3.147.1 Optimal result

Integrand size = 15, antiderivative size = 77

$$\int \frac{\sin(ax)}{(b+c \sin(ax))^2} dx = -\frac{2c \arctan\left(\frac{c+b \tan\left(\frac{ax}{2}\right)}{\sqrt{b^2-c^2}}\right)}{a(b^2-c^2)^{3/2}} - \frac{b \cos(ax)}{a(b^2-c^2)(b+c \sin(ax))}$$

output `-2*c*arctan((c+b*tan(1/2*a*x))/(b^2-c^2)^(1/2))/a/(b^2-c^2)^(3/2)-b*cos(a*x)/a/(b^2-c^2)/(b+c*sin(a*x))`

#### 3.147.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{\sin(ax)}{(b+c \sin(ax))^2} dx = -\frac{2c \arctan\left(\frac{c+b \tan\left(\frac{ax}{2}\right)}{\sqrt{b^2-c^2}}\right)}{(b^2-c^2)^{3/2}} + \frac{b \cos(ax)}{(b-c)(b+c)(b+c \sin(ax))}$$

input `Integrate[Sin[a*x]/(b + c*Sin[a*x])^2,x]`

output `-(((2*c*ArcTan[(c + b*Tan[(a*x)/2])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(3/2) + (b*Cos[a*x])/((b - c)*(b + c)*(b + c*Sin[a*x])))/a)`

**3.147.3 Rubi [A] (verified)**

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$ , Rules used = {3042, 3233, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sin(ax)}{(c \sin(ax) + b)^2} dx \\
 & \quad \downarrow \text{3042} \\
 & \int \frac{\sin(ax)}{(c \sin(ax) + b)^2} dx \\
 & \quad \downarrow \text{3233} \\
 & -\frac{\int \frac{c}{b+c \sin(ax)} dx}{b^2 - c^2} - \frac{b \cos(ax)}{a(b^2 - c^2)(c \sin(ax) + b)} \\
 & \quad \downarrow \text{27} \\
 & -\frac{c \int \frac{1}{b+c \sin(ax)} dx}{b^2 - c^2} - \frac{b \cos(ax)}{a(b^2 - c^2)(c \sin(ax) + b)} \\
 & \quad \downarrow \text{3042} \\
 & -\frac{c \int \frac{1}{b+c \sin(ax)} dx}{b^2 - c^2} - \frac{b \cos(ax)}{a(b^2 - c^2)(c \sin(ax) + b)} \\
 & \quad \downarrow \text{3139} \\
 & -\frac{2c \int \frac{1}{b \tan^2(\frac{ax}{2}) + 2c \tan(\frac{ax}{2}) + b} d \tan(\frac{ax}{2})}{a(b^2 - c^2)} - \frac{b \cos(ax)}{a(b^2 - c^2)(c \sin(ax) + b)} \\
 & \quad \downarrow \text{1083} \\
 & \frac{4c \int \frac{1}{-(2c+2b \tan(\frac{ax}{2}))^2 - 4(b^2 - c^2)} d(2c + 2b \tan(\frac{ax}{2}))}{a(b^2 - c^2)} - \frac{b \cos(ax)}{a(b^2 - c^2)(c \sin(ax) + b)} \\
 & \quad \downarrow \text{217} \\
 & -\frac{2c \arctan\left(\frac{2b \tan(\frac{ax}{2}) + 2c}{2\sqrt{b^2 - c^2}}\right)}{a(b^2 - c^2)^{3/2}} - \frac{b \cos(ax)}{a(b^2 - c^2)(c \sin(ax) + b)}
 \end{aligned}$$

input `Int[Sin[a*x]/(b + c*Sin[a*x])^2,x]`

---

3.147.  $\int \frac{\sin(ax)}{(b+c \sin(ax))^2} dx$

output  $(-2*c*ArcTan[(2*c + 2*b*Tan[(a*x)/2])/(2*sqrt[b^2 - c^2])])/(a*(b^2 - c^2)^{(3/2)}) - (b*cos[a*x])/(a*(b^2 - c^2)*(b + c*sin[a*x]))$

### 3.147.3.1 Defintions of rubi rules used

rule 27  $Int[(a_)*(F_x_), x\_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] \&\& !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]$

rule 217  $Int[((a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^{-1})*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] \&\& PosQ[a/b] \& \& (LtQ[a, 0] || LtQ[b, 0])$

rule 1083  $Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x\_Symbol] := Simp[-2 Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x]$

rule 3042  $Int[u_, x\_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]$

rule 3139  $Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^{-1}, x\_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d) Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] \&\& NeQ[a^2 - b^2, 0]$

rule 3233  $Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^m * ((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x\_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^{m+1}/(f*(m+1)*(a^2 - b^2))), x] + Simp[1/((m+1)*(a^2 - b^2)) Int[(a + b*Sin[e + f*x])^{m+1}*Simp[(a*c - b*d)*(m+1) - (b*c - a*d)*(m+2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] \&\& NeQ[b*c - a*d, 0] \&\& NeQ[a^2 - b^2, 0] \&\& LtQ[m, -1] \&\& IntegerQ[2*m]$

### 3.147.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

| method            | result                                                                                                                                                                                                                                                                                                                                 | size |
|-------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| derivativedivides | $\frac{-8c \tan\left(\frac{ax}{2}\right) - 8b}{(4b^2 - 4c^2)\left(b \tan^2\left(\frac{ax}{2}\right) + 2c \tan\left(\frac{ax}{2}\right) + b\right)} - \frac{8c \arctan\left(\frac{2b \tan\left(\frac{ax}{2}\right) + 2c}{2\sqrt{b^2 - c^2}}\right)}{(4b^2 - 4c^2)\sqrt{b^2 - c^2}}$                                                     | 107  |
| default           | $\frac{-8c \tan\left(\frac{ax}{2}\right) - 8b}{(4b^2 - 4c^2)\left(b \tan^2\left(\frac{ax}{2}\right) + 2c \tan\left(\frac{ax}{2}\right) + b\right)} - \frac{8c \arctan\left(\frac{2b \tan\left(\frac{ax}{2}\right) + 2c}{2\sqrt{b^2 - c^2}}\right)}{(4b^2 - 4c^2)\sqrt{b^2 - c^2}}$                                                     | 107  |
| risch             | $\frac{2ib(c - ib e^{iax})}{c(-b^2 + c^2)a(c e^{2iax} - c + 2ib e^{iax})} - \frac{ic \ln\left(e^{iax} + \frac{i(b\sqrt{b^2 - c^2} + b^2 - c^2)}{\sqrt{b^2 - c^2}c}\right)}{\sqrt{b^2 - c^2}(b+c)(b-c)a} + \frac{ic \ln\left(e^{iax} + \frac{i(b\sqrt{b^2 - c^2} - b^2 + c^2)}{\sqrt{b^2 - c^2}c}\right)}{\sqrt{b^2 - c^2}(b+c)(b-c)a}$ | 213  |

input `int(sin(a*x)/(b+c*sin(a*x))^2,x,method=_RETURNVERBOSE)`

output `1/a*(4*(-2*c*tan(1/2*a*x)-2*b)/(4*b^2-4*c^2)/(b*tan(1/2*a*x)^2+2*c*tan(1/2*a*x)+b)-8*c/(4*b^2-4*c^2)/(b^2-c^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*a*x)+2*c)/(b^2-c^2)^(1/2)))`

### 3.147.5 Fracas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 312, normalized size of antiderivative = 4.05

$$\int \frac{\sin(ax)}{(b + c \sin(ax))^2} dx = \left[ \frac{(c^2 \sin(ax) + bc)\sqrt{-b^2 + c^2} \log\left(\frac{(2b^2 - c^2) \cos(ax)^2 - 2bc \sin(ax) - b^2 - c^2 + 2(b \cos(ax) \sin(ax) + c \cos(ax))\sqrt{-b^2 + c^2}}{c^2 \cos(ax)^2 - 2bc \sin(ax) - b^2 - c^2}\right) - 2(b^3 - 2ab^2c + abc^2 + (ab^4c - 2ab^2c^3 + ac^5) \sin(ax))}{2(ab^5 - 2ab^3c^2 + abc^4 + (ab^4c - 2ab^2c^3 + ac^5) \sin(ax))} \right]$$

input `integrate(sin(a*x)/(b+c*sin(a*x))^2,x, algorithm="fricas")`

output `[1/2*((c^2*sin(a*x) + b*c)*sqrt(-b^2 + c^2)*log(((2*b^2 - c^2)*cos(a*x)^2 - 2*b*c*sin(a*x) - b^2 - c^2 + 2*(b*cos(a*x)*sin(a*x) + c*cos(a*x))*sqrt(-b^2 + c^2)))/(c^2*cos(a*x)^2 - 2*b*c*sin(a*x) - b^2 - c^2)) - 2*(b^3 - b*c^2*cos(a*x))/(a*b^5 - 2*a*b^3*c^2 + a*b*c^4 + (a*b^4*c - 2*a*b^2*c^3 + a*c^5)*sin(a*x)), ((c^2*sin(a*x) + b*c)*sqrt(b^2 - c^2)*arctan(-(b*sin(a*x) + c)/(sqrt(b^2 - c^2)*cos(a*x))) - (b^3 - b*c^2*cos(a*x))/(a*b^5 - 2*a*b^3*c^2 + a*b*c^4 + (a*b^4*c - 2*a*b^2*c^3 + a*c^5)*sin(a*x))]`

**3.147.6 Sympy [F(-1)]**

Timed out.

$$\int \frac{\sin(ax)}{(b + c \sin(ax))^2} dx = \text{Timed out}$$

input `integrate(sin(a*x)/(b+c*sin(a*x))**2,x)`output `Timed out`**3.147.7 Maxima [F(-2)]**

Exception generated.

$$\int \frac{\sin(ax)}{(b + c \sin(ax))^2} dx = \text{Exception raised: ValueError}$$

input `integrate(sin(a*x)/(b+c*sin(a*x))^2,x, algorithm="maxima")`output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` f or more de`**3.147.8 Giac [A] (verification not implemented)**

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int \frac{\sin(ax)}{(b + c \sin(ax))^2} dx$$

$$= - \frac{2 \left( \frac{\left( \pi \lfloor \frac{ax}{2\pi} + \frac{1}{2} \rfloor \operatorname{sgn}(b) + \arctan \left( \frac{b \tan(\frac{1}{2} ax) + c}{\sqrt{b^2 - c^2}} \right) \right) c}{(b^2 - c^2)^{\frac{3}{2}}} + \frac{c \tan(\frac{1}{2} ax) + b}{(b \tan(\frac{1}{2} ax)^2 + 2c \tan(\frac{1}{2} ax) + b)(b^2 - c^2)} \right)}{a}$$

input `integrate(sin(a*x)/(b+c*sin(a*x))^2,x, algorithm="giac")`



output  $-2*((\pi*\text{floor}(1/2*a*x/\pi + 1/2)*\text{sgn}(b) + \arctan((b*\tan(1/2*a*x) + c)/\sqrt{b^2 - c^2}))*c/(b^2 - c^2)^{(3/2)} + (c*\tan(1/2*a*x) + b)/((b*\tan(1/2*a*x)^2 + 2*c*\tan(1/2*a*x) + b)*(b^2 - c^2)))/a$

### 3.147.9 Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.73

$$\int \frac{\sin(ax)}{(b + c \sin(ax))^2} dx = -\frac{\frac{2b}{b^2-c^2} + \frac{2c \tan(\frac{ax}{2})}{b^2-c^2}}{a \left( b \tan\left(\frac{ax}{2}\right)^2 + 2c \tan\left(\frac{ax}{2}\right) + b \right)} - \frac{2c \operatorname{atan}\left(\frac{\left(\frac{2c^2}{(b+c)^{3/2}(b-c)^{3/2}} + \frac{2bc \tan(\frac{ax}{2})}{(b+c)^{3/2}(b-c)^{3/2}}\right)(b^2-c^2)}{2c}\right)}{a(b+c)^{3/2}(b-c)^{3/2}}$$

input `int(sin(a*x)/(b + c*sin(a*x))^2,x)`

output  $-((2*b)/(b^2 - c^2) + (2*c*\tan((a*x)/2))/(b^2 - c^2))/(a*(b + 2*c*\tan((a*x)/2) + b*\tan((a*x)/2)^2) - (2*c*\operatorname{atan}(((2*c^2)/((b + c)^{(3/2})*(b - c)^{(3/2})) + (2*b*c*\tan((a*x)/2))/((b + c)^{(3/2})*(b - c)^{(3/2}))* (b^2 - c^2))/(2*c)))/(a*(b + c)^{(3/2})*(b - c)^{(3/2}))$

## 3.148 $\int \sin(\log(x)) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.148.1 Optimal result . . . . .                            | 857 |
| 3.148.2 Mathematica [A] (verified) . . . . .                | 857 |
| 3.148.3 Rubi [A] (verified) . . . . .                       | 858 |
| 3.148.4 Maple [A] (verified) . . . . .                      | 858 |
| 3.148.5 Fricas [A] (verification not implemented) . . . . . | 859 |
| 3.148.6 Sympy [A] (verification not implemented) . . . . .  | 859 |
| 3.148.7 Maxima [A] (verification not implemented) . . . . . | 859 |
| 3.148.8 Giac [A] (verification not implemented) . . . . .   | 860 |
| 3.148.9 Mupad [B] (verification not implemented) . . . . .  | 860 |

### 3.148.1 Optimal result

Integrand size = 3, antiderivative size = 17

$$\int \sin(\log(x)) dx = -\frac{1}{2}x \cos(\log(x)) + \frac{1}{2}x \sin(\log(x))$$

output `-1/2*x*cos(ln(x))+1/2*x*sin(ln(x))`

### 3.148.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sin(\log(x)) dx = -\frac{1}{2}x \cos(\log(x)) + \frac{1}{2}x \sin(\log(x))$$

input `Integrate[Sin[Log[x]],x]`

output `-1/2*(x*Cos[Log[x]]) + (x*Sin[Log[x]])/2`

### 3.148.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {4978}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(\log(x)) dx$$

↓ 4978

$$\frac{1}{2}x \sin(\log(x)) - \frac{1}{2}x \cos(\log(x))$$

input `Int[Sin[Log[x]],x]`

output `-1/2*(x*Cos[Log[x]]) + (x*Sin[Log[x]])/2`

#### 3.148.3.1 Defintions of rubi rules used

rule 4978 `Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] := Simp[x*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] - Simp[b*d*n*x*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b^2*d^2*n^2 + 1, 0]`

### 3.148.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

| method        | result                                                                                                                                                             | size |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| parallelrisch | $-\frac{x(\cos(\ln(x)) - \sin(\ln(x)))}{2}$                                                                                                                        | 13   |
| lookup        | $-\frac{x \cos(\ln(x))}{2} + \frac{x \sin(\ln(x))}{2}$                                                                                                             | 14   |
| default       | $-\frac{x \cos(\ln(x))}{2} + \frac{x \sin(\ln(x))}{2}$                                                                                                             | 14   |
| risch         | $\left(-\frac{1}{4} - \frac{i}{4}\right) x x^i + \left(-\frac{1}{4} + \frac{i}{4}\right) x x^{-i}$                                                                 | 22   |
| norman        | $\frac{x \tan\left(\frac{\ln(x)}{2}\right) - \frac{x}{2} + \frac{x \left(\tan^2\left(\frac{\ln(x)}{2}\right)\right)}{2}}{1 + \tan^2\left(\frac{\ln(x)}{2}\right)}$ | 34   |

input `int(sin(ln(x)),x,method=_RETURNVERBOSE)`

output `-1/2*x*(cos(ln(x))-sin(ln(x)))`

### 3.148.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(\log(x)) dx = -\frac{1}{2} x \cos(\log(x)) + \frac{1}{2} x \sin(\log(x))$$

input `integrate(sin(log(x)),x, algorithm="fricas")`

output `-1/2*x*cos(log(x)) + 1/2*x*sin(log(x))`

### 3.148.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sin(\log(x)) dx = \frac{x \sin(\log(x))}{2} - \frac{x \cos(\log(x))}{2}$$

input `integrate(sin(ln(x)),x)`

output `x*sin(log(x))/2 - x*cos(log(x))/2`

### 3.148.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \sin(\log(x)) dx = -\frac{1}{2} x(\cos(\log(x)) - \sin(\log(x)))$$

input `integrate(sin(log(x)),x, algorithm="maxima")`

output `-1/2*x*(cos(log(x)) - sin(log(x)))`

**3.148.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(\log(x)) dx = -\frac{1}{2} x \cos(\log(x)) + \frac{1}{2} x \sin(\log(x))$$

input `integrate(sin(log(x)),x, algorithm="giac")`output `-1/2*x*cos(log(x)) + 1/2*x*sin(log(x))`**3.148.9 Mupad [B] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(\log(x)) dx = -\frac{\sqrt{2} x \cos\left(\frac{\pi}{4} + \ln(x)\right)}{2}$$

input `int(sin(log(x)),x)`output `-(2^(1/2)*x*cos(pi/4 + log(x)))/2`

## 3.149 $\int \cos(\log(x)) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.149.1 Optimal result . . . . .                            | 861 |
| 3.149.2 Mathematica [A] (verified) . . . . .                | 861 |
| 3.149.3 Rubi [A] (verified) . . . . .                       | 862 |
| 3.149.4 Maple [A] (verified) . . . . .                      | 862 |
| 3.149.5 Fricas [A] (verification not implemented) . . . . . | 863 |
| 3.149.6 Sympy [A] (verification not implemented) . . . . .  | 863 |
| 3.149.7 Maxima [A] (verification not implemented) . . . . . | 863 |
| 3.149.8 Giac [A] (verification not implemented) . . . . .   | 864 |
| 3.149.9 Mupad [B] (verification not implemented) . . . . .  | 864 |

### 3.149.1 Optimal result

Integrand size = 3, antiderivative size = 17

$$\int \cos(\log(x)) dx = \frac{1}{2}x \cos(\log(x)) + \frac{1}{2}x \sin(\log(x))$$

output `1/2*x*cos(ln(x))+1/2*x*sin(ln(x))`

### 3.149.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(\log(x)) dx = \frac{1}{2}x \cos(\log(x)) + \frac{1}{2}x \sin(\log(x))$$

input `Integrate[Cos[Log[x]],x]`

output `(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2`

### 3.149.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {4979}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(\log(x)) dx$$

↓ 4979

$$\frac{1}{2}x \sin(\log(x)) + \frac{1}{2}x \cos(\log(x))$$

input `Int[Cos[Log[x]],x]`

output `(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2`

#### 3.149.3.1 Defintions of rubi rules used

rule 4979 `Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] := Simp[x*(Cos[d*(a + b*Log[c*x^n]])/(b^2*d^2*n^2 + 1)), x] + Simp[b*d*n*x*(Sin[d*(a + b*Log[c*x^n]])/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b^2*d^2*n^2 + 1, 0]`

### 3.149.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

| method        | result                                                                     | size |
|---------------|----------------------------------------------------------------------------|------|
| parallelrisch | $\frac{x(\cos(\ln(x))+\sin(\ln(x)))}{2}$                                   | 11   |
| lookup        | $\frac{x \cos(\ln(x))}{2} + \frac{x \sin(\ln(x))}{2}$                      | 14   |
| default       | $\frac{x \cos(\ln(x))}{2} + \frac{x \sin(\ln(x))}{2}$                      | 14   |
| risch         | $(\frac{1}{4} - \frac{i}{4}) x x^i + (\frac{1}{4} + \frac{i}{4}) x x^{-i}$ | 22   |

input `int(cos(ln(x)),x,method=_RETURNVERBOSE)`

output `1/2*x*(cos(ln(x))+sin(ln(x)))`

### 3.149.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x)) dx = \frac{1}{2} x \cos(\log(x)) + \frac{1}{2} x \sin(\log(x))$$

input `integrate(cos(log(x)),x, algorithm="fricas")`

output `1/2*x*cos(log(x)) + 1/2*x*sin(log(x))`

### 3.149.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cos(\log(x)) dx = \frac{x \sin(\log(x))}{2} + \frac{x \cos(\log(x))}{2}$$

input `integrate(cos(ln(x)),x)`

output `x*sin(log(x))/2 + x*cos(log(x))/2`

### 3.149.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \cos(\log(x)) dx = \frac{1}{2} x(\cos(\log(x)) + \sin(\log(x)))$$

input `integrate(cos(log(x)),x, algorithm="maxima")`

output `1/2*x*(cos(log(x)) + sin(log(x)))`



**3.149.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x)) dx = \frac{1}{2} x \cos(\log(x)) + \frac{1}{2} x \sin(\log(x))$$

input `integrate(cos(log(x)),x, algorithm="giac")`output `1/2*x*cos(log(x)) + 1/2*x*sin(log(x))`**3.149.9 Mupad [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(\log(x)) dx = \frac{\sqrt{2} x \sin\left(\frac{\pi}{4} + \ln(x)\right)}{2}$$

input `int(cos(log(x)),x)`output `(2^(1/2)*x*sin(pi/4 + log(x)))/2`

## 3.150 $\int e^x dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.150.1 Optimal result . . . . .                            | 865 |
| 3.150.2 Mathematica [A] (verified) . . . . .                | 865 |
| 3.150.3 Rubi [A] (verified) . . . . .                       | 866 |
| 3.150.4 Maple [A] (verified) . . . . .                      | 866 |
| 3.150.5 Fricas [A] (verification not implemented) . . . . . | 867 |
| 3.150.6 Sympy [A] (verification not implemented) . . . . .  | 867 |
| 3.150.7 Maxima [A] (verification not implemented) . . . . . | 867 |
| 3.150.8 Giac [A] (verification not implemented) . . . . .   | 868 |
| 3.150.9 Mupad [B] (verification not implemented) . . . . .  | 868 |

### 3.150.1 Optimal result

Integrand size = 3, antiderivative size = 3

$$\int e^x dx = e^x$$

output `exp(x)`

### 3.150.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int e^x dx = e^x$$

input `Integrate[E^x,x]`

output `E^x`

### 3.150.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x dx$$

↓ 2624

$$e^x$$

input `Int [E^x,x]`

output `E^x`

#### 3.150.3.1 Defintions of rubi rules used

rule 2624 `Int[((F_)^(v_))^(n_), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;`  
`FreeQ[{F, n}, x] && LinearQ[v, x]`

### 3.150.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

| method            | result     | size |
|-------------------|------------|------|
| gospers           | $e^x$      | 3    |
| lookup            | $e^x$      | 3    |
| derivativedivides | $e^x$      | 3    |
| default           | $e^x$      | 3    |
| norman            | $e^x$      | 3    |
| risch             | $e^x$      | 3    |
| parallelrisc      | $e^x$      | 3    |
| meijerg           | $-1 + e^x$ | 5    |

input `int (exp(x), x, method=_RETURNVERBOSE)`

output `exp(x)`

### 3.150.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x dx = e^x$$

input `integrate(exp(x),x, algorithm="fricas")`

output `e^x`

### 3.150.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x dx = e^x$$

input `integrate(exp(x),x)`

output `exp(x)`

### 3.150.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x dx = e^x$$

input `integrate(exp(x),x, algorithm="maxima")`

output `e^x`

**3.150.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x dx = e^x$$

input `integrate(exp(x),x, algorithm="giac")`

output `e^x`

**3.150.9 Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int e^x dx = e^x$$

input `int(exp(x),x)`

output `exp(x)`

## 3.151 $\int a^x dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.151.1 Optimal result . . . . .                            | 869 |
| 3.151.2 Mathematica [A] (verified) . . . . .                | 869 |
| 3.151.3 Rubi [A] (verified) . . . . .                       | 870 |
| 3.151.4 Maple [A] (verified) . . . . .                      | 870 |
| 3.151.5 Fricas [A] (verification not implemented) . . . . . | 871 |
| 3.151.6 Sympy [A] (verification not implemented) . . . . .  | 871 |
| 3.151.7 Maxima [A] (verification not implemented) . . . . . | 871 |
| 3.151.8 Giac [A] (verification not implemented) . . . . .   | 872 |
| 3.151.9 Mupad [B] (verification not implemented) . . . . .  | 872 |

### 3.151.1 Optimal result

Integrand size = 3, antiderivative size = 8

$$\int a^x dx = \frac{a^x}{\log(a)}$$

output  $a^x/\ln(a)$

### 3.151.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x dx = \frac{a^x}{\log(a)}$$

input `Integrate[a^x,x]`

output  $a^x/\text{Log}[a]$

**3.151.3 Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int a^x dx$$

$$\downarrow 2624$$

$$\frac{a^x}{\log(a)}$$

input `Int[a^x,x]`

output `a^x/Log[a]`

**3.151.3.1 Defintions of rubi rules used**

rule 2624 `Int[((F_)^(v_))^(n_), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;`  
`FreeQ[{F, n}, x] && LinearQ[v, x]`

**3.151.4 Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

| method            | result                           | size |
|-------------------|----------------------------------|------|
| gospers           | $\frac{a^x}{\ln(a)}$             | 9    |
| derivativedivides | $\frac{a^x}{\ln(a)}$             | 9    |
| default           | $\frac{a^x}{\ln(a)}$             | 9    |
| risch             | $\frac{a^x}{\ln(a)}$             | 9    |
| parallelrisch     | $\frac{a^x}{\ln(a)}$             | 9    |
| norman            | $\frac{e^{x \ln(a)}}{\ln(a)}$    | 11   |
| meijerg           | $-\frac{1-e^{x \ln(a)}}{\ln(a)}$ | 16   |

input `int(a^x,x,method=_RETURNVERBOSE)`

output `a^x/ln(a)`

### 3.151.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x dx = \frac{a^x}{\log(a)}$$

input `integrate(a^x,x, algorithm="fricas")`

output `a^x/log(a)`

### 3.151.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x dx = \begin{cases} \frac{a^x}{\log(a)} & \text{for } \log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

input `integrate(a**x,x)`

output `Piecewise((a**x/log(a), Ne(log(a), 0)), (x, True))`

### 3.151.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x dx = \frac{a^x}{\log(a)}$$

input `integrate(a^x,x, algorithm="maxima")`

output `a^x/log(a)`



**3.151.8 Giac [A] (verification not implemented)**

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x dx = \frac{a^x}{\log(a)}$$

input `integrate(a^x,x, algorithm="giac")`

output `a^x/log(a)`

**3.151.9 Mupad [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int a^x dx = \frac{a^x}{\ln(a)}$$

input `int(a^x,x)`

output `a^x/log(a)`

## 3.152 $\int e^{ax} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.152.1 Optimal result . . . . .                            | 873 |
| 3.152.2 Mathematica [A] (verified) . . . . .                | 873 |
| 3.152.3 Rubi [A] (verified) . . . . .                       | 874 |
| 3.152.4 Maple [A] (verified) . . . . .                      | 874 |
| 3.152.5 Fricas [A] (verification not implemented) . . . . . | 875 |
| 3.152.6 Sympy [A] (verification not implemented) . . . . .  | 875 |
| 3.152.7 Maxima [A] (verification not implemented) . . . . . | 875 |
| 3.152.8 Giac [A] (verification not implemented) . . . . .   | 876 |
| 3.152.9 Mupad [B] (verification not implemented) . . . . .  | 876 |

### 3.152.1 Optimal result

Integrand size = 5, antiderivative size = 9

$$\int e^{ax} dx = \frac{e^{ax}}{a}$$

output `exp(a*x)/a`

### 3.152.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int e^{ax} dx = \frac{e^{ax}}{a}$$

input `Integrate[E^(a*x),x]`

output `E^(a*x)/a`

### 3.152.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{ax} dx$$

$$\downarrow 2624$$

$$\frac{e^{ax}}{a}$$

input `Int[E^(a*x), x]`

output `E^(a*x)/a`

#### 3.152.3.1 Defintions of rubi rules used

rule 2624 `Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;`  
`FreeQ[{F, n}, x] && LinearQ[v, x]`

### 3.152.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

| method            | result                | size |
|-------------------|-----------------------|------|
| gosper            | $\frac{e^{ax}}{a}$    | 9    |
| derivativedivides | $\frac{e^{ax}}{a}$    | 9    |
| default           | $\frac{e^{ax}}{a}$    | 9    |
| norman            | $\frac{e^{ax}}{a}$    | 9    |
| risch             | $\frac{e^{ax}}{a}$    | 9    |
| parallelrisc      | $\frac{e^{ax}}{a}$    | 9    |
| meijerg           | $-\frac{1-e^{ax}}{a}$ | 14   |

input `int(exp(a*x),x,method=_RETURNVERBOSE)`

output `exp(a*x)/a`

### 3.152.5 Fracas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{ax} dx = \frac{e^{(ax)}}{a}$$

input `integrate(exp(a*x),x, algorithm="fricas")`

output `e^(a*x)/a`

### 3.152.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int e^{ax} dx = \begin{cases} \frac{e^{ax}}{a} & \text{for } a \neq 0 \\ x & \text{otherwise} \end{cases}$$

input `integrate(exp(a*x),x)`

output `Piecewise((exp(a*x)/a, Ne(a, 0)), (x, True))`

### 3.152.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{ax} dx = \frac{e^{(ax)}}{a}$$

input `integrate(exp(a*x),x, algorithm="maxima")`

output `e^(a*x)/a`

**3.152.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{ax} dx = \frac{e^{(ax)}}{a}$$

input `integrate(exp(a*x),x, algorithm="giac")`

output `e^(a*x)/a`

**3.152.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{ax} dx = \frac{e^{ax}}{a}$$

input `int(exp(a*x),x)`

output `exp(a*x)/a`

### 3.153 $\int \frac{e^{ax}}{x} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.153.1 Optimal result . . . . .                            | 877 |
| 3.153.2 Mathematica [A] (verified) . . . . .                | 877 |
| 3.153.3 Rubi [A] (verified) . . . . .                       | 878 |
| 3.153.4 Maple [A] (verified) . . . . .                      | 878 |
| 3.153.5 Fricas [A] (verification not implemented) . . . . . | 879 |
| 3.153.6 Sympy [A] (verification not implemented) . . . . .  | 879 |
| 3.153.7 Maxima [A] (verification not implemented) . . . . . | 879 |
| 3.153.8 Giac [A] (verification not implemented) . . . . .   | 880 |
| 3.153.9 Mupad [B] (verification not implemented) . . . . .  | 880 |

#### 3.153.1 Optimal result

Integrand size = 9, antiderivative size = 4

$$\int \frac{e^{ax}}{x} dx = \text{ExpIntegralEi}(ax)$$

output `Ei(a*x)`

#### 3.153.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}}{x} dx = \text{ExpIntegralEi}(ax)$$

input `Integrate[E^(a*x)/x,x]`

output `ExpIntegralEi[a*x]`

### 3.153.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{ax}}{x} dx$$

↓ 2609

$$\text{ExpIntegralEi}(ax)$$

input `Int [E^(a*x)/x, x]`

output `ExpIntegralEi [a*x]`

#### 3.153.3.1 Defintions of rubi rules used

rule 2609 `Int[(F_)^((g_)*((e_.) + (f_)*(x_)))/((c_.) + (d_)*(x_)), x_Symbol] := Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

### 3.153.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 2.25

| method            | result                                    | size |
|-------------------|-------------------------------------------|------|
| derivativedivides | $-Ei_1(-ax)$                              | 9    |
| default           | $-Ei_1(-ax)$                              | 9    |
| risch             | $-Ei_1(-ax)$                              | 9    |
| meijerg           | $\ln(x) + \ln(-a) - \ln(-ax) - Ei_1(-ax)$ | 23   |

input `int(exp(a*x)/x,x,method=_RETURNVERBOSE)`

output `-Ei(1, -a*x)`

**3.153.5 Fricas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}}{x} dx = \text{Ei}(ax)$$

input `integrate(exp(a*x)/x,x, algorithm="fricas")`

output `Ei(a*x)`

**3.153.6 Sympy [A] (verification not implemented)**

Time = 0.47 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^{ax}}{x} dx = \text{Ei}(ax)$$

input `integrate(exp(a*x)/x,x)`

output `Ei(a*x)`

**3.153.7 Maxima [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}}{x} dx = \text{Ei}(ax)$$

input `integrate(exp(a*x)/x,x, algorithm="maxima")`

output `Ei(a*x)`



**3.153.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}}{x} dx = \text{Ei}(ax)$$

input `integrate(exp(a*x)/x,x, algorithm="giac")`

output `Ei(a*x)`

**3.153.9 Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}}{x} dx = \text{ei}(ax)$$

input `int(exp(a*x)/x,x)`

output `ei(a*x)`

### 3.154 $\int \frac{1}{a+be^{mx}} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.154.1 Optimal result . . . . .                            | 881 |
| 3.154.2 Mathematica [A] (verified) . . . . .                | 881 |
| 3.154.3 Rubi [A] (verified) . . . . .                       | 882 |
| 3.154.4 Maple [A] (verified) . . . . .                      | 883 |
| 3.154.5 Fricas [A] (verification not implemented) . . . . . | 883 |
| 3.154.6 Sympy [A] (verification not implemented) . . . . .  | 884 |
| 3.154.7 Maxima [A] (verification not implemented) . . . . . | 884 |
| 3.154.8 Giac [A] (verification not implemented) . . . . .   | 884 |
| 3.154.9 Mupad [B] (verification not implemented) . . . . .  | 885 |

#### 3.154.1 Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{1}{a + be^{mx}} dx = \frac{x}{a} - \frac{\log(a + be^{mx})}{am}$$

output `x/a-ln(a+b*exp(m*x))/a/m`

#### 3.154.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{1}{a + be^{mx}} dx = \frac{\log(e^{mx})}{am} - \frac{\log(a^2m + abe^{mx}m)}{am}$$

input `Integrate[(a + b*E^(m*x))^(-1), x]`

output `Log[E^(m*x)]/(a*m) - Log[a^2*m + a*b*E^(m*x)*m]/(a*m)`

**3.154.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c}
 \int \frac{1}{a + be^{mx}} dx \\
 \downarrow \text{2720} \\
 \int \frac{e^{-mx}}{a + be^{mx}} de^{mx} \\
 \downarrow \text{47} \\
 \frac{\int e^{-mx} de^{mx}}{a} - \frac{b \int \frac{1}{a + be^{mx}} de^{mx}}{a} \\
 \downarrow \text{14} \\
 \frac{\log(e^{mx})}{a} - \frac{b \int \frac{1}{a + be^{mx}} de^{mx}}{a} \\
 \downarrow \text{16} \\
 \frac{\log(e^{mx})}{a} - \frac{\log(a + be^{mx})}{a} \\
 m
 \end{array}$$

input `Int[(a + b*E^(m*x))^(-1),x]`

output `(Log[E^(m*x)]/a - Log[a + b*E^(m*x)]/a)/m`

**3.154.3.1 Defintions of rubi rules used**

rule 14 `Int[(c_.)/(x_), x_Symbol] := Simp[a*Log[x], x] /; FreeQ[a, x]`

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

```
rule 47 Int[1/((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[b/(b*c
- a*d) Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d) Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

```
rule 2720 Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### 3.154.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

| method            | result                                                          | size |
|-------------------|-----------------------------------------------------------------|------|
| parallelrisch     | $-\frac{-mx + \ln(a + be^{mx})}{am}$                            | 23   |
| norman            | $\frac{x}{a} - \frac{\ln(a + be^{mx})}{am}$                     | 24   |
| risch             | $\frac{x}{a} - \frac{\ln(e^{mx} + \frac{a}{b})}{am}$            | 26   |
| derivativedivides | $\frac{-\frac{\ln(a + be^{mx})}{a} + \frac{\ln(e^{mx})}{a}}{m}$ | 29   |
| default           | $\frac{-\frac{\ln(a + be^{mx})}{a} + \frac{\ln(e^{mx})}{a}}{m}$ | 29   |

```
input int(1/(a+b*exp(m*x)),x,method=_RETURNVERBOSE)
```

```
output -(-m*x+ln(a+b*exp(m*x)))/a/m
```

### 3.154.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{a + be^{mx}} dx = \frac{mx - \log(be^{mx} + a)}{am}$$

```
input integrate(1/(a+b*exp(m*x)),x, algorithm="fricas")
```

output  $(m*x - \log(b*e^{(m*x)} + a))/(a*m)$

### 3.154.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a + be^{mx}} dx = \frac{x}{a} - \frac{\log\left(\frac{a}{b} + e^{mx}\right)}{am}$$

input `integrate(1/(a+b*exp(m*x)),x)`

output  $x/a - \log(a/b + \exp(m*x))/(a*m)$

### 3.154.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{a + be^{mx}} dx = \frac{x}{a} - \frac{\log\left( be^{(mx)} + a \right)}{am}$$

input `integrate(1/(a+b*exp(m*x)),x, algorithm="maxima")`

output  $x/a - \log(b*e^{(m*x)} + a)/(a*m)$

### 3.154.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{a + be^{mx}} dx = \frac{mx}{a} - \frac{\log(|be^{(mx)}+a|)}{m}$$

input `integrate(1/(a+b*exp(m*x)),x, algorithm="giac")`

output  $(m*x/a - \log(\text{abs}(b*e^{(m*x)} + a)))/a/m$

**3.154.9 Mupad [B] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{a + b e^{m x}} dx = -\frac{\ln(a + b e^{m x}) - m x}{a m}$$

input `int(1/(a + b*exp(m*x)),x)`

output `-(log(a + b*exp(m*x)) - m*x)/(a*m)`

### 3.155 $\int \frac{e^{2x}}{1+e^x} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.155.1 Optimal result . . . . .                            | 886 |
| 3.155.2 Mathematica [A] (verified) . . . . .                | 886 |
| 3.155.3 Rubi [A] (verified) . . . . .                       | 887 |
| 3.155.4 Maple [A] (verified) . . . . .                      | 888 |
| 3.155.5 Fricas [A] (verification not implemented) . . . . . | 888 |
| 3.155.6 Sympy [A] (verification not implemented) . . . . .  | 889 |
| 3.155.7 Maxima [A] (verification not implemented) . . . . . | 889 |
| 3.155.8 Giac [A] (verification not implemented) . . . . .   | 889 |
| 3.155.9 Mupad [B] (verification not implemented) . . . . .  | 890 |

#### 3.155.1 Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{e^{2x}}{1+e^x} dx = e^x - \log(1+e^x)$$

output `exp(x)-ln(1+exp(x))`

#### 3.155.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{1+e^x} dx = e^x - \log(1+e^x)$$

input `Integrate[E^(2*x)/(1 + E^x), x]`

output `E^x - Log[1 + E^x]`

**3.155.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{e^{2x}}{e^x + 1} dx \\ & \quad \downarrow \text{2678} \\ & \int \frac{e^x}{e^x + 1} de^x \\ & \quad \downarrow \text{49} \\ & \int \left( \frac{1}{-e^x - 1} + 1 \right) de^x \\ & \quad \downarrow \text{2009} \\ & e^x - \log(e^x + 1) \end{aligned}$$

input `Int[E^(2*x)/(1 + E^x),x]`

output `E^x - Log[1 + E^x]`

**3.155.3.1 Defintions of rubi rules used**

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`



```
rule 2678 Int[((a_) + (b_)*(F_)^((e_)*((c_) + (d_)*(x_)))^(p_)*(G_)^((h_)*((f_
.) + (g_)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])) Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

### 3.155.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

| method  | result               | size |
|---------|----------------------|------|
| default | $e^x - \ln(1 + e^x)$ | 11   |
| norman  | $e^x - \ln(1 + e^x)$ | 11   |
| risch   | $e^x - \ln(1 + e^x)$ | 11   |

```
input int(exp(2*x)/(1+exp(x)),x,method=_RETURNVERBOSE)
```

```
output exp(x)-ln(1+exp(x))
```

### 3.155.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x} dx = e^x - \log(e^x + 1)$$

```
input integrate(exp(2*x)/(1+exp(x)),x, algorithm="fracas")
```

```
output e^x - log(e^x + 1)
```

**3.155.6 Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{e^{2x}}{1+e^x} dx = e^x - \log(e^x + 1)$$

input `integrate(exp(2*x)/(1+exp(x)),x)`output `exp(x) - log(exp(x) + 1)`**3.155.7 Maxima [A] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x} dx = e^x - \log(e^x + 1)$$

input `integrate(exp(2*x)/(1+exp(x)),x, algorithm="maxima")`output `e^x - log(e^x + 1)`**3.155.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x} dx = e^x - \log(e^x + 1)$$

input `integrate(exp(2*x)/(1+exp(x)),x, algorithm="giac")`output `e^x - log(e^x + 1)`

**3.155.9 Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x} dx = e^x - \ln(e^x + 1)$$

input `int(exp(2*x)/(exp(x) + 1),x)`

output `exp(x) - log(exp(x) + 1)`

### 3.156 $\int e^{2x+ax} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.156.1 Optimal result . . . . .                            | 891 |
| 3.156.2 Mathematica [A] (verified) . . . . .                | 891 |
| 3.156.3 Rubi [A] (verified) . . . . .                       | 892 |
| 3.156.4 Maple [A] (verified) . . . . .                      | 892 |
| 3.156.5 Fricas [A] (verification not implemented) . . . . . | 893 |
| 3.156.6 Sympy [A] (verification not implemented) . . . . .  | 893 |
| 3.156.7 Maxima [A] (verification not implemented) . . . . . | 893 |
| 3.156.8 Giac [A] (verification not implemented) . . . . .   | 894 |
| 3.156.9 Mupad [B] (verification not implemented) . . . . .  | 894 |

#### 3.156.1 Optimal result

Integrand size = 9, antiderivative size = 13

$$\int e^{2x+ax} dx = \frac{e^{(2+a)x}}{2+a}$$

output `exp((2+a)*x)/(2+a)`

#### 3.156.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{2x+ax} dx = \frac{e^{(2+a)x}}{2+a}$$

input `Integrate[E^(2*x + a*x),x]`

output `E^((2 + a)*x)/(2 + a)`

### 3.156.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{ax+2x} dx$$

$$\downarrow \text{2624}$$

$$\frac{e^{(a+2)x}}{a+2}$$

input `Int[E^(2*x + a*x), x]`

output `E^((2 + a)*x)/(2 + a)`

#### 3.156.3.1 Defintions of rubi rules used

rule 2624 `Int[((F_)^(v_))^(n_), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;`  
`FreeQ[{F, n}, x] && LinearQ[v, x]`

### 3.156.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

| method            | result                        | size |
|-------------------|-------------------------------|------|
| risch             | $\frac{e^{(2+a)x}}{2+a}$      | 13   |
| parallelrisc      | $\frac{e^{(2+a)x}}{2+a}$      | 13   |
| gospers           | $\frac{e^{ax+2x}}{2+a}$       | 15   |
| derivativedivides | $\frac{e^{ax+2x}}{2+a}$       | 15   |
| default           | $\frac{e^{ax+2x}}{2+a}$       | 15   |
| norman            | $\frac{e^{ax+2x}}{2+a}$       | 15   |
| meijerg           | $\frac{1-e^{-x(-a-2)}}{-a-2}$ | 22   |

input `int(exp(a*x+2*x),x,method=_RETURNVERBOSE)`

output `exp((2+a)*x)/(2+a)`

### 3.156.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int e^{2x+ax} dx = \frac{e^{(a+2)x}}{a+2}$$

input `integrate(exp(a*x+2*x),x, algorithm="fricas")`

output `e^((a + 2)*x)/(a + 2)`

### 3.156.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int e^{2x+ax} dx = \begin{cases} \frac{e^{ax+2x}}{a+2} & \text{for } a \neq -2 \\ x & \text{otherwise} \end{cases}$$

input `integrate(exp(a*x+2*x),x)`

output `Piecewise((exp(a*x + 2*x)/(a + 2), Ne(a, -2)), (x, True))`

### 3.156.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int e^{2x+ax} dx = \frac{e^{(a+2)x}}{a+2}$$

input `integrate(exp(a*x+2*x),x, algorithm="maxima")`

output `e^(a*x + 2*x)/(a + 2)`

**3.156.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int e^{2x+ax} dx = \frac{e^{(ax+2x)}}{a+2}$$

input `integrate(exp(a*x+2*x),x, algorithm="giac")`

output `e^(a*x + 2*x)/(a + 2)`

**3.156.9 Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int e^{2x+ax} dx = \frac{e^{2x+ax}}{a+2}$$

input `int(exp(2*x + a*x),x)`

output `exp(2*x + a*x)/(a + 2)`

### 3.157 $\int \frac{1}{be^{-mx} + ae^{mx}} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.157.1 Optimal result . . . . .                            | 895 |
| 3.157.2 Mathematica [A] (verified) . . . . .                | 895 |
| 3.157.3 Rubi [A] (verified) . . . . .                       | 896 |
| 3.157.4 Maple [A] (verified) . . . . .                      | 897 |
| 3.157.5 Fricas [A] (verification not implemented) . . . . . | 897 |
| 3.157.6 Sympy [A] (verification not implemented) . . . . .  | 897 |
| 3.157.7 Maxima [A] (verification not implemented) . . . . . | 898 |
| 3.157.8 Giac [A] (verification not implemented) . . . . .   | 898 |
| 3.157.9 Mupad [B] (verification not implemented) . . . . .  | 898 |

#### 3.157.1 Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{1}{be^{-mx} + ae^{mx}} dx = \frac{\arctan\left(\frac{\sqrt{ae^{mx}}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{bm}}$$

output `arctan(exp(m*x)*a^(1/2)/b^(1/2))/m/a^(1/2)/b^(1/2)`

#### 3.157.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{be^{-mx} + ae^{mx}} dx = \frac{\arctan\left(\frac{\sqrt{ae^{mx}}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{bm}}$$

input `Integrate[(b/E^(m*x) + a*E^(m*x))^-1, x]`

output `ArcTan[(Sqrt[a]*E^(m*x))/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*m)`



### 3.157.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2720, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ae^{mx} + be^{-mx}} dx$$

↓ 2720

$$\int \frac{1}{e^{2mx}a+b} de^{mx}$$

↓ 218

$$\frac{\arctan\left(\frac{\sqrt{a}e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}m}$$

input `Int[(b/E^(m*x) + a*E^(m*x))^(−1),x]`

output `ArcTan[(Sqrt[a]*E^(m*x))/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*m)`

#### 3.157.3.1 Defintions of rubi rules used

rule 218 `Int[((a_) + (b_.)*(x_)^2)^(−1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]`

rule 2720 `Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

**3.157.4 Maple [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

| method            | result                                                                                                                                 | size |
|-------------------|----------------------------------------------------------------------------------------------------------------------------------------|------|
| derivativedivides | $\frac{\arctan\left(\frac{ae^{mx}}{\sqrt{ab}}\right)}{m\sqrt{ab}}$                                                                     | 22   |
| default           | $\frac{\arctan\left(\frac{ae^{mx}}{\sqrt{ab}}\right)}{m\sqrt{ab}}$                                                                     | 22   |
| risch             | $-\frac{\ln\left(e^{mx}-\frac{b}{\sqrt{-ab}}\right)}{2\sqrt{-ab}m} + \frac{\ln\left(e^{mx}+\frac{b}{\sqrt{-ab}}\right)}{2\sqrt{-ab}m}$ | 53   |

input `int(1/(b/exp(m*x)+a*exp(m*x)),x,method=_RETURNVERBOSE)`output `1/m/(a*b)^(1/2)*arctan(a*exp(m*x)/(a*b)^(1/2))`**3.157.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.74

$$\int \frac{1}{be^{-mx} + ae^{mx}} dx = \left[ -\frac{\sqrt{-ab} \log\left(\frac{ae^{(2mx)} - 2\sqrt{-ab}e^{(mx)} - b}{ae^{(2mx)} + b}\right)}{2abm}, -\frac{\sqrt{ab} \arctan\left(\frac{\sqrt{ab}e^{(-mx)}}{a}\right)}{abm} \right]$$

input `integrate(1/(b/exp(m*x)+a*exp(m*x)),x, algorithm="fricas")`output `[-1/2*sqrt(-a*b)*log((a*e^(2*m*x) - 2*sqrt(-a*b)*e^(m*x) - b)/(a*e^(2*m*x) + b))/(a*b*m), -sqrt(a*b)*arctan(sqrt(a*b)*e^(-m*x)/a)/(a*b*m)]`**3.157.6 Sympy [A] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{be^{-mx} + ae^{mx}} dx = \frac{\text{RootSum}(4z^2ab + 1, (i \mapsto i \log(2ib + e^{mx})))}{m}$$

input `integrate(1/(b/exp(m*x)+a*exp(m*x)),x)`output `RootSum(4*_z**2*a*b + 1, Lambda(_i, _i*log(2*_i*b + exp(m*x))))/m`

**3.157.7 Maxima [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{be^{-mx} + ae^{mx}} dx = -\frac{\arctan\left(\frac{be^{-mx}}{\sqrt{ab}}\right)}{\sqrt{ab}m}$$

input `integrate(1/(b/exp(m*x)+a*exp(m*x)),x, algorithm="maxima")`output `-arctan(b*e^(-m*x)/sqrt(a*b))/(sqrt(a*b)*m)`**3.157.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{be^{-mx} + ae^{mx}} dx = \frac{\arctan\left(\frac{ae^{mx}}{\sqrt{ab}}\right)}{\sqrt{ab}m}$$

input `integrate(1/(b/exp(m*x)+a*exp(m*x)),x, algorithm="giac")`output `arctan(a*e^(m*x)/sqrt(a*b))/(sqrt(a*b)*m)`**3.157.9 Mupad [B] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{be^{-mx} + ae^{mx}} dx = \frac{\operatorname{atan}\left(\frac{ae^{mx}}{\sqrt{ab}}\right)}{m\sqrt{ab}}$$

input `int(1/(a*exp(m*x) + b*exp(-m*x)),x)`output `atan((a*exp(m*x))/(a*b)^(1/2))/(m*(a*b)^(1/2))`

## 3.158 $\int e^{ax} x dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.158.1 Optimal result . . . . .                            | 899 |
| 3.158.2 Mathematica [A] (verified) . . . . .                | 899 |
| 3.158.3 Rubi [A] (verified) . . . . .                       | 900 |
| 3.158.4 Maple [A] (verified) . . . . .                      | 901 |
| 3.158.5 Fricas [A] (verification not implemented) . . . . . | 901 |
| 3.158.6 Sympy [A] (verification not implemented) . . . . .  | 902 |
| 3.158.7 Maxima [A] (verification not implemented) . . . . . | 902 |
| 3.158.8 Giac [A] (verification not implemented) . . . . .   | 902 |
| 3.158.9 Mupad [B] (verification not implemented) . . . . .  | 903 |

### 3.158.1 Optimal result

Integrand size = 7, antiderivative size = 21

$$\int e^{ax} x dx = -\frac{e^{ax}}{a^2} + \frac{e^{ax} x}{a}$$

output `-exp(a*x)/a^2+exp(a*x)*x/a`

### 3.158.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int e^{ax} x dx = \frac{e^{ax}(-1 + ax)}{a^2}$$

input `Integrate[E^(a*x)*x,x]`

output `(E^(a*x)*(-1 + a*x))/a^2`

**3.158.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x e^{ax} dx$$

$$\downarrow \text{2607}$$

$$\frac{x e^{ax}}{a} - \frac{\int e^{ax} dx}{a}$$

$$\downarrow \text{2624}$$

$$\frac{x e^{ax}}{a} - \frac{e^{ax}}{a^2}$$

input `Int[E^(a*x)*x,x]`

output `-(E^(a*x)/a^2) + (E^(a*x)*x)/a`

**3.158.3.1 Defintions of rubi rules used**

rule 2607 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2624 `Int[((F_)^(v_))^(n_.), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /; FreeQ[{F, n}, x] && LinearQ[v, x]`

**3.158.4 Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

| method           | result                                     | size |
|------------------|--------------------------------------------|------|
| gospers          | $\frac{(ax-1)e^{ax}}{a^2}$                 | 14   |
| risch            | $\frac{(ax-1)e^{ax}}{a^2}$                 | 14   |
| derivativdivides | $\frac{ae^{ax}x - e^{ax}}{a^2}$            | 19   |
| default          | $\frac{ae^{ax}x - e^{ax}}{a^2}$            | 19   |
| meijerg          | $\frac{1 - \frac{(-2ax+2)e^{ax}}{2}}{a^2}$ | 19   |
| parallelrisc     | $\frac{ae^{ax}x - e^{ax}}{a^2}$            | 19   |
| norman           | $-\frac{e^{ax}}{a^2} + \frac{e^{ax}x}{a}$  | 20   |
| parts            | $-\frac{e^{ax}}{a^2} + \frac{e^{ax}x}{a}$  | 20   |

input `int(exp(a*x)*x,x,method=_RETURNVERBOSE)`output `(a*x-1)*exp(a*x)/a^2`**3.158.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int e^{ax} x dx = \frac{(ax-1)e^{(ax)}}{a^2}$$

input `integrate(exp(a*x)*x,x, algorithm="fricas")`output `(a*x - 1)*e^(a*x)/a^2`

**3.158.6 Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int e^{ax} x dx = \begin{cases} \frac{(ax-1)e^{ax}}{a^2} & \text{for } a^2 \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

input `integrate(exp(a*x)*x,x)`output `Piecewise(((a*x - 1)*exp(a*x)/a**2, Ne(a**2, 0)), (x**2/2, True))`**3.158.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int e^{ax} x dx = \frac{(ax - 1)e^{(ax)}}{a^2}$$

input `integrate(exp(a*x)*x,x, algorithm="maxima")`output `(a*x - 1)*e^(a*x)/a^2`**3.158.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int e^{ax} x dx = \frac{(ax - 1)e^{(ax)}}{a^2}$$

input `integrate(exp(a*x)*x,x, algorithm="giac")`output `(a*x - 1)*e^(a*x)/a^2`

**3.158.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int e^{ax} x dx = \frac{e^{ax} (ax - 1)}{a^2}$$

input `int(x*exp(a*x),x)`

output `(exp(a*x)*(a*x - 1))/a^2`



### 3.159 $\int e^x x^{20} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.159.1 Optimal result . . . . .                            | 904 |
| 3.159.2 Mathematica [A] (verified) . . . . .                | 905 |
| 3.159.3 Rubi [A] (verified) . . . . .                       | 905 |
| 3.159.4 Maple [A] (verified) . . . . .                      | 908 |
| 3.159.5 Fracas [A] (verification not implemented) . . . . . | 909 |
| 3.159.6 Sympy [A] (verification not implemented) . . . . .  | 909 |
| 3.159.7 Maxima [A] (verification not implemented) . . . . . | 910 |
| 3.159.8 Giac [A] (verification not implemented) . . . . .   | 910 |
| 3.159.9 Mupad [B] (verification not implemented) . . . . .  | 911 |

#### 3.159.1 Optimal result

Integrand size = 7, antiderivative size = 163

$$\int e^x x^{20} dx = 2432902008176640000e^x - 2432902008176640000e^x x + 1216451004088320000e^x x^2 - 405483668029440000e^x x^3 + 101370917007360000e^x x^4 - 20274183401472000e^x x^5 + 3379030566912000e^x x^6 - 482718652416000e^x x^7 + 60339831552000e^x x^8 - 6704425728000e^x x^9 + 670442572800e^x x^{10} - 60949324800e^x x^{11} + 5079110400e^x x^{12} - 390700800e^x x^{13} + 27907200e^x x^{14} - 1860480e^x x^{15} + 116280e^x x^{16} - 6840e^x x^{17} + 380e^x x^{18} - 20e^x x^{19} + e^x x^{20}$$

```
output 2432902008176640000*exp(x)-2432902008176640000*exp(x)*x+121645100408832000
0*exp(x)*x^2-405483668029440000*exp(x)*x^3+101370917007360000*exp(x)*x^4-2
0274183401472000*exp(x)*x^5+3379030566912000*exp(x)*x^6-482718652416000*ex
p(x)*x^7+60339831552000*exp(x)*x^8-6704425728000*exp(x)*x^9+670442572800*e
xp(x)*x^10-60949324800*exp(x)*x^11+5079110400*exp(x)*x^12-390700800*exp(x)
*x^13+27907200*exp(x)*x^14-1860480*exp(x)*x^15+116280*exp(x)*x^16-6840*exp
(x)*x^17+380*exp(x)*x^18-20*exp(x)*x^19+exp(x)*x^20
```

**3.159.2 Mathematica [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.63

$$\int e^x x^{20} dx = e^x (2432902008176640000 - 2432902008176640000x + 1216451004088320000x^2 - 405483668029440000x^3 + 101370917007360000x^4 - 20274183401472000x^5 + 3379030566912000x^6 - 482718652416000x^7 + 60339831552000x^8 - 6704425728000x^9 + 670442572800x^{10} - 60949324800x^{11} + 5079110400x^{12} - 390700800x^{13} + 27907200x^{14} - 1860480x^{15} + 116280x^{16} - 6840x^{17} + 380x^{18} - 20x^{19} + x^{20})$$

input `Integrate[E^x*x^20,x]`

output `E^x*(2432902008176640000 - 2432902008176640000*x + 1216451004088320000*x^2 - 405483668029440000*x^3 + 101370917007360000*x^4 - 20274183401472000*x^5 + 3379030566912000*x^6 - 482718652416000*x^7 + 60339831552000*x^8 - 6704425728000*x^9 + 670442572800*x^10 - 60949324800*x^11 + 5079110400*x^12 - 390700800*x^13 + 27907200*x^14 - 1860480*x^15 + 116280*x^16 - 6840*x^17 + 380*x^18 - 20*x^19 + x^20)`

**3.159.3 Rubi [A] (verified)**

Time = 1.07 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.23, number of steps used = 21, number of rules used = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 3.000$ , Rules used = {2607, 2607, 2607, 2607, 2607, 2607, 2607, 2607, 2607, 2607, 2607, 2607, 2607, 2607, 2607, 2607, 2607, 2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int e^x x^{20} dx \\ & \quad \downarrow \text{2607} \\ & e^x x^{20} - 20 \int e^x x^{19} dx \\ & \quad \downarrow \text{2607} \\ & e^x x^{20} - 20 \left( e^x x^{19} - 19 \int e^x x^{18} dx \right) \end{aligned}$$





$$20(e^x x^{19} - 19(e^x x^{18} - 18(e^x x^{17} - 17(e^x x^{16} - 16(e^x x^{15} - 15(e^x x^{14} - 14(e^x x^{13} - 13(e^x x^{12} - 12(e^x x^{11} - 11(e^x x^{10} - 10(e^x x^9 - 9(e^x x^8 - 8(e^x x^7 - 7(e^x x^6 - 6(e^x x^5 - 5(e^x x^4 - 4(e^x x^3 - 3(e^x x^2 - 2(-e^x + E^x x)))))))))))))))))$$

input `Int [E^x*x^20, x]`

output `E^x*x^20 - 20*(E^x*x^19 - 19*(E^x*x^18 - 18*(E^x*x^17 - 17*(E^x*x^16 - 16*(E^x*x^15 - 15*(E^x*x^14 - 14*(E^x*x^13 - 13*(E^x*x^12 - 12*(E^x*x^11 - 11*(E^x*x^10 - 10*(E^x*x^9 - 9*(E^x*x^8 - 8*(E^x*x^7 - 7*(E^x*x^6 - 6*(E^x*x^5 - 5*(E^x*x^4 - 4*(E^x*x^3 - 3*(E^x*x^2 - 2*(-E^x + E^x*x)))))))))))))))))`

### 3.159.3.1 Defintions of rubi rules used

rule 2607 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2624 `Int[((F_)^(v_))^(n_), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /; FreeQ[{F, n}, x] && LinearQ[v, x]`

### 3.159.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.63

| method        | result                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| gospers       | $(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 4568400000x^{12} - 47827200000x^{11} + 448224000000x^{10} - 3777600000000x^9 + 30720000000000x^8 - 236160000000000x^7 + 1747200000000000x^6 - 12288000000000000x^5 + 80640000000000000x^4 - 491520000000000000x^3 + 2949120000000000000x^2 - 16790400000000000000x - 104947200000000000000)$                                                                                                                                                                                                                                                                                                                                                                                                               |
| risch         | $(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 4568400000x^{12} - 47827200000x^{11} + 448224000000x^{10} - 3777600000000x^9 + 30720000000000x^8 - 236160000000000x^7 + 1747200000000000x^6 - 12288000000000000x^5 + 80640000000000000x^4 - 491520000000000000x^3 + 2949120000000000000x^2 - 16790400000000000000x - 104947200000000000000)$                                                                                                                                                                                                                                                                                                                                                                                                               |
| meijerg       | $-2432902008176640000 + \frac{(21x^{20} - 420x^{19} + 7980x^{18} - 143640x^{17} + 2441880x^{16} - 39070080x^{15} + 586051200x^{14} - 8208000000x^{13} + 101280000000x^{12} - 1136640000000x^{11} + 12288000000000x^{10} - 128832000000000x^9 + 1307520000000000x^8 - 12883200000000000x^7 + 122880000000000000x^6 - 1012800000000000000x^5 + 7980000000000000000x^4 - 42000000000000000000x^3 + 210000000000000000000x^2 - 1050000000000000000000x + 10494720000000000000000)}$                                                                                                                                                                                                                                                                                                                                             |
| default       | $2432902008176640000 e^x - 2432902008176640000 e^x x + 1216451004088320000 e^x x^2 - 4051353501294976000 e^x x^3 + 1012800000000000000 e^x x^4 - 20256000000000000000 e^x x^5 + 364608000000000000000 e^x x^6 - 6243840000000000000000 e^x x^7 + 100392000000000000000000 e^x x^8 - 1606560000000000000000000 e^x x^9 + 25703040000000000000000000 e^x x^{10} - 411248640000000000000000000 e^x x^{11} + 6579974400000000000000000000 e^x x^{12} - 105279616000000000000000000000 e^x x^{13} + 1764633600000000000000000000000 e^x x^{14} - 29427264000000000000000000000000 e^x x^{15} + 490860160000000000000000000000000 e^x x^{16} - 8264163200000000000000000000000000 e^x x^{17} + 13939276800000000000000000000000000 e^x x^{18} - 239001600000000000000000000000000000 e^x x^{19} + 4051353501294976000 e^x x^{20}$ |
| parallelrisch | $2432902008176640000 e^x - 2432902008176640000 e^x x + 1216451004088320000 e^x x^2 - 4051353501294976000 e^x x^3 + 1012800000000000000 e^x x^4 - 20256000000000000000 e^x x^5 + 364608000000000000000 e^x x^6 - 6243840000000000000000 e^x x^7 + 100392000000000000000000 e^x x^8 - 1606560000000000000000000 e^x x^9 + 25703040000000000000000000 e^x x^{10} - 411248640000000000000000000 e^x x^{11} + 6579974400000000000000000000 e^x x^{12} - 105279616000000000000000000000 e^x x^{13} + 1764633600000000000000000000000 e^x x^{14} - 29427264000000000000000000000000 e^x x^{15} + 490860160000000000000000000000000 e^x x^{16} - 8264163200000000000000000000000000 e^x x^{17} + 13939276800000000000000000000000000 e^x x^{18} - 239001600000000000000000000000000000 e^x x^{19} + 4051353501294976000 e^x x^{20}$ |
| parts         | $2432902008176640000 e^x - 2432902008176640000 e^x x + 1216451004088320000 e^x x^2 - 4051353501294976000 e^x x^3 + 1012800000000000000 e^x x^4 - 20256000000000000000 e^x x^5 + 364608000000000000000 e^x x^6 - 6243840000000000000000 e^x x^7 + 100392000000000000000000 e^x x^8 - 1606560000000000000000000 e^x x^9 + 25703040000000000000000000 e^x x^{10} - 411248640000000000000000000 e^x x^{11} + 6579974400000000000000000000 e^x x^{12} - 105279616000000000000000000000 e^x x^{13} + 1764633600000000000000000000000 e^x x^{14} - 29427264000000000000000000000000 e^x x^{15} + 490860160000000000000000000000000 e^x x^{16} - 8264163200000000000000000000000000 e^x x^{17} + 13939276800000000000000000000000000 e^x x^{18} - 239001600000000000000000000000000000 e^x x^{19} + 4051353501294976000 e^x x^{20}$ |

input `int (exp(x)*x^20, x, method=_RETURNVERBOSE)`

output  $(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 5079110400x^{12} - 60949324800x^{11} + 670442572800x^{10} - 6704425728000x^9 + 60339831552000x^8 - 482718652416000x^7 + 3379030566912000x^6 - 20274183401472000x^5 + 101370917007360000x^4 - 405483668029440000x^3 + 1216451004088320000x^2 - 2432902008176640000x + 2432902008176640000) \cdot \exp(x)$

### 3.159.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.62

$$\int e^x x^{20} dx = (x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 5079110400x^{12} - 60949324800x^{11} + 670442572800x^{10} - 6704425728000x^9 + 60339831552000x^8 - 482718652416000x^7 + 3379030566912000x^6 - 20274183401472000x^5 + 101370917007360000x^4 - 405483668029440000x^3 + 1216451004088320000x^2 - 2432902008176640000x + 2432902008176640000) e^x$$

input `integrate(exp(x)*x^20,x, algorithm="fricas")`

output  $(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 5079110400x^{12} - 60949324800x^{11} + 670442572800x^{10} - 6704425728000x^9 + 60339831552000x^8 - 482718652416000x^7 + 3379030566912000x^6 - 20274183401472000x^5 + 101370917007360000x^4 - 405483668029440000x^3 + 1216451004088320000x^2 - 2432902008176640000x + 2432902008176640000) \cdot e^x$

### 3.159.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.63

$$\int e^x x^{20} dx = (x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 5079110400x^{12} - 60949324800x^{11} + 670442572800x^{10} - 6704425728000x^9 + 60339831552000x^8 - 482718652416000x^7 + 3379030566912000x^6 - 20274183401472000x^5 + 101370917007360000x^4 - 405483668029440000x^3 + 1216451004088320000x^2 - 2432902008176640000x + 2432902008176640000) e^x$$

input `integrate(exp(x)*x**20,x)`

```
output (x**20 - 20*x**19 + 380*x**18 - 6840*x**17 + 116280*x**16 - 1860480*x**15
+ 27907200*x**14 - 390700800*x**13 + 5079110400*x**12 - 60949324800*x**11
+ 670442572800*x**10 - 6704425728000*x**9 + 60339831552000*x**8 - 48271865
2416000*x**7 + 3379030566912000*x**6 - 20274183401472000*x**5 + 1013709170
07360000*x**4 - 405483668029440000*x**3 + 1216451004088320000*x**2 - 24329
02008176640000*x + 2432902008176640000)*exp(x)
```

### 3.159.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.62

$$\int e^x x^{20} dx$$

$$= (x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 5079110400x^{12} - 60949324800x^{11} + 670442572800x^{10} - 6704425728000x^9 + 60339831552000x^8 - 482718652416000x^7 + 3379030566912000x^6 - 20274183401472000x^5 + 101370917007360000x^4 - 405483668029440000x^3 + 1216451004088320000x^2 - 2432902008176640000x + 2432902008176640000)e^x$$

```
input integrate(exp(x)*x^20,x, algorithm="maxima")
```

```
output (x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^15 + 2790
7200*x^14 - 390700800*x^13 + 5079110400*x^12 - 60949324800*x^11 + 67044257
2800*x^10 - 6704425728000*x^9 + 60339831552000*x^8 - 482718652416000*x^7 +
3379030566912000*x^6 - 20274183401472000*x^5 + 101370917007360000*x^4 - 4
05483668029440000*x^3 + 1216451004088320000*x^2 - 2432902008176640000*x +
2432902008176640000)*e^x
```

### 3.159.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.62

$$\int e^x x^{20} dx$$

$$= (x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 5079110400x^{12} - 60949324800x^{11} + 670442572800x^{10} - 6704425728000x^9 + 60339831552000x^8 - 482718652416000x^7 + 3379030566912000x^6 - 20274183401472000x^5 + 101370917007360000x^4 - 405483668029440000x^3 + 1216451004088320000x^2 - 2432902008176640000x + 2432902008176640000)e^x$$

```
input integrate(exp(x)*x^20,x, algorithm="giac")
```

output  $(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 5079110400x^{12} - 60949324800x^{11} + 670442572800x^{10} - 6704425728000x^9 + 60339831552000x^8 - 482718652416000x^7 + 3379030566912000x^6 - 20274183401472000x^5 + 101370917007360000x^4 - 405483668029440000x^3 + 1216451004088320000x^2 - 2432902008176640000x + 2432902008176640000)e^x$

### 3.159.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.62

$$\int e^x x^{20} dx = e^x (x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 5079110400x^{12} - 60949324800x^{11} + 670442572800x^{10} - 6704425728000x^9 + 60339831552000x^8 - 482718652416000x^7 + 3379030566912000x^6 - 20274183401472000x^5 + 101370917007360000x^4 - 405483668029440000x^3 + 1216451004088320000x^2 - 2432902008176640000x + 2432902008176640000)$$

input `int(x^20*exp(x),x)`

output  $\exp(x) \cdot (1216451004088320000x^2 - 2432902008176640000x - 405483668029440000x^3 + 101370917007360000x^4 - 20274183401472000x^5 + 3379030566912000x^6 - 482718652416000x^7 + 60339831552000x^8 - 6704425728000x^9 + 670442572800x^{10} - 60949324800x^{11} + 5079110400x^{12} - 390700800x^{13} + 27907200x^{14} - 1860480x^{15} + 116280x^{16} - 6840x^{17} + 380x^{18} - 20x^{19} + x^{20} + 2432902008176640000)$



## 3.160 $\int a^x b^{-x} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.160.1 Optimal result . . . . .                            | 912 |
| 3.160.2 Mathematica [A] (verified) . . . . .                | 912 |
| 3.160.3 Rubi [A] (verified) . . . . .                       | 913 |
| 3.160.4 Maple [A] (verified) . . . . .                      | 914 |
| 3.160.5 Fricas [A] (verification not implemented) . . . . . | 914 |
| 3.160.6 Sympy [F(-2)] . . . . .                             | 914 |
| 3.160.7 Maxima [F(-2)] . . . . .                            | 915 |
| 3.160.8 Giac [C] (verification not implemented) . . . . .   | 915 |
| 3.160.9 Mupad [B] (verification not implemented) . . . . .  | 916 |

### 3.160.1 Optimal result

Integrand size = 9, antiderivative size = 18

$$\int a^x b^{-x} dx = \frac{a^x b^{-x}}{\log(a) - \log(b)}$$

output  $a^x/(b^x)/(\ln(a)-\ln(b))$

### 3.160.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int a^x b^{-x} dx = \frac{a^x b^{-x}}{\log(a) - \log(b)}$$

input `Integrate[a^x/b^x,x]`

output  $a^x/(b^x*(\text{Log}[a] - \text{Log}[b]))$

**3.160.3 Rubi [A] (verified)**

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int a^x b^{-x} dx \\ \downarrow 2725 \\ \int e^{x(\log(a) - \log(b))} dx \\ \downarrow 2624 \\ \frac{a^x b^{-x}}{\log(a) - \log(b)} \end{array}$$

input `Int[a^x/b^x,x]`

output `a^x/(b^x*(Log[a] - Log[b]))`

**3.160.3.1 Defintions of rubi rules used**

rule 2624 `Int[((F_)^(v_))^(n_), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;`  
`FreeQ[{F, n}, x] && LinearQ[v, x]`

rule 2725 `Int[(u_)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},`  
`Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,`  
`x] && LeQ[Exponent[z, x], 2]]] /; FreeQ[{F, G}, x]`

**3.160.4 Maple [A] (verified)**

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

| method        | result                                                                                                           | size |
|---------------|------------------------------------------------------------------------------------------------------------------|------|
| gospers       | $\frac{a^x b^{-x}}{\ln(a) - \ln(b)}$                                                                             | 19   |
| risch         | $\frac{a^x b^{-x}}{\ln(a) - \ln(b)}$                                                                             | 19   |
| parallelrisch | $\frac{a^x b^{-x}}{\ln(a) - \ln(b)}$                                                                             | 19   |
| norman        | $\frac{e^{x \ln(a)} e^{-x \ln(b)}}{\ln(a) - \ln(b)}$                                                             | 23   |
| meijerg       | $-\frac{1 - e^{x \ln(a) \left(1 - \frac{\ln(b)}{\ln(a)}\right)}}{\ln(a) \left(1 - \frac{\ln(b)}{\ln(a)}\right)}$ | 38   |

input `int(a^x/(b^x),x,method=_RETURNVERBOSE)`output `a^x/(b^x)/(ln(a)-ln(b))`**3.160.5 Fracas [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int a^x b^{-x} dx = \frac{a^x}{b^x (\log(a) - \log(b))}$$

input `integrate(a^x/(b^x),x, algorithm="fracas")`output `a^x/(b^x*(log(a) - log(b)))`**3.160.6 Sympy [F(-2)]**

Exception generated.

$$\int a^x b^{-x} dx = \text{Exception raised: TypeError}$$

input `integrate(a**x/(b**x),x)`

output Exception raised: TypeError >> Invalid NaN comparison

### 3.160.7 Maxima [F(-2)]

Exception generated.

$$\int a^x b^{-x} dx = \text{Exception raised: ValueError}$$

input `integrate(a^x/(b^x),x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(-log(b)/log(a)>0)', see `assume?` for more

### 3.160.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 216, normalized size of antiderivative = 12.00

$$\int a^x b^{-x} dx = 2 \left( \frac{2(\log(|a|) - \log(|b|)) \cos(-\frac{1}{2} \pi x \operatorname{sgn}(a) + \frac{1}{2} \pi x \operatorname{sgn}(b))}{(\pi \operatorname{sgn}(a) - \pi \operatorname{sgn}(b))^2 + 4(\log(|a|) - \log(|b|))^2} - \frac{(\pi \operatorname{sgn}(a) - \pi \operatorname{sgn}(b)) \sin(-\frac{1}{2} \pi x \operatorname{sgn}(a) + \frac{1}{2} \pi x \operatorname{sgn}(b))}{(\pi \operatorname{sgn}(a) - \pi \operatorname{sgn}(b))^2 + 4(\log(|a|) - \log(|b|))^2} \right) + i \left( \frac{e^{(\frac{1}{2} i \pi x \operatorname{sgn}(a) - \frac{1}{2} i \pi x \operatorname{sgn}(b))}}{i \pi \operatorname{sgn}(a) - i \pi \operatorname{sgn}(b) + 2 \log(|a|) - 2 \log(|b|)} - \frac{e^{(-\frac{1}{2} i \pi x \operatorname{sgn}(a) + \frac{1}{2} i \pi x \operatorname{sgn}(b))}}{-i \pi \operatorname{sgn}(a) + i \pi \operatorname{sgn}(b) + 2 \log(|a|) - 2 \log(|b|)} \right)$$

input `integrate(a^x/(b^x),x, algorithm="giac")`

output `2*(2*(log(abs(a)) - log(abs(b)))*cos(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2) - (pi*sgn(a) - pi*sgn(b))*sin(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2))*e^(x*(log(abs(a)) - log(abs(b)))) + I*(I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b))/(I*pi*sgn(a) - I*pi*sgn(b) + 2*log(abs(a)) - 2*log(abs(b))) - I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*x*sgn(b))/(-I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a)) - 2*log(abs(b))))*e^(x*(log(abs(a)) - log(abs(b))))`

**3.160.9 Mupad [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int a^x b^{-x} dx = \frac{a^x}{b^x (\ln(a) - \ln(b))}$$

input `int(a^x/b^x,x)`

output `a^x/(b^x*(log(a) - log(b)))`

## 3.161 $\int a^x b^x dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.161.1 Optimal result . . . . .                            | 917 |
| 3.161.2 Mathematica [A] (verified) . . . . .                | 917 |
| 3.161.3 Rubi [A] (verified) . . . . .                       | 918 |
| 3.161.4 Maple [A] (verified) . . . . .                      | 919 |
| 3.161.5 Fricas [A] (verification not implemented) . . . . . | 919 |
| 3.161.6 Sympy [B] (verification not implemented) . . . . .  | 919 |
| 3.161.7 Maxima [F(-2)] . . . . .                            | 920 |
| 3.161.8 Giac [C] (verification not implemented) . . . . .   | 920 |
| 3.161.9 Mupad [B] (verification not implemented) . . . . .  | 921 |

### 3.161.1 Optimal result

Integrand size = 7, antiderivative size = 14

$$\int a^x b^x dx = \frac{a^x b^x}{\log(a) + \log(b)}$$

output `ax*bx/(ln(a)+ln(b))`

### 3.161.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int a^x b^x dx = \frac{a^x b^x}{\log(a) + \log(b)}$$

input `Integrate[ax*bx,x]`

output `(ax*bx)/(Log[a] + Log[b])`

**3.161.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int a^x b^x dx \\ \downarrow 2725 \\ \int e^{x(\log(a)+\log(b))} dx \\ \downarrow 2624 \\ \frac{a^x b^x}{\log(a) + \log(b)} \end{array}$$

input `Int[a^x*b^x,x]`

output `(a^x*b^x)/(Log[a] + Log[b])`

**3.161.3.1 Defintions of rubi rules used**

rule 2624 `Int[((F_)^(v_))^(n_), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;`  
`FreeQ[{F, n}, x] && LinearQ[v, x]`

rule 2725 `Int[(u_)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},`  
`Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,`  
`x] && LeQ[Exponent[z, x], 2]] /; FreeQ[{F, G}, x]`

**3.161.4 Maple [A] (verified)**

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

| method        | result                                                                                                     | size |
|---------------|------------------------------------------------------------------------------------------------------------|------|
| gospers       | $\frac{a^x b^x}{\ln(a)+\ln(b)}$                                                                            | 15   |
| risch         | $\frac{a^x b^x}{\ln(a)+\ln(b)}$                                                                            | 15   |
| parallelrisch | $\frac{a^x b^x}{\ln(a)+\ln(b)}$                                                                            | 15   |
| norman        | $\frac{e^{x \ln(a)} e^{x \ln(b)}}{\ln(a)+\ln(b)}$                                                          | 19   |
| meijerg       | $-\frac{1-e^{x \ln(b) \left(1+\frac{\ln(a)}{\ln(b)}\right)}}{\ln(b) \left(1+\frac{\ln(a)}{\ln(b)}\right)}$ | 36   |

input `int(a^x*b^x,x,method=_RETURNVERBOSE)`

output `a^x*b^x/(ln(a)+ln(b))`

**3.161.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int a^x b^x dx = \frac{a^x b^x}{\log(a) + \log(b)}$$

input `integrate(a^x*b^x,x, algorithm="fricas")`

output `a^x*b^x/(log(a) + log(b))`

**3.161.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 31 vs.  $2(12) = 24$ .

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int a^x b^x dx = \begin{cases} \frac{a^x b^x}{\log(a)+\log(b)} & \text{for } a \neq \frac{1}{b} \\ \frac{b^x \left(\frac{1}{b}\right)^x}{\log\left(\frac{1}{b}\right)+\log(b)} & \text{otherwise} \end{cases}$$



input `integrate(a**x*b**x,x)`

output `Piecewise((a**x*b**x/(log(a) + log(b)), Ne(a, 1/b)), (b**x*(1/b)**x/(log(1/b) + log(b)), True))`

### 3.161.7 Maxima [F(-2)]

Exception generated.

$$\int a^x b^x dx = \text{Exception raised: ValueError}$$

input `integrate(a^x*b^x,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(log(b)/log(a)>0)', see `assume?` for more`

### 3.161.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 16.93

$$\int a^x b^x dx = 2 \left( \frac{2(\log(|a|) + \log(|b|)) \cos(-\frac{1}{2} \pi x \operatorname{sgn}(a) - \frac{1}{2} \pi x \operatorname{sgn}(b) + \pi x)}{(2\pi - \pi \operatorname{sgn}(a) - \pi \operatorname{sgn}(b))^2 + 4(\log(|a|) + \log(|b|))^2} + \frac{(2\pi - \pi \operatorname{sgn}(a) - \pi \operatorname{sgn}(b)) \sin(-\frac{1}{2} \pi x \operatorname{sgn}(a) - \frac{1}{2} \pi x \operatorname{sgn}(b) + \pi x)}{(2\pi - \pi \operatorname{sgn}(a) - \pi \operatorname{sgn}(b))^2 + 4(\log(|a|) + \log(|b|))^2} \right) + i \left( \frac{i e^{(\frac{1}{2} i \pi x \operatorname{sgn}(a) + \frac{1}{2} i \pi x \operatorname{sgn}(b) - i \pi x)}}{-2i\pi + i\pi \operatorname{sgn}(a) + i\pi \operatorname{sgn}(b) + 2\log(|a|) + 2\log(|b|)} - \frac{i e^{(-\frac{1}{2} i \pi x \operatorname{sgn}(a) - \frac{1}{2} i \pi x \operatorname{sgn}(b) + i \pi x)}}{2i\pi - i\pi \operatorname{sgn}(a) - i\pi \operatorname{sgn}(b) + 2\log(|a|) + 2\log(|b|)} \right)$$

input `integrate(a^x*b^x,x, algorithm="giac")`

```
output 2*(2*(log(abs(a)) + log(abs(b)))*cos(-1/2*pi*x*sgn(a) - 1/2*pi*x*sgn(b) +
pi*x)/((2*pi - pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) + log(abs(b)))^2)
+ (2*pi - pi*sgn(a) - pi*sgn(b))*sin(-1/2*pi*x*sgn(a) - 1/2*pi*x*sgn(b) +
pi*x)/((2*pi - pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) + log(abs(b)))^2
))*e^(x*(log(abs(a)) + log(abs(b)))) + I*(I*e^(1/2*I*pi*x*sgn(a) + 1/2*I*pi
*x*sgn(b) - I*pi*x)/(-2*I*pi + I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a))
+ 2*log(abs(b))) - I*e^(-1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b) + I*pi*x)/(
2*I*pi - I*pi*sgn(a) - I*pi*sgn(b) + 2*log(abs(a)) + 2*log(abs(b))))*e^(x*
(log(abs(a)) + log(abs(b))))
```

### 3.161.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int a^x b^x dx = \frac{a^x b^x}{\ln(a) + \ln(b)}$$

```
input int(a^x*b^x,x)
```

```
output (a^x*b^x)/(log(a) + log(b))
```

### 3.162 $\int \frac{a^x}{x^2} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.162.1 Optimal result . . . . .                            | 922 |
| 3.162.2 Mathematica [A] (verified) . . . . .                | 922 |
| 3.162.3 Rubi [A] (verified) . . . . .                       | 923 |
| 3.162.4 Maple [A] (verified) . . . . .                      | 924 |
| 3.162.5 Fricas [A] (verification not implemented) . . . . . | 924 |
| 3.162.6 Sympy [F] . . . . .                                 | 924 |
| 3.162.7 Maxima [A] (verification not implemented) . . . . . | 925 |
| 3.162.8 Giac [F] . . . . .                                  | 925 |
| 3.162.9 Mupad [B] (verification not implemented) . . . . .  | 925 |

#### 3.162.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \frac{a^x}{x^2} dx = -\frac{a^x}{x} + \text{ExpIntegralEi}(x \log(a)) \log(a)$$

output `-a^x/x+Ei(x*ln(a))*ln(a)`

#### 3.162.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a^x}{x^2} dx = -\frac{a^x}{x} + \text{ExpIntegralEi}(x \log(a)) \log(a)$$

input `Integrate[a^x/x^2,x]`

output `-(a^x/x) + ExpIntegralEi[x*Log[a]]*Log[a]`

### 3.162.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \frac{a^x}{x^2} dx \\ \downarrow \text{2608} \\ \log(a) \int \frac{a^x}{x} dx - \frac{a^x}{x} \\ \downarrow \text{2609} \\ \log(a) \text{ExpIntegralEi}(x \log(a)) - \frac{a^x}{x} \end{array}$$

input `Int[a^x/x^2,x]`

output `-(a^x/x) + ExpIntegralEi[x*Log[a]]*Log[a]`

#### 3.162.3.1 Defintions of rubi rules used

rule 2608 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1))), x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2609 `Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

**3.162.4 Maple [A] (verified)**

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

| method  | result                                                                                                                                                                                          |
|---------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| risch   | $-\frac{a^x}{x} - \ln(a) \operatorname{Ei}_1(-x \ln(a))$                                                                                                                                        |
| meijerg | $-\ln(a) \left( \frac{1}{x \ln(a)} + 1 - \ln(x) - i\pi - \ln(\ln(a)) - \frac{2+2x \ln(a)}{2x \ln(a)} + \frac{e^{x \ln(a)}}{x \ln(a)} + \ln(-x \ln(a)) + \operatorname{Ei}_1(-x \ln(a)) \right)$ |

input `int(a^x/x^2,x,method=_RETURNVERBOSE)`output `-a^x/x-ln(a)*Ei(1,-x*ln(a))`**3.162.5 Fracas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{a^x}{x^2} dx = \frac{x \operatorname{Ei}(x \log(a)) \log(a) - a^x}{x}$$

input `integrate(a^x/x^2,x, algorithm="fracas")`output `(x*Ei(x*log(a))*log(a) - a^x)/x`**3.162.6 Sympy [F]**

$$\int \frac{a^x}{x^2} dx = \int \frac{a^x}{x^2} dx$$

input `integrate(a**x/x**2,x)`output `Integral(a**x/x**2, x)`

**3.162.7 Maxima [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{a^x}{x^2} dx = \Gamma(-1, -x \log(a)) \log(a)$$

input `integrate(a^x/x^2,x, algorithm="maxima")`output `gamma(-1, -x*log(a))*log(a)`**3.162.8 Giac [F]**

$$\int \frac{a^x}{x^2} dx = \int \frac{a^x}{x^2} dx$$

input `integrate(a^x/x^2,x, algorithm="giac")`output `integrate(a^x/x^2, x)`**3.162.9 Mupad [B] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{a^x}{x^2} dx = -\ln(a) \exp(-x \ln(a)) - \frac{a^x}{x}$$

input `int(a^x/x^2,x)`output `-log(a)*expint(-x*log(a)) - a^x/x`

### 3.163 $\int \frac{a^x x}{(1+bx)^2} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.163.1 Optimal result . . . . .                            | 926 |
| 3.163.2 Mathematica [A] (verified) . . . . .                | 926 |
| 3.163.3 Rubi [A] (verified) . . . . .                       | 927 |
| 3.163.4 Maple [A] (verified) . . . . .                      | 928 |
| 3.163.5 Fricas [A] (verification not implemented) . . . . . | 928 |
| 3.163.6 Sympy [F] . . . . .                                 | 928 |
| 3.163.7 Maxima [F] . . . . .                                | 929 |
| 3.163.8 Giac [F] . . . . .                                  | 929 |
| 3.163.9 Mupad [F(-1)] . . . . .                             | 929 |

#### 3.163.1 Optimal result

Integrand size = 12, antiderivative size = 64

$$\int \frac{a^x x}{(1+bx)^2} dx = \frac{a^x}{b^2(1+bx)} + \frac{a^{-1/b} \text{ExpIntegralEi}\left(\frac{(1+bx)\log(a)}{b}\right)}{b^2} - \frac{a^{-1/b} \text{ExpIntegralEi}\left(\frac{(1+bx)\log(a)}{b}\right) \log(a)}{b^3}$$

```
output a^x/b^2/(b*x+1)+Ei((b*x+1)*ln(a)/b)/(a^(1/b))/b^2-Ei((b*x+1)*ln(a)/b)*ln(a)/(a^(1/b))/b^3
```

#### 3.163.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.67

$$\int \frac{a^x x}{(1+bx)^2} dx = \frac{\frac{a^x b}{1+bx} + a^{-1/b} \text{ExpIntegralEi}\left(\frac{(1+bx)\log(a)}{b}\right) (b - \log(a))}{b^3}$$

```
input Integrate[(a^x*x)/(1 + b*x)^2,x]
```

```
output ((a^x*b)/(1 + b*x) + (ExpIntegralEi[((1 + b*x)*Log[a])/b]*(b - Log[a]))/a^b^(-1))/b^3
```

**3.163.3 Rubi [A] (verified)**

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x a^x}{(b x + 1)^2} dx$$

$$\downarrow 2629$$

$$\int \left( \frac{a^x}{b(b x + 1)} - \frac{a^x}{b(b x + 1)^2} \right) dx$$

$$\downarrow 2009$$

$$-\frac{a^{-1/b} \log(a) \operatorname{ExpIntegralEi}\left(\frac{(b x + 1) \log(a)}{b}\right)}{b^3} + \frac{a^{-1/b} \operatorname{ExpIntegralEi}\left(\frac{(b x + 1) \log(a)}{b}\right)}{b^2} + \frac{a^x}{b^2(b x + 1)}$$

input `Int[(a^x*x)/(1 + b*x)^2,x]`

output `a^x/(b^2*(1 + b*x)) + ExpIntegralEi[((1 + b*x)*Log[a])/b]/(a^b^(-1)*b^2) - (ExpIntegralEi[((1 + b*x)*Log[a])/b]*Log[a])/b/(a^b^(-1)*b^3)`

**3.163.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2629 `Int[(F_)^(v_)*(Px_)*((d_) + (e_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`



**3.163.4 Maple [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.23

| method | result                                                                                                                                                                                                                                                         | size |
|--------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch  | $-\frac{a^{-\frac{1}{b}} \operatorname{Ei}_1\left(-x \ln(a) - \frac{\ln(a)}{b}\right)}{b^2} + \frac{\ln(a)a^x}{b^3\left(x \ln(a) + \frac{\ln(a)}{b}\right)} + \frac{\ln(a)a^{-\frac{1}{b}} \operatorname{Ei}_1\left(-x \ln(a) - \frac{\ln(a)}{b}\right)}{b^3}$ | 79   |

input `int(a^x*x/(b*x+1)^2,x,method=_RETURNVERBOSE)`output `-1/b^2*a^(-1/b)*Ei(1,-x*ln(a)-ln(a)/b)+ln(a)/b^3*a^x/(x*ln(a)+ln(a)/b)+ln(a)/b^3*a^(-1/b)*Ei(1,-x*ln(a)-ln(a)/b)`**3.163.5 Fracas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{a^x x}{(1+bx)^2} dx = \frac{a^x b + \frac{(b^2 x - (bx+1) \log(a) + b) \operatorname{Ei}\left(\frac{(bx+1) \log(a)}{b}\right)}{a^{\frac{1}{b}}}}{b^4 x + b^3}$$

input `integrate(a^x*x/(b*x+1)^2,x, algorithm="fracas")`output `(a^x*b + (b^2*x - (b*x + 1)*log(a) + b)*Ei((b*x + 1)*log(a)/b)/a^(1/b))/(b^4*x + b^3)`**3.163.6 Sympy [F]**

$$\int \frac{a^x x}{(1+bx)^2} dx = \int \frac{a^x x}{(bx+1)^2} dx$$

input `integrate(a**x*x/(b*x+1)**2,x)`output `Integral(a**x*x/(b*x + 1)**2, x)`

**3.163.7 Maxima [F]**

$$\int \frac{a^x x}{(1+bx)^2} dx = \int \frac{a^x x}{(bx+1)^2} dx$$

input `integrate(a^x*x/(b*x+1)^2,x, algorithm="maxima")`

output `a^x*x/(b^2*x^2*log(a) + 2*b*x*log(a) + log(a)) + integrate((b*x - 1)*a^x/(b^3*x^3*log(a) + 3*b^2*x^2*log(a) + 3*b*x*log(a) + log(a)), x)`

**3.163.8 Giac [F]**

$$\int \frac{a^x x}{(1+bx)^2} dx = \int \frac{a^x x}{(bx+1)^2} dx$$

input `integrate(a^x*x/(b*x+1)^2,x, algorithm="giac")`

output `integrate(a^x*x/(b*x + 1)^2, x)`

**3.163.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{a^x x}{(1+bx)^2} dx = \int \frac{a^x x}{(bx+1)^2} dx$$

input `int((a^x*x)/(b*x + 1)^2,x)`

output `int((a^x*x)/(b*x + 1)^2, x)`

### 3.164 $\int \frac{e^{ax}x}{(1+ax)^2} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.164.1 Optimal result . . . . .                            | 930 |
| 3.164.2 Mathematica [A] (verified) . . . . .                | 930 |
| 3.164.3 Rubi [A] (verified) . . . . .                       | 931 |
| 3.164.4 Maple [A] (verified) . . . . .                      | 931 |
| 3.164.5 Fricas [A] (verification not implemented) . . . . . | 932 |
| 3.164.6 Sympy [A] (verification not implemented) . . . . .  | 932 |
| 3.164.7 Maxima [A] (verification not implemented) . . . . . | 932 |
| 3.164.8 Giac [B] (verification not implemented) . . . . .   | 933 |
| 3.164.9 Mupad [B] (verification not implemented) . . . . .  | 933 |

#### 3.164.1 Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{e^{ax}x}{(1+ax)^2} dx = \frac{e^{ax}}{a^2(1+ax)}$$

output `exp(a*x)/a^2/(a*x+1)`

#### 3.164.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}x}{(1+ax)^2} dx = \frac{e^{ax}}{a^2(1+ax)}$$

input `Integrate[(E^(a*x)*x)/(1 + a*x)^2,x]`

output `E^(a*x)/(a^2*(1 + a*x))`

**3.164.3 Rubi [A] (verified)**

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {2627}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x e^{ax}}{(ax+1)^2} dx$$

$$\downarrow \text{2627}$$

$$\frac{e^{ax}}{a^2(ax+1)}$$

input `Int[(E^(a*x)*x)/(1 + a*x)^2,x]`

output `E^(a*x)/(a^2*(1 + a*x))`

**3.164.3.1 Defintions of rubi rules used**

rule 2627 `Int[(F_)^(v_)*((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_)), x_Symbol] :=  
Simp[g*(d + e*x)^(m + 1)*(F^v/(D[v, x]*e*Log[F])), x] /; FreeQ[{F, d, e, f,  
, g, m}, x] && LinearQ[v, x] && EqQ[e*g*(m + 1) - D[v, x]*(e*f - d*g)*Log[F], 0]`

**3.164.4 Maple [A] (verified)**

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

| method            | result                     | size |
|-------------------|----------------------------|------|
| gospers           | $\frac{e^{ax}}{a^2(ax+1)}$ | 16   |
| derivativedivides | $\frac{e^{ax}}{a^2(ax+1)}$ | 16   |
| default           | $\frac{e^{ax}}{a^2(ax+1)}$ | 16   |
| norman            | $\frac{e^{ax}}{a^2(ax+1)}$ | 16   |
| risch             | $\frac{e^{ax}}{a^2(ax+1)}$ | 16   |
| parallelrisch     | $\frac{e^{ax}}{a^2(ax+1)}$ | 16   |

input `int(exp(a*x)*x/(a*x+1)^2,x,method=_RETURNVERBOSE)`

output `exp(a*x)/a^2/(a*x+1)`

### 3.164.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}x}{(1+ax)^2} dx = \frac{e^{(ax)}}{a^3x + a^2}$$

input `integrate(exp(a*x)*x/(a*x+1)^2,x, algorithm="fricas")`

output `e^(a*x)/(a^3*x + a^2)`

### 3.164.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{e^{ax}x}{(1+ax)^2} dx = \frac{e^{ax}}{a^3x + a^2}$$

input `integrate(exp(a*x)*x/(a*x+1)**2,x)`

output `exp(a*x)/(a**3*x + a**2)`

### 3.164.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{e^{ax}x}{(1+ax)^2} dx = \frac{e^{(ax)}}{a^3x + a^2}$$

input `integrate(exp(a*x)*x/(a*x+1)^2,x, algorithm="maxima")`

output `e^(a*x)/(a^3*x + a^2)`

**3.164.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 45 vs.  $2(15) = 30$ .

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.81

$$\int \frac{e^{ax}x}{(1+ax)^2} dx = -\frac{e^{-(ax+1)\left(\frac{1}{ax+1}-1\right)}}{(ax+1)a^2\left(\frac{1}{ax+1}-1\right)-a^2}$$

input `integrate(exp(a*x)*x/(a*x+1)^2,x, algorithm="giac")`

output `-e^(-(a*x + 1)*(1/(a*x + 1) - 1))/((a*x + 1)*a^2*(1/(a*x + 1) - 1) - a^2)`

**3.164.9 Mupad [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{e^{ax}x}{(1+ax)^2} dx = \frac{e^{ax}}{a^2(ax+1)}$$

input `int((x*exp(a*x))/(a*x + 1)^2,x)`

output `exp(a*x)/(a^2*(a*x + 1))`

## 3.165 $\int k^{x^2} x dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.165.1 Optimal result . . . . .                            | 934 |
| 3.165.2 Mathematica [A] (verified) . . . . .                | 934 |
| 3.165.3 Rubi [A] (verified) . . . . .                       | 935 |
| 3.165.4 Maple [A] (verified) . . . . .                      | 936 |
| 3.165.5 Fricas [A] (verification not implemented) . . . . . | 936 |
| 3.165.6 Sympy [A] (verification not implemented) . . . . .  | 937 |
| 3.165.7 Maxima [A] (verification not implemented) . . . . . | 937 |
| 3.165.8 Giac [A] (verification not implemented) . . . . .   | 937 |
| 3.165.9 Mupad [B] (verification not implemented) . . . . .  | 938 |

### 3.165.1 Optimal result

Integrand size = 7, antiderivative size = 13

$$\int k^{x^2} x dx = \frac{k^{x^2}}{2 \log(k)}$$

output  $1/2*k^{(x^2)}/\ln(k)$

### 3.165.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int k^{x^2} x dx = \frac{k^{x^2}}{2 \log(k)}$$

input `Integrate[k^x^2*x,x]`

output  $k^{x^2}/(2*\text{Log}[k])$

**3.165.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x k^{x^2} dx$$

$$\downarrow 2638$$

$$\frac{k^{x^2}}{2 \log(k)}$$

input `Int [k^x^2*x, x]`

output `k^x^2/(2*Log[k])`

**3.165.3.1 Defintions of rubi rules used**

rule 2638 `Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^(n_)))*((e_.) + (f_.)*(x_)^(m_.), x_Symbol] := Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n *Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]`



**3.165.4 Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

| method           | result                              | size |
|------------------|-------------------------------------|------|
| gospers          | $\frac{k^{x^2}}{2 \ln(k)}$          | 12   |
| derivativdivides | $\frac{k^{x^2}}{2 \ln(k)}$          | 12   |
| default          | $\frac{k^{x^2}}{2 \ln(k)}$          | 12   |
| risch            | $\frac{k^{x^2}}{2 \ln(k)}$          | 12   |
| parallelrisch    | $\frac{k^{x^2}}{2 \ln(k)}$          | 12   |
| norman           | $\frac{e^{\ln(k)x^2}}{2 \ln(k)}$    | 14   |
| meijerg          | $-\frac{1-e^{\ln(k)x^2}}{2 \ln(k)}$ | 18   |

input `int(k^(x^2)*x,x,method=_RETURNVERBOSE)`output `1/2*k^(x^2)/ln(k)`**3.165.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int k^{x^2} x dx = \frac{k^{(x^2)}}{2 \log(k)}$$

input `integrate(k^(x^2)*x,x, algorithm="fricas")`output `1/2*k^(x^2)/log(k)`

**3.165.6 Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int k^{x^2} x dx = \begin{cases} \frac{k^{x^2}}{2 \log(k)} & \text{for } \log(k) \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

input `integrate(k**(x**2)*x,x)`output `Piecewise((k**(x**2)/(2*log(k)), Ne(log(k), 0)), (x**2/2, True))`**3.165.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int k^{x^2} x dx = \frac{k^{(x^2)}}{2 \log(k)}$$

input `integrate(k^(x^2)*x,x, algorithm="maxima")`output `1/2*k^(x^2)/log(k)`**3.165.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int k^{x^2} x dx = \frac{k^{(x^2)}}{2 \log(k)}$$

input `integrate(k^(x^2)*x,x, algorithm="giac")`output `1/2*k^(x^2)/log(k)`

**3.165.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int k^{x^2} x dx = \frac{k^{x^2}}{2 \ln(k)}$$

input `int(k^(x^2)*x,x)`

output `k^(x^2)/(2*log(k))`

### 3.166 $\int e^{x^2} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.166.1 Optimal result . . . . .                            | 939 |
| 3.166.2 Mathematica [A] (verified) . . . . .                | 939 |
| 3.166.3 Rubi [A] (verified) . . . . .                       | 940 |
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| 3.166.5 Fricas [A] (verification not implemented) . . . . . | 941 |
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| 3.166.8 Giac [C] (verification not implemented) . . . . .   | 942 |
| 3.166.9 Mupad [B] (verification not implemented) . . . . .  | 942 |

#### 3.166.1 Optimal result

Integrand size = 5, antiderivative size = 11

$$\int e^{x^2} dx = \frac{1}{2}\sqrt{\pi}\operatorname{erfi}(x)$$

output `1/2*erfi(x)*Pi^(1/2)`

#### 3.166.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{x^2} dx = \frac{1}{2}\sqrt{\pi}\operatorname{erfi}(x)$$

input `Integrate[E^x^2,x]`

output `(Sqrt[Pi]*Erfi[x])/2`

**3.166.3 Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} dx$$

↓ 2633

$$\frac{1}{2}\sqrt{\pi}\operatorname{erfi}(x)$$

input `Int [E^x^2,x]`

output `(Sqrt [Pi]*Erfi [x])/2`

**3.166.3.1 Defintions of rubi rules used**

rule 2633 `Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt [Pi]*(Erfi[(c + d*x)*Rt [b*Log [F], 2]]/(2*d*Rt [b*Log [F], 2])), x] /; FreeQ[{ F, a, b, c, d}, x] && PosQ[b]`

**3.166.4 Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

| method  | result                                       | size |
|---------|----------------------------------------------|------|
| default | $\frac{\operatorname{erfi}(x)\sqrt{\pi}}{2}$ | 8    |
| meijerg | $\frac{\operatorname{erfi}(x)\sqrt{\pi}}{2}$ | 8    |
| risch   | $\frac{\operatorname{erfi}(x)\sqrt{\pi}}{2}$ | 8    |

input `int (exp (x^2) ,x,method=_RETURNVERBOSE)`

output `1/2*erfi(x)*Pi^(1/2)`

**3.166.5 Fricas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int e^{x^2} dx = \frac{1}{2} \sqrt{\pi} \operatorname{erfi}(x)$$

input `integrate(exp(x^2),x, algorithm="fricas")`

output `1/2*sqrt(pi)*erfi(x)`

**3.166.6 Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{x^2} dx = \frac{\sqrt{\pi} \operatorname{erfi}(x)}{2}$$

input `integrate(exp(x**2),x)`

output `sqrt(pi)*erfi(x)/2`

**3.166.7 Maxima [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int e^{x^2} dx = -\frac{1}{2}i \sqrt{\pi} \operatorname{erf}(ix)$$

input `integrate(exp(x^2),x, algorithm="maxima")`

output `-1/2*I*sqrt(pi)*erf(I*x)`

**3.166.8 Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int e^{x^2} dx = \frac{1}{2}i\sqrt{\pi} \operatorname{erf}(-ix)$$

input `integrate(exp(x^2),x, algorithm="giac")`

output `1/2*I*sqrt(pi)*erf(-I*x)`

**3.166.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int e^{x^2} dx = \frac{\sqrt{\pi} \operatorname{erfi}(x)}{2}$$

input `int(exp(x^2),x)`

output `(pi^(1/2)*erfi(x))/2`

## 3.167 $\int e^{x^2} x dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.167.1 Optimal result . . . . .                            | 943 |
| 3.167.2 Mathematica [A] (verified) . . . . .                | 943 |
| 3.167.3 Rubi [A] (verified) . . . . .                       | 944 |
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| 3.167.5 Fricas [A] (verification not implemented) . . . . . | 945 |
| 3.167.6 Sympy [A] (verification not implemented) . . . . .  | 946 |
| 3.167.7 Maxima [A] (verification not implemented) . . . . . | 946 |
| 3.167.8 Giac [A] (verification not implemented) . . . . .   | 946 |
| 3.167.9 Mupad [B] (verification not implemented) . . . . .  | 947 |

### 3.167.1 Optimal result

Integrand size = 7, antiderivative size = 9

$$\int e^{x^2} x dx = \frac{e^{x^2}}{2}$$

output `1/2*exp(x^2)`

### 3.167.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int e^{x^2} x dx = \frac{e^{x^2}}{2}$$

input `Integrate[E^x^2*x,x]`

output `E^x^2/2`



**3.167.3 Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} x dx$$

$$\downarrow \text{2638}$$

$$\frac{e^{x^2}}{2}$$

input `Int [E^x^2*x,x]`

output `E^x^2/2`

**3.167.3.1 Defintions of rubi rules used**

rule 2638 `Int[(F_)^((a_) + (b_)*((c_) + (d_)*(x_))^(n_))*((e_) + (f_)*(x_))^(m_ .), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n *Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ [d*e - c*f, 0]`

**3.167.4 Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

| method            | result                                                                                                                                 | size |
|-------------------|----------------------------------------------------------------------------------------------------------------------------------------|------|
| gospers           | $\frac{e^{x^2}}{2}$                                                                                                                    | 7    |
| derivativedivides | $\frac{e^{x^2}}{2}$                                                                                                                    | 7    |
| default           | $\frac{e^{x^2}}{2}$                                                                                                                    | 7    |
| norman            | $\frac{e^{x^2}}{2}$                                                                                                                    | 7    |
| risch             | $\frac{e^{x^2}}{2}$                                                                                                                    | 7    |
| parallelrisch     | $\frac{e^{x^2}}{2}$                                                                                                                    | 7    |
| meijerg           | $-\frac{1}{2} + \frac{e^{x^2}}{2}$                                                                                                     | 9    |
| parts             | $\frac{\operatorname{erfi}(x)\sqrt{\pi}x}{2} - \frac{\sqrt{\pi}\left(x \operatorname{erfi}(x) - \frac{e^{x^2}}{\sqrt{\pi}}\right)}{2}$ | 29   |

input `int(exp(x^2)*x,x,method=_RETURNVERBOSE)`output `1/2*exp(x^2)`**3.167.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x dx = \frac{1}{2} e^{(x^2)}$$

input `integrate(exp(x^2)*x,x, algorithm="fricas")`output `1/2*e^(x^2)`

**3.167.6 Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int e^{x^2} x dx = \frac{e^{x^2}}{2}$$

input `integrate(exp(x**2)*x,x)`

output `exp(x**2)/2`

**3.167.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x dx = \frac{1}{2} e^{(x^2)}$$

input `integrate(exp(x^2)*x,x, algorithm="maxima")`

output `1/2*e^(x^2)`

**3.167.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x dx = \frac{1}{2} e^{(x^2)}$$

input `integrate(exp(x^2)*x,x, algorithm="giac")`

output `1/2*e^(x^2)`

**3.167.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x dx = \frac{e^{x^2}}{2}$$

input `int(x*exp(x^2),x)`

output `exp(x^2)/2`

$$3.168 \quad \int \frac{e^{\frac{1}{x}}(1+x)}{x^4} dx$$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.168.1 Optimal result . . . . .                            | 948 |
| 3.168.2 Mathematica [A] (verified) . . . . .                | 948 |
| 3.168.3 Rubi [A] (verified) . . . . .                       | 949 |
| 3.168.4 Maple [A] (verified) . . . . .                      | 950 |
| 3.168.5 Fricas [A] (verification not implemented) . . . . . | 950 |
| 3.168.6 Sympy [A] (verification not implemented) . . . . .  | 951 |
| 3.168.7 Maxima [C] (verification not implemented) . . . . . | 951 |
| 3.168.8 Giac [A] (verification not implemented) . . . . .   | 951 |
| 3.168.9 Mupad [B] (verification not implemented) . . . . .  | 952 |

### 3.168.1 Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \frac{e^{\frac{1}{x}}(1+x)}{x^4} dx = -e^{\frac{1}{x}} - \frac{e^{\frac{1}{x}}}{x^2} + \frac{e^{\frac{1}{x}}}{x}$$

output `-exp(1/x)-exp(1/x)/x^2+exp(1/x)/x`

### 3.168.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{e^{\frac{1}{x}}(1+x)}{x^4} dx = \frac{e^{\frac{1}{x}}(-1+x-x^2)}{x^2}$$

input `Integrate[(E^x^(-1))*(1 + x))/x^4,x]`

output `(E^x^(-1))*(-1 + x - x^2))/x^2`

**3.168.3 Rubi [A] (verified)**

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{x}}(x+1)}{x^4} dx$$

$$\downarrow \text{7293}$$

$$\int \left( \frac{e^{\frac{1}{x}}}{x^4} + \frac{e^{\frac{1}{x}}}{x^3} \right) dx$$

$$\downarrow \text{2009}$$

$$-\frac{e^{\frac{1}{x}}}{x^2} - e^{\frac{1}{x}} + \frac{e^{\frac{1}{x}}}{x}$$

input `Int[(E^x^(-1))*(1 + x))/x^4,x]`

output `-E^x^(-1) - E^x^(-1)/x^2 + E^x^(-1)/x`

**3.168.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

rule 7293 `Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]  
]`

**3.168.4 Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

| method            | result                                                                                                                              | size |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------------|------|
| gosper            | $-\frac{(x^2-x+1)e^{\frac{1}{x}}}{x^2}$                                                                                             | 18   |
| risch             | $-\frac{(x^2-x+1)e^{\frac{1}{x}}}{x^2}$                                                                                             | 18   |
| derivativedivides | $-e^{\frac{1}{x}} - \frac{e^{\frac{1}{x}}}{x^2} + \frac{e^{\frac{1}{x}}}{x}$                                                        | 25   |
| default           | $-e^{\frac{1}{x}} - \frac{e^{\frac{1}{x}}}{x^2} + \frac{e^{\frac{1}{x}}}{x}$                                                        | 25   |
| parallelrisch     | $-\frac{x^2e^{\frac{1}{x}}-xe^{\frac{1}{x}}+e^{\frac{1}{x}}}{x^2}$                                                                  | 26   |
| norman            | $\frac{x^2e^{\frac{1}{x}}-xe^{\frac{1}{x}}-x^3e^{\frac{1}{x}}}{x^3}$                                                                | 30   |
| meijerg           | $1 - \frac{\left(\frac{3}{x^2} - \frac{6}{x} + 6\right)e^{\frac{1}{x}}}{3} + \frac{\left(2 - \frac{2}{x}\right)e^{\frac{1}{x}}}{2}$ | 34   |

input `int(exp(1/x)*(1+x)/x^4,x,method=_RETURNVERBOSE)`output `-(x^2-x+1)*exp(1/x)/x^2`**3.168.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{e^{\frac{1}{x}}(1+x)}{x^4} dx = -\frac{(x^2-x+1)e^{\frac{1}{x}}}{x^2}$$

input `integrate(exp(1/x)*(1+x)/x^4,x, algorithm="fricas")`output `-(x^2 - x + 1)*e^(1/x)/x^2`

**3.168.6 Sympy [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{e^{\frac{1}{x}}(1+x)}{x^4} dx = \frac{(-x^2 + x - 1)e^{\frac{1}{x}}}{x^2}$$

input `integrate(exp(1/x)*(1+x)/x**4,x)`

output `(-x**2 + x - 1)*exp(1/x)/x**2`

**3.168.7 Maxima [C] (verification not implemented)**

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{e^{\frac{1}{x}}(1+x)}{x^4} dx = -\Gamma\left(3, -\frac{1}{x}\right) + \Gamma\left(2, -\frac{1}{x}\right)$$

input `integrate(exp(1/x)*(1+x)/x^4,x, algorithm="maxima")`

output `-gamma(3, -1/x) + gamma(2, -1/x)`

**3.168.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{e^{\frac{1}{x}}(1+x)}{x^4} dx = \frac{e^{\frac{1}{x}}}{x} - \frac{e^{\frac{1}{x}}}{x^2} - e^{\frac{1}{x}}$$

input `integrate(exp(1/x)*(1+x)/x^4,x, algorithm="giac")`

output `e^(1/x)/x - e^(1/x)/x^2 - e^(1/x)`



**3.168.9 Mupad [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{e^{\frac{1}{x}}(1+x)}{x^4} dx = -\frac{e^{1/x}(x^2 - x + 1)}{x^2}$$

input `int((exp(1/x)*(x + 1))/x^4,x)`

output `-(exp(1/x)*(x^2 - x + 1))/x^2`

$$3.169 \quad \int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2} dx$$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.169.1 Optimal result . . . . .                            | 953 |
| 3.169.2 Mathematica [A] (verified) . . . . .                | 953 |
| 3.169.3 Rubi [F] . . . . .                                  | 954 |
| 3.169.4 Maple [A] (verified) . . . . .                      | 955 |
| 3.169.5 Fricas [A] (verification not implemented) . . . . . | 955 |
| 3.169.6 Sympy [A] (verification not implemented) . . . . .  | 955 |
| 3.169.7 Maxima [A] (verification not implemented) . . . . . | 956 |
| 3.169.8 Giac [A] (verification not implemented) . . . . .   | 956 |
| 3.169.9 Mupad [F(-1)] . . . . .                             | 956 |

### 3.169.1 Optimal result

Integrand size = 37, antiderivative size = 25

$$\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2} dx = -\frac{e^{1-e^{x^2}x}}{-1+e^{x^2}x}$$

output `-exp(1-exp(x^2)*x)/(-1+exp(x^2)*x)`

### 3.169.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2} dx = -\frac{e^{1-e^{x^2}x}}{-1+e^{x^2}x}$$

input `Integrate[(E^(1 - E^x^2*x + 2*x^2))*(x + 2*x^3))/(1 - E^x^2*x)^2,x]`

output `-(E^(1 - E^x^2*x)/(-1 + E^x^2*x))`

---


$$3.169. \quad \int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2} dx$$

**3.169.3 Rubi [F]**

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x^2 - e^{x^2}x + 1}(2x^3 + x)}{(1 - e^{x^2}x)^2} dx$$

↓ 2027

$$\int \frac{e^{2x^2 - e^{x^2}x + 1}x(2x^2 + 1)}{(1 - e^{x^2}x)^2} dx$$

↓ 7293

$$\int \left( \frac{e^{2x^2 - e^{x^2}x + 1}x}{(e^{x^2}x - 1)^2} + \frac{2e^{2x^2 - e^{x^2}x + 1}x^3}{(e^{x^2}x - 1)^2} \right) dx$$

↓ 2009

$$\int \frac{e^{2x^2 - e^{x^2}x + 1}x}{(e^{x^2}x - 1)^2} dx + 2 \int \frac{e^{2x^2 - e^{x^2}x + 1}x^3}{(e^{x^2}x - 1)^2} dx$$

input `Int[(E^(1 - E^x^2*x + 2*x^2))*(x + 2*x^3))/(1 - E^x^2*x)^2,x]`

output `$Aborted`

**3.169.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2027 `Int[(F_x_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^p_., x_Symbol] := Int[x^(p*r)*(a + b*x^(s - r))^p*F_x, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] && PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])`

rule 7293 `Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]`

---

3.169.  $\int \frac{e^{1 - e^{x^2}x + 2x^2}(x + 2x^3)}{(1 - e^{x^2}x)^2} dx$

**3.169.4 Maple [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

| method | result                           | size |
|--------|----------------------------------|------|
| risch  | $-\frac{e^{1-e^2x}}{-1+e^{x^2}}$ | 23   |

```
input int(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x,method=_RETURNVER
BOSE)
```

```
output -exp(1-exp(x^2)*x)/(-1+exp(x^2)*x)
```

**3.169.5 Fracas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2} dx = -\frac{e^{(2x^2-xe^{x^2})+1}}{xe^{3x^2}-e^{2x^2}}$$

```
input integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x, algorithm=
"fricas")
```

```
output -e^(2*x^2 - x*e^(x^2) + 1)/(x*e^(3*x^2) - e^(2*x^2))
```

**3.169.6 Sympy [A] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2} dx = -\frac{e^{2x^2-xe^{x^2}+1}}{xe^{3x^2}-e^{2x^2}}$$

```
input integrate(exp(1-exp(x**2)*x+2*x**2)*(2*x**3+x)/(1-exp(x**2)*x)**2,x)
```

```
output -exp(2*x**2 - x*exp(x**2) + 1)/(x*exp(3*x**2) - exp(2*x**2))
```

---

3.169.  $\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2} dx$

**3.169.7 Maxima [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2} dx = -\frac{e^{(-xe^{(x^2)}+1)}}{xe^{(x^2)}-1}$$

input `integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x, algorithm="maxima")`

output `-e^(-x*e^(x^2) + 1)/(x*e^(x^2) - 1)`

**3.169.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2} dx = -\frac{e^{(2x^2-xe^{(x^2)}+1)}}{xe^{(3x^2)}-e^{(2x^2)}}$$

input `integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x, algorithm="giac")`

output `-e^(2*x^2 - x*e^(x^2) + 1)/(x*e^(3*x^2) - e^(2*x^2))`

**3.169.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2} dx = \int \frac{e^{2x^2-xe^{x^2}+1}(2x^3+x)}{(xe^{x^2}-1)^2} dx$$

input `int((exp(2*x^2 - x*exp(x^2) + 1)*(x + 2*x^3))/(x*exp(x^2) - 1)^2,x)`

output `int((exp(2*x^2 - x*exp(x^2) + 1)*(x + 2*x^3))/(x*exp(x^2) - 1)^2, x)`

---

3.169.  $\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2} dx$

# 3.170 $\int e^{e^{e^{e^x}}} dx$

|                                     |     |
|-------------------------------------|-----|
| 3.170.1 Optimal result . . . . .    | 957 |
| 3.170.2 Mathematica [N/A] . . . . . | 957 |
| 3.170.3 Rubi [N/A] . . . . .        | 958 |
| 3.170.4 Maple [N/A] . . . . .       | 959 |
| 3.170.5 Fricas [N/A] . . . . .      | 959 |
| 3.170.6 Sympy [N/A] . . . . .       | 959 |
| 3.170.7 Maxima [N/A] . . . . .      | 960 |
| 3.170.8 Giac [N/A] . . . . .        | 960 |
| 3.170.9 Mupad [N/A] . . . . .       | 960 |

## 3.170.1 Optimal result

Integrand size = 9, antiderivative size = 9

$$\int e^{e^{e^{e^x}}} dx = \text{Int}\left(e^{e^{e^{e^x}}}, x\right)$$

output `CannotIntegrate(exp(exp(exp(exp(x)))) , x)`

## 3.170.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int e^{e^{e^{e^x}}} dx = \int e^{e^{e^{e^x}}} dx$$

input `Integrate[E^E^E^E^x, x]`

output `Integrate[E^E^E^E^x, x]`

**3.170.3 Rubi [N/A]**

Not integrable

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {2720, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{e^{e^{e^x}}} dx$$

↓ 2720

$$\int e^{e^{e^{e^x}} - x} de^x$$

↓ 7299

$$\int e^{e^{e^{e^x}} - x} de^x$$

input `Int[E^E^E^E^x,x]`

output `$Aborted`

**3.170.3.1 Defintions of rubi rules used**

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x)) *(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 7299 `Int[u_, x_] := CannotIntegrate[u, x]`

**3.170.4 Maple [N/A]**

Not integrable

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int e^{e^{e^{e^x}}} dx$$

input `int(exp(exp(exp(exp(x))))),x)`output `int(exp(exp(exp(exp(x))))),x)`**3.170.5 Fricas [N/A]**

Not integrable

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int e^{e^{e^{e^x}}} dx = \int e^{\left(e^{\left(e^{\left(e^x\right)}\right)}\right)} dx$$

input `integrate(exp(exp(exp(exp(x))))),x, algorithm="fricas")`output `integral(e^(e^(e^(e^x))), x)`**3.170.6 Sympy [N/A]**

Not integrable

Time = 0.52 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{e^{e^{e^x}}} dx = \int e^{e^{e^{e^x}}} dx$$

input `integrate(exp(exp(exp(exp(x))))),x)`output `Integral(exp(exp(exp(exp(x))))), x)`



**3.170.7 Maxima [N/A]**

Not integrable

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int e^{e^{e^{e^x}}} dx = \int e^{\left(e^{\left(e^{e^x}\right)}\right)} dx$$

input `integrate(exp(exp(exp(exp(x)))),x, algorithm="maxima")`output `integrate(e^(e^(e^(e^x))), x)`**3.170.8 Giac [N/A]**

Not integrable

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int e^{e^{e^{e^x}}} dx = \int e^{\left(e^{\left(e^{e^x}\right)}\right)} dx$$

input `integrate(exp(exp(exp(exp(x)))),x, algorithm="giac")`output `integrate(e^(e^(e^(e^x))), x)`**3.170.9 Mupad [N/A]**

Not integrable

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int e^{e^{e^{e^x}}} dx = \int e^{e^{e^{e^x}}} dx$$

input `int(exp(exp(exp(exp(x)))),x)`output `int(exp(exp(exp(exp(x)))), x)`

### 3.171 $\int e^x \log(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.171.1 Optimal result . . . . .                            | 961 |
| 3.171.2 Mathematica [A] (verified) . . . . .                | 961 |
| 3.171.3 Rubi [A] (verified) . . . . .                       | 962 |
| 3.171.4 Maple [A] (verified) . . . . .                      | 963 |
| 3.171.5 Fricas [A] (verification not implemented) . . . . . | 963 |
| 3.171.6 Sympy [A] (verification not implemented) . . . . .  | 963 |
| 3.171.7 Maxima [A] (verification not implemented) . . . . . | 964 |
| 3.171.8 Giac [A] (verification not implemented) . . . . .   | 964 |
| 3.171.9 Mupad [B] (verification not implemented) . . . . .  | 964 |

#### 3.171.1 Optimal result

Integrand size = 6, antiderivative size = 11

$$\int e^x \log(x) dx = -\text{ExpIntegralEi}(x) + e^x \log(x)$$

output `-Ei(x)+exp(x)*ln(x)`

#### 3.171.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^x \log(x) dx = -\text{ExpIntegralEi}(x) + e^x \log(x)$$

input `Integrate[E^x*Log[x],x]`

output `-ExpIntegralEi[x] + E^x*Log[x]`

**3.171.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3034, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int e^x \log(x) dx \\ \downarrow \text{3034} \\ e^x \log(x) - \int \frac{e^x}{x} dx \\ \downarrow \text{2609} \\ e^x \log(x) - \text{ExpIntegralEi}(x) \end{array}$$

input `Int [E^x*Log [x] ,x]`

output `-ExpIntegralEi [x] + E^x*Log [x]`

**3.171.3.1 Defintions of rubi rules used**

rule 2609 `Int[(F_)^((g_.)*(e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

rule 3034 `Int[Log[u_]*(v_), x_Symbol] := With[{w = IntHide[v, x]}, Simp[Log[u] w, x] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x]] /; InverseFunctionFreeQ[u, x]`

**3.171.4 Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

| method | result                         | size |
|--------|--------------------------------|------|
| risch  | $e^x \ln(x) + \text{Ei}_1(-x)$ | 12   |

input `int(exp(x)*ln(x),x,method=_RETURNVERBOSE)`output `exp(x)*ln(x)+Ei(1,-x)`**3.171.5 Fracas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^x \log(x) dx = e^x \log(x) - \text{Ei}(x)$$

input `integrate(exp(x)*log(x),x, algorithm="fricas")`output `e^x*log(x) - Ei(x)`**3.171.6 Sympy [A] (verification not implemented)**

Time = 1.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^x \log(x) dx = e^x \log(x) - \text{Ei}(x)$$

input `integrate(exp(x)*ln(x),x)`output `exp(x)*log(x) - Ei(x)`

**3.171.7 Maxima [A] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^x \log(x) dx = e^x \log(x) - \text{Ei}(x)$$

input `integrate(exp(x)*log(x),x, algorithm="maxima")`output `e^x*log(x) - Ei(x)`**3.171.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^x \log(x) dx = e^x \log(x) - \text{Ei}(x)$$

input `integrate(exp(x)*log(x),x, algorithm="giac")`output `e^x*log(x) - Ei(x)`**3.171.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^x \log(x) dx = e^x \ln(x) - \text{ei}(x)$$

input `int(exp(x)*log(x),x)`output `exp(x)*log(x) - ei(x)`

### 3.172 $\int e^x x \log(x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.172.1 Optimal result . . . . .                            | 965 |
| 3.172.2 Mathematica [A] (verified) . . . . .                | 965 |
| 3.172.3 Rubi [A] (verified) . . . . .                       | 966 |
| 3.172.4 Maple [A] (verified) . . . . .                      | 967 |
| 3.172.5 Fricas [A] (verification not implemented) . . . . . | 967 |
| 3.172.6 Sympy [A] (verification not implemented) . . . . .  | 967 |
| 3.172.7 Maxima [A] (verification not implemented) . . . . . | 968 |
| 3.172.8 Giac [A] (verification not implemented) . . . . .   | 968 |
| 3.172.9 Mupad [B] (verification not implemented) . . . . .  | 968 |

#### 3.172.1 Optimal result

Integrand size = 7, antiderivative size = 22

$$\int e^x x \log(x) dx = -e^x + \text{ExpIntegralEi}(x) - e^x \log(x) + e^x x \log(x)$$

output `-exp(x)+Ei(x)-exp(x)*ln(x)+exp(x)*x*ln(x)`

#### 3.172.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int e^x x \log(x) dx = -e^x + \text{ExpIntegralEi}(x) + e^x(-1 + x) \log(x)$$

input `Integrate[E^x*x*Log[x],x]`

output `-E^x + ExpIntegralEi[x] + E^x*(-1 + x)*Log[x]`

**3.172.3 Rubi [A] (verified)**

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {3034, 2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int e^x x \log(x) dx \\ & \quad \downarrow \text{3034} \\ & - \int \frac{e^x(x-1)}{x} dx - e^x \log(x) + e^x x \log(x) \\ & \quad \downarrow \text{2629} \\ & - \int \left( e^x - \frac{e^x}{x} \right) dx - e^x \log(x) + e^x x \log(x) \\ & \quad \downarrow \text{2009} \\ & \text{ExpIntegralEi}(x) - e^x - e^x \log(x) + e^x x \log(x) \end{aligned}$$

input `Int[E^x*x*Log[x],x]`

output `-E^x + ExpIntegralEi[x] - E^x*Log[x] + E^x*x*Log[x]`

**3.172.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2629 `Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] := Int[ExpandIntegrand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

rule 3034 `Int[Log[u_]*(v_), x_Symbol] := With[{w = IntHide[v, x]}, Simp[Log[u] w, x] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x]] /; InverseFunctionFreeQ[u, x]`

**3.172.4 Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

| method | result                                        | size |
|--------|-----------------------------------------------|------|
| risch  | $(-1 + x) e^x \ln(x) - \text{Ei}_1(-x) - e^x$ | 21   |

input `int(exp(x)*x*ln(x),x,method=_RETURNVERBOSE)`output `(-1+x)*exp(x)*ln(x)-Ei(1,-x)-exp(x)`**3.172.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int e^x x \log(x) dx = (x - 1)e^x \log(x) + \text{Ei}(x) - e^x$$

input `integrate(exp(x)*x*log(x),x, algorithm="fracas")`output `(x - 1)*e^x*log(x) + Ei(x) - e^x`**3.172.6 Sympy [A] (verification not implemented)**

Time = 1.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int e^x x \log(x) dx = (xe^x - e^x) \log(x) - e^x + \text{Ei}(x)$$

input `integrate(exp(x)*x*ln(x),x)`output `(x*exp(x) - exp(x))*log(x) - exp(x) + Ei(x)`



**3.172.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int e^x x \log(x) dx = (x - 1)e^x \log(x) + \text{Ei}(x) - e^x$$

input `integrate(exp(x)*x*log(x),x, algorithm="maxima")`output `(x - 1)*e^x*log(x) + Ei(x) - e^x`**3.172.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int e^x x \log(x) dx = (x - 1)e^x \log(x) + \text{Ei}(x) - e^x$$

input `integrate(exp(x)*x*log(x),x, algorithm="giac")`output `(x - 1)*e^x*log(x) + Ei(x) - e^x`**3.172.9 Mupad [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int e^x x \log(x) dx = \text{ei}(x) - \frac{x e^x + x e^x \ln(x) - x^2 e^x \ln(x)}{x}$$

input `int(x*exp(x)*log(x),x)`output `ei(x) - (x*exp(x) + x*exp(x)*log(x) - x^2*exp(x)*log(x))/x`

### 3.173 $\int e^{2x} \log(e^x) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.173.1 Optimal result . . . . .                            | 969 |
| 3.173.2 Mathematica [A] (verified) . . . . .                | 969 |
| 3.173.3 Rubi [A] (verified) . . . . .                       | 970 |
| 3.173.4 Maple [A] (verified) . . . . .                      | 971 |
| 3.173.5 Fracas [A] (verification not implemented) . . . . . | 971 |
| 3.173.6 Sympy [A] (verification not implemented) . . . . .  | 971 |
| 3.173.7 Maxima [A] (verification not implemented) . . . . . | 972 |
| 3.173.8 Giac [A] (verification not implemented) . . . . .   | 972 |
| 3.173.9 Mupad [B] (verification not implemented) . . . . .  | 972 |

#### 3.173.1 Optimal result

Integrand size = 10, antiderivative size = 23

$$\int e^{2x} \log(e^x) dx = -\frac{e^{2x}}{4} + \frac{1}{2}e^{2x} \log(e^x)$$

output `-1/4*exp(2*x)+1/2*exp(2*x)*ln(exp(x))`

#### 3.173.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int e^{2x} \log(e^x) dx = \frac{1}{4}e^{2x}(-1 + 2 \log(e^x))$$

input `Integrate[E^(2*x)*Log[E^x],x]`

output `(E^(2*x)*(-1 + 2*Log[E^x]))/4`

**3.173.3 Rubi [A] (verified)**

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {3034, 27, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int e^{2x} \log(e^x) dx \\ & \quad \downarrow \text{3034} \\ & \frac{1}{2} e^{2x} \log(e^x) - \int \frac{e^{2x}}{2} dx \\ & \quad \downarrow \text{27} \\ & \frac{1}{2} e^{2x} \log(e^x) - \frac{\int e^{2x} dx}{2} \\ & \quad \downarrow \text{2624} \\ & \frac{1}{2} e^{2x} \log(e^x) - \frac{e^{2x}}{4} \end{aligned}$$

input `Int[E^(2*x)*Log[E^x],x]`

output `-1/4*E^(2*x) + (E^(2*x)*Log[E^x])/2`

**3.173.3.1 Defintions of rubi rules used**

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 2624 `Int[((F_)^(v_))^(n_), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /; FreeQ[{F, n}, x] && LinearQ[v, x]`

rule 3034 `Int[Log[u_]*(v_), x_Symbol] := With[{w = IntHide[v, x]}, Simp[Log[u] w, x] - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x] /; InverseFunctionFreeQ[u, x]`

**3.173.4 Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

| method       | result                                                                | size |
|--------------|-----------------------------------------------------------------------|------|
| norman       | $-\frac{e^{2x}}{4} + \frac{e^{2x} \ln(e^x)}{2}$                       | 17   |
| risch        | $-\frac{e^{2x}}{4} + \frac{e^{2x} \ln(e^x)}{2}$                       | 17   |
| parallelrisc | $-\frac{e^{2x}}{4} + \frac{e^{2x} \ln(e^x)}{2}$                       | 17   |
| default      | $\frac{e^{2x}x}{2} - \frac{e^{2x}}{4} + \frac{e^{2x}(\ln(e^x)-x)}{2}$ | 28   |

input `int(exp(2*x)*ln(exp(x)),x,method=_RETURNVERBOSE)`output `-1/4*exp(x)^2+1/2*exp(x)^2*ln(exp(x))`**3.173.5 Fracas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int e^{2x} \log(e^x) dx = \frac{1}{4} (2x - 1)e^{(2x)}$$

input `integrate(exp(2*x)*log(exp(x)),x, algorithm="fricas")`output `1/4*(2*x - 1)*e^(2*x)`**3.173.6 Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.43

$$\int e^{2x} \log(e^x) dx = \frac{(2x - 1)e^{2x}}{4}$$

input `integrate(exp(2*x)*ln(exp(x)),x)`output `(2*x - 1)*exp(2*x)/4`

**3.173.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int e^{2x} \log(e^x) dx = \frac{1}{4} (2x - 1)e^{(2x)}$$

input `integrate(exp(2*x)*log(exp(x)),x, algorithm="maxima")`output `1/4*(2*x - 1)*e^(2*x)`**3.173.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int e^{2x} \log(e^x) dx = \frac{1}{4} (2x - 1)e^{(2x)}$$

input `integrate(exp(2*x)*log(exp(x)),x, algorithm="giac")`output `1/4*(2*x - 1)*e^(2*x)`**3.173.9 Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.48

$$\int e^{2x} \log(e^x) dx = \frac{e^{2x} (2x - 1)}{4}$$

input `int(log(exp(x))*exp(2*x),x)`output `(exp(2*x)*(2*x - 1))/4`

### 3.174 $\int (2x + \sqrt{2}x^2) dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.174.1 Optimal result . . . . .                            | 973 |
| 3.174.2 Mathematica [A] (verified) . . . . .                | 973 |
| 3.174.3 Rubi [A] (verified) . . . . .                       | 974 |
| 3.174.4 Maple [A] (verified) . . . . .                      | 974 |
| 3.174.5 Fricas [A] (verification not implemented) . . . . . | 975 |
| 3.174.6 Sympy [A] (verification not implemented) . . . . .  | 975 |
| 3.174.7 Maxima [A] (verification not implemented) . . . . . | 975 |
| 3.174.8 Giac [A] (verification not implemented) . . . . .   | 976 |
| 3.174.9 Mupad [B] (verification not implemented) . . . . .  | 976 |

#### 3.174.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int (2x + \sqrt{2}x^2) dx = x^2 + \frac{\sqrt{2}x^3}{3}$$

output `x^2+1/3*x^3*2^(1/2)`

#### 3.174.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (2x + \sqrt{2}x^2) dx = x^2 + \frac{\sqrt{2}x^3}{3}$$

input `Integrate[2*x + Sqrt[2]*x^2,x]`

output `x^2 + (Sqrt[2]*x^3)/3`

**3.174.3 Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sqrt{2}x^2 + 2x) dx$$

$$\downarrow \text{2009}$$

$$\frac{\sqrt{2}x^3}{3} + x^2$$

input `Int[2*x + Sqrt[2]*x^2,x]`

output `x^2 + (Sqrt[2]*x^3)/3`

**3.174.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

**3.174.4 Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

| method        | result                                                          | size |
|---------------|-----------------------------------------------------------------|------|
| norman        | $x^2 + \frac{x^3\sqrt{2}}{3}$                                   | 13   |
| risch         | $x^2 + \frac{x^3\sqrt{2}}{3}$                                   | 13   |
| parallelrisch | $x^2 + \frac{x^3\sqrt{2}}{3}$                                   | 13   |
| parts         | $x^2 + \frac{x^3\sqrt{2}}{3}$                                   | 13   |
| default       | $\sqrt{2} \left( \frac{x^3}{3} + \frac{x^2\sqrt{2}}{2} \right)$ | 19   |
| gosper        | $\frac{x^2(2x+3\sqrt{2})(x\sqrt{2}+2)}{6x+6\sqrt{2}}$           | 29   |

input `int(2*x+x^2*2^(1/2),x,method=_RETURNVERBOSE)`

output `x^2+1/3*x^3*2^(1/2)`

### 3.174.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (2x + \sqrt{2}x^2) dx = \frac{1}{3} \sqrt{2}x^3 + x^2$$

input `integrate(2*x+x^2*2^(1/2),x, algorithm="fricas")`

output `1/3*sqrt(2)*x^3 + x^2`

### 3.174.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (2x + \sqrt{2}x^2) dx = \frac{\sqrt{2}x^3}{3} + x^2$$

input `integrate(2*x+x**2*2**(1/2),x)`

output `sqrt(2)*x**3/3 + x**2`

### 3.174.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (2x + \sqrt{2}x^2) dx = \frac{1}{3} \sqrt{2}x^3 + x^2$$

input `integrate(2*x+x^2*2^(1/2),x, algorithm="maxima")`

output `1/3*sqrt(2)*x^3 + x^2`



**3.174.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (2x + \sqrt{2}x^2) dx = \frac{1}{3} \sqrt{2}x^3 + x^2$$

input `integrate(2*x+x^2*2^(1/2),x, algorithm="giac")`

output `1/3*sqrt(2)*x^3 + x^2`

**3.174.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (2x + \sqrt{2}x^2) dx = \frac{x^2 (\sqrt{2}x + 3)}{3}$$

input `int(2*x + 2^(1/2)*x^2,x)`

output `(x^2*(2^(1/2)*x + 3))/3`

### 3.175 $\int \frac{\log(x)}{\sqrt{b+ax}} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.175.1 Optimal result . . . . .                            | 977 |
| 3.175.2 Mathematica [A] (verified) . . . . .                | 977 |
| 3.175.3 Rubi [A] (verified) . . . . .                       | 978 |
| 3.175.4 Maple [A] (verified) . . . . .                      | 979 |
| 3.175.5 Fricas [A] (verification not implemented) . . . . . | 980 |
| 3.175.6 Sympy [C] (verification not implemented) . . . . .  | 980 |
| 3.175.7 Maxima [A] (verification not implemented) . . . . . | 981 |
| 3.175.8 Giac [A] (verification not implemented) . . . . .   | 982 |
| 3.175.9 Mupad [B] (verification not implemented) . . . . .  | 982 |

#### 3.175.1 Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \frac{\log(x)}{\sqrt{b+ax}} dx = -\frac{4\sqrt{b+ax}}{a} + \frac{4\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)}{a} + \frac{2\sqrt{b+ax}\log(x)}{a}$$

output `4*arctanh((a*x+b)^(1/2)/b^(1/2))*b^(1/2)/a-4*(a*x+b)^(1/2)/a+2*ln(x)*(a*x+b)^(1/2)/a`

#### 3.175.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int \frac{\log(x)}{\sqrt{b+ax}} dx = \frac{4\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right) + 2\sqrt{b+ax}(-2 + \log(x))}{a}$$

input `Integrate[Log[x]/Sqrt[b + a*x],x]`

output `(4*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/Sqrt[b]] + 2*Sqrt[b + a*x]*(-2 + Log[x]))/a`

**3.175.3 Rubi [A] (verified)**

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2756, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\log(x)}{\sqrt{ax+b}} dx \\
 & \quad \downarrow \text{2756} \\
 & \frac{2\log(x)\sqrt{ax+b}}{a} - \frac{2\int \frac{\sqrt{b+ax}}{x} dx}{a} \\
 & \quad \downarrow \text{60} \\
 & \frac{2\log(x)\sqrt{ax+b}}{a} - \frac{2\left(b\int \frac{1}{x\sqrt{b+ax}} dx + 2\sqrt{ax+b}\right)}{a} \\
 & \quad \downarrow \text{73} \\
 & \frac{2\log(x)\sqrt{ax+b}}{a} - \frac{2\left(\frac{2b\int \frac{1}{\frac{b+ax}{a} - \frac{b}{a}} d\sqrt{b+ax}}{a} + 2\sqrt{ax+b}\right)}{a} \\
 & \quad \downarrow \text{221} \\
 & \frac{2\log(x)\sqrt{ax+b}}{a} - \frac{2\left(2\sqrt{ax+b} - 2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\right)}{a}
 \end{aligned}$$

input `Int[Log[x]/Sqrt[b + a*x],x]`

output `(-2*(2*Sqrt[b + a*x] - 2*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/Sqrt[b]]))/a + (2*Sqrt[b + a*x]*Log[x])/a`

3.175.3.1 Defintions of rubi rules used

rule 60 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(b*(m + n + 1)) Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 2756 `Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(q_.), x_Symbol] := Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x] - Simp[b*n*(p/(e*(q + 1))) Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q, -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] && !IGtQ[q, 0]) || (EqQ[p, 2] && NeQ[q, 1]))`

3.175.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

| method            | result                                                                                                                     | size |
|-------------------|----------------------------------------------------------------------------------------------------------------------------|------|
| derivativedivides | $\frac{2\sqrt{ax+b} \ln(x) - 4\sqrt{ax+b} + 4\sqrt{b} \operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$ | 43   |
| default           | $\frac{2\sqrt{ax+b} \ln(x) - 4\sqrt{ax+b} + 4\sqrt{b} \operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$ | 43   |

input `int(ln(x)/(a*x+b)^(1/2),x,method=_RETURNVERBOSE)`

output  $2/a*((a*x+b)^{(1/2)}*\ln(x)-2*(a*x+b)^{(1/2)}+2*b^{(1/2)}*\operatorname{arctanh}((a*x+b)^{(1/2)}/b^{(1/2)}))$

### 3.175.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.56

$$\int \frac{\log(x)}{\sqrt{b+ax}} dx$$

$$= \left[ \frac{2 \left( \sqrt{ax+b}(\log(x) - 2) + \sqrt{b} \log\left(\frac{ax+2\sqrt{ax+b}\sqrt{b+2b}}{x}\right) \right)}{a}, \frac{2 \left( \sqrt{ax+b}(\log(x) - 2) - 2\sqrt{-b} \arctan\left(\frac{\sqrt{ax+b}}{b}\right) \right)}{a} \right]$$

input `integrate(log(x)/(a*x+b)^(1/2),x, algorithm="fricas")`

output  $[2*(\sqrt{a*x + b}*(\log(x) - 2) + \sqrt{b}*\log((a*x + 2*\sqrt{a*x + b})*\sqrt{b} + 2*b)/x))/a, 2*(\sqrt{a*x + b}*(\log(x) - 2) - 2*\sqrt{-b}*\arctan(\sqrt{a*x + b}*\sqrt{-b}/b))/a]$

### 3.175.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 1166, normalized size of antiderivative = 20.46

$$\int \frac{\log(x)}{\sqrt{b+ax}} dx = \text{Too large to display}$$

input `integrate(ln(x)/(a*x+b)**(1/2),x)`

output `Piecewise((4*sqrt(b)*acoth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(-1 + b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a), (Abs(x + b/a) < 1) & (1/Abs(x + b/a) < 1) & (Abs(b/(a*(x + b/a))) > 1)), (4*sqrt(b)*atanh(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(1 - b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) - 2*I*pi*sqrt(x + b/a)/sqrt(a), (Abs(x + b/a) < 1) & (1/Abs(x + b/a) < 1)), (4*sqrt(b)*acoth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*log(b/a)/sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(-1 + b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) + 2*I*pi*sqrt(x + b/a)/sqrt(a), (Abs(x + b/a) < 1) & (Abs(b/(a*(x + b/a))) > 1)), (4*sqrt(b)*atanh(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*log(b/a)/sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(1 - b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a), Abs(x + b/a) < 1), (4*sqrt(b)*acoth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*log(b/a)/sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(-1 + b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) + 2*I*pi*sqrt(x + b/a)/sqrt(a), (1/Abs(x + b/a) < 1) & (Abs(b/(a*(x + b/a))) > 1)), (4*sqrt(b)*atanh(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*log(b/a)/sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(...`

### 3.175.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{\log(x)}{\sqrt{b+ax}} dx = \frac{2 \left( \sqrt{ax+b} \log(x) - \sqrt{b} \log\left(\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right) - 2\sqrt{ax+b} \right)}{a}$$

input `integrate(log(x)/(a*x+b)^(1/2),x, algorithm="maxima")`

output `2*(sqrt(a*x + b)*log(x) - sqrt(b)*log((sqrt(a*x + b) - sqrt(b))/(sqrt(a*x + b) + sqrt(b))) - 2*sqrt(a*x + b))/a`

**3.175.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{\log(x)}{\sqrt{b+ax}} dx = -\frac{2 \left( \frac{2b \arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \sqrt{ax+b} \log(x) + 2\sqrt{ax+b} \right)}{a}$$

input `integrate(log(x)/(a*x+b)^(1/2),x, algorithm="giac")`output `-2*(2*b*arctan(sqrt(a*x + b)/sqrt(-b))/sqrt(-b) - sqrt(a*x + b)*log(x) + 2*sqrt(a*x + b))/a`**3.175.9 Mupad [B] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{\log(x)}{\sqrt{b+ax}} dx = \frac{2\sqrt{b} \ln\left(\frac{2b+ax+2\sqrt{b}\sqrt{b+ax}}{x}\right)}{a} + \frac{2(\ln(x) - 2)\sqrt{b+ax}}{a}$$

input `int(log(x)/(b + a*x)^(1/2),x)`output `(2*b^(1/2)*log((2*b + a*x + 2*b^(1/2)*(b + a*x)^(1/2))/x))/a + (2*(log(x) - 2)*(b + a*x)^(1/2))/a`

### 3.176 $\int \sqrt{a + bx}\sqrt{c + dx} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
| 3.176.1 Optimal result . . . . .                            | 983 |
| 3.176.2 Mathematica [A] (verified) . . . . .                | 983 |
| 3.176.3 Rubi [A] (verified) . . . . .                       | 984 |
| 3.176.4 Maple [A] (verified) . . . . .                      | 985 |
| 3.176.5 Fricas [A] (verification not implemented) . . . . . | 986 |
| 3.176.6 Sympy [F] . . . . .                                 | 986 |
| 3.176.7 Maxima [F(-2)] . . . . .                            | 986 |
| 3.176.8 Giac [B] (verification not implemented) . . . . .   | 987 |
| 3.176.9 Mupad [B] (verification not implemented) . . . . .  | 987 |

#### 3.176.1 Optimal result

Integrand size = 19, antiderivative size = 116

$$\int \sqrt{a + bx}\sqrt{c + dx} dx = \frac{(bc - ad)\sqrt{a + bx}\sqrt{c + dx}}{4bd} + \frac{(a + bx)^{3/2}\sqrt{c + dx}}{2b} - \frac{(bc - ad)^2 \operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}$$

output `-1/4*(-a*d+b*c)^2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(3/2)/d^(3/2)+1/2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b+1/4*(-a*d+b*c)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b/d`

#### 3.176.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.82

$$\int \sqrt{a + bx}\sqrt{c + dx} dx = \frac{\sqrt{a + bx}\sqrt{c + dx}(bc + ad + 2bdx)}{4bd} - \frac{(bc - ad)^2 \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{4b^{3/2}d^{3/2}}$$

input `Integrate[Sqrt[a + b*x]*Sqrt[c + d*x],x]`

output `(Sqrt[a + b*x]*Sqrt[c + d*x]*(b*c + a*d + 2*b*d*x))/(4*b*d) - ((b*c - a*d)^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*b^(3/2)*d^(3/2))`



**3.176.3 Rubi [A] (verified)**

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$ , Rules used = {60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \sqrt{a+bx}\sqrt{c+dx} \, dx \\
 & \quad \downarrow 60 \\
 & \frac{(bc-ad) \int \frac{\sqrt{a+bx}}{\sqrt{c+dx}} \, dx}{4b} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} \\
 & \quad \downarrow 60 \\
 & \frac{(bc-ad) \left( \frac{\sqrt{a+bx}\sqrt{c+dx}}{d} - \frac{(bc-ad) \int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}} \, dx}{2d} \right)}{4b} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} \\
 & \quad \downarrow 66 \\
 & \frac{(bc-ad) \left( \frac{\sqrt{a+bx}\sqrt{c+dx}}{d} - \frac{(bc-ad) \int \frac{1}{b-\frac{d(a+bx)}{c+dx}} d \frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{d} \right)}{4b} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} \\
 & \quad \downarrow 221 \\
 & \frac{(bc-ad) \left( \frac{\sqrt{a+bx}\sqrt{c+dx}}{d} - \frac{(bc-ad) \operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{bd}^{3/2}} \right)}{4b} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}
 \end{aligned}$$

input `Int[Sqrt[a + b*x]*Sqrt[c + d*x],x]`

output `((a + b*x)^(3/2)*Sqrt[c + d*x])/(2*b) + ((b*c - a*d)*((Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])]))/(Sqrt[b]*d^(3/2)))/(4*b)`

3.176.3.1 Defintions of rubi rules used

```
rule 60 Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1))) Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

```
rule 66 Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] := Simp[
2 Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && !GtQ[c - a*(d/b), 0]
```

```
rule 221 Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.176.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.21

| method  | result                                                                                                                                                                                                                                                                                 | size |
|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default | $\frac{\sqrt{bx+a}(dx+c)^{\frac{3}{2}}}{2d} - \frac{(-ad+bc) \left( \frac{\sqrt{dx+c}\sqrt{bx+a}}{b} - \frac{(ad-bc)\sqrt{(bx+a)(dx+c)} \ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}} + \sqrt{bdx^2+(ad+bc)x+ac}\right)}{2b\sqrt{dx+c}\sqrt{bx+a}\sqrt{bd}} \right)}{4d}$ | 140  |

```
input int((b*x+a)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
output 1/2/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)-1/4*(-a*d+b*c)/d*(1/b*(d*x+c)^(1/2)*(b*x
+a)^(1/2)-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1
/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2)
)/(b*d)^(1/2))
```

**3.176.5 Fricas [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.59

$$\int \sqrt{a+bx}\sqrt{c+dx} dx = \left[ \frac{(b^2c^2 - 2abcd + a^2d^2)\sqrt{bd} \log\left(8b^2d^2x^2 + b^2c^2 + 6abcd + a^2d^2 - 4(2bdx + bc + ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}\right)}{16b^2d^2} \right]$$

input `integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")`output `[1/16*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x + b^2*c*d + a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2), 1/8*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(2*b^2*d^2*x + b^2*c*d + a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2)]`**3.176.6 Sympy [F]**

$$\int \sqrt{a+bx}\sqrt{c+dx} dx = \int \sqrt{a+bx}\sqrt{c+dx} dx$$

input `integrate((b*x+a)**(1/2)*(d*x+c)**(1/2),x)`output `Integral(sqrt(a + b*x)*sqrt(c + d*x), x)`**3.176.7 Maxima [F(-2)]**

Exception generated.

$$\int \sqrt{a+bx}\sqrt{c+dx} dx = \text{Exception raised: ValueError}$$

input `integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a\*d-b\*c>0)', see `assume?` for more detail

### 3.176.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(90) = 180.

Time = 0.31 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.00

$$\int \sqrt{a+bx}\sqrt{c+dx} dx = \frac{4 \left( \frac{(b^2c-abd) \log\left(\frac{-\sqrt{bd}\sqrt{bx+a} + \sqrt{b^2c+(bx+a)bd-abd}}{\sqrt{bd}}\right) - \sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}}{b^2} \right) a|b| - \left( \sqrt{b^2c+(bx+a)bd-abd} \left( 2bx+2a + \frac{bcd-5ad}{d^2} \right) \right)}{4b}$$

input `integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")`

output 
$$-1/4*(4*((b^2*c - a*b*d)*\log(\text{abs}(-\text{sqrt}(b*d))*\text{sqrt}(b*x + a) + \text{sqrt}(b^2*c + (b*x + a)*b*d - a*b*d)))/\text{sqrt}(b*d) - \text{sqrt}(b^2*c + (b*x + a)*b*d - a*b*d)*\text{sqrt}(b*x + a))*a*\text{abs}(b)/b^2 - (\text{sqrt}(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*\text{sqrt}(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*\log(\text{abs}(-\text{sqrt}(b*d))*\text{sqrt}(b*x + a) + \text{sqrt}(b^2*c + (b*x + a)*b*d - a*b*d)))/(\text{sqrt}(b*d)*d))*\text{abs}(b)/b^2)/b$$

### 3.176.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.76

$$\int \sqrt{a+bx}\sqrt{c+dx} dx = \left( \frac{x}{2} + \frac{ad+bc}{4bd} \right) \sqrt{a+bx}\sqrt{c+dx} - \frac{\ln\left(ad+bc+2bdx+2\sqrt{b}\sqrt{d}\sqrt{a+bx}\sqrt{c+dx}\right) (ad-bc)^2}{8b^{3/2}d^{3/2}}$$

input `int((a + b*x)^(1/2)*(c + d*x)^(1/2),x)`

output  $(x/2 + (a*d + b*c)/(4*b*d))*(a + b*x)^{1/2}*(c + d*x)^{1/2} - (\log(a*d + b*c + 2*b*d*x + 2*b^{1/2}*d^{1/2}*(a + b*x)^{1/2}*(c + d*x)^{1/2})*(a*d - b*c)^2)/(8*b^{3/2}*d^{3/2})$

### 3.177 $\int \sqrt{a + bx} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
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| 3.177.2 Mathematica [A] (verified) . . . . .                | 989 |
| 3.177.3 Rubi [A] (verified) . . . . .                       | 990 |
| 3.177.4 Maple [A] (verified) . . . . .                      | 990 |
| 3.177.5 Fricas [A] (verification not implemented) . . . . . | 991 |
| 3.177.6 Sympy [A] (verification not implemented) . . . . .  | 991 |
| 3.177.7 Maxima [A] (verification not implemented) . . . . . | 991 |
| 3.177.8 Giac [A] (verification not implemented) . . . . .   | 992 |
| 3.177.9 Mupad [B] (verification not implemented) . . . . .  | 992 |

#### 3.177.1 Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \sqrt{a + bx} dx = \frac{2(a + bx)^{3/2}}{3b}$$

output `2/3*(b*x+a)^(3/2)/b`

#### 3.177.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{a + bx} dx = \frac{2(a + bx)^{3/2}}{3b}$$

input `Integrate[Sqrt[a + b*x],x]`

output `(2*(a + b*x)^(3/2))/(3*b)`

**3.177.3 Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a + bx} dx$$

$$\downarrow 17$$

$$\frac{2(a + bx)^{3/2}}{3b}$$

input `Int[Sqrt[a + b*x], x]`

output `(2*(a + b*x)^(3/2))/(3*b)`

**3.177.3.1 Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**3.177.4 Maple [A] (verified)**

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

| method            | result                             | size |
|-------------------|------------------------------------|------|
| gospers           | $\frac{2(bx+a)^{\frac{3}{2}}}{3b}$ | 13   |
| derivativedivides | $\frac{2(bx+a)^{\frac{3}{2}}}{3b}$ | 13   |
| default           | $\frac{2(bx+a)^{\frac{3}{2}}}{3b}$ | 13   |
| trager            | $\frac{2(bx+a)^{\frac{3}{2}}}{3b}$ | 13   |
| risch             | $\frac{2(bx+a)^{\frac{3}{2}}}{3b}$ | 13   |
| pseudoelliptic    | $\frac{2(bx+a)^{\frac{3}{2}}}{3b}$ | 13   |

input `int((b*x+a)^(1/2),x,method=_RETURNVERBOSE)`

output `2/3*(b*x+a)^(3/2)/b`

### 3.177.5 Fracas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{a + bx} dx = \frac{2(bx + a)^{\frac{3}{2}}}{3b}$$

input `integrate((b*x+a)^(1/2),x, algorithm="fricas")`

output `2/3*(b*x + a)^(3/2)/b`

### 3.177.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{a + bx} dx = \frac{2(a + bx)^{\frac{3}{2}}}{3b}$$

input `integrate((b*x+a)**(1/2),x)`

output `2*(a + b*x)**(3/2)/(3*b)`

### 3.177.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{a + bx} dx = \frac{2(bx + a)^{\frac{3}{2}}}{3b}$$

input `integrate((b*x+a)^(1/2),x, algorithm="maxima")`

output `2/3*(b*x + a)^(3/2)/b`



**3.177.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{a + bx} dx = \frac{2(bx + a)^{\frac{3}{2}}}{3b}$$

input `integrate((b*x+a)^(1/2),x, algorithm="giac")`

output `2/3*(b*x + a)^(3/2)/b`

**3.177.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \sqrt{a + bx} dx = \frac{2(a + bx)^{\frac{3}{2}}}{3b}$$

input `int((a + b*x)^(1/2),x)`

output `(2*(a + b*x)^(3/2))/(3*b)`

## 3.178 $\int x\sqrt{a+bx} dx$

|                                                             |     |
|-------------------------------------------------------------|-----|
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| 3.178.2 Mathematica [A] (verified) . . . . .                | 993 |
| 3.178.3 Rubi [A] (verified) . . . . .                       | 994 |
| 3.178.4 Maple [A] (verified) . . . . .                      | 995 |
| 3.178.5 Fricas [A] (verification not implemented) . . . . . | 995 |
| 3.178.6 Sympy [B] (verification not implemented) . . . . .  | 996 |
| 3.178.7 Maxima [A] (verification not implemented) . . . . . | 996 |
| 3.178.8 Giac [B] (verification not implemented) . . . . .   | 997 |
| 3.178.9 Mupad [B] (verification not implemented) . . . . .  | 997 |

### 3.178.1 Optimal result

Integrand size = 11, antiderivative size = 34

$$\int x\sqrt{a+bx} dx = -\frac{2a(a+bx)^{3/2}}{3b^2} + \frac{2(a+bx)^{5/2}}{5b^2}$$

output `-2/3*a*(b*x+a)^(3/2)/b^2+2/5*(b*x+a)^(5/2)/b^2`

### 3.178.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int x\sqrt{a+bx} dx = \frac{2\sqrt{a+bx}(-2a^2+abx+3b^2x^2)}{15b^2}$$

input `Integrate[x*Sqrt[a + b*x],x]`

output `(2*Sqrt[a + b*x]*(-2*a^2 + a*b*x + 3*b^2*x^2))/(15*b^2)`

**3.178.3 Rubi [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx} dx$$

$$\downarrow \text{53}$$

$$\int \left( \frac{(a+bx)^{3/2}}{b} - \frac{a\sqrt{a+bx}}{b} \right) dx$$

$$\downarrow \text{2009}$$

$$\frac{2(a+bx)^{5/2}}{5b^2} - \frac{2a(a+bx)^{3/2}}{3b^2}$$

input `Int[x*Sqrt[a + b*x],x]`

output `(-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2)`

**3.178.3.1 Defintions of rubi rules used**

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int  
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},  
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])  
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

**3.178.4 Maple [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

| method            | result                                                                        | size |
|-------------------|-------------------------------------------------------------------------------|------|
| gospers           | $-\frac{2(bx+a)^{\frac{3}{2}}(-3bx+2a)}{15b^2}$                               | 21   |
| pseudoelliptic    | $-\frac{2(bx+a)^{\frac{3}{2}}(-3bx+2a)}{15b^2}$                               | 21   |
| derivativedivides | $\frac{2(bx+a)^{\frac{5}{2}}}{5} - \frac{2a(bx+a)^{\frac{3}{2}}}{3}$<br>$b^2$ | 26   |
| default           | $\frac{2(bx+a)^{\frac{5}{2}}}{5} - \frac{2a(bx+a)^{\frac{3}{2}}}{3}$<br>$b^2$ | 26   |
| trager            | $-\frac{2(-3x^2b^2-axb+2a^2)\sqrt{bx+a}}{15b^2}$                              | 32   |
| risch             | $-\frac{2(-3x^2b^2-axb+2a^2)\sqrt{bx+a}}{15b^2}$                              | 32   |

input `int(x*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)`output `-2/15*(b*x+a)^(3/2)*(-3*b*x+2*a)/b^2`**3.178.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int x\sqrt{a+bx} dx = \frac{2(3b^2x^2 + abx - 2a^2)\sqrt{bx+a}}{15b^2}$$

input `integrate(x*(b*x+a)^(1/2),x, algorithm="fricas")`output `2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2`

**3.178.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 202 vs.  $2(31) = 62$ .

Time = 0.67 (sec) , antiderivative size = 202, normalized size of antiderivative = 5.94

$$\int x\sqrt{a+bx} dx = -\frac{4a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x} + \frac{4a^{\frac{9}{2}}}{15a^2b^2+15ab^3x} - \frac{2a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x} \\ + \frac{4a^{\frac{7}{2}}bx}{15a^2b^2+15ab^3x} + \frac{8a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x} + \frac{6a^{\frac{3}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}$$

input `integrate(x*(b*x+a)**(1/2),x)`

output `-4*a**(9/2)*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 4*a**(9/2)/(15*a**2*b**2 + 15*a*b**3*x) - 2*a**(7/2)*b*x*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 4*a**(7/2)*b*x/(15*a**2*b**2 + 15*a*b**3*x) + 8*a**(5/2)*b**2*x**2*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 6*a**(3/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x)`

**3.178.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int x\sqrt{a+bx} dx = \frac{2(bx+a)^{\frac{5}{2}}}{5b^2} - \frac{2(bx+a)^{\frac{3}{2}}a}{3b^2}$$

input `integrate(x*(b*x+a)^(1/2),x, algorithm="maxima")`

output `2/5*(b*x + a)^(5/2)/b^2 - 2/3*(b*x + a)^(3/2)*a/b^2`

**3.178.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 66 vs.  $2(26) = 52$ .

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int x\sqrt{a+bx} dx = \frac{2 \left( \frac{5((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+aa})a}{b} + \frac{3(bx+a)^{\frac{5}{2}} - 10(bx+a)^{\frac{3}{2}}a + 15\sqrt{bx+aa^2}}{b} \right)}{15b}$$

input `integrate(x*(b*x+a)^(1/2),x, algorithm="giac")`

output `2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b)/b`

**3.178.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int x\sqrt{a+bx} dx = -\frac{10a(a+bx)^{3/2} - 6(a+bx)^{5/2}}{15b^2}$$

input `int(x*(a + b*x)^(1/2),x)`

output `-(10*a*(a + b*x)^(3/2) - 6*(a + b*x)^(5/2))/(15*b^2)`

### 3.179 $\int x^2 \sqrt{a + bx} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.179.1 Optimal result . . . . .                            | 998  |
| 3.179.2 Mathematica [A] (verified) . . . . .                | 998  |
| 3.179.3 Rubi [A] (verified) . . . . .                       | 999  |
| 3.179.4 Maple [A] (verified) . . . . .                      | 1000 |
| 3.179.5 Fricas [A] (verification not implemented) . . . . . | 1000 |
| 3.179.6 Sympy [B] (verification not implemented) . . . . .  | 1001 |
| 3.179.7 Maxima [A] (verification not implemented) . . . . . | 1002 |
| 3.179.8 Giac [B] (verification not implemented) . . . . .   | 1002 |
| 3.179.9 Mupad [B] (verification not implemented) . . . . .  | 1003 |

#### 3.179.1 Optimal result

Integrand size = 13, antiderivative size = 53

$$\int x^2 \sqrt{a + bx} dx = \frac{2a^2(a + bx)^{3/2}}{3b^3} - \frac{4a(a + bx)^{5/2}}{5b^3} + \frac{2(a + bx)^{7/2}}{7b^3}$$

output  $2/3*a^2*(b*x+a)^(3/2)/b^3-4/5*a*(b*x+a)^(5/2)/b^3+2/7*(b*x+a)^(7/2)/b^3$

#### 3.179.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int x^2 \sqrt{a + bx} dx = \frac{2(a + bx)^{3/2} (8a^2 - 12abx + 15b^2x^2)}{105b^3}$$

input `Integrate[x^2*Sqrt[a + b*x],x]`

output  $(2*(a + b*x)^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3)$

**3.179.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{a + bx} dx$$

↓ 53

$$\int \left( \frac{a^2 \sqrt{a + bx}}{b^2} + \frac{(a + bx)^{5/2}}{b^2} - \frac{2a(a + bx)^{3/2}}{b^2} \right) dx$$

↓ 2009

$$\frac{2a^2(a + bx)^{3/2}}{3b^3} + \frac{2(a + bx)^{7/2}}{7b^3} - \frac{4a(a + bx)^{5/2}}{5b^3}$$

input `Int[x^2*Sqrt[a + b*x],x]`

output `(2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) + (2*(a + b*x)^(7/2))/(7*b^3)`

**3.179.3.1 Defintions of rubi rules used**

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`



**3.179.4 Maple [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

| method            | result                                                                                                                | size |
|-------------------|-----------------------------------------------------------------------------------------------------------------------|------|
| gospers           | $\frac{2(bx+a)^{\frac{3}{2}}(15x^2b^2-12axb+8a^2)}{105b^3}$                                                           | 32   |
| pseudoelliptic    | $\frac{2(bx+a)^{\frac{3}{2}}(15x^2b^2-12axb+8a^2)}{105b^3}$                                                           | 32   |
| derivativedivides | $\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7} - \frac{4a(bx+a)^{\frac{5}{2}}}{5} + \frac{2a^2(bx+a)^{\frac{3}{2}}}{3}}{b^3}$ | 38   |
| default           | $\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7} - \frac{4a(bx+a)^{\frac{5}{2}}}{5} + \frac{2a^2(bx+a)^{\frac{3}{2}}}{3}}{b^3}$ | 38   |
| trager            | $\frac{2(15x^3b^3+3ab^2x^2-4a^2bx+8a^3)\sqrt{bx+a}}{105b^3}$                                                          | 43   |
| risch             | $\frac{2(15x^3b^3+3ab^2x^2-4a^2bx+8a^3)\sqrt{bx+a}}{105b^3}$                                                          | 43   |

input `int(x^2*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)`output `2/105*(b*x+a)^(3/2)*(15*b^2*x^2-12*a*b*x+8*a^2)/b^3`**3.179.5 Fracas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int x^2 \sqrt{a+bx} dx = \frac{2(15b^3x^3 + 3ab^2x^2 - 4a^2bx + 8a^3)\sqrt{bx+a}}{105b^3}$$

input `integrate(x^2*(b*x+a)^(1/2),x, algorithm="fricas")`output `2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3`

**3.179.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 666 vs.  $2(49) = 98$ .

Time = 1.00 (sec) , antiderivative size = 666, normalized size of antiderivative = 12.57

$$\int x^2 \sqrt{a+bx} dx = \frac{16a^{\frac{23}{2}} \sqrt{1 + \frac{bx}{a}}}{105a^8b^3 + 315a^7b^4x + 315a^6b^5x^2 + 105a^5b^6x^3} - \frac{16a^{\frac{23}{2}}}{105a^8b^3 + 315a^7b^4x + 315a^6b^5x^2 + 105a^5b^6x^3} + \frac{40a^{\frac{21}{2}} bx \sqrt{1 + \frac{bx}{a}}}{105a^8b^3 + 315a^7b^4x + 315a^6b^5x^2 + 105a^5b^6x^3} - \frac{48a^{\frac{21}{2}} bx}{105a^8b^3 + 315a^7b^4x + 315a^6b^5x^2 + 105a^5b^6x^3} + \frac{30a^{\frac{19}{2}} b^2 x^2 \sqrt{1 + \frac{bx}{a}}}{105a^8b^3 + 315a^7b^4x + 315a^6b^5x^2 + 105a^5b^6x^3} - \frac{48a^{\frac{19}{2}} b^2 x^2}{105a^8b^3 + 315a^7b^4x + 315a^6b^5x^2 + 105a^5b^6x^3} + \frac{40a^{\frac{17}{2}} b^3 x^3 \sqrt{1 + \frac{bx}{a}}}{105a^8b^3 + 315a^7b^4x + 315a^6b^5x^2 + 105a^5b^6x^3} - \frac{16a^{\frac{17}{2}} b^3 x^3}{105a^8b^3 + 315a^7b^4x + 315a^6b^5x^2 + 105a^5b^6x^3} + \frac{100a^{\frac{15}{2}} b^4 x^4 \sqrt{1 + \frac{bx}{a}}}{105a^8b^3 + 315a^7b^4x + 315a^6b^5x^2 + 105a^5b^6x^3} - \frac{100a^{\frac{15}{2}} b^4 x^4}{105a^8b^3 + 315a^7b^4x + 315a^6b^5x^2 + 105a^5b^6x^3} + \frac{96a^{\frac{13}{2}} b^5 x^5 \sqrt{1 + \frac{bx}{a}}}{105a^8b^3 + 315a^7b^4x + 315a^6b^5x^2 + 105a^5b^6x^3} - \frac{96a^{\frac{13}{2}} b^5 x^5}{105a^8b^3 + 315a^7b^4x + 315a^6b^5x^2 + 105a^5b^6x^3} + \frac{30a^{\frac{11}{2}} b^6 x^6 \sqrt{1 + \frac{bx}{a}}}{105a^8b^3 + 315a^7b^4x + 315a^6b^5x^2 + 105a^5b^6x^3} - \frac{30a^{\frac{11}{2}} b^6 x^6}{105a^8b^3 + 315a^7b^4x + 315a^6b^5x^2 + 105a^5b^6x^3}$$

input `integrate(x**2*(b*x+a)**(1/2),x)`

output

```

16*a**(23/2)*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b
**5*x**2 + 105*a**5*b**6*x**3) - 16*a**(23/2)/(105*a**8*b**3 + 315*a**7*b
**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 40*a**(21/2)*b*x*sqrt(1
+ b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5
b**6*x**3) - 48*a**(21/2)*b*x/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6
b**5*x**2 + 105*a**5*b**6*x**3) + 30*a**(19/2)*b**2*x**2*sqrt(1 + b*x/a)/(
105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3)
- 48*a**(19/2)*b**2*x**2/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5
*x**2 + 105*a**5*b**6*x**3) + 40*a**(17/2)*b**3*x**3*sqrt(1 + b*x/a)/(105*
a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 1
6*a**(17/2)*b**3*x**3/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**
2 + 105*a**5*b**6*x**3) + 100*a**(15/2)*b**4*x**4*sqrt(1 + b*x/a)/(105*a**
8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 96*a
**(13/2)*b**5*x**5*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*
a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 30*a**(11/2)*b**6*x**6*sqrt(1 + b*x
/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*
x**3)

```

### 3.179.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int x^2 \sqrt{a + bx} dx = \frac{2(bx + a)^{\frac{7}{2}}}{7b^3} - \frac{4(bx + a)^{\frac{5}{2}}a}{5b^3} + \frac{2(bx + a)^{\frac{3}{2}}a^2}{3b^3}$$

input `integrate(x^2*(b*x+a)^(1/2),x, algorithm="maxima")`

output  $2/7*(b*x + a)^{(7/2)}/b^3 - 4/5*(b*x + a)^{(5/2)}*a/b^3 + 2/3*(b*x + a)^{(3/2)}*a^2/b^3$

### 3.179.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs.  $2(41) = 82$ .

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.75

$$\int x^2 \sqrt{a + bx} dx = \frac{2 \left( \frac{7 \left( 3(bx+a)^{\frac{5}{2}} - 10(bx+a)^{\frac{3}{2}}a + 15\sqrt{bx+aa^2} \right) a}{b^2} + \frac{3 \left( 5(bx+a)^{\frac{7}{2}} - 21(bx+a)^{\frac{5}{2}}a + 35(bx+a)^{\frac{3}{2}}a^2 - 35\sqrt{bx+aa^3} \right)}{b^2} \right)}{105b}$$

input `integrate(x^2*(b*x+a)^(1/2),x, algorithm="giac")`

output `2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)/b^2)/b`

### 3.179.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int x^2 \sqrt{a + bx} dx = \frac{30(a + bx)^{7/2} - 84a(a + bx)^{5/2} + 70a^2(a + bx)^{3/2}}{105b^3}$$

input `int(x^2*(a + b*x)^(1/2),x)`

output `(30*(a + b*x)^(7/2) - 84*a*(a + b*x)^(5/2) + 70*a^2*(a + b*x)^(3/2))/(105*b^3)`

### 3.180 $\int \frac{\sqrt{a+bx}}{x} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.180.1 Optimal result . . . . .                            | 1004 |
| 3.180.2 Mathematica [A] (verified) . . . . .                | 1004 |
| 3.180.3 Rubi [A] (verified) . . . . .                       | 1005 |
| 3.180.4 Maple [A] (verified) . . . . .                      | 1006 |
| 3.180.5 Fricas [A] (verification not implemented) . . . . . | 1006 |
| 3.180.6 Sympy [B] (verification not implemented) . . . . .  | 1007 |
| 3.180.7 Maxima [A] (verification not implemented) . . . . . | 1007 |
| 3.180.8 Giac [A] (verification not implemented) . . . . .   | 1007 |
| 3.180.9 Mupad [B] (verification not implemented) . . . . .  | 1008 |

#### 3.180.1 Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{\sqrt{a+bx}}{x} dx = 2\sqrt{a+bx} - 2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

output `-2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x+a)^(1/2)`

#### 3.180.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx}}{x} dx = 2\sqrt{a+bx} - 2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input `Integrate[Sqrt[a + b*x]/x,x]`

output `2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]`

**3.180.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{\sqrt{a+bx}}{x} dx \\ & \quad \downarrow 60 \\ & a \int \frac{1}{x\sqrt{a+bx}} dx + 2\sqrt{a+bx} \\ & \quad \downarrow 73 \\ & \frac{2a \int \frac{1}{\frac{a+bx}{b} - \frac{a}{b}} d\sqrt{a+bx}}{b} + 2\sqrt{a+bx} \\ & \quad \downarrow 221 \\ & 2\sqrt{a+bx} - 2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) \end{aligned}$$

input `Int[Sqrt[a + b*x]/x,x]`

output `2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]`

**3.180.3.1 Defintions of rubi rules used**

rule 60 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(b*(m + n + 1)) Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

```
rule 73 Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
rule 221 Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### 3.180.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

| method            | result                                                                                       | size |
|-------------------|----------------------------------------------------------------------------------------------|------|
| derivativedivides | $-2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) \sqrt{a} + 2\sqrt{bx+a}$ | 28   |
| default           | $-2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) \sqrt{a} + 2\sqrt{bx+a}$ | 28   |
| pseudoelliptic    | $-2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) \sqrt{a} + 2\sqrt{bx+a}$ | 28   |

```
input int((b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
output -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x+a)^(1/2)
```

### 3.180.5 Fracas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int \frac{\sqrt{a+bx}}{x} dx = \left[ \sqrt{a} \log\left(\frac{bx - 2\sqrt{bx+a}\sqrt{a} + 2a}{x}\right) + 2\sqrt{bx+a}, 2\sqrt{-a} \arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + 2\sqrt{bx+a} \right]$$

```
input integrate((b*x+a)^(1/2)/x,x, algorithm="fracas")
```

```
output [sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a), 2
*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)]
```

**3.180.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 68 vs.  $2(31) = 62$ .

Time = 0.79 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{a+bx}}{x} dx = -2\sqrt{a} \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) + \frac{2a}{\sqrt{b}\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{2\sqrt{b}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

input `integrate((b*x+a)**(1/2)/x,x)`

output `-2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*a/(sqrt(b)*sqrt(x)*sqrt(a/(b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(a/(b*x) + 1)`

**3.180.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+bx}}{x} dx = \sqrt{a} \log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right) + 2\sqrt{bx+a}$$

input `integrate((b*x+a)^(1/2)/x,x, algorithm="maxima")`

output `sqrt(a)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2*sqrt(b*x + a)`

**3.180.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+bx}}{x} dx = \frac{2a \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{bx+a}$$

input `integrate((b*x+a)^(1/2)/x,x, algorithm="giac")`

output `2*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)`



**3.180.9 Mupad [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+bx}}{x} dx = 2\sqrt{a+bx} - 2\sqrt{a} \operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

input `int((a + b*x)^(1/2)/x,x)`

output `2*(a + b*x)^(1/2) - 2*a^(1/2)*atanh((a + b*x)^(1/2)/a^(1/2))`

### 3.181 $\int \frac{\sqrt{a+bx}}{x^2} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.181.1 Optimal result . . . . .                            | 1009 |
| 3.181.2 Mathematica [A] (verified) . . . . .                | 1009 |
| 3.181.3 Rubi [A] (verified) . . . . .                       | 1010 |
| 3.181.4 Maple [A] (verified) . . . . .                      | 1011 |
| 3.181.5 Fricas [A] (verification not implemented) . . . . . | 1012 |
| 3.181.6 Sympy [A] (verification not implemented) . . . . .  | 1012 |
| 3.181.7 Maxima [A] (verification not implemented) . . . . . | 1012 |
| 3.181.8 Giac [A] (verification not implemented) . . . . .   | 1013 |
| 3.181.9 Mupad [B] (verification not implemented) . . . . .  | 1013 |

#### 3.181.1 Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{\sqrt{a+bx}}{x^2} dx = -\frac{\sqrt{a+bx}}{x} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output `-b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)-(b*x+a)^(1/2)/x`

#### 3.181.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx}}{x^2} dx = -\frac{\sqrt{a+bx}}{x} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input `Integrate[Sqrt[a + b*x]/x^2,x]`

output `-(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]`

**3.181.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{a+bx}}{x^2} dx \\
 & \quad \downarrow \text{51} \\
 & \frac{1}{2}b \int \frac{1}{x\sqrt{a+bx}} dx - \frac{\sqrt{a+bx}}{x} \\
 & \quad \downarrow \text{73} \\
 & \int \frac{1}{\frac{a+bx}{b} - \frac{a}{b}} d\sqrt{a+bx} - \frac{\sqrt{a+bx}}{x} \\
 & \quad \downarrow \text{221} \\
 & -\frac{\operatorname{barctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{a+bx}}{x}
 \end{aligned}$$

input `Int[Sqrt[a + b*x]/x^2,x]`

output `-(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]`

## 3.181.3.1 Defintions of rubi rules used

```
rule 51 Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

```
rule 73 Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

```
rule 221 Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

## 3.181.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

| method            | result                                                                                                                        | size |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------|------|
| risch             | $-\frac{b \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{bx+a}}{x}$                 | 32   |
| pseudoelliptic    | $-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx + \sqrt{bx+a}\sqrt{a}}{x\sqrt{a}}$                  | 36   |
| derivativedivides | $2b\left(-\frac{\sqrt{bx+a}}{2bx} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$ | 37   |
| default           | $2b\left(-\frac{\sqrt{bx+a}}{2bx} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$ | 37   |

```
input int((b*x+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
output -b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)-(b*x+a)^(1/2)/x
```

**3.181.5 Fracas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt{a+bx}}{x^2} dx = \left[ \frac{\sqrt{abx} \log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a+2a}}{x}\right) - 2\sqrt{bx+aa}}{2ax}, \frac{\sqrt{-abx} \arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) - \sqrt{bx+aa}}{ax} \right]$$

input `integrate((b*x+a)^(1/2)/x^2,x, algorithm="fricas")`output `[1/2*(sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a)*a)/(a*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(b*x + a)*a)/(a*x)]`**3.181.6 Sympy [A] (verification not implemented)**

Time = 0.97 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a+bx}}{x^2} dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}} - \frac{b \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

input `integrate((b*x+a)**(1/2)/x**2,x)`output `-sqrt(b)*sqrt(a/(b*x) + 1)/sqrt(x) - b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)`**3.181.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a+bx}}{x^2} dx = \frac{b \log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{bx+a}}{x}$$

input `integrate((b*x+a)^(1/2)/x^2,x, algorithm="maxima")`

output  $\frac{1}{2}b \log\left(\frac{\sqrt{bx+a} - \sqrt{a}}{\sqrt{bx+a} + \sqrt{a}}\right) / \sqrt{a} - \sqrt{bx+a}/x$

### 3.181.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+bx}}{x^2} dx = \frac{b^2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right) - \frac{\sqrt{bx+a}}{x}}{b}$$

input `integrate((b*x+a)^(1/2)/x^2,x, algorithm="giac")`

output  $(b^2 \arctan(\sqrt{bx+a}/\sqrt{-a})/\sqrt{-a} - \sqrt{bx+a} \cdot b/x)/b$

### 3.181.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+bx}}{x^2} dx = -\frac{\sqrt{a+bx}}{x} - \frac{b \operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input `int((a + b*x)^(1/2)/x^2,x)`

output  $-(a + b*x)^{(1/2)}/x - (b \operatorname{atanh}((a + b*x)^{(1/2)}/a^{(1/2)}))/a^{(1/2)}$

$$3.182 \quad \int \frac{1}{\sqrt{a+bx}} dx$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.182.1 Optimal result . . . . .                            | 1014 |
| 3.182.2 Mathematica [A] (verified) . . . . .                | 1014 |
| 3.182.3 Rubi [A] (verified) . . . . .                       | 1015 |
| 3.182.4 Maple [A] (verified) . . . . .                      | 1015 |
| 3.182.5 Fricas [A] (verification not implemented) . . . . . | 1016 |
| 3.182.6 Sympy [A] (verification not implemented) . . . . .  | 1016 |
| 3.182.7 Maxima [A] (verification not implemented) . . . . . | 1016 |
| 3.182.8 Giac [A] (verification not implemented) . . . . .   | 1017 |
| 3.182.9 Mupad [B] (verification not implemented) . . . . .  | 1017 |

### 3.182.1 Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{\sqrt{a+bx}} dx = \frac{2\sqrt{a+bx}}{b}$$

output `2*(b*x+a)^(1/2)/b`

### 3.182.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx}} dx = \frac{2\sqrt{a+bx}}{b}$$

input `Integrate[1/Sqrt[a + b*x],x]`

output `(2*Sqrt[a + b*x])/b`

**3.182.3 Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx}} dx$$

↓ 17

$$\frac{2\sqrt{a+bx}}{b}$$

input `Int[1/Sqrt[a + b*x],x]`

output `(2*Sqrt[a + b*x])/b`

**3.182.3.1 Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_.))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**3.182.4 Maple [A] (verified)**

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

| method            | result                   | size |
|-------------------|--------------------------|------|
| gospers           | $\frac{2\sqrt{bx+a}}{b}$ | 13   |
| derivativedivides | $\frac{2\sqrt{bx+a}}{b}$ | 13   |
| default           | $\frac{2\sqrt{bx+a}}{b}$ | 13   |
| trager            | $\frac{2\sqrt{bx+a}}{b}$ | 13   |
| risch             | $\frac{2\sqrt{bx+a}}{b}$ | 13   |
| pseudoelliptic    | $\frac{2\sqrt{bx+a}}{b}$ | 13   |



input `int(1/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)`

output `2*(b*x+a)^(1/2)/b`

### 3.182.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+bx}} dx = \frac{2\sqrt{bx+a}}{b}$$

input `integrate(1/(b*x+a)^(1/2),x, algorithm="fricas")`

output `2*sqrt(b*x + a)/b`

### 3.182.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{a+bx}} dx = \frac{2\sqrt{a+bx}}{b}$$

input `integrate(1/(b*x+a)**(1/2),x)`

output `2*sqrt(a + b*x)/b`

### 3.182.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+bx}} dx = \frac{2\sqrt{bx+a}}{b}$$

input `integrate(1/(b*x+a)^(1/2),x, algorithm="maxima")`

output `2*sqrt(b*x + a)/b`

**3.182.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+bx}} dx = \frac{2\sqrt{bx+a}}{b}$$

input `integrate(1/(b*x+a)^(1/2),x, algorithm="giac")`

output `2*sqrt(b*x + a)/b`

**3.182.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+bx}} dx = \frac{2\sqrt{a+bx}}{b}$$

input `int(1/(a + b*x)^(1/2),x)`

output `(2*(a + b*x)^(1/2))/b`

### 3.183 $\int \frac{x}{\sqrt{a+bx}} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.183.1 Optimal result . . . . .                            | 1018 |
| 3.183.2 Mathematica [A] (verified) . . . . .                | 1018 |
| 3.183.3 Rubi [A] (verified) . . . . .                       | 1019 |
| 3.183.4 Maple [A] (verified) . . . . .                      | 1020 |
| 3.183.5 Fricas [A] (verification not implemented) . . . . . | 1020 |
| 3.183.6 Sympy [B] (verification not implemented) . . . . .  | 1021 |
| 3.183.7 Maxima [A] (verification not implemented) . . . . . | 1021 |
| 3.183.8 Giac [A] (verification not implemented) . . . . .   | 1021 |
| 3.183.9 Mupad [B] (verification not implemented) . . . . .  | 1022 |

#### 3.183.1 Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{x}{\sqrt{a+bx}} dx = -\frac{2a\sqrt{a+bx}}{b^2} + \frac{2(a+bx)^{3/2}}{3b^2}$$

output  $2/3*(b*x+a)^{(3/2)}/b^2-2*a*(b*x+a)^{(1/2)}/b^2$

#### 3.183.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{a+bx}} dx = \frac{2(-2a+bx)\sqrt{a+bx}}{3b^2}$$

input `Integrate[x/Sqrt[a + b*x],x]`

output  $(2*(-2*a + b*x)*\text{Sqrt}[a + b*x])/(3*b^2)$

**3.183.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx}} dx$$

↓ 53

$$\int \left( \frac{\sqrt{a+bx}}{b} - \frac{a}{b\sqrt{a+bx}} \right) dx$$

↓ 2009

$$\frac{2(a+bx)^{3/2}}{3b^2} - \frac{2a\sqrt{a+bx}}{b^2}$$

input `Int[x/Sqrt[a + b*x],x]`

output `(-2*a*Sqrt[a + b*x])/b^2 + (2*(a + b*x)^(3/2))/(3*b^2)`

**3.183.3.1 Defintions of rubi rules used**

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

**3.183.4 Maple [A] (verified)**

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

| method            | result                                                        | size |
|-------------------|---------------------------------------------------------------|------|
| gospers           | $-\frac{2\sqrt{bx+a}(-bx+2a)}{3b^2}$                          | 21   |
| trager            | $-\frac{2\sqrt{bx+a}(-bx+2a)}{3b^2}$                          | 21   |
| risch             | $-\frac{2\sqrt{bx+a}(-bx+2a)}{3b^2}$                          | 21   |
| pseudoelliptic    | $-\frac{2\sqrt{bx+a}(-bx+2a)}{3b^2}$                          | 21   |
| derivativedivides | $\frac{2(bx+a)^{\frac{3}{2}}}{3} - \frac{2a\sqrt{bx+a}}{b^2}$ | 26   |
| default           | $\frac{2(bx+a)^{\frac{3}{2}}}{3} - \frac{2a\sqrt{bx+a}}{b^2}$ | 26   |

input `int(x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)`output `-2/3*(b*x+a)^(1/2)*(-b*x+2*a)/b^2`**3.183.5 Fracas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \frac{x}{\sqrt{a+bx}} dx = \frac{2\sqrt{bx+a}(bx-2a)}{3b^2}$$

input `integrate(x/(b*x+a)^(1/2),x, algorithm="fracas")`output `2/3*sqrt(b*x + a)*(b*x - 2*a)/b^2`

**3.183.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 162 vs.  $2(29) = 58$ .

Time = 0.61 (sec) , antiderivative size = 162, normalized size of antiderivative = 5.06

$$\int \frac{x}{\sqrt{a+bx}} dx = -\frac{4a^{\frac{7}{2}}\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x} + \frac{4a^{\frac{7}{2}}}{3a^2b^2+3ab^3x} - \frac{2a^{\frac{5}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x} \\ + \frac{4a^{\frac{5}{2}}bx}{3a^2b^2+3ab^3x} + \frac{2a^{\frac{3}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}$$

input `integrate(x/(b*x+a)**(1/2),x)`

output `-4*a**(7/2)*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x) + 4*a**(7/2)/(3*a**2*b**2 + 3*a*b**3*x) - 2*a**(5/2)*b*x*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x) + 4*a**(5/2)*b*x/(3*a**2*b**2 + 3*a*b**3*x) + 2*a**(3/2)*b**2*x**2*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x)`

**3.183.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{x}{\sqrt{a+bx}} dx = \frac{2(bx+a)^{\frac{3}{2}}}{3b^2} - \frac{2\sqrt{bx+aa}}{b^2}$$

input `integrate(x/(b*x+a)^(1/2),x, algorithm="maxima")`

output `2/3*(b*x + a)^(3/2)/b^2 - 2*sqrt(b*x + a)*a/b^2`

**3.183.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{a+bx}} dx = \frac{2\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+aa}\right)}{3b^2}$$

input `integrate(x/(b*x+a)^(1/2),x, algorithm="giac")`

output `2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/b^2`

### 3.183.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt{a+bx}} dx = -\frac{6a\sqrt{a+bx} - 2(a+bx)^{3/2}}{3b^2}$$

input `int(x/(a + b*x)^(1/2),x)`

output `-(6*a*(a + b*x)^(1/2) - 2*(a + b*x)^(3/2))/(3*b^2)`

### 3.184 $\int \frac{x^2}{\sqrt{a+bx}} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.184.1 Optimal result . . . . .                            | 1023 |
| 3.184.2 Mathematica [A] (verified) . . . . .                | 1023 |
| 3.184.3 Rubi [A] (verified) . . . . .                       | 1024 |
| 3.184.4 Maple [A] (verified) . . . . .                      | 1025 |
| 3.184.5 Fricas [A] (verification not implemented) . . . . . | 1025 |
| 3.184.6 Sympy [B] (verification not implemented) . . . . .  | 1026 |
| 3.184.7 Maxima [A] (verification not implemented) . . . . . | 1027 |
| 3.184.8 Giac [A] (verification not implemented) . . . . .   | 1027 |
| 3.184.9 Mupad [B] (verification not implemented) . . . . .  | 1028 |

#### 3.184.1 Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \frac{x^2}{\sqrt{a+bx}} dx = \frac{2a^2\sqrt{a+bx}}{b^3} - \frac{4a(a+bx)^{3/2}}{3b^3} + \frac{2(a+bx)^{5/2}}{5b^3}$$

output `-4/3*a*(b*x+a)^(3/2)/b^3+2/5*(b*x+a)^(5/2)/b^3+2*a^2*(b*x+a)^(1/2)/b^3`

#### 3.184.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{a+bx}} dx = \frac{2\sqrt{a+bx}(8a^2 - 4abx + 3b^2x^2)}{15b^3}$$

input `Integrate[x^2/Sqrt[a + b*x],x]`

output `(2*Sqrt[a + b*x]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3)`



**3.184.3 Rubi [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx}} dx$$

↓ 53

$$\int \left( \frac{a^2}{b^2\sqrt{a+bx}} - \frac{2a\sqrt{a+bx}}{b^2} + \frac{(a+bx)^{3/2}}{b^2} \right) dx$$

↓ 2009

$$\frac{2a^2\sqrt{a+bx}}{b^3} + \frac{2(a+bx)^{5/2}}{5b^3} - \frac{4a(a+bx)^{3/2}}{3b^3}$$

input `Int[x^2/Sqrt[a + b*x],x]`

output `(2*a^2*Sqrt[a + b*x])/b^3 - (4*a*(a + b*x)^(3/2))/(3*b^3) + (2*(a + b*x)^(5/2))/(5*b^3)`

**3.184.3.1 Defintions of rubi rules used**

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

**3.184.4 Maple [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

| method            | result                                                                                   | size |
|-------------------|------------------------------------------------------------------------------------------|------|
| gospers           | $\frac{2\sqrt{bx+a}(3x^2b^2-4axb+8a^2)}{15b^3}$                                          | 32   |
| trager            | $\frac{2\sqrt{bx+a}(3x^2b^2-4axb+8a^2)}{15b^3}$                                          | 32   |
| risch             | $\frac{2\sqrt{bx+a}(3x^2b^2-4axb+8a^2)}{15b^3}$                                          | 32   |
| pseudoelliptic    | $\frac{2\sqrt{bx+a}(3x^2b^2-4axb+8a^2)}{15b^3}$                                          | 32   |
| derivativedivides | $\frac{2(bx+a)^{\frac{5}{2}}}{5} - \frac{4a(bx+a)^{\frac{3}{2}}}{b^3} + 2a^2\sqrt{bx+a}$ | 37   |
| default           | $\frac{2(bx+a)^{\frac{5}{2}}}{5} - \frac{4a(bx+a)^{\frac{3}{2}}}{b^3} + 2a^2\sqrt{bx+a}$ | 37   |

input `int(x^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)`output `2/15*(b*x+a)^(1/2)*(3*b^2*x^2-4*a*b*x+8*a^2)/b^3`**3.184.5 Fracas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{\sqrt{a+bx}} dx = \frac{2(3b^2x^2 - 4abx + 8a^2)\sqrt{bx+a}}{15b^3}$$

input `integrate(x^2/(b*x+a)^(1/2),x, algorithm="fracas")`output `2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x + a)/b^3`

**3.184.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 600 vs.  $2(48) = 96$ .

Time = 0.99 (sec) , antiderivative size = 600, normalized size of antiderivative = 11.76

$$\int \frac{x^2}{\sqrt{a+bx}} dx = \frac{16a^{\frac{21}{2}} \sqrt{1 + \frac{bx}{a}}}{15a^8b^3 + 45a^7b^4x + 45a^6b^5x^2 + 15a^5b^6x^3} - \frac{16a^{\frac{21}{2}}}{15a^8b^3 + 45a^7b^4x + 45a^6b^5x^2 + 15a^5b^6x^3} + \frac{40a^{\frac{19}{2}} bx \sqrt{1 + \frac{bx}{a}}}{15a^8b^3 + 45a^7b^4x + 45a^6b^5x^2 + 15a^5b^6x^3} - \frac{48a^{\frac{19}{2}} bx}{15a^8b^3 + 45a^7b^4x + 45a^6b^5x^2 + 15a^5b^6x^3} + \frac{30a^{\frac{17}{2}} b^2x^2 \sqrt{1 + \frac{bx}{a}}}{15a^8b^3 + 45a^7b^4x + 45a^6b^5x^2 + 15a^5b^6x^3} - \frac{48a^{\frac{17}{2}} b^2x^2}{15a^8b^3 + 45a^7b^4x + 45a^6b^5x^2 + 15a^5b^6x^3} + \frac{10a^{\frac{15}{2}} b^3x^3 \sqrt{1 + \frac{bx}{a}}}{15a^8b^3 + 45a^7b^4x + 45a^6b^5x^2 + 15a^5b^6x^3} - \frac{16a^{\frac{15}{2}} b^3x^3}{15a^8b^3 + 45a^7b^4x + 45a^6b^5x^2 + 15a^5b^6x^3} + \frac{10a^{\frac{13}{2}} b^4x^4 \sqrt{1 + \frac{bx}{a}}}{15a^8b^3 + 45a^7b^4x + 45a^6b^5x^2 + 15a^5b^6x^3} - \frac{10a^{\frac{13}{2}} b^4x^4}{15a^8b^3 + 45a^7b^4x + 45a^6b^5x^2 + 15a^5b^6x^3} + \frac{6a^{\frac{11}{2}} b^5x^5 \sqrt{1 + \frac{bx}{a}}}{15a^8b^3 + 45a^7b^4x + 45a^6b^5x^2 + 15a^5b^6x^3} - \frac{6a^{\frac{11}{2}} b^5x^5}{15a^8b^3 + 45a^7b^4x + 45a^6b^5x^2 + 15a^5b^6x^3}$$

input `integrate(x**2/(b*x+a)**(1/2), x)`

output  $16*a^{(21/2)}*\sqrt{1 + b*x/a}/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) - 16*a^{(21/2)}/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) + 40*a^{(19/2)}*b*x*\sqrt{1 + b*x/a}/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) - 48*a^{(19/2)}*b*x/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) + 30*a^{(17/2)}*b^{**2}*x^{**2}*\sqrt{1 + b*x/a}/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) - 48*a^{(17/2)}*b^{**2}*x^{**2}/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) + 10*a^{(15/2)}*b^{**3}*x^{**3}*\sqrt{1 + b*x/a}/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) - 16*a^{(15/2)}*b^{**3}*x^{**3}/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) + 10*a^{(13/2)}*b^{**4}*x^{**4}*\sqrt{1 + b*x/a}/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3}) + 6*a^{(11/2)}*b^{**5}*x^{**5}*\sqrt{1 + b*x/a}/(15*a^{**8}*b^{**3} + 45*a^{**7}*b^{**4}*x + 45*a^{**6}*b^{**5}*x^{**2} + 15*a^{**5}*b^{**6}*x^{**3})$

### 3.184.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{\sqrt{a+bx}} dx = \frac{2(bx+a)^{\frac{5}{2}}}{5b^3} - \frac{4(bx+a)^{\frac{3}{2}}a}{3b^3} + \frac{2\sqrt{bx+aa^2}}{b^3}$$

input `integrate(x^2/(b*x+a)^(1/2),x, algorithm="maxima")`

output  $2/5*(b*x + a)^{(5/2)}/b^3 - 4/3*(b*x + a)^{(3/2)}*a/b^3 + 2*\sqrt{b*x + a}*a^2/b^3$

### 3.184.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{a+bx}} dx = \frac{2\left(3(bx+a)^{\frac{5}{2}} - 10(bx+a)^{\frac{3}{2}}a + 15\sqrt{bx+aa^2}\right)}{15b^3}$$

input `integrate(x^2/(b*x+a)^(1/2),x, algorithm="giac")`

output  $2/15*(3*(b*x + a)^{(5/2)} - 10*(b*x + a)^{(3/2)}*a + 15*\sqrt{b*x + a}*a^2)/b^3$

**3.184.9 Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{a+bx}} dx = \frac{6(a+bx)^{5/2} - 20a(a+bx)^{3/2} + 30a^2\sqrt{a+bx}}{15b^3}$$

input `int(x^2/(a + b*x)^(1/2),x)`

output `(6*(a + b*x)^(5/2) - 20*a*(a + b*x)^(3/2) + 30*a^2*(a + b*x)^(1/2))/(15*b^3)`

### 3.185 $\int \frac{1}{x\sqrt{a+bx}} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.185.1 Optimal result . . . . .                            | 1029 |
| 3.185.2 Mathematica [A] (verified) . . . . .                | 1029 |
| 3.185.3 Rubi [A] (verified) . . . . .                       | 1030 |
| 3.185.4 Maple [A] (verified) . . . . .                      | 1031 |
| 3.185.5 Fricas [A] (verification not implemented) . . . . . | 1031 |
| 3.185.6 Sympy [A] (verification not implemented) . . . . .  | 1031 |
| 3.185.7 Maxima [A] (verification not implemented) . . . . . | 1032 |
| 3.185.8 Giac [A] (verification not implemented) . . . . .   | 1032 |
| 3.185.9 Mupad [B] (verification not implemented) . . . . .  | 1032 |

#### 3.185.1 Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{1}{x\sqrt{a+bx}} dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output `-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)`

#### 3.185.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx}} dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input `Integrate[1/(x*Sqrt[a + b*x]),x]`

output `(-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]`

**3.185.3 Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx}} dx$$

↓ 73

$$\frac{2 \int \frac{1}{\frac{a+bx}{b} - \frac{a}{b}} d\sqrt{a+bx}}{b}$$

↓ 221

$$-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input `Int[1/(x*Sqrt[a + b*x]),x]`

output `(-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]`

**3.185.3.1 Defintions of rubi rules used**

rule 73 `Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

**3.185.4 Maple [A] (verified)**

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

| method            | result                                                                                | size |
|-------------------|---------------------------------------------------------------------------------------|------|
| derivativedivides | $-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$ | 18   |
| default           | $-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$ | 18   |
| pseudoelliptic    | $-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$ | 18   |

input `int(1/x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)`output `-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)`**3.185.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.43

$$\int \frac{1}{x\sqrt{a+bx}} dx = \left[ \frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a} \right]$$

input `integrate(1/x/(b*x+a)^(1/2),x, algorithm="fracas")`output `[log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a)/a]`**3.185.6 Sympy [A] (verification not implemented)**

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\sqrt{a+bx}} dx = -\frac{2 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

input `integrate(1/x/(b*x+a)**(1/2),x)`



output `-2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)`

### 3.185.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\sqrt{a+bx}} dx = \frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

input `integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")`

output `log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)`

### 3.185.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{a+bx}} dx = \frac{2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input `integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")`

output `2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)`

### 3.185.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\sqrt{a+bx}} dx = -\frac{2 \operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input `int(1/(x*(a + b*x)^(1/2)),x)`

output `-(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)`

### 3.186 $\int \frac{1}{x^2\sqrt{a+bx}} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.186.1 Optimal result . . . . .                            | 1033 |
| 3.186.2 Mathematica [A] (verified) . . . . .                | 1033 |
| 3.186.3 Rubi [A] (verified) . . . . .                       | 1034 |
| 3.186.4 Maple [A] (verified) . . . . .                      | 1035 |
| 3.186.5 Fricas [A] (verification not implemented) . . . . . | 1036 |
| 3.186.6 Sympy [A] (verification not implemented) . . . . .  | 1036 |
| 3.186.7 Maxima [A] (verification not implemented) . . . . . | 1036 |
| 3.186.8 Giac [A] (verification not implemented) . . . . .   | 1037 |
| 3.186.9 Mupad [B] (verification not implemented) . . . . .  | 1037 |

#### 3.186.1 Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{1}{x^2\sqrt{a+bx}} dx = -\frac{\sqrt{a+bx}}{ax} + \frac{\operatorname{barctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

output `b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)-(b*x+a)^(1/2)/a/x`

#### 3.186.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+bx}} dx = -\frac{\sqrt{a+bx}}{ax} + \frac{\operatorname{barctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

input `Integrate[1/(x^2*Sqrt[a + b*x]),x]`

output `-(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)`

**3.186.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{x^2 \sqrt{a+bx}} dx \\
 & \quad \downarrow \text{52} \\
 & -\frac{b \int \frac{1}{x \sqrt{a+bx}} dx}{2a} - \frac{\sqrt{a+bx}}{ax} \\
 & \quad \downarrow \text{73} \\
 & -\frac{\int \frac{1}{\frac{a+bx}{b} - \frac{a}{b}} d\sqrt{a+bx}}{a} - \frac{\sqrt{a+bx}}{ax} \\
 & \quad \downarrow \text{221} \\
 & \frac{\text{barctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\sqrt{a+bx}}{ax}
 \end{aligned}$$

input `Int[1/(x^2*sqrt[a + b*x]),x]`

output `-(sqrt[a + b*x]/(a*x)) + (b*ArcTanh[sqrt[a + b*x]/sqrt[a]])/a^(3/2)`

## 3.186.3.1 Defintions of rubi rules used

rule 52 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1))) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

## 3.186.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

| method            | result                                                                                                                                | size |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------------|------|
| risch             | $\frac{b \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{\sqrt{bx+a}}{ax}$                  | 34   |
| pseudoelliptic    | $\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx - \sqrt{bx+a}\sqrt{a}}{a^{\frac{3}{2}}x}$                    | 36   |
| derivativedivides | $2b\left(-\frac{\sqrt{bx+a}}{2abx} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$ | 40   |
| default           | $2b\left(-\frac{\sqrt{bx+a}}{2abx} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$ | 40   |

input `int(1/x^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)`

output `b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)-(b*x+a)^(1/2)/a/x`

**3.186.5 Fricas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^2\sqrt{a+bx}} dx = \left[ \frac{\sqrt{abx} \log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a+2a}}{x}\right) - 2\sqrt{bx+aa}}{2a^2x}, \right. \\ \left. - \frac{\sqrt{-abx} \arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + \sqrt{bx+aa}}{a^2x} \right]$$

input `integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="fricas")`output `[1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a)*a)/(a^2*x), -(sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + sqrt(b*x + a)*a)/(a^2*x)]`**3.186.6 Sympy [A] (verification not implemented)**

Time = 1.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2\sqrt{a+bx}} dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a\sqrt{x}} + \frac{b \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}}$$

input `integrate(1/x**2/(b*x+a)**(1/2),x)`output `-sqrt(b)*sqrt(a/(b*x) + 1)/(a*sqrt(x)) + b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/a**(3/2)`**3.186.7 Maxima [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^2\sqrt{a+bx}} dx = -\frac{\sqrt{bx+ab}}{(bx+a)a-a^2} - \frac{b \log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2a^{\frac{3}{2}}}$$

input `integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="maxima")`

output `-sqrt(b*x + a)*b/((b*x + a)*a - a^2) - 1/2*b*log((sqrt(b*x + a) - sqrt(a))  
/(sqrt(b*x + a) + sqrt(a)))/a^(3/2)`

### 3.186.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^2 \sqrt{a+bx}} dx = -\frac{\frac{b^2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-aa}} + \frac{\sqrt{bx+ab}}{ax}}{b}$$

input `integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="giac")`

output `-(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))  
/b`

### 3.186.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2 \sqrt{a+bx}} dx = \frac{b \operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\sqrt{a+bx}}{ax}$$

input `int(1/(x^2*(a + b*x)^(1/2)),x)`

output `(b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(3/2) - (a + b*x)^(1/2)/(a*x)`

### 3.187 $\int (a + bx)^{p/2} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.187.1 Optimal result . . . . .                            | 1038 |
| 3.187.2 Mathematica [A] (verified) . . . . .                | 1038 |
| 3.187.3 Rubi [A] (verified) . . . . .                       | 1039 |
| 3.187.4 Maple [A] (verified) . . . . .                      | 1039 |
| 3.187.5 Fricas [A] (verification not implemented) . . . . . | 1040 |
| 3.187.6 Sympy [A] (verification not implemented) . . . . .  | 1040 |
| 3.187.7 Maxima [A] (verification not implemented) . . . . . | 1040 |
| 3.187.8 Giac [A] (verification not implemented) . . . . .   | 1041 |
| 3.187.9 Mupad [B] (verification not implemented) . . . . .  | 1041 |

#### 3.187.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int (a + bx)^{p/2} dx = \frac{2(a + bx)^{\frac{2+p}{2}}}{b(2 + p)}$$

output `2*(b*x+a)^(1+1/2*p)/b/(2+p)`

#### 3.187.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int (a + bx)^{p/2} dx = \frac{(a + bx)^{1+\frac{p}{2}}}{b(1 + \frac{p}{2})}$$

input `Integrate[(a + b*x)^(p/2),x]`

output `(a + b*x)^(1 + p/2)/(b*(1 + p/2))`

**3.187.3 Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + bx)^{p/2} dx$$

$$\downarrow 17$$

$$\frac{2(a + bx)^{\frac{p+2}{2}}}{b(p + 2)}$$

input `Int[(a + b*x)^(p/2), x]`

output `(2*(a + b*x)^((2 + p)/2))/(b*(2 + p))`

**3.187.3.1 Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**3.187.4 Maple [A] (verified)**

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

| method  | result                                       | size |
|---------|----------------------------------------------|------|
| gospers | $\frac{2(bx+a)(bx+a)^{\frac{p}{2}}}{b(2+p)}$ | 25   |
| risch   | $\frac{2(bx+a)(bx+a)^{\frac{p}{2}}}{b(2+p)}$ | 25   |

input `int(((b*x+a)^(1/2))^p,x,method=_RETURNVERBOSE)`

output `2*(b*x+a)*((b*x+a)^(1/2))^p/b/(2+p)`



**3.187.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a + bx)^{p/2} dx = \frac{2(bx + a)\sqrt{bx + a}^p}{bp + 2b}$$

input `integrate(((b*x+a)^(1/2))^p,x, algorithm="fricas")`output `2*(b*x + a)*sqrt(b*x + a)^p/(b*p + 2*b)`**3.187.6 Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int (a + bx)^{p/2} dx = \begin{cases} \frac{(a+bx)^{\frac{p}{2}+1}}{\frac{p}{2}+1} & \text{for } p \neq -2 \\ \log(a + bx) & \text{otherwise} \end{cases} \frac{1}{b}$$

input `integrate(((b*x+a)**(1/2))**p,x)`output `Piecewise(((a + b*x)**(p/2 + 1)/(p/2 + 1), Ne(p, -2)), (log(a + b*x), True)))/b`**3.187.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (a + bx)^{p/2} dx = \frac{2(bx + a)^{\frac{1}{2}p+1}}{b(p + 2)}$$

input `integrate(((b*x+a)^(1/2))^p,x, algorithm="maxima")`output `2*(b*x + a)^(1/2*p + 1)/(b*(p + 2))`

**3.187.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (a + bx)^{p/2} dx = \frac{2 (bx + a)^{\frac{1}{2}p+1}}{b(p+2)}$$

input `integrate(((b*x+a)^(1/2))^p,x, algorithm="giac")`output `2*(b*x + a)^(1/2*p + 1)/(b*(p + 2))`**3.187.9 Mupad [B] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (a + bx)^{p/2} dx = \frac{2 (a + bx)^{\frac{p}{2}+1}}{b (p + 2)}$$

input `int((a + b*x)^(p/2),x)`output `(2*(a + b*x)^(p/2 + 1))/(b*(p + 2))`

### 3.188 $\int x(a + bx)^{p/2} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.188.1 Optimal result . . . . .                            | 1042 |
| 3.188.2 Mathematica [A] (verified) . . . . .                | 1042 |
| 3.188.3 Rubi [A] (verified) . . . . .                       | 1043 |
| 3.188.4 Maple [A] (verified) . . . . .                      | 1044 |
| 3.188.5 Fricas [A] (verification not implemented) . . . . . | 1044 |
| 3.188.6 Sympy [B] (verification not implemented) . . . . .  | 1044 |
| 3.188.7 Maxima [A] (verification not implemented) . . . . . | 1045 |
| 3.188.8 Giac [A] (verification not implemented) . . . . .   | 1045 |
| 3.188.9 Mupad [B] (verification not implemented) . . . . .  | 1046 |

#### 3.188.1 Optimal result

Integrand size = 13, antiderivative size = 48

$$\int x(a + bx)^{p/2} dx = -\frac{2a(a + bx)^{\frac{2+p}{2}}}{b^2(2 + p)} + \frac{2(a + bx)^{\frac{4+p}{2}}}{b^2(4 + p)}$$

output  $-2*a*(b*x+a)^{(1+1/2*p)}/b^2/(2+p)+2*(b*x+a)^{(2+1/2*p)}/b^2/(4+p)$

#### 3.188.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int x(a + bx)^{p/2} dx = \frac{2(a + bx)^{1+\frac{p}{2}}(-2a + b(2 + p)x)}{b^2(2 + p)(4 + p)}$$

input `Integrate[x*(a + b*x)^(p/2),x]`

output  $(2*(a + b*x)^{(1 + p/2)}*(-2*a + b*(2 + p)*x))/(b^2*(2 + p)*(4 + p))$

**3.188.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a + bx)^{p/2} dx$$

$$\downarrow \text{53}$$

$$\int \left( \frac{(a + bx)^{\frac{p}{2}+1}}{b} - \frac{a(a + bx)^{p/2}}{b} \right) dx$$

$$\downarrow \text{2009}$$

$$\frac{2(a + bx)^{\frac{p+4}{2}}}{b^2(p + 4)} - \frac{2a(a + bx)^{\frac{p+2}{2}}}{b^2(p + 2)}$$

input `Int[x*(a + b*x)^(p/2),x]`

output `(-2*a*(a + b*x)^((2 + p)/2))/(b^2*(2 + p)) + (2*(a + b*x)^((4 + p)/2))/(b^2*(4 + p))`

**3.188.3.1 Defintions of rubi rules used**

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

### 3.188.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

| method  | result                                                                     | size |
|---------|----------------------------------------------------------------------------|------|
| gospers | $-\frac{2(bx+a)^{\frac{p}{2}}(-xpb-2bx+2a)(bx+a)}{b^2(p^2+6p+8)}$          | 43   |
| risch   | $-\frac{2(-x^2b^2p-xapb-2x^2b^2+2a^2)(bx+a)^{\frac{p}{2}}}{b^2(4+p)(2+p)}$ | 54   |

input `int(x*((b*x+a)^(1/2))^p,x,method=_RETURNVERBOSE)`

output `-2*((b*x+a)^(1/2))^p*(-b*p*x-2*b*x+2*a)*(b*x+a)/b^2/(p^2+6*p+8)`

### 3.188.5 Fracas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int x(a+bx)^{p/2} dx = \frac{2(abpx + (b^2p + 2b^2)x^2 - 2a^2)\sqrt{bx+a}^p}{b^2p^2 + 6b^2p + 8b^2}$$

input `integrate(x*((b*x+a)^(1/2))^p,x, algorithm="fracas")`

output `2*(a*b*p*x + (b^2*p + 2*b^2)*x^2 - 2*a^2)*sqrt(b*x + a)^p/(b^2*p^2 + 6*b^2*p + 8*b^2)`

### 3.188.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(37) = 74.

Time = 0.32 (sec) , antiderivative size = 216, normalized size of antiderivative = 4.50

$$\int x(a+bx)^{p/2} dx = \begin{cases} \frac{a^{\frac{p}{2}}x^2}{2} & \text{for } b = 0 \\ \frac{a \log(\frac{a}{b}+x)}{ab^2+b^3x} + \frac{a}{ab^2+b^3x} + \frac{bx \log(\frac{a}{b}+x)}{ab^2+b^3x} & \text{for } p = -4 \\ -\frac{a \log(\frac{a}{b}+x)}{b^2} + \frac{x}{b} & \text{for } p = -2 \\ -\frac{4a^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} + \frac{2abpx(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} + \frac{2b^2px^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} + \frac{4b^2x^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} & \text{otherwise} \end{cases}$$

input `integrate(x*((b*x+a)**(1/2))**p,x)`

output `Piecewise((a**(p/2)*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) + a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(p, -4)), (-a*log(a/b + x)/b**2 + x/b, Eq(p, -2)), (-4*a**2*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2) + 2*a*b*p*x*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2) + 2*b**2*p*x**2*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2) + 4*b**2*x**2*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2), True))`

### 3.188.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int x(a + bx)^{p/2} dx = \frac{2(b^2(p+2)x^2 + abpx - 2a^2)(bx + a)^{\frac{1}{2}p}}{(p^2 + 6p + 8)b^2}$$

input `integrate(x*((b*x+a)^(1/2))^p,x, algorithm="maxima")`

output `2*(b^2*(p + 2)*x^2 + a*b*p*x - 2*a^2)*(b*x + a)^(1/2*p)/((p^2 + 6*p + 8)*b^2)`

### 3.188.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.79

$$\int x(a + bx)^{p/2} dx = \frac{\left( (bx + a)^{\frac{1}{2}p} b^2 p x^2 + (bx + a)^{\frac{1}{2}p} abpx + 2(bx + a)^{\frac{1}{2}p} b^2 x^2 - 2(bx + a)^{\frac{1}{2}p} a^2 \right)}{b^2 p^2 + 6 b^2 p + 8 b^2}$$

input `integrate(x*((b*x+a)^(1/2))^p,x, algorithm="giac")`

output `2*((b*x + a)^(1/2*p)*b^2*p*x^2 + (b*x + a)^(1/2*p)*a*b*p*x + 2*(b*x + a)^(1/2*p)*b^2*x^2 - 2*(b*x + a)^(1/2*p)*a^2)/(b^2*p^2 + 6*b^2*p + 8*b^2)`

**3.188.9 Mupad [B] (verification not implemented)**

Time = 0.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.96

$$\int x(a+bx)^{p/2} dx = \begin{cases} -\frac{a \ln(a+bx) - bx}{b^2} & \text{if } p = -2 \\ \frac{\ln(a+bx) + \frac{a}{a+bx}}{b^2} & \text{if } p = -4 \\ \frac{2 \left( \frac{(a+bx)^{\frac{p}{2}+2}}{p+4} - \frac{a(a+bx)^{\frac{p}{2}+1}}{p+2} \right)}{b^2} & \text{if } p \neq -2 \wedge p \neq -4 \end{cases}$$

input `int(x*(a + b*x)^(p/2),x)`output `piecewise(p == -2, -(a*log(a + b*x) - b*x)/b^2, p == -4, (log(a + b*x) + a/(a + b*x))/b^2, p ~= -2 & p ~= -4, (2*((a + b*x)^(p/2 + 2)/(p + 4) - (a*(a + b*x)^(p/2 + 1))/(p + 2)))/b^2)`

### 3.189 $\int \arctan\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.189.1 Optimal result . . . . .                            | 1047 |
| 3.189.2 Mathematica [A] (verified) . . . . .                | 1047 |
| 3.189.3 Rubi [A] (verified) . . . . .                       | 1048 |
| 3.189.4 Maple [A] (verified) . . . . .                      | 1050 |
| 3.189.5 Fricas [A] (verification not implemented) . . . . . | 1050 |
| 3.189.6 Sympy [B] (verification not implemented) . . . . .  | 1051 |
| 3.189.7 Maxima [A] (verification not implemented) . . . . . | 1051 |
| 3.189.8 Giac [A] (verification not implemented) . . . . .   | 1052 |
| 3.189.9 Mupad [B] (verification not implemented) . . . . .  | 1052 |

#### 3.189.1 Optimal result

Integrand size = 18, antiderivative size = 55

$$\int \arctan\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right) dx = \frac{\arctan(1-\sqrt{2}x)}{\sqrt{2}} - x \arctan(1-\sqrt{2}x) - \frac{\log(1-\sqrt{2}x+x^2)}{2\sqrt{2}}$$

output `x*arctan(-1+x*2^(1/2))-1/2*arctan(-1+x*2^(1/2))*2^(1/2)-1/4*ln(1+x^2-x*2^(1/2))*2^(1/2)`

#### 3.189.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \arctan\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right) dx = \frac{1}{4}\left(2(\sqrt{2}-2x)\arctan(1-\sqrt{2}x) - \sqrt{2}\log(1-\sqrt{2}x+x^2)\right)$$

input `Integrate[ArcTan[(-Sqrt[2] + 2*x)/Sqrt[2]],x]`

output `(2*(Sqrt[2] - 2*x)*ArcTan[1 - Sqrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2])/4`



**3.189.3 Rubi [A] (verified)**

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {5726, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \arctan\left(\frac{2x - \sqrt{2}}{\sqrt{2}}\right) dx \\
 & \quad \downarrow \text{5726} \\
 & - \int \frac{x}{\sqrt{2}(x^2 - \sqrt{2}x + 1)} dx - x \arctan(1 - \sqrt{2}x) \\
 & \quad \downarrow \text{27} \\
 & - \frac{\int \frac{x}{x^2 - \sqrt{2}x + 1} dx}{\sqrt{2}} - x \arctan(1 - \sqrt{2}x) \\
 & \quad \downarrow \text{1142} \\
 & - \frac{\int \frac{1}{x^2 - \sqrt{2}x + 1} dx}{\sqrt{2}} + \frac{1}{2} \int -\frac{\sqrt{2}(1 - \sqrt{2}x)}{x^2 - \sqrt{2}x + 1} dx - x \arctan(1 - \sqrt{2}x) \\
 & \quad \downarrow \text{25} \\
 & - \frac{\int \frac{1}{x^2 - \sqrt{2}x + 1} dx}{\sqrt{2}} - \frac{1}{2} \int \frac{\sqrt{2}(1 - \sqrt{2}x)}{x^2 - \sqrt{2}x + 1} dx - x \arctan(1 - \sqrt{2}x) \\
 & \quad \downarrow \text{27} \\
 & - \frac{\int \frac{1}{x^2 - \sqrt{2}x + 1} dx}{\sqrt{2}} - \frac{\int \frac{1 - \sqrt{2}x}{x^2 - \sqrt{2}x + 1} dx}{\sqrt{2}} - x \arctan(1 - \sqrt{2}x) \\
 & \quad \downarrow \text{1082} \\
 & - \frac{\int \frac{1}{(1 - \sqrt{2}x)^2 - 1} d(1 - \sqrt{2}x) - \frac{\int \frac{1 - \sqrt{2}x}{x^2 - \sqrt{2}x + 1} dx}{\sqrt{2}}}{\sqrt{2}} - x \arctan(1 - \sqrt{2}x) \\
 & \quad \downarrow \text{217} \\
 & - \frac{-\frac{\int \frac{1 - \sqrt{2}x}{x^2 - \sqrt{2}x + 1} dx}{\sqrt{2}} - \arctan(1 - \sqrt{2}x)}{\sqrt{2}} - x \arctan(1 - \sqrt{2}x)
 \end{aligned}$$

---

3.189.  $\int \arctan\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right) dx$

$$\downarrow 1103$$

$$-\frac{\frac{1}{2} \log(x^2 - \sqrt{2}x + 1) - \arctan(1 - \sqrt{2}x)}{\sqrt{2}} - x \arctan(1 - \sqrt{2}x)$$

input `Int[ArcTan[(-Sqrt[2] + 2*x)/Sqrt[2]],x]`

output `-(x*ArcTan[1 - Sqrt[2]*x]) - (-ArcTan[1 - Sqrt[2]*x] + Log[1 - Sqrt[2]*x + x^2]/2)/Sqrt[2]`

### 3.189.3.1 Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

rule 1082 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Simp[-2/b Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x]`

rule 1103 `Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

rule 1142 `Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(2*c*d - b*e)/(2*c) Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c) Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]`

rule 5726 `Int[ArcTan[u_], x_Symbol] := Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]`

### 3.189.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

| method            | result                                                                                                                                                                                                                                                                            |
|-------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| derivativedivides | $\frac{\sqrt{2} \left( (-1+x\sqrt{2}) \arctan(-1+x\sqrt{2}) - \frac{\ln\left(\frac{(-1+x\sqrt{2})^2+1}{2}\right)}{2} \right)}{2}$                                                                                                                                                 |
| default           | $\frac{\sqrt{2} \left( (-1+x\sqrt{2}) \arctan(-1+x\sqrt{2}) - \frac{\ln\left(\frac{(-1+x\sqrt{2})^2+1}{2}\right)}{2} \right)}{2}$                                                                                                                                                 |
| parts             | $x \arctan\left(\frac{(2x-\sqrt{2})\sqrt{2}}{2}\right) - 2\sqrt{2} \left( \frac{\ln(1+x^2-x\sqrt{2})}{8} + \frac{\arctan\left(\frac{(2x-\sqrt{2})\sqrt{2}}{2}\right)}{4} \right)$                                                                                                 |
| risch             | $\frac{ix \ln\left(1 + \frac{i(-2x+\sqrt{2})\sqrt{2}}{2}\right)}{2} - \frac{ix \ln\left(1 - \frac{i(-2x+\sqrt{2})\sqrt{2}}{2}\right)}{2} - \frac{\sqrt{2} \ln(4-4x\sqrt{2}+4x^2)}{4} - \frac{\sqrt{2} \arctan\left(\frac{(2x-\sqrt{2})\sqrt{2}}{2}\right)}{2}$                    |
| parallelrisc      | $\frac{\sqrt{2} \ln(1+x^2-x\sqrt{2})x^2+6\sqrt{2} \arctan\left(\frac{(2x-\sqrt{2})\sqrt{2}}{2}\right)x^2-4x^3 \arctan\left(\frac{(2x-\sqrt{2})\sqrt{2}}{2}\right)+\ln(1+x^2-x\sqrt{2})\sqrt{2}+2\sqrt{2} \arctan\left(\frac{(2x-\sqrt{2})\sqrt{2}}{2}\right)}{4x\sqrt{2}-4x^2-4}$ |

input `int(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x,method=_RETURNVERBOSE)`

output `1/2*2^(1/2)*((-1+x*2^(1/2))*arctan(-1+x*2^(1/2))-1/2*ln((-1+x*2^(1/2))^2+1))`

### 3.189.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

$$\int \arctan\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right) dx = \frac{1}{2} (2x - \sqrt{2}) \arctan(\sqrt{2}x - 1) - \frac{1}{4} \sqrt{2} \log(x^2 - \sqrt{2}x + 1)$$

input `integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x, algorithm="fricas")`

---

3.189.  $\int \arctan\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right) dx$

output  $1/2*(2*x - \text{sqrt}(2))*\arctan(\text{sqrt}(2)*x - 1) - 1/4*\text{sqrt}(2)*\log(x^2 - \text{sqrt}(2)*x + 1)$

### 3.189.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs.  $2(48) = 96$ .

Time = 0.48 (sec) , antiderivative size = 230, normalized size of antiderivative = 4.18

$$\int \arctan\left(\frac{-\sqrt{2} + 2x}{\sqrt{2}}\right) dx = \frac{4x^3 \operatorname{atan}(\sqrt{2}x - 1)}{4x^2 - 4\sqrt{2}x + 4} - \frac{\sqrt{2}x^2 \log(x^2 - \sqrt{2}x + 1)}{4x^2 - 4\sqrt{2}x + 4} - \frac{6\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}x - 1)}{4x^2 - 4\sqrt{2}x + 4} + \frac{2x \log(x^2 - \sqrt{2}x + 1)}{4x^2 - 4\sqrt{2}x + 4} + \frac{8x \operatorname{atan}(\sqrt{2}x - 1)}{4x^2 - 4\sqrt{2}x + 4} - \frac{\sqrt{2} \log(x^2 - \sqrt{2}x + 1)}{4x^2 - 4\sqrt{2}x + 4} - \frac{2\sqrt{2} \operatorname{atan}(\sqrt{2}x - 1)}{4x^2 - 4\sqrt{2}x + 4}$$

input `integrate(atan(1/2*(2*x-2**(1/2))*2**(1/2)),x)`

output  $4*x**3*\operatorname{atan}(\text{sqrt}(2)*x - 1)/(4*x**2 - 4*\text{sqrt}(2)*x + 4) - \text{sqrt}(2)*x**2*\log(x**2 - \text{sqrt}(2)*x + 1)/(4*x**2 - 4*\text{sqrt}(2)*x + 4) - 6*\text{sqrt}(2)*x**2*\operatorname{atan}(\text{sqrt}(2)*x - 1)/(4*x**2 - 4*\text{sqrt}(2)*x + 4) + 2*x*\log(x**2 - \text{sqrt}(2)*x + 1)/(4*x**2 - 4*\text{sqrt}(2)*x + 4) + 8*x*\operatorname{atan}(\text{sqrt}(2)*x - 1)/(4*x**2 - 4*\text{sqrt}(2)*x + 4) - \text{sqrt}(2)*\log(x**2 - \text{sqrt}(2)*x + 1)/(4*x**2 - 4*\text{sqrt}(2)*x + 4) - 2*\text{sqrt}(2)*\operatorname{atan}(\text{sqrt}(2)*x - 1)/(4*x**2 - 4*\text{sqrt}(2)*x + 4)$

### 3.189.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \arctan\left(\frac{-\sqrt{2} + 2x}{\sqrt{2}}\right) dx = \frac{1}{4} \sqrt{2} \left( \sqrt{2} (2x - \sqrt{2}) \arctan\left(\frac{1}{2} \sqrt{2} (2x - \sqrt{2})\right) - \log\left(\frac{1}{2} (2x - \sqrt{2})^2 + 1\right) \right)$$

input `integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x, algorithm="maxima")`

output `1/4*sqrt(2)*(sqrt(2)*(2*x - sqrt(2))*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - log(1/2*(2*x - sqrt(2))^2 + 1))`

### 3.189.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \arctan\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right) dx$$

$$= \frac{1}{4}\sqrt{2}\left(\sqrt{2}(2x-\sqrt{2})\arctan\left(\frac{1}{2}\sqrt{2}(2x-\sqrt{2})\right) - \log\left(\frac{1}{2}(2x-\sqrt{2})^2+1\right)\right)$$

input `integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x, algorithm="giac")`

output `1/4*sqrt(2)*(sqrt(2)*(2*x - sqrt(2))*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - log(1/2*(2*x - sqrt(2))^2 + 1))`

### 3.189.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \arctan\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right) dx$$

$$= \operatorname{atan}\left(\frac{\sqrt{2}(2x-\sqrt{2})}{2}\right)\left(x-\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}\ln\left((2x-\sqrt{2})^2+2\right)}{4}$$

input `int(atan((2^(1/2)*(2*x - 2^(1/2))))/2),x)`

output `atan((2^(1/2)*(2*x - 2^(1/2))))/2*(x - 2^(1/2)/2) - (2^(1/2)*log((2*x - 2^(1/2))^2 + 2))/4`

$$3.190 \quad \int \frac{1}{\sqrt{-1+x^2}} dx$$

|                                                             |      |
|-------------------------------------------------------------|------|
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| 3.190.2 Mathematica [B] (verified) . . . . .                | 1053 |
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| 3.190.4 Maple [A] (verified) . . . . .                      | 1055 |
| 3.190.5 Fricas [A] (verification not implemented) . . . . . | 1055 |
| 3.190.6 Sympy [A] (verification not implemented) . . . . .  | 1055 |
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| 3.190.8 Giac [B] (verification not implemented) . . . . .   | 1056 |
| 3.190.9 Mupad [B] (verification not implemented) . . . . .  | 1056 |

### 3.190.1 Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{\sqrt{-1+x^2}} dx = \operatorname{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

output `arctanh(x/(x^2-1)^(1/2))`

### 3.190.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 38 vs.  $2(12) = 24$ .

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.17

$$\int \frac{1}{\sqrt{-1+x^2}} dx = -\frac{1}{2} \log\left(1 - \frac{x}{\sqrt{-1+x^2}}\right) + \frac{1}{2} \log\left(1 + \frac{x}{\sqrt{-1+x^2}}\right)$$

input `Integrate[1/Sqrt[-1 + x^2],x]`

output `-1/2*Log[1 - x/Sqrt[-1 + x^2]] + Log[1 + x/Sqrt[-1 + x^2]]/2`

**3.190.3 Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2 - 1}} dx$$

↓ 224

$$\int \frac{1}{1 - \frac{x^2}{x^2 - 1}} d \frac{x}{\sqrt{x^2 - 1}}$$

↓ 219

$$\operatorname{arctanh}\left(\frac{x}{\sqrt{x^2 - 1}}\right)$$

input `Int[1/Sqrt[-1 + x^2],x]`

output `ArcTanh[x/Sqrt[-1 + x^2]]`

**3.190.3.1 Defintions of rubi rules used**

rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

rule 224 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

**3.190.4 Maple [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

| method         | result                                                                                                      | size |
|----------------|-------------------------------------------------------------------------------------------------------------|------|
| default        | $\ln(x + \sqrt{x^2 - 1})$                                                                                   | 11   |
| pseudoelliptic | $\operatorname{arctanh}\left(\frac{\sqrt{x^2-1}}{x}\right)$                                                 | 13   |
| trager         | $-\ln(-\sqrt{x^2-1} + x)$                                                                                   | 15   |
| meijerg        | $\frac{\sqrt{-\operatorname{signum}(x^2-1)} \operatorname{arcsin}(x)}{\sqrt{\operatorname{signum}(x^2-1)}}$ | 22   |

input `int(1/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)`output `ln(x+(x^2-1)^(1/2))`**3.190.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{-1+x^2}} dx = -\log(-x + \sqrt{x^2-1})$$

input `integrate(1/(x^2-1)^(1/2),x, algorithm="fracas")`output `-log(-x + sqrt(x^2 - 1))`**3.190.6 Sympy [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-1+x^2}} dx = \log(x + \sqrt{x^2-1})$$

input `integrate(1/(x**2-1)**(1/2),x)`output `log(x + sqrt(x**2 - 1))`



**3.190.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{-1+x^2}} dx = \log(2x + 2\sqrt{x^2-1})$$

input `integrate(1/(x^2-1)^(1/2),x, algorithm="maxima")`

output `log(2*x + 2*sqrt(x^2 - 1))`

**3.190.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{-1+x^2}} dx = \frac{1}{2} \sqrt{x^2-1}x + \frac{1}{2} \log(|-x + \sqrt{x^2-1}|)$$

input `integrate(1/(x^2-1)^(1/2),x, algorithm="giac")`

output `1/2*sqrt(x^2 - 1)*x + 1/2*log(abs(-x + sqrt(x^2 - 1)))`

**3.190.9 Mupad [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-1+x^2}} dx = \ln(x + \sqrt{x^2-1})$$

input `int(1/(x^2 - 1)^(1/2),x)`

output `log(x + (x^2 - 1)^(1/2))`

### 3.191 $\int \sqrt{x}\sqrt{1+x} dx$

|                                                             |       |
|-------------------------------------------------------------|-------|
| 3.191.1 Optimal result . . . . .                            | .1057 |
| 3.191.2 Mathematica [A] (verified) . . . . .                | .1057 |
| 3.191.3 Rubi [A] (verified) . . . . .                       | .1058 |
| 3.191.4 Maple [A] (verified) . . . . .                      | .1059 |
| 3.191.5 Fracas [A] (verification not implemented) . . . . . | .1059 |
| 3.191.6 Sympy [C] (verification not implemented) . . . . .  | .1060 |
| 3.191.7 Maxima [B] (verification not implemented) . . . . . | .1060 |
| 3.191.8 Giac [A] (verification not implemented) . . . . .   | .1061 |
| 3.191.9 Mupad [B] (verification not implemented) . . . . .  | .1061 |

#### 3.191.1 Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \sqrt{x}\sqrt{1+x} dx = \frac{1}{4}\sqrt{x}\sqrt{1+x} + \frac{1}{2}x^{3/2}\sqrt{1+x} - \frac{\operatorname{arcsinh}(\sqrt{x})}{4}$$

output `-1/4*arcsinh(x^(1/2))+1/2*x^(3/2)*(1+x)^(1/2)+1/4*x^(1/2)*(1+x)^(1/2)`

#### 3.191.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \sqrt{x}\sqrt{1+x} dx = \frac{1}{4}\left(\sqrt{x}\sqrt{1+x}(1+2x) + \log\left(-\sqrt{x} + \sqrt{1+x}\right)\right)$$

input `Integrate[Sqrt[x]*Sqrt[1+x],x]`

output `(Sqrt[x]*Sqrt[1+x]*(1+2*x) + Log[-Sqrt[x] + Sqrt[1+x]])/4`

**3.191.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \sqrt{x}\sqrt{x+1} dx \\
 & \quad \downarrow 60 \\
 & \frac{1}{4} \int \frac{\sqrt{x}}{\sqrt{x+1}} dx + \frac{1}{2} \sqrt{x+1} x^{3/2} \\
 & \quad \downarrow 60 \\
 & \frac{1}{4} \left( \sqrt{x}\sqrt{x+1} - \frac{1}{2} \int \frac{1}{\sqrt{x}\sqrt{x+1}} dx \right) + \frac{1}{2} \sqrt{x+1} x^{3/2} \\
 & \quad \downarrow 63 \\
 & \frac{1}{4} \left( \sqrt{x}\sqrt{x+1} - \int \frac{1}{\sqrt{x+1}} d\sqrt{x} \right) + \frac{1}{2} \sqrt{x+1} x^{3/2} \\
 & \quad \downarrow 222 \\
 & \frac{1}{4} \left( \sqrt{x}\sqrt{x+1} - \operatorname{arcsinh}(\sqrt{x}) \right) + \frac{1}{2} \sqrt{x+1} x^{3/2}
 \end{aligned}$$

input `Int[Sqrt[x]*Sqrt[1 + x],x]`

output `(x^(3/2)*Sqrt[1 + x])/2 + (Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]])/4`

**3.191.3.1 Defintions of rubi rules used**

rule 60 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(b*(m + n + 1)) Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

```
rule 63 Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] := Simp[2/b S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

```
rule 222 Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

### 3.191.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

| method  | result                                                                                                                                                       | size |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| meijerg | $-\frac{-\frac{\sqrt{\pi}\sqrt{x}(3+6x)\sqrt{1+x} + \sqrt{\pi}\operatorname{arcsinh}(\sqrt{x})}{6}}{2\sqrt{\pi}}$                                            | 34   |
| risch   | $\frac{(1+2x)\sqrt{x}\sqrt{1+x}}{4} - \frac{\sqrt{x(1+x)}\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{8\sqrt{1+x}\sqrt{x}}$                                   | 45   |
| default | $\frac{\sqrt{x}(1+x)^{\frac{3}{2}}}{2} - \frac{\sqrt{x}\sqrt{1+x}}{4} - \frac{\sqrt{x(1+x)}\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{8\sqrt{1+x}\sqrt{x}}$ | 50   |

```
input int(x^(1/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
output -1/2/Pi^(1/2)*(-1/6*Pi^(1/2)*x^(1/2)*(3+6*x)*(1+x)^(1/2)+1/2*Pi^(1/2)*arcs
inh(x^(1/2)))
```

### 3.191.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \sqrt{x}\sqrt{1+x} dx = \frac{1}{4}(2x+1)\sqrt{x+1}\sqrt{x} + \frac{1}{8}\log\left(2\sqrt{x+1}\sqrt{x} - 2x - 1\right)$$

```
input integrate(x^(1/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
output 1/4*(2*x + 1)*sqrt(x + 1)*sqrt(x) + 1/8*log(2*sqrt(x + 1)*sqrt(x) - 2*x -
1)
```

**3.191.6 Sympy [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 1.78 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.77

$$\int \sqrt{x}\sqrt{1+x} dx = \begin{cases} -\frac{\operatorname{acosh}(\sqrt{x+1})}{4} + \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{x}} - \frac{3(x+1)^{\frac{3}{2}}}{4\sqrt{x}} + \frac{\sqrt{x+1}}{4\sqrt{x}} & \text{for } |x+1| > 1 \\ \frac{i \operatorname{asin}(\sqrt{x+1})}{4} - \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{-x}} + \frac{3i(x+1)^{\frac{3}{2}}}{4\sqrt{-x}} - \frac{i\sqrt{x+1}}{4\sqrt{-x}} & \text{otherwise} \end{cases}$$

input `integrate(x**(1/2)*(1+x)**(1/2),x)`

output `Piecewise((-acosh(sqrt(x + 1))/4 + (x + 1)**(5/2)/(2*sqrt(x)) - 3*(x + 1)**(3/2)/(4*sqrt(x)) + sqrt(x + 1)/(4*sqrt(x)), Abs(x + 1) > 1), (I*asin(sqrt(x + 1))/4 - I*(x + 1)**(5/2)/(2*sqrt(-x)) + 3*I*(x + 1)**(3/2)/(4*sqrt(-x)) - I*sqrt(x + 1)/(4*sqrt(-x)), True))`

**3.191.7 Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 71 vs.  $2(27) = 54$ .

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \sqrt{x}\sqrt{1+x} dx = \frac{\frac{(x+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}} + \frac{\sqrt{x+1}}{\sqrt{x}}}{4\left(\frac{(x+1)^2}{x^2} - \frac{2(x+1)}{x} + 1\right)} - \frac{1}{8} \log\left(\frac{\sqrt{x+1}}{\sqrt{x}} + 1\right) + \frac{1}{8} \log\left(\frac{\sqrt{x+1}}{\sqrt{x}} - 1\right)$$

input `integrate(x^(1/2)*(1+x)^(1/2),x, algorithm="maxima")`

output `1/4*((x + 1)^(3/2)/x^(3/2) + sqrt(x + 1)/sqrt(x))/((x + 1)^2/x^2 - 2*(x + 1)/x + 1) - 1/8*log(sqrt(x + 1)/sqrt(x) + 1) + 1/8*log(sqrt(x + 1)/sqrt(x) - 1)`

**3.191.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \sqrt{x}\sqrt{1+x} dx = \frac{1}{4}(2x-3)\sqrt{x+1}\sqrt{x} + \sqrt{x+1}\sqrt{x} + \frac{1}{4}\log(\sqrt{x+1}-\sqrt{x})$$

input `integrate(x^(1/2)*(1+x)^(1/2),x, algorithm="giac")`output `1/4*(2*x - 3)*sqrt(x + 1)*sqrt(x) + sqrt(x + 1)*sqrt(x) + 1/4*log(sqrt(x + 1) - sqrt(x))`**3.191.9 Mupad [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \sqrt{x}\sqrt{1+x} dx = \sqrt{x} \left( \frac{x}{2} + \frac{1}{4} \right) \sqrt{x+1} - \frac{\ln(x + \sqrt{x}\sqrt{x+1} + \frac{1}{2})}{8}$$

input `int(x^(1/2)*(x + 1)^(1/2),x)`output `x^(1/2)*(x/2 + 1/4)*(x + 1)^(1/2) - log(x + x^(1/2)*(x + 1)^(1/2) + 1/2)/8`

### 3.192 $\int \sin(\sqrt{x}) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.192.1 Optimal result . . . . .                            | 1062 |
| 3.192.2 Mathematica [A] (verified) . . . . .                | 1062 |
| 3.192.3 Rubi [A] (verified) . . . . .                       | 1063 |
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| 3.192.5 Fricas [A] (verification not implemented) . . . . . | 1065 |
| 3.192.6 Sympy [A] (verification not implemented) . . . . .  | 1065 |
| 3.192.7 Maxima [A] (verification not implemented) . . . . . | 1065 |
| 3.192.8 Giac [A] (verification not implemented) . . . . .   | 1066 |
| 3.192.9 Mupad [B] (verification not implemented) . . . . .  | 1066 |

#### 3.192.1 Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \sin(\sqrt{x}) dx = -2\sqrt{x} \cos(\sqrt{x}) + 2 \sin(\sqrt{x})$$

output `2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)`

#### 3.192.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sin(\sqrt{x}) dx = -2\sqrt{x} \cos(\sqrt{x}) + 2 \sin(\sqrt{x})$$

input `Integrate[Sin[Sqrt[x]],x]`

output `-2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]`

**3.192.3 Rubi [A] (verified)**

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$ , Rules used = {3842, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \sin(\sqrt{x}) \, dx \\
 & \quad \downarrow \text{3842} \\
 & 2 \int \sqrt{x} \sin(\sqrt{x}) \, d\sqrt{x} \\
 & \quad \downarrow \text{3042} \\
 & 2 \int \sqrt{x} \sin(\sqrt{x}) \, d\sqrt{x} \\
 & \quad \downarrow \text{3777} \\
 & 2 \left( \int \cos(\sqrt{x}) \, d\sqrt{x} - \sqrt{x} \cos(\sqrt{x}) \right) \\
 & \quad \downarrow \text{3042} \\
 & 2 \left( \int \sin\left(\sqrt{x} + \frac{\pi}{2}\right) \, d\sqrt{x} - \sqrt{x} \cos(\sqrt{x}) \right) \\
 & \quad \downarrow \text{3117} \\
 & 2(\sin(\sqrt{x}) - \sqrt{x} \cos(\sqrt{x}))
 \end{aligned}$$

input `Int[Sin[Sqrt[x]], x]`

output `2*(-(Sqrt[x]*Cos[Sqrt[x]]) + Sin[Sqrt[x]])`



## 3.192.3.1 Defintions of rubi rules used

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3117 `Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

rule 3777 `Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

rule 3842 `Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_Symbol] := Simp[1/(n*f) Subst[Int[x^(1/n - 1)*(a + b*SIN[c + d*x])^p, x], (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[1/n]`

## 3.192.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

| method            | result                                                                                                         | size |
|-------------------|----------------------------------------------------------------------------------------------------------------|------|
| derivativedivides | $2 \sin(\sqrt{x}) - 2 \cos(\sqrt{x}) \sqrt{x}$                                                                 | 17   |
| default           | $2 \sin(\sqrt{x}) - 2 \cos(\sqrt{x}) \sqrt{x}$                                                                 | 17   |
| meijerg           | $4\sqrt{\pi} \left( -\frac{\sqrt{x} \cos(\sqrt{x})}{2\sqrt{\pi}} + \frac{\sin(\sqrt{x})}{2\sqrt{\pi}} \right)$ | 28   |

input `int(sin(x^(1/2)),x,method=_RETURNVERBOSE)`

output `2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)`

**3.192.5 Fricas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin(\sqrt{x}) dx = -2\sqrt{x} \cos(\sqrt{x}) + 2 \sin(\sqrt{x})$$

input `integrate(sin(x^(1/2)),x, algorithm="fricas")`

output `-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))`

**3.192.6 Sympy [A] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sin(\sqrt{x}) dx = -2\sqrt{x} \cos(\sqrt{x}) + 2 \sin(\sqrt{x})$$

input `integrate(sin(x**(1/2)),x)`

output `-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))`

**3.192.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin(\sqrt{x}) dx = -2\sqrt{x} \cos(\sqrt{x}) + 2 \sin(\sqrt{x})$$

input `integrate(sin(x^(1/2)),x, algorithm="maxima")`

output `-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))`

**3.192.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin(\sqrt{x}) dx = -2\sqrt{x} \cos(\sqrt{x}) + 2 \sin(\sqrt{x})$$

input `integrate(sin(x^(1/2)),x, algorithm="giac")`

output `-2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))`

**3.192.9 Mupad [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \sin(\sqrt{x}) dx = 2 \sin(\sqrt{x}) - 2\sqrt{x} \cos(\sqrt{x})$$

input `int(sin(x^(1/2)),x)`

output `2*sin(x^(1/2)) - 2*x^(1/2)*cos(x^(1/2))`

### 3.193 $\int \frac{x}{(1-x^2)^{9/8}} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.193.1 Optimal result . . . . .                            | 1067 |
| 3.193.2 Mathematica [A] (verified) . . . . .                | 1067 |
| 3.193.3 Rubi [A] (verified) . . . . .                       | 1068 |
| 3.193.4 Maple [A] (verified) . . . . .                      | 1069 |
| 3.193.5 Fricas [A] (verification not implemented) . . . . . | 1069 |
| 3.193.6 Sympy [A] (verification not implemented) . . . . .  | 1070 |
| 3.193.7 Maxima [A] (verification not implemented) . . . . . | 1070 |
| 3.193.8 Giac [A] (verification not implemented) . . . . .   | 1070 |
| 3.193.9 Mupad [B] (verification not implemented) . . . . .  | 1071 |

#### 3.193.1 Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x}{(1-x^2)^{9/8}} dx = \frac{4}{\sqrt[8]{1-x^2}}$$

output `4/(-x^2+1)^(1/8)`

#### 3.193.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-x^2)^{9/8}} dx = \frac{4}{\sqrt[8]{1-x^2}}$$

input `Integrate[x/(1 - x^2)^(9/8), x]`

output `4/(1 - x^2)^(1/8)`

**3.193.3 Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-x^2)^{9/8}} dx$$

↓ 241

$$\frac{4}{\sqrt[8]{1-x^2}}$$

input `Int[x/(1 - x^2)^(9/8),x]`

output `4/(1 - x^2)^(1/8)`

**3.193.3.1 Defintions of rubi rules used**

rule 241 `Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]`

**3.193.4 Maple [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

| method            | result                                          | size |
|-------------------|-------------------------------------------------|------|
| derivativedivides | $\frac{4}{(-x^2+1)^{\frac{1}{8}}}$              | 12   |
| default           | $\frac{4}{(-x^2+1)^{\frac{1}{8}}}$              | 12   |
| risch             | $\frac{4}{(-x^2+1)^{\frac{1}{8}}}$              | 12   |
| pseudoelliptic    | $\frac{4}{(-x^2+1)^{\frac{1}{8}}}$              | 12   |
| meijerg           | $\frac{x^2 {}_2F_1(1, \frac{9}{8}; 2; x^2)}{2}$ | 15   |
| gospers           | $-\frac{4(-1+x)(1+x)}{(-x^2+1)^{\frac{9}{8}}}$  | 18   |
| trager            | $-\frac{4(-x^2+1)^{\frac{7}{8}}}{x^2-1}$        | 19   |

input `int(x/(-x^2+1)^(9/8),x,method=_RETURNVERBOSE)`output `4/(-x^2+1)^(1/8)`**3.193.5 Fricas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{x}{(1-x^2)^{9/8}} dx = -\frac{4(-x^2+1)^{\frac{7}{8}}}{x^2-1}$$

input `integrate(x/(-x^2+1)^(9/8),x, algorithm="fricas")`output `-4*(-x^2 + 1)^(7/8)/(x^2 - 1)`

**3.193.6 Sympy [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{x}{(1-x^2)^{9/8}} dx = \frac{4}{\sqrt[8]{1-x^2}}$$

input `integrate(x/(-x**2+1)**(9/8),x)`output `4/(1 - x**2)**(1/8)`**3.193.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{(1-x^2)^{9/8}} dx = \frac{4}{(-x^2+1)^{1/8}}$$

input `integrate(x/(-x^2+1)^(9/8),x, algorithm="maxima")`output `4/(-x^2 + 1)^(1/8)`**3.193.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{(1-x^2)^{9/8}} dx = \frac{4}{(-x^2+1)^{1/8}}$$

input `integrate(x/(-x^2+1)^(9/8),x, algorithm="giac")`output `4/(-x^2 + 1)^(1/8)`

**3.193.9 Mupad [B] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{(1-x^2)^{9/8}} dx = \frac{4}{(1-x^2)^{1/8}}$$

input `int(x/(1 - x^2)^(9/8),x)`

output `4/(1 - x^2)^(1/8)`



### 3.194 $\int \frac{x}{\sqrt{1-x^4}} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.194.1 Optimal result . . . . .                            | 1072 |
| 3.194.2 Mathematica [A] (verified) . . . . .                | 1072 |
| 3.194.3 Rubi [A] (verified) . . . . .                       | 1073 |
| 3.194.4 Maple [A] (verified) . . . . .                      | 1074 |
| 3.194.5 Fricas [B] (verification not implemented) . . . . . | 1074 |
| 3.194.6 Sympy [C] (verification not implemented) . . . . .  | 1075 |
| 3.194.7 Maxima [B] (verification not implemented) . . . . . | 1075 |
| 3.194.8 Giac [A] (verification not implemented) . . . . .   | 1075 |
| 3.194.9 Mupad [B] (verification not implemented) . . . . .  | 1076 |

#### 3.194.1 Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{x}{\sqrt{1-x^4}} dx = \frac{\arcsin(x^2)}{2}$$

output `1/2*arcsin(x^2)`

#### 3.194.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1-x^4}} dx = \frac{\arcsin(x^2)}{2}$$

input `Integrate[x/Sqrt[1 - x^4],x]`

output `ArcSin[x^2]/2`

**3.194.3 Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {807, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-x^4}} dx$$

↓ 807

$$\frac{1}{2} \int \frac{1}{\sqrt{1-x^4}} dx^2$$

↓ 223

$$\frac{\arcsin(x^2)}{2}$$

input `Int[x/Sqrt[1 - x^4],x]`

output `ArcSin[x^2]/2`

**3.194.3.1 Defintions of rubi rules used**

rule 223 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

rule 807 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Simp[1/k Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]`

**3.194.4 Maple [A] (verified)**

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

| method         | result                                                                          | size |
|----------------|---------------------------------------------------------------------------------|------|
| default        | $\frac{\arcsin(x^2)}{2}$                                                        | 7    |
| meijerg        | $\frac{\arcsin(x^2)}{2}$                                                        | 7    |
| elliptic       | $\frac{\arcsin(x^2)}{2}$                                                        | 7    |
| pseudoelliptic | $\frac{\arcsin(x^2)}{2}$                                                        | 7    |
| trager         | $\frac{\text{RootOf}(\_Z^2+1) \ln(\text{RootOf}(\_Z^2+1)\sqrt{-x^4+1+x^2})}{2}$ | 30   |

input `int(x/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)`

output `1/2*arcsin(x^2)`

**3.194.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(6) = 12.

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{x}{\sqrt{1-x^4}} dx = -\arctan\left(\frac{\sqrt{-x^4+1}-1}{x^2}\right)$$

input `integrate(x/(-x^4+1)^(1/2),x, algorithm="fracas")`

output `-arctan((sqrt(-x^4 + 1) - 1)/x^2)`

**3.194.6 Sympy [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \frac{x}{\sqrt{1-x^4}} dx = \begin{cases} -\frac{i \operatorname{acosh}(x^2)}{2} & \text{for } |x^4| > 1 \\ \frac{\operatorname{asin}(x^2)}{2} & \text{otherwise} \end{cases}$$

input `integrate(x/(-x**4+1)**(1/2),x)`

output `Piecewise((-I*acosh(x**2)/2, Abs(x**4) > 1), (asin(x**2)/2, True))`

**3.194.7 Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{x}{\sqrt{1-x^4}} dx = -\frac{1}{2} \arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

input `integrate(x/(-x^4+1)^(1/2),x, algorithm="maxima")`

output `-1/2*arctan(sqrt(-x^4 + 1)/x^2)`

**3.194.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{\sqrt{1-x^4}} dx = \frac{1}{2} \arcsin(x^2)$$

input `integrate(x/(-x^4+1)^(1/2),x, algorithm="giac")`

output `1/2*arcsin(x^2)`

**3.194.9 Mupad [B] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{x}{\sqrt{1-x^4}} dx = \frac{\operatorname{atan}\left(\frac{x^2}{\sqrt{1-x^4}}\right)}{2}$$

input `int(x/(1 - x^4)^(1/2),x)`

output `atan(x^2/(1 - x^4)^(1/2))/2`

### 3.195 $\int \frac{1}{x\sqrt{1+x^4}} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.195.1 Optimal result . . . . .                            | 1077 |
| 3.195.2 Mathematica [A] (verified) . . . . .                | 1077 |
| 3.195.3 Rubi [A] (verified) . . . . .                       | 1078 |
| 3.195.4 Maple [A] (verified) . . . . .                      | 1079 |
| 3.195.5 Fricas [B] (verification not implemented) . . . . . | 1079 |
| 3.195.6 Sympy [A] (verification not implemented) . . . . .  | 1080 |
| 3.195.7 Maxima [B] (verification not implemented) . . . . . | 1080 |
| 3.195.8 Giac [B] (verification not implemented) . . . . .   | 1080 |
| 3.195.9 Mupad [B] (verification not implemented) . . . . .  | 1081 |

#### 3.195.1 Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{x\sqrt{1+x^4}} dx = -\frac{1}{2}\operatorname{arctanh}\left(\sqrt{1+x^4}\right)$$

output `-1/2*arctanh((x^4+1)^(1/2))`

#### 3.195.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+x^4}} dx = -\frac{1}{2}\operatorname{arctanh}\left(\sqrt{1+x^4}\right)$$

input `Integrate[1/(x*Sqrt[1 + x^4]),x]`

output `-1/2*ArcTanh[Sqrt[1 + x^4]]`

**3.195.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {798, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{x\sqrt{x^4+1}} dx \\ & \quad \downarrow \text{798} \\ & \frac{1}{4} \int \frac{1}{x^4\sqrt{x^4+1}} dx^4 \\ & \quad \downarrow \text{73} \\ & \frac{1}{2} \int \frac{1}{x^8-1} d\sqrt{x^4+1} \\ & \quad \downarrow \text{220} \\ & -\frac{1}{2} \operatorname{arctanh}(\sqrt{x^4+1}) \end{aligned}$$

input `Int[1/(x*Sqrt[1 + x^4]),x]`

output `-1/2*ArcTanh[Sqrt[1 + x^4]]`

**3.195.3.1 Defintions of rubi rules used**

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 220 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

```
rule 798 Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[1/n Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### 3.195.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

| method         | result                                                                                                           | size |
|----------------|------------------------------------------------------------------------------------------------------------------|------|
| default        | $-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$                                           | 11   |
| elliptic       | $-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$                                           | 11   |
| pseudoelliptic | $-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$                                           | 11   |
| trager         | $\frac{\ln\left(\frac{-1+\sqrt{x^4+1}}{x^2}\right)}{2}$                                                          | 17   |
| meijerg        | $\frac{(-2\ln(2)+4\ln(x))\sqrt{\pi}-2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{x^4+1}}{2}\right)}{4\sqrt{\pi}}$ | 37   |

```
input int(1/x/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
output -1/2*arctanh(1/(x^4+1)^(1/2))
```

### 3.195.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs.  $2(10) = 20$ .

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{x\sqrt{1+x^4}} dx = -\frac{1}{4} \log\left(\sqrt{x^4+1}+1\right) + \frac{1}{4} \log\left(\sqrt{x^4+1}-1\right)$$

```
input integrate(1/x/(x^4+1)^(1/2),x, algorithm="fracas")
```

```
output -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```



**3.195.6 Sympy [A] (verification not implemented)**

Time = 0.48 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{x\sqrt{1+x^4}} dx = -\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2}$$

input `integrate(1/x/(x**4+1)**(1/2),x)`

output `-asinh(x**(-2))/2`

**3.195.7 Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{x\sqrt{1+x^4}} dx = -\frac{1}{4} \log\left(\sqrt{x^4+1}+1\right) + \frac{1}{4} \log\left(\sqrt{x^4+1}-1\right)$$

input `integrate(1/x/(x^4+1)^(1/2),x, algorithm="maxima")`

output `-1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)`

**3.195.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{x\sqrt{1+x^4}} dx = -\frac{1}{4} \log\left(\sqrt{x^4+1}+1\right) + \frac{1}{4} \log\left(\sqrt{x^4+1}-1\right)$$

input `integrate(1/x/(x^4+1)^(1/2),x, algorithm="giac")`

output `-1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)`

**3.195.9 Mupad [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{1+x^4}} dx = -\frac{\operatorname{atanh}(\sqrt{x^4+1})}{2}$$

input `int(1/(x*(x^4 + 1)^(1/2)),x)`

output `-atanh((x^4 + 1)^(1/2))/2`

### 3.196 $\int \frac{x}{\sqrt{1+x^2+x^4}} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.196.1 Optimal result . . . . .                            | 1082 |
| 3.196.2 Mathematica [A] (verified) . . . . .                | 1082 |
| 3.196.3 Rubi [A] (verified) . . . . .                       | 1083 |
| 3.196.4 Maple [A] (verified) . . . . .                      | 1084 |
| 3.196.5 Fricas [A] (verification not implemented) . . . . . | 1084 |
| 3.196.6 Sympy [F] . . . . .                                 | 1084 |
| 3.196.7 Maxima [F] . . . . .                                | 1085 |
| 3.196.8 Giac [A] (verification not implemented) . . . . .   | 1085 |
| 3.196.9 Mupad [B] (verification not implemented) . . . . .  | 1085 |

#### 3.196.1 Optimal result

Integrand size = 14, antiderivative size = 18

$$\int \frac{x}{\sqrt{1+x^2+x^4}} dx = \frac{1}{2} \operatorname{arcsinh} \left( \frac{1+2x^2}{\sqrt{3}} \right)$$

output `1/2*arcsinh(1/3*(2*x^2+1)*3^(1/2))`

#### 3.196.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{x}{\sqrt{1+x^2+x^4}} dx = -\frac{1}{2} \log \left( -1 - 2x^2 + 2\sqrt{1+x^2+x^4} \right)$$

input `Integrate[x/Sqrt[1 + x^2 + x^4],x]`

output `-1/2*Log[-1 - 2*x^2 + 2*Sqrt[1 + x^2 + x^4]]`

**3.196.3 Rubi [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {1432, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{x}{\sqrt{x^4 + x^2 + 1}} dx \\ & \quad \downarrow \text{1432} \\ & \frac{1}{2} \int \frac{1}{\sqrt{x^4 + x^2 + 1}} dx^2 \\ & \quad \downarrow \text{1090} \\ & \frac{\int \frac{1}{\sqrt{\frac{x^4}{3} + 1}} d(2x^2 + 1)}{2\sqrt{3}} \\ & \quad \downarrow \text{222} \\ & \frac{1}{2} \operatorname{arcsinh}\left(\frac{2x^2 + 1}{\sqrt{3}}\right) \end{aligned}$$

input `Int[x/Sqrt[1 + x^2 + x^4], x]`

output `ArcSinh[(1 + 2*x^2)/Sqrt[3]]/2`

**3.196.3.1 Defintions of rubi rules used**

rule 222 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]`

rule 1090 `Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[1/(2*c*(-4*(c/(b^2 - 4*a*c)))^p) Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]`

rule 1432 `Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[1/2 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]`

**3.196.4 Maple [A] (verified)**

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

| method         | result                                                                                         | size |
|----------------|------------------------------------------------------------------------------------------------|------|
| default        | $\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x^2+\frac{1}{2}\right)}{3}\right)}{2}$ | 14   |
| elliptic       | $\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x^2+\frac{1}{2}\right)}{3}\right)}{2}$ | 14   |
| pseudoelliptic | $\frac{\operatorname{arcsinh}\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)}{2}$                      | 16   |
| trager         | $-\frac{\ln\left(-2x^2+2\sqrt{x^4+x^2+1}-1\right)}{2}$                                         | 23   |

input `int(x/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)`output `1/2*arcsinh(2/3*3^(1/2)*(x^2+1/2))`**3.196.5 Fricas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x}{\sqrt{1+x^2+x^4}} dx = -\frac{1}{2} \log\left(-2x^2 + 2\sqrt{x^4+x^2+1} - 1\right)$$

input `integrate(x/(x^4+x^2+1)^(1/2),x, algorithm="fricas")`output `-1/2*log(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1)`**3.196.6 Sympy [F]**

$$\int \frac{x}{\sqrt{1+x^2+x^4}} dx = \int \frac{x}{\sqrt{(x^2-x+1)(x^2+x+1)}} dx$$

input `integrate(x/(x**4+x**2+1)**(1/2),x)`

output `Integral(x/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)`

### 3.196.7 Maxima [F]

$$\int \frac{x}{\sqrt{1+x^2+x^4}} dx = \int \frac{x}{\sqrt{x^4+x^2+1}} dx$$

input `integrate(x/(x^4+x^2+1)^(1/2),x, algorithm="maxima")`

output `integrate(x/sqrt(x^4 + x^2 + 1), x)`

### 3.196.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x}{\sqrt{1+x^2+x^4}} dx = -\frac{1}{2} \log \left( -2x^2 + 2\sqrt{x^4+x^2+1} - 1 \right)$$

input `integrate(x/(x^4+x^2+1)^(1/2),x, algorithm="giac")`

output `-1/2*log(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1)`

### 3.196.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1+x^2+x^4}} dx = \frac{\ln \left( \sqrt{x^4+x^2+1} + x^2 + \frac{1}{2} \right)}{2}$$

input `int(x/(x^2 + x^4 + 1)^(1/2),x)`

output `log((x^2 + x^4 + 1)^(1/2) + x^2 + 1/2)/2`

### 3.197 $\int \frac{1}{x\sqrt{-1+x^2-x^4}} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.197.1 Optimal result . . . . .                            | 1086 |
| 3.197.2 Mathematica [A] (verified) . . . . .                | 1086 |
| 3.197.3 Rubi [A] (verified) . . . . .                       | 1087 |
| 3.197.4 Maple [A] (verified) . . . . .                      | 1088 |
| 3.197.5 Fricas [C] (verification not implemented) . . . . . | 1088 |
| 3.197.6 Sympy [F] . . . . .                                 | 1089 |
| 3.197.7 Maxima [C] (verification not implemented) . . . . . | 1089 |
| 3.197.8 Giac [F] . . . . .                                  | 1089 |
| 3.197.9 Mupad [B] (verification not implemented) . . . . .  | 1090 |

#### 3.197.1 Optimal result

Integrand size = 18, antiderivative size = 30

$$\int \frac{1}{x\sqrt{-1+x^2-x^4}} dx = -\frac{1}{2} \arctan\left(\frac{2-x^2}{2\sqrt{-1+x^2-x^4}}\right)$$

output `-1/2*arctan(1/2*(-x^2+2)/(-x^4+x^2-1)^(1/2))`

#### 3.197.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\sqrt{-1+x^2-x^4}} dx = \frac{1}{2} \arctan\left(\frac{-1+\frac{x^2}{2}}{\sqrt{-1+x^2-x^4}}\right)$$

input `Integrate[1/(x*Sqrt[-1 + x^2 - x^4]),x]`

output `ArcTan[(-1 + x^2/2)/Sqrt[-1 + x^2 - x^4]]/2`

**3.197.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1434, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{x\sqrt{-x^4+x^2-1}} dx \\ & \quad \downarrow \text{1434} \\ & \frac{1}{2} \int \frac{1}{x^2\sqrt{-x^4+x^2-1}} dx^2 \\ & \quad \downarrow \text{1154} \\ & - \int \frac{1}{-x^4-4} d\left(-\frac{2-x^2}{\sqrt{-x^4+x^2-1}}\right) \\ & \quad \downarrow \text{217} \\ & -\frac{1}{2} \arctan\left(\frac{2-x^2}{2\sqrt{-x^4+x^2-1}}\right) \end{aligned}$$

input `Int[1/(x*Sqrt[-1 + x^2 - x^4]),x]`

output `-1/2*ArcTan[(2 - x^2)/(2*Sqrt[-1 + x^2 - x^4])]`

**3.197.3.1 Defintions of rubi rules used**

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & (LtQ[a, 0] || LtQ[b, 0])`

rule 1154 `Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2 Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]`



```
rule 1434 Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp
[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

### 3.197.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

| method         | result                                                                                                                            | size |
|----------------|-----------------------------------------------------------------------------------------------------------------------------------|------|
| default        | $\frac{\arctan\left(\frac{x^2-2}{2\sqrt{-x^4+x^2-1}}\right)}{2}$                                                                  | 23   |
| elliptic       | $\frac{\arctan\left(\frac{x^2-2}{2\sqrt{-x^4+x^2-1}}\right)}{2}$                                                                  | 23   |
| pseudoelliptic | $\frac{\arctan\left(\frac{x^2-2}{2\sqrt{-x^4+x^2-1}}\right)}{2}$                                                                  | 23   |
| trager         | $\frac{\text{RootOf}(-Z^2+1) \ln\left(\frac{-\text{RootOf}(-Z^2+1)x^2+2\sqrt{-x^4+x^2-1}+2\text{RootOf}(-Z^2+1)}{x^2}\right)}{2}$ | 48   |

```
input int(1/x/(-x^4+x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
output 1/2*arctan(1/2*(x^2-2)/(-x^4+x^2-1)^(1/2))
```

### 3.197.5 Fracas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.83

$$\int \frac{1}{x\sqrt{-1+x^2-x^4}} dx = \frac{1}{4}i \log\left(\frac{x^2+2i\sqrt{-x^4+x^2-1}-2}{2x^2}\right) - \frac{1}{4}i \log\left(\frac{x^2-2i\sqrt{-x^4+x^2-1}-2}{2x^2}\right)$$

```
input integrate(1/x/(-x^4+x^2-1)^(1/2),x, algorithm="fracas")
```

output  $\frac{1}{4}i \log\left(\frac{1}{2}(x^2 + 2i\sqrt{-x^4 + x^2 - 1}) - 2\right)/x^2 - \frac{1}{4}i \log\left(\frac{1}{2}(x^2 - 2i\sqrt{-x^4 + x^2 - 1}) - 2\right)/x^2$

### 3.197.6 Sympy [F]

$$\int \frac{1}{x\sqrt{-1 + x^2 - x^4}} dx = \int \frac{1}{x\sqrt{-x^4 + x^2 - 1}} dx$$

input `integrate(1/x/(-x**4+x**2-1)**(1/2),x)`

output `Integral(1/(x*sqrt(-x**4 + x**2 - 1)), x)`

### 3.197.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int \frac{1}{x\sqrt{-1 + x^2 - x^4}} dx = -\frac{1}{2}i \operatorname{arsinh}\left(-\frac{1}{3}\sqrt{3} + \frac{2\sqrt{3}}{3x^2}\right)$$

input `integrate(1/x/(-x^4+x^2-1)^(1/2),x, algorithm="maxima")`

output `-1/2*I*arcsinh(-1/3*sqrt(3) + 2/3*sqrt(3)/x^2)`

### 3.197.8 Giac [F]

$$\int \frac{1}{x\sqrt{-1 + x^2 - x^4}} dx = \int \frac{1}{\sqrt{-x^4 + x^2 - 1}x} dx$$

input `integrate(1/x/(-x^4+x^2-1)^(1/2),x, algorithm="giac")`

output `integrate(1/(sqrt(-x^4 + x^2 - 1)*x), x)`

**3.197.9 Mupad [B] (verification not implemented)**

Time = 0.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt{-1+x^2-x^4}} dx = \frac{\ln\left(\frac{1}{x^2}\right) \text{li}}{2} + \frac{\ln\left(x^2 - 2 + \sqrt{-x^4 + x^2 - 1}\right) \text{li}}{2}$$

input `int(1/(x*(x^2 - x^4 - 1)^(1/2)),x)`output `(log(1/x^2)*1i)/2 + (log((x^2 - x^4 - 1)^(1/2)*2i + x^2 - 2)*1i)/2`

**3.198**      $\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}} dx$

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 3.198.2 Mathematica [A] (verified) . . . . . 1091  
 3.198.3 Rubi [A] (verified) . . . . . 1092  
 3.198.4 Maple [A] (verified) . . . . . 1092  
 3.198.5 Fricas [A] (verification not implemented) . . . . . 1093  
 3.198.6 Sympy [F] . . . . . 1093  
 3.198.7 Maxima [A] (verification not implemented) . . . . . 1093  
 3.198.8 Giac [B] (verification not implemented) . . . . . 1094  
 3.198.9 Mupad [B] (verification not implemented) . . . . . 1094

**3.198.1 Optimal result**

Integrand size = 20, antiderivative size = 17

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}} dx = \frac{\sqrt{1+x^2}}{1-x}$$

output `(x^2+1)^(1/2)/(1-x)`

**3.198.2 Mathematica [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}} dx = \frac{\sqrt{1+x^2}}{1-x}$$

input `Integrate[(1 + x)/((1 - x)^2*Sqrt[1 + x^2]),x]`

output `Sqrt[1 + x^2]/(1 - x)`

### 3.198.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$ , Rules used = {677}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{(1-x)^2 \sqrt{x^2+1}} dx$$

↓ 677

$$\frac{\sqrt{x^2+1}}{1-x}$$

input `Int[(1 + x)/((1 - x)^2*Sqrt[1 + x^2]),x]`

output `Sqrt[1 + x^2]/(1 - x)`

#### 3.198.3.1 Defintions of rubi rules used

rule 677 `Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 + a*e^2)), x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[Simplify[m + 2*p + 3], 0] && EqQ[c*d*f + a*e*g, 0]`

### 3.198.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

| method  | result                             | size |
|---------|------------------------------------|------|
| gospers | $-\frac{\sqrt{x^2+1}}{-1+x}$       | 15   |
| trager  | $-\frac{\sqrt{x^2+1}}{-1+x}$       | 15   |
| risch   | $-\frac{\sqrt{x^2+1}}{-1+x}$       | 15   |
| default | $-\frac{\sqrt{(-1+x)^2+2x}}{-1+x}$ | 19   |

input `int((1+x)/(1-x)^2/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output `-(x^2+1)^(1/2)/(-1+x)`

### 3.198.5 Fracas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}} dx = -\frac{x + \sqrt{x^2+1} - 1}{x-1}$$

input `integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x, algorithm="fricas")`

output `-(x + sqrt(x^2 + 1) - 1)/(x - 1)`

### 3.198.6 Sympy [F]

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}} dx = \int \frac{x+1}{(x-1)^2\sqrt{x^2+1}} dx$$

input `integrate((1+x)/(1-x)**2/(x**2+1)**(1/2),x)`

output `Integral((x + 1)/((x - 1)**2*sqrt(x**2 + 1)), x)`

### 3.198.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}} dx = -\frac{\sqrt{x^2+1}}{x-1}$$

input `integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x, algorithm="maxima")`

output `-sqrt(x^2 + 1)/(x - 1)`

**3.198.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 35 vs.  $2(14) = 28$ .

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}} dx = -\frac{\sqrt{\frac{2}{x-1} + \frac{2}{(x-1)^2} + 1}}{\operatorname{sgn}\left(\frac{1}{x-1}\right)} + \operatorname{sgn}\left(\frac{1}{x-1}\right)$$

input `integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x, algorithm="giac")`

output `-sqrt(2/(x - 1) + 2/(x - 1)^2 + 1)/sgn(1/(x - 1)) + sgn(1/(x - 1))`

**3.198.9 Mupad [B] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}} dx = -\frac{\sqrt{x^2+1}}{x-1}$$

input `int((x + 1)/((x^2 + 1)^(1/2)*(x - 1)^2),x)`

output `-(x^2 + 1)^(1/2)/(x - 1)`

### 3.199 $\int \frac{1}{\sqrt{1+x^2}} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.199.1 Optimal result . . . . .                            | 1095 |
| 3.199.2 Mathematica [B] (verified) . . . . .                | 1095 |
| 3.199.3 Rubi [A] (verified) . . . . .                       | 1096 |
| 3.199.4 Maple [A] (verified) . . . . .                      | 1096 |
| 3.199.5 Fricas [B] (verification not implemented) . . . . . | 1097 |
| 3.199.6 Sympy [A] (verification not implemented) . . . . .  | 1097 |
| 3.199.7 Maxima [A] (verification not implemented) . . . . . | 1097 |
| 3.199.8 Giac [B] (verification not implemented) . . . . .   | 1098 |
| 3.199.9 Mupad [B] (verification not implemented) . . . . .  | 1098 |

#### 3.199.1 Optimal result

Integrand size = 9, antiderivative size = 2

$$\int \frac{1}{\sqrt{1+x^2}} dx = \operatorname{arcsinh}(x)$$

output `arcsinh(x)`

#### 3.199.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16 vs.  $2(2) = 4$ .

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 8.00

$$\int \frac{1}{\sqrt{1+x^2}} dx = -\log(-x + \sqrt{1+x^2})$$

input `Integrate[1/Sqrt[1 + x^2],x]`

output `-Log[-x + Sqrt[1 + x^2]]`



**3.199.3 Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2 + 1}} dx$$

↓ 222

$$\operatorname{arcsinh}(x)$$

input `Int[1/Sqrt[1 + x^2], x]`

output `ArcSinh[x]`

**3.199.3.1 Defintions of rubi rules used**

rule 222 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]`

**3.199.4 Maple [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

| method         | result                                                      | size |
|----------------|-------------------------------------------------------------|------|
| default        | $\operatorname{arcsinh}(x)$                                 | 3    |
| meijerg        | $\operatorname{arcsinh}(x)$                                 | 3    |
| pseudoelliptic | $\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}}{x}\right)$ | 13   |
| trager         | $-\ln(x - \sqrt{x^2 + 1})$                                  | 15   |

input `int(1/(x^2+1)^(1/2), x, method=_RETURNVERBOSE)`

output `arcsinh(x)`

**3.199.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 14 vs.  $2(2) = 4$ .

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 7.00

$$\int \frac{1}{\sqrt{1+x^2}} dx = -\log(-x + \sqrt{x^2+1})$$

input `integrate(1/(x^2+1)^(1/2),x, algorithm="fracas")`

output `-log(-x + sqrt(x^2 + 1))`

**3.199.6 Sympy [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}} dx = \operatorname{asinh}(x)$$

input `integrate(1/(x**2+1)**(1/2),x)`

output `asinh(x)`

**3.199.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}} dx = \operatorname{arsinh}(x)$$

input `integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")`

output `arcsinh(x)`

**3.199.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 25 vs.  $2(2) = 4$ .

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 12.50

$$\int \frac{1}{\sqrt{1+x^2}} dx = \frac{1}{2} \sqrt{x^2+1} x - \frac{1}{2} \log(-x + \sqrt{x^2+1})$$

input `integrate(1/(x^2+1)^(1/2),x, algorithm="giac")`

output `1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))`

**3.199.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}} dx = \operatorname{asinh}(x)$$

input `int(1/(x^2 + 1)^(1/2),x)`

output `asinh(x)`

$$3.200 \quad \int \frac{\sqrt{x}\sqrt{1+x} + \sqrt{x}\sqrt{2+x} + \sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}} dx$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.200.1 Optimal result . . . . .                            | 1099 |
| 3.200.2 Mathematica [A] (verified) . . . . .                | 1099 |
| 3.200.3 Rubi [A] (verified) . . . . .                       | 1100 |
| 3.200.4 Maple [A] (verified) . . . . .                      | 1101 |
| 3.200.5 Fricas [A] (verification not implemented) . . . . . | 1101 |
| 3.200.6 Sympy [A] (verification not implemented) . . . . .  | 1102 |
| 3.200.7 Maxima [A] (verification not implemented) . . . . . | 1102 |
| 3.200.8 Giac [F] . . . . .                                  | 1102 |
| 3.200.9 Mupad [B] (verification not implemented) . . . . .  | 1103 |

### 3.200.1 Optimal result

Integrand size = 65, antiderivative size = 20

$$\int \frac{\sqrt{x}\sqrt{1+x} + \sqrt{x}\sqrt{2+x} + \sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}} dx = \sqrt{x} + \sqrt{1+x} + \sqrt{2+x}$$

output `x^(1/2)+(1+x)^(1/2)+(2+x)^(1/2)`

### 3.200.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{x}\sqrt{1+x} + \sqrt{x}\sqrt{2+x} + \sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}} dx = \frac{1}{2} (2\sqrt{x} + 2\sqrt{1+x} + 2\sqrt{2+x})$$

input `Integrate[(Sqrt[x]*Sqrt[1 + x] + Sqrt[x]*Sqrt[2 + x] + Sqrt[1 + x]*Sqrt[2 + x])/(2*Sqrt[x]*Sqrt[1 + x]*Sqrt[2 + x]),x]`

output `(2*Sqrt[x] + 2*Sqrt[1 + x] + 2*Sqrt[2 + x])/2`

### 3.200.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {27, 2035, 7239, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{\sqrt{x}\sqrt{x+1} + \sqrt{x+2}\sqrt{x+1} + \sqrt{x}\sqrt{x+2}}{2\sqrt{x}\sqrt{x+1}\sqrt{x+2}} dx \\ & \quad \downarrow 27 \\ & \frac{1}{2} \int \frac{\sqrt{x}\sqrt{x+1} + \sqrt{x+2}\sqrt{x+1} + \sqrt{x}\sqrt{x+2}}{\sqrt{x}\sqrt{x+1}\sqrt{x+2}} dx \\ & \quad \downarrow 2035 \\ & \int \frac{\sqrt{x}\sqrt{x+1} + \sqrt{x+2}\sqrt{x+1} + \sqrt{x}\sqrt{x+2}}{\sqrt{x+1}\sqrt{x+2}} d\sqrt{x} \\ & \quad \downarrow 7239 \\ & \int \left( \sqrt{x} \left( \frac{1}{\sqrt{x+2}} + \frac{1}{\sqrt{x+1}} \right) + 1 \right) d\sqrt{x} \\ & \quad \downarrow 2009 \\ & \sqrt{x} + \sqrt{x+1} + \sqrt{x+2} \end{aligned}$$

input `Int[(Sqrt[x]*Sqrt[1 + x] + Sqrt[x]*Sqrt[2 + x] + Sqrt[1 + x]*Sqrt[2 + x])/ (2*Sqrt[x]*Sqrt[1 + x]*Sqrt[2 + x]),x]`

output `Sqrt[x] + Sqrt[1 + x] + Sqrt[2 + x]`

#### 3.200.3.1 Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2035 `Int[(Fx_)*(x_)^(m_), x_Symbol] := With[{k = Denominator[m]}, Simp[k Subst [Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x] /; FractionQ[m] && AlgebraicFunctionQ[Fx, x]`

rule 7239 `Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifierIntegrandQ[v, u, x]`

### 3.200.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

| method  | result                               | size |
|---------|--------------------------------------|------|
| default | $\sqrt{x} + \sqrt{1+x} + \sqrt{2+x}$ | 15   |

input `int(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x,method=_RETURNVERBOSE)`

output `x^(1/2)+(1+x)^(1/2)+(2+x)^(1/2)`

### 3.200.5 Fracas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{x}\sqrt{1+x} + \sqrt{x}\sqrt{2+x} + \sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}} dx = \sqrt{x+2} + \sqrt{x+1} + \sqrt{x}$$

input `integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x, algorithm="fracas")`

output `sqrt(x + 2) + sqrt(x + 1) + sqrt(x)`

**3.200.6 Sympy [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{x}\sqrt{1+x} + \sqrt{x}\sqrt{2+x} + \sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}} dx = \sqrt{x} + \sqrt{x+1} + \sqrt{x+2}$$

input `integrate(1/2*(x**(1/2)*(1+x)**(1/2)+x**(1/2)*(2+x)**(1/2)+(1+x)**(1/2)*(2+x)**(1/2))/x**(1/2)/(1+x)**(1/2)/(2+x)**(1/2),x)`

output `sqrt(x) + sqrt(x + 1) + sqrt(x + 2)`

**3.200.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{x}\sqrt{1+x} + \sqrt{x}\sqrt{2+x} + \sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}} dx = \sqrt{x+2} + \sqrt{x+1} + \sqrt{x}$$

input `integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x, algorithm="maxima")`

output `sqrt(x + 2) + sqrt(x + 1) + sqrt(x)`

**3.200.8 Giac [F]**

$$\begin{aligned} & \int \frac{\sqrt{x}\sqrt{1+x} + \sqrt{x}\sqrt{2+x} + \sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}} dx \\ &= \int \frac{\sqrt{x+2}\sqrt{x+1} + \sqrt{x+2}\sqrt{x} + \sqrt{x+1}\sqrt{x}}{2\sqrt{x+2}\sqrt{x+1}\sqrt{x}} dx \end{aligned}$$

input `integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x, algorithm="giac")`

output `integrate(1/2*(sqrt(x + 2)*sqrt(x + 1) + sqrt(x + 2)*sqrt(x) + sqrt(x + 1)*sqrt(x))/(sqrt(x + 2)*sqrt(x + 1)*sqrt(x)), x)`

**3.200.9 Mupad [B] (verification not implemented)**

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{x}\sqrt{1+x} + \sqrt{x}\sqrt{2+x} + \sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}} dx = \sqrt{x+1} + \sqrt{x+2} + \sqrt{x}$$

input `int(((x^(1/2)*(x + 1)^(1/2))/2 + (x^(1/2)*(x + 2)^(1/2))/2 + ((x + 1)^(1/2)*(x + 2)^(1/2))/2)/(x^(1/2)*(x + 1)^(1/2)*(x + 2)^(1/2)),x)`

output `(x + 1)^(1/2) + (x + 2)^(1/2) + x^(1/2)`



$$3.201 \quad \int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}} dx$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.201.1 Optimal result . . . . .                            | 1104 |
| 3.201.2 Mathematica [A] (verified) . . . . .                | 1104 |
| 3.201.3 Rubi [A] (verified) . . . . .                       | 1105 |
| 3.201.4 Maple [A] (verified) . . . . .                      | 1106 |
| 3.201.5 Fricas [A] (verification not implemented) . . . . . | 1106 |
| 3.201.6 Sympy [A] (verification not implemented) . . . . .  | 1107 |
| 3.201.7 Maxima [A] (verification not implemented) . . . . . | 1107 |
| 3.201.8 Giac [F] . . . . .                                  | 1107 |
| 3.201.9 Mupad [B] (verification not implemented) . . . . .  | 1108 |

### 3.201.1 Optimal result

Integrand size = 68, antiderivative size = 24

$$\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}} dx = -\sqrt{1+x^3} + \sqrt{1-2x+x^5}$$

output `-(x^3+1)^(1/2)+(x^5-2*x+1)^(1/2)`

### 3.201.2 Mathematica [A] (verified)

Time = 14.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}} dx = -\sqrt{1+x^3} + \sqrt{1-2x+x^5}$$

input `Integrate[(-2*Sqrt[1 + x^3] + 5*x^4*Sqrt[1 + x^3] - 3*x^2*Sqrt[1 - 2*x + x^5])/(2*Sqrt[1 + x^3]*Sqrt[1 - 2*x + x^5]),x]`

output `-Sqrt[1 + x^3] + Sqrt[1 - 2*x + x^5]`

**3.201.3 Rubi [A] (verified)**

Time = 0.50 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {27, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2\sqrt{x^3+1} - 3\sqrt{x^5-2x+1}x^2 + 5\sqrt{x^3+1}x^4}{2\sqrt{x^3+1}\sqrt{x^5-2x+1}} dx$$

↓ 27

$$\frac{1}{2} \int -\frac{-5\sqrt{x^3+1}x^4 + 3\sqrt{x^5-2x+1}x^2 + 2\sqrt{x^3+1}}{\sqrt{x^3+1}\sqrt{x^5-2x+1}} dx$$

↓ 25

$$-\frac{1}{2} \int \frac{-5\sqrt{x^3+1}x^4 + 3\sqrt{x^5-2x+1}x^2 + 2\sqrt{x^3+1}}{\sqrt{x^3+1}\sqrt{x^5-2x+1}} dx$$

↓ 7293

$$-\frac{1}{2} \int \left( -\frac{5x^4}{\sqrt{x^5-2x+1}} + \frac{3x^2}{\sqrt{x^3+1}} + \frac{2}{\sqrt{x^5-2x+1}} \right) dx$$

↓ 2009

$$\frac{1}{2} \left( 2\sqrt{x^5-2x+1} - 2\sqrt{x^3+1} \right)$$

input `Int[(-2*Sqrt[1 + x^3] + 5*x^4*Sqrt[1 + x^3] - 3*x^2*Sqrt[1 - 2*x + x^5])/(2*Sqrt[1 + x^3]*Sqrt[1 - 2*x + x^5]),x]`

output `(-2*Sqrt[1 + x^3] + 2*Sqrt[1 - 2*x + x^5])/2`

**3.201.3.1 Defintions of rubi rules used**

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

---

3.201.  $\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}} dx$

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 7293 `Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]  
]`

### 3.201.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

| method   | result                                                              | size |
|----------|---------------------------------------------------------------------|------|
| default  | $-\sqrt{x^3 + 1} + \sqrt{x^5 - 2x + 1}$                             | 21   |
| elliptic | $-\sqrt{x^3 + 1} + \frac{(-1+x)(x^4+x^3+x^2+x-1)}{\sqrt{x^5-2x+1}}$ | 37   |

input `int(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2))/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x,method=_RETURNVERBOSE)`

output `-(x^3+1)^(1/2)+(x^5-2*x+1)^(1/2)`

### 3.201.5 Fracas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-2\sqrt{1+x^3} + 5x^4\sqrt{1+x^3} - 3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}} dx = \sqrt{x^5 - 2x + 1} - \sqrt{x^3 + 1}$$

input `integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2))/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x, algorithm="fracas")`

output `sqrt(x^5 - 2*x + 1) - sqrt(x^3 + 1)`

**3.201.6 Sympy [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{-2\sqrt{1+x^3} + 5x^4\sqrt{1+x^3} - 3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}} dx = -\sqrt{x^3+1} + \sqrt{x^5-2x+1}$$

input `integrate(1/2*(-2*(x**3+1)**(1/2)+5*x**4*(x**3+1)**(1/2)-3*x**2*(x**5-2*x+1)**(1/2))/(x**3+1)**(1/2)/(x**5-2*x+1)**(1/2),x)`

output `-sqrt(x**3 + 1) + sqrt(x**5 - 2*x + 1)`

**3.201.7 Maxima [A] (verification not implemented)**

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{-2\sqrt{1+x^3} + 5x^4\sqrt{1+x^3} - 3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}} dx$$

$$= \sqrt{x^4+x^3+x^2+x-1}\sqrt{x-1} - \sqrt{x^3+1}$$

input `integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2)))/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x, algorithm="maxima")`

output `sqrt(x^4 + x^3 + x^2 + x - 1)*sqrt(x - 1) - sqrt(x^3 + 1)`

**3.201.8 Giac [F]**

$$\int \frac{-2\sqrt{1+x^3} + 5x^4\sqrt{1+x^3} - 3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}} dx$$

$$= \int \frac{5\sqrt{x^3+1}x^4 - 3\sqrt{x^5-2x+1}x^2 - 2\sqrt{x^3+1}}{2\sqrt{x^5-2x+1}\sqrt{x^3+1}} dx$$

input `integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2)))/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x, algorithm="giac")`

output `integrate(1/2*(5*sqrt(x^3 + 1)*x^4 - 3*sqrt(x^5 - 2*x + 1)*x^2 - 2*sqrt(x^3 + 1)))/(sqrt(x^5 - 2*x + 1)*sqrt(x^3 + 1)), x)`

**3.201.9 Mupad [B] (verification not implemented)**

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-2\sqrt{1+x^3} + 5x^4\sqrt{1+x^3} - 3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}} dx = \sqrt{x^5 - 2x + 1} - \sqrt{x^3 + 1}$$

input `int(-((x^3 + 1)^(1/2) + (3*x^2*(x^5 - 2*x + 1)^(1/2))/2 - (5*x^4*(x^3 + 1)^(1/2))/2)/((x^3 + 1)^(1/2)*(x^5 - 2*x + 1)^(1/2)),x)`

output `(x^5 - 2*x + 1)^(1/2) - (x^3 + 1)^(1/2)`

$$3.202 \quad \int \left( \frac{10}{\sqrt{-4+x^2}} + \frac{1}{\sqrt{-1+x^2}} \right) dx$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.202.1 Optimal result . . . . .                            | 1109 |
| 3.202.2 Mathematica [B] (verified) . . . . .                | 1109 |
| 3.202.3 Rubi [A] (verified) . . . . .                       | 1110 |
| 3.202.4 Maple [A] (verified) . . . . .                      | 1110 |
| 3.202.5 Fricas [A] (verification not implemented) . . . . . | 1111 |
| 3.202.6 Sympy [A] (verification not implemented) . . . . .  | 1111 |
| 3.202.7 Maxima [A] (verification not implemented) . . . . . | 1111 |
| 3.202.8 Giac [B] (verification not implemented) . . . . .   | 1112 |
| 3.202.9 Mupad [B] (verification not implemented) . . . . .  | 1112 |

### 3.202.1 Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \left( \frac{10}{\sqrt{-4+x^2}} + \frac{1}{\sqrt{-1+x^2}} \right) dx = 10 \operatorname{arctanh} \left( \frac{x}{\sqrt{-4+x^2}} \right) + \operatorname{arctanh} \left( \frac{x}{\sqrt{-1+x^2}} \right)$$

output `10*arctanh(x/(x^2-4)^(1/2))+arctanh(x/(x^2-1)^(1/2))`

### 3.202.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 71 vs.  $2(27) = 54$ .

Time = 0.00 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.63

$$\int \left( \frac{10}{\sqrt{-4+x^2}} + \frac{1}{\sqrt{-1+x^2}} \right) dx = -5 \log \left( 1 - \frac{x}{\sqrt{-4+x^2}} \right) + 5 \log \left( 1 + \frac{x}{\sqrt{-4+x^2}} \right) \\ - \frac{1}{2} \log \left( 1 - \frac{x}{\sqrt{-1+x^2}} \right) + \frac{1}{2} \log \left( 1 + \frac{x}{\sqrt{-1+x^2}} \right)$$

input `Integrate[10/Sqrt[-4 + x^2] + 1/Sqrt[-1 + x^2],x]`

output `-5*Log[1 - x/Sqrt[-4 + x^2]] + 5*Log[1 + x/Sqrt[-4 + x^2]] - Log[1 - x/Sqrt[-1 + x^2]]/2 + Log[1 + x/Sqrt[-1 + x^2]]/2`

---


$$3.202. \quad \int \left( \frac{10}{\sqrt{-4+x^2}} + \frac{1}{\sqrt{-1+x^2}} \right) dx$$

### 3.202.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$ , Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left( \frac{1}{\sqrt{x^2-1}} + \frac{10}{\sqrt{x^2-4}} \right) dx$$

↓ 2009

$$10 \operatorname{arctanh} \left( \frac{x}{\sqrt{x^2-4}} \right) + \operatorname{arctanh} \left( \frac{x}{\sqrt{x^2-1}} \right)$$

input `Int[10/Sqrt[-4 + x^2] + 1/Sqrt[-1 + x^2], x]`

output `10*ArcTanh[x/Sqrt[-4 + x^2]] + ArcTanh[x/Sqrt[-1 + x^2]]`

#### 3.202.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

### 3.202.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

| method  | result                                                                                                                                                                                                                             | size |
|---------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default | $\ln(x + \sqrt{x^2-1}) + 10 \ln(x + \sqrt{x^2-4})$                                                                                                                                                                                 | 24   |
| meijerg | $\frac{10 \sqrt{-\operatorname{signum}(-1 + \frac{x^2}{4})} \arcsin(\frac{x}{2})}{\sqrt{\operatorname{signum}(-1 + \frac{x^2}{4})}} + \frac{\sqrt{-\operatorname{signum}(x^2-1)} \arcsin(x)}{\sqrt{\operatorname{signum}(x^2-1)}}$ | 51   |

input `int(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2), x, method=_RETURNVERBOSE)`

output `ln(x+(x^2-1)^(1/2))+10*ln(x+(x^2-4)^(1/2))`

**3.202.5 Fracas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \left( \frac{10}{\sqrt{-4+x^2}} + \frac{1}{\sqrt{-1+x^2}} \right) dx = -\log(-x + \sqrt{x^2-1}) - 10 \log(-x + \sqrt{x^2-4})$$

input `integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x, algorithm="fricas")`output `-log(-x + sqrt(x^2 - 1)) - 10*log(-x + sqrt(x^2 - 4))`**3.202.6 Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \left( \frac{10}{\sqrt{-4+x^2}} + \frac{1}{\sqrt{-1+x^2}} \right) dx = 10 \log(x + \sqrt{x^2-4}) + \log(x + \sqrt{x^2-1})$$

input `integrate(10/(x**2-4)**(1/2)+1/(x**2-1)**(1/2),x)`output `10*log(x + sqrt(x**2 - 4)) + log(x + sqrt(x**2 - 1))`**3.202.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \left( \frac{10}{\sqrt{-4+x^2}} + \frac{1}{\sqrt{-1+x^2}} \right) dx = \log(2x + 2\sqrt{x^2-1}) + 10 \log(2x + 2\sqrt{x^2-4})$$

input `integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x, algorithm="maxima")`output `log(2*x + 2*sqrt(x^2 - 1)) + 10*log(2*x + 2*sqrt(x^2 - 4))`



**3.202.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 51 vs.  $2(23) = 46$ .

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.89

$$\int \left( \frac{10}{\sqrt{-4+x^2}} + \frac{1}{\sqrt{-1+x^2}} \right) dx = \frac{1}{2} \sqrt{x^2-1}x + 5\sqrt{x^2-4}x + \frac{1}{2} \log \left( \left| -x + \sqrt{x^2-1} \right| \right) + 20 \log \left( \left| -x + \sqrt{x^2-4} \right| \right)$$

input `integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x, algorithm="giac")`

output `1/2*sqrt(x^2 - 1)*x + 5*sqrt(x^2 - 4)*x + 1/2*log(abs(-x + sqrt(x^2 - 1))) + 20*log(abs(-x + sqrt(x^2 - 4)))`

**3.202.9 Mupad [B] (verification not implemented)**

Time = 0.74 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \left( \frac{10}{\sqrt{-4+x^2}} + \frac{1}{\sqrt{-1+x^2}} \right) dx = \ln \left( x + \sqrt{x^2-1} \right) + 10 \ln \left( x + \sqrt{x^2-4} \right)$$

input `int(1/(x^2 - 1)^(1/2) + 10/(x^2 - 4)^(1/2),x)`

output `log(x + (x^2 - 1)^(1/2)) + 10*log(x + (x^2 - 4)^(1/2))`

### 3.203 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.203.1 Optimal result . . . . .                            | 1113 |
| 3.203.2 Mathematica [A] (verified) . . . . .                | 1113 |
| 3.203.3 Rubi [A] (verified) . . . . .                       | 1114 |
| 3.203.4 Maple [C] (verified) . . . . .                      | 1116 |
| 3.203.5 Fricas [A] (verification not implemented) . . . . . | 1117 |
| 3.203.6 Sympy [C] (verification not implemented) . . . . .  | 1117 |
| 3.203.7 Maxima [F] . . . . .                                | 1118 |
| 3.203.8 Giac [F] . . . . .                                  | 1118 |
| 3.203.9 Mupad [F(-1)] . . . . .                             | 1118 |

#### 3.203.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} dx = 2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a} \arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)$$

output `-2*arctan((x+(a^2+x^2)^(1/2))^(1/2)/a^(1/2))*a^(1/2)-2*arctanh((x+(a^2+x^2)^(1/2))^(1/2)/a^(1/2))*a^(1/2)+2*(x+(a^2+x^2)^(1/2))^(1/2)`

#### 3.203.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} dx = 2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a} \arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)$$

input `Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]`

output `2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]`

**3.203.3 Rubi [A] (verified)**

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2544, 25, 363, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{\sqrt{a^2+x^2}+x}}{x} dx \\
 & \quad \downarrow \text{2544} \\
 & \int -\frac{(\sqrt{a^2+x^2}+x)^2+a^2}{\sqrt{\sqrt{a^2+x^2}+x} \left(a^2 - (\sqrt{a^2+x^2}+x)^2\right)} d(\sqrt{a^2+x^2}+x) \\
 & \quad \downarrow \text{25} \\
 & -\int \frac{a^2 + (x + \sqrt{a^2+x^2})^2}{\sqrt{x + \sqrt{a^2+x^2}} \left(a^2 - (x + \sqrt{a^2+x^2})^2\right)} d(x + \sqrt{a^2+x^2}) \\
 & \quad \downarrow \text{363} \\
 & 2\sqrt{\sqrt{a^2+x^2}+x} - 2a^2 \int \frac{1}{\sqrt{x + \sqrt{a^2+x^2}} \left(a^2 - (x + \sqrt{a^2+x^2})^2\right)} d(x + \sqrt{a^2+x^2}) \\
 & \quad \downarrow \text{266} \\
 & 2\sqrt{\sqrt{a^2+x^2}+x} - 4a^2 \int \frac{1}{a^2 - (x + \sqrt{a^2+x^2})^2} d\sqrt{x + \sqrt{a^2+x^2}} \\
 & \quad \downarrow \text{756} \\
 & 2\sqrt{\sqrt{a^2+x^2}+x} - 4a^2 \left( \frac{\int \frac{1}{a-x-\sqrt{a^2+x^2}} d\sqrt{x + \sqrt{a^2+x^2}}}{2a} + \frac{\int \frac{1}{a+x+\sqrt{a^2+x^2}} d\sqrt{x + \sqrt{a^2+x^2}}}{2a} \right) \\
 & \quad \downarrow \text{216} \\
 & 2\sqrt{\sqrt{a^2+x^2}+x} - 4a^2 \left( \frac{\int \frac{1}{a-x-\sqrt{a^2+x^2}} d\sqrt{x + \sqrt{a^2+x^2}}}{2a} + \frac{\arctan\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{2a^{3/2}} \right) \\
 & \quad \downarrow \text{219}
 \end{aligned}$$

$$2\sqrt{\sqrt{a^2+x^2}+x} - 4a^2 \left( \frac{\arctan\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{2a^{3/2}} \right)$$

input `Int[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]`

output `2*Sqrt[x + Sqrt[a^2 + x^2]] - 4*a^2*(ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]/(2*a^(3/2))) + ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]/(2*a^(3/2))`

### 3.203.3.1 Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 216 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

rule 266 `Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{k = Denominator[m]}, Simp[k/c Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && IntBinomialQ[a, b, c, 2, m, p, x]`

rule 363 `Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3)) Int[(e*x)^(m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 3, 0]`

```
rule 756 Int[((a_) + (b_)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a) Int[1/(r - s*x^2), x], x]
+ Simp[r/(2*a) Int[1/(r + s*x^2), x], x] /; FreeQ[{a, b}, x] && !GtQ[a
/b, 0]
```

```
rule 2544 Int[((g_) + (h_)*(x_)^(m_))*((e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^
2])^(n_), x_Symbol] := Simp[1/(2^(m + 1)*e^(m + 1)) Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && I
ntegerQ[m]
```

### 3.203.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.30

| method  | result                                                                                                                      | size |
|---------|-----------------------------------------------------------------------------------------------------------------------------|------|
| meijerg | $2\sqrt{2}\sqrt{x} {}_3F_2\left(-\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}; \frac{1}{2}, \frac{3}{4}; -\frac{a^2}{x^2}\right)$ | 25   |

```
input int((x+(a^2+x^2)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
output 2*2^(1/2)*x^(1/2)*hypergeom([-1/4, -1/4, 1/4], [1/2, 3/4], -a^2/x^2)
```

**3.203.5 Fricas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.63

$$\int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x} dx$$

$$= \left[ -2\sqrt{a} \arctan\left(\frac{\sqrt{x + \sqrt{a^2 + x^2}}}{\sqrt{a}}\right) + \sqrt{a} \log\left(\frac{a^2 + \sqrt{a^2 + x^2}a - ((a-x)\sqrt{a} + \sqrt{a^2 + x^2}\sqrt{a})\sqrt{x + \sqrt{a^2 + x^2}}}{x} + 2\sqrt{x + \sqrt{a^2 + x^2}}, 2\sqrt{-a} \arctan\left(\frac{\sqrt{-a}\sqrt{x + \sqrt{a^2 + x^2}}}{a}\right) + \sqrt{-a} \log\left(-\frac{a^2 - \sqrt{a^2 + x^2}a + (\sqrt{-a}(a+x) - \sqrt{a^2 + x^2}\sqrt{-a})\sqrt{x + \sqrt{a^2 + x^2}}}{x} + 2\sqrt{x + \sqrt{a^2 + x^2}}\right) \right]$$

input `integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="fricas")`output `[-2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) + sqrt(a)*log((a^2 + sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + sqrt(a^2 + x^2)))/x) + 2*sqrt(x + sqrt(a^2 + x^2)), 2*sqrt(-a)*arctan(sqrt(-a)*sqrt(x + sqrt(a^2 + x^2))/a) + sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a + (sqrt(-a)*(a + x) - sqrt(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x) + 2*sqrt(x + sqrt(a^2 + x^2))]`**3.203.6 Sympy [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 1.83 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x} dx = \frac{\sqrt{x}\Gamma^2\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right) {}_3F_2\left(\begin{matrix} -\frac{1}{4}, -\frac{1}{4}, \frac{1}{4} \\ \frac{1}{2}, \frac{3}{4} \end{matrix} \middle| \frac{a^2 e^{i\pi}}{x^2} \right)}{8\pi\Gamma\left(\frac{3}{4}\right)}$$

input `integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)`

output `sqrt(x)*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2, 3/4), a**2*exp_polar(I*pi)/x**2)/(8*pi*gamma(3/4))`

### 3.203.7 Maxima [F]

$$\int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x} dx = \int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x} dx$$

input `integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="maxima")`

output `integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)`

### 3.203.8 Giac [F]

$$\int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x} dx = \int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x} dx$$

input `integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="giac")`

output `integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)`

### 3.203.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x} dx = \int \frac{\sqrt{x + \sqrt{a^2 + x^2}}}{x} dx$$

input `int((x + (a^2 + x^2)^(1/2))^(1/2)/x,x)`

output `int((x + (a^2 + x^2)^(1/2))^(1/2)/x, x)`

$$3.204 \quad \int \frac{3x^2}{2(1+x^3+\sqrt{1+x^3})} dx$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.204.1 Optimal result . . . . .                            | 1119 |
| 3.204.2 Mathematica [A] (verified) . . . . .                | 1119 |
| 3.204.3 Rubi [A] (verified) . . . . .                       | 1120 |
| 3.204.4 Maple [A] (verified) . . . . .                      | 1121 |
| 3.204.5 Fricas [B] (verification not implemented) . . . . . | 1121 |
| 3.204.6 Sympy [B] (verification not implemented) . . . . .  | 1122 |
| 3.204.7 Maxima [B] (verification not implemented) . . . . . | 1122 |
| 3.204.8 Giac [A] (verification not implemented) . . . . .   | 1122 |
| 3.204.9 Mupad [B] (verification not implemented) . . . . .  | 1123 |

### 3.204.1 Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{3x^2}{2(1+x^3+\sqrt{1+x^3})} dx = \log(1+\sqrt{1+x^3})$$

output `ln(1+(x^3+1)^(1/2))`

### 3.204.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{3x^2}{2(1+x^3+\sqrt{1+x^3})} dx = \log(1+\sqrt{1+x^3})$$

input `Integrate[(3*x^2)/(2*(1 + x^3 + Sqrt[1 + x^3])),x]`

output `Log[1 + Sqrt[1 + x^3]]`



### 3.204.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$ , Rules used = {27, 2586, 7267, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{3x^2}{2(x^3 + \sqrt{x^3 + 1} + 1)} dx \\ & \quad \downarrow 27 \\ & \frac{3}{2} \int \frac{x^2}{x^3 + \sqrt{x^3 + 1} + 1} dx \\ & \quad \downarrow 2586 \\ & \frac{1}{2} \int \frac{1}{x^3 + \sqrt{x^3 + 1} + 1} dx^3 \\ & \quad \downarrow 7267 \\ & \int \frac{1}{\sqrt{x^3 + 1} + 1} d\sqrt{x^3 + 1} \\ & \quad \downarrow 16 \\ & \log(\sqrt{x^3 + 1} + 1) \end{aligned}$$

input `Int[(3*x^2)/(2*(1 + x^3 + Sqrt[1 + x^3])),x]`

output `Log[1 + Sqrt[1 + x^3]]`

#### 3.204.3.1 Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

```
rule 2586 Int[(x_)^(m_)/((c_) + (d_)*(x_)^(n_) + (e_)*Sqrt[(a_) + (b_)*(x_)^(n_)
], x_Symbol] :> Simp[1/n Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
+ b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

```
rule 7267 Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]] Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2])], x
] /; !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]
```

### 3.204.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

| method   | result                                                                                                                           | size |
|----------|----------------------------------------------------------------------------------------------------------------------------------|------|
| trager   | $\frac{\ln(-x^3 - 2\sqrt{x^3 + 1} - 2)}{2}$                                                                                      | 20   |
| default  | $-\frac{\ln(1+x)}{2} + \frac{3\ln(x)}{2} - \frac{\ln(x^2-x+1)}{2} + \frac{\ln(x^3+1)}{2} + \operatorname{arctanh}(\sqrt{x^3+1})$ | 39   |
| elliptic | $\frac{(1+\sqrt{x^3+1})\sqrt{x^3+1}\left(\frac{3\ln(x)}{2} + \operatorname{arctanh}(\sqrt{x^3+1})\right)}{1+x^3+\sqrt{x^3+1}}$   | 45   |

```
input int(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
output 1/2*ln(-x^3-2*(x^3+1)^(1/2)-2)
```

### 3.204.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs.  $2(10) = 20$ .

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{3x^2}{2(1+x^3+\sqrt{1+x^3})} dx = \frac{3}{2} \log(x) + \frac{1}{2} \log(\sqrt{x^3+1}+1) - \frac{1}{2} \log(\sqrt{x^3+1}-1)$$

```
input integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x, algorithm="fracas")
```

```
output 3/2*log(x) + 1/2*log(sqrt(x^3 + 1) + 1) - 1/2*log(sqrt(x^3 + 1) - 1)
```

---

3.204.  $\int \frac{3x^2}{2(1+x^3+\sqrt{1+x^3})} dx$

**3.204.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 48 vs.  $2(10) = 20$ .

Time = 49.82 (sec) , antiderivative size = 48, normalized size of antiderivative = 4.00

$$\int \frac{3x^2}{2(1+x^3+\sqrt{1+x^3})} dx = -\frac{\log(2\sqrt{x^3+1})}{2} + \frac{\log(2\sqrt{x^3+1}+2)}{2} + \frac{\log(3x^3+3\sqrt{x^3+1}+3)}{2}$$

input `integrate(3/2*x**2/(1+x**3+(x**3+1)**(1/2)),x)`

output `-log(2*sqrt(x**3 + 1))/2 + log(2*sqrt(x**3 + 1) + 2)/2 + log(3*x**3 + 3*sqrt(x**3 + 1) + 3)/2`

**3.204.7 Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 40 vs.  $2(10) = 20$ .

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.33

$$\int \frac{3x^2}{2(1+x^3+\sqrt{1+x^3})} dx = -\frac{1}{2} \log(x^2 - x + 1) + \log\left(\frac{x^3 + \sqrt{x^2 - x + 1}\sqrt{x + 1} + 1}{\sqrt{x + 1}}\right)$$

input `integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x, algorithm="maxima")`

output `-1/2*log(x^2 - x + 1) + log((x^3 + sqrt(x^2 - x + 1)*sqrt(x + 1) + 1)/sqrt(x + 1))`

**3.204.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{3x^2}{2(1+x^3+\sqrt{1+x^3})} dx = \log(\sqrt{x^3+1}+1)$$

input `integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x, algorithm="giac")`

output `log(sqrt(x^3 + 1) + 1)`

---

3.204.  $\int \frac{3x^2}{2(1+x^3+\sqrt{1+x^3})} dx$

**3.204.9 Mupad [B] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 14.08

$$\int \frac{3x^2}{2(1+x^3+\sqrt{1+x^3})} dx = \frac{3 \ln(x)}{2} + \frac{3 \left( \frac{3}{2} + \frac{\sqrt{3}1i}{2} \right) \sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}1i}{2}}} \sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}1i}{2}}} \sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}1i}{2}}} \Pi \left( \frac{3}{2} + \frac{\sqrt{3}1i}{2}; \operatorname{asin} \left( \sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}1i}{2}}} \right) \middle| -\frac{\frac{3}{2}+\frac{\sqrt{3}1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}1i}{2}} \right)}{\sqrt{x^3 + \left( -\left( -\frac{1}{2} + \frac{\sqrt{3}1i}{2} \right) \left( \frac{1}{2} + \frac{\sqrt{3}1i}{2} \right) - 1 \right) x - \left( -\frac{1}{2} + \frac{\sqrt{3}1i}{2} \right) \left( \frac{1}{2} + \frac{\sqrt{3}1i}{2} \right)}$$

input `int((3*x^2)/(2*((x^3 + 1)^(1/2) + x^3 + 1)),x)`

output `(3*log(x))/2 + (3*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))/(x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)`

### 3.205 $\int \frac{1}{\sqrt{-\alpha^2+2hr^2}} dr$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.205.1 Optimal result . . . . .                            | 1124 |
| 3.205.2 Mathematica [A] (verified) . . . . .                | 1124 |
| 3.205.3 Rubi [A] (verified) . . . . .                       | 1125 |
| 3.205.4 Maple [A] (verified) . . . . .                      | 1126 |
| 3.205.5 Fricas [A] (verification not implemented) . . . . . | 1126 |
| 3.205.6 Sympy [C] (verification not implemented) . . . . .  | 1127 |
| 3.205.7 Maxima [A] (verification not implemented) . . . . . | 1127 |
| 3.205.8 Giac [A] (verification not implemented) . . . . .   | 1127 |
| 3.205.9 Mupad [B] (verification not implemented) . . . . .  | 1128 |

#### 3.205.1 Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{1}{\sqrt{-\alpha^2+2hr^2}} dr = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{hr}}{\sqrt{-\alpha^2+2hr^2}}\right)}{\sqrt{2}\sqrt{h}}$$

output `1/2*arctanh(r*2^(1/2)*h^(1/2)/(2*h*r^2-alpha^2)^(1/2))*2^(1/2)/h^(1/2)`

#### 3.205.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-\alpha^2+2hr^2}} dr = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{hr}}{\sqrt{-\alpha^2+2hr^2}}\right)}{\sqrt{2}\sqrt{h}}$$

input `Integrate[1/Sqrt[-alpha^2 + 2*h*r^2],r]`

output `ArcTanh[(Sqrt[2]*Sqrt[h]*r)/Sqrt[-alpha^2 + 2*h*r^2]]/(Sqrt[2]*Sqrt[h])`

### 3.205.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2hr^2 - \alpha^2}} dr$$

↓ 224

$$\int \frac{1}{1 - \frac{2hr^2}{2hr^2 - \alpha^2}} d \frac{r}{\sqrt{2hr^2 - \alpha^2}}$$

↓ 219

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{hr}}{\sqrt{2hr^2 - \alpha^2}}\right)}{\sqrt{2}\sqrt{h}}$$

input `Int[1/Sqrt[-alpha^2 + 2*h*r^2],r]`

output `ArcTanh[(Sqrt[2]*Sqrt[h]*r)/Sqrt[-alpha^2 + 2*h*r^2]]/(Sqrt[2]*Sqrt[h])`

#### 3.205.3.1 Defintions of rubi rules used

rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

rule 224 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

**3.205.4 Maple [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

| method         | result                                                                                                          | size |
|----------------|-----------------------------------------------------------------------------------------------------------------|------|
| default        | $\frac{\ln\left(\sqrt{h}r\sqrt{2+\sqrt{2hr^2-\alpha^2}}\right)\sqrt{2}}{2\sqrt{h}}$                             | 33   |
| pseudoelliptic | $\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2hr^2-\alpha^2}\sqrt{2}}{2r\sqrt{h}}\right)}{2\sqrt{h}}$ | 35   |

input `int(1/(2*h*r^2-alpha^2)^(1/2),r,method=_RETURNVERBOSE)`output `1/2*ln(h^(1/2)*r*2^(1/2)+(2*h*r^2-alpha^2)^(1/2))*2^(1/2)/h^(1/2)`**3.205.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.12

$$\int \frac{1}{\sqrt{-\alpha^2 + 2hr^2}} dr = \left[ \frac{\sqrt{2} \log\left(4hr^2 + 2\sqrt{2}\sqrt{2hr^2 - \alpha^2}\sqrt{hr} - \alpha^2\right)}{4\sqrt{h}}, \right. \\ \left. -\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{h}} \arctan\left(\frac{\sqrt{2}hr\sqrt{-\frac{1}{h}}}{\sqrt{2hr^2 - \alpha^2}}\right) \right]$$

input `integrate(1/(2*h*r^2-alpha^2)^(1/2),r, algorithm="fracas")`output `[1/4*sqrt(2)*log(4*h*r^2 + 2*sqrt(2)*sqrt(2*h*r^2 - alpha^2)*sqrt(h)*r - alpha^2)/sqrt(h), -1/2*sqrt(2)*sqrt(-1/h)*arctan(sqrt(2)*h*r*sqrt(-1/h)/sqrt(2*h*r^2 - alpha^2))]`

**3.205.6 Sympy [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{-\alpha^2 + 2hr^2}} dr = \begin{cases} \frac{\sqrt{2} \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{hr}}{\alpha}\right)}{2\sqrt{h}} & \text{for } \left|\frac{hr^2}{\alpha^2}\right| > \frac{1}{2} \\ -\frac{\sqrt{2}i \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{hr}}{\alpha}\right)}{2\sqrt{h}} & \text{otherwise} \end{cases}$$

input `integrate(1/(2*h*r**2-alpha**2)**(1/2),r)`

output `Piecewise((sqrt(2)*acosh(sqrt(2)*sqrt(h)*r/alpha)/(2*sqrt(h)), Abs(h*r**2/alpha**2) > 1/2), (-sqrt(2)*I*asin(sqrt(2)*sqrt(h)*r/alpha)/(2*sqrt(h)), True))`

**3.205.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{-\alpha^2 + 2hr^2}} dr = \frac{\sqrt{2} \log\left(4hr + 2\sqrt{2}\sqrt{2hr^2 - \alpha^2}\sqrt{h}\right)}{2\sqrt{h}}$$

input `integrate(1/(2*h*r^2-alpha^2)^(1/2),r, algorithm="maxima")`

output `1/2*sqrt(2)*log(4*h*r + 2*sqrt(2)*sqrt(2*h*r^2 - alpha^2)*sqrt(h))/sqrt(h)`

**3.205.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{-\alpha^2 + 2hr^2}} dr = \frac{\sqrt{2}\alpha^2 \log\left(\left|-\sqrt{2}\sqrt{hr} + \sqrt{2hr^2 - \alpha^2}\right|\right)}{4\sqrt{h}} + \frac{1}{2}\sqrt{2hr^2 - \alpha^2}r$$

input `integrate(1/(2*h*r^2-alpha^2)^(1/2),r, algorithm="giac")`

output `1/4*sqrt(2)*alpha^2*log(abs(-sqrt(2)*sqrt(h)*r + sqrt(2*h*r^2 - alpha^2)))/sqrt(h) + 1/2*sqrt(2*h*r^2 - alpha^2)*r`



**3.205.9 Mupad [B] (verification not implemented)**

Time = 0.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{-\alpha^2 + 2hr^2}} dr = \frac{\sqrt{2} \ln\left(\sqrt{2hr^2 - \alpha^2} + \sqrt{2}\sqrt{hr}\right)}{2\sqrt{h}}$$

input `int(1/(2*h*r^2 - alpha^2)^(1/2),r)`output `(2^(1/2)*log((2*h*r^2 - alpha^2)^(1/2) + 2^(1/2)*h^(1/2)*r))/(2*h^(1/2))`

$$3.206 \quad \int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}} dr$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.206.1 Optimal result . . . . .                            | 1129 |
| 3.206.2 Mathematica [A] (verified) . . . . .                | 1129 |
| 3.206.3 Rubi [A] (verified) . . . . .                       | 1130 |
| 3.206.4 Maple [A] (verified) . . . . .                      | 1131 |
| 3.206.5 Fricas [A] (verification not implemented) . . . . . | 1131 |
| 3.206.6 Sympy [A] (verification not implemented) . . . . .  | 1132 |
| 3.206.7 Maxima [A] (verification not implemented) . . . . . | 1132 |
| 3.206.8 Giac [A] (verification not implemented) . . . . .   | 1132 |
| 3.206.9 Mupad [B] (verification not implemented) . . . . .  | 1133 |

### 3.206.1 Optimal result

Integrand size = 25, antiderivative size = 46

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}} dr = \frac{\arctan\left(\frac{\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}}{\sqrt{\alpha^2 + \epsilon^2}}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

output `arctan((2*h*r^2-alpha^2-epsilon^2)^(1/2)/(alpha^2+epsilon^2)^(1/2))/(alpha^2+epsilon^2)^(1/2)`

### 3.206.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}} dr = \frac{\arctan\left(\frac{\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}}{\sqrt{\alpha^2 + \epsilon^2}}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

input `Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]),r]`

output `ArcTan[Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]/Sqrt[alpha^2 + epsilon^2]]/Sqrt[alpha^2 + epsilon^2]`

### 3.206.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {243, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}} dr \\ & \quad \downarrow \text{243} \\ & \frac{1}{2} \int \frac{1}{r^2\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}} dr^2 \\ & \quad \downarrow \text{73} \\ & \frac{\int \frac{1}{\frac{r^4}{2h} + \frac{\alpha^2 + \epsilon^2}{2h}} d\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}}{2h} \\ & \quad \downarrow \text{218} \\ & \frac{\arctan\left(\frac{\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}}{\sqrt{\alpha^2 + \epsilon^2}}\right)}{\sqrt{\alpha^2 + \epsilon^2}} \end{aligned}$$

input `Int[1/(r*sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]),r]`

output `ArcTan[Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]/Sqrt[alpha^2 + epsilon^2]]/Sqrt[alpha^2 + epsilon^2]`

#### 3.206.3.1 Defintions of rubi rules used

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 218 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]`

rule 243 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && IntegerQ[(m - 1)/2]`

### 3.206.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

| method         | result                                                                                                                                               | size |
|----------------|------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| pseudoelliptic | $\frac{\arctan\left(\frac{\sqrt{2hr^2-\alpha^2-\epsilon^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$                         | 41   |
| default        | $-\frac{\ln\left(\frac{-2\alpha^2-2\epsilon^2+2\sqrt{-\alpha^2-\epsilon^2}\sqrt{2hr^2-\alpha^2-\epsilon^2}}{r}\right)}{\sqrt{-\alpha^2-\epsilon^2}}$ | 66   |

input `int(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r,method=_RETURNVERBOSE)`

output `arctan((2*h*r^2-alpha^2-epsilon^2)^(1/2)/(alpha^2+epsilon^2)^(1/2))/(alpha^2+epsilon^2)^(1/2)`

### 3.206.5 Fracas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}} dr = -\frac{\arctan\left(\frac{\sqrt{\alpha^2 + \epsilon^2}}{\sqrt{2hr^2 - \alpha^2 - \epsilon^2}}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

input `integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fracas")`

output `-arctan(sqrt(alpha^2 + epsilon^2)/sqrt(2*h*r^2 - alpha^2 - epsilon^2))/sqrt(alpha^2 + epsilon^2)`

**3.206.6 Sympy [A] (verification not implemented)**

Time = 0.62 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}} dr = -\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{\operatorname{polar\_lift}(-\alpha^2 - \epsilon^2)}}{2\sqrt{hr}}\right)}{\sqrt{\operatorname{polar\_lift}(-\alpha^2 - \epsilon^2)}}$$

input `integrate(1/r/(2*h*r**2-alpha**2-epsilon**2)**(1/2),r)`output `-asinh(sqrt(2)*sqrt(polar_lift(-alpha**2 - epsilon**2))/(2*sqrt(h)*r))/sqrt(polar_lift(-alpha**2 - epsilon**2))`**3.206.7 Maxima [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}} dr = -\frac{\arcsin\left(\frac{\sqrt{2}\alpha^2}{2\sqrt{(\alpha^2 + \epsilon^2)hr}} + \frac{\sqrt{2}\epsilon^2}{2\sqrt{(\alpha^2 + \epsilon^2)hr}}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

input `integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="maxima")`output `-arcsin(1/2*sqrt(2)*alpha^2/(sqrt((alpha^2 + epsilon^2)*h)*r) + 1/2*sqrt(2)*epsilon^2/(sqrt((alpha^2 + epsilon^2)*h)*r))/sqrt(alpha^2 + epsilon^2)`**3.206.8 Giac [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}} dr = \frac{\arctan\left(\frac{\sqrt{2hr^2 - \alpha^2 - \epsilon^2}}{\sqrt{\alpha^2 + \epsilon^2}}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

input `integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="giac")`output `arctan(sqrt(2*h*r^2 - alpha^2 - epsilon^2)/sqrt(alpha^2 + epsilon^2))/sqrt(alpha^2 + epsilon^2)`

**3.206.9 Mupad [B] (verification not implemented)**

Time = 0.58 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}} dr = \frac{\operatorname{atan}\left(\frac{\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2}}{\sqrt{\alpha^2 + \epsilon^2}}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

input `int(1/(r*(2*h*r^2 - alpha^2 - epsilon^2)^(1/2)),r)`

output `atan((2*h*r^2 - alpha^2 - epsilon^2)^(1/2)/(alpha^2 + epsilon^2)^(1/2))/(alpha^2 + epsilon^2)^(1/2)`

$$3.207 \quad \int \frac{1}{r\sqrt{-\alpha^2 - 2kr + 2hr^2}} dr$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.207.1 Optimal result . . . . .                            | 1134 |
| 3.207.2 Mathematica [A] (verified) . . . . .                | 1134 |
| 3.207.3 Rubi [A] (verified) . . . . .                       | 1135 |
| 3.207.4 Maple [A] (verified) . . . . .                      | 1136 |
| 3.207.5 Fricas [A] (verification not implemented) . . . . . | 1136 |
| 3.207.6 Sympy [F] . . . . .                                 | 1136 |
| 3.207.7 Maxima [A] (verification not implemented) . . . . . | 1137 |
| 3.207.8 Giac [A] (verification not implemented) . . . . .   | 1137 |
| 3.207.9 Mupad [B] (verification not implemented) . . . . .  | 1137 |

### 3.207.1 Optimal result

Integrand size = 24, antiderivative size = 37

$$\int \frac{1}{r\sqrt{-\alpha^2 - 2kr + 2hr^2}} dr = -\frac{\arctan\left(\frac{\alpha^2 + kr}{\alpha\sqrt{-\alpha^2 - 2kr + 2hr^2}}\right)}{\alpha}$$

output `-arctan((alpha^2+k*r)/alpha/(2*h*r^2-alpha^2-2*k*r)^(1/2))/alpha`

### 3.207.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{1}{r\sqrt{-\alpha^2 - 2kr + 2hr^2}} dr = -\frac{2 \arctan\left(\frac{\sqrt{2}\sqrt{hr} - \sqrt{-\alpha^2 - 2kr + 2hr^2}}{\alpha}\right)}{\alpha}$$

input `Integrate[1/(r*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2]),r]`

output `(-2*ArcTan[(Sqrt[2]*Sqrt[h]*r - Sqrt[-alpha^2 - 2*k*r + 2*h*r^2])/alpha])/alpha`

### 3.207.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{r\sqrt{-\alpha^2 + 2hr^2 - 2kr}} dr$$

↓ 1154

$$-2 \int \frac{1}{-4\alpha^2 - \frac{4(\alpha^2 + kr)^2}{-\alpha^2 + 2hr^2 - 2kr}} d\left(-\frac{2(\alpha^2 + kr)}{\sqrt{-\alpha^2 + 2hr^2 - 2kr}}\right)$$

↓ 217

$$-\frac{\arctan\left(\frac{\alpha^2 + kr}{\alpha\sqrt{-\alpha^2 + 2hr^2 - 2kr}}\right)}{\alpha}$$

input `Int[1/(r*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2]),r]`

output `-(ArcTan[(alpha^2 + k*r)/(alpha*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2]])/alpha)`

#### 3.207.3.1 Defintions of rubi rules used

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])`

rule 1154 `Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2 Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]`



**3.207.4 Maple [A] (verified)**

Time = 0.45 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

| method  | result                                                                                                          | size |
|---------|-----------------------------------------------------------------------------------------------------------------|------|
| default | $-\frac{\ln\left(\frac{-2\alpha^2-2kr+2\sqrt{-\alpha^2}\sqrt{2hr^2-\alpha^2-2kr}}{r}\right)}{\sqrt{-\alpha^2}}$ | 52   |

input `int(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r,method=_RETURNVERBOSE)`output `-1/(-alpha^2)^(1/2)*ln((-2*alpha^2-2*k*r+2*(-alpha^2)^(1/2)*(2*h*r^2-alpha^2-2*k*r)^(1/2))/r)`**3.207.5 Fracas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int \frac{1}{r\sqrt{-\alpha^2 - 2kr + 2hr^2}} dr = -\frac{\arctan\left(\frac{\sqrt{2hr^2-\alpha^2-2kr}(\alpha^2+kr)}{2\alpha hr^2-\alpha^3-2\alpha kr}\right)}{\alpha}$$

input `integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="fricas")`output `-arctan(sqrt(2*h*r^2 - alpha^2 - 2*k*r)*(alpha^2 + k*r)/(2*alpha*h*r^2 - alpha^3 - 2*alpha*k*r))/alpha`**3.207.6 Sympy [F]**

$$\int \frac{1}{r\sqrt{-\alpha^2 - 2kr + 2hr^2}} dr = \int \frac{1}{r\sqrt{-\alpha^2 + 2hr^2 - 2kr}} dr$$

input `integrate(1/r/(2*h*r**2-alpha**2-2*k*r)**(1/2),r)`output `Integral(1/(r*sqrt(-alpha**2 + 2*h*r**2 - 2*k*r)), r)`

**3.207.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{1}{r\sqrt{-\alpha^2 - 2kr + 2hr^2}} dr = -\frac{\arcsin\left(\frac{k}{\sqrt{2\alpha^2 h + k^2}} + \frac{\alpha^2}{\sqrt{2\alpha^2 h + k^2}r}\right)}{\alpha}$$

input `integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="maxima")`output `-arcsin(k/sqrt(2*alpha^2*h + k^2) + alpha^2/(sqrt(2*alpha^2*h + k^2)*r))/alpha`**3.207.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{1}{r\sqrt{-\alpha^2 - 2kr + 2hr^2}} dr = \frac{2 \arctan\left(-\frac{\sqrt{2}\sqrt{hr}-\sqrt{2hr^2-\alpha^2-2kr}}{\alpha}\right)}{\alpha}$$

input `integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="giac")`output `2*arctan(-(sqrt(2)*sqrt(h)*r - sqrt(2*h*r^2 - alpha^2 - 2*k*r))/alpha)/alpha`**3.207.9 Mupad [B] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

$$\int \frac{1}{r\sqrt{-\alpha^2 - 2kr + 2hr^2}} dr = -\frac{\ln\left(\frac{\sqrt{-\alpha^2}\sqrt{-\alpha^2+2hr^2-2kr}}{r} - \frac{\alpha^2}{r} - k\right)}{\sqrt{-\alpha^2}}$$

input `int(1/(r*(2*h*r^2 - 2*k*r - alpha^2)^(1/2)),r)`output `-log(((alpha^2)^(1/2)*(2*h*r^2 - 2*k*r - alpha^2)^(1/2))/r - alpha^2/r - k)/(alpha^2)^(1/2)`

**3.208**  $\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}} dr$

3.208.1 Optimal result . . . . . 1138  
 3.208.2 Mathematica [A] (verified) . . . . . 1138  
 3.208.3 Rubi [A] (verified) . . . . . 1139  
 3.208.4 Maple [A] (verified) . . . . . 1140  
 3.208.5 Fricas [A] (verification not implemented) . . . . . 1140  
 3.208.6 Sympy [F] . . . . . 1140  
 3.208.7 Maxima [A] (verification not implemented) . . . . . 1141  
 3.208.8 Giac [A] (verification not implemented) . . . . . 1141  
 3.208.9 Mupad [B] (verification not implemented) . . . . . 1141

**3.208.1 Optimal result**

Integrand size = 29, antiderivative size = 61

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}} dr = -\frac{\arctan\left(\frac{\alpha^2+\epsilon^2+kr}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

output `-arctan((alpha^2+epsilon^2+k*r)/(alpha^2+epsilon^2)^(1/2)/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2))/(alpha^2+epsilon^2)^(1/2)`

**3.208.2 Mathematica [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}} dr = -\frac{2\arctan\left(\frac{\sqrt{2}\sqrt{hr}-\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

input `Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2]),r]`

output `(-2*ArcTan[(Sqrt[2]*Sqrt[h]*r - Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2])/Sqrt[alpha^2 + epsilon^2]])/Sqrt[alpha^2 + epsilon^2]`

### 3.208.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr}} dr$$

↓ 1154

$$-2 \int \frac{1}{-\frac{4(\alpha^2 + \epsilon^2 + kr)^2}{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr} - 4(\alpha^2 + \epsilon^2)} d\left(-\frac{2(\alpha^2 + \epsilon^2 + kr)}{\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr}}\right)$$

↓ 217

$$-\frac{\arctan\left(\frac{\alpha^2 + \epsilon^2 + kr}{\sqrt{\alpha^2 + \epsilon^2}\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr}}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

input `Int[1/(r*Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2]),r]`

output `-(ArcTan[(alpha^2 + epsilon^2 + k*r)/(Sqrt[alpha^2 + epsilon^2]*Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2]])/Sqrt[alpha^2 + epsilon^2]`

#### 3.208.3.1 Defintions of rubi rules used

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])`

rule 1154 `Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2 Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]`

**3.208.4 Maple [A] (verified)**

Time = 0.48 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.21

| method  | result                                                                                                                                                       | size |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default | $-\frac{\ln\left(\frac{-2\alpha^2-2\epsilon^2-2kr+2\sqrt{-\alpha^2-\epsilon^2}\sqrt{2hr^2-\alpha^2-\epsilon^2-2kr}}{r}\right)}{\sqrt{-\alpha^2-\epsilon^2}}$ | 74   |

input `int(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r,method=_RETURNVERBOSE)`output `-1/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2-2*k*r+2*(-alpha^2-epsilon^2)^(1/2)*(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2))/r)`**3.208.5 Fracas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 - 2kr + 2hr^2}} dr = -\frac{\arctan\left(-\frac{\sqrt{2hr^2-\alpha^2-\epsilon^2-2kr}(\alpha^2+\epsilon^2+kr)\sqrt{\alpha^2+\epsilon^2}}{\alpha^4+2\alpha^2\epsilon^2+\epsilon^4-2(\alpha^2+\epsilon^2)hr^2+2(\alpha^2+\epsilon^2)kr}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

input `integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r, algorithm="fracas")`output `-arctan(-sqrt(2*h*r^2 - alpha^2 - epsilon^2 - 2*k*r)*(alpha^2 + epsilon^2 + k*r)*sqrt(alpha^2 + epsilon^2)/(alpha^4 + 2*alpha^2*epsilon^2 + epsilon^4 - 2*(alpha^2 + epsilon^2)*h*r^2 + 2*(alpha^2 + epsilon^2)*k*r))/sqrt(alpha^2 + epsilon^2)`**3.208.6 Sympy [F]**

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 - 2kr + 2hr^2}} dr = \int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr}} dr$$

input `integrate(1/r/(2*h*r**2-alpha**2-epsilon**2-2*k*r)**(1/2),r)`output `Integral(1/(r*sqrt(-alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r)), r)`

**3.208.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 - 2kr + 2hr^2}} dr = -\frac{\arcsin\left(\frac{k}{\sqrt{2(\alpha^2 + \epsilon^2)h + k^2}} + \frac{\alpha^2}{\sqrt{2(\alpha^2 + \epsilon^2)h + k^2}r} + \frac{\epsilon^2}{\sqrt{2(\alpha^2 + \epsilon^2)h + k^2}r}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

input `integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r, algorithm="maxima")`

output `-arcsin(k/sqrt(2*(alpha^2 + epsilon^2)*h + k^2) + alpha^2/(sqrt(2*(alpha^2 + epsilon^2)*h + k^2)*r) + epsilon^2/(sqrt(2*(alpha^2 + epsilon^2)*h + k^2)*r))/sqrt(alpha^2 + epsilon^2)`

**3.208.8 Giac [A] (verification not implemented)**

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 - 2kr + 2hr^2}} dr = \frac{2 \arctan\left(-\frac{\sqrt{2}\sqrt{hr}-\sqrt{2hr^2-\alpha^2-\epsilon^2-2kr}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

input `integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r, algorithm="giac")`

output `2*arctan(-(sqrt(2)*sqrt(h)*r - sqrt(2*h*r^2 - alpha^2 - epsilon^2 - 2*k*r))/sqrt(alpha^2 + epsilon^2))/sqrt(alpha^2 + epsilon^2)`

**3.208.9 Mupad [B] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 - 2kr + 2hr^2}} dr = -\frac{\ln\left(\frac{\sqrt{-\alpha^2 - \epsilon^2}\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr}}{r} - \frac{\alpha^2 + \epsilon^2}{r} - k\right)}{\sqrt{-\alpha^2 - \epsilon^2}}$$

input `int(1/(r*(2*h*r^2 - 2*k*r - alpha^2 - epsilon^2)^(1/2)),r)`

output `-log((- alpha^2 - epsilon^2)^(1/2)*(2*h*r^2 - 2*k*r - alpha^2 - epsilon^2)^(1/2))/r - (alpha^2 + epsilon^2)/r - k/(- alpha^2 - epsilon^2)^(1/2)`

$$3.209 \quad \int \frac{r}{\sqrt{-\alpha^2 + 2er^2}} dr$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.209.1 Optimal result . . . . .                            | 1143 |
| 3.209.2 Mathematica [A] (verified) . . . . .                | 1143 |
| 3.209.3 Rubi [A] (verified) . . . . .                       | 1144 |
| 3.209.4 Maple [A] (verified) . . . . .                      | 1144 |
| 3.209.5 Fricas [A] (verification not implemented) . . . . . | 1145 |
| 3.209.6 Sympy [A] (verification not implemented) . . . . .  | 1145 |
| 3.209.7 Maxima [A] (verification not implemented) . . . . . | 1146 |
| 3.209.8 Giac [A] (verification not implemented) . . . . .   | 1146 |
| 3.209.9 Mupad [B] (verification not implemented) . . . . .  | 1146 |

### 3.209.1 Optimal result

Integrand size = 18, antiderivative size = 23

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2}} dr = \frac{\sqrt{-\alpha^2 + 2er^2}}{2e}$$

output `1/2*(2*e*r^2-alpha^2)^(1/2)/e`

### 3.209.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2}} dr = \frac{\sqrt{-\alpha^2 + 2er^2}}{2e}$$

input `Integrate[r/Sqrt[-alpha^2 + 2*e*r^2],r]`

output `Sqrt[-alpha^2 + 2*e*r^2]/(2*e)`



**3.209.3 Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{r}{\sqrt{2er^2 - \alpha^2}} dr$$

↓ 241

$$\frac{\sqrt{2er^2 - \alpha^2}}{2e}$$

input `Int[r/Sqrt[-alpha^2 + 2*e*r^2],r]`

output `Sqrt[-alpha^2 + 2*e*r^2]/(2*e)`

**3.209.3.1 Defintions of rubi rules used**

rule 241 `Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]`

**3.209.4 Maple [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

| method            | result                                                 | size |
|-------------------|--------------------------------------------------------|------|
| gospers           | $\frac{\sqrt{2er^2 - \alpha^2}}{2e}$                   | 20   |
| derivativedivides | $\frac{\sqrt{2er^2 - \alpha^2}}{2e}$                   | 20   |
| default           | $\frac{\sqrt{2er^2 - \alpha^2}}{2e}$                   | 20   |
| trager            | $\frac{\sqrt{2er^2 - \alpha^2}}{2e}$                   | 20   |
| pseudoelliptic    | $\frac{\sqrt{2er^2 - \alpha^2}}{2e}$                   | 20   |
| risch             | $-\frac{-2er^2 + \alpha^2}{2e\sqrt{2er^2 - \alpha^2}}$ | 30   |

input `int(r/(2*e*r^2-alpha^2)^(1/2),r,method=_RETURNVERBOSE)`

output `1/2*(2*e*r^2-alpha^2)^(1/2)/e`

### 3.209.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2}} dr = \frac{\sqrt{2er^2 - \alpha^2}}{2e}$$

input `integrate(r/(2*e*r^2-alpha^2)^(1/2),r, algorithm="fricas")`

output `1/2*sqrt(2*e*r^2 - alpha^2)/e`

### 3.209.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2}} dr = \begin{cases} \frac{\sqrt{-\alpha^2 + 2er^2}}{2e} & \text{for } e \neq 0 \\ \frac{r^2}{2\sqrt{-\alpha^2}} & \text{otherwise} \end{cases}$$

input `integrate(r/(2*e*r**2-alpha**2)**(1/2),r)`

output `Piecewise((sqrt(-alpha**2 + 2*e*r**2)/(2*e), Ne(e, 0)), (r**2/(2*sqrt(-alpha**2)), True))`

**3.209.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2}} dr = \frac{\sqrt{2er^2 - \alpha^2}}{2e}$$

input `integrate(r/(2*e*r^2-alpha^2)^(1/2),r, algorithm="maxima")`output `1/2*sqrt(2*e*r^2 - alpha^2)/e`**3.209.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2}} dr = \frac{\sqrt{2er^2 - \alpha^2}}{2e}$$

input `integrate(r/(2*e*r^2-alpha^2)^(1/2),r, algorithm="giac")`output `1/2*sqrt(2*e*r^2 - alpha^2)/e`**3.209.9 Mupad [B] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2}} dr = \frac{\sqrt{2er^2 - \alpha^2}}{2e}$$

input `int(r/(2*e*r^2 - alpha^2)^(1/2),r)`output `(2*e*r^2 - alpha^2)^(1/2)/(2*e)`

$$3.210 \quad \int \frac{r}{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}} dr$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.210.1 Optimal result . . . . .                            | 1147 |
| 3.210.2 Mathematica [A] (verified) . . . . .                | 1147 |
| 3.210.3 Rubi [A] (verified) . . . . .                       | 1148 |
| 3.210.4 Maple [A] (verified) . . . . .                      | 1148 |
| 3.210.5 Fricas [A] (verification not implemented) . . . . . | 1149 |
| 3.210.6 Sympy [A] (verification not implemented) . . . . .  | 1149 |
| 3.210.7 Maxima [A] (verification not implemented) . . . . . | 1150 |
| 3.210.8 Giac [A] (verification not implemented) . . . . .   | 1150 |
| 3.210.9 Mupad [B] (verification not implemented) . . . . .  | 1150 |

### 3.210.1 Optimal result

Integrand size = 23, antiderivative size = 28

$$\int \frac{r}{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}} dr = \frac{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}}{2e}$$

output `1/2*(2*e*r^2-alpha^2-epsilon^2)^(1/2)/e`

### 3.210.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{r}{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}} dr = \frac{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}}{2e}$$

input `Integrate[r/Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2],r]`

output `Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2]/(2*e)`

**3.210.3 Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - \epsilon^2}} dr$$

↓ 241

$$\frac{\sqrt{-\alpha^2 + 2er^2 - \epsilon^2}}{2e}$$

input `Int[r/Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2],r]`

output `Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2]/(2*e)`

**3.210.3.1 Defintions of rubi rules used**

rule 241 `Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]`

**3.210.4 Maple [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

| method            | result                                                                             | size |
|-------------------|------------------------------------------------------------------------------------|------|
| gospers           | $\frac{\sqrt{2e r^2 - \alpha^2 - \epsilon^2}}{2e}$                                 | 25   |
| derivativedivides | $\frac{\sqrt{2e r^2 - \alpha^2 - \epsilon^2}}{2e}$                                 | 25   |
| default           | $\frac{\sqrt{2e r^2 - \alpha^2 - \epsilon^2}}{2e}$                                 | 25   |
| trager            | $\frac{\sqrt{2e r^2 - \alpha^2 - \epsilon^2}}{2e}$                                 | 25   |
| pseudoelliptic    | $\frac{\sqrt{2e r^2 - \alpha^2 - \epsilon^2}}{2e}$                                 | 25   |
| risch             | $-\frac{-2e r^2 + \alpha^2 + \epsilon^2}{2e\sqrt{2e r^2 - \alpha^2 - \epsilon^2}}$ | 38   |

input `int(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r,method=_RETURNVERBOSE)`

output `1/2*(2*e*r^2-alpha^2-epsilon^2)^(1/2)/e`

### 3.210.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{r}{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}} dr = \frac{\sqrt{2er^2 - \alpha^2 - \epsilon^2}}{2e}$$

input `integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fricas")`

output `1/2*sqrt(2*e*r^2 - alpha^2 - epsilon^2)/e`

### 3.210.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{r}{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}} dr = \begin{cases} \frac{\sqrt{-\alpha^2 + 2er^2 - \epsilon^2}}{2e} & \text{for } e \neq 0 \\ \frac{r^2}{2\sqrt{-\alpha^2 - \epsilon^2}} & \text{otherwise} \end{cases}$$

input `integrate(r/(2*e*r**2-alpha**2-epsilon**2)**(1/2),r)`

output `Piecewise((sqrt(-alpha**2 + 2*e*r**2 - epsilon**2)/(2*e), Ne(e, 0)), (r**2/(2*sqrt(-alpha**2 - epsilon**2)), True))`

**3.210.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{r}{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}} dr = \frac{\sqrt{2er^2 - \alpha^2 - \epsilon^2}}{2e}$$

input `integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="maxima")`output `1/2*sqrt(2*e*r^2 - alpha^2 - epsilon^2)/e`**3.210.8 Giac [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{r}{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}} dr = \frac{\sqrt{2er^2 - \alpha^2 - \epsilon^2}}{2e}$$

input `integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="giac")`output `1/2*sqrt(2*e*r^2 - alpha^2 - epsilon^2)/e`**3.210.9 Mupad [B] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{r}{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}} dr = \frac{\sqrt{-\alpha^2 - \epsilon^2 + 2er^2}}{2e}$$

input `int(r/(2*e*r^2 - alpha^2 - epsilon^2)^(1/2),r)`output `(2*e*r^2 - alpha^2 - epsilon^2)^(1/2)/(2*e)`

### 3.211 $\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr^4}} dr$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.211.1 Optimal result . . . . .                            | 1151 |
| 3.211.2 Mathematica [A] (verified) . . . . .                | 1151 |
| 3.211.3 Rubi [A] (verified) . . . . .                       | 1152 |
| 3.211.4 Maple [A] (verified) . . . . .                      | 1153 |
| 3.211.5 Fricas [A] (verification not implemented) . . . . . | 1153 |
| 3.211.6 Sympy [F] . . . . .                                 | 1154 |
| 3.211.7 Maxima [F(-2)] . . . . .                            | 1154 |
| 3.211.8 Giac [A] (verification not implemented) . . . . .   | 1155 |
| 3.211.9 Mupad [B] (verification not implemented) . . . . .  | 1155 |

#### 3.211.1 Optimal result

Integrand size = 24, antiderivative size = 56

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}} dr = -\frac{\arctan\left(\frac{e-2kr^2}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}$$

output `-1/4*arctan(1/2*(-2*k*r^2+e)*2^(1/2)/k^(1/2)/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2))*2^(1/2)/k^(1/2)`

#### 3.211.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}} dr = -\frac{\arctan\left(\frac{\sqrt{2}\sqrt{kr^2}}{\sqrt{-\alpha^2-\sqrt{-\alpha^2+2er^2-2kr^4}}}\right)}{\sqrt{2}\sqrt{k}}$$

input `Integrate[r/Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4],r]`

output `-(ArcTan[(Sqrt[2]*Sqrt[k]*r^2)/(Sqrt[-alpha^2] - Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4])]/(Sqrt[2]*Sqrt[k]))`



### 3.211.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1432, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}} dr \\
 & \quad \downarrow \text{1432} \\
 & \frac{1}{2} \int \frac{1}{\sqrt{-2kr^4 + 2er^2 - \alpha^2}} dr^2 \\
 & \quad \downarrow \text{1092} \\
 & \int \frac{1}{-8k - r^4} d \frac{2(e - 2kr^2)}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}} \\
 & \quad \downarrow \text{217} \\
 & \frac{\arctan\left(\frac{e - 2kr^2}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2 + 2er^2 - 2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}
 \end{aligned}$$

input `Int[r/Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4],r]`

output `-1/2*ArcTan[(e - 2*k*r^2)/(Sqrt[2]*Sqrt[k]*Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4])]/(Sqrt[2]*Sqrt[k])`

#### 3.211.3.1 Defintions of rubi rules used

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & (LtQ[a, 0] || LtQ[b, 0])`

rule 1092 `Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2 Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x]`

rule 1432 `Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[1/2  
Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]`

### 3.211.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

| method         | result                                                                                                                             | size |
|----------------|------------------------------------------------------------------------------------------------------------------------------------|------|
| pseudoelliptic | $-\frac{\arctan\left(\frac{(-2kr^2+e)\sqrt{2}}{2\sqrt{k}\sqrt{-2kr^4+2er^2-\alpha^2}}\right)\sqrt{2}}{4\sqrt{k}}$                  | 46   |
| default        | $\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{k}\left(r^2-\frac{e}{2k}\right)}{\sqrt{-2kr^4+2er^2-\alpha^2}}\right)}{4\sqrt{k}}$ | 47   |
| elliptic       | $\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{k}\left(r^2-\frac{e}{2k}\right)}{\sqrt{-2kr^4+2er^2-\alpha^2}}\right)}{4\sqrt{k}}$ | 47   |

input `int(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r,method=_RETURNVERBOSE)`

output `-1/4*arctan(1/2*(-2*k*r^2+e)*2^(1/2)/k^(1/2)/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2))*2^(1/2)/k^(1/2)`

### 3.211.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.71

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}} dr$$

$$= \left[ -\frac{\sqrt{2}\sqrt{-k} \log(-8k^2r^4 + 8ekr^2 - 2\alpha^2k + 2\sqrt{2}\sqrt{-2kr^4 + 2er^2 - \alpha^2}(2kr^2 - e)\sqrt{-k} - e^2)}{8k}, \right. \\ \left. -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-2kr^4+2er^2-\alpha^2}(2kr^2-e)\sqrt{k}}{2(2k^2r^4-2ekr^2+\alpha^2k)}\right)}{4\sqrt{k}} \right]$$

input `integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r, algorithm="fracas")`

output `[-1/8*sqrt(2)*sqrt(-k)*log(-8*k^2*r^4 + 8*e*k*r^2 - 2*alpha^2*k + 2*sqrt(2)*sqrt(-2*k*r^4 + 2*e*r^2 - alpha^2)*(2*k*r^2 - e)*sqrt(-k) - e^2)/k, -1/4*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-2*k*r^4 + 2*e*r^2 - alpha^2)*(2*k*r^2 - e)*sqrt(k)/(2*k^2*r^4 - 2*e*k*r^2 + alpha^2*k))/sqrt(k)]`

### 3.211.6 Sympy [F]

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}} dr = \int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}} dr$$

input `integrate(r/(-2*k*r**4+2*e*r**2-alpha**2)**(1/2),r)`

output `Integral(r/sqrt(-alpha**2 + 2*e*r**2 - 2*k*r**4), r)`

### 3.211.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}} dr = \text{Exception raised: ValueError}$$

input `integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(2*alpha^2*k-e^2>0)', see `assume?` for mor`

**3.211.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}} dr = -\frac{\sqrt{2} \log\left(\left|\sqrt{2}\left(\sqrt{2}\sqrt{-k}r^2 - \sqrt{-2kr^4 + 2er^2 - \alpha^2}\right)\sqrt{-k} + e\right|\right)}{4\sqrt{-k}}$$

input `integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r, algorithm="giac")`output `-1/4*sqrt(2)*log(abs(sqrt(2)*(sqrt(2)*sqrt(-k)*r^2 - sqrt(-2*k*r^4 + 2*e*r^2 - alpha^2))*sqrt(-k) + e))/sqrt(-k)`**3.211.9 Mupad [B] (verification not implemented)**

Time = 0.95 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}} dr = \frac{\sqrt{2} \ln\left(\sqrt{-\alpha^2 - 2kr^4 + 2er^2} + \frac{\sqrt{2}(e - 2kr^2)}{2\sqrt{-k}}\right)}{4\sqrt{-k}}$$

input `int(r/(2*e*r^2 - 2*k*r^4 - alpha^2)^(1/2),r)`output `(2^(1/2)*log((2*e*r^2 - 2*k*r^4 - alpha^2)^(1/2) + (2^(1/2)*(e - 2*k*r^2))/(2*(-k)^(1/2))))/(4*(-k)^(1/2))`

### 3.212 $\int \frac{r}{\sqrt{-\alpha^2 - 2kr + 2er^2}} dr$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.212.1 Optimal result . . . . .                            | 1156 |
| 3.212.2 Mathematica [A] (verified) . . . . .                | 1156 |
| 3.212.3 Rubi [A] (verified) . . . . .                       | 1157 |
| 3.212.4 Maple [A] (verified) . . . . .                      | 1158 |
| 3.212.5 Fricas [A] (verification not implemented) . . . . . | 1158 |
| 3.212.6 Sympy [B] (verification not implemented) . . . . .  | 1159 |
| 3.212.7 Maxima [F(-2)] . . . . .                            | 1160 |
| 3.212.8 Giac [A] (verification not implemented) . . . . .   | 1160 |
| 3.212.9 Mupad [B] (verification not implemented) . . . . .  | 1160 |

#### 3.212.1 Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{r}{\sqrt{-\alpha^2 - 2kr + 2er^2}} dr = \frac{\sqrt{-\alpha^2 - 2kr + 2er^2}}{2e} - \frac{k \operatorname{arctanh}\left(\frac{k-2er}{\sqrt{2}\sqrt{e}\sqrt{-\alpha^2 - 2kr + 2er^2}}\right)}{2\sqrt{2}e^{3/2}}$$

output `-1/4*k*arctanh(1/2*(-2*e*r+k)*2^(1/2)/e^(1/2)/(2*e*r^2-alpha^2-2*k*r)^(1/2)))/e^(3/2)*2^(1/2)+1/2*(2*e*r^2-alpha^2-2*k*r)^(1/2)/e`

#### 3.212.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$\int \frac{r}{\sqrt{-\alpha^2 - 2kr + 2er^2}} dr = \frac{1}{2} \left( \frac{\sqrt{-\alpha^2 - 2kr + 2er^2}}{e} - \frac{\sqrt{2}k \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{er}}{\sqrt{-\alpha^2 - \sqrt{-\alpha^2 - 2kr + 2er^2}}}\right)}{e^{3/2}} \right)$$

input `Integrate[r/Sqrt[-alpha^2 - 2*k*r + 2*e*r^2],r]`

output `(Sqrt[-alpha^2 - 2*k*r + 2*e*r^2]/e - (Sqrt[2]*k*ArcTanh[(Sqrt[2]*Sqrt[e]*r)/(Sqrt[-alpha^2] - Sqrt[-alpha^2 - 2*k*r + 2*e*r^2])])/e^(3/2))/2`

### 3.212.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr}} dr \\
 & \quad \downarrow 1160 \\
 & \frac{k \int \frac{1}{\sqrt{-\alpha^2 + 2er^2 - 2kr}} dr}{2e} + \frac{\sqrt{-\alpha^2 + 2er^2 - 2kr}}{2e} \\
 & \quad \downarrow 1092 \\
 & \frac{k \int \frac{1}{8e^{-\frac{4(k-2er)^2}{-\alpha^2 + 2er^2 - 2kr}}} d\left(-\frac{2(k-2er)}{\sqrt{-\alpha^2 + 2er^2 - 2kr}}\right)}{e} + \frac{\sqrt{-\alpha^2 + 2er^2 - 2kr}}{2e} \\
 & \quad \downarrow 219 \\
 & \frac{\sqrt{-\alpha^2 + 2er^2 - 2kr}}{2e} - \frac{k \operatorname{arctanh}\left(\frac{k-2er}{\sqrt{2}\sqrt{e}\sqrt{-\alpha^2 + 2er^2 - 2kr}}\right)}{2\sqrt{2}e^{3/2}}
 \end{aligned}$$

input `Int[r/Sqrt[-alpha^2 - 2*k*r + 2*e*r^2],r]`

output `Sqrt[-alpha^2 - 2*k*r + 2*e*r^2]/(2*e) - (k*ArcTanh[(k - 2*e*r)/(Sqrt[2]*Sqrt[e]*Sqrt[-alpha^2 - 2*k*r + 2*e*r^2])])/(2*Sqrt[2]*e^(3/2))`

#### 3.212.3.1 Defintions of rubi rules used

rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

rule 1092 `Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2 Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x]`

```
rule 1160 Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c) Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[p, -1]
```

### 3.212.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

| method  | result                                                                                                                                                                                       | size |
|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default | $\frac{\sqrt{2e r^2 - \alpha^2 - 2kr}}{2e} + \frac{k \ln\left(\frac{(2er-k)\sqrt{2} + \sqrt{2e r^2 - \alpha^2 - 2kr}}{2\sqrt{e}}\right)\sqrt{2}}{4e^{\frac{3}{2}}}$                          | 70   |
| risch   | $-\frac{-2e r^2 + \alpha^2 + 2kr}{2e\sqrt{2e r^2 - \alpha^2 - 2kr}} + \frac{k \ln\left(\frac{(2er-k)\sqrt{2} + \sqrt{2e r^2 - \alpha^2 - 2kr}}{2\sqrt{e}}\right)\sqrt{2}}{4e^{\frac{3}{2}}}$ | 84   |

```
input int(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r,method=_RETURNVERBOSE)
```

```
output 1/2*(2*e*r^2-alpha^2-2*k*r)^(1/2)/e+1/4*k/e^(3/2)*ln(1/2*(2*e*r-k)*2^(1/2)
/e^(1/2)+(2*e*r^2-alpha^2-2*k*r)^(1/2))*2^(1/2)
```

### 3.212.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.35

$$\int \frac{r}{\sqrt{-\alpha^2 - 2kr + 2er^2}} dr$$

$$= \left[ \frac{\sqrt{2}\sqrt{ek} \log(8e^2r^2 - 2\alpha^2e - 8ekr + 2\sqrt{2}\sqrt{2er^2 - \alpha^2 - 2kr}(2er - k)\sqrt{e} + k^2) + 4\sqrt{2er^2 - \alpha^2 - 2kr}}{8e^2} \right. \\ \left. - \frac{\sqrt{2}\sqrt{-ek} \arctan\left(\frac{\sqrt{2}\sqrt{2er^2 - \alpha^2 - 2kr}(2er - k)\sqrt{-e}}{2(2e^2r^2 - \alpha^2e - 2ekr)}\right) - 2\sqrt{2er^2 - \alpha^2 - 2kre}}{4e^2} \right]$$

```
input integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="fricas")
```

```
output [1/8*(sqrt(2)*sqrt(e)*k*log(8*e^2*r^2 - 2*alpha^2*e - 8*e*k*r + 2*sqrt(2)*
sqrt(2*e*r^2 - alpha^2 - 2*k*r)*(2*e*r - k)*sqrt(e) + k^2) + 4*sqrt(2*e*r^
2 - alpha^2 - 2*k*r)*e)/e^2, -1/4*(sqrt(2)*sqrt(-e)*k*arctan(1/2*sqrt(2)*s
qrt(2*e*r^2 - alpha^2 - 2*k*r)*(2*e*r - k)*sqrt(-e)/(2*e^2*r^2 - alpha^2*e
- 2*e*k*r)) - 2*sqrt(2*e*r^2 - alpha^2 - 2*k*r)*e)/e^2]
```

### 3.212.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs.  $2(71) = 142$ .

Time = 0.41 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.12

$$\int \frac{r}{\sqrt{-\alpha^2 - 2kr + 2er^2}} dr$$

$$= \begin{cases} \frac{k \begin{cases} \frac{\sqrt{2} \log\left(\frac{2\sqrt{2}\sqrt{e}\sqrt{-\alpha^2+2er^2-2kr+4er-2k}}{2\sqrt{e}}\right)}{2\sqrt{e}} & \text{for } \alpha^2 + \frac{k^2}{2e} \neq 0 \\ \frac{\sqrt{2}\left(r - \frac{k}{2e}\right) \log\left(r - \frac{k}{2e}\right)}{2\sqrt{e}\left(r - \frac{k}{2e}\right)^2} & \text{otherwise} \end{cases}}{2e} + \frac{\sqrt{-\alpha^2+2er^2-2kr}}{2e} & \text{for } e \neq 0 \\ \frac{\alpha^2\sqrt{-\alpha^2-2kr} + \frac{(-\alpha^2-2kr)^{\frac{3}{2}}}{3}}{2k^2} & \text{for } k \neq 0 \\ \frac{r^2}{2\sqrt{-\alpha^2}} & \text{otherwise} \end{cases}$$

```
input integrate(r/(2*e*r**2-alpha**2-2*k*r)**(1/2),r)
```

```
output Piecewise((k*Piecewise((sqrt(2)*log(2*sqrt(2)*sqrt(e)*sqrt(-alpha**2 + 2*e
*r**2 - 2*k*r) + 4*e*r - 2*k)/(2*sqrt(e)), Ne(alpha**2 + k**2/(2*e), 0)),
(sqrt(2)*(r - k/(2*e))*log(r - k/(2*e))/(2*sqrt(e*(r - k/(2*e))**2))), True
))/(2*e) + sqrt(-alpha**2 + 2*e*r**2 - 2*k*r)/(2*e), Ne(e, 0)), ((alpha**2
*sqrt(-alpha**2 - 2*k*r) + (-alpha**2 - 2*k*r)**(3/2)/3)/(2*k**2), Ne(k, 0
)), (r**2/(2*sqrt(-alpha**2)), True))
```



**3.212.7 Maxima [F(-2)]**

Exception generated.

$$\int \frac{r}{\sqrt{-\alpha^2 - 2kr + 2er^2}} dr = \text{Exception raised: ValueError}$$

```
input integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="maxima")
```

```
output Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(k^2+2*alpha^2*e>0)', see `assume
?` for mor
```

**3.212.8 Giac [A] (verification not implemented)**

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{r}{\sqrt{-\alpha^2 - 2kr + 2er^2}} dr = -\frac{\sqrt{2}k \log(|\sqrt{2}(\sqrt{2}\sqrt{er} - \sqrt{2er^2 - \alpha^2 - 2kr})\sqrt{e} - k|)}{4e^{\frac{3}{2}}} + \frac{\sqrt{2er^2 - \alpha^2 - 2kr}}{2e}$$

```
input integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="giac")
```

```
output -1/4*sqrt(2)*k*log(abs(sqrt(2)*(sqrt(2)*sqrt(e)*r - sqrt(2*e*r^2 - alpha^2
- 2*k*r))*sqrt(e) - k))/e^(3/2) + 1/2*sqrt(2*e*r^2 - alpha^2 - 2*k*r)/e
```

**3.212.9 Mupad [B] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

$$\int \frac{r}{\sqrt{-\alpha^2 - 2kr + 2er^2}} dr = \frac{\sqrt{-\alpha^2 + 2er^2 - 2kr}}{2e} + \frac{\sqrt{2}k \ln\left(\sqrt{-\alpha^2 + 2er^2 - 2kr} - \frac{\sqrt{2}(k-2er)}{2\sqrt{e}}\right)}{4e^{3/2}}$$

input `int(r/(2*e*r^2 - 2*k*r - alpha^2)^(1/2),r)`

output  $(2*e*r^2 - 2*k*r - \alpha^2)^{(1/2)}/(2*e) + (2^{(1/2)}*k*\log((2*e*r^2 - 2*k*r - \alpha^2)^{(1/2)} - (2^{(1/2)}*(k - 2*e*r))/(2*e^{(1/2)})))/(4*e^{(3/2)})$

**3.213**  $\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}} dr$

3.213.1 Optimal result . . . . . 1162  
 3.213.2 Mathematica [A] (verified) . . . . . 1162  
 3.213.3 Rubi [A] (verified) . . . . . 1163  
 3.213.4 Maple [A] (verified) . . . . . 1164  
 3.213.5 Fricas [A] (verification not implemented) . . . . . 1164  
 3.213.6 Sympy [F] . . . . . 1165  
 3.213.7 Maxima [F(-2)] . . . . . 1165  
 3.213.8 Giac [A] (verification not implemented) . . . . . 1165  
 3.213.9 Mupad [B] (verification not implemented) . . . . . 1166

**3.213.1 Optimal result**

Integrand size = 26, antiderivative size = 44

$$\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}} dr = -\frac{\arctan\left(\frac{\alpha^2-hr^2}{\alpha\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)}{2\alpha}$$

output `-1/2*arctan((-h*r^2+alpha^2)/alpha/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2))/alpha`

**3.213.2 Mathematica [A] (verified)**

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}} dr = -\frac{\arctan\left(\frac{\sqrt{2}\sqrt{-kr^2}}{\alpha} - \frac{\sqrt{-\alpha^2+2hr^2-2kr^4}}{\alpha}\right)}{\alpha}$$

input `Integrate[1/(r*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]),r]`

output `-(ArcTan[(Sqrt[2]*Sqrt[-k]*r^2)/alpha - Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]/alpha]/alpha)`

### 3.213.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$ , Rules used = {1434, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{r\sqrt{-\alpha^2 + 2hr^2 - 2kr^4}} dr \\ & \quad \downarrow 1434 \\ & \frac{1}{2} \int \frac{1}{r^2\sqrt{-2kr^4 + 2hr^2 - \alpha^2}} dr^2 \\ & \quad \downarrow 1154 \\ & - \int \frac{1}{-r^4 - 4\alpha^2} d\left(-\frac{2(\alpha^2 - hr^2)}{\sqrt{-2kr^4 + 2hr^2 - \alpha^2}}\right) \\ & \quad \downarrow 217 \\ & -\frac{\arctan\left(\frac{\alpha^2 - hr^2}{\alpha\sqrt{-\alpha^2 + 2hr^2 - 2kr^4}}\right)}{2\alpha} \end{aligned}$$

input `Int[1/(r*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]),r]`

output `-1/2*ArcTan[(alpha^2 - h*r^2)/(alpha*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4])]/alpha`

#### 3.213.3.1 Defintions of rubi rules used

rule 217 `Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & (LtQ[a, 0] || LtQ[b, 0])`

rule 1154 `Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] :> Simp[-2 Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]`

rule 1434 `Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp  
[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free  
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]`

### 3.213.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

| method         | result                                                                                                                     | size |
|----------------|----------------------------------------------------------------------------------------------------------------------------|------|
| default        | $-\frac{\ln\left(\frac{-2\alpha^2+2hr^2+2\sqrt{-\alpha^2}\sqrt{-2kr^4+2hr^2-\alpha^2}}{r^2}\right)}{2\sqrt{-\alpha^2}}$    | 56   |
| elliptic       | $-\frac{\ln\left(\frac{-2\alpha^2+2hr^2+2\sqrt{-\alpha^2}\sqrt{-2kr^4+2hr^2-\alpha^2}}{r^2}\right)}{2\sqrt{-\alpha^2}}$    | 56   |
| pseudoelliptic | $-\frac{\ln(2)+\ln\left(\frac{hr^2+\sqrt{-\alpha^2}\sqrt{-2kr^4+2hr^2-\alpha^2-\alpha^2}}{r^2}\right)}{2\sqrt{-\alpha^2}}$ | 57   |

input `int(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r,method=_RETURNVERBOSE)`

output `-1/2/(-alpha^2)^(1/2)*ln((-2*alpha^2+2*h*r^2+2*(-alpha^2)^(1/2)*(-2*k*r^4+  
2*h*r^2-alpha^2)^(1/2))/r^2)`

### 3.213.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int \frac{1}{r\sqrt{-\alpha^2 + 2hr^2 - 2kr^4}} dr = -\frac{\arctan\left(\frac{\sqrt{-2kr^4+2hr^2-\alpha^2}(hr^2-\alpha^2)}{2\alpha kr^4-2\alpha hr^2+\alpha^3}\right)}{2\alpha}$$

input `integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r, algorithm="fracas")`

output `-1/2*arctan(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2)*(h*r^2 - alpha^2)/(2*alpha*  
k*r^4 - 2*alpha*h*r^2 + alpha^3))/alpha`

**3.213.6 Sympy [F]**

$$\int \frac{1}{r\sqrt{-\alpha^2 + 2hr^2 - 2kr^4}} dr = \int \frac{1}{r\sqrt{-\alpha^2 + 2hr^2 - 2kr^4}} dr$$

input `integrate(1/r/(-2*k*r**4+2*h*r**2-alpha**2)**(1/2),r)`

output `Integral(1/(r*sqrt(-alpha**2 + 2*h*r**2 - 2*k*r**4)), r)`

**3.213.7 Maxima [F(-2)]**

Exception generated.

$$\int \frac{1}{r\sqrt{-\alpha^2 + 2hr^2 - 2kr^4}} dr = \text{Exception raised: ValueError}$$

input `integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(2*alpha^2*k-h^2>0)', see `assume ?` for mor`

**3.213.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{1}{r\sqrt{-\alpha^2 + 2hr^2 - 2kr^4}} dr = \frac{\arctan\left(-\frac{\sqrt{2}\sqrt{-kr^2 - \sqrt{-2kr^4 + 2hr^2 - \alpha^2}}}{\alpha}\right)}{\alpha}$$

input `integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r, algorithm="giac")`

output `arctan(-(sqrt(2)*sqrt(-k)*r^2 - sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2))/alpha)/alpha`

**3.213.9 Mupad [B] (verification not implemented)**

Time = 0.40 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \frac{1}{r\sqrt{-\alpha^2 + 2hr^2 - 2kr^4}} dr = -\frac{\ln\left(\frac{1}{r^2}\right) + \ln\left(hr^2 - \alpha^2 + \sqrt{-\alpha^2}\sqrt{-\alpha^2 - 2kr^4 + 2hr^2}\right)}{2\sqrt{-\alpha^2}}$$

input `int(1/(r*(2*h*r^2 - 2*k*r^4 - alpha^2)^(1/2)),r)`output `-(log(1/r^2) + log(h*r^2 - alpha^2 + (-alpha^2)^(1/2)*(2*h*r^2 - 2*k*r^4 - alpha^2)^(1/2)))/(2*(-alpha^2)^(1/2))`

**3.214**  $\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} dr$

3.214.1 Optimal result . . . . . 1167  
 3.214.2 Mathematica [A] (verified) . . . . . 1167  
 3.214.3 Rubi [A] (verified) . . . . . 1168  
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 3.214.5 Fricas [A] (verification not implemented) . . . . . 1169  
 3.214.6 Sympy [F] . . . . . 1170  
 3.214.7 Maxima [F(-2)] . . . . . 1170  
 3.214.8 Giac [A] (verification not implemented) . . . . . 1170  
 3.214.9 Mupad [B] (verification not implemented) . . . . . 1171

**3.214.1 Optimal result**

Integrand size = 31, antiderivative size = 68

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} dr = -\frac{\arctan\left(\frac{\alpha^2 + \epsilon^2 - hr^2}{\sqrt{\alpha^2 + \epsilon^2}\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}}\right)}{2\sqrt{\alpha^2 + \epsilon^2}}$$

output `-1/2*arctan((-h*r^2+alpha^2+epsilon^2)/(alpha^2+epsilon^2)^(1/2)/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2))/(alpha^2+epsilon^2)^(1/2)`

**3.214.2 Mathematica [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} dr = -\frac{\arctan\left(\frac{\sqrt{2}\sqrt{-kr^2 - \sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}}}{\sqrt{\alpha^2 + \epsilon^2}}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

input `Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4]),r]`

output `-(ArcTan[(Sqrt[2]*Sqrt[-k]*r^2 - Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4])/Sqrt[alpha^2 + epsilon^2]]/Sqrt[alpha^2 + epsilon^2])`



### 3.214.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {1434, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} dr \\
 & \quad \downarrow \text{1434} \\
 & \frac{1}{2} \int \frac{1}{r^2\sqrt{-2kr^4 + 2hr^2 - \alpha^2 - \epsilon^2}} dr^2 \\
 & \quad \downarrow \text{1154} \\
 & - \int \frac{1}{-r^4 - 4(\alpha^2 + \epsilon^2)} d\left(-\frac{2(\alpha^2 + \epsilon^2 - hr^2)}{\sqrt{-2kr^4 + 2hr^2 - \alpha^2 - \epsilon^2}}\right) \\
 & \quad \downarrow \text{217} \\
 & -\frac{\arctan\left(\frac{\alpha^2 + \epsilon^2 - hr^2}{\sqrt{\alpha^2 + \epsilon^2}\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}}\right)}{2\sqrt{\alpha^2 + \epsilon^2}}
 \end{aligned}$$

input `Int[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4]),r]`

output `-1/2*ArcTan[(alpha^2 + epsilon^2 - h*r^2)/(Sqrt[alpha^2 + epsilon^2]*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4])/Sqrt[alpha^2 + epsilon^2]`

#### 3.214.3.1 Defintions of rubi rules used

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & (LtQ[a, 0] || LtQ[b, 0])`

rule 1154 `Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Simp[-2 Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]`

rule 1434 `Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp  
[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free  
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]`

### 3.214.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.15

| method         | result                                                                                                                                                                 | size |
|----------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default        | $-\frac{\ln\left(\frac{-2\alpha^2-2\epsilon^2+2hr^2+2\sqrt{-\alpha^2-\epsilon^2}\sqrt{-2kr^4+2hr^2-\alpha^2-\epsilon^2}}{r^2}\right)}{2\sqrt{-\alpha^2-\epsilon^2}}$   | 78   |
| elliptic       | $-\frac{\ln\left(\frac{-2\alpha^2-2\epsilon^2+2hr^2+2\sqrt{-\alpha^2-\epsilon^2}\sqrt{-2kr^4+2hr^2-\alpha^2-\epsilon^2}}{r^2}\right)}{2\sqrt{-\alpha^2-\epsilon^2}}$   | 78   |
| pseudoelliptic | $-\frac{\ln(2)+\ln\left(\frac{hr^2+\sqrt{-\alpha^2-\epsilon^2}\sqrt{-2kr^4+2hr^2-\alpha^2-\epsilon^2}-\alpha^2-\epsilon^2}{r^2}\right)}{2\sqrt{-\alpha^2-\epsilon^2}}$ | 79   |

input `int(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r,method=_RETURNVERBOSE)`

output `-1/2/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2+2*h*r^2+2*(-alpha^2-epsilon^2)^(1/2)*(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2))/r^2)`

### 3.214.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.56

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} dr = -\frac{\arctan\left(\frac{\sqrt{-2kr^4+2hr^2-\alpha^2-\epsilon^2}(hr^2-\alpha^2-\epsilon^2)\sqrt{\alpha^2+\epsilon^2}}{2(\alpha^2+\epsilon^2)kr^4+\alpha^4+2\alpha^2\epsilon^2+\epsilon^4-2(\alpha^2+\epsilon^2)hr^2}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

input `integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fricas")`

output `-1/2*arctan(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2 - epsilon^2)*(h*r^2 - alpha^2 - epsilon^2)*sqrt(alpha^2 + epsilon^2)/(2*(alpha^2 + epsilon^2)*k*r^4 + alpha^4 + 2*alpha^2*epsilon^2 + epsilon^4 - 2*(alpha^2 + epsilon^2)*h*r^2))/sqrt(alpha^2 + epsilon^2)`

---

3.214.  $\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}} dr$

### 3.214.6 Sympy [F]

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} dr = \int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} dr$$

input `integrate(1/r/(-2*k*r**4+2*h*r**2-alpha**2-epsilon**2)**(1/2),r)`

output `Integral(1/(r*sqrt(-alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r**4)), r)`

### 3.214.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} dr = \text{Exception raised: ValueError}$$

input `integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(2*epsilon^2*k+2*alpha^2*k>0)', see `assume`

### 3.214.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} dr = \frac{\arctan\left(\frac{-\sqrt{2}\sqrt{-kr^2 - \sqrt{-2kr^4 + 2hr^2 - \alpha^2 - \epsilon^2}}}{\sqrt{\alpha^2 + \epsilon^2}}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

input `integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="giac")`

output `arctan(-(sqrt(2)*sqrt(-k)*r^2 - sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2 - epsilon^2))/sqrt(alpha^2 + epsilon^2))/sqrt(alpha^2 + epsilon^2)`

**3.214.9 Mupad [B] (verification not implemented)**

Time = 0.45 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr^4}} dr = -\frac{\ln\left(h - \frac{\alpha^2 + \epsilon^2}{r^2} + \frac{\sqrt{-\alpha^2 - \epsilon^2}\sqrt{-\alpha^2 - \epsilon^2 - 2kr^4 + 2hr^2}}{r^2}\right)}{2\sqrt{-\alpha^2 - \epsilon^2}}$$

input `int(1/(r*(2*h*r^2 - 2*k*r^4 - alpha^2 - epsilon^2)^(1/2)),r)`output `-log(h - (alpha^2 + epsilon^2)/r^2 + ((- alpha^2 - epsilon^2)^(1/2)*(2*h*r^2 - 2*k*r^4 - alpha^2 - epsilon^2)^(1/2))/r^2)/(2*(- alpha^2 - epsilon^2)^(1/2))`

### 3.215 $\int a \cos(5 + 3x) \sin^2(5 + 3x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.215.1 Optimal result . . . . .                            | 1172 |
| 3.215.2 Mathematica [A] (verified) . . . . .                | 1172 |
| 3.215.3 Rubi [A] (verified) . . . . .                       | 1173 |
| 3.215.4 Maple [A] (verified) . . . . .                      | 1174 |
| 3.215.5 Fricas [A] (verification not implemented) . . . . . | 1174 |
| 3.215.6 Sympy [A] (verification not implemented) . . . . .  | 1175 |
| 3.215.7 Maxima [A] (verification not implemented) . . . . . | 1175 |
| 3.215.8 Giac [A] (verification not implemented) . . . . .   | 1175 |
| 3.215.9 Mupad [B] (verification not implemented) . . . . .  | 1176 |

#### 3.215.1 Optimal result

Integrand size = 16, antiderivative size = 13

$$\int a \cos(5 + 3x) \sin^2(5 + 3x) dx = \frac{1}{9} a \sin^3(5 + 3x)$$

output `1/9*a*sin(5+3*x)^3`

#### 3.215.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int a \cos(5 + 3x) \sin^2(5 + 3x) dx = \frac{1}{9} a \sin^3(5 + 3x)$$

input `Integrate[a*Cos[5 + 3*x]*Sin[5 + 3*x]^2,x]`

output `(a*Sin[5 + 3*x]^3)/9`

### 3.215.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {27, 3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int a \sin^2(3x + 5) \cos(3x + 5) dx \\
 & \quad \downarrow \text{27} \\
 & a \int \cos(3x + 5) \sin^2(3x + 5) dx \\
 & \quad \downarrow \text{3042} \\
 & a \int \cos(3x + 5) \sin(3x + 5)^2 dx \\
 & \quad \downarrow \text{3044} \\
 & \frac{1}{3} a \int \sin^2(3x + 5) d \sin(3x + 5) \\
 & \quad \downarrow \text{15} \\
 & \frac{1}{9} a \sin^3(3x + 5)
 \end{aligned}$$

input `Int[a*Cos[5 + 3*x]*Sin[5 + 3*x]^2,x]`

output `(a*Ssin[5 + 3*x]^3)/9`

#### 3.215.3.1 Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

```
rule 3042 Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

```
rule 3044 Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] := Simp[1/(a*f) Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

### 3.215.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

| method            | result                                                                                     | size |
|-------------------|--------------------------------------------------------------------------------------------|------|
| derivativedivides | $\frac{a(\sin^3(5+3x))}{9}$                                                                | 12   |
| default           | $\frac{a(\sin^3(5+3x))}{9}$                                                                | 12   |
| risch             | $\frac{a \sin(5+3x)}{12} - \frac{a \sin(15+9x)}{36}$                                       | 20   |
| parallelrisch     | $a \left( -\frac{\sin(15+9x)}{36} + \frac{\sin(5+3x)}{12} \right)$                         | 20   |
| norman            | $\frac{8a(\tan^3(\frac{5}{2} + \frac{3x}{2}))}{9(1+\tan^2(\frac{5}{2} + \frac{3x}{2}))^3}$ | 24   |

```
input int(a*cos(5+3*x)*sin(5+3*x)^2,x,method=_RETURNVERBOSE)
```

```
output 1/9*a*sin(5+3*x)^3
```

### 3.215.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int a \cos(5 + 3x) \sin^2(5 + 3x) dx = -\frac{1}{9} (a \cos(3x + 5)^2 - a) \sin(3x + 5)$$

```
input integrate(a*cos(5+3*x)*sin(5+3*x)^2,x, algorithm="fricas")
```

```
output -1/9*(a*cos(3*x + 5)^2 - a)*sin(3*x + 5)
```

**3.215.6 Sympy [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int a \cos(5 + 3x) \sin^2(5 + 3x) dx = \frac{a \sin^3(3x + 5)}{9}$$

input `integrate(a*cos(5+3*x)*sin(5+3*x)**2,x)`output `a*sin(3*x + 5)**3/9`**3.215.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int a \cos(5 + 3x) \sin^2(5 + 3x) dx = \frac{1}{9} a \sin(3x + 5)^3$$

input `integrate(a*cos(5+3*x)*sin(5+3*x)^2,x, algorithm="maxima")`output `1/9*a*sin(3*x + 5)^3`**3.215.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int a \cos(5 + 3x) \sin^2(5 + 3x) dx = \frac{1}{9} a \sin(3x + 5)^3$$

input `integrate(a*cos(5+3*x)*sin(5+3*x)^2,x, algorithm="giac")`output `1/9*a*sin(3*x + 5)^3`



**3.215.9 Mupad [B] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int a \cos(5 + 3x) \sin^2(5 + 3x) dx = \frac{a \sin(3x + 5)^3}{9}$$

input `int(a*cos(3*x + 5)*sin(3*x + 5)^2,x)`

output `(a*sin(3*x + 5)^3)/9`

### 3.216 $\int \frac{\log(x^2)}{x^3} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.216.1 Optimal result . . . . .                            | 1177 |
| 3.216.2 Mathematica [A] (verified) . . . . .                | 1177 |
| 3.216.3 Rubi [A] (verified) . . . . .                       | 1178 |
| 3.216.4 Maple [A] (verified) . . . . .                      | 1178 |
| 3.216.5 Fricas [A] (verification not implemented) . . . . . | 1179 |
| 3.216.6 Sympy [A] (verification not implemented) . . . . .  | 1179 |
| 3.216.7 Maxima [A] (verification not implemented) . . . . . | 1179 |
| 3.216.8 Giac [A] (verification not implemented) . . . . .   | 1180 |
| 3.216.9 Mupad [B] (verification not implemented) . . . . .  | 1180 |

#### 3.216.1 Optimal result

Integrand size = 8, antiderivative size = 19

$$\int \frac{\log(x^2)}{x^3} dx = -\frac{1}{2x^2} - \frac{\log(x^2)}{2x^2}$$

output `-1/2/x^2-1/2*ln(x^2)/x^2`

#### 3.216.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\log(x^2)}{x^3} dx = -\frac{1}{2x^2} - \frac{\log(x^2)}{2x^2}$$

input `Integrate[Log[x^2]/x^3,x]`

output `-1/2*1/x^2 - Log[x^2]/(2*x^2)`

### 3.216.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x^2)}{x^3} dx$$

$$\downarrow \text{2741}$$

$$-\frac{1}{2x^2} - \frac{\log(x^2)}{2x^2}$$

input `Int [Log[x^2]/x^3,x]`

output `-1/2*1/x^2 - Log[x^2]/(2*x^2)`

#### 3.216.3.1 Defintions of rubi rules used

rule 2741 `Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol] :=  
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(  
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]`

### 3.216.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

| method        | result                                        | size |
|---------------|-----------------------------------------------|------|
| parallelrisch | $-\frac{1+\ln(x^2)}{2x^2}$                    | 12   |
| norman        | $-\frac{\frac{1}{2}-\frac{\ln(x^2)}{2}}{x^2}$ | 13   |
| default       | $-\frac{1}{2x^2} - \frac{\ln(x^2)}{2x^2}$     | 16   |
| risch         | $-\frac{1}{2x^2} - \frac{\ln(x^2)}{2x^2}$     | 16   |
| parts         | $-\frac{1}{2x^2} - \frac{\ln(x^2)}{2x^2}$     | 16   |

input `int(ln(x^2)/x^3,x,method=_RETURNVERBOSE)`

output `-1/2/x^2*(1+ln(x^2))`

### 3.216.5 Fracas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{\log(x^2)}{x^3} dx = -\frac{\log(x^2) + 1}{2x^2}$$

input `integrate(log(x^2)/x^3,x, algorithm="fricas")`

output `-1/2*(log(x^2) + 1)/x^2`

### 3.216.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\log(x^2)}{x^3} dx = -\frac{\log(x^2)}{2x^2} - \frac{1}{2x^2}$$

input `integrate(ln(x**2)/x**3,x)`

output `-log(x**2)/(2*x**2) - 1/(2*x**2)`

### 3.216.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\log(x^2)}{x^3} dx = -\frac{\log(x^2)}{2x^2} - \frac{1}{2x^2}$$

input `integrate(log(x^2)/x^3,x, algorithm="maxima")`

output `-1/2*log(x^2)/x^2 - 1/2/x^2`

**3.216.8 Giac [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\log(x^2)}{x^3} dx = -\frac{\log(x^2)}{2x^2} - \frac{1}{2x^2}$$

input `integrate(log(x^2)/x^3,x, algorithm="giac")`

output `-1/2*log(x^2)/x^2 - 1/2/x^2`

**3.216.9 Mupad [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{\log(x^2)}{x^3} dx = -\frac{\ln(x^2) + 1}{2x^2}$$

input `int(log(x^2)/x^3,x)`

output `-(log(x^2) + 1)/(2*x^2)`

## 3.217 $\int x \sin(a + x) dx$

|                                                             |       |
|-------------------------------------------------------------|-------|
| 3.217.1 Optimal result . . . . .                            | .1181 |
| 3.217.2 Mathematica [A] (verified) . . . . .                | .1181 |
| 3.217.3 Rubi [A] (verified) . . . . .                       | .1182 |
| 3.217.4 Maple [A] (verified) . . . . .                      | .1183 |
| 3.217.5 Fricas [A] (verification not implemented) . . . . . | .1183 |
| 3.217.6 Sympy [A] (verification not implemented) . . . . .  | .1184 |
| 3.217.7 Maxima [A] (verification not implemented) . . . . . | .1184 |
| 3.217.8 Giac [A] (verification not implemented) . . . . .   | .1184 |
| 3.217.9 Mupad [B] (verification not implemented) . . . . .  | .1185 |

### 3.217.1 Optimal result

Integrand size = 6, antiderivative size = 12

$$\int x \sin(a + x) dx = -x \cos(a + x) + \sin(a + x)$$

output `-x*cos(a+x)+sin(a+x)`

### 3.217.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x \sin(a + x) dx = -x \cos(a + x) + \sin(a + x)$$

input `Integrate[x*Sin[a + x],x]`

output `-(x*Cos[a + x]) + Sin[a + x]`

**3.217.3 Rubi [A] (verified)**

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c}
 \int x \sin(a+x) dx \\
 \downarrow \text{3042} \\
 \int x \sin(a+x) dx \\
 \downarrow \text{3777} \\
 \int \cos(a+x) dx - x \cos(a+x) \\
 \downarrow \text{3042} \\
 \int \sin\left(a+x+\frac{\pi}{2}\right) dx - x \cos(a+x) \\
 \downarrow \text{3117} \\
 \sin(a+x) - x \cos(a+x)
 \end{array}$$

input `Int[x*Sin[a + x],x]`

output `-(x*Cos[a + x]) + Sin[a + x]`

**3.217.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3117 `Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

```
rule 3777 Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-
(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### 3.217.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

| method            | result                                                                                                                                                                                                                        | size |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch             | $-x \cos(a + x) + \sin(a + x)$                                                                                                                                                                                                | 13   |
| parallelrisch     | $-x \cos(a + x) + \sin(a + x)$                                                                                                                                                                                                | 13   |
| parts             | $-x \cos(a + x) + \sin(a + x)$                                                                                                                                                                                                | 13   |
| derivativedivides | $a \cos(a + x) + \sin(a + x) - (a + x) \cos(a + x)$                                                                                                                                                                           | 21   |
| default           | $a \cos(a + x) + \sin(a + x) - (a + x) \cos(a + x)$                                                                                                                                                                           | 21   |
| norman            | $\frac{x(\tan^2(\frac{a}{2} + \frac{x}{2}) - x + 2 \tan(\frac{a}{2} + \frac{x}{2}))}{1 + \tan^2(\frac{a}{2} + \frac{x}{2})}$                                                                                                  | 42   |
| meijerg           | $2 \sin(a) \sqrt{\pi} \left( -\frac{1}{2\sqrt{\pi}} + \frac{\cos(x)}{2\sqrt{\pi}} + \frac{x \sin(x)}{2\sqrt{\pi}} \right) + 2 \cos(a) \sqrt{\pi} \left( -\frac{x \cos(x)}{2\sqrt{\pi}} + \frac{\sin(x)}{2\sqrt{\pi}} \right)$ | 53   |

```
input int(x*sin(a+x),x,method=_RETURNVERBOSE)
```

```
output -x*cos(a+x)+sin(a+x)
```

### 3.217.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x \sin(a + x) dx = -x \cos(a + x) + \sin(a + x)$$

```
input integrate(x*sin(a+x),x, algorithm="fricas")
```

```
output -x*cos(a + x) + sin(a + x)
```



**3.217.6 Sympy [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int x \sin(a + x) dx = -x \cos(a + x) + \sin(a + x)$$

input `integrate(x*sin(a+x),x)`

output `-x*cos(a + x) + sin(a + x)`

**3.217.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int x \sin(a + x) dx = -(a + x) \cos(a + x) + a \cos(a + x) + \sin(a + x)$$

input `integrate(x*sin(a+x),x, algorithm="maxima")`

output `-(a + x)*cos(a + x) + a*cos(a + x) + sin(a + x)`

**3.217.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x \sin(a + x) dx = -x \cos(a + x) + \sin(a + x)$$

input `integrate(x*sin(a+x),x, algorithm="giac")`

output `-x*cos(a + x) + sin(a + x)`

**3.217.9 Mupad [B] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x \sin(a + x) dx = \sin(a + x) - x \cos(a + x)$$

input `int(x*sin(a + x),x)`

output `sin(a + x) - x*cos(a + x)`

**3.218**       $\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)} dx$

3.218.1 Optimal result . . . . . 1186  
 3.218.2 Mathematica [A] (verified) . . . . . 1186  
 3.218.3 Rubi [A] (verified) . . . . . 1187  
 3.218.4 Maple [A] (verified) . . . . . 1187  
 3.218.5 Fricas [A] (verification not implemented) . . . . . 1188  
 3.218.6 Sympy [A] (verification not implemented) . . . . . 1188  
 3.218.7 Maxima [A] (verification not implemented) . . . . . 1188  
 3.218.8 Giac [A] (verification not implemented) . . . . . 1189  
 3.218.9 Mupad [B] (verification not implemented) . . . . . 1189

**3.218.1 Optimal result**

Integrand size = 20, antiderivative size = 11

$$\int \frac{e^{-x}(-1 + (1 - x)\log(x))}{\log^2(x)} dx = \frac{e^{-x}x}{\log(x)}$$

output `x/exp(x)/ln(x)`

**3.218.2 Mathematica [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{e^{-x}(-1 + (1 - x)\log(x))}{\log^2(x)} dx = \frac{e^{-x}x}{\log(x)}$$

input `Integrate[(-1 + (1 - x)*Log[x])/(E^x*Log[x]^2), x]`

output `x/(E^x*Log[x])`

### 3.218.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$ , Rules used = {2630}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-x}((1-x)\log(x) - 1)}{\log^2(x)} dx$$

↓ 2630

$$\frac{e^{-x}x}{\log(x)}$$

input `Int[(-1 + (1 - x)*Log[x])/(E^x*Log[x]^2),x]`

output `x/(E^x*Log[x])`

#### 3.218.3.1 Defintions of rubi rules used

rule 2630 `Int[Log[(d_.)*(x_)^(n_.)*(F_)^(v_.)*((e_) + Log[(d_.)*(x_)*(h_.)*((f_.) + (g_.)*(x_))], x_Symbol] := Simp[e*x*F^v*(Log[d*x]^(n + 1)/(n + 1)), x] /; FreeQ[{F, d, e, f, g, h, n}, x] && LinearQ[v, x] && EqQ[e, f*h*(n + 1)] && EqQ[g*h*(n + 1), D[v, x]*e*Log[F]] && NeQ[n, -1]`

### 3.218.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

| method        | result                    | size |
|---------------|---------------------------|------|
| norman        | $\frac{x e^{-x}}{\ln(x)}$ | 11   |
| risch         | $\frac{x e^{-x}}{\ln(x)}$ | 11   |
| parallelrisch | $\frac{x e^{-x}}{\ln(x)}$ | 11   |

input `int((-1+(1-x)*ln(x))/exp(x)/ln(x)^2,x,method=_RETURNVERBOSE)`

output `x/exp(x)/ln(x)`

### 3.218.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{e^{-x}(-1 + (1-x)\log(x))}{\log^2(x)} dx = \frac{xe^{-x}}{\log(x)}$$

input `integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x, algorithm="fricas")`

output `x*e^(-x)/log(x)`

### 3.218.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{e^{-x}(-1 + (1-x)\log(x))}{\log^2(x)} dx = \frac{xe^{-x}}{\log(x)}$$

input `integrate((-1+(1-x)*ln(x))/exp(x)/ln(x)**2,x)`

output `x*exp(-x)/log(x)`

### 3.218.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{e^{-x}(-1 + (1-x)\log(x))}{\log^2(x)} dx = \frac{xe^{-x}}{\log(x)}$$

input `integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x, algorithm="maxima")`

output `x*e^(-x)/log(x)`

**3.218.8 Giac [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{e^{-x}(-1 + (1-x)\log(x))}{\log^2(x)} dx = \frac{x e^{-x}}{\log(x)}$$

input `integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x, algorithm="giac")`output `x*e^(-x)/log(x)`**3.218.9 Mupad [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{e^{-x}(-1 + (1-x)\log(x))}{\log^2(x)} dx = \frac{x e^{-x}}{\ln(x)}$$

input `int(-(exp(-x)*(log(x)*(x - 1) + 1))/log(x)^2,x)`output `(x*exp(-x))/log(x)`

### 3.219 $\int \frac{x^3}{b+ax^2} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.219.1 Optimal result . . . . .                            | 1190 |
| 3.219.2 Mathematica [A] (verified) . . . . .                | 1190 |
| 3.219.3 Rubi [A] (verified) . . . . .                       | 1191 |
| 3.219.4 Maple [A] (verified) . . . . .                      | 1192 |
| 3.219.5 Fricas [A] (verification not implemented) . . . . . | 1192 |
| 3.219.6 Sympy [A] (verification not implemented) . . . . .  | 1192 |
| 3.219.7 Maxima [A] (verification not implemented) . . . . . | 1193 |
| 3.219.8 Giac [A] (verification not implemented) . . . . .   | 1193 |
| 3.219.9 Mupad [B] (verification not implemented) . . . . .  | 1193 |

#### 3.219.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{x^3}{b+ax^2} dx = \frac{x^2}{2a} - \frac{b \log(b+ax^2)}{2a^2}$$

output `1/2*x^2/a-1/2*b*ln(a*x^2+b)/a^2`

#### 3.219.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{b+ax^2} dx = \frac{x^2}{2a} - \frac{b \log(b+ax^2)}{2a^2}$$

input `Integrate[x^3/(b + a*x^2),x]`

output `x^2/(2*a) - (b*Log[b + a*x^2])/(2*a^2)`

**3.219.3 Rubi [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{x^3}{ax^2 + b} dx \\ & \quad \downarrow \text{243} \\ & \frac{1}{2} \int \frac{x^2}{ax^2 + b} dx^2 \\ & \quad \downarrow \text{49} \\ & \frac{1}{2} \int \left( \frac{1}{a} - \frac{b}{a(ax^2 + b)} \right) dx^2 \\ & \quad \downarrow \text{2009} \\ & \frac{1}{2} \left( \frac{x^2}{a} - \frac{b \log(ax^2 + b)}{a^2} \right) \end{aligned}$$

input `Int[x^3/(b + a*x^2),x]`

output `(x^2/a - (b*Log[b + a*x^2])/a^2)/2`

**3.219.3.1 Defintions of rubi rules used**

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 243 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && IntegerQ[(m - 1)/2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`



**3.219.4 Maple [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

| method       | result                                       | size |
|--------------|----------------------------------------------|------|
| parallelrisc | $-\frac{-ax^2+b\ln(ax^2+b)}{2a^2}$           | 23   |
| default      | $\frac{x^2}{2a} - \frac{b\ln(ax^2+b)}{2a^2}$ | 24   |
| norman       | $\frac{x^2}{2a} - \frac{b\ln(ax^2+b)}{2a^2}$ | 24   |
| risc         | $\frac{x^2}{2a} - \frac{b\ln(ax^2+b)}{2a^2}$ | 24   |

input `int(x^3/(a*x^2+b),x,method=_RETURNVERBOSE)`output  $-1/2*(-a*x^2+b*\ln(a*x^2+b))/a^2$ **3.219.5 Fracas [A] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{b+ax^2} dx = \frac{ax^2 - b \log(ax^2 + b)}{2a^2}$$

input `integrate(x^3/(a*x^2+b),x, algorithm="fricas")`output  $1/2*(a*x^2 - b*\log(a*x^2 + b))/a^2$ **3.219.6 Sympy [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{b+ax^2} dx = \frac{x^2}{2a} - \frac{b \log(ax^2 + b)}{2a^2}$$

input `integrate(x**3/(a*x**2+b),x)`output  $x**2/(2*a) - b*\log(a*x**2 + b)/(2*a**2)$

**3.219.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{b + ax^2} dx = \frac{x^2}{2a} - \frac{b \log(ax^2 + b)}{2a^2}$$

input `integrate(x^3/(a*x^2+b),x, algorithm="maxima")`output `1/2*x^2/a - 1/2*b*log(a*x^2 + b)/a^2`**3.219.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{b + ax^2} dx = \frac{x^2}{2a} - \frac{b \log(|ax^2 + b|)}{2a^2}$$

input `integrate(x^3/(a*x^2+b),x, algorithm="giac")`output `1/2*x^2/a - 1/2*b*log(abs(a*x^2 + b))/a^2`**3.219.9 Mupad [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{b + ax^2} dx = -\frac{b \ln(ax^2 + b) - ax^2}{2a^2}$$

input `int(x^3/(b + a*x^2),x)`output `-(b*log(b + a*x^2) - a*x^2)/(2*a^2)`

$$3.220 \quad \int \frac{\sqrt{x}}{(1+x)^{7/2}} dx$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.220.1 Optimal result . . . . .                            | 1194 |
| 3.220.2 Mathematica [A] (verified) . . . . .                | 1194 |
| 3.220.3 Rubi [A] (verified) . . . . .                       | 1195 |
| 3.220.4 Maple [A] (verified) . . . . .                      | 1196 |
| 3.220.5 Fricas [B] (verification not implemented) . . . . . | 1196 |
| 3.220.6 Sympy [C] (verification not implemented) . . . . .  | 1197 |
| 3.220.7 Maxima [A] (verification not implemented) . . . . . | 1197 |
| 3.220.8 Giac [B] (verification not implemented) . . . . .   | 1198 |
| 3.220.9 Mupad [B] (verification not implemented) . . . . .  | 1198 |

### 3.220.1 Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{\sqrt{x}}{(1+x)^{7/2}} dx = \frac{2x^{3/2}}{5(1+x)^{5/2}} + \frac{4x^{3/2}}{15(1+x)^{3/2}}$$

output `2/5*x^(3/2)/(1+x)^(5/2)+4/15*x^(3/2)/(1+x)^(3/2)`

### 3.220.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{x}}{(1+x)^{7/2}} dx = \frac{2x^{3/2}(5+2x)}{15(1+x)^{5/2}}$$

input `Integrate[Sqrt[x]/(1+x)^(7/2),x]`

output `(2*x^(3/2)*(5+2*x))/(15*(1+x)^(5/2))`

### 3.220.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(x+1)^{7/2}} dx$$

$$\downarrow 55$$

$$\frac{2}{5} \int \frac{\sqrt{x}}{(x+1)^{5/2}} dx + \frac{2x^{3/2}}{5(x+1)^{5/2}}$$

$$\downarrow 48$$

$$\frac{4x^{3/2}}{15(x+1)^{3/2}} + \frac{2x^{3/2}}{5(x+1)^{5/2}}$$

input `Int[Sqrt[x]/(1 + x)^(7/2), x]`

output `(2*x^(3/2))/(5*(1 + x)^(5/2)) + (4*x^(3/2))/(15*(1 + x)^(3/2))`

#### 3.220.3.1 Defintions of rubi rules used

rule 48 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]`

rule 55 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(Simplify[m + n + 2]/((b*c - a*d)*(m + 1))) Int[(a + b*x)^Simplify[m + 1]*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])`

**3.220.4 Maple [A] (verified)**

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.48

| method  | result                                                                                                               | size |
|---------|----------------------------------------------------------------------------------------------------------------------|------|
| gospers | $\frac{2x^{\frac{3}{2}}(5+2x)}{15(1+x)^{\frac{5}{2}}}$                                                               | 16   |
| meijerg | $\frac{2x^{\frac{3}{2}}(5+2x)}{15(1+x)^{\frac{5}{2}}}$                                                               | 16   |
| risch   | $\frac{2x^{\frac{3}{2}}(5+2x)}{15(1+x)^{\frac{5}{2}}}$                                                               | 16   |
| default | $-\frac{2\sqrt{x}}{5(1+x)^{\frac{5}{2}}} + \frac{2\sqrt{x}}{15(1+x)^{\frac{3}{2}}} + \frac{4\sqrt{x}}{15\sqrt{1+x}}$ | 32   |

input `int(x^(1/2)/(1+x)^(7/2),x,method=_RETURNVERBOSE)`

output `2/15*x^(3/2)*(5+2*x)/(1+x)^(5/2)`

**3.220.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 50 vs.  $2(21) = 42$ .

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{x}}{(1+x)^{7/2}} dx = \frac{2(2x^3 + (2x^2 + 5x)\sqrt{x+1}\sqrt{x} + 6x^2 + 6x + 2)}{15(x^3 + 3x^2 + 3x + 1)}$$

input `integrate(x^(1/2)/(1+x)^(7/2),x, algorithm="fracas")`

output `2/15*(2*x^3 + (2*x^2 + 5*x)*sqrt(x + 1)*sqrt(x) + 6*x^2 + 6*x + 2)/(x^3 + 3*x^2 + 3*x + 1)`

**3.220.6 Sympy [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 167, normalized size of antiderivative = 5.06

$$\int \frac{\sqrt{x}}{(1+x)^{7/2}} dx = \begin{cases} \frac{4i\sqrt{-1+\frac{1}{x+1}}(x+1)^2}{-15x+15(x+1)^2-15} - \frac{2i\sqrt{-1+\frac{1}{x+1}}(x+1)}{-15x+15(x+1)^2-15} - \frac{8i\sqrt{-1+\frac{1}{x+1}}}{-15x+15(x+1)^2-15} + \frac{6i\sqrt{-1+\frac{1}{x+1}}}{(x+1)(-15x+15(x+1)^2-15)} & \text{for } \frac{1}{|x+1|} > 1 \\ \frac{4\sqrt{1-\frac{1}{x+1}}}{15} + \frac{2\sqrt{1-\frac{1}{x+1}}}{15(x+1)} - \frac{2\sqrt{1-\frac{1}{x+1}}}{5(x+1)^2} & \text{other} \end{cases}$$

input `integrate(x**(1/2)/(1+x)**(7/2), x)`

output `Piecewise((4*I*sqrt(-1 + 1/(x + 1))*(x + 1)**2/(-15*x + 15*(x + 1)**2 - 15) - 2*I*sqrt(-1 + 1/(x + 1))*(x + 1)/(-15*x + 15*(x + 1)**2 - 15) - 8*I*sqrt(-1 + 1/(x + 1))/(-15*x + 15*(x + 1)**2 - 15) + 6*I*sqrt(-1 + 1/(x + 1))/((x + 1)*(-15*x + 15*(x + 1)**2 - 15)), 1/Abs(x + 1) > 1), (4*sqrt(1 - 1/(x + 1))/15 + 2*sqrt(1 - 1/(x + 1))/(15*(x + 1)) - 2*sqrt(1 - 1/(x + 1))/(5*(x + 1)**2), True))`

**3.220.7 Maxima [A] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{x}}{(1+x)^{7/2}} dx = \frac{2x^{\frac{5}{2}} \left( \frac{5(x+1)}{x} - 3 \right)}{15(x+1)^{\frac{5}{2}}}$$

input `integrate(x^(1/2)/(1+x)^(7/2), x, algorithm="maxima")`

output `2/15*x^(5/2)*(5*(x + 1)/x - 3)/(x + 1)^(5/2)`

**3.220.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 66 vs.  $2(21) = 42$ .

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{x}}{(1+x)^{7/2}} dx = \frac{8 \left( 15 (\sqrt{x+1} - \sqrt{x})^6 - 5 (\sqrt{x+1} - \sqrt{x})^4 + 5 (\sqrt{x+1} - \sqrt{x})^2 + 1 \right)}{15 \left( (\sqrt{x+1} - \sqrt{x})^2 + 1 \right)^5}$$

input `integrate(x^(1/2)/(1+x)^(7/2),x, algorithm="giac")`

output `8/15*(15*(sqrt(x + 1) - sqrt(x))^6 - 5*(sqrt(x + 1) - sqrt(x))^4 + 5*(sqrt(x + 1) - sqrt(x))^2 + 1)/((sqrt(x + 1) - sqrt(x))^2 + 1)^5`

**3.220.9 Mupad [B] (verification not implemented)**

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{x}}{(1+x)^{7/2}} dx = \frac{2x^{3/2}(2x+5)}{15(x+1)^{5/2}}$$

input `int(x^(1/2)/(x + 1)^(7/2),x)`

output `(2*x^(3/2)*(2*x + 5))/(15*(x + 1)^(5/2))`

## 3.221 $\int \frac{1}{x(1+x)} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.221.1 Optimal result . . . . .                            | 1199 |
| 3.221.2 Mathematica [A] (verified) . . . . .                | 1199 |
| 3.221.3 Rubi [A] (verified) . . . . .                       | 1200 |
| 3.221.4 Maple [A] (verified) . . . . .                      | 1201 |
| 3.221.5 Fricas [A] (verification not implemented) . . . . . | 1201 |
| 3.221.6 Sympy [A] (verification not implemented) . . . . .  | 1201 |
| 3.221.7 Maxima [A] (verification not implemented) . . . . . | 1202 |
| 3.221.8 Giac [A] (verification not implemented) . . . . .   | 1202 |
| 3.221.9 Mupad [B] (verification not implemented) . . . . .  | 1202 |

### 3.221.1 Optimal result

Integrand size = 9, antiderivative size = 9

$$\int \frac{1}{x(1+x)} dx = \log(x) - \log(1+x)$$

output `ln(x)-ln(1+x)`

### 3.221.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x)} dx = \log(x) - \log(1+x)$$

input `Integrate[1/(x*(1 + x)),x]`

output `Log[x] - Log[1 + x]`



**3.221.3 Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{x(x+1)} dx \\ & \quad \downarrow 47 \\ & \int \frac{1}{x} dx - \int \frac{1}{x+1} dx \\ & \quad \downarrow 14 \\ & \log(x) - \int \frac{1}{x+1} dx \\ & \quad \downarrow 16 \\ & \log(x) - \log(x+1) \end{aligned}$$

input `Int[1/(x*(1 + x)),x]`

output `Log[x] - Log[1 + x]`

**3.221.3.1 Defintions of rubi rules used**

rule 14 `Int[(a_.)/(x_), x_Symbol] := Simp[a*Log[x], x] /; FreeQ[a, x]`

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 47 `Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Simp[b/(b*c - a*d) Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d) Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x]`

**3.221.4 Maple [A] (verified)**

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

| method        | result              | size |
|---------------|---------------------|------|
| default       | $\ln(x) - \ln(1+x)$ | 10   |
| norman        | $\ln(x) - \ln(1+x)$ | 10   |
| meijerg       | $\ln(x) - \ln(1+x)$ | 10   |
| risch         | $\ln(x) - \ln(1+x)$ | 10   |
| parallelrisch | $\ln(x) - \ln(1+x)$ | 10   |

input `int(1/x/(1+x),x,method=_RETURNVERBOSE)`output `ln(x)-ln(1+x)`**3.221.5 Fracas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x)} dx = -\log(x+1) + \log(x)$$

input `integrate(1/x/(1+x),x, algorithm="fricas")`output `-log(x + 1) + log(x)`**3.221.6 Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{x(1+x)} dx = \log(x) - \log(x+1)$$

input `integrate(1/x/(1+x),x)`output `log(x) - log(x + 1)`

**3.221.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+x)} dx = -\log(x+1) + \log(x)$$

input `integrate(1/x/(1+x),x, algorithm="maxima")`output  `-log(x + 1) + log(x)`**3.221.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{1}{x(1+x)} dx = -\log(|x+1|) + \log(|x|)$$

input `integrate(1/x/(1+x),x, algorithm="giac")`output  `-log(abs(x + 1)) + log(abs(x))`**3.221.9 Mupad [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(1+x)} dx = -\ln\left(\frac{1}{x} + 1\right)$$

input `int(1/(x*(x + 1)),x)`output  `-log(1/x + 1)`

$$3.222 \quad \int \frac{1}{\sqrt{x}(-1+2x)} dx$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.222.1 Optimal result . . . . .                            | 1203 |
| 3.222.2 Mathematica [A] (verified) . . . . .                | 1203 |
| 3.222.3 Rubi [A] (verified) . . . . .                       | 1204 |
| 3.222.4 Maple [A] (verified) . . . . .                      | 1205 |
| 3.222.5 Fricas [B] (verification not implemented) . . . . . | 1205 |
| 3.222.6 Sympy [B] (verification not implemented) . . . . .  | 1205 |
| 3.222.7 Maxima [B] (verification not implemented) . . . . . | 1206 |
| 3.222.8 Giac [B] (verification not implemented) . . . . .   | 1206 |
| 3.222.9 Mupad [B] (verification not implemented) . . . . .  | 1207 |

### 3.222.1 Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{\sqrt{x}(-1+2x)} dx = -\sqrt{2}\operatorname{arctanh}(\sqrt{2}\sqrt{x})$$

output `-arctanh(2^(1/2)*x^(1/2))*2^(1/2)`

### 3.222.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}(-1+2x)} dx = -\sqrt{2}\operatorname{arctanh}(\sqrt{2}\sqrt{x})$$

input `Integrate[1/(Sqrt[x]*(-1 + 2*x)),x]`

output `-(Sqrt[2]*ArcTanh[Sqrt[2]*Sqrt[x]])`

**3.222.3 Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}(2x-1)} dx$$

$$\downarrow 73$$

$$2 \int \frac{1}{2x-1} d\sqrt{x}$$

$$\downarrow 220$$

$$-\sqrt{2} \operatorname{arctanh}(\sqrt{2}\sqrt{x})$$

input `Int [1/(Sqrt [x]*(-1 + 2*x)),x]`

output `-(Sqrt [2]*ArcTanh [Sqrt [2]*Sqrt [x]])`

**3.222.3.1 Defintions of rubi rules used**

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 220 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

**3.222.4 Maple [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

| method           | result                                                                                                                                         | size |
|------------------|------------------------------------------------------------------------------------------------------------------------------------------------|------|
| derivativdivides | $-\operatorname{arctanh}(\sqrt{2}\sqrt{x})\sqrt{2}$                                                                                            | 14   |
| default          | $-\operatorname{arctanh}(\sqrt{2}\sqrt{x})\sqrt{2}$                                                                                            | 14   |
| meijerg          | $-\operatorname{arctanh}(\sqrt{2}\sqrt{x})\sqrt{2}$                                                                                            | 14   |
| trager           | $\frac{\operatorname{RootOf}(-Z^2-2)\ln\left(-\frac{2\operatorname{RootOf}(-Z^2-2)x+\operatorname{RootOf}(-Z^2-2)-4\sqrt{x}}{2x-1}\right)}{2}$ | 40   |

input `int(1/x^(1/2)/(2*x-1),x,method=_RETURNVERBOSE)`

output `-arctanh(2^(1/2)*x^(1/2))*2^(1/2)`

**3.222.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(13) = 26.

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{x}(-1+2x)} dx = \frac{1}{2} \sqrt{2} \log\left(-\frac{2\sqrt{2}\sqrt{x}-2x-1}{2x-1}\right)$$

input `integrate(1/x^(1/2)/(-1+2*x),x, algorithm="fracas")`

output `1/2*sqrt(2)*log(-(2*sqrt(2)*sqrt(x) - 2*x - 1)/(2*x - 1))`

**3.222.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{x}(-1+2x)} dx = \frac{\sqrt{2} \log\left(\sqrt{x} - \frac{\sqrt{2}}{2}\right)}{2} - \frac{\sqrt{2} \log\left(\sqrt{x} + \frac{\sqrt{2}}{2}\right)}{2}$$

input `integrate(1/x**(1/2)/(-1+2*x),x)`

output `sqrt(2)*log(sqrt(x) - sqrt(2)/2)/2 - sqrt(2)*log(sqrt(x) + sqrt(2)/2)/2`

### 3.222.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs.  $2(13) = 26$ .

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{x}(-1+2x)} dx = \frac{1}{2} \sqrt{2} \log \left( -\frac{\sqrt{2} - 2\sqrt{x}}{\sqrt{2} + 2\sqrt{x}} \right)$$

input `integrate(1/x^(1/2)/(-1+2*x),x, algorithm="maxima")`

output `1/2*sqrt(2)*log(-(sqrt(2) - 2*sqrt(x))/(sqrt(2) + 2*sqrt(x)))`

### 3.222.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs.  $2(13) = 26$ .

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int \frac{1}{\sqrt{x}(-1+2x)} dx = -\frac{1}{2} \sqrt{2} \log \left( \frac{1}{2} \sqrt{2} + \sqrt{x} \right) + \frac{1}{2} \sqrt{2} \log \left( \left| -\frac{1}{2} \sqrt{2} + \sqrt{x} \right| \right)$$

input `integrate(1/x^(1/2)/(-1+2*x),x, algorithm="giac")`

output `-1/2*sqrt(2)*log(1/2*sqrt(2) + sqrt(x)) + 1/2*sqrt(2)*log(abs(-1/2*sqrt(2) + sqrt(x)))`

**3.222.9 Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{x}(-1+2x)} dx = -\sqrt{2} \operatorname{atanh}(\sqrt{2x})$$

input `int(1/(x^(1/2)*(2*x - 1)),x)`

output `-2^(1/2)*atanh((2*x)^(1/2))`



### 3.223 $\int \sqrt{x}(1+x^2) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.223.1 Optimal result . . . . .                            | 1208 |
| 3.223.2 Mathematica [A] (verified) . . . . .                | 1208 |
| 3.223.3 Rubi [A] (verified) . . . . .                       | 1209 |
| 3.223.4 Maple [A] (verified) . . . . .                      | 1210 |
| 3.223.5 Fricas [A] (verification not implemented) . . . . . | 1210 |
| 3.223.6 Sympy [A] (verification not implemented) . . . . .  | 1210 |
| 3.223.7 Maxima [A] (verification not implemented) . . . . . | 1211 |
| 3.223.8 Giac [A] (verification not implemented) . . . . .   | 1211 |
| 3.223.9 Mupad [B] (verification not implemented) . . . . .  | 1211 |

#### 3.223.1 Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \sqrt{x}(1+x^2) dx = \frac{2x^{3/2}}{3} + \frac{2x^{7/2}}{7}$$

output `2/3*x^(3/2)+2/7*x^(7/2)`

#### 3.223.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \sqrt{x}(1+x^2) dx = \frac{2}{21}x^{3/2}(7+3x^2)$$

input `Integrate[Sqrt[x]*(1+x^2),x]`

output `(2*x^(3/2)*(7+3*x^2))/21`

**3.223.3 Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(x^2 + 1) dx$$

$$\downarrow \text{244}$$

$$\int (x^{5/2} + \sqrt{x}) dx$$

$$\downarrow \text{2009}$$

$$\frac{2x^{7/2}}{7} + \frac{2x^{3/2}}{3}$$

input `Int[Sqrt[x]*(1 + x^2),x]`

output `(2*x^(3/2))/3 + (2*x^(7/2))/7`

**3.223.3.1 Defintions of rubi rules used**

rule 244 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p, 0]`

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

**3.223.4 Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

| method           | result                                                    | size |
|------------------|-----------------------------------------------------------|------|
| derivativdivides | $\frac{2x^{\frac{3}{2}}}{3} + \frac{2x^{\frac{7}{2}}}{7}$ | 12   |
| default          | $\frac{2x^{\frac{3}{2}}}{3} + \frac{2x^{\frac{7}{2}}}{7}$ | 12   |
| gosper           | $\frac{2x^{\frac{3}{2}}(3x^2+7)}{21}$                     | 13   |
| trager           | $\frac{2x^{\frac{3}{2}}(3x^2+7)}{21}$                     | 13   |
| risch            | $\frac{2x^{\frac{3}{2}}(3x^2+7)}{21}$                     | 13   |

input `int(x^(1/2)*(x^2+1),x,method=_RETURNVERBOSE)`output `2/3*x^(3/2)+2/7*x^(7/2)`**3.223.5 Fracas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \sqrt{x}(1+x^2) dx = \frac{2}{21} (3x^3 + 7x)\sqrt{x}$$

input `integrate(x^(1/2)*(x^2+1),x, algorithm="fracas")`output `2/21*(3*x^3 + 7*x)*sqrt(x)`**3.223.6 Sympy [A] (verification not implemented)**

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \sqrt{x}(1+x^2) dx = \frac{2x^{\frac{7}{2}}}{7} + \frac{2x^{\frac{3}{2}}}{3}$$

input `integrate(x**(1/2)*(x**2+1),x)`output `2*x**(7/2)/7 + 2*x**(3/2)/3`

**3.223.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \sqrt{x}(1+x^2) dx = \frac{2}{7}x^{\frac{7}{2}} + \frac{2}{3}x^{\frac{3}{2}}$$

input `integrate(x^(1/2)*(x^2+1),x, algorithm="maxima")`output `2/7*x^(7/2) + 2/3*x^(3/2)`**3.223.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \sqrt{x}(1+x^2) dx = \frac{2}{7}x^{\frac{7}{2}} + \frac{2}{3}x^{\frac{3}{2}}$$

input `integrate(x^(1/2)*(x^2+1),x, algorithm="giac")`output `2/7*x^(7/2) + 2/3*x^(3/2)`**3.223.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \sqrt{x}(1+x^2) dx = \frac{2x^{3/2}(3x^2+7)}{21}$$

input `int(x^(1/2)*(x^2 + 1),x)`output `(2*x^(3/2)*(3*x^2 + 7))/21`

**3.224**  $\int \frac{\sqrt[3]{-a+x}}{x} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.224.1 Optimal result . . . . .                            | 1212 |
| 3.224.2 Mathematica [A] (verified) . . . . .                | 1212 |
| 3.224.3 Rubi [A] (verified) . . . . .                       | 1213 |
| 3.224.4 Maple [A] (verified) . . . . .                      | 1215 |
| 3.224.5 Fricas [A] (verification not implemented) . . . . . | 1215 |
| 3.224.6 Sympy [C] (verification not implemented) . . . . .  | 1216 |
| 3.224.7 Maxima [A] (verification not implemented) . . . . . | 1217 |
| 3.224.8 Giac [A] (verification not implemented) . . . . .   | 1217 |
| 3.224.9 Mupad [B] (verification not implemented) . . . . .  | 1218 |

**3.224.1 Optimal result**

Integrand size = 13, antiderivative size = 88

$$\int \frac{\sqrt[3]{-a+x}}{x} dx = 3\sqrt[3]{-a+x} + \sqrt{3}\sqrt[3]{a} \arctan\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{-a+x}}{\sqrt{3}\sqrt[3]{a}}\right) + \frac{1}{2}\sqrt[3]{a} \log(x) - \frac{3}{2}\sqrt[3]{a} \log(\sqrt[3]{a} + \sqrt[3]{-a+x})$$

output `3*(-a+x)^(1/3)+1/2*a^(1/3)*ln(x)-3/2*a^(1/3)*ln(a^(1/3)+(-a+x)^(1/3))+a^(1/3)*arctan(1/3*(a^(1/3)-2*(-a+x)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)`

**3.224.2 Mathematica [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt[3]{-a+x}}{x} dx = 3\sqrt[3]{-a+x} + \sqrt{3}\sqrt[3]{a} \arctan\left(\frac{1 - \frac{2\sqrt[3]{-a+x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \sqrt[3]{a} \log(\sqrt[3]{a} + \sqrt[3]{-a+x}) + \frac{1}{2}\sqrt[3]{a} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{-a+x} + (-a+x)^{2/3})$$

input `Integrate[(-a + x)^(1/3)/x,x]`

output  $3*(-a + x)^{(1/3)} + \text{Sqrt}[3]*a^{(1/3)}*\text{ArcTan}[(1 - (2*(-a + x)^{(1/3}))/a^{(1/3)})/\text{Sqrt}[3]] - a^{(1/3)}*\text{Log}[a^{(1/3)} + (-a + x)^{(1/3)}] + (a^{(1/3)}*\text{Log}[a^{(2/3)} - a^{(1/3)}*(-a + x)^{(1/3)} + (-a + x)^{(2/3)}])/2$

### 3.224.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {60, 70, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt[3]{x-a}}{x} dx \\
 & \quad \downarrow 60 \\
 & 3\sqrt[3]{x-a} - a \int \frac{1}{x(x-a)^{2/3}} dx \\
 & \quad \downarrow 70 \\
 & 3\sqrt[3]{x-a} - a \left( \frac{3 \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{x-a}} d\sqrt[3]{x-a}}{2a^{2/3}} + \frac{3 \int \frac{1}{a^{2/3} - \sqrt[3]{x-a} \sqrt[3]{a+(x-a)^{2/3}}} d\sqrt[3]{x-a}}{2\sqrt[3]{a}} - \frac{\log(x)}{2a^{2/3}} \right) \\
 & \quad \downarrow 16 \\
 & 3\sqrt[3]{x-a} - a \left( \frac{3 \int \frac{1}{a^{2/3} - \sqrt[3]{x-a} \sqrt[3]{a+(x-a)^{2/3}}} d\sqrt[3]{x-a}}{2\sqrt[3]{a}} - \frac{\log(x)}{2a^{2/3}} + \frac{3 \log(\sqrt[3]{x-a} + \sqrt[3]{a})}{2a^{2/3}} \right) \\
 & \quad \downarrow 1082 \\
 & 3\sqrt[3]{x-a} - a \left( \frac{3 \int \frac{1}{-(x-a)^{2/3} - 3} d\left(1 - \frac{2\sqrt[3]{x-a}}{\sqrt[3]{a}}\right)}{a^{2/3}} - \frac{\log(x)}{2a^{2/3}} + \frac{3 \log(\sqrt[3]{x-a} + \sqrt[3]{a})}{2a^{2/3}} \right) \\
 & \quad \downarrow 217
 \end{aligned}$$

$$3\sqrt[3]{x-a} - a \left( \frac{\sqrt{3} \arctan \left( \frac{1 - 2\sqrt[3]{x-a}}{\sqrt[3]{a}} \right)}{a^{2/3}} - \frac{\log(x)}{2a^{2/3}} + \frac{3 \log(\sqrt[3]{x-a} + \sqrt[3]{a})}{2a^{2/3}} \right)$$

input `Int[(-a + x)^(1/3)/x,x]`

output `3*(-a + x)^(1/3) - a*(-((Sqrt[3]*ArcTan[(1 - (2*(-a + x)^(1/3))/a^(1/3)]/Sqrt[3]))/a^(2/3)) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) + (-a + x)^(1/3)])/(2*a^(2/3)))`

### 3.224.3.1 Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 60 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(b*(m + n + 1)) Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

rule 70 `Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (Simp[3/(2*b*q) Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Simp[3/(2*b*q^2) Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]`

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

```
rule 1082 Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c))] /; Fre
eQ[{a, b, c}, x]
```

### 3.224.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

| method            | result                                                                                                                                                                                                                                                                                                                                                             |
|-------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| derivativedivides | $3(-a+x)^{\frac{1}{3}} - 3 \left( \frac{\ln(a^{\frac{1}{3}} + (-a+x)^{\frac{1}{3}})}{3a^{\frac{2}{3}}} - \frac{\ln((-a+x)^{\frac{2}{3}} - a^{\frac{1}{3}}(-a+x)^{\frac{1}{3}} + a^{\frac{2}{3}})}{6a^{\frac{2}{3}}} + \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(\frac{2(-a+x)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{3a^{\frac{2}{3}}}\right)$ |
| default           | $3(-a+x)^{\frac{1}{3}} - 3 \left( \frac{\ln(a^{\frac{1}{3}} + (-a+x)^{\frac{1}{3}})}{3a^{\frac{2}{3}}} - \frac{\ln((-a+x)^{\frac{2}{3}} - a^{\frac{1}{3}}(-a+x)^{\frac{1}{3}} + a^{\frac{2}{3}})}{6a^{\frac{2}{3}}} + \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(\frac{2(-a+x)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{3a^{\frac{2}{3}}}\right)$ |

```
input int((-a+x)^(1/3)/x,x,method=_RETURNVERBOSE)
```

```
output 3*(-a+x)^(1/3)-3*(1/3/a^(2/3)*ln(a^(1/3)+(-a+x)^(1/3))-1/6/a^(2/3)*ln((-a+
x)^(2/3)-a^(1/3)*(-a+x)^(1/3)+a^(2/3))+1/3/a^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/a^(1/3)*(-a+x)^(1/3)-1)))*a
```

### 3.224.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt[3]{-a+x}}{x} dx = \sqrt{3}(-a)^{\frac{1}{3}} \arctan\left(-\frac{\sqrt{3}a - 2\sqrt{3}(-a)^{\frac{2}{3}}(-a+x)^{\frac{1}{3}}}{3a}\right) - \frac{1}{2}(-a)^{\frac{1}{3}} \log\left((-a)^{\frac{2}{3}} + (-a)^{\frac{1}{3}}(-a+x)^{\frac{1}{3}} + (-a+x)^{\frac{2}{3}}\right) + (-a)^{\frac{1}{3}} \log\left(-(-a)^{\frac{1}{3}} + (-a+x)^{\frac{1}{3}}\right) + 3(-a+x)^{\frac{1}{3}}$$

---

3.224.  $\int \frac{\sqrt[3]{-a+x}}{x} dx$



input `integrate((-a+x)^(1/3)/x,x, algorithm="fricas")`

output `sqrt(3)*(-a)^(1/3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-a)^(2/3)*(-a + x)^(1/3))/a) - 1/2*(-a)^(1/3)*log((-a)^(2/3) + (-a)^(1/3)*(-a + x)^(1/3) + (-a + x)^(2/3)) + (-a)^(1/3)*log(-(-a)^(1/3) + (-a + x)^(1/3)) + 3*(-a + x)^(1/3)`

### 3.224.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt[3]{-a+x}}{x} dx = \frac{4\sqrt[3]{a}e^{-\frac{i\pi}{3}} \log\left(1 - \frac{\sqrt[3]{-a+x}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)} - \frac{4\sqrt[3]{a} \log\left(1 - \frac{\sqrt[3]{-a+x}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{4\sqrt[3]{a}e^{\frac{i\pi}{3}} \log\left(1 - \frac{\sqrt[3]{-a+x}e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{4\sqrt[3]{-a+x}\Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{7}{3}\right)}$$

input `integrate((-a+x)**(1/3)/x,x)`

output `4*a**(1/3)*exp(-I*pi/3)*log(1 - (-a + x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) - 4*a**(1/3)*log(1 - (-a + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*a**(1/3)*exp(I*pi/3)*log(1 - (-a + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*(-a + x)**(1/3)*gamma(4/3)/gamma(7/3)`

**3.224.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt[3]{-a+x}}{x} dx = -\sqrt{3}a^{\frac{1}{3}} \arctan\left(-\frac{\sqrt{3}\left(a^{\frac{1}{3}} - 2(-a+x)^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right) + \frac{1}{2}a^{\frac{1}{3}} \log\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}(-a+x)^{\frac{1}{3}} + (-a+x)^{\frac{2}{3}}\right) - a^{\frac{1}{3}} \log\left(a^{\frac{1}{3}} + (-a+x)^{\frac{1}{3}}\right) + 3(-a+x)^{\frac{1}{3}}$$

input `integrate((-a+x)^(1/3)/x,x, algorithm="maxima")`output `-sqrt(3)*a^(1/3)*arctan(-1/3*sqrt(3)*(a^(1/3) - 2*(-a + x)^(1/3))/a^(1/3)) + 1/2*a^(1/3)*log(a^(2/3) - a^(1/3)*(-a + x)^(1/3) + (-a + x)^(2/3)) - a^(1/3)*log(a^(1/3) + (-a + x)^(1/3)) + 3*(-a + x)^(1/3)`**3.224.8 Giac [A] (verification not implemented)**

Time = 0.49 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt[3]{-a+x}}{x} dx = -\sqrt{3}(-a)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left((-a)^{\frac{1}{3}} + 2(-a+x)^{\frac{1}{3}}\right)}{3(-a)^{\frac{1}{3}}}\right) - \frac{1}{2}(-a)^{\frac{1}{3}} \log\left((-a)^{\frac{2}{3}} + (-a)^{\frac{1}{3}}(-a+x)^{\frac{1}{3}} + (-a+x)^{\frac{2}{3}}\right) + (-a)^{\frac{1}{3}} \log\left(\left| -(-a)^{\frac{1}{3}} + (-a+x)^{\frac{1}{3}} \right|\right) + 3(-a+x)^{\frac{1}{3}}$$

input `integrate((-a+x)^(1/3)/x,x, algorithm="giac")`output `-sqrt(3)*(-a)^(1/3)*arctan(1/3*sqrt(3)*((-a)^(1/3) + 2*(-a + x)^(1/3))/(-a)^(1/3)) - 1/2*(-a)^(1/3)*log((-a)^(2/3) + (-a)^(1/3)*(-a + x)^(1/3) + (-a + x)^(2/3)) + (-a)^(1/3)*log(abs(-(-a)^(1/3) + (-a + x)^(1/3))) + 3*(-a + x)^(1/3)`

**3.224.9 Mupad [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt[3]{-a+x}}{x} dx = (-a)^{1/3} \ln \left( -9(-a)^{4/3} \right. \\ \left. -9a(x-a)^{1/3} \right) + 3(x-a)^{1/3} + \frac{(-a)^{1/3} \ln \left( \frac{9(-a)^{4/3}(-1+\sqrt{3}1i)}{2} + 9a(x-a)^{1/3} \right) (-1+\sqrt{3}1i)}{2} - \frac{(-a)^{1/3} \ln \left( \frac{9(-a)^{4/3}(-1-\sqrt{3}1i)}{2} + 9a(x-a)^{1/3} \right) (-1-\sqrt{3}1i)}{2}$$

input `int((x - a)^(1/3)/x,x)`output `(-a)^(1/3)*log(- 9*(-a)^(4/3) - 9*a*(x - a)^(1/3)) + 3*(x - a)^(1/3) + ((-a)^(1/3)*log((9*(-a)^(4/3)*(3^(1/2)*1i - 1))/2 + 9*a*(x - a)^(1/3))*(3^(1/2)*1i - 1))/2 - ((-a)^(1/3)*log((9*(-a)^(4/3)*(3^(1/2)*1i + 1))/2 - 9*a*(x - a)^(1/3))*(3^(1/2)*1i + 1))/2`

## 3.225 $\int x \sinh(x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.225.1 Optimal result . . . . .                            | 1219 |
| 3.225.2 Mathematica [A] (verified) . . . . .                | 1219 |
| 3.225.3 Rubi [C] (verified) . . . . .                       | 1220 |
| 3.225.4 Maple [A] (verified) . . . . .                      | 1221 |
| 3.225.5 Fricas [A] (verification not implemented) . . . . . | 1222 |
| 3.225.6 Sympy [A] (verification not implemented) . . . . .  | 1222 |
| 3.225.7 Maxima [B] (verification not implemented) . . . . . | 1222 |
| 3.225.8 Giac [A] (verification not implemented) . . . . .   | 1223 |
| 3.225.9 Mupad [B] (verification not implemented) . . . . .  | 1223 |

### 3.225.1 Optimal result

Integrand size = 4, antiderivative size = 9

$$\int x \sinh(x) dx = x \cosh(x) - \sinh(x)$$

output `x*cosh(x)-sinh(x)`

### 3.225.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x \sinh(x) dx = x \cosh(x) - \sinh(x)$$

input `Integrate[x*Sinh[x],x]`

output `x*Cosh[x] - Sinh[x]`

**3.225.3 Rubi [C] (verified)**

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00, number of steps used = 5, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$ , Rules used = {3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x \sinh(x) dx \\
 & \quad \downarrow \text{3042} \\
 & \int -ix \sin(ix) dx \\
 & \quad \downarrow \text{26} \\
 & -i \int x \sin(ix) dx \\
 & \quad \downarrow \text{3777} \\
 & -i(ix \cosh(x) - i \int \cosh(x) dx) \\
 & \quad \downarrow \text{3042} \\
 & -i \left( ix \cosh(x) - i \int \sin \left( ix + \frac{\pi}{2} \right) dx \right) \\
 & \quad \downarrow \text{3117} \\
 & -i(ix \cosh(x) - i \sinh(x))
 \end{aligned}$$

input `Int[x*Sinh[x],x]`

output `(-I)*(I*x*Cosh[x] - I*Sinh[x])`

## 3.225.3.1 Defintions of rubi rules used

rule 26 `Int[(Complex[0, a_])*(Fx_), x_Symbol] := Simp[(Complex[Identity[0], a]) Int[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3117 `Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

rule 3777 `Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

## 3.225.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

| method        | result                                                                  | size |
|---------------|-------------------------------------------------------------------------|------|
| default       | $x \cosh(x) - \sinh(x)$                                                 | 10   |
| meijerg       | $x \cosh(x) - \sinh(x)$                                                 | 10   |
| parallelrisch | $x \cosh(x) - \sinh(x)$                                                 | 10   |
| parts         | $x \cosh(x) - \sinh(x)$                                                 | 10   |
| risch         | $(-\frac{1}{2} + \frac{x}{2}) e^x + (\frac{1}{2} + \frac{x}{2}) e^{-x}$ | 20   |

input `int(x*sinh(x),x,method=_RETURNVERBOSE)`

output `x*cosh(x)-sinh(x)`

**3.225.5 Fricas [A] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x \sinh(x) dx = x \cosh(x) - \sinh(x)$$

input `integrate(x*sinh(x),x, algorithm="fricas")`

output `x*cosh(x) - sinh(x)`

**3.225.6 Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int x \sinh(x) dx = x \cosh(x) - \sinh(x)$$

input `integrate(x*sinh(x),x)`

output `x*cosh(x) - sinh(x)`

**3.225.7 Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(9) = 18.

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.78

$$\int x \sinh(x) dx = \frac{1}{2} x^2 \sinh(x) + \frac{1}{4} (x^2 + 2x + 2) e^{-x} - \frac{1}{4} (x^2 - 2x + 2) e^x$$

input `integrate(x*sinh(x),x, algorithm="maxima")`

output `1/2*x^2*sinh(x) + 1/4*(x^2 + 2*x + 2)*e^(-x) - 1/4*(x^2 - 2*x + 2)*e^x`

**3.225.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int x \sinh(x) dx = \frac{1}{2} (x + 1)e^{(-x)} + \frac{1}{2} (x - 1)e^x$$

input `integrate(x*sinh(x),x, algorithm="giac")`

output `1/2*(x + 1)*e^(-x) + 1/2*(x - 1)*e^x`

**3.225.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x \sinh(x) dx = x \cosh(x) - \sinh(x)$$

input `int(x*sinh(x),x)`

output `x*cosh(x) - sinh(x)`



## 3.226 $\int x \cosh(x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.226.1 Optimal result . . . . .                            | 1224 |
| 3.226.2 Mathematica [A] (verified) . . . . .                | 1224 |
| 3.226.3 Rubi [A] (verified) . . . . .                       | 1225 |
| 3.226.4 Maple [A] (verified) . . . . .                      | 1226 |
| 3.226.5 Fricas [A] (verification not implemented) . . . . . | 1227 |
| 3.226.6 Sympy [A] (verification not implemented) . . . . .  | 1227 |
| 3.226.7 Maxima [B] (verification not implemented) . . . . . | 1227 |
| 3.226.8 Giac [A] (verification not implemented) . . . . .   | 1228 |
| 3.226.9 Mupad [B] (verification not implemented) . . . . .  | 1228 |

### 3.226.1 Optimal result

Integrand size = 4, antiderivative size = 9

$$\int x \cosh(x) dx = -\cosh(x) + x \sinh(x)$$

output `-cosh(x)+x*sinh(x)`

### 3.226.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x \cosh(x) dx = -\cosh(x) + x \sinh(x)$$

input `Integrate[x*Cosh[x],x]`

output `-Cosh[x] + x*Sinh[x]`

**3.226.3 Rubi [A] (verified)**

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$ , Rules used = {3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x \cosh(x) dx \\
 & \quad \downarrow \text{3042} \\
 & \int x \sin\left(\frac{\pi}{2} + ix\right) dx \\
 & \quad \downarrow \text{3777} \\
 & x \sinh(x) - i \int -i \sinh(x) dx \\
 & \quad \downarrow \text{26} \\
 & x \sinh(x) - \int \sinh(x) dx \\
 & \quad \downarrow \text{3042} \\
 & x \sinh(x) - \int -i \sin(ix) dx \\
 & \quad \downarrow \text{26} \\
 & x \sinh(x) + i \int \sin(ix) dx \\
 & \quad \downarrow \text{3118} \\
 & x \sinh(x) - \cosh(x)
 \end{aligned}$$

input `Int[x*Cosh[x],x]`

output `-Cosh[x] + x*Sinh[x]`

## 3.226.3.1 Defintions of rubi rules used

rule 26 `Int[(Complex[0, a_])*(Fx_), x_Symbol] := Simp[(Complex[Identity[0], a]) Int[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3118 `Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

rule 3777 `Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

## 3.226.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

| method       | result                                                                                                             | size |
|--------------|--------------------------------------------------------------------------------------------------------------------|------|
| default      | $-\cosh(x) + x \sinh(x)$                                                                                           | 10   |
| parts        | $-\cosh(x) + x \sinh(x)$                                                                                           | 10   |
| risch        | $\left(-\frac{1}{2} + \frac{x}{2}\right) e^x + \left(-\frac{1}{2} - \frac{x}{2}\right) e^{-x}$                     | 20   |
| parallelrisc | $\frac{2-2x \tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1}$                                     | 21   |
| meijerg      | $-2\sqrt{\pi} \left(-\frac{1}{2\sqrt{\pi}} + \frac{\cosh(x)}{2\sqrt{\pi}} - \frac{x \sinh(x)}{2\sqrt{\pi}}\right)$ | 27   |

input `int(x*cosh(x), x, method=_RETURNVERBOSE)`

output `-cosh(x)+x*sinh(x)`

**3.226.5 Fricas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x \cosh(x) dx = x \sinh(x) - \cosh(x)$$

input `integrate(x*cosh(x),x, algorithm="fricas")`

output `x*sinh(x) - cosh(x)`

**3.226.6 Sympy [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int x \cosh(x) dx = x \sinh(x) - \cosh(x)$$

input `integrate(x*cosh(x),x)`

output `x*sinh(x) - cosh(x)`

**3.226.7 Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 34 vs.  $2(9) = 18$ .

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.78

$$\int x \cosh(x) dx = \frac{1}{2} x^2 \cosh(x) - \frac{1}{4} (x^2 + 2x + 2) e^{(-x)} - \frac{1}{4} (x^2 - 2x + 2) e^x$$

input `integrate(x*cosh(x),x, algorithm="maxima")`

output `1/2*x^2*cosh(x) - 1/4*(x^2 + 2*x + 2)*e^(-x) - 1/4*(x^2 - 2*x + 2)*e^x`

**3.226.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int x \cosh(x) dx = -\frac{1}{2}(x+1)e^{(-x)} + \frac{1}{2}(x-1)e^x$$

input `integrate(x*cosh(x),x, algorithm="giac")`

output `-1/2*(x + 1)*e^(-x) + 1/2*(x - 1)*e^x`

**3.226.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x \cosh(x) dx = x \sinh(x) - \cosh(x)$$

input `int(x*cosh(x),x)`

output `x*sinh(x) - cosh(x)`

## 3.227 $\int \tanh(2x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.227.1 Optimal result . . . . .                            | 1229 |
| 3.227.2 Mathematica [A] (verified) . . . . .                | 1229 |
| 3.227.3 Rubi [A] (verified) . . . . .                       | 1230 |
| 3.227.4 Maple [A] (verified) . . . . .                      | 1231 |
| 3.227.5 Fricas [B] (verification not implemented) . . . . . | 1231 |
| 3.227.6 Sympy [A] (verification not implemented) . . . . .  | 1231 |
| 3.227.7 Maxima [A] (verification not implemented) . . . . . | 1232 |
| 3.227.8 Giac [A] (verification not implemented) . . . . .   | 1232 |
| 3.227.9 Mupad [B] (verification not implemented) . . . . .  | 1232 |

### 3.227.1 Optimal result

Integrand size = 4, antiderivative size = 9

$$\int \tanh(2x) dx = \frac{1}{2} \log(\cosh(2x))$$

output `1/2*ln(cosh(2*x))`

### 3.227.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \tanh(2x) dx = \frac{1}{2} \log(\cosh(2x))$$

input `Integrate[Tanh[2*x],x]`

output `Log[Cosh[2*x]]/2`

**3.227.3 Rubi [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$ , Rules used = {3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \tanh(2x) dx \\ & \quad \downarrow \text{3042} \\ & \int -i \tan(2ix) dx \\ & \quad \downarrow \text{26} \\ & -i \int \tan(2ix) dx \\ & \quad \downarrow \text{3956} \\ & \frac{1}{2} \log(\cosh(2x)) \end{aligned}$$

input `Int [Tanh[2*x] , x]`

output `Log[Cosh[2*x]]/2`

**3.227.3.1 Defintions of rubi rules used**

rule 26 `Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a]) Int[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]`

rule 3042 `Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3956 `Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

**3.227.4 Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

| method            | result                                              | size |
|-------------------|-----------------------------------------------------|------|
| derivativedivides | $\frac{\ln(\cosh(2x))}{2}$                          | 8    |
| default           | $\frac{\ln(\cosh(2x))}{2}$                          | 8    |
| risch             | $-x + \frac{\ln(e^{4x}+1)}{2}$                      | 14   |
| parallelrisch     | $-x + \ln\left(\frac{1}{\sqrt{1-\tanh(2x)}}\right)$ | 16   |

input `int(sinh(2*x)/cosh(2*x),x,method=_RETURNVERBOSE)`

output `1/2*ln(cosh(2*x))`

**3.227.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 26 vs.  $2(7) = 14$ .

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.89

$$\int \tanh(2x) dx = -x + \frac{1}{2} \log\left(\frac{2 \cosh(2x)}{\cosh(2x) - \sinh(2x)}\right)$$

input `integrate(sinh(2*x)/cosh(2*x),x, algorithm="fricas")`

output `-x + 1/2*log(2*cosh(2*x)/(cosh(2*x) - sinh(2*x)))`

**3.227.6 Sympy [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \tanh(2x) dx = \frac{\log(\cosh(2x))}{2}$$

input `integrate(sinh(2*x)/cosh(2*x),x)`

output `log(cosh(2*x))/2`



**3.227.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \tanh(2x) dx = \frac{1}{2} \log(\cosh(2x))$$

input `integrate(sinh(2*x)/cosh(2*x),x, algorithm="maxima")`output `1/2*log(cosh(2*x))`**3.227.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int \tanh(2x) dx = -x + \frac{1}{2} \log(e^{4x} + 1)$$

input `integrate(sinh(2*x)/cosh(2*x),x, algorithm="giac")`output `-x + 1/2*log(e^(4*x) + 1)`**3.227.9 Mupad [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \tanh(2x) dx = \frac{\ln(\cosh(2x))}{2}$$

input `int(sinh(2*x)/cosh(2*x),x)`output `log(cosh(2*x))/2`

$$3.228 \quad \int \frac{-1+i\mathbf{eps} \sinh(x)}{ia-x+i\mathbf{eps} \cosh(x)} dx$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.228.1 Optimal result . . . . .                            | 1233 |
| 3.228.2 Mathematica [A] (verified) . . . . .                | 1233 |
| 3.228.3 Rubi [A] (verified) . . . . .                       | 1234 |
| 3.228.4 Maple [A] (verified) . . . . .                      | 1234 |
| 3.228.5 Fricas [B] (verification not implemented) . . . . . | 1235 |
| 3.228.6 Sympy [B] (verification not implemented) . . . . .  | 1235 |
| 3.228.7 Maxima [A] (verification not implemented) . . . . . | 1235 |
| 3.228.8 Giac [B] (verification not implemented) . . . . .   | 1236 |
| 3.228.9 Mupad [B] (verification not implemented) . . . . .  | 1236 |

### 3.228.1 Optimal result

Integrand size = 28, antiderivative size = 12

$$\int \frac{-1 + i\mathbf{eps} \sinh(x)}{ia - x + i\mathbf{eps} \cosh(x)} dx = \log(a + ix + \mathbf{eps} \cosh(x))$$

output `ln(a+I*x+eps*cosh(x))`

### 3.228.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{-1 + i\mathbf{eps} \sinh(x)}{ia - x + i\mathbf{eps} \cosh(x)} dx = \log(a + ix + \mathbf{eps} \cosh(x))$$

input `Integrate[(-1 + I*eps*Sinh[x])/(I*a - x + I*eps*Cosh[x]), x]`

output `Log[a + I*x + eps*Cosh[x]]`

### 3.228.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$ , Rules used = {7235}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-1 + i \text{eps} \sinh(x)}{ia + i \text{eps} \cosh(x) - x} dx$$

↓ 7235

$$\log(a + \text{eps} \cosh(x) + ix)$$

input `Int[(-1 + I*eps*Sinh[x])/(I*a - x + I*eps*Cosh[x]), x]`

output `Log[a + I*x + eps*Cosh[x]]`

#### 3.228.3.1 Defintions of rubi rules used

rule 7235 `Int[(u_)/(y_), x_Symbol] := With[{q = DerivativeDivides[y, u, x]}, Simp[q*Log[RemoveContent[y, x]], x] /; !FalseQ[q]]`

### 3.228.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

| method            | result                                                            | size |
|-------------------|-------------------------------------------------------------------|------|
| derivativedivides | $\ln(ia - x + i \text{eps} \cosh(x))$                             | 16   |
| default           | $\ln(ia - x + i \text{eps} \cosh(x))$                             | 16   |
| risch             | $-x + \ln\left(1 + \frac{2(ix+a)e^x}{\text{eps}} + e^{2x}\right)$ | 25   |

input `int((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)), x, method=_RETURNVERBOSE)`

output `ln(I*a-x+I*eps*cosh(x))`

**3.228.5 Fricas [B] (verification not implemented)**

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 26 vs.  $2(10) = 20$ .

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{-1 + i\text{eps} \sinh(x)}{ia - x + i\text{eps} \cosh(x)} dx = -x + \log\left(\frac{\text{epse}^{(2x)} + 2(a + ix)e^x + \text{eps}}{\text{eps}}\right)$$

input `integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x, algorithm="fricas")`

output `-x + log((eps*e^(2*x) + 2*(a + I*x)*e^x + eps)/eps)`

**3.228.6 Sympy [B] (verification not implemented)**

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 22 vs.  $2(10) = 20$ .

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{-1 + i\text{eps} \sinh(x)}{ia - x + i\text{eps} \cosh(x)} dx = -x + \log\left(e^{2x} + 1 + \frac{(2a + 2ix)e^x}{\text{eps}}\right)$$

input `integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x)`

output `-x + log(exp(2*x) + 1 + (2*a + 2*I*x)*exp(x)/eps)`

**3.228.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{-1 + i\text{eps} \sinh(x)}{ia - x + i\text{eps} \cosh(x)} dx = \log(i \text{eps} \cosh(x) + ia - x)$$

input `integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x, algorithm="maxima")`

output `log(I*eps*cosh(x) + I*a - x)`

---

3.228.  $\int \frac{-1+i\text{eps} \sinh(x)}{ia-x+i\text{eps} \cosh(x)} dx$

**3.228.8 Giac [B] (verification not implemented)**

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 23 vs.  $2(10) = 20$ .

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{-1 + i\text{eps} \sinh(x)}{ia - x + i\text{eps} \cosh(x)} dx = -x + \log(\text{epse}^{(2x)} + 2ae^x + 2ixe^x + \text{eps})$$

input `integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x, algorithm="giac")`

output `-x + log(eps*e^(2*x) + 2*a*e^x + 2*I*x*e^x + eps)`

**3.228.9 Mupad [B] (verification not implemented)**

Time = 0.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{-1 + i\text{eps} \sinh(x)}{ia - x + i\text{eps} \cosh(x)} dx = \ln(x - a \text{ li} - \text{eps} \cosh(x) \text{ li})$$

input `int((eps*sinh(x)*1i - 1)/(a*1i - x + eps*cosh(x)*1i),x)`

output `log(x - a*1i - eps*cosh(x)*1i)`

### 3.229 $\int \cos^2(x) \sin(3 + 2x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.229.1 Optimal result . . . . .                            | 1237 |
| 3.229.2 Mathematica [A] (verified) . . . . .                | 1237 |
| 3.229.3 Rubi [A] (verified) . . . . .                       | 1238 |
| 3.229.4 Maple [A] (verified) . . . . .                      | 1239 |
| 3.229.5 Fricas [A] (verification not implemented) . . . . . | 1239 |
| 3.229.6 Sympy [B] (verification not implemented) . . . . .  | 1240 |
| 3.229.7 Maxima [A] (verification not implemented) . . . . . | 1240 |
| 3.229.8 Giac [A] (verification not implemented) . . . . .   | 1240 |
| 3.229.9 Mupad [B] (verification not implemented) . . . . .  | 1241 |

#### 3.229.1 Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \cos^2(x) \sin(3 + 2x) dx = -\frac{1}{4} \cos(3 + 2x) - \frac{1}{16} \cos(3 + 4x) + \frac{1}{4} x \sin(3)$$

output `-1/4*cos(3+2*x)-1/16*cos(3+4*x)+1/4*x*sin(3)`

#### 3.229.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \cos^2(x) \sin(3 + 2x) dx = -\frac{1}{4} \cos(3 + 2x) - \frac{1}{16} \cos(3 + 4x) + \frac{1}{4} x \sin(3)$$

input `Integrate[Cos[x]^2*Sin[3 + 2*x],x]`

output `-1/4*Cos[3 + 2*x] - Cos[3 + 4*x]/16 + (x*Sin[3])/4`

**3.229.3 Rubi [A] (verified)**

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2x + 3) \cos^2(x) dx$$

$$\downarrow \text{5085}$$

$$\int \left( \frac{1}{2} \sin(2x + 3) + \frac{1}{4} \sin(4x + 3) + \frac{\sin(3)}{4} \right) dx$$

$$\downarrow \text{2009}$$

$$\frac{1}{4} x \sin(3) - \frac{1}{4} \cos(2x + 3) - \frac{1}{16} \cos(4x + 3)$$

input `Int[Cos[x]^2*Sin[3 + 2*x],x]`

output `-1/4*Cos[3 + 2*x] - Cos[3 + 4*x]/16 + (x*Sin[3])/4`

**3.229.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5085 `Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] := Int[ExpandTrigReduce[Sin[v]^p *Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))`

**3.229.4 Maple [A] (verified)**

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

| method        | result                                                                                                                                                                                                                                                                                  |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| default       | $-\frac{\cos(3+2x)}{4} - \frac{\cos(4x+3)}{16} + \frac{x \sin(3)}{4}$                                                                                                                                                                                                                   |
| risch         | $-\frac{\cos(3+2x)}{4} - \frac{\cos(4x+3)}{16} + \frac{x \sin(3)}{4}$                                                                                                                                                                                                                   |
| parallelrisch | $-\frac{\cos(3+2x)}{4} - \frac{\cos(4x+3)}{16} - \frac{\cos(3)}{16} + \frac{1}{8} + \frac{x \sin(3)}{4}$                                                                                                                                                                                |
| norman        | $\frac{-2(\tan^2(\frac{x}{2})+(\tan^3(\frac{x}{2}))x+x \tan(\frac{x}{2})(\tan^2(\frac{3}{2}+x))-3(\tan^3(\frac{x}{2})) \tan(\frac{3}{2}+x)+2(\tan^2(\frac{x}{2}))(\tan^2(\frac{3}{2}+x))+3 \tan(\frac{x}{2}) \tan(\frac{3}{2}+x))}{(1+\tan^2(\frac{x}{2}))^2(1+\tan^2(\frac{3}{2}+x))}$ |

input `int(cos(x)^2*sin(3+2*x),x,method=_RETURNVERBOSE)`output `-1/4*cos(3+2*x)-1/16*cos(4*x+3)+1/4*x*sin(3)`**3.229.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \cos^2(x) \sin(3 + 2x) dx = -\frac{1}{2} \cos(3) \cos(x)^4 + \frac{1}{4} x \sin(3) + \frac{1}{4} (2 \cos(x)^3 \sin(3) + \cos(x) \sin(3)) \sin(x)$$

input `integrate(cos(x)^2*sin(3+2*x),x, algorithm="fracas")`output `-1/2*cos(3)*cos(x)^4 + 1/4*x*sin(3) + 1/4*(2*cos(x)^3*sin(3) + cos(x)*sin(3))*sin(x)`



**3.229.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 75 vs.  $2(22) = 44$ .

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \cos^2(x) \sin(3 + 2x) dx = -\frac{x \sin^2(x) \sin(2x + 3)}{4} - \frac{x \sin(x) \cos(x) \cos(2x + 3)}{2} + \frac{x \sin(2x + 3) \cos^2(x)}{4} - \frac{\sin(x) \sin(2x + 3) \cos(x)}{4} - \frac{\cos^2(x) \cos(2x + 3)}{2}$$

input `integrate(cos(x)**2*sin(3+2*x),x)`

output `-x*sin(x)**2*sin(2*x + 3)/4 - x*sin(x)*cos(x)*cos(2*x + 3)/2 + x*sin(2*x + 3)*cos(x)**2/4 - sin(x)*sin(2*x + 3)*cos(x)/4 - cos(x)**2*cos(2*x + 3)/2`

**3.229.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \cos^2(x) \sin(3 + 2x) dx = \frac{1}{4} x \sin(3) - \frac{1}{16} \cos(4x + 3) - \frac{1}{4} \cos(2x + 3)$$

input `integrate(cos(x)^2*sin(3+2*x),x, algorithm="maxima")`

output `1/4*x*sin(3) - 1/16*cos(4*x + 3) - 1/4*cos(2*x + 3)`

**3.229.8 Giac [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \cos^2(x) \sin(3 + 2x) dx = \frac{1}{4} x \sin(3) - \frac{1}{16} \cos(4x + 3) - \frac{1}{4} \cos(2x + 3)$$

input `integrate(cos(x)^2*sin(3+2*x),x, algorithm="giac")`

output `1/4*x*sin(3) - 1/16*cos(4*x + 3) - 1/4*cos(2*x + 3)`

**3.229.9 Mupad [B] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \cos^2(x) \sin(3 + 2x) dx = \frac{x \sin(3)}{4} - \frac{\cos(4x + 3)}{16} - \frac{\cos(2x + 3)}{4}$$

input `int(sin(2*x + 3)*cos(x)^2,x)`

output `(x*sin(3))/4 - cos(4*x + 3)/16 - cos(2*x + 3)/4`

### 3.230 $\int x \arctan(x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.230.1 Optimal result . . . . .                            | 1242 |
| 3.230.2 Mathematica [A] (verified) . . . . .                | 1242 |
| 3.230.3 Rubi [A] (verified) . . . . .                       | 1243 |
| 3.230.4 Maple [A] (verified) . . . . .                      | 1244 |
| 3.230.5 Fricas [A] (verification not implemented) . . . . . | 1244 |
| 3.230.6 Sympy [A] (verification not implemented) . . . . .  | 1245 |
| 3.230.7 Maxima [A] (verification not implemented) . . . . . | 1245 |
| 3.230.8 Giac [A] (verification not implemented) . . . . .   | 1245 |
| 3.230.9 Mupad [B] (verification not implemented) . . . . .  | 1246 |

#### 3.230.1 Optimal result

Integrand size = 4, antiderivative size = 21

$$\int x \arctan(x) dx = -\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{1}{2}x^2 \arctan(x)$$

output `-1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)`

#### 3.230.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int x \arctan(x) dx = \frac{1}{2}(-x + (1 + x^2) \arctan(x))$$

input `Integrate[x*ArcTan[x],x]`

output `(-x + (1 + x^2)*ArcTan[x])/2`

### 3.230.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$ , Rules used = {5361, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(x) dx$$

$$\downarrow \text{5361}$$

$$\frac{1}{2}x^2 \arctan(x) - \frac{1}{2} \int \frac{x^2}{x^2 + 1} dx$$

$$\downarrow \text{262}$$

$$\frac{1}{2} \left( \int \frac{1}{x^2 + 1} dx - x \right) + \frac{1}{2}x^2 \arctan(x)$$

$$\downarrow \text{216}$$

$$\frac{1}{2}x^2 \arctan(x) + \frac{1}{2}(\arctan(x) - x)$$

input `Int[x*ArcTan[x],x]`

output `(x^2*ArcTan[x])/2 + (-x + ArcTan[x])/2`

#### 3.230.3.1 Defintions of rubi rules used

rule 216 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

rule 262 `Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[c*(c*x)^(m-1)*((a + b*x^2)^(p+1)/(b*(m+2*p+1))), x] - Simp[a*c^2*((m-1)/(b*(m+2*p+1)) Int[(c*x)^(m-2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[m, 2-1] && NeQ[m+2*p+1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]`

```
rule 5361 Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
  Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
  1)) Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
  x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
  & IntegerQ[m])) && NeQ[m, -1]
```

### 3.230.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

| method        | result                                                                                       | size |
|---------------|----------------------------------------------------------------------------------------------|------|
| default       | $-\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{x^2 \arctan(x)}{2}$                             | 16   |
| meijerg       | $-\frac{x}{2} + \frac{(3x^2+3) \arctan(x)}{6}$                                               | 16   |
| parallelrisch | $-\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{x^2 \arctan(x)}{2}$                             | 16   |
| parts         | $-\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{x^2 \arctan(x)}{2}$                             | 16   |
| risch         | $-\frac{ix^2 \ln(ix+1)}{4} + \frac{ix^2 \ln(-ix+1)}{4} - \frac{x}{2} + \frac{\arctan(x)}{2}$ | 35   |

```
input int(x*arctan(x), x, method=_RETURNVERBOSE)
```

```
output -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

### 3.230.5 Fracas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x \arctan(x) dx = \frac{1}{2} (x^2 + 1) \arctan(x) - \frac{1}{2} x$$

```
input integrate(x*arctan(x), x, algorithm="fricas")
```

```
output 1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

**3.230.6 Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x \arctan(x) dx = \frac{x^2 \operatorname{atan}(x)}{2} - \frac{x}{2} + \frac{\operatorname{atan}(x)}{2}$$

input `integrate(x*atan(x),x)`output `x**2*atan(x)/2 - x/2 + atan(x)/2`**3.230.7 Maxima [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x \arctan(x) dx = \frac{1}{2} x^2 \arctan(x) - \frac{1}{2} x + \frac{1}{2} \arctan(x)$$

input `integrate(x*arctan(x),x, algorithm="maxima")`output `1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)`**3.230.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x \arctan(x) dx = \frac{1}{2} x^2 \arctan(x) - \frac{1}{2} x + \frac{1}{2} \arctan(x)$$

input `integrate(x*arctan(x),x, algorithm="giac")`output `1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)`

**3.230.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int x \arctan(x) dx = \operatorname{atan}(x) \left( \frac{x^2}{2} + \frac{1}{2} \right) - \frac{x}{2}$$

input `int(x*atan(x),x)`

output `atan(x)*(x^2/2 + 1/2) - x/2`

### 3.231 $\int x \cot^{-1}(x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.231.1 Optimal result . . . . .                            | 1247 |
| 3.231.2 Mathematica [A] (verified) . . . . .                | 1247 |
| 3.231.3 Rubi [A] (verified) . . . . .                       | 1248 |
| 3.231.4 Maple [A] (verified) . . . . .                      | 1249 |
| 3.231.5 Fricas [A] (verification not implemented) . . . . . | 1249 |
| 3.231.6 Sympy [A] (verification not implemented) . . . . .  | 1250 |
| 3.231.7 Maxima [A] (verification not implemented) . . . . . | 1250 |
| 3.231.8 Giac [A] (verification not implemented) . . . . .   | 1250 |
| 3.231.9 Mupad [B] (verification not implemented) . . . . .  | 1251 |

#### 3.231.1 Optimal result

Integrand size = 4, antiderivative size = 21

$$\int x \cot^{-1}(x) dx = \frac{x}{2} + \frac{1}{2}x^2 \cot^{-1}(x) - \frac{\arctan(x)}{2}$$

output `1/2*x+1/2*x^2*arccot(x)-1/2*arctan(x)`

#### 3.231.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int x \cot^{-1}(x) dx = \frac{1}{2}(x + x^2 \cot^{-1}(x) - \arctan(x))$$

input `Integrate[x*ArcCot[x],x]`

output `(x + x^2*ArcCot[x] - ArcTan[x])/2`



**3.231.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$ , Rules used = {5362, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int x \cot^{-1}(x) dx \\ & \quad \downarrow \text{5362} \\ & \frac{1}{2} \int \frac{x^2}{x^2+1} dx + \frac{1}{2} x^2 \cot^{-1}(x) \\ & \quad \downarrow \text{262} \\ & \frac{1}{2} \left( x - \int \frac{1}{x^2+1} dx \right) + \frac{1}{2} x^2 \cot^{-1}(x) \\ & \quad \downarrow \text{216} \\ & \frac{1}{2} (x - \arctan(x)) + \frac{1}{2} x^2 \cot^{-1}(x) \end{aligned}$$

input `Int[x*ArcCot[x],x]`

output `(x^2*ArcCot[x])/2 + (x - ArcTan[x])/2`

**3.231.3.1 Defintions of rubi rules used**

rule 216 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

rule 262 `Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[c*(c*x)^(m-1)*((a + b*x^2)^(p+1)/(b*(m+2*p+1))), x] - Simp[a*c^2*((m-1)/(b*(m+2*p+1)) Int[(c*x)^(m-2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[m, 2-1] && NeQ[m+2*p+1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]`

```
rule 5362 Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
  Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
  1)) Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
  x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
  & IntegerQ[m])) && NeQ[m, -1]
```

### 3.231.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

| method       | result                                                                                                                        | size |
|--------------|-------------------------------------------------------------------------------------------------------------------------------|------|
| default      | $\frac{x}{2} + \frac{x^2 \operatorname{arccot}(x)}{2} - \frac{\operatorname{arctan}(x)}{2}$                                   | 16   |
| parallelrisc | $\frac{x^2 \operatorname{arccot}(x)}{2} + \frac{x}{2} + \frac{\operatorname{arccot}(x)}{2}$                                   | 16   |
| parts        | $\frac{x}{2} + \frac{x^2 \operatorname{arccot}(x)}{2} - \frac{\operatorname{arctan}(x)}{2}$                                   | 16   |
| risc         | $\frac{ix^2 \ln(ix+1)}{4} - \frac{ix^2 \ln(-ix+1)}{4} + \frac{\pi x^2}{4} + \frac{x}{2} - \frac{\operatorname{arctan}(x)}{2}$ | 41   |

input `int(x*arccot(x),x,method=_RETURNVERBOSE)`

output `1/2*x+1/2*x^2*arccot(x)-1/2*arctan(x)`

### 3.231.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x \cot^{-1}(x) dx = \frac{1}{2} (x^2 + 1) \operatorname{arccot}(x) + \frac{1}{2} x$$

input `integrate(x*arccot(x),x, algorithm="fricas")`

output `1/2*(x^2 + 1)*arccot(x) + 1/2*x`

**3.231.6 Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x \cot^{-1}(x) dx = \frac{x^2 \operatorname{acot}(x)}{2} + \frac{x}{2} + \frac{\operatorname{acot}(x)}{2}$$

input `integrate(x*acot(x),x)`output `x**2*acot(x)/2 + x/2 + acot(x)/2`**3.231.7 Maxima [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x \cot^{-1}(x) dx = \frac{1}{2} x^2 \operatorname{arccot}(x) + \frac{1}{2} x - \frac{1}{2} \operatorname{arctan}(x)$$

input `integrate(x*arccot(x),x, algorithm="maxima")`output `1/2*x^2*arccot(x) + 1/2*x - 1/2*arctan(x)`**3.231.8 Giac [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x \cot^{-1}(x) dx = \frac{1}{2} x^2 \operatorname{arctan}\left(\frac{1}{x}\right) + \frac{1}{2} x + \frac{1}{2} \operatorname{arctan}\left(\frac{1}{x}\right)$$

input `integrate(x*arccot(x),x, algorithm="giac")`output `1/2*x^2*arctan(1/x) + 1/2*x + 1/2*arctan(1/x)`

**3.231.9 Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x \cot^{-1}(x) dx = \frac{x}{2} - \frac{\operatorname{atan}(x)}{2} + \frac{x^2 \operatorname{acot}(x)}{2}$$

input `int(x*acot(x),x)`

output `x/2 - atan(x)/2 + (x^2*acot(x))/2`

### 3.232 $\int x \log(a + x^2) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.232.1 Optimal result . . . . .                            | 1252 |
| 3.232.2 Mathematica [A] (verified) . . . . .                | 1252 |
| 3.232.3 Rubi [A] (verified) . . . . .                       | 1253 |
| 3.232.4 Maple [A] (verified) . . . . .                      | 1254 |
| 3.232.5 Fricas [A] (verification not implemented) . . . . . | 1254 |
| 3.232.6 Sympy [A] (verification not implemented) . . . . .  | 1255 |
| 3.232.7 Maxima [A] (verification not implemented) . . . . . | 1255 |
| 3.232.8 Giac [A] (verification not implemented) . . . . .   | 1255 |
| 3.232.9 Mupad [B] (verification not implemented) . . . . .  | 1256 |

#### 3.232.1 Optimal result

Integrand size = 8, antiderivative size = 23

$$\int x \log(a + x^2) dx = -\frac{x^2}{2} + \frac{1}{2}(a + x^2) \log(a + x^2)$$

output `-1/2*x^2+1/2*(x^2+a)*ln(x^2+a)`

#### 3.232.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x \log(a + x^2) dx = \frac{1}{2}(-x^2 + (a + x^2) \log(a + x^2))$$

input `Integrate[x*Log[a + x^2],x]`

output `(-x^2 + (a + x^2)*Log[a + x^2])/2`

**3.232.3 Rubi [A] (verified)**

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {2904, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int x \log(a + x^2) dx \\ & \quad \downarrow \text{2904} \\ & \frac{1}{2} \int \log(x^2 + a) dx^2 \\ & \quad \downarrow \text{2836} \\ & \frac{1}{2} \int \log(x^2 + a) d(x^2 + a) \\ & \quad \downarrow \text{2732} \\ & \frac{1}{2} ((a + x^2) \log(a + x^2) - a - x^2) \end{aligned}$$

input `Int[x*Log[a + x^2],x]`

output `(-a - x^2 + (a + x^2)*Log[a + x^2])/2`

**3.232.3.1 Defintions of rubi rules used**

rule 2732 `Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]`

rule 2836 `Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.), x_Symbol] :> Simp[1/e Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]`

```
rule 2904 Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
& !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

### 3.232.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

| method            | result                                                                          | size |
|-------------------|---------------------------------------------------------------------------------|------|
| derivativedivides | $\frac{(x^2+a)\ln(x^2+a)}{2} - \frac{x^2}{2} - \frac{a}{2}$                     | 23   |
| default           | $\frac{(x^2+a)\ln(x^2+a)}{2} - \frac{x^2}{2} - \frac{a}{2}$                     | 23   |
| norman            | $-\frac{x^2}{2} + \frac{\ln(x^2+a)a}{2} + \frac{\ln(x^2+a)x^2}{2}$              | 27   |
| risch             | $-\frac{x^2}{2} + \frac{\ln(x^2+a)a}{2} + \frac{\ln(x^2+a)x^2}{2}$              | 27   |
| parts             | $-\frac{x^2}{2} + \frac{\ln(x^2+a)a}{2} + \frac{\ln(x^2+a)x^2}{2}$              | 27   |
| parallelrisch     | $\frac{\ln(x^2+a)x^2}{2} - \frac{x^2}{2} + \frac{\ln(x^2+a)a}{2} + \frac{a}{2}$ | 30   |

input `int(x*ln(x^2+a),x,method=_RETURNVERBOSE)`

output `1/2*(x^2+a)*ln(x^2+a)-1/2*x^2-1/2*a`

### 3.232.5 Fracas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x \log(a + x^2) dx = -\frac{1}{2}x^2 + \frac{1}{2}(x^2 + a) \log(x^2 + a)$$

input `integrate(x*log(x^2+a),x, algorithm="fricas")`

output `-1/2*x^2 + 1/2*(x^2 + a)*log(x^2 + a)`

**3.232.6 Sympy [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int x \log(a + x^2) dx = \frac{a \log(a + x^2)}{2} + \frac{x^2 \log(a + x^2)}{2} - \frac{x^2}{2}$$

input `integrate(x*ln(x**2+a),x)`output `a*log(a + x**2)/2 + x**2*log(a + x**2)/2 - x**2/2`**3.232.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x \log(a + x^2) dx = -\frac{1}{2}x^2 + \frac{1}{2}(x^2 + a) \log(x^2 + a) - \frac{1}{2}a$$

input `integrate(x*log(x^2+a),x, algorithm="maxima")`output `-1/2*x^2 + 1/2*(x^2 + a)*log(x^2 + a) - 1/2*a`**3.232.8 Giac [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x \log(a + x^2) dx = -\frac{1}{2}x^2 + \frac{1}{2}(x^2 + a) \log(x^2 + a) - \frac{1}{2}a$$

input `integrate(x*log(x^2+a),x, algorithm="giac")`output `-1/2*x^2 + 1/2*(x^2 + a)*log(x^2 + a) - 1/2*a`



**3.232.9 Mupad [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int x \log(a + x^2) dx = \frac{a \ln(x + \sqrt{-a})}{2} + \frac{x^2 \ln(x^2 + a)}{2} + \frac{a \ln(x - \sqrt{-a})}{2} - \frac{x^2}{2}$$

input `int(x*log(a + x^2),x)`

output `(a*log(x + (-a)^(1/2)))/2 + (x^2*log(a + x^2))/2 + (a*log(x - (-a)^(1/2)))/2 - x^2/2`

### 3.233 $\int \cos(x) \sin(a + x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.233.1 Optimal result . . . . .                            | 1257 |
| 3.233.2 Mathematica [A] (verified) . . . . .                | 1257 |
| 3.233.3 Rubi [A] (verified) . . . . .                       | 1258 |
| 3.233.4 Maple [A] (verified) . . . . .                      | 1259 |
| 3.233.5 Fricas [A] (verification not implemented) . . . . . | 1259 |
| 3.233.6 Sympy [B] (verification not implemented) . . . . .  | 1259 |
| 3.233.7 Maxima [A] (verification not implemented) . . . . . | 1260 |
| 3.233.8 Giac [A] (verification not implemented) . . . . .   | 1260 |
| 3.233.9 Mupad [B] (verification not implemented) . . . . .  | 1260 |

#### 3.233.1 Optimal result

Integrand size = 7, antiderivative size = 18

$$\int \cos(x) \sin(a + x) dx = -\frac{1}{4} \cos(a + 2x) + \frac{1}{2} x \sin(a)$$

output `-1/4*cos(a+2*x)+1/2*x*sin(a)`

#### 3.233.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \cos(x) \sin(a + x) dx = \frac{1}{4} (-\cos(a + 2x) + 2x \sin(a))$$

input `Integrate[Cos[x]*Sin[a + x],x]`

output `(-Cos[a + 2*x] + 2*x*Sin[a])/4`

**3.233.3 Rubi [A] (verified)**

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x) \sin(a+x) dx$$

$$\downarrow \text{5085}$$

$$\int \left( \frac{1}{2} \sin(a+2x) + \frac{\sin(a)}{2} \right) dx$$

$$\downarrow \text{2009}$$

$$\frac{1}{2}x \sin(a) - \frac{1}{4} \cos(a+2x)$$

input `Int[Cos[x]*Sin[a + x],x]`

output `-1/4*Cos[a + 2*x] + (x*Sin[a])/2`

**3.233.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5085 `Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] := Int[ExpandTrigReduce[Sin[v]^p *Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))`

**3.233.4 Maple [A] (verified)**

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

| method        | result                                                                                                                                                                                                                                                                                                        | size |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default       | $-\frac{\cos(a+2x)}{4} + \frac{x \sin(a)}{2}$                                                                                                                                                                                                                                                                 | 15   |
| risch         | $-\frac{\cos(a+2x)}{4} + \frac{x \sin(a)}{2}$                                                                                                                                                                                                                                                                 | 15   |
| parallelrisch | $-\frac{\cos(a+2x)}{4} + \frac{\cos(a)}{4} + \frac{x \sin(a)}{2}$                                                                                                                                                                                                                                             | 19   |
| norman        | $\frac{x \tan(\frac{a}{2} + \frac{x}{2}) + x \tan(\frac{x}{2}) (\tan^2(\frac{a}{2} + \frac{x}{2})) + 2 \tan(\frac{x}{2}) \tan(\frac{a}{2} + \frac{x}{2}) - x \tan(\frac{x}{2}) - x (\tan^2(\frac{x}{2})) \tan(\frac{a}{2} + \frac{x}{2})}{(1 + \tan^2(\frac{x}{2})) (1 + \tan^2(\frac{a}{2} + \frac{x}{2}))}$ | 91   |

input `int(cos(x)*sin(a+x),x,method=_RETURNVERBOSE)`output `-1/4*cos(a+2*x)+1/2*x*sin(a)`**3.233.5 Fracas [A] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \cos(x) \sin(a+x) dx = -\frac{1}{2} \cos(a+x)^2 \cos(a) - \frac{1}{2} \cos(a+x) \sin(a+x) \sin(a) + \frac{1}{2} x \sin(a)$$

input `integrate(cos(x)*sin(a+x),x, algorithm="fricas")`output `-1/2*cos(a + x)^2*cos(a) - 1/2*cos(a + x)*sin(a + x)*sin(a) + 1/2*x*sin(a)`**3.233.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(14) = 28.

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \cos(x) \sin(a+x) dx = -\frac{x \sin(x) \cos(a+x)}{2} + \frac{x \sin(a+x) \cos(x)}{2} - \frac{\cos(x) \cos(a+x)}{2}$$

input `integrate(cos(x)*sin(a+x),x)`output `-x*sin(x)*cos(a + x)/2 + x*sin(a + x)*cos(x)/2 - cos(x)*cos(a + x)/2`

**3.233.7 Maxima [A] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \cos(x) \sin(a+x) dx = \frac{1}{2} x \sin(a) - \frac{1}{4} \cos(a+2x)$$

input `integrate(cos(x)*sin(a+x),x, algorithm="maxima")`output `1/2*x*sin(a) - 1/4*cos(a + 2*x)`**3.233.8 Giac [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \cos(x) \sin(a+x) dx = \frac{1}{2} x \sin(a) - \frac{1}{4} \cos(a+2x)$$

input `integrate(cos(x)*sin(a+x),x, algorithm="giac")`output `1/2*x*sin(a) - 1/4*cos(a + 2*x)`**3.233.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \cos(x) \sin(a+x) dx = \frac{x \sin(a)}{2} - \frac{\cos(a+2x)}{4}$$

input `int(sin(a + x)*cos(x),x)`output `(x*sin(a))/2 - cos(a + 2*x)/4`

### 3.234 $\int \cos(a + x) \sin(x) dx$

|                                                             |       |
|-------------------------------------------------------------|-------|
| 3.234.1 Optimal result . . . . .                            | .1261 |
| 3.234.2 Mathematica [A] (verified) . . . . .                | .1261 |
| 3.234.3 Rubi [A] (verified) . . . . .                       | 1262  |
| 3.234.4 Maple [A] (verified) . . . . .                      | 1263  |
| 3.234.5 Fricas [A] (verification not implemented) . . . . . | 1263  |
| 3.234.6 Sympy [B] (verification not implemented) . . . . .  | 1263  |
| 3.234.7 Maxima [A] (verification not implemented) . . . . . | 1264  |
| 3.234.8 Giac [A] (verification not implemented) . . . . .   | 1264  |
| 3.234.9 Mupad [B] (verification not implemented) . . . . .  | 1264  |

#### 3.234.1 Optimal result

Integrand size = 7, antiderivative size = 18

$$\int \cos(a + x) \sin(x) dx = -\frac{1}{4} \cos(a + 2x) - \frac{1}{2} x \sin(a)$$

output `-1/4*cos(a+2*x)-1/2*x*sin(a)`

#### 3.234.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \cos(a + x) \sin(x) dx = \frac{1}{4} (-\cos(a + 2x) - 2x \sin(a))$$

input `Integrate[Cos[a + x]*Sin[x],x]`

output `(-Cos[a + 2*x] - 2*x*Sin[a])/4`

**3.234.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x) \cos(a+x) dx$$

$$\downarrow \text{5085}$$

$$\int \left( \frac{1}{2} \sin(a+2x) - \frac{\sin(a)}{2} \right) dx$$

$$\downarrow \text{2009}$$

$$-\frac{1}{2}x \sin(a) - \frac{1}{4} \cos(a+2x)$$

input `Int[Cos[a + x]*Sin[x],x]`

output `-1/4*Cos[a + 2*x] - (x*Sin[a])/2`

**3.234.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5085 `Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] := Int[ExpandTrigReduce[Sin[v]^p *Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))`

**3.234.4 Maple [A] (verified)**

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

| method        | result                                                                                                                                                                                                                                                                                                                                                                                                                                           | size |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default       | $-\frac{\cos(a+2x)}{4} - \frac{x \sin(a)}{2}$                                                                                                                                                                                                                                                                                                                                                                                                    | 15   |
| risch         | $-\frac{\cos(a+2x)}{4} - \frac{x \sin(a)}{2}$                                                                                                                                                                                                                                                                                                                                                                                                    | 15   |
| parallelrisch | $-\frac{\cos(a+2x)}{4} + \frac{\cos(a)}{4} - \frac{x \sin(a)}{2}$                                                                                                                                                                                                                                                                                                                                                                                | 19   |
| meijerg       | $\frac{\cos(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}} - \frac{\cos(2x)}{\sqrt{\pi}}\right)}{4} - \frac{\sin(a)\sqrt{\pi}\left(\frac{2x}{\sqrt{\pi}} - \frac{\sin(2x)}{\sqrt{\pi}}\right)}{4}$                                                                                                                                                                                                                                                       | 45   |
| norman        | $\frac{x \tan\left(\frac{x}{2}\right) + x \left(\tan^2\left(\frac{x}{2}\right)\right) \tan\left(\frac{a}{2} + \frac{x}{2}\right) + 2 \tan\left(\frac{x}{2}\right) \tan\left(\frac{a}{2} + \frac{x}{2}\right) - x \tan\left(\frac{a}{2} + \frac{x}{2}\right) - x \tan\left(\frac{x}{2}\right) \left(\tan^2\left(\frac{a}{2} + \frac{x}{2}\right)\right)}{(1 + \tan^2\left(\frac{x}{2}\right))(1 + \tan^2\left(\frac{a}{2} + \frac{x}{2}\right))}$ | 91   |

input `int(cos(a+x)*sin(x),x,method=_RETURNVERBOSE)`output `-1/4*cos(a+2*x)-1/2*x*sin(a)`**3.234.5 Fricas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \cos(a+x) \sin(x) dx = -\frac{1}{2} \cos(a+x)^2 \cos(a) - \frac{1}{2} \cos(a+x) \sin(a+x) \sin(a) - \frac{1}{2} x \sin(a)$$

input `integrate(cos(a+x)*sin(x),x, algorithm="fricas")`output `-1/2*cos(a + x)^2*cos(a) - 1/2*cos(a + x)*sin(a + x)*sin(a) - 1/2*x*sin(a)`**3.234.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(15) = 30.

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \cos(a+x) \sin(x) dx = \frac{x \sin(x) \cos(a+x)}{2} - \frac{x \sin(a+x) \cos(x)}{2} - \frac{\cos(x) \cos(a+x)}{2}$$



input `integrate(cos(a+x)*sin(x),x)`

output `x*sin(x)*cos(a + x)/2 - x*sin(a + x)*cos(x)/2 - cos(x)*cos(a + x)/2`

### 3.234.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \cos(a+x) \sin(x) dx = -\frac{1}{2} x \sin(a) - \frac{1}{4} \cos(a+2x)$$

input `integrate(cos(a+x)*sin(x),x, algorithm="maxima")`

output `-1/2*x*sin(a) - 1/4*cos(a + 2*x)`

### 3.234.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \cos(a+x) \sin(x) dx = -\frac{1}{2} x \sin(a) - \frac{1}{4} \cos(a+2x)$$

input `integrate(cos(a+x)*sin(x),x, algorithm="giac")`

output `-1/2*x*sin(a) - 1/4*cos(a + 2*x)`

### 3.234.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \cos(a+x) \sin(x) dx = -\frac{\cos(a+2x)}{4} - \frac{x \sin(a)}{2}$$

input `int(cos(a + x)*sin(x),x)`

output `- cos(a + 2*x)/4 - (x*sin(a))/2`

### 3.235 $\int \sqrt{1 + \sin(x)} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.235.1 Optimal result . . . . .                            | 1265 |
| 3.235.2 Mathematica [B] (verified) . . . . .                | 1265 |
| 3.235.3 Rubi [A] (verified) . . . . .                       | 1266 |
| 3.235.4 Maple [A] (verified) . . . . .                      | 1267 |
| 3.235.5 Fricas [B] (verification not implemented) . . . . . | 1267 |
| 3.235.6 Sympy [F] . . . . .                                 | 1267 |
| 3.235.7 Maxima [F] . . . . .                                | 1268 |
| 3.235.8 Giac [B] (verification not implemented) . . . . .   | 1268 |
| 3.235.9 Mupad [B] (verification not implemented) . . . . .  | 1268 |

#### 3.235.1 Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \sqrt{1 + \sin(x)} dx = -\frac{2 \cos(x)}{\sqrt{1 + \sin(x)}}$$

output `-2*cos(x)/(1+sin(x))^(1/2)`

#### 3.235.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs.  $2(12) = 24$ .

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.33

$$\int \sqrt{1 + \sin(x)} dx = \frac{2(-\cos(\frac{x}{2}) + \sin(\frac{x}{2})) \sqrt{1 + \sin(x)}}{\cos(\frac{x}{2}) + \sin(\frac{x}{2})}$$

input `Integrate[Sqrt[1 + Sin[x]],x]`

output `(2*(-Cos[x/2] + Sin[x/2])*Sqrt[1 + Sin[x]])/(Cos[x/2] + Sin[x/2])`

**3.235.3 Rubi [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(x) + 1} dx$$

↓ 3042

$$\int \sqrt{\sin(x) + 1} dx$$

↓ 3125

$$-\frac{2 \cos(x)}{\sqrt{\sin(x) + 1}}$$

input `Int[Sqrt[1 + Sin[x]],x]`

output `(-2*Cos[x])/Sqrt[1 + Sin[x]]`

**3.235.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3125 `Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[-2*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

**3.235.4 Maple [A] (verified)**

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

| method  | result                                                                       | size |
|---------|------------------------------------------------------------------------------|------|
| default | $\frac{2(\sin(x)-1)\sqrt{\sin(x)+1}}{\cos(x)}$                               | 17   |
| risch   | $-\frac{i\sqrt{2}\sqrt{2\sin(x)+2}(e^{ix}-i)(i+e^{ix})}{e^{2ix}-1+2ie^{ix}}$ | 48   |

input `int((sin(x)+1)^(1/2),x,method=_RETURNVERBOSE)`

output `2*(sin(x)-1)*(sin(x)+1)^(1/2)/cos(x)`

**3.235.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 24 vs.  $2(10) = 20$ .

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \sqrt{1 + \sin(x)} dx = -\frac{2(\cos(x) - \sin(x) + 1)\sqrt{\sin(x) + 1}}{\cos(x) + \sin(x) + 1}$$

input `integrate((1+sin(x))^(1/2),x, algorithm="fracas")`

output `-2*(cos(x) - sin(x) + 1)*sqrt(sin(x) + 1)/(cos(x) + sin(x) + 1)`

**3.235.6 Sympy [F]**

$$\int \sqrt{1 + \sin(x)} dx = \int \sqrt{\sin(x) + 1} dx$$

input `integrate((1+sin(x))**(1/2),x)`

output `Integral(sqrt(sin(x) + 1), x)`

**3.235.7 Maxima [F]**

$$\int \sqrt{1 + \sin(x)} dx = \int \sqrt{\sin(x) + 1} dx$$

input `integrate((1+sin(x))^(1/2),x, algorithm="maxima")`

output `integrate(sqrt(sin(x) + 1), x)`

**3.235.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20.

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \sqrt{1 + \sin(x)} dx = 2\sqrt{2}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}x\right)\right)\sin\left(-\frac{1}{4}\pi + \frac{1}{2}x\right)$$

input `integrate((1+sin(x))^(1/2),x, algorithm="giac")`

output `2*sqrt(2)*sgn(cos(-1/4*pi + 1/2*x))*sin(-1/4*pi + 1/2*x)`

**3.235.9 Mupad [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \sqrt{1 + \sin(x)} dx = \frac{2(\sin(x) - 1)\sqrt{\sin(x) + 1}}{\cos(x)}$$

input `int((sin(x) + 1)^(1/2),x)`

output `(2*(sin(x) - 1)*(sin(x) + 1)^(1/2))/cos(x)`

### 3.236 $\int \sqrt{1 - \sin(x)} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.236.1 Optimal result . . . . .                            | 1269 |
| 3.236.2 Mathematica [B] (verified) . . . . .                | 1269 |
| 3.236.3 Rubi [A] (verified) . . . . .                       | 1270 |
| 3.236.4 Maple [A] (verified) . . . . .                      | 1271 |
| 3.236.5 Fricas [B] (verification not implemented) . . . . . | 1271 |
| 3.236.6 Sympy [F] . . . . .                                 | 1271 |
| 3.236.7 Maxima [F] . . . . .                                | 1272 |
| 3.236.8 Giac [B] (verification not implemented) . . . . .   | 1272 |
| 3.236.9 Mupad [B] (verification not implemented) . . . . .  | 1272 |

#### 3.236.1 Optimal result

Integrand size = 10, antiderivative size = 14

$$\int \sqrt{1 - \sin(x)} dx = \frac{2 \cos(x)}{\sqrt{1 - \sin(x)}}$$

output `2*cos(x)/(1-sin(x))^(1/2)`

#### 3.236.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(14) = 28.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \sqrt{1 - \sin(x)} dx = \frac{2(\cos(\frac{x}{2}) + \sin(\frac{x}{2})) \sqrt{1 - \sin(x)}}{\cos(\frac{x}{2}) - \sin(\frac{x}{2})}$$

input `Integrate[Sqrt[1 - Sin[x]],x]`

output `(2*(Cos[x/2] + Sin[x/2])*Sqrt[1 - Sin[x]])/(Cos[x/2] - Sin[x/2])`

**3.236.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1 - \sin(x)} dx$$

$$\downarrow \text{3042}$$

$$\int \sqrt{1 - \sin(x)} dx$$

$$\downarrow \text{3125}$$

$$\frac{2 \cos(x)}{\sqrt{1 - \sin(x)}}$$

input `Int[Sqrt[1 - Sin[x]],x]`

output `(2*Cos[x])/Sqrt[1 - Sin[x]]`

**3.236.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3125 `Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[-2*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

**3.236.4 Maple [A] (verified)**

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

| method  | result                                                                                                                                             | size |
|---------|----------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default | $-\frac{2(\sin(x)-1)(\sin(x)+1)}{\cos(x)\sqrt{-\sin(x)+1}}$                                                                                        | 23   |
| risch   | $\frac{i\sqrt{2-2\sin(x)}\sqrt{-i(2ie^{2ix}-e^{3ix}+e^{ix})}\sqrt{2}(e^{ix}-i)(i+e^{ix})}{(2ie^{ix}-e^{2ix}+1)\sqrt{i(e^{3ix}-2ie^{2ix}-e^{ix})}}$ | 102  |

input `int((-sin(x)+1)^(1/2),x,method=_RETURNVERBOSE)`

output `-2*(sin(x)-1)*(sin(x)+1)/cos(x)/(-sin(x)+1)^(1/2)`

**3.236.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \sqrt{1 - \sin(x)} dx = \frac{2(\cos(x) + \sin(x) + 1)\sqrt{-\sin(x) + 1}}{\cos(x) - \sin(x) + 1}$$

input `integrate((1-sin(x))^(1/2),x, algorithm="fracas")`

output `2*(cos(x) + sin(x) + 1)*sqrt(-sin(x) + 1)/(cos(x) - sin(x) + 1)`

**3.236.6 Sympy [F]**

$$\int \sqrt{1 - \sin(x)} dx = \int \sqrt{1 - \sin(x)} dx$$

input `integrate((1-sin(x))**(1/2),x)`

output `Integral(sqrt(1 - sin(x)), x)`



**3.236.7 Maxima [F]**

$$\int \sqrt{1 - \sin(x)} dx = \int \sqrt{-\sin(x) + 1} dx$$

input `integrate((1-sin(x))^(1/2),x, algorithm="maxima")`

output `integrate(sqrt(-sin(x) + 1), x)`

**3.236.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 35 vs.  $2(12) = 24$ .

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

$$\begin{aligned} & \int \sqrt{1 - \sin(x)} dx \\ &= -2\sqrt{2} \left( \cos\left(-\frac{1}{4}\pi + \frac{1}{2}x\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}x\right)\right) - \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}x\right)\right) \right) \end{aligned}$$

input `integrate((1-sin(x))^(1/2),x, algorithm="giac")`

output `-2*sqrt(2)*(cos(-1/4*pi + 1/2*x)*sgn(sin(-1/4*pi + 1/2*x)) - sgn(sin(-1/4*pi + 1/2*x)))`

**3.236.9 Mupad [B] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \sqrt{1 - \sin(x)} dx = \frac{2\sqrt{1 - \sin(x)}(\sin(x) + 1)}{\cos(x)}$$

input `int((1 - sin(x))^(1/2),x)`

output `(2*(1 - sin(x))^(1/2)*(sin(x) + 1))/cos(x)`

### 3.237 $\int \sqrt{1 + \cos(x)} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.237.1 Optimal result . . . . .                            | 1273 |
| 3.237.2 Mathematica [A] (verified) . . . . .                | 1273 |
| 3.237.3 Rubi [A] (verified) . . . . .                       | 1274 |
| 3.237.4 Maple [B] (verified) . . . . .                      | 1275 |
| 3.237.5 Fricas [A] (verification not implemented) . . . . . | 1275 |
| 3.237.6 Sympy [B] (verification not implemented) . . . . .  | 1275 |
| 3.237.7 Maxima [A] (verification not implemented) . . . . . | 1276 |
| 3.237.8 Giac [A] (verification not implemented) . . . . .   | 1276 |
| 3.237.9 Mupad [B] (verification not implemented) . . . . .  | 1276 |

#### 3.237.1 Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \sqrt{1 + \cos(x)} dx = \frac{2 \sin(x)}{\sqrt{1 + \cos(x)}}$$

output `2*sin(x)/(1+cos(x))^(1/2)`

#### 3.237.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \sqrt{1 + \cos(x)} dx = 2\sqrt{1 + \cos(x)} \tan\left(\frac{x}{2}\right)$$

input `Integrate[Sqrt[1 + Cos[x]],x]`

output `2*Sqrt[1 + Cos[x]]*Tan[x/2]`

**3.237.3 Rubi [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(x) + 1} dx$$

$$\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(x + \frac{\pi}{2}\right) + 1} dx$$

$$\downarrow \text{3125}$$

$$\frac{2 \sin(x)}{\sqrt{\cos(x) + 1}}$$

input `Int[Sqrt[1 + Cos[x]],x]`

output `(2*Sin[x])/Sqrt[1 + Cos[x]]`

**3.237.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 3125 `Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos [c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq Q[a^2 - b^2, 0]`

**3.237.4 Maple [B] (verified)**

Leaf count of result is larger than twice the leaf count of optimal. 21 vs.  $2(10) = 20$ .

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

| method  | result                                                                      | size |
|---------|-----------------------------------------------------------------------------|------|
| default | $\frac{4 \cos(\frac{x}{2}) \sin(\frac{x}{2}) \sqrt{2}}{\sqrt{2 \cos(x)+2}}$ | 22   |
| risch   | $-\frac{i\sqrt{2} \sqrt{(e^{ix}+1)^2 e^{-ix} (e^{ix}-1)}}{e^{ix}+1}$        | 40   |

input `int((cos(x)+1)^(1/2),x,method=_RETURNVERBOSE)`

output `2*cos(1/2*x)*sin(1/2*x)*2^(1/2)/(cos(1/2*x)^2)^(1/2)`

**3.237.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \sqrt{1 + \cos(x)} dx = \frac{2 \sin(x)}{\sqrt{\cos(x) + 1}}$$

input `integrate((1+cos(x))^(1/2),x, algorithm="fracas")`

output `2*sin(x)/sqrt(cos(x) + 1)`

**3.237.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 36 vs.  $2(12) = 24$ .

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 3.00

$$\int \sqrt{1 + \cos(x)} dx = 2\sqrt{1 - \frac{\tan^2(\frac{x}{2})}{\tan^2(\frac{x}{2}) + 1} + \frac{1}{\tan^2(\frac{x}{2}) + 1}} \tan\left(\frac{x}{2}\right)$$

input `integrate((1+cos(x))**(1/2),x)`

output `2*sqrt(1 - tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(tan(x/2)**2 + 1))*tan(x/2)`

**3.237.7 Maxima [A] (verification not implemented)**

Time = 0.33 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \sqrt{1 + \cos(x)} dx = 2\sqrt{2} \sin\left(\frac{1}{2}x\right)$$

input `integrate((1+cos(x))^(1/2),x, algorithm="maxima")`output `2*sqrt(2)*sin(1/2*x)`**3.237.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \sqrt{1 + \cos(x)} dx = 2\sqrt{2} \operatorname{sgn}\left(\cos\left(\frac{1}{2}x\right)\right) \sin\left(\frac{1}{2}x\right)$$

input `integrate((1+cos(x))^(1/2),x, algorithm="giac")`output `2*sqrt(2)*sgn(cos(1/2*x))*sin(1/2*x)`**3.237.9 Mupad [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \sqrt{1 + \cos(x)} dx = \frac{2 \sin(x)}{\sqrt{\cos(x) + 1}}$$

input `int((cos(x) + 1)^(1/2),x)`output `(2*sin(x))/(cos(x) + 1)^(1/2)`

### 3.238 $\int \sqrt{1 - \cos(x)} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.238.1 Optimal result . . . . .                            | 1277 |
| 3.238.2 Mathematica [A] (verified) . . . . .                | 1277 |
| 3.238.3 Rubi [A] (verified) . . . . .                       | 1278 |
| 3.238.4 Maple [A] (verified) . . . . .                      | 1279 |
| 3.238.5 Fricas [A] (verification not implemented) . . . . . | 1279 |
| 3.238.6 Sympy [F] . . . . .                                 | 1279 |
| 3.238.7 Maxima [A] (verification not implemented) . . . . . | 1280 |
| 3.238.8 Giac [A] (verification not implemented) . . . . .   | 1280 |
| 3.238.9 Mupad [B] (verification not implemented) . . . . .  | 1280 |

#### 3.238.1 Optimal result

Integrand size = 10, antiderivative size = 14

$$\int \sqrt{1 - \cos(x)} dx = -\frac{2 \sin(x)}{\sqrt{1 - \cos(x)}}$$

output `-2*sin(x)/(1-cos(x))^(1/2)`

#### 3.238.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \sqrt{1 - \cos(x)} dx = -2\sqrt{1 - \cos(x)} \cot\left(\frac{x}{2}\right)$$

input `Integrate[Sqrt[1 - Cos[x]],x]`

output `-2*Sqrt[1 - Cos[x]]*Cot[x/2]`

**3.238.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1 - \cos(x)} dx$$

$$\downarrow \text{3042}$$

$$\int \sqrt{1 - \sin\left(x + \frac{\pi}{2}\right)} dx$$

$$\downarrow \text{3125}$$

$$-\frac{2 \sin(x)}{\sqrt{1 - \cos(x)}}$$

input `Int[Sqrt[1 - Cos[x]],x]`

output `(-2*Sin[x])/Sqrt[1 - Cos[x]]`

**3.238.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 3125 `Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos [c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq Q[a^2 - b^2, 0]`

**3.238.4 Maple [A] (verified)**

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

| method  | result                                                                       | size |
|---------|------------------------------------------------------------------------------|------|
| default | $-\frac{4 \sin(\frac{x}{2}) \cos(\frac{x}{2}) \sqrt{2}}{\sqrt{2-2 \cos(x)}}$ | 22   |
| risch   | $-\frac{i\sqrt{2} \sqrt{-(e^{ix}-1)^2 e^{-ix} (e^{ix}+1)}}{e^{ix}-1}$        | 41   |

input `int((1-cos(x))^(1/2),x,method=_RETURNVERBOSE)`output `-2*sin(1/2*x)*cos(1/2*x)*2^(1/2)/(sin(1/2*x)^2)^(1/2)`**3.238.5 Fricas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \sqrt{1 - \cos(x)} dx = -\frac{2(\cos(x) + 1)\sqrt{-\cos(x) + 1}}{\sin(x)}$$

input `integrate((1-cos(x))^(1/2),x, algorithm="fricas")`output `-2*(cos(x) + 1)*sqrt(-cos(x) + 1)/sin(x)`**3.238.6 Sympy [F]**

$$\int \sqrt{1 - \cos(x)} dx = \int \sqrt{1 - \cos(x)} dx$$

input `integrate((1-cos(x))**(1/2),x)`output `Integral(sqrt(1 - cos(x)), x)`



**3.238.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \sqrt{1 - \cos(x)} dx = -\frac{2\sqrt{2}}{\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1}}$$

input `integrate((1-cos(x))^(1/2),x, algorithm="maxima")`output `-2*sqrt(2)/sqrt(sin(x)^2/(cos(x) + 1)^2 + 1)`**3.238.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \sqrt{1 - \cos(x)} dx = -2\sqrt{2} \left( \cos\left(\frac{1}{2}x\right) \operatorname{sgn}\left(\sin\left(\frac{1}{2}x\right)\right) - \operatorname{sgn}\left(\sin\left(\frac{1}{2}x\right)\right) \right)$$

input `integrate((1-cos(x))^(1/2),x, algorithm="giac")`output `-2*sqrt(2)*(cos(1/2*x)*sgn(sin(1/2*x)) - sgn(sin(1/2*x)))`**3.238.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \sqrt{1 - \cos(x)} dx = -\frac{2 \sin(x)}{\sqrt{1 - \cos(x)}}$$

input `int((1 - cos(x))^(1/2),x)`output `-(2*sin(x))/(1 - cos(x))^(1/2)`

$$3.239 \quad \int \frac{1}{-\sqrt{-1+x}+\sqrt{x}} dx$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.239.1 Optimal result . . . . .                            | 1281 |
| 3.239.2 Mathematica [A] (verified) . . . . .                | 1281 |
| 3.239.3 Rubi [A] (verified) . . . . .                       | 1282 |
| 3.239.4 Maple [A] (verified) . . . . .                      | 1283 |
| 3.239.5 Fricas [A] (verification not implemented) . . . . . | 1283 |
| 3.239.6 Sympy [B] (verification not implemented) . . . . .  | 1283 |
| 3.239.7 Maxima [F] . . . . .                                | 1284 |
| 3.239.8 Giac [A] (verification not implemented) . . . . .   | 1284 |
| 3.239.9 Mupad [B] (verification not implemented) . . . . .  | 1284 |

### 3.239.1 Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}} dx = \frac{2}{3}(-1+x)^{3/2} + \frac{2x^{3/2}}{3}$$

output `2/3*(-1+x)^(3/2)+2/3*x^(3/2)`

### 3.239.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}} dx = \frac{2}{3}((-1+x)^{3/2} + x^{3/2})$$

input `Integrate[(-Sqrt[-1 + x] + Sqrt[x])^(-1),x]`

output `(2*((-1 + x)^(3/2) + x^(3/2)))/3`

**3.239.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {2531, 15, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{1}{\sqrt{x} - \sqrt{x-1}} dx \\ & \quad \downarrow \text{2531} \\ & \int \sqrt{x-1} dx + \int \sqrt{x} dx \\ & \quad \downarrow \text{15} \\ & \int \sqrt{x-1} dx + \frac{2x^{3/2}}{3} \\ & \quad \downarrow \text{17} \\ & \frac{2x^{3/2}}{3} + \frac{2}{3}(x-1)^{3/2} \end{aligned}$$

input `Int[(-Sqrt[-1 + x] + Sqrt[x])^(-1), x]`

output `(2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3`

**3.239.3.1 Defintions of rubi rules used**

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1))/(m + 1), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

rule 2531 `Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symbol] := Simp[-b/(a*d) Int[u*x^n, x], x] + Simp[1/(a*c) Int[u*Sqrt[a + b*x^(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d^2, 0]`

### 3.239.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

| method  | result                                                         | size |
|---------|----------------------------------------------------------------|------|
| default | $\frac{2(-1+x)^{\frac{3}{2}}}{3} + \frac{2x^{\frac{3}{2}}}{3}$ | 14   |

input `int(1/(-(-1+x)^(1/2)+x^(1/2)),x,method=_RETURNVERBOSE)`

output `2/3*(-1+x)^(3/2)+2/3*x^(3/2)`

### 3.239.5 Fracas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{-\sqrt{-1+x} + \sqrt{x}} dx = \frac{2}{3} (x-1)^{\frac{3}{2}} + \frac{2}{3} x^{\frac{3}{2}}$$

input `integrate(1/(-(-1+x)^(1/2)+x^(1/2)),x, algorithm="fricas")`

output `2/3*(x - 1)^(3/2) + 2/3*x^(3/2)`

### 3.239.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs.  $2(17) = 34$ .

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.00

$$\int \frac{1}{-\sqrt{-1+x} + \sqrt{x}} dx = \frac{2\sqrt{x}\sqrt{x-1}}{-3\sqrt{x} + 3\sqrt{x-1}} - \frac{4x}{-3\sqrt{x} + 3\sqrt{x-1}} + \frac{2}{-3\sqrt{x} + 3\sqrt{x-1}}$$

input `integrate(1/(-(-1+x)**(1/2)+x**(1/2)),x)`

output `2*sqrt(x)*sqrt(x - 1)/(-3*sqrt(x) + 3*sqrt(x - 1)) - 4*x/(-3*sqrt(x) + 3*sqrt(x - 1)) + 2/(-3*sqrt(x) + 3*sqrt(x - 1))`

### 3.239.7 Maxima [F]

$$\int \frac{1}{-\sqrt{-1+x} + \sqrt{x}} dx = \int -\frac{1}{\sqrt{x-1} - \sqrt{x}} dx$$

input `integrate(1/(-(-1+x)^(1/2)+x^(1/2)),x, algorithm="maxima")`

output `-integrate(1/(sqrt(x - 1) - sqrt(x)), x)`

### 3.239.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{-\sqrt{-1+x} + \sqrt{x}} dx = \frac{2}{3} (x-1)^{\frac{3}{2}} + \frac{2}{3} x^{\frac{3}{2}}$$

input `integrate(1/(-(-1+x)^(1/2)+x^(1/2)),x, algorithm="giac")`

output `2/3*(x - 1)^(3/2) + 2/3*x^(3/2)`

### 3.239.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{-\sqrt{-1+x} + \sqrt{x}} dx = \frac{2x\sqrt{x-1}}{3} - \frac{2\sqrt{x-1}}{3} + \frac{2x^{3/2}}{3}$$

input `int(-1/((x - 1)^(1/2) - x^(1/2)),x)`

output `(2*x*(x - 1)^(1/2))/3 - (2*(x - 1)^(1/2))/3 + (2*x^(3/2))/3`

### 3.240 $\int \frac{1}{1-\sqrt{1+x}} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.240.1 Optimal result . . . . .                            | 1285 |
| 3.240.2 Mathematica [A] (verified) . . . . .                | 1285 |
| 3.240.3 Rubi [A] (warning: unable to verify) . . . . .      | 1286 |
| 3.240.4 Maple [A] (verified) . . . . .                      | 1287 |
| 3.240.5 Fricas [A] (verification not implemented) . . . . . | 1287 |
| 3.240.6 Sympy [A] (verification not implemented) . . . . .  | 1288 |
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| 3.240.8 Giac [A] (verification not implemented) . . . . .   | 1288 |
| 3.240.9 Mupad [B] (verification not implemented) . . . . .  | 1289 |

#### 3.240.1 Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{1}{1-\sqrt{1+x}} dx = -2\sqrt{1+x} - 2\log(1-\sqrt{1+x})$$

output `-2*ln(1-(1+x)^(1/2))-2*(1+x)^(1/2)`

#### 3.240.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{1-\sqrt{1+x}} dx = -2\sqrt{1+x} - 2\log(-1+\sqrt{1+x})$$

input `Integrate[(1 - Sqrt[1 + x])^(-1), x]`

output `-2*Sqrt[1 + x] - 2*Log[-1 + Sqrt[1 + x]]`

**3.240.3 Rubi [A] (warning: unable to verify)**

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {239, 774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{1 - \sqrt{x+1}} dx \\
 & \quad \downarrow \text{239} \\
 & \int \frac{1}{1 - \sqrt{x+1}} d(x+1) \\
 & \quad \downarrow \text{774} \\
 & 2 \int -\frac{\sqrt{x+1}}{x} d\sqrt{x+1} \\
 & \quad \downarrow \text{49} \\
 & 2 \int \left(-1 - \frac{1}{x}\right) d\sqrt{x+1} \\
 & \quad \downarrow \text{2009} \\
 & 2(-x - \log(-x) - 1)
 \end{aligned}$$

input `Int[(1 - Sqrt[1 + x])^(-1),x]`

output `2*(-1 - x - Log[-x])`

**3.240.3.1 Defintions of rubi rules used**

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int  
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]  
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 239 `Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1]  
] Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin  
earQ[v, x] && NeQ[v, x]`

```
rule 774 Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[n]},
Simp[k Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, p}, x] && FractionQ[n]
```

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

### 3.240.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

| method            | result                                                               | size |
|-------------------|----------------------------------------------------------------------|------|
| derivativedivides | $-2\sqrt{1+x} - 2\ln(-1 + \sqrt{1+x})$                               | 19   |
| trager            | $-2\sqrt{1+x} - \ln(2\sqrt{1+x} - 2 - x)$                            | 24   |
| default           | $-\ln(x) - 2\sqrt{1+x} - \ln(-1 + \sqrt{1+x}) + \ln(1 + \sqrt{1+x})$ | 31   |

```
input int(1/(1-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
output -2*(1+x)^(1/2)-2*ln(-1+(1+x)^(1/2))
```

### 3.240.5 Fracas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1}{1 - \sqrt{1+x}} dx = -2\sqrt{x+1} - 2 \log(\sqrt{x+1} - 1)$$

```
input integrate(1/(1-(1+x)^(1/2)),x, algorithm="fracas")
```

```
output -2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)
```



**3.240.6 Sympy [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{1 - \sqrt{1+x}} dx = -2\sqrt{x+1} - 2\log(\sqrt{x+1} - 1)$$

input `integrate(1/(1-(1+x)**(1/2)),x)`output `-2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)`**3.240.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1}{1 - \sqrt{1+x}} dx = -2\sqrt{x+1} - 2\log(\sqrt{x+1} - 1)$$

input `integrate(1/(1-(1+x)^(1/2)),x, algorithm="maxima")`output `-2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)`**3.240.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1}{1 - \sqrt{1+x}} dx = -2\sqrt{x+1} - 2\log\left(\left|\sqrt{x+1} - 1\right|\right)$$

input `integrate(1/(1-(1+x)^(1/2)),x, algorithm="giac")`output `-2*sqrt(x + 1) - 2*log(abs(sqrt(x + 1) - 1))`

**3.240.9 Mupad [B] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1}{1 - \sqrt{1+x}} dx = -2 \ln(\sqrt{x+1} - 1) - 2\sqrt{x+1}$$

input `int(-1/((x + 1)^(1/2) - 1),x)`

output `- 2*log((x + 1)^(1/2) - 1) - 2*(x + 1)^(1/2)`

### 3.241 $\int \frac{x}{\sqrt{36+x^4}} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.241.1 Optimal result . . . . .                            | 1290 |
| 3.241.2 Mathematica [A] (verified) . . . . .                | 1290 |
| 3.241.3 Rubi [A] (verified) . . . . .                       | 1291 |
| 3.241.4 Maple [A] (verified) . . . . .                      | 1292 |
| 3.241.5 Fricas [A] (verification not implemented) . . . . . | 1292 |
| 3.241.6 Sympy [A] (verification not implemented) . . . . .  | 1292 |
| 3.241.7 Maxima [B] (verification not implemented) . . . . . | 1293 |
| 3.241.8 Giac [A] (verification not implemented) . . . . .   | 1293 |
| 3.241.9 Mupad [B] (verification not implemented) . . . . .  | 1293 |

#### 3.241.1 Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{x}{\sqrt{36+x^4}} dx = \frac{1}{2} \operatorname{arcsinh}\left(\frac{x^2}{6}\right)$$

output `1/2*arcsinh(1/6*x^2)`

#### 3.241.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{\sqrt{36+x^4}} dx = \frac{1}{2} \log\left(x^2 + \sqrt{36+x^4}\right)$$

input `Integrate[x/Sqrt[36 + x^4],x]`

output `Log[x^2 + Sqrt[36 + x^4]]/2`

**3.241.3 Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {807, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^4 + 36}} dx$$

↓ 807

$$\frac{1}{2} \int \frac{1}{\sqrt{x^4 + 36}} dx^2$$

↓ 222

$$\frac{1}{2} \operatorname{arcsinh}\left(\frac{x^2}{6}\right)$$

input `Int[x/Sqrt[36 + x^4],x]`

output `ArcSinh[x^2/6]/2`

**3.241.3.1 Defintions of rubi rules used**

rule 222 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]`

rule 807 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Simp[1/k Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]`

**3.241.4 Maple [A] (verified)**

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

| method         | result                                                       | size |
|----------------|--------------------------------------------------------------|------|
| default        | $\frac{\operatorname{arcsinh}\left(\frac{x^2}{6}\right)}{2}$ | 9    |
| meijerg        | $\frac{\operatorname{arcsinh}\left(\frac{x^2}{6}\right)}{2}$ | 9    |
| elliptic       | $\frac{\operatorname{arcsinh}\left(\frac{x^2}{6}\right)}{2}$ | 9    |
| pseudoelliptic | $\frac{\operatorname{arcsinh}\left(\frac{x^2}{6}\right)}{2}$ | 9    |
| trager         | $\frac{\ln\left(x^2 + \sqrt{x^4 + 36}\right)}{2}$            | 15   |

input `int(x/(x^4+36)^(1/2),x,method=_RETURNVERBOSE)`output `1/2*arcsinh(1/6*x^2)`**3.241.5 Fricas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{x}{\sqrt{36+x^4}} dx = -\frac{1}{2} \log\left(-x^2 + \sqrt{x^4 + 36}\right)$$

input `integrate(x/(x^4+36)^(1/2),x, algorithm="fricas")`output `-1/2*log(-x^2 + sqrt(x^4 + 36))`**3.241.6 Sympy [A] (verification not implemented)**

Time = 0.49 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{x}{\sqrt{36+x^4}} dx = \frac{\operatorname{asinh}\left(\frac{x^2}{6}\right)}{2}$$

input `integrate(x/(x**4+36)**(1/2),x)`

output `asinh(x**2/6)/2`

### 3.241.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs.  $2(8) = 16$ .

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.75

$$\int \frac{x}{\sqrt{36+x^4}} dx = \frac{1}{4} \log \left( \frac{\sqrt{x^4+36}}{x^2} + 1 \right) - \frac{1}{4} \log \left( \frac{\sqrt{x^4+36}}{x^2} - 1 \right)$$

input `integrate(x/(x^4+36)^(1/2),x, algorithm="maxima")`

output `1/4*log(sqrt(x^4 + 36)/x^2 + 1) - 1/4*log(sqrt(x^4 + 36)/x^2 - 1)`

### 3.241.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{x}{\sqrt{36+x^4}} dx = -\frac{1}{2} \log \left( -x^2 + \sqrt{x^4+36} \right)$$

input `integrate(x/(x^4+36)^(1/2),x, algorithm="giac")`

output `-1/2*log(-x^2 + sqrt(x^4 + 36))`

### 3.241.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{36+x^4}} dx = \frac{\operatorname{asinh}\left(\frac{x^2}{6}\right)}{2}$$

input `int(x/(x^4 + 36)^(1/2),x)`

output `asinh(x^2/6)/2`

### 3.242 $\int \frac{1}{\sqrt[3]{x+\sqrt{x}}} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.242.1 Optimal result . . . . .                            | 1295 |
| 3.242.2 Mathematica [A] (verified) . . . . .                | 1295 |
| 3.242.3 Rubi [A] (verified) . . . . .                       | 1296 |
| 3.242.4 Maple [A] (verified) . . . . .                      | 1297 |
| 3.242.5 Fracas [A] (verification not implemented) . . . . . | 1297 |
| 3.242.6 Sympy [F] . . . . .                                 | 1298 |
| 3.242.7 Maxima [A] (verification not implemented) . . . . . | 1298 |
| 3.242.8 Giac [A] (verification not implemented) . . . . .   | 1298 |
| 3.242.9 Mupad [B] (verification not implemented) . . . . .  | 1299 |

#### 3.242.1 Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{1}{\sqrt[3]{x+\sqrt{x}}} dx = 6\sqrt[6]{x} - 3\sqrt[3]{x} + 2\sqrt{x} - 6 \log(1 + \sqrt[6]{x})$$

output `6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)`

#### 3.242.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt[3]{x+\sqrt{x}}} dx = (6 - 3\sqrt[6]{x} + 2\sqrt[3]{x}) \sqrt[6]{x} - 6 \log(1 + \sqrt[6]{x})$$

input `Integrate[(x^(1/3) + Sqrt[x])^(-1),x]`

output `(6 - 3*x^(1/6) + 2*x^(1/3))*x^(1/6) - 6*Log[1 + x^(1/6)]`



### 3.242.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2027, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{\sqrt{x} + \sqrt[3]{x}} dx \\
 & \quad \downarrow \text{2027} \\
 & \int \frac{1}{(\sqrt[6]{x} + 1)\sqrt[3]{x}} dx \\
 & \quad \downarrow \text{798} \\
 & 6 \int \frac{\sqrt{x}}{\sqrt[6]{x} + 1} d\sqrt[6]{x} \\
 & \quad \downarrow \text{49} \\
 & 6 \int \left( \sqrt[3]{x} - \sqrt[6]{x} + \frac{1}{-\sqrt[6]{x} - 1} + 1 \right) d\sqrt[6]{x} \\
 & \quad \downarrow \text{2009} \\
 & 6 \left( \frac{\sqrt{x}}{3} - \frac{\sqrt[3]{x}}{2} + \sqrt[6]{x} - \log(\sqrt[6]{x} + 1) \right)
 \end{aligned}$$

input `Int[(x^(1/3) + Sqrt[x])^(-1),x]`

output `6*(x^(1/6) - x^(1/3)/2 + Sqrt[x]/3 - Log[1 + x^(1/6)])`

#### 3.242.3.1 Defintions of rubi rules used

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 798 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[1/n Subst  
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,  
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2027 `Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] := Int[x^  
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &  
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])`

### 3.242.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

| method            | result                                                                                                                                                                 |
|-------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| derivativedivides | $6x^{\frac{1}{6}} - 3x^{\frac{1}{3}} - 6 \ln(1 + x^{\frac{1}{6}}) + 2\sqrt{x}$                                                                                         |
| meijerg           | $\frac{x^{\frac{1}{6}}(4x^{\frac{1}{3}} - 6x^{\frac{1}{6}} + 12)}{2} - 6 \ln(1 + x^{\frac{1}{6}})$                                                                     |
| default           | $-\ln(x^{\frac{1}{3}} + x^{\frac{1}{6}} + 1) + 2 \ln(x^{\frac{1}{6}} - 1) - 2 \ln(1 + x^{\frac{1}{6}}) + \ln(x^{\frac{1}{3}} - x^{\frac{1}{6}} + 1) + 2\sqrt{x} + \ln$ |

input `int(1/(x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)`

output `6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)`

### 3.242.5 Fracas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}} dx = 2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6 \log(x^{\frac{1}{6}} + 1)$$

input `integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")`

output `2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)`

**3.242.6 Sympy [F]**

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}} dx = \int \frac{1}{\sqrt[3]{x} + \sqrt{x}} dx$$

input `integrate(1/(x**(1/3)+x**(1/2)),x)`

output `Integral(1/(x**(1/3) + sqrt(x)), x)`

**3.242.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}} dx = 2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

input `integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")`

output `2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)`

**3.242.8 Giac [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}} dx = 2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

input `integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="giac")`

output `2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)`

**3.242.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}} dx = 2\sqrt{x} - 6 \ln(x^{1/6} + 1) - 3x^{1/3} + 6x^{1/6}$$

input `int(1/(x^(1/2) + x^(1/3)),x)`

output `2*x^(1/2) - 6*log(x^(1/6) + 1) - 3*x^(1/3) + 6*x^(1/6)`

## 3.243 $\int \log(2 + 3x^2) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.243.1 Optimal result . . . . .                            | 1300 |
| 3.243.2 Mathematica [A] (verified) . . . . .                | 1300 |
| 3.243.3 Rubi [A] (verified) . . . . .                       | 1301 |
| 3.243.4 Maple [A] (verified) . . . . .                      | 1302 |
| 3.243.5 Fricas [A] (verification not implemented) . . . . . | 1302 |
| 3.243.6 Sympy [A] (verification not implemented) . . . . .  | 1303 |
| 3.243.7 Maxima [A] (verification not implemented) . . . . . | 1303 |
| 3.243.8 Giac [A] (verification not implemented) . . . . .   | 1303 |
| 3.243.9 Mupad [B] (verification not implemented) . . . . .  | 1304 |

### 3.243.1 Optimal result

Integrand size = 8, antiderivative size = 33

$$\int \log(2 + 3x^2) dx = -2x + 2\sqrt{\frac{2}{3}} \arctan\left(\sqrt{\frac{3}{2}}x\right) + x \log(2 + 3x^2)$$

output `-2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)`

### 3.243.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \log(2 + 3x^2) dx = -2x + 2\sqrt{\frac{2}{3}} \arctan\left(\sqrt{\frac{3}{2}}x\right) + x \log(2 + 3x^2)$$

input `Integrate[Log[2 + 3*x^2],x]`

output `-2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]`

**3.243.3 Rubi [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {2898, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \log(3x^2 + 2) dx \\ & \quad \downarrow \text{2898} \\ & x \log(3x^2 + 2) - 6 \int \frac{x^2}{3x^2 + 2} dx \\ & \quad \downarrow \text{262} \\ & x \log(3x^2 + 2) - 6 \left( \frac{x}{3} - \frac{2}{3} \int \frac{1}{3x^2 + 2} dx \right) \\ & \quad \downarrow \text{216} \\ & x \log(3x^2 + 2) - 6 \left( \frac{x}{3} - \frac{1}{3} \sqrt{\frac{2}{3}} \arctan \left( \sqrt{\frac{3}{2}} x \right) \right) \end{aligned}$$

input `Int[Log[2 + 3*x^2], x]`

output `-6*(x/3 - (Sqrt[2/3]*ArcTan[Sqrt[3/2]*x])/3) + x*Log[2 + 3*x^2]`

**3.243.3.1 Defintions of rubi rules used**

rule 216 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

rule 262 `Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[c*(c*x)^(m-1)*((a + b*x^2)^(p+1)/(b*(m+2*p+1))), x] - Simp[a*c^2*((m-1)/(b*(m+2*p+1)) Int[(c*x)^(m-2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[m, 2-1] && NeQ[m+2*p+1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]`

rule 2898 `Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] := Simp[x*Log[c*(d + e*x^n)^p], x] - Simp[e*n*p Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]`

### 3.243.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

| method  | result                                                                                | size |
|---------|---------------------------------------------------------------------------------------|------|
| default | $-2x + x \ln(3x^2 + 2) + \frac{2 \arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$ | 27   |
| risch   | $-2x + x \ln(3x^2 + 2) + \frac{2 \arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$ | 27   |
| parts   | $-2x + x \ln(3x^2 + 2) + \frac{2 \arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$ | 27   |

input `int(ln(3*x^2+2),x,method=_RETURNVERBOSE)`

output `-2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)`

### 3.243.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \log(2 + 3x^2) dx = \frac{2}{3} \sqrt{3}\sqrt{2} \arctan\left(\frac{1}{2} \sqrt{3}\sqrt{2}x\right) + x \log(3x^2 + 2) - 2x$$

input `integrate(log(3*x^2+2),x, algorithm="fracas")`

output `2/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x) + x*log(3*x^2 + 2) - 2*x`

**3.243.6 Sympy [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \log(2 + 3x^2) dx = x \log(3x^2 + 2) - 2x + \frac{2\sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{3}$$

input `integrate(ln(3*x**2+2),x)`output `x*log(3*x**2 + 2) - 2*x + 2*sqrt(6)*atan(sqrt(6)*x/2)/3`**3.243.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \log(2 + 3x^2) dx = x \log(3x^2 + 2) + \frac{2}{3} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6}x\right) - 2x$$

input `integrate(log(3*x^2+2),x, algorithm="maxima")`output `x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x`**3.243.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \log(2 + 3x^2) dx = x \log(3x^2 + 2) + \frac{2}{3} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6}x\right) - 2x$$

input `integrate(log(3*x^2+2),x, algorithm="giac")`output `x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x`



**3.243.9 Mupad [B] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \log(2 + 3x^2) dx = \frac{2\sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{3} - 2x + x \ln(3x^2 + 2)$$

input `int(log(3*x^2 + 2),x)`

output `(2*6^(1/2)*atan((6^(1/2)*x)/2))/3 - 2*x + x*log(3*x^2 + 2)`

## 3.244 $\int \cot(x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.244.1 Optimal result . . . . .                            | 1305 |
| 3.244.2 Mathematica [B] (verified) . . . . .                | 1305 |
| 3.244.3 Rubi [A] (verified) . . . . .                       | 1306 |
| 3.244.4 Maple [A] (verified) . . . . .                      | 1307 |
| 3.244.5 Fricas [B] (verification not implemented) . . . . . | 1307 |
| 3.244.6 Sympy [A] (verification not implemented) . . . . .  | 1308 |
| 3.244.7 Maxima [A] (verification not implemented) . . . . . | 1308 |
| 3.244.8 Giac [A] (verification not implemented) . . . . .   | 1308 |
| 3.244.9 Mupad [B] (verification not implemented) . . . . .  | 1309 |

### 3.244.1 Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \cot(x) dx = \log(\sin(x))$$

output `ln(sin(x))`

### 3.244.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7 vs.  $2(3) = 6$ .

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \cot(x) dx = \log(\cos(x)) + \log(\tan(x))$$

input `Integrate[Cot[x], x]`

output `Log[Cos[x]] + Log[Tan[x]]`

**3.244.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$ , Rules used = {3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \cot(x) dx \\ \downarrow 3042 \\ \int -\tan\left(x + \frac{\pi}{2}\right) dx \\ \downarrow 25 \\ -\int \tan\left(x + \frac{\pi}{2}\right) dx \\ \downarrow 3956 \\ \log(\sin(x)) \end{array}$$

input `Int[Cot[x], x]`

output `Log[Sin[x]]`

**3.244.3.1 Defintions of rubi rules used**

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3956 `Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

**3.244.4 Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

| method            | result                                                      | size |
|-------------------|-------------------------------------------------------------|------|
| lookup            | $\ln(\sin(x))$                                              | 4    |
| default           | $\ln(\sin(x))$                                              | 4    |
| derivativedivides | $-\frac{\ln(\cot^2(x)+1)}{2}$                               | 10   |
| parallelsch       | $\ln\left(\frac{1}{\sqrt{\sec^2(x)}}\right) + \ln(\tan(x))$ | 12   |
| norman            | $-\frac{\ln(1+\tan^2(x))}{2} + \ln(\tan(x))$                | 14   |
| risch             | $-ix + \ln(e^{2ix} - 1)$                                    | 14   |

input `int(cot(x),x,method=_RETURNVERBOSE)`

output `ln(sin(x))`

**3.244.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 11 vs.  $2(3) = 6$ .

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \cot(x) dx = \frac{1}{2} \log\left(-\frac{1}{2} \cos(2x) + \frac{1}{2}\right)$$

input `integrate(cot(x),x, algorithm="fricas")`

output `1/2*log(-1/2*cos(2*x) + 1/2)`

**3.244.6 Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x) dx = \log(\sin(x))$$

input `integrate(cot(x),x)`

output `log(sin(x))`

**3.244.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x) dx = \log(\sin(x))$$

input `integrate(cot(x),x, algorithm="maxima")`

output `log(sin(x))`

**3.244.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int \cot(x) dx = \log(|\sin(x)|)$$

input `integrate(cot(x),x, algorithm="giac")`

output `log(abs(sin(x)))`

**3.244.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cot(x) dx = \ln(\sin(x))$$

input `int(cot(x),x)`

output `log(sin(x))`

## 3.245 $\int \cot^4(x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.245.1 Optimal result . . . . .                            | 1310 |
| 3.245.2 Mathematica [C] (verified) . . . . .                | 1310 |
| 3.245.3 Rubi [A] (verified) . . . . .                       | 1311 |
| 3.245.4 Maple [A] (verified) . . . . .                      | 1312 |
| 3.245.5 Fricas [B] (verification not implemented) . . . . . | 1313 |
| 3.245.6 Sympy [A] (verification not implemented) . . . . .  | 1313 |
| 3.245.7 Maxima [A] (verification not implemented) . . . . . | 1313 |
| 3.245.8 Giac [B] (verification not implemented) . . . . .   | 1314 |
| 3.245.9 Mupad [B] (verification not implemented) . . . . .  | 1314 |

### 3.245.1 Optimal result

Integrand size = 4, antiderivative size = 12

$$\int \cot^4(x) dx = x + \cot(x) - \frac{\cot^3(x)}{3}$$

output `x+cot(x)-1/3*cot(x)^3`

### 3.245.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \cot^4(x) dx = -\frac{1}{3} \cot^3(x) \operatorname{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, -\tan^2(x)\right)$$

input `Integrate[Cot[x]^4,x]`

output `-1/3*(Cot[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[x]^2])`

**3.245.3 Rubi [A] (verified)**

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$ , Rules used = {3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \cot^4(x) dx \\
 & \quad \downarrow \text{3042} \\
 & \int \tan\left(x + \frac{\pi}{2}\right)^4 dx \\
 & \quad \downarrow \text{3954} \\
 & - \int \cot^2(x) dx - \frac{1}{3} \cot^3(x) \\
 & \quad \downarrow \text{3042} \\
 & - \int \tan\left(x + \frac{\pi}{2}\right)^2 dx - \frac{1}{3} \cot^3(x) \\
 & \quad \downarrow \text{3954} \\
 & \int 1 dx - \frac{1}{3} \cot^3(x) + \cot(x) \\
 & \quad \downarrow \text{24} \\
 & x - \frac{1}{3} \cot^3(x) + \cot(x)
 \end{aligned}$$

input `Int[Cot[x]^4,x]`

output `x + Cot[x] - Cot[x]^3/3`



## 3.245.3.1 Defintions of rubi rules used

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3954 `Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2 Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]`

## 3.245.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

| method            | result                                                                              | size |
|-------------------|-------------------------------------------------------------------------------------|------|
| parallelrisch     | $x + \cot(x) - \frac{(\cot^3(x))}{3}$                                               | 11   |
| derivativedivides | $-\frac{(\cot^3(x))}{3} + \cot(x) - \frac{\pi}{2} + \operatorname{arccot}(\cot(x))$ | 16   |
| default           | $-\frac{(\cot^3(x))}{3} + \cot(x) - \frac{\pi}{2} + \operatorname{arccot}(\cot(x))$ | 16   |
| norman            | $\frac{-\frac{1}{3} + \tan^2(x) + x(\tan^3(x))}{\tan(x)^3}$                         | 18   |
| risch             | $x + \frac{4i(3e^{4ix} - 3e^{2ix} + 2)}{3(e^{2ix} - 1)^3}$                          | 31   |

input `int(cot(x)^4, x, method=_RETURNVERBOSE)`

output `x+cot(x)-1/3*cot(x)^3`

**3.245.5 Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 48 vs.  $2(10) = 20$ .

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 4.00

$$\int \cot^4(x) dx = \frac{4 \cos(2x)^2 + 3(x \cos(2x) - x) \sin(2x) + 2 \cos(2x) - 2}{3(\cos(2x) - 1) \sin(2x)}$$

input `integrate(cot(x)^4,x, algorithm="fricas")`

output `1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)/((cos(2*x) - 1)*sin(2*x))`

**3.245.6 Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \cot^4(x) dx = x + \frac{\cos(x)}{\sin(x)} - \frac{\cos^3(x)}{3 \sin^3(x)}$$

input `integrate(cot(x)**4,x)`

output `x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)`

**3.245.7 Maxima [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \cot^4(x) dx = x + \frac{3 \tan(x)^2 - 1}{3 \tan(x)^3}$$

input `integrate(cot(x)^4,x, algorithm="maxima")`

output `x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3`

**3.245.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 34 vs.  $2(10) = 20$ .

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.83

$$\int \cot^4(x) dx = \frac{1}{24} \tan\left(\frac{1}{2}x\right)^3 + x + \frac{15 \tan\left(\frac{1}{2}x\right)^2 - 1}{24 \tan\left(\frac{1}{2}x\right)^3} - \frac{5}{8} \tan\left(\frac{1}{2}x\right)$$

input `integrate(cot(x)^4,x, algorithm="giac")`

output `1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 5/8*tan(1/2*x)`

**3.245.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \cot^4(x) dx = -\frac{\cot(x)^3}{3} + \cot(x) + x$$

input `int(cot(x)^4,x)`

output `x + cot(x) - cot(x)^3/3`

## 3.246 $\int \tanh(x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.246.1 Optimal result . . . . .                            | 1315 |
| 3.246.2 Mathematica [A] (verified) . . . . .                | 1315 |
| 3.246.3 Rubi [A] (verified) . . . . .                       | 1316 |
| 3.246.4 Maple [A] (verified) . . . . .                      | 1317 |
| 3.246.5 Fricas [B] (verification not implemented) . . . . . | 1317 |
| 3.246.6 Sympy [B] (verification not implemented) . . . . .  | 1317 |
| 3.246.7 Maxima [A] (verification not implemented) . . . . . | 1318 |
| 3.246.8 Giac [B] (verification not implemented) . . . . .   | 1318 |
| 3.246.9 Mupad [B] (verification not implemented) . . . . .  | 1318 |

### 3.246.1 Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \tanh(x) dx = \log(\cosh(x))$$

output `ln(cosh(x))`

### 3.246.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x) dx = \log(\cosh(x))$$

input `Integrate[Tanh[x], x]`

output `Log[Cosh[x]]`

**3.246.3 Rubi [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$ , Rules used = {3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \tanh(x) dx \\ \downarrow 3042 \\ \int -i \tan(ix) dx \\ \downarrow 26 \\ -i \int \tan(ix) dx \\ \downarrow 3956 \\ \log(\cosh(x)) \end{array}$$

input `Int [Tanh[x], x]`

output `Log[Cosh[x]]`

**3.246.3.1 Defintions of rubi rules used**

rule 26 `Int[(Complex[0, a_])*(Fx_), x_Symbol] := Simp[(Complex[Identity[0], a]) Int[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3956 `Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

**3.246.4 Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

| method            | result                   | size |
|-------------------|--------------------------|------|
| lookup            | $\ln(\cosh(x))$          | 4    |
| derivativedivides | $\ln(\cosh(x))$          | 4    |
| default           | $\ln(\cosh(x))$          | 4    |
| risch             | $-x + \ln(1 + e^{2x})$   | 12   |
| parallelrisch     | $-\ln(1 - \tanh(x)) - x$ | 14   |

input `int(tanh(x),x,method=_RETURNVERBOSE)`

output `ln(cosh(x))`

**3.246.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 18 vs.  $2(3) = 6$ .

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 6.00

$$\int \tanh(x) dx = -x + \log\left(\frac{2 \cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

input `integrate(tanh(x),x, algorithm="fracas")`

output `-x + log(2*cosh(x)/(cosh(x) - sinh(x)))`

**3.246.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 7 vs.  $2(3) = 6$ .

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \tanh(x) dx = x - \log(\tanh(x) + 1)$$

input `integrate(tanh(x),x)`

output `x - log(tanh(x) + 1)`

**3.246.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x) dx = \log(\cosh(x))$$

input `integrate(tanh(x),x, algorithm="maxima")`

output `log(cosh(x))`

**3.246.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 11 vs.  $2(3) = 6$ .

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \tanh(x) dx = -x + \log(e^{2x} + 1)$$

input `integrate(tanh(x),x, algorithm="giac")`

output `-x + log(e^(2*x) + 1)`

**3.246.9 Mupad [B] (verification not implemented)**

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \tanh(x) dx = \ln(\cosh(x))$$

input `int(tanh(x),x)`

output `log(cosh(x))`

## 3.247 $\int \coth(x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.247.1 Optimal result . . . . .                            | 1319 |
| 3.247.2 Mathematica [B] (verified) . . . . .                | 1319 |
| 3.247.3 Rubi [A] (verified) . . . . .                       | 1320 |
| 3.247.4 Maple [A] (verified) . . . . .                      | 1321 |
| 3.247.5 Fricas [B] (verification not implemented) . . . . . | 1321 |
| 3.247.6 Sympy [B] (verification not implemented) . . . . .  | 1321 |
| 3.247.7 Maxima [A] (verification not implemented) . . . . . | 1322 |
| 3.247.8 Giac [B] (verification not implemented) . . . . .   | 1322 |
| 3.247.9 Mupad [B] (verification not implemented) . . . . .  | 1322 |

### 3.247.1 Optimal result

Integrand size = 2, antiderivative size = 3

$$\int \coth(x) dx = \log(\sinh(x))$$

output `ln(sinh(x))`

### 3.247.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \coth(x) dx = \log(\cosh(x)) + \log(\tanh(x))$$

input `Integrate[Coth[x], x]`

output `Log[Cosh[x]] + Log[Tanh[x]]`



**3.247.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$ , Rules used = {3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \coth(x) dx \\ & \quad \downarrow \text{3042} \\ & \int -i \tan\left(\frac{\pi}{2} + ix\right) dx \\ & \quad \downarrow \text{26} \\ & -i \int \tan\left(ix + \frac{\pi}{2}\right) dx \\ & \quad \downarrow \text{3956} \\ & \log(\sinh(x)) \end{aligned}$$

input `Int[Coth[x],x]`

output `Log[Sinh[x]]`

**3.247.3.1 Defintions of rubi rules used**

rule 26 `Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a]) Int[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]`

rule 3042 `Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3956 `Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

**3.247.4 Maple [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

| method            | result                                  | size |
|-------------------|-----------------------------------------|------|
| lookup            | $\ln(\sinh(x))$                         | 4    |
| derivativedivides | $\ln(\sinh(x))$                         | 4    |
| default           | $\ln(\sinh(x))$                         | 4    |
| risch             | $-x + \ln(e^{2x} - 1)$                  | 12   |
| parallelrisch     | $\ln(\tanh(x)) - \ln(1 - \tanh(x)) - x$ | 17   |

input `int(coth(x),x,method=_RETURNVERBOSE)`

output `ln(sinh(x))`

**3.247.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 18 vs.  $2(3) = 6$ .

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 6.00

$$\int \coth(x) dx = -x + \log\left(\frac{2 \sinh(x)}{\cosh(x) - \sinh(x)}\right)$$

input `integrate(coth(x),x, algorithm="fricas")`

output `-x + log(2*sinh(x)/(cosh(x) - sinh(x)))`

**3.247.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 12 vs.  $2(3) = 6$ .

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

$$\int \coth(x) dx = x - \log(\tanh(x) + 1) + \log(\tanh(x))$$

input `integrate(coth(x),x)`

output `x - log(tanh(x) + 1) + log(tanh(x))`

**3.247.7 Maxima [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \coth(x) dx = \log(\sinh(x))$$

input `integrate(coth(x),x, algorithm="maxima")`

output `log(sinh(x))`

**3.247.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(3) = 6.

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

$$\int \coth(x) dx = -x + \log(|e^{(2x)} - 1|)$$

input `integrate(coth(x),x, algorithm="giac")`

output `-x + log(abs(e^(2*x) - 1))`

**3.247.9 Mupad [B] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \coth(x) dx = \ln(\sinh(x))$$

input `int(coth(x),x)`

output `log(sinh(x))`

## 3.248 $\int b^x dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.248.1 Optimal result . . . . .                            | 1323 |
| 3.248.2 Mathematica [A] (verified) . . . . .                | 1323 |
| 3.248.3 Rubi [A] (verified) . . . . .                       | 1324 |
| 3.248.4 Maple [A] (verified) . . . . .                      | 1324 |
| 3.248.5 Fricas [A] (verification not implemented) . . . . . | 1325 |
| 3.248.6 Sympy [A] (verification not implemented) . . . . .  | 1325 |
| 3.248.7 Maxima [A] (verification not implemented) . . . . . | 1325 |
| 3.248.8 Giac [A] (verification not implemented) . . . . .   | 1326 |
| 3.248.9 Mupad [B] (verification not implemented) . . . . .  | 1326 |

### 3.248.1 Optimal result

Integrand size = 3, antiderivative size = 8

$$\int b^x dx = \frac{b^x}{\log(b)}$$

output  $b^x/\ln(b)$

### 3.248.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int b^x dx = \frac{b^x}{\log(b)}$$

input `Integrate[b^x,x]`

output  $b^x/\text{Log}[b]$

### 3.248.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int b^x dx$$

$$\downarrow 2624$$

$$\frac{b^x}{\log(b)}$$

input `Int [b^x, x]`

output `b^x/Log[b]`

#### 3.248.3.1 Defintions of rubi rules used

rule 2624 `Int[((F_)^(v_))^(n_), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;`  
`FreeQ[{F, n}, x] && LinearQ[v, x]`

### 3.248.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

| method            | result                           | size |
|-------------------|----------------------------------|------|
| gospers           | $\frac{b^x}{\ln(b)}$             | 9    |
| derivativedivides | $\frac{b^x}{\ln(b)}$             | 9    |
| default           | $\frac{b^x}{\ln(b)}$             | 9    |
| risch             | $\frac{b^x}{\ln(b)}$             | 9    |
| parallelrisch     | $\frac{b^x}{\ln(b)}$             | 9    |
| norman            | $\frac{e^{x \ln(b)}}{\ln(b)}$    | 11   |
| meijerg           | $-\frac{1-e^{x \ln(b)}}{\ln(b)}$ | 16   |

input `int(b^x,x,method=_RETURNVERBOSE)`

output `b^x/ln(b)`

### 3.248.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int b^x dx = \frac{b^x}{\log(b)}$$

input `integrate(b^x,x, algorithm="fricas")`

output `b^x/log(b)`

### 3.248.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int b^x dx = \begin{cases} \frac{b^x}{\log(b)} & \text{for } \log(b) \neq 0 \\ x & \text{otherwise} \end{cases}$$

input `integrate(b**x,x)`

output `Piecewise((b**x/log(b), Ne(log(b), 0)), (x, True))`

### 3.248.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int b^x dx = \frac{b^x}{\log(b)}$$

input `integrate(b^x,x, algorithm="maxima")`

output `b^x/log(b)`

**3.248.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int b^x dx = \frac{b^x}{\log(b)}$$

input `integrate(b^x,x, algorithm="giac")`

output `b^x/log(b)`

**3.248.9 Mupad [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int b^x dx = \frac{b^x}{\ln(b)}$$

input `int(b^x,x)`

output `b^x/log(b)`

**3.249**  $\int \sqrt{2 + \frac{1}{x^4} + x^4} dx$

3.249.1 Optimal result . . . . . 1327  
 3.249.2 Mathematica [A] (verified) . . . . . 1327  
 3.249.3 Rubi [A] (verified) . . . . . 1328  
 3.249.4 Maple [A] (verified) . . . . . 1329  
 3.249.5 Fricas [A] (verification not implemented) . . . . . 1330  
 3.249.6 Sympy [F] . . . . . 1330  
 3.249.7 Maxima [A] (verification not implemented) . . . . . 1330  
 3.249.8 Giac [A] (verification not implemented) . . . . . 1331  
 3.249.9 Mupad [F(-1)] . . . . . 1331

**3.249.1 Optimal result**

Integrand size = 12, antiderivative size = 49

$$\int \sqrt{2 + \frac{1}{x^4} + x^4} dx = -\frac{x\sqrt{2 + \frac{1}{x^4} + x^4}}{1 + x^4} + \frac{x^5\sqrt{2 + \frac{1}{x^4} + x^4}}{3(1 + x^4)}$$

output `-x*(2+1/x^4+x^4)^(1/2)/(x^4+1)+1/3*x^5*(2+1/x^4+x^4)^(1/2)/(x^4+1)`

**3.249.2 Mathematica [A] (verified)**

Time = 1.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \sqrt{2 + \frac{1}{x^4} + x^4} dx = \frac{x(-3 + x^4)\sqrt{2 + \frac{1}{x^4} + x^4}}{3(1 + x^4)}$$

input `Integrate[Sqrt[2 + x^(-4) + x^4],x]`

output `(x*(-3 + x^4)*Sqrt[2 + x^(-4) + x^4])/(3*(1 + x^4))`



**3.249.3 Rubi [A] (verified)**

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {1689, 1384, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \sqrt{x^4 + \frac{1}{x^4} + 2} dx \\
 & \quad \downarrow \text{1689} \\
 & \frac{x^2 \sqrt{x^4 + \frac{1}{x^4} + 2} \int \frac{\sqrt{x^8 + 2x^4 + 1}}{x^2} dx}{\sqrt{x^8 + 2x^4 + 1}} \\
 & \quad \downarrow \text{1384} \\
 & \frac{x^2 \sqrt{x^4 + \frac{1}{x^4} + 2} \int \frac{x^4 + 1}{x^2} dx}{x^4 + 1} \\
 & \quad \downarrow \text{802} \\
 & \frac{x^2 \sqrt{x^4 + \frac{1}{x^4} + 2} \int (x^2 + \frac{1}{x^2}) dx}{x^4 + 1} \\
 & \quad \downarrow \text{2009} \\
 & \frac{x^2 \left( \frac{x^3}{3} - \frac{1}{x} \right) \sqrt{x^4 + \frac{1}{x^4} + 2}}{x^4 + 1}
 \end{aligned}$$

input `Int[Sqrt[2 + x^(-4) + x^4],x]`

output `(x^2*(-x^(-1) + x^3/3)*Sqrt[2 + x^(-4) + x^4])/(1 + x^4)`

## 3.249.3.1 Defintions of rubi rules used

- rule 802 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]`
- rule 1384 `Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])) Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n - 1)] && NeQ[u, x^(2*n - 1)] && !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])`
- rule 1689 `Int[((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_), x_Symbol] := Simp[x^(n*FracPart[p])*((a + b/x^n + c*x^n)^FracPart[p]/(b + a*x^n + c*x^(2*n))^FracPart[p]) Int[(b + a*x^n + c*x^(2*n))^p/x^(n*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[mn, -n] && !IntegerQ[p] && PosQ[n]`
- rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

## 3.249.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

| method  | result                                                                                           | size |
|---------|--------------------------------------------------------------------------------------------------|------|
| gosper  | $\frac{x(x^4-3)\sqrt{\frac{x^8+2x^4+1}{x^4}}}{3x^4+3}$                                           | 32   |
| default | $\frac{x(x^4-3)\sqrt{\frac{x^8+2x^4+1}{x^4}}}{3x^4+3}$                                           | 32   |
| risch   | $\frac{\sqrt{\frac{(x^4+1)^2}{x^4}} x^5}{3x^4+3} - \frac{\sqrt{\frac{(x^4+1)^2}{x^4}} x}{x^4+1}$ | 50   |

input `int((2+1/x^4+x^4)^(1/2),x,method=_RETURNVERBOSE)`

output `1/3*x*(x^4-3)*((x^8+2*x^4+1)/x^4)^(1/2)/(x^4+1)`

**3.249.5 Fricas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.20

$$\int \sqrt{2 + \frac{1}{x^4} + x^4} dx = \frac{x^4 - 3}{3x}$$

input `integrate((2+1/x^4+x^4)^(1/2),x, algorithm="fricas")`output `1/3*(x^4 - 3)/x`**3.249.6 Sympy [F]**

$$\int \sqrt{2 + \frac{1}{x^4} + x^4} dx = \int \sqrt{x^4 + 2 + \frac{1}{x^4}} dx$$

input `integrate((2+1/x**4+x**4)**(1/2),x)`output `Integral(sqrt(x**4 + 2 + x**(-4)), x)`**3.249.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.20

$$\int \sqrt{2 + \frac{1}{x^4} + x^4} dx = \frac{x^4 - 3}{3x}$$

input `integrate((2+1/x^4+x^4)^(1/2),x, algorithm="maxima")`output `1/3*(x^4 - 3)/x`

**3.249.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.22

$$\int \sqrt{2 + \frac{1}{x^4} + x^4} dx = \frac{1}{3}x^3 - \frac{1}{x}$$

input `integrate((2+1/x^4+x^4)^(1/2),x, algorithm="giac")`output `1/3*x^3 - 1/x`**3.249.9 Mupad [F(-1)]**

Timed out.

$$\int \sqrt{2 + \frac{1}{x^4} + x^4} dx = \int \sqrt{\frac{1}{x^4} + x^4 + 2} dx$$

input `int((1/x^4 + x^4 + 2)^(1/2),x)`output `int((1/x^4 + x^4 + 2)^(1/2), x)`

### 3.250 $\int \frac{1+2x}{2+3x} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.250.1 Optimal result . . . . .                            | 1332 |
| 3.250.2 Mathematica [A] (verified) . . . . .                | 1332 |
| 3.250.3 Rubi [A] (verified) . . . . .                       | 1333 |
| 3.250.4 Maple [A] (verified) . . . . .                      | 1334 |
| 3.250.5 Fricas [A] (verification not implemented) . . . . . | 1334 |
| 3.250.6 Sympy [A] (verification not implemented) . . . . .  | 1334 |
| 3.250.7 Maxima [A] (verification not implemented) . . . . . | 1335 |
| 3.250.8 Giac [A] (verification not implemented) . . . . .   | 1335 |
| 3.250.9 Mupad [B] (verification not implemented) . . . . .  | 1335 |

#### 3.250.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1+2x}{2+3x} dx = \frac{2x}{3} - \frac{1}{9} \log(2+3x)$$

output `2/3*x-1/9*ln(2+3*x)`

#### 3.250.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1+2x}{2+3x} dx = \frac{1}{9}(4+6x - \log(2+3x))$$

input `Integrate[(1 + 2*x)/(2 + 3*x), x]`

output `(4 + 6*x - Log[2 + 3*x])/9`

**3.250.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+1}{3x+2} dx$$

↓ 49

$$\int \left( \frac{2}{3} - \frac{1}{3(3x+2)} \right) dx$$

↓ 2009

$$\frac{2x}{3} - \frac{1}{9} \log(3x+2)$$

input `Int[(1 + 2*x)/(2 + 3*x), x]`

output `(2*x)/3 - Log[2 + 3*x]/9`

**3.250.3.1 Defintions of rubi rules used**

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

**3.250.4 Maple [A] (verified)**

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

| method      | result                                          | size |
|-------------|-------------------------------------------------|------|
| parallelsch | $\frac{2x}{3} - \frac{\ln(\frac{2}{3}+x)}{9}$   | 11   |
| default     | $\frac{2x}{3} - \frac{\ln(2+3x)}{9}$            | 13   |
| norman      | $\frac{2x}{3} - \frac{\ln(2+3x)}{9}$            | 13   |
| meijerg     | $-\frac{\ln(1+\frac{3x}{2})}{9} + \frac{2x}{3}$ | 13   |
| risch       | $\frac{2x}{3} - \frac{\ln(2+3x)}{9}$            | 13   |

input `int((1+2*x)/(2+3*x),x,method=_RETURNVERBOSE)`output `2/3*x-1/9*ln(2/3+x)`**3.250.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1+2x}{2+3x} dx = \frac{2}{3}x - \frac{1}{9} \log(3x+2)$$

input `integrate((1+2*x)/(2+3*x),x, algorithm="fracas")`output `2/3*x - 1/9*log(3*x + 2)`**3.250.6 Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1+2x}{2+3x} dx = \frac{2x}{3} - \frac{\log(3x+2)}{9}$$

input `integrate((1+2*x)/(2+3*x),x)`output `2*x/3 - log(3*x + 2)/9`

**3.250.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1+2x}{2+3x} dx = \frac{2}{3}x - \frac{1}{9} \log(3x+2)$$

input `integrate((1+2*x)/(2+3*x),x, algorithm="maxima")`output `2/3*x - 1/9*log(3*x + 2)`**3.250.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{1+2x}{2+3x} dx = \frac{2}{3}x - \frac{1}{9} \log(|3x+2|)$$

input `integrate((1+2*x)/(2+3*x),x, algorithm="giac")`output `2/3*x - 1/9*log(abs(3*x + 2))`**3.250.9 Mupad [B] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1+2x}{2+3x} dx = \frac{2x}{3} - \frac{\ln\left(x + \frac{2}{3}\right)}{9}$$

input `int((2*x + 1)/(3*x + 2),x)`output `(2*x)/3 - log(x + 2/3)/9`



## 3.251 $\int x \log \left( x + \sqrt{1 + x^2} \right) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.251.1 Optimal result . . . . .                            | 1336 |
| 3.251.2 Mathematica [A] (verified) . . . . .                | 1336 |
| 3.251.3 Rubi [A] (verified) . . . . .                       | 1337 |
| 3.251.4 Maple [A] (verified) . . . . .                      | 1338 |
| 3.251.5 Fricas [A] (verification not implemented) . . . . . | 1338 |
| 3.251.6 Sympy [F] . . . . .                                 | 1339 |
| 3.251.7 Maxima [F] . . . . .                                | 1339 |
| 3.251.8 Giac [A] (verification not implemented) . . . . .   | 1339 |
| 3.251.9 Mupad [B] (verification not implemented) . . . . .  | 1340 |

### 3.251.1 Optimal result

Integrand size = 14, antiderivative size = 40

$$\int x \log \left( x + \sqrt{1 + x^2} \right) dx = -\frac{1}{4}x\sqrt{1 + x^2} + \frac{\operatorname{arcsinh}(x)}{4} + \frac{1}{2}x^2 \log \left( x + \sqrt{1 + x^2} \right)$$

output `1/4*arcsinh(x)+1/2*x^2*ln(x+(x^2+1)^(1/2))-1/4*x*(x^2+1)^(1/2)`

### 3.251.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int x \log \left( x + \sqrt{1 + x^2} \right) dx = \frac{1}{4} \left( -x\sqrt{1 + x^2} + \operatorname{arcsinh}(x) + 2x^2 \log \left( x + \sqrt{1 + x^2} \right) \right)$$

input `Integrate[x*Log[x + Sqrt[1 + x^2]],x]`

output `(-(x*Sqrt[1 + x^2]) + ArcSinh[x] + 2*x^2*Log[x + Sqrt[1 + x^2]])/4`

**3.251.3 Rubi [A] (verified)**

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3016, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int x \log(\sqrt{x^2+1}+x) dx \\ & \quad \downarrow \text{3016} \\ & \frac{1}{2}x^2 \log(\sqrt{x^2+1}+x) - \frac{1}{2} \int \frac{x^2}{\sqrt{x^2+1}} dx \\ & \quad \downarrow \text{262} \\ & \frac{1}{2} \left( \frac{1}{2} \int \frac{1}{\sqrt{x^2+1}} dx - \frac{1}{2}x\sqrt{x^2+1} \right) + \frac{1}{2}x^2 \log(\sqrt{x^2+1}+x) \\ & \quad \downarrow \text{222} \\ & \frac{1}{2} \left( \frac{\operatorname{arcsinh}(x)}{2} - \frac{1}{2}x\sqrt{x^2+1} \right) + \frac{1}{2}x^2 \log(\sqrt{x^2+1}+x) \end{aligned}$$

input `Int[x*Log[x + Sqrt[1 + x^2]],x]`

output `(-1/2*(x*Sqrt[1 + x^2]) + ArcSinh[x]/2)/2 + (x^2*Log[x + Sqrt[1 + x^2]])/2`

**3.251.3.1 Defintions of rubi rules used**

rule 222 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]`

rule 262 `Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[c*(c*x)^(m-1)*((a + b*x^2)^(p+1)/(b*(m+2*p+1))), x] - Simp[a*c^2*((m-1)/(b*(m+2*p+1))) Int[(c*x)^(m-2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[m, 2-1] && NeQ[m+2*p+1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]`

```
rule 3016 Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]]*((g_.)*(x_)
^(m_.), x_Symbol] := Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[a + c*x^2]]/(
g*(m + 1))), x] - Simp[a*c*(f^2/(g*(m + 1))) Int[(g*x)^(m + 1)/(d*e*(a +
c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

### 3.251.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

| method | result                                                                                                                                                                         | size |
|--------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| parts  | $\frac{x^2 \ln(x + \sqrt{x^2 + 1})}{2} + \frac{x^3 \sqrt{x^2 + 1}}{8} - \frac{x \sqrt{x^2 + 1}}{8} + \frac{\operatorname{arcsinh}(x)}{4} - \frac{x(x^2 + 1)^{\frac{3}{2}}}{8}$ | 53   |

```
input int(x*ln(x+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
output 1/2*x^2*ln(x+(x^2+1)^(1/2))+1/8*x^3*(x^2+1)^(1/2)-1/8*x*(x^2+1)^(1/2)+1/4*
arcsinh(x)-1/8*x*(x^2+1)^(3/2)
```

### 3.251.5 Fracas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int x \log(x + \sqrt{1 + x^2}) dx = \frac{1}{4} (2x^2 + 1) \log(x + \sqrt{x^2 + 1}) - \frac{1}{4} \sqrt{x^2 + 1} x$$

```
input integrate(x*log(x+(x^2+1)^(1/2)),x, algorithm="fracas")
```

```
output 1/4*(2*x^2 + 1)*log(x + sqrt(x^2 + 1)) - 1/4*sqrt(x^2 + 1)*x
```

**3.251.6 Sympy [F]**

$$\int x \log(x + \sqrt{1 + x^2}) dx = \int x \log(x + \sqrt{x^2 + 1}) dx$$

input `integrate(x*ln(x+(x**2+1)**(1/2)),x)`

output `Integral(x*log(x + sqrt(x**2 + 1)), x)`

**3.251.7 Maxima [F]**

$$\int x \log(x + \sqrt{1 + x^2}) dx = \int x \log(x + \sqrt{x^2 + 1}) dx$$

input `integrate(x*log(x+(x^2+1)^(1/2)),x, algorithm="maxima")`

output `1/2*x^2*log(x + sqrt(x^2 + 1)) - 1/4*x^2 - integrate(1/2*x^2/(x^3 + (x^2 + 1)^(3/2) + x), x) + 1/4*log(x^2 + 1)`

**3.251.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int x \log(x + \sqrt{1 + x^2}) dx = \frac{1}{2} x^2 \log(x + \sqrt{x^2 + 1}) - \frac{1}{4} \sqrt{x^2 + 1} x - \frac{1}{4} \log(-x + \sqrt{x^2 + 1})$$

input `integrate(x*log(x+(x^2+1)^(1/2)),x, algorithm="giac")`

output `1/2*x^2*log(x + sqrt(x^2 + 1)) - 1/4*sqrt(x^2 + 1)*x - 1/4*log(-x + sqrt(x^2 + 1))`

**3.251.9 Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int x \log(x + \sqrt{1 + x^2}) dx = x \ln(x + \sqrt{x^2 + 1}) \left( \frac{x}{2} + \frac{1}{4x} \right) - \frac{x \sqrt{x^2 + 1}}{4}$$

input `int(x*log(x + (x^2 + 1)^(1/2)),x)`output `x*log(x + (x^2 + 1)^(1/2))*(x/2 + 1/(4*x)) - (x*(x^2 + 1)^(1/2))/4`

### 3.252 $\int x(1 + e^x \sin(x))^2 dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.252.1 Optimal result . . . . .                            | 1341 |
| 3.252.2 Mathematica [A] (verified) . . . . .                | 1341 |
| 3.252.3 Rubi [A] (verified) . . . . .                       | 1342 |
| 3.252.4 Maple [A] (verified) . . . . .                      | 1343 |
| 3.252.5 Fricas [A] (verification not implemented) . . . . . | 1343 |
| 3.252.6 Sympy [A] (verification not implemented) . . . . .  | 1343 |
| 3.252.7 Maxima [A] (verification not implemented) . . . . . | 1344 |
| 3.252.8 Giac [A] (verification not implemented) . . . . .   | 1344 |
| 3.252.9 Mupad [B] (verification not implemented) . . . . .  | 1345 |

#### 3.252.1 Optimal result

Integrand size = 12, antiderivative size = 128

$$\begin{aligned} \int x(1 + e^x \sin(x))^2 dx = & -\frac{3e^{2x}}{32} + \frac{1}{8}e^{2x}x + \frac{x^2}{2} + e^x \cos(x) - e^x x \cos(x) - \frac{1}{32}e^{2x} \cos(2x) \\ & + e^x x \sin(x) + \frac{1}{16}e^{2x} \cos(x) \sin(x) - \frac{1}{4}e^{2x} x \cos(x) \sin(x) \\ & - \frac{1}{16}e^{2x} \sin^2(x) + \frac{1}{4}e^{2x} x \sin^2(x) + \frac{1}{32}e^{2x} \sin(2x) \end{aligned}$$

output `-3/32*exp(2*x)+1/8*exp(2*x)*x+1/2*x^2+exp(x)*cos(x)-exp(x)*x*cos(x)-1/32*exp(2*x)*cos(2*x)+exp(x)*x*sin(x)+1/16*exp(2*x)*cos(x)*sin(x)-1/4*exp(2*x)*x*cos(x)*sin(x)-1/16*exp(2*x)*sin(x)^2+1/4*exp(2*x)*x*sin(x)^2+1/32*exp(2*x)*sin(2*x)`

#### 3.252.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.52

$$\begin{aligned} \int x(1 + e^x \sin(x))^2 dx = & \frac{1}{8}(4x^2 + e^{2x}(-1 + 2x) - 8e^x(-1 + x) \cos(x) - e^{2x} x \cos(2x) \\ & + 8e^x x \sin(x) - e^{2x}(-1 + 2x) \cos(x) \sin(x)) \end{aligned}$$

input `Integrate[x*(1 + E^x*Sin[x])^2,x]`

output `(4*x^2 + E^(2*x)*(-1 + 2*x) - 8*E^x*(-1 + x)*Cos[x] - E^(2*x)*x*Cos[2*x] + 8*E^x*x*Sin[x] - E^(2*x)*(-1 + 2*x)*Cos[x]*Sin[x])/8`

**3.252.3 Rubi [A] (verified)**

Time = 0.37 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(e^x \sin(x) + 1)^2 dx$$

$$\downarrow \text{7293}$$

$$\int (x + e^{2x}x \sin^2(x) + 2e^x x \sin(x)) dx$$

$$\downarrow \text{2009}$$

$$\frac{x^2}{2} + \frac{1}{8}e^{2x}x - \frac{3e^{2x}}{32} + \frac{1}{4}e^{2x}x \sin^2(x) - \frac{1}{16}e^{2x} \sin^2(x) + e^x x \sin(x) + \frac{1}{32}e^{2x} \sin(2x) - e^x x \cos(x) + e^x \cos(x) - \frac{1}{32}e^{2x} \cos(2x) - \frac{1}{4}e^{2x}x \sin(x) \cos(x) + \frac{1}{16}e^{2x} \sin(x) \cos(x)$$

input `Int[x*(1 + E^x*Sin[x])^2,x]`

output `(-3*E^(2*x))/32 + (E^(2*x)*x)/8 + x^2/2 + E^x*Cos[x] - E^x*x*Cos[x] - (E^(2*x)*Cos[2*x])/32 + E^x*x*Sin[x] + (E^(2*x)*Cos[x]*Sin[x])/16 - (E^(2*x)*x*Cos[x]*Sin[x])/4 - (E^(2*x)*Sin[x]^2)/16 + (E^(2*x)*x*Sin[x]^2)/4 + (E^(2*x)*Sin[2*x])/32`

**3.252.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 7293 `Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]`

**3.252.4 Maple [A] (verified)**

Time = 0.42 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.49

| method  | result                                                                                                                                                                                                   |
|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| default | $\frac{e^{2x}x}{4} - \frac{e^{2x}}{8} - \frac{x e^{2x} \cos(2x)}{8} + \frac{(-\frac{x}{4} + \frac{1}{8})e^{2x} \sin(2x)}{2} + 2(-\frac{x}{2} + \frac{1}{2}) e^x \cos(x) + e^x x \sin(x) + \frac{x^2}{2}$ |
| parts   | $\frac{e^{2x}x}{4} - \frac{e^{2x}}{8} - \frac{x e^{2x} \cos(2x)}{8} + \frac{(-\frac{x}{4} + \frac{1}{8})e^{2x} \sin(2x)}{2} + 2(-\frac{x}{2} + \frac{1}{2}) e^x \cos(x) + e^x x \sin(x) + \frac{x^2}{2}$ |
| risch   | $\frac{x^2}{2} + (-\frac{1}{8} + \frac{x}{4}) e^{2x} + (-\frac{1}{64} + \frac{i}{64}) (-1 + i + 4x) e^{(2+2i)x} + (-\frac{1}{4} - \frac{i}{4}) (-1 + i + 2x) e^{(1+i)x} + (-\frac{1}{4}$                 |

input `int(x*(1+exp(x)*sin(x))^2,x,method=_RETURNVERBOSE)`output `1/4*exp(x)^2*x-1/8*exp(x)^2-1/8*x*exp(2*x)*cos(2*x)+1/2*(-1/4*x+1/8)*exp(2*x)*sin(2*x)+2*(-1/2*x+1/2)*exp(x)*cos(x)+exp(x)*x*sin(x)+1/2*x^2`**3.252.5 Fracas [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.43

$$\int x(1 + e^x \sin(x))^2 dx = -(x-1) \cos(x) e^x + \frac{1}{2} x^2 - \frac{1}{8} (2x \cos(x))^2 - 3x + 1) e^{(2x)} - \frac{1}{8} ((2x-1) \cos(x) e^{(2x)} - 8x e^x) \sin(x)$$

input `integrate(x*(1+exp(x)*sin(x))^2,x, algorithm="fricas")`output `-(x-1)*cos(x)*e^x + 1/2*x^2 - 1/8*(2*x*cos(x))^2 - 3*x + 1)*e^(2*x) - 1/8*((2*x-1)*cos(x)*e^(2*x) - 8*x*e^x)*sin(x)`**3.252.6 Sympy [A] (verification not implemented)**

Time = 0.58 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.85

$$\int x(1 + e^x \sin(x))^2 dx = \frac{x^2}{2} + \frac{3xe^{2x} \sin^2(x)}{8} - \frac{xe^{2x} \sin(x) \cos(x)}{4} + \frac{xe^{2x} \cos^2(x)}{8} + xe^x \sin(x) - xe^x \cos(x) - \frac{e^{2x} \sin^2(x)}{8} + \frac{e^{2x} \sin(x) \cos(x)}{8} - \frac{e^{2x} \cos^2(x)}{8} + e^x \cos(x)$$



input `integrate(x*(1+exp(x)*sin(x))**2,x)`

output `x**2/2 + 3*x*exp(2*x)*sin(x)**2/8 - x*exp(2*x)*sin(x)*cos(x)/4 + x*exp(2*x)*cos(x)**2/8 + x*exp(x)*sin(x) - x*exp(x)*cos(x) - exp(2*x)*sin(x)**2/8 + exp(2*x)*sin(x)*cos(x)/8 - exp(2*x)*cos(x)**2/8 + exp(x)*cos(x)`

### 3.252.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.45

$$\int x(1 + e^x \sin(x))^2 dx = -\frac{1}{8} x \cos(2x) e^{(2x)} - (x-1) \cos(x) e^x - \frac{1}{16} (2x-1) e^{(2x)} \sin(2x) + x e^x \sin(x) + \frac{1}{2} x^2 + \frac{1}{8} (2x-1) e^{(2x)}$$

input `integrate(x*(1+exp(x)*sin(x))^2,x, algorithm="maxima")`

output `-1/8*x*cos(2*x)*e^(2*x) - (x - 1)*cos(x)*e^x - 1/16*(2*x - 1)*e^(2*x)*sin(2*x) + x*e^x*sin(x) + 1/2*x^2 + 1/8*(2*x - 1)*e^(2*x)`

### 3.252.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.45

$$\int x(1 + e^x \sin(x))^2 dx = \frac{1}{2} x^2 - \frac{1}{16} (2x \cos(2x) + (2x-1) \sin(2x)) e^{(2x)} + \frac{1}{8} (2x-1) e^{(2x)} - ((x-1) \cos(x) - x \sin(x)) e^x$$

input `integrate(x*(1+exp(x)*sin(x))^2,x, algorithm="giac")`

output `1/2*x^2 - 1/16*(2*x*cos(2*x) + (2*x - 1)*sin(2*x))*e^(2*x) + 1/8*(2*x - 1)*e^(2*x) - ((x - 1)*cos(x) - x*sin(x))*e^x`

**3.252.9 Mupad [B] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.54

$$\int x(1 + e^x \sin(x))^2 dx = \frac{3xe^{2x}}{8} - \frac{e^{2x}}{8} + e^x \cos(x) + \frac{x^2}{2} - \frac{xe^{2x} \cos(x)^2}{4} + \frac{e^{2x} \cos(x) \sin(x)}{8} - xe^x \cos(x) + xe^x \sin(x) - \frac{xe^{2x} \cos(x) \sin(x)}{4}$$

input `int(x*(exp(x)*sin(x) + 1)^2,x)`

output `(3*x*exp(2*x))/8 - exp(2*x)/8 + exp(x)*cos(x) + x^2/2 - (x*exp(2*x)*cos(x)^2)/4 + (exp(2*x)*cos(x)*sin(x))/8 - x*exp(x)*cos(x) + x*exp(x)*sin(x) - (x*exp(2*x)*cos(x)*sin(x))/4`

### 3.253 $\int e^x x \cos(x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.253.1 Optimal result . . . . .                            | 1346 |
| 3.253.2 Mathematica [A] (verified) . . . . .                | 1346 |
| 3.253.3 Rubi [A] (verified) . . . . .                       | 1347 |
| 3.253.4 Maple [A] (verified) . . . . .                      | 1348 |
| 3.253.5 Fricas [A] (verification not implemented) . . . . . | 1348 |
| 3.253.6 Sympy [A] (verification not implemented) . . . . .  | 1348 |
| 3.253.7 Maxima [A] (verification not implemented) . . . . . | 1349 |
| 3.253.8 Giac [A] (verification not implemented) . . . . .   | 1349 |
| 3.253.9 Mupad [B] (verification not implemented) . . . . .  | 1349 |

#### 3.253.1 Optimal result

Integrand size = 7, antiderivative size = 30

$$\int e^x x \cos(x) dx = \frac{1}{2}e^x x \cos(x) - \frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x)$$

output `1/2*exp(x)*x*cos(x)-1/2*exp(x)*sin(x)+1/2*exp(x)*x*sin(x)`

#### 3.253.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int e^x x \cos(x) dx = \frac{1}{2}e^x (x \cos(x) + (-1 + x) \sin(x))$$

input `Integrate[E^x*x*Cos[x],x]`

output `(E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2`

**3.253.3 Rubi [A] (verified)**

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {4969, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x x \cos(x) dx$$

$$\downarrow 4969$$

$$-\int \left( \frac{1}{2} e^x \cos(x) + \frac{1}{2} e^x \sin(x) \right) dx + \frac{1}{2} e^x x \sin(x) + \frac{1}{2} e^x x \cos(x)$$

$$\downarrow 2009$$

$$-\frac{1}{2} e^x \sin(x) + \frac{1}{2} e^x x \sin(x) + \frac{1}{2} e^x x \cos(x)$$

input `Int[E^x*x*Cos[x],x]`

output `(E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2`

**3.253.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 4969 `Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.), x_Symbol] := Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^n, x]}, Simp[(f*x)^m u, x] - Simp[f*m Int[(f*x)^(m - 1)*u, x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]`

**3.253.4 Maple [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.53

| method        | result                                                                                                                                                                                             | size |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| parallelrisch | $\frac{e^x((-1+x)\sin(x)+x\cos(x))}{2}$                                                                                                                                                            | 16   |
| default       | $\frac{e^x x \cos(x)}{2} - \left(-\frac{x}{2} + \frac{1}{2}\right) e^x \sin(x)$                                                                                                                    | 20   |
| risch         | $\left(\frac{1}{8} - \frac{i}{8}\right) (-1 + i + 2x) e^{(1+i)x} + \left(\frac{1}{8} + \frac{i}{8}\right) (-1 - i + 2x) e^{(1-i)x}$                                                                | 36   |
| norman        | $\frac{e^x x \tan\left(\frac{x}{2}\right) + \frac{e^x x}{2} - e^x \tan\left(\frac{x}{2}\right) - \frac{e^x x \left(\tan^2\left(\frac{x}{2}\right)\right)}{1 + \tan^2\left(\frac{x}{2}\right)}}{2}$ | 45   |

input `int(exp(x)*x*cos(x),x,method=_RETURNVERBOSE)`output `1/2*exp(x)*((-1+x)*sin(x)+x*cos(x))`**3.253.5 Fricas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x \cos(x) dx = \frac{1}{2} x \cos(x) e^x + \frac{1}{2} (x - 1) e^x \sin(x)$$

input `integrate(exp(x)*x*cos(x),x, algorithm="fricas")`output `1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)`**3.253.6 Sympy [A] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int e^x x \cos(x) dx = \frac{x e^x \sin(x)}{2} + \frac{x e^x \cos(x)}{2} - \frac{e^x \sin(x)}{2}$$

input `integrate(exp(x)*x*cos(x),x)`output `x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2`

**3.253.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x \cos(x) dx = \frac{1}{2} x \cos(x) e^x + \frac{1}{2} (x - 1) e^x \sin(x)$$

input `integrate(exp(x)*x*cos(x),x, algorithm="maxima")`output `1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)`**3.253.8 Giac [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.50

$$\int e^x x \cos(x) dx = \frac{1}{2} (x \cos(x) + (x - 1) \sin(x)) e^x$$

input `integrate(exp(x)*x*cos(x),x, algorithm="giac")`output `1/2*(x*cos(x) + (x - 1)*sin(x))*e^x`**3.253.9 Mupad [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int e^x x \cos(x) dx = \frac{e^x (x \cos(x) - \sin(x) + x \sin(x))}{2}$$

input `int(x*exp(x)*cos(x),x)`output `(exp(x)*(x*cos(x) - sin(x) + x*sin(x)))/2`

## 3.254 $\int \frac{1}{(-3+x)^4} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.254.1 Optimal result . . . . .                            | 1350 |
| 3.254.2 Mathematica [A] (verified) . . . . .                | 1350 |
| 3.254.3 Rubi [A] (verified) . . . . .                       | 1351 |
| 3.254.4 Maple [A] (verified) . . . . .                      | 1351 |
| 3.254.5 Fricas [B] (verification not implemented) . . . . . | 1352 |
| 3.254.6 Sympy [B] (verification not implemented) . . . . .  | 1352 |
| 3.254.7 Maxima [A] (verification not implemented) . . . . . | 1353 |
| 3.254.8 Giac [A] (verification not implemented) . . . . .   | 1353 |
| 3.254.9 Mupad [B] (verification not implemented) . . . . .  | 1353 |

### 3.254.1 Optimal result

Integrand size = 5, antiderivative size = 11

$$\int \frac{1}{(-3+x)^4} dx = \frac{1}{3(3-x)^3}$$

output 1/3/(3-x)^3

### 3.254.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-3+x)^4} dx = -\frac{1}{3(-3+x)^3}$$

input Integrate[(-3 + x)^(-4), x]

output -1/3\*1/(-3 + x)^3

**3.254.3 Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-3)^4} dx$$

↓ 17

$$\frac{1}{3(3-x)^3}$$

input `Int[(-3 + x)^(-4), x]`

output `1/(3*(3 - x)^3)`

**3.254.3.1 Defintions of rubi rules used**

rule 17 `Int[(c_.)*((a_.) + (b_.)*(x_.))^(m_.), x_Symbol] := Simp[c*((a + b*x)^(m + 1))/(b*(m + 1)), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]`

**3.254.4 Maple [A] (verified)**

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

| method       | result                                               | size |
|--------------|------------------------------------------------------|------|
| gospers      | $-\frac{1}{3(-3+x)^3}$                               | 8    |
| default      | $-\frac{1}{3(-3+x)^3}$                               | 8    |
| norman       | $-\frac{1}{3(-3+x)^3}$                               | 8    |
| risch        | $-\frac{1}{3(-3+x)^3}$                               | 8    |
| parallelrisc | $-\frac{1}{3(-3+x)^3}$                               | 8    |
| meijerg      | $\frac{x(\frac{1}{3}x^2-x+3)}{243(1-\frac{x}{3})^3}$ | 21   |



input `int(1/(-3+x)^4,x,method=_RETURNVERBOSE)`

output `-1/3/(-3+x)^3`

### 3.254.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs.  $2(7) = 14$ .

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1}{(-3+x)^4} dx = -\frac{1}{3(x^3 - 9x^2 + 27x - 27)}$$

input `integrate(1/(-3+x)^4,x, algorithm="fricas")`

output `-1/3/(x^3 - 9*x^2 + 27*x - 27)`

### 3.254.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs.  $2(7) = 14$ .

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1}{(-3+x)^4} dx = -\frac{1}{3x^3 - 27x^2 + 81x - 81}$$

input `integrate(1/(-3+x)**4,x)`

output `-1/(3*x**3 - 27*x**2 + 81*x - 81)`

**3.254.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{(-3+x)^4} dx = -\frac{1}{3(x-3)^3}$$

input `integrate(1/(-3+x)^4,x, algorithm="maxima")`output `-1/3/(x - 3)^3`**3.254.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{(-3+x)^4} dx = -\frac{1}{3(x-3)^3}$$

input `integrate(1/(-3+x)^4,x, algorithm="giac")`output `-1/3/(x - 3)^3`**3.254.9 Mupad [B] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1}{(-3+x)^4} dx = -\frac{1}{3(x-3)^3}$$

input `int(1/(x - 3)^4,x)`output `-1/(3*(x - 3)^3)`

### 3.255 $\int \frac{x}{-1+x^3} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.255.1 Optimal result . . . . .                            | 1354 |
| 3.255.2 Mathematica [A] (verified) . . . . .                | 1354 |
| 3.255.3 Rubi [A] (verified) . . . . .                       | 1355 |
| 3.255.4 Maple [A] (verified) . . . . .                      | 1357 |
| 3.255.5 Fricas [A] (verification not implemented) . . . . . | 1357 |
| 3.255.6 Sympy [A] (verification not implemented) . . . . .  | 1357 |
| 3.255.7 Maxima [A] (verification not implemented) . . . . . | 1358 |
| 3.255.8 Giac [A] (verification not implemented) . . . . .   | 1358 |
| 3.255.9 Mupad [B] (verification not implemented) . . . . .  | 1359 |

#### 3.255.1 Optimal result

Integrand size = 9, antiderivative size = 40

$$\int \frac{x}{-1+x^3} dx = \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3} \log(1-x) - \frac{1}{6} \log(1+x+x^2)$$

output `1/3*ln(1-x)-1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)`

#### 3.255.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x}{-1+x^3} dx = \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3} \log(1-x) - \frac{1}{6} \log(1+x+x^2)$$

input `Integrate[x/(-1 + x^3),x]`

output `ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^2]/6`

**3.255.3 Rubi [A] (verified)**

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$ , Rules used = {821, 16, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x}{x^3 - 1} dx \\
 & \quad \downarrow \text{821} \\
 & \frac{1}{3} \int \frac{1}{x-1} dx - \frac{1}{3} \int -\frac{1-x}{x^2+x+1} dx \\
 & \quad \downarrow \text{16} \\
 & \frac{1}{3} \log(1-x) - \frac{1}{3} \int -\frac{1-x}{x^2+x+1} dx \\
 & \quad \downarrow \text{25} \\
 & \frac{1}{3} \int \frac{1-x}{x^2+x+1} dx + \frac{1}{3} \log(1-x) \\
 & \quad \downarrow \text{1142} \\
 & \frac{1}{3} \left( \frac{3}{2} \int \frac{1}{x^2+x+1} dx - \frac{1}{2} \int \frac{2x+1}{x^2+x+1} dx \right) + \frac{1}{3} \log(1-x) \\
 & \quad \downarrow \text{1083} \\
 & \frac{1}{3} \left( -\frac{1}{2} \int \frac{2x+1}{x^2+x+1} dx - 3 \int \frac{1}{-(2x+1)^2-3} d(2x+1) \right) + \frac{1}{3} \log(1-x) \\
 & \quad \downarrow \text{217} \\
 & \frac{1}{3} \left( \sqrt{3} \arctan \left( \frac{2x+1}{\sqrt{3}} \right) - \frac{1}{2} \int \frac{2x+1}{x^2+x+1} dx \right) + \frac{1}{3} \log(1-x) \\
 & \quad \downarrow \text{1103} \\
 & \frac{1}{3} \left( \sqrt{3} \arctan \left( \frac{2x+1}{\sqrt{3}} \right) - \frac{1}{2} \log(x^2+x+1) \right) + \frac{1}{3} \log(1-x)
 \end{aligned}$$

input `Int[x/(-1 + x^3), x]`

output `Log[1 - x]/3 + (Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[1 + x + x^2]/2)/3`

## 3.255.3.1 Defintions of rubi rules used

- rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`
- rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`
- rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & (LtQ[a, 0] || LtQ[b, 0])`
- rule 821 `Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-1) Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3]) Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]`
- rule 1083 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Simp[-2 Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x]`
- rule 1103 `Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`
- rule 1142 `Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(2*c*d - b*e)/(2*c) Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c) Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]`

**3.255.4 Maple [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

| method  | result                                                                                                                                                                                                                                                 | size |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch   | $\frac{\ln(-1+x)}{3} - \frac{\ln(x^2+x+1)}{6} + \frac{\sqrt{3} \arctan\left(\frac{2\sqrt{3}(x+\frac{1}{2})}{3}\right)}{3}$                                                                                                                             | 31   |
| default | $\frac{\ln(-1+x)}{3} - \frac{\ln(x^2+x+1)}{6} + \frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$                                                                                                                                        | 33   |
| meijerg | $\frac{x^2 \left( \ln\left(1-(x^3)^{\frac{1}{3}}\right) - \frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2} + \sqrt{3} \arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right) \right)}{3(x^3)^{\frac{2}{3}}}$ | 63   |

input `int(x/(x^3-1),x,method=_RETURNVERBOSE)`output `1/3*ln(-1+x)-1/6*ln(x^2+x+1)+1/3*3^(1/2)*arctan(2/3*3^(1/2)*(x+1/2))`**3.255.5 Fracas [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x}{-1+x^3} dx = \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) - \frac{1}{6} \log(x^2+x+1) + \frac{1}{3} \log(x-1)$$

input `integrate(x/(x^3-1),x, algorithm="fracas")`output `1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log(x - 1)`**3.255.6 Sympy [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{x}{-1+x^3} dx = \frac{\log(x-1)}{3} - \frac{\log(x^2+x+1)}{6} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

input `integrate(x/(x**3-1),x)`

output `log(x - 1)/3 - log(x**2 + x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3`

### 3.255.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x}{-1+x^3} dx = \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) - \frac{1}{6} \log(x^2+x+1) + \frac{1}{3} \log(x-1)$$

input `integrate(x/(x^3-1),x, algorithm="maxima")`

output `1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log(x - 1)`

### 3.255.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x}{-1+x^3} dx = \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) - \frac{1}{6} \log(x^2+x+1) + \frac{1}{3} \log(|x-1|)$$

input `integrate(x/(x^3-1),x, algorithm="giac")`

output `1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log(abs(x - 1))`

**3.255.9 Mupad [B] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{x}{-1+x^3} dx = \frac{\ln(x-1)}{3} - \ln\left(x + \frac{1}{2} - \frac{\sqrt{3}1i}{2}\right) \left(\frac{1}{6} + \frac{\sqrt{3}1i}{6}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3}1i}{2}\right) \left(-\frac{1}{6} + \frac{\sqrt{3}1i}{6}\right)$$

input `int(x/(x^3 - 1),x)`output `log(x - 1)/3 - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/6) + log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/6)`



### 3.256 $\int \frac{x}{-1+x^4} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.256.1 Optimal result . . . . .                            | 1360 |
| 3.256.2 Mathematica [B] (verified) . . . . .                | 1360 |
| 3.256.3 Rubi [A] (verified) . . . . .                       | 1361 |
| 3.256.4 Maple [A] (verified) . . . . .                      | 1362 |
| 3.256.5 Fricas [B] (verification not implemented) . . . . . | 1362 |
| 3.256.6 Sympy [B] (verification not implemented) . . . . .  | 1362 |
| 3.256.7 Maxima [B] (verification not implemented) . . . . . | 1363 |
| 3.256.8 Giac [B] (verification not implemented) . . . . .   | 1363 |
| 3.256.9 Mupad [B] (verification not implemented) . . . . .  | 1364 |

#### 3.256.1 Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \frac{x}{-1+x^4} dx = -\frac{1}{2} \operatorname{arctanh}(x^2)$$

output `-1/2*arctanh(x^2)`

#### 3.256.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs.  $2(8) = 16$ .

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.88

$$\int \frac{x}{-1+x^4} dx = \frac{1}{4} \log(1-x^2) - \frac{1}{4} \log(1+x^2)$$

input `Integrate[x/(-1 + x^4), x]`

output `Log[1 - x^2]/4 - Log[1 + x^2]/4`

**3.256.3 Rubi [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {807, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^4 - 1} dx$$

↓ 807

$$\frac{1}{2} \int \frac{1}{x^4 - 1} dx^2$$

↓ 220

$$-\frac{1}{2} \operatorname{arctanh}(x^2)$$

input `Int[x/(-1 + x^4), x]`

output `-1/2*ArcTanh[x^2]`

**3.256.3.1 Defintions of rubi rules used**

rule 220 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

rule 807 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Simp[1/k Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]`

**3.256.4 Maple [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

| method        | result                                                            | size |
|---------------|-------------------------------------------------------------------|------|
| meijerg       | $-\frac{\operatorname{arctanh}(x^2)}{2}$                          | 7    |
| risch         | $-\frac{\ln(x^2+1)}{4} + \frac{\ln(x^2-1)}{4}$                    | 18   |
| default       | $\frac{\ln(-1+x)}{4} + \frac{\ln(1+x)}{4} - \frac{\ln(x^2+1)}{4}$ | 22   |
| norman        | $\frac{\ln(-1+x)}{4} + \frac{\ln(1+x)}{4} - \frac{\ln(x^2+1)}{4}$ | 22   |
| parallelrisch | $\frac{\ln(-1+x)}{4} + \frac{\ln(1+x)}{4} - \frac{\ln(x^2+1)}{4}$ | 22   |

input `int(x/(x^4-1),x,method=_RETURNVERBOSE)`

output `-1/2*arctanh(x^2)`

**3.256.5 Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{x}{-1+x^4} dx = -\frac{1}{4} \log(x^2+1) + \frac{1}{4} \log(x^2-1)$$

input `integrate(x/(x^4-1),x, algorithm="fricas")`

output `-1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)`

**3.256.6 Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{x}{-1+x^4} dx = \frac{\log(x^2-1)}{4} - \frac{\log(x^2+1)}{4}$$

input `integrate(x/(x**4-1),x)`

output `log(x**2 - 1)/4 - log(x**2 + 1)/4`

### 3.256.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs.  $2(6) = 12$ .

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{x}{-1+x^4} dx = -\frac{1}{4} \log(x^2+1) + \frac{1}{4} \log(x^2-1)$$

input `integrate(x/(x^4-1),x, algorithm="maxima")`

output `-1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)`

### 3.256.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs.  $2(6) = 12$ .

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{x}{-1+x^4} dx = -\frac{1}{4} \log(x^2+1) + \frac{1}{4} \log(|x^2-1|)$$

input `integrate(x/(x^4-1),x, algorithm="giac")`

output `-1/4*log(x^2 + 1) + 1/4*log(abs(x^2 - 1))`

**3.256.9 Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{-1+x^4} dx = -\frac{\operatorname{atanh}(x^2)}{2}$$

input `int(x/(x^4 - 1),x)`

output `-atanh(x^2)/2`

### 3.257 $\int \frac{(1+x^3) \log(x)}{2+x^4} dx$

|                                              |      |
|----------------------------------------------|------|
| 3.257.1 Optimal result . . . . .             | 1365 |
| 3.257.2 Mathematica [A] (verified) . . . . . | 1365 |
| 3.257.3 Rubi [A] (verified) . . . . .        | 1366 |
| 3.257.4 Maple [B] (verified) . . . . .       | 1367 |
| 3.257.5 Fricas [F] . . . . .                 | 1368 |
| 3.257.6 Sympy [F] . . . . .                  | 1368 |
| 3.257.7 Maxima [F] . . . . .                 | 1369 |
| 3.257.8 Giac [F] . . . . .                   | 1369 |
| 3.257.9 Mupad [F(-1)] . . . . .              | 1369 |

#### 3.257.1 Optimal result

Integrand size = 15, antiderivative size = 227

$$\int \frac{(1+x^3) \log(x)}{2+x^4} dx = \frac{1}{8}(2+i\sqrt[4]{-2}) \log(x) \log\left(1-\frac{(1+i)x}{2^{3/4}}\right) + \frac{1}{16}(4+(1-i)2^{3/4}) \log(x) \log\left(1+\frac{(1+i)x}{2^{3/4}}\right) + \frac{1}{8}(2+\sqrt[4]{-2}) \log(x) \log\left(1-\frac{(-1)^{3/4}x}{\sqrt[4]{2}}\right) + \frac{1}{8}(2-\sqrt[4]{-2}) \log(x) \log\left(1+\frac{(-1)^{3/4}x}{\sqrt[4]{2}}\right)$$

output

```
1/8*(2+I*(-2)^(1/4))*ln(x)*ln(1-(1/2+1/2*I)*x*2^(1/4))+1/16*(4+(1-I)*2^(3/4))*ln(x)*ln(1+(1/2+1/2*I)*x*2^(1/4))+1/8*(2+(-2)^(1/4))*ln(x)*ln(1-1/2*(-1)^(3/4)*x*2^(3/4))+1/8*(2-(-2)^(1/4))*ln(x)*ln(1+1/2*(-1)^(3/4)*x*2^(3/4))+1/16*(4+(1-I)*2^(3/4))*polylog(2,(-1/2-1/2*I)*x*2^(1/4))+1/8*(2+I*(-2)^(1/4))*polylog(2,(1/2+1/2*I)*x*2^(1/4))+1/8*(2-(-2)^(1/4))*polylog(2,-1/2*(-1)^(3/4)*x*2^(3/4))+1/8*(2+(-2)^(1/4))*polylog(2,1/2*(-1)^(3/4)*x*2^(3/4))
```

#### 3.257.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.85

$$\int \frac{(1+x^3) \log(x)}{2+x^4} dx = \frac{1}{8} \left( (2+i\sqrt[4]{-2}) \log(x) \log\left(1-\sqrt[4]{-\frac{1}{2}}x\right) + \left(2+\frac{1-i}{\sqrt[4]{2}}\right) \log(x) \log\left(1+\sqrt[4]{-\frac{1}{2}}x\right) - (-2+\sqrt[4]{-2}) \log(x) \log\left(1-\frac{(1-i)x}{2^{3/4}}\right) + (2+\sqrt[4]{-2}) \log(x) \log\left(1+\frac{(1-i)x}{2^{3/4}}\right) + \left(2+\frac{1-i}{\sqrt[4]{2}}\right) \text{PolyLog}\left(2,-\frac{(1+i)x}{2^{3/4}}\right) + (2+\sqrt[4]{-2}) \text{PolyLog}\left(2,\frac{(1+i)x}{2^{3/4}}\right) \right)$$

input `Integrate[((1 + x^3)*Log[x])/(2 + x^4), x]`

output `((2 + I*(-2)^(1/4))*Log[x]*Log[1 - (-1/2)^(1/4)*x] + (2 + (1 - I)/2^(1/4))*Log[x]*Log[1 + (-1/2)^(1/4)*x] - (-2 + (-2)^(1/4))*Log[x]*Log[1 - ((1 - I)*x)/2^(3/4)] + (2 + (-2)^(1/4))*Log[x]*Log[1 + ((1 - I)*x)/2^(3/4)] + (2 + (1 - I)/2^(1/4))*PolyLog[2, ((-1 - I)*x)/2^(3/4)] + (2 + (-2)^(1/4))*PolyLog[2, ((-1 + I)*x)/2^(3/4)] - (-2 + (-2)^(1/4))*PolyLog[2, ((1 - I)*x)/2^(3/4)] + (2 + I*(-2)^(1/4))*PolyLog[2, ((1 + I)*x)/2^(3/4)])/8`

### 3.257.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3 + 1) \log(x)}{x^4 + 2} dx$$

↓ 2804

$$\int \left( \frac{(\sqrt[4]{-2} - 2) \log(x)}{8(\sqrt[4]{-2} - x)} + \frac{(\sqrt[4]{-2} - 2i) \log(x)}{8(\sqrt[4]{-2} - ix)} + \frac{(\sqrt[4]{-2} + 2i) \log(x)}{8(\sqrt[4]{-2} + ix)} + \frac{(2 + \sqrt[4]{-2}) \log(x)}{8(x + \sqrt[4]{-2})} \right) dx$$

↓ 2009

$$\begin{aligned} & \frac{1}{8}(2 - i\sqrt[4]{-2}) \text{PolyLog} \left( 2, -\frac{(1+i)x}{2^{3/4}} \right) + \frac{1}{8}(2 + i\sqrt[4]{-2}) \text{PolyLog} \left( 2, \frac{(1+i)x}{2^{3/4}} \right) + \\ & \frac{1}{8}(2 - \sqrt[4]{-2}) \text{PolyLog} \left( 2, -\frac{(-1)^{3/4}x}{\sqrt[4]{2}} \right) + \frac{1}{8}(2 + \sqrt[4]{-2}) \text{PolyLog} \left( 2, \frac{(-1)^{3/4}x}{\sqrt[4]{2}} \right) + \\ & \frac{1}{8}(2 + i\sqrt[4]{-2}) \log(x) \log \left( 1 - \frac{(1+i)x}{2^{3/4}} \right) + \frac{1}{8}(2 - i\sqrt[4]{-2}) \log(x) \log \left( 1 + \frac{(1+i)x}{2^{3/4}} \right) + \\ & \frac{1}{8}(2 + \sqrt[4]{-2}) \log(x) \log \left( 1 - \frac{(-1)^{3/4}x}{\sqrt[4]{2}} \right) + \frac{1}{8}(2 - \sqrt[4]{-2}) \log(x) \log \left( \frac{(-1)^{3/4}x}{\sqrt[4]{2}} + 1 \right) \end{aligned}$$

input `Int[((1 + x^3)*Log[x])/(2 + x^4), x]`

```
output ((2 + I*(-2)^(1/4))*Log[x]*Log[1 - ((1 + I)*x)/2^(3/4)]/8 + ((2 - I*(-2)^(1/4))*Log[x]*Log[1 + ((1 + I)*x)/2^(3/4)]/8 + ((2 + (-2)^(1/4))*Log[x]*Log[1 - ((-1)^(3/4)*x)/2^(1/4)]/8 + ((2 - (-2)^(1/4))*Log[x]*Log[1 + ((-1)^(3/4)*x)/2^(1/4)]/8 + ((2 - I*(-2)^(1/4))*PolyLog[2, ((-1 - I)*x)/2^(3/4)]/8 + ((2 + I*(-2)^(1/4))*PolyLog[2, ((1 + I)*x)/2^(3/4)]/8 + ((2 - (-2)^(1/4))*PolyLog[2, -(((1)^(3/4)*x)/2^(1/4)]/8 + ((2 + (-2)^(1/4))*PolyLog[2, ((-1)^(3/4)*x)/2^(1/4)]/8
```

**3.257.3.1 Defintions of rubi rules used**

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 2804 Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

**3.257.4 Maple [B] (verified)**

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(171) = 342.

Time = 0.37 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.74

| method  | result                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|---------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| default | $\frac{\left(\left(\frac{2^{\frac{3}{4}}}{2} + i\frac{2^{\frac{3}{4}}}{2}\right)^3 + 1\right) \left(\ln(x) \ln\left(\frac{\frac{2^{\frac{3}{4}}}{2} + i\frac{2^{\frac{3}{4}}}{2} - x}{\frac{2^{\frac{3}{4}}}{2} + i\frac{2^{\frac{3}{4}}}{2}}\right) + \operatorname{dilog}\left(\frac{\frac{2^{\frac{3}{4}}}{2} + i\frac{2^{\frac{3}{4}}}{2} - x}{\frac{2^{\frac{3}{4}}}{2} + i\frac{2^{\frac{3}{4}}}{2}}\right)\right)}{4\left(\frac{2^{\frac{3}{4}}}{2} + i\frac{2^{\frac{3}{4}}}{2}\right)^3} + \frac{\left(\left(\frac{i2^{\frac{3}{4}}}{2} - \frac{2^{\frac{3}{4}}}{2}\right)^3 + 1\right) \left(\ln(x) \ln\left(\frac{\frac{i2^{\frac{3}{4}}}{2} - \frac{2^{\frac{3}{4}}}{2} - x}{\frac{i2^{\frac{3}{4}}}{2} - \frac{2^{\frac{3}{4}}}{2}}\right) + \operatorname{dilog}\left(\frac{\frac{i2^{\frac{3}{4}}}{2} - \frac{2^{\frac{3}{4}}}{2} - x}{\frac{i2^{\frac{3}{4}}}{2} - \frac{2^{\frac{3}{4}}}{2}}\right)\right)}{4\left(\frac{i2^{\frac{3}{4}}}{2} - \frac{2^{\frac{3}{4}}}{2}\right)^3}$ |
| parts   | Expression too large to display                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| risch   | Expression too large to display                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |

```
input int((x^3+1)*ln(x)/(x^4+2),x,method=_RETURNVERBOSE)
```

3.257.  $\int \frac{(1+x^3) \log(x)}{2+x^4} dx$



```
output 1/4*((1/2*2^(3/4)+1/2*I*2^(3/4))^3+1)/(1/2*2^(3/4)+1/2*I*2^(3/4))^3*(ln(x)
*ln((1/2*2^(3/4)+1/2*I*2^(3/4)-x)/(1/2*2^(3/4)+1/2*I*2^(3/4))))+dilog((1/2*
2^(3/4)+1/2*I*2^(3/4)-x)/(1/2*2^(3/4)+1/2*I*2^(3/4))))+1/4*((1/2*I*2^(3/4)
-1/2*2^(3/4))^3+1)/(1/2*I*2^(3/4)-1/2*2^(3/4))^3*(ln(x)*ln((1/2*I*2^(3/4)-
1/2*2^(3/4)-x)/(1/2*I*2^(3/4)-1/2*2^(3/4))))+dilog((1/2*I*2^(3/4)-1/2*2^(3/
4)-x)/(1/2*I*2^(3/4)-1/2*2^(3/4))))+1/4*((-1/2*2^(3/4)-1/2*I*2^(3/4))^3+1)
/(-1/2*2^(3/4)-1/2*I*2^(3/4))^3*(ln(x)*ln((-1/2*2^(3/4)-1/2*I*2^(3/4)-x)/(
-1/2*2^(3/4)-1/2*I*2^(3/4))))+dilog((-1/2*2^(3/4)-1/2*I*2^(3/4)-x)/(-1/2*2^
(3/4)-1/2*I*2^(3/4))))+1/4*((-1/2*I*2^(3/4)+1/2*2^(3/4))^3+1)/(-1/2*I*2^(3
/4)+1/2*2^(3/4))^3*(ln(x)*ln((-1/2*I*2^(3/4)+1/2*2^(3/4)-x)/(-1/2*I*2^(3/4)
)+1/2*2^(3/4))))+dilog((-1/2*I*2^(3/4)+1/2*2^(3/4)-x)/(-1/2*I*2^(3/4)+1/2*2
^(3/4))))
```

### 3.257.5 Fracas [F]

$$\int \frac{(1+x^3)\log(x)}{2+x^4} dx = \int \frac{(x^3+1)\log(x)}{x^4+2} dx$$

```
input integrate((x^3+1)*log(x)/(x^4+2),x, algorithm="fricas")
```

```
output integral((x^3 + 1)*log(x)/(x^4 + 2), x)
```

### 3.257.6 Sympy [F]

$$\int \frac{(1+x^3)\log(x)}{2+x^4} dx = \int \frac{(x+1)(x^2-x+1)\log(x)}{x^4+2} dx$$

```
input integrate((x**3+1)*ln(x)/(x**4+2),x)
```

```
output Integral((x + 1)*(x**2 - x + 1)*log(x)/(x**4 + 2), x)
```

**3.257.7 Maxima [F]**

$$\int \frac{(1+x^3)\log(x)}{2+x^4} dx = \int \frac{(x^3+1)\log(x)}{x^4+2} dx$$

input `integrate((x^3+1)*log(x)/(x^4+2),x, algorithm="maxima")`

output `integrate((x^3 + 1)*log(x)/(x^4 + 2), x)`

**3.257.8 Giac [F]**

$$\int \frac{(1+x^3)\log(x)}{2+x^4} dx = \int \frac{(x^3+1)\log(x)}{x^4+2} dx$$

input `integrate((x^3+1)*log(x)/(x^4+2),x, algorithm="giac")`

output `integrate((x^3 + 1)*log(x)/(x^4 + 2), x)`

**3.257.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{(1+x^3)\log(x)}{2+x^4} dx = \int \frac{\ln(x)(x^3+1)}{x^4+2} dx$$

input `int((log(x)*(x^3 + 1))/(x^4 + 2),x)`

output `int((log(x)*(x^3 + 1))/(x^4 + 2), x)`

### 3.258 $\int (\log(x) + \log(1 + x) + \log(2 + x)) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.258.1 Optimal result . . . . .                            | 1370 |
| 3.258.2 Mathematica [A] (verified) . . . . .                | 1370 |
| 3.258.3 Rubi [A] (verified) . . . . .                       | 1371 |
| 3.258.4 Maple [A] (verified) . . . . .                      | 1371 |
| 3.258.5 Fricas [A] (verification not implemented) . . . . . | 1372 |
| 3.258.6 Sympy [A] (verification not implemented) . . . . .  | 1372 |
| 3.258.7 Maxima [A] (verification not implemented) . . . . . | 1372 |
| 3.258.8 Giac [A] (verification not implemented) . . . . .   | 1373 |
| 3.258.9 Mupad [B] (verification not implemented) . . . . .  | 1373 |

#### 3.258.1 Optimal result

Integrand size = 11, antiderivative size = 24

$$\int (\log(x) + \log(1 + x) + \log(2 + x)) dx = -3x + x \log(x) + (1 + x) \log(1 + x) + (2 + x) \log(2 + x)$$

output `-3*x+x*ln(x)+(1+x)*ln(1+x)+(2+x)*ln(2+x)`

#### 3.258.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (\log(x) + \log(1 + x) + \log(2 + x)) dx = -3x + x \log(x) + (1 + x) \log(1 + x) + (2 + x) \log(2 + x)$$

input `Integrate[Log[x] + Log[1 + x] + Log[2 + x],x]`

output `-3*x + x*Log[x] + (1 + x)*Log[1 + x] + (2 + x)*Log[2 + x]`

### 3.258.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\log(x) + \log(x + 1) + \log(x + 2)) dx$$

$$\downarrow \text{2009}$$

$$-3x + x \log(x) + (x + 1) \log(x + 1) + (x + 2) \log(x + 2)$$

input `Int[Log[x] + Log[1 + x] + Log[2 + x], x]`

output `-3*x + x*Log[x] + (1 + x)*Log[1 + x] + (2 + x)*Log[2 + x]`

#### 3.258.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

### 3.258.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

| method        | result                                                                      | size |
|---------------|-----------------------------------------------------------------------------|------|
| default       | $-3x + x \ln(x) + (1 + x) \ln(1 + x) - 3 + (2 + x) \ln(2 + x)$              | 26   |
| parts         | $-3x + x \ln(x) + (1 + x) \ln(1 + x) - 3 + (2 + x) \ln(2 + x)$              | 26   |
| risch         | $-3x + x \ln(x) + \ln(1 + x)x + \ln(1 + x) + \ln(2 + x)x + 2 \ln(2 + x)$    | 31   |
| parallelrisch | $x \ln(x) + \ln(1 + x)x + \ln(2 + x)x - 3x + \ln(1 + x) + 2 \ln(2 + x) + 9$ | 32   |

input `int(ln(x)+ln(1+x)+ln(2+x),x,method=_RETURNVERBOSE)`

output `-3*x+x*ln(x)+(1+x)*ln(1+x)-3+(2+x)*ln(2+x)`

**3.258.5 Fricas [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (\log(x) + \log(1+x) + \log(2+x)) dx = (x+2) \log(x+2) + (x+1) \log(x+1) + x \log(x) - 3x$$

input `integrate(log(x)+log(1+x)+log(2+x),x, algorithm="fricas")`output `(x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x`**3.258.6 Sympy [A] (verification not implemented)**

Time = 0.70 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int (\log(x) + \log(1+x) + \log(2+x)) dx = x \log(x) - 3x + \left(x + \frac{1}{2}\right) \log(x+1) + (x+1) \log(x+2) + \frac{\log(x+1)}{2} + \log(x+2)$$

input `integrate(ln(x)+ln(1+x)+ln(2+x),x)`output `x*log(x) - 3*x + (x + 1/2)*log(x + 1) + (x + 1)*log(x + 2) + log(x + 1)/2 + log(x + 2)`**3.258.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int (\log(x) + \log(1+x) + \log(2+x)) dx = (x+2) \log(x+2) + (x+1) \log(x+1) + x \log(x) - 3x - 3$$

input `integrate(log(x)+log(1+x)+log(2+x),x, algorithm="maxima")`output `(x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x - 3`

**3.258.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int (\log(x) + \log(1+x) + \log(2+x)) dx = (x+2)\log(x+2) + (x+1)\log(x+1) + x\log(x) - 3x - 3$$

input `integrate(log(x)+log(1+x)+log(2+x),x, algorithm="giac")`output `(x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x - 3`**3.258.9 Mupad [B] (verification not implemented)**

Time = 0.44 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int (\log(x) + \log(1+x) + \log(2+x)) dx = \ln(x+1) - 3x + 2\ln(x+2) + x\ln(x+1) + x\ln(x) + \frac{\ln(x+2)(x^3 + 3x^2 + 2x)}{(x+1)(x+2)}$$

input `int(log(x + 1) + log(x + 2) + log(x),x)`output `log(x + 1) - 3*x + 2*log(x + 2) + x*log(x + 1) + x*log(x) + (log(x + 2)*(2*x + 3*x^2 + x^3))/((x + 1)*(x + 2))`

### 3.259 $\int \frac{1}{5+x^3} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.259.1 Optimal result . . . . .                            | 1374 |
| 3.259.2 Mathematica [A] (verified) . . . . .                | 1374 |
| 3.259.3 Rubi [A] (verified) . . . . .                       | 1375 |
| 3.259.4 Maple [C] (verified) . . . . .                      | 1377 |
| 3.259.5 Fricas [A] (verification not implemented) . . . . . | 1377 |
| 3.259.6 Sympy [A] (verification not implemented) . . . . .  | 1378 |
| 3.259.7 Maxima [A] (verification not implemented) . . . . . | 1378 |
| 3.259.8 Giac [A] (verification not implemented) . . . . .   | 1379 |
| 3.259.9 Mupad [B] (verification not implemented) . . . . .  | 1379 |

#### 3.259.1 Optimal result

Integrand size = 7, antiderivative size = 78

$$\int \frac{1}{5+x^3} dx = -\frac{\arctan\left(\frac{\sqrt[3]{5-2x}}{\sqrt{3}\sqrt[3]{5}}\right)}{\sqrt{3}5^{2/3}} + \frac{\log(\sqrt[3]{5}+x)}{3\ 5^{2/3}} - \frac{\log(5^{2/3}-\sqrt[3]{5}x+x^2)}{6\ 5^{2/3}}$$

output `1/15*ln(5^(1/3)+x)*5^(1/3)-1/30*ln(5^(2/3)-5^(1/3)*x+x^2)*5^(1/3)-1/15*arctan(1/15*(5^(1/3)-2*x)*5^(2/3)*3^(1/2))*5^(1/3)*3^(1/2)`

#### 3.259.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{1}{5+x^3} dx = \frac{2\sqrt{3}\arctan\left(\frac{-5+2\ 5^{2/3}x}{5\sqrt{3}}\right) + 2\log(5+5^{2/3}x) - \log(5-5^{2/3}x+\sqrt[3]{5}x^2)}{6\ 5^{2/3}}$$

input `Integrate[(5 + x^3)^(-1),x]`

output `(2*sqrt[3]*ArcTan[(-5 + 2*5^(2/3)*x)/(5*sqrt[3])]) + 2*Log[5 + 5^(2/3)*x] - Log[5 - 5^(2/3)*x + 5^(1/3)*x^2])/(6*5^(2/3))`

**3.259.3 Rubi [A] (verified)**

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {750, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{x^3+5} dx \\
 & \quad \downarrow \text{750} \\
 & \frac{\int \frac{2\sqrt[3]{5}-x}{x^2-\sqrt[3]{5}x+5^{2/3}} dx}{3 \cdot 5^{2/3}} + \frac{\int \frac{1}{x+\sqrt[3]{5}} dx}{3 \cdot 5^{2/3}} \\
 & \quad \downarrow \text{16} \\
 & \frac{\int \frac{2\sqrt[3]{5}-x}{x^2-\sqrt[3]{5}x+5^{2/3}} dx}{3 \cdot 5^{2/3}} + \frac{\log(x+\sqrt[3]{5})}{3 \cdot 5^{2/3}} \\
 & \quad \downarrow \text{1142} \\
 & \frac{\frac{3}{2}\sqrt[3]{5} \int \frac{1}{x^2-\sqrt[3]{5}x+5^{2/3}} dx - \frac{1}{2} \int \frac{\sqrt[3]{5}-2x}{x^2-\sqrt[3]{5}x+5^{2/3}} dx}{3 \cdot 5^{2/3}} + \frac{\log(x+\sqrt[3]{5})}{3 \cdot 5^{2/3}} \\
 & \quad \downarrow \text{25} \\
 & \frac{\frac{3}{2}\sqrt[3]{5} \int \frac{1}{x^2-\sqrt[3]{5}x+5^{2/3}} dx + \frac{1}{2} \int \frac{\sqrt[3]{5}-2x}{x^2-\sqrt[3]{5}x+5^{2/3}} dx}{3 \cdot 5^{2/3}} + \frac{\log(x+\sqrt[3]{5})}{3 \cdot 5^{2/3}} \\
 & \quad \downarrow \text{1082} \\
 & \frac{\frac{1}{2} \int \frac{\sqrt[3]{5}-2x}{x^2-\sqrt[3]{5}x+5^{2/3}} dx + 3 \int \frac{1}{-\left(1-\frac{2x}{\sqrt[3]{5}}\right)^2} d\left(1-\frac{2x}{\sqrt[3]{5}}\right)}{3 \cdot 5^{2/3}} + \frac{\log(x+\sqrt[3]{5})}{3 \cdot 5^{2/3}} \\
 & \quad \downarrow \text{217} \\
 & \frac{\frac{1}{2} \int \frac{\sqrt[3]{5}-2x}{x^2-\sqrt[3]{5}x+5^{2/3}} dx - \sqrt{3} \arctan\left(\frac{1-\frac{2x}{\sqrt[3]{5}}}{\sqrt{3}}\right)}{3 \cdot 5^{2/3}} + \frac{\log(x+\sqrt[3]{5})}{3 \cdot 5^{2/3}} \\
 & \quad \downarrow \text{1103}
 \end{aligned}$$



$$\frac{-\sqrt{3} \arctan\left(\frac{1-\frac{2x}{\sqrt[3]{5}}}{\sqrt{3}}\right) - \frac{1}{2} \log\left(x^2 - \sqrt[3]{5}x + 5^{2/3}\right)}{3 \cdot 5^{2/3}} + \frac{\log\left(x + \sqrt[3]{5}\right)}{3 \cdot 5^{2/3}}$$

input `Int[(5 + x^3)^(-1), x]`

output `Log[5^(1/3) + x]/(3*5^(2/3)) + (-Sqrt[3]*ArcTan[(1 - (2*x)/5^(1/3))/Sqrt[3]]) - Log[5^(2/3) - 5^(1/3)*x + x^2]/2/(3*5^(2/3))`

### 3.259.3.1 Defintions of rubi rules used

rule 16 `Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] := Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]`

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

rule 750 `Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Simp[1/(3*Rt[a, 3]^2) Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2) Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]`

rule 1082 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Simp[-2/b Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x]`

rule 1103 `Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

```
rule 1142 Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(2*c*d - b*e)/(2*c) Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

### 3.259.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.28

| method  | result                                                                                                                                                                                                                                                                                                                                                                                                                                           | size |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| risch   | $\frac{\sum_{-R=\text{RootOf}(-Z^3+5)} \frac{\ln(x-R)}{-R^2}}{3}$                                                                                                                                                                                                                                                                                                                                                                                | 22   |
| default | $\frac{\ln(5^{\frac{1}{3}}+x)5^{\frac{1}{3}}}{15} - \frac{\ln(5^{\frac{2}{3}}-5^{\frac{1}{3}}x+x^2)5^{\frac{1}{3}}}{30} + \frac{5^{\frac{1}{3}}\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(\frac{25^{\frac{2}{3}}x-1}{5}\right)}{3}\right)}{15}$                                                                                                                                                                                                   | 54   |
| meijerg | $\frac{5^{\frac{1}{3}} \left( \frac{x \ln\left(1 + \frac{5^{\frac{2}{3}}(x^3)^{\frac{1}{3}}}{5}\right)}{(x^3)^{\frac{1}{3}}} - \frac{x \ln\left(1 - \frac{5^{\frac{2}{3}}(x^3)^{\frac{1}{3}}}{5} + \frac{5^{\frac{1}{3}}(x^3)^{\frac{2}{3}}}{5}\right)}{2(x^3)^{\frac{1}{3}}} + \frac{x\sqrt{3} \arctan\left(\frac{\sqrt{3}5^{\frac{2}{3}}(x^3)^{\frac{1}{3}}}{10-5^{\frac{2}{3}}(x^3)^{\frac{1}{3}}}\right)}{(x^3)^{\frac{1}{3}}} \right)}{15}$ | 96   |

```
input int(1/(x^3+5),x,method=_RETURNVERBOSE)
```

```
output 1/3*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3+5))
```

### 3.259.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \frac{1}{5+x^3} dx = \frac{1}{15} \cdot 25^{\frac{1}{6}} \sqrt{3} \arctan\left(\frac{1}{75} \cdot 25^{\frac{1}{6}} \left(2 \cdot 25^{\frac{2}{3}} \sqrt{3}x - 5 \cdot 25^{\frac{1}{3}} \sqrt{3}\right)\right) - \frac{1}{150} \cdot 25^{\frac{2}{3}} \log\left(5x^2 - 25^{\frac{2}{3}}x + 5 \cdot 25^{\frac{1}{3}}\right) + \frac{1}{75} \cdot 25^{\frac{2}{3}} \log\left(5x + 25^{\frac{2}{3}}\right)$$

```
input integrate(1/(x^3+5),x, algorithm="fricas")
```

output  $1/15*25^{(1/6)}*\sqrt{3}*\arctan(1/75*25^{(1/6)}*(2*25^{(2/3)}*\sqrt{3}*x - 5*25^{(1/3)}*\sqrt{3})) - 1/150*25^{(2/3)}*\log(5*x^2 - 25^{(2/3)}*x + 5*25^{(1/3)}) + 1/75*25^{(2/3)}*\log(5*x + 25^{(2/3)})$

### 3.259.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{1}{5+x^3} dx = \frac{\sqrt[3]{5} \log(x + \sqrt[3]{5})}{15} - \frac{\sqrt[3]{5} \log(x^2 - \sqrt[3]{5}x + 5^{2/3})}{30} + \frac{\sqrt{3} \cdot \sqrt[3]{5} \operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{5}x}{15} - \frac{\sqrt{3}}{3}\right)}{15}$$

input `integrate(1/(x**3+5),x)`

output  $5^{(1/3)}*\log(x + 5^{(1/3)})/15 - 5^{(1/3)}*\log(x^2 - 5^{(1/3)}*x + 5^{(2/3)})/30 + \sqrt{3}*5^{(1/3)}*\operatorname{atan}(2*\sqrt{3}*5^{(2/3)}*x/15 - \sqrt{3}/3)/15$

### 3.259.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int \frac{1}{5+x^3} dx = \frac{1}{15} \cdot 5^{1/3} \sqrt{3} \arctan\left(\frac{1}{15} \cdot 5^{2/3} \sqrt{3} (2x - 5^{1/3})\right) - \frac{1}{30} \cdot 5^{1/3} \log(x^2 - 5^{1/3}x + 5^{2/3}) + \frac{1}{15} \cdot 5^{1/3} \log(x + 5^{1/3})$$

input `integrate(1/(x^3+5),x, algorithm="maxima")`

output  $1/15*5^{(1/3)}*\sqrt{3}*\arctan(1/15*5^{(2/3)}*\sqrt{3}*(2*x - 5^{(1/3)})) - 1/30*5^{(1/3)}*\log(x^2 - 5^{(1/3)}*x + 5^{(2/3)}) + 1/15*5^{(1/3)}*\log(x + 5^{(1/3)})$

**3.259.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \frac{1}{5+x^3} dx = \frac{1}{15} \cdot 5^{\frac{1}{3}} \sqrt{3} \arctan \left( \frac{1}{15} \cdot 5^{\frac{2}{3}} \sqrt{3} (2x - 5^{\frac{1}{3}}) \right) - \frac{1}{30} \cdot 5^{\frac{1}{3}} \log \left( x^2 - 5^{\frac{1}{3}} x + 5^{\frac{2}{3}} \right) + \frac{1}{15} \cdot 5^{\frac{1}{3}} \log \left( \left| x + 5^{\frac{1}{3}} \right| \right)$$

input `integrate(1/(x^3+5),x, algorithm="giac")`output `1/15*5^(1/3)*sqrt(3)*arctan(1/15*5^(2/3)*sqrt(3)*(2*x - 5^(1/3))) - 1/30*5^(1/3)*log(x^2 - 5^(1/3)*x + 5^(2/3)) + 1/15*5^(1/3)*log(abs(x + 5^(1/3)))`**3.259.9 Mupad [B] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \frac{1}{5+x^3} dx = \frac{5^{1/3} \ln(x + 5^{1/3})}{15} + \frac{5^{1/3} \ln \left( x + \frac{5^{1/3}(-1+\sqrt{3}i)}{2} \right) (-1 + \sqrt{3}i)}{30} - \frac{5^{1/3} \ln \left( x - \frac{5^{1/3}(1+\sqrt{3}i)}{2} \right) (1 + \sqrt{3}i)}{30}$$

input `int(1/(x^3 + 5),x)`output `(5^(1/3)*log(x + 5^(1/3)))/15 + (5^(1/3)*log(x + (5^(1/3)*(3^(1/2)*1i - 1)/2)*(3^(1/2)*1i - 1)))/30 - (5^(1/3)*log(x - (5^(1/3)*(3^(1/2)*1i + 1)/2)*(3^(1/2)*1i + 1)))/30`

### 3.260 $\int \frac{1}{\sqrt{1+x^2}} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.260.1 Optimal result . . . . .                            | 1380 |
| 3.260.2 Mathematica [B] (verified) . . . . .                | 1380 |
| 3.260.3 Rubi [A] (verified) . . . . .                       | 1381 |
| 3.260.4 Maple [A] (verified) . . . . .                      | 1381 |
| 3.260.5 Fricas [B] (verification not implemented) . . . . . | 1382 |
| 3.260.6 Sympy [A] (verification not implemented) . . . . .  | 1382 |
| 3.260.7 Maxima [A] (verification not implemented) . . . . . | 1382 |
| 3.260.8 Giac [B] (verification not implemented) . . . . .   | 1383 |
| 3.260.9 Mupad [B] (verification not implemented) . . . . .  | 1383 |

#### 3.260.1 Optimal result

Integrand size = 9, antiderivative size = 2

$$\int \frac{1}{\sqrt{1+x^2}} dx = \operatorname{arcsinh}(x)$$

output `arcsinh(x)`

#### 3.260.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16 vs.  $2(2) = 4$ .

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 8.00

$$\int \frac{1}{\sqrt{1+x^2}} dx = -\log(-x + \sqrt{1+x^2})$$

input `Integrate[1/Sqrt[1 + x^2],x]`

output `-Log[-x + Sqrt[1 + x^2]]`

**3.260.3 Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2 + 1}} dx$$

↓ 222

$$\operatorname{arcsinh}(x)$$

input `Int[1/Sqrt[1 + x^2], x]`

output `ArcSinh[x]`

**3.260.3.1 Defintions of rubi rules used**

rule 222 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]`

**3.260.4 Maple [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

| method         | result                                                      | size |
|----------------|-------------------------------------------------------------|------|
| default        | $\operatorname{arcsinh}(x)$                                 | 3    |
| meijerg        | $\operatorname{arcsinh}(x)$                                 | 3    |
| trager         | $\ln(x + \sqrt{x^2 + 1})$                                   | 11   |
| pseudoelliptic | $\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}}{x}\right)$ | 13   |

input `int(1/(x^2+1)^(1/2), x, method=_RETURNVERBOSE)`

output `arcsinh(x)`

**3.260.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 14 vs.  $2(2) = 4$ .

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 7.00

$$\int \frac{1}{\sqrt{1+x^2}} dx = -\log(-x + \sqrt{x^2+1})$$

input `integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")`

output `-log(-x + sqrt(x^2 + 1))`

**3.260.6 Sympy [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}} dx = \operatorname{asinh}(x)$$

input `integrate(1/(x**2+1)**(1/2),x)`

output `asinh(x)`

**3.260.7 Maxima [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}} dx = \operatorname{arsinh}(x)$$

input `integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")`

output `arcsinh(x)`

**3.260.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 25 vs.  $2(2) = 4$ .

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 12.50

$$\int \frac{1}{\sqrt{1+x^2}} dx = \frac{1}{2} \sqrt{x^2+1}x - \frac{1}{2} \log(-x + \sqrt{x^2+1})$$

input `integrate(1/(x^2+1)^(1/2),x, algorithm="giac")`

output `1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))`

**3.260.9 Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}} dx = \operatorname{asinh}(x)$$

input `int(1/(x^2 + 1)^(1/2),x)`

output `asinh(x)`



### 3.261 $\int \sqrt{3 + x^2} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.261.1 Optimal result . . . . .                            | 1384 |
| 3.261.2 Mathematica [A] (verified) . . . . .                | 1384 |
| 3.261.3 Rubi [A] (verified) . . . . .                       | 1385 |
| 3.261.4 Maple [A] (verified) . . . . .                      | 1386 |
| 3.261.5 Fricas [A] (verification not implemented) . . . . . | 1386 |
| 3.261.6 Sympy [A] (verification not implemented) . . . . .  | 1387 |
| 3.261.7 Maxima [A] (verification not implemented) . . . . . | 1387 |
| 3.261.8 Giac [A] (verification not implemented) . . . . .   | 1387 |
| 3.261.9 Mupad [B] (verification not implemented) . . . . .  | 1388 |

#### 3.261.1 Optimal result

Integrand size = 9, antiderivative size = 27

$$\int \sqrt{3 + x^2} dx = \frac{1}{2}x\sqrt{3 + x^2} + \frac{3}{2}\operatorname{arcsinh}\left(\frac{x}{\sqrt{3}}\right)$$

output `3/2*arcsinh(1/3*x*3^(1/2))+1/2*x*(x^2+3)^(1/2)`

#### 3.261.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \sqrt{3 + x^2} dx = \frac{1}{2}x\sqrt{3 + x^2} - \frac{3}{2}\log\left(-x + \sqrt{3 + x^2}\right)$$

input `Integrate[Sqrt[3 + x^2],x]`

output `(x*Sqrt[3 + x^2])/2 - (3*Log[-x + Sqrt[3 + x^2]])/2`

**3.261.3 Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2 + 3} dx$$

$$\downarrow \text{211}$$

$$\frac{3}{2} \int \frac{1}{\sqrt{x^2 + 3}} dx + \frac{1}{2} \sqrt{x^2 + 3} x$$

$$\downarrow \text{222}$$

$$\frac{3}{2} \operatorname{arcsinh}\left(\frac{x}{\sqrt{3}}\right) + \frac{1}{2} \sqrt{x^2 + 3} x$$

input `Int[Sqrt[3 + x^2], x]`

output `(x*Sqrt[3 + x^2])/2 + (3*ArcSinh[x/Sqrt[3]])/2`

**3.261.3.1 Defintions of rubi rules used**

rule 211 `Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[x*((a + b*x^2)^p/(2*p + 1)), x] + Simp[2*a*(p/(2*p + 1)) Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])`

rule 222 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]`

**3.261.4 Maple [A] (verified)**

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

| method         | result                                                                                                                                                                | size |
|----------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default        | $\frac{3 \operatorname{arcsinh}\left(\frac{x\sqrt{3}}{3}\right)}{2} + \frac{x\sqrt{x^2+3}}{2}$                                                                        | 21   |
| risch          | $\frac{3 \operatorname{arcsinh}\left(\frac{x\sqrt{3}}{3}\right)}{2} + \frac{x\sqrt{x^2+3}}{2}$                                                                        | 21   |
| trager         | $\frac{x\sqrt{x^2+3}}{2} + \frac{3 \ln(x+\sqrt{x^2+3})}{2}$                                                                                                           | 24   |
| meijerg        | $-\frac{3 \left( -\frac{2\sqrt{\pi} x \sqrt{3} \sqrt{\frac{x^2}{3}+1}}{3} - 2\sqrt{\pi} \operatorname{arcsinh}\left(\frac{x\sqrt{3}}{3}\right) \right)}{4\sqrt{\pi}}$ | 37   |
| pseudoelliptic | $\frac{x\sqrt{x^2+3}}{2} + \frac{3 \ln\left(\frac{x+\sqrt{x^2+3}}{x}\right)}{4} - \frac{3 \ln\left(\frac{\sqrt{x^2+3}-x}{x}\right)}{4}$                               | 46   |

input `int((x^2+3)^(1/2),x,method=_RETURNVERBOSE)`output `3/2*arcsinh(1/3*x*3^(1/2))+1/2*x*(x^2+3)^(1/2)`**3.261.5 Fricas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sqrt{3+x^2} dx = \frac{1}{2} \sqrt{x^2+3}x - \frac{3}{2} \log(-x + \sqrt{x^2+3})$$

input `integrate((x^2+3)^(1/2),x, algorithm="fricas")`output `1/2*sqrt(x^2 + 3)*x - 3/2*log(-x + sqrt(x^2 + 3))`

**3.261.6 Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \sqrt{3+x^2} dx = \frac{x\sqrt{x^2+3}}{2} + \frac{3 \operatorname{asinh}\left(\frac{\sqrt{3}x}{3}\right)}{2}$$

input `integrate((x**2+3)**(1/2),x)`output `x*sqrt(x**2 + 3)/2 + 3*asinh(sqrt(3)*x/3)/2`**3.261.7 Maxima [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \sqrt{3+x^2} dx = \frac{1}{2} \sqrt{x^2+3}x + \frac{3}{2} \operatorname{arsinh}\left(\frac{1}{3} \sqrt{3}x\right)$$

input `integrate((x^2+3)^(1/2),x, algorithm="maxima")`output `1/2*sqrt(x^2 + 3)*x + 3/2*arcsinh(1/3*sqrt(3)*x)`**3.261.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sqrt{3+x^2} dx = \frac{1}{2} \sqrt{x^2+3}x - \frac{3}{2} \log\left(-x + \sqrt{x^2+3}\right)$$

input `integrate((x^2+3)^(1/2),x, algorithm="giac")`output `1/2*sqrt(x^2 + 3)*x - 3/2*log(-x + sqrt(x^2 + 3))`

**3.261.9 Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \sqrt{3+x^2} dx = \frac{3 \operatorname{asinh}\left(\frac{\sqrt{3}x}{3}\right)}{2} + \frac{x\sqrt{x^2+3}}{2}$$

input `int((x^2 + 3)^(1/2),x)`

output `(3*asinh((3^(1/2)*x)/3))/2 + (x*(x^2 + 3)^(1/2))/2`

## 3.262 $\int \frac{x}{(1+x)^2} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.262.1 Optimal result . . . . .                            | 1389 |
| 3.262.2 Mathematica [A] (verified) . . . . .                | 1389 |
| 3.262.3 Rubi [A] (verified) . . . . .                       | 1390 |
| 3.262.4 Maple [A] (verified) . . . . .                      | 1391 |
| 3.262.5 Fricas [A] (verification not implemented) . . . . . | 1391 |
| 3.262.6 Sympy [A] (verification not implemented) . . . . .  | 1391 |
| 3.262.7 Maxima [A] (verification not implemented) . . . . . | 1392 |
| 3.262.8 Giac [A] (verification not implemented) . . . . .   | 1392 |
| 3.262.9 Mupad [B] (verification not implemented) . . . . .  | 1392 |

### 3.262.1 Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{x}{(1+x)^2} dx = \frac{1}{1+x} + \log(1+x)$$

output `1/(1+x)+ln(1+x)`

### 3.262.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)^2} dx = \frac{1}{1+x} + \log(1+x)$$

input `Integrate[x/(1 + x)^2,x]`

output `(1 + x)^(-1) + Log[1 + x]`

**3.262.3 Rubi [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)^2} dx$$

↓ 49

$$\int \left( \frac{1}{x+1} - \frac{1}{(x+1)^2} \right) dx$$

↓ 2009

$$\frac{1}{x+1} + \log(x+1)$$

input `Int[x/(1 + x)^2,x]`

output `(1 + x)^(-1) + Log[1 + x]`

**3.262.3.1 Defintions of rubi rules used**

rule 49 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

**3.262.4 Maple [A] (verified)**

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

| method        | result                             | size |
|---------------|------------------------------------|------|
| default       | $\frac{1}{1+x} + \ln(1+x)$         | 11   |
| norman        | $\frac{1}{1+x} + \ln(1+x)$         | 11   |
| risch         | $\frac{1}{1+x} + \ln(1+x)$         | 11   |
| meijerg       | $-\frac{x}{1+x} + \ln(1+x)$        | 14   |
| parallelrisch | $\frac{\ln(1+x)x+1+\ln(1+x)}{1+x}$ | 19   |

input `int(x/(1+x)^2,x,method=_RETURNVERBOSE)`output `1/(1+x)+ln(1+x)`**3.262.5 Fracas [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{x}{(1+x)^2} dx = \frac{(x+1)\log(x+1)+1}{x+1}$$

input `integrate(x/(1+x)^2,x, algorithm="fricas")`output `((x + 1)*log(x + 1) + 1)/(x + 1)`**3.262.6 Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x)^2} dx = \log(x+1) + \frac{1}{x+1}$$

input `integrate(x/(1+x)**2,x)`output `log(x + 1) + 1/(x + 1)`



**3.262.7 Maxima [A] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)^2} dx = \frac{1}{x+1} + \log(x+1)$$

input `integrate(x/(1+x)^2,x, algorithm="maxima")`output `1/(x + 1) + log(x + 1)`**3.262.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x}{(1+x)^2} dx = \frac{1}{x+1} + \log(|x+1|)$$

input `integrate(x/(1+x)^2,x, algorithm="giac")`output `1/(x + 1) + log(abs(x + 1))`**3.262.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)^2} dx = \ln(x+1) + \frac{1}{x+1}$$

input `int(x/(x + 1)^2,x)`output `log(x + 1) + 1/(x + 1)`

## 3.263 $\int \arcsin(x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.263.1 Optimal result . . . . .                            | 1393 |
| 3.263.2 Mathematica [A] (verified) . . . . .                | 1393 |
| 3.263.3 Rubi [A] (verified) . . . . .                       | 1394 |
| 3.263.4 Maple [A] (verified) . . . . .                      | 1395 |
| 3.263.5 Fricas [A] (verification not implemented) . . . . . | 1395 |
| 3.263.6 Sympy [A] (verification not implemented) . . . . .  | 1395 |
| 3.263.7 Maxima [A] (verification not implemented) . . . . . | 1396 |
| 3.263.8 Giac [A] (verification not implemented) . . . . .   | 1396 |
| 3.263.9 Mupad [B] (verification not implemented) . . . . .  | 1396 |

### 3.263.1 Optimal result

Integrand size = 2, antiderivative size = 16

$$\int \arcsin(x) dx = \sqrt{1-x^2} + x \arcsin(x)$$

output `x*arcsin(x)+(-x^2+1)^(1/2)`

### 3.263.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \arcsin(x) dx = \sqrt{1-x^2} + x \arcsin(x)$$

input `Integrate[ArcSin[x],x]`

output `Sqrt[1 - x^2] + x*ArcSin[x]`

**3.263.3 Rubi [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {5130, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arcsin(x) dx$$

$$\downarrow \text{5130}$$

$$x \arcsin(x) - \int \frac{x}{\sqrt{1-x^2}} dx$$

$$\downarrow \text{241}$$

$$x \arcsin(x) + \sqrt{1-x^2}$$

input `Int[ArcSin[x], x]`

output `Sqrt[1 - x^2] + x*ArcSin[x]`

**3.263.3.1 Defintions of rubi rules used**

rule 241 `Int[(x_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]`

rule 5130 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_), x_Symbol] := Simp[x*(a + b*ArcSin[c*x])^n, x] - Simp[b*c*n Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]`

**3.263.4 Maple [A] (verified)**

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

| method  | result                           | size |
|---------|----------------------------------|------|
| lookup  | $\arcsin(x) x + \sqrt{-x^2 + 1}$ | 15   |
| default | $\arcsin(x) x + \sqrt{-x^2 + 1}$ | 15   |
| parts   | $\arcsin(x) x + \sqrt{-x^2 + 1}$ | 15   |

input `int(arcsin(x),x,method=_RETURNVERBOSE)`

output `arcsin(x)*x+(-x^2+1)^(1/2)`

**3.263.5 Fricas [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \arcsin(x) dx = x \arcsin(x) + \sqrt{-x^2 + 1}$$

input `integrate(arcsin(x),x, algorithm="fricas")`

output `x*arcsin(x) + sqrt(-x^2 + 1)`

**3.263.6 Sympy [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \arcsin(x) dx = x \operatorname{asin}(x) + \sqrt{1 - x^2}$$

input `integrate(asin(x),x)`

output `x*asin(x) + sqrt(1 - x**2)`

**3.263.7 Maxima [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \arcsin(x) dx = x \arcsin(x) + \sqrt{-x^2 + 1}$$

input `integrate(arcsin(x),x, algorithm="maxima")`

output `x*arcsin(x) + sqrt(-x^2 + 1)`

**3.263.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \arcsin(x) dx = x \arcsin(x) + \sqrt{-x^2 + 1}$$

input `integrate(arcsin(x),x, algorithm="giac")`

output `x*arcsin(x) + sqrt(-x^2 + 1)`

**3.263.9 Mupad [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \arcsin(x) dx = x \arcsin(x) + \sqrt{1 - x^2}$$

input `int(asin(x),x)`

output `x*asin(x) + (1 - x^2)^(1/2)`

### 3.264 $\int x^2 \arcsin(x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.264.1 Optimal result . . . . .                            | 1397 |
| 3.264.2 Mathematica [A] (verified) . . . . .                | 1397 |
| 3.264.3 Rubi [A] (verified) . . . . .                       | 1398 |
| 3.264.4 Maple [A] (verified) . . . . .                      | 1399 |
| 3.264.5 Fricas [A] (verification not implemented) . . . . . | 1399 |
| 3.264.6 Sympy [A] (verification not implemented) . . . . .  | 1400 |
| 3.264.7 Maxima [A] (verification not implemented) . . . . . | 1400 |
| 3.264.8 Giac [A] (verification not implemented) . . . . .   | 1400 |
| 3.264.9 Mupad [B] (verification not implemented) . . . . .  | 1401 |

#### 3.264.1 Optimal result

Integrand size = 6, antiderivative size = 40

$$\int x^2 \arcsin(x) dx = \frac{\sqrt{1-x^2}}{3} - \frac{1}{9}(1-x^2)^{3/2} + \frac{1}{3}x^3 \arcsin(x)$$

output `-1/9*(-x^2+1)^(3/2)+1/3*x^3*arcsin(x)+1/3*(-x^2+1)^(1/2)`

#### 3.264.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int x^2 \arcsin(x) dx = \frac{1}{9}(\sqrt{1-x^2}(2+x^2) + 3x^3 \arcsin(x))$$

input `Integrate[x^2*ArcSin[x],x]`

output `(Sqrt[1 - x^2]*(2 + x^2) + 3*x^3*ArcSin[x])/9`

**3.264.3 Rubi [A] (verified)**

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {5138, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^2 \arcsin(x) dx \\
 & \quad \downarrow \text{5138} \\
 & \frac{1}{3}x^3 \arcsin(x) - \frac{1}{3} \int \frac{x^3}{\sqrt{1-x^2}} dx \\
 & \quad \downarrow \text{243} \\
 & \frac{1}{3}x^3 \arcsin(x) - \frac{1}{6} \int \frac{x^2}{\sqrt{1-x^2}} dx^2 \\
 & \quad \downarrow \text{53} \\
 & \frac{1}{3}x^3 \arcsin(x) - \frac{1}{6} \int \left( \frac{1}{\sqrt{1-x^2}} - \sqrt{1-x^2} \right) dx^2 \\
 & \quad \downarrow \text{2009} \\
 & \frac{1}{3}x^3 \arcsin(x) + \frac{1}{6} \left( 2\sqrt{1-x^2} - \frac{2}{3}(1-x^2)^{3/2} \right)
 \end{aligned}$$

input `Int[x^2*ArcSin[x],x]`

output `(2*sqrt[1 - x^2] - (2*(1 - x^2)^(3/2))/3)/6 + (x^3*ArcSin[x])/3`

**3.264.3.1 Defintions of rubi rules used**

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]`

rule 243 `Int[(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && IntegerQ[(m - 1)/2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5138 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*((d_)*(x_))^(m_), x_Symbol] := Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n/(d*(m + 1))) Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]`

### 3.264.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

| method  | result                                                                              | size |
|---------|-------------------------------------------------------------------------------------|------|
| default | $\frac{x^3 \arcsin(x)}{3} + \frac{x^2 \sqrt{-x^2+1}}{9} + \frac{2\sqrt{-x^2+1}}{9}$ | 34   |
| parts   | $\frac{x^3 \arcsin(x)}{3} + \frac{x^2 \sqrt{-x^2+1}}{9} + \frac{2\sqrt{-x^2+1}}{9}$ | 34   |

input `int(x^2*arcsin(x),x,method=_RETURNVERBOSE)`

output `1/3*x^3*arcsin(x)+1/9*x^2*(-x^2+1)^(1/2)+2/9*(-x^2+1)^(1/2)`

### 3.264.5 Fracas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int x^2 \arcsin(x) dx = \frac{1}{3} x^3 \arcsin(x) + \frac{1}{9} (x^2 + 2) \sqrt{-x^2 + 1}$$

input `integrate(x^2*arcsin(x),x, algorithm="fricas")`

output `1/3*x^3*arcsin(x) + 1/9*(x^2 + 2)*sqrt(-x^2 + 1)`



**3.264.6 Sympy [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int x^2 \arcsin(x) dx = \frac{x^3 \arcsin(x)}{3} + \frac{x^2 \sqrt{1-x^2}}{9} + \frac{2\sqrt{1-x^2}}{9}$$

input `integrate(x**2*asin(x),x)`output `x**3*asin(x)/3 + x**2*sqrt(1 - x**2)/9 + 2*sqrt(1 - x**2)/9`**3.264.7 Maxima [A] (verification not implemented)**

Time = 0.46 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int x^2 \arcsin(x) dx = \frac{1}{3} x^3 \arcsin(x) + \frac{1}{9} \sqrt{-x^2+1} x^2 + \frac{2}{9} \sqrt{-x^2+1}$$

input `integrate(x^2*arcsin(x),x, algorithm="maxima")`output `1/3*x^3*arcsin(x) + 1/9*sqrt(-x^2 + 1)*x^2 + 2/9*sqrt(-x^2 + 1)`**3.264.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int x^2 \arcsin(x) dx = \frac{1}{3} (x^2 - 1)x \arcsin(x) + \frac{1}{3} x \arcsin(x) - \frac{1}{9} (-x^2 + 1)^{\frac{3}{2}} + \frac{1}{3} \sqrt{-x^2 + 1}$$

input `integrate(x^2*arcsin(x),x, algorithm="giac")`output `1/3*(x^2 - 1)*x*arcsin(x) + 1/3*x*arcsin(x) - 1/9*(-x^2 + 1)^(3/2) + 1/3*sqrt(-x^2 + 1)`

**3.264.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int x^2 \arcsin(x) dx = \frac{x^3 \arcsin(x)}{3} + \frac{\sqrt{1-x^2}(x^2+2)}{9}$$

input `int(x^2*asin(x),x)`

output `(x^3*asin(x))/3 + ((1 - x^2)^(1/2)*(x^2 + 2))/9`

$$3.265 \quad \int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)} dx$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.265.1 Optimal result . . . . .                            | 1402 |
| 3.265.2 Mathematica [A] (verified) . . . . .                | 1402 |
| 3.265.3 Rubi [A] (verified) . . . . .                       | 1403 |
| 3.265.4 Maple [A] (verified) . . . . .                      | 1404 |
| 3.265.5 Fricas [A] (verification not implemented) . . . . . | 1404 |
| 3.265.6 Sympy [F] . . . . .                                 | 1405 |
| 3.265.7 Maxima [A] (verification not implemented) . . . . . | 1405 |
| 3.265.8 Giac [A] (verification not implemented) . . . . .   | 1405 |
| 3.265.9 Mupad [B] (verification not implemented) . . . . .  | 1406 |

### 3.265.1 Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)} dx = -\log(\cos(x)-\sin(x)) + \log(2\cos(x)-\sin(x))$$

output `-ln(cos(x)-sin(x))+ln(2*cos(x)-sin(x))`

### 3.265.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)} dx = 2\left(-\frac{1}{2}\log(\cos(x)-\sin(x)) + \frac{1}{2}\log(2\cos(x)-\sin(x))\right)$$

input `Integrate[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]`

output `2*(-1/2*Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]/2)`

**3.265.3 Rubi [A] (verified)**

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {3042, 4889, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sec^2(x)}{-3 \tan(x) + \sec^2(x) + 1} dx \\
 & \quad \downarrow \text{3042} \\
 & \int \frac{\sec(x)^2}{-3 \tan(x) + \sec(x)^2 + 1} dx \\
 & \quad \downarrow \text{4889} \\
 & \int \frac{1}{\tan^2(x) - 3 \tan(x) + 2} d \tan(x) \\
 & \quad \downarrow \text{1081} \\
 & \int \left( \frac{1}{\tan(x) - 2} + \frac{1}{1 - \tan(x)} \right) d \tan(x) \\
 & \quad \downarrow \text{2009} \\
 & \log(2 - \tan(x)) - \log(1 - \tan(x))
 \end{aligned}$$

input `Int[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]`

output `-Log[1 - Tan[x]] + Log[2 - Tan[x]]`

**3.265.3.1 Defintions of rubi rules used**

rule 1081 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2 + c*x)), x], x], x] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

```
rule 3042 Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

```
rule 4889 Int[u_, x_Symbol] := With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1] Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /; !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] && !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

### 3.265.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

| method  | result                                                                    | size |
|---------|---------------------------------------------------------------------------|------|
| default | $\ln(\tan(x) - 2) - \ln(\tan(x) - 1)$                                     | 14   |
| risch   | $\ln\left(e^{2ix} + \frac{3}{5} - \frac{4i}{5}\right) - \ln(e^{2ix} - i)$ | 23   |

```
input int(sec(x)^2/(1+sec(x)^2-3*tan(x)),x,method=_RETURNVERBOSE)
```

```
output ln(tan(x)-2)-ln(tan(x)-1)
```

### 3.265.5 Fracas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{\sec^2(x)}{1 + \sec^2(x) - 3 \tan(x)} dx = \frac{1}{2} \log\left(\frac{3}{4} \cos^2(x) - \cos(x) \sin(x) + \frac{1}{4}\right) - \frac{1}{2} \log(-2 \cos(x) \sin(x) + 1)$$

```
input integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="fricas")
```

```
output 1/2*log(3/4*cos(x)^2 - cos(x)*sin(x) + 1/4) - 1/2*log(-2*cos(x)*sin(x) + 1
)
```

**3.265.6 Sympy [F]**

$$\int \frac{\sec^2(x)}{1 + \sec^2(x) - 3 \tan(x)} dx = \int \frac{\sec^2(x)}{-3 \tan(x) + \sec^2(x) + 1} dx$$

input `integrate(sec(x)**2/(1+sec(x)**2-3*tan(x)),x)`

output `Integral(sec(x)**2/(-3*tan(x) + sec(x)**2 + 1), x)`

**3.265.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{\sec^2(x)}{1 + \sec^2(x) - 3 \tan(x)} dx = -\log(\tan(x) - 1) + \log(\tan(x) - 2)$$

input `integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="maxima")`

output `-log(tan(x) - 1) + log(tan(x) - 2)`

**3.265.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\sec^2(x)}{1 + \sec^2(x) - 3 \tan(x)} dx = -\log(|\tan(x) - 1|) + \log(|\tan(x) - 2|)$$

input `integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="giac")`

output `-log(abs(tan(x) - 1)) + log(abs(tan(x) - 2))`

**3.265.9 Mupad [B] (verification not implemented)**

Time = 0.78 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.43

$$\int \frac{\sec^2(x)}{1 + \sec^2(x) - 3 \tan(x)} dx = -2 \operatorname{atanh}(2 \tan(x) - 3)$$

input `int(1/(cos(x)^2*(1/cos(x)^2 - 3*tan(x) + 1)),x)`

output `-2*atanh(2*tan(x) - 3)`

### 3.266 $\int \cos^2(x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.266.1 Optimal result . . . . .                            | 1407 |
| 3.266.2 Mathematica [A] (verified) . . . . .                | 1407 |
| 3.266.3 Rubi [A] (verified) . . . . .                       | 1408 |
| 3.266.4 Maple [A] (verified) . . . . .                      | 1409 |
| 3.266.5 Fricas [A] (verification not implemented) . . . . . | 1409 |
| 3.266.6 Sympy [A] (verification not implemented) . . . . .  | 1409 |
| 3.266.7 Maxima [A] (verification not implemented) . . . . . | 1410 |
| 3.266.8 Giac [A] (verification not implemented) . . . . .   | 1410 |
| 3.266.9 Mupad [B] (verification not implemented) . . . . .  | 1410 |

#### 3.266.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cos^2(x) dx = \frac{x}{2} + \frac{1}{2} \cos(x) \sin(x)$$

output `1/2*x+1/2*cos(x)*sin(x)`

#### 3.266.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(x) dx = \frac{x}{2} + \frac{1}{4} \sin(2x)$$

input `Integrate[Cos[x]^2,x]`

output `x/2 + Sin[2*x]/4`



**3.266.3 Rubi [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$ , Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \cos^2(x) dx \\ & \quad \downarrow \text{3042} \\ & \int \sin\left(x + \frac{\pi}{2}\right)^2 dx \\ & \quad \downarrow \text{3115} \\ & \frac{\int 1 dx}{2} + \frac{1}{2} \sin(x) \cos(x) \\ & \quad \downarrow \text{24} \\ & \frac{x}{2} + \frac{1}{2} \sin(x) \cos(x) \end{aligned}$$

input `Int[Cos[x]^2,x]`

output `x/2 + (Cos[x]*Sin[x])/2`

**3.266.3.1 Defintions of rubi rules used**

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3115 `Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sine[c + d*x])^(n - 1)/(d*n), x] + Simp[b^2*((n - 1)/n) Int[(b*Sine[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

**3.266.4 Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

| method        | result                                                                                                                                                  | size |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default       | $\frac{x}{2} + \frac{\cos(x) \sin(x)}{2}$                                                                                                               | 11   |
| risch         | $\frac{x}{2} + \frac{\sin(2x)}{4}$                                                                                                                      | 11   |
| parallelrisch | $\frac{x}{2} + \frac{\sin(2x)}{4}$                                                                                                                      | 11   |
| norman        | $\frac{x(\tan^2(\frac{x}{2})) + \frac{x}{2} - (\tan^3(\frac{x}{2})) + \frac{x(\tan^4(\frac{x}{2}))}{2} + \tan(\frac{x}{2})}{(1+\tan^2(\frac{x}{2}))^2}$ | 45   |

input `int(1/sec(x)^2,x,method=_RETURNVERBOSE)`output `1/2*x+1/2*cos(x)*sin(x)`**3.266.5 Fracas [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x) dx = \frac{1}{2} \cos(x) \sin(x) + \frac{1}{2} x$$

input `integrate(1/sec(x)^2,x, algorithm="fricas")`output `1/2*cos(x)*sin(x) + 1/2*x`**3.266.6 Sympy [A] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x) dx = \frac{x}{2} + \frac{\sin(x) \cos(x)}{2}$$

input `integrate(1/sec(x)**2,x)`output `x/2 + sin(x)*cos(x)/2`

**3.266.7 Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x) dx = \frac{1}{2}x + \frac{1}{4}\sin(2x)$$

input `integrate(1/sec(x)^2,x, algorithm="maxima")`output `1/2*x + 1/4*sin(2*x)`**3.266.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \cos^2(x) dx = \frac{1}{2}x + \frac{\tan(x)}{2(\tan(x)^2 + 1)}$$

input `integrate(1/sec(x)^2,x, algorithm="giac")`output `1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)`**3.266.9 Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x) dx = \frac{x}{2} + \frac{\sin(2x)}{4}$$

input `int(cos(x)^2,x)`output `x/2 + sin(2*x)/4`

$$3.267 \quad \int \frac{-2-3x+5x^2}{(-2+x)x^2} dx$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.267.1 Optimal result . . . . .                            | 1411 |
| 3.267.2 Mathematica [A] (verified) . . . . .                | 1411 |
| 3.267.3 Rubi [A] (verified) . . . . .                       | 1412 |
| 3.267.4 Maple [A] (verified) . . . . .                      | 1413 |
| 3.267.5 Fricas [A] (verification not implemented) . . . . . | 1413 |
| 3.267.6 Sympy [A] (verification not implemented) . . . . .  | 1413 |
| 3.267.7 Maxima [A] (verification not implemented) . . . . . | 1414 |
| 3.267.8 Giac [A] (verification not implemented) . . . . .   | 1414 |
| 3.267.9 Mupad [B] (verification not implemented) . . . . .  | 1414 |

### 3.267.1 Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2} dx = -\frac{1}{x} + 3 \log(2-x) + 2 \log(x)$$

output `-1/x+3*ln(2-x)+2*ln(x)`

### 3.267.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-2-3x+5x^2}{(-2+x)x^2} dx = -\frac{1}{x} + 3 \log(2-x) + 2 \log(x)$$

input `Integrate[(-2 - 3*x + 5*x^2)/((-2 + x)*x^2),x]`

output `-x^(-1) + 3*Log[2 - x] + 2*Log[x]`

**3.267.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2 - 3x - 2}{(x-2)x^2} dx$$

↓ 1195

$$\int \left( \frac{1}{x^2} + \frac{2}{x} + \frac{3}{x-2} \right) dx$$

↓ 2009

$$-\frac{1}{x} + 3 \log(2-x) + 2 \log(x)$$

input `Int[(-2 - 3*x + 5*x^2)/((-2 + x)*x^2), x]`

output `-x^(-1) + 3*Log[2 - x] + 2*Log[x]`

**3.267.3.1 Defintions of rubi rules used**

rule 1195 `Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_)^(n_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]`

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

**3.267.4 Maple [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

| method       | result                                                                           | size |
|--------------|----------------------------------------------------------------------------------|------|
| default      | $-\frac{1}{x} + 2 \ln(x) + 3 \ln(-2 + x)$                                        | 17   |
| norman       | $-\frac{1}{x} + 2 \ln(x) + 3 \ln(-2 + x)$                                        | 17   |
| risch        | $-\frac{1}{x} + 2 \ln(x) + 3 \ln(-2 + x)$                                        | 17   |
| parallelrisc | $\frac{2x \ln(x) + 3 \ln(-2+x)x - 1}{x}$                                         | 19   |
| meijerg      | $-\frac{1}{x} + 2 \ln(x) - 2 \ln(2) + 2i\pi + 3 \ln\left(1 - \frac{x}{2}\right)$ | 27   |

input `int((5*x^2-3*x-2)/(-2+x)/x^2,x,method=_RETURNVERBOSE)`output `-1/x+2*ln(x)+3*ln(-2+x)`**3.267.5 Fracas [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-2 - 3x + 5x^2}{(-2 + x)x^2} dx = \frac{3x \log(x - 2) + 2x \log(x) - 1}{x}$$

input `integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="fricas")`output `(3*x*log(x - 2) + 2*x*log(x) - 1)/x`**3.267.6 Sympy [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{-2 - 3x + 5x^2}{(-2 + x)x^2} dx = 2 \log(x) + 3 \log(x - 2) - \frac{1}{x}$$

input `integrate((5*x**2-3*x-2)/(-2+x)/x**2,x)`output `2*log(x) + 3*log(x - 2) - 1/x`

---

3.267.  $\int \frac{-2-3x+5x^2}{(-2+x)x^2} dx$

**3.267.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{-2 - 3x + 5x^2}{(-2 + x)x^2} dx = -\frac{1}{x} + 3 \log(x - 2) + 2 \log(x)$$

input `integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="maxima")`output `-1/x + 3*log(x - 2) + 2*log(x)`**3.267.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-2 - 3x + 5x^2}{(-2 + x)x^2} dx = -\frac{1}{x} + 3 \log(|x - 2|) + 2 \log(|x|)$$

input `integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="giac")`output `-1/x + 3*log(abs(x - 2)) + 2*log(abs(x))`**3.267.9 Mupad [B] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{-2 - 3x + 5x^2}{(-2 + x)x^2} dx = 3 \ln(x - 2) + 2 \ln(x) - \frac{1}{x}$$

input `int(-(3*x - 5*x^2 + 2)/(x^2*(x - 2)),x)`output `3*log(x - 2) + 2*log(x) - 1/x`

### 3.268 $\int \frac{1}{\sqrt{9+4x^2}} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.268.1 Optimal result . . . . .                            | 1415 |
| 3.268.2 Mathematica [A] (verified) . . . . .                | 1415 |
| 3.268.3 Rubi [A] (verified) . . . . .                       | 1416 |
| 3.268.4 Maple [A] (verified) . . . . .                      | 1416 |
| 3.268.5 Fricas [B] (verification not implemented) . . . . . | 1417 |
| 3.268.6 Sympy [A] (verification not implemented) . . . . .  | 1417 |
| 3.268.7 Maxima [A] (verification not implemented) . . . . . | 1417 |
| 3.268.8 Giac [B] (verification not implemented) . . . . .   | 1418 |
| 3.268.9 Mupad [B] (verification not implemented) . . . . .  | 1418 |

#### 3.268.1 Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{1}{\sqrt{9+4x^2}} dx = \frac{1}{2} \operatorname{arcsinh}\left(\frac{2x}{3}\right)$$

output `1/2*arcsinh(2/3*x)`

#### 3.268.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{9+4x^2}} dx = -\frac{1}{2} \log\left(-2x + \sqrt{9+4x^2}\right)$$

input `Integrate[1/Sqrt[9 + 4*x^2],x]`

output `-1/2*Log[-2*x + Sqrt[9 + 4*x^2]]`



**3.268.3 Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4x^2 + 9}} dx$$

↓ 222

$$\frac{1}{2} \operatorname{arcsinh}\left(\frac{2x}{3}\right)$$

input `Int[1/Sqrt[9 + 4*x^2],x]`

output `ArcSinh[(2*x)/3]/2`

**3.268.3.1 Defintions of rubi rules used**

rule 222 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]`

**3.268.4 Maple [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

| method         | result                                                                                                                 | size |
|----------------|------------------------------------------------------------------------------------------------------------------------|------|
| default        | $\frac{\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}$                                                            | 7    |
| meijerg        | $\frac{\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}$                                                            | 7    |
| trager         | $-\frac{\ln(2x - \sqrt{4x^2 + 9})}{2}$                                                                                 | 19   |
| pseudoelliptic | $-\frac{\ln\left(\frac{\sqrt{4x^2 + 9} - 2x}{x}\right)}{4} + \frac{\ln\left(\frac{\sqrt{4x^2 + 9} + 2x}{x}\right)}{4}$ | 42   |

input `int(1/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)`

output `1/2*arcsinh(2/3*x)`

### 3.268.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs.  $2(6) = 12$ .

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{9+4x^2}} dx = -\frac{1}{2} \log(-2x + \sqrt{4x^2+9})$$

input `integrate(1/(4*x^2+9)^(1/2),x, algorithm="fricas")`

output `-1/2*log(-2*x + sqrt(4*x^2 + 9))`

### 3.268.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{9+4x^2}} dx = \frac{\operatorname{asinh}\left(\frac{2x}{3}\right)}{2}$$

input `integrate(1/(4*x**2+9)**(1/2),x)`

output `asinh(2*x/3)/2`

### 3.268.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{9+4x^2}} dx = \frac{1}{2} \operatorname{arsinh}\left(\frac{2}{3}x\right)$$

input `integrate(1/(4*x^2+9)^(1/2),x, algorithm="maxima")`

output `1/2*arcsinh(2/3*x)`

**3.268.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(6) = 12.

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.90

$$\int \frac{1}{\sqrt{9+4x^2}} dx = \frac{1}{2} \sqrt{4x^2+9} - \frac{9}{4} \log(-2x + \sqrt{4x^2+9})$$

input `integrate(1/(4*x^2+9)^(1/2),x, algorithm="giac")`

output `1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))`

**3.268.9 Mupad [B] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{9+4x^2}} dx = \frac{\operatorname{asinh}\left(\frac{2x}{3}\right)}{2}$$

input `int(1/(4*x^2 + 9)^(1/2),x)`

output `asinh((2*x)/3)/2`

$$3.269 \quad \int \frac{1}{\sqrt{4+x^2}} dx$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.269.1 Optimal result . . . . .                            | 1419 |
| 3.269.2 Mathematica [B] (verified) . . . . .                | 1419 |
| 3.269.3 Rubi [A] (verified) . . . . .                       | 1420 |
| 3.269.4 Maple [A] (verified) . . . . .                      | 1420 |
| 3.269.5 Fricas [B] (verification not implemented) . . . . . | 1421 |
| 3.269.6 Sympy [A] (verification not implemented) . . . . .  | 1421 |
| 3.269.7 Maxima [A] (verification not implemented) . . . . . | 1421 |
| 3.269.8 Giac [B] (verification not implemented) . . . . .   | 1422 |
| 3.269.9 Mupad [B] (verification not implemented) . . . . .  | 1422 |

### 3.269.1 Optimal result

Integrand size = 9, antiderivative size = 6

$$\int \frac{1}{\sqrt{4+x^2}} dx = \operatorname{arcsinh}\left(\frac{x}{2}\right)$$

output `arcsinh(1/2*x)`

### 3.269.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16 vs.  $2(6) = 12$ .

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{1}{\sqrt{4+x^2}} dx = -\log\left(-x + \sqrt{4+x^2}\right)$$

input `Integrate[1/Sqrt[4 + x^2],x]`

output `-Log[-x + Sqrt[4 + x^2]]`

**3.269.3 Rubi [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2 + 4}} dx$$

↓ 222

$$\operatorname{arcsinh}\left(\frac{x}{2}\right)$$

input `Int[1/Sqrt[4 + x^2], x]`

output `ArcSinh[x/2]`

**3.269.3.1 Defintions of rubi rules used**

rule 222 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]`

**3.269.4 Maple [A] (verified)**

Time = 0.13 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

| method         | result                                                      | size |
|----------------|-------------------------------------------------------------|------|
| default        | $\operatorname{arcsinh}\left(\frac{x}{2}\right)$            | 5    |
| meijerg        | $\operatorname{arcsinh}\left(\frac{x}{2}\right)$            | 5    |
| pseudoelliptic | $\operatorname{arctanh}\left(\frac{\sqrt{x^2+4}}{x}\right)$ | 13   |
| trager         | $-\ln\left(x - \sqrt{x^2 + 4}\right)$                       | 15   |

input `int(1/(x^2+4)^(1/2), x, method=_RETURNVERBOSE)`

output `arcsinh(1/2*x)`

**3.269.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 14 vs.  $2(4) = 8$ .

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

$$\int \frac{1}{\sqrt{4+x^2}} dx = -\log\left(-x + \sqrt{x^2+4}\right)$$

input `integrate(1/(x^2+4)^(1/2),x, algorithm="fricas")`

output `-log(-x + sqrt(x^2 + 4))`

**3.269.6 Sympy [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{4+x^2}} dx = \operatorname{asinh}\left(\frac{x}{2}\right)$$

input `integrate(1/(x**2+4)**(1/2),x)`

output `asinh(x/2)`

**3.269.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{4+x^2}} dx = \operatorname{arsinh}\left(\frac{1}{2}x\right)$$

input `integrate(1/(x^2+4)^(1/2),x, algorithm="maxima")`

output `arcsinh(1/2*x)`

**3.269.8 Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 25 vs.  $2(4) = 8$ .

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 4.17

$$\int \frac{1}{\sqrt{4+x^2}} dx = \frac{1}{2} \sqrt{x^2+4}x - 2 \log(-x + \sqrt{x^2+4})$$

input `integrate(1/(x^2+4)^(1/2),x, algorithm="giac")`

output `1/2*sqrt(x^2 + 4)*x - 2*log(-x + sqrt(x^2 + 4))`

**3.269.9 Mupad [B] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{4+x^2}} dx = \operatorname{asinh}\left(\frac{x}{2}\right)$$

input `int(1/(x^2 + 4)^(1/2),x)`

output `asinh(x/2)`

### 3.270 $\int \frac{1}{10-12x+9x^2} dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.270.1 Optimal result . . . . .                            | 1423 |
| 3.270.2 Mathematica [A] (verified) . . . . .                | 1423 |
| 3.270.3 Rubi [A] (verified) . . . . .                       | 1424 |
| 3.270.4 Maple [A] (verified) . . . . .                      | 1425 |
| 3.270.5 Fricas [A] (verification not implemented) . . . . . | 1425 |
| 3.270.6 Sympy [A] (verification not implemented) . . . . .  | 1425 |
| 3.270.7 Maxima [A] (verification not implemented) . . . . . | 1426 |
| 3.270.8 Giac [A] (verification not implemented) . . . . .   | 1426 |
| 3.270.9 Mupad [B] (verification not implemented) . . . . .  | 1426 |

#### 3.270.1 Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{1}{10-12x+9x^2} dx = -\frac{\arctan\left(\frac{2-3x}{\sqrt{6}}\right)}{3\sqrt{6}}$$

output `-1/18*arctan(1/6*(2-3*x)*6^(1/2))*6^(1/2)`

#### 3.270.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{10-12x+9x^2} dx = \frac{\arctan\left(\frac{-2+3x}{\sqrt{6}}\right)}{3\sqrt{6}}$$

input `Integrate[(10 - 12*x + 9*x^2)^(-1),x]`

output `ArcTan[(-2 + 3*x)/Sqrt[6]]/(3*Sqrt[6])`



**3.270.3 Rubi [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{9x^2 - 12x + 10} dx$$

↓ 1083

$$-2 \int \frac{1}{-(18x - 12)^2 - 216} d(18x - 12)$$

↓ 217

$$\frac{\arctan\left(\frac{18x-12}{6\sqrt{6}}\right)}{3\sqrt{6}}$$

input `Int[(10 - 12*x + 9*x^2)^(-1),x]`

output `ArcTan[(-12 + 18*x)/(6*Sqrt[6])]/(3*Sqrt[6])`

**3.270.3.1 Defintions of rubi rules used**

rule 217 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])`

rule 1083 `Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Simp[-2 Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x]`

**3.270.4 Maple [A] (verified)**

Time = 0.63 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

| method  | result                                                                | size |
|---------|-----------------------------------------------------------------------|------|
| default | $\frac{\sqrt{6} \arctan\left(\frac{(18x-12)\sqrt{6}}{36}\right)}{18}$ | 17   |
| risch   | $\frac{\sqrt{6} \arctan\left(\frac{(-2+3x)\sqrt{6}}{6}\right)}{18}$   | 17   |

input `int(1/(9*x^2-12*x+10),x,method=_RETURNVERBOSE)`output `1/18*6^(1/2)*arctan(1/36*(18*x-12)*6^(1/2))`**3.270.5 Fracas [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{1}{10 - 12x + 9x^2} dx = \frac{1}{18} \sqrt{6} \arctan\left(\frac{1}{6} \sqrt{6}(3x - 2)\right)$$

input `integrate(1/(9*x^2-12*x+10),x, algorithm="fricas")`output `1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))`**3.270.6 Sympy [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{10 - 12x + 9x^2} dx = \frac{\sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6}x}{2} - \frac{\sqrt{6}}{3}\right)}{18}$$

input `integrate(1/(9*x**2-12*x+10),x)`output `sqrt(6)*atan(sqrt(6)*x/2 - sqrt(6)/3)/18`

**3.270.7 Maxima [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{1}{10 - 12x + 9x^2} dx = \frac{1}{18} \sqrt{6} \arctan \left( \frac{1}{6} \sqrt{6}(3x - 2) \right)$$

input `integrate(1/(9*x^2-12*x+10),x, algorithm="maxima")`output `1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))`**3.270.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{1}{10 - 12x + 9x^2} dx = \frac{1}{18} \sqrt{6} \arctan \left( \frac{1}{6} \sqrt{6}(3x - 2) \right)$$

input `integrate(1/(9*x^2-12*x+10),x, algorithm="giac")`output `1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))`**3.270.9 Mupad [B] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{1}{10 - 12x + 9x^2} dx = \frac{\sqrt{6} \operatorname{atan} \left( \frac{\sqrt{6}(3x-2)}{6} \right)}{18}$$

input `int(1/(9*x^2 - 12*x + 10),x)`output `(6^(1/2)*atan((6^(1/2)*(3*x - 2))/6))/18`

$$3.271 \quad \int \frac{1}{x^4 - 2x^5 + 2x^6 - 2x^7 + x^8} dx$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.271.1 Optimal result . . . . .                            | 1427 |
| 3.271.2 Mathematica [A] (verified) . . . . .                | 1427 |
| 3.271.3 Rubi [A] (verified) . . . . .                       | 1428 |
| 3.271.4 Maple [A] (verified) . . . . .                      | 1429 |
| 3.271.5 Fricas [A] (verification not implemented) . . . . . | 1429 |
| 3.271.6 Sympy [A] (verification not implemented) . . . . .  | 1430 |
| 3.271.7 Maxima [A] (verification not implemented) . . . . . | 1430 |
| 3.271.8 Giac [A] (verification not implemented) . . . . .   | 1430 |
| 3.271.9 Mupad [B] (verification not implemented) . . . . .  | 1431 |

### 3.271.1 Optimal result

Integrand size = 24, antiderivative size = 53

$$\int \frac{1}{x^4 - 2x^5 + 2x^6 - 2x^7 + x^8} dx = \frac{1}{2(1-x)} - \frac{1}{3x^3} - \frac{1}{x^2} - \frac{2}{x} - \frac{5}{2} \log(1-x) + 2 \log(x) + \frac{1}{4} \log(1+x^2)$$

output `1/2/(1-x)-1/3/x^3-1/x^2-2/x-5/2*ln(1-x)+2*ln(x)+1/4*ln(x^2+1)`

### 3.271.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^4 - 2x^5 + 2x^6 - 2x^7 + x^8} dx = -\frac{1}{2(-1+x)} - \frac{1}{3x^3} - \frac{1}{x^2} - \frac{2}{x} - \frac{5}{2} \log(1-x) + 2 \log(x) + \frac{1}{4} \log(1+x^2)$$

input `Integrate[(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8)^(-1),x]`

output `-1/2*1/(-1 + x) - 1/(3*x^3) - x^(-2) - 2/x - (5*Log[1 - x])/2 + 2*Log[x] + Log[1 + x^2]/4`

---

3.271.  $\int \frac{1}{x^4 - 2x^5 + 2x^6 - 2x^7 + x^8} dx$

**3.271.3 Rubi [A] (verified)**

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2026, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 - 2x^7 + 2x^6 - 2x^5 + x^4} dx$$

↓ 2026

$$\int \frac{1}{x^4(x^4 - 2x^3 + 2x^2 - 2x + 1)} dx$$

↓ 2462

$$\int \left( \frac{1}{x^4} + \frac{2}{x^3} + \frac{x}{2(x^2 + 1)} + \frac{2}{x^2} - \frac{5}{2(x-1)} + \frac{1}{2(x-1)^2} + \frac{2}{x} \right) dx$$

↓ 2009

$$-\frac{1}{3x^3} - \frac{1}{x^2} + \frac{1}{4} \log(x^2 + 1) + \frac{1}{2(1-x)} - \frac{2}{x} - \frac{5}{2} \log(1-x) + 2 \log(x)$$

input `Int[(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8)^(-1),x]`

output `1/(2*(1 - x)) - 1/(3*x^3) - x^(-2) - 2/x - (5*Log[1 - x])/2 + 2*Log[x] + Log[1 + x^2]/4`

**3.271.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2026 `Int[(Fx_.)*(Px_)^(p_.), x_Symbol] := With[{r = Expon[Px, x, Min]}, Int[x^(p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && IntegerQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])`

```
rule 2462 Int[(u_.)*(Px_)^(p_), x_Symbol] := With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /; !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

### 3.271.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

| method        | result                                                                                                                                             | size |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------|------|
| default       | $-\frac{1}{2(-1+x)} - \frac{5 \ln(-1+x)}{2} + \frac{\ln(x^2+1)}{4} - \frac{1}{3x^3} - \frac{1}{x^2} - \frac{2}{x} + 2 \ln(x)$                      | 42   |
| norman        | $\frac{\frac{1}{3}+x^2-\frac{5}{2}x^3+\frac{2}{3}x}{x^3(-1+x)} + 2 \ln(x) - \frac{5 \ln(-1+x)}{2} + \frac{\ln(x^2+1)}{4}$                          | 42   |
| risch         | $\frac{\frac{1}{3}+x^2-\frac{5}{2}x^3+\frac{2}{3}x}{x^3(-1+x)} + 2 \ln(x) - \frac{5 \ln(-1+x)}{2} + \frac{\ln(x^2+1)}{4}$                          | 42   |
| parallelrisch | $\frac{24x^4 \ln(x) - 30 \ln(-1+x)x^4 + 3 \ln(x^2+1)x^4 + 4 - 24x^3 \ln(x) + 30 \ln(-1+x)x^3 - 3 \ln(x^2+1)x^3 - 30x^3 + 12x^2 + 8x}{12x^3(-1+x)}$ | 80   |

```
input int(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x,method=_RETURNVERBOSE)
```

```
output -1/2/(-1+x)-5/2*ln(-1+x)+1/4*ln(x^2+1)-1/3/x^3-1/x^2-2/x+2*ln(x)
```

### 3.271.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \frac{1}{x^4 - 2x^5 + 2x^6 - 2x^7 + x^8} dx = \frac{30x^3 - 12x^2 - 3(x^4 - x^3) \log(x^2 + 1) + 30(x^4 - x^3) \log(x - 1) - 24(x^4 - x^3) \log(x) - 8x - 4}{12(x^4 - x^3)}$$

```
input integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x, algorithm="fracas")
```

```
output -1/12*(30*x^3 - 12*x^2 - 3*(x^4 - x^3)*log(x^2 + 1) + 30*(x^4 - x^3)*log(x
- 1) - 24*(x^4 - x^3)*log(x) - 8*x - 4)/(x^4 - x^3)
```

**3.271.6 Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^4 - 2x^5 + 2x^6 - 2x^7 + x^8} dx = 2 \log(x) - \frac{5 \log(x-1)}{2} + \frac{\log(x^2+1)}{4} + \frac{-15x^3 + 6x^2 + 4x + 2}{6x^4 - 6x^3}$$

input `integrate(1/(x**8-2*x**7+2*x**6-2*x**5+x**4),x)`output `2*log(x) - 5*log(x - 1)/2 + log(x**2 + 1)/4 + (-15*x**3 + 6*x**2 + 4*x + 2)/(6*x**4 - 6*x**3)`**3.271.7 Maxima [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^4 - 2x^5 + 2x^6 - 2x^7 + x^8} dx = -\frac{15x^3 - 6x^2 - 4x - 2}{6(x^4 - x^3)} + \frac{1}{4} \log(x^2 + 1) - \frac{5}{2} \log(x - 1) + 2 \log(x)$$

input `integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x, algorithm="maxima")`output `-1/6*(15*x^3 - 6*x^2 - 4*x - 2)/(x^4 - x^3) + 1/4*log(x^2 + 1) - 5/2*log(x - 1) + 2*log(x)`**3.271.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^4 - 2x^5 + 2x^6 - 2x^7 + x^8} dx = -\frac{15x^3 - 6x^2 - 4x - 2}{6(x-1)x^3} + \frac{1}{4} \log(x^2 + 1) - \frac{5}{2} \log(|x-1|) + 2 \log(|x|)$$

input `integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x, algorithm="giac")`

output `-1/6*(15*x^3 - 6*x^2 - 4*x - 2)/((x - 1)*x^3) + 1/4*log(x^2 + 1) - 5/2*log(abs(x - 1)) + 2*log(abs(x))`

### 3.271.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4 - 2x^5 + 2x^6 - 2x^7 + x^8} dx = \frac{\ln(x^2 + 1)}{4} - \frac{5 \ln(x - 1)}{2} + 2 \ln(x) - \frac{-\frac{5x^3}{2} + x^2 + \frac{2x}{3} + \frac{1}{3}}{x^3 - x^4}$$

input `int(1/(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8),x)`

output `log(x^2 + 1)/4 - (5*log(x - 1))/2 + 2*log(x) - ((2*x)/3 + x^2 - (5*x^3)/2 + 1/3)/(x^3 - x^4)`



$$3.272 \quad \int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)} dx$$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.272.1 Optimal result . . . . .                            | 1432 |
| 3.272.2 Mathematica [A] (verified) . . . . .                | 1432 |
| 3.272.3 Rubi [A] (verified) . . . . .                       | 1433 |
| 3.272.4 Maple [A] (verified) . . . . .                      | 1434 |
| 3.272.5 Fricas [A] (verification not implemented) . . . . . | 1434 |
| 3.272.6 Sympy [B] (verification not implemented) . . . . .  | 1435 |
| 3.272.7 Maxima [A] (verification not implemented) . . . . . | 1436 |
| 3.272.8 Giac [A] (verification not implemented) . . . . .   | 1436 |
| 3.272.9 Mupad [B] (verification not implemented) . . . . .  | 1436 |

### 3.272.1 Optimal result

Integrand size = 29, antiderivative size = 49

$$\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)} dx = ax + \frac{1}{12}(27a+9b+3c+d)\log(3-x) - \frac{1}{3}d\log(x) - \frac{1}{4}(a-b+c-d)\log(1+x)$$

output `a*x+1/12*(27*a+9*b+3*c+d)*ln(3-x)-1/3*d*ln(x)-1/4*(a-b+c-d)*ln(1+x)`

### 3.272.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)} dx = ax + \frac{1}{12}(27a+9b+3c+d)\log(3-x) - \frac{1}{3}d\log(x) + \frac{1}{4}(-a+b-c+d)\log(1+x)$$

input `Integrate[(d + c*x + b*x^2 + a*x^3)/((-3 + x)*x*(1 + x)),x]`

output `a*x + ((27*a + 9*b + 3*c + d)*Log[3 - x])/12 - (d*Log[x])/3 + ((-a + b - c + d)*Log[1 + x])/4`

**3.272.3 Rubi [A] (verified)**

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^3 + bx^2 + cx + d}{(x-3)x(x+1)} dx$$

↓ 2115

$$\int \left( \frac{27a + 9b + 3c + d}{12(x-3)} + \frac{-a + b - c + d}{4(x+1)} + a - \frac{d}{3x} \right) dx$$

↓ 2009

$$\frac{1}{12} \log(3-x)(27a + 9b + 3c + d) - \frac{1}{4} \log(x+1)(a - b + c - d) + ax - \frac{1}{3} d \log(x)$$

input `Int[(d + c*x + b*x^2 + a*x^3)/((-3 + x)*x*(1 + x)),x]`

output `a*x + ((27*a + 9*b + 3*c + d)*Log[3 - x])/12 - (d*Log[x])/3 - ((a - b + c - d)*Log[1 + x])/4`

**3.272.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2115 `Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]`

**3.272.4 Maple [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

| method        | result                                                                                                                                                                                                             |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| default       | $ax - \frac{d \ln(x)}{3} + \left(-\frac{a}{4} + \frac{b}{4} - \frac{c}{4} + \frac{d}{4}\right) \ln(1+x) + \left(\frac{9a}{4} + \frac{3b}{4} + \frac{c}{4} + \frac{d}{12}\right) \ln(-3+x)$                         |
| norman        | $ax - \frac{d \ln(x)}{3} + \left(-\frac{a}{4} + \frac{b}{4} - \frac{c}{4} + \frac{d}{4}\right) \ln(1+x) + \left(\frac{9a}{4} + \frac{3b}{4} + \frac{c}{4} + \frac{d}{12}\right) \ln(-3+x)$                         |
| parallelrisch | $ax - \frac{d \ln(x)}{3} - \frac{\ln(1+x)a}{4} + \frac{\ln(1+x)b}{4} - \frac{\ln(1+x)c}{4} + \frac{\ln(1+x)d}{4} + \frac{9 \ln(-3+x)a}{4} + \frac{3 \ln(-3+x)b}{4} + \frac{\ln(-3+x)c}{4} + \frac{\ln(-3+x)d}{4}$  |
| risch         | $ax + \frac{9 \ln(3-x)a}{4} + \frac{3 \ln(3-x)b}{4} + \frac{\ln(3-x)c}{4} + \frac{\ln(3-x)d}{12} - \frac{d \ln(x)}{3} - \frac{\ln(-1-x)a}{4} + \frac{\ln(-1-x)b}{4} - \frac{\ln(-1-x)c}{4} + \frac{\ln(-1-x)d}{4}$ |

input `int((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x,method=_RETURNVERBOSE)`output `a*x-1/3*d*ln(x)+(-1/4*a+1/4*b-1/4*c+1/4*d)*ln(1+x)+(9/4*a+3/4*b+1/4*c+1/12*d)*ln(-3+x)`**3.272.5 Fracas [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{d + cx + bx^2 + ax^3}{(-3+x)x(1+x)} dx = ax - \frac{1}{4}(a - b + c - d) \log(x + 1) + \frac{1}{12}(27a + 9b + 3c + d) \log(x - 3) - \frac{1}{3}d \log(x)$$

input `integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x, algorithm="fricas")`output `a*x - 1/4*(a - b + c - d)*log(x + 1) + 1/12*(27*a + 9*b + 3*c + d)*log(x - 3) - 1/3*d*log(x)`

### 3.272.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(49) = 98.

Time = 55.06 (sec) , antiderivative size = 762, normalized size of antiderivative = 15.55

$$\int \frac{d + cx + bx^2 + ax^3}{(-3 + x)x(1 + x)} dx = ax - \frac{d \log(x)}{3} + \frac{(a - b + c - d) \log\left(x + \frac{-1512a^2d + 1134a^2(a - b + c - d) - 864abd + 648ab(a - b + c - d) - 432acd + 324ac(a - b + c - d) - 144ad^2 + 81a(a - b + c - d)}{1215a^3 - 567a^2b + 1593a^2c - 2691a^2d}\right)}{1215} + \frac{(27a + 9b + 3c + d) \log\left(x + \frac{-1512a^2d - 378a^2(27a + 9b + 3c + d) - 864abd - 216ab(27a + 9b + 3c + d) - 432acd - 108ac(27a + 9b + 3c + d)}{1215}\right)}{1215}$$

input `integrate((a*x**3+b*x**2+c*x+d)/(-3+x)/x/(1+x), x)`

output

```
a*x - d*log(x)/3 - (a - b + c - d)*log(x + (-1512*a**2*d + 1134*a**2*(a - b + c - d) - 864*a*b*d + 648*a*b*(a - b + c - d) - 432*a*c*d + 324*a*c*(a - b + c - d) - 144*a*d**2 + 81*a*(a - b + c - d)**2 - 216*b**2*d + 162*b**2*(a - b + c - d) - 288*b*d**2 + 108*b*d*(a - b + c - d) + 81*b*(a - b + c - d)**2 - 72*c**2*d + 54*c**2*(a - b + c - d) + 144*c*d**2 - 72*c*d*(a - b + c - d) - 27*c*(a - b + c - d)**2 - 136*d**3 - 54*d**2*(a - b + c - d) + 117*d*(a - b + c - d)**2)/(1215*a**3 - 567*a**2*b + 1593*a**2*c - 2691*a**2*d - 567*a*b**2 + 378*a*b*c - 1638*a*b*d + 405*a*c**2 - 702*a*c*d - 351*a*d**2 - 81*b**3 - 27*b**2*c - 207*b**2*d + 81*b*c**2 - 270*b*c*d - 27*b*d**2 + 27*c**3 - 27*c**2*d - 99*c*d**2 + 35*d**3))/4 + (27*a + 9*b + 3*c + d)*log(x + (-1512*a**2*d - 378*a**2*(27*a + 9*b + 3*c + d) - 864*a*b*d - 216*a*b*(27*a + 9*b + 3*c + d) - 432*a*c*d - 108*a*c*(27*a + 9*b + 3*c + d) - 144*a*d**2 + 9*a*(27*a + 9*b + 3*c + d)**2 - 216*b**2*d - 54*b**2*(27*a + 9*b + 3*c + d) - 288*b*d**2 - 36*b*d*(27*a + 9*b + 3*c + d) + 9*b*(27*a + 9*b + 3*c + d)**2 - 72*c**2*d - 18*c**2*(27*a + 9*b + 3*c + d) + 144*c*d**2 + 24*c*d*(27*a + 9*b + 3*c + d) - 3*c*(27*a + 9*b + 3*c + d)**2 - 136*d**3 + 18*d**2*(27*a + 9*b + 3*c + d) + 13*d*(27*a + 9*b + 3*c + d)**2)/(1215*a**3 - 567*a**2*b + 1593*a**2*c - 2691*a**2*d - 567*a*b**2 + 378*a*b*c - 1638*a*b*d + 405*a*c**2 - 702*a*c*d - 351*a*d**2 - 81*b**3 - 27*b**2*c - 207*b**2*d + 81*b*c**2 - 270*b*c*d - 27*b*d**2 + 27*c**3 - 27*c**2*d...
```

**3.272.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{d + cx + bx^2 + ax^3}{(-3 + x)x(1 + x)} dx = ax - \frac{1}{4}(a - b + c - d) \log(x + 1) + \frac{1}{12}(27a + 9b + 3c + d) \log(x - 3) - \frac{1}{3}d \log(x)$$

input `integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x, algorithm="maxima")`output `a*x - 1/4*(a - b + c - d)*log(x + 1) + 1/12*(27*a + 9*b + 3*c + d)*log(x - 3) - 1/3*d*log(x)`**3.272.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{d + cx + bx^2 + ax^3}{(-3 + x)x(1 + x)} dx = ax - \frac{1}{4}(a - b + c - d) \log(|x + 1|) + \frac{1}{12}(27a + 9b + 3c + d) \log(|x - 3|) - \frac{1}{3}d \log(|x|)$$

input `integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x, algorithm="giac")`output `a*x - 1/4*(a - b + c - d)*log(abs(x + 1)) + 1/12*(27*a + 9*b + 3*c + d)*log(abs(x - 3)) - 1/3*d*log(abs(x))`**3.272.9 Mupad [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{d + cx + bx^2 + ax^3}{(-3 + x)x(1 + x)} dx = \ln(x - 3) \left( \frac{9a}{4} + \frac{3b}{4} + \frac{c}{4} + \frac{d}{12} \right) - \ln(x + 1) \left( \frac{a}{4} - \frac{b}{4} + \frac{c}{4} - \frac{d}{4} \right) + ax - \frac{d \ln(x)}{3}$$

input `int((d + c*x + a*x^3 + b*x^2)/(x*(x + 1)*(x - 3)),x)`

output `log(x - 3)*((9*a)/4 + (3*b)/4 + c/4 + d/12) - log(x + 1)*(a/4 - b/4 + c/4 - d/4) + a*x - (d*log(x))/3`

**3.273**  $\int \frac{1}{(2-\log(1+x^2))^5} dx$

|                                     |      |
|-------------------------------------|------|
| 3.273.1 Optimal result . . . . .    | 1438 |
| 3.273.2 Mathematica [N/A] . . . . . | 1438 |
| 3.273.3 Rubi [N/A] . . . . .        | 1439 |
| 3.273.4 Maple [N/A] . . . . .       | 1439 |
| 3.273.5 Fricas [N/A] . . . . .      | 1440 |
| 3.273.6 Sympy [N/A] . . . . .       | 1440 |
| 3.273.7 Maxima [N/A] . . . . .      | 1441 |
| 3.273.8 Giac [N/A] . . . . .        | 1441 |
| 3.273.9 Mupad [N/A] . . . . .       | 1441 |

**3.273.1 Optimal result**

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{(2 - \log(1 + x^2))^5} dx = \text{Int}\left(\frac{1}{(2 - \log(1 + x^2))^5}, x\right)$$

output `Unintegrable(1/(2-ln(x^2+1))^5,x)`

**3.273.2 Mathematica [N/A]**

Not integrable

Time = 2.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{(2 - \log(1 + x^2))^5} dx = \int \frac{1}{(2 - \log(1 + x^2))^5} dx$$

input `Integrate[(2 - Log[1 + x^2])^(-5), x]`

output `Integrate[(2 - Log[1 + x^2])^(-5), x]`

**3.273.3 Rubi [N/A]**

Not integrable

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {2902}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2 - \log(x^2 + 1))^5} dx$$

↓ 2902

$$\int \frac{1}{(2 - \log(x^2 + 1))^5} dx$$

input `Int[(2 - Log[1 + x^2])^(-5),x]`output `$Aborted`**3.273.3.1 Defintions of rubi rules used**

rule 2902 `Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.)]*(b_.))^(q_), x_Symbol] := Unintegrable[(a + b*Log[c*(d + e*x^n)^p]]^q, x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]`

**3.273.4 Maple [N/A]**

Not integrable

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2 - \ln(x^2 + 1))^5} dx$$

input `int(1/(2-ln(x^2+1))^5,x)`output `int(1/(2-ln(x^2+1))^5,x)`



**3.273.5 Fricas [N/A]**

Not integrable

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.50

$$\int \frac{1}{(2 - \log(1 + x^2))^5} dx = \int -\frac{1}{(\log(x^2 + 1) - 2)^5} dx$$

input `integrate(1/(2-log(x^2+1))^5,x, algorithm="fricas")`output `integral(-1/(log(x^2 + 1)^5 - 10*log(x^2 + 1)^4 + 40*log(x^2 + 1)^3 - 80*log(x^2 + 1)^2 + 80*log(x^2 + 1) - 32), x)`**3.273.6 Sympy [N/A]**

Not integrable

Time = 7.44 (sec) , antiderivative size = 270, normalized size of antiderivative = 22.50

$$\int \frac{1}{(2 - \log(1 + x^2))^5} dx$$

$$= -\frac{\int \frac{120x^2}{x^8 \log(x^2+1)-2x^8} dx + \int \frac{30x^4}{x^8 \log(x^2+1)-2x^8} dx + \int \frac{x^8}{x^8 \log(x^2+1)-2x^8} dx + \int \frac{105}{x^8 \log(x^2+1)-2x^8} dx}{384}$$

$$+ \frac{\frac{2x^8}{3} + \frac{7x^6}{6} + \frac{5x^4}{2} + \frac{9x^2}{2} + \left(\frac{x^8}{48} - \frac{5x^4}{24} - \frac{x^2}{2} - \frac{5}{16}\right) \log(x^2 + 1)^3 + \left(-\frac{x^8}{12} + \frac{x^6}{24} + \frac{11x^4}{8} + \frac{25x^2}{8} + \frac{15}{8}\right) \log(x^2 + 1)^2 + \left(\frac{x^8}{4} - \frac{5x^6}{6} - \frac{19x^4}{6} - \frac{13x^2}{2} - \frac{15}{4}\right) \log(x^2 + 1) + \frac{5}{2}}{8x^7 \log(x^2 + 1)^4 - 64x^7 \log(x^2 + 1)^3 + 192x^7 \log(x^2 + 1)^2 - 256x^7 \log(x^2 + 1) + 128x^7}$$

input `integrate(1/(2-ln(x**2+1))**5,x)`output `-(Integral(120*x**2/(x**8*log(x**2 + 1) - 2*x**8), x) + Integral(30*x**4/(x**8*log(x**2 + 1) - 2*x**8), x) + Integral(x**8/(x**8*log(x**2 + 1) - 2*x**8), x) + Integral(105/(x**8*log(x**2 + 1) - 2*x**8), x))/384 + (2*x**8/3 + 7*x**6/6 + 5*x**4/2 + 9*x**2/2 + (x**8/48 - 5*x**4/24 - x**2/2 - 5/16)*log(x**2 + 1)**3 + (-x**8/12 + x**6/24 + 11*x**4/8 + 25*x**2/8 + 15/8)*log(x**2 + 1)**2 + (x**8/4 - x**6/6 - 19*x**4/6 - 13*x**2/2 - 15/4)*log(x**2 + 1) + 5/2)/(8*x**7*log(x**2 + 1)**4 - 64*x**7*log(x**2 + 1)**3 + 192*x**7*log(x**2 + 1)**2 - 256*x**7*log(x**2 + 1) + 128*x**7)`

**3.273.7 Maxima [N/A]**

Not integrable

Time = 0.31 (sec) , antiderivative size = 207, normalized size of antiderivative = 17.25

$$\int \frac{1}{(2 - \log(1 + x^2))^5} dx = \int -\frac{1}{(\log(x^2 + 1) - 2)^5} dx$$

```
input integrate(1/(2-log(x^2+1))^5,x, algorithm="maxima")
```

```
output 1/384*(32*x^8 + 56*x^6 + 120*x^4 + (x^8 - 10*x^4 - 24*x^2 - 15)*log(x^2 +
1)^3 - 2*(2*x^8 - x^6 - 33*x^4 - 75*x^2 - 45)*log(x^2 + 1)^2 + 216*x^2 + 4
*(3*x^8 - 2*x^6 - 38*x^4 - 78*x^2 - 45)*log(x^2 + 1) + 120)/(x^7*log(x^2 +
1)^4 - 8*x^7*log(x^2 + 1)^3 + 24*x^7*log(x^2 + 1)^2 - 32*x^7*log(x^2 + 1)
+ 16*x^7) - integrate(1/384*(x^8 + 30*x^4 + 120*x^2 + 105)/(x^8*log(x^2 +
1) - 2*x^8), x)
```

**3.273.8 Giac [N/A]**

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{(2 - \log(1 + x^2))^5} dx = \int -\frac{1}{(\ln(x^2 + 1) - 2)^5} dx$$

```
input integrate(1/(2-log(x^2+1))^5,x, algorithm="giac")
```

```
output integrate(-1/(log(x^2 + 1) - 2)^5, x)
```

**3.273.9 Mupad [N/A]**

Not integrable

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{(2 - \log(1 + x^2))^5} dx = \int -\frac{1}{(\ln(x^2 + 1) - 2)^5} dx$$

input `int(-1/(log(x^2 + 1) - 2)^5,x)`

output `int(-1/(log(x^2 + 1) - 2)^5, x)`

**3.274**  $\int \left( \frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2+\log(x)}{(x+\log^2(x))^2} + \frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)} \right) dx$

3.274.1 Optimal result . . . . . 1443  
 3.274.2 Mathematica [A] (verified) . . . . . 1443  
 3.274.3 Rubi [A] (verified) . . . . . 1444  
 3.274.4 Maple [A] (verified) . . . . . 1444  
 3.274.5 Fricas [A] (verification not implemented) . . . . . 1445  
 3.274.6 Sympy [A] (verification not implemented) . . . . . 1445  
 3.274.7 Maxima [A] (verification not implemented) . . . . . 1446  
 3.274.8 Giac [A] (verification not implemented) . . . . . 1446  
 3.274.9 Mupad [B] (verification not implemented) . . . . . 1446

**3.274.1 Optimal result**

Integrand size = 54, antiderivative size = 28

$$\int \left( \frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2 + \log(x)}{(x + \log^2(x))^2} + \frac{1 + \frac{1}{x} + \frac{2\log(x)}{x}}{x + \log^2(x)} \right) dx$$

$$= e^{x^2} \log(x) - \frac{\log(x)}{x + \log^2(x)} + \log(x + \log^2(x))$$

output `exp(x^2)*ln(x)-ln(x)/(x+ln(x)^2)+ln(x+ln(x)^2)`

**3.274.2 Mathematica [A] (verified)**

Time = 37.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left( \frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2 + \log(x)}{(x + \log^2(x))^2} + \frac{1 + \frac{1}{x} + \frac{2\log(x)}{x}}{x + \log^2(x)} \right) dx$$

$$= e^{x^2} \log(x) - \frac{\log(x)}{x + \log^2(x)} + \log(x + \log^2(x))$$

input `Integrate[E^x^2/x + 2*E^x^2*x*Log[x] + (-2 + Log[x])/(x + Log[x]^2)^2 + (1 + x^(-1) + (2*Log[x])/x)/(x + Log[x]^2), x]`

output `E^x^2*Log[x] - Log[x]/(x + Log[x]^2) + Log[x + Log[x]^2]`

---

3.274.  $\int \left( \frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2+\log(x)}{(x+\log^2(x))^2} + \frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)} \right) dx$

**3.274.3 Rubi [A] (verified)**

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.019$ , Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left( \frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{\log(x) - 2}{(x + \log^2(x))^2} + \frac{\frac{1}{x} + \frac{2\log(x)}{x} + 1}{x + \log^2(x)} \right) dx$$

↓ 2009

$$e^{x^2} \log(x) - \frac{\log(x)}{x + \log^2(x)} + \log(x + \log^2(x))$$

input `Int[E^x^2/x + 2*E^x^2*x*Log[x] + (-2 + Log[x])/(x + Log[x]^2)^2 + (1 + x^(-1) + (2*Log[x])/x)/(x + Log[x]^2), x]`

output `E^x^2*Log[x] - Log[x]/(x + Log[x]^2) + Log[x + Log[x]^2]`

**3.274.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

**3.274.4 Maple [A] (verified)**

Time = 0.62 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

| method       | result                                                                                                                 | size |
|--------------|------------------------------------------------------------------------------------------------------------------------|------|
| default      | $e^{x^2} \ln(x) - \frac{\ln(x)}{x + \ln(x)^2} + \ln(x + \ln(x)^2)$                                                     | 28   |
| risch        | $e^{x^2} \ln(x) - \frac{\ln(x)}{x + \ln(x)^2} + \ln(x + \ln(x)^2)$                                                     | 28   |
| parts        | $e^{x^2} \ln(x) - \frac{\ln(x)}{x + \ln(x)^2} + \ln(x + \ln(x)^2)$                                                     | 28   |
| parallelrisc | $\frac{e^{x^2} \ln(x)^3 + e^{x^2} x \ln(x) + \ln(x)^2 \ln(x + \ln(x)^2) + \ln(x + \ln(x)^2) x - \ln(x)}{x + \ln(x)^2}$ | 53   |

---

3.274.  $\int \left( \frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2 + \log(x)}{(x + \log^2(x))^2} + \frac{1 + \frac{1}{x} + \frac{2\log(x)}{x}}{x + \log^2(x)} \right) dx$

```
input int(exp(x^2)/x+2*exp(x^2)*x*ln(x)+(-2+ln(x))/(x+ln(x)^2)+(1+1/x+2*ln(x)/x)/(x+ln(x)^2),x,method=_RETURNVERBOSE)
```

```
output exp(x^2)*ln(x)-ln(x)/(x+ln(x)^2)+ln(x+ln(x)^2)
```

### 3.274.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \left( \frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2 + \log(x)}{(x + \log^2(x))^2} + \frac{1 + \frac{1}{x} + \frac{2\log(x)}{x}}{x + \log^2(x)} \right) dx$$

$$= \frac{e^{(x^2)} \log(x)^3 + (\log(x)^2 + x) \log(\log(x)^2 + x) + (xe^{(x^2)} - 1) \log(x)}{\log(x)^2 + x}$$

```
input integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)+(1+1/x+2*log(x)/x)/(x+log(x)^2),x, algorithm="fricas")
```

```
output (e^(x^2)*log(x)^3 + (log(x)^2 + x)*log(log(x)^2 + x) + (x*e^(x^2) - 1)*log(x))/(log(x)^2 + x)
```

### 3.274.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left( \frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2 + \log(x)}{(x + \log^2(x))^2} + \frac{1 + \frac{1}{x} + \frac{2\log(x)}{x}}{x + \log^2(x)} \right) dx$$

$$= e^{x^2} \log(x) + \log(x + \log(x)^2) - \frac{\log(x)}{x + \log(x)^2}$$

```
input integrate(exp(x**2)/x+2*exp(x**2)*x*ln(x)+(-2+ln(x))/(x+ln(x)**2)**2+(1+1/x+2*ln(x)/x)/(x+ln(x)**2),x)
```

```
output exp(x**2)*log(x) + log(x + log(x)**2) - log(x)/(x + log(x)**2)
```

---


$$3.274. \quad \int \left( \frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2 + \log(x)}{(x + \log^2(x))^2} + \frac{1 + \frac{1}{x} + \frac{2\log(x)}{x}}{x + \log^2(x)} \right) dx$$

**3.274.7 Maxima [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \left( \frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2 + \log(x)}{(x + \log^2(x))^2} + \frac{1 + \frac{1}{x} + \frac{2\log(x)}{x}}{x + \log^2(x)} \right) dx$$

$$= e^{(x^2)} \log(x) - \frac{\log(x)}{\log(x)^2 + x} + \log(\log(x)^2 + x)$$

```
input integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)^2+(1+1/x
+2*log(x)/x)/(x+log(x)^2),x, algorithm="maxima")
```

```
output e^(x^2)*log(x) - log(x)/(log(x)^2 + x) + log(log(x)^2 + x)
```

**3.274.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \left( \frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2 + \log(x)}{(x + \log^2(x))^2} + \frac{1 + \frac{1}{x} + \frac{2\log(x)}{x}}{x + \log^2(x)} \right) dx$$

$$= e^{(x^2)} \log(x) - \frac{3 \log(x)}{\log(x)^2 + x} + \log(\log(x)^2 + x)$$

```
input integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)^2+(1+1/x
+2*log(x)/x)/(x+log(x)^2),x, algorithm="giac")
```

```
output e^(x^2)*log(x) - 3*log(x)/(log(x)^2 + x) + log(log(x)^2 + x)
```

**3.274.9 Mupad [B] (verification not implemented)**

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \left( \frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2 + \log(x)}{(x + \log^2(x))^2} + \frac{1 + \frac{1}{x} + \frac{2\log(x)}{x}}{x + \log^2(x)} \right) dx$$

$$= \ln(\ln(x)^2 + x) + e^{x^2} \ln(x) - \frac{\ln(x)}{\ln(x)^2 + x}$$

---


$$3.274. \quad \int \left( \frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2 + \log(x)}{(x + \log^2(x))^2} + \frac{1 + \frac{1}{x} + \frac{2\log(x)}{x}}{x + \log^2(x)} \right) dx$$

input `int((log(x) - 2)/(x + log(x)^2)^2 + ((2*log(x))/x + 1/x + 1)/(x + log(x)^2) + exp(x^2)/x + 2*x*exp(x^2)*log(x),x)`

output `log(x + log(x)^2) + exp(x^2)*log(x) - log(x)/(x + log(x)^2)`

---

3.274.  $\int \left( \frac{e^{x^2}}{x} + 2e^{x^2} x \log(x) + \frac{-2+\log(x)}{(x+\log^2(x))^2} + \frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)} \right) dx$



### 3.275 $\int e^{\frac{x}{2}+xz} x^4 \sin^4(\pi z) dz$

|                                                             |      |
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#### 3.275.1 Optimal result

Integrand size = 21, antiderivative size = 199

$$\int e^{\frac{x}{2}+xz} x^4 \sin^4(\pi z) dz = \frac{24e^{\frac{x}{2}+xz} \pi^4 x^3}{64\pi^4 + 20\pi^2 x^2 + x^4} - \frac{24e^{\frac{x}{2}+xz} \pi^3 x^4 \cos(\pi z) \sin(\pi z)}{64\pi^4 + 20\pi^2 x^2 + x^4} + \frac{12e^{\frac{x}{2}+xz} \pi^2 x^5 \sin^2(\pi z)}{64\pi^4 + 20\pi^2 x^2 + x^4} - \frac{4e^{\frac{x}{2}+xz} \pi x^4 \cos(\pi z) \sin^3(\pi z)}{16\pi^2 + x^2} + \frac{e^{\frac{x}{2}+xz} x^5 \sin^4(\pi z)}{16\pi^2 + x^2}$$

output `24*exp(1/2*x+x*z)*Pi^4*x^3/(64*Pi^4+20*Pi^2*x^2+x^4)-24*exp(1/2*x+x*z)*Pi^3*x^4*cos(Pi*z)*sin(Pi*z)/(64*Pi^4+20*Pi^2*x^2+x^4)+12*exp(1/2*x+x*z)*Pi^2*x^5*sin(Pi*z)^2/(64*Pi^4+20*Pi^2*x^2+x^4)-4*exp(1/2*x+x*z)*Pi*x^4*cos(Pi*z)*sin(Pi*z)^3/(16*Pi^2+x^2)+exp(1/2*x+x*z)*x^5*sin(Pi*z)^4/(16*Pi^2+x^2)`

#### 3.275.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.68

$$\int e^{\frac{x}{2}+xz} x^4 \sin^4(\pi z) dz = \frac{e^{x(\frac{1}{2}+z)} x^4 (192\pi^4 + 60\pi^2 x^2 + 3x^4 - 4x^2(16\pi^2 + x^2) \cos(2\pi z) + x^2(4\pi^2 + x^2) \cos(4\pi z) - 128\pi^3 x \sin(2\pi z))}{8(64\pi^4 x + 20\pi^2 x^3 + x^5)}$$

input `Integrate[E^(x/2 + x*z)*x^4*Sin[Pi*z]^4,z]`

output  $(E^{(x*(1/2 + z))*x^4*(192*Pi^4 + 60*Pi^2*x^2 + 3*x^4 - 4*x^2*(16*Pi^2 + x^2)*Cos[2*Pi*z] + x^2*(4*Pi^2 + x^2)*Cos[4*Pi*z] - 128*Pi^3*x*Sin[2*Pi*z] - 8*Pi*x^3*Sin[2*Pi*z] + 16*Pi^3*x*Sin[4*Pi*z] + 4*Pi*x^3*Sin[4*Pi*z]))/(8*(64*Pi^4*x + 20*Pi^2*x^3 + x^5))$

### 3.275.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {27, 4934, 4934, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{xz + \frac{x}{2}} \sin^4(\pi z) dz$$

$$\downarrow 27$$

$$x^4 \int e^{xz + \frac{x}{2}} \sin^4(\pi z) dz$$

$$\downarrow 4934$$

$$x^4 \left( \frac{12\pi^2 \int e^{xz + \frac{x}{2}} \sin^2(\pi z) dz}{x^2 + 16\pi^2} + \frac{x e^{xz + \frac{x}{2}} \sin^4(\pi z)}{x^2 + 16\pi^2} - \frac{4\pi e^{xz + \frac{x}{2}} \sin^3(\pi z) \cos(\pi z)}{x^2 + 16\pi^2} \right)$$

$$\downarrow 4934$$

$$x^4 \left( \frac{12\pi^2 \left( \frac{2\pi^2 \int e^{xz + \frac{x}{2}} dz}{x^2 + 4\pi^2} + \frac{x e^{xz + \frac{x}{2}} \sin^2(\pi z)}{x^2 + 4\pi^2} - \frac{2\pi e^{xz + \frac{x}{2}} \sin(\pi z) \cos(\pi z)}{x^2 + 4\pi^2} \right)}{x^2 + 16\pi^2} + \frac{x e^{xz + \frac{x}{2}} \sin^4(\pi z)}{x^2 + 16\pi^2} - \frac{4\pi e^{xz + \frac{x}{2}} \sin^3(\pi z) \cos(\pi z)}{x^2 + 16\pi^2} \right)$$

$$\downarrow 2624$$

$$x^4 \left( \frac{x e^{xz + \frac{x}{2}} \sin^4(\pi z)}{x^2 + 16\pi^2} - \frac{4\pi e^{xz + \frac{x}{2}} \sin^3(\pi z) \cos(\pi z)}{x^2 + 16\pi^2} + \frac{12\pi^2 \left( \frac{2\pi^2 e^{xz + \frac{x}{2}}}{x(x^2 + 4\pi^2)} + \frac{x e^{xz + \frac{x}{2}} \sin^2(\pi z)}{x^2 + 4\pi^2} - \frac{2\pi e^{xz + \frac{x}{2}} \sin(\pi z) \cos(\pi z)}{x^2 + 4\pi^2} \right)}{x^2 + 16\pi^2} \right)$$

input  $\text{Int}[E^{(x/2 + x*z)}*x^4*\text{Sin}[Pi*z]^4, z]$

```
output x^4*((-4*E^(x/2 + x*z)*Pi*Cos[Pi*z]*Sin[Pi*z]^3)/(16*Pi^2 + x^2) + (E^(x/2 + x*z)*x*Sin[Pi*z]^4)/(16*Pi^2 + x^2) + (12*Pi^2*((2*E^(x/2 + x*z)*Pi^2)/(x*(4*Pi^2 + x^2)) - (2*E^(x/2 + x*z)*Pi*Cos[Pi*z]*Sin[Pi*z])/(4*Pi^2 + x^2) + (E^(x/2 + x*z)*x*Sin[Pi*z]^2)/(4*Pi^2 + x^2)))/(16*Pi^2 + x^2)
```

### 3.275.3.1 Defintions of rubi rules used

```
rule 27 Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

```
rule 2624 Int[((F_)^(v_))^(n_), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /; FreeQ[{F, n}, x] && LinearQ[v, x]
```

```
rule 4934 Int[(F_)^((c_)*((a_) + (b_)*(x_)))*Sin[(d_) + (e_)*(x_)]^(n_), x_Symbol] := Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + (-Simp[e*n*F^(c*(a + b*x))*Cos[d + e*x]*(Sin[d + e*x]^(n - 1))/(e^2*n^2 + b^2*c^2*Log[F]^2)), x] + Simp[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2) Int[F^(c*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

### 3.275.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.63

| method        | result                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| default       | $x^4 \left( \frac{3e^{x(\frac{1}{2}+z)}}{8x} + \frac{x e^{\frac{1}{2}x+xz} \cos(4\pi z)}{128\pi^2+8x^2} + \frac{\pi e^{\frac{1}{2}x+xz} \sin(4\pi z)}{32\pi^2+2x^2} - \frac{x e^{\frac{1}{2}x+xz} \cos(2\pi z)}{2(4\pi^2+x^2)} - \frac{\pi e^{\frac{1}{2}x+xz} \sin(2\pi z)}{4\pi^2+x^2} \right)$                                                                                                                                        |
| risch         | $\frac{3x^3 e^{\frac{x(1+2z)}{2}}}{8} + \frac{x^5 e^{\frac{1}{2}x+xz} \cos(4\pi z)}{128\pi^2+8x^2} + \frac{x^4 e^{\frac{1}{2}x+xz} \pi \sin(4\pi z)}{32\pi^2+2x^2} - \frac{x^5 e^{\frac{1}{2}x+xz} \cos(2\pi z)}{2(4\pi^2+x^2)} - \frac{x^4 e^{\frac{1}{2}x+xz} \pi \sin(2\pi z)}{4\pi^2+x^2}$                                                                                                                                           |
| parallelrisch | $\frac{x^3 e^{\frac{x(1+2z)}{2}} (-128\pi^3 x \sin(2\pi z) + 16\pi^3 x \sin(4\pi z) + 4\pi^2 x^2 \cos(4\pi z) - 64\pi^2 x^2 \cos(2\pi z) - 8x^3 \pi \sin(2\pi z) + 4x^3 \pi \sin(4\pi z) + x^4 \cos(4\pi z))}{512\pi^4 + 160\pi^2 x^2 + 8x^4}$                                                                                                                                                                                           |
| norman        | $\frac{24 e^{\frac{1}{2}x+xz} \pi^4 x^3}{64\pi^4+20\pi^2 x^2+x^4} - \frac{48\pi^3 x^4 e^{\frac{1}{2}x+xz} \tan(\frac{\pi z}{2})}{64\pi^4+20\pi^2 x^2+x^4} + \frac{48\pi^3 x^4 e^{\frac{1}{2}x+xz} (\tan^7(\frac{\pi z}{2}))}{64\pi^4+20\pi^2 x^2+x^4} + \frac{16(9\pi^4+10\pi^2 x^2+x^4) x^3 e^{\frac{1}{2}x+xz} (\tan^4(\frac{\pi z}{2}))}{64\pi^4+20\pi^2 x^2+x^4} + \frac{24 e^{\frac{1}{2}x+xz} \pi^4 x^3}{64\pi^4+20\pi^2 x^2+x^4}$ |

```
input int(x^4*exp(1/2*x+x*z)*sin(Pi*z)^4,z,method=_RETURNVERBOSE)
```

3.275.  $\int e^{\frac{x}{2}+xz} x^4 \sin^4(\pi z) dz$

output  $x^4 \cdot (3/8 \cdot \exp(x \cdot (1/2+z)) / x + 1/8 \cdot x / (16 \cdot \pi^2 + x^2) \cdot \exp(1/2 \cdot x + x \cdot z) \cdot \cos(4 \cdot \pi \cdot z) + 1/2 \cdot \pi / (16 \cdot \pi^2 + x^2) \cdot \exp(1/2 \cdot x + x \cdot z) \cdot \sin(4 \cdot \pi \cdot z) - 1/2 \cdot x / (4 \cdot \pi^2 + x^2) \cdot \exp(1/2 \cdot x + x \cdot z) \cdot \cos(2 \cdot \pi \cdot z) - \pi / (4 \cdot \pi^2 + x^2) \cdot \exp(1/2 \cdot x + x \cdot z) \cdot \sin(2 \cdot \pi \cdot z))$

### 3.275.5 Fracas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.73

$$\int e^{\frac{x}{2} + xz} x^4 \sin^4(\pi z) dz$$

$$= \frac{4 \left( (4 \pi^3 x^4 + \pi x^6) \cos(\pi z)^3 - (10 \pi^3 x^4 + \pi x^6) \cos(\pi z) \right) e^{(xz + \frac{1}{2}x)} \sin(\pi z) + (24 \pi^4 x^3 + 16 \pi^2 x^5 + x^7 + (4 \pi^2 x^5 + x^7) \cos(\pi z)^2) e^{(xz + \frac{1}{2}x)}}{64 \pi^4 + 20 \pi^2 x^2 + x^4}$$

input `integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z, algorithm="fricas")`

output  $(4 \cdot ((4 \cdot \pi^3 \cdot x^4 + \pi \cdot x^6) \cdot \cos(\pi \cdot z)^3 - (10 \cdot \pi^3 \cdot x^4 + \pi \cdot x^6) \cdot \cos(\pi \cdot z)) \cdot e^{(x \cdot z + 1/2 \cdot x)} \cdot \sin(\pi \cdot z) + (24 \cdot \pi^4 \cdot x^3 + 16 \cdot \pi^2 \cdot x^5 + x^7 + (4 \cdot \pi^2 \cdot x^5 + x^7) \cdot \cos(\pi \cdot z)^2 - 2 \cdot (10 \cdot \pi^2 \cdot x^5 + x^7) \cdot \cos(\pi \cdot z)^2) \cdot e^{(x \cdot z + 1/2 \cdot x)}) / (64 \cdot \pi^4 + 20 \cdot \pi^2 \cdot x^2 + x^4)$

### 3.275.6 Sympy [A] (verification not implemented)

Time = 151.38 (sec) , antiderivative size = 1277, normalized size of antiderivative = 6.42

$$\int e^{\frac{x}{2} + xz} x^4 \sin^4(\pi z) dz = \text{Too large to display}$$

input `integrate(x**4*exp(1/2*x+x*z)*sin(pi*z)**4,z)`

```

output x**4*Piecewise((3*z*sin(pi*z)**4/8 + 3*z*sin(pi*z)**2*cos(pi*z)**2/4 + 3*z
*cos(pi*z)**4/8 - 5*sin(pi*z)**3*cos(pi*z)/(8*pi) - 3*sin(pi*z)*cos(pi*z)*
*3/(8*pi), Eq(x, 0)), (z*exp(-4*I*pi*z)*sin(pi*z)**4/16 - I*z*exp(-4*I*pi*
z)*sin(pi*z)**3*cos(pi*z)/4 - 3*z*exp(-4*I*pi*z)*sin(pi*z)**2*cos(pi*z)**2
/8 + I*z*exp(-4*I*pi*z)*sin(pi*z)*cos(pi*z)**3/4 + z*exp(-4*I*pi*z)*cos(pi
*z)**4/16 + 7*I*exp(-4*I*pi*z)*sin(pi*z)**4/(24*pi) + 11*exp(-4*I*pi*z)*si
n(pi*z)**3*cos(pi*z)/(48*pi) + 5*exp(-4*I*pi*z)*sin(pi*z)*cos(pi*z)**3/(48
*pi) - I*exp(-4*I*pi*z)*cos(pi*z)**4/(24*pi), Eq(x, -4*I*pi)), (-z*exp(-2*
I*pi*z)*sin(pi*z)**4/4 + I*z*exp(-2*I*pi*z)*sin(pi*z)**3*cos(pi*z)/2 + I*z
*exp(-2*I*pi*z)*sin(pi*z)*cos(pi*z)**3/2 + z*exp(-2*I*pi*z)*cos(pi*z)**4/4
- 5*I*exp(-2*I*pi*z)*sin(pi*z)**4/(24*pi) + exp(-2*I*pi*z)*sin(pi*z)**3*c
os(pi*z)/(3*pi) - I*exp(-2*I*pi*z)*sin(pi*z)**2*cos(pi*z)**2/(2*pi) - I*ex
p(-2*I*pi*z)*cos(pi*z)**4/(8*pi), Eq(x, -2*I*pi)), (-z*exp(2*I*pi*z)*sin(p
i*z)**4/4 - I*z*exp(2*I*pi*z)*sin(pi*z)**3*cos(pi*z)/2 - I*z*exp(2*I*pi*z)
*sin(pi*z)*cos(pi*z)**3/2 + z*exp(2*I*pi*z)*cos(pi*z)**4/4 + 5*I*exp(2*I*p
i*z)*sin(pi*z)**4/(24*pi) + exp(2*I*pi*z)*sin(pi*z)**3*cos(pi*z)/(3*pi) +
I*exp(2*I*pi*z)*sin(pi*z)**2*cos(pi*z)**2/(2*pi) + I*exp(2*I*pi*z)*cos(pi*
z)**4/(8*pi), Eq(x, 2*I*pi)), (z*exp(4*I*pi*z)*sin(pi*z)**4/16 + I*z*exp(4
*I*pi*z)*sin(pi*z)**3*cos(pi*z)/4 - 3*z*exp(4*I*pi*z)*sin(pi*z)**2*cos(pi*
z)**2/8 - I*z*exp(4*I*pi*z)*sin(pi*z)*cos(pi*z)**3/4 + z*exp(4*I*pi*z)*...

```

### 3.275.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

$$\int e^{\frac{x}{2}+xz} x^4 \sin^4(\pi z) dz$$

$$= \frac{\left( (4\pi^2 x^2 + x^4) \cos(4\pi z) e^{(xz + \frac{1}{2}x)} - 4(16\pi^2 x^2 + x^4) \cos(2\pi z) e^{(xz + \frac{1}{2}x)} + 4(4\pi^3 x + \pi x^3) e^{(xz + \frac{1}{2}x)} \sin(4\pi z) \right)}{8(64\pi^4 x + 20\pi^2 x^3 + x^5)}$$

```
input integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z, algorithm="maxima")
```

```

output 1/8*((4*pi^2*x^2 + x^4)*cos(4*pi*z)*e^(x*z + 1/2*x) - 4*(16*pi^2*x^2 + x^4
)*cos(2*pi*z)*e^(x*z + 1/2*x) + 4*(4*pi^3*x + pi*x^3)*e^(x*z + 1/2*x)*sin(
4*pi*z) - 8*(16*pi^3*x + pi*x^3)*e^(x*z + 1/2*x)*sin(2*pi*z) + 3*(64*pi^4
+ 20*pi^2*x^2 + x^4)*e^(x*z + 1/2*x))*x^4/(64*pi^4*x + 20*pi^2*x^3 + x^5)

```

**3.275.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.57

$$\int e^{\frac{x}{2}+xz} x^4 \sin^4(\pi z) dz$$

$$= \frac{1}{8} \left( \left( \frac{x \cos(4\pi z)}{16\pi^2 + x^2} + \frac{4\pi \sin(4\pi z)}{16\pi^2 + x^2} \right) e^{(xz+\frac{1}{2}x)} - 4 \left( \frac{x \cos(2\pi z)}{4\pi^2 + x^2} + \frac{2\pi \sin(2\pi z)}{4\pi^2 + x^2} \right) e^{(xz+\frac{1}{2}x)} + \frac{3e^{(xz+\frac{1}{2}x)}}{x} \right) x^4$$

input `integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z, algorithm="giac")`output `1/8*((x*cos(4*pi*z)/(16*pi^2 + x^2) + 4*pi*sin(4*pi*z)/(16*pi^2 + x^2))*e^(x*z + 1/2*x) - 4*(x*cos(2*pi*z)/(4*pi^2 + x^2) + 2*pi*sin(2*pi*z)/(4*pi^2 + x^2))*e^(x*z + 1/2*x) + 3*e^(x*z + 1/2*x)/x)*x^4`**3.275.9 Mupad [B] (verification not implemented)**

Time = 1.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int e^{\frac{x}{2}+xz} x^4 \sin^4(\pi z) dz$$

$$= \frac{x^3 e^{\frac{x}{2}+xz} \left( 24\Pi^4 - \frac{x^4 \cos(2\Pi z)}{2} + \frac{x^4 \cos(4\Pi z)}{8} + \frac{3x^4}{8} + \frac{15\Pi^2 x^2}{2} - \Pi x^3 \sin(2\Pi z) - 16\Pi^3 x \sin(2\Pi z) + \frac{\Pi x^3 \sin(4\Pi z)}{2} \right)}{64\Pi^4 + 20\Pi^2 x^2 + x^4}$$

input `int(x^4*exp(x/2 + x*z)*sin(Pi*z)^4,z)`output `(x^3*exp(x/2 + x*z)*(24*Pi^4 - (x^4*cos(2*Pi*z))/2 + (x^4*cos(4*Pi*z))/8 + (3*x^4)/8 + (15*Pi^2*x^2)/2 - Pi*x^3*sin(2*Pi*z) - 16*Pi^3*x*sin(2*Pi*z) + (Pi*x^3*sin(4*Pi*z))/2 + 2*Pi^3*x*sin(4*Pi*z) - 8*Pi^2*x^2*cos(2*Pi*z) + (Pi^2*x^2*cos(4*Pi*z))/2))/(64*Pi^4 + x^4 + 20*Pi^2*x^2)`

## 3.276 $\int \operatorname{erf}(x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.276.1 Optimal result . . . . .                            | 1454 |
| 3.276.2 Mathematica [A] (verified) . . . . .                | 1454 |
| 3.276.3 Rubi [A] (verified) . . . . .                       | 1455 |
| 3.276.4 Maple [A] (verified) . . . . .                      | 1455 |
| 3.276.5 Fricas [A] (verification not implemented) . . . . . | 1456 |
| 3.276.6 Sympy [A] (verification not implemented) . . . . .  | 1456 |
| 3.276.7 Maxima [A] (verification not implemented) . . . . . | 1456 |
| 3.276.8 Giac [A] (verification not implemented) . . . . .   | 1457 |
| 3.276.9 Mupad [B] (verification not implemented) . . . . .  | 1457 |

### 3.276.1 Optimal result

Integrand size = 2, antiderivative size = 18

$$\int \operatorname{erf}(x) dx = \frac{e^{-x^2}}{\sqrt{\pi}} + x\operatorname{erf}(x)$$

output `x*erf(x)+1/exp(x^2)/Pi^(1/2)`

### 3.276.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \operatorname{erf}(x) dx = \frac{e^{-x^2}}{\sqrt{\pi}} + x\operatorname{erf}(x)$$

input `Integrate[Erf[x],x]`

output `1/(E^x^2*Sqrt[Pi]) + x*Erf[x]`

**3.276.3 Rubi [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {6903}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{erf}(x) dx$$

$$\downarrow 6903$$

$$x \operatorname{erf}(x) + \frac{e^{-x^2}}{\sqrt{\pi}}$$

input `Int[Erf[x], x]`

output `1/(E^x^2*Sqrt[Pi]) + x*Erf[x]`

**3.276.3.1 Defintions of rubi rules used**

rule 6903 `Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] := Simp[(a + b*x)*(Erf[a + b*x]/b), x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]`

**3.276.4 Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

| method       | result                                                                    | size |
|--------------|---------------------------------------------------------------------------|------|
| default      | $x \operatorname{erf}(x) + \frac{e^{-x^2}}{\sqrt{\pi}}$                   | 16   |
| parts        | $x \operatorname{erf}(x) + \frac{e^{-x^2}}{\sqrt{\pi}}$                   | 16   |
| parallelrisc | $\frac{x\sqrt{\pi} \operatorname{erf}(x) + e^{-x^2}}{\sqrt{\pi}}$         | 19   |
| meijerg      | $\frac{-2 + 2e^{-x^2} + 2x\sqrt{\pi} \operatorname{erf}(x)}{2\sqrt{\pi}}$ | 24   |

input `int(erf(x), x, method=_RETURNVERBOSE)`



output `x*erf(x)+1/Pi^(1/2)*exp(-x^2)`

### 3.276.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \operatorname{erf}(x) dx = \frac{\pi x \operatorname{erf}(x) + \sqrt{\pi} e^{-x^2}}{\pi}$$

input `integrate(erf(x),x, algorithm="fricas")`

output `(pi*x*erf(x) + sqrt(pi)*e^(-x^2))/pi`

### 3.276.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \operatorname{erf}(x) dx = x \operatorname{erf}(x) + \frac{e^{-x^2}}{\sqrt{\pi}}$$

input `integrate(erf(x),x)`

output `x*erf(x) + exp(-x**2)/sqrt(pi)`

### 3.276.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \operatorname{erf}(x) dx = x \operatorname{erf}(x) + \frac{e^{(-x^2)}}{\sqrt{\pi}}$$

input `integrate(erf(x),x, algorithm="maxima")`

output `x*erf(x) + e^(-x^2)/sqrt(pi)`

**3.276.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \operatorname{erf}(x) dx = x \operatorname{erf}(x) + \frac{e^{(-x^2)}}{\sqrt{\pi}}$$

input `integrate(erf(x),x, algorithm="giac")`

output `x*erf(x) + e^(-x^2)/sqrt(pi)`

**3.276.9 Mupad [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \operatorname{erf}(x) dx = \frac{e^{-x^2}}{\sqrt{\pi}} + x \operatorname{erf}(x)$$

input `int(erf(x),x)`

output `exp(-x^2)/pi^(1/2) + x*erf(x)`

## 3.277 $\int \operatorname{erf}(a + x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.277.1 Optimal result . . . . .                            | 1458 |
| 3.277.2 Mathematica [A] (verified) . . . . .                | 1458 |
| 3.277.3 Rubi [A] (verified) . . . . .                       | 1459 |
| 3.277.4 Maple [A] (verified) . . . . .                      | 1459 |
| 3.277.5 Fricas [A] (verification not implemented) . . . . . | 1460 |
| 3.277.6 Sympy [A] (verification not implemented) . . . . .  | 1460 |
| 3.277.7 Maxima [A] (verification not implemented) . . . . . | 1460 |
| 3.277.8 Giac [A] (verification not implemented) . . . . .   | 1461 |
| 3.277.9 Mupad [B] (verification not implemented) . . . . .  | 1461 |

### 3.277.1 Optimal result

Integrand size = 4, antiderivative size = 24

$$\int \operatorname{erf}(a + x) dx = \frac{e^{-(a+x)^2}}{\sqrt{\pi}} + (a + x)\operatorname{erf}(a + x)$$

output `(a+x)*erf(a+x)+1/exp((a+x)^2)/Pi^(1/2)`

### 3.277.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \operatorname{erf}(a + x) dx = \frac{e^{-(a+x)^2}}{\sqrt{\pi}} + (a + x)\operatorname{erf}(a + x)$$

input `Integrate[Erf[a + x],x]`

output `1/(E^(a + x)^2*Sqrt[Pi]) + (a + x)*Erf[a + x]`

**3.277.3 Rubi [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {6903}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{erf}(a+x) dx$$

$$\downarrow 6903$$

$$(a+x)\operatorname{erf}(a+x) + \frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

input `Int[Erf[a + x], x]`

output `1/(E^(a + x)^2*Sqrt[Pi]) + (a + x)*Erf[a + x]`

**3.277.3.1 Defintions of rubi rules used**

rule 6903 `Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] := Simp[(a + b*x)*(Erf[a + b*x]/b), x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]`

**3.277.4 Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

| method            | result                                                                                                                                     | size |
|-------------------|--------------------------------------------------------------------------------------------------------------------------------------------|------|
| derivativedivides | $(a+x)\operatorname{erf}(a+x) + \frac{e^{-(a+x)^2}}{\sqrt{\pi}}$                                                                           | 22   |
| default           | $(a+x)\operatorname{erf}(a+x) + \frac{e^{-(a+x)^2}}{\sqrt{\pi}}$                                                                           | 22   |
| parallelrisc      | $\frac{x \operatorname{erf}(a+x)\sqrt{\pi} + a \operatorname{erf}(a+x)\sqrt{\pi} + e^{-(a+x)^2}}{\sqrt{\pi}}$                              | 32   |
| parts             | $x \operatorname{erf}(a+x) - \frac{2\left(-\frac{e^{-a^2-2ax-x^2}}{2} - \frac{a \operatorname{erf}(a+x)\sqrt{\pi}}{2}\right)}{\sqrt{\pi}}$ | 42   |

input `int(erf(a+x), x, method=_RETURNVERBOSE)`

output  $(a+x)*\text{erf}(a+x)+1/\text{Pi}^{(1/2)}*\text{exp}(-(a+x)^2)$

### 3.277.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \text{erf}(a+x) dx = \frac{(\pi a + \pi x) \text{erf}(a+x) + \sqrt{\pi} e^{(-a^2 - 2ax - x^2)}}{\pi}$$

input `integrate(erf(a+x),x, algorithm="fricas")`

output  $((\text{pi}*a + \text{pi}*x)*\text{erf}(a+x) + \text{sqrt}(\text{pi})*\text{e}^{(-a^2 - 2*a*x - x^2)})/\text{pi}$

### 3.277.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \text{erf}(a+x) dx = a \text{erf}(a+x) + x \text{erf}(a+x) + \frac{e^{-a^2} e^{-x^2} e^{-2ax}}{\sqrt{\pi}}$$

input `integrate(erf(a+x),x)`

output  $a*\text{erf}(a+x) + x*\text{erf}(a+x) + \text{exp}(-a**2)*\text{exp}(-x**2)*\text{exp}(-2*a*x)/\text{sqrt}(\text{pi})$

### 3.277.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \text{erf}(a+x) dx = (a+x) \text{erf}(a+x) + \frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

input `integrate(erf(a+x),x, algorithm="maxima")`

output  $(a+x)*\text{erf}(a+x) + \text{e}^{-(a+x)^2}/\text{sqrt}(\text{pi})$

**3.277.8 Giac [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \operatorname{erf}(a+x) dx = x \operatorname{erf}(a+x) + \frac{\sqrt{\pi} a \operatorname{erf}(a+x) + e^{(-a^2-2ax-x^2)}}{\sqrt{\pi}}$$

input `integrate(erf(a+x),x, algorithm="giac")`output `x*erf(a + x) + (sqrt(pi)*a*erf(a + x) + e^(-a^2 - 2*a*x - x^2))/sqrt(pi)`**3.277.9 Mupad [B] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \operatorname{erf}(a+x) dx = \operatorname{erf}(a+x) (a+x) + \frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

input `int(erf(a + x),x)`output `erf(a + x)*(a + x) + exp(-(a + x)^2)/pi^(1/2)`

**3.278**  $\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}} dx$

3.278.1 Optimal result . . . . . 1462  
 3.278.2 Mathematica [C] (warning: unable to verify) . . . . . 1462  
 3.278.3 Rubi [F] . . . . . 1463  
 3.278.4 Maple [F(-1)] . . . . . 1464  
 3.278.5 Fracas [B] (verification not implemented) . . . . . 1464  
 3.278.6 Sympy [F] . . . . . 1465  
 3.278.7 Maxima [F] . . . . . 1465  
 3.278.8 Giac [F] . . . . . 1465  
 3.278.9 Mupad [F(-1)] . . . . . 1466

**3.278.1 Optimal result**

Integrand size = 59, antiderivative size = 94

$$\int \frac{-8 - 8x - x^2 - 3x^3 + 7x^4 + 4x^5 + 2x^6}{(-1 + 2x^2)^2 \sqrt{1 + 2x^2 + 4x^3 + x^4}} dx$$

$$= \frac{(1 + 2x)\sqrt{1 + 2x^2 + 4x^3 + x^4}}{2(-1 + 2x^2)} - \operatorname{arctanh}\left(\frac{x(2 + x)(7 - x + 27x^2 + 33x^3)}{(2 + 37x^2 + 31x^3)\sqrt{1 + 2x^2 + 4x^3 + x^4}}\right)$$

output `-arctanh(x*(2+x)*(33*x^3+27*x^2-x+7)/(31*x^3+37*x^2+2)/(x^4+4*x^3+2*x^2+1)^(1/2))+1/2*(1+2*x)*(x^4+4*x^3+2*x^2+1)^(1/2)/(2*x^2-1)`

**3.278.2 Mathematica [C] (warning: unable to verify)**

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 16.14 (sec) , antiderivative size = 5141, normalized size of antiderivative = 54.69

$$\int \frac{-8 - 8x - x^2 - 3x^3 + 7x^4 + 4x^5 + 2x^6}{(-1 + 2x^2)^2 \sqrt{1 + 2x^2 + 4x^3 + x^4}} dx = \text{Result too large to show}$$

input `Integrate[(-8 - 8*x - x^2 - 3*x^3 + 7*x^4 + 4*x^5 + 2*x^6)/((-1 + 2*x^2)^2*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]),x]`

output `Result too large to show`

---

3.278.  $\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}} dx$

**3.278.3 Rubi [F]**

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^6 + 4x^5 + 7x^4 - 3x^3 - x^2 - 8x - 8}{(2x^2 - 1)^2 \sqrt{x^4 + 4x^3 + 2x^2 + 1}} dx$$

↓ 7293

$$\int \left( \frac{x^2}{2\sqrt{x^4 + 4x^3 + 2x^2 + 1}} + \frac{x}{\sqrt{x^4 + 4x^3 + 2x^2 + 1}} + \frac{2x + 15}{4(2x^2 - 1)\sqrt{x^4 + 4x^3 + 2x^2 + 1}} + \frac{-17x - 13}{2(2x^2 - 1)^2 \sqrt{x^4 + 4x^3 + 2x^2 + 1}} \right) dx$$

↓ 2009

$$\begin{aligned} & \frac{9}{4} \int \frac{1}{\sqrt{x^4 + 4x^3 + 2x^2 + 1}} dx - \frac{13}{4} \int \frac{1}{(\sqrt{2} - 2x)^2 \sqrt{x^4 + 4x^3 + 2x^2 + 1}} dx + \\ & \int \frac{x}{\sqrt{x^4 + 4x^3 + 2x^2 + 1}} dx + \frac{1}{2} \int \frac{x^2}{\sqrt{x^4 + 4x^3 + 2x^2 + 1}} dx - \\ & \frac{13}{4} \int \frac{1}{(2x + \sqrt{2})^2 \sqrt{x^4 + 4x^3 + 2x^2 + 1}} dx - \frac{1}{8} (15 + \sqrt{2}) \int \frac{1}{(1 - \sqrt{2}x) \sqrt{x^4 + 4x^3 + 2x^2 + 1}} dx - \\ & \frac{13}{8} \int \frac{1}{(1 - \sqrt{2}x) \sqrt{x^4 + 4x^3 + 2x^2 + 1}} dx - \frac{1}{8} (15 - \sqrt{2}) \int \frac{1}{(\sqrt{2}x + 1) \sqrt{x^4 + 4x^3 + 2x^2 + 1}} dx - \\ & \frac{13}{8} \int \frac{1}{(\sqrt{2}x + 1) \sqrt{x^4 + 4x^3 + 2x^2 + 1}} dx - \frac{17}{2} \int \frac{x}{(2x^2 - 1)^2 \sqrt{x^4 + 4x^3 + 2x^2 + 1}} dx \end{aligned}$$

input `Int[(-8 - 8*x - x^2 - 3*x^3 + 7*x^4 + 4*x^5 + 2*x^6)/((-1 + 2*x^2)^2*sqrt[1 + 2*x^2 + 4*x^3 + x^4]),x]`

output `$Aborted`

**3.278.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 7293 `Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]`

---

3.278.  $\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2 \sqrt{1+2x^2+4x^3+x^4}} dx$



**3.278.4 Maple [F(-1)]**

Timed out.

hanged

input `int((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+1)^(1/2),x)`

output `int((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+1)^(1/2),x)`

**3.278.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(88) = 176.

Time = 0.32 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.90

$$\int \frac{-8 - 8x - x^2 - 3x^3 + 7x^4 + 4x^5 + 2x^6}{(-1 + 2x^2)^2 \sqrt{1 + 2x^2 + 4x^3 + x^4}} dx$$

$$= \frac{(2x^2 - 1) \log\left(\frac{1025x^{10} + 6138x^9 + 12307x^8 + 10188x^7 + 4503x^6 + 3134x^5 + 1589x^4 + 140x^3 + 176x^2 - (1023x^8 + 4104x^7 + 5084x^6 + 2182x^5 + 805x^4 + 624x^3 + 10x^2 + 28x) \sqrt{x^4 + 4x^3 + 2x^2 + 1} + 2}{32x^{10} - 80x^8 + 80x^6 - 40x^4 + 10x^2 - 1}\right) + \sqrt{x^4 + 4x^3 + 2x^2 + 1} (2x + 1)}{2(2x^2 - 1)}$$

input `integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+1)^(1/2),x, algorithm="fracas")`

output `1/2*((2*x^2 - 1)*log((1025*x^10 + 6138*x^9 + 12307*x^8 + 10188*x^7 + 4503*x^6 + 3134*x^5 + 1589*x^4 + 140*x^3 + 176*x^2 - (1023*x^8 + 4104*x^7 + 5084*x^6 + 2182*x^5 + 805*x^4 + 624*x^3 + 10*x^2 + 28*x)*sqrt(x^4 + 4*x^3 + 2*x^2 + 1) + 2)/(32*x^10 - 80*x^8 + 80*x^6 - 40*x^4 + 10*x^2 - 1)) + sqrt(x^4 + 4*x^3 + 2*x^2 + 1)*(2*x + 1))/(2*x^2 - 1)`

**3.278.6 Sympy [F]**

$$\int \frac{-8 - 8x - x^2 - 3x^3 + 7x^4 + 4x^5 + 2x^6}{(-1 + 2x^2)^2 \sqrt{1 + 2x^2 + 4x^3 + x^4}} dx$$

$$= \int \frac{2x^6 + 4x^5 + 7x^4 - 3x^3 - x^2 - 8x - 8}{\sqrt{(x+1)(x^3 + 3x^2 - x + 1)}(2x^2 - 1)^2} dx$$

input `integrate((2*x**6+4*x**5+7*x**4-3*x**3-x**2-8*x-8)/(2*x**2-1)**2/(x**4+4*x**3+2*x**2+1)**(1/2),x)`

output `Integral((2*x**6 + 4*x**5 + 7*x**4 - 3*x**3 - x**2 - 8*x - 8)/(sqrt((x + 1)*(x**3 + 3*x**2 - x + 1))*(2*x**2 - 1)**2), x)`

**3.278.7 Maxima [F]**

$$\int \frac{-8 - 8x - x^2 - 3x^3 + 7x^4 + 4x^5 + 2x^6}{(-1 + 2x^2)^2 \sqrt{1 + 2x^2 + 4x^3 + x^4}} dx$$

$$= \int \frac{2x^6 + 4x^5 + 7x^4 - 3x^3 - x^2 - 8x - 8}{\sqrt{x^4 + 4x^3 + 2x^2 + 1}(2x^2 - 1)^2} dx$$

input `integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+1)^(1/2),x, algorithm="maxima")`

output `integrate((2*x^6 + 4*x^5 + 7*x^4 - 3*x^3 - x^2 - 8*x - 8)/(sqrt(x^4 + 4*x^3 + 2*x^2 + 1)*(2*x^2 - 1)^2), x)`

**3.278.8 Giac [F]**

$$\int \frac{-8 - 8x - x^2 - 3x^3 + 7x^4 + 4x^5 + 2x^6}{(-1 + 2x^2)^2 \sqrt{1 + 2x^2 + 4x^3 + x^4}} dx$$

$$= \int \frac{2x^6 + 4x^5 + 7x^4 - 3x^3 - x^2 - 8x - 8}{\sqrt{x^4 + 4x^3 + 2x^2 + 1}(2x^2 - 1)^2} dx$$

input `integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+1)^(1/2),x, algorithm="giac")`

output `integrate((2*x^6 + 4*x^5 + 7*x^4 - 3*x^3 - x^2 - 8*x - 8)/(sqrt(x^4 + 4*x^3 + 2*x^2 + 1)*(2*x^2 - 1)^2), x)`

### 3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{-8 - 8x - x^2 - 3x^3 + 7x^4 + 4x^5 + 2x^6}{(-1 + 2x^2)^2 \sqrt{1 + 2x^2 + 4x^3 + x^4}} dx$$

$$= \int -\frac{2x^6 - 4x^5 - 7x^4 + 3x^3 + x^2 + 8x + 8}{(2x^2 - 1)^2 \sqrt{x^4 + 4x^3 + 2x^2 + 1}} dx$$

input `int(-(8*x + x^2 + 3*x^3 - 7*x^4 - 4*x^5 - 2*x^6 + 8)/((2*x^2 - 1)^2*(2*x^2 + 4*x^3 + x^4 + 1)^(1/2)),x)`

output `int(-(8*x + x^2 + 3*x^3 - 7*x^4 - 4*x^5 - 2*x^6 + 8)/((2*x^2 - 1)^2*(2*x^2 + 4*x^3 + x^4 + 1)^(1/2)), x)`

**3.279** 
$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}} dy$$

3.279.1 Optimal result . . . . . 1467  
 3.279.2 Mathematica [C] (warning: unable to verify) . . . . . 1467  
 3.279.3 Rubi [F] . . . . . 1468  
 3.279.4 Maple [C] (verified) . . . . . 1469  
 3.279.5 Fricas [A] (verification not implemented) . . . . . 1470  
 3.279.6 Sympy [F] . . . . . 1470  
 3.279.7 Maxima [F] . . . . . 1471  
 3.279.8 Giac [F] . . . . . 1471  
 3.279.9 Mupad [F(-1)] . . . . . 1471

**3.279.1 Optimal result**

Integrand size = 47, antiderivative size = 142

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}} dy = -\frac{1}{4} \operatorname{arctanh}\left(\frac{(1-3y)\sqrt{1-5y-5y^2}}{(1-5y)\sqrt{1-y-y^2}}\right) - \frac{1}{2} \operatorname{arctanh}\left(\frac{(4+3y)\sqrt{1-5y-5y^2}}{(6+5y)\sqrt{1-y-y^2}}\right) + \frac{9}{4} \operatorname{arctanh}\left(\frac{(11+7y)\sqrt{1-5y-5y^2}}{3(7+5y)\sqrt{1-y-y^2}}\right)$$

output `-1/4*arctanh((1-3*y)*(-5*y^2-5*y+1)^(1/2)/(1-5*y)/(-y^2-y+1)^(1/2))-1/2*arctanh((4+3*y)*(-5*y^2-5*y+1)^(1/2)/(6+5*y)/(-y^2-y+1)^(1/2))+9/4*arctanh(1/3*(11+7*y)*(-5*y^2-5*y+1)^(1/2)/(7+5*y)/(-y^2-y+1)^(1/2))`

**3.279.2 Mathematica [C] (warning: unable to verify)**

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 3.38 (sec) , antiderivative size = 630, normalized size of antiderivative = 4.44

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}} dy = \left(-1 - \frac{2}{\sqrt{5}}\right) (1 + \sqrt{5} + 2y)^2 \sqrt{\frac{5+3\sqrt{5}+10y}{5+5\sqrt{5}+10y}} \left(20\left(-4\sqrt{\frac{-5+3\sqrt{5}-10y}{1+\sqrt{5}+2y}} \sqrt{\frac{-1+\sqrt{5}-2y}{1+\sqrt{5}+2y}} + \sqrt{5}\sqrt{\frac{-5+3\sqrt{5}-10y}{1+\sqrt{5}+2y}} \sqrt{\frac{-1+\sqrt{5}-2y}{1+\sqrt{5}+2y}}\right)\right)$$

---

3.279. 
$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}} dy$$

input `Integrate[((1 + 2*y)*Sqrt[1 - 5*y - 5*y^2])/(y*(1 + y)*(2 + y)*Sqrt[1 - y - y^2]),y]`

output `((-1 - 2/Sqrt[5])*(1 + Sqrt[5] + 2*y)^2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(5 + 5*Sqrt[5] + 10*y)]*(20*(-4*Sqrt[(-5 + 3*Sqrt[5] - 10*y)/(1 + Sqrt[5] + 2*y)]*Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*y)] + Sqrt[5]*Sqrt[(-5 + 3*Sqrt[5] - 10*y)/(1 + Sqrt[5] + 2*y)]*Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*y)] + 5*Sqrt[-((-5 + Sqrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))]*Sqrt[-((-3 + Sqrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))] - 2*Sqrt[5]*Sqrt[-((-5 + Sqrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))]*Sqrt[-((-3 + Sqrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))])*EllipticF[ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y))]/Sqrt[15]], 15/16] + Sqrt[(-5 + 3*Sqrt[5] - 10*y)/(1 + Sqrt[5] + 2*y)]*Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*y)]*(9*Sqrt[5]*EllipticPi[5/8 - Sqrt[5]/8, ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y))]/Sqrt[15]], 15/16] + (-20 + 9*Sqrt[5])*EllipticPi[-3*(-5 + Sqrt[5])/8, ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y))]/Sqrt[15]], 15/16] + 2*Sqrt[5]*EllipticPi[(3*(5 + Sqrt[5]))/8, ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y))]/Sqrt[15]], 15/16])))/(16*Sqrt[1 - 5*y - 5*y^2]*Sqrt[1 - y - y^2])`

### 3.279.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2y+1)\sqrt{-5y^2-5y+1}}{y(y+1)(y+2)\sqrt{-y^2-y+1}} dy$$

↓ 7293

$$\int \left( \frac{\sqrt{-5y^2-5y+1}}{2y\sqrt{-y^2-y+1}} + \frac{\sqrt{-5y^2-5y+1}}{(y+1)\sqrt{-y^2-y+1}} - \frac{3\sqrt{-5y^2-5y+1}}{2(y+2)\sqrt{-y^2-y+1}} \right) dy$$

↓ 2009

$$\frac{1}{2} \int \frac{\sqrt{-5y^2-5y+1}}{y\sqrt{-y^2-y+1}} dy + \int \frac{\sqrt{-5y^2-5y+1}}{(y+1)\sqrt{-y^2-y+1}} dy - \frac{3}{2} \int \frac{\sqrt{-5y^2-5y+1}}{(y+2)\sqrt{-y^2-y+1}} dy$$

input `Int[((1 + 2*y)*Sqrt[1 - 5*y - 5*y^2])/(y*(1 + y)*(2 + y)*Sqrt[1 - y - y^2]),y]`

output \$Aborted

### 3.279.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 7293 `Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]  
]`

### 3.279.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 3.29 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.48

| method   | result                                                                                                                                                                                                                                                                                                                                                                                                            |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| default  | $\frac{300\sqrt{-5y^2-5y+1}\sqrt{-y^2-y+1}\sqrt{-\frac{10y+5+3\sqrt{5}}{-10y-5+3\sqrt{5}}}\left(-10y-5+3\sqrt{5}\right)^2\sqrt{\frac{-2y+\sqrt{5}-1}{-10y-5+3\sqrt{5}}}\sqrt{5}\sqrt{\frac{2y+1+\sqrt{5}}{-10y-5+3\sqrt{5}}}\left(\Pi\left(2\sqrt{-\frac{10y+5+3\sqrt{5}}{-10y-5+3\sqrt{5}}}\right)\right)}{\sqrt{5y^4+10y^3-y^2-6y+1}\sqrt{(10y+5+3\sqrt{5})(-10y-5+3\sqrt{5})(-2y+\sqrt{5}-1)(2y+1+\sqrt{5})}}$ |
| elliptic | Expression too large to display                                                                                                                                                                                                                                                                                                                                                                                   |

input `int((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y,method=_  
RETURNVERBOSE)`

output 
$$\begin{aligned} & -300*(-5*y^2-5*y+1)^{(1/2)}*(-y^2-y+1)^{(1/2)}*(-(10*y+5+3*5^{(1/2)})/(-10*y-5+3 \\ & *5^{(1/2)}))^{(1/2)}*(-10*y-5+3*5^{(1/2)})^2*((-2*y+5^{(1/2)}-1)/(-10*y-5+3*5^{(1/2)} \\ & ))^{(1/2)}*5^{(1/2)}*((2*y+1+5^{(1/2)})/(-10*y-5+3*5^{(1/2)}))^{(1/2)}*(\text{EllipticPi}( \\ & 2*(-(10*y+5+3*5^{(1/2)})/(-10*y-5+3*5^{(1/2)}))^{(1/2)},-1/4*(3*5^{(1/2)}-5)/(5+3* \\ & 5^{(1/2)}),1/4)+2*\text{EllipticPi}(2*(-(10*y+5+3*5^{(1/2)})/(-10*y-5+3*5^{(1/2)}))^{(1/2)} \\ & ),-1/4*(5+3*5^{(1/2)})/(3*5^{(1/2)}-5),1/4)-3*\text{EllipticPi}(2*(-(10*y+5+3*5^{(1/2)} \\ & ))/(-10*y-5+3*5^{(1/2)}))^{(1/2)},-1/4*(5+5^{(1/2)})/(5^{(1/2)}-5),1/4))/(5*y^4+10 \\ & *y^3-y^2-6*y+1)^{(1/2)}((10*y+5+3*5^{(1/2)})*(-10*y-5+3*5^{(1/2)})*(-2*y+5^{(1/2)} \\ & )-1)*(2*y+1+5^{(1/2)}))^{(1/2)}/(3*5^{(1/2)}-5)/(5+3*5^{(1/2)})/(5+5^{(1/2)})/(5^{(1/2)}-5) \end{aligned}$$

**3.279.5 Fricas [A] (verification not implemented)**

Time = 0.32 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.57

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}} dy$$

$$= \frac{9}{8} \log \left( -\frac{235y^4 + 935y^3 - 3(35y^2 + 104y + 77)\sqrt{-y^2 - y + 1}\sqrt{-5y^2 - 5y + 1} + 1086y^2 + 131y - 281}{y^4 + 8y^3 + 24y^2 + 32y + 16} \right)$$

$$+ \frac{1}{4} \log \left( \frac{35y^4 + 125y^3 + (15y^2 + 38y + 24)\sqrt{-y^2 - y + 1}\sqrt{-5y^2 - 5y + 1} + 131y^2 + 16y - 26}{y^4 + 4y^3 + 6y^2 + 4y + 1} \right)$$

$$+ \frac{1}{8} \log \left( \frac{35y^4 + 15y^3 + (15y^2 - 8y + 1)\sqrt{-y^2 - y + 1}\sqrt{-5y^2 - 5y + 1} - 34y^2 + 11y - 1}{y^4} \right)$$

```
input integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2), y, algorithm="fricas")
```

```
output 9/8*log(-(235*y^4 + 935*y^3 - 3*(35*y^2 + 104*y + 77)*sqrt(-y^2 - y + 1)*sqrt(-5*y^2 - 5*y + 1) + 1086*y^2 + 131*y - 281)/(y^4 + 8*y^3 + 24*y^2 + 32*y + 16)) + 1/4*log((35*y^4 + 125*y^3 + (15*y^2 + 38*y + 24)*sqrt(-y^2 - y + 1)*sqrt(-5*y^2 - 5*y + 1) + 131*y^2 + 16*y - 26)/(y^4 + 4*y^3 + 6*y^2 + 4*y + 1)) + 1/8*log((35*y^4 + 15*y^3 + (15*y^2 - 8*y + 1)*sqrt(-y^2 - y + 1)*sqrt(-5*y^2 - 5*y + 1) - 34*y^2 + 11*y - 1)/y^4)
```

**3.279.6 Sympy [F]**

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}} dy = \int \frac{(2y+1)\sqrt{-5y^2-5y+1}}{y(y+1)(y+2)\sqrt{-y^2-y+1}} dy$$

```
input integrate((1+2*y)*(-5*y**2-5*y+1)**(1/2)/y/(1+y)/(2+y)/(-y**2-y+1)**(1/2), y)
```

```
output Integral((2*y + 1)*sqrt(-5*y**2 - 5*y + 1)/(y*(y + 1)*(y + 2)*sqrt(-y**2 - y + 1)), y)
```

**3.279.7 Maxima [F]**

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}} dy = \int \frac{\sqrt{-5y^2-5y+1}(2y+1)}{\sqrt{-y^2-y+1}(y+2)(y+1)y} dy$$

input `integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y, algorithm="maxima")`

output `integrate(sqrt(-5*y^2 - 5*y + 1)*(2*y + 1)/(sqrt(-y^2 - y + 1)*(y + 2)*(y + 1)*y), y)`

**3.279.8 Giac [F]**

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}} dy = \int \frac{\sqrt{-5y^2-5y+1}(2y+1)}{\sqrt{-y^2-y+1}(y+2)(y+1)y} dy$$

input `integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y, algorithm="giac")`

output `integrate(sqrt(-5*y^2 - 5*y + 1)*(2*y + 1)/(sqrt(-y^2 - y + 1)*(y + 2)*(y + 1)*y), y)`

**3.279.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}} dy = \int \frac{(2y+1)\sqrt{-5y^2-5y+1}}{y(y+1)(y+2)\sqrt{-y^2-y+1}} dy$$

input `int(((2*y + 1)*(1 - 5*y^2 - 5*y)^(1/2))/(y*(y + 1)*(y + 2)*(1 - y^2 - y)^(1/2)),y)`

output `int(((2*y + 1)*(1 - 5*y^2 - 5*y)^(1/2))/(y*(y + 1)*(y + 2)*(1 - y^2 - y)^(1/2)), y)`

---

3.279.  $\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}} dy$



$$3.280 \quad \int \frac{x \left( -\sqrt{-4+x^2} + x^2 \sqrt{-4+x^2} - 4\sqrt{-1+x^2} + x^2 \sqrt{-1+x^2} \right)}{(4-5x^2+x^4) \left( 1 + \sqrt{-4+x^2} + \sqrt{-1+x^2} \right)} dx$$

|                                                   |      |
|---------------------------------------------------|------|
| 3.280.1 Optimal result                            | 1472 |
| 3.280.2 Mathematica [A] (verified)                | 1472 |
| 3.280.3 Rubi [A] (verified)                       | 1473 |
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### 3.280.1 Optimal result

Integrand size = 85, antiderivative size = 21

$$\int \frac{x \left( -\sqrt{-4+x^2} + x^2 \sqrt{-4+x^2} - 4\sqrt{-1+x^2} + x^2 \sqrt{-1+x^2} \right)}{(4-5x^2+x^4) \left( 1 + \sqrt{-4+x^2} + \sqrt{-1+x^2} \right)} dx$$

$$= \log \left( 1 + \sqrt{-4+x^2} + \sqrt{-1+x^2} \right)$$

output  $\ln(1+(x^2-4)^{(1/2)}+(x^2-1)^{(1/2)})$

### 3.280.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{x \left( -\sqrt{-4+x^2} + x^2 \sqrt{-4+x^2} - 4\sqrt{-1+x^2} + x^2 \sqrt{-1+x^2} \right)}{(4-5x^2+x^4) \left( 1 + \sqrt{-4+x^2} + \sqrt{-1+x^2} \right)} dx$$

$$= 2 \operatorname{arctanh} \left( 1 - \frac{2}{3} \sqrt{-4+x^2} + \frac{2}{3} \sqrt{-1+x^2} \right)$$

input `Integrate[(x*(-Sqrt[-4 + x^2] + x^2*Sqrt[-4 + x^2] - 4*Sqrt[-1 + x^2] + x^2*Sqrt[-1 + x^2]))/((4 - 5*x^2 + x^4)*(1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2])),x]`

output  $2*\operatorname{ArcTanh}[1 - (2*\operatorname{Sqrt}[-4 + x^2])/3 + (2*\operatorname{Sqrt}[-1 + x^2])/3]$

---


$$3.280. \quad \int \frac{x \left( -\sqrt{-4+x^2} + x^2 \sqrt{-4+x^2} - 4\sqrt{-1+x^2} + x^2 \sqrt{-1+x^2} \right)}{(4-5x^2+x^4) \left( 1 + \sqrt{-4+x^2} + \sqrt{-1+x^2} \right)} dx$$

**3.280.3 Rubi [A] (verified)**

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.012$ , Rules used = {7235}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \left( \sqrt{x^2 - 4}x^2 + \sqrt{x^2 - 1}x^2 - \sqrt{x^2 - 4} - 4\sqrt{x^2 - 1} \right)}{(x^4 - 5x^2 + 4) \left( \sqrt{x^2 - 4} + \sqrt{x^2 - 1} + 1 \right)} dx$$

↓ 7235

$$\log \left( \sqrt{x^2 - 4} + \sqrt{x^2 - 1} + 1 \right)$$

input `Int[(x*(-Sqrt[-4 + x^2] + x^2*Sqrt[-4 + x^2] - 4*Sqrt[-1 + x^2] + x^2*Sqrt[-1 + x^2]))/((4 - 5*x^2 + x^4)*(1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2])),x]`

output `Log[1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2]]`

**3.280.3.1 Defintions of rubi rules used**

rule 7235 `Int[(u_)/(y_), x_Symbol] := With[{q = DerivativeDivides[y, u, x]}, Simp[q*Log[RemoveContent[y, x]], x] /; !FalseQ[q]]`

---

3.280.  $\int \frac{x \left( -\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2} \right)}{(4-5x^2+x^4) \left( 1+\sqrt{-4+x^2}+\sqrt{-1+x^2} \right)} dx$

### 3.280.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(17) = 34.

Time = 0.40 (sec) , antiderivative size = 250, normalized size of antiderivative = 11.90

| method   | result                                                                                                                                                                                                                                                                                                                                         |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| elliptic | $\sqrt{(x^2-4)(x^2-1)} \left( \frac{\ln(x^2-5)}{4} + \frac{\ln(-\frac{5}{2}+x^2+\sqrt{x^4-5x^2+4})}{4} + \frac{\operatorname{arctanh}\left(\frac{5x^2-17}{4\sqrt{(x^2-5)^2+5x^2-21}}\right)}{4} + \frac{\operatorname{arctanh}\left(\frac{8+2\sqrt{5}(x-\sqrt{5})}{4\sqrt{(x-\sqrt{5})^2+2\sqrt{5}(x-\sqrt{5})+4}}\right)}{4} + \dots \right)$ |
| default  | $\frac{\ln(x^2-5)}{4} - \frac{\sqrt{(-2+x)^2+4x-8}+2\ln\left(x+\sqrt{(-2+x)^2+4x-8}\right)}{4(2+\sqrt{5})(-2+\sqrt{5})} - \frac{\sqrt{(2+x)^2-4x-8}-2\ln\left(x+\sqrt{(2+x)^2-4x-8}\right)}{4(2+\sqrt{5})(-2+\sqrt{5})} + \frac{\sqrt{(x-\sqrt{5})^2}}{\sqrt{x^2-4}\sqrt{x^2}}$                                                                |

```
input int(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(1/2))
/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
output ((x^2-4)*(x^2-1)^(1/2)/(x^2-4)^(1/2)/(x^2-1)^(1/2)*(1/4*ln(x^2-5)+1/4*ln(-5/2+x^2+(x^4-5*x^2+4)^(1/2))+1/4*arctanh(1/4*(5*x^2-17)/((x^2-5)^2+5*x^2-21)^(1/2))+1/4*arctanh(1/4*(8+2*5^(1/2)*(x-5^(1/2))))/((x-5^(1/2))^2+2*5^(1/2)*(x-5^(1/2))+4)^(1/2))+1/4*arctanh(1/4*(8-2*5^(1/2)*(x+5^(1/2))))/((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+4)^(1/2))-1/4*arctanh(1/2*(2+2*5^(1/2)*(x-5^(1/2))))/((x-5^(1/2))^2+2*5^(1/2)*(x-5^(1/2))+1)^(1/2))-1/4*arctanh(1/2*(2-2*5^(1/2)*(x+5^(1/2))))/((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+1)^(1/2)))
```

### 3.280.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(17) = 34.

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 7.71

$$\int \frac{x(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2})}{(4-5x^2+x^4)(1+\sqrt{-4+x^2}+\sqrt{-1+x^2})} dx$$

$$= -\frac{1}{4} \log(4x^4 - (4x^2 - 11)\sqrt{x^2 - 1}\sqrt{x^2 - 4} - 21x^2 + 23)$$

$$- \frac{1}{4} \log(x^2 - \sqrt{x^2 - 1}(x + 2) + 2x - 1) + \frac{1}{4} \log(x^2 - \sqrt{x^2 - 4}(x + 1) + x - 4)$$

$$- \frac{1}{4} \log(x^2 - \sqrt{x^2 - 4}(x - 1) - x - 4) + \frac{1}{4} \log(x^2 - \sqrt{x^2 - 1}(x - 2) - 2x - 1)$$

$$+ \frac{1}{4} \log(x^2 - 5) + \frac{1}{4} \log(-x^2 + \sqrt{x^2 - 1}\sqrt{x^2 - 4} + 7)$$

---

3.280.  $\int \frac{x(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2})}{(4-5x^2+x^4)(1+\sqrt{-4+x^2}+\sqrt{-1+x^2})} dx$

```
input integrate(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x, algorithm="fricas")
```

```
output -1/4*log(4*x^4 - (4*x^2 - 11)*sqrt(x^2 - 1)*sqrt(x^2 - 4) - 21*x^2 + 23) - 1/4*log(x^2 - sqrt(x^2 - 1)*(x + 2) + 2*x - 1) + 1/4*log(x^2 - sqrt(x^2 - 4)*(x + 1) + x - 4) - 1/4*log(x^2 - sqrt(x^2 - 4)*(x - 1) - x - 4) + 1/4*log(x^2 - sqrt(x^2 - 1)*(x - 2) - 2*x - 1) + 1/4*log(x^2 - 5) + 1/4*log(-x^2 + sqrt(x^2 - 1)*sqrt(x^2 - 4) + 7)
```

### 3.280.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2})}{(4-5x^2+x^4)(1+\sqrt{-4+x^2}+\sqrt{-1+x^2})} dx = \text{Timed out}$$

```
input integrate(x*(-(x**2-4)**(1/2)+x**2*(x**2-4)**(1/2)-4*(x**2-1)**(1/2)+x**2*(x**2-1)**(1/2))/(x**4-5*x**2+4)/(1+(x**2-4)**(1/2)+(x**2-1)**(1/2)),x)
```

output Timed out

### 3.280.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs.  $2(17) = 34$ .

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 8.14

$$\begin{aligned} & \int \frac{x(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2})}{(4-5x^2+x^4)(1+\sqrt{-4+x^2}+\sqrt{-1+x^2})} dx \\ &= \frac{1}{4} \log(x+1) + \frac{3}{8} \log(x-1) + \frac{1}{8} \log(x-2) \\ &+ \frac{1}{4} \log\left(\frac{2x^4+4(x^2-3)\sqrt{x+1}\sqrt{x-1}-7x^2+2((x^2-1)\sqrt{x+1}\sqrt{x-1}\sqrt{x-2}+(2x^2-3)\sqrt{x-2})}{2((x^2-1)\sqrt{x+1}\sqrt{x-1}\sqrt{x-2}+(2x^2-3)\sqrt{x-2})}\right) \\ &+ \frac{1}{4} \log\left(\frac{(x^2-1)\sqrt{x+1}\sqrt{x-1}+2x^2-3}{(x^2-1)\sqrt{x-1}}\right) \end{aligned}$$

---

3.280.  $\int \frac{x(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2})}{(4-5x^2+x^4)(1+\sqrt{-4+x^2}+\sqrt{-1+x^2})} dx$

```
input integrate(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x, algorithm="maxima")
```

```
output 1/4*log(x + 1) + 3/8*log(x - 1) + 1/8*log(x - 2) + 1/4*log(1/2*(2*x^4 + 4*(x^2 - 3)*sqrt(x + 1)*sqrt(x - 1) - 7*x^2 + 2*((x^2 - 1)*sqrt(x + 1)*sqrt(x - 1)*sqrt(x - 2) + (2*x^2 - 3)*sqrt(x - 2))*sqrt(x + 2) + 3)/((x^2 - 1)*sqrt(x + 1)*sqrt(x - 1)*sqrt(x - 2) + (2*x^2 - 3)*sqrt(x - 2))) + 1/4*log(((x^2 - 1)*sqrt(x + 1)*sqrt(x - 1) + 2*x^2 - 3)/((x^2 - 1)*sqrt(x - 1)))
```

### 3.280.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs.  $2(17) = 34$ .

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.62

$$\int \frac{x(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2})}{(4-5x^2+x^4)(1+\sqrt{-4+x^2}+\sqrt{-1+x^2})} dx$$

$$= -\frac{1}{2} \log(\sqrt{x^2-1}-\sqrt{x^2-4}+1) - \frac{1}{2} \log(\sqrt{x^2-1}-\sqrt{x^2-4})$$

$$+ \frac{1}{2} \log(\sqrt{x^2-1}+2) + \frac{1}{2} \log(|-\sqrt{x^2-1}+\sqrt{x^2-4}-3|)$$

```
input integrate(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x, algorithm="giac")
```

```
output -1/2*log(sqrt(x^2 - 1) - sqrt(x^2 - 4) + 1) - 1/2*log(sqrt(x^2 - 1) - sqrt(x^2 - 4)) + 1/2*log(sqrt(x^2 - 1) + 2) + 1/2*log(abs(-sqrt(x^2 - 1) + sqrt(x^2 - 4) - 3))
```

---

3.280. 
$$\int \frac{x(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2})}{(4-5x^2+x^4)(1+\sqrt{-4+x^2}+\sqrt{-1+x^2})} dx$$

**3.280.9 Mupad [B] (verification not implemented)**

Time = 2.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 8.19

$$\int \frac{x(-\sqrt{-4+x^2} + x^2\sqrt{-4+x^2} - 4\sqrt{-1+x^2} + x^2\sqrt{-1+x^2})}{(4-5x^2+x^4)(1+\sqrt{-4+x^2}+\sqrt{-1+x^2})} dx$$

$$= \frac{\ln(x-\sqrt{5})}{4} - \operatorname{atanh}\left(\frac{\sqrt{3}-\sqrt{x^2-1}}{\sqrt{x^2-4}}\right) + \frac{\operatorname{atanh}\left(\frac{\sqrt{x^2-1}}{2}\right)}{2}$$

$$+ \frac{\ln(x+\sqrt{5})}{4} - \frac{7 \operatorname{atanh}\left(\frac{4(\sqrt{3}-\sqrt{x^2-1})}{\sqrt{x^2-4}\left(\frac{(\sqrt{3}-\sqrt{x^2-1})^2}{x^2-4}+1\right)}\right)}{4}$$

$$+ \frac{5 \operatorname{atanh}\left(\frac{12150(\sqrt{3}-\sqrt{x^2-1})}{\sqrt{x^2-4}\left(\frac{6075(\sqrt{3}-\sqrt{x^2-1})^2}{2(x^2-4)}+\frac{6075}{2}\right)}\right)}{4} - \frac{\operatorname{atanh}(\sqrt{x^2-4})}{2}$$

```
input int(-(x*(4*(x^2 - 1)^(1/2) + (x^2 - 4)^(1/2) - x^2*(x^2 - 1)^(1/2) - x^2*(x^2 - 4)^(1/2)))/((x^4 - 5*x^2 + 4)*((x^2 - 1)^(1/2) + (x^2 - 4)^(1/2) + 1)),x)
```

```
output log(x - 5^(1/2))/4 - atanh((3^(1/2) - (x^2 - 1)^(1/2))/(x^2 - 4)^(1/2)) + atanh((x^2 - 1)^(1/2)/2)/2 + log(x + 5^(1/2))/4 - (7*atanh((4*(3^(1/2) - (x^2 - 1)^(1/2)))/((x^2 - 4)^(1/2)*((3^(1/2) - (x^2 - 1)^(1/2))^2/(x^2 - 4) + 1))))/4 + (5*atanh((12150*(3^(1/2) - (x^2 - 1)^(1/2)))/((x^2 - 4)^(1/2)*((6075*(3^(1/2) - (x^2 - 1)^(1/2))^2/(2*(x^2 - 4)) + 6075/2)))))/4 - atanh((x^2 - 4)^(1/2))/2
```

---

3.280.  $\int \frac{x(-\sqrt{-4+x^2} + x^2\sqrt{-4+x^2} - 4\sqrt{-1+x^2} + x^2\sqrt{-1+x^2})}{(4-5x^2+x^4)(1+\sqrt{-4+x^2}+\sqrt{-1+x^2})} dx$

**3.281**  $\int \left( \sqrt{9 - 4\sqrt{2}x} - \sqrt{2}\sqrt{1 + 4x + 2x^2 + x^4} \right) dx$

3.281.1 Optimal result . . . . . 1478  
 3.281.2 Mathematica [C] (warning: unable to verify) . . . . . 1479  
 3.281.3 Rubi [F] . . . . . 1479  
 3.281.4 Maple [A] (warning: unable to verify) . . . . . 1480  
 3.281.5 Fricas [F] . . . . . 1481  
 3.281.6 Sympy [F] . . . . . 1482  
 3.281.7 Maxima [F] . . . . . 1482  
 3.281.8 Giac [F] . . . . . 1482  
 3.281.9 Mupad [F(-1)] . . . . . 1483

**3.281.1 Optimal result**

Integrand size = 40, antiderivative size = 4030

$$\int \left( \sqrt{9 - 4\sqrt{2}x} - \sqrt{2}\sqrt{1 + 4x + 2x^2 + x^4} \right) dx = \text{Too large to display}$$

output `1/2*x^2*(-1+2*2^(1/2))-2^(1/2)*(-1/3*(x^4+2*x^2+4*x+1)^(1/2)+1/3*(1+x)*(x^4+2*x^2+4*x+1)^(1/2)+4*I*(-13+3*33^(1/2))^(1/3)*(x^4+2*x^2+4*x+1)^(1/2)/(4*2^(2/3))*(-I+3^(1/2))-2*I*(-13+3*33^(1/2))^(1/3)+6*I*x*(-13+3*33^(1/2))^(1/3)+2^(1/3)*(3^(1/2)+I)*(-13+3*33^(1/2))^(2/3))-8*2^(2/3)*EllipticE((26-6*33^(1/2)+6*x*(-13+3*33^(1/2))+(-13-13*I*3^(1/2)+9*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)+4*I*(3^(1/2)+I)*(-26+6*33^(1/2))^(2/3))^(1/2)/((39+13*I*3^(1/2)-9*I*11^(1/2)-9*33^(1/2)+4*(3-I*3^(1/2))*(-26+6*33^(1/2))^(1/3))/(39-13*I*3^(1/2)+9*I*11^(1/2)-9*33^(1/2)+4*(3+I*3^(1/2))*(-26+6*33^(1/2))^(1/3)))^(1/2)/(26-6*33^(1/2)+6*x*(-13+3*33^(1/2))+(-13+13*I*3^(1/2)-9*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-4-4*I*3^(1/2))*(-26+6*33^(1/2))^(2/3))^(1/2),((84+28*I*3^(1/2)-12*I*11^(1/2)-12*33^(1/2)+(3-I*3^(1/2)-3*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3))/(84-28*I*3^(1/2)+12*I*11^(1/2)-12*33^(1/2)+(3+I*3^(1/2)+3*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)))^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)*3^(1/2)/(-13+3*33^(1/2)+4*(-26+6*33^(1/2))^(1/3))^(1/2)*(I*(-19899+x*(59697-10335*33^(1/2))+3445*33^(1/2))+(-26+6*33^(1/2))^(2/3))*(-2574+466*33^(1/2))+(-26+6*33^(1/2))^(1/3)*(-19899+3445*33^(1/2)))/(-39-13*I*3^(1/2)+9*I*11^(1/2)+9*33^(1/2)+4*I*(3+I*3^(1/2))*(-26+6*33^(1/2))^(1/3))/(26-6*33^(1/2)+6*x*(-13+3*33^(1/2))+(-13+13*I*3^(1/2)-9*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-4-4*I*3^(1/2))*(-26+6*33^(1/2))^(2/3))^(1/2)/(4*2^(2/3)-(-13+3*33^(1/2))^(1/3)+3*x*(-13+3*3...`

3.281.  $\int \left( \sqrt{9 - 4\sqrt{2}x} - \sqrt{2}\sqrt{1 + 4x + 2x^2 + x^4} \right) dx$

**3.281.2 Mathematica [C] (warning: unable to verify)**

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 13.55 (sec) , antiderivative size = 3168, normalized size of antiderivative = 0.79

$$\int \left( \sqrt{9 - 4\sqrt{2}x} - \sqrt{2}\sqrt{1 + 4x + 2x^2 + x^4} \right) dx = \text{Result too large to show}$$

input `Integrate[Sqrt[9 - 4*Sqrt[2]]*x - Sqrt[2]*Sqrt[1 + 4*x + 2*x^2 + x^4],x]`

output `(Sqrt[9 - 4*Sqrt[2]]*x^2)/2 - (Sqrt[2]*x*Sqrt[1 + 4*x + 2*x^2 + x^4])/3 - (2*Sqrt[2]*((6*(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])^2*(-(EllipticF[ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0]) - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])])]/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]), ((Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))*Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0]) + EllipticPi[(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])/(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0]) - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])])]/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]), ((Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0]))*Sqrt[(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0]))]*(-1 - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])*Sqrt[(x - Root[1 + ...`

**3.281.3 Rubi [F]**

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left( \sqrt{9 - 4\sqrt{2}x} - \sqrt{2}\sqrt{x^4 + 2x^2 + 4x + 1} \right) dx$$

↓ 2009

---

3.281.  $\int \left( \sqrt{9 - 4\sqrt{2}x} - \sqrt{2}\sqrt{1 + 4x + 2x^2 + x^4} \right) dx$



$$\frac{1}{2}\sqrt{9-4\sqrt{2}x^2}-\sqrt{2}\int\sqrt{x^4+2x^2+4x+1}dx$$

input `Int[Sqrt[9 - 4*Sqrt[2]]*x - Sqrt[2]*Sqrt[1 + 4*x + 2*x^2 + x^4],x]`

output `$Aborted`

### 3.281.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

### 3.281.4 Maple [A] (warning: unable to verify)

Time = 2.31 (sec) , antiderivative size = 4640, normalized size of antiderivative = 1.15

| method   | result                          | size |
|----------|---------------------------------|------|
| default  | Expression too large to display | 4640 |
| parts    | Expression too large to display | 4640 |
| elliptic | Expression too large to display | 4646 |

input `int(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x,method=_RETURNVERBOSE)`



**3.281.6 Sympy [F]**

$$\int \left( \sqrt{9 - 4\sqrt{2}x} - \sqrt{2}\sqrt{1 + 4x + 2x^2 + x^4} \right) dx$$

$$= \int \left( x(-1 + 2\sqrt{2}) - \sqrt{2}\sqrt{x^4 + 2x^2 + 4x + 1} \right) dx$$

input `integrate(-2**(1/2)*(x**4+2*x**2+4*x+1)**(1/2)+x*(-1+2*2**(1/2)),x)`

output `Integral(x*(-1 + 2*sqrt(2)) - sqrt(2)*sqrt(x**4 + 2*x**2 + 4*x + 1), x)`

**3.281.7 Maxima [F]**

$$\int \left( \sqrt{9 - 4\sqrt{2}x} - \sqrt{2}\sqrt{1 + 4x + 2x^2 + x^4} \right) dx = \int x(2\sqrt{2} - 1) - \sqrt{2}\sqrt{x^4 + 2x^2 + 4x + 1} dx$$

input `integrate(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x, algorithm="maxima")`

output `1/2*x^2*(2*sqrt(2) - 1) - sqrt(2)*integrate(sqrt(x^3 - x^2 + 3*x + 1)*sqrt(x + 1), x)`

**3.281.8 Giac [F]**

$$\int \left( \sqrt{9 - 4\sqrt{2}x} - \sqrt{2}\sqrt{1 + 4x + 2x^2 + x^4} \right) dx = \int x(2\sqrt{2} - 1) - \sqrt{2}\sqrt{x^4 + 2x^2 + 4x + 1} dx$$

input `integrate(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x, algorithm="giac")`

output `integrate(x*(2*sqrt(2) - 1) - sqrt(2)*sqrt(x^4 + 2*x^2 + 4*x + 1), x)`

---

3.281.  $\int \left( \sqrt{9 - 4\sqrt{2}x} - \sqrt{2}\sqrt{1 + 4x + 2x^2 + x^4} \right) dx$

**3.281.9 Mupad [F(-1)]**

Timed out.

$$\int \left( \sqrt{9 - 4\sqrt{2}x} - \sqrt{2}\sqrt{1 + 4x + 2x^2 + x^4} \right) dx$$

$$= \int x \left( 2\sqrt{2} - 1 \right) - \sqrt{2}\sqrt{x^4 + 2x^2 + 4x + 1} dx$$

input `int(x*(2*2^(1/2) - 1) - 2^(1/2)*(4*x + 2*x^2 + x^4 + 1)^(1/2), x)`output `int(x*(2*2^(1/2) - 1) - 2^(1/2)*(4*x + 2*x^2 + x^4 + 1)^(1/2), x)`

**3.282**  $\int \frac{e^{-\frac{x}{y}} \left( \pi^2 (-3mc^8 + 4mc^9 + 24mc^6x - 48mc^7x - 144mc^5x^2 - 24mc^2x^3 + 176mc^3x^3 + 3x^4 + 12mcx^4) + 12mc^3\pi^2(3mc - 12mc^2 - 8x)x^2 \log\left(\frac{x}{mc^2}\right) \right)}{384x^2}$

3.282.1 Optimal result . . . . . 1484  
 3.282.2 Mathematica [A] (verified) . . . . . 1485  
 3.282.3 Rubi [A] (verified) . . . . . 1485  
 3.282.4 Maple [C] (warning: unable to verify) . . . . . 1487  
 3.282.5 Fricas [A] (verification not implemented) . . . . . 1488  
 3.282.6 Sympy [A] (verification not implemented) . . . . . 1488  
 3.282.7 Maxima [F] . . . . . 1489  
 3.282.8 Giac [A] (verification not implemented) . . . . . 1490  
 3.282.9 Mupad [B] (verification not implemented) . . . . . 1490

**3.282.1 Optimal result**

Integrand size = 107, antiderivative size = 330

$$\int \frac{e^{-\frac{x}{y}} \left( \pi^2 (-3mc^8 + 4mc^9 + 24mc^6x - 48mc^7x - 144mc^5x^2 - 24mc^2x^3 + 176mc^3x^3 + 3x^4 + 12mcx^4) + 12mc^3\pi^2(3mc - 12mc^2 - 8x)x^2 \log\left(\frac{x}{mc^2}\right) \right)}{384x^2}$$

$$= \frac{e^{-\frac{x}{y}}(3 - 4mc)mc^8\pi^2}{384x} + \frac{3}{8}e^{-\frac{x}{y}}mc^5\pi^2y + \frac{1}{48}e^{-\frac{x}{y}}(3 - 22mc)mc^2\pi^2xy$$

$$- \frac{1}{128}e^{-\frac{x}{y}}(1 + 4mc)\pi^2x^2y + \frac{1}{48}e^{-\frac{x}{y}}(3 - 22mc)mc^2\pi^2y^2$$

$$+ \frac{1}{4}e^{-\frac{x}{y}}mc^3\pi^2y^2 - \frac{1}{64}e^{-\frac{x}{y}}(1 + 4mc)\pi^2xy^2 - \frac{1}{64}e^{-\frac{x}{y}}(1 + 4mc)\pi^2y^3$$

$$+ \frac{1}{16}(1 - 2mc)mc^6\pi^2 \text{ExpIntegralEi}\left(-\frac{x}{y}\right) + \frac{(3 - 4mc)mc^8\pi^2 \text{ExpIntegralEi}\left(-\frac{x}{y}\right)}{384y}$$

$$+ \frac{1}{32}mc^3\pi^2(3mc - 12mc^2 - 8y)y \text{ExpIntegralEi}\left(-\frac{x}{y}\right)$$

$$- \frac{1}{32}e^{-\frac{x}{y}}mc^3\pi^2(3(1 - 4mc)mc - 8x)y \log\left(\frac{x}{mc^2}\right) + \frac{1}{4}e^{-\frac{x}{y}}mc^3\pi^2y^2 \log\left(\frac{x}{mc^2}\right)$$

```
output 1/384*(3-4*mc)*mc^8*Pi^2/exp(x/y)/x+3/8*mc^5*Pi^2*y/exp(x/y)+1/48*(3-22*mc)
)*mc^2*Pi^2*x*y/exp(x/y)-1/128*(1+4*mc)*Pi^2*x^2*y/exp(x/y)+1/48*(3-22*mc)
)*mc^2*Pi^2*y^2/exp(x/y)+1/4*mc^3*Pi^2*y^2/exp(x/y)-1/64*(1+4*mc)*Pi^2*x*y^
2/exp(x/y)-1/64*(1+4*mc)*Pi^2*y^3/exp(x/y)+1/16*(1-2*mc)*mc^6*Pi^2*Ei(-x/y)
)+1/384*(3-4*mc)*mc^8*Pi^2*Ei(-x/y)/y+1/32*mc^3*Pi^2*(-12*mc^2+3*mc-8*y)*y
)*Ei(-x/y)-1/32*mc^3*Pi^2*(3*(1-4*mc)*mc-8*x)*y*ln(x/mc^2)/exp(x/y)+1/4*mc^
3*Pi^2*y^2*ln(x/mc^2)/exp(x/y)
```

3.282.  
 $\int \frac{e^{-\frac{x}{y}} \left( \pi^2 (-3mc^8 + 4mc^9 + 24mc^6x - 48mc^7x - 144mc^5x^2 - 24mc^2x^3 + 176mc^3x^3 + 3x^4 + 12mcx^4) + 12mc^3\pi^2(3mc - 12mc^2 - 8x)x^2 \log\left(\frac{x}{mc^2}\right) \right)}{384x^2}$

### 3.282.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\frac{x}{y}} (\pi^2 (-3mc^8 + 4mc^9 + 24mc^6x - 48mc^7x - 144mc^5x^2 - 24mc^2x^3 + 176mc^3x^3 + 3x^4 + 12mcx^4) + 12mc^3\pi^2(3mc - 12mc^2 - 8x)x^2 \text{Log}[x/mc^2])}{384x^2} dx$$

$$= \frac{1}{384} \pi^2 \left( -\frac{mc^3(-3mc^5 + 4mc^6 - 24mc^3y + 48mc^4y - 36mcy^2 + 144mc^2y^2 + 96y^3) \text{ExpIntegralEi}\left(-\frac{x}{y}\right)}{y} \right.$$

$$\left. + \frac{e^{-\frac{x}{y}} (3mc^8 - 4mc^9 + 144mc^5xy + 24mc^2xy(x+y) - 16mc^3xy(11x+5y) - 3xy(x^2+2xy+2y^2) - 12mc^3x^2 \text{Log}[x/mc^2])}{x} \right)$$

input `Integrate[(Pi^2*(-3*mc^8 + 4*mc^9 + 24*mc^6*x - 48*mc^7*x - 144*mc^5*x^2 - 24*mc^2*x^3 + 176*mc^3*x^3 + 3*x^4 + 12*mc*x^4) + 12*mc^3*Pi^2*(3*mc - 12*mc^2 - 8*x)*x^2*Log[x/mc^2])/(384*E^(x/y)*x^2), x]`

output `(Pi^2*(-((mc^3*(-3*mc^5 + 4*mc^6 - 24*mc^3*y + 48*mc^4*y - 36*mc*y^2 + 144*mc^2*y^2 + 96*y^3)*ExpIntegralEi[-(x/y)])/y) + (3*mc^8 - 4*mc^9 + 144*mc^5*x*y + 24*mc^2*x*y*(x + y) - 16*mc^3*x*y*(11*x + 5*y) - 3*x*y*(x^2 + 2*x*y + 2*y^2) - 12*mc*x*y*(x^2 + 2*x*y + 2*y^2) + 12*mc^3*x*y*(-3*mc + 12*mc^2 + 8*(x + y))*Log[x/mc^2])/(E^(x/y)*x))/384`

### 3.282.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$ , Rules used = {27, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{x}{y}} (12\pi^2 mc^3 x^2 (-12mc^2 + 3mc - 8x) \log\left(\frac{x}{mc^2}\right) + \pi^2 (4mc^9 - 3mc^8 - 48mc^7x + 24mc^6x - 144mc^5x^2 + 176mc^3x^3 + 3x^4 + 12mcx^4))}{384x^2} dx$$

↓ 27

$$\frac{1}{384} \int -\frac{e^{-\frac{x}{y}} (\pi^2 ((3 - 4mc)mc^8 + 48xmc^7 - 24xmc^6 + 144x^2mc^5 - 176x^3mc^3 + 24x^3mc^2 - 12x^4mc - 3x^4) - 12mc^3\pi^2(3mc - 12mc^2 - 8x)x^2 \log\left(\frac{x}{mc^2}\right))}{x^2} dx$$

3.282.

$$\int \frac{e^{-\frac{x}{y}} (\pi^2 (-3mc^8 + 4mc^9 + 24mc^6x - 48mc^7x - 144mc^5x^2 - 24mc^2x^3 + 176mc^3x^3 + 3x^4 + 12mcx^4) + 12mc^3\pi^2(3mc - 12mc^2 - 8x)x^2 \log\left(\frac{x}{mc^2}\right))}{384x^2} dx$$

↓ 25

$$-\frac{1}{384} \int \frac{e^{-\frac{x}{y}} (\pi^2 ((3 - 4mc)mc^8 + 48xmc^7 - 24xmc^6 + 144x^2mc^5 - 176x^3mc^3 + 24x^3mc^2 - 12x^4mc - 3x^4) - 12x^2)}{x^2}$$

↓ 7293

$$-\frac{1}{384} \int \left( 12e^{-\frac{x}{y}} \pi^2 (12mc^2 - 3mc + 8x) \log\left(\frac{x}{mc^2}\right) mc^3 + \frac{e^{-\frac{x}{y}} \pi^2 (mc^2 - x) ((3 - 4mc)mc^6 - (21 - 44mc)xmc^4 - 12x^2)}{x^2} \right)$$

↓ 2009

$$\frac{1}{384} \left( \frac{\pi^2 (3 - 4mc) mc^8 \text{ExpIntegralEi}\left(-\frac{x}{y}\right)}{y} + 24\pi^2 (1 - 2mc) mc^6 \text{ExpIntegralEi}\left(-\frac{x}{y}\right) + 12\pi^2 mc^3 y (-12mc^2 + 12x) \log\left(\frac{x}{mc^2}\right) \right)$$

```
input Int[(Pi^2*(-3*mc^8 + 4*mc^9 + 24*mc^6*x - 48*mc^7*x - 144*mc^5*x^2 - 24*mc^2*x^3 + 176*mc^3*x^3 + 3*x^4 + 12*mc*x^4) + 12*mc^3*Pi^2*(3*mc - 12*mc^2 - 8*x)*x^2*Log[x/mc^2])/(384*E^(x/y)*x^2),x]
```

```
output (((3 - 4*mc)*mc^8*Pi^2)/(E^(x/y)*x) + (144*mc^5*Pi^2*y)/E^(x/y) + (8*(3 - 22*mc)*mc^2*Pi^2*x*y)/E^(x/y) - (3*(1 + 4*mc)*Pi^2*x^2*y)/E^(x/y) + (8*(3 - 22*mc)*mc^2*Pi^2*y^2)/E^(x/y) + (96*mc^3*Pi^2*y^2)/E^(x/y) - (6*(1 + 4*mc)*Pi^2*x*y^2)/E^(x/y) - (6*(1 + 4*mc)*Pi^2*y^3)/E^(x/y) + 24*(1 - 2*mc)*mc^6*Pi^2*ExpIntegralEi[-(x/y)] + ((3 - 4*mc)*mc^8*Pi^2*ExpIntegralEi[-(x/y)])/y + 12*mc^3*Pi^2*(3*mc - 12*mc^2 - 8*y)*y*ExpIntegralEi[-(x/y)] - (12*mc^3*Pi^2*(3*(1 - 4*mc)*mc - 8*x)*y*Log[x/mc^2])/E^(x/y) + (96*mc^3*Pi^2*y^2*Log[x/mc^2])/E^(x/y))/384
```

3.282.3.1 Defintions of rubi rules used

```
rule 25 Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

```
rule 27 Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

---

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$$\int \frac{e^{-\frac{x}{y}} (\pi^2 (-3mc^8 + 4mc^9 + 24mc^6x - 48mc^7x - 144mc^5x^2 - 24mc^2x^3 + 176mc^3x^3 + 3x^4 + 12mcx^4) + 12mc^3\pi^2(3mc - 12mc^2 - 8x)x^2 \log(\frac{x}{mc^2}))}{384x^2}$$

```
rule 7293 Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### 3.282.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.70 (sec) , antiderivative size = 1356, normalized size of antiderivative = 4.11

| method | result                          | size |
|--------|---------------------------------|------|
| risch  | Expression too large to display | 1356 |

```
input int(1/384*(Pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*mc^3*x
^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*Pi^2*(-12*mc^2+3*mc-8*x)*x^2*ln(x/
mc^2))/exp(x/y)/x^2,x,method=_RETURNVERBOSE)
```

```
output 1/8*I*y*Pi^3*mc^3*csgn(I/mc^2*x)^2*csgn(I*x)*exp(-x/y)*x+1/8*I*y*Pi^3*mc^3
*csgn(I/mc^2)*csgn(I/mc^2*x)^2*exp(-x/y)*x-1/4*I*y*Pi^3*mc^3*csgn(I*mc)*csgn(I*mc^2)^2*exp(-x/y)*x+1/8*I*y*Pi^3*mc^3*csgn(I*mc)^2*csgn(I*mc^2)*exp(-x/y)*x+3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I/mc^2)*csgn(I/mc^2*x)*csgn(I*x)-3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I/mc^2)*csgn(I/mc^2*x)*csgn(I*x)-1/8*I*y^2*Pi^3*mc^3*csgn(I/mc^2)*csgn(I/mc^2*x)*csgn(I*x)*exp(-x/y)-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I*mc)^2*csgn(I*mc^2)+3/32*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I*mc)*csgn(I*mc^2)^2-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I/mc^2*x)^2*csgn(I*x)+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I/mc^2)*csgn(I/mc^2*x)^2+1/8*I*y*Pi^3*mc^3*csgn(I*mc^2)^3*exp(-x/y)*x-1/8*I*y*Pi^3*mc^3*csgn(I/mc^2*x)^3*exp(-x/y)*x-1/4*I*y^2*Pi^3*mc^3*csgn(I*mc)*csgn(I*mc^2)^2*exp(-x/y)+1/8*I*y^2*Pi^3*mc^3*csgn(I*mc)^2*csgn(I*mc^2)*exp(-x/y)+1/8*I*y^2*Pi^3*mc^3*csgn(I/mc^2*x)^2*csgn(I*x)*exp(-x/y)+1/8*I*y^2*Pi^3*mc^3*csgn(I/mc^2)*csgn(I/mc^2*x)^2*exp(-x/y)+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I*mc)^2*csgn(I*mc^2)-3/8*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I*mc)*csgn(I*mc^2)^2+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I/mc^2*x)^2*csgn(I*x)-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I/mc^2)*csgn(I/mc^2*x)^2-1/8*I*y*Pi^3*mc^3*csgn(I/mc^2)*csgn(I/mc^2*x)*csgn(I*x)*exp(-x/y)*x-1/128/y*Pi^2*mc^8*Ei(1,x/y)+1/96/y*Pi^2*mc^9*Ei(1,x/y)-1/128*y*Pi^2*exp(-x/y)*x^2-1/64*y^2*Pi^2*x*exp(-x/y)-1/16*y^3*Pi^2*mc*exp(-x/y)+1/16*y^2*Pi^2*mc^2*exp(-x/y)+3/8*y*Pi^2*exp(-x/y)*mc^5-1/96*Pi^2*mc^9/x*e...
```



**3.282.5 Fracas [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.82

$$\int \frac{e^{-\frac{x}{y}} (\pi^2 (-3mc^8 + 4mc^9 + 24mc^6x - 48mc^7x - 144mc^5x^2 - 24mc^2x^3 + 176mc^3x^3 + 3x^4 + 12mcx^4) + 12(8\pi^2mc^3xy^3 + (8\pi^2mc^3x^2 + 3\pi^2(4mc^5 - mc^4)x)y^2)e^{-\frac{x}{y}} \log\left(\frac{x}{mc^2}\right) - (96\pi^2mc^3xy^3 + 36\pi^2(4mc^5 - mc^4)x^2))}{384x^2} dx$$

```
input integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*mc^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2*log(x/mc^2))/exp(x/y)/x^2,x, algorithm="fricas")
```

```
output 1/384*(12*(8*pi^2*mc^3*x*y^3 + (8*pi^2*mc^3*x^2 + 3*pi^2*(4*mc^5 - mc^4)*x)*y^2)*e^(-x/y)*log(x/mc^2) - (96*pi^2*mc^3*x*y^3 + 36*pi^2*(4*mc^5 - mc^4)*x*y^2 + 24*pi^2*(2*mc^7 - mc^6)*x*y + pi^2*(4*mc^9 - 3*mc^8)*x)*Ei(-x/y) - (6*pi^2*(4*mc + 1)*x*y^4 + pi^2*(4*mc^9 - 3*mc^8)*y + 2*(3*pi^2*(4*mc + 1)*x^2 + 4*pi^2*(10*mc^3 - 3*mc^2)*x)*y^3 - (144*pi^2*mc^5*x - 3*pi^2*(4*mc + 1)*x^3 - 8*pi^2*(22*mc^3 - 3*mc^2)*x^2)*y^2)*e^(-x/y))/(x*y)
```

**3.282.6 Sympy [A] (verification not implemented)**

Time = 6.17 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\frac{x}{y}} (\pi^2 (-3mc^8 + 4mc^9 + 24mc^6x - 48mc^7x - 144mc^5x^2 - 24mc^2x^3 + 176mc^3x^3 + 3x^4 + 12mcx^4) + 12(8\pi^2mc^3xy^3 + (8\pi^2mc^3x^2 + 3\pi^2(4mc^5 - mc^4)x)y^2)e^{-\frac{x}{y}} \log\left(\frac{x}{mc^2}\right) - (96\pi^2mc^3xy^3 + 36\pi^2(4mc^5 - mc^4)x^2))}{384x^2} dx$$

$$= -\frac{\pi^2 mc^9 E_2\left(\frac{x}{y}\right)}{96x} + \frac{\pi^2 mc^8 E_2\left(\frac{x}{y}\right)}{128x} - \frac{\pi^2 mc^7 Ei\left(-\frac{x}{y}\right)}{8} + \frac{\pi^2 mc^6 Ei\left(-\frac{x}{y}\right)}{16}$$

$$+ \frac{3\pi^2 mc^5 ye^{-\frac{x}{y}}}{8} - \frac{3\pi^2 mc^5 \left(y Ei\left(-\frac{x}{y}\right) - ye^{-\frac{x}{y}} \log\left(\frac{x}{mc^2}\right)\right)}{8}$$

$$+ \frac{3\pi^2 mc^4 \left(y Ei\left(-\frac{x}{y}\right) - ye^{-\frac{x}{y}} \log\left(\frac{x}{mc^2}\right)\right)}{32} + \frac{11\pi^2 mc^3 \left(-xye^{-\frac{x}{y}} - y^2 e^{-\frac{x}{y}}\right)}{24}$$

$$- \frac{\pi^2 mc^3 \left(y^2 Ei\left(-\frac{x}{y}\right) - y^2 e^{-\frac{x}{y}} + \left(-xye^{-\frac{x}{y}} - y^2 e^{-\frac{x}{y}}\right) \log\left(\frac{x}{mc^2}\right)\right)}{4}$$

$$- \frac{\pi^2 mc^2 \left(-xye^{-\frac{x}{y}} - y^2 e^{-\frac{x}{y}}\right)}{16} + \frac{\pi^2 mc \left(-x^2 ye^{-\frac{x}{y}} - 2xy^2 e^{-\frac{x}{y}} - 2y^3 e^{-\frac{x}{y}}\right)}{32}$$

$$+ \frac{\pi^2 \left(-x^2 ye^{-\frac{x}{y}} - 2xy^2 e^{-\frac{x}{y}} - 2y^3 e^{-\frac{x}{y}}\right)}{128}$$

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$$\int \frac{e^{-\frac{x}{y}} (\pi^2 (-3mc^8 + 4mc^9 + 24mc^6x - 48mc^7x - 144mc^5x^2 - 24mc^2x^3 + 176mc^3x^3 + 3x^4 + 12mcx^4) + 12mc^3\pi^2(3mc - 12mc^2 - 8x)x^2 \log\left(\frac{x}{mc^2}\right) - (96\pi^2mc^3xy^3 + 36\pi^2(4mc^5 - mc^4)x^2))}{384x^2} dx$$

input `integrate(1/384*(pi**2*(4*mc**9-3*mc**8-48*mc**7*x+24*mc**6*x-144*mc**5*x**2+176*mc**3*x**3-24*mc**2*x**3+12*mc*x**4+3*x**4)+12*mc**3*pi**2*(-12*mc**2+3*mc-8*x)*x**2*ln(x/mc**2))/exp(x/y)/x**2,x)`

output `-pi**2*mc**9*expint(2, x/y)/(96*x) + pi**2*mc**8*expint(2, x/y)/(128*x) - pi**2*mc**7*Ei(-x/y)/8 + pi**2*mc**6*Ei(-x/y)/16 + 3*pi**2*mc**5*y*exp(-x/y)/8 - 3*pi**2*mc**5*(y*Ei(-x/y) - y*exp(-x/y)*log(x/mc**2))/8 + 3*pi**2*mc**4*(y*Ei(-x/y) - y*exp(-x/y)*log(x/mc**2))/32 + 11*pi**2*mc**3*(-x*y*exp(-x/y) - y**2*exp(-x/y))/24 - pi**2*mc**3*(y**2*Ei(-x/y) - y**2*exp(-x/y) + (-x*y*exp(-x/y) - y**2*exp(-x/y))*log(x/mc**2))/4 - pi**2*mc**2*(-x*y*exp(-x/y) - y**2*exp(-x/y))/16 + pi**2*mc*(-x**2*y*exp(-x/y) - 2*x*y**2*exp(-x/y) - 2*y**3*exp(-x/y))/32 + pi**2*(-x**2*y*exp(-x/y) - 2*x*y**2*exp(-x/y) - 2*y**3*exp(-x/y))/128`

### 3.282.7 Maxima [F]

$$\int \frac{e^{-\frac{x}{y}} \left( \pi^2 (-3mc^8 + 4mc^9 + 24mc^6x - 48mc^7x - 144mc^5x^2 - 24mc^2x^3 + 176mc^3x^3 + 3x^4 + 12mcx^4) + 12mc^3\pi^2 (3mc - 12mc^2 - 8x)x^2 \log\left(\frac{x}{mc^2}\right) \right)}{384x^2} dx$$

$$= \int -\frac{(12\pi^2(12mc^2 - 3mc + 8x)mc^3x^2 \log\left(\frac{x}{mc^2}\right) - \pi^2(4mc^9 - 3mc^8 - 48mc^7x + 24mc^6x - 144mc^5x^2 - 24mc^2x^3 + 176mc^3x^3 + 3x^4 + 12mcx^4))}{384x^2} dx$$

input `integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*mc^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2*log(x/mc^2))/exp(x/y)/x^2,x, algorithm="maxima")`

output `-1/96*pi^2*mc^9*gamma(-1, x/y)/y - 1/8*pi^2*mc^7*Ei(-x/y) + 1/128*pi^2*mc^8*gamma(-1, x/y)/y + 3/8*pi^2*mc^5*y*e^(-x/y)*log(x/mc^2) + 1/16*pi^2*mc^6*Ei(-x/y) - 3/8*pi^2*mc^5*y*Ei(-x/y) + 3/8*pi^2*mc^5*y*e^(-x/y) - 3/32*pi^2*mc^4*y*e^(-x/y)*log(x/mc^2) + 3/32*pi^2*mc^4*y*Ei(-x/y) - 11/24*pi^2*(x*y + y^2)*mc^3*e^(-x/y) + 1/4*pi^2*((x*y + y^2)*e^(-x/y)*log(x) + integrate((2*x^2*log(mc) - x*y - y^2)*e^(-x/y)/x, x))*mc^3 + 1/16*pi^2*(x*y + y^2)*mc^2*e^(-x/y) - 1/32*pi^2*(x^2*y + 2*x*y^2 + 2*y^3)*mc*e^(-x/y) - 1/128*pi^2*(x^2*y + 2*x*y^2 + 2*y^3)*e^(-x/y)`

**3.282.8 Giac [A] (verification not implemented)**

Time = 0.29 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.43

$$\int \frac{e^{-\frac{x}{y}} (\pi^2 (-3mc^8 + 4mc^9 + 24mc^6x - 48mc^7x - 144mc^5x^2 - 24mc^2x^3 + 176mc^3x^3 + 3x^4 + 12mcx^4) + 12mc^3\pi^2(-12mc^2+3mc-8x)x^2 \log(x/mc^2))}{384x^2} dx$$

$$= \frac{4\pi^2 mc^9 x \operatorname{Ei}\left(-\frac{x}{y}\right) + 4\pi^2 mc^9 y e^{\left(-\frac{x}{y}\right)} - 3\pi^2 mc^8 x \operatorname{Ei}\left(-\frac{x}{y}\right) + 48\pi^2 mc^7 xy \operatorname{Ei}\left(-\frac{x}{y}\right) - 3\pi^2 mc^8 y e^{\left(-\frac{x}{y}\right)} - 144\pi^2 mc^5 x^2 \operatorname{Ei}\left(-\frac{x}{y}\right) + 144\pi^2 mc^5 xy^2 \operatorname{Ei}\left(-\frac{x}{y}\right) - 144\pi^2 mc^5 x^2 y^2 \operatorname{Ei}\left(-\frac{x}{y}\right) + 36\pi^2 mc^4 x^2 y^2 \operatorname{Ei}\left(-\frac{x}{y}\right) \log(x/mc^2) - 96\pi^2 mc^3 x^2 y^2 \operatorname{Ei}\left(-\frac{x}{y}\right) \log(x/mc^2) - 96\pi^2 mc^3 x^2 y^3 \operatorname{Ei}\left(-\frac{x}{y}\right) \log(x/mc^2) - 36\pi^2 mc^4 x^2 y^2 \operatorname{Ei}\left(-\frac{x}{y}\right) + 96\pi^2 mc^3 x^2 y^3 \operatorname{Ei}\left(-\frac{x}{y}\right) + 176\pi^2 mc^3 x^2 y^2 \operatorname{Ei}\left(-\frac{x}{y}\right) + 80\pi^2 mc^3 x^2 y^3 \operatorname{Ei}\left(-\frac{x}{y}\right) - 24\pi^2 mc^2 x^2 y^2 \operatorname{Ei}\left(-\frac{x}{y}\right) + 12\pi^2 mc^2 x^3 y^2 \operatorname{Ei}\left(-\frac{x}{y}\right) - 24\pi^2 mc^2 x^2 y^3 \operatorname{Ei}\left(-\frac{x}{y}\right) + 24\pi^2 mc^2 x^2 y^3 \operatorname{Ei}\left(-\frac{x}{y}\right) + 24\pi^2 mc^2 x^2 y^4 \operatorname{Ei}\left(-\frac{x}{y}\right) + 3\pi^2 x^2 y^3 \operatorname{Ei}\left(-\frac{x}{y}\right) + 6\pi^2 x^2 y^3 \operatorname{Ei}\left(-\frac{x}{y}\right) + 6\pi^2 x^2 y^4 \operatorname{Ei}\left(-\frac{x}{y}\right)}{x^2 y}$$

```
input integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*mc^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2*log(x/mc^2))/exp(x/y)/x^2,x, algorithm="giac")
```

```
output -1/384*(4*pi^2*mc^9*x*Ei(-x/y) + 4*pi^2*mc^9*y*e^(-x/y) - 3*pi^2*mc^8*x*Ei(-x/y) + 48*pi^2*mc^7*x*y*Ei(-x/y) - 3*pi^2*mc^8*y*e^(-x/y) - 144*pi^2*mc^5*x*y^2*e^(-x/y)*log(x/mc^2) - 24*pi^2*mc^6*x*y*Ei(-x/y) + 144*pi^2*mc^5*x*y^2*Ei(-x/y) - 144*pi^2*mc^5*x^2*y^2*e^(-x/y) + 36*pi^2*mc^4*x*y^2*e^(-x/y)*log(x/mc^2) - 96*pi^2*mc^3*x^2*y^2*e^(-x/y)*log(x/mc^2) - 96*pi^2*mc^3*x^2*y^3*e^(-x/y)*log(x/mc^2) - 36*pi^2*mc^4*x*y^2*Ei(-x/y) + 96*pi^2*mc^3*x^2*y^3*Ei(-x/y) + 176*pi^2*mc^3*x^2*y^2*e^(-x/y) + 80*pi^2*mc^3*x^2*y^3*e^(-x/y) - 24*pi^2*mc^2*x^2*y^2*e^(-x/y) + 12*pi^2*mc^2*x^3*y^2*e^(-x/y) - 24*pi^2*mc^2*x^2*y^3*e^(-x/y) + 24*pi^2*mc^2*x^2*y^3*e^(-x/y) + 24*pi^2*mc^2*x^2*y^4*e^(-x/y) + 3*pi^2*x^2*y^3*e^(-x/y) + 6*pi^2*x^2*y^3*e^(-x/y) + 6*pi^2*x^2*y^4*e^(-x/y))/(x*y)
```

**3.282.9 Mupad [B] (verification not implemented)**

Time = 0.76 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.80

$$\int \frac{e^{-\frac{x}{y}} (\pi^2 (-3mc^8 + 4mc^9 + 24mc^6x - 48mc^7x - 144mc^5x^2 - 24mc^2x^3 + 176mc^3x^3 + 3x^4 + 12mcx^4) + 12mc^3\pi^2(8x - 3mc + 12mc^2)x^2 \log(x/mc^2))}{384x^2} dx$$

$$= \operatorname{Ei}\left(-\frac{x}{y}\right) \left( \frac{\Pi^2 mc^8}{128} - \frac{\Pi^2 mc^9}{96} + \frac{\Pi^2 mc^6}{16} - \frac{\Pi^2 mc^7}{8} + y \left( \frac{3\Pi^2 mc^4}{32} - \frac{3\Pi^2 mc^5}{8} \right) - \frac{\Pi^2 mc^3 y^2}{4} \right) + \frac{2\Pi^2 x^2 y e^{-\frac{x}{y}} (-72 mc^5 + 40 mc^3 y - 12 mc^2 y + 12 mc y^2 + 3 y^2) + 2\Pi^2 x^3 y e^{-\frac{x}{y}} (88 mc^3 - 12 mc^2 + 12 mc y)}{384x^2}$$

```
input int((exp(-x/y))*((Pi^2*(176*mc^3*x^3 - 24*mc^2*x^3 - 144*mc^5*x^2 + 12*mc*x^4 + 24*mc^6*x - 48*mc^7*x - 3*mc^8 + 4*mc^9 + 3*x^4))/384 - (Pi^2*mc^3*x^2*log(x/mc^2)*(8*x - 3*mc + 12*mc^2))/32))/x^2,x)
```

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$$\int \frac{e^{-\frac{x}{y}} (\pi^2 (-3mc^8 + 4mc^9 + 24mc^6x - 48mc^7x - 144mc^5x^2 - 24mc^2x^3 + 176mc^3x^3 + 3x^4 + 12mcx^4) + 12mc^3\pi^2(3mc - 12mc^2 - 8x)x^2 \log(x/mc^2))}{384x^2} dx$$

output

$$\frac{e^{-x/y} \left( \left( \frac{\pi^2 mc^8}{128} - \frac{\pi^2 mc^9}{96} \right) / y + \frac{\pi^2 mc^6}{16} - \frac{\pi^2 mc^7}{8} + y \left( \frac{3\pi^2 mc^4}{32} - \frac{3\pi^2 mc^5}{8} \right) - \frac{\pi^2 mc^3 y^2}{4} - (2\pi^2 x^2 y \exp(-x/y) (12mc^2 y^2 - 12mc^2 y + 40mc^3 y - 72mc^5 + 3y^2) + 2\pi^2 x^3 y \exp(-x/y) (3y + 12mc y - 12mc^2 + 88mc^3) + \pi^2 mc^8 x \exp(-x/y) (4mc - 3) + 3\pi^2 x^4 y \exp(-x/y) (4mc + 1) - 96\pi^2 mc^3 x^3 y \log(x/mc^2) \exp(-x/y) - 12\pi^2 mc^3 x^2 y \log(x/mc^2) \exp(-x/y) (8y - 3mc + 12mc^2)) \right)}{384x^2}$$

### 3.283 $\int \sec(x) \sin(2x) dx$

|                                                             |      |
|-------------------------------------------------------------|------|
| 3.283.1 Optimal result . . . . .                            | 1492 |
| 3.283.2 Mathematica [A] (verified) . . . . .                | 1492 |
| 3.283.3 Rubi [A] (verified) . . . . .                       | 1493 |
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| 3.283.7 Maxima [A] (verification not implemented) . . . . . | 1495 |
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#### 3.283.1 Optimal result

Integrand size = 7, antiderivative size = 4

$$\int \sec(x) \sin(2x) dx = -2 \cos(x)$$

output `-2*cos(x)`

#### 3.283.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sec(x) \sin(2x) dx = -2 \cos(x)$$

input `Integrate[Sec[x]*Sin[2*x],x]`

output `-2*Cos[x]`

**3.283.3 Rubi [A] (verified)**

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {3042, 4775, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c}
 \int \sin(2x) \sec(x) dx \\
 \downarrow \text{3042} \\
 \int \frac{\sin(2x)}{\cos(x)} dx \\
 \downarrow \text{4775} \\
 2 \int \sin(x) dx \\
 \downarrow \text{3042} \\
 2 \int \sin(x) dx \\
 \downarrow \text{3118} \\
 -2 \cos(x)
 \end{array}$$

input `Int[Sec[x]*Sin[2*x],x]`

output `-2*Cos[x]`

**3.283.3.1 Defintions of rubi rules used**

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 3118 `Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

```
rule 4775 Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p Int[(e*cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

### 3.283.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

| method             | result       | size |
|--------------------|--------------|------|
| derivativeldivides | $-2 \cos(x)$ | 5    |
| default            | $-2 \cos(x)$ | 5    |
| risch              | $-2 \cos(x)$ | 5    |

```
input int(sin(2*x)/cos(x),x,method=_RETURNVERBOSE)
```

```
output -2*cos(x)
```

### 3.283.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sec(x) \sin(2x) dx = -2 \cos(x)$$

```
input integrate(sin(2*x)/cos(x),x, algorithm="fricas")
```

```
output -2*cos(x)
```

**3.283.6 Sympy [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \sec(x) \sin(2x) dx = -2 \cos(x)$$

input `integrate(sin(2*x)/cos(x),x)`

output `-2*cos(x)`

**3.283.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sec(x) \sin(2x) dx = -2 \cos(x)$$

input `integrate(sin(2*x)/cos(x),x, algorithm="maxima")`

output `-2*cos(x)`

**3.283.8 Giac [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sec(x) \sin(2x) dx = -2 \cos(x)$$

input `integrate(sin(2*x)/cos(x),x, algorithm="giac")`

output `-2*cos(x)`



**3.283.9 Mupad [B] (verification not implemented)**

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sec(x) \sin(2x) dx = -2 \cos(x)$$

input `int(sin(2*x)/cos(x),x)`

output `-2*cos(x)`

**3.284**  $\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}} dx$

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 3.284.2 Mathematica [A] (verified) . . . . . 1497  
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 3.284.5 Fricas [B] (verification not implemented) . . . . . 1500  
 3.284.6 Sympy [A] (verification not implemented) . . . . . 1500  
 3.284.7 Maxima [F] . . . . . 1501  
 3.284.8 Giac [A] (verification not implemented) . . . . . 1501  
 3.284.9 Mupad [B] (verification not implemented) . . . . . 1502

**3.284.1 Optimal result**

Integrand size = 71, antiderivative size = 71

$$\int \frac{3 + 3x - 4x^2 - 4x^3 - 7x^6 + 4x^7 + 10x^8 + 7x^{13}}{1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}} dx$$

$$= \frac{1}{2} \left( (1 + \sqrt{2}) \log \left( 1 + x + \sqrt{2}x + \sqrt{2}x^2 - x^7 \right) - (-1 + \sqrt{2}) \log \left( -1 + (-1 + \sqrt{2})x + \sqrt{2}x^2 + x^7 \right) \right)$$

output `-1/2*ln(-1+x^7+x*(2^(1/2)-1)+x^2*2^(1/2))*(2^(1/2)-1)+1/2*ln(1+x-x^7+x*2^(1/2)+x^2*2^(1/2))*(1+2^(1/2))`

**3.284.2 Mathematica [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{3 + 3x - 4x^2 - 4x^3 - 7x^6 + 4x^7 + 10x^8 + 7x^{13}}{1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}} dx$$

$$= \frac{1}{2} \left( (1 + \sqrt{2}) \log \left( 1 + x + \sqrt{2}x + \sqrt{2}x^2 - x^7 \right) - (-1 + \sqrt{2}) \log \left( -1 + (-1 + \sqrt{2})x + \sqrt{2}x^2 + x^7 \right) \right)$$

input `Integrate[(3 + 3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^13)/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14), x]`

---

3.284.  $\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}} dx$

output  $((1 + \text{Sqrt}[2]) \cdot \text{Log}[1 + x + \text{Sqrt}[2] \cdot x + \text{Sqrt}[2] \cdot x^2 - x^7] - (-1 + \text{Sqrt}[2]) \cdot \text{Log}[-1 + (-1 + \text{Sqrt}[2]) \cdot x + \text{Sqrt}[2] \cdot x^2 + x^7]) / 2$

### 3.284.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{7x^{13} + 10x^8 + 4x^7 - 7x^6 - 4x^3 - 4x^2 + 3x + 3}{x^{14} - 2x^8 - 2x^7 - 2x^4 - 4x^3 - x^2 + 2x + 1} dx$$

↓ 2525

$$\frac{1}{14} \int \frac{28(5x^8 + 6x^7 + x^2 + 2x + 1)}{x^{14} - 2x^8 - 2x^7 - 2x^4 - 4x^3 - x^2 + 2x + 1} dx + \frac{1}{2} \log(x^{14} - 2x^8 - 2x^7 - 2x^4 - 4x^3 - x^2 + 2x + 1)$$

↓ 27

$$2 \int \frac{5x^8 + 6x^7 + x^2 + 2x + 1}{x^{14} - 2x^8 - 2x^7 - 2x^4 - 4x^3 - x^2 + 2x + 1} dx + \frac{1}{2} \log(x^{14} - 2x^8 - 2x^7 - 2x^4 - 4x^3 - x^2 + 2x + 1)$$

↓ 7293

$$2 \int \left( \frac{5x^8}{x^{14} - 2x^8 - 2x^7 - 2x^4 - 4x^3 - x^2 + 2x + 1} + \frac{6x^7}{x^{14} - 2x^8 - 2x^7 - 2x^4 - 4x^3 - x^2 + 2x + 1} + \frac{1}{x^{14} - 2x^8 - 2x^7 - 2x^4 - 4x^3 - x^2 + 2x + 1} \right) dx + \frac{1}{2} \log(x^{14} - 2x^8 - 2x^7 - 2x^4 - 4x^3 - x^2 + 2x + 1)$$

↓ 2009

$$2 \left( \int \frac{1}{x^{14} - 2x^8 - 2x^7 - 2x^4 - 4x^3 - x^2 + 2x + 1} dx + 2 \int \frac{x}{x^{14} - 2x^8 - 2x^7 - 2x^4 - 4x^3 - x^2 + 2x + 1} dx + \int \frac{1}{x^{14} - 2x^8 - 2x^7 - 2x^4 - 4x^3 - x^2 + 2x + 1} dx \right) + \frac{1}{2} \log(x^{14} - 2x^8 - 2x^7 - 2x^4 - 4x^3 - x^2 + 2x + 1)$$

input  $\text{Int}[(3 + 3x - 4x^2 - 4x^3 - 7x^6 + 4x^7 + 10x^8 + 7x^{13}) / (1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}), x]$

output  $\$Aborted$

---

3.284.  $\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}} dx$

## 3.284.3.1 Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2525 `Int[(Pm_)/(Qn_), x_Symbol] := With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Simp[Coeff[Pm, x, m]*(Log[Qn]/(n*Coeff[Qn, x, n])), x] + Simp[1/(n*Coeff[Qn, x, n]) Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x]/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]`

rule 7293 `Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]`

## 3.284.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

| method  | result                                                                                                                                                                                                                                                  |
|---------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| default | $\left(\frac{1}{2} + \frac{\sqrt{2}}{2}\right) \ln(x^7 - x^2\sqrt{2} + (-1 - \sqrt{2})x - 1) + \left(-\frac{\sqrt{2}}{2} + \frac{1}{2}\right) \ln(-1 + x^7 + x(\sqrt{2} - 1) + x^2\sqrt{2})$                                                            |
| risch   | $\frac{\ln(x^7 - x^2\sqrt{2} + (-1 - \sqrt{2})x - 1)}{2} + \frac{\ln(x^7 - x^2\sqrt{2} + (-1 - \sqrt{2})x - 1)\sqrt{2}}{2} - \frac{\ln(-1 + x^7 + x(\sqrt{2} - 1) + x^2\sqrt{2})\sqrt{2}}{2} + \frac{\ln(-1 + x^7 + x(\sqrt{2} - 1) + x^2\sqrt{2})}{2}$ |

input `int((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2*x^4-4*x^3-x^2+2*x+1),x,method=_RETURNVERBOSE)`

output  $(1/2+1/2*2^{(1/2)})*\ln(x^7-x^2*2^{(1/2)}+(-1-2^{(1/2)})*x-1)+(-1/2*2^{(1/2)}+1/2)*\ln(-1+x^7+x*(2^{(1/2)}-1)+x^2*2^{(1/2)})$

---

3.284.  $\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}} dx$

**3.284.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(56) = 112.

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.93

$$\int \frac{3 + 3x - 4x^2 - 4x^3 - 7x^6 + 4x^7 + 10x^8 + 7x^{13}}{1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}} dx$$

$$= \frac{1}{2} \sqrt{2} \log \left( \frac{x^{14} - 2x^8 - 2x^7 + 2x^4 + 4x^3 + 3x^2 - 2\sqrt{2}(x^9 + x^8 - x^3 - 2x^2 - x) + 2x + 1}{x^{14} - 2x^8 - 2x^7 - 2x^4 - 4x^3 - x^2 + 2x + 1} \right)$$

$$+ \frac{1}{2} \log(x^{14} - 2x^8 - 2x^7 - 2x^4 - 4x^3 - x^2 + 2x + 1)$$

input `integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2*x^4-4*x^3-x^2+2*x+1),x, algorithm="fracas")`

output `1/2*sqrt(2)*log((x^14 - 2*x^8 - 2*x^7 + 2*x^4 + 4*x^3 + 3*x^2 - 2*sqrt(2)*(x^9 + x^8 - x^3 - 2*x^2 - x) + 2*x + 1)/(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1)) + 1/2*log(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1)`

**3.284.6 Sympy [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{3 + 3x - 4x^2 - 4x^3 - 7x^6 + 4x^7 + 10x^8 + 7x^{13}}{1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}} dx$$

$$= \left( \frac{1}{2} + \frac{\sqrt{2}}{2} \right) \log \left( x^7 - \sqrt{2}x^2 - 2x \left( \frac{1}{2} + \frac{\sqrt{2}}{2} \right) - 1 \right)$$

$$+ \left( \frac{1}{2} - \frac{\sqrt{2}}{2} \right) \log \left( x^7 + \sqrt{2}x^2 - 2x \left( \frac{1}{2} - \frac{\sqrt{2}}{2} \right) - 1 \right)$$

input `integrate((7*x**13+10*x**8+4*x**7-7*x**6-4*x**3-4*x**2+3*x+3)/(x**14-2*x**8-2*x**7-2*x**4-4*x**3-x**2+2*x+1),x)`

output `(1/2 + sqrt(2)/2)*log(x**7 - sqrt(2)*x**2 - 2*x*(1/2 + sqrt(2)/2) - 1) + (1/2 - sqrt(2)/2)*log(x**7 + sqrt(2)*x**2 - 2*x*(1/2 - sqrt(2)/2) - 1)`

**3.284.7 Maxima [F]**

$$\int \frac{3 + 3x - 4x^2 - 4x^3 - 7x^6 + 4x^7 + 10x^8 + 7x^{13}}{1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}} dx$$

$$= \int \frac{7x^{13} + 10x^8 + 4x^7 - 7x^6 - 4x^3 - 4x^2 + 3x + 3}{x^{14} - 2x^8 - 2x^7 - 2x^4 - 4x^3 - x^2 + 2x + 1} dx$$

input `integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2*x^4-4*x^3-x^2+2*x+1),x, algorithm="maxima")`

output `integrate((7*x^13 + 10*x^8 + 4*x^7 - 7*x^6 - 4*x^3 - 4*x^2 + 3*x + 3)/(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1), x)`

**3.284.8 Giac [A] (verification not implemented)**

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

$$\int \frac{3 + 3x - 4x^2 - 4x^3 - 7x^6 + 4x^7 + 10x^8 + 7x^{13}}{1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}} dx$$

$$= -\frac{1}{2} \sqrt{2} \log \left( \left| x^7 + \sqrt{2}x^2 + \sqrt{2}x - x - 1 \right| \right) + \frac{1}{2} \sqrt{2} \log \left( \left| x^7 - \sqrt{2}x^2 - \sqrt{2}x - x - 1 \right| \right)$$

$$+ \frac{1}{2} \log \left( \left| x^{14} - 2x^8 - 2x^7 - 2x^4 - 4x^3 - x^2 + 2x + 1 \right| \right)$$

input `integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2*x^4-4*x^3-x^2+2*x+1),x, algorithm="giac")`

output `-1/2*sqrt(2)*log(abs(x^7 + sqrt(2)*x^2 + sqrt(2)*x - x - 1)) + 1/2*sqrt(2)*log(abs(x^7 - sqrt(2)*x^2 - sqrt(2)*x - x - 1)) + 1/2*log(abs(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1))`

**3.284.9 Mupad [B] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.45

$$\int \frac{3 + 3x - 4x^2 - 4x^3 - 7x^6 + 4x^7 + 10x^8 + 7x^{13}}{1 + 2x - x^2 - 4x^3 - 2x^4 - 2x^7 - 2x^8 + x^{14}} dx$$

$$= \frac{\ln(\sqrt{2}x - x + \sqrt{2}x^2 + x^7 - 1)}{2} + \frac{\ln(x^7 - \sqrt{2}x - \sqrt{2}x^2 - x - 1)}{2}$$

$$- \frac{\sqrt{2} \ln(\sqrt{2}x - x + \sqrt{2}x^2 + x^7 - 1)}{2} + \frac{\sqrt{2} \ln(x^7 - \sqrt{2}x - \sqrt{2}x^2 - x - 1)}{2}$$

input `int(-(3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^13 + 3)/(x^2 - 2*x + 4*x^3 + 2*x^4 + 2*x^7 + 2*x^8 - x^14 - 1),x)`

output `log(2^(1/2)*x - x + 2^(1/2)*x^2 + x^7 - 1)/2 + log(x^7 - 2^(1/2)*x - 2^(1/2)*x^2 - x - 1)/2 - (2^(1/2)*log(2^(1/2)*x - x + 2^(1/2)*x^2 + x^7 - 1))/2 + (2^(1/2)*log(x^7 - 2^(1/2)*x - 2^(1/2)*x^2 - x - 1))/2`

## APPENDIX

|                                            |      |
|--------------------------------------------|------|
| 4.1 Listing of Grading functions . . . . . | 1503 |
|--------------------------------------------|------|

## 4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*           Small rewrite of logic in main function to make it*)
(*           match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
```



```

(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A"," "}
          ,(*ELSE*)
          finalresult={"B","Both result and optimal contain complex but leaf count
        ]
        ,(*ELSE*)
        finalresult={"C","Result contains complex when optimal does not."}
      ]
      ,(*ELSE*)(*result does not contains complex*)
      If[leafCountResult<=2*leafCountOptimal,
        finalresult={"A"," "}
        ,(*ELSE*)
        finalresult={"B","Leaf count is larger than twice the leaf count of optimal.$
      ]
    ]
    ,(*ELSE*)(*expnResult>expnOptimal*)
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      finalresult={"C","Result contains higher order function than in optimal. Order "<
      ,
      finalresult={"F","Contains unresolved integral."}
    ]
  ];

  finalresult
]

```

```

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
    If[ListQ[expn],
      Max[Map[ExpnType,expn]],
      If[Head[expn]===Power,
        If[IntegerQ[expn[[2]]],
          ExpnType[expn[[1]]],
          If[Head[expn[[2]]]===Rational,
            If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
              1,
              Max[ExpnType[expn[[1]],2]],
            Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]]],
          If[Head[expn]===Plus || Head[expn]===Times,
            Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
            If[ElementaryFunctionQ[Head[expn]],
              Max[3,ExpnType[expn[[1]]],
            If[SpecialFunctionQ[Head[expn]],
              Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
            If[HypergeometricFunctionQ[Head[expn]],
              Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
            If[AppellFunctionQ[Head[expn]],
              Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
            If[Head[expn]===RootSum,
              Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
            If[Head[expn]===Integrate || Head[expn]===Int,
              Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
            9]]]]]]]]]]

```

```

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, CsCh,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsCh
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1}, func]

```

### 4.1.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems
#Nasser 4/07/2022 add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

      leaf_count_result:=leafcount(result);
#do NOT call ExpnType() if leaf size is too large. Recursion problem
if leaf_count_result > 500000 then
      return "B","result has leaf size over 500,000. Avoiding possible recursion issues";
fi;

      leaf_count_optimal := leafcount(optimal);
      ExpnType_result := ExpnType(result);
      ExpnType_optimal := ExpnType(optimal);

      if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
      return "F","Result contains unresolved integral";
fi;

```

```

if ExpnType_result<=ExpnType_optimal then
  if debug then
    print("ExpnType_result<=ExpnType_optimal");
  fi;
  if is_contains_complex(result) then
    if is_contains_complex(optimal) then
      if debug then
        print("both result and optimal complex");
      fi;
      if leaf_count_result<=2*leaf_count_optimal then
        return "A"," ";
      else
        return "B",cat("Both result and optimal contain complex but leaf count of
                        convert(leaf_count_result,string)," vs. $2 (" ,
                        convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
        end if
      else #result contains complex but optimal is not
        if debug then
          print("result contains complex but optimal is not");
        fi;
        return "C","Result contains complex when optimal does not.";
      fi;
    else # result do not contain complex
      # this assumes optimal do not as well. No check is needed here.
      if debug then
        print("result do not contain complex, this assumes optimal do not as well");
      fi;
      if leaf_count_result<=2*leaf_count_optimal then
        if debug then
          print("leaf_count_result<=2*leaf_count_optimal");
        fi;
        return "A"," ";
      else
        if debug then
          print("leaf_count_result>2*leaf_count_optimal");
        fi;
        return "B",cat("Leaf count of result is larger than twice the leaf count of o
                        convert(leaf_count_result,string)," $ vs. $2(" ,
                        convert(leaf_count_optimal,string)," )=" ,convert(2*leaf_cou
        fi;
      fi;
    fi;
  fi;

```

```

else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C",cat("Result contains higher order function than in optimal. Order ",
    convert(ExpnType_result,string)," vs. order ",
    convert(ExpnType_optimal,string),".");
fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  end if
end proc:

```

```

elif type(expn, ``~`) then
  if type(op(2,expn), 'integer') then
    ExpnType(op(1,expn))
  elif type(op(2,expn), 'rational') then
    if type(op(1,expn), 'rational') then
      1
    else
      max(2, ExpnType(op(1,expn)))
    end if
  else
    max(3, ExpnType(op(1,expn)), ExpnType(op(2,expn)))
  end if
elif type(expn, ``+`) or type(expn, ``*`) then
  max(ExpnType(op(1,expn)), max(ExpnType(rest(expn))))
elif ElementaryFunctionQ(op(0,expn)) then
  max(3, ExpnType(op(1,expn)))
elif SpecialFunctionQ(op(0,expn)) then
  max(4, apply(max, map(ExpnType, [op(expn)])))
elif HypergeometricFunctionQ(op(0,expn)) then
  max(5, apply(max, map(ExpnType, [op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6, apply(max, map(ExpnType, [op(expn)])))
elif op(0,expn)='int' then
  max(8, apply(max, map(ExpnType, [op(expn)]))) else
  9
end if
end proc:

```

```

ElementaryFunctionQ := proc(func)
  member(func, [
    exp, log, ln,
    sin, cos, tan, cot, sec, csc,
    arcsin, arccos, arctan, arccot, arcsec, arccsc,
    sinh, cosh, tanh, coth, sech, csch,
    arcsinh, arccosh, arctanh, arccoth, arcsech, arccsch])
end proc:

```

```

SpecialFunctionQ := proc(func)
  member(func, [
    erf, erfc, erfi,
    FresnelS, FresnelC,
    Ei, Ei, Li, Si, Ci, Shi, Chi,

```

```

        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
    member(func, [Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
    member(func, [AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
    if nops(u)=2 then
        op(2,u)
    else
        apply(op(0,u),op(2..nops(u),u))
    end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
    MmaTranslator[Mma] [LeafCount] (u);
end proc:

```

### 4.1.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#          Port of original Maple grading function by
#          Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#          added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):

```



```
if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
        return True
    else:
        return False
else:
    return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
    ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
    ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```

return 1
elif isinstance(expn,list):
    return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
elif is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
    else:
        return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
elif isinstance(expn,Pow): #type(expn,``^`)
    if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
        return expnType(expn.args[0]) #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or type(expn,``*`)
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]]
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

*#main function*

```

def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation = ""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation = ""
                    else:
                        grade = "B"
                        grade_annotation = "Both result and optimal contain complex but leaf count of result is large"
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation = "Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result)
        else:
            grade = "C"
            grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType_result)

    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

```

```
return grade, grade_annotation
```

#### 4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#      Albert Rich to use with Sagemath. This is used to
#      grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#      'arctan2', 'floor', 'abs', 'log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#      issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```

        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arcoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral',
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func , " is special_function")
        else:
            print ("func ", func , " is NOT special_function")

    return m

```

```

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric', 'hypergeometric_M', 'hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']  #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=", expn)

    if not hasattr(expn, 'parent'):
        return False

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-try:
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception, AttributeError in is_atom")
        print ("caught exception" , type(error).__name__ )
        return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:  #isinstance(expn,list):

```

```

    return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
elif is_sqrt(expn):
    if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
        return 1
    else:
        return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
elif expn.operator() == operator.pow: #instance(expn,Pow)
    if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
        return expnType(expn.operands()[0]) #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational: #instance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #instance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #instance(expn,Add) or inst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

```

```

if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)",type(result))
    print("type(optimal)",type(optimal))

leaf_count_result = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

expnType_result = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = " "
            else:
                grade = "B"
                grade_annotation = "Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation = "Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = " "
            else:
                grade = "B"
                grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal." + str(leaf
    else:
        grade = "C"
        grade_annotation = "Result contains higher order function than in optimal. Order " + str(expnType_resu

print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

```



```
return grade, grade_annotation
```