Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

0-Independent-test-suites/8-Moses-Problems

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [113]. This is test number [8].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (113)	0.00 (0)
Mathematica	100.00 (113)	0.00 (0)
Maple	100.00 (113)	0.00 (0)
Fricas	99.12 (112)	0.88 (1)
Giac	98.23 (111)	1.77 (2)
Maxima	98.23 (111)	1.77 (2)
Sympy	94.69 (107)	5.31 (6)
Mupad	93.81 (106)	6.19 (7)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

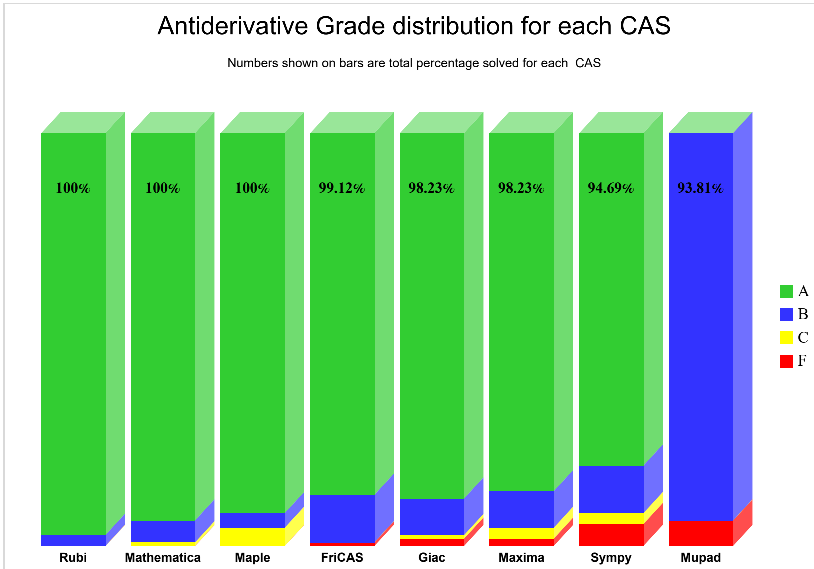
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

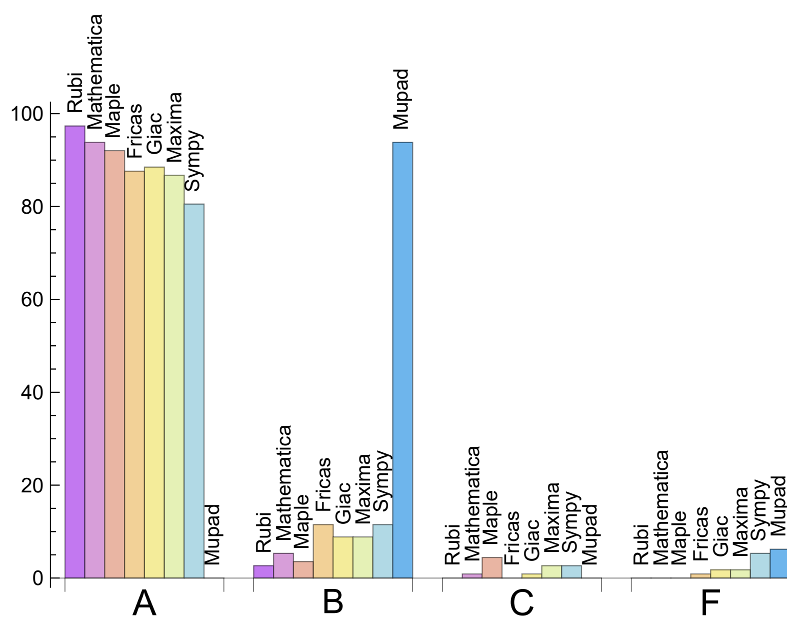
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.345	2.655	0.000	0.000
Mathematica	93.805	5.310	0.885	0.000
Maple	92.035	3.540	4.425	0.000
Giac	88.496	8.850	0.885	1.770
Fricas	87.611	11.504	0.000	0.885
Maxima	86.726	8.850	2.655	1.770
Sympy	80.531	11.504	2.655	5.310
Mupad	0.000	93.805	0.000	6.195

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Fricas	1	100.00	0.00	0.00
Giac	2	100.00	0.00	0.00
Maxima	2	100.00	0.00	0.00
Sympy	6	100.00	0.00	0.00
Mupad	7	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Mathematica	0.05
Mupad	0.11
Maple	0.13
Rubi	0.16
Maxima	0.22
Fricas	0.24
Giac	0.28
Sympy	1.29

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	20.01	1.08	13.00	0.88
Rubi	21.19	1.06	17.00	1.00
Giac	23.58	1.15	14.00	0.85
Maxima	23.66	1.16	14.00	0.88
Mathematica	24.39	1.13	16.00	1.00
Fricas	24.87	1.16	14.00	0.95
Mupad	28.25	1.65	12.00	0.83
Sympy	30.84	1.68	15.00	0.83

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

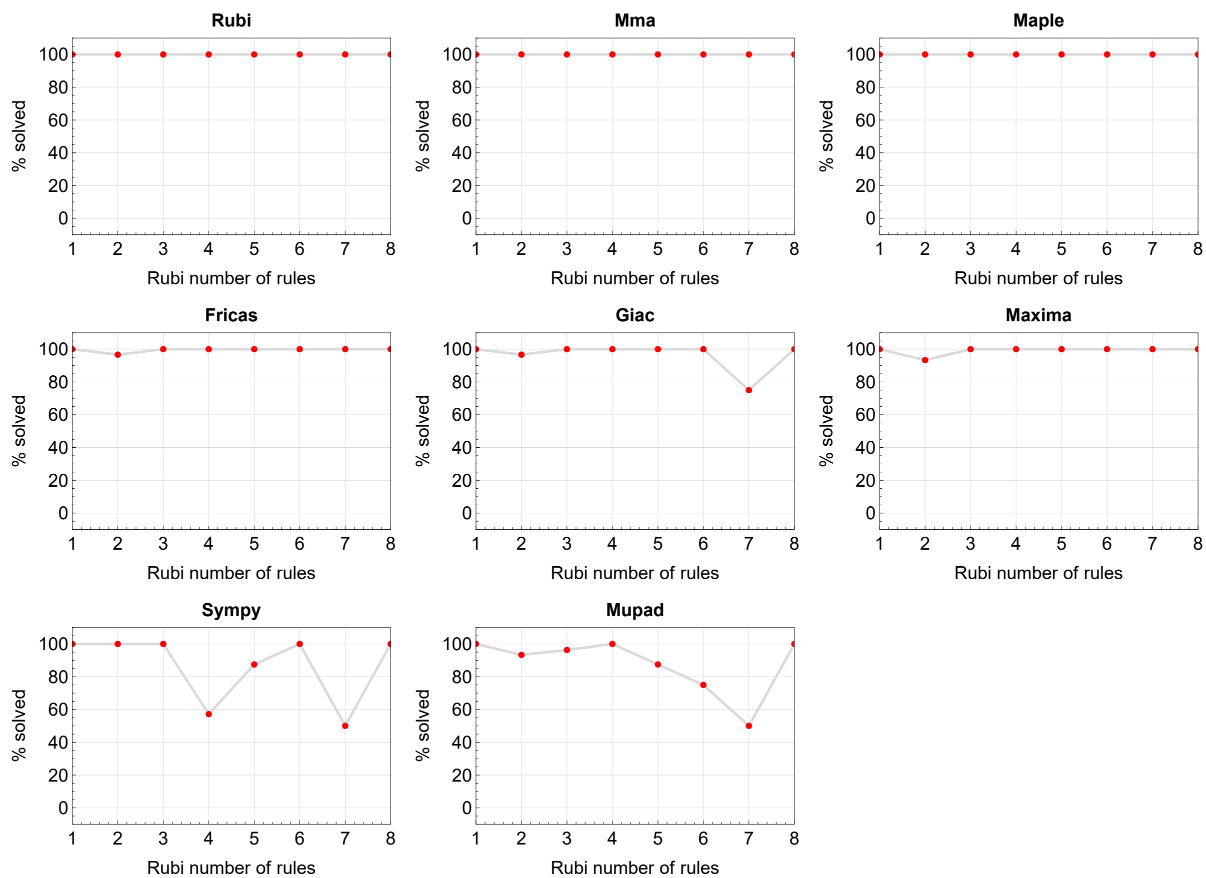


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

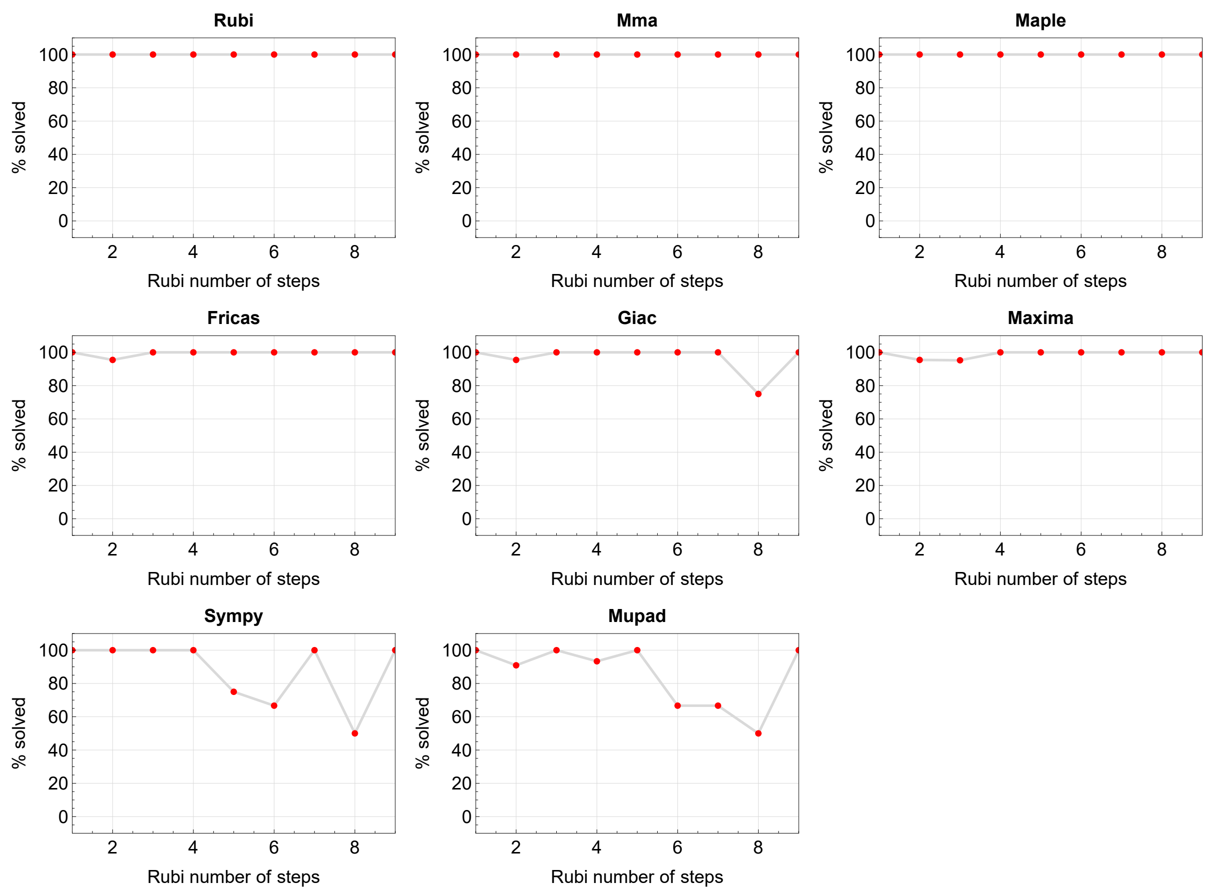


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

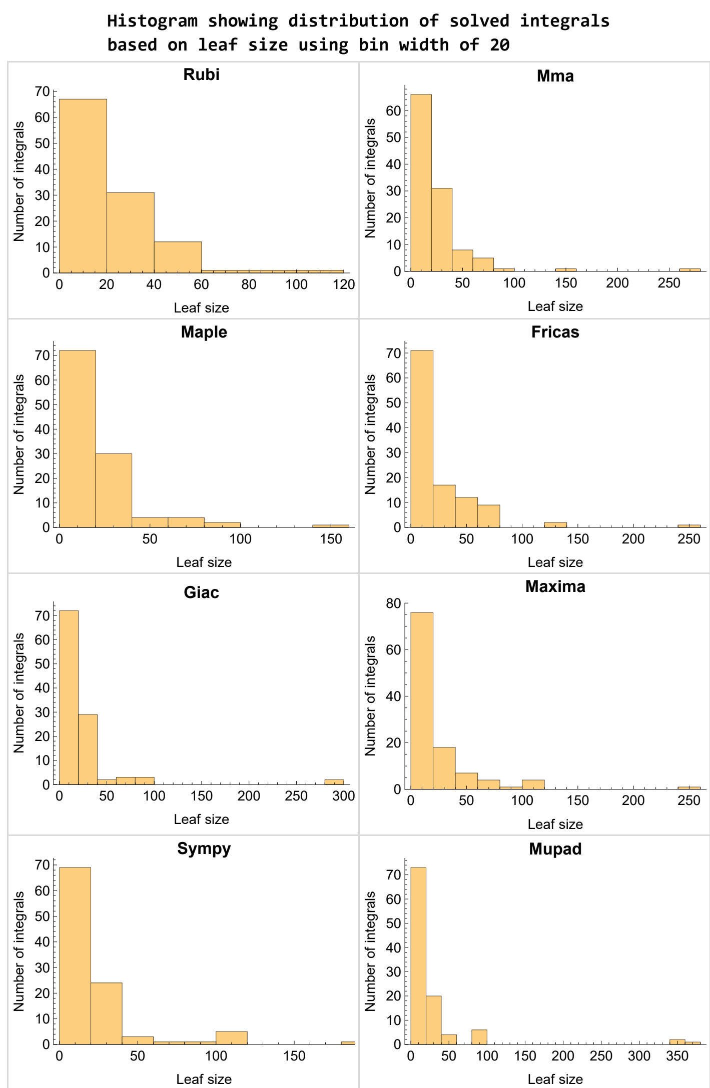


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

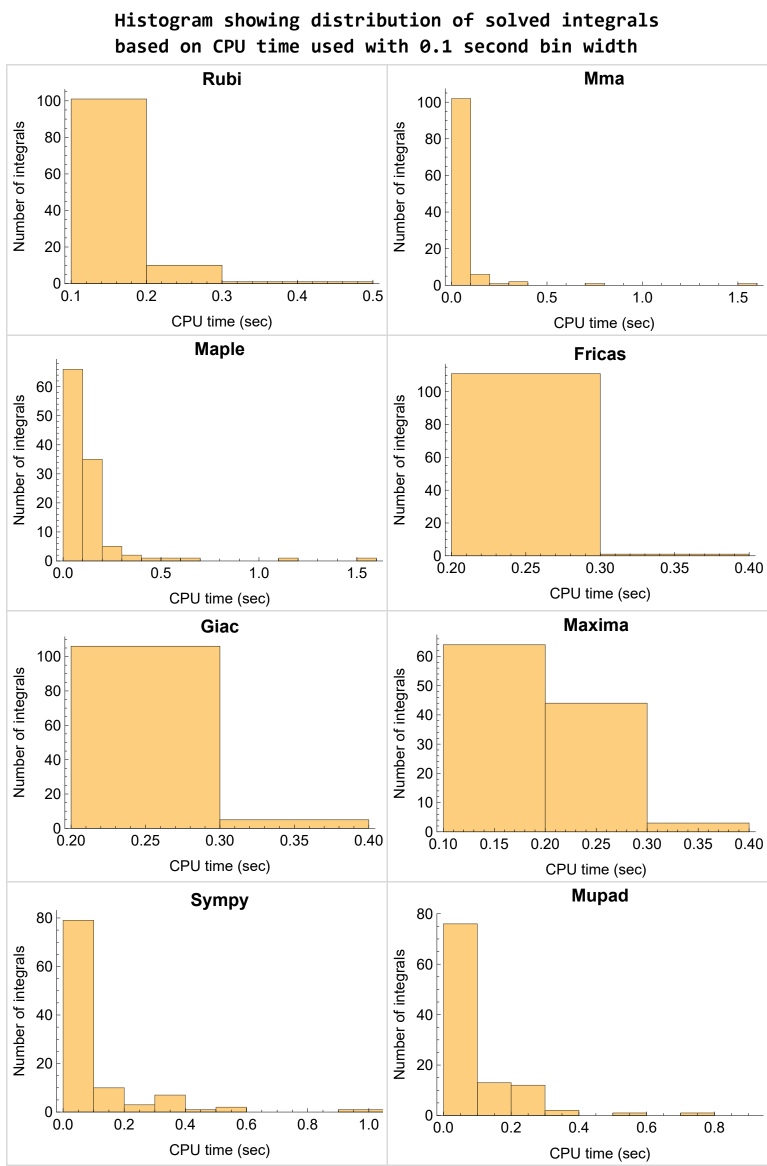


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

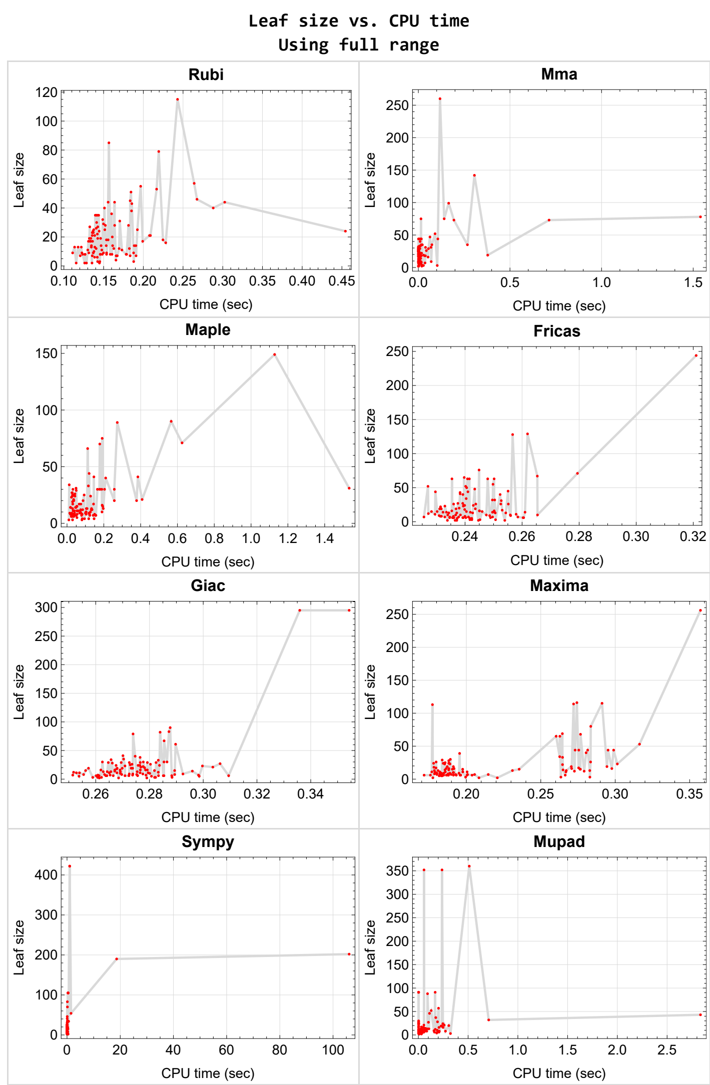


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {23}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

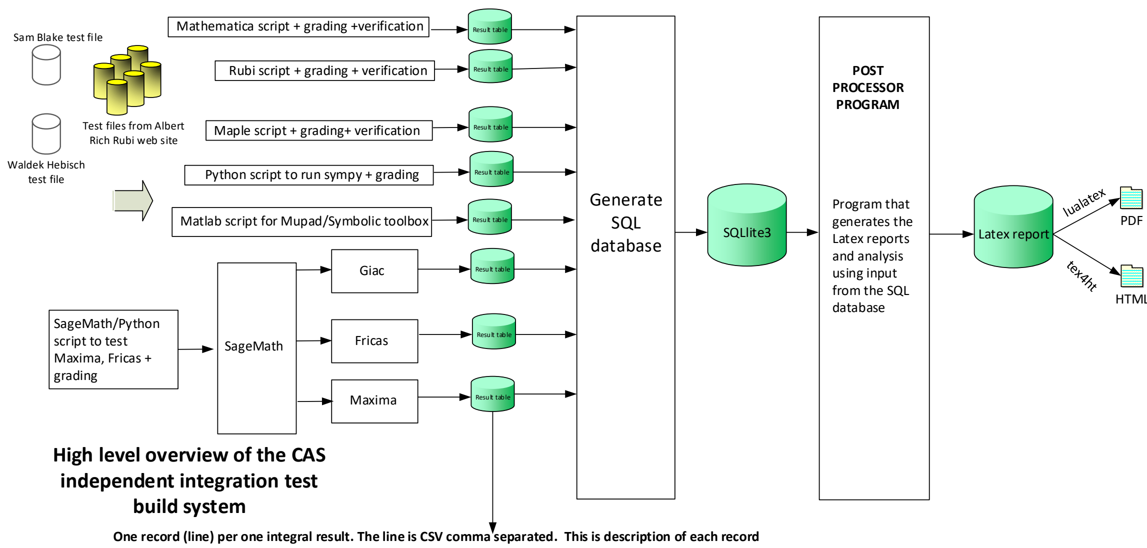
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

The following fields are present only in ***Rubi*** *Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade { 70, 71, 72 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade { 31, 42, 70, 71, 72, 84 }

C grade { 1 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade { 36, 48, 57, 70 }

C grade { 32, 42, 68, 71, 72 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 38, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade { 1, 9, 37, 39, 40, 42, 69, 70, 73, 74, 83, 84, 100 }

C grade { }

F normal fail { 32 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 39, 41, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade { 36, 38, 40, 42, 68, 69, 70, 71, 72, 87 }

C grade { 10, 11, 47 }

F normal fail { 32, 67 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 44, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade { 1, 25, 36, 40, 43, 45, 69, 70, 71, 72 }

C grade { 47 }

F normal fail { 32, 42 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 39, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

C grade { }

F normal fail { }

F(-1) timedout fail { 10, 11, 32, 38, 40, 42, 69 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 41, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 70, 74, 75, 76, 77, 78, 79, 80, 81, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade { 7, 18, 37, 39, 50, 51, 56, 66, 73, 82, 83, 84, 100 }

C grade { 38, 71, 72 }

F normal fail { 35, 36, 40, 42, 69, 87 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	22	11	16	48	19	34	10
N.S.	1	1.00	1.83	0.92	1.33	4.00	1.58	2.83	0.83
time (sec)	N/A	0.183	0.001	0.058	0.271	0.243	0.025	0.266	0.002

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	19	14	15	12
N.S.	1	1.00	1.00	0.92	1.15	1.46	1.08	1.15	0.92
time (sec)	N/A	0.124	0.006	0.136	0.277	0.231	0.045	0.256	0.170

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	11	11	14	15	11	10
N.S.	1	1.00	0.74	0.58	0.58	0.74	0.79	0.58	0.53
time (sec)	N/A	0.122	0.012	0.073	0.198	0.242	0.074	0.277	0.030

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.128	0.001	0.030	0.208	0.245	0.032	0.261	0.031

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.78	0.67	0.67	0.56	0.67	0.67
time (sec)	N/A	0.129	0.001	0.032	0.204	0.235	0.029	0.260	0.019

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.155	0.007	0.155	0.193	0.261	0.020	0.288	0.024

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	22	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	1.69	0.69	0.69
time (sec)	N/A	0.117	0.002	0.103	0.195	0.231	0.068	0.292	0.029

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	12	11	13	15	11	11
N.S.	1	1.00	0.74	0.63	0.58	0.68	0.79	0.58	0.58
time (sec)	N/A	0.146	0.010	0.085	0.201	0.237	0.098	0.280	0.018

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	14	8	6	6
N.S.	1	1.00	1.00	0.88	0.75	1.75	1.00	0.75	0.75
time (sec)	N/A	0.157	0.007	0.157	0.196	0.232	0.022	0.285	0.054

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	15	3	3	3	0
N.S.	1	1.00	1.00	1.00	3.75	0.75	0.75	0.75	0.00
time (sec)	N/A	0.177	0.018	0.147	0.235	0.242	0.281	0.282	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	13	2	2	2	0
N.S.	1	1.00	1.00	1.50	6.50	1.00	1.00	1.00	0.00
time (sec)	N/A	0.145	0.019	0.081	0.231	0.254	0.389	0.279	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	7	7	5	7	7
N.S.	1	1.00	1.00	1.00	0.88	0.88	0.62	0.88	0.88
time (sec)	N/A	0.119	0.010	0.104	0.193	0.241	0.032	0.298	0.042

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.00	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.134	0.036	0.043	0.201	0.236	0.035	0.309	0.191

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	19	19	20	19	21
N.S.	1	1.00	0.93	0.75	0.68	0.68	0.71	0.68	0.75
time (sec)	N/A	0.180	0.044	0.036	0.191	0.237	0.039	0.273	0.056

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.68	0.73	0.73
time (sec)	N/A	0.134	0.013	0.031	0.188	0.235	0.037	0.279	0.046

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.143	0.001	0.049	0.188	0.260	0.034	0.283	0.017

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.78	0.67	0.67	0.56	0.67	0.67
time (sec)	N/A	0.130	0.000	0.033	0.191	0.249	0.029	0.272	0.002

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	22	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	1.69	0.69	0.69
time (sec)	N/A	0.119	0.001	0.104	0.180	0.256	0.070	0.272	0.002

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	5	5	5
N.S.	1	1.00	1.00	1.00	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.144	0.013	0.029	0.182	0.242	0.030	0.299	0.034

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56
time (sec)	N/A	0.110	0.001	0.037	0.183	0.237	0.032	0.281	0.042

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.142	0.008	0.076	0.184	0.240	0.045	0.263	0.067

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	7	7	7	7	7
N.S.	1	1.00	1.00	1.00	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.130	0.001	0.032	0.190	0.226	0.046	0.251	0.017

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	9	19	8	7	7	8	7	7
N.S.	1	0.90	1.90	0.80	0.70	0.70	0.80	0.70	0.70
time (sec)	N/A	0.187	0.029	0.150	0.201	0.259	0.285	0.261	0.209

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	13	15	15	34	15	12
N.S.	1	1.00	0.78	0.57	0.65	0.65	1.48	0.65	0.52
time (sec)	N/A	0.138	0.016	0.109	0.191	0.230	0.555	0.289	0.032

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	17	17	17	19	9
N.S.	1	1.00	1.92	0.77	1.31	1.31	1.31	1.46	0.69
time (sec)	N/A	0.129	0.004	0.127	0.275	0.253	0.063	0.268	0.157

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	14	13	13	15	13	13
N.S.	1	1.00	1.00	0.78	0.72	0.72	0.83	0.72	0.72
time (sec)	N/A	0.157	0.038	0.049	0.283	0.251	0.062	0.264	0.090

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	20	19	76	22	19	19
N.S.	1	1.00	1.00	0.65	0.61	2.45	0.71	0.61	0.61
time (sec)	N/A	0.176	0.039	0.087	0.294	0.245	0.078	0.270	0.235

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	11	8	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.38	1.00	1.00	1.00
time (sec)	N/A	0.176	0.016	0.041	0.182	0.258	0.040	0.289	0.211

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	15	14	14	10	13	12
N.S.	1	1.00	1.00	1.25	1.17	1.17	0.83	1.08	1.00
time (sec)	N/A	0.137	0.004	0.054	0.186	0.233	0.033	0.266	0.066

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	18	20	17	16	16	15	16	16
N.S.	1	0.90	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.218	0.001	0.144	0.186	0.245	0.122	0.267	0.207

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	55	260	41	40	40	46	40	52
N.S.	1	1.12	5.31	0.84	0.82	0.82	0.94	0.82	1.06
time (sec)	N/A	0.195	0.118	0.146	0.280	0.239	0.078	0.275	0.129

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	142	41	0	0	54	0	0
N.S.	1	1.00	1.23	0.36	0.00	0.00	0.47	0.00	0.00
time (sec)	N/A	0.243	0.306	0.385	0.000	0.000	1.501	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	21	22	17	16	16	20	16	16
N.S.	1	0.95	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.210	0.018	0.075	0.192	0.250	0.096	0.277	0.248

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	13	15	15	34	15	12
N.S.	1	1.00	0.78	0.57	0.65	0.65	1.48	0.65	0.52
time (sec)	N/A	0.134	0.002	0.105	0.198	0.228	0.559	0.282	0.002

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	36	33	25	24	24	0	24	24
N.S.	1	1.12	1.03	0.78	0.75	0.75	0.00	0.75	0.75
time (sec)	N/A	0.161	0.006	0.093	0.188	0.240	0.000	0.284	0.002

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	75	75	80	55	0	61	57
N.S.	1	1.00	1.70	1.70	1.82	1.25	0.00	1.39	1.30
time (sec)	N/A	0.154	0.141	0.191	0.284	0.250	0.000	0.290	0.203

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	44	30	44	63	105	29	91
N.S.	1	1.00	1.26	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.141	0.106	0.257	0.273	0.250	0.398	0.279	0.169

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	85	52	44	113	44	190	90	0
N.S.	1	1.13	0.69	0.59	1.51	0.59	2.53	1.20	0.00
time (sec)	N/A	0.158	0.091	0.120	0.177	0.241	18.654	0.288	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	44	30	44	63	105	29	91
N.S.	1	1.00	1.26	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.139	0.005	0.165	0.276	0.235	0.391	0.272	0.002

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	78	90	114	129	0	295	0
N.S.	1	1.00	1.53	1.76	2.24	2.53	0.00	5.78	0.00
time (sec)	N/A	0.182	1.540	0.566	0.272	0.262	0.000	0.336	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.145	0.002	0.061	0.191	0.256	0.018	0.269	0.029

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	57	99	149	116	244	0	0	0
N.S.	1	1.16	2.02	3.04	2.37	4.98	0.00	0.00	0.00
time (sec)	N/A	0.264	0.166	1.128	0.274	0.321	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	9	9	3	30	4
N.S.	1	1.00	0.67	0.56	1.00	1.00	0.33	3.33	0.44
time (sec)	N/A	0.158	0.003	0.054	0.189	0.249	0.098	0.285	0.184

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	7	7	6	6	5	6	6
N.S.	1	1.00	0.64	0.64	0.55	0.55	0.45	0.55	0.55
time (sec)	N/A	0.150	0.001	0.026	0.185	0.236	0.028	0.267	0.016

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	8	8	5	30	8
N.S.	1	1.00	1.00	1.00	0.89	0.89	0.56	3.33	0.89
time (sec)	N/A	0.151	0.070	0.100	0.188	0.232	0.033	0.287	0.084

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.00	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.169	0.002	0.040	0.187	0.239	0.032	0.285	0.162

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	7	8	9	7
N.S.	1	1.00	1.00	0.73	0.82	0.64	0.73	0.82	0.64
time (sec)	N/A	0.127	0.001	0.018	0.185	0.235	0.081	0.274	0.018

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	8	2	2	2	2	2
N.S.	1	1.00	1.00	4.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.130	0.001	0.033	0.221	0.237	0.366	0.269	0.008

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	45	40	33	34	34	41	35	46
N.S.	1	1.10	0.98	0.80	0.83	0.83	1.00	0.85	1.12
time (sec)	N/A	0.183	0.010	0.116	0.263	0.240	0.061	0.270	0.112

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	79	75	66	65	65	83	67	88
N.S.	1	1.68	1.60	1.40	1.38	1.38	1.77	1.43	1.87
time (sec)	N/A	0.220	0.014	0.112	0.260	0.240	0.127	0.285	0.092

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	30	39	27	70	41	21
N.S.	1	1.00	1.00	1.43	1.86	1.29	3.33	1.95	1.00
time (sec)	N/A	0.134	0.005	0.189	0.195	0.237	0.151	0.270	0.260

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.135	0.000	0.019	0.179	0.243	0.031	0.275	0.035

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	43	29	34	33	24	32	38	24
N.S.	1	1.08	0.72	0.85	0.82	0.60	0.80	0.95	0.60
time (sec)	N/A	0.201	0.009	0.012	0.265	0.243	0.093	0.277	0.002

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	5	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	0.71	1.00	1.00
time (sec)	N/A	0.122	0.001	0.122	0.206	0.238	0.026	0.265	0.019

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	5	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.62	1.00	1.00
time (sec)	N/A	0.176	0.016	0.119	0.179	0.231	0.035	0.275	0.272

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	15	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.145	0.028	0.095	0.264	0.242	0.069	0.259	0.323

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	9	3	2	2	3	2
N.S.	1	1.00	1.00	4.50	1.50	1.00	1.00	1.50	1.00
time (sec)	N/A	0.124	0.001	0.015	0.200	0.237	0.173	0.261	0.009

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	15	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	1.07	1.00	1.00
time (sec)	N/A	0.158	0.006	0.194	0.188	0.261	0.069	0.267	0.061

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	13	13	11	14	8	11	15
N.S.	1	1.00	0.76	0.76	0.65	0.82	0.47	0.65	0.88
time (sec)	N/A	0.194	0.006	0.030	0.186	0.250	0.044	0.268	0.057

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	29	25	24	24	26	24	28
N.S.	1	1.00	0.76	0.66	0.63	0.63	0.68	0.63	0.74
time (sec)	N/A	0.179	0.055	0.037	0.189	0.240	0.042	0.276	0.193

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	7	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.176	0.003	0.082	0.215	0.233	0.070	0.279	0.019

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	21	22	17	16	16	20	16	16
N.S.	1	0.95	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.208	0.006	0.066	0.192	0.245	0.107	0.279	0.002

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	7	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.180	0.002	0.055	0.180	0.250	0.061	0.274	0.002

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	17	22	22	22	17
N.S.	1	1.00	1.00	0.82	0.61	0.79	0.79	0.79	0.61
time (sec)	N/A	0.158	0.001	0.024	0.180	0.231	0.040	0.278	0.031

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	15	8	9	9
N.S.	1	1.00	1.00	0.91	0.82	1.36	0.73	0.82	0.82
time (sec)	N/A	0.170	0.004	0.190	0.183	0.240	0.121	0.278	0.033

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	15	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.141	0.003	0.083	0.283	0.245	0.091	0.282	0.002

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	0	3	3	3	43
N.S.	1	1.00	1.00	1.33	0.00	1.00	1.00	1.00	14.33
time (sec)	N/A	0.189	0.103	0.118	0.000	0.251	0.133	0.288	2.835

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	20	53	15	12	21	13
N.S.	1	1.00	1.00	1.25	3.31	0.94	0.75	1.31	0.81
time (sec)	N/A	0.229	0.053	0.257	0.316	0.246	0.055	0.304	0.029

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	73	89	115	128	0	295	0
N.S.	1	1.00	1.38	1.68	2.17	2.42	0.00	5.57	0.00
time (sec)	N/A	0.215	0.194	0.273	0.291	0.257	0.000	0.354	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	46	35	71	69	67	202	83	360
N.S.	1	2.88	2.19	4.44	4.31	4.19	12.62	5.19	22.50
time (sec)	N/A	0.275	0.268	0.625	0.265	0.265	106.059	0.287	0.512

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	C	B	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	44	35	70	68	26	422	82	352
N.S.	1	2.75	2.19	4.38	4.25	1.62	26.38	5.12	22.00
time (sec)	N/A	0.299	0.022	0.178	0.277	0.255	0.991	0.284	0.238

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	C	B	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	44	35	40	65	26	422	79	352
N.S.	1	2.75	2.19	2.50	4.06	1.62	26.38	4.94	22.00
time (sec)	N/A	0.159	0.016	0.210	0.263	0.253	1.009	0.274	0.057

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	44	30	44	63	105	29	91
N.S.	1	1.00	1.26	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.144	0.016	0.204	0.282	0.241	0.382	0.277	0.002

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	12	26	19	12	12
N.S.	1	1.00	1.14	0.93	0.86	1.86	1.36	0.86	0.86
time (sec)	N/A	0.194	0.007	0.197	0.279	0.244	0.022	0.283	0.073

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	8	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.62	0.85	0.85
time (sec)	N/A	0.133	0.005	0.132	0.267	0.229	0.032	0.265	0.028

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.00	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.163	0.002	0.039	0.188	0.240	0.032	0.276	0.002

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	19	24	23	23	20	30	24
N.S.	1	1.00	0.79	1.00	0.96	0.96	0.83	1.25	1.00
time (sec)	N/A	0.442	0.379	0.128	0.301	0.239	0.048	0.278	0.247

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	8	8	7	8	8
N.S.	1	1.00	1.00	1.00	0.89	0.89	0.78	0.89	0.89
time (sec)	N/A	0.135	0.015	0.026	0.182	0.236	0.029	0.255	0.033

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	24	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.96	0.76	0.76	0.76	0.76
time (sec)	N/A	0.192	0.004	0.037	0.185	0.244	0.039	0.257	0.228

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.127	0.004	0.124	0.266	0.236	0.038	0.271	0.054

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	14	16	13	12	12	10	12	12
N.S.	1	0.88	1.00	0.81	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.134	0.006	0.133	0.273	0.227	0.040	0.252	0.021

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	16	20	36	16	10
N.S.	1	1.00	1.00	0.79	1.14	1.43	2.57	1.14	0.71
time (sec)	N/A	0.191	0.003	0.055	0.298	0.238	0.178	0.262	0.190

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	44	30	44	63	105	29	91
N.S.	1	1.00	1.26	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.147	0.001	0.176	0.295	0.242	0.391	0.269	0.002

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	35	16	15	45	34	21	37
N.S.	1	1.00	2.06	0.94	0.88	2.65	2.00	1.24	2.18
time (sec)	N/A	0.134	0.074	0.191	0.276	0.255	0.180	0.277	0.175

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	12	11	13	15	11	11
N.S.	1	1.00	0.74	0.63	0.58	0.68	0.79	0.58	0.58
time (sec)	N/A	0.146	0.007	0.074	0.177	0.254	0.088	0.253	0.002

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	3	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.50	1.00
time (sec)	N/A	0.113	0.000	0.010	0.178	0.234	0.033	0.262	0.007

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	40	73	31	256	71	0	33	32
N.S.	1	0.89	1.62	0.69	5.69	1.58	0.00	0.73	0.71
time (sec)	N/A	0.276	0.713	1.533	0.357	0.279	0.000	0.283	0.707

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	16	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	1.14	0.71
time (sec)	N/A	0.148	0.002	0.054	0.179	0.237	0.023	0.269	0.003

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	14	11	11	10	14	11	10
N.S.	1	1.00	0.82	0.65	0.65	0.59	0.82	0.65	0.59
time (sec)	N/A	0.131	0.013	0.040	0.182	0.234	0.077	0.254	0.026

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	25	31	20	19	27	19	27	27
N.S.	1	1.09	1.35	0.87	0.83	1.17	0.83	1.17	1.17
time (sec)	N/A	0.161	0.069	0.378	0.271	0.239	0.255	0.306	0.101

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	10	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	1.25	0.62	0.75	0.75
time (sec)	N/A	0.158	0.002	0.079	0.176	0.265	0.034	0.264	0.027

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	5	5	5
N.S.	1	1.00	1.00	1.00	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.140	0.001	0.023	0.184	0.242	0.032	0.254	0.003

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.160	0.002	0.033	0.191	0.238	0.037	0.280	0.047

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	10	10	7	8	6
N.S.	1	1.00	0.67	0.75	0.83	0.83	0.58	0.67	0.50
time (sec)	N/A	0.157	0.006	0.069	0.179	0.247	0.163	0.270	0.208

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.152	0.002	0.142	0.172	0.239	0.021	0.261	0.003

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.136	0.000	0.019	0.181	0.234	0.038	0.269	0.003

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.149	0.001	0.036	0.196	0.236	0.032	0.264	0.003

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	28	47	21	22	32	15	20	20
N.S.	1	1.17	1.96	0.88	0.92	1.33	0.62	0.83	0.83
time (sec)	N/A	0.153	0.063	0.408	0.265	0.250	0.348	0.278	0.306

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	14	13	19	15	13	13
N.S.	1	1.00	1.00	0.70	0.65	0.95	0.75	0.65	0.65
time (sec)	N/A	0.158	0.006	0.046	0.267	0.232	0.049	0.263	0.106

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	44	30	44	63	105	29	91
N.S.	1	1.00	1.26	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.140	0.013	0.180	0.299	0.248	0.403	0.282	0.002

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	18	18	15	14	14	24	14	14
N.S.	1	0.90	0.90	0.75	0.70	0.70	1.20	0.70	0.70
time (sec)	N/A	0.153	0.035	0.053	0.265	0.250	0.051	0.296	0.073

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	40	33	27	26	32	31	26	26
N.S.	1	1.21	1.00	0.82	0.79	0.97	0.94	0.79	0.79
time (sec)	N/A	0.151	0.011	0.028	0.283	0.250	0.054	0.280	0.002

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	19	31	15	19	19
N.S.	1	1.00	1.00	0.95	0.90	1.48	0.71	0.90	0.90
time (sec)	N/A	0.134	0.000	0.035	0.187	0.253	0.016	0.274	0.003

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	24	40	19	24	24
N.S.	1	1.00	1.00	0.96	0.92	1.54	0.73	0.92	0.92
time (sec)	N/A	0.142	0.000	0.039	0.184	0.252	0.018	0.268	0.002

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	25	43	22	25	25
N.S.	1	1.00	1.00	0.96	0.93	1.59	0.81	0.93	0.93
time (sec)	N/A	0.141	0.000	0.043	0.186	0.238	0.017	0.273	0.002

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	30	52	26	30	30
N.S.	1	1.00	1.00	0.97	0.94	1.62	0.81	0.94	0.94
time (sec)	N/A	0.147	0.000	0.049	0.189	0.240	0.018	0.262	0.003

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	25	24	23	41	20	23	23
N.S.	1	1.00	1.04	1.00	0.96	1.71	0.83	0.96	0.96
time (sec)	N/A	0.144	0.000	0.043	0.189	0.238	0.015	0.300	0.003

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	30	29	28	50	24	28	28
N.S.	1	1.00	1.03	1.00	0.97	1.72	0.83	0.97	0.97
time (sec)	N/A	0.148	0.000	0.050	0.188	0.241	0.017	0.266	0.003

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	18	18	17	18	18
N.S.	1	1.00	1.00	1.00	0.95	0.95	0.89	0.95	0.95
time (sec)	N/A	0.132	0.000	0.030	0.178	0.240	0.012	0.268	0.003

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	24	23	23	20	23	23
N.S.	1	1.00	1.00	1.00	0.96	0.96	0.83	0.96	0.96
time (sec)	N/A	0.134	0.000	0.029	0.185	0.233	0.012	0.271	0.003

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	24	44	24	24	24
N.S.	1	1.00	1.00	1.00	0.96	1.76	0.96	0.96	0.96
time (sec)	N/A	0.137	0.000	0.030	0.178	0.230	0.012	0.276	0.003

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	30	29	52	27	29	29
N.S.	1	1.00	1.00	1.00	0.97	1.73	0.90	0.97	0.97
time (sec)	N/A	0.141	0.000	0.032	0.184	0.227	0.013	0.265	0.003

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	28	27	26	26	24	26	26
N.S.	1	1.00	1.04	1.00	0.96	0.96	0.89	0.96	0.96
time (sec)	N/A	0.137	0.000	0.027	0.180	0.236	0.012	0.273	0.002

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [1] had the largest ratio of [1.250000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.00	4	1.250
2	A	3	3	1.00	11	0.273
3	A	3	3	1.00	11	0.273
4	A	2	2	1.00	2	1.000
5	A	1	1	1.00	7	0.143
6	A	4	3	1.00	7	0.429
7	A	1	1	1.00	11	0.091
8	A	1	1	1.00	6	0.167
9	A	5	4	1.00	7	0.571
10	A	4	3	1.00	4	0.750
11	A	2	2	1.00	6	0.333
12	A	1	1	1.00	6	0.167
13	A	1	1	1.00	16	0.062
14	A	2	2	1.00	7	0.286
15	A	1	1	1.00	14	0.071
16	A	4	3	1.00	5	0.600
17	A	1	1	1.00	7	0.143
18	A	1	1	1.00	11	0.091
19	A	3	2	1.00	11	0.182
20	A	1	1	1.00	5	0.200
21	A	2	2	1.00	6	0.333
22	A	2	2	1.00	9	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	5	4	0.90	14	0.286
24	A	2	2	1.00	9	0.222
25	A	3	3	1.00	7	0.429
26	A	3	2	1.00	15	0.133
27	A	3	2	1.00	17	0.118
28	A	4	3	1.00	13	0.231
29	A	1	1	1.00	5	0.200
30	A	6	5	0.90	8	0.625
31	A	9	8	1.12	11	0.727
32	A	2	2	1.00	12	0.167
33	A	7	6	0.95	6	1.000
34	A	2	2	1.00	9	0.222
35	A	5	4	1.12	13	0.308
36	A	5	4	1.00	15	0.267
37	A	3	3	1.00	15	0.200
38	A	7	6	1.13	13	0.462
39	A	3	3	1.00	15	0.200
40	A	6	5	1.00	29	0.172
41	A	3	3	1.00	4	0.750
42	A	8	7	1.16	19	0.368
43	A	2	2	1.00	6	0.333
44	A	2	2	1.00	5	0.400
45	A	1	1	1.00	10	0.100
46	A	2	2	1.00	13	0.154
47	A	1	1	1.00	5	0.200
48	A	1	1	1.00	7	0.143
49	A	8	7	1.10	9	0.778
50	A	9	8	1.68	7	1.143
51	A	1	1	1.00	27	0.037
52	A	1	1	1.00	4	0.250
53	A	5	4	1.08	6	0.667
54	A	2	2	1.00	10	0.200
55	A	2	2	1.00	9	0.222
56	A	3	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	1	1	1.00	4	0.250
58	A	2	2	1.00	7	0.286
59	A	2	2	1.00	11	0.182
60	A	2	2	1.00	9	0.222
61	A	5	5	1.00	4	1.250
62	A	7	6	0.95	6	1.000
63	A	5	5	1.00	4	1.250
64	A	2	2	1.00	6	0.333
65	A	4	3	1.00	9	0.333
66	A	3	2	1.00	12	0.167
67	A	3	2	1.00	20	0.100
68	A	4	4	1.00	10	0.400
69	A	8	7	1.00	30	0.233
70	B	8	7	2.88	39	0.179
71	B	6	6	2.75	48	0.125
72	B	5	5	2.75	31	0.161
73	A	3	3	1.00	15	0.200
74	A	5	5	1.00	4	1.250
75	A	2	2	1.00	11	0.182
76	A	2	2	1.00	13	0.154
77	A	2	2	1.00	33	0.061
78	A	1	1	1.00	7	0.143
79	A	3	3	1.00	8	0.375
80	A	3	2	1.00	9	0.222
81	A	4	3	0.88	11	0.273
82	A	5	5	1.00	8	0.625
83	A	3	3	1.00	15	0.200
84	A	3	3	1.00	16	0.188
85	A	1	1	1.00	6	0.167
86	A	1	1	1.00	3	0.333
87	A	5	4	0.89	17	0.235
88	A	3	3	1.00	4	0.750
89	A	2	2	1.00	11	0.182
90	A	4	3	1.09	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	4	3	1.00	7	0.429
92	A	3	2	1.00	11	0.182
93	A	4	3	1.00	13	0.231
94	A	2	2	1.00	8	0.250
95	A	4	3	1.00	7	0.429
96	A	1	1	1.00	4	0.250
97	A	4	3	1.00	5	0.600
98	A	4	3	1.17	17	0.176
99	A	4	3	1.00	16	0.188
100	A	3	3	1.00	15	0.200
101	A	4	3	0.90	15	0.200
102	A	3	3	1.21	8	0.375
103	A	1	1	1.00	20	0.050
104	A	1	1	1.00	25	0.040
105	A	1	1	1.00	26	0.038
106	A	1	1	1.00	31	0.032
107	A	1	1	1.00	24	0.042
108	A	1	1	1.00	29	0.034
109	A	1	1	1.00	18	0.056
110	A	1	1	1.00	23	0.043
111	A	1	1	1.00	24	0.042
112	A	1	1	1.00	29	0.034
113	A	1	1	1.00	27	0.037

CHAPTER 3

Listing of integrals

3.1 $\int \cot^4(x)\,dx$

3.1.1 Optimal result

Integrand size = 4, antiderivative size = 12

$$\int \cot^4(x)\,dx = x + \cot(x) - \frac{\cot^3(x)}{3}$$

output
```
x+cot(x)-1/3*cot(x)^3
```

3.1.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \cot^4(x)\,dx = -\frac{1}{3}\cot^3(x)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(x)\right)$$

input
```
Integrate[Cot[x]^4,x]
```

output
```
-1/3*(Cot[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[x]^2])
```

3.1.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(x)\, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x+\frac{\pi}{2}\right)^4 dx$$

$$\Big\downarrow \text{3954}$$

$$-\int \cot^2(x)dx-\frac{1}{3}\cot^3(x)$$

$$\Big\downarrow \text{3042}$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)^2 dx-\frac{1}{3}\cot^3(x)$$

$$\Big\downarrow \text{3954}$$

$$\int 1dx-\frac{1}{3}\cot^3(x)+\cot(x)$$

$$\Big\downarrow \text{24}$$

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

input
```
Int[Cot[x]^4,x]
```

output
```
x + Cot[x] - Cot[x]^3/3
```

3.1.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.1.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$x+\cot\left(x\right)-\frac{\left(\cot^{3}\left(x\right)\right)}{3}$	11
derivativedivides	$-\frac{\left(\cot^{3}\left(x\right)\right)}{3}+\cot\left(x\right)-\frac{\pi}{2}+\operatorname{arccot}\left(\cot\left(x\right)\right)$	16
default	$-\frac{\left(\cot^{3}\left(x\right)\right)}{3}+\cot\left(x\right)-\frac{\pi}{2}+\operatorname{arccot}\left(\cot\left(x\right)\right)$	16
norman	$\frac{-\frac{1}{3}+\tan^{2}\left(x\right)+x\left(\tan^{3}\left(x\right)\right)}{\tan\left(x\right)^{3}}$	18
risch	$x+\frac{4 i \left(3\,{\mathrm e}^{4 i x}-3\,{\mathrm e}^{2 i x}+2\right)}{3 \left({\mathrm e}^{2 i x}-1\right)^{3}}$	31

input
```
int(cot(x)^4,x,method=_RETURNVERBOSE)
```

output
```
x+cot(x)-1/3*cot(x)^3
```

3.1.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(10) = 20.

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 4.00

$$\int \cot^4(x)\,dx = \frac{4\cos(2x)^2 + 3(x\cos(2x) - x)\sin(2x) + 2\cos(2x) - 2}{3(\cos(2x) - 1)\sin(2x)}$$

input
```
integrate(cot(x)^4,x, algorithm="fricas")
```

output
```
1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)/((cos(2*
x) - 1)*sin(2*x))
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \cot^4(x)\,dx = x + \frac{\cos(x)}{\sin(x)} - \frac{\cos^3(x)}{3\sin^3(x)}$$

input
```
integrate(cot(x)**4,x)
```

output
```
x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \cot^4(x)\,dx = x + \frac{3\tan(x)^2 - 1}{3\tan(x)^3}$$

input
```
integrate(cot(x)^4,x, algorithm="maxima")
```

output
```
x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3
```

3.1.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(10) = 20$.

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.83

$$\int \cot^4(x)\,dx = \frac{1}{24}\tan\left(\frac{1}{2}x\right)^3 + x + \frac{15\tan\left(\frac{1}{2}x\right)^2 - 1}{24\tan\left(\frac{1}{2}x\right)^3} - \frac{5}{8}\tan\left(\frac{1}{2}x\right)$$

input
```
integrate(cot(x)^4,x, algorithm="giac")
```

output
```
1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 5/8*tan(
1/2*x)
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \cot^4(x)\,dx = -\frac{\cot(x)^3}{3} + \cot(x) + x$$

input
```
int(cot(x)^4,x)
```

output
```
x + cot(x) - cot(x)^3/3
```

3.2 $\int \frac{1}{x^4(1+x^2)}\,dx$

3.2.1 Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{1}{x^4\,(1+x^2)}\,dx = -\frac{1}{3x^3} + \frac{1}{x} + \arctan(x)$$

output
```
-1/3/x^3+1/x+arctan(x)
```

3.2.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\,(1+x^2)}\,dx = -\frac{1}{3x^3} + \frac{1}{x} + \arctan(x)$$

input
```
Integrate[1/(x^4*(1 + x^2)),x]
```

output
```
-1/3*1/x^3 + x^(-1) + ArcTan[x]
```

3.2.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^2+1)} \, dx$$

$$\Big\downarrow \text{ 264}$$

$$-\int \frac{1}{x^2 (x^2+1)} dx - \frac{1}{3x^3}$$

$$\Big\downarrow \text{ 264}$$

$$\int \frac{1}{x^2+1} dx - \frac{1}{3x^3} + \frac{1}{x}$$

$$\Big\downarrow \text{ 216}$$

$$\arctan(x) - \frac{1}{3x^3} + \frac{1}{x}$$

input
```
Int[1/(x^4*(1 + x^2)),x]
```

output
```
-1/3*1/x^3 + x^(-1) + ArcTan[x]
```

3.2.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

3.2.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{1}{3x^3}+\frac{1}{x}+\arctan(x)$	12
meijerg	$-\frac{1}{3x^3}+\frac{1}{x}+\arctan(x)$	12
risch	$\frac{x^2-\frac{1}{3}}{x^3}+\arctan(x)$	13
parallelrisch	$-\frac{3i\ln(x-i)x^3-3i\ln(x+i)x^3+2-6x^2}{6x^3}$	35

input
```
int(1/x^4/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3+1/x+arctan(x)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^4(1+x^2)}\,dx = \frac{3\,x^3\arctan(x)+3\,x^2-1}{3\,x^3}$$

input
```
integrate(1/x^4/(x^2+1),x, algorithm="fricas")
```

output
```
1/3*(3*x^3*arctan(x) + 3*x^2 - 1)/x^3
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4(1+x^2)}\,dx = \operatorname{atan}(x)+\frac{3x^2-1}{3x^3}$$

input
```
integrate(1/x**4/(x**2+1),x)
```

output
```
atan(x) + (3*x**2 - 1)/(3*x**3)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^4 (1+x^2)}\,dx = \frac{3\,x^2-1}{3\,x^3} + \arctan\left(x\right)$$

input
```
integrate(1/x^4/(x^2+1),x, algorithm="maxima")
```

output
```
1/3*(3*x^2 - 1)/x^3 + arctan(x)
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^4 (1+x^2)}\,dx = \frac{3\,x^2-1}{3\,x^3} + \arctan\left(x\right)$$

input
```
integrate(1/x^4/(x^2+1),x, algorithm="giac")
```

output
```
1/3*(3*x^2 - 1)/x^3 + arctan(x)
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (1+x^2)}\,dx = \mathrm{atan}(x) + \frac{x^2-\frac{1}{3}}{x^3}$$

input
```
int(1/(x^4*(x^2 + 1)),x)
```

output
```
atan(x) + (x^2 - 1/3)/x^3
```

3.3 $\int \frac{x+x^2}{\sqrt{x}} \, dx$

3.3.1 Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{x+x^2}{\sqrt{x}} \, dx = \frac{2x^{3/2}}{3} + \frac{2x^{5/2}}{5}$$

output
```
2/3*x^(3/2)+2/5*x^(5/2)
```

3.3.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{x+x^2}{\sqrt{x}} \, dx = \frac{2}{15} x^{3/2} (5+3x)$$

input
```
Integrate[(x + x^2)/Sqrt[x],x]
```

output
```
(2*x^(3/2)*(5 + 3*x))/15
```

3.3.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {9, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x}{\sqrt{x}} \, dx$$
$$\Big\downarrow \; 9$$
$$\int \sqrt{x}(x+1)dx$$
$$\Big\downarrow \; 53$$
$$\int \left(x^{3/2}+\sqrt{x}\right) dx$$
$$\Big\downarrow \; 2009$$
$$\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{3}$$

input
```
Int[(x + x^2)/Sqrt[x],x]
```

output
```
(2*x^(3/2))/3 + (2*x^(5/2))/5
```

3.3.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.3.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}(5+3x)}{15}$	11
trager	$\frac{2x^{\frac{3}{2}}(5+3x)}{15}$	11
risch	$\frac{2x^{\frac{3}{2}}(5+3x)}{15}$	11
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3}+\frac{2x^{\frac{5}{2}}}{5}$	12
default	$\frac{2x^{\frac{3}{2}}}{3}+\frac{2x^{\frac{5}{2}}}{5}$	12

input
```
int((x^2+x)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*x^(3/2)*(5+3*x)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{x+x^2}{\sqrt{x}}\,dx = \frac{2}{15}\left(3\,x^2+5\,x\right)\sqrt{x}$$

input
```
integrate((x^2+x)/x^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*x^2 + 5*x)*sqrt(x)
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x+x^2}{\sqrt{x}}\,dx=\frac{2x^{\frac{5}{2}}}{5}+\frac{2x^{\frac{3}{2}}}{3}$$

input
```
integrate((x**2+x)/x**(1/2),x)
```

output
```
2*x**(5/2)/5 + 2*x**(3/2)/3
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{x+x^2}{\sqrt{x}}\,dx=\frac{2}{5}\,x^{\frac{5}{2}}+\frac{2}{3}\,x^{\frac{3}{2}}$$

input
```
integrate((x^2+x)/x^(1/2),x, algorithm="maxima")
```

output
```
2/5*x^(5/2) + 2/3*x^(3/2)
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{x+x^2}{\sqrt{x}}\,dx=\frac{2}{5}\,x^{\frac{5}{2}}+\frac{2}{3}\,x^{\frac{3}{2}}$$

input
```
integrate((x^2+x)/x^(1/2),x, algorithm="giac")
```

output
```
2/5*x^(5/2) + 2/3*x^(3/2)
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int \frac{x+x^2}{\sqrt{x}}\,dx = \frac{2\,x^{3/2}\,(3\,x+5)}{15}$$

input
```
int((x + x^2)/x^(1/2),x)
```

output
```
(2*x^(3/2)*(3*x + 5))/15
```

3.4 $\int \cos(x)\,dx$

3.4.1 Optimal result

Integrand size = 2, antiderivative size = 2

$$\int \cos(x)\,dx = \sin(x)$$

output
```
sin(x)
```

3.4.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

input
```
Integrate[Cos[x],x]
```

output
```
Sin[x]
```

3.4.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\sin(x)$$

input
```
Int[Cos[x],x]
```

output
```
Sin[x]
```

3.4.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.4.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
lookup	$\sin \left(x \right)$	3
default	$\sin \left(x \right)$	3
meijerg	$\sin \left(x \right)$	3
risch	$\sin \left(x \right)$	3
parallelrisch	$\sin \left(x \right)$	3
norman	$\frac{2 \tan \left(\frac{x}{2}\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}$	17

input
```
int(cos(x),x,method=_RETURNVERBOSE)
```

output
```
sin(x)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\, dx = \sin \left(x\right)$$

input
```
integrate(cos(x),x, algorithm="fricas")
```

output
```
sin(x)
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\, dx = \sin \left(x\right)$$

input
```
integrate(cos(x),x)
```

output
```
sin(x)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

input
```
integrate(cos(x),x, algorithm="maxima")
```

output
```
sin(x)
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

input
```
integrate(cos(x),x, algorithm="giac")
```

output
```
sin(x)
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \cos(x)\,dx = \sin(x)$$

input
```
int(cos(x),x)
```

output
```
sin(x)
```

3.5 $\int e^{x^2} x \, dx$

3.5.1 Optimal result

Integrand size = 7, antiderivative size = 9

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

output
```
1/2*exp(x^2)
```

3.5.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

input
```
Integrate[E^x^2*x,x]
```

output
```
E^x^2/2
```

3.5.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} x\,dx$$

$$\Big\downarrow \ 2638$$

$$\frac{e^{x^2}}{2}$$

input
```
Int[E^x^2*x,x]
```

output
```
E^x^2/2
```

3.5.3.1 Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

3.5.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{e^{x^2}}{2}$	7
derivativedivides	$\frac{e^{x^2}}{2}$	7
default	$\frac{e^{x^2}}{2}$	7
norman	$\frac{e^{x^2}}{2}$	7
risch	$\frac{e^{x^2}}{2}$	7
parallelrisch	$\frac{e^{x^2}}{2}$	7
meijerg	$-\frac{1}{2}+\frac{e^{x^2}}{2}$	9
parts	$\frac{\operatorname{erfi}(x)\sqrt{\pi}\,x}{2}-\frac{\sqrt{\pi}\left(x\operatorname{erfi}(x)-\frac{e^{x^2}}{\sqrt{\pi}}\right)}{2}$	29

input
```
int(exp(x^2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x^2)
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2}x\,dx=\frac{1}{2}\,e^{\left(x^2\right)}$$

input
```
integrate(exp(x^2)*x,x, algorithm="fricas")
```

output
```
1/2*e^(x^2)
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int e^{x^2} x\,dx = \frac{e^{x^2}}{2}$$

input
```
integrate(exp(x**2)*x,x)
```

output
```
exp(x**2)/2
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x\,dx = \frac{1}{2}\,e^{(x^2)}$$

input
```
integrate(exp(x^2)*x,x, algorithm="maxima")
```

output
```
1/2*e^(x^2)
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x\,dx = \frac{1}{2}\,e^{(x^2)}$$

input
```
integrate(exp(x^2)*x,x, algorithm="giac")
```

output
```
1/2*e^(x^2)
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x\,dx = \frac{\mathrm{e}^{x^2}}{2}$$

input
```
int(x*exp(x^2),x)
```

output
```
exp(x^2)/2
```

3.6 $\int \sec^2(x)\tan(x)\,dx$

3.6.1 Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \sec^2(x)\tan(x)\,dx = \frac{\sec^2(x)}{2}$$

output
```
1/2*sec(x)^2
```

3.6.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\tan(x)\,dx = \frac{\sec^2(x)}{2}$$

input
```
Integrate[Sec[x]^2*Tan[x],x]
```

output
```
Sec[x]^2/2
```

3.6.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\sec^2(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(x)\sec(x)^2 dx$$

$$\Big\downarrow \text{ 3086}$$

$$\int \sec(x)d\sec(x)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\sec^2(x)}{2}$$

input
```
Int[Sec[x]^2*Tan[x],x]
```

output
```
Sec[x]^2/2
```

3.6.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.6.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\sec^2(x))}{2}$	7
default	$\frac{(\sec^2(x))}{2}$	7
risch	$\frac{2\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^2}$	17

input
```
int(sec(x)^2*tan(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sec(x)^2
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2\,\cos{(x)}^2}$$

input
```
integrate(sec(x)^2*tan(x),x, algorithm="fricas")
```

output
```
1/2/cos(x)^2
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2\cos^2{(x)}}$$

input
```
integrate(sec(x)**2*tan(x),x)
```

output
```
1/(2*cos(x)**2)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2}\,\tan{(x)}^2$$

input
```
integrate(sec(x)^2*tan(x),x, algorithm="maxima")
```

output
```
1/2*tan(x)^2
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2\,\cos{(x)}^2}$$

input
```
integrate(sec(x)^2*tan(x),x, algorithm="giac")
```

output
```
1/2/cos(x)^2
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{\tan(x)^2}{2}$$

input
```
int(tan(x)/cos(x)^2,x)
```

output
```
tan(x)^2/2
```

3.7 $\int x\sqrt{1+x^2}\,dx$

3.7.1 Optimal result

Integrand size = 11, antiderivative size = 13

$$\int x\sqrt{1+x^2}\,dx=\frac{1}{3}\left(1+x^2\right)^{3/2}$$

output
```
1/3*(x^2+1)^(3/2)
```

3.7.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int x\sqrt{1+x^2}\,dx=\frac{1}{3}\left(1+x^2\right)^{3/2}$$

input
```
Integrate[x*Sqrt[1 + x^2],x]
```

output
```
(1 + x^2)^(3/2)/3
```

3.7.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{x^2+1}\,dx$$

$$\Big\downarrow \text{ 241}$$

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

input
```
Int[x*Sqrt[1 + x^2],x]
```

output
```
(1 + x^2)^(3/2)/3
```

3.7.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

3.7.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
derivativedivides	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
default	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
risch	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
pseudoelliptic	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
trager	$\left(\frac{x^2}{3}+\frac{1}{3}\right)\sqrt{x^2+1}$	16
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2x^2+2\right)\sqrt{x^2+1}}{3}}{4\sqrt{\pi}}$	31

input
```
int(x*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^2+1)^(3/2)
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x*(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(x^2 + 1)^(3/2)
```

3.7.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(8) = 16.

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int x\sqrt{1+x^2}\,dx = \frac{x^2\sqrt{x^2+1}}{3} + \frac{\sqrt{x^2+1}}{3}$$

input
```
integrate(x*(x**2+1)**(1/2),x)
```

output
```
x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x*(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(x^2 + 1)^(3/2)
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x*(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/3*(x^2 + 1)^(3/2)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{(x^2+1)^{3/2}}{3}$$

input
```
int(x*(x^2 + 1)^(1/2),x)
```

output
```
(x^2 + 1)^(3/2)/3
```

3.8 $\int e^x \sin(x)\,dx$

3.8.1 Optimal result

Integrand size = 6, antiderivative size = 19

$$\int e^x \sin(x)\,dx = -\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x\sin(x)$$

output
```
-1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

3.8.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int e^x \sin(x)\,dx = \frac{1}{2}e^x(-\cos(x) + \sin(x))$$

input
```
Integrate[E^x*Sin[x],x]
```

output
```
(E^x*(-Cos[x] + Sin[x]))/2
```

3.8.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sin(x)\,dx$$

$$\Big\downarrow \text{4932}$$

$$\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^x\cos(x)$$

input
```
Int[E^x*Sin[x],x]
```

output
```
-1/2*(E^x*Cos[x]) + (E^x*Sin[x])/2
```

3.8.3.1 Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.8.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{{\mathrm e}^{x} \left(-\cos \left(x \right)+\sin \left(x \right)\right)}{2}$	12
default	$-\frac{{\mathrm e}^{x} \cos \left(x \right)}{2}+\frac{{\mathrm e}^{x} \sin \left(x \right)}{2}$	14
norman	$\frac{{\mathrm e}^{x} \tan \left(\frac{x}{2}\right)+\frac{{\mathrm e}^{x} \left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\frac{{\mathrm e}^{x}}{2}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	34
risch	$-\frac{{\mathrm e}^{\left(1+i\right) x}}{4}-\frac{i {\mathrm e}^{\left(1+i\right) x}}{4}-\frac{{\mathrm e}^{\left(1-i\right) x}}{4}+\frac{i {\mathrm e}^{\left(1-i\right) x}}{4}$	36

input
```
int(exp(x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)*(-cos(x)+sin(x))
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sin(x)\,dx = -\frac{1}{2}\cos(x)\,e^x + \frac{1}{2}\,e^x\sin(x)$$

input
```
integrate(exp(x)*sin(x),x, algorithm="fricas")
```

output
```
-1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int e^x \sin(x)\,dx = \frac{e^x\sin(x)}{2} - \frac{e^x\cos(x)}{2}$$

input
```
integrate(exp(x)*sin(x),x)
```

output
```
exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{1}{2}(\cos(x) - \sin(x))e^x$$

input
```
integrate(exp(x)*sin(x),x, algorithm="maxima")
```

output
```
-1/2*(cos(x) - sin(x))*e^x
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{1}{2}\,(\cos{(x)} - \sin{(x)})e^x$$

input
```
integrate(exp(x)*sin(x),x, algorithm="giac")
```

output
```
-1/2*(cos(x) - sin(x))*e^x
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{e^x\,(\cos{(x)} - \sin{(x)})}{2}$$

input
```
int(exp(x)*sin(x),x)
```

output
```
-(exp(x)*(cos(x) - sin(x)))/2
```

3.9 $\int \cot(x)\csc^3(x)\,dx$

3.9.1 Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \cot(x)\csc^3(x)\,dx = -\frac{1}{3}\csc^3(x)$$

output
```
-1/3*csc(x)^3
```

3.9.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cot(x)\csc^3(x)\,dx = -\frac{1}{3}\csc^3(x)$$

input
```
Integrate[Cot[x]*Csc[x]^3,x]
```

output
```
-1/3*Csc[x]^3
```

3.9.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 25, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x)\csc^3(x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan\left(x-\frac{\pi}{2}\right)\left(-\sec\left(x-\frac{\pi}{2}\right)^3\right)dx$$

$$\Big\downarrow \ 25$$

$$-\int \sec\left(x-\frac{\pi}{2}\right)^3\tan\left(x-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \ 3086$$

$$-\int \csc^2(x)d\csc(x)$$

$$\Big\downarrow \ 15$$

$$-\frac{1}{3}\csc^3(x)$$

input
```
Int[Cot[x]*Csc[x]^3,x]
```

output
```
-1/3*Csc[x]^3
```

3.9.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.9.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{(\csc^3(x))}{3}$	7
default	$-\frac{(\csc^3(x))}{3}$	7
parallelrisch	$-\frac{(\csc^3(x))}{3}$	7
risch	$\frac{8i e^{3ix}}{3(e^{2ix}-1)^3}$	18
norman	$\frac{-\frac{1}{24}-\frac{(\tan^2(\frac{x}{2}))}{8}-\frac{(\tan^4(\frac{x}{2}))}{8}-\frac{(\tan^6(\frac{x}{2}))}{24}}{\tan(\frac{x}{2})^3}$	34

input
```
int(cos(x)*csc(x)^2/sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*csc(x)^3
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \cot(x)\csc^3(x)\,dx = \frac{1}{3\left(\cos{(x)}^2-1\right)\sin{(x)}}$$

input
```
integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="fricas")
```

output
```
1/3/((cos(x)^2 - 1)*sin(x))
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cot(x)\csc^3(x)\,dx = -\frac{1}{3\sin^3(x)}$$

input
```
integrate(cos(x)*csc(x)**2/sin(x)**2,x)
```

output
```
-1/(3*sin(x)**3)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cot(x)\csc^3(x)\,dx = -\frac{1}{3\,\sin(x)^3}$$

input
```
integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="maxima")
```

output
```
-1/3/sin(x)^3
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cot(x)\csc^3(x)\,dx = -\frac{1}{3\,\sin(x)^3}$$

input
```
integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="giac")
```

output
```
-1/3/sin(x)^3
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cot(x)\csc^3(x)\,dx = -\frac{1}{3\sin(x)^3}$$

input
```
int(cos(x)/sin(x)^4,x)
```

output
```
-1/(3*sin(x)^3)
```

3.10 $\int \sin(e^x)\,dx$

3.10.1 Optimal result

Integrand size = 4, antiderivative size = 4

$$\int \sin(e^x)\,dx = \mathrm{Si}(e^x)$$

output
```
Si(exp(x))
```

3.10.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sin(e^x)\,dx = \mathrm{Si}(e^x)$$

input
```
Integrate[Sin[E^x],x]
```

output
```
SinIntegral[E^x]
```

3.10.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {2720, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e^x)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int e^{-x}\sin(e^x)\,de^x$$

$$\Big\downarrow \text{ 3042}$$

$$\int e^{-x}\sin(e^x)\,de^x$$

$$\Big\downarrow \text{ 3780}$$

$$\text{Si}(e^x)$$

input
```
Int[Sin[E^x],x]
```

output
```
SinIntegral[E^x]
```

3.10.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

3.10.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\operatorname{Si}(e^{x})$	4
default	$\operatorname{Si}(e^{x})$	4
risch	$-\frac{\pi\ \operatorname{csgn}(e^{x})}{2}+\operatorname{Si}(e^{x})$	11

input
```
int(sin(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
Si(exp(x))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \sin(e^{x})\ dx = \operatorname{Si}(e^{x})$$

input
```
integrate(sin(exp(x)),x, algorithm="fricas")
```

output
```
sin_integral(e^x)
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \sin(e^{x})\ dx = \operatorname{Si}(e^{x})$$

input
```
integrate(sin(exp(x)),x)
```

output
```
Si(exp(x))
```

3.10.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \sin(e^x)\,dx = -\frac{1}{2}i\,\mathrm{Ei}(i\,e^x) + \frac{1}{2}i\,\mathrm{Ei}(-i\,e^x)$$

input
```
integrate(sin(exp(x)),x, algorithm="maxima")
```

output
```
-1/2*I*Ei(I*e^x) + 1/2*I*Ei(-I*e^x)
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \sin(e^x)\,dx = \mathrm{Si}(e^x)$$

input
```
integrate(sin(exp(x)),x, algorithm="giac")
```

output
```
sin_integral(e^x)
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int \sin(e^x)\,dx = \mathrm{sinint}(\mathrm{e}^x)$$

input
```
int(sin(exp(x)),x)
```

output
```
sinint(exp(x))
```

3.11 $\int \frac{\sin(y)}{y} \, dy$

3.11.1 Optimal result

Integrand size = 6, antiderivative size = 2

$$\int \frac{\sin(y)}{y} \, dy = \operatorname{Si}(y)$$

output
```
Si(y)
```

3.11.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sin(y)}{y} \, dy = \operatorname{Si}(y)$$

input
```
Integrate[Sin[y]/y,y]
```

output
```
SinIntegral[y]
```

3.11.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(y)}{y} \, dy$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(y)}{y} dy$$

$$\Big\downarrow \; 3780$$

$$\text{Si}(y)$$

input
```
Int[Sin[y]/y,y]
```

output
```
SinIntegral[y]
```

3.11.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

3.11.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\mathrm{Si}(y)$	3
meijerg	$\mathrm{Si}(y)$	3
risch	$-\frac{\pi\ \mathrm{csgn}(y)}{2}+\mathrm{Si}(y)$	9

input
```
int(sin(y)/y,y,method=_RETURNVERBOSE)
```

output
```
Si(y)
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sin(y)}{y}\,dy = \mathrm{Si}(y)$$

input
```
integrate(sin(y)/y,y, algorithm="fricas")
```

output
```
sin_integral(y)
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sin(y)}{y}\,dy = \mathrm{Si}(y)$$

input
```
integrate(sin(y)/y,y)
```

output
```
Si(y)
```

3.11.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{\sin(y)}{y}\,dy = -\frac{1}{2}i\,\mathrm{Ei}(i\,y) + \frac{1}{2}i\,\mathrm{Ei}(-i\,y)$$

input
```
integrate(sin(y)/y,y, algorithm="maxima")
```

output
```
-1/2*I*Ei(I*y) + 1/2*I*Ei(-I*y)
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{\sin(y)}{y}\,dy = \mathrm{Si}\,(y)$$

input
```
integrate(sin(y)/y,y, algorithm="giac")
```

output
```
sin_integral(y)
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(y)}{y}\,dy = \mathrm{sinint}(y)$$

input
```
int(sin(y)/y,y)
```

output
```
sinint(y)
```

3.12 $\int (e^x + \sin(x))\,dx$

3.12.1 Optimal result

Integrand size = 6, antiderivative size = 8

$$\int (e^x + \sin(x))\,dx = e^x - \cos(x)$$

output
```
exp(x)-cos(x)
```

3.12.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int (e^x + \sin(x))\,dx = e^x - \cos(x)$$

input
```
Integrate[E^x + Sin[x],x]
```

output
```
E^x - Cos[x]
```

3.12.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e^x + \sin(x))\, dx$$

$$\Big\downarrow \text{2009}$$

$$e^x - \cos(x)$$

input
```
Int[E^x + Sin[x],x]
```

output
```
E^x - Cos[x]
```

3.12.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.12.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

method	result	size
default	$e^{x} - \cos\left(x\right)$	8
risch	$e^{x} - \cos\left(x\right)$	8
parts	$e^{x} - \cos\left(x\right)$	8
parallelrisch	$-\cos\left(x\right) + e^{x} - 1$	9
meijerg	$-1 + e^{x} + \sqrt{\pi}\left(\frac{1}{\sqrt{\pi}} - \frac{\cos(x)}{\sqrt{\pi}}\right)$	20
norman	$\frac{e^{x}\left(\tan^{2}\left(\frac{x}{2}\right)\right) - 2 + e^{x}}{1 + \tan^{2}\left(\frac{x}{2}\right)}$	25

input
```
int(exp(x)+sin(x),x,method=_RETURNVERBOSE)
```

output
```
exp(x)-cos(x)
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int (e^x + \sin(x)) \, dx = -\cos(x) + e^x$$

input
```
integrate(exp(x)+sin(x),x, algorithm="fricas")
```

output
```
-cos(x) + e^x
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int (e^x + \sin(x)) \, dx = e^x - \cos(x)$$

input
```
integrate(exp(x)+sin(x),x)
```

output
```
exp(x) - cos(x)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int (e^x + \sin(x)) \, dx = -\cos(x) + e^x$$

input
```
integrate(exp(x)+sin(x),x, algorithm="maxima")
```

output
```
-cos(x) + e^x
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int (e^x + \sin(x)) \, dx = -\cos(x) + e^x$$

input
```
integrate(exp(x)+sin(x),x, algorithm="giac")
```

output
```
-cos(x) + e^x
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int (e^x + \sin(x)) \, dx = \mathrm{e}^x - \cos(x)$$

input
```
int(exp(x) + sin(x),x)
```

output
```
exp(x) - cos(x)
```

3.13 $\int \left(e^{x^2}+2e^{x^2}x^2\right)\,dx$

3.13.1 Optimal result

Integrand size = 16, antiderivative size = 7

$$\int \left(e^{x^2}+2e^{x^2}x^2\right)\,dx=e^{x^2}x$$

output
```
exp(x^2)*x
```

3.13.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \left(e^{x^2}+2e^{x^2}x^2\right)\,dx=e^{x^2}x$$

input
```
Integrate[E^x^2 + 2*E^x^2*x^2,x]
```

output
```
E^x^2*x
```

3.13.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2e^{x^2}x^2+e^{x^2}\right) dx$$

$$\Big\downarrow \; 2009$$

$$e^{x^2}x$$

input
```
Int[E^x^2 + 2*E^x^2*x^2,x]
```

output
```
E^x^2*x
```

3.13.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.13.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

method	result	size
gosper	$\mathrm{e}^{x^2}x$	7
default	$\mathrm{e}^{x^2}x$	7
norman	$\mathrm{e}^{x^2}x$	7
risch	$\mathrm{e}^{x^2}x$	7
parallelrisch	$\mathrm{e}^{x^2}x$	7
meijerg	$i\left(-ix\,\mathrm{e}^{x^2}+\frac{i\,\mathrm{erfi}(x)\sqrt{\pi}}{2}\right)+\frac{\mathrm{erfi}(x)\sqrt{\pi}}{2}$	29
parts	$\mathrm{erfi}\left(x\right)\sqrt{\pi}\,x^2+\frac{\mathrm{erfi}(x)\sqrt{\pi}}{2}-2\sqrt{\pi}\left(\frac{x^2\,\mathrm{erfi}(x)}{2}-\frac{\frac{\mathrm{e}^{x^2}x}{2}-\frac{\mathrm{erfi}(x)\sqrt{\pi}}{4}}{\sqrt{\pi}}\right)$	51

input
```
int(2*exp(x^2)*x^2+exp(x^2),x,method=_RETURNVERBOSE)
```

output
```
exp(x^2)*x
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \left(e^{x^2}+2e^{x^2}x^2\right)\,dx=xe^{\left(x^2\right)}$$

input
```
integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="fricas")
```

output
```
x*e^(x^2)
```

3.13.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \left(e^{x^2}+2e^{x^2}x^2\right)\,dx=xe^{x^2}$$

input
```
integrate(exp(x**2)+2*exp(x**2)*x**2,x)
```

output
```
x*exp(x**2)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \left(e^{x^2}+2e^{x^2}x^2\right)\,dx=xe^{\left(x^2\right)}$$

input
```
integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="maxima")
```

output
```
x*e^(x^2)
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \left(e^{x^2} + 2e^{x^2}x^2\right) dx = xe^{(x^2)}$$

input
```
integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="giac")
```

output
```
x*e^(x^2)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int \left(e^{x^2} + 2e^{x^2}x^2\right) dx = x\,\mathrm{e}^{x^2}$$

input
```
int(exp(x^2) + 2*x^2*exp(x^2),x)
```

output
```
x*exp(x^2)
```

3.14 $\int (e^x + x)^2 \, dx$

3.14.1 Optimal result

Integrand size = 7, antiderivative size = 28

$$\int (e^x + x)^2 \, dx = -2e^x + \frac{e^{2x}}{2} + 2e^x x + \frac{x^3}{3}$$

output
```
-2*exp(x)+1/2*exp(2*x)+2*exp(x)*x+1/3*x^3
```

3.14.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (e^x + x)^2 \, dx = \frac{e^{2x}}{2} + \frac{x^3}{3} + e^x(-2 + 2x)$$

input
```
Integrate[(E^x + x)^2,x]
```

output
```
E^(2*x)/2 + x^3/3 + E^x*(-2 + 2*x)
```

3.14.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+e^x)^2\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(x^2+2e^x x+e^{2x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3}{3}+2e^x x-2e^x+\frac{e^{2x}}{2}$$

input
```
Int[(E^x + x)^2,x]
```

output
```
-2*E^x + E^(2*x)/2 + 2*E^x*x + x^3/3
```

3.14.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.14.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{x^3}{3}+(-2+2x)\,\mathrm{e}^x+\frac{\mathrm{e}^{2x}}{2}$	21
default	$-2\,\mathrm{e}^x+\frac{\mathrm{e}^{2x}}{2}+2\,\mathrm{e}^x x+\frac{x^3}{3}$	22
norman	$-2\,\mathrm{e}^x+\frac{\mathrm{e}^{2x}}{2}+2\,\mathrm{e}^x x+\frac{x^3}{3}$	22
parallelrisch	$-2\,\mathrm{e}^x+\frac{\mathrm{e}^{2x}}{2}+2\,\mathrm{e}^x x+\frac{x^3}{3}$	22
parts	$-2\,\mathrm{e}^x+\frac{\mathrm{e}^{2x}}{2}+2\,\mathrm{e}^x x+\frac{x^3}{3}$	22

input
```
int((exp(x)+x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+(-2+2*x)*exp(x)+1/2*exp(2*x)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int (e^x+x)^2\,dx=\frac{1}{3}\,x^3+2\,(x-1)e^x+\frac{1}{2}\,e^{(2\,x)}$$

input
```
integrate((exp(x)+x)^2,x, algorithm="fricas")
```

output
```
1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int (e^x+x)^2\,dx=\frac{x^3}{3}+\frac{(4x-4)\,e^x}{2}+\frac{e^{2x}}{2}$$

input
```
integrate((exp(x)+x)**2,x)
```

output
```
x**3/3 + (4*x - 4)*exp(x)/2 + exp(2*x)/2
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int (e^x + x)^2 \, dx = \frac{1}{3} x^3 + 2 (x - 1)e^x + \frac{1}{2} e^{(2x)}$$

input
```
integrate((exp(x)+x)^2,x, algorithm="maxima")
```

output
```
1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int (e^x + x)^2 \, dx = \frac{1}{3} x^3 + 2 (x - 1)e^x + \frac{1}{2} e^{(2x)}$$

input
```
integrate((exp(x)+x)^2,x, algorithm="giac")
```

output
```
1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int (e^x + x)^2 \, dx = \frac{e^{2x}}{2} - 2 e^x + 2 x e^x + \frac{x^3}{3}$$

input
```
int((x + exp(x))^2,x)
```

output
```
exp(2*x)/2 - 2*exp(x) + 2*x*exp(x) + x^3/3
```

3.15 $\int (2e^x + e^{2x} + x^2) \, dx$

3.15.1 Optimal result

Integrand size = 14, antiderivative size = 22

$$\int (2e^x + e^{2x} + x^2) \, dx = 2e^x + \frac{e^{2x}}{2} + \frac{x^3}{3}$$

output
```
2*exp(x)+1/2*exp(2*x)+1/3*x^3
```

3.15.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (2e^x + e^{2x} + x^2) \, dx = 2e^x + \frac{e^{2x}}{2} + \frac{x^3}{3}$$

input
```
Integrate[2*E^x + E^(2*x) + x^2,x]
```

output
```
2*E^x + E^(2*x)/2 + x^3/3
```

3.15.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2+2e^x+e^{2x}\right) dx$$
$$\Big\downarrow \; 2009$$
$$\frac{x^3}{3}+2e^x+\frac{e^{2x}}{2}$$

input
```
Int[2*E^x + E^(2*x) + x^2,x]
```

output
```
2*E^x + E^(2*x)/2 + x^3/3
```

3.15.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.15.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
default	$2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}+\frac{x^{3}}{3}$	17
norman	$2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}+\frac{x^{3}}{3}$	17
risch	$2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}+\frac{x^{3}}{3}$	17
parallelrisch	$2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}+\frac{x^{3}}{3}$	17
parts	$2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}+\frac{x^{3}}{3}$	17

input
```
int(2*exp(x)+exp(2*x)+x^2,x,method=_RETURNVERBOSE)
```

output
```
2*exp(x)+1/2*exp(2*x)+1/3*x^3
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \left(2e^x + e^{2x} + x^2\right) \, dx = \frac{1}{3} \, x^3 + \frac{1}{2} \, e^{(2 \, x)} + 2 \, e^x$$

input
```
integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="fricas")
```

output
```
1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \left(2e^x + e^{2x} + x^2\right) \, dx = \frac{x^3}{3} + \frac{e^{2x}}{2} + 2e^x$$

input
```
integrate(2*exp(x)+exp(2*x)+x**2,x)
```

output
```
x**3/3 + exp(2*x)/2 + 2*exp(x)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \left(2e^x + e^{2x} + x^2\right) \, dx = \frac{1}{3} \, x^3 + \frac{1}{2} \, e^{(2 \, x)} + 2 \, e^x$$

input
```
integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="maxima")
```

output
```
1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \left(2e^x + e^{2x} + x^2\right) dx = \frac{1}{3}x^3 + \frac{1}{2}e^{(2x)} + 2e^x$$

input
```
integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="giac")
```

output
```
1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

3.15.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \left(2e^x + e^{2x} + x^2\right) dx = \frac{e^{2x}}{2} + 2e^x + \frac{x^3}{3}$$

input
```
int(exp(2*x) + 2*exp(x) + x^2,x)
```

output
```
exp(2*x)/2 + 2*exp(x) + x^3/3
```

3.16 $\int \cos(x)\sin(x)\,dx$

3.16.1 Optimal result

Integrand size = 5, antiderivative size = 8

$$\int \cos(x)\sin(x)\,dx = \frac{\sin^2(x)}{2}$$

output
```
1/2*sin(x)^2
```

3.16.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos^2(x)$$

input
```
Integrate[Cos[x]*Sin[x],x]
```

output
```
-1/2*Cos[x]^2
```

3.16.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\cos(x)dx$$

$$\Big\downarrow \text{3044}$$

$$\int \sin(x)d\sin(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{\sin^2(x)}{2}$$

input
```
Int[Cos[x]*Sin[x],x]
```

output
```
Sin[x]^2/2
```

3.16.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.16.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\sin^2(x))}{2}$	7
default	$\frac{(\sin^2(x))}{2}$	7
risch	$-\frac{\cos(2x)}{4}$	7
parallelrisch	$\frac{1}{4}-\frac{\cos(2x)}{4}$	9
norman	$\frac{2(\tan^2(\frac{x}{2}))}{(1+\tan^2(\frac{x}{2}))^2}$	19
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{4}$	19

input
```
int(cos(x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x)^2
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\,\cos\left(x\right)^2$$

input
```
integrate(cos(x)*sin(x),x, algorithm="fricas")
```

output
```
-1/2*cos(x)^2
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \cos(x)\sin(x)\,dx = \frac{\sin^2{(x)}}{2}$$

input
```
integrate(cos(x)*sin(x),x)
```

output
```
sin(x)**2/2
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos{(x)}^2$$

input
```
integrate(cos(x)*sin(x),x, algorithm="maxima")
```

output
```
-1/2*cos(x)^2
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos{(x)}^2$$

input
```
integrate(cos(x)*sin(x),x, algorithm="giac")
```

output
```
-1/2*cos(x)^2
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = \frac{\sin(x)^2}{2}$$

input
```
int(cos(x)*sin(x),x)
```

output
```
sin(x)^2/2
```

3.17 $\int e^{x^2} x \, dx$

3.17.1 Optimal result

Integrand size = 7, antiderivative size = 9

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

output
```
1/2*exp(x^2)
```

3.17.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

input
```
Integrate[E^x^2*x,x]
```

output
```
E^x^2/2
```

3.17.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} x \, dx$$
$$\Big\downarrow \text{ 2638}$$
$$\frac{e^{x^2}}{2}$$

input
```
Int[E^x^2*x,x]
```

output
```
E^x^2/2
```

3.17.3.1 Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

3.17.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{e^{x^2}}{2}$	7
derivativedivides	$\frac{e^{x^2}}{2}$	7
default	$\frac{e^{x^2}}{2}$	7
norman	$\frac{e^{x^2}}{2}$	7
risch	$\frac{e^{x^2}}{2}$	7
parallelrisch	$\frac{e^{x^2}}{2}$	7
meijerg	$-\frac{1}{2}+\frac{e^{x^2}}{2}$	9
parts	$\frac{\operatorname{erfi}(x)\sqrt{\pi}\,x}{2}-\frac{\sqrt{\pi}\left(x\operatorname{erfi}(x)-\frac{e^{x^2}}{\sqrt{\pi}}\right)}{2}$	29

input
```
int(exp(x^2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x^2)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2}x\,dx=\frac{1}{2}\,e^{\left(x^2\right)}$$

input
```
integrate(exp(x^2)*x,x, algorithm="fricas")
```

output
```
1/2*e^(x^2)
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int e^{x^2} x\,dx = \frac{e^{x^2}}{2}$$

input
```
integrate(exp(x**2)*x,x)
```

output
```
exp(x**2)/2
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x\,dx = \frac{1}{2}\,e^{(x^2)}$$

input
```
integrate(exp(x^2)*x,x, algorithm="maxima")
```

output
```
1/2*e^(x^2)
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x\,dx = \frac{1}{2}\,e^{(x^2)}$$

input
```
integrate(exp(x^2)*x,x, algorithm="giac")
```

output
```
1/2*e^(x^2)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int e^{x^2} x\, dx = \frac{e^{x^2}}{2}$$

input
```
int(x*exp(x^2),x)
```

output
```
exp(x^2)/2
```

3.18 $\int x\sqrt{1+x^2}\,dx$

3.18.1 Optimal result

Integrand size = 11, antiderivative size = 13

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(1+x^2\right)^{3/2}$$

output
```
1/3*(x^2+1)^(3/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(1+x^2\right)^{3/2}$$

input
```
Integrate[x*Sqrt[1 + x^2],x]
```

output
```
(1 + x^2)^(3/2)/3
```

3.18.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{241\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{x^2+1}\,dx$$

$$\Big\downarrow \text{ 241}$$

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

input
```
Int[x*Sqrt[1 + x^2],x]
```

output
```
(1 + x^2)^(3/2)/3
```

3.18.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

3.18.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
derivativedivides	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
default	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
risch	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
pseudoelliptic	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
trager	$\left(\frac{x^2}{3}+\frac{1}{3}\right)\sqrt{x^2+1}$	16
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2x^2+2\right)\sqrt{x^2+1}}{3}}{4\sqrt{\pi}}$	31

input
```
int(x*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^2+1)^(3/2)
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x*(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(x^2 + 1)^(3/2)
```

3.18.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int x\sqrt{1+x^2}\,dx = \frac{x^2\sqrt{x^2+1}}{3} + \frac{\sqrt{x^2+1}}{3}$$

input
```
integrate(x*(x**2+1)**(1/2),x)
```

output
```
x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x*(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(x^2 + 1)^(3/2)
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x*(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/3*(x^2 + 1)^(3/2)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x\sqrt{1+x^2}\,dx = \frac{(x^2+1)^{3/2}}{3}$$

input
```
int(x*(x^2 + 1)^(1/2),x)
```

output
```
(x^2 + 1)^(3/2)/3
```

3.19 $\int \frac{e^x}{1+e^x} dx$

3.19.1 Optimal result

Integrand size = 11, antiderivative size = 6

$$\int \frac{e^x}{1+e^x} dx = \log(1+e^x)$$

output
```
ln(1+exp(x))
```

3.19.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{1+e^x} dx = \log(1+e^x)$$

input
```
Integrate[E^x/(1 + E^x),x]
```

output
```
Log[1 + E^x]
```

3.19.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2676, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{e^x+1}dx$$
$$\Big\downarrow \text{ 2676}$$
$$\int \frac{1}{e^x+1}de^x$$
$$\Big\downarrow \text{ 16}$$
$$\log \left(e^x+1\right)$$

input
```
Int[E^x/(1 + E^x),x]
```

output
```
Log[1 + E^x]
```

3.19.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

3.19.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\ln(1+e^x)$	6
default	$\ln(1+e^x)$	6
norman	$\ln(1+e^x)$	6
risch	$\ln(1+e^x)$	6
parallelrisch	$\ln(1+e^x)$	6

input
```
int(1/(1+exp(x))*exp(x),x,method=_RETURNVERBOSE)
```

output
```
ln(1+exp(x))
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x}\,dx = \log(e^x+1)$$

input
```
integrate(exp(x)/(1+exp(x)),x, algorithm="fricas")
```

output
```
log(e^x + 1)
```

3.19.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x}\,dx = \log(e^x+1)$$

input
```
integrate(exp(x)/(1+exp(x)),x)
```

output
```
log(exp(x) + 1)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x}\,dx = \log\left(e^x+1\right)$$

input
```
integrate(exp(x)/(1+exp(x)),x, algorithm="maxima")
```

output
```
log(e^x + 1)
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x}\,dx = \log\left(e^x+1\right)$$

input
```
integrate(exp(x)/(1+exp(x)),x, algorithm="giac")
```

output
```
log(e^x + 1)
```

3.19.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x}\,dx = \ln\left(\mathrm{e}^x+1\right)$$

input
```
int(exp(x)/(exp(x) + 1),x)
```

output
```
log(exp(x) + 1)
```

3.20 $\int x^{3/2}\,dx$

3.20.1 Optimal result

Integrand size = 5, antiderivative size = 9

$$\int x^{3/2}\,dx = \frac{2x^{5/2}}{5}$$

output
```
2/5*x^(5/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x^{3/2}\,dx = \frac{2x^{5/2}}{5}$$

input
```
Integrate[x^(3/2),x]
```

output
```
(2*x^(5/2))/5
```

3.20.3 Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\,dx$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2x^{5/2}}{5}$$

input
```
Int[x^(3/2),x]
```

output
```
(2*x^(5/2))/5
```

3.20.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

3.20.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}}{5}$	6
derivativedivides	$\frac{2x^{\frac{5}{2}}}{5}$	6
default	$\frac{2x^{\frac{5}{2}}}{5}$	6
trager	$\frac{2x^{\frac{5}{2}}}{5}$	6
risch	$\frac{2x^{\frac{5}{2}}}{5}$	6

input
```
int(x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^(5/2)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int x^{3/2}\,dx = \frac{2}{5}\,x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2),x, algorithm="fricas")
```

output
```
2/5*x^(5/2)
```

3.20.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int x^{3/2}\,dx = \frac{2x^{\frac{5}{2}}}{5}$$

input
```
integrate(x**(3/2),x)
```

output
```
2*x**(5/2)/5
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int x^{3/2}\,dx = \frac{2}{5}\,x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2),x, algorithm="maxima")
```

output
```
2/5*x^(5/2)
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int x^{3/2}\,dx = \frac{2}{5}\,x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2),x, algorithm="giac")
```

output
```
2/5*x^(5/2)
```

3.20.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int x^{3/2}\,dx = \frac{2\,x^{5/2}}{5}$$

input
```
int(x^(3/2),x)
```

output
```
(2*x^(5/2))/5
```

3.21 $\int \cos(3+2x)\,dx$

3.21.1 Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \cos(3+2x)\,dx = \frac{1}{2}\sin(3+2x)$$

output
```
1/2*sin(3+2*x)
```

3.21.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(3+2x)\,dx = \frac{1}{2}\sin(3+2x)$$

input
```
Integrate[Cos[3 + 2*x],x]
```

output
```
Sin[3 + 2*x]/2
```

3.21.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(2x+3)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(2x+\frac{\pi}{2}+3\right)dx$$

$$\Big\downarrow \; 3117$$

$$\frac{1}{2}\sin(2x+3)$$

input
```
Int[Cos[3 + 2*x],x]
```

output
```
Sin[3 + 2*x]/2
```

3.21.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.21.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\sin(3+2x)}{2}$	9
default	$\frac{\sin(3+2x)}{2}$	9
risch	$\frac{\sin(3+2x)}{2}$	9
parallelrisch	$\frac{\sin(3+2x)}{2}$	9
norman	$\frac{\tan(\frac{3}{2}+x)}{1+\tan^2(\frac{3}{2}+x)}$	16
meijerg	$\frac{\cos(3)\sin(2x)}{2}-\frac{\sin(3)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{2}$	30

input
```
int(cos(3+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(3+2*x)
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \cos(3+2x)\,dx = \frac{1}{2}\sin(2x+3)$$

input
```
integrate(cos(3+2*x),x, algorithm="fricas")
```

output
```
1/2*sin(2*x + 3)
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \cos(3+2x)\,dx = \frac{\sin(2x+3)}{2}$$

input
```
integrate(cos(3+2*x),x)
```

output
```
sin(2*x + 3)/2
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \cos(3+2x)\,dx = \frac{1}{2}\,\sin\left(2\,x+3\right)$$

input
```
integrate(cos(3+2*x),x, algorithm="maxima")
```

output
```
1/2*sin(2*x + 3)
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \cos(3+2x)\,dx = \frac{1}{2}\,\sin\left(2\,x+3\right)$$

input
```
integrate(cos(3+2*x),x, algorithm="giac")
```

output
```
1/2*sin(2*x + 3)
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \cos(3+2x)\,dx = \frac{\sin\left(2\,x+3\right)}{2}$$

input
```
int(cos(2*x + 3),x)
```

output
```
sin(2*x + 3)/2
```

3.22 $\int 2e^{2x}yz\,dx$

3.22.1 Optimal result

Integrand size = 9, antiderivative size = 8

$$\int 2e^{2x}yz\,dx = e^{2x}yz$$

output
```
exp(2*x)*y*z
```

3.22.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int 2e^{2x}yz\,dx = e^{2x}yz$$

input
```
Integrate[2*E^(2*x)*y*z,x]
```

output
```
E^(2*x)*y*z
```

3.22.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {27, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 2e^{2x}yz\,dx$$
$$\Big\downarrow \text{ 27}$$
$$2yz\int e^{2x}dx$$
$$\Big\downarrow \text{ 2624}$$
$$e^{2x}yz$$

input
```
Int[2*E^(2*x)*y*z,x]
```

output
```
E^(2*x)*y*z
```

3.22.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.22.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

method	result	size
gosper	$e^{2x}yz$	8
derivativedivides	$e^{2x}yz$	8
default	$e^{2x}yz$	8
norman	$e^{2x}yz$	8
risch	$e^{2x}yz$	8
parallelrisch	$e^{2x}yz$	8
parts	$e^{2x}yz$	8
meijerg	$-yz(1-e^{2x})$	13

input
```
int(2*exp(2*x)*y*z,x,method=_RETURNVERBOSE)
```

output
```
exp(2*x)*y*z
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int 2e^{2x}yz\,dx = yze^{(2\,x)}$$

input
```
integrate(2*exp(2*x)*y*z,x, algorithm="fricas")
```

output
```
y*z*e^(2*x)
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int 2e^{2x}yz\,dx = yze^{2x}$$

input
```
integrate(2*exp(2*x)*y*z,x)
```

output
```
y*z*exp(2*x)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int 2e^{2x}yz\,dx = yze^{(2\,x)}$$

input
```
integrate(2*exp(2*x)*y*z,x, algorithm="maxima")
```

output
```
y*z*e^(2*x)
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int 2e^{2x}yz\,dx = yze^{(2\,x)}$$

input
```
integrate(2*exp(2*x)*y*z,x, algorithm="giac")
```

output
```
y*z*e^(2*x)
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int 2e^{2x}yz\,dx = y\,z\,\mathrm{e}^{2\,x}$$

input
```
int(2*y*z*exp(2*x),x)
```

output
```
y*z*exp(2*x)
```

3.23 $\int e^x \cos^2(e^x) \sin(e^x) \, dx$

3.23.1 Optimal result

Integrand size = 14, antiderivative size = 10

$$\int e^x \cos^2(e^x) \sin(e^x) \, dx = -\frac{1}{3} \cos^3(e^x)$$

output
```
-1/3*cos(exp(x))^3
```

3.23.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int e^x \cos^2(e^x) \sin(e^x) \, dx = -\frac{1}{4} \cos(e^x) - \frac{1}{12} \cos(3e^x)$$

input
```
Integrate[E^x*Cos[E^x]^2*Sin[E^x],x]
```

output
```
-1/4*Cos[E^x] - Cos[3*E^x]/12
```

3.23.3 Rubi [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2720, 3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sin(e^x) \cos^2(e^x)\, dx$$

$$\Big\downarrow \; 2720$$

$$\int \sin(e^x) \cos^2(e^x)\, de^x$$

$$\Big\downarrow \; 3042$$

$$\int \sin(e^x) \cos(e^x)^2\, de^x$$

$$\Big\downarrow \; 3045$$

$$-\int e^{2x} d\cos(e^x)$$

$$\Big\downarrow \; 15$$

$$-\frac{e^{3x}}{3}$$

input
```
Int[E^x*Cos[E^x]^2*Sin[E^x],x]
```

output
```
-1/3*E^(3*x)
```

3.23.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

3.23.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\left(\cos^{3}\left(\mathrm{e}^{x}\right)\right)}{3}$	8
default	$-\frac{\left(\cos^{3}\left(\mathrm{e}^{x}\right)\right)}{3}$	8
risch	$-\frac{\cos\left(\mathrm{e}^{x}\right)}{4}-\frac{\cos\left(3\,\mathrm{e}^{x}\right)}{12}$	14
parallelrisch	$\frac{1}{3}-\frac{\cos\left(\mathrm{e}^{x}\right)}{4}-\frac{\cos\left(3\,\mathrm{e}^{x}\right)}{12}$	15
norman	$\frac{-2\left(\tan^{4}\left(\frac{\mathrm{e}^{x}}{2}\right)\right)-\frac{2}{3}}{\left(1+\tan^{2}\left(\frac{\mathrm{e}^{x}}{2}\right)\right)^{3}}$	24

input
```
int(exp(x)*cos(exp(x))^2*sin(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*cos(exp(x))^3
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^x \cos^2(e^x) \sin(e^x)\, dx = -\frac{1}{3} \cos(e^x)^3$$

input
```
integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="fricas")
```

output
```
-1/3*cos(e^x)^3
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int e^x \cos^2(e^x) \sin(e^x)\, dx = -\frac{\cos^3(e^x)}{3}$$

input
```
integrate(exp(x)*cos(exp(x))**2*sin(exp(x)),x)
```

output
```
-cos(exp(x))**3/3
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^x \cos^2(e^x) \sin(e^x)\, dx = -\frac{1}{3} \cos(e^x)^3$$

input
```
integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="maxima")
```

output
```
-1/3*cos(e^x)^3
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^x \cos^2(e^x)\sin(e^x)\,dx = -\frac{1}{3}\cos(e^x)^3$$

input
```
integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="giac")
```

output
```
-1/3*cos(e^x)^3
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^x \cos^2(e^x)\sin(e^x)\,dx = -\frac{\cos(e^x)^3}{3}$$

input
```
int(cos(exp(x))^2*sin(exp(x))*exp(x),x)
```

output
```
-cos(exp(x))^3/3
```

3.24 $\int x\sqrt{1+x}\,dx$

3.24.1 Optimal result

Integrand size = 9, antiderivative size = 23

$$\int x\sqrt{1+x}\,dx=-\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}$$

output
```
-2/3*(1+x)^(3/2)+2/5*(1+x)^(5/2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int x\sqrt{1+x}\,dx=\frac{2}{15}(1+x)^{3/2}(-5+3(1+x))$$

input
```
Integrate[x*Sqrt[1 + x],x]
```

output
```
(2*(1 + x)^(3/2)*(-5 + 3*(1 + x)))/15
```

3.24.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{x+1}\,dx$$

$$\Big\downarrow \; 53$$

$$\int \left((x+1)^{3/2}-\sqrt{x+1}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

input
```
Int[x*Sqrt[1 + x],x]
```

output
```
(-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

3.24.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.24.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{2(1+x)^{\frac{3}{2}}(-2+3x)}{15}$	13
derivativedivides	$-\frac{2(1+x)^{\frac{3}{2}}}{3}+\frac{2(1+x)^{\frac{5}{2}}}{5}$	16
default	$-\frac{2(1+x)^{\frac{3}{2}}}{3}+\frac{2(1+x)^{\frac{5}{2}}}{5}$	16
risch	$\frac{2(3x^2+x-2)\sqrt{1+x}}{15}$	16
trager	$\left(\frac{2}{5}x^2+\frac{2}{15}x-\frac{4}{15}\right)\sqrt{1+x}$	17
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,(1+x)^{\frac{3}{2}}(2-3x)}{15}}{2\sqrt{\pi}}$	27

input
```
int(x*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(1+x)^(3/2)*(-2+3*x)
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int x\sqrt{1+x}\,dx=\frac{2}{15}\left(3\,x^2+x-2\right)\sqrt{x+1}$$

input
```
integrate(x*(1+x)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int x\sqrt{1+x}\,dx = \frac{2x^2\sqrt{x+1}}{5} + \frac{2x\sqrt{x+1}}{15} - \frac{4\sqrt{x+1}}{15}$$

input
```
integrate(x*(1+x)**(1/2),x)
```

output
```
2*x**2*sqrt(x + 1)/5 + 2*x*sqrt(x + 1)/15 - 4*sqrt(x + 1)/15
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int x\sqrt{1+x}\,dx = \frac{2}{5}\,(x+1)^{\frac{5}{2}} - \frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

input
```
integrate(x*(1+x)^(1/2),x, algorithm="maxima")
```

output
```
2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int x\sqrt{1+x}\,dx = \frac{2}{5}\,(x+1)^{\frac{5}{2}} - \frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

input
```
integrate(x*(1+x)^(1/2),x, algorithm="giac")
```

output
```
2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

3.24.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int x\sqrt{1+x}\,dx = \frac{2\,(3\,x-2)\,\left(x+1\right)^{3/2}}{15}$$

input
```
int(x*(x + 1)^(1/2),x)
```

output
```
(2*(3*x - 2)*(x + 1)^(3/2))/15
```

3.25 $\int \frac{1}{-1+x^4}\,dx$

3.25.1 Optimal result

Integrand size = 7, antiderivative size = 13

$$\int \frac{1}{-1+x^4}\,dx=-\frac{\arctan(x)}{2}-\frac{\operatorname{arctanh}(x)}{2}$$

output
```
-1/2*arctan(x)-1/2*arctanh(x)
```

3.25.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{1}{-1+x^4}\,dx=-\frac{\arctan(x)}{2}+\frac{1}{4}\log(1-x)-\frac{1}{4}\log(1+x)$$

input
```
Integrate[(-1 + x^4)^(-1),x]
```

output
```
-1/2*ArcTan[x] + Log[1 - x]/4 - Log[1 + x]/4
```

3.25.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4-1}\,dx$$

$$\Big\downarrow \text{756}$$

$$-\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+1}dx$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\arctan(x)}{2}-\frac{\text{arctanh}(x)}{2}$$

input
```
Int[(-1 + x^4)^(-1),x]
```

output
```
-1/2*ArcTan[x] - ArcTanh[x]/2
```

3.25.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

3.25.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\arctan(x)}{2}-\frac{\operatorname{arctanh}(x)}{2}$	10
risch	$-\frac{\arctan(x)}{2}-\frac{\ln(1+x)}{4}+\frac{\ln(-1+x)}{4}$	18
parallelrisch	$\frac{i\ln(x-i)}{4}-\frac{i\ln(x+i)}{4}-\frac{\ln(1+x)}{4}+\frac{\ln(-1+x)}{4}$	30
meijerg	$\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}$	38

input
```
int(1/(x^4-1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(x)-1/2*arctanh(x)
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{-1+x^4}\,dx=-\frac{1}{2}\arctan(x)-\frac{1}{4}\log(x+1)+\frac{1}{4}\log(x-1)$$

input
```
integrate(1/(x^4-1),x, algorithm="fricas")
```

output
```
-1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{-1+x^4}\,dx = \frac{\log{(x-1)}}{4} - \frac{\log{(x+1)}}{4} - \frac{\operatorname{atan}{(x)}}{2}$$

input
```
integrate(1/(x**4-1),x)
```

output
```
log(x - 1)/4 - log(x + 1)/4 - atan(x)/2
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{-1+x^4}\,dx = -\frac{1}{2}\arctan\left(x\right) - \frac{1}{4}\log\left(x+1\right) + \frac{1}{4}\log\left(x-1\right)$$

input
```
integrate(1/(x^4-1),x, algorithm="maxima")
```

output
```
-1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

3.25.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{1}{-1+x^4}\,dx = -\frac{1}{2}\arctan\left(x\right) - \frac{1}{4}\log\left(|x+1|\right) + \frac{1}{4}\log\left(|x-1|\right)$$

input
```
integrate(1/(x^4-1),x, algorithm="giac")
```

output
```
-1/2*arctan(x) - 1/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{-1+x^4} \, dx = -\frac{\operatorname{atan}(x)}{2} - \frac{\operatorname{atanh}(x)}{2}$$

input
```
int(1/(x^4 - 1),x)
```

output
```
- atan(x)/2 - atanh(x)/2
```

3.26 $\int \frac{e^x}{2+3e^{2x}}\,dx$

3.26.1 Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{e^x}{2+3e^{2x}}\,dx = \frac{\arctan\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

output
```
1/6*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

3.26.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{2+3e^{2x}}\,dx = \frac{\arctan\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

input
```
Integrate[E^x/(2 + 3*E^(2*x)),x]
```

output
```
ArcTan[Sqrt[3/2]*E^x]/Sqrt[6]
```

3.26.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2679, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{3e^{2x}+2}dx$$
$$\Big\downarrow \text{2679}$$
$$\int \frac{1}{3e^{2x}+2}de^x$$
$$\Big\downarrow \text{216}$$
$$\frac{\arctan\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

input
```
Int[E^x/(2 + 3*E^(2*x)),x]
```

output
```
ArcTan[Sqrt[3/2]*E^x]/Sqrt[6]
```

3.26.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

3.26.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\arctan\left(\frac{e^{x}\sqrt{6}}{2}\right)\sqrt{6}}{6}$	14
risch	$\frac{i\sqrt{6}\,\ln\left(e^{x}+\frac{i\sqrt{6}}{3}\right)}{12}-\frac{i\sqrt{6}\,\ln\left(e^{x}-\frac{i\sqrt{6}}{3}\right)}{12}$	34

input
```
int(exp(x)/(2+3*exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{e^{x}}{2+3e^{2x}}\,dx = \frac{1}{6}\,\sqrt{6}\arctan\left(\frac{1}{2}\,\sqrt{6}e^{x}\right)$$

input
```
integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="fricas")
```

output
```
1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

3.26.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{x}}{2+3e^{2x}}\,dx = \operatorname{RootSum}\left(24z^{2}+1,\left(i\mapsto i\log\left(4i+e^{x}\right)\right)\right)$$

input
```
integrate(exp(x)/(2+3*exp(2*x)),x)
```

output
```
RootSum(24*_z**2 + 1, Lambda(_i, _i*log(4*_i + exp(x))))
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{e^x}{2+3e^{2x}}\,dx = \frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^x\right)$$

input
```
integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="maxima")
```

output
```
1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{e^x}{2+3e^{2x}}\,dx = \frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^x\right)$$

input
```
integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="giac")
```

output
```
1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

3.26.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{e^x}{2+3e^{2x}}\,dx = \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,e^x}{2}\right)}{6}$$

input
```
int(exp(x)/(3*exp(2*x) + 2),x)
```

output
```
(6^(1/2)*atan((6^(1/2)*exp(x))/2))/6
```

3.27 $\int \frac{e^{2x}}{A+Be^{4x}}\,dx$

3.27.1 Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{e^{2x}}{A+Be^{4x}}\,dx=\frac{\arctan\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

output
```
1/2*arctan(exp(2*x)*B^(1/2)/A^(1/2))/A^(1/2)/B^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{A+Be^{4x}}\,dx=\frac{\arctan\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

input
```
Integrate[E^(2*x)/(A + B*E^(4*x)),x]
```

output
```
ArcTan[(Sqrt[B]*E^(2*x))/Sqrt[A]]/(2*Sqrt[A]*Sqrt[B])
```

3.27.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2679, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x}}{A+Be^{4x}}dx$$

$$\Big\downarrow \text{2679}$$

$$\frac{1}{2}\int \frac{1}{A+Be^{4x}}de^{2x}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

input
```
Int[E^(2*x)/(A + B*E^(4*x)),x]
```

output
```
ArcTan[(Sqrt[B]*E^(2*x))/Sqrt[A]]/(2*Sqrt[A]*Sqrt[B])
```

3.27.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

3.27.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\arctan\left(\frac{B\,\mathrm{e}^{2x}}{\sqrt{AB}}\right)}{2\sqrt{AB}}$	20
risch	$-\frac{\ln\left(\mathrm{e}^{2x}-\frac{A}{\sqrt{-AB}}\right)}{4\sqrt{-AB}}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{A}{\sqrt{-AB}}\right)}{4\sqrt{-AB}}$	47

input
```
int(exp(2*x)/(A+B*exp(4*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2/(A*B)^(1/2)*arctan(B*exp(x)^2/(A*B)^(1/2))
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int \frac{e^{2x}}{A+Be^{4x}}\,dx = \left[-\frac{\sqrt{-AB}\log\left(\frac{Be^{(4\,x)}-2\,\sqrt{-AB}e^{(2\,x)}-A}{Be^{(4\,x)}+A}\right)}{4\,AB}, -\frac{\sqrt{AB}\arctan\left(\frac{\sqrt{AB}e^{(-2\,x)}}{B}\right)}{2\,AB}\right]$$

input
```
integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-A*B)*log((B*e^(4*x) - 2*sqrt(-A*B)*e^(2*x) - A)/(B*e^(4*x) + A
))/(A*B), -1/2*sqrt(A*B)*arctan(sqrt(A*B)*e^(-2*x)/B)/(A*B)]
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{e^{2x}}{A+Be^{4x}}\,dx = \operatorname{RootSum}\left(16z^2AB+1, \left(i \mapsto i\log\left(4iA+e^{2x}\right)\right)\right)$$

input
```
integrate(exp(2*x)/(A+B*exp(4*x)),x)
```

output
```
RootSum(16*_z**2*A*B + 1, Lambda(_i, _i*log(4*_i*A + exp(2*x))))
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{e^{2x}}{A+Be^{4x}}\,dx = \frac{\arctan\left(\frac{Be^{(2x)}}{\sqrt{AB}}\right)}{2\sqrt{AB}}$$

input
```
integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="maxima")
```

output
```
1/2*arctan(B*e^(2*x)/sqrt(A*B))/sqrt(A*B)
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{e^{2x}}{A+Be^{4x}}\,dx = \frac{\arctan\left(\frac{Be^{(2x)}}{\sqrt{AB}}\right)}{2\sqrt{AB}}$$

input
```
integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="giac")
```

output
```
1/2*arctan(B*e^(2*x)/sqrt(A*B))/sqrt(A*B)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{e^{2x}}{A+Be^{4x}}\,dx = \frac{\operatorname{atan}\left(\frac{B\,e^{2\,x}}{\sqrt{A\,B}}\right)}{2\sqrt{A\,B}}$$

input
```
int(exp(2*x)/(A + B*exp(4*x)),x)
```

output
```
atan((B*exp(2*x))/(A*B)^(1/2))/(2*(A*B)^(1/2))
```

3.28 $\int \frac{e^{1+x}}{1+e^x} \, dx$

3.28.1 Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{e^{1+x}}{1+e^x} \, dx = e \log (1+e^x)$$

output
```
exp(1)*ln(1+exp(x))
```

3.28.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{e^{1+x}}{1+e^x} \, dx = e \log (1+e^x)$$

input
```
Integrate[E^(1 + x)/(1 + E^x),x]
```

output
```
E*Log[1 + E^x]
```

3.28.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2677, 2676, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{x+1}}{e^x+1}dx$$

$$\Big\downarrow \ 2677$$

$$e\int \frac{e^x}{1+e^x}dx$$

$$\Big\downarrow \ 2676$$

$$e\int \frac{1}{1+e^x}de^x$$

$$\Big\downarrow \ 16$$

$$e\log(e^x+1)$$

input
```
Int[E^(1 + x)/(1 + E^x),x]
```

output
```
E*Log[1 + E^x]
```

3.28.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

rule 2677
```
Int[((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.))^(p_.)*((G_)^((
h_.)*((f_.) + (g_.)*(x_))))^(m_.), x_Symbol] :> Simp[(G^(h*(f + g*x)))^m/(F
^(e*(c + d*x)))^n   Int[(F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p,
x], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d*e*n*Lo
g[F], g*h*m*Log[G]]
```

3.28.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$\mathrm{e}\ln\left(1+\mathrm{e}^{x}\right)$	9
norman	$\mathrm{e}\ln\left(1+\mathrm{e}^{x}\right)$	9
risch	$\mathrm{e}\ln\left(1+\mathrm{e}^{x}\right)$	9

input
```
int(exp(1+x)/(1+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
exp(1)*ln(1+exp(x))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int \frac{e^{1+x}}{1+e^x}\,dx = e\log\left(e + e^{(x+1)}\right)$$

input
```
integrate(exp(1+x)/(1+exp(x)),x, algorithm="fricas")
```

output
```
e*log(e + e^(x + 1))
```

3.28.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{e^{1+x}}{1+e^x}\,dx = e\log\left(e^x+1\right)$$

input
```
integrate(exp(1+x)/(1+exp(x)),x)
```

output
```
E*log(exp(x) + 1)
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{e^{1+x}}{1+e^x}\,dx = e\log\left(e^x+1\right)$$

input
```
integrate(exp(1+x)/(1+exp(x)),x, algorithm="maxima")
```

output
```
e*log(e^x + 1)
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{e^{1+x}}{1+e^x}\,dx = e\log\left(e^x+1\right)$$

input
```
integrate(exp(1+x)/(1+exp(x)),x, algorithm="giac")
```

output
```
e*log(e^x + 1)
```

3.28.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{e^{1+x}}{1+e^x}\,dx = \mathrm{e}\,\ln\left(\mathrm{e}^x+1\right)$$

input
```
int(exp(x + 1)/(exp(x) + 1),x)
```

output
```
exp(1)*log(exp(x) + 1)
```

3.29 $\int (10e)^x \, dx$

3.29.1 Optimal result

Integrand size = 5, antiderivative size = 12

$$\int (10e)^x \, dx = \frac{(10e)^x}{1 + \log(10)}$$

output
```
(10*exp(1))^x/(1+ln(10))
```

3.29.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (10e)^x \, dx = \frac{(10e)^x}{\log(10e)}$$

input
```
Integrate[(10*E)^x,x]
```

output
```
(10*E)^x/Log[10*E]
```

3.29.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (10e)^x\,dx$$

$$\Big\downarrow \text{2624}$$

$$\frac{(10e)^x}{1+\log(10)}$$

input
```
Int[(10*E)^x,x]
```

output
```
(10*E)^x/(1 + Log[10])
```

3.29.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.29.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

method	result	size
gosper	$\frac{(10\,\mathrm{e})^{x}}{\ln(10\,\mathrm{e})}$	15
parallelrisch	$\frac{(10\,\mathrm{e})^{x}}{\ln(10\,\mathrm{e})}$	15
norman	$\frac{\mathrm{e}^{x\ln(10\,\mathrm{e})}}{1+\ln(10)}$	16
risch	$\frac{5^{x}2^{x}\mathrm{e}^{x}}{1+\ln(2)+\ln(5)}$	18
meijerg	$-\frac{1-\mathrm{e}^{x(1+\ln(10))}}{1+\ln(10)}$	20

input
```
int((10*exp(1))^x,x,method=_RETURNVERBOSE)
```

output
```
1/ln(10*exp(1))*(10*exp(1))^x
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (10e)^x\,dx = \frac{e^{(x\log(10)+x)}}{\log(10)+1}$$

input
```
integrate((10*exp(1))^x,x, algorithm="fricas")
```

output
```
e^(x*log(10) + x)/(log(10) + 1)
```

3.29.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (10e)^x\,dx = \frac{(10e)^x}{1+\log(10)}$$

input
```
integrate((10*exp(1))**x,x)
```

output
```
(10*E)**x/(1 + log(10))
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (10e)^x\,dx = \frac{(10\,e)^x}{\log(10\,e)}$$

input
```
integrate((10*exp(1))^x,x, algorithm="maxima")
```

output
```
(10*e)^x/log(10*e)
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int (10e)^x\,dx = \frac{(10\,e)^x}{\log(10)+1}$$

input
```
integrate((10*exp(1))^x,x, algorithm="giac")
```

output
```
(10*e)^x/(log(10) + 1)
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (10e)^x\,dx = \frac{10^x\,\mathrm{e}^x}{\ln(10)+1}$$

input
```
int((10*exp(1))^x,x)
```

output
```
(10^x*exp(x))/(log(10) + 1)
```

3.30 $\int x^3 \sin(x^2)\,dx$

3.30.1 Optimal result

Integrand size = 8, antiderivative size = 20

$$\int x^3 \sin(x^2)\,dx = -\frac{1}{2}x^2\cos(x^2) + \frac{\sin(x^2)}{2}$$

output
```
-1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

3.30.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^3 \sin(x^2)\,dx = -\frac{1}{2}x^2\cos(x^2) + \frac{\sin(x^2)}{2}$$

input
```
Integrate[x^3*Sin[x^2],x]
```

output
```
-1/2*(x^2*Cos[x^2]) + Sin[x^2]/2
```

3.30.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3860, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sin\left(x^2\right) dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2}\int x^2 \sin\left(x^2\right) dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int x^2 \sin\left(x^2\right) dx^2$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{2}\left(\int \cos\left(x^2\right) dx^2 - x^2 \cos\left(x^2\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \sin\left(x^2+\frac{\pi}{2}\right) dx^2 - x^2 \cos\left(x^2\right)\right)$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{2}\left(\sin\left(x^2\right) - x^2 \cos\left(x^2\right)\right)$$

input
```
Int[x^3*Sin[x^2],x]
```

output
```
(-(x^2*Cos[x^2]) + Sin[x^2])/2
```

3.30.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.30.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{x^{2}\cos\left(x^{2}\right)}{2}+\frac{\sin\left(x^{2}\right)}{2}$	17
default	$-\frac{x^{2}\cos\left(x^{2}\right)}{2}+\frac{\sin\left(x^{2}\right)}{2}$	17
risch	$-\frac{x^{2}\cos\left(x^{2}\right)}{2}+\frac{\sin\left(x^{2}\right)}{2}$	17
parallelrisch	$-\frac{x^{2}\cos\left(x^{2}\right)}{2}+\frac{\sin\left(x^{2}\right)}{2}$	17
meijerg	$\sqrt{\pi}\,\left(-\frac{x^{2}\cos\left(x^{2}\right)}{2\sqrt{\pi}}+\frac{\sin\left(x^{2}\right)}{2\sqrt{\pi}}\right)$	27
norman	$\frac{-\frac{x^{2}}{2}+\frac{x^{2}\left(\tan^{2}\left(\frac{x^{2}}{2}\right)\right)}{2}+\tan\left(\frac{x^{2}}{2}\right)}{1+\tan^{2}\left(\frac{x^{2}}{2}\right)}$	39
parts	$\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)x^{3}}{2}-\frac{3\pi^{2}\left(\frac{2\,\mathrm{S}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)\sqrt{2}\,x^{3}}{3\pi^{\frac{3}{2}}}+\frac{2x^{2}\cos\left(x^{2}\right)}{3\pi^{2}}-\frac{2\sin\left(x^{2}\right)}{3\pi^{2}}\right)}{4}$	69

input
```
int(x^3*sin(x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin \left(x^2\right) \, dx = -\frac{1}{2} \, x^2 \cos \left(x^2\right) + \frac{1}{2} \, \sin \left(x^2\right)$$

input
```
integrate(x^3*sin(x^2),x, algorithm="fricas")
```

output
```
-1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

3.30.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int x^3 \sin \left(x^2\right) \, dx = -\frac{x^2 \cos \left(x^2\right)}{2} + \frac{\sin \left(x^2\right)}{2}$$

input
```
integrate(x**3*sin(x**2),x)
```

output
```
-x**2*cos(x**2)/2 + sin(x**2)/2
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin \left(x^2\right) \, dx = -\frac{1}{2} \, x^2 \cos \left(x^2\right) + \frac{1}{2} \, \sin \left(x^2\right)$$

input
```
integrate(x^3*sin(x^2),x, algorithm="maxima")
```

output
```
-1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin\left(x^2\right)\,dx = -\frac{1}{2}\,x^2\cos\left(x^2\right) + \frac{1}{2}\,\sin\left(x^2\right)$$

input
```
integrate(x^3*sin(x^2),x, algorithm="giac")
```

output
```
-1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin\left(x^2\right)\,dx = \frac{\sin\left(x^2\right)}{2} - \frac{x^2\,\cos\left(x^2\right)}{2}$$

input
```
int(x^3*sin(x^2),x)
```

output
```
sin(x^2)/2 - (x^2*cos(x^2))/2
```

3.31 $\int \frac{x^7}{1+x^{12}}\,dx$

3.31.1 Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{x^7}{1+x^{12}}\,dx=-\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\log\left(1-x^4+x^8\right)$$

output
```
-1/12*ln(x^4+1)+1/24*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(
1/2)
```

3.31.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 260 vs. 2(49) = 98.

Time = 0.12 (sec) , antiderivative size = 260, normalized size of antiderivative = 5.31

$$\begin{aligned}\int \frac{x^7}{1+x^{12}}\,dx=\frac{1}{24}\Bigg(&2\sqrt{3}\arctan\left(\frac{1+\sqrt{3}-2\sqrt{2}x}{1-\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1-\sqrt{3}+2\sqrt{2}x}{1+\sqrt{3}}\right)\\&+2\sqrt{3}\arctan\left(\frac{-1+\sqrt{3}+2\sqrt{2}x}{1+\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+\sqrt{3}+2\sqrt{2}x}{-1+\sqrt{3}}\right)\\&-2\log\left(1-\sqrt{2}x+x^2\right)-2\log\left(1+\sqrt{2}x+x^2\right)\\&+\log\left(2+\sqrt{2}x-\sqrt{6}x+2x^2\right)+\log\left(2+\sqrt{2}\left(-1+\sqrt{3}\right)x+2x^2\right)\\&+\log\left(2-\left(\sqrt{2}+\sqrt{6}\right)x+2x^2\right)+\log\left(2+\left(\sqrt{2}+\sqrt{6}\right)x+2x^2\right)\Bigg)\end{aligned}$$

input
```
Integrate[x^7/(1 + x^12),x]
```

output
```
(2*Sqrt[3]*ArcTan[(1 + Sqrt[3] - 2*Sqrt[2]*x)/(1 - Sqrt[3])] - 2*Sqrt[3]*A
rcTan[(1 - Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] + 2*Sqrt[3]*ArcTan[(-1 +
Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] - 2*Sqrt[3]*ArcTan[(1 + Sqrt[3] + 2*
Sqrt[2]*x)/(-1 + Sqrt[3])] - 2*Log[1 - Sqrt[2]*x + x^2] - 2*Log[1 + Sqrt[2
]*x + x^2] + Log[2 + Sqrt[2]*x - Sqrt[6]*x + 2*x^2] + Log[2 + Sqrt[2]*(-1
+ Sqrt[3])*x + 2*x^2] + Log[2 - (Sqrt[2] + Sqrt[6])*x + 2*x^2] + Log[2 + (
Sqrt[2] + Sqrt[6])*x + 2*x^2])/24
```

3.31.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {807, 821, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{x^{12}+1}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{4}\int \frac{x^4}{x^{12}+1}dx^4$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{4}\left(\frac{1}{3}\int \frac{x^4+1}{x^8-x^4+1}dx^4-\frac{1}{3}\int \frac{1}{x^4+1}dx^4\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{4}\left(\frac{1}{3}\int \frac{x^4+1}{x^8-x^4+1}dx^4-\frac{1}{3}\log\left(x^4+1\right)\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^8-x^4+1}dx^4+\frac{1}{2}\int -\frac{1-2x^4}{x^8-x^4+1}dx^4\right)-\frac{1}{3}\log\left(x^4+1\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^8-x^4+1}dx^4-\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4\right)-\frac{1}{3}\log\left(x^4+1\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(-3\int \frac{1}{-x^8-3}d(2x^4-1)-\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4\right)-\frac{1}{3}\log\left(x^4+1\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x^4-1}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{1-2x^4}{x^8-x^4+1}dx^4\right)-\frac{1}{3}\log\left(x^4+1\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x^4-1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^8-x^4+1\right)\right)-\frac{1}{3}\log\left(x^4+1\right)\right)$$

input
```
Int[x^7/(1 + x^12),x]
```

output
```
(-1/3*Log[1 + x^4] + (Sqrt[3]*ArcTan[(-1 + 2*x^4)/Sqrt[3]] + Log[1 - x^4 +
 x^8]/2)/3)/4
```

3.31.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.31.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln\left(x^{8}-x^{4}+1\right)}{24}+\frac{\arctan\left(\frac{\left(2x^{4}-1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln\left(x^{4}+1\right)}{12}$	41
risch	$\frac{\ln\left(4x^{8}-4x^{4}+4\right)}{24}+\frac{\arctan\left(\frac{\left(2x^{4}-1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln\left(x^{4}+1\right)}{12}$	43
meijerg	$-\frac{x^{8}\ln\left(1+\left(x^{12}\right)^{\frac{1}{3}}\right)}{12\left(x^{12}\right)^{\frac{2}{3}}}+\frac{x^{8}\ln\left(1-\left(x^{12}\right)^{\frac{1}{3}}+\left(x^{12}\right)^{\frac{2}{3}}\right)}{24\left(x^{12}\right)^{\frac{2}{3}}}+\frac{x^{8}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{12}\right)^{\frac{1}{3}}}{2-\left(x^{12}\right)^{\frac{1}{3}}}\right)}{12\left(x^{12}\right)^{\frac{2}{3}}}$	80

input
```
int(x^7/(x^12+1),x,method=_RETURNVERBOSE)
```

output
```
1/24*ln(x^8-x^4+1)+1/12*arctan(1/3*(2*x^4-1)*3^(1/2))*3^(1/2)-1/12*ln(x^4+
1)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+x^{12}}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4-1\right)\right) + \frac{1}{24}\,\log\left(x^8-x^4+1\right) - \frac{1}{12}\,\log\left(x^4+1\right)$$

input
```
integrate(x^7/(x^12+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1
/12*log(x^4 + 1)
```

3.31.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{x^7}{1+x^{12}}\,dx = -\frac{\log\left(x^4+1\right)}{12} + \frac{\log\left(x^8-x^4+1\right)}{24} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x**7/(x**12+1),x)
```

output
```
-log(x**4 + 1)/12 + log(x**8 - x**4 + 1)/24 + sqrt(3)*atan(2*sqrt(3)*x**4/
3 - sqrt(3)/3)/12
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+x^{12}}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4-1\right)\right) + \frac{1}{24}\,\log\left(x^8-x^4+1\right) - \frac{1}{12}\,\log\left(x^4+1\right)$$

input
```
integrate(x^7/(x^12+1),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1
/12*log(x^4 + 1)
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{1+x^{12}}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) + \frac{1}{24}\,\log\left(x^8-x^4+1\right) - \frac{1}{12}\,\log\left(x^4+1\right)$$

input
```
integrate(x^7/(x^12+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1
/12*log(x^4 + 1)
```

3.31.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{x^7}{1+x^{12}}\,dx = -\frac{\ln\left(x^4+1\right)}{12} - \ln\left(x^4-\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(-\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right) + \ln\left(x^4+\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right)$$

input
```
int(x^7/(x^12 + 1),x)
```

output
```
log((3^(1/2)*1i)/2 + x^4 - 1/2)*((3^(1/2)*1i)/24 + 1/24) - log(x^4 - (3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/24 - 1/24) - log(x^4 + 1)/12
```

3.32 $\int x^{3a} \sin(x^{2a}) \, dx$

3.32.1 Optimal result

Integrand size = 12, antiderivative size = 115

$$\int x^{3a} \sin(x^{2a}) \, dx = \frac{i x^{1+3a} (-i x^{2a})^{-\frac{1+3a}{2a}} \Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right), -i x^{2a}\right)}{4a} - \frac{i x^{1+3a} (i x^{2a})^{-\frac{1+3a}{2a}} \Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right), i x^{2a}\right)}{4a}$$

output
```
1/4*I*x^(1+3*a)*GAMMA(3/2+1/2/a,-I*x^(2*a))/a/((-I*x^(2*a))^(1/2*(1+3*a)/a
))-1/4*I*x^(1+3*a)*GAMMA(3/2+1/2/a,I*x^(2*a))/a/((I*x^(2*a))^(1/2*(1+3*a)/
a))
```

3.32.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.23

$$\int x^{3a} \sin(x^{2a}) \, dx = -\frac{x^{1+a} (x^{4a})^{-\frac{1+a}{2a}} \left(4a (x^{4a})^{\frac{1+a}{2a}} \cos(x^{2a}) + (1+a) (i x^{2a})^{\frac{1+a}{2a}} \Gamma\left(\frac{1+a}{2a}, -i x^{2a}\right) + (1+a) (-i x^{2a})^{\frac{1+a}{2a}} \Gamma\left(\frac{1+a}{2a}, i x^{2a}\right.\right.}{8a^2}$$

input
```
Integrate[x^(3*a)*Sin[x^(2*a)],x]
```

output
```
-1/8*(x^(1 + a)*(4*a*(x^(4*a))^((1 + a)/(2*a))*Cos[x^(2*a)] + (1 + a)*(I*x
^(2*a))^((1 + a)/(2*a))*Gamma[(1 + a)/(2*a), (-I)*x^(2*a)] + (1 + a)*((-I)
*x^(2*a))^((1 + a)/(2*a))*Gamma[(1 + a)/(2*a), I*x^(2*a)]))/(a^2*(x^(4*a))
^((1 + a)/(2*a)))
```

3.32.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3904, 2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3a} \sin \left(x^{2a}\right) \, dx$$

$$\Big\downarrow \text{ 3904}$$

$$\frac{1}{2} i \int e^{-i x^{2a}} x^{3a} dx - \frac{1}{2} i \int e^{i x^{2a}} x^{3a} dx$$

$$\Big\downarrow \text{ 2648}$$

$$\frac{i x^{3a+1} \left(-i x^{2a}\right)^{-\frac{3a+1}{2a}} \Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right), -i x^{2a}\right)}{4a} - \frac{i x^{3a+1} \left(i x^{2a}\right)^{-\frac{3a+1}{2a}} \Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right), i x^{2a}\right)}{4a}$$

input
```
Int[x^(3*a)*Sin[x^(2*a)],x]
```

output
```
((I/4)*x^(1 + 3*a)*Gamma[(3 + a^(-1))/2, (-I)*x^(2*a)])/(a*((-I)*x^(2*a))^
((1 + 3*a)/(2*a))) - ((I/4)*x^(1 + 3*a)*Gamma[(3 + a^(-1))/2, I*x^(2*a)])/
(a*(I*x^(2*a))^((1 + 3*a)/(2*a)))
```

3.32.3.1 Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

rule 3904
```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[I/2
  Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Simp[I/2   Int[(e*x)^m*E^(c*I
+ d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

3.32.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.36

method	result	size
meijerg	$\frac{x^{5a+1}\,{}_1F_2\left(\frac{5}{4}+\frac{1}{4a};\frac{3}{2},\frac{9}{4}+\frac{1}{4a};-\frac{x^{4a}}{4}\right)}{5a+1}$	41

input
```
int(x^(3*a)*sin(x^(2*a)),x,method=_RETURNVERBOSE)
```

output
```
1/(5*a+1)*x^(5*a+1)*hypergeom([5/4+1/4/a],[3/2,9/4+1/4/a],-1/4*x^(4*a))
```

3.32.5 Fricas [F]

$$\int x^{3a}\sin\left(x^{2a}\right)\,dx=\int x^{3\,a}\sin\left(x^{2\,a}\right)\,dx$$

input
```
integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="fricas")
```

output
```
integral(x^(3*a)*sin(x^(2*a)), x)
```

3.32.6 Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int x^{3a}\sin\left(x^{2a}\right)\,dx=\frac{x^{5a+1}\Gamma\left(\frac{5}{4}+\frac{1}{4a}\right)\,{}_1F_2\left(\left.\begin{matrix}\frac{5}{4}+\frac{1}{4a}\\ \frac{3}{2},\frac{9}{4}+\frac{1}{4a}\end{matrix}\right|-\frac{x^{4a}}{4}\right)}{4a\Gamma\left(\frac{9}{4}+\frac{1}{4a}\right)}$$

input
```
integrate(x**(3*a)*sin(x**(2*a)),x)
```

output
```
x**(5*a + 1)*gamma(5/4 + 1/(4*a))*hyper((5/4 + 1/(4*a),), (3/2, 9/4 + 1/(4
*a)), -x**(4*a)/4)/(4*a*gamma(9/4 + 1/(4*a)))
```

3.32.7 Maxima [F]

$$\int x^{3a}\sin\left(x^{2a}\right)\,dx=\int x^{3\,a}\sin\left(x^{2\,a}\right)\,dx$$

input
```
integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="maxima")
```

output
```
-1/2*(x*x^a*cos(x^(2*a)) - (a + 1)*integrate(x^a*cos(x^(2*a)), x))/a
```

3.32.8 Giac [F]

$$\int x^{3a}\sin\left(x^{2a}\right)\,dx=\int x^{3\,a}\sin\left(x^{2\,a}\right)\,dx$$

input
```
integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="giac")
```

output
```
integrate(x^(3*a)*sin(x^(2*a)), x)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int x^{3a}\sin\left(x^{2a}\right)\,dx=\int x^{3\,a}\,\sin\left(x^{2\,a}\right)\,dx$$

input
```
int(x^(3*a)*sin(x^(2*a)),x)
```

output
```
int(x^(3*a)*sin(x^(2*a)), x)
```

3.33 $\int \cos(\sqrt{x})\,dx$

3.33.1 Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\sin\left(\sqrt{x}\right)$$

output
```
2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

3.33.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\sin\left(\sqrt{x}\right)$$

input
```
Integrate[Cos[Sqrt[x]],x]
```

output
```
2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

3.33.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3843, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos\left(\sqrt{x}\right) \, dx$$

$$\Big\downarrow \text{3843}$$

$$2 \int \sqrt{x} \cos\left(\sqrt{x}\right) d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2 \int \sqrt{x} \sin\left(\sqrt{x} + \frac{\pi}{2}\right) d\sqrt{x}$$

$$\Big\downarrow \text{3777}$$

$$2\left(\int -\sin\left(\sqrt{x}\right) d\sqrt{x} + \sqrt{x} \sin\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\sqrt{x} \sin\left(\sqrt{x}\right) - \int \sin\left(\sqrt{x}\right) d\sqrt{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\sqrt{x} \sin\left(\sqrt{x}\right) - \int \sin\left(\sqrt{x}\right) d\sqrt{x}\right)$$

$$\Big\downarrow \text{3118}$$

$$2\left(\sqrt{x} \sin\left(\sqrt{x}\right) + \cos\left(\sqrt{x}\right)\right)$$

input
```
Int[Cos[Sqrt[x]],x]
```

output
```
2*(Cos[Sqrt[x]] + Sqrt[x]*Sin[Sqrt[x]])
```

3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3843
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

3.33.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$2\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\sqrt{x}$	17
default	$2\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\sqrt{x}$	17
meijerg	$4\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(\sqrt{x})}{2\sqrt{\pi}}+\frac{\sqrt{x}\,\sin(\sqrt{x})}{2\sqrt{\pi}}\right)$	33

input
```
int(cos(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

input
```
integrate(cos(x^(1/2)),x, algorithm="fricas")
```

output
```
2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

input
```
integrate(cos(x**(1/2)),x)
```

output
```
2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

input
```
integrate(cos(x^(1/2)),x, algorithm="maxima")
```

output
```
2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

input
```
integrate(cos(x^(1/2)),x, algorithm="giac")
```

output
```
2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.33.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\,\sin\left(\sqrt{x}\right)$$

input
```
int(cos(x^(1/2)),x)
```

output
```
2*cos(x^(1/2)) + 2*x^(1/2)*sin(x^(1/2))
```

3.34 $\int x\sqrt{1+x}\,dx$

3.34.1 Optimal result

Integrand size = 9, antiderivative size = 23

$$\int x\sqrt{1+x}\,dx=-\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}$$

output
```
-2/3*(1+x)^(3/2)+2/5*(1+x)^(5/2)
```

3.34.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int x\sqrt{1+x}\,dx=\frac{2}{15}(1+x)^{3/2}(-5+3(1+x))$$

input
```
Integrate[x*Sqrt[1 + x],x]
```

output
```
(2*(1 + x)^(3/2)*(-5 + 3*(1 + x)))/15
```

3.34.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{x+1}\,dx$$
$$\Big\downarrow \text{ 53}$$
$$\int \left((x+1)^{3/2}-\sqrt{x+1}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

input
```
Int[x*Sqrt[1 + x],x]
```

output
```
(-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

3.34.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.34.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{2(1+x)^{\frac{3}{2}}(-2+3x)}{15}$	13
derivativedivides	$-\frac{2(1+x)^{\frac{3}{2}}}{3}+\frac{2(1+x)^{\frac{5}{2}}}{5}$	16
default	$-\frac{2(1+x)^{\frac{3}{2}}}{3}+\frac{2(1+x)^{\frac{5}{2}}}{5}$	16
risch	$\frac{2(3x^2+x-2)\sqrt{1+x}}{15}$	16
trager	$\left(\frac{2}{5}x^2+\frac{2}{15}x-\frac{4}{15}\right)\sqrt{1+x}$	17
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,(1+x)^{\frac{3}{2}}(2-3x)}{15}}{2\sqrt{\pi}}$	27

input
```
int(x*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(1+x)^(3/2)*(-2+3*x)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int x\sqrt{1+x}\,dx=\frac{2}{15}\left(3\,x^2+x-2\right)\sqrt{x+1}$$

input
```
integrate(x*(1+x)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int x\sqrt{1+x}\,dx=\frac{2x^2\sqrt{x+1}}{5}+\frac{2x\sqrt{x+1}}{15}-\frac{4\sqrt{x+1}}{15}$$

input
```
integrate(x*(1+x)**(1/2),x)
```

output
```
2*x**2*sqrt(x + 1)/5 + 2*x*sqrt(x + 1)/15 - 4*sqrt(x + 1)/15
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int x\sqrt{1+x}\,dx=\frac{2}{5}\,(x+1)^{\frac{5}{2}}-\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

input
```
integrate(x*(1+x)^(1/2),x, algorithm="maxima")
```

output
```
2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int x\sqrt{1+x}\,dx=\frac{2}{5}\,(x+1)^{\frac{5}{2}}-\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

input
```
integrate(x*(1+x)^(1/2),x, algorithm="giac")
```

output
```
2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

3.34.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int x\sqrt{1+x}\,dx = \frac{2\,(3\,x-2)\,\left(x+1\right)^{3/2}}{15}$$

input
```
int(x*(x + 1)^(1/2),x)
```

output
```
(2*(3*x - 2)*(x + 1)^(3/2))/15
```

3.35 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$

3.35.1 Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)$$

output
```
6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)
```

3.35.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = \left(6-3\sqrt[6]{x}+2\sqrt[3]{x}\right)\sqrt[6]{x}-6\log\left(1+\sqrt[6]{x}\right)$$

input
```
Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
```

output
```
(6 - 3*x^(1/6) + 2*x^(1/3))*x^(1/6) - 6*Log[1 + x^(1/6)]
```

3.35.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2027, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}+\sqrt[3]{x}}dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{1}{\left(\sqrt[6]{x}+1\right)\sqrt[3]{x}}dx$$
$$\Big\downarrow \text{798}$$
$$6\int \frac{\sqrt{x}}{\sqrt[6]{x}+1}d\sqrt[6]{x}$$
$$\Big\downarrow \text{49}$$
$$6\int \left(\sqrt[3]{x}-\sqrt[6]{x}+\frac{1}{-\sqrt[6]{x}-1}+1\right)d\sqrt[6]{x}$$
$$\Big\downarrow \text{2009}$$
$$6\left(\frac{\sqrt{x}}{3}-\frac{\sqrt[3]{x}}{2}+\sqrt[6]{x}-\log\left(\sqrt[6]{x}+1\right)\right)$$

input
```
Int[(x^(1/3) + Sqrt[x])^(-1),x]
```

output
```
6*(x^(1/6) - x^(1/3)/2 + Sqrt[x]/3 - Log[1 + x^(1/6)])
```

3.35.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.35.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result
derivativedivides	$6x^{\frac{1}{6}}-3x^{\frac{1}{3}}-6\ln\left(1+x^{\frac{1}{6}}\right)+2\sqrt{x}$
meijerg	$\frac{x^{\frac{1}{6}}\left(4x^{\frac{1}{3}}-6x^{\frac{1}{6}}+12\right)}{2}-6\ln\left(1+x^{\frac{1}{6}}\right)$
default	$-\ln\left(x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)+2\ln\left(x^{\frac{1}{6}}-1\right)-2\ln\left(1+x^{\frac{1}{6}}\right)+\ln\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)+2\sqrt{x}+\ln$

input
```
int(1/(x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\,\sqrt{x}-3\,x^{\frac{1}{3}}+6\,x^{\frac{1}{6}}-6\,\log\left(x^{\frac{1}{6}}+1\right)$$

input
```
integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

output
```
2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

3.35.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = \int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$$

input
```
integrate(1/(x**(1/3)+x**(1/2)),x)
```

output
```
Integral(1/(x**(1/3) + sqrt(x)), x)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\log\left(x^{\frac{1}{6}}+1\right)$$

input
```
integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

output
```
2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\log\left(x^{\frac{1}{6}}+1\right)$$

input
```
integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="giac")
```

output
```
2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 6\ln\left(x^{1/6}+1\right) - 3\,x^{1/3} + 6\,x^{1/6}$$

input
```
int(1/(x^(1/2) + x^(1/3)),x)
```

output
```
2*x^(1/2) - 6*log(x^(1/6) + 1) - 3*x^(1/3) + 6*x^(1/6)
```

3.36 $\int \sqrt{\frac{1+x}{3+2x}}\,dx$

3.36.1 Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \sqrt{\frac{1+x}{3+2x}}\,dx=\frac{1}{2}\sqrt{1+x}\sqrt{3+2x}-\frac{\operatorname{arcsinh}\left(\sqrt{2}\sqrt{1+x}\right)}{2\sqrt{2}}$$

output
```
-1/4*arcsinh(2^(1/2)*(1+x)^(1/2))*2^(1/2)+1/2*(1+x)^(1/2)*(3+2*x)^(1/2)
```

3.36.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int \sqrt{\frac{1+x}{3+2x}}\,dx=\frac{\sqrt{\frac{1+x}{3+2x}}\left(2\sqrt{1+x}(3+2x)-\sqrt{6+4x}\operatorname{arctanh}\left(\frac{\sqrt{3+2x}}{\sqrt{2}\sqrt{1+x}}\right)\right)}{4\sqrt{1+x}}$$

input
```
Integrate[Sqrt[(1 + x)/(3 + 2*x)],x]
```

output
```
(Sqrt[(1 + x)/(3 + 2*x)]*(2*Sqrt[1 + x]*(3 + 2*x) - Sqrt[6 + 4*x]*ArcTanh[
Sqrt[3 + 2*x]/(Sqrt[2]*Sqrt[1 + x])]))/(4*Sqrt[1 + x])
```

3.36.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2050, 60, 64, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x+1}{2x+3}}dx$$

$$\Big\downarrow \text{2050}$$

$$\int \frac{\sqrt{x+1}}{\sqrt{2x+3}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\sqrt{x+1}\sqrt{2x+3}-\frac{1}{4}\int \frac{1}{\sqrt{x+1}\sqrt{2x+3}}dx$$

$$\Big\downarrow \text{64}$$

$$\frac{1}{2}\sqrt{x+1}\sqrt{2x+3}-\frac{1}{2}\int \frac{1}{\sqrt{2(x+1)+1}}d\sqrt{x+1}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\sqrt{x+1}\sqrt{2x+3}-\frac{\text{arcsinh}\left(\sqrt{2}\sqrt{x+1}\right)}{2\sqrt{2}}$$

input
```
Int[Sqrt[(1 + x)/(3 + 2*x)],x]
```

output
```
(Sqrt[1 + x]*Sqrt[3 + 2*x])/2 - ArcSinh[Sqrt[2]*Sqrt[1 + x]]/(2*Sqrt[2])
```

3.36.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 64
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[2/b   Subst[Int[1/Sqrt[c - a*(d/b) + d*(x^2/b)], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[c - a*(d/b), 0] && ( !GtQ[a - c*(b/d), 0]
 || PosQ[b])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

3.36.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(30) = 60$.

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

method	result
default	$-\frac{\sqrt{\frac{1+x}{3+2x}}\,(3+2x)\left(\ln\left(\frac{5\sqrt{2}}{4}+x\sqrt{2}+\sqrt{2x^2+5x+3}\right)\sqrt{2}-4\sqrt{2x^2+5x+3}\right)}{8\sqrt{(3+2x)(1+x)}}$
risch	$\frac{(3+2x)\sqrt{\frac{1+x}{3+2x}}}{2}-\frac{\ln\left(\frac{\left(\frac{5}{2}+2x\right)\sqrt{2}}{2}+\sqrt{2x^2+5x+3}\right)\sqrt{2}\,\sqrt{\frac{1+x}{3+2x}}\,\sqrt{(3+2x)(1+x)}}{8(1+x)}$
trager	$3\left(\frac{1}{2}+\frac{x}{3}\right)\sqrt{-\frac{-1-x}{3+2x}}-\frac{\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\mathrm{RootOf}\left(\_Z^2-2\right)x+8\sqrt{-\frac{-1-x}{3+2x}}\,x+5\,\mathrm{RootOf}\left(\_Z^2-2\right)+12\sqrt{-\frac{-1-x}{3+2x}}\right)}{8}$

input
```
int(((1+x)/(3+2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*((1+x)/(3+2*x))^(1/2)*(3+2*x)*(ln(5/4*2^(1/2)+x*2^(1/2)+(2*x^2+5*x+3)
^(1/2))*2^(1/2)-4*(2*x^2+5*x+3)^(1/2))/((3+2*x)*(1+x))^(1/2)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \sqrt{\frac{1+x}{3+2x}}\,dx = \frac{1}{2}\,(2\,x+3)\sqrt{\frac{x+1}{2\,x+3}} + \frac{1}{8}\,\sqrt{2}\log\left(2\,\sqrt{2}(2\,x+3)\sqrt{\frac{x+1}{2\,x+3}} - 4\,x - 5\right)$$

input
```
integrate(((1+x)/(3+2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*x + 3)*sqrt((x + 1)/(2*x + 3)) + 1/8*sqrt(2)*log(2*sqrt(2)*(2*x + 3
)*sqrt((x + 1)/(2*x + 3)) - 4*x - 5)
```

3.36.6 Sympy [F]

$$\int \sqrt{\frac{1+x}{3+2x}}\,dx = \int \sqrt{\frac{x+1}{2x+3}}\,dx$$

input
```
integrate(((1+x)/(3+2*x))**(1/2),x)
```

output
```
Integral(sqrt((x + 1)/(2*x + 3)), x)
```

3.36.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(30) = 60.

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.82

$$\int \sqrt{\frac{1+x}{3+2x}}\,dx = \frac{1}{8}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-2\,\sqrt{\frac{x+1}{2\,x+3}}}{\sqrt{2}+2\,\sqrt{\frac{x+1}{2\,x+3}}}\right) - \frac{\sqrt{\frac{x+1}{2\,x+3}}}{2\left(\frac{2\,(x+1)}{2\,x+3}-1\right)}$$

input
```
integrate(((1+x)/(3+2*x))^(1/2),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*log(-(sqrt(2) - 2*sqrt((x + 1)/(2*x + 3)))/(sqrt(2) + 2*sqrt((
x + 1)/(2*x + 3)))) - 1/2*sqrt((x + 1)/(2*x + 3))/(2*(x + 1)/(2*x + 3) - 1
)
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(30) = 60.

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \sqrt{\frac{1+x}{3+2x}}\,dx = \frac{1}{8}\sqrt{2}\log\left(\left|-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2+5x+3}\right)-5\right|\right)\operatorname{sgn}(2x+3) + \frac{1}{2}\sqrt{2x^2+5x+3}\operatorname{sgn}(2x+3)$$

input
```
integrate(((1+x)/(3+2*x))^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*log(abs(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 + 5*x + 3)) - 5))*s
gn(2*x + 3) + 1/2*sqrt(2*x^2 + 5*x + 3)*sgn(2*x + 3)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \sqrt{\frac{1+x}{3+2x}}\,dx = -\frac{\sqrt{2}\operatorname{atanh}\left(\sqrt{2}\sqrt{\frac{x+1}{2x+3}}\right)}{4} - \frac{\sqrt{\frac{x+1}{2x+3}}}{2\left(\frac{2x+2}{2x+3}-1\right)}$$

input
```
int(((x + 1)/(2*x + 3))^(1/2),x)
```

output
```
- (2^(1/2)*atanh(2^(1/2)*((x + 1)/(2*x + 3))^(1/2)))/4 - ((x + 1)/(2*x + 3
))^(1/2)/(2*((2*x + 2)/(2*x + 3) - 1))
```

3.37 $\int \frac{x^4}{(1-x^2)^{5/2}}\,dx$

3.37.1 Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \arcsin(x)$$

output
```
1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

3.37.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{x(-3+4x^2)}{3(1-x^2)^{3/2}} + 2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

input
```
Integrate[x^4/(1 - x^2)^(5/2),x]
```

output
```
(x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

3.37.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {252, 252, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(1-x^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{x^3}{3(1-x^2)^{3/2}} - \int \frac{x^2}{(1-x^2)^{3/2}} dx$$

$$\Big\downarrow \text{ 252}$$

$$\int \frac{1}{\sqrt{1-x^2}} dx - \frac{x}{\sqrt{1-x^2}} + \frac{x^3}{3(1-x^2)^{3/2}}$$

$$\Big\downarrow \text{ 223}$$

$$\arcsin(x) - \frac{x}{\sqrt{1-x^2}} + \frac{x^3}{3(1-x^2)^{3/2}}$$

input
```
Int[x^4/(1 - x^2)^(5/2),x]
```

output
```
x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

3.37.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

3.37.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x^{3}}{3\left(-x^{2}+1\right)^{\frac{3}{2}}}+\arcsin\left(x\right)-\frac{x}{\sqrt{-x^{2}+1}}$	30
risch	$-\frac{x\left(4x^{2}-3\right)}{3\left(x^{2}-1\right)\sqrt{-x^{2}+1}}+\arcsin\left(x\right)$	30
meijerg	$-\frac{2i\left(-\frac{i\sqrt{\pi}\,x\left(-20x^{2}+15\right)}{10\left(-x^{2}+1\right)^{\frac{3}{2}}}+\frac{3i\sqrt{\pi}\,\arcsin\left(x\right)}{2}\right)}{3\sqrt{\pi}}$	39
pseudoelliptic	$\frac{\left(x^{2}-1\right)\sqrt{-x^{2}+1}\,\arctan\left(\frac{\sqrt{-x^{2}+1}}{x}\right)+\frac{4x^{3}}{3}-x}{\left(-x^{2}+1\right)^{\frac{3}{2}}}$	49
trager	$\frac{\left(4x^{2}-3\right)x\sqrt{-x^{2}+1}}{3\left(x^{2}-1\right)^{2}}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)$	54

input
```
int(x^4/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-1/(-x^2+1)^(1/2)*x
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(29) = 58.

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = -\frac{6\,(x^4-2\,x^2+1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-(4\,x^3-3\,x)\sqrt{-x^2+1}}{3\,(x^4-2\,x^2+1)}$$

input
```
integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*s
qrt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

3.37.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(26) = 52.

Time = 0.40 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.00

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3} + \frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3} - \frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3} - \frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3} + \frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

input
```
integrate(x**4/(-x**2+1)**(5/2),x)
```

output
```
3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x
**2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**
*4 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{1}{3}\,x\left(\frac{3\,x^2}{(-x^2+1)^{\frac{3}{2}}} - \frac{2}{(-x^2+1)^{\frac{3}{2}}}\right) - \frac{x}{3\,\sqrt{-x^2+1}} + \arcsin(x)$$

input
```
integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
```

output
```
1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
 + arcsin(x)
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{(4\,x^2-3)\sqrt{-x^2+1}x}{3\,(x^2-1)^2} + \arcsin(x)$$

input
```
integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

output
```
1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.60

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \operatorname{asin}(x) + \frac{3\sqrt{1-x^2}}{4\,(x-1)} + \frac{3\sqrt{1-x^2}}{4\,(x+1)} - \sqrt{1-x^2}\left(\frac{1}{12\,(x-1)} - \frac{1}{12\,(x-1)^2}\right) - \sqrt{1-x^2}\left(\frac{1}{12\,(x+1)} + \frac{1}{12\,(x+1)^2}\right)$$

input
```
int(x^4/(1 - x^2)^(5/2),x)
```

output
```
asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1)
) - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*
(1/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

3.38 $\int \sqrt{x}(1+x)^{5/2}\,dx$

3.38.1 Optimal result

Integrand size = 13, antiderivative size = 75

$$\int \sqrt{x}(1+x)^{5/2}\,dx = \frac{5}{64}\sqrt{x}\sqrt{1+x}+\frac{5}{32}x^{3/2}\sqrt{1+x} + \frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}-\frac{5\operatorname{arcsinh}\left(\sqrt{x}\right)}{64}$$

output
```
5/24*x^(3/2)*(1+x)^(3/2)+1/4*x^(3/2)*(1+x)^(5/2)-5/64*arcsinh(x^(1/2))+5/3
2*x^(3/2)*(1+x)^(1/2)+5/64*x^(1/2)*(1+x)^(1/2)
```

3.38.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(1+x)^{5/2}\,dx = \frac{1}{192}\sqrt{x}\sqrt{1+x}\left(15+118x+136x^2+48x^3\right)+\frac{5}{64}\log\left(-\sqrt{x}+\sqrt{1+x}\right)$$

input
```
Integrate[Sqrt[x]*(1 + x)^(5/2),x]
```

output
```
(Sqrt[x]*Sqrt[1 + x]*(15 + 118*x + 136*x^2 + 48*x^3))/192 + (5*Log[-Sqrt[x
] + Sqrt[1 + x]])/64
```

3.38.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {60, 60, 60, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(x+1)^{5/2}\,dx$$

$$\Big\downarrow \; 60$$

$$\frac{5}{8}\int \sqrt{x}(x+1)^{3/2}dx+\frac{1}{4}x^{3/2}(x+1)^{5/2}$$

$$\Big\downarrow \; 60$$

$$\frac{5}{8}\left(\frac{1}{2}\int \sqrt{x}\sqrt{x+1}dx+\frac{1}{3}x^{3/2}(x+1)^{3/2}\right)+\frac{1}{4}x^{3/2}(x+1)^{5/2}$$

$$\Big\downarrow \; 60$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{x+1}}dx+\frac{1}{2}\sqrt{x+1}x^{3/2}\right)+\frac{1}{3}x^{3/2}(x+1)^{3/2}\right)+\frac{1}{4}x^{3/2}(x+1)^{5/2}$$

$$\Big\downarrow \; 60$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\left(\sqrt{x}\sqrt{x+1}-\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx\right)+\frac{1}{2}\sqrt{x+1}x^{3/2}\right)+\frac{1}{3}x^{3/2}(x+1)^{3/2}\right)+\frac{1}{4}x^{3/2}(x+1)^{5/2}$$

$$\Big\downarrow \; 63$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\left(\sqrt{x}\sqrt{x+1}-\int \frac{1}{\sqrt{x+1}}d\sqrt{x}\right)+\frac{1}{2}\sqrt{x+1}x^{3/2}\right)+\frac{1}{3}x^{3/2}(x+1)^{3/2}\right)+\frac{1}{4}x^{3/2}(x+1)^{5/2}$$

$$\Big\downarrow \; 222$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\left(\sqrt{x}\sqrt{x+1}-\text{arcsinh}\left(\sqrt{x}\right)\right)+\frac{1}{2}\sqrt{x+1}x^{3/2}\right)+\frac{1}{3}x^{3/2}(x+1)^{3/2}\right)+\frac{1}{4}x^{3/2}(x+1)^{5/2}$$

input
```
Int[Sqrt[x]*(1 + x)^(5/2),x]
```

output
```
(x^(3/2)*(1 + x)^(5/2))/4 + (5*((x^(3/2)*(1 + x)^(3/2))/3 + ((x^(3/2)*Sqrt
[1 + x])/2 + (Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]])/4)/2))/8
```

3.38.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.38.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.59

method	result	size
meijerg	$-\frac{15\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\left(48x^3+136x^2+118x+15\right)\sqrt{1+x}}{360}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{x}\right)}{24}\right)}{8\sqrt{\pi}}$	44
risch	$\frac{\left(48x^3+136x^2+118x+15\right)\sqrt{x}\,\sqrt{1+x}}{192}-\frac{5\sqrt{x(1+x)}\,\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{128\sqrt{1+x}\,\sqrt{x}}$	55
default	$\frac{\sqrt{x}\,(1+x)^{\frac{7}{2}}}{4}-\frac{\sqrt{x}\,(1+x)^{\frac{5}{2}}}{24}-\frac{5\sqrt{x}\,(1+x)^{\frac{3}{2}}}{96}-\frac{5\sqrt{x}\,\sqrt{1+x}}{64}-\frac{5\sqrt{x(1+x)}\,\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{128\sqrt{1+x}\,\sqrt{x}}$	70

input
```
int(x^(1/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-15/8/Pi^(1/2)*(-1/360*Pi^(1/2)*x^(1/2)*(48*x^3+136*x^2+118*x+15)*(1+x)^(1
/2)+1/24*Pi^(1/2)*arcsinh(x^(1/2)))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.59

$$\int \sqrt{x}(1+x)^{5/2}\,dx = \frac{1}{192}\left(48\,x^3+136\,x^2+118\,x+15\right)\sqrt{x+1}\sqrt{x} + \frac{5}{128}\,\log\left(2\,\sqrt{x+1}\sqrt{x}-2\,x-1\right)$$

input
```
integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="fricas")
```

output
```
1/192*(48*x^3 + 136*x^2 + 118*x + 15)*sqrt(x + 1)*sqrt(x) + 5/128*log(2*sq
rt(x + 1)*sqrt(x) - 2*x - 1)
```

3.38.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.65 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.53

$$\int \sqrt{x}(1+x)^{5/2}\,dx = \begin{cases} -\frac{5\,\mathrm{acosh}\left(\sqrt{x+1}\right)}{64} + \frac{(x+1)^{\frac{9}{2}}}{4\sqrt{x}} - \frac{7(x+1)^{\frac{7}{2}}}{24\sqrt{x}} - \frac{(x+1)^{\frac{5}{2}}}{96\sqrt{x}} - \frac{5(x+1)^{\frac{3}{2}}}{192\sqrt{x}} + \frac{5\sqrt{x+1}}{64\sqrt{x}} & \text{for } |x+1|>1 \\ \frac{5i\,\mathrm{asin}\left(\sqrt{x+1}\right)}{64} - \frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{-x}} + \frac{7i(x+1)^{\frac{7}{2}}}{24\sqrt{-x}} + \frac{i(x+1)^{\frac{5}{2}}}{96\sqrt{-x}} + \frac{5i(x+1)^{\frac{3}{2}}}{192\sqrt{-x}} - \frac{5i\sqrt{x+1}}{64\sqrt{-x}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)*(1+x)**(5/2),x)
```

output
```
Piecewise((-5*acosh(sqrt(x + 1))/64 + (x + 1)**(9/2)/(4*sqrt(x)) - 7*(x +
1)**(7/2)/(24*sqrt(x)) - (x + 1)**(5/2)/(96*sqrt(x)) - 5*(x + 1)**(3/2)/(1
92*sqrt(x)) + 5*sqrt(x + 1)/(64*sqrt(x)), Abs(x + 1) > 1), (5*I*asin(sqrt(
x + 1))/64 - I*(x + 1)**(9/2)/(4*sqrt(-x)) + 7*I*(x + 1)**(7/2)/(24*sqrt(-
x)) + I*(x + 1)**(5/2)/(96*sqrt(-x)) + 5*I*(x + 1)**(3/2)/(192*sqrt(-x)) -
 5*I*sqrt(x + 1)/(64*sqrt(-x)), True))
```

3.38.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(47) = 94.

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.51

$$\int \sqrt{x}(1+x)^{5/2}\,dx = \frac{\frac{15\,(x+1)^{\frac{7}{2}}}{x^{\frac{7}{2}}} + \frac{73\,(x+1)^{\frac{5}{2}}}{x^{\frac{5}{2}}} - \frac{55\,(x+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}} + \frac{15\,\sqrt{x+1}}{\sqrt{x}}}{192\left(\frac{(x+1)^4}{x^4} - \frac{4\,(x+1)^3}{x^3} + \frac{6\,(x+1)^2}{x^2} - \frac{4\,(x+1)}{x} + 1\right)} - \frac{5}{128}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}} + 1\right) + \frac{5}{128}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}} - 1\right)$$

input
```
integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="maxima")
```

output
```
1/192*(15*(x + 1)^(7/2)/x^(7/2) + 73*(x + 1)^(5/2)/x^(5/2) - 55*(x + 1)^(3
/2)/x^(3/2) + 15*sqrt(x + 1)/sqrt(x))/((x + 1)^4/x^4 - 4*(x + 1)^3/x^3 + 6
*(x + 1)^2/x^2 - 4*(x + 1)/x + 1) - 5/128*log(sqrt(x + 1)/sqrt(x) + 1) + 5
/128*log(sqrt(x + 1)/sqrt(x) - 1)
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int \sqrt{x}(1+x)^{5/2}\,dx = \frac{1}{192}\left(2\left(4\left(6\,x-19\right)(x+1)+163\right)(x+1)-279\right)\sqrt{x+1}\sqrt{x} + \frac{1}{8}\left(2\left(4\,x-9\right)(x+1)+33\right)\sqrt{x+1}\sqrt{x} + \frac{3}{4}\left(2\,x-3\right)\sqrt{x+1}\sqrt{x} + \sqrt{x+1}\sqrt{x} + \frac{5}{64}\log\left(\sqrt{x+1}-\sqrt{x}\right)$$

input
```
integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="giac")
```

output
```
1/192*(2*(4*(6*x - 19)*(x + 1) + 163)*(x + 1) - 279)*sqrt(x + 1)*sqrt(x) +
 1/8*(2*(4*x - 9)*(x + 1) + 33)*sqrt(x + 1)*sqrt(x) + 3/4*(2*x - 3)*sqrt(x
 + 1)*sqrt(x) + sqrt(x + 1)*sqrt(x) + 5/64*log(sqrt(x + 1) - sqrt(x))
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(1+x)^{5/2}\,dx = \int \sqrt{x}\,(x+1)^{5/2}\,dx$$

input
```
int(x^(1/2)*(x + 1)^(5/2),x)
```

output
```
int(x^(1/2)*(x + 1)^(5/2), x)
```

3.39 $\int \frac{x^4}{\left(1-x^2\right)^{5/2}}\,dx$

3.39.1 Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{x^4}{\left(1-x^2\right)^{5/2}}\,dx=\frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\arcsin(x)$$

output
```
1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

3.39.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{\left(1-x^2\right)^{5/2}}\,dx=\frac{x\left(-3+4x^2\right)}{3\left(1-x^2\right)^{3/2}}+2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

input
```
Integrate[x^4/(1 - x^2)^(5/2),x]
```

output
```
(x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

3.39.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {252, 252, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(1-x^2)^{5/2}} \, dx$$
$$\Big\downarrow \; 252$$
$$\frac{x^3}{3(1-x^2)^{3/2}} - \int \frac{x^2}{(1-x^2)^{3/2}} dx$$
$$\Big\downarrow \; 252$$
$$\int \frac{1}{\sqrt{1-x^2}} dx - \frac{x}{\sqrt{1-x^2}} + \frac{x^3}{3(1-x^2)^{3/2}}$$
$$\Big\downarrow \; 223$$
$$\arcsin(x) - \frac{x}{\sqrt{1-x^2}} + \frac{x^3}{3(1-x^2)^{3/2}}$$

input
```
Int[x^4/(1 - x^2)^(5/2),x]
```

output
```
x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

3.39.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

3.39.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x^3}{3(-x^2+1)^{\frac{3}{2}}}+\arcsin(x)-\frac{x}{\sqrt{-x^2+1}}$	30
risch	$-\frac{x(4x^2-3)}{3(x^2-1)\sqrt{-x^2+1}}+\arcsin(x)$	30
meijerg	$-\frac{2i\left(-\frac{i\sqrt{\pi}\,x(-20x^2+15)}{10(-x^2+1)^{\frac{3}{2}}}+\frac{3i\sqrt{\pi}\,\arcsin(x)}{2}\right)}{3\sqrt{\pi}}$	39
pseudoelliptic	$\frac{(x^2-1)\sqrt{-x^2+1}\,\arctan\left(\frac{\sqrt{-x^2+1}}{x}\right)+\frac{4x^3}{3}-x}{(-x^2+1)^{\frac{3}{2}}}$	49
trager	$\frac{(4x^2-3)x\sqrt{-x^2+1}}{3(x^2-1)^2}+\operatorname{RootOf}(\_Z^2+1)\ln\left(\operatorname{RootOf}(\_Z^2+1)\sqrt{-x^2+1}+x\right)$	54

input
```
int(x^4/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-1/(-x^2+1)^(1/2)*x
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(29) = 58.

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = -\frac{6\,(x^4-2\,x^2+1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-(4\,x^3-3\,x)\sqrt{-x^2+1}}{3\,(x^4-2\,x^2+1)}$$

input
```
integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*s
qrt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

3.39.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(26) = 52.

Time = 0.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.00

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3} + \frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3} - \frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3} - \frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3} + \frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

input
```
integrate(x**4/(-x**2+1)**(5/2),x)
```

output
```
3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x
**2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x*
*4 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{1}{3}\,x\left(\frac{3\,x^2}{(-x^2+1)^{\frac{3}{2}}} - \frac{2}{(-x^2+1)^{\frac{3}{2}}}\right) - \frac{x}{3\sqrt{-x^2+1}} + \arcsin(x)$$

input
```
integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
```

output
```
1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
 + arcsin(x)
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{(4\,x^2-3)\sqrt{-x^2+1}x}{3\,(x^2-1)^2} + \arcsin(x)$$

input
```
integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

output
```
1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

3.39.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.60

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \operatorname{asin}(x) + \frac{3\sqrt{1-x^2}}{4\,(x-1)} + \frac{3\sqrt{1-x^2}}{4\,(x+1)} - \sqrt{1-x^2}\left(\frac{1}{12\,(x-1)} - \frac{1}{12\,(x-1)^2}\right) - \sqrt{1-x^2}\left(\frac{1}{12\,(x+1)} + \frac{1}{12\,(x+1)^2}\right)$$

input
```
int(x^4/(1 - x^2)^(5/2),x)
```

output
```
asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1)
) - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*
(1/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

3.40 $\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2} \, dy$

3.40.1 Optimal result

Integrand size = 29, antiderivative size = 51

$$\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2} \, dy = B \arctan\left(\frac{By}{\sqrt{A^2+B^2-B^2y^2}}\right) + A \operatorname{arctanh}\left(\frac{Ay}{\sqrt{A^2+B^2-B^2y^2}}\right)$$

output
```
B*arctan(B*y/(-B^2*y^2+A^2+B^2)^(1/2))+A*arctanh(A*y/(-B^2*y^2+A^2+B^2)^(1
/2))
```

3.40.2 Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2} \, dy = -2B \arctan\left(\frac{-\frac{\sqrt{A^2+B^2}}{B}+\frac{\sqrt{A^2+B^2-B^2y^2}}{B}}{y}\right) + A \operatorname{arctanh}\left(\frac{\sqrt{A^2+B^2-B^2y^2}}{Ay}\right)$$

input
```
Integrate[Sqrt[A^2 + B^2 - B^2*y^2]/(1 - y^2),y]
```

output
```
-2*B*ArcTan[(-(Sqrt[A^2 + B^2]/B) + Sqrt[A^2 + B^2 - B^2*y^2]/B)/y] + A*Ar
cTanh[Sqrt[A^2 + B^2 - B^2*y^2]/(A*y)]
```

3.40.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {301, 224, 216, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{A^2-B^2y^2+B^2}}{1-y^2}dy$$

$$\Big\downarrow \text{ 301}$$

$$A^2\int \frac{1}{(1-y^2)\sqrt{A^2+B^2-B^2y^2}}dy+B^2\int \frac{1}{\sqrt{A^2+B^2-B^2y^2}}dy$$

$$\Big\downarrow \text{ 224}$$

$$A^2\int \frac{1}{(1-y^2)\sqrt{A^2+B^2-B^2y^2}}dy+B^2\int \frac{1}{\frac{B^2y^2}{A^2+B^2-B^2y^2}+1}d\frac{y}{\sqrt{A^2+B^2-B^2y^2}}$$

$$\Big\downarrow \text{ 216}$$

$$A^2\int \frac{1}{(1-y^2)\sqrt{A^2+B^2-B^2y^2}}dy+B\arctan\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

$$\Big\downarrow \text{ 291}$$

$$A^2\int \frac{1}{1-\frac{A^2y^2}{A^2+B^2-B^2y^2}}d\frac{y}{\sqrt{A^2+B^2-B^2y^2}}+B\arctan\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$B\arctan\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)+A\text{arctanh}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

input
```
Int[Sqrt[A^2 + B^2 - B^2*y^2]/(1 - y^2),y]
```

output
```
B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]] + A*ArcTanh[(A*y)/Sqrt[A^2 + B^2
 - B^2*y^2]]
```

3.40.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

3.40.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.76

method	result
pseudoelliptic	$\frac{A \ln\left(\frac{A y+\sqrt{-B^{2} y^{2}+A^{2}+B^{2}}}{y}\right)}{2}-\frac{A \ln\left(\frac{A y-\sqrt{-B^{2} y^{2}+A^{2}+B^{2}}}{y}\right)}{2}-B \arctan\left(\frac{\sqrt{-B^{2} y^{2}+A^{2}+B^{2}}}{B y}\right)$
default	$-\frac{\sqrt{-B^{2} \left(y-1\right)^{2}-2 B^{2} \left(y-1\right)+A^{2}}}{2}+\frac{B^{2} \arctan\left(\frac{\sqrt{B^{2}}\, y}{\sqrt{-B^{2} \left(y-1\right)^{2}-2 B^{2} \left(y-1\right)+A^{2}}}\right)}{2 \sqrt{B^{2}}}+\frac{A^{2} \ln\left(\frac{2 A^{2}-2 B^{2} \left(y-1\right)+2 \sqrt{A^{2}}\, \sqrt{-B^{2} (y}}{y-1}\right)}{2 \sqrt{A^{2}}}$

input
```
int((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y,method=_RETURNVERBOSE)
```

output
```
1/2*A*ln((A*y+(-B^2*y^2+A^2+B^2)^(1/2))/y)-1/2*A*ln((A*y-(-B^2*y^2+A^2+B^2
)^(1/2))/y)-B*arctan(1/B/y*(-B^2*y^2+A^2+B^2)^(1/2))
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(47) = 94$.

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.53

$$\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy = -B\arctan\left(\frac{\sqrt{-B^2y^2+A^2+B^2}}{By}\right) + \frac{1}{4}A\log\left(-\frac{(A^2-B^2)y^2+2\sqrt{-B^2y^2+A^2+B^2}Ay+A^2+B^2}{y^2}\right) - \frac{1}{4}A\log\left(-\frac{(A^2-B^2)y^2-2\sqrt{-B^2y^2+A^2+B^2}Ay+A^2+B^2}{y^2}\right)$$

input
```
integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y, algorithm="fricas")
```

output
```
-B*arctan(sqrt(-B^2*y^2 + A^2 + B^2)/(B*y)) + 1/4*A*log(-((A^2 - B^2)*y^2
+ 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2) - 1/4*A*log(-((A^2 -
B^2)*y^2 - 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2)
```

3.40.6 Sympy [F]

$$\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy = -\int \frac{\sqrt{A^2-B^2y^2+B^2}}{y^2-1}\,dy$$

input
```
integrate((-B**2*y**2+A**2+B**2)**(1/2)/(-y**2+1),y)
```

output
```
-Integral(sqrt(A**2 - B**2*y**2 + B**2)/(y**2 - 1), y)
```

3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(47) = 94$.

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.24

$$\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy = B\arcsin\left(\frac{B^2y}{\sqrt{A^2B^2+B^4}}\right) - \frac{1}{2}A\log\left(B^2+\frac{A^2}{y+1}+\frac{\sqrt{-B^2y^2+A^2+B^2}A}{y+1}\right) + \frac{1}{2}A\log\left(-B^2+\frac{2A^2}{|2y-2|}+\frac{2\sqrt{-B^2y^2+A^2+B^2}A}{|2y-2|}\right)$$

input
```
integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y, algorithm="maxima")
```

output
```
B*arcsin(B^2*y/sqrt(A^2*B^2 + B^4)) - 1/2*A*log(B^2 + A^2/(y + 1) + sqrt(-
B^2*y^2 + A^2 + B^2)*A/(y + 1)) + 1/2*A*log(-B^2 + 2*A^2/abs(2*y - 2) + 2*
sqrt(-B^2*y^2 + A^2 + B^2)*A/abs(2*y - 2))
```

3.40.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(47) = 94$.

Time = 0.34 (sec) , antiderivative size = 295, normalized size of antiderivative = 5.78

$$\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy = -\frac{\left(\pi\mathrm{sgn}(y) - 2\arctan\left(-\frac{B^2y\left(\frac{\left(\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|\right)^2}{B^4y^2}-1\right)}{2\left(\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|\right)}\right)\right)B^2}{2|B|} + \frac{AB\log\left(\left|-\left(\frac{B^2y}{\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|}-\frac{\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|}{B^2y}\right)B+2A\right|\right)}{2|B|} - \frac{AB\log\left(\left|-\left(\frac{B^2y}{\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|}-\frac{\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|}{B^2y}\right)B-2A\right|\right)}{2|B|}$$

input
```
integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y, algorithm="giac")
```

output
```
-1/2*(pi*sgn(y) - 2*arctan(-1/2*B^2*y*((sqrt(A^2 + B^2)*B + sqrt(-B^2*y^2
+ A^2 + B^2)*abs(B))^2/(B^4*y^2) - 1)/(sqrt(A^2 + B^2)*B + sqrt(-B^2*y^2 +
 A^2 + B^2)*abs(B))))*B^2/abs(B) + 1/2*A*B*log(abs(-(B^2*y/(sqrt(A^2 + B^2
)*B + sqrt(-B^2*y^2 + A^2 + B^2)*abs(B)) - (sqrt(A^2 + B^2)*B + sqrt(-B^2*
y^2 + A^2 + B^2)*abs(B))/(B^2*y))*B + 2*A))/abs(B) - 1/2*A*B*log(abs(-(B^2
*y/(sqrt(A^2 + B^2)*B + sqrt(-B^2*y^2 + A^2 + B^2)*abs(B)) - (sqrt(A^2 + B
^2)*B + sqrt(-B^2*y^2 + A^2 + B^2)*abs(B))/(B^2*y))*B - 2*A))/abs(B)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy$$
$$= \begin{cases} -\int \frac{\sqrt{-B^2\,y^2}}{y^2-1}\,dy & \text{if} \quad A^2+B^2=0 \\ -\ln\left(2\,y\,\sqrt{-B^2}+2\,\sqrt{A^2-B^2\,y^2+B^2}\right)\,\sqrt{-B^2}-\operatorname{atan}\left(\frac{y\,\sqrt{A^2}\,1\mathrm{i}}{\sqrt{A^2-B^2\,y^2+B^2}}\right)\,\sqrt{A^2}\,1\mathrm{i} & \text{if} \quad A^2+B^2\neq 0 \end{cases}$$

input
```
int(-(A^2 + B^2 - B^2*y^2)^(1/2)/(y^2 - 1),y)
```

output
```
piecewise(A^2 + B^2 == 0, -int((-B^2*y^2)^(1/2)/(y^2 - 1), y), A^2 + B^2 ~
= 0, - atan((y*(A^2)^(1/2)*1i)/(A^2 + B^2 - B^2*y^2)^(1/2))*(A^2)^(1/2)*1i
 - log(2*y*(-B^2)^(1/2) + 2*(A^2 + B^2 - B^2*y^2)^(1/2))*(-B^2)^(1/2))
```

3.41 $\int \sin^2(x)\,dx$

3.41.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{1}{2}\cos(x)\sin(x)$$

output
```
1/2*x-1/2*cos(x)*sin(x)
```

3.41.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{1}{4}\sin(2x)$$

input
```
Integrate[Sin[x]^2,x]
```

output
```
x/2 - Sin[2*x]/4
```

3.41.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(x)^2 dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[Sin[x]^2,x]
```

output
```
x/2 - (Cos[x]*Sin[x])/2
```

3.41.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.41.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}-\frac{\sin(2x)}{4}$	11
parallelrisch	$\frac{x}{2}-\frac{\sin(2x)}{4}$	11
meijerg	$\frac{\sqrt{\pi}\left(\frac{2x}{\sqrt{\pi}}-\frac{\sin(2x)}{\sqrt{\pi}}\right)}{4}$	22
norman	$\frac{\tan^3\left(\frac{x}{2}\right)+x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}-\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

input
```
int(sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x-1/2*cos(x)*sin(x)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = -\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

input
```
integrate(sin(x)^2,x, algorithm="fricas")
```

output
```
-1/2*cos(x)*sin(x) + 1/2*x
```

3.41.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{x}{2}-\frac{\sin(x)\cos(x)}{2}$$

input
```
integrate(sin(x)**2,x)
```

output
```
x/2 - sin(x)*cos(x)/2
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{1}{2}\,x - \frac{1}{4}\,\sin\left(2\,x\right)$$

input
```
integrate(sin(x)^2,x, algorithm="maxima")
```

output
```
1/2*x - 1/4*sin(2*x)
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{1}{2}\,x - \frac{1}{4}\,\sin\left(2\,x\right)$$

input
```
integrate(sin(x)^2,x, algorithm="giac")
```

output
```
1/2*x - 1/4*sin(2*x)
```

3.41.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\,dx = \frac{x}{2} - \frac{\sin\left(2\,x\right)}{4}$$

input
```
int(sin(x)^2,x)
```

output
```
x/2 - sin(2*x)/4
```

3.42 $\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx$

3.42.1 Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx = -B\arctan\left(\frac{B\cos(x)}{\sqrt{A^2+B^2\sin^2(x)}}\right) - A\text{arctanh}\left(\frac{A\cos(x)}{\sqrt{A^2+B^2\sin^2(x)}}\right)$$

output
```
-B*arctan(B*cos(x)/(A^2+B^2*sin(x)^2)^(1/2))-A*arctanh(A*cos(x)/(A^2+B^2*s
in(x)^2)^(1/2))
```

3.42.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 99 vs. 2(49) = 98.

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.02

$$\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx = -\sqrt{A^2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{A^2}\cos(x)}{\sqrt{2A^2+B^2-B^2\cos(2x)}}\right) + \sqrt{-B^2}\log\left(\sqrt{2}\sqrt{-B^2}\cos(x) + \sqrt{2A^2+B^2-B^2\cos(2x)}\right)$$

input
```
Integrate[Csc[x]*Sqrt[A^2 + B^2*Sin[x]^2],x]
```

output
```
-(Sqrt[A^2]*ArcTanh[(Sqrt[2]*Sqrt[A^2]*Cos[x])/Sqrt[2*A^2 + B^2 - B^2*Cos[
2*x]]]) + Sqrt[-B^2]*Log[Sqrt[2]*Sqrt[-B^2]*Cos[x] + Sqrt[2*A^2 + B^2 - B^
2*Cos[2*x]]]
```

3.42.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 3665, 301, 224, 216, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{A^2+B^2\sin(x)^2}}{\sin(x)}dx$$

$$\Big\downarrow \ 3665$$

$$-\int \frac{\sqrt{A^2+B^2-B^2\cos^2(x)}}{1-\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \ 301$$

$$A^2\left(-\int \frac{1}{(1-\cos^2(x))\sqrt{A^2+B^2-B^2\cos^2(x)}}d\cos(x)\right)-B^2\int \frac{1}{\sqrt{A^2+B^2-B^2\cos^2(x)}}d\cos(x)$$

$$\Big\downarrow \ 224$$

$$A^2\left(-\int \frac{1}{(1-\cos^2(x))\sqrt{A^2+B^2-B^2\cos^2(x)}}d\cos(x)\right)-B^2\int \frac{1}{\frac{B^2\cos^2(x)}{A^2+B^2-B^2\cos^2(x)}+1}d\frac{\cos(x)}{\sqrt{A^2+B^2-B^2\cos^2(x)}}$$

$$\Big\downarrow \ 216$$

$$A^2\left(-\int \frac{1}{(1-\cos^2(x))\sqrt{A^2+B^2-B^2\cos^2(x)}}d\cos(x)\right)-B\arctan\left(\frac{B\cos(x)}{\sqrt{A^2-B^2\cos^2(x)+B^2}}\right)$$

$$\Big\downarrow \text{291}$$

$$A^2\left(-\int \frac{1}{1-\frac{A^2\cos^2(x)}{A^2+B^2-B^2\cos^2(x)}}d\frac{\cos(x)}{\sqrt{A^2+B^2-B^2\cos^2(x)}}\right)-B\arctan\left(\frac{B\cos(x)}{\sqrt{A^2-B^2\cos^2(x)+B^2}}\right)$$

$$\Big\downarrow \text{219}$$

$$-B\arctan\left(\frac{B\cos(x)}{\sqrt{A^2-B^2\cos^2(x)+B^2}}\right)-A\text{arctanh}\left(\frac{A\cos(x)}{\sqrt{A^2-B^2\cos^2(x)+B^2}}\right)$$

input
```
Int[Csc[x]*Sqrt[A^2 + B^2*Sin[x]^2],x]
```

output
```
-(B*ArcTan[(B*Cos[x])/Sqrt[A^2 + B^2 - B^2*Cos[x]^2]]) - A*ArcTanh[(A*Cos[
x])/Sqrt[A^2 + B^2 - B^2*Cos[x]^2]]
```

3.42.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.42.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.04

method	result
default	$-\frac{\sqrt{\left(A^{2}+B^{2}\left(\sin^{2}\left(x\right)\right)\right)\left(\cos^{2}\left(x\right)\right)}\left(A\ \operatorname{csgn}\left(A\right)\ln\left(-\frac{A^{2}\left(\sin^{2}\left(x\right)\right)-B^{2}\left(\sin^{2}\left(x\right)\right)-2\ \operatorname{csgn}\left(A\right)A\sqrt{\left(A^{2}+B^{2}\left(\sin^{2}\left(x\right)\right)\right)\left(\cos^{2}\left(x\right)\right)}-2A^{2}}{\sin\left(x\right)^{2}}\right)-B\ \mathrm{c}\right.}{2\cos\left(x\right)\sqrt{A^{2}+B^{2}\left(\sin^{2}\left(x\right)\right)}}$

input
```
int((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((A^2+B^2*sin(x)^2)*cos(x)^2)^(1/2)*(A*csgn(A)*ln(-(A^2*sin(x)^2-B^2*
sin(x)^2-2*csgn(A)*A*((A^2+B^2*sin(x)^2)*cos(x)^2)^(1/2)-2*A^2)/sin(x)^2)-
B*csgn(B)*arctan(1/2*csgn(B)/B*(2*B^2*sin(x)^2+A^2-B^2)/((A^2+B^2*sin(x)^2
)*cos(x)^2)^(1/2)))/cos(x)/(A^2+B^2*sin(x)^2)^(1/2)
```

3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(45) = 90$.

Time = 0.32 (sec) , antiderivative size = 244, normalized size of antiderivative = 4.98

$$\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx$$
$$= \frac{1}{2}B\arctan\left(-\frac{(A^4+2A^2B^2+B^4)\cos(x)\sin(x)-2\left(2B^3\cos(x)^3-(A^2B+B^3)\cos(x)\right)\sqrt{-B^2\cos(x)^2+A^2+B^2}}{4B^4\cos(x)^4+A^4+2A^2B^2+B^4-(A^4+6A^2B^2+5B^4)\cos(x)^2}\right)$$
$$-\frac{1}{2}B\arctan\left(\frac{\sin(x)}{\cos(x)}\right)-\frac{1}{2}A\log\left(-B^2\cos(x)^2+AB\cos(x)\sin(x)+A^2+B^2\right.$$
$$\left.+\sqrt{-B^2\cos(x)^2+A^2+B^2}(A\cos(x)+B\sin(x))\right)+\frac{1}{2}A\log\left(-B^2\cos(x)^2\right.$$
$$\left.-AB\cos(x)\sin(x)+A^2+B^2-\sqrt{-B^2\cos(x)^2+A^2+B^2}(A\cos(x)-B\sin(x))\right)$$

input
```
integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="fricas")
```

output
```
1/2*B*arctan(-((A^4 + 2*A^2*B^2 + B^4)*cos(x)*sin(x) - 2*(2*B^3*cos(x)^3 -
 (A^2*B + B^3)*cos(x))*sqrt(-B^2*cos(x)^2 + A^2 + B^2))/(4*B^4*cos(x)^4 +
A^4 + 2*A^2*B^2 + B^4 - (A^4 + 6*A^2*B^2 + 5*B^4)*cos(x)^2)) - 1/2*B*arcta
n(sin(x)/cos(x)) - 1/2*A*log(-B^2*cos(x)^2 + A*B*cos(x)*sin(x) + A^2 + B^2
 + sqrt(-B^2*cos(x)^2 + A^2 + B^2)*(A*cos(x) + B*sin(x))) + 1/2*A*log(-B^2
*cos(x)^2 - A*B*cos(x)*sin(x) + A^2 + B^2 - sqrt(-B^2*cos(x)^2 + A^2 + B^2
)*(A*cos(x) - B*sin(x)))
```

3.42.6 Sympy [F]

$$\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx = \int \frac{\sqrt{A^2+B^2\sin^2(x)}}{\sin(x)}\,dx$$

input
```
integrate((A**2+B**2*sin(x)**2)**(1/2)/sin(x),x)
```

output
```
Integral(sqrt(A**2 + B**2*sin(x)**2)/sin(x), x)
```

3.42.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.37

$$\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx = -B\arcsin\left(\frac{B^2\cos(x)}{\sqrt{A^2B^2+B^4}}\right) - \frac{1}{2}A\log\left(B^2 - \frac{A^2}{\cos(x)-1} - \frac{\sqrt{-B^2\cos(x)^2+A^2+B^2}A}{\cos(x)-1}\right) + \frac{1}{2}A\log\left(-B^2 + \frac{A^2}{\cos(x)+1} + \frac{\sqrt{-B^2\cos(x)^2+A^2+B^2}A}{\cos(x)+1}\right)$$

input
```
integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="maxima")
```

output
```
-B*arcsin(B^2*cos(x)/sqrt(A^2*B^2 + B^4)) - 1/2*A*log(B^2 - A^2/(cos(x) -
1) - sqrt(-B^2*cos(x)^2 + A^2 + B^2)*A/(cos(x) - 1)) + 1/2*A*log(-B^2 + A^
2/(cos(x) + 1) + sqrt(-B^2*cos(x)^2 + A^2 + B^2)*A/(cos(x) + 1))
```

3.42.8 Giac [F]

$$\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx = \int \frac{\sqrt{B^2\sin(x)^2+A^2}}{\sin(x)}\,dx$$

input
```
integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="giac")
```

output
```
integrate(sqrt(B^2*sin(x)^2 + A^2)/sin(x), x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx=\int \frac{\sqrt{A^2+B^2\sin\left(x\right)^2}}{\sin\left(x\right)}\,dx$$

input
```
int((B^2*sin(x)^2 + A^2)^(1/2)/sin(x),x)
```

output
```
int((B^2*sin(x)^2 + A^2)^(1/2)/sin(x), x)
```

3.43 $\int \frac{1}{1+\cos(x)}\,dx$

3.43.1 Optimal result

Integrand size = 6, antiderivative size = 9

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{1+\cos(x)}$$

output
```
sin(x)/(1+cos(x))
```

3.43.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

input
```
Integrate[(1 + Cos[x])^(-1),x]
```

output
```
Tan[x/2]
```

3.43.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \text{3127}$$

$$\frac{\sin(x)}{\cos(x)+1}$$

input
```
Int[(1 + Cos[x])^(-1),x]
```

output
```
Sin[x]/(1 + Cos[x])
```

3.43.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

3.43.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

method	result	size
default	$\tan\left(\frac{x}{2}\right)$	5
norman	$\tan\left(\frac{x}{2}\right)$	5
parallelrisch	$\tan\left(\frac{x}{2}\right)$	5
risch	$\frac{2i}{e^{ix}+1}$	13

input
```
int(1/(cos(x)+1),x,method=_RETURNVERBOSE)
```

output
```
tan(1/2*x)
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{\cos(x)+1}$$

input
```
integrate(1/(1+cos(x)),x, algorithm="fricas")
```

output
```
sin(x)/(cos(x) + 1)
```

3.43.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.33

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

input
```
integrate(1/(1+cos(x)),x)
```

output
```
tan(x/2)
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{\cos(x)+1}$$

input
```
integrate(1/(1+cos(x)),x, algorithm="maxima")
```

output
```
sin(x)/(cos(x) + 1)
```

3.43.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(9) = 18.

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 3.33

$$\int \frac{1}{1+\cos(x)}\,dx = -\frac{2\tan\left(\frac{1}{2}\,x\right)}{(x^2+1)\left(\frac{x^2-1}{x^2+1}-1\right)}$$

input
```
integrate(1/(1+cos(x)),x, algorithm="giac")
```

output
```
-2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

3.43.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

input
```
int(1/(cos(x) + 1),x)
```

output
```
tan(x/2)
```

3.44 $\int e^x x \, dx$

3.44.1 Optimal result

Integrand size = 5, antiderivative size = 11

$$\int e^x x \, dx = -e^x + e^x x$$

output
```
-exp(x)+exp(x)*x
```

3.44.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int e^x x \, dx = e^x(-1 + x)$$

input
```
Integrate[E^x*x,x]
```

output
```
E^x*(-1 + x)
```

3.44.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x x \, dx$$

$$\Big\downarrow \text{ 2607}$$

$$e^x x - \int e^x dx$$

$$\Big\downarrow \text{ 2624}$$

$$e^x x - e^x$$

input
```
Int[E^x*x,x]
```

output
```
-E^x + E^x*x
```

3.44.3.1 Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.44.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

method	result	size
gosper	$(-1+x)\,\mathrm{e}^{x}$	7
risch	$(-1+x)\,\mathrm{e}^{x}$	7
default	$-\mathrm{e}^{x}+\mathrm{e}^{x}x$	10
norman	$-\mathrm{e}^{x}+\mathrm{e}^{x}x$	10
parallelrisch	$-\mathrm{e}^{x}+\mathrm{e}^{x}x$	10
parts	$-\mathrm{e}^{x}+\mathrm{e}^{x}x$	10
meijerg	$1-\frac{(-2x+2)\mathrm{e}^{x}}{2}$	12

input
```
int(exp(x)*x,x,method=_RETURNVERBOSE)
```

output
```
(-1+x)*exp(x)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int e^x x\,dx = (x-1)e^x$$

input
```
integrate(exp(x)*x,x, algorithm="fricas")
```

output
```
(x - 1)*e^x
```

3.44.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.45

$$\int e^x x\,dx = (x-1)\,e^x$$

input
```
integrate(exp(x)*x,x)
```

output
```
(x - 1)*exp(x)
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int e^x x\,dx = (x-1)e^x$$

input
```
integrate(exp(x)*x,x, algorithm="maxima")
```

output
```
(x - 1)*e^x
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int e^x x\,dx = (x-1)e^x$$

input
```
integrate(exp(x)*x,x, algorithm="giac")
```

output
```
(x - 1)*e^x
```

3.44.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int e^x x\,dx = \mathrm{e}^x\,(x-1)$$

input
```
int(x*exp(x),x)
```

output
```
exp(x)*(x - 1)
```

3.45 $\int \frac{e^x x}{(1+x)^2} \, dx$

3.45.1 Optimal result

Integrand size = 10, antiderivative size = 9

$$\int \frac{e^x x}{(1+x)^2} \, dx = \frac{e^x}{1+x}$$

output
```
exp(x)/(1+x)
```

3.45.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{e^x x}{(1+x)^2} \, dx = \frac{e^x}{1+x}$$

input
```
Integrate[(E^x*x)/(1 + x)^2,x]
```

output
```
E^x/(1 + x)
```

3.45.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2627}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x x}{(x+1)^2} \, dx$$

$$\Big\downarrow \text{2627}$$

$$\frac{e^x}{x+1}$$

input
```
Int[(E^x*x)/(1 + x)^2,x]
```

output
```
E^x/(1 + x)
```

3.45.3.1 Defintions of rubi rules used

rule 2627
```
Int[(F_)^(v_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)), x_Symbol] :>
 Simp[g*(d + e*x)^(m + 1)*(F^v/(D[v, x]*e*Log[F])), x] /; FreeQ[{F, d, e, f
, g, m}, x] && LinearQ[v, x] && EqQ[e*g*(m + 1) - D[v, x]*(e*f - d*g)*Log[F
], 0]
```

3.45.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{{\mathrm e}^{x}}{1+x}$	9
default	$\frac{{\mathrm e}^{x}}{1+x}$	9
norman	$\frac{{\mathrm e}^{x}}{1+x}$	9
risch	$\frac{{\mathrm e}^{x}}{1+x}$	9
parallelrisch	$\frac{{\mathrm e}^{x}}{1+x}$	9

input
```
int(exp(x)*x/(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
exp(x)/(1+x)
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{e^x x}{(1+x)^2} \, dx = \frac{e^x}{x+1}$$

input
```
integrate(exp(x)*x/(1+x)^2,x, algorithm="fricas")
```

output
```
e^x/(x + 1)
```

3.45.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int \frac{e^x x}{(1+x)^2} \, dx = \frac{e^x}{x+1}$$

input
```
integrate(exp(x)*x/(1+x)**2,x)
```

output
```
exp(x)/(x + 1)
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{e^x x}{(1+x)^2} \, dx = \frac{e^x}{x+1}$$

input
```
integrate(exp(x)*x/(1+x)^2,x, algorithm="maxima")
```

output
```
e^x/(x + 1)
```

3.45.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(8) = 16$.

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 3.33

$$\int \frac{e^x x}{(1+x)^2} \, dx = -\frac{e^{\left(-(x+1)\left(\frac{1}{x+1}-1\right)\right)}}{(x+1)\left(\frac{1}{x+1}-1\right)-1}$$

input
```
integrate(exp(x)*x/(1+x)^2,x, algorithm="giac")
```

output
```
-e^(-(x + 1)*(1/(x + 1) - 1))/((x + 1)*(1/(x + 1) - 1) - 1)
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{e^x x}{(1+x)^2} \, dx = \frac{e^x}{x+1}$$

input
```
int((x*exp(x))/(x + 1)^2,x)
```

output
```
exp(x)/(x + 1)
```

3.46 $\int e^{x^2}(1+2x^2)\,dx$

3.46.1 Optimal result

Integrand size = 13, antiderivative size = 7

$$\int e^{x^2}\left(1+2x^2\right)\,dx=e^{x^2}x$$

output
```
exp(x^2)*x
```

3.46.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int e^{x^2}\left(1+2x^2\right)\,dx=e^{x^2}x$$

input
```
Integrate[E^x^2*(1 + 2*x^2),x]
```

output
```
E^x^2*x
```

3.46.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2}\left(2x^2+1\right)\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(2e^{x^2}x^2+e^{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$e^{x^2}x$$

input
```
Int[E^x^2*(1 + 2*x^2),x]
```

output
```
E^x^2*x
```

3.46.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

3.46.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

method	result	size
gosper	$e^{x^2} x$	7
default	$e^{x^2} x$	7
norman	$e^{x^2} x$	7
risch	$e^{x^2} x$	7
parallelrisch	$e^{x^2} x$	7
meijerg	$i\left(-ix\,e^{x^2}+\frac{i\,\mathrm{erfi}(x)\sqrt{\pi}}{2}\right)+\frac{\mathrm{erfi}(x)\sqrt{\pi}}{2}$	29
parts	$\mathrm{erfi}\left(x\right)\sqrt{\pi}\,x^2+\frac{\mathrm{erfi}(x)\sqrt{\pi}}{2}-2\sqrt{\pi}\left(\frac{x^2\,\mathrm{erfi}(x)}{2}-\frac{\frac{e^{x^2}x}{2}-\frac{\mathrm{erfi}(x)\sqrt{\pi}}{4}}{\sqrt{\pi}}\right)$	51

input
```
int(exp(x^2)*(2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
exp(x^2)*x
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int e^{x^2}\left(1+2x^2\right)\,dx = xe^{\left(x^2\right)}$$

input
```
integrate(exp(x^2)*(2*x^2+1),x, algorithm="fricas")
```

output
```
x*e^(x^2)
```

3.46.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int e^{x^2} \left(1+2x^2\right) \, dx = xe^{x^2}$$

input
```
integrate(exp(x**2)*(2*x**2+1),x)
```

output
```
x*exp(x**2)
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int e^{x^2} \left(1+2x^2\right) \, dx = xe^{\left(x^2\right)}$$

input
```
integrate(exp(x^2)*(2*x^2+1),x, algorithm="maxima")
```

output
```
x*e^(x^2)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int e^{x^2} \left(1+2x^2\right) \, dx = xe^{\left(x^2\right)}$$

input
```
integrate(exp(x^2)*(2*x^2+1),x, algorithm="giac")
```

output
```
x*e^(x^2)
```

3.46.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int e^{x^2} \left(1+2x^2\right) \, dx = x \, e^{x^2}$$

input
```
int(exp(x^2)*(2*x^2 + 1),x)
```

output
```
x*exp(x^2)
```

3.47 $\int e^{x^2} dx$

3.47.1 Optimal result

Integrand size = 5, antiderivative size = 11

$$\int e^{x^2} dx = \frac{1}{2}\sqrt{\pi}\text{erfi}(x)$$

output
```
1/2*erfi(x)*Pi^(1/2)
```

3.47.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{x^2} dx = \frac{1}{2}\sqrt{\pi}\text{erfi}(x)$$

input
```
Integrate[E^x^2,x]
```

output
```
(Sqrt[Pi]*Erfi[x])/2
```

3.47.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2}\,dx$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{1}{2}\sqrt{\pi}\text{erfi}(x)$$

input
```
Int[E^x^2,x]
```

output
```
(Sqrt[Pi]*Erfi[x])/2
```

3.47.3.1 Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

3.47.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\text{erfi}(x)\sqrt{\pi}}{2}$	8
meijerg	$\frac{\text{erfi}(x)\sqrt{\pi}}{2}$	8
risch	$\frac{\text{erfi}(x)\sqrt{\pi}}{2}$	8

input
```
int(exp(x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*erfi(x)*Pi^(1/2)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int e^{x^2}\,dx = \frac{1}{2}\,\sqrt{\pi}\,\mathrm{erfi}\,(x)$$

input
```
integrate(exp(x^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(pi)*erfi(x)
```

3.47.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{x^2}\,dx = \frac{\sqrt{\pi}\,\mathrm{erfi}\,(x)}{2}$$

input
```
integrate(exp(x**2),x)
```

output
```
sqrt(pi)*erfi(x)/2
```

3.47.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int e^{x^2}\,dx = -\frac{1}{2}i\,\sqrt{\pi}\,\mathrm{erf}\,(i\,x)$$

input
```
integrate(exp(x^2),x, algorithm="maxima")
```

output
```
-1/2*I*sqrt(pi)*erf(I*x)
```

3.47.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int e^{x^2} dx = \frac{1}{2} i \sqrt{\pi} \operatorname{erf}(-i x)$$

input
```
integrate(exp(x^2),x, algorithm="giac")
```

output
```
1/2*I*sqrt(pi)*erf(-I*x)
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int e^{x^2} dx = \frac{\sqrt{\pi} \operatorname{erfi}(x)}{2}$$

input
```
int(exp(x^2),x)
```

output
```
(pi^(1/2)*erfi(x))/2
```

3.48 $\int \frac{e^x}{x} dx$

3.48.1 Optimal result

Integrand size = 7, antiderivative size = 2

$$\int \frac{e^x}{x} dx = \text{ExpIntegralEi}(x)$$

output
```
Ei(x)
```

3.48.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{x} dx = \text{ExpIntegralEi}(x)$$

input
```
Integrate[E^x/x,x]
```

output
```
ExpIntegralEi[x]
```

3.48.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{x} \, dx$$
$$\Big\downarrow \text{ 2609}$$
$$\text{ExpIntegralEi}(x)$$

input
```
Int[E^x/x,x]
```

output
```
ExpIntegralEi[x]
```

3.48.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

3.48.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(2) = 4.

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 4.00

method	result	size
default	$-\mathrm{Ei}_1(-x)$	8
risch	$-\mathrm{Ei}_1(-x)$	8
meijerg	$\ln(x) + i\pi - \ln(-x) - \mathrm{Ei}_1(-x)$	21

input
```
int(exp(x)/x,x,method=_RETURNVERBOSE)
```

output
```
-Ei(1,-x)
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{x} \, dx = \operatorname{Ei}(x)$$

input
```
integrate(exp(x)/x,x, algorithm="fricas")
```

output
```
Ei(x)
```

3.48.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{x} \, dx = \operatorname{Ei}\left(x\right)$$

input
```
integrate(exp(x)/x,x)
```

output
```
Ei(x)
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{x} \, dx = \operatorname{Ei}(x)$$

input
```
integrate(exp(x)/x,x, algorithm="maxima")
```

output
```
Ei(x)
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{x}\,dx = \operatorname{Ei}(x)$$

input
```
integrate(exp(x)/x,x, algorithm="giac")
```

output
```
Ei(x)
```

3.48.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{x}\,dx = \operatorname{ei}(x)$$

input
```
int(exp(x)/x,x)
```

output
```
ei(x)
```

3.49 $\int \frac{x}{1+x^3} \, dx$

3.49.1 Optimal result

Integrand size = 9, antiderivative size = 41

$$\int \frac{x}{1+x^3} \, dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right)$$

output
```
-1/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{x}{1+x^3} \, dx = \frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log(1+x) + \frac{1}{6}\log\left(1-x+x^2\right)$$

input
```
Integrate[x/(1 + x^3),x]
```

output
```
ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

3.49.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {821, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^3+1}dx$$

$$\Big\downarrow \text{ 821}$$

$$\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\int \frac{1}{x+1}dx$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{3}\int \frac{x+1}{x^2-x+1}dx-\frac{1}{3}\log(x+1)$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{3}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-3\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)-\frac{1}{3}\log(x+1)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^2-x+1\right)\right)-\frac{1}{3}\log(x+1)$$

input
```
Int[x/(1 + x^3),x]
```

output
```
-1/3*Log[1 + x] + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + Log[1 - x + x^2]/2
)/3
```

3.49.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.49.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	33
default	$\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{\ln(1+x)}{3}$	35
meijerg	$-\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}+\frac{x^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{2}{3}}}$	80

input
```
int(x/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+x^3}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)$$

input
```
integrate(x/(x^3+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log
(x + 1)
```

3.49.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x^3}\,dx = -\frac{\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(x/(x**3+1),x)
```

output
```
-log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)
/3)/3
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x}{1+x^3} \, dx = \frac{1}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2 x-1)\right)+\frac{1}{6} \log \left(x^2-x+1\right)-\frac{1}{3} \log (x+1)$$

input
```
integrate(x/(x^3+1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log
(x + 1)
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x}{1+x^3} \, dx = \frac{1}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2 x-1)\right)+\frac{1}{6} \log \left(x^2-x+1\right)-\frac{1}{3} \log (|x+1|)$$

input
```
integrate(x/(x^3+1),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log
(abs(x + 1))
```

3.49.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{x}{1+x^3}\,dx = -\frac{\ln(x+1)}{3} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(x/(x^3 + 1),x)
```

output
```
log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6) - log(x - (3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x + 1)/3
```

3.50 $\int \frac{1}{-1+x^6} \, dx$

3.50.1 Optimal result

Integrand size = 7, antiderivative size = 47

$$\int \frac{1}{-1+x^6} \, dx = -\frac{\arctan\left(\frac{\sqrt{3}x}{1-x^2}\right)}{2\sqrt{3}} - \frac{\operatorname{arctanh}(x)}{3} - \frac{1}{6}\operatorname{arctanh}\left(\frac{x}{1+x^2}\right)$$

output
```
-1/3*arctanh(x)-1/6*arctanh(x/(x^2+1))-1/6*arctan(x*3^(1/2)/(-x^2+1))*3^(1
/2)
```

3.50.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int \frac{1}{-1+x^6} \, dx = \frac{1}{12}\left(-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) + 2\log(1-x) - 2\log(1+x) + \log\left(1-x+x^2\right) - \log\left(1+x+x^2\right)\right)$$

input
```
Integrate[(-1 + x^6)^(-1),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3
]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])/12
```

3.50.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6-1}\,dx$$

$$\Big\downarrow \text{754}$$

$$-\frac{1}{3}\int \frac{1}{1-x^2}dx-\frac{1}{3}\int \frac{2-x}{2\left(x^2-x+1\right)}dx-\frac{1}{3}\int \frac{x+2}{2\left(x^2+x+1\right)}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}\int \frac{1}{1-x^2}dx-\frac{1}{6}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{6}\int \frac{x+2}{x^2+x+1}dx$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{6}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{6}\int \frac{x+2}{x^2+x+1}dx-\frac{\text{arctanh}(x)}{3}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{6}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)-\frac{\text{arctanh}(x)}{3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)-\frac{\text{arctanh}(x)}{3}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{6}\left(3\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{6}\left(3\int \frac{1}{-(2x+1)^2-3}d(2x+1)-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)-\frac{\text{arctanh}(x)}{3}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{6}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+$$
$$\frac{1}{6}\left(-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right)-\frac{\operatorname{arctanh}(x)}{3}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{6}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+$$
$$\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2+x+1\right)\right)-\frac{\operatorname{arctanh}(x)}{3}$$

input
```
Int[(-1 + x^6)^(-1),x]
```

output
```
-1/3*ArcTanh[x] + (-(Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]]) + Log[1 - x + x^2
]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]) - Log[1 + x + x^2]/2)/6
```

3.50.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.50.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

method	result
default	$\frac{\ln(-1+x)}{6}-\frac{\ln(x^2+x+1)}{12}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(1+x)}{6}$
risch	$\frac{\ln(4x^2-4x+4)}{12}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(1+x)}{6}-\frac{\ln(x^2+x+1)}{12}-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{6}+\frac{\ln(-1+x)}{6}$
meijerg	$\frac{x\left(\ln\left(1-(x^6)^{\frac{1}{6}}\right)-\ln\left(1+(x^6)^{\frac{1}{6}}\right)+\frac{\ln\left(1-(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{2-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6}{2+(x^6)}\right.\right.}{6(x^6)^{\frac{1}{6}}}$

input
```
int(1/(x^6-1),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(-1+x)-1/12*ln(x^2+x+1)-1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/12
*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/6*ln(1+x)
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{1}{-1+x^6}\,dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{12}\log\left(x^2+x+1\right) + \frac{1}{12}\log\left(x^2-x+1\right) - \frac{1}{6}\log\left(x+1\right) + \frac{1}{6}\log\left(x-1\right)$$

input
```
integrate(1/(x^6-1),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x +
 1) + 1/6*log(x - 1)
```

3.50.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(36) = 72.

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.77

$$\int \frac{1}{-1+x^6}\,dx = \frac{\log\left(x-1\right)}{6} - \frac{\log\left(x+1\right)}{6} + \frac{\log\left(x^2-x+1\right)}{12} - \frac{\log\left(x^2+x+1\right)}{12} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(1/(x**6-1),x)
```

output
```
log(x - 1)/6 - log(x + 1)/6 + log(x**2 - x + 1)/12 - log(x**2 + x + 1)/12
- sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 +
 sqrt(3)/3)/6
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{1}{-1+x^6}\,dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{12}\log\left(x^2+x+1\right) + \frac{1}{12}\log\left(x^2-x+1\right) - \frac{1}{6}\log\left(x+1\right) + \frac{1}{6}\log\left(x-1\right)$$

input
```
integrate(1/(x^6-1),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x +
 1) + 1/6*log(x - 1)
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{1}{-1+x^6}\,dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{12}\log\left(x^2+x+1\right) + \frac{1}{12}\log\left(x^2-x+1\right) - \frac{1}{6}\log\left(|x+1|\right) + \frac{1}{6}\log\left(|x-1|\right)$$

input
```
integrate(1/(x^6-1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(abs
(x + 1)) + 1/6*log(abs(x - 1))
```

3.50.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.87

$$\int \frac{1}{-1+x^6}\,dx = -\frac{\operatorname{atanh}(x)}{3} - \operatorname{atan}\left(\frac{x\,1\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}} + \frac{\sqrt{3}\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6} + \frac{1}{6}\mathrm{i}\right) - \operatorname{atan}\left(\frac{x\,1\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}} - \frac{\sqrt{3}\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6} - \frac{1}{6}\mathrm{i}\right)$$

input
```
int(1/(x^6 - 1),x)
```

output
```
- atanh(x)/3 - atan((x*1i)/(3^(1/2)*1i + 1) + (3^(1/2)*x)/(3^(1/2)*1i + 1)
)*(3^(1/2)/6 + 1i/6) - atan((x*1i)/(3^(1/2)*1i - 1) - (3^(1/2)*x)/(3^(1/2)
*1i - 1))*(3^(1/2)/6 - 1i/6)
```

3.50. $\int \frac{1}{-1+x^6} dx$

3.51 $\int \frac{1}{A^4-A^2B^2+(-A^2+B^2)x^2}\,dx$

3.51.1 Optimal result

Integrand size = 27, antiderivative size = 21

$$\int \frac{1}{A^4-A^2B^2+(-A^2+B^2)\,x^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{x}{A}\right)}{A\,(A^2-B^2)}$$

output
```
arctanh(x/A)/A/(A^2-B^2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{A^4-A^2B^2+(-A^2+B^2)\,x^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{x}{A}\right)}{A\,(A^2-B^2)}$$

input
```
Integrate[(A^4 - A^2*B^2 + (-A^2 + B^2)*x^2)^(-1),x]
```

output
```
ArcTanh[x/A]/(A*(A^2 - B^2))
```

3.51.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{A^4 + x^2 (B^2 - A^2) - A^2 B^2} \, dx$$

$$\Big\downarrow \; 221$$

$$\frac{\text{arctanh}\left(\frac{x}{A}\right)}{A (A^2 - B^2)}$$

input
```
Int[(A^4 - A^2*B^2 + (-A^2 + B^2)*x^2)^(-1),x]
```

output
```
ArcTanh[x/A]/(A*(A^2 - B^2))
```

3.51.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.51.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

method	result	size
parallelrisch	$\frac{-\ln(A-x)+\ln(A+x)}{2A(A^2-B^2)}$	30
default	$\frac{-\frac{\ln(A-x)}{2A}+\frac{\ln(A+x)}{2A}}{A^2-B^2}$	34
norman	$-\frac{\ln(A-x)}{2A(A^2-B^2)}+\frac{\ln(A+x)}{2A(A^2-B^2)}$	44
risch	$-\frac{\ln(A-x)}{2A(A^2-B^2)}+\frac{\ln(A+x)}{2A(A^2-B^2)}$	44

input
```
int(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-ln(A-x)+ln(A+x))/A/(A^2-B^2)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{1}{A^4 - A^2B^2 + (-A^2 + B^2)\,x^2}\,dx = \frac{\log(A + x) - \log(-A + x)}{2\,(A^3 - AB^2)}$$

input
```
integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="fricas")
```

output
```
1/2*(log(A + x) - log(-A + x))/(A^3 - A*B^2)
```

3.51.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(12) = 24.

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.33

$$\int \frac{1}{A^4 - A^2B^2 + (-A^2 + B^2)\,x^2}\,dx = -\frac{\log\left(-\frac{A^3}{(A-B)(A+B)} + \frac{AB^2}{(A-B)(A+B)} + x\right)}{2A\,(A - B)\,(A + B)} + \frac{\log\left(\frac{A^3}{(A-B)(A+B)} - \frac{AB^2}{(A-B)(A+B)} + x\right)}{2A\,(A - B)\,(A + B)}$$

input
```
integrate(1/(A**4-A**2*B**2+(-A**2+B**2)*x**2),x)
```

output
```
-log(-A**3/((A - B)*(A + B)) + A*B**2/((A - B)*(A + B)) + x)/(2*A*(A - B)*
(A + B)) + log(A**3/((A - B)*(A + B)) - A*B**2/((A - B)*(A + B)) + x)/(2*A
*(A - B)*(A + B))
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{1}{A^4 - A^2B^2 + (-A^2 + B^2)\,x^2}\,dx = \frac{\log(A + x)}{2\,(A^3 - AB^2)} - \frac{\log(-A + x)}{2\,(A^3 - AB^2)}$$

input
```
integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="maxima")
```

output
```
1/2*log(A + x)/(A^3 - A*B^2) - 1/2*log(-A + x)/(A^3 - A*B^2)
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \frac{1}{A^4 - A^2B^2 + (-A^2 + B^2)\,x^2}\,dx = \frac{\log(|A + x|)}{2\,(A^3 - AB^2)} - \frac{\log(|-A + x|)}{2\,(A^3 - AB^2)}$$

input
```
integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="giac")
```

output
```
1/2*log(abs(A + x))/(A^3 - A*B^2) - 1/2*log(abs(-A + x))/(A^3 - A*B^2)
```

3.51.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{A^4 - A^2B^2 + (-A^2 + B^2)\,x^2}\,dx = -\frac{\operatorname{atanh}\left(\frac{x}{A}\right)}{A\,B^2 - A^3}$$

input
```
int(-1/(x^2*(A^2 - B^2) - A^4 + A^2*B^2),x)
```

output
```
-atanh(x/A)/(A*B^2 - A^3)
```

3.52 $\int x \log(x)\,dx$

3.52.1 Optimal result

Integrand size = 4, antiderivative size = 17

$$\int x \log(x)\,dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

output
```
-1/4*x^2+1/2*x^2*ln(x)
```

3.52.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x \log(x)\,dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

input
```
Integrate[x*Log[x],x]
```

output
```
-1/4*x^2 + (x^2*Log[x])/2
```

3.52.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\log(x)\,dx$$

$$\Big\downarrow \; 2741$$

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

input
```
Int[x*Log[x],x]
```

output
```
-1/4*x^2 + (x^2*Log[x])/2
```

3.52.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

3.52.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
norman	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
risch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
parallelrisch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
parts	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14

input
```
int(x*ln(x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2+1/2*x^2*ln(x)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right) - \frac{1}{4}\,x^2$$

input
```
integrate(x*log(x),x, algorithm="fricas")
```

output
```
1/2*x^2*log(x) - 1/4*x^2
```

3.52.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x\log(x)\,dx = \frac{x^2\log\left(x\right)}{2} - \frac{x^2}{4}$$

input
```
integrate(x*ln(x),x)
```

output
```
x**2*log(x)/2 - x**2/4
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right) - \frac{1}{4}\,x^2$$

input
```
integrate(x*log(x),x, algorithm="maxima")
```

output
```
1/2*x^2*log(x) - 1/4*x^2
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right) - \frac{1}{4}\,x^2$$

input
```
integrate(x*log(x),x, algorithm="giac")
```

output
```
1/2*x^2*log(x) - 1/4*x^2
```

3.52.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int x\log(x)\,dx = \frac{x^2\left(\ln\left(x\right) - \frac{1}{2}\right)}{2}$$

input
```
int(x*log(x),x)
```

output
```
(x^2*(log(x) - 1/2))/2
```

3.53 $\int x^2 \arcsin(x)\,dx$

3.53.1 Optimal result

Integrand size = 6, antiderivative size = 40

$$\int x^2 \arcsin(x)\,dx = \frac{\sqrt{1-x^2}}{3} - \frac{1}{9}\left(1-x^2\right)^{3/2} + \frac{1}{3}x^3 \arcsin(x)$$

output
```
-1/9*(-x^2+1)^(3/2)+1/3*x^3*arcsin(x)+1/3*(-x^2+1)^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int x^2 \arcsin(x)\,dx = \frac{1}{9}\left(\sqrt{1-x^2}\left(2+x^2\right) + 3x^3 \arcsin(x)\right)$$

input
```
Integrate[x^2*ArcSin[x],x]
```

output
```
(Sqrt[1 - x^2]*(2 + x^2) + 3*x^3*ArcSin[x])/9
```

3.53.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5138, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arcsin(x)\,dx$$

$$\Big\downarrow \text{5138}$$

$$\frac{1}{3}x^3\arcsin(x)-\frac{1}{3}\int\frac{x^3}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{3}x^3\arcsin(x)-\frac{1}{6}\int\frac{x^2}{\sqrt{1-x^2}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}x^3\arcsin(x)-\frac{1}{6}\int\left(\frac{1}{\sqrt{1-x^2}}-\sqrt{1-x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\arcsin(x)+\frac{1}{6}\left(2\sqrt{1-x^2}-\frac{2}{3}\left(1-x^2\right)^{3/2}\right)$$

input
```
Int[x^2*ArcSin[x],x]
```

output
```
(2*Sqrt[1 - x^2] - (2*(1 - x^2)^(3/2))/3)/6 + (x^3*ArcSin[x])/3
```

3.53.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5138
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

3.53.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{x^3 \arcsin(x)}{3}+\frac{x^2\sqrt{-x^2+1}}{9}+\frac{2\sqrt{-x^2+1}}{9}$	34
parts	$\frac{x^3 \arcsin(x)}{3}+\frac{x^2\sqrt{-x^2+1}}{9}+\frac{2\sqrt{-x^2+1}}{9}$	34

input
```
int(x^2*arcsin(x),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arcsin(x)+1/9*x^2*(-x^2+1)^(1/2)+2/9*(-x^2+1)^(1/2)
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int x^2 \arcsin(x)\,dx=\frac{1}{3}\,x^3 \arcsin\left(x\right)+\frac{1}{9}\,\left(x^2+2\right)\sqrt{-x^2+1}$$

input
```
integrate(x^2*arcsin(x),x, algorithm="fricas")
```

output
```
1/3*x^3*arcsin(x) + 1/9*(x^2 + 2)*sqrt(-x^2 + 1)
```

3.53.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int x^2 \arcsin(x)\,dx = \frac{x^3 \operatorname{asin}(x)}{3} + \frac{x^2\sqrt{1-x^2}}{9} + \frac{2\sqrt{1-x^2}}{9}$$

input
```
integrate(x**2*asin(x),x)
```

output
```
x**3*asin(x)/3 + x**2*sqrt(1 - x**2)/9 + 2*sqrt(1 - x**2)/9
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int x^2 \arcsin(x)\,dx = \frac{1}{3}\,x^3 \arcsin(x) + \frac{1}{9}\,\sqrt{-x^2+1}x^2 + \frac{2}{9}\,\sqrt{-x^2+1}$$

input
```
integrate(x^2*arcsin(x),x, algorithm="maxima")
```

output
```
1/3*x^3*arcsin(x) + 1/9*sqrt(-x^2 + 1)*x^2 + 2/9*sqrt(-x^2 + 1)
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int x^2 \arcsin(x)\,dx = \frac{1}{3}\,(x^2-1)x \arcsin(x) + \frac{1}{3}\,x \arcsin(x) - \frac{1}{9}\,(-x^2+1)^{\frac{3}{2}} + \frac{1}{3}\,\sqrt{-x^2+1}$$

input
```
integrate(x^2*arcsin(x),x, algorithm="giac")
```

output
```
1/3*(x^2 - 1)*x*arcsin(x) + 1/3*x*arcsin(x) - 1/9*(-x^2 + 1)^(3/2) + 1/3*s
qrt(-x^2 + 1)
```

3.53.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int x^2 \arcsin(x)\, dx = \frac{x^3 \operatorname{asin}(x)}{3} + \frac{\sqrt{1-x^2}\,(x^2+2)}{9}$$

input
```
int(x^2*asin(x),x)
```

output
```
(x^3*asin(x))/3 + ((1 - x^2)^(1/2)*(x^2 + 2))/9
```

3.54 $\int \frac{1}{1+2x+x^2}\,dx$

3.54.1 Optimal result

Integrand size = 10, antiderivative size = 7

$$\int \frac{1}{1+2x+x^2}\,dx = -\frac{1}{1+x}$$

output
```
-1/(1+x)
```

3.54.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+2x+x^2}\,dx = -\frac{1}{1+x}$$

input
```
Integrate[(1 + 2*x + x^2)^(-1),x]
```

output
```
-(1 + x)^(-1)
```

3.54.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1077, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2+2x+1} \, dx$$

$$\Big\downarrow \ 1077$$

$$\int \frac{1}{(x+1)^2}dx$$

$$\Big\downarrow \ 17$$

$$-\frac{1}{x+1}$$

input
```
Int[(1 + 2*x + x^2)^(-1),x]
```

output
```
-(1 + x)^(-1)
```

3.54.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1077
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int
[(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] &&
 IntegerQ[p]
```

3.54.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
gosper	$-\frac{1}{1+x}$	8
default	$-\frac{1}{1+x}$	8
norman	$-\frac{1}{1+x}$	8
meijerg	$\frac{x}{1+x}$	8
risch	$-\frac{1}{1+x}$	8
parallelrisch	$-\frac{1}{1+x}$	8

input
```
int(1/(x^2+2*x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/(1+x)
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+2x+x^2}\,dx=-\frac{1}{x+1}$$

input
```
integrate(1/(x^2+2*x+1),x, algorithm="fricas")
```

output
```
-1/(x + 1)
```

3.54.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{1+2x+x^2}\,dx=-\frac{1}{x+1}$$

input
```
integrate(1/(x**2+2*x+1),x)
```

output
```
-1/(x + 1)
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+2x+x^2}\,dx = -\frac{1}{x+1}$$

input
```
integrate(1/(x^2+2*x+1),x, algorithm="maxima")
```

output
```
-1/(x + 1)
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+2x+x^2}\,dx = -\frac{1}{x+1}$$

input
```
integrate(1/(x^2+2*x+1),x, algorithm="giac")
```

output
```
-1/(x + 1)
```

3.54.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+2x+x^2}\,dx = -\frac{1}{x+1}$$

input
```
int(1/(2*x + x^2 + 1),x)
```

output
```
-1/(x + 1)
```

3.55 $\int \frac{\log(x)}{(1+\log(x))^2} dx$

3.55.1 Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \frac{\log(x)}{(1+\log(x))^2} dx = \frac{x}{1+\log(x)}$$

output
```
x/(1+ln(x))
```

3.55.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{(1+\log(x))^2} dx = \frac{x}{1+\log(x)}$$

input
```
Integrate[Log[x]/(1 + Log[x])^2,x]
```

output
```
x/(1 + Log[x])
```

3.55.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2807, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)}{(\log(x)+1)^2} \, dx$$

$$\Big\downarrow \text{2807}$$

$$\int \left(\frac{1}{\log(x)+1}-\frac{1}{(\log(x)+1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x}{\log(x)+1}$$

input
```
Int[Log[x]/(1 + Log[x])^2,x]
```

output
```
x/(1 + Log[x])
```

3.55.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2807
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(Log[(c_.)*(x_)^(n_.)]*(e_.
) + (d_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*x^n])^p*(d +
 e*Log[c*x^n])^q, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IntegerQ[p] &&
IntegerQ[q]
```

3.55.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{x}{1+\ln(x)}$	9
norman	$\frac{x}{1+\ln(x)}$	9
risch	$\frac{x}{1+\ln(x)}$	9
parallelrisch	$\frac{x}{1+\ln(x)}$	9

input
```
int(ln(x)/(1+ln(x))^2,x,method=_RETURNVERBOSE)
```

output
```
x/(1+ln(x))
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{(1+\log(x))^2}\,dx = \frac{x}{\log{(x)}+1}$$

input
```
integrate(log(x)/(1+log(x))^2,x, algorithm="fricas")
```

output
```
x/(log(x) + 1)
```

3.55.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{\log(x)}{(1+\log(x))^2}\,dx = \frac{x}{\log{(x)}+1}$$

input
```
integrate(ln(x)/(1+ln(x))**2,x)
```

output
```
x/(log(x) + 1)
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{(1+\log(x))^2}\,dx = \frac{x}{\log(x)+1}$$

input
```
integrate(log(x)/(1+log(x))^2,x, algorithm="maxima")
```

output
```
x/(log(x) + 1)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{(1+\log(x))^2}\,dx = \frac{x}{\log(x)+1}$$

input
```
integrate(log(x)/(1+log(x))^2,x, algorithm="giac")
```

output
```
x/(log(x) + 1)
```

3.55.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log(x)}{(1+\log(x))^2}\,dx = \frac{x}{\ln(x)+1}$$

input
```
int(log(x)/(log(x) + 1)^2,x)
```

output
```
x/(log(x) + 1)
```

3.56 $\int \frac{1}{x(1+\log^2(x))} \, dx$

3.56.1 Optimal result

Integrand size = 12, antiderivative size = 3

$$\int \frac{1}{x(1+\log^2(x))} \, dx = \arctan(\log(x))$$

output
```
arctan(ln(x))
```

3.56.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+\log^2(x))} \, dx = \arctan(\log(x))$$

input
```
Integrate[1/(x*(1 + Log[x]^2)),x]
```

output
```
ArcTan[Log[x]]
```

3.56.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3039, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\log^2(x)+1\right)}\,dx$$

$$\Big\downarrow \ 3039$$

$$\int \frac{1}{\log^2(x)+1}d\log(x)$$

$$\Big\downarrow \ 216$$

$$\arctan(\log(x))$$

input
```
Int[1/(x*(1 + Log[x]^2)),x]
```

output
```
ArcTan[Log[x]]
```

3.56.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

3.56.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\arctan\left(\ln\left(x\right)\right)$	4
default	$\arctan\left(\ln\left(x\right)\right)$	4
risch	$\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$	20
parallelrisch	$\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$	20

input
```
int(1/x/(1+ln(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(ln(x))
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

input
```
integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
```

output
```
arctan(log(x))
```

3.56.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \operatorname{RootSum}\left(4z^2+1,\left(i\mapsto i\log\left(2i+\log\left(x\right)\right)\right)\right)$$

input
```
integrate(1/x/(1+ln(x)**2),x)
```

output
```
RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

input
```
integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
```

output
```
arctan(log(x))
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

input
```
integrate(1/x/(1+log(x)^2),x, algorithm="giac")
```

output
```
arctan(log(x))
```

3.56.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \mathrm{atan}(\ln\left(x\right))$$

input
```
int(1/(x*(log(x)^2 + 1)),x)
```

output
```
atan(log(x))
```

3.57 $\int \frac{1}{\log(x)}\,dx$

3.57.1 Optimal result

Integrand size = 4, antiderivative size = 2

$$\int \frac{1}{\log(x)}\,dx = \text{LogIntegral}(x)$$

output
```
Li(x)
```

3.57.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(x)}\,dx = \text{LogIntegral}(x)$$

input
```
Integrate[Log[x]^(-1),x]
```

output
```
LogIntegral[x]
```

3.57.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2735}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log(x)} \, dx$$

$$\Big\downarrow \text{2735}$$

$$\text{LogIntegral}(x)$$

input
```
Int[Log[x]^(-1),x]
```

output
```
LogIntegral[x]
```

3.57.3.1 Defintions of rubi rules used

rule 2735
```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

3.57.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs. 2(2) = 4.

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 4.50

method	result	size
default	$-\operatorname{Ei}_1(-\ln(x))$	9
risch	$-\operatorname{Ei}_1(-\ln(x))$	9

input
```
int(1/ln(x),x,method=_RETURNVERBOSE)
```

output
```
-Ei(1,-ln(x))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(x)}\,dx = \text{log\_integral}\,(x)$$

input
```
integrate(1/log(x),x, algorithm="fricas")
```

output
```
log_integral(x)
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(x)}\,dx = \text{li}\,(x)$$

input
```
integrate(1/ln(x),x)
```

output
```
li(x)
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{\log(x)}\,dx = \text{Ei}(\log\,(x))$$

input
```
integrate(1/log(x),x, algorithm="maxima")
```

output
```
Ei(log(x))
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{\log(x)}\,dx = \mathrm{Ei}(\log{(x)})$$

input
```
integrate(1/log(x),x, algorithm="giac")
```

output
```
Ei(log(x))
```

3.57.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(x)}\,dx = \mathrm{logint}(x)$$

input
```
int(1/log(x),x)
```

output
```
logint(x)
```

3.58 $\int x(\cos(x)+\sin(x))\,dx$

3.58.1 Optimal result

Integrand size = 7, antiderivative size = 14

$$\int x(\cos(x)+\sin(x))\,dx=\cos(x)-x\cos(x)+\sin(x)+x\sin(x)$$

output
```
cos(x)-x*cos(x)+sin(x)+x*sin(x)
```

3.58.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x(\cos(x)+\sin(x))\,dx=\cos(x)-x\cos(x)+\sin(x)+x\sin(x)$$

input
```
Integrate[x*(Cos[x] + Sin[x]),x]
```

output
```
Cos[x] - x*Cos[x] + Sin[x] + x*Sin[x]
```

3.58.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(\sin(x)+\cos(x))\,dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int (x\sin(x)+x\cos(x))dx$$

$$\Big\downarrow \text{ 2009}$$

$$x\sin(x)+\sin(x)-x\cos(x)+\cos(x)$$

input
```
Int[x*(Cos[x] + Sin[x]),x]
```

output
```
Cos[x] - x*Cos[x] + Sin[x] + x*Sin[x]
```

3.58.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

3.58.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$\cos(x) - x\cos(x) + \sin(x) + x\sin(x)$	15
parts	$\cos(x) - x\cos(x) + \sin(x) + x\sin(x)$	15
risch	$(1-x)\cos(x) + (1+x)\sin(x)$	16
parallelrisch	$(1-x)\cos(x) + 1 + (1+x)\sin(x)$	17
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)-x+2x\tan\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+2}{1+\tan^2\left(\frac{x}{2}\right)}$	38
meijerg	$2\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(x)}{2\sqrt{\pi}}+\frac{x\sin(x)}{2\sqrt{\pi}}\right)+2\sqrt{\pi}\left(-\frac{x\cos(x)}{2\sqrt{\pi}}+\frac{\sin(x)}{2\sqrt{\pi}}\right)$	49

input
```
int(x*(cos(x)+sin(x)),x,method=_RETURNVERBOSE)
```

output
```
cos(x)-x*cos(x)+sin(x)+x*sin(x)
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x(\cos(x)+\sin(x))\,dx = -(x-1)\cos(x) + (x+1)\sin(x)$$

input
```
integrate(x*(cos(x)+sin(x)),x, algorithm="fricas")
```

output
```
-(x - 1)*cos(x) + (x + 1)*sin(x)
```

3.58.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int x(\cos(x)+\sin(x))\,dx = x\sin(x) - x\cos(x) + \sin(x) + \cos(x)$$

input
```
integrate(x*(cos(x)+sin(x)),x)
```

output
```
x*sin(x) - x*cos(x) + sin(x) + cos(x)
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x(\cos(x)+\sin(x))\,dx = -x\cos{(x)} + x\sin{(x)} + \cos{(x)} + \sin{(x)}$$

input
```
integrate(x*(cos(x)+sin(x)),x, algorithm="maxima")
```

output
```
-x*cos(x) + x*sin(x) + cos(x) + sin(x)
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x(\cos(x)+\sin(x))\,dx = -x\cos{(x)} + x\sin{(x)} + \cos{(x)} + \sin{(x)}$$

input
```
integrate(x*(cos(x)+sin(x)),x, algorithm="giac")
```

output
```
-x*cos(x) + x*sin(x) + cos(x) + sin(x)
```

3.58.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x(\cos(x)+\sin(x))\,dx = \cos{(x)} + \sin{(x)} - x\,\cos{(x)} + x\,\sin{(x)}$$

input
```
int(x*(cos(x) + sin(x)),x)
```

output
```
cos(x) + sin(x) - x*cos(x) + x*sin(x)
```

3.59 $\int e^{-x}(e^x+x)\,dx$

3.59.1 Optimal result

Integrand size = 11, antiderivative size = 17

$$\int e^{-x}(e^x+x)\,dx=-e^{-x}+x-e^{-x}x$$

output
```
-1/exp(x)+x-x/exp(x)
```

3.59.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int e^{-x}(e^x+x)\,dx=e^{-x}(-1-x)+x$$

input
```
Integrate[(E^x + x)/E^x,x]
```

output
```
(-1 - x)/E^x + x
```

3.59.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x}(x+e^x)\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(e^{-x}x+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-e^{-x}x+x-e^{-x}$$

input
```
Int[(E^x + x)/E^x,x]
```

output
```
-E^(-x) + x - x/E^x
```

3.59.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.59.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
risch	$x + (-1 - x)\,\mathrm{e}^{-x}$	13
norman	$(-1 + \mathrm{e}^{x}x - x)\,\mathrm{e}^{-x}$	15
parallelrisch	$(-1 + \mathrm{e}^{x}x - x)\,\mathrm{e}^{-x}$	15
default	$-x\,\mathrm{e}^{-x} - \mathrm{e}^{-x} + x$	16
parts	$-x\,\mathrm{e}^{-x} - \mathrm{e}^{-x} + x$	16

input
```
int((exp(x)+x)/exp(x),x,method=_RETURNVERBOSE)
```

output
```
x+(-1-x)*exp(-x)
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int e^{-x}(e^x + x)\ dx = (xe^x - x - 1)e^{(-x)}$$

input
```
integrate((exp(x)+x)/exp(x),x, algorithm="fricas")
```

output
```
(x*e^x - x - 1)*e^(-x)
```

3.59.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.47

$$\int e^{-x}(e^x + x)\ dx = x + (-x - 1)\,e^{-x}$$

input
```
integrate((exp(x)+x)/exp(x),x)
```

output
```
x + (-x - 1)*exp(-x)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int e^{-x}(e^x+x)\,dx = -(x+1)e^{(-x)}+x$$

input
```
integrate((exp(x)+x)/exp(x),x, algorithm="maxima")
```

output
```
-(x + 1)*e^(-x) + x
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int e^{-x}(e^x+x)\,dx = -(x+1)e^{(-x)}+x$$

input
```
integrate((exp(x)+x)/exp(x),x, algorithm="giac")
```

output
```
-(x + 1)*e^(-x) + x
```

3.59.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int e^{-x}(e^x+x)\,dx = x-\mathrm{e}^{-x}-x\,\mathrm{e}^{-x}$$

input
```
int(exp(-x)*(x + exp(x)),x)
```

output
```
x - exp(-x) - x*exp(-x)
```

3.60 $\int (1+e^x)^2 x \, dx$

3.60.1 Optimal result

Integrand size = 9, antiderivative size = 38

$$\int (1+e^x)^2 x \, dx = -2e^x - \frac{e^{2x}}{4} + 2e^x x + \frac{1}{2}e^{2x}x + \frac{x^2}{2}$$

output
```
-2*exp(x)-1/4*exp(2*x)+2*exp(x)*x+1/2*exp(2*x)*x+1/2*x^2
```

3.60.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int (1+e^x)^2 x \, dx = \frac{1}{4}\left(8e^x(-1+x) + 2x^2 + e^{2x}(-1+2x)\right)$$

input
```
Integrate[(1 + E^x)^2*x,x]
```

output
```
(8*E^x*(-1 + x) + 2*x^2 + E^(2*x)*(-1 + 2*x))/4
```

3.60.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e^x+1)^2 x \, dx$$

$$\Big\downarrow \text{2614}$$

$$\int \left(2e^x x+e^{2x} x+x\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}+2e^x x+\frac{1}{2}e^{2x} x-2e^x-\frac{e^{2x}}{4}$$

input
```
Int[(1 + E^x)^2*x,x]
```

output
```
-2*E^x - E^(2*x)/4 + 2*E^x*x + (E^(2*x)*x)/2 + x^2/2
```

3.60.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

3.60.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{x^2}{2}+\left(-\frac{1}{4}+\frac{x}{2}\right)\mathrm{e}^{2x}+(-2+2x)\,\mathrm{e}^{x}$	25
default	$-2\,\mathrm{e}^{x}-\frac{\mathrm{e}^{2x}}{4}+2\,\mathrm{e}^{x}x+\frac{\mathrm{e}^{2x}x}{2}+\frac{x^2}{2}$	29
norman	$-2\,\mathrm{e}^{x}-\frac{\mathrm{e}^{2x}}{4}+2\,\mathrm{e}^{x}x+\frac{\mathrm{e}^{2x}x}{2}+\frac{x^2}{2}$	29
parallelrisch	$-2\,\mathrm{e}^{x}-\frac{\mathrm{e}^{2x}}{4}+2\,\mathrm{e}^{x}x+\frac{\mathrm{e}^{2x}x}{2}+\frac{x^2}{2}$	29
parts	$-2\,\mathrm{e}^{x}-\frac{\mathrm{e}^{2x}}{4}+2\,\mathrm{e}^{x}x+\frac{\mathrm{e}^{2x}x}{2}+\frac{x^2}{2}$	29

input
```
int((1+exp(x))^2*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+(-1/4+1/2*x)*exp(x)^2+(-2+2*x)*exp(x)
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int (1+e^x)^2\,x\,dx = \frac{1}{2}\,x^2+\frac{1}{4}\,(2\,x-1)e^{(2\,x)}+2\,(x-1)e^x$$

input
```
integrate((1+exp(x))^2*x,x, algorithm="fricas")
```

output
```
1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

3.60.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int (1+e^x)^2\,x\,dx = \frac{x^2}{2}+\frac{(2x-1)\,e^{2x}}{4}+\frac{(8x-8)\,e^x}{4}$$

input
```
integrate((1+exp(x))**2*x,x)
```

output
```
x**2/2 + (2*x - 1)*exp(2*x)/4 + (8*x - 8)*exp(x)/4
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int (1+e^x)^2\,x\,dx = \frac{1}{2}\,x^2 + \frac{1}{4}\,(2\,x-1)e^{(2\,x)} + 2\,(x-1)e^x$$

input
```
integrate((1+exp(x))^2*x,x, algorithm="maxima")
```

output
```
1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int (1+e^x)^2\,x\,dx = \frac{1}{2}\,x^2 + \frac{1}{4}\,(2\,x-1)e^{(2\,x)} + 2\,(x-1)e^x$$

input
```
integrate((1+exp(x))^2*x,x, algorithm="giac")
```

output
```
1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

3.60.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int (1+e^x)^2\,x\,dx = \frac{x\,\mathrm{e}^{2\,x}}{2} - 2\,\mathrm{e}^x - \frac{\mathrm{e}^{2\,x}}{4} + 2\,x\,\mathrm{e}^x + \frac{x^2}{2}$$

input
```
int(x*(exp(x) + 1)^2,x)
```

output
```
(x*exp(2*x))/2 - 2*exp(x) - exp(2*x)/4 + 2*x*exp(x) + x^2/2
```

3.61 $\int x\cos(x)\,dx$

3.61.1 Optimal result

Integrand size = 4, antiderivative size = 7

$$\int x\cos(x)\,dx = \cos(x) + x\sin(x)$$

output
```
cos(x)+x*sin(x)
```

3.61.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = \cos(x) + x\sin(x)$$

input
```
Integrate[x*Cos[x],x]
```

output
```
Cos[x] + x*Sin[x]
```

3.61.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sin\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\int -\sin(x)dx + x\sin(x)$$

$$\Big\downarrow \text{25}$$

$$x\sin(x) - \int \sin(x)dx$$

$$\Big\downarrow \text{3042}$$

$$x\sin(x) - \int \sin(x)dx$$

$$\Big\downarrow \text{3118}$$

$$x\sin(x) + \cos(x)$$

input
```
Int[x*Cos[x],x]
```

output
```
Cos[x] + x*Sin[x]
```

3.61.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.61.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
default	$\cos\left(x\right)+x\sin\left(x\right)$	8
risch	$\cos\left(x\right)+x\sin\left(x\right)$	8
parts	$\cos\left(x\right)+x\sin\left(x\right)$	8
parallelrisch	$x\sin\left(x\right)+\cos\left(x\right)+1$	9
norman	$\frac{2x\tan\left(\frac{x}{2}\right)+2}{1+\tan^{2}\left(\frac{x}{2}\right)}$	21
meijerg	$2\sqrt{\pi}\,\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos\left(x\right)}{2\sqrt{\pi}}+\frac{x\sin\left(x\right)}{2\sqrt{\pi}}\right)$	27

input
```
int(x*cos(x),x,method=_RETURNVERBOSE)
```

output
```
cos(x)+x*sin(x)
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

input
```
integrate(x*cos(x),x, algorithm="fricas")
```

output
```
x*sin(x) + cos(x)
```

3.61.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

input
```
integrate(x*cos(x),x)
```

output
```
x*sin(x) + cos(x)
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

input
```
integrate(x*cos(x),x, algorithm="maxima")
```

output
```
x*sin(x) + cos(x)
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

input
```
integrate(x*cos(x),x, algorithm="giac")
```

output
```
x*sin(x) + cos(x)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = \cos(x) + x\,\sin(x)$$

input
```
int(x*cos(x),x)
```

output
```
cos(x) + x*sin(x)
```

3.62 $\int \cos(\sqrt{x})\,dx$

3.62.1 Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \cos(\sqrt{x})\,dx = 2\cos(\sqrt{x}) + 2\sqrt{x}\sin(\sqrt{x})$$

output
```
2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

3.62.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \cos(\sqrt{x})\,dx = 2\cos(\sqrt{x}) + 2\sqrt{x}\sin(\sqrt{x})$$

input
```
Integrate[Cos[Sqrt[x]],x]
```

output
```
2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

3.62.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3843, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos\left(\sqrt{x}\right)\,dx$$

$$\Big\downarrow \text{3843}$$

$$2\int \sqrt{x}\cos\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int \sqrt{x}\sin\left(\sqrt{x}+\frac{\pi}{2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3777}$$

$$2\left(\int -\sin\left(\sqrt{x}\right)d\sqrt{x}+\sqrt{x}\sin\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\sqrt{x}\sin\left(\sqrt{x}\right)-\int \sin\left(\sqrt{x}\right)d\sqrt{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$2\left(\sqrt{x}\sin\left(\sqrt{x}\right)-\int \sin\left(\sqrt{x}\right)d\sqrt{x}\right)$$

$$\Big\downarrow \text{3118}$$

$$2\left(\sqrt{x}\sin\left(\sqrt{x}\right)+\cos\left(\sqrt{x}\right)\right)$$

input
```
Int[Cos[Sqrt[x]],x]
```

output
```
2*(Cos[Sqrt[x]] + Sqrt[x]*Sin[Sqrt[x]])
```

3.62.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3843
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Simp[1/(n*f)   Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x],
x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Intege
rQ[1/n]
```

3.62.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$2\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\sqrt{x}$	17
default	$2\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\sqrt{x}$	17
meijerg	$4\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(\sqrt{x})}{2\sqrt{\pi}}+\frac{\sqrt{x}\,\sin(\sqrt{x})}{2\sqrt{\pi}}\right)$	33

input
```
int(cos(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\,\sqrt{x}\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

input
```
integrate(cos(x^(1/2)),x, algorithm="fricas")
```

output
```
2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.62.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

input
```
integrate(cos(x**(1/2)),x)
```

output
```
2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\,\sqrt{x}\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

input
```
integrate(cos(x^(1/2)),x, algorithm="maxima")
```

output
```
2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\, dx = 2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

input
```
integrate(cos(x^(1/2)),x, algorithm="giac")
```

output
```
2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.62.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\, dx = 2\,\cos\left(\sqrt{x}\right) + 2\sqrt{x}\,\sin\left(\sqrt{x}\right)$$

input
```
int(cos(x^(1/2)),x)
```

output
```
2*cos(x^(1/2)) + 2*x^(1/2)*sin(x^(1/2))
```

3.63 $\int x \cos(x)\,dx$

3.63.1 Optimal result

Integrand size = 4, antiderivative size = 7

$$\int x\cos(x)\,dx=\cos(x)+x\sin(x)$$

output
```
cos(x)+x*sin(x)
```

3.63.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx=\cos(x)+x\sin(x)$$

input
```
Integrate[x*Cos[x],x]
```

output
```
Cos[x] + x*Sin[x]
```

3.63.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sin\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\int -\sin(x)dx+x\sin(x)$$

$$\Big\downarrow \text{25}$$

$$x\sin(x)-\int \sin(x)dx$$

$$\Big\downarrow \text{3042}$$

$$x\sin(x)-\int \sin(x)dx$$

$$\Big\downarrow \text{3118}$$

$$x\sin(x)+\cos(x)$$

input
```
Int[x*Cos[x],x]
```

output
```
Cos[x] + x*Sin[x]
```

3.63.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.63.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
default	$\cos\left(x\right)+x\sin\left(x\right)$	8
risch	$\cos\left(x\right)+x\sin\left(x\right)$	8
parts	$\cos\left(x\right)+x\sin\left(x\right)$	8
parallelrisch	$x\sin\left(x\right)+\cos\left(x\right)+1$	9
norman	$\frac{2x\tan\left(\frac{x}{2}\right)+2}{1+\tan^{2}\left(\frac{x}{2}\right)}$	21
meijerg	$2\sqrt{\pi}\,\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos\left(x\right)}{2\sqrt{\pi}}+\frac{x\sin\left(x\right)}{2\sqrt{\pi}}\right)$	27

input
```
int(x*cos(x),x,method=_RETURNVERBOSE)
```

output
```
cos(x)+x*sin(x)
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

input
```
integrate(x*cos(x),x, algorithm="fricas")
```

output
```
x*sin(x) + cos(x)
```

3.63.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

input
```
integrate(x*cos(x),x)
```

output
```
x*sin(x) + cos(x)
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

input
```
integrate(x*cos(x),x, algorithm="maxima")
```

output
```
x*sin(x) + cos(x)
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

input
```
integrate(x*cos(x),x, algorithm="giac")
```

output
```
x*sin(x) + cos(x)
```

3.63.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int x\cos(x)\,dx = \cos(x) + x\,\sin(x)$$

input
```
int(x*cos(x),x)
```

output
```
cos(x) + x*sin(x)
```

3.64 $\int x \log^2(x)\,dx$

3.64.1 Optimal result

Integrand size = 6, antiderivative size = 28

$$\int x \log^2(x)\,dx = \frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)$$

output
```
1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

3.64.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int x \log^2(x)\,dx = \frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)$$

input
```
Integrate[x*Log[x]^2,x]
```

output
```
x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

3.64.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log^2(x) \, dx$$

$$\Big\downarrow \text{2742}$$

$$\frac{1}{2}x^2 \log^2(x) - \int x \log(x) dx$$

$$\Big\downarrow \text{2741}$$

$$\frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

input
```
Int[x*Log[x]^2,x]
```

output
```
x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

3.64.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

3.64.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
norman	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
risch	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
parallelrisch	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
parts	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23

input
```
int(x*ln(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx=\frac{1}{2}\,x^2\log{(x)}^2-\frac{1}{2}\,x^2\log{(x)}+\frac{1}{4}\,x^2$$

input
```
integrate(x*log(x)^2,x, algorithm="fricas")
```

output
```
1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

3.64.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x\log^2(x)\,dx=\frac{x^2\log{(x)}^2}{2}-\frac{x^2\log{(x)}}{2}+\frac{x^2}{4}$$

input
```
integrate(x*ln(x)**2,x)
```

output
```
x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x \log^2(x)\, dx = \frac{1}{4} \left(2 \log(x)^2 - 2 \log(x) + 1\right)x^2$$

input
```
integrate(x*log(x)^2,x, algorithm="maxima")
```

output
```
1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int x \log^2(x)\, dx = \frac{1}{2} x^2 \log(x)^2 - \frac{1}{2} x^2 \log(x) + \frac{1}{4} x^2$$

input
```
integrate(x*log(x)^2,x, algorithm="giac")
```

output
```
1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

3.64.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int x \log^2(x)\, dx = \frac{x^2 \left(2 \ln(x)^2 - 2 \ln(x) + 1\right)}{4}$$

input
```
int(x*log(x)^2,x)
```

output
```
(x^2*(2*log(x)^2 - 2*log(x) + 1))/4
```

3.65 $\int \cos(x)\left(1+\sin^3(x)\right)\,dx$

3.65.1 Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \cos(x)\left(1+\sin^3(x)\right)\,dx=\sin(x)+\frac{\sin^4(x)}{4}$$

output
```
sin(x)+1/4*sin(x)^4
```

3.65.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cos(x)\left(1+\sin^3(x)\right)\,dx=\sin(x)+\frac{\sin^4(x)}{4}$$

input
```
Integrate[Cos[x]*(1 + Sin[x]^3),x]
```

output
```
Sin[x] + Sin[x]^4/4
```

3.65.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3702, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sin^3(x)+1)\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sin(x)^3+1)\cos(x)dx$$

$$\Big\downarrow \text{3702}$$

$$\int (\sin^3(x)+1)\,d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin^4(x)}{4}+\sin(x)$$

input
```
Int[Cos[x]*(1 + Sin[x]^3),x]
```

output
```
Sin[x] + Sin[x]^4/4
```

3.65.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.65.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\sin(x)+\frac{(\sin^4(x))}{4}$	10
default	$\sin(x)+\frac{(\sin^4(x))}{4}$	10
risch	$\sin(x)+\frac{\cos(4x)}{32}-\frac{\cos(2x)}{8}$	16
parallelrisch	$-\frac{\cos(2x)}{8}+\frac{\cos(4x)}{32}+\frac{3}{32}+\sin(x)$	17

input
```
int(cos(x)*(1+sin(x)^3),x,method=_RETURNVERBOSE)
```

output
```
sin(x)+1/4*sin(x)^4
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \cos(x)\left(1+\sin^3(x)\right)\,dx=\frac{1}{4}\cos(x)^4-\frac{1}{2}\cos(x)^2+\sin(x)$$

input
```
integrate(cos(x)*(1+sin(x)^3),x, algorithm="fricas")
```

output
```
1/4*cos(x)^4 - 1/2*cos(x)^2 + sin(x)
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \cos(x)\left(1+\sin^3(x)\right)\,dx=\frac{\sin^4(x)}{4}+\sin(x)$$

input
```
integrate(cos(x)*(1+sin(x)**3),x)
```

output
```
sin(x)**4/4 + sin(x)
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos(x)\left(1+\sin^3(x)\right)\,dx=\frac{1}{4}\sin{(x)}^4+\sin{(x)}$$

input
```
integrate(cos(x)*(1+sin(x)^3),x, algorithm="maxima")
```

output
```
1/4*sin(x)^4 + sin(x)
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos(x)\left(1+\sin^3(x)\right)\,dx=\frac{1}{4}\sin{(x)}^4+\sin{(x)}$$

input
```
integrate(cos(x)*(1+sin(x)^3),x, algorithm="giac")
```

output
```
1/4*sin(x)^4 + sin(x)
```

3.65.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \cos(x)\left(1+\sin^3(x)\right)\,dx=\frac{\sin{(x)}^4}{4}+\sin{(x)}$$

input
```
int(cos(x)*(sin(x)^3 + 1),x)
```

output
```
sin(x) + sin(x)^4/4
```

3.66 $\int \frac{1}{x(1+\log^2(x))} \, dx$

3.66.1 Optimal result

Integrand size = 12, antiderivative size = 3

$$\int \frac{1}{x\left(1+\log^2(x)\right)} \, dx = \arctan(\log(x))$$

output
```
arctan(ln(x))
```

3.66.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)} \, dx = \arctan(\log(x))$$

input
```
Integrate[1/(x*(1 + Log[x]^2)),x]
```

output
```
ArcTan[Log[x]]
```

3.66.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3039, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\log^2(x)+1\right)}dx$$
$$\Big\downarrow \text{3039}$$
$$\int \frac{1}{\log^2(x)+1}d\log(x)$$
$$\Big\downarrow \text{216}$$
$$\arctan(\log(x))$$

input
```
Int[1/(x*(1 + Log[x]^2)),x]
```

output
```
ArcTan[Log[x]]
```

3.66.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

3.66.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\arctan(\ln(x))$	4
default	$\arctan(\ln(x))$	4
risch	$\frac{i\ln(\ln(x)+i)}{2} - \frac{i\ln(\ln(x)-i)}{2}$	20
parallelrisch	$\frac{i\ln(\ln(x)+i)}{2} - \frac{i\ln(\ln(x)-i)}{2}$	20

input
```
int(1/x/(1+ln(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(ln(x))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+\log^2(x))}\,dx = \arctan(\log(x))$$

input
```
integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
```

output
```
arctan(log(x))
```

3.66.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \frac{1}{x(1+\log^2(x))}\,dx = \operatorname{RootSum}\left(4z^2+1, \left(i \mapsto i\log(2i+\log(x))\right)\right)$$

input
```
integrate(1/x/(1+ln(x)**2),x)
```

output
```
RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

input
```
integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
```

output
```
arctan(log(x))
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \arctan\left(\log\left(x\right)\right)$$

input
```
integrate(1/x/(1+log(x)^2),x, algorithm="giac")
```

output
```
arctan(log(x))
```

3.66.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx = \operatorname{atan}(\ln\left(x\right))$$

input
```
int(1/(x*(log(x)^2 + 1)),x)
```

output
```
atan(log(x))
```

3.67 $\int \frac{1}{\sqrt{1-x^2}(1+\arcsin(x)^2)}\,dx$

3.67.1 Optimal result

Integrand size = 20, antiderivative size = 3

$$\int \frac{1}{\sqrt{1-x^2}\,(1+\arcsin(x)^2)}\,dx = \arctan(\arcsin(x))$$

output
```
arctan(arcsin(x))
```

3.67.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\,(1+\arcsin(x)^2)}\,dx = \arctan(\arcsin(x))$$

input
```
Integrate[1/(Sqrt[1 - x^2]*(1 + ArcSin[x]^2)),x]
```

output
```
ArcTan[ArcSin[x]]
```

3.67.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7247, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^2}\left(\arcsin(x)^2+1\right)}dx$$

$$\Big\downarrow \text{ 7247}$$

$$\int \frac{1}{\arcsin(x)^2+1}d\arcsin(x)$$

$$\Big\downarrow \text{ 216}$$

$$\arctan(\arcsin(x))$$

input
```
Int[1/(Sqrt[1 - x^2]*(1 + ArcSin[x]^2)),x]
```

output
```
ArcTan[ArcSin[x]]
```

3.67.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 7247
```
Int[(u_.)*((a_.) + (b_.)*(y_)^(n_))^(p_), x_Symbol] :> With[{q = Derivative
Divides[y, u, x]}, Simp[q   Subst[Int[(a + b*x^n)^p, x], x, y], x] /;  !Fal
seQ[q]] /; FreeQ[{a, b, n, p}, x]
```

3.67.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\arctan(\arcsin(x))$	4
default	$\arctan(\arcsin(x))$	4

input
```
int(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(arcsin(x))
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\,(1+\arcsin(x)^2)}\,dx = \arctan(\arcsin(x))$$

input
```
integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
arctan(arcsin(x))
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\,(1+\arcsin(x)^2)}\,dx = \operatorname{atan}(\operatorname{asin}(x))$$

input
```
integrate(1/(1+asin(x)**2)/(-x**2+1)**(1/2),x)
```

output
```
atan(asin(x))
```

3.67.7 Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\,(1+\arcsin(x)^2)}\,dx = \int \frac{1}{\sqrt{-x^2+1}\left(\arcsin\left(x\right)^2+1\right)}\,dx$$

input
```
integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^2 + 1)*(arcsin(x)^2 + 1)), x)
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\,(1+\arcsin(x)^2)}\,dx = \arctan\left(\arcsin\left(x\right)\right)$$

input
```
integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
arctan(arcsin(x))
```

3.67.9 Mupad [B] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 43, normalized size of antiderivative = 14.33

$$\int \frac{1}{\sqrt{1-x^2}\,(1+\arcsin(x)^2)}\,dx = \frac{\ln\left(\frac{-1+\mathrm{asin}(x)\,1\mathrm{i}}{\sqrt{1-x^2}}\right)\,1\mathrm{i}}{2} - \frac{\ln\left(\frac{1+\mathrm{asin}(x)\,1\mathrm{i}}{\sqrt{1-x^2}}\right)\,1\mathrm{i}}{2}$$

input
```
int(1/((1 - x^2)^(1/2)*(asin(x)^2 + 1)),x)
```

output
```
(log((asin(x)*1i - 1)/(1 - x^2)^(1/2))*1i)/2 - (log((asin(x)*1i + 1)/(1 -
x^2)^(1/2))*1i)/2
```

3.68 $\int \frac{\sin(x)}{\cos(x)+\sin(x)} dx$

3.68.1 Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{\sin(x)}{\cos(x)+\sin(x)} dx = \frac{x}{2} - \frac{1}{2}\log(\cos(x)+\sin(x))$$

output
```
1/2*x-1/2*ln(cos(x)+sin(x))
```

3.68.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\cos(x)+\sin(x)} dx = \frac{x}{2} - \frac{1}{2}\log(\cos(x)+\sin(x))$$

input
```
Integrate[Sin[x]/(Cos[x] + Sin[x]),x]
```

output
```
x/2 - Log[Cos[x] + Sin[x]]/2
```

3.68.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3576, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\sin(x)+\cos(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)}{\sin(x)+\cos(x)}dx$$

$$\Big\downarrow \text{ 3576}$$

$$\frac{x}{2}-\frac{1}{2}\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x}{2}-\frac{1}{2}\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}dx$$

$$\Big\downarrow \text{ 3612}$$

$$\frac{x}{2}-\frac{1}{2}\log(\sin(x)+\cos(x))$$

input
```
Int[Sin[x]/(Cos[x] + Sin[x]),x]
```

output
```
x/2 - Log[Cos[x] + Sin[x]]/2
```

3.68.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3576
```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Simp[a/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.68.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

method	result	size
risch	$\frac{x}{2}+\frac{ix}{2}-\frac{\ln\left(\mathrm{e}^{2ix}+i\right)}{2}$	20
default	$-\frac{\ln(\tan(x)+1)}{2}+\frac{\ln(1+\tan^2(x))}{4}+\frac{\arctan(\tan(x))}{2}$	23
parallelrisch	$\frac{x}{2}+\ln\left(\sqrt{\frac{1}{\cos(x)+1}}\right)+\ln\left(\frac{1}{\sqrt{-\frac{\cos(x)+\sin(x)}{\cos(x)+1}}}\right)$	30
norman	$\frac{\frac{x}{2}+\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}}{1+\tan^2\left(\frac{x}{2}\right)}+\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{\ln\left(\tan^2\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)-1\right)}{2}$	54

input
```
int(sin(x)/(cos(x)+sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*I*x-1/2*ln(exp(2*I*x)+I)
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx = \frac{1}{2}\,x - \frac{1}{4}\,\log\left(2\,\cos\left(x\right)\sin\left(x\right) + 1\right)$$

input
```
integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="fricas")
```

output
```
1/2*x - 1/4*log(2*cos(x)*sin(x) + 1)
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx = \frac{x}{2} - \frac{\log\left(\sin\left(x\right)+\cos\left(x\right)\right)}{2}$$

input
```
integrate(sin(x)/(cos(x)+sin(x)),x)
```

output
```
x/2 - log(sin(x) + cos(x))/2
```

3.68.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(12) = 24.

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.31

$$\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx = \arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right) - \frac{1}{2}\log\left(-\frac{2\sin\left(x\right)}{\cos\left(x\right)+1} + \frac{\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}} - 1\right) + \frac{1}{2}\log\left(\frac{\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}} + 1\right)$$

input
```
integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="maxima")
```

output
```
arctan(sin(x)/(cos(x) + 1)) - 1/2*log(-2*sin(x)/(cos(x) + 1) + sin(x)^2/(c
os(x) + 1)^2 - 1) + 1/2*log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx = \frac{1}{2}x + \frac{1}{4}\log\left(\tan\left(x\right)^{2}+1\right) - \frac{1}{2}\log\left(\left|\tan\left(x\right)+1\right|\right)$$

input
```
integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="giac")
```

output
```
1/2*x + 1/4*log(tan(x)^2 + 1) - 1/2*log(abs(tan(x) + 1))
```

3.68.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx = \frac{x}{2}-\frac{\ln\left(\cos\left(x-\frac{\pi}{4}\right)\right)}{2}$$

input
```
int(sin(x)/(cos(x) + sin(x)),x)
```

output
```
x/2 - log(cos(x - pi/4))/2
```

3.69 $\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy$

3.69.1 Optimal result

Integrand size = 30, antiderivative size = 53

$$\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy = -B\arctan\left(\frac{By}{\sqrt{A^2+B^2-B^2y^2}}\right) - A\text{arctanh}\left(\frac{Ay}{\sqrt{A^2+B^2-B^2y^2}}\right)$$

output
```
-B*arctan(B*y/(-B^2*y^2+A^2+B^2)^(1/2))-A*arctanh(A*y/(-B^2*y^2+A^2+B^2)^(
1/2))
```

3.69.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy = 2B\arctan\left(\frac{By}{\sqrt{A^2+B^2}-\sqrt{A^2+B^2-B^2y^2}}\right) - A\text{arctanh}\left(\frac{\sqrt{A^2+B^2-B^2y^2}}{Ay}\right)$$

input
```
Integrate[-(Sqrt[A^2 + B^2*(1 - y^2)]/(1 - y^2)),y]
```

output
```
2*B*ArcTan[(B*y)/(Sqrt[A^2 + B^2] - Sqrt[A^2 + B^2 - B^2*y^2])] - A*ArcTan
h[Sqrt[A^2 + B^2 - B^2*y^2]/(A*y)]
```

3.69.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {25, 2074, 301, 224, 216, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{\sqrt{A^2+B^2\left(1-y^2\right)}}{1-y^2}dy$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\sqrt{A^2+B^2\left(1-y^2\right)}}{1-y^2}dy$$

$$\Big\downarrow \text{ 2074}$$

$$-\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}dy$$

$$\Big\downarrow \text{ 301}$$

$$A^2\left(-\int \frac{1}{\left(1-y^2\right)\sqrt{A^2+B^2-B^2y^2}}dy\right)-B^2\int \frac{1}{\sqrt{A^2+B^2-B^2y^2}}dy$$

$$\Big\downarrow \text{ 224}$$

$$A^2\left(-\int \frac{1}{\left(1-y^2\right)\sqrt{A^2+B^2-B^2y^2}}dy\right)-B^2\int \frac{1}{\frac{B^2y^2}{A^2+B^2-B^2y^2}+1}d\frac{y}{\sqrt{A^2+B^2-B^2y^2}}$$

$$\Big\downarrow \text{ 216}$$

$$A^2\left(-\int \frac{1}{\left(1-y^2\right)\sqrt{A^2+B^2-B^2y^2}}dy\right)-B\arctan\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

$$\Big\downarrow \text{ 291}$$

$$A^2\left(-\int \frac{1}{1-\frac{A^2y^2}{A^2+B^2-B^2y^2}}d\frac{y}{\sqrt{A^2+B^2-B^2y^2}}\right)-B\arctan\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$-B\arctan\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)-A\text{arctanh}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

input
```
Int[-(Sqrt[A^2 + B^2*(1 - y^2)]/(1 - y^2)),y]
```

output
```
-(B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]]) - A*ArcTanh[(A*y)/Sqrt[A^2 +
B^2 - B^2*y^2]]
```

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 2074
```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

3.69.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.68

method	result
pseudoelliptic	$-\frac{A\ln\left(\frac{Ay+\sqrt{-B^2y^2+A^2+B^2}}{y}\right)}{2}+\frac{A\ln\left(\frac{Ay-\sqrt{-B^2y^2+A^2+B^2}}{y}\right)}{2}+B\arctan\left(\frac{\sqrt{-B^2y^2+A^2+B^2}}{By}\right)$
default	$\frac{\sqrt{-B^2(y-1)^2-2B^2(y-1)+A^2}}{2}-\frac{B^2\arctan\left(\frac{\sqrt{B^2}\,y}{\sqrt{-B^2(y-1)^2-2B^2(y-1)+A^2}}\right)}{2\sqrt{B^2}}-\frac{A^2\ln\left(\frac{2A^2-2B^2(y-1)+2\sqrt{A^2}\,\sqrt{-B^2(y-1}}{y-1}\right.}{2\sqrt{A^2}}$

input
```
int(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y,method=_RETURNVERBOSE)
```

output
```
-1/2*A*ln((A*y+(-B^2*y^2+A^2+B^2)^(1/2))/y)+1/2*A*ln((A*y-(-B^2*y^2+A^2+B^
2)^(1/2))/y)+B*arctan(1/B/y*(-B^2*y^2+A^2+B^2)^(1/2))
```

3.69.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.42

$$\begin{aligned}&\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy\\&=B\arctan\left(\frac{\sqrt{-B^2y^2+A^2+B^2}}{By}\right)\\&\quad-\frac{1}{4}A\log\left(-\frac{(A^2-B^2)y^2+2\sqrt{-B^2y^2+A^2+B^2}Ay+A^2+B^2}{y^2}\right)\\&\quad+\frac{1}{4}A\log\left(-\frac{(A^2-B^2)y^2-2\sqrt{-B^2y^2+A^2+B^2}Ay+A^2+B^2}{y^2}\right)\end{aligned}$$

input
```
integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="fricas")
```

output
```
B*arctan(sqrt(-B^2*y^2 + A^2 + B^2)/(B*y)) - 1/4*A*log(-((A^2 - B^2)*y^2 +
 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2) + 1/4*A*log(-((A^2 - B
^2)*y^2 - 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2)
```

3.69.6 Sympy [F]

$$\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy = \int \frac{\sqrt{A^2-B^2y^2+B^2}}{(y-1)(y+1)}\,dy$$

input
```
integrate(-(A**2+B**2*(-y**2+1))**(1/2)/(-y**2+1),y)
```

output
```
Integral(sqrt(A**2 - B**2*y**2 + B**2)/((y - 1)*(y + 1)), y)
```

3.69.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(49) = 98$.

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.17

$$\begin{aligned}\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy = &-B\arcsin\left(\frac{B^2y}{\sqrt{A^2B^2+B^4}}\right)\\ &+\frac{1}{2}A\log\left(B^2+\frac{A^2}{y+1}+\frac{\sqrt{-B^2y^2+A^2+B^2}A}{y+1}\right)\\ &-\frac{1}{2}A\log\left(-B^2+\frac{2A^2}{|2y-2|}+\frac{2\sqrt{-B^2y^2+A^2+B^2}A}{|2y-2|}\right)\end{aligned}$$

input
```
integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="maxima")
```

output
```
-B*arcsin(B^2*y/sqrt(A^2*B^2 + B^4)) + 1/2*A*log(B^2 + A^2/(y + 1) + sqrt(
-B^2*y^2 + A^2 + B^2)*A/(y + 1)) - 1/2*A*log(-B^2 + 2*A^2/abs(2*y - 2) + 2
*sqrt(-B^2*y^2 + A^2 + B^2)*A/abs(2*y - 2))
```

3.69.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(49) = 98$.

Time = 0.35 (sec) , antiderivative size = 295, normalized size of antiderivative = 5.57

$$\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy$$
$$=\frac{\left(\pi\operatorname{sgn}(y)-2\arctan\left(-\frac{B^2y\left(\frac{\left(\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|\right)^2}{B^4y^2}-1\right)}{2\left(\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|\right)}\right)\right)B^2}{2|B|}$$
$$-\frac{AB\log\left(\left|-\left(\frac{B^2y}{\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|}-\frac{\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|}{B^2y}\right)B+2A\right|\right)}{2|B|}$$
$$+\frac{AB\log\left(\left|-\left(\frac{B^2y}{\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|}-\frac{\sqrt{A^2+B^2}B+\sqrt{-B^2y^2+A^2+B^2}|B|}{B^2y}\right)B-2A\right|\right)}{2|B|}$$

input
```
integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="giac")
```

output
```
1/2*(pi*sgn(y) - 2*arctan(-1/2*B^2*y*((sqrt(A^2 + B^2)*B + sqrt(-B^2*y^2 +
 A^2 + B^2)*abs(B))^2/(B^4*y^2) - 1)/(sqrt(A^2 + B^2)*B + sqrt(-B^2*y^2 +
A^2 + B^2)*abs(B))))*B^2/abs(B) - 1/2*A*B*log(abs(-(B^2*y/(sqrt(A^2 + B^2)
*B + sqrt(-B^2*y^2 + A^2 + B^2)*abs(B)) - (sqrt(A^2 + B^2)*B + sqrt(-B^2*y
^2 + A^2 + B^2)*abs(B))/(B^2*y))*B + 2*A))/abs(B) + 1/2*A*B*log(abs(-(B^2*
y/(sqrt(A^2 + B^2)*B + sqrt(-B^2*y^2 + A^2 + B^2)*abs(B)) - (sqrt(A^2 + B^
2)*B + sqrt(-B^2*y^2 + A^2 + B^2)*abs(B))/(B^2*y))*B - 2*A))/abs(B)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy$$
$$=\begin{cases}\int\frac{\sqrt{-B^2\,y^2}}{y^2-1}\,dy & \text{if }\ A^2+B^2=0\\ \ln\left(2\,y\,\sqrt{-B^2}+2\,\sqrt{A^2-B^2\,y^2+B^2}\right)\,\sqrt{-B^2}+\operatorname{atan}\left(\frac{y\,\sqrt{A^2}\,1\mathrm{i}}{\sqrt{A^2-B^2\,y^2+B^2}}\right)\,\sqrt{A^2}\,1\mathrm{i} & \text{if }\ A^2+B^2\neq 0\end{cases}$$

input
```
int((A^2 - B^2*(y^2 - 1))^(1/2)/(y^2 - 1),y)
```

output
```
piecewise(A^2 + B^2 == 0, int((-B^2*y^2)^(1/2)/(y^2 - 1), y), A^2 + B^2 ~=
 0, atan((y*(A^2)^(1/2)*1i)/(A^2 + B^2 - B^2*y^2)^(1/2))*(A^2)^(1/2)*1i +
log(2*y*(-B^2)^(1/2) + 2*(A^2 + B^2 - B^2*y^2)^(1/2))*(-B^2)^(1/2))
```

3.69. $\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy$

3.70 $\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz$

3.70.1 Optimal result

Integrand size = 39, antiderivative size = 16

$$\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz = -Bz - A\operatorname{arctanh}\left(\frac{A\tan(z)}{B}\right)$$

output
```
-B*z-A*arctanh(A*tan(z)/B)
```

3.70.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz = -\frac{B(A^2+B^2)\left(Bz + A\operatorname{arctanh}\left(\frac{A\tan(z)}{B}\right)\right)}{A^2B+B^3}$$

input
```
Integrate[((-A^2 - B^2)*Cos[z]^2)/(B*(1 - ((A^2 + B^2)*Sin[z]^2)/B^2)),z]
```

output
```
-((B*(A^2 + B^2)*(B*z + A*ArcTanh[(A*Tan[z])/B]))/(A^2*B + B^3))
```

3.70.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {27, 3042, 3670, 27, 303, 216, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}dz$$

$$\Big\downarrow \text{27}$$

$$-\frac{(A^2+B^2)\int \frac{\cos^2(z)}{1-\frac{(A^2+B^2)\sin^2(z)}{B^2}}dz}{B}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A^2+B^2)\int \frac{\cos(z)^2}{1-\frac{(A^2+B^2)\sin(z)^2}{B^2}}dz}{B}$$

$$\Big\downarrow \text{3670}$$

$$-\frac{(A^2+B^2)\int \frac{B^2}{(\tan^2(z)+1)(B^2-A^2\tan^2(z))}d\tan(z)}{B}$$

$$\Big\downarrow \text{27}$$

$$-B(A^2+B^2)\int \frac{1}{(\tan^2(z)+1)(B^2-A^2\tan^2(z))}d\tan(z)$$

$$\Big\downarrow \text{303}$$

$$-B(A^2+B^2)\left(\frac{A^2\int \frac{1}{B^2-A^2\tan^2(z)}d\tan(z)}{A^2+B^2}+\frac{\int \frac{1}{\tan^2(z)+1}d\tan(z)}{A^2+B^2}\right)$$

$$\Big\downarrow \text{216}$$

$$-B(A^2+B^2)\left(\frac{A^2\int \frac{1}{B^2-A^2\tan^2(z)}d\tan(z)}{A^2+B^2}+\frac{\arctan(\tan(z))}{A^2+B^2}\right)$$

$$\Big\downarrow \text{221}$$

$$-B\left(A^2+B^2\right)\left(\frac{\arctan(\tan(z))}{A^2+B^2}+\frac{A\text{arctanh}\left(\frac{A\tan(z)}{B}\right)}{B\left(A^2+B^2\right)}\right)$$

input
```
Int[((-A^2 - B^2)*Cos[z]^2)/(B*(1 - ((A^2 + B^2)*Sin[z]^2)/B^2)),z]
```

output
```
-(B*(A^2 + B^2)*(ArcTan[Tan[z]]/(A^2 + B^2) + (A*ArcTanh[(A*Tan[z])/B])/(B
*(A^2 + B^2))))
```

3.70.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.70.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(16) = 32$.

Time = 0.62 (sec) , antiderivative size = 71, normalized size of antiderivative = 4.44

method	result
parallelrisch	$\frac{(-A^2-B^2)\left(A\ln\left(\frac{A\sin(z)+B\cos(z)}{\cos(z)+1}\right)-A\ln\left(\frac{A\sin(z)-B\cos(z)}{\cos(z)+1}\right)+2Bz\right)}{2A^2+2B^2}$
default	$(-A^2-B^2)\,B\left(\frac{A\ln(A\tan(z)+B)}{2B(A^2+B^2)}+\frac{\arctan(\tan(z))}{A^2+B^2}-\frac{A\ln(A\tan(z)-B)}{2B(A^2+B^2)}\right)$
norman	$\frac{-Bz-2Bz(\tan^2(\frac{z}{2}))-Bz(\tan^4(\frac{z}{2}))}{(1+\tan^2(\frac{z}{2}))^2}-\frac{A\ln(-B(\tan^2(\frac{z}{2}))+2A\tan(\frac{z}{2})+B)}{2}+\frac{A\ln(B(\tan^2(\frac{z}{2}))+2A\tan(\frac{z}{2})-B)}{2}$
risch	$-\frac{Bz\,A^2}{A^2+B^2}-\frac{B^3z}{A^2+B^2}-\frac{A^3\ln\left(e^{2iz}-\frac{-iB+A}{iB+A}\right)}{2(A^2+B^2)}-\frac{A\ln\left(e^{2iz}-\frac{-iB+A}{iB+A}\right)B^2}{2(A^2+B^2)}+\frac{A^3\ln\left(e^{2iz}-\frac{iB+A}{-iB+A}\right)}{2A^2+2B^2}+\frac{A\ln\left(e^{2iz}-\frac{iB+}{-iB+}\right.}{2A^2+2B^2}$

input
```
int((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z,method=_RETURNVERBO
SE)
```

output
```
(-A^2-B^2)*(A*ln((A*sin(z)+B*cos(z))/(cos(z)+1))-A*ln((A*sin(z)-B*cos(z))/
(cos(z)+1))+2*B*z)/(2*A^2+2*B^2)
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 4.19

$$\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz = -Bz-\frac{1}{4}A\log\left(2\,AB\cos(z)\sin(z)-\left(A^2-B^2\right)\cos(z)^2+A^2\right) + \frac{1}{4}A\log\left(-2\,AB\cos(z)\sin(z)-\left(A^2-B^2\right)\cos(z)^2+A^2\right)$$

input
```
integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="f
ricas")
```

output
```
-B*z - 1/4*A*log(2*A*B*cos(z)*sin(z) - (A^2 - B^2)*cos(z)^2 + A^2) + 1/4*A
*log(-2*A*B*cos(z)*sin(z) - (A^2 - B^2)*cos(z)^2 + A^2)
```

3.70.6 Sympy [A] (verification not implemented)

Time = 106.06 (sec) , antiderivative size = 202, normalized size of antiderivative = 12.62

$$\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz = \frac{(-A^2-B^2)\left(\begin{cases} z \\ \frac{z\sin^2(z)}{2}+\frac{z\cos^2(z)}{2}+\frac{\sin(z)\cos(z)}{2} \\ \frac{AB\log\left(-\frac{A}{B}+\tan\left(\frac{z}{2}\right)-\frac{\sqrt{A^2+B^2}}{B}\right)}{2A^2+2B^2}+\frac{AB\log\left(-\frac{A}{B}+\tan\left(\frac{z}{2}\right)+\frac{\sqrt{A^2+B^2}}{B}\right)}{2A^2+2B^2}-\frac{AB\log\left(\frac{A}{B}+\tan\left(\frac{z}{2}\right)-\frac{\sqrt{A^2+B^2}}{B}\right)}{2A^2+2B^2}-\end{cases}\right)}{B}$$

input
```
integrate((-A**2-B**2)*cos(z)**2/B/(1-(A**2+B**2)*sin(z)**2/B**2),z)
```

output
```
(-A**2 - B**2)*Piecewise((z, Eq(A, 0) & Eq(B, 0)), (z*sin(z)**2/2 + z*cos(
z)**2/2 + sin(z)*cos(z)/2, Eq(A, I*B) | Eq(A, -I*B)), (A*B*log(-A/B + tan(
z/2) - sqrt(A**2 + B**2)/B)/(2*A**2 + 2*B**2) + A*B*log(-A/B + tan(z/2) +
sqrt(A**2 + B**2)/B)/(2*A**2 + 2*B**2) - A*B*log(A/B + tan(z/2) - sqrt(A**
2 + B**2)/B)/(2*A**2 + 2*B**2) - A*B*log(A/B + tan(z/2) + sqrt(A**2 + B**2
)/B)/(2*A**2 + 2*B**2) + 2*B**2*z/(2*A**2 + 2*B**2), True))/B
```

3.70.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(16) = 32.

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 4.31

$$\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz = -\frac{(A^2+B^2)\left(\frac{2B^2z}{A^2+B^2}+\frac{AB\log(A\tan(z)+B)}{A^2+B^2}-\frac{AB\log(A\tan(z)-B)}{A^2+B^2}\right)}{2B}$$

input
```
integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="m
axima")
```

output
```
-1/2*(A^2 + B^2)*(2*B^2*z/(A^2 + B^2) + A*B*log(A*tan(z) + B)/(A^2 + B^2)
- A*B*log(A*tan(z) - B)/(A^2 + B^2))/B
```

3.70.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 5.19

$$\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz = -\frac{\left(\frac{A^3B\log(|A\tan(z)+B|)}{A^4+A^2B^2}-\frac{A^3B\log(|A\tan(z)-B|)}{A^4+A^2B^2}+\frac{2B^2z}{A^2+B^2}\right)(A^2+B^2)}{2B}$$

input
```
integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="g
iac")
```

output
```
-1/2*(A^3*B*log(abs(A*tan(z) + B))/(A^4 + A^2*B^2) - A^3*B*log(abs(A*tan(z
) - B))/(A^4 + A^2*B^2) + 2*B^2*z/(A^2 + B^2))*(A^2 + B^2)/B
```

3.70.9 Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 360, normalized size of antiderivative = 22.50

$$\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz = -A\,\mathrm{atanh}\left(\frac{2A^{13}\tan(z)}{2A^{12}B+6A^{10}B^3+6A^8B^5+2A^6B^7}+\frac{2A^7B^6\tan(z)}{2A^{12}B+6A^{10}B^3+6A^8B^5+2A^6B^7}+\frac{6A^9B^4\tan(z)}{2A^{12}B+6A^{10}B^3+6A^8B^5+2A^6B^7}+\frac{6A^{11}B^2\tan(z)}{2A^{12}B+6A^{10}B^3+6A^8B^5+2A^6B^7}\right) - B\,\mathrm{atan}\left(\frac{2A^4B^9\tan(z)}{2A^{10}B^3+6A^8B^5+6A^6B^7+2A^4B^9}+\frac{6A^6B^7\tan(z)}{2A^{10}B^3+6A^8B^5+6A^6B^7+2A^4B^9}+\frac{6A^8B^5\tan(z)}{2A^{10}B^3+6A^8B^5+6A^6B^7+2A^4B^9}+\frac{2A^{10}B^3\tan(z)}{2A^{10}B^3+6A^8B^5+6A^6B^7+2A^4B^9}\right)$$

input
```
int((cos(z)^2*(A^2 + B^2))/(B*((sin(z)^2*(A^2 + B^2))/B^2 - 1)),z)
```

output

```
- A*atanh((2*A^13*tan(z))/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3)
+ (2*A^7*B^6*tan(z))/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*
A^9*B^4*tan(z))/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^11*
B^2*tan(z))/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3)) - B*atan((2*A
^4*B^9*tan(z))/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^6*B
^7*tan(z))/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^8*B^5*t
an(z))/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (2*A^10*B^3*tan(
z))/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3))
```

3.70. $\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz$

3.71 $\int \frac{-A^2-B^2}{B(1+w^2)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw$

3.71.1 Optimal result

Integrand size = 48, antiderivative size = 16

$$\int \frac{-A^2-B^2}{B\left(1+w^2\right)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw=-B\arctan(w)-A\operatorname{arctanh}\left(\frac{Aw}{B}\right)$$

output
```
-B*arctan(w)-A*arctanh(A*w/B)
```

3.71.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \frac{-A^2-B^2}{B\left(1+w^2\right)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw=-\frac{B(A^2+B^2)\left(B\arctan(w)+A\operatorname{arctanh}\left(\frac{Aw}{B}\right)\right)}{A^2B+B^3}$$

input
```
Integrate[(-A^2 - B^2)/(B*(1 + w^2)^2*(1 - ((A^2 + B^2)*w^2)/(B^2*(1 + w^2
)))),w]
```

output
```
-((B*(A^2 + B^2)*(B*ArcTan[w] + A*ArcTanh[(A*w)/B]))/(A^2*B + B^3))
```

3.71.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 44 vs. $2(16) = 32$.

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {27, 7239, 27, 303, 216, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-A^2-B^2}{B(w^2+1)^2\left(1-\frac{w^2(A^2+B^2)}{B^2(w^2+1)}\right)}dw$$

$$\Big\downarrow \text{27}$$

$$-\frac{(A^2+B^2)\int \frac{1}{(w^2+1)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(w^2+1)}\right)}dw}{B}$$

$$\Big\downarrow \text{7239}$$

$$-\frac{(A^2+B^2)\int \frac{B^2}{(w^2+1)(B^2-A^2w^2)}dw}{B}$$

$$\Big\downarrow \text{27}$$

$$-B(A^2+B^2)\int \frac{1}{(w^2+1)(B^2-A^2w^2)}dw$$

$$\Big\downarrow \text{303}$$

$$-B(A^2+B^2)\left(\frac{A^2\int \frac{1}{B^2-A^2w^2}dw}{A^2+B^2}+\frac{\int \frac{1}{w^2+1}dw}{A^2+B^2}\right)$$

$$\Big\downarrow \text{216}$$

$$-B(A^2+B^2)\left(\frac{A^2\int \frac{1}{B^2-A^2w^2}dw}{A^2+B^2}+\frac{\arctan(w)}{A^2+B^2}\right)$$

$$\Big\downarrow \text{221}$$

$$-B(A^2+B^2)\left(\frac{\arctan(w)}{A^2+B^2}+\frac{A\text{arctanh}\left(\frac{Aw}{B}\right)}{B(A^2+B^2)}\right)$$

input
```
Int[(-A^2 - B^2)/(B*(1 + w^2)^2*(1 - ((A^2 + B^2)*w^2)/(B^2*(1 + w^2)))),w
]
```

3.71. $\int \frac{-A^2-B^2}{B(1+w^2)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}dw$

output
```
-(B*(A^2 + B^2)*(ArcTan[w]/(A^2 + B^2) + (A*ArcTanh[(A*w)/B])/(B*(A^2 + B^
2))))
```

3.71.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

3.71.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.38

method	result
parallelrisch	$\frac{\left(-A^{2}-B^{2}\right)\left(-i B^{2} \ln \left(w-i\right)+i B^{2} \ln \left(w+i\right)-A B \ln \left(A w-B\right)+A B \ln \left(A w+B\right)\right)}{2 B \left(A^{2}+B^{2}\right)}$
default	$\left(-A^{2}-B^{2}\right) B \left(\frac{\arctan \left(w\right)}{A^{2}+B^{2}}+\frac{A \ln \left(A w+B\right)}{2 B \left(A^{2}+B^{2}\right)}-\frac{A \ln \left(A w-B\right)}{2 B \left(A^{2}+B^{2}\right)}\right)$
risch	$-\frac{A^{3} \ln \left(-A w-B\right)}{2 \left(A^{2}+B^{2}\right)}-\frac{A \ln \left(-A w-B\right) B^{2}}{2 \left(A^{2}+B^{2}\right)}-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(A^{4}+2 A^{2} B^{2}+B^{4}\right) \_Z^{2}+B^{4}\right)} \_R \ln \left(\left(\left(-A^{6}-B^{2} A^{4}+A^{2} B^{4}+B^{6}\right.\right.\right.}{2 B}$

input
```
int((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w,method=_RETURNV
ERBOSE)
```

output
```
1/2*(-A^2-B^2)/B*(-I*B^2*ln(w-I)+I*B^2*ln(w+I)-A*B*ln(A*w-B)+A*B*ln(A*w+B)
)/(A^2+B^2)
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{-A^2-B^2}{B(1+w^2)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw = -B\arctan(w)-\frac{1}{2}A\log(Aw+B)+\frac{1}{2}A\log(Aw-B)$$

input
```
integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorith
m="fricas")
```

output
```
-B*arctan(w) - 1/2*A*log(A*w + B) + 1/2*A*log(A*w - B)
```

3.71.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 422, normalized size of antiderivative = 26.38

$$\int \frac{-A^2-B^2}{B(1+w^2)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw$$
$$= \left(A^2B+B^3\right)\left(-\frac{A\log\left(w+\frac{-\frac{A^9}{B(A^2+B^2)^3}-\frac{A^7B}{(A^2+B^2)^3}+\frac{A^5B^3}{(A^2+B^2)^3}+\frac{A^5}{B(A^2+B^2)}+\frac{A^3B^5}{(A^2+B^2)^3}+\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B(A^2+B^2)}\right.$$
$$+\frac{A\log\left(w+\frac{\frac{A^9}{B(A^2+B^2)^3}+\frac{A^7B}{(A^2+B^2)^3}-\frac{A^5B^3}{(A^2+B^2)^3}-\frac{A^5}{B(A^2+B^2)}-\frac{A^3B^5}{(A^2+B^2)^3}-\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B(A^2+B^2)}$$
$$+\frac{i\log\left(w+\frac{-\frac{iA^6B^2}{(A^2+B^2)^3}-\frac{iA^4B^4}{(A^2+B^2)^3}-\frac{iA^4}{A^2+B^2}+\frac{iA^2B^6}{(A^2+B^2)^3}+\frac{iB^8}{(A^2+B^2)^3}-\frac{iB^4}{A^2+B^2}}{A^2}\right)}{2(A^2+B^2)}$$
$$\left.-\frac{i\log\left(w+\frac{\frac{iA^6B^2}{(A^2+B^2)^3}+\frac{iA^4B^4}{(A^2+B^2)^3}+\frac{iA^4}{A^2+B^2}-\frac{iA^2B^6}{(A^2+B^2)^3}-\frac{iB^8}{(A^2+B^2)^3}+\frac{iB^4}{A^2+B^2}}{A^2}\right)}{2(A^2+B^2)}\right)$$

input
```
integrate((-A**2-B**2)/B/(w**2+1)**2/(1-(A**2+B**2)*w**2/B**2/(w**2+1)),w)
```

output
```
(A**2*B + B**3)*(-A*log(w + (-A**9/(B*(A**2 + B**2)**3) - A**7*B/(A**2 + B
**2)**3 + A**5*B**3/(A**2 + B**2)**3 + A**5/(B*(A**2 + B**2)) + A**3*B**5/
(A**2 + B**2)**3 + A*B**3/(A**2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + A*log
(w + (A**9/(B*(A**2 + B**2)**3) + A**7*B/(A**2 + B**2)**3 - A**5*B**3/(A**
2 + B**2)**3 - A**5/(B*(A**2 + B**2)) - A**3*B**5/(A**2 + B**2)**3 - A*B**
3/(A**2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + I*log(w + (-I*A**6*B**2/(A**2
 + B**2)**3 - I*A**4*B**4/(A**2 + B**2)**3 - I*A**4/(A**2 + B**2) + I*A**2
*B**6/(A**2 + B**2)**3 + I*B**8/(A**2 + B**2)**3 - I*B**4/(A**2 + B**2))/A
**2)/(2*(A**2 + B**2)) - I*log(w + (I*A**6*B**2/(A**2 + B**2)**3 + I*A**4*
B**4/(A**2 + B**2)**3 + I*A**4/(A**2 + B**2) - I*A**2*B**6/(A**2 + B**2)**
3 - I*B**8/(A**2 + B**2)**3 + I*B**4/(A**2 + B**2))/A**2)/(2*(A**2 + B**2)
))
```

3.71. $\int \frac{-A^2-B^2}{B(1+w^2)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw$

3.71.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.25

$$\int \frac{-A^2-B^2}{B\left(1+w^2\right)^2\left(1-\frac{\left(A^2+B^2\right)w^2}{B^2\left(1+w^2\right)}\right)}\,dw = -\frac{\left(A^2+B^2\right)\left(\frac{2\,B^2\arctan\left(w\right)}{A^2+B^2}+\frac{AB\log\left(Aw+B\right)}{A^2+B^2}-\frac{AB\log\left(Aw-B\right)}{A^2+B^2}\right)}{2\,B}$$

input
```
integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorith
m="maxima")
```

output
```
-1/2*(A^2 + B^2)*(2*B^2*arctan(w)/(A^2 + B^2) + A*B*log(A*w + B)/(A^2 + B^
2) - A*B*log(A*w - B)/(A^2 + B^2))/B
```

3.71.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 5.12

$$\int \frac{-A^2-B^2}{B\left(1+w^2\right)^2\left(1-\frac{\left(A^2+B^2\right)w^2}{B^2\left(1+w^2\right)}\right)}\,dw = -\frac{\left(\frac{A^3B\log\left(|Aw+B|\right)}{A^4+A^2B^2}-\frac{A^3B\log\left(|Aw-B|\right)}{A^4+A^2B^2}+\frac{2\,B^2\arctan\left(w\right)}{A^2+B^2}\right)\left(A^2+B^2\right)}{2\,B}$$

input
```
integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorith
m="giac")
```

output
```
-1/2*(A^3*B*log(abs(A*w + B))/(A^4 + A^2*B^2) - A^3*B*log(abs(A*w - B))/(A
^4 + A^2*B^2) + 2*B^2*arctan(w)/(A^2 + B^2))*(A^2 + B^2)/B
```

3.71.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 352, normalized size of antiderivative = 22.00

$$\int \frac{-A^2-B^2}{B(1+w^2)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw = -A\,\mathrm{atanh}\left(\frac{2\,A^{13}\,w}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7} + \frac{2\,A^7\,B^6\,w}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7} + \frac{6\,A^9\,B^4\,w}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7} + \frac{6\,A^{11}\,B^2\,w}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7}\right) - B\,\mathrm{atan}\left(\frac{2\,A^4\,B^9\,w}{2\,A^{10}\,B^3+6\,A^8\,B^5+6\,A^6\,B^7+2\,A^4\,B^9} + \frac{6\,A^6\,B^7\,w}{2\,A^{10}\,B^3+6\,A^8\,B^5+6\,A^6\,B^7+2\,A^4\,B^9} + \frac{6\,A^8\,B^5\,w}{2\,A^{10}\,B^3+6\,A^8\,B^5+6\,A^6\,B^7+2\,A^4\,B^9} + \frac{2\,A^{10}\,B^3\,w}{2\,A^{10}\,B^3+6\,A^8\,B^5+6\,A^6\,B^7+2\,A^4\,B^9}\right)$$

input
```
int((A^2 + B^2)/(B*(w^2 + 1)^2*((w^2*(A^2 + B^2))/(B^2*(w^2 + 1)) - 1)),w)
```

output
```
- A*atanh((2*A^13*w)/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (2*
A^7*B^6*w)/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^9*B^4*w)
/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^11*B^2*w)/(2*A^12*
B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3)) - B*atan((2*A^4*B^9*w)/(2*A^4*B^9
 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^6*B^7*w)/(2*A^4*B^9 + 6*A^6*
B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^8*B^5*w)/(2*A^4*B^9 + 6*A^6*B^7 + 6*A
^8*B^5 + 2*A^10*B^3) + (2*A^10*B^3*w)/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 +
 2*A^10*B^3))
```

3.72 $\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)}\,dw$

3.72.1 Optimal result

Integrand size = 31, antiderivative size = 16

$$\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)}\,dw = -B\arctan(w) - A\operatorname{arctanh}\left(\frac{Aw}{B}\right)$$

output
```
-B*arctan(w)-A*arctanh(A*w/B)
```

3.72.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)}\,dw = -\frac{B(A^2+B^2)\left(B\arctan(w)+A\operatorname{arctanh}\left(\frac{Aw}{B}\right)\right)}{A^2B+B^3}$$

input
```
Integrate[-((B*(A^2 + B^2))/((1 + w^2)*(B^2 - A^2*w^2))),w]
```

output
```
-((B*(A^2 + B^2)*(B*ArcTan[w] + A*ArcTanh[(A*w)/B]))/(A^2*B + B^3))
```

3.72.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 44 vs. $2(16) = 32$.

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {25, 27, 303, 216, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{B(A^2+B^2)}{(w^2+1)(B^2-A^2w^2)}dw$$
$$\Big\downarrow \; 25$$
$$-\int \frac{B(A^2+B^2)}{(w^2+1)(B^2-A^2w^2)}dw$$
$$\Big\downarrow \; 27$$
$$-B(A^2+B^2)\int \frac{1}{(w^2+1)(B^2-A^2w^2)}dw$$
$$\Big\downarrow \; 303$$
$$-B(A^2+B^2)\left(\frac{A^2\int \frac{1}{B^2-A^2w^2}dw}{A^2+B^2}+\frac{\int \frac{1}{w^2+1}dw}{A^2+B^2}\right)$$
$$\Big\downarrow \; 216$$
$$-B(A^2+B^2)\left(\frac{A^2\int \frac{1}{B^2-A^2w^2}dw}{A^2+B^2}+\frac{\arctan(w)}{A^2+B^2}\right)$$
$$\Big\downarrow \; 221$$
$$-B(A^2+B^2)\left(\frac{\arctan(w)}{A^2+B^2}+\frac{A\text{arctanh}\left(\frac{Aw}{B}\right)}{B(A^2+B^2)}\right)$$

input
```
Int[-((B*(A^2 + B^2))/((1 + w^2)*(B^2 - A^2*w^2))),w]
```

output
```
-(B*(A^2 + B^2)*(ArcTan[w]/(A^2 + B^2) + (A*ArcTanh[(A*w)/B])/(B*(A^2 + B^
2))))
```

3.72.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

3.72.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.50

method	result
parallelrisch	$\frac{i\ln(w-i)B}{2}-\frac{i\ln(w+i)B}{2}+\frac{A\ln(Aw-B)}{2}-\frac{A\ln(Aw+B)}{2}$
default	$-(A^2+B^2)\,B\left(\frac{\arctan(w)}{A^2+B^2}+\frac{A\ln(Aw+B)}{2B(A^2+B^2)}-\frac{A\ln(Aw-B)}{2B(A^2+B^2)}\right)$
risch	$-\frac{A\ln(-Aw-B)}{2}+\frac{A\ln(-Aw+B)}{2}-\frac{A^2B\left(\sum_{\_R=\text{RootOf}\left(1+\left(A^4+2A^2B^2+B^4\right)\_Z^2\right)}\_R\ln\left(\left(\left(-A^6-B^2A^4+A^2B^4+B^6\right)\_\right.\right.\right.}{2}$

input
```
int(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w,method=_RETURNVERBOSE)
```

output
```
1/2*I*ln(w-I)*B-1/2*I*ln(w+I)*B+1/2*A*ln(A*w-B)-1/2*A*ln(A*w+B)
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)}\,dw = -B\arctan(w) - \frac{1}{2}A\log(Aw+B) + \frac{1}{2}A\log(Aw-B)$$

input
```
integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w, algorithm="fricas")
```

output
```
-B*arctan(w) - 1/2*A*log(A*w + B) + 1/2*A*log(A*w - B)
```

3.72.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 422, normalized size of antiderivative = 26.38

$$\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)}\,dw$$
$$= (A^2B+B^3)\left(-\frac{A\log\left(w+\frac{-\frac{A^9}{B(A^2+B^2)^3}-\frac{A^7B}{(A^2+B^2)^3}+\frac{A^5B^3}{(A^2+B^2)^3}+\frac{A^5}{B(A^2+B^2)}+\frac{A^3B^5}{(A^2+B^2)^3}+\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B(A^2+B^2)}\right.$$
$$+\frac{A\log\left(w+\frac{\frac{A^9}{B(A^2+B^2)^3}+\frac{A^7B}{(A^2+B^2)^3}-\frac{A^5B^3}{(A^2+B^2)^3}-\frac{A^5}{B(A^2+B^2)}-\frac{A^3B^5}{(A^2+B^2)^3}-\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B(A^2+B^2)}$$
$$+\frac{i\log\left(w+\frac{-\frac{iA^6B^2}{(A^2+B^2)^3}-\frac{iA^4B^4}{(A^2+B^2)^3}-\frac{iA^4}{A^2+B^2}+\frac{iA^2B^6}{(A^2+B^2)^3}+\frac{iB^8}{(A^2+B^2)^3}-\frac{iB^4}{A^2+B^2}}{A^2}\right)}{2(A^2+B^2)}$$
$$\left.-\frac{i\log\left(w+\frac{\frac{iA^6B^2}{(A^2+B^2)^3}+\frac{iA^4B^4}{(A^2+B^2)^3}+\frac{iA^4}{A^2+B^2}-\frac{iA^2B^6}{(A^2+B^2)^3}-\frac{iB^8}{(A^2+B^2)^3}+\frac{iB^4}{A^2+B^2}}{A^2}\right)}{2(A^2+B^2)}\right)$$

input
```
integrate(-B*(A**2+B**2)/(w**2+1)/(-A**2*w**2+B**2),w)
```

output
```
(A**2*B + B**3)*(-A*log(w + (-A**9/(B*(A**2 + B**2)**3) - A**7*B/(A**2 + B
**2)**3 + A**5*B**3/(A**2 + B**2)**3 + A**5/(B*(A**2 + B**2)) + A**3*B**5/
(A**2 + B**2)**3 + A*B**3/(A**2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + A*log
(w + (A**9/(B*(A**2 + B**2)**3) + A**7*B/(A**2 + B**2)**3 - A**5*B**3/(A**
2 + B**2)**3 - A**5/(B*(A**2 + B**2)) - A**3*B**5/(A**2 + B**2)**3 - A*B**
3/(A**2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + I*log(w + (-I*A**6*B**2/(A**2
 + B**2)**3 - I*A**4*B**4/(A**2 + B**2)**3 - I*A**4/(A**2 + B**2) + I*A**2
*B**6/(A**2 + B**2)**3 + I*B**8/(A**2 + B**2)**3 - I*B**4/(A**2 + B**2))/A
**2)/(2*(A**2 + B**2)) - I*log(w + (I*A**6*B**2/(A**2 + B**2)**3 + I*A**4*
B**4/(A**2 + B**2)**3 + I*A**4/(A**2 + B**2) - I*A**2*B**6/(A**2 + B**2)**
3 - I*B**8/(A**2 + B**2)**3 + I*B**4/(A**2 + B**2))/A**2)/(2*(A**2 + B**2)
))
```

3.72.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.06

$$\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)}\,dw$$
$$= -\frac{1}{2}\left(A^2+B^2\right)B\left(\frac{A\log\left(Aw+B\right)}{A^2B+B^3}-\frac{A\log\left(Aw-B\right)}{A^2B+B^3}+\frac{2\arctan\left(w\right)}{A^2+B^2}\right)$$

input
```
integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w, algorithm="maxima")
```

output
```
-1/2*(A^2 + B^2)*B*(A*log(A*w + B)/(A^2*B + B^3) - A*log(A*w - B)/(A^2*B +
 B^3) + 2*arctan(w)/(A^2 + B^2))
```

3.72.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 4.94

$$\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)}\,dw = -\frac{1}{2}\left(\frac{A^3\log(|Aw+B|)}{A^4B+A^2B^3}-\frac{A^3\log(|Aw-B|)}{A^4B+A^2B^3}+\frac{2\arctan(w)}{A^2+B^2}\right)(A^2+B^2)B$$

input
```
integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w, algorithm="giac")
```

output
```
-1/2*(A^3*log(abs(A*w + B))/(A^4*B + A^2*B^3) - A^3*log(abs(A*w - B))/(A^4
*B + A^2*B^3) + 2*arctan(w)/(A^2 + B^2))*(A^2 + B^2)*B
```

3.72.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 352, normalized size of antiderivative = 22.00

$$\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)}\,dw = -A\,\mathrm{atanh}\left(\frac{2\,A^{13}\,w}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7}+\frac{2\,A^7\,B^6\,w}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7}+\frac{6\,A^9\,B^4\,w}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7}+\frac{6\,A^{11}\,B^2\,w}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7}\right) - B\,\mathrm{atan}\left(\frac{2\,A^4\,B^9\,w}{2\,A^{10}\,B^3+6\,A^8\,B^5+6\,A^6\,B^7+2\,A^4\,B^9}+\frac{6\,A^6\,B^7\,w}{2\,A^{10}\,B^3+6\,A^8\,B^5+6\,A^6\,B^7+2\,A^4\,B^9}+\frac{6\,A^8\,B^5\,w}{2\,A^{10}\,B^3+6\,A^8\,B^5+6\,A^6\,B^7+2\,A^4\,B^9}+\frac{2\,A^{10}\,B^3\,w}{2\,A^{10}\,B^3+6\,A^8\,B^5+6\,A^6\,B^7+2\,A^4\,B^9}\right)$$

input
```
int(-(B*(A^2 + B^2))/((w^2 + 1)*(B^2 - A^2*w^2)),w)
```

output
```
- A*atanh((2*A^13*w)/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (2*
A^7*B^6*w)/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^9*B^4*w)
/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^11*B^2*w)/(2*A^12*
B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3)) - B*atan((2*A^4*B^9*w)/(2*A^4*B^9
 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^6*B^7*w)/(2*A^4*B^9 + 6*A^6*
B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^8*B^5*w)/(2*A^4*B^9 + 6*A^6*B^7 + 6*A
^8*B^5 + 2*A^10*B^3) + (2*A^10*B^3*w)/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 +
 2*A^10*B^3))
```

3.72. $\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)}\,dw$

3.73 $\int \frac{x^4}{(1-x^2)^{5/2}} \, dx$

3.73.1 Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{x^4}{(1-x^2)^{5/2}} \, dx = \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \arcsin(x)$$

output
```
1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

3.73.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}} \, dx = \frac{x(-3+4x^2)}{3(1-x^2)^{3/2}} + 2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

input
```
Integrate[x^4/(1 - x^2)^(5/2),x]
```

output
```
(x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

3.73.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {252, 252, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 252$$

$$\frac{x^3}{3(1-x^2)^{3/2}}-\int \frac{x^2}{(1-x^2)^{3/2}}dx$$

$$\Big\downarrow \ 252$$

$$\int \frac{1}{\sqrt{1-x^2}}dx-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3(1-x^2)^{3/2}}$$

$$\Big\downarrow \ 223$$

$$\arcsin(x)-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3(1-x^2)^{3/2}}$$

input
```
Int[x^4/(1 - x^2)^(5/2),x]
```

output
```
x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

3.73.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

3.73.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x^{3}}{3(-x^{2}+1)^{\frac{3}{2}}}+\arcsin\left(x\right)-\frac{x}{\sqrt{-x^{2}+1}}$	30
risch	$-\frac{x(4x^{2}-3)}{3(x^{2}-1)\sqrt{-x^{2}+1}}+\arcsin\left(x\right)$	30
meijerg	$-\frac{2i\left(-\frac{i\sqrt{\pi}\,x\left(-20x^{2}+15\right)}{10\left(-x^{2}+1\right)^{\frac{3}{2}}}+\frac{3i\sqrt{\pi}\,\arcsin(x)}{2}\right)}{3\sqrt{\pi}}$	39
pseudoelliptic	$\frac{(x^{2}-1)\sqrt{-x^{2}+1}\,\arctan\left(\frac{\sqrt{-x^{2}+1}}{x}\right)+\frac{4x^{3}}{3}-x}{(-x^{2}+1)^{\frac{3}{2}}}$	49
trager	$\frac{(4x^{2}-3)x\sqrt{-x^{2}+1}}{3(x^{2}-1)^{2}}+\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)$	54

input
```
int(x^4/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-1/(-x^2+1)^(1/2)*x
```

3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(29) = 58.

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = -\frac{6\left(x^{4}-2\,x^{2}+1\right)\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)-\left(4\,x^{3}-3\,x\right)\sqrt{-x^{2}+1}}{3\left(x^{4}-2\,x^{2}+1\right)}$$

input
```
integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*s
qrt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

3.73.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(26) = 52.

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.00

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3} + \frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3} - \frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3} - \frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3} + \frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

input
```
integrate(x**4/(-x**2+1)**(5/2),x)
```

output
```
3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x
**2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x*
*4 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{1}{3}\,x\left(\frac{3\,x^2}{(-x^2+1)^{\frac{3}{2}}} - \frac{2}{(-x^2+1)^{\frac{3}{2}}}\right) - \frac{x}{3\,\sqrt{-x^2+1}} + \arcsin(x)$$

input
```
integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
```

output
```
1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
 + arcsin(x)
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{(4\,x^2-3)\sqrt{-x^2+1}x}{3\,(x^2-1)^2} + \arcsin(x)$$

input
```
integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

output
```
1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

3.73.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.60

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \operatorname{asin}(x) + \frac{3\sqrt{1-x^2}}{4\,(x-1)} + \frac{3\sqrt{1-x^2}}{4\,(x+1)} - \sqrt{1-x^2}\left(\frac{1}{12\,(x-1)} - \frac{1}{12\,(x-1)^2}\right) - \sqrt{1-x^2}\left(\frac{1}{12\,(x+1)} + \frac{1}{12\,(x+1)^2}\right)$$

input

```
int(x^4/(1 - x^2)^(5/2),x)
```

output

```
asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1)
) - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*
(1/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

3.74 $\int \tan^4(y)\,dy$

3.74.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \tan^4(y)\,dy = y - \tan(y) + \frac{\tan^3(y)}{3}$$

output
```
y-tan(y)+1/3*tan(y)^3
```

3.74.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \tan^4(y)\,dy = \arctan(\tan(y)) - \tan(y) + \frac{\tan^3(y)}{3}$$

input
```
Integrate[Tan[y]^4,y]
```

output
```
ArcTan[Tan[y]] - Tan[y] + Tan[y]^3/3
```

3.74.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(y)\,dy$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(y)^4dy$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^3(y)}{3}-\int \tan^2(y)dy$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^3(y)}{3}-\int \tan(y)^2dy$$

$$\Big\downarrow \text{3954}$$

$$\int 1dy+\frac{\tan^3(y)}{3}-\tan(y)$$

$$\Big\downarrow \text{24}$$

$$y+\frac{\tan^3(y)}{3}-\tan(y)$$

input
```
Int[Tan[y]^4,y]
```

output
```
y - Tan[y] + Tan[y]^3/3
```

3.74.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.74.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$y-\tan(y)+\frac{(\tan^3(y))}{3}$	13
parallelrisch	$y-\frac{4\tan(y)}{3}+\frac{\tan(y)(\sec^2(y))}{3}$	15
risch	$y-\frac{4i(3\,e^{4iy}+3\,e^{2iy}+2)}{3(e^{2iy}+1)^3}$	31
norman	$\frac{y(\tan^6(\frac{y}{2}))-y-\frac{20(\tan^3(\frac{y}{2}))}{3}+2(\tan^5(\frac{y}{2}))+3y(\tan^2(\frac{y}{2}))-3y(\tan^4(\frac{y}{2}))+2\tan(\frac{y}{2})}{(\tan^2(\frac{y}{2})-1)^3}$	64

input
```
int(sin(y)^4/cos(y)^4,y,method=_RETURNVERBOSE)
```

output
```
y-tan(y)+1/3*tan(y)^3
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \tan^4(y)\,dy = \frac{3\,y\cos(y)^3-\left(4\,\cos(y)^2-1\right)\sin(y)}{3\,\cos(y)^3}$$

input
```
integrate(sin(y)^4/cos(y)^4,y, algorithm="fricas")
```

output
```
1/3*(3*y*cos(y)^3 - (4*cos(y)^2 - 1)*sin(y))/cos(y)^3
```

3.74.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \tan^4(y)\,dy = y + \frac{\sin^3(y)}{3\cos^3(y)} - \frac{\sin(y)}{\cos(y)}$$

input
```
integrate(sin(y)**4/cos(y)**4,y)
```

output
```
y + sin(y)**3/(3*cos(y)**3) - sin(y)/cos(y)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(y)\,dy = \frac{1}{3}\tan(y)^3 + y - \tan(y)$$

input
```
integrate(sin(y)^4/cos(y)^4,y, algorithm="maxima")
```

output
```
1/3*tan(y)^3 + y - tan(y)
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(y)\,dy = \frac{1}{3}\tan(y)^3 + y - \tan(y)$$

input
```
integrate(sin(y)^4/cos(y)^4,y, algorithm="giac")
```

output
```
1/3*tan(y)^3 + y - tan(y)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \tan^4(y)\,dy = \frac{\tan(y)^3}{3} - \tan(y) + y$$

input
```
int(sin(y)^4/cos(y)^4,y)
```

output
```
y - tan(y) + tan(y)^3/3
```

3.75 $\int \frac{z^4}{1+z^2}\,dz$

3.75.1 Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{z^4}{1+z^2}\,dz=-z+\frac{z^3}{3}+\arctan(z)$$

output
```
-z+1/3*z^3+arctan(z)
```

3.75.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{z^4}{1+z^2}\,dz=-z+\frac{z^3}{3}+\arctan(z)$$

input
```
Integrate[z^4/(1 + z^2),z]
```

output
```
-z + z^3/3 + ArcTan[z]
```

3.75.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{z^4}{z^2+1}\,dz$$

$$\big\downarrow \text{ 254}$$

$$\int \left(z^2+\frac{1}{z^2+1}-1\right)dz$$

$$\big\downarrow \text{ 2009}$$

$$\arctan(z)+\frac{z^3}{3}-z$$

input
```
Int[z^4/(1 + z^2),z]
```

output
```
-z + z^3/3 + ArcTan[z]
```

3.75.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.75.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$-z+\frac{z^{3}}{3}+\arctan \left(z \right)$	12
risch	$-z+\frac{z^{3}}{3}+\arctan \left(z \right)$	12
meijerg	$-\frac{z \left(-5 z^{2}+15\right)}{15}+\arctan \left(z \right)$	14
parallelrisch	$\frac{z^{3}}{3}-z+\frac{i \ln \left(z +i \right)}{2}-\frac{i \ln \left(z -i \right)}{2}$	26

input
```
int(z^4/(z^2+1),z,method=_RETURNVERBOSE)
```

output
```
-z+1/3*z^3+arctan(z)
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{z^4}{1+z^2}\,dz=\frac{1}{3}\,z^{3}-z+\arctan \left(z\right)$$

input
```
integrate(z^4/(z^2+1),z, algorithm="fricas")
```

output
```
1/3*z^3 - z + arctan(z)
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{z^4}{1+z^2}\,dz=\frac{z^{3}}{3}-z+\operatorname{atan}{\left(z \right)}$$

input
```
integrate(z**4/(z**2+1),z)
```

output
```
z**3/3 - z + atan(z)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{z^4}{1+z^2}\,dz = \frac{1}{3}\,z^3 - z + \arctan\left(z\right)$$

input
```
integrate(z^4/(z^2+1),z, algorithm="maxima")
```

output
```
1/3*z^3 - z + arctan(z)
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{z^4}{1+z^2}\,dz = \frac{1}{3}\,z^3 - z + \arctan\left(z\right)$$

input
```
integrate(z^4/(z^2+1),z, algorithm="giac")
```

output
```
1/3*z^3 - z + arctan(z)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{z^4}{1+z^2}\,dz = \mathrm{atan}(z) - z + \frac{z^3}{3}$$

input
```
int(z^4/(z^2 + 1),z)
```

output
```
atan(z) - z + z^3/3
```

3.76 $\int e^{x^2}(1+2x^2)\,dx$

3.76.1 Optimal result

Integrand size = 13, antiderivative size = 7

$$\int e^{x^2}\left(1+2x^2\right)\,dx=e^{x^2}x$$

output
```
exp(x^2)*x
```

3.76.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int e^{x^2}\left(1+2x^2\right)\,dx=e^{x^2}x$$

input
```
Integrate[E^x^2*(1 + 2*x^2),x]
```

output
```
E^x^2*x
```

3.76.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2}\left(2x^2+1\right)\,dx$$

$$\Big\downarrow \ 2656$$

$$\int \left(2e^{x^2}x^2+e^{x^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$e^{x^2}x$$

input
```
Int[E^x^2*(1 + 2*x^2),x]
```

output
```
E^x^2*x
```

3.76.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

3.76.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

method	result	size
gosper	$e^{x^2} x$	7
default	$e^{x^2} x$	7
norman	$e^{x^2} x$	7
risch	$e^{x^2} x$	7
parallelrisch	$e^{x^2} x$	7
meijerg	$i\left(-ix\, e^{x^2}+\frac{i\, \mathrm{erfi}(x)\sqrt{\pi}}{2}\right)+\frac{\mathrm{erfi}(x)\sqrt{\pi}}{2}$	29
parts	$\mathrm{erfi}\left(x\right)\sqrt{\pi}\, x^{2}+\frac{\mathrm{erfi}(x)\sqrt{\pi}}{2}-2\sqrt{\pi}\left(\frac{x^{2}\, \mathrm{erfi}(x)}{2}-\frac{\frac{e^{x^{2}} x}{2}-\frac{\mathrm{erfi}(x)\sqrt{\pi}}{4}}{\sqrt{\pi}}\right)$	51

input
```
int(exp(x^2)*(2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
exp(x^2)*x
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int e^{x^2}\left(1+2x^2\right)\,dx = xe^{\left(x^2\right)}$$

input
```
integrate(exp(x^2)*(2*x^2+1),x, algorithm="fricas")
```

output
```
x*e^(x^2)
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int e^{x^2}\left(1+2x^2\right)\,dx = xe^{x^2}$$

input
```
integrate(exp(x**2)*(2*x**2+1),x)
```

output
```
x*exp(x**2)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int e^{x^2}\left(1+2x^2\right)\,dx = xe^{\left(x^2\right)}$$

input
```
integrate(exp(x^2)*(2*x^2+1),x, algorithm="maxima")
```

output
```
x*e^(x^2)
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int e^{x^2}\left(1+2x^2\right)\,dx = xe^{\left(x^2\right)}$$

input
```
integrate(exp(x^2)*(2*x^2+1),x, algorithm="giac")
```

output
```
x*e^(x^2)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

$$\int e^{x^2}\left(1+2x^2\right)\,dx = x\,\mathrm{e}^{x^2}$$

input
```
int(exp(x^2)*(2*x^2 + 1),x)
```

output
```
x*exp(x^2)
```

3.77 $\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2}\,dx$

3.77.1 Optimal result

Integrand size = 33, antiderivative size = 24

$$\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2}\,dx = e^{x^2}x + \frac{e^{x^2}}{2(1+x^2)}$$

output
```
exp(x^2)*x+1/2*exp(x^2)/(x^2+1)
```

3.77.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2}\,dx = e^{x^2}\left(x + \frac{1}{2(1+x^2)}\right)$$

input
```
Integrate[(E^x^2*(1 + 4*x^2 + x^3 + 5*x^4 + 2*x^6))/(1 + x^2)^2,x]
```

output
```
E^x^2*(x + 1/(2*(1 + x^2)))
```

3.77.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{x^2}(2x^6+5x^4+x^3+4x^2+1)}{(x^2+1)^2}dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(2e^{x^2}x^2+\frac{e^{x^2}x}{x^2+1}-\frac{e^{x^2}x}{(x^2+1)^2}+e^{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$e^{x^2}x+\frac{e^{x^2}}{2(x^2+1)}$$

input
```
Int[(E^x^2*(1 + 4*x^2 + x^3 + 5*x^4 + 2*x^6))/(1 + x^2)^2,x]
```

output
```
E^x^2*x + E^x^2/(2*(1 + x^2))
```

3.77.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.77.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{(2x^3+2x+1)e^{x^2}}{2x^2+2}$	24
risch	$\frac{(2x^3+2x+1)e^{x^2}}{2x^2+2}$	24
norman	$\frac{x^3e^{x^2}+e^{x^2}x+\frac{e^{x^2}}{2}}{x^2+1}$	30
parallelrisch	$\frac{2x^3e^{x^2}+2e^{x^2}x+e^{x^2}}{2x^2+2}$	31

input
```
int(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*x^3+2*x+1)*exp(x^2)/(x^2+1)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2}\,dx = \frac{(2\,x^3+2\,x+1)e^{(x^2)}}{2\,(x^2+1)}$$

input
```
integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="frica
s")
```

output
```
1/2*(2*x^3 + 2*x + 1)*e^(x^2)/(x^2 + 1)
```

3.77.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2}\,dx = \frac{(2x^3+2x+1)\,e^{x^2}}{2x^2+2}$$

input
```
integrate(exp(x**2)*(2*x**6+5*x**4+x**3+4*x**2+1)/(x**2+1)**2,x)
```

output
```
(2*x**3 + 2*x + 1)*exp(x**2)/(2*x**2 + 2)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2}\,dx = \frac{(2\,x^3+2\,x+1)e^{(x^2)}}{2\,(x^2+1)}$$

input
```
integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="maxim
a")
```

output
```
1/2*(2*x^3 + 2*x + 1)*e^(x^2)/(x^2 + 1)
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2}\,dx = \frac{2\,x^3e^{(x^2)}+2\,xe^{(x^2)}+e^{(x^2)}}{2\,(x^2+1)}$$

input
```
integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="giac"
)
```

output
```
1/2*(2*x^3*e^(x^2) + 2*x*e^(x^2) + e^(x^2))/(x^2 + 1)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2}\,dx = \frac{e^{x^2}\,(2\,x^3+2\,x+1)}{2\,(x^2+1)}$$

input
```
int((exp(x^2)*(4*x^2 + x^3 + 5*x^4 + 2*x^6 + 1))/(x^2 + 1)^2,x)
```

output
```
(exp(x^2)*(2*x + 2*x^3 + 1))/(2*(x^2 + 1))
```

3.78 $\int e^{-1-x} \, dx$

3.78.1 Optimal result

Integrand size = 7, antiderivative size = 9

$$\int e^{-1-x} \, dx = -e^{-1-x}$$

output
```
-exp(-1-x)
```

3.78.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int e^{-1-x} \, dx = -e^{-1-x}$$

input
```
Integrate[E^(-1 - x),x]
```

output
```
-E^(-1 - x)
```

3.78.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x-1}\,dx$$
$$\Big\downarrow \text{2624}$$
$$-e^{-x-1}$$

input
```
Int[E^(-1 - x),x]
```

output
```
-E^(-1 - x)
```

3.78.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

3.78.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\mathrm{e}^{-1-x}$	9
derivativedivides	$-\mathrm{e}^{-1-x}$	9
default	$-\mathrm{e}^{-1-x}$	9
norman	$-\mathrm{e}^{-1-x}$	9
risch	$-\mathrm{e}^{-1-x}$	9
parallelrisch	$-\mathrm{e}^{-1-x}$	9
meijerg	$\mathrm{e}^{-1}(1-\mathrm{e}^{-x})$	12

input
```
int(exp(-1-x),x,method=_RETURNVERBOSE)
```

output
```
-exp(-1-x)
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{-1-x}\,dx = -e^{(-x-1)}$$

input
```
integrate(exp(-1-x),x, algorithm="fricas")
```

output
```
-e^(-x - 1)
```

3.78.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int e^{-1-x}\,dx = -e^{-x-1}$$

input
```
integrate(exp(-1-x),x)
```

output
```
-exp(-x - 1)
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{-1-x}\,dx = -e^{(-x-1)}$$

input
```
integrate(exp(-1-x),x, algorithm="maxima")
```

output
```
-e^(-x - 1)
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{-1-x}\,dx=-e^{(-x-1)}$$

input
```
integrate(exp(-1-x),x, algorithm="giac")
```

output
```
-e^(-x - 1)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int e^{-1-x}\,dx=-\mathrm{e}^{-x-1}$$

input
```
int(exp(- x - 1),x)
```

output
```
-exp(- x - 1)
```

3.79 $\int \left(\frac{1}{x}+x\right)\log(x)\,dx$

3.79.1 Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \left(\frac{1}{x}+x\right)\log(x)\,dx=-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)+\frac{\log^2(x)}{2}$$

output
```
-1/4*x^2+1/2*x^2*ln(x)+1/2*ln(x)^2
```

3.79.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{x}+x\right)\log(x)\,dx=-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)+\frac{\log^2(x)}{2}$$

input
```
Integrate[(x^(-1) + x)*Log[x],x]
```

output
```
-1/4*x^2 + (x^2*Log[x])/2 + Log[x]^2/2
```

3.79.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2027, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x+\frac{1}{x}\right)\log(x)\,dx$$
$$\Big\downarrow \text{ 2027}$$
$$\int \frac{(x^2+1)\log(x)}{x}dx$$
$$\Big\downarrow \text{ 2793}$$
$$\int \left(x\log(x)+\frac{\log(x)}{x}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)+\frac{\log^2(x)}{2}$$

input
```
Int[(x^(-1) + x)*Log[x],x]
```

output
```
-1/4*x^2 + (x^2*Log[x])/2 + Log[x]^2/2
```

3.79.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

3.79.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}+\frac{\ln(x)^2}{2}$	20
norman	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}+\frac{\ln(x)^2}{2}$	20
risch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}+\frac{\ln(x)^2}{2}$	20
parallelrisch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}+\frac{\ln(x)^2}{2}$	20
parts	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}+\frac{\ln(x)^2}{2}$	20

input
```
int((1/x+x)*ln(x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2+1/2*x^2*ln(x)+1/2*ln(x)^2
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(\frac{1}{x}+x\right)\log(x)\,dx=\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2+\frac{1}{2}\,\log\left(x\right)^2$$

input
```
integrate((1/x+x)*log(x),x, algorithm="fricas")
```

output
```
1/2*x^2*log(x) - 1/4*x^2 + 1/2*log(x)^2
```

3.79.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(\frac{1}{x}+x\right)\log(x)\,dx=\frac{x^2\log(x)}{2}-\frac{x^2}{4}+\frac{\log(x)^2}{2}$$

input
```
integrate((1/x+x)*ln(x),x)
```

output
```
x**2*log(x)/2 - x**2/4 + log(x)**2/2
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \left(\frac{1}{x}+x\right)\log(x)\,dx=-\frac{1}{4}x^2+\frac{1}{2}\left(x^2+2\log(x)\right)\log(x)-\frac{1}{2}\log(x)^2$$

input
```
integrate((1/x+x)*log(x),x, algorithm="maxima")
```

output
```
-1/4*x^2 + 1/2*(x^2 + 2*log(x))*log(x) - 1/2*log(x)^2
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(\frac{1}{x}+x\right)\log(x)\,dx=\frac{1}{2}x^2\log(x)-\frac{1}{4}x^2+\frac{1}{2}\log(x)^2$$

input
```
integrate((1/x+x)*log(x),x, algorithm="giac")
```

output
```
1/2*x^2*log(x) - 1/4*x^2 + 1/2*log(x)^2
```

3.79.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(\frac{1}{x}+x\right)\log(x)\,dx=\frac{x^2\,\ln\left(x\right)}{2}-\frac{x^2}{4}+\frac{{\ln\left(x\right)}^2}{2}$$

input
```
int(log(x)*(x + 1/x),x)
```

output
```
(x^2*log(x))/2 + log(x)^2/2 - x^2/4
```

3.80 $\int \frac{x}{1+x^4}\,dx$

3.80.1 Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \frac{x}{1+x^4}\,dx=\frac{\arctan(x^2)}{2}$$

output
```
1/2*arctan(x^2)
```

3.80.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x^4}\,dx=\frac{\arctan(x^2)}{2}$$

input
```
Integrate[x/(1 + x^4),x]
```

output
```
ArcTan[x^2]/2
```

3.80.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {807, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^4+1}dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2}\int \frac{1}{x^4+1}dx^2$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(x^2\right)}{2}$$

input
```
Int[x/(1 + x^4),x]
```

output
```
ArcTan[x^2]/2
```

3.80.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

3.80.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\arctan(x^2)}{2}$	7
meijerg	$\frac{\arctan(x^2)}{2}$	7
risch	$\frac{\arctan(x^2)}{2}$	7
parallelrisch	$\frac{i\ln(x^2+i)}{4} - \frac{i\ln(x^2-i)}{4}$	22

input
```
int(x/(x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x^2)
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\arctan\left(x^2\right)$$

input
```
integrate(x/(x^4+1),x, algorithm="fricas")
```

output
```
1/2*arctan(x^2)
```

3.80.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{x}{1+x^4}\,dx = \frac{\operatorname{atan}\left(x^2\right)}{2}$$

input
```
integrate(x/(x**4+1),x)
```

output
```
atan(x**2)/2
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\arctan\left(x^2\right)$$

input
```
integrate(x/(x^4+1),x, algorithm="maxima")
```

output
```
1/2*arctan(x^2)
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\arctan\left(x^2\right)$$

input
```
integrate(x/(x^4+1),x, algorithm="giac")
```

output
```
1/2*arctan(x^2)
```

3.80.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{1+x^4}\,dx = \frac{\mathrm{atan}(x^2)}{2}$$

input
```
int(x/(x^4 + 1),x)
```

output
```
atan(x^2)/2
```

3.81 $\int \frac{x^5}{1+x^4} dx$

3.81.1 Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{x^5}{1+x^4} dx = \frac{x^2}{2} - \frac{\arctan(x^2)}{2}$$

output
```
1/2*x^2-1/2*arctan(x^2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{1+x^4} dx = \frac{x^2}{2} - \frac{\arctan(x^2)}{2}$$

input
```
Integrate[x^5/(1 + x^4),x]
```

output
```
x^2/2 - ArcTan[x^2]/2
```

3.81.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {807, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^4+1}\,dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2}\int \frac{x^4}{x^4+1}dx^2$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{2}\left(x^2-\int \frac{1}{x^4+1}dx^2\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}\left(x^2-\arctan\left(x^2\right)\right)$$

input
```
Int[x^5/(1 + x^4),x]
```

output
```
(x^2 - ArcTan[x^2])/2
```

3.81.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

3.81.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x^2}{2} - \frac{\arctan(x^2)}{2}$	13
meijerg	$\frac{x^2}{2} - \frac{\arctan(x^2)}{2}$	13
risch	$\frac{x^2}{2} - \frac{\arctan(x^2)}{2}$	13
parallelrisch	$\frac{x^2}{2} + \frac{i\ln(x^2-i)}{4} - \frac{i\ln(x^2+i)}{4}$	27

input
```
int(x^5/(x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-1/2*arctan(x^2)
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{1+x^4}\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,\arctan\left(x^2\right)$$

input
```
integrate(x^5/(x^4+1),x, algorithm="fricas")
```

output
```
1/2*x^2 - 1/2*arctan(x^2)
```

3.81.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x^5}{1+x^4}\,dx = \frac{x^2}{2} - \frac{\operatorname{atan}(x^2)}{2}$$

input
```
integrate(x**5/(x**4+1),x)
```

output
```
x**2/2 - atan(x**2)/2
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{1+x^4}\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,\arctan\left(x^2\right)$$

input
```
integrate(x^5/(x^4+1),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/2*arctan(x^2)
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{1+x^4}\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,\arctan\left(x^2\right)$$

input
```
integrate(x^5/(x^4+1),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/2*arctan(x^2)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{1+x^4}\,dx = \frac{x^2}{2} - \frac{\operatorname{atan}(x^2)}{2}$$

input
```
int(x^5/(x^4 + 1),x)
```

output
```
x^2/2 - atan(x^2)/2
```

3.82 $\int \frac{1}{1+\tan^2(x)}\,dx$

3.82.1 Optimal result

Integrand size = 8, antiderivative size = 14

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)$$

output
```
1/2*x+1/2*cos(x)*sin(x)
```

3.82.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{x}{2}+\frac{1}{4}\sin(2x)$$

input
```
Integrate[(1 + Tan[x]^2)^(-1),x]
```

output
```
x/2 + Sin[2*x]/4
```

3.82.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 4140, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\tan^2(x)+1} \, dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\tan(x)^2+1}dx$$
$$\Big\downarrow \text{ 4140}$$
$$\int \cos^2(x)dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sin\left(x+\frac{\pi}{2}\right)^2 dx$$
$$\Big\downarrow \text{ 3115}$$
$$\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)$$
$$\Big\downarrow \text{ 24}$$
$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[(1 + Tan[x]^2)^(-1),x]
```

output
```
x/2 + (Cos[x]*Sin[x])/2
```

3.82.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

3.82.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
derivativedivides	$\frac{\tan(x)}{2+2(\tan^2(x))}+\frac{\arctan(\tan(x))}{2}$	19
default	$\frac{\tan(x)}{2+2(\tan^2(x))}+\frac{\arctan(\tan(x))}{2}$	19
parallelrisch	$\frac{x(\tan^2(x))+x+\tan(x)}{2+2(\tan^2(x))}$	21
norman	$\frac{\frac{x}{2}+\frac{x\left(\tan^2(x)\right)}{2}+\frac{\tan(x)}{2}}{1+\tan^2(x)}$	25

input
```
int(1/(1+tan(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/4*sin(2*x)
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{x\tan(x)^2 + x + \tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

input
```
integrate(1/(1+tan(x)^2),x, algorithm="fricas")
```

output
```
1/2*(x*tan(x)^2 + x + tan(x))/(tan(x)^2 + 1)
```

3.82.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(10) = 20.

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.57

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{x\tan^2(x)}{2\tan^2(x)+2} + \frac{x}{2\tan^2(x)+2} + \frac{\tan(x)}{2\tan^2(x)+2}$$

input
```
integrate(1/(1+tan(x)**2),x)
```

output
```
x*tan(x)**2/(2*tan(x)**2 + 2) + x/(2*tan(x)**2 + 2) + tan(x)/(2*tan(x)**2
+ 2)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{1}{2}x + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

input
```
integrate(1/(1+tan(x)^2),x, algorithm="maxima")
```

output
```
1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{1}{2}\,x + \frac{\tan(x)}{2\left(\tan(x)^2+1\right)}$$

input
```
integrate(1/(1+tan(x)^2),x, algorithm="giac")
```

output
```
1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

3.82.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{1+\tan^2(x)}\,dx = \frac{x}{2} + \frac{\sin(2\,x)}{4}$$

input
```
int(1/(tan(x)^2 + 1),x)
```

output
```
x/2 + sin(2*x)/4
```

3.83 $\int \frac{x^4}{(1-x^2)^{5/2}}\,dx$

3.83.1 Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \arcsin(x)$$

output
```
1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{x(-3+4x^2)}{3(1-x^2)^{3/2}} + 2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

input
```
Integrate[x^4/(1 - x^2)^(5/2),x]
```

output
```
(x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

3.83.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {252, 252, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(1-x^2)^{5/2}} \, dx$$
$$\Big\downarrow \text{252}$$
$$\frac{x^3}{3(1-x^2)^{3/2}} - \int \frac{x^2}{(1-x^2)^{3/2}}dx$$
$$\Big\downarrow \text{252}$$
$$\int \frac{1}{\sqrt{1-x^2}}dx - \frac{x}{\sqrt{1-x^2}} + \frac{x^3}{3(1-x^2)^{3/2}}$$
$$\Big\downarrow \text{223}$$
$$\arcsin(x) - \frac{x}{\sqrt{1-x^2}} + \frac{x^3}{3(1-x^2)^{3/2}}$$

input
```
Int[x^4/(1 - x^2)^(5/2),x]
```

output
```
x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

3.83.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

3.83.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x^3}{3(-x^2+1)^{\frac{3}{2}}}+\arcsin(x)-\frac{x}{\sqrt{-x^2+1}}$	30
risch	$-\frac{x(4x^2-3)}{3(x^2-1)\sqrt{-x^2+1}}+\arcsin(x)$	30
meijerg	$-\frac{2i\left(-\frac{i\sqrt{\pi}\,x\left(-20x^2+15\right)}{10\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{3i\sqrt{\pi}\,\arcsin(x)}{2}\right)}{3\sqrt{\pi}}$	39
pseudoelliptic	$\frac{(x^2-1)\sqrt{-x^2+1}\,\arctan\left(\frac{\sqrt{-x^2+1}}{x}\right)+\frac{4x^3}{3}-x}{(-x^2+1)^{\frac{3}{2}}}$	49
trager	$\frac{(4x^2-3)x\sqrt{-x^2+1}}{3(x^2-1)^2}+\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)$	54

input
```
int(x^4/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-1/(-x^2+1)^(1/2)*x
```

3.83.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(29) = 58.

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = -\frac{6\,(x^4-2\,x^2+1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-(4\,x^3-3\,x)\sqrt{-x^2+1}}{3\,(x^4-2\,x^2+1)}$$

input
```
integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*s
qrt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

3.83.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(26) = 52.

Time = 0.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.00

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3} + \frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3} - \frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3} - \frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3} + \frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

input
```
integrate(x**4/(-x**2+1)**(5/2),x)
```

output
```
3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x
**2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x*
*4 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{1}{3}\,x\left(\frac{3\,x^2}{(-x^2+1)^{\frac{3}{2}}} - \frac{2}{(-x^2+1)^{\frac{3}{2}}}\right) - \frac{x}{3\,\sqrt{-x^2+1}} + \arcsin(x)$$

input
```
integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
```

output
```
1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
 + arcsin(x)
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{(4\,x^2-3)\sqrt{-x^2+1}x}{3\,(x^2-1)^2} + \arcsin(x)$$

input
```
integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

output
```
1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

3.83.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.60

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \operatorname{asin}(x) + \frac{3\sqrt{1-x^2}}{4\ (x-1)} + \frac{3\sqrt{1-x^2}}{4\ (x+1)} - \sqrt{1-x^2}\left(\frac{1}{12\ (x-1)} - \frac{1}{12\,(x-1)^2}\right) - \sqrt{1-x^2}\left(\frac{1}{12\ (x+1)} + \frac{1}{12\,(x+1)^2}\right)$$

input
```
int(x^4/(1 - x^2)^(5/2),x)
```

output
```
asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1)
) - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*
(1/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

3.84 $\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx$

3.84.1 Optimal result

Integrand size = 16, antiderivative size = 17

$$\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx = -\frac{x}{\sqrt{1-x^2}}+\arcsin(x)$$

output
```
arcsin(x)-x/(-x^2+1)^(1/2)
```

3.84.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(17) = 34.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx = -\frac{x}{\sqrt{1-x^2}}+2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

input
```
Integrate[-(x^2/(1 - x^2)^(3/2)),x]
```

output
```
-(x/Sqrt[1 - x^2]) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

3.84.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {25, 252, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{x^2}{(1-x^2)^{3/2}}dx$$

$$\Big\downarrow \ 252$$

$$\int \frac{1}{\sqrt{1-x^2}}dx-\frac{x}{\sqrt{1-x^2}}$$

$$\Big\downarrow \ 223$$

$$\arcsin(x)-\frac{x}{\sqrt{1-x^2}}$$

input
```
Int[-(x^2/(1 - x^2)^(3/2)),x]
```

output
```
-(x/Sqrt[1 - x^2]) + ArcSin[x]
```

3.84.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

3.84.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\arcsin(x)-\frac{x}{\sqrt{-x^2+1}}$	16
risch	$\arcsin(x)-\frac{x}{\sqrt{-x^2+1}}$	16
meijerg	$-\frac{i\left(-\frac{i\sqrt{\pi}\,x}{\sqrt{-x^2+1}}+i\sqrt{\pi}\,\arcsin(x)\right)}{\sqrt{\pi}}$	32
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-x^2+1}}{x}\right)\sqrt{-x^2+1}+x}{\sqrt{-x^2+1}}$	38
trager	$\frac{x\sqrt{-x^2+1}}{x^2-1}+\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)$	46

input
```
int(-1/(-x^2+1)^(3/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
arcsin(x)-1/(-x^2+1)^(1/2)*x
```

3.84.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.65

$$\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx = -\frac{2\,(x^2-1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\sqrt{-x^2+1}x}{x^2-1}$$

input
```
integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-(2*(x^2 - 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - sqrt(-x^2 + 1)*x)/(x^2 - 1)
```

3.84.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(12) = 24.

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

$$\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx = \frac{x^2\,\mathrm{asin}\,(x)}{x^2-1} + \frac{x\sqrt{1-x^2}}{x^2-1} - \frac{\mathrm{asin}\,(x)}{x^2-1}$$

input
```
integrate(-x**2/(-x**2+1)**(3/2),x)
```

output
```
x**2*asin(x)/(x**2 - 1) + x*sqrt(1 - x**2)/(x**2 - 1) - asin(x)/(x**2 - 1)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx = -\frac{x}{\sqrt{-x^2+1}} + \arcsin\,(x)$$

input
```
integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
-x/sqrt(-x^2 + 1) + arcsin(x)
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx = \frac{\sqrt{-x^2+1}x}{x^2-1} + \arcsin\,(x)$$

input
```
integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="giac")
```

output
```
sqrt(-x^2 + 1)*x/(x^2 - 1) + arcsin(x)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx = \operatorname{asin}(x) + \frac{\sqrt{1-x^2}}{2\,(x-1)} + \frac{\sqrt{1-x^2}}{2\,(x+1)}$$

input
```
int(-x^2/(1 - x^2)^(3/2),x)
```

output
```
asin(x) + (1 - x^2)^(1/2)/(2*(x - 1)) + (1 - x^2)^(1/2)/(2*(x + 1))
```

3.85 $\int e^x \sin(x) \, dx$

3.85.1 Optimal result

Integrand size = 6, antiderivative size = 19

$$\int e^x \sin(x) \, dx = -\frac{1}{2} e^x \cos(x) + \frac{1}{2} e^x \sin(x)$$

output
```
-1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

3.85.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int e^x \sin(x) \, dx = \frac{1}{2} e^x (-\cos(x) + \sin(x))$$

input
```
Integrate[E^x*Sin[x],x]
```

output
```
(E^x*(-Cos[x] + Sin[x]))/2
```

3.85.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sin(x) \, dx$$

$$\Big\downarrow \text{4932}$$

$$\frac{1}{2} e^x \sin(x) - \frac{1}{2} e^x \cos(x)$$

input
```
Int[E^x*Sin[x],x]
```

output
```
-1/2*(E^x*Cos[x]) + (E^x*Sin[x])/2
```

3.85.3.1 Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.85.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{e^{x} \left(-\cos \left(x \right)+\sin \left(x \right)\right)}{2}$	12
default	$-\frac{e^{x} \cos \left(x \right)}{2}+\frac{e^{x} \sin \left(x \right)}{2}$	14
norman	$\frac{e^{x} \tan \left(\frac{x}{2}\right)+\frac{e^{x} \left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\frac{e^{x}}{2}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	34
risch	$-\frac{e^{\left(1+i\right) x}}{4}-\frac{i e^{\left(1+i\right) x}}{4}-\frac{e^{\left(1-i\right) x}}{4}+\frac{i e^{\left(1-i\right) x}}{4}$	36

input
```
int(exp(x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)*(-cos(x)+sin(x))
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sin(x)\, dx = -\frac{1}{2} \cos(x)\, e^x + \frac{1}{2}\, e^x \sin(x)$$

input
```
integrate(exp(x)*sin(x),x, algorithm="fricas")
```

output
```
-1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

3.85.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int e^x \sin(x)\, dx = \frac{e^x \sin(x)}{2} - \frac{e^x \cos(x)}{2}$$

input
```
integrate(exp(x)*sin(x),x)
```

output
```
exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\, dx = -\frac{1}{2}\, (\cos(x) - \sin(x))e^x$$

input
```
integrate(exp(x)*sin(x),x, algorithm="maxima")
```

output
```
-1/2*(cos(x) - sin(x))*e^x
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{1}{2}\,(\cos(x) - \sin(x))e^x$$

input
```
integrate(exp(x)*sin(x),x, algorithm="giac")
```

output
```
-1/2*(cos(x) - sin(x))*e^x
```

3.85.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x \sin(x)\,dx = -\frac{e^x\,(\cos(x) - \sin(x))}{2}$$

input
```
int(exp(x)*sin(x),x)
```

output
```
-(exp(x)*(cos(x) - sin(x)))/2
```

3.86 $\int \frac{1}{x} dx$

3.86.1 Optimal result

Integrand size = 3, antiderivative size = 2

$$\int \frac{1}{x} dx = \log(x)$$

output
```
ln(x)
```

3.86.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} dx = \log(x)$$

input
```
Integrate[x^(-1),x]
```

output
```
Log[x]
```

3.86.3 Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x} \, dx$$
$$\Big\downarrow \text{14}$$
$$\log(x)$$

input
```
Int[x^(-1),x]
```

output
```
Log[x]
```

3.86.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

3.86.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\ln(x)$	3
norman	$\ln(x)$	3
risch	$\ln(x)$	3
parallelrisch	$\ln(x)$	3

input
```
int(1/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} \, dx = \log(x)$$

input
```
integrate(1/x,x, algorithm="fricas")
```

output
```
log(x)
```

3.86.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} \, dx = \log(x)$$

input
```
integrate(1/x,x)
```

output
```
log(x)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x} \, dx = \log(x)$$

input
```
integrate(1/x,x, algorithm="maxima")
```

output
```
log(x)
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

$$\int \frac{1}{x}\,dx = \log\left(|x|\right)$$

input
```
integrate(1/x,x, algorithm="giac")
```

output
```
log(abs(x))
```

3.86.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{x}\,dx = \ln\left(x\right)$$

input
```
int(1/x,x)
```

output
```
log(x)
```

3.87 $\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt$

3.87.1 Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt = -\frac{1}{12}\log(\cos(t)-\sin(t))-\frac{1}{4}\log(\cos(t)+\sin(t)) + \frac{1}{3}\log(2\cos(t)+\sin(t))-\frac{1}{2(1+\tan(t))}$$

output
```
-1/12*ln(cos(t)-sin(t))-1/4*ln(cos(t)+sin(t))+1/3*ln(2*cos(t)+sin(t))-1/2/
(1+tan(t))
```

3.87.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt = -\frac{\cos(t)(\log(\cos(t)-\sin(t))+3\log(\cos(t)+\sin(t))-4\log(2\cos(t)+\sin(t)))+(-6+\log(\cos(t)-\sin(}{12(\cos(t)+\sin(t))}$$

input
```
Integrate[Sec[2*t]/(1 + Sec[t]^2 + 3*Tan[t]),t]
```

output
```
-1/12*(Cos[t]*(Log[Cos[t] - Sin[t]] + 3*Log[Cos[t] + Sin[t]] - 4*Log[2*Cos
[t] + Sin[t]]) + (-6 + Log[Cos[t] - Sin[t]] + 3*Log[Cos[t] + Sin[t]] - 4*L
og[2*Cos[t] + Sin[t]])*Sin[t])/(Cos[t] + Sin[t])
```

3.87.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 4889, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(2t)}{3\tan(t)+\sec^2(t)+1}dt$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sec(2t)}{3\tan(t)+\sec(t)^2+1}dt$$
$$\Big\downarrow \text{ 4889}$$
$$\int \frac{1}{(\tan(t)+1)^2\left(-\tan^2(t)-\tan(t)+2\right)}d\tan(t)$$
$$\Big\downarrow \text{ 1141}$$
$$-\int \left(\frac{1}{4(\tan(t)+1)}-\frac{1}{3(\tan(t)+2)}-\frac{1}{2(\tan(t)+1)^2}-\frac{1}{12(1-\tan(t))}\right)d\tan(t)$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{1}{2(\tan(t)+1)}-\frac{1}{12}\log(1-\tan(t))-\frac{1}{4}\log(\tan(t)+1)+\frac{1}{3}\log(\tan(t)+2)$$

input
```
Int[Sec[2*t]/(1 + Sec[t]^2 + 3*Tan[t]),t]
```

output
```
-1/12*Log[1 - Tan[t]] - Log[1 + Tan[t]]/4 + Log[2 + Tan[t]]/3 - 1/(2*(1 +
Tan[t]))
```

3.87.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.87.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\ln(\tan(t)+2)}{3}-\frac{1}{2(1+\tan(t))}-\frac{\ln(1+\tan(t))}{4}-\frac{\ln(\tan(t)-1)}{12}$	31
risch	$-\frac{1}{2(\mathrm{e}^{2it}+i)}-\frac{\ln(\mathrm{e}^{2it}-i)}{12}+\frac{\ln(\mathrm{e}^{2it}+\frac{3}{5}+\frac{4i}{5})}{3}-\frac{\ln(\mathrm{e}^{2it}+i)}{4}$	48

input
```
int(sec(2*t)/(1+sec(t)^2+3*tan(t)),t,method=_RETURNVERBOSE)
```

output
```
1/3*ln(tan(t)+2)-1/2/(1+tan(t))-1/4*ln(1+tan(t))-1/12*ln(tan(t)-1)
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58

$$\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt$$
$$=\frac{4(\cos(t)+\sin(t))\log\left(\frac{3}{4}\cos(t)^2+\cos(t)\sin(t)+\frac{1}{4}\right)-3(\cos(t)+\sin(t))\log(2\cos(t)\sin(t)+1)-(\mathrm{c}}{24(\cos(t)+\sin(t))}$$

input
```
integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="fricas")
```

output
```
1/24*(4*(cos(t) + sin(t))*log(3/4*cos(t)^2 + cos(t)*sin(t) + 1/4) - 3*(cos
(t) + sin(t))*log(2*cos(t)*sin(t) + 1) - (cos(t) + sin(t))*log(-2*cos(t)*s
in(t) + 1) - 6*cos(t) + 6*sin(t))/(cos(t) + sin(t))
```

3.87.6 Sympy [F]

$$\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt = \int \frac{\sec(2t)}{3\tan(t)+\sec^2(t)+1}\,dt$$

input
```
integrate(sec(2*t)/(1+sec(t)**2+3*tan(t)),t)
```

output
```
Integral(sec(2*t)/(3*tan(t) + sec(t)**2 + 1), t)
```

3.87.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(37) = 74$.

Time = 0.36 (sec) , antiderivative size = 256, normalized size of antiderivative = 5.69

$$\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt$$
$$= \frac{3\left(\cos(2t)^2+\sin(2t)^2+2\sin(2t)+1\right)\log\left(953674316406250\,(3\cos(2t)+\sin(2t)+4)\cos(4t)+238\right.}{}$$

input
```
integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="maxima")
```

output
```
1/48*(3*(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)*log(953674316406250*(3*
cos(2*t) + sin(2*t) + 4)*cos(4*t) + 2384185791015625*cos(4*t)^2 + 95367431
6406250*cos(2*t)^2 - 953674316406250*(cos(2*t) - 3*sin(2*t) + 3)*sin(4*t)
+ 2384185791015625*sin(4*t)^2 + 953674316406250*sin(2*t)^2 + 2861022949218
750*cos(2*t) - 953674316406250*sin(2*t) + 2384185791015625) - 6*(cos(2*t)^
2 + sin(2*t)^2 + 2*sin(2*t) + 1)*log(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t)
+ 1) + 5*(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)*log(1/5*(5*cos(2*t)^2
+ 5*sin(2*t)^2 + 6*cos(2*t) + 8*sin(2*t) + 5)/(cos(2*t)^2 + sin(2*t)^2 - 2
*sin(2*t) + 1)) - 24*cos(2*t))/(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt = -\frac{1}{2(\tan(t)+1)} + \frac{1}{3}\log(|\tan(t)+2|) - \frac{1}{4}\log(|\tan(t)+1|) - \frac{1}{12}\log(|\tan(t)-1|)$$

input
```
integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="giac")
```

output
```
-1/2/(tan(t) + 1) + 1/3*log(abs(tan(t) + 2)) - 1/4*log(abs(tan(t) + 1)) -
1/12*log(abs(tan(t) - 1))
```

3.87.9 Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

$$\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt = \frac{\ln(\tan(t)+2)}{3} - \frac{\ln(\tan(t)+1)}{4} - \frac{\ln(\tan(t)-1)}{12} - \frac{1}{2(\tan(t)+1)}$$

input
```
int(1/(cos(2*t)*(3*tan(t) + 1/cos(t)^2 + 1)),t)
```

output
```
log(tan(t) + 2)/3 - log(tan(t) + 1)/4 - log(tan(t) - 1)/12 - 1/(2*(tan(t)
+ 1))
```

3.88 $\int \cos^2(x)\,dx$

3.88.1 Optimal result

Integrand size = 4, antiderivative size = 14

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x)$$

output
```
1/2*x+1/2*cos(x)*sin(x)
```

3.88.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{1}{4}\sin(2x)$$

input
```
Integrate[Cos[x]^2,x]
```

output
```
x/2 + Sin[2*x]/4
```

3.88.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \; 3115$$

$$\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)$$

$$\Big\downarrow \; 24$$

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[Cos[x]^2,x]
```

output
```
x/2 + (Cos[x]*Sin[x])/2
```

3.88.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.88.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
parallelrisch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}-\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

input
```
int(1/sec(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*cos(x)*sin(x)
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

input
```
integrate(1/sec(x)^2,x, algorithm="fricas")
```

output
```
1/2*cos(x)*sin(x) + 1/2*x
```

3.88.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

input
```
integrate(1/sec(x)**2,x)
```

output
```
x/2 + sin(x)*cos(x)/2
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{1}{2}\,x + \frac{1}{4}\,\sin{(2\,x)}$$

input
```
integrate(1/sec(x)^2,x, algorithm="maxima")
```

output
```
1/2*x + 1/4*sin(2*x)
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \cos^2(x)\,dx = \frac{1}{2}\,x + \frac{\tan{(x)}}{2\,(\tan{(x)}^2 + 1)}$$

input
```
integrate(1/sec(x)^2,x, algorithm="giac")
```

output
```
1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

3.88.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin{(2\,x)}}{4}$$

input
```
int(cos(x)^2,x)
```

output
```
x/2 + sin(2*x)/4
```

3.89 $\int \frac{1+x^2}{\sqrt{x}}\,dx$

3.89.1 Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1+x^2}{\sqrt{x}}\,dx = 2\sqrt{x}+\frac{2x^{5/2}}{5}$$

output
```
2/5*x^(5/2)+2*x^(1/2)
```

3.89.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1+x^2}{\sqrt{x}}\,dx = \frac{2}{5}\sqrt{x}\left(5+x^2\right)$$

input
```
Integrate[(1 + x^2)/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(5 + x^2))/5
```

3.89.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{\sqrt{x}}\,dx$$

$$\Big\downarrow \ 244$$

$$\int \left(x^{3/2}+\frac{1}{\sqrt{x}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2x^{5/2}}{5}+2\sqrt{x}$$

input
```
Int[(1 + x^2)/Sqrt[x],x]
```

output
```
2*Sqrt[x] + (2*x^(5/2))/5
```

3.89.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.89.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{2\sqrt{x}\,(x^2+5)}{5}$	11
risch	$\frac{2\sqrt{x}\,(x^2+5)}{5}$	11
derivativedivides	$\frac{2x^{\frac{5}{2}}}{5}+2\sqrt{x}$	12
default	$\frac{2x^{\frac{5}{2}}}{5}+2\sqrt{x}$	12
trager	$\left(\frac{2x^2}{5}+2\right)\sqrt{x}$	12

input
```
int((x^2+1)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^(1/2)*(x^2+5)
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{1+x^2}{\sqrt{x}}\,dx = \frac{2}{5}\left(x^2+5\right)\sqrt{x}$$

input
```
integrate((x^2+1)/x^(1/2),x, algorithm="fricas")
```

output
```
2/5*(x^2 + 5)*sqrt(x)
```

3.89.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1+x^2}{\sqrt{x}}\,dx = \frac{2x^{\frac{5}{2}}}{5}+2\sqrt{x}$$

input
```
integrate((x**2+1)/x**(1/2),x)
```

output
```
2*x**(5/2)/5 + 2*sqrt(x)
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1+x^2}{\sqrt{x}}\,dx = \frac{2}{5}\,x^{\frac{5}{2}} + 2\,\sqrt{x}$$

input
```
integrate((x^2+1)/x^(1/2),x, algorithm="maxima")
```

output
```
2/5*x^(5/2) + 2*sqrt(x)
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1+x^2}{\sqrt{x}}\,dx = \frac{2}{5}\,x^{\frac{5}{2}} + 2\,\sqrt{x}$$

input
```
integrate((x^2+1)/x^(1/2),x, algorithm="giac")
```

output
```
2/5*x^(5/2) + 2*sqrt(x)
```

3.89.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{1+x^2}{\sqrt{x}}\,dx = \frac{2\,\sqrt{x}\,(x^2+5)}{5}$$

input
```
int((x^2 + 1)/x^(1/2),x)
```

output
```
(2*x^(1/2)*(x^2 + 5))/5
```

3.90 $\int \frac{x}{\sqrt{5+2x+x^2}}\,dx$

3.90.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{x}{\sqrt{5+2x+x^2}}\,dx=\sqrt{5+2x+x^2}-\operatorname{arcsinh}\left(\frac{1+x}{2}\right)$$

output
```
-arcsinh(1/2+1/2*x)+(x^2+2*x+5)^(1/2)
```

3.90.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{x}{\sqrt{5+2x+x^2}}\,dx=\sqrt{5+2x+x^2}+\log\left(-1-x+\sqrt{5+2x+x^2}\right)$$

input
```
Integrate[x/Sqrt[5 + 2*x + x^2],x]
```

output
```
Sqrt[5 + 2*x + x^2] + Log[-1 - x + Sqrt[5 + 2*x + x^2]]
```

3.90.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^2+2x+5}}dx$$

$$\Big\downarrow \text{ 1160}$$

$$\sqrt{x^2+2x+5}-\int \frac{1}{\sqrt{x^2+2x+5}}dx$$

$$\Big\downarrow \text{ 1090}$$

$$\sqrt{x^2+2x+5}-\frac{1}{4}\int \frac{1}{\sqrt{\frac{1}{16}(2x+2)^2+1}}d(2x+2)$$

$$\Big\downarrow \text{ 222}$$

$$\sqrt{x^2+2x+5}-\operatorname{arcsinh}\left(\frac{1}{4}(2x+2)\right)$$

input
```
Int[x/Sqrt[5 + 2*x + x^2],x]
```

output
```
Sqrt[5 + 2*x + x^2] - ArcSinh[(2 + 2*x)/4]
```

3.90.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

3.90.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$-\operatorname{arcsinh}\left(\frac{1}{2}+\frac{x}{2}\right)+\sqrt{x^{2}+2x+5}$	20
risch	$-\operatorname{arcsinh}\left(\frac{1}{2}+\frac{x}{2}\right)+\sqrt{x^{2}+2x+5}$	20
trager	$\sqrt{x^{2}+2x+5}-\ln\left(1+x+\sqrt{x^{2}+2x+5}\right)$	28

input
```
int(x/(x^2+2*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arcsinh(1/2+1/2*x)+(x^2+2*x+5)^(1/2)
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x}{\sqrt{5+2x+x^2}}\,dx=\sqrt{x^2+2\,x+5}+\log\left(-x+\sqrt{x^2+2\,x+5}-1\right)$$

input
```
integrate(x/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x^2 + 2*x + 5) + log(-x + sqrt(x^2 + 2*x + 5) - 1)
```

3.90.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{5+2x+x^2}}\,dx = \sqrt{x^2+2x+5} - \operatorname{asinh}\left(\frac{x}{2}+\frac{1}{2}\right)$$

input
```
integrate(x/(x**2+2*x+5)**(1/2),x)
```

output
```
sqrt(x**2 + 2*x + 5) - asinh(x/2 + 1/2)
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{5+2x+x^2}}\,dx = \sqrt{x^2+2\,x+5} - \operatorname{arsinh}\left(\frac{1}{2}\,x+\frac{1}{2}\right)$$

input
```
integrate(x/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 + 2*x + 5) - arcsinh(1/2*x + 1/2)
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x}{\sqrt{5+2x+x^2}}\,dx = \sqrt{x^2+2\,x+5} + \log\left(-x+\sqrt{x^2+2\,x+5}-1\right)$$

input
```
integrate(x/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

output
```
sqrt(x^2 + 2*x + 5) + log(-x + sqrt(x^2 + 2*x + 5) - 1)
```

3.90.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x}{\sqrt{5+2x+x^2}}\,dx = \sqrt{x^2+2\,x+5} - \ln\left(x+\sqrt{x^2+2\,x+5}+1\right)$$

input
```
int(x/(2*x + x^2 + 5)^(1/2),x)
```

output
```
(2*x + x^2 + 5)^(1/2) - log(x + (2*x + x^2 + 5)^(1/2) + 1)
```

3.91 $\int \cos(x)\sin^2(x)\,dx$

3.91.1 Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \cos(x)\sin^2(x)\,dx = \frac{\sin^3(x)}{3}$$

output
```
1/3*sin(x)^3
```

3.91.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin^2(x)\,dx = \frac{\sin^3(x)}{3}$$

input
```
Integrate[Cos[x]*Sin[x]^2,x]
```

output
```
Sin[x]^3/3
```

3.91.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x)\cos(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(x)^2\cos(x)dx$$

$$\Big\downarrow \; 3044$$

$$\int \sin^2(x)d\sin(x)$$

$$\Big\downarrow \; 15$$

$$\frac{\sin^3(x)}{3}$$

input
```
Int[Cos[x]*Sin[x]^2,x]
```

output
```
Sin[x]^3/3
```

3.91.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.91.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\sin^3(x))}{3}$	7
default	$\frac{(\sin^3(x))}{3}$	7
risch	$\frac{\sin(x)}{4} - \frac{\sin(3x)}{12}$	12
parallelrisch	$\frac{\sin(x)}{4} - \frac{\sin(3x)}{12}$	12
norman	$\frac{8(\tan^3(\frac{x}{2}))}{3(1+\tan^2(\frac{x}{2}))^3}$	19

input
```
int(sin(x)^2*cos(x),x,method=_RETURNVERBOSE)
```

output
```
1/3*sin(x)^3
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \cos(x) \sin^2(x) \, dx = -\frac{1}{3} \left(\cos(x)^2 - 1\right) \sin(x)$$

input
```
integrate(cos(x)*sin(x)^2,x, algorithm="fricas")
```

output
```
-1/3*(cos(x)^2 - 1)*sin(x)
```

3.91.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \cos(x) \sin^2(x) \, dx = \frac{\sin^3(x)}{3}$$

input
```
integrate(cos(x)*sin(x)**2,x)
```

output
```
sin(x)**3/3
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin^2(x)\,dx = \frac{1}{3}\sin{(x)}^3$$

input
```
integrate(cos(x)*sin(x)^2,x, algorithm="maxima")
```

output
```
1/3*sin(x)^3
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin^2(x)\,dx = \frac{1}{3}\sin{(x)}^3$$

input
```
integrate(cos(x)*sin(x)^2,x, algorithm="giac")
```

output
```
1/3*sin(x)^3
```

3.91.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin^2(x)\,dx = \frac{\sin{(x)}^3}{3}$$

input
```
int(cos(x)*sin(x)^2,x)
```

output
```
sin(x)^3/3
```

3.92 $\int \frac{e^x}{1+e^x} \, dx$

3.92.1 Optimal result

Integrand size = 11, antiderivative size = 6

$$\int \frac{e^x}{1+e^x} \, dx = \log \left(1+e^x\right)$$

output
```
ln(1+exp(x))
```

3.92.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{1+e^x} \, dx = \log \left(1+e^x\right)$$

input
```
Integrate[E^x/(1 + E^x),x]
```

output
```
Log[1 + E^x]
```

3.92.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2676, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{e^x+1}dx$$

$$\Big\downarrow \ 2676$$

$$\int \frac{1}{e^x+1}de^x$$

$$\Big\downarrow \ 16$$

$$\log(e^x+1)$$

input
```
Int[E^x/(1 + E^x),x]
```

output
```
Log[1 + E^x]
```

3.92.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

3.92.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\ln(1+e^x)$	6
default	$\ln(1+e^x)$	6
norman	$\ln(1+e^x)$	6
risch	$\ln(1+e^x)$	6
parallelrisch	$\ln(1+e^x)$	6

input
```
int(1/(1+exp(x))*exp(x),x,method=_RETURNVERBOSE)
```

output
```
ln(1+exp(x))
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x}\,dx = \log(e^x+1)$$

input
```
integrate(exp(x)/(1+exp(x)),x, algorithm="fricas")
```

output
```
log(e^x + 1)
```

3.92.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x}\,dx = \log(e^x+1)$$

input
```
integrate(exp(x)/(1+exp(x)),x)
```

output
```
log(exp(x) + 1)
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x}\,dx = \log\left(e^x+1\right)$$

input
```
integrate(exp(x)/(1+exp(x)),x, algorithm="maxima")
```

output
```
log(e^x + 1)
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x}\,dx = \log\left(e^x+1\right)$$

input
```
integrate(exp(x)/(1+exp(x)),x, algorithm="giac")
```

output
```
log(e^x + 1)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{1+e^x}\,dx = \ln\left(\mathrm{e}^x+1\right)$$

input
```
int(exp(x)/(exp(x) + 1),x)
```

output
```
log(exp(x) + 1)
```

3.93 $\int \frac{e^{2x}}{1+e^x} dx$

3.93.1 Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{e^{2x}}{1+e^x} dx = e^x - \log(1+e^x)$$

output
```
exp(x)-ln(1+exp(x))
```

3.93.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{1+e^x} dx = e^x - \log(1+e^x)$$

input
```
Integrate[E^(2*x)/(1 + E^x),x]
```

output
```
E^x - Log[1 + E^x]
```

3.93.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x}}{e^x+1}dx$$

$$\Big\downarrow \text{2678}$$

$$\int \frac{e^x}{e^x+1}de^x$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{1}{-e^x-1}+1\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$e^x-\log \left(e^x+1\right)$$

input
```
Int[E^(2*x)/(1 + E^x),x]
```

output
```
E^x - Log[1 + E^x]
```

3.93.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

3.93.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$e^{x}-\ln \left(1+e^{x}\right)$	11
norman	$e^{x}-\ln \left(1+e^{x}\right)$	11
risch	$e^{x}-\ln \left(1+e^{x}\right)$	11

input
```
int(exp(2*x)/(1+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
exp(x)-ln(1+exp(x))
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x+1\right)$$

input
```
integrate(exp(2*x)/(1+exp(x)),x, algorithm="fricas")
```

output
```
e^x - log(e^x + 1)
```

3.93.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x+1\right)$$

input
```
integrate(exp(2*x)/(1+exp(x)),x)
```

output
```
exp(x) - log(exp(x) + 1)
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x+1\right)$$

input
```
integrate(exp(2*x)/(1+exp(x)),x, algorithm="maxima")
```

output
```
e^x - log(e^x + 1)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x+1\right)$$

input
```
integrate(exp(2*x)/(1+exp(x)),x, algorithm="giac")
```

output
```
e^x - log(e^x + 1)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x} \, dx = \mathrm{e}^x - \ln\left(\mathrm{e}^x + 1\right)$$

input
```
int(exp(2*x)/(exp(x) + 1),x)
```

output
```
exp(x) - log(exp(x) + 1)
```

3.94 $\int \frac{1}{1-\cos(x)}\,dx$

3.94.1 Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\sin(x)}{1-\cos(x)}$$

output
```
-sin(x)/(1-cos(x))
```

3.94.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx = -\cot\left(\frac{x}{2}\right)$$

input
```
Integrate[(1 - Cos[x])^(-1),x]
```

output
```
-Cot[x/2]
```

3.94.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cos(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{ 3127}$$
$$-\frac{\sin(x)}{1-\cos(x)}$$

input
```
Int[(1 - Cos[x])^(-1),x]
```

output
```
-(Sin[x]/(1 - Cos[x]))
```

3.94.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

3.94.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
norman	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
parallelrisch	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
risch	$-\frac{2i}{e^{ix}-1}$	13

input
```
int(1/(1-cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/tan(1/2*x)
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\cos\left(x\right)+1}{\sin\left(x\right)}$$

input
```
integrate(1/(1-cos(x)),x, algorithm="fricas")
```

output
```
-(cos(x) + 1)/sin(x)
```

3.94.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(1-cos(x)),x)
```

output
```
-1/tan(x/2)
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\cos(x)+1}{\sin(x)}$$

input
```
integrate(1/(1-cos(x)),x, algorithm="maxima")
```

output
```
-(cos(x) + 1)/sin(x)
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

input
```
integrate(1/(1-cos(x)),x, algorithm="giac")
```

output
```
-1/tan(1/2*x)
```

3.94.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{1-\cos(x)}\,dx = -\cot\left(\frac{x}{2}\right)$$

input
```
int(-1/(cos(x) - 1),x)
```

output
```
-cot(x/2)
```

3.95 $\int \sec^2(x)\tan(x)\,dx$

3.95.1 Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \sec^2(x)\tan(x)\,dx = \frac{\sec^2(x)}{2}$$

output
```
1/2*sec(x)^2
```

3.95.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\tan(x)\,dx = \frac{\sec^2(x)}{2}$$

input
```
Integrate[Sec[x]^2*Tan[x],x]
```

output
```
Sec[x]^2/2
```

3.95.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\sec^2(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(x)\sec(x)^2dx$$

$$\Big\downarrow \text{ 3086}$$

$$\int \sec(x)d\sec(x)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\sec^2(x)}{2}$$

input
```
Int[Sec[x]^2*Tan[x],x]
```

output
```
Sec[x]^2/2
```

3.95.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.95.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{(\sec^2(x))}{2}$	7
default	$\frac{(\sec^2(x))}{2}$	7
risch	$\frac{2\,e^{2ix}}{(e^{2ix}+1)^2}$	17

input
```
int(sec(x)^2*tan(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sec(x)^2
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2\,\cos(x)^2}$$

input
```
integrate(sec(x)^2*tan(x),x, algorithm="fricas")
```

output
```
1/2/cos(x)^2
```

3.95.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2\cos^2(x)}$$

input
```
integrate(sec(x)**2*tan(x),x)
```

output
```
1/(2*cos(x)**2)
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2}\tan{(x)}^2$$

input
```
integrate(sec(x)^2*tan(x),x, algorithm="maxima")
```

output
```
1/2*tan(x)^2
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2\cos{(x)}^2}$$

input
```
integrate(sec(x)^2*tan(x),x, algorithm="giac")
```

output
```
1/2/cos(x)^2
```

3.95.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sec^2(x)\tan(x)\,dx = \frac{\tan(x)^2}{2}$$

input
```
int(tan(x)/cos(x)^2,x)
```

output
```
tan(x)^2/2
```

3.96 $\int x \log(x) \, dx$

3.96.1 Optimal result

Integrand size = 4, antiderivative size = 17

$$\int x \log(x) \, dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

output
```
-1/4*x^2+1/2*x^2*ln(x)
```

3.96.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x \log(x) \, dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

input
```
Integrate[x*Log[x],x]
```

output
```
-1/4*x^2 + (x^2*Log[x])/2
```

3.96.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\log(x)\,dx$$

$$\Big\downarrow \text{ 2741}$$

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

input
```
Int[x*Log[x],x]
```

output
```
-1/4*x^2 + (x^2*Log[x])/2
```

3.96.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

3.96.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
norman	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
risch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
parallelrisch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
parts	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14

input
```
int(x*ln(x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2+1/2*x^2*ln(x)
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right) - \frac{1}{4}\,x^2$$

input
```
integrate(x*log(x),x, algorithm="fricas")
```

output
```
1/2*x^2*log(x) - 1/4*x^2
```

3.96.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x\log(x)\,dx = \frac{x^2\log\left(x\right)}{2} - \frac{x^2}{4}$$

input
```
integrate(x*ln(x),x)
```

output
```
x**2*log(x)/2 - x**2/4
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right) - \frac{1}{4}\,x^2$$

input
```
integrate(x*log(x),x, algorithm="maxima")
```

output
```
1/2*x^2*log(x) - 1/4*x^2
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\log(x)\,dx = \frac{1}{2}\,x^2\log\left(x\right) - \frac{1}{4}\,x^2$$

input
```
integrate(x*log(x),x, algorithm="giac")
```

output
```
1/2*x^2*log(x) - 1/4*x^2
```

3.96.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.53

$$\int x\log(x)\,dx = \frac{x^2\,\left(\ln\left(x\right) - \frac{1}{2}\right)}{2}$$

input
```
int(x*log(x),x)
```

output
```
(x^2*(log(x) - 1/2))/2
```

3.97 $\int \cos(x)\sin(x)\,dx$

3.97.1 Optimal result

Integrand size = 5, antiderivative size = 8

$$\int \cos(x)\sin(x)\,dx = \frac{\sin^2(x)}{2}$$

output
```
1/2*sin(x)^2
```

3.97.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos^2(x)$$

input
```
Integrate[Cos[x]*Sin[x],x]
```

output
```
-1/2*Cos[x]^2
```

3.97.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\cos(x)dx$$

$$\Big\downarrow \text{3044}$$

$$\int \sin(x)d\sin(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{\sin^2(x)}{2}$$

input
```
Int[Cos[x]*Sin[x],x]
```

output
```
Sin[x]^2/2
```

3.97.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.97.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\left(\sin^{2}\left(x\right)\right)}{2}$	7
default	$\frac{\left(\sin^{2}\left(x\right)\right)}{2}$	7
risch	$-\frac{\cos\left(2x\right)}{4}$	7
parallelrisch	$\frac{1}{4}-\frac{\cos\left(2x\right)}{4}$	9
norman	$\frac{2\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}}$	19
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(2x\right)}{\sqrt{\pi}}\right)}{4}$	19

input
```
int(cos(x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x)^2
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\,\cos\left(x\right)^{2}$$

input
```
integrate(cos(x)*sin(x),x, algorithm="fricas")
```

output
```
-1/2*cos(x)^2
```

3.97.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \cos(x)\sin(x)\,dx = \frac{\sin^2(x)}{2}$$

input
```
integrate(cos(x)*sin(x),x)
```

output
```
sin(x)**2/2
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos(x)^2$$

input
```
integrate(cos(x)*sin(x),x, algorithm="maxima")
```

output
```
-1/2*cos(x)^2
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = -\frac{1}{2}\cos(x)^2$$

input
```
integrate(cos(x)*sin(x),x, algorithm="giac")
```

output
```
-1/2*cos(x)^2
```

3.97.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \cos(x)\sin(x)\,dx = \frac{\sin{(x)}^2}{2}$$

input
```
int(cos(x)*sin(x),x)
```

output
```
sin(x)^2/2
```

3.98 $\int \frac{1+x}{\sqrt{2x-x^2}} \, dx$

3.98.1 Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \frac{1+x}{\sqrt{2x-x^2}} \, dx = -\sqrt{2x-x^2} - 2\arcsin(1-x)$$

output
```
2*arcsin(-1+x)-(-x^2+2*x)^(1/2)
```

3.98.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \frac{1+x}{\sqrt{2x-x^2}} \, dx = \frac{(-2+x)x - 4\sqrt{-2+x}\sqrt{x}\log\left(\sqrt{-2+x}-\sqrt{x}\right)}{\sqrt{-((-2+x)x)}}$$

input
```
Integrate[(1 + x)/Sqrt[2*x - x^2],x]
```

output
```
((-2 + x)*x - 4*Sqrt[-2 + x]*Sqrt[x]*Log[Sqrt[-2 + x] - Sqrt[x]])/Sqrt[-((
-2 + x)*x)]
```

3.98.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1160, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{\sqrt{2x-x^2}}dx$$
$$\Big\downarrow \text{1160}$$
$$2\int \frac{1}{\sqrt{2x-x^2}}dx-\sqrt{2x-x^2}$$
$$\Big\downarrow \text{1090}$$
$$-\int \frac{1}{\sqrt{1-\frac{1}{4}(2-2x)^2}}d(2-2x)-\sqrt{2x-x^2}$$
$$\Big\downarrow \text{223}$$
$$-2\arcsin\left(\frac{1}{2}(2-2x)\right)-\sqrt{2x-x^2}$$

input
```
Int[(1 + x)/Sqrt[2*x - x^2],x]
```

output
```
-Sqrt[2*x - x^2] - 2*ArcSin[(2 - 2*x)/2]
```

3.98.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

3.98.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$2\arcsin\left(-1+x\right)-\sqrt{-x^{2}+2x}$	21
risch	$\frac{x(-2+x)}{\sqrt{-x(-2+x)}}+2\arcsin\left(-1+x\right)$	21
pseudoelliptic	$-\sqrt{-x\left(-2+x\right)}-4\arctan\left(\frac{\sqrt{-x(-2+x)}}{x}\right)$	27
trager	$-\sqrt{-x^{2}+2x}+2\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+2x}+x-1\right)$	45
meijerg	$2\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)+\frac{2i\left(\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{1-\frac{x}{2}}}{2}-i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)\right)}{\sqrt{\pi}}$	54

input
```
int((1+x)/(-x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsin(-1+x)-(-x^2+2*x)^(1/2)
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{-x^2+2\,x}-4\,\arctan\left(\frac{\sqrt{-x^2+2\,x}}{x}\right)$$

input
```
integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-x^2 + 2*x) - 4*arctan(sqrt(-x^2 + 2*x)/x)
```

3.98.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{-x^2+2x}+2\operatorname{asin}(x-1)$$

input
```
integrate((1+x)/(-x**2+2*x)**(1/2),x)
```

output
```
-sqrt(-x**2 + 2*x) + 2*asin(x - 1)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{-x^2+2\,x}-2\,\arcsin(-x+1)$$

input
```
integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 2*x) - 2*arcsin(-x + 1)
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{-x^2+2\,x}+2\,\arcsin(x-1)$$

input
```
integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 2*x) + 2*arcsin(x - 1)
```

3.98.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1+x}{\sqrt{2x-x^2}}\,dx = 2\,\mathrm{asin}(x-1) - \sqrt{2\,x-x^2}$$

input
```
int((x + 1)/(2*x - x^2)^(1/2),x)
```

output
```
2*asin(x - 1) - (2*x - x^2)^(1/2)
```

3.99 $\int \frac{2e^x}{2+3e^{2x}}\,dx$

3.99.1 Optimal result

Integrand size = 16, antiderivative size = 20

$$\int \frac{2e^x}{2+3e^{2x}}\,dx = \sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}e^x\right)$$

output
```
1/3*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

3.99.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{2e^x}{2+3e^{2x}}\,dx = \sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}e^x\right)$$

input
```
Integrate[(2*E^x)/(2 + 3*E^(2*x)),x]
```

output
```
Sqrt[2/3]*ArcTan[Sqrt[3/2]*E^x]
```

3.99.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {27, 2679, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2e^x}{3e^{2x}+2} \, dx$$
$$\Big\downarrow \ 27$$
$$2\int \frac{e^x}{2+3e^{2x}}dx$$
$$\Big\downarrow \ 2679$$
$$2\int \frac{1}{2+3e^{2x}}de^x$$
$$\Big\downarrow \ 216$$
$$\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}e^x\right)$$

input
```
Int[(2*E^x)/(2 + 3*E^(2*x)),x]
```

output
```
Sqrt[2/3]*ArcTan[Sqrt[3/2]*E^x]
```

3.99.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

3.99.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\arctan\left(\frac{e^{x}\sqrt{6}}{2}\right)\sqrt{6}}{3}$	14
risch	$\frac{i\sqrt{6}\,\ln\left(e^{x}+\frac{i\sqrt{6}}{3}\right)}{6}-\frac{i\sqrt{6}\,\ln\left(e^{x}-\frac{i\sqrt{6}}{3}\right)}{6}$	34

input
```
int(2*exp(x)/(2+3*exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{2e^x}{2+3e^{2x}}\,dx = \frac{1}{3}\,\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{3}\sqrt{2}e^x\right)$$

input
```
integrate(2*exp(x)/(2+3*exp(2*x)),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*e^x)
```

3.99.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{2e^x}{2+3e^{2x}}\,dx = \operatorname{RootSum}\left(6z^2+1,\left(i\mapsto i\log\left(2i+e^x\right)\right)\right)$$

input
```
integrate(2*exp(x)/(2+3*exp(2*x)),x)
```

output
```
RootSum(6*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{2e^x}{2+3e^{2x}}\,dx = \frac{1}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^x\right)$$

input
```
integrate(2*exp(x)/(2+3*exp(2*x)),x, algorithm="maxima")
```

output
```
1/3*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{2e^x}{2+3e^{2x}}\,dx = \frac{1}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^x\right)$$

input
```
integrate(2*exp(x)/(2+3*exp(2*x)),x, algorithm="giac")
```

output
```
1/3*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

3.99.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{2e^x}{2+3e^{2x}}\,dx = \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,\mathrm{e}^x}{2}\right)}{3}$$

input
```
int((2*exp(x))/(3*exp(2*x) + 2),x)
```

output
```
(6^(1/2)*atan((6^(1/2)*exp(x))/2))/3
```

3.100 $\int \frac{x^4}{(1-x^2)^{5/2}}\,dx$

3.100.1 Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \arcsin(x)$$

output
```
1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

3.100.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{x(-3+4x^2)}{3(1-x^2)^{3/2}} + 2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

input
```
Integrate[x^4/(1 - x^2)^(5/2),x]
```

output
```
(x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

3.100.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {252, 252, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{252}$$
$$\frac{x^3}{3(1-x^2)^{3/2}}-\int \frac{x^2}{(1-x^2)^{3/2}}dx$$
$$\Big\downarrow \text{252}$$
$$\int \frac{1}{\sqrt{1-x^2}}dx-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3(1-x^2)^{3/2}}$$
$$\Big\downarrow \text{223}$$
$$\arcsin(x)-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3(1-x^2)^{3/2}}$$

input
```
Int[x^4/(1 - x^2)^(5/2),x]
```

output
```
x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

3.100.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

3.100.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x^{3}}{3\left(-x^{2}+1\right)^{\frac{3}{2}}}+\arcsin\left(x\right)-\frac{x}{\sqrt{-x^{2}+1}}$	30
risch	$-\frac{x\left(4x^{2}-3\right)}{3\left(x^{2}-1\right)\sqrt{-x^{2}+1}}+\arcsin\left(x\right)$	30
meijerg	$-\frac{2i\left(-\frac{i\sqrt{\pi}\,x\left(-20x^{2}+15\right)}{10\left(-x^{2}+1\right)^{\frac{3}{2}}}+\frac{3i\sqrt{\pi}\,\arcsin\left(x\right)}{2}\right)}{3\sqrt{\pi}}$	39
pseudoelliptic	$\frac{\left(x^{2}-1\right)\sqrt{-x^{2}+1}\,\arctan\left(\frac{\sqrt{-x^{2}+1}}{x}\right)+\frac{4x^{3}}{3}-x}{\left(-x^{2}+1\right)^{\frac{3}{2}}}$	49
trager	$\frac{\left(4x^{2}-3\right)x\sqrt{-x^{2}+1}}{3\left(x^{2}-1\right)^{2}}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)$	54

input
```
int(x^4/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-1/(-x^2+1)^(1/2)*x
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(29) = 58.

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int \frac{x^4}{\left(1-x^2\right)^{5/2}}\,dx=-\frac{6\left(x^{4}-2\,x^{2}+1\right)\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)-\left(4\,x^{3}-3\,x\right)\sqrt{-x^{2}+1}}{3\left(x^{4}-2\,x^{2}+1\right)}$$

input
```
integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*s
qrt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

3.100.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(26) = 52.

Time = 0.40 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.00

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{3x^4 \operatorname{asin}(x)}{3x^4-6x^2+3} + \frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3} - \frac{6x^2 \operatorname{asin}(x)}{3x^4-6x^2+3} - \frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3} + \frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

input
```
integrate(x**4/(-x**2+1)**(5/2),x)
```

output
```
3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x
**2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x*
*4 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{1}{3}\,x\left(\frac{3\,x^2}{(-x^2+1)^{\frac{3}{2}}} - \frac{2}{(-x^2+1)^{\frac{3}{2}}}\right) - \frac{x}{3\,\sqrt{-x^2+1}} + \arcsin(x)$$

input
```
integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
```

output
```
1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
 + arcsin(x)
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{(4\,x^2-3)\sqrt{-x^2+1}x}{3\,(x^2-1)^2} + \arcsin(x)$$

input
```
integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

output
```
1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

3.100.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.60

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \operatorname{asin}(x) + \frac{3\sqrt{1-x^2}}{4\,(x-1)} + \frac{3\sqrt{1-x^2}}{4\,(x+1)} - \sqrt{1-x^2}\left(\frac{1}{12\,(x-1)} - \frac{1}{12\,(x-1)^2}\right) - \sqrt{1-x^2}\left(\frac{1}{12\,(x+1)} + \frac{1}{12\,(x+1)^2}\right)$$

input
```
int(x^4/(1 - x^2)^(5/2),x)
```

output
```
asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1)
) - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*
(1/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

3.101 $\int \frac{e^{6x}}{1+e^{4x}}\,dx$

3.101.1 Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{e^{6x}}{1+e^{4x}}\,dx = \frac{e^{2x}}{2} - \frac{1}{2}\arctan\left(e^{2x}\right)$$

output
```
1/2*exp(2*x)-1/2*arctan(exp(2*x))
```

3.101.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{e^{6x}}{1+e^{4x}}\,dx = \frac{1}{2}\left(e^{2x} - \arctan\left(e^{2x}\right)\right)$$

input
```
Integrate[E^(6*x)/(1 + E^(4*x)),x]
```

output
```
(E^(2*x) - ArcTan[E^(2*x)])/2
```

3.101.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2678, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{6x}}{e^{4x}+1}dx$$
$$\Big\downarrow \text{2678}$$
$$\frac{1}{2}\int \frac{e^{4x}}{1+e^{4x}}de^{2x}$$
$$\Big\downarrow \text{262}$$
$$\frac{1}{2}\left(e^{2x}-\int \frac{1}{1+e^{4x}}de^{2x}\right)$$
$$\Big\downarrow \text{216}$$
$$\frac{1}{2}\left(e^{2x}-\arctan\left(e^{2x}\right)\right)$$

input
```
Int[E^(6*x)/(1 + E^(4*x)),x]
```

output
```
(E^(2*x) - ArcTan[E^(2*x)])/2
```

3.101.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

3.101.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{e^{2x}}{2}-\frac{\arctan\left(e^{2x}\right)}{2}$	15
risch	$\frac{e^{2x}}{2}+\frac{i\ln\left(e^{2x}-i\right)}{4}-\frac{i\ln\left(e^{2x}+i\right)}{4}$	30

input
```
int(exp(6*x)/(exp(4*x)+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)^2-1/2*arctan(exp(x)^2)
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{e^{6x}}{1+e^{4x}}\,dx = -\frac{1}{2}\arctan\left(e^{(2x)}\right)+\frac{1}{2}e^{(2x)}$$

input
```
integrate(exp(6*x)/(1+exp(4*x)),x, algorithm="fricas")
```

output
```
-1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

3.101.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{e^{6x}}{1+e^{4x}}\,dx = \frac{e^{2x}}{2} + \operatorname{RootSum}\left(16z^2+1,\left(i\mapsto i\log\left(-4i+e^{2x}\right)\right)\right)$$

input
```
integrate(exp(6*x)/(1+exp(4*x)),x)
```

output
```
exp(2*x)/2 + RootSum(16*_z**2 + 1, Lambda(_i, _i*log(-4*_i + exp(2*x))))
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{e^{6x}}{1+e^{4x}}\,dx = -\frac{1}{2}\arctan\left(e^{(2x)}\right) + \frac{1}{2}e^{(2x)}$$

input
```
integrate(exp(6*x)/(1+exp(4*x)),x, algorithm="maxima")
```

output
```
-1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{e^{6x}}{1+e^{4x}}\,dx = -\frac{1}{2}\arctan\left(e^{(2x)}\right) + \frac{1}{2}e^{(2x)}$$

input
```
integrate(exp(6*x)/(1+exp(4*x)),x, algorithm="giac")
```

output
```
-1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

3.101.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{e^{6x}}{1+e^{4x}}\,dx = \frac{\mathrm{e}^{2\,x}}{2} - \frac{\operatorname{atan}(\mathrm{e}^{2\,x})}{2}$$

input
```
int(exp(6*x)/(exp(4*x) + 1),x)
```

output
```
exp(2*x)/2 - atan(exp(2*x))/2
```

3.102 $\int \log(2+3x^2)\,dx$

3.102.1 Optimal result

Integrand size = 8, antiderivative size = 33

$$\int \log(2+3x^2)\,dx = -2x + 2\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}x\right) + x\log(2+3x^2)$$

output
```
-2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)
```

3.102.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \log(2+3x^2)\,dx = -2x + 2\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}x\right) + x\log(2+3x^2)$$

input
```
Integrate[Log[2 + 3*x^2],x]
```

output
```
-2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

3.102.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2898, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(3x^2+2\right)dx$$

$$\Big\downarrow \text{ 2898}$$

$$x\log\left(3x^2+2\right)-6\int \frac{x^2}{3x^2+2}dx$$

$$\Big\downarrow \text{ 262}$$

$$x\log\left(3x^2+2\right)-6\left(\frac{x}{3}-\frac{2}{3}\int \frac{1}{3x^2+2}dx\right)$$

$$\Big\downarrow \text{ 216}$$

$$x\log\left(3x^2+2\right)-6\left(\frac{x}{3}-\frac{1}{3}\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}x\right)\right)$$

input
```
Int[Log[2 + 3*x^2],x]
```

output
```
-6*(x/3 - (Sqrt[2/3]*ArcTan[Sqrt[3/2]*x])/3) + x*Log[2 + 3*x^2]
```

3.102.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

3.102.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
default	$-2x+x\ln\left(3x^{2}+2\right)+\frac{2\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$	27
risch	$-2x+x\ln\left(3x^{2}+2\right)+\frac{2\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$	27
parts	$-2x+x\ln\left(3x^{2}+2\right)+\frac{2\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$	27

input
```
int(ln(3*x^2+2),x,method=_RETURNVERBOSE)
```

output
```
-2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \log\left(2+3x^{2}\right)\,dx=\frac{2}{3}\,\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{3}\sqrt{2}x\right)+x\log\left(3\,x^{2}+2\right)-2\,x$$

input
```
integrate(log(3*x^2+2),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x) + x*log(3*x^2 + 2) - 2*x
```

3.102.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \log \left(2+3x^2\right)\,dx = x\log \left(3x^2+2\right)-2x+\frac{2\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}x}{2}\right)}{3}$$

input
```
integrate(ln(3*x**2+2),x)
```

output
```
x*log(3*x**2 + 2) - 2*x + 2*sqrt(6)*atan(sqrt(6)*x/2)/3
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \log \left(2+3x^2\right)\,dx = x\log \left(3\,x^2+2\right)+\frac{2}{3}\,\sqrt{6}\arctan \left(\frac{1}{2}\,\sqrt{6}x\right)-2\,x$$

input
```
integrate(log(3*x^2+2),x, algorithm="maxima")
```

output
```
x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \log \left(2+3x^2\right)\,dx = x\log \left(3\,x^2+2\right)+\frac{2}{3}\,\sqrt{6}\arctan \left(\frac{1}{2}\,\sqrt{6}x\right)-2\,x$$

input
```
integrate(log(3*x^2+2),x, algorithm="giac")
```

output
```
x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

3.102.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \log \left(2+3x^2\right)\,dx=\frac{2\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{2}\right)}{3}-2\,x+x\,\ln\left(3\,x^2+2\right)$$

input
```
int(log(3*x^2 + 2),x)
```

output
```
(2*6^(1/2)*atan((6^(1/2)*x)/2))/3 - 2*x + x*log(3*x^2 + 2)
```

3.103 $\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx$

3.103.1 Optimal result

Integrand size = 20, antiderivative size = 21

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx=\frac{x}{r\sqrt{-a^2+2Hr^2}}$$

output
```
x/r/(2*H*r^2-a^2)^(1/2)
```

3.103.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx=\frac{x}{r\sqrt{-a^2+2Hr^2}}$$

input
```
Integrate[1/(r*Sqrt[-a^2 + 2*H*r^2]),x]
```

output
```
x/(r*Sqrt[-a^2 + 2*H*r^2])
```

3.103.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{r\sqrt{2Hr^2-a^2}}\,dx$$
$$\Big\downarrow \; 24$$
$$\frac{x}{r\sqrt{2Hr^2-a^2}}$$

input
```
Int[1/(r*Sqrt[-a^2 + 2*H*r^2]),x]
```

output
```
x/(r*Sqrt[-a^2 + 2*H*r^2])
```

3.103.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

3.103.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{x}{r\sqrt{2H\,r^2-a^2}}$	20
norman	$\frac{x}{r\sqrt{2H\,r^2-a^2}}$	20
parallelrisch	$\frac{x}{r\sqrt{2H\,r^2-a^2}}$	20

input
```
int(1/r/(2*H*r^2-a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x/r/(2*H*r^2-a^2)^(1/2)
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx = \frac{\sqrt{2\,Hr^2-a^2}x}{2\,Hr^3-a^2r}$$

input
```
integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(2*H*r^2 - a^2)*x/(2*H*r^3 - a^2*r)
```

3.103.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx = \frac{x}{r\sqrt{2Hr^2-a^2}}$$

input
```
integrate(1/r/(2*H*r**2-a**2)**(1/2),x)
```

output
```
x/(r*sqrt(2*H*r**2 - a**2))
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx = \frac{x}{\sqrt{2\,Hr^2-a^2}r}$$

input
```
integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="maxima")
```

output
```
x/(sqrt(2*H*r^2 - a^2)*r)
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx = \frac{x}{\sqrt{2\,Hr^2-a^2r}}$$

input
```
integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="giac")
```

output
```
x/(sqrt(2*H*r^2 - a^2)*r)
```

3.103.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx = \frac{x}{r\,\sqrt{2\,H\,r^2-a^2}}$$

input
```
int(1/(r*(2*H*r^2 - a^2)^(1/2)),x)
```

output
```
x/(r*(2*H*r^2 - a^2)^(1/2))
```

3.104 $\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx$

3.104.1 Optimal result

Integrand size = 25, antiderivative size = 26

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx=\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

output
```
x/r/(2*H*r^2-a^2-e^2)^(1/2)
```

3.104.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx=\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

input
```
Integrate[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2]),x]
```

output
```
x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2])
```

3.104.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx$$
$$\Big\downarrow \; 24$$
$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

input
```
Int[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2]),x]
```

output
```
x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2])
```

3.104.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

3.104.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{x}{r\sqrt{2H\,r^2-a^2-e^2}}$	25
norman	$\frac{x}{r\sqrt{2H\,r^2-a^2-e^2}}$	25
parallelrisch	$\frac{x}{r\sqrt{2H\,r^2-a^2-e^2}}$	25

input
```
int(1/r/(2*H*r^2-a^2-e^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x/r/(2*H*r^2-a^2-e^2)^(1/2)
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx = \frac{\sqrt{2\,Hr^2-a^2-e^2}x}{2\,Hr^3-(a^2+e^2)r}$$

input
```
integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(2*H*r^2 - a^2 - e^2)*x/(2*H*r^3 - (a^2 + e^2)*r)
```

3.104.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx = \frac{x}{r\sqrt{2Hr^2-a^2-e^2}}$$

input
```
integrate(1/r/(2*H*r**2-a**2-e**2)**(1/2),x)
```

output
```
x/(r*sqrt(2*H*r**2 - a**2 - e**2))
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx = \frac{x}{\sqrt{2\,Hr^2-a^2-e^2}r}$$

input
```
integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")
```

output
```
x/(sqrt(2*H*r^2 - a^2 - e^2)*r)
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx = \frac{x}{\sqrt{2\,Hr^2-a^2-e^2}r}$$

input
```
integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="giac")
```

output
```
x/(sqrt(2*H*r^2 - a^2 - e^2)*r)
```

3.104.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx = \frac{x}{r\,\sqrt{-a^2-e^2+2\,H\,r^2}}$$

input
```
int(1/(r*(2*H*r^2 - a^2 - e^2)^(1/2)),x)
```

output
```
x/(r*(2*H*r^2 - a^2 - e^2)^(1/2))
```

3.105 $\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx$

3.105.1 Optimal result

Integrand size = 26, antiderivative size = 27

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

output
```
x/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2)
```

3.105.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

input
```
Integrate[1/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4]),x]
```

output
```
x/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4])
```

3.105.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx$$
$$\Big\downarrow \; 24$$
$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

input
```
Int[1/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4]),x]
```

output
```
x/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4])
```

3.105.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

3.105.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{x}{r\sqrt{-2K\,r^{4}+2H\,r^{2}-a^{2}}}$	26
norman	$\frac{x}{r\sqrt{-2K\,r^{4}+2H\,r^{2}-a^{2}}}$	26
parallelrisch	$\frac{x}{r\sqrt{-2K\,r^{4}+2H\,r^{2}-a^{2}}}$	26

input
```
int(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2)
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx = -\frac{\sqrt{-2\,Kr^4+2\,Hr^2-a^2}x}{2\,Kr^5-2\,Hr^3+a^2r}$$

input
```
integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*x/(2*K*r^5 - 2*H*r^3 + a^2*r)
```

3.105.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\sqrt{2Hr^2-2Kr^4-a^2}}$$

input
```
integrate(1/r/(-2*K*r**4+2*H*r**2-a**2)**(1/2),x)
```

output
```
x/(r*sqrt(2*H*r**2 - 2*K*r**4 - a**2))
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2}r}$$

input
```
integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="maxima")
```

output
```
x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*r)
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx=\frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2}r}$$

input
```
integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="giac")
```

output
```
x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*r)
```

3.105.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx=\frac{x}{r\,\sqrt{-a^2-2\,K\,r^4+2\,H\,r^2}}$$

input
```
int(1/(r*(2*H*r^2 - 2*K*r^4 - a^2)^(1/2)),x)
```

output
```
x/(r*(2*H*r^2 - 2*K*r^4 - a^2)^(1/2))
```

3.106 $\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx$

3.106.1 Optimal result

Integrand size = 31, antiderivative size = 32

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

output
```
x/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

input
```
Integrate[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4]),x]
```

output
```
x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4])
```

3.106.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx$$
$$\Big\downarrow \; 24$$
$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

input
```
Int[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4]),x]
```

output
```
x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4])
```

3.106.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

3.106.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{x}{r\sqrt{-2K\,r^{4}+2H\,r^{2}-a^{2}-e^{2}}}$	31
norman	$\frac{x}{r\sqrt{-2K\,r^{4}+2H\,r^{2}-a^{2}-e^{2}}}$	31
parallelrisch	$\frac{x}{r\sqrt{-2K\,r^{4}+2H\,r^{2}-a^{2}-e^{2}}}$	31

input
```
int(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2)
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = -\frac{\sqrt{-2\,Kr^4+2\,Hr^2-a^2-e^2}x}{2\,Kr^5-2\,Hr^3+(a^2+e^2)r}$$

input
```
integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*x/(2*K*r^5 - 2*H*r^3 + (a^2 + e^2)*r
)
```

3.106.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\sqrt{2Hr^2-2Kr^4-a^2-e^2}}$$

input
```
integrate(1/r/(-2*K*r**4+2*H*r**2-a**2-e**2)**(1/2),x)
```

output
```
x/(r*sqrt(2*H*r**2 - 2*K*r**4 - a**2 - e**2))
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2-e^2}r}$$

input
```
integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")
```

output
```
x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2-e^2}r}$$

input
```
integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="giac")
```

output
```
x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r)
```

3.106.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\,\sqrt{-a^2-e^2-2\,K\,r^4+2\,H\,r^2}}$$

input
```
int(1/(r*(2*H*r^2 - 2*K*r^4 - a^2 - e^2)^(1/2)),x)
```

output
```
x/(r*(2*H*r^2 - 2*K*r^4 - a^2 - e^2)^(1/2))
```

3.107 $\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx$

3.107.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

output
```
x/r/(-a^2-2*r*(-H*r+K))^(1/2)
```

3.107.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2-2Kr+2Hr^2}}$$

input
```
Integrate[1/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2]),x]
```

output
```
x/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2])
```

3.107.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{24\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr}}\,dx$$
$$\Big\downarrow \; 24$$
$$\frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

input
```
Int[1/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2]),x]
```

output
```
x/(r*Sqrt[-a^2 - 2*r*(K - H*r)])
```

3.107.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

3.107.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{x}{r\sqrt{2H\,r^2-2Kr-a^2}}$	24
norman	$\frac{x}{r\sqrt{2H\,r^2-2Kr-a^2}}$	24
parallelrisch	$\frac{x}{r\sqrt{2H\,r^2-2Kr-a^2}}$	24

input
```
int(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/r/(2*H*r^2-2*K*r-a^2)^(1/2)*x
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx = \frac{\sqrt{2\,Hr^2-a^2-2\,Kr}x}{2\,Hr^3-a^2r-2\,Kr^2}$$

input
```
integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(2*H*r^2 - a^2 - 2*K*r)*x/(2*H*r^3 - a^2*r - 2*K*r^2)
```

3.107.6 Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\sqrt{2Hr^2-2Kr-a^2}}$$

input
```
integrate(1/r/(2*H*r**2-2*K*r-a**2)**(1/2),x)
```

output
```
x/(r*sqrt(2*H*r**2 - 2*K*r - a**2))
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx = \frac{x}{\sqrt{2\,Hr^2-a^2-2\,Kr}r}$$

input
```
integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="maxima")
```

output
```
x/(sqrt(2*H*r^2 - a^2 - 2*K*r)*r)
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx = \frac{x}{\sqrt{2\,Hr^2-a^2-2\,Kr}r}$$

input
```
integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="giac")
```

output
```
x/(sqrt(2*H*r^2 - a^2 - 2*K*r)*r)
```

3.107.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\,\sqrt{-a^2+2\,H\,r^2-2\,K\,r}}$$

input
```
int(1/(r*(2*H*r^2 - 2*K*r - a^2)^(1/2)),x)
```

output
```
x/(r*(2*H*r^2 - 2*K*r - a^2)^(1/2))
```

3.108 $\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx$

3.108.1 Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2-e^2-2r(K-Hr)}}$$

output
```
x/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}$$

input
```
Integrate[1/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]),x]
```

output
```
x/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2])
```

3.108.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr}}dx$$
$$\Big\downarrow \text{ 24}$$
$$\frac{x}{r\sqrt{-a^2-e^2-2r(K-Hr)}}$$

input
```
Int[1/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]),x]
```

output
```
x/(r*Sqrt[-a^2 - e^2 - 2*r*(K - H*r)])
```

3.108.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

3.108.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{x}{r\sqrt{2H\,r^2-2Kr-a^2-e^2}}$	29
norman	$\frac{x}{r\sqrt{2H\,r^2-2Kr-a^2-e^2}}$	29
parallelrisch	$\frac{x}{r\sqrt{2H\,r^2-2Kr-a^2-e^2}}$	29

input
```
int(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.72

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{\sqrt{2\,Hr^2-a^2-e^2-2\,Kr}x}{2\,Hr^3-2\,Kr^2-(a^2+e^2)r}$$

input
```
integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)*x/(2*H*r^3 - 2*K*r^2 - (a^2 + e^2)*r)
```

3.108.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\sqrt{2Hr^2-2Kr-a^2-e^2}}$$

input
```
integrate(1/r/(2*H*r**2-2*K*r-a**2-e**2)**(1/2),x)
```

output
```
x/(r*sqrt(2*H*r**2 - 2*K*r - a**2 - e**2))
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{x}{\sqrt{2\,Hr^2-a^2-e^2-2\,Kr}r}$$

input
```
integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="maxima")
```

output
```
x/(sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)*r)
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{x}{\sqrt{2\,Hr^2-a^2-e^2-2\,Krr}}$$

input
```
integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="giac")
```

output
```
x/(sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)*r)
```

3.108.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\,\sqrt{-a^2-e^2+2\,H\,r^2-2\,K\,r}}$$

input
```
int(1/(r*(2*H*r^2 - 2*K*r - a^2 - e^2)^(1/2)),x)
```

output
```
x/(r*(2*H*r^2 - 2*K*r - a^2 - e^2)^(1/2))
```

3.109 $\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx$

3.109.1 Optimal result

Integrand size = 18, antiderivative size = 19

$$\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx=\frac{rx}{\sqrt{-a^2+2er^2}}$$

output
```
r*x/(-a^2+2*exp(1)*r^2)^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx=\frac{rx}{\sqrt{-a^2+2er^2}}$$

input
```
Integrate[r/Sqrt[-a^2 + 2*E*r^2],x]
```

output
```
(r*x)/Sqrt[-a^2 + 2*E*r^2]
```

3.109.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{r}{\sqrt{2er^2-a^2}}\,dx$$

$$\Big\downarrow \; 24$$

$$\frac{rx}{\sqrt{2er^2-a^2}}$$

input
```
Int[r/Sqrt[-a^2 + 2*E*r^2],x]
```

output
```
(r*x)/Sqrt[-a^2 + 2*E*r^2]
```

3.109.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

3.109.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{rx}{\sqrt{-a^2+2\,\mathrm{e}r^2}}$	19
norman	$\frac{rx}{\sqrt{-a^2+2\,\mathrm{e}r^2}}$	19
parallelrisch	$\frac{rx}{\sqrt{-a^2+2\,\mathrm{e}r^2}}$	19

input
```
int(r/(-a^2+2*exp(1)*r^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
r*x/(-a^2+2*exp(1)*r^2)^(1/2)
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx = \frac{rx}{\sqrt{2\,r^2e-a^2}}$$

input
```
integrate(r/(-a^2+2*exp(1)*r^2)^(1/2),x, algorithm="fricas")
```

output
```
r*x/sqrt(2*r^2*e - a^2)
```

3.109.6 Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx = \frac{rx}{\sqrt{-a^2+2er^2}}$$

input
```
integrate(r/(-a**2+2*exp(1)*r**2)**(1/2),x)
```

output
```
r*x/sqrt(-a**2 + 2*E*r**2)
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx = \frac{rx}{\sqrt{2\,r^2e-a^2}}$$

input
```
integrate(r/(-a^2+2*exp(1)*r^2)^(1/2),x, algorithm="maxima")
```

output
```
r*x/sqrt(2*r^2*e - a^2)
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx=\frac{rx}{\sqrt{2\,r^2e-a^2}}$$

input
```
integrate(r/(-a^2+2*exp(1)*r^2)^(1/2),x, algorithm="giac")
```

output
```
r*x/sqrt(2*r^2*e - a^2)
```

3.109.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx=\frac{r\,x}{\sqrt{2\,r^2\,\mathrm{e}-a^2}}$$

input
```
int(r/(2*r^2*exp(1) - a^2)^(1/2),x)
```

output
```
(r*x)/(2*r^2*exp(1) - a^2)^(1/2)
```

3.110 $\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx$

3.110.1 Optimal result

Integrand size = 23, antiderivative size = 24

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx = \frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

output
```
r*x/(-a^2-e^2+2*exp(1)*r^2)^(1/2)
```

3.110.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx = \frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

input
```
Integrate[r/Sqrt[-a^2 - e^2 + 2*E*r^2],x]
```

output
```
(r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2]
```

3.110.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx$$
$$\Big\downarrow \; 24$$
$$\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

input
```
Int[r/Sqrt[-a^2 - e^2 + 2*E*r^2],x]
```

output
```
(r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2]
```

3.110.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

3.110.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{rx}{\sqrt{-a^2-e^2+2\,\mathrm{e}r^2}}$	24
norman	$\frac{rx}{\sqrt{-a^2-e^2+2\,\mathrm{e}r^2}}$	24
parallelrisch	$\frac{rx}{\sqrt{-a^2-e^2+2\,\mathrm{e}r^2}}$	24

input
```
int(r/(-a^2-e^2+2*exp(1)*r^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
r*x/(-a^2-e^2+2*exp(1)*r^2)^(1/2)
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx=\frac{rx}{\sqrt{2\,r^2e-a^2-e^2}}$$

input
```
integrate(r/(-a^2-e^2+2*exp(1)*r^2)^(1/2),x, algorithm="fricas")
```

output
```
r*x/sqrt(2*r^2*e - a^2 - e^2)
```

3.110.6 Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx=\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

input
```
integrate(r/(-a**2-e**2+2*exp(1)*r**2)**(1/2),x)
```

output
```
r*x/sqrt(-a**2 - e**2 + 2*E*r**2)
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx=\frac{rx}{\sqrt{2\,r^2e-a^2-e^2}}$$

input
```
integrate(r/(-a^2-e^2+2*exp(1)*r^2)^(1/2),x, algorithm="maxima")
```

output
```
r*x/sqrt(2*r^2*e - a^2 - e^2)
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2 - e^2 + 2er^2}}\, dx = \frac{rx}{\sqrt{2\, r^2 e - a^2 - e^2}}$$

input
```
integrate(r/(-a^2-e^2+2*exp(1)*r^2)^(1/2),x, algorithm="giac")
```

output
```
r*x/sqrt(2*r^2*e - a^2 - e^2)
```

3.110.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2 - e^2 + 2er^2}}\, dx = \frac{r\, x}{\sqrt{-a^2 - e^2 + 2\, \mathrm{e}\, r^2}}$$

input
```
int(r/(2*r^2*exp(1) - a^2 - e^2)^(1/2),x)
```

output
```
(r*x)/(2*r^2*exp(1) - a^2 - e^2)^(1/2)
```

3.111 $\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx$

3.111.1 Optimal result

Integrand size = 24, antiderivative size = 25

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx=\frac{rx}{\sqrt{-a^2+2er^2-2Kr^4}}$$

output
```
r*x/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2)
```

3.111.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx=\frac{rx}{\sqrt{-a^2+2er^2-2Kr^4}}$$

input
```
Integrate[r/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4],x]
```

output
```
(r*x)/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4]
```

3.111.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{24\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{r}{\sqrt{-a^2-2Kr^4+2er^2}}\,dx$$
$$\Big\downarrow 24$$
$$\frac{rx}{\sqrt{-a^2-2Kr^4+2er^2}}$$

input
```
Int[r/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4],x]
```

output
```
(r*x)/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4]
```

3.111.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

3.111.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{rx}{\sqrt{-a^2+2\,\mathrm{e}r^2-2K\,r^4}}$	25
norman	$\frac{rx}{\sqrt{-a^2+2\,\mathrm{e}r^2-2K\,r^4}}$	25
parallelrisch	$\frac{rx}{\sqrt{-a^2+2\,\mathrm{e}r^2-2K\,r^4}}$	25

input
```
int(r/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
r*x/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2)
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx = -\frac{\sqrt{-2\,Kr^4+2\,r^2e-a^2}rx}{2\,Kr^4-2\,r^2e+a^2}$$

input
```
integrate(r/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-2*K*r^4 + 2*r^2*e - a^2)*r*x/(2*K*r^4 - 2*r^2*e + a^2)
```

3.111.6 Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx = \frac{rx}{\sqrt{-2Kr^4-a^2+2er^2}}$$

input
```
integrate(r/(-a**2+2*exp(1)*r**2-2*K*r**4)**(1/2),x)
```

output
```
r*x/sqrt(-2*K*r**4 - a**2 + 2*E*r**2)
```

3.111.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx = \frac{rx}{\sqrt{-2\,Kr^4+2\,r^2e-a^2}}$$

input
```
integrate(r/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="maxima")
```

output
```
r*x/sqrt(-2*K*r^4 + 2*r^2*e - a^2)
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx=\frac{rx}{\sqrt{-2\,Kr^4+2\,r^2e-a^2}}$$

input
```
integrate(r/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="giac")
```

output
```
r*x/sqrt(-2*K*r^4 + 2*r^2*e - a^2)
```

3.111.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx=\frac{r\,x}{\sqrt{-a^2-2\,K\,r^4+2\,\mathrm{e}\,r^2}}$$

input
```
int(r/(2*r^2*exp(1) - 2*K*r^4 - a^2)^(1/2),x)
```

output
```
(r*x)/(2*r^2*exp(1) - 2*K*r^4 - a^2)^(1/2)
```

3.112 $\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx$

3.112.1 Optimal result

Integrand size = 29, antiderivative size = 30

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx=\frac{rx}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}$$

output
```
r*x/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2)
```

3.112.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx=\frac{rx}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}$$

input
```
Integrate[r/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4],x]
```

output
```
(r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4]
```

3.112.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{r}{\sqrt{-a^2-e^2-2Kr^4+2er^2}}\,dx$$
$$\Big\downarrow \; 24$$
$$\frac{rx}{\sqrt{-a^2-e^2-2Kr^4+2er^2}}$$

input
```
Int[r/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4],x]
```

output
```
(r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4]
```

3.112.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

3.112.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{rx}{\sqrt{-a^{2}-e^{2}+2\,\mathrm{e}r^{2}-2K\,r^{4}}}$	30
norman	$\frac{rx}{\sqrt{-a^{2}-e^{2}+2\,\mathrm{e}r^{2}-2K\,r^{4}}}$	30
parallelrisch	$\frac{rx}{\sqrt{-a^{2}-e^{2}+2\,\mathrm{e}r^{2}-2K\,r^{4}}}$	30

input
```
int(r/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
r*x/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2)
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx = -\frac{\sqrt{-2\,Kr^4+2\,r^2e-a^2-e^2}rx}{2\,Kr^4-2\,r^2e+a^2+e^2}$$

input
```
integrate(r/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-2*K*r^4 + 2*r^2*e - a^2 - e^2)*r*x/(2*K*r^4 - 2*r^2*e + a^2 + e^2)
```

3.112.6 Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx = \frac{rx}{\sqrt{-2Kr^4-a^2-e^2+2er^2}}$$

input
```
integrate(r/(-a**2-e**2+2*exp(1)*r**2-2*K*r**4)**(1/2),x)
```

output
```
r*x/sqrt(-2*K*r**4 - a**2 - e**2 + 2*E*r**2)
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx = \frac{rx}{\sqrt{-2\,Kr^4+2\,r^2e-a^2-e^2}}$$

input
```
integrate(r/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="maxima")
```

output
```
r*x/sqrt(-2*K*r^4 + 2*r^2*e - a^2 - e^2)
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx = \frac{rx}{\sqrt{-2\,Kr^4+2\,r^2e-a^2-e^2}}$$

input
```
integrate(r/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="giac")
```

output
```
r*x/sqrt(-2*K*r^4 + 2*r^2*e - a^2 - e^2)
```

3.112.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx = \frac{r\,x}{\sqrt{-a^2-e^2-2\,K\,r^4+2\,\mathrm{e}\,r^2}}$$

input
```
int(r/(2*r^2*exp(1) - 2*K*r^4 - a^2 - e^2)^(1/2),x)
```

output
```
(r*x)/(2*r^2*exp(1) - 2*K*r^4 - a^2 - e^2)^(1/2)
```

3.113 $\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx$

3.113.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx=\frac{rx}{\sqrt{-a^2-e^2-2r(K-Hr)}}$$

output
```
r*x/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)
```

3.113.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx=\frac{rx}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}$$

input
```
Integrate[r/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2],x]
```

output
```
(r*x)/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]
```

3.113.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{r}{\sqrt{-a^2-e^2+2Hr^2-2Kr}}\,dx$$
$$\Big\downarrow \ 24$$
$$\frac{rx}{\sqrt{-a^2-e^2-2r(K-Hr)}}$$

input
```
Int[r/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2],x]
```

output
```
(r*x)/Sqrt[-a^2 - e^2 - 2*r*(K - H*r)]
```

3.113.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

3.113.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{rx}{\sqrt{2H\,r^2-2Kr-a^2-e^2}}$	27
norman	$\frac{rx}{\sqrt{2H\,r^2-2Kr-a^2-e^2}}$	27
parallelrisch	$\frac{rx}{\sqrt{2H\,r^2-2Kr-a^2-e^2}}$	27

input
```
int(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
r*x/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{rx}{\sqrt{2\,Hr^2-a^2-e^2-2\,Kr}}$$

input
```
integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="fricas")
```

output
```
r*x/sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)
```

3.113.6 Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{rx}{\sqrt{2Hr^2-2Kr-a^2-e^2}}$$

input
```
integrate(r/(2*H*r**2-2*K*r-a**2-e**2)**(1/2),x)
```

output
```
r*x/sqrt(2*H*r**2 - 2*K*r - a**2 - e**2)
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{rx}{\sqrt{2\,Hr^2-a^2-e^2-2\,Kr}}$$

input
```
integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="maxima")
```

output
```
r*x/sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{rx}{\sqrt{2\,Hr^2-a^2-e^2-2\,Kr}}$$

input
```
integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="giac")
```

output
```
r*x/sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)
```

3.113.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{r\,x}{\sqrt{-a^2-e^2+2\,H\,r^2-2\,K\,r}}$$

input
```
int(r/(2*H*r^2 - 2*K*r - a^2 - e^2)^(1/2),x)
```

output
```
(r*x)/(2*H*r^2 - 2*K*r - a^2 - e^2)^(1/2)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                 #is checked before calling the grading function that is passed.
                 #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```