Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.3-Miscellaneous/51-1.3.1-Rational-functions

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [494]. This is test number [51].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (494)	0.00 (0)
Rubi	99.39 (491)	0.61 (3)
Maple	98.99 (489)	1.01 (5)
Mupad	98.18 (485)	1.82 (9)
Fricas	92.31 (456)	7.69 (38)
Sympy	88.26 (436)	11.74 (58)
Giac	86.23 (426)	13.77 (68)
Maxima	82.59 (408)	17.41 (86)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

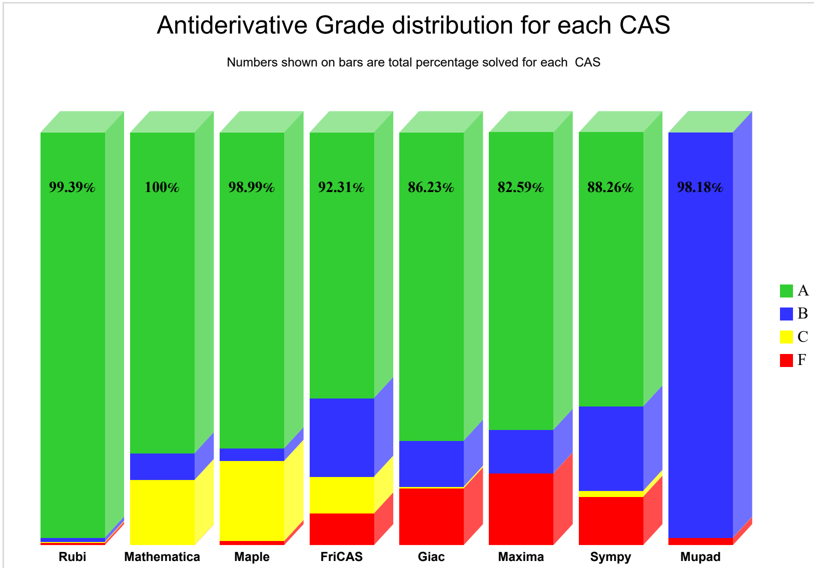
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

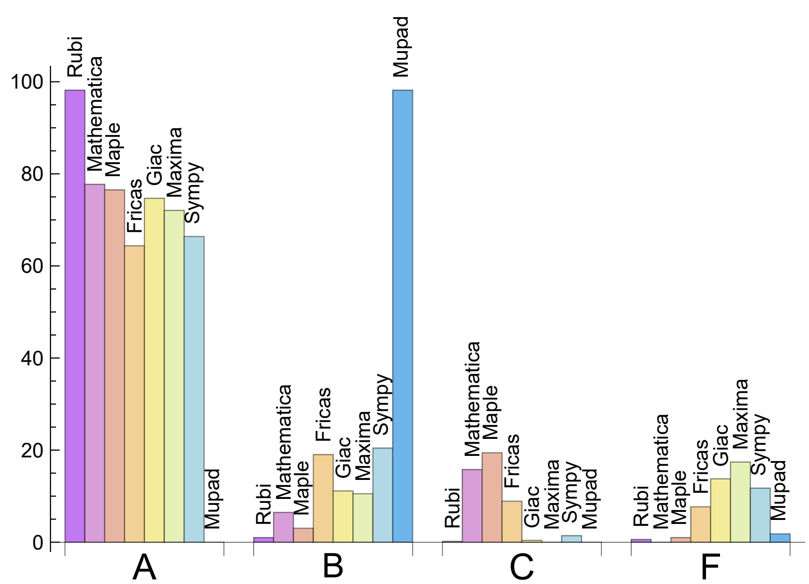
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.178	1.012	0.202	0.607
Mathematica	77.733	6.478	15.789	0.000
Maple	76.518	3.036	19.433	1.012
Giac	74.696	11.134	0.405	13.765
Maxima	72.065	10.526	0.000	17.409
Sympy	66.397	20.445	1.417	11.741
Fricas	64.372	19.028	8.907	7.692
Mupad	0.000	98.178	0.000	1.822

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	3	100.00	0.00	0.00
Maple	5	100.00	0.00	0.00
Mupad	9	0.00	100.00	0.00
Fricas	38	10.53	76.32	13.16
Sympy	58	8.62	91.38	0.00
Giac	68	98.53	0.00	1.47
Maxima	86	96.51	0.00	3.49

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Mathematica	0.05
Maxima	0.23
Rubi	0.36
Giac	0.47
Maple	0.49
Fricas	1.36
Sympy	1.47
Mupad	5.01

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	63.27	0.99	28.00	0.86
Mathematica	81.37	1.26	38.00	1.00
Maxima	116.03	1.79	28.00	0.88
Rubi	119.70	1.05	34.00	1.00
Giac	184.22	1.78	29.00	0.92
Sympy	198.72	2.51	39.00	0.91
Mupad	450.36	2.59	41.00	0.95
Fricas	14149.51	68.45	39.00	1.14

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

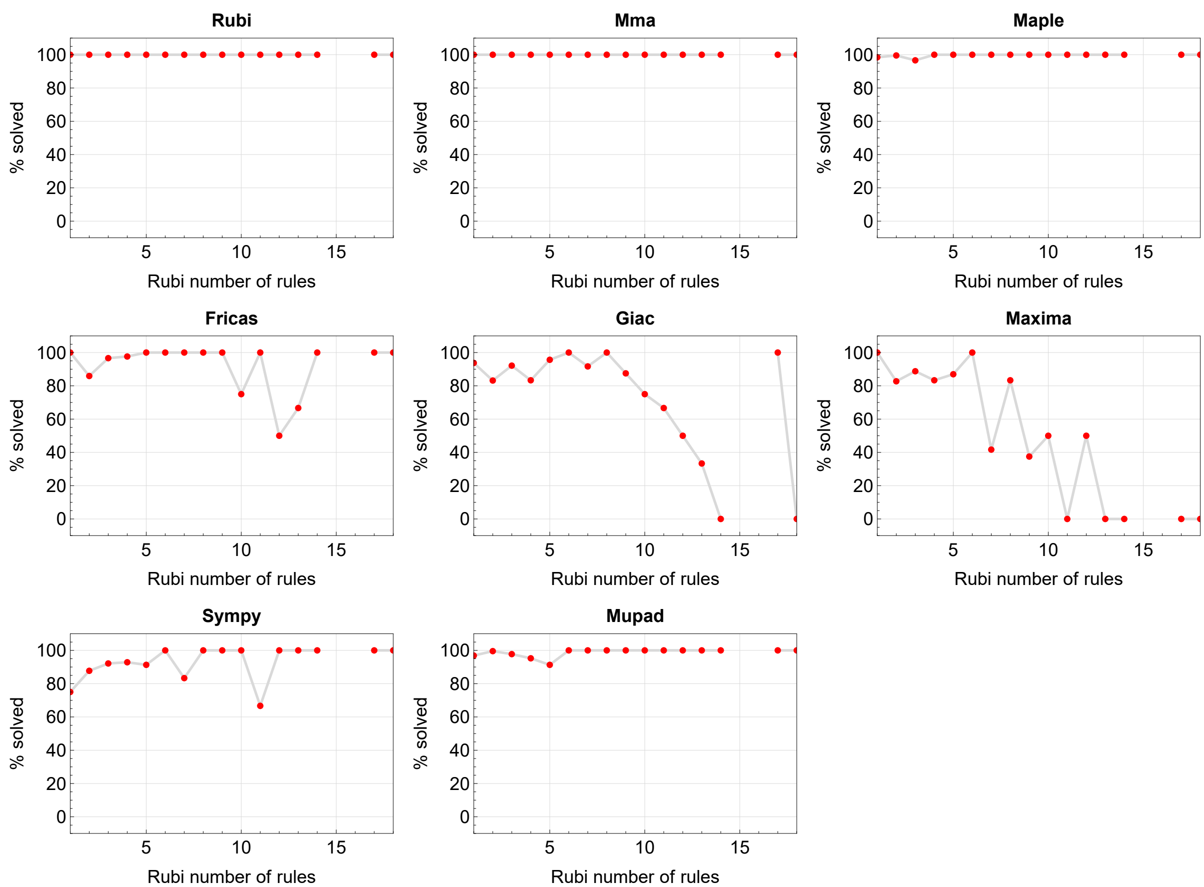


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

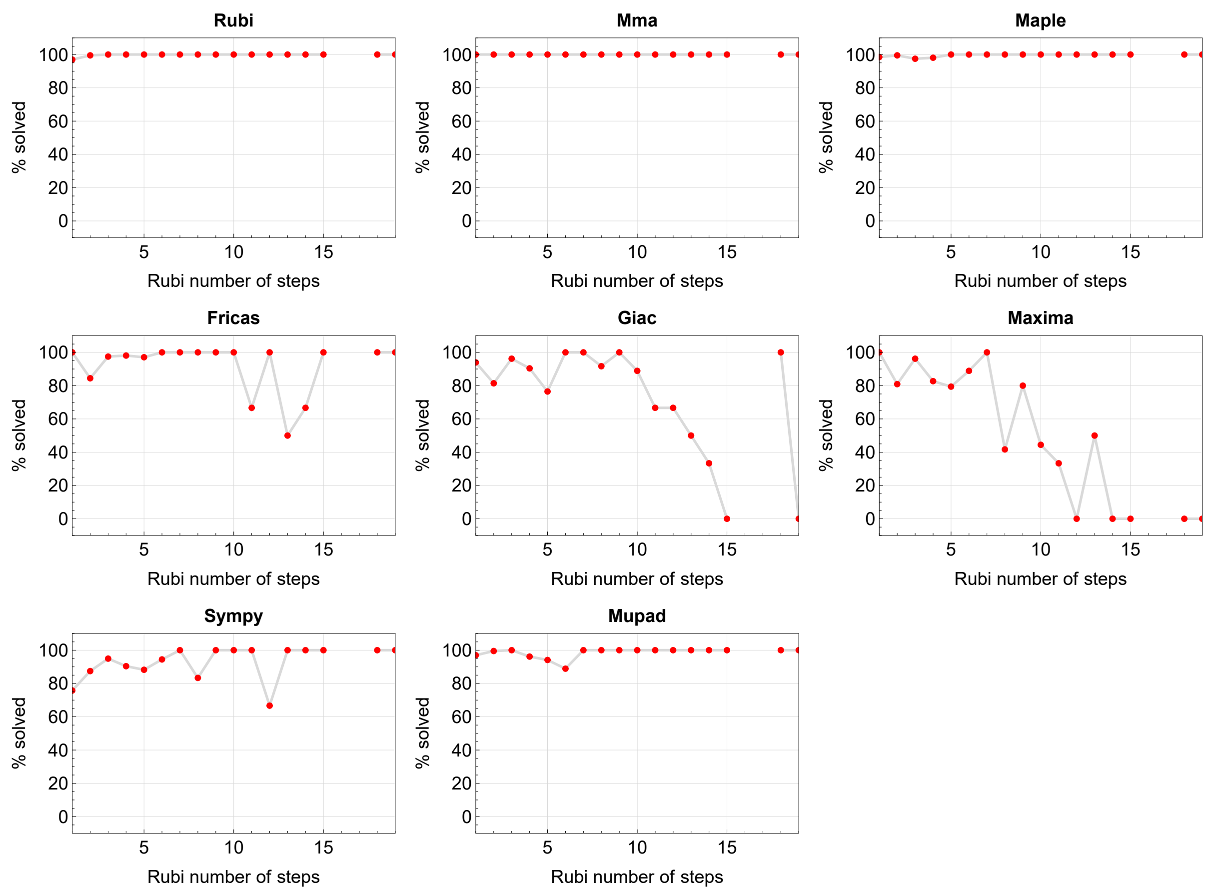


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

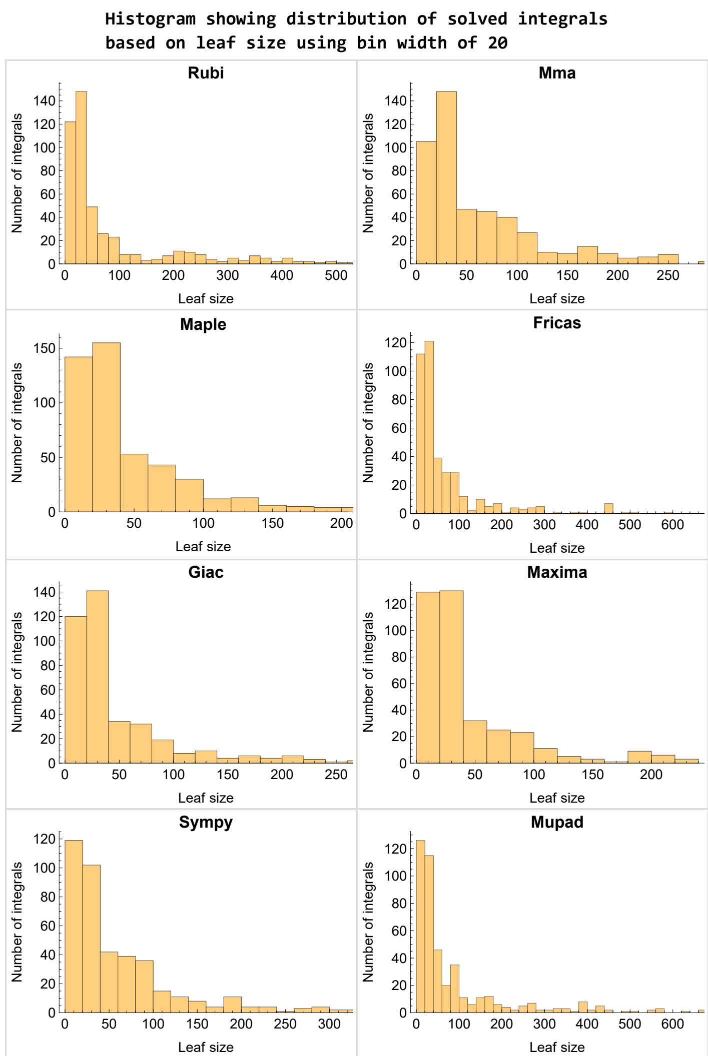


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

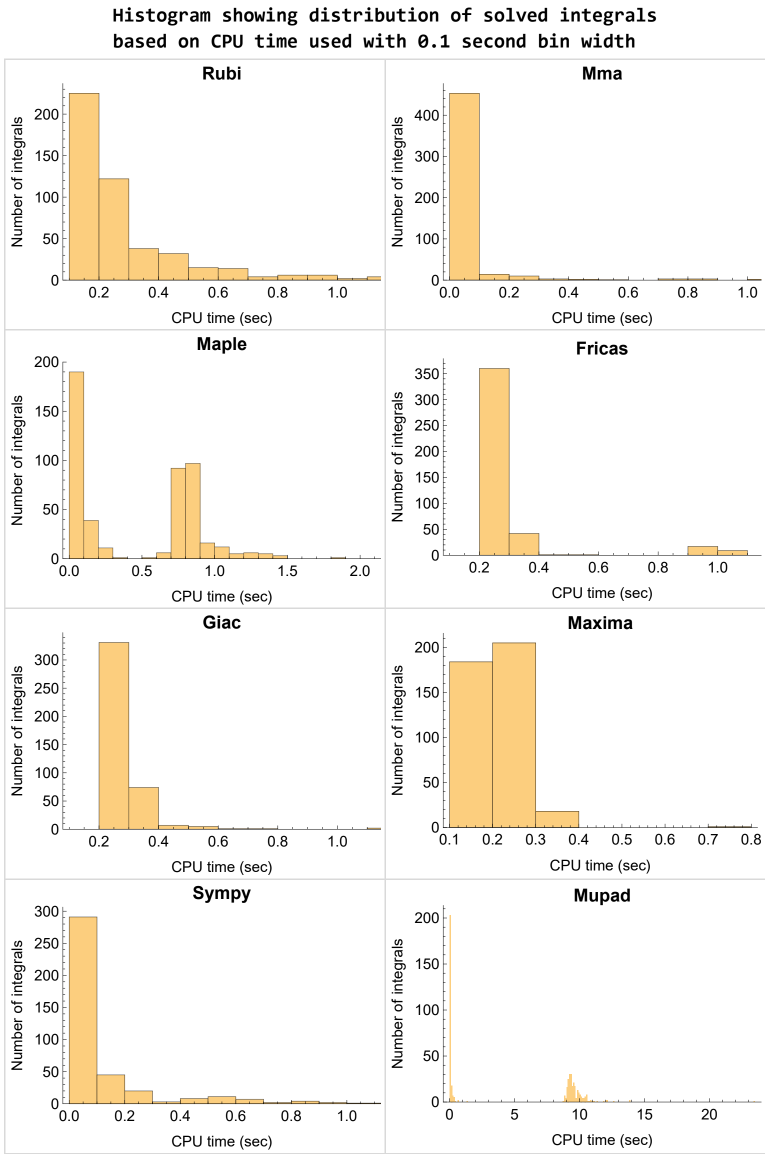


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

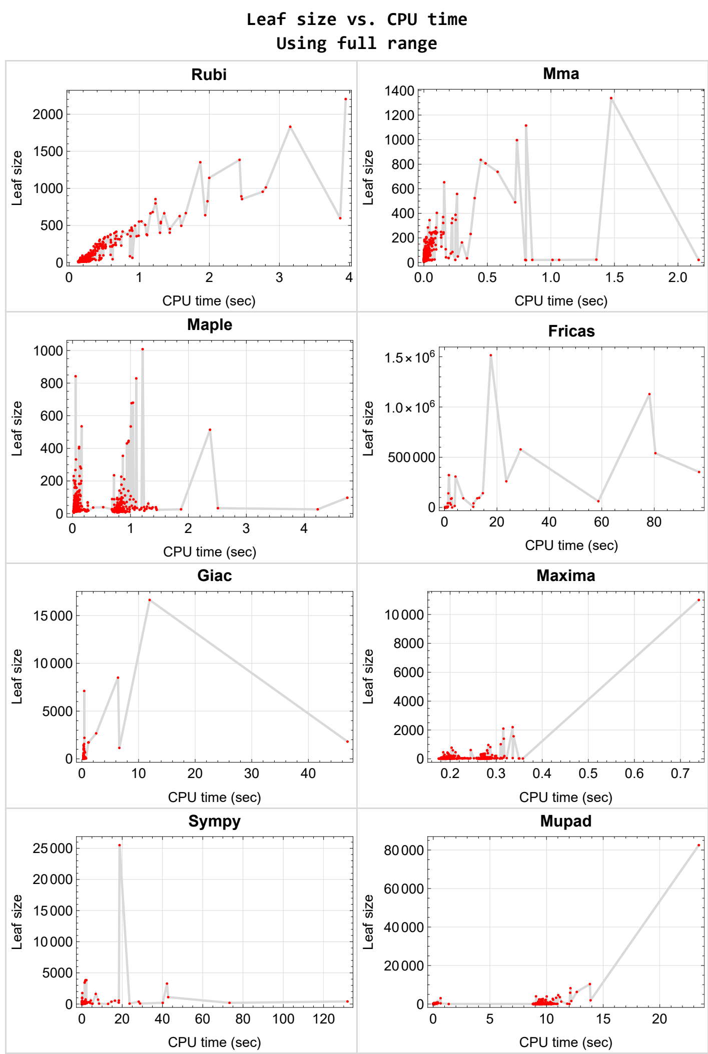


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{56, 128, 129, 135, 155, 156\}$

Mathematica $\{32\}$

Maple $\{1, 174\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

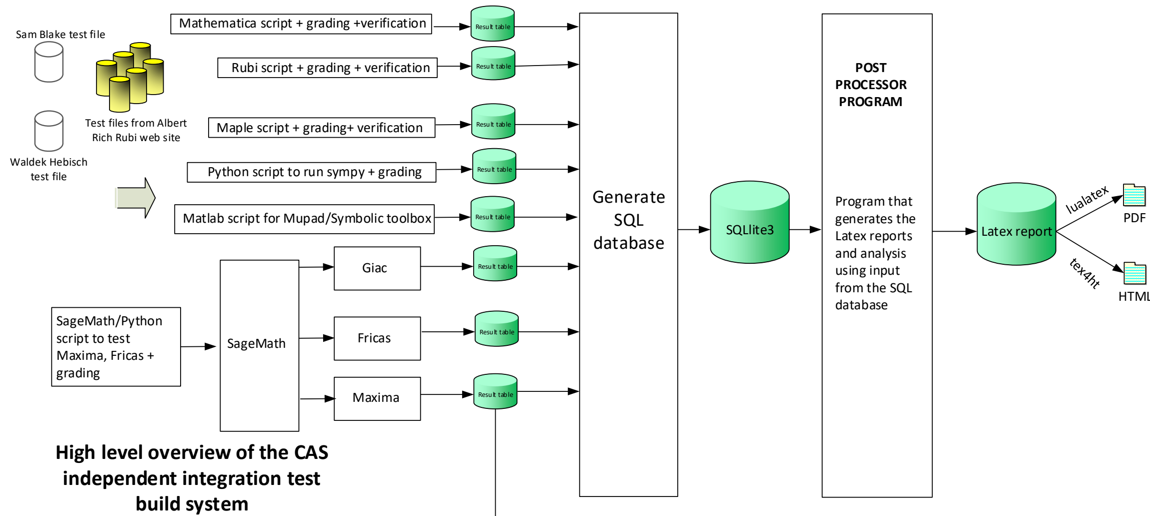
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492 }

B grade { 77, 221, 222, 233, 424 }

C grade { 174 }

F normal fail { 393, 493, 494 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 102, 106, 113, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 177, 178, 179, 180, 181, 182, 183, 186, 187, 188, 189, 190, 191, 193, 194, 208, 216, 217, 218, 219, 220, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 492, 493, 494 }

B grade { 65, 88, 92, 95, 101, 160, 161, 162, 192, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 221, 222, 421 }

C grade { 12, 13, 14, 32, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 173, 174, 175, 176, 184, 185, 227, 250, 251, 252, 253, 254, 255, 256, 257, 387, 388, 389, 390, 391, 392, 393, 491 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 2, 5, 6, 7, 8, 10, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 101, 102, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 172, 173, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 200, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 339, 340, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 399, 400, 405, 406, 407, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 492, 493, 494 }

B grade { 3, 4, 9, 15, 63, 64, 92, 95, 163, 171, 194, 197, 201, 205, 222 }

C grade { 1, 12, 13, 14, 27, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 174, 250, 251, 252, 253, 254, 255, 256, 257, 334, 337, 338, 341, 342, 367, 368, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 401, 402, 403, 404, 408, 409, 410, 411, 455, 491 }

F normal fail { 29, 30, 31, 32, 176 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 5, 6, 9, 10, 11, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 69, 70, 71, 72, 73, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 96, 98, 99, 100, 101, 102, 106, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 164, 165, 166, 167, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 208, 209, 216, 217, 218, 219, 220, 224, 225, 226, 228, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 339, 340, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 415, 416, 417, 418, 419, 420, 422, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 490, 492 }

B grade { 3, 4, 7, 8, 12, 13, 14, 15, 37, 38, 43, 44, 63, 64, 65, 66, 67, 68, 74, 75, 76, 77, 87, 91, 92, 93, 94, 95, 97, 120, 121, 122, 146, 154, 160, 161, 162, 163, 168, 169, 170, 171, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 221, 222, 223, 229, 233, 250, 251, 252, 253, 254, 255, 256, 257, 268, 277, 343, 344, 387, 388, 389, 390, 421, 423, 424, 453, 455, 477, 484, 493, 494 }

C grade { 49, 50, 55, 56, 61, 62, 103, 104, 105, 107, 108, 109, 111, 112, 113, 114, 115, 127, 134, 138, 334, 337, 338, 341, 342, 367, 368, 391, 392, 393, 394, 395, 396, 397, 398, 401, 402, 403, 404, 408, 409, 410, 411, 491 }

F normal fail { 29, 30, 31, 32 }

F(-1) timedout fail { 19, 20, 110, 128, 129, 135, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 227, 399, 400, 405, 406, 407, 412, 413, 414 }

F(-2) exception fail { 136, 137, 139, 140, 141 }

2.1.5 Maxima

A grade { 2, 5, 6, 10, 11, 15, 16, 17, 18, 21, 22, 23, 24, 25, 27, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 90, 93, 96, 97, 98, 99, 100, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 160, 164, 165, 166, 168, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 208, 209, 216, 217, 218, 219, 220, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 492 }

B grade { 3, 4, 7, 8, 9, 19, 20, 63, 64, 65, 66, 67, 68, 74, 87, 88, 91, 92, 94, 95, 101, 102, 161, 162, 163, 167, 169, 170, 171, 194, 197, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 221, 222, 421, 424, 477, 493, 494 }

C grade { }

F normal fail { 1, 12, 13, 14, 29, 30, 31, 32, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 227, 228, 250, 251, 252, 253, 254, 255, 256, 257, 335, 336, 337, 338, 387, 388, 389, 390, 391, 392, 393, 491 }

F(-1) timedout fail { }

F(-2) exception fail { 26, 489, 490 }

2.1.6 Giac

A grade { 1, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 101, 102, 106, 113, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 160, 164, 165, 166, 168, 169, 170, 172, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 189, 190, 191, 193, 194, 196, 197, 199, 200, 201, 206, 208, 209, 210, 212, 214, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 486, 487, 488, 489, 490, 491, 492 }

B grade { 2, 3, 4, 9, 15, 19, 20, 43, 44, 63, 64, 65, 74, 92, 95, 116, 120, 121, 122, 159, 161, 162, 163, 173, 174, 175, 182, 188, 192, 195, 198, 202, 203, 204, 205, 207, 211, 213, 215, 234, 235, 236, 237, 238, 268, 282, 324, 337, 338, 359, 421, 424, 485, 493, 494 }

C grade { 55, 56 }

F normal fail { 29, 30, 31, 32, 49, 50, 61, 62, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 115, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 167, 171, 176, 239, 240, 241, 250, 251, 252, 253, 254, 255, 256, 257, 387, 388, 389, 390, 391, 392, 393 }

F(-1) timedout fail { }

F(-2) exception fail { 227 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 173, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494 }

C grade { }

F normal fail { }

F(-1) timedout fail { 31, 100, 101, 102, 167, 171, 174, 176, 227 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 5, 10, 11, 12, 13, 14, 16, 17, 21, 22, 23, 24, 25, 27, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 51, 52, 53, 54, 57, 58, 59, 60, 61, 69, 70, 71, 72, 73, 75, 76, 93, 96, 97, 98, 99, 100, 103, 104, 105, 106, 110, 111, 112, 113, 116, 117, 118, 119, 120, 123, 124, 125, 126, 127, 130, 131, 132, 133, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 164, 165, 166, 177, 178, 219, 220, 224, 225, 226, 228, 229, 230, 231, 232, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 402, 403, 404, 408, 409, 410, 411, 415, 416, 417, 418, 419, 420, 422, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 491, 492 }

B grade { 3, 4, 6, 7, 8, 9, 15, 26, 28, 50, 55, 56, 62, 63, 64, 65, 66, 67, 68, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 92, 94, 95, 101, 102, 121, 122, 128, 129, 134, 135, 160, 161, 162, 167, 168, 169, 170, 172, 173, 179, 184, 185, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 215, 216, 217, 221, 222, 223, 233, 251, 255, 268, 339, 421, 423, 424, 459, 477, 489, 490, 493, 494 }

C grade { 32, 89, 90, 91, 257, 279, 458 }

F normal fail { 2, 29, 30, 31, 174 }

F(-1) timedout fail { 18, 19, 20, 38, 44, 107, 108, 109, 114, 115, 136, 137, 138, 139, 140, 141, 142, 163, 171, 175, 176, 180, 181, 182, 183, 186, 190, 209, 218, 227, 234, 235, 236, 237, 238, 239, 240, 241, 337, 338, 340, 341, 342, 398, 399, 400, 401, 405, 406, 407, 412, 413, 414 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	77	88	143	43	0	76	60	55	51
N.S.	1	1.14	1.86	0.56	0.00	0.99	0.78	0.71	0.66
time (sec)	N/A	0.250	0.035	0.113	0.000	0.246	0.152	0.300	0.118

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	25	25	26	25	43	0	73	52
N.S.	1	0.83	0.83	0.87	0.83	1.43	0.00	2.43	1.73
time (sec)	N/A	0.162	0.013	0.051	0.192	0.234	0.000	0.282	9.335

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	98	216	97	107	97	97
N.S.	1	1.00	1.00	7.00	15.43	6.93	7.64	6.93	6.93
time (sec)	N/A	0.146	0.001	0.034	0.197	0.258	0.033	0.290	9.169

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	65	99	64	66	64	64
N.S.	1	1.00	1.00	4.64	7.07	4.57	4.71	4.57	4.57
time (sec)	N/A	0.147	0.002	0.032	0.198	0.247	0.027	0.338	0.017

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	14	35	13	31	31	32	31	31
N.S.	1	0.40	1.00	0.37	0.89	0.89	0.91	0.89	0.89
time (sec)	N/A	0.144	0.000	0.026	0.191	0.230	0.018	0.286	0.022

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	24	24	26	12	26
N.S.	1	1.00	1.00	0.93	1.71	1.71	1.86	0.86	1.86
time (sec)	N/A	0.149	0.003	0.035	0.200	0.237	0.078	0.298	0.021

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	57	57	61	12	59
N.S.	1	1.00	1.00	0.93	4.07	4.07	4.36	0.86	4.21
time (sec)	N/A	0.150	0.005	0.075	0.202	0.255	0.171	0.296	9.285

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	90	90	97	12	92
N.S.	1	1.00	1.00	0.93	6.43	6.43	6.93	0.86	6.57
time (sec)	N/A	0.149	0.002	0.107	0.201	0.240	0.251	0.277	9.019

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	93	159	159	204	156	175	166	149
N.S.	1	1.11	1.89	1.89	2.43	1.86	2.08	1.98	1.77
time (sec)	N/A	0.297	0.016	0.035	0.211	0.250	0.044	0.276	9.303

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	64	82	82	93	80	87	83	79
N.S.	1	1.14	1.46	1.46	1.66	1.43	1.55	1.48	1.41
time (sec)	N/A	0.245	0.007	0.029	0.209	0.254	0.030	0.297	0.026

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	28	28	31	28	28
N.S.	1	1.00	1.00	0.91	0.88	0.88	0.97	0.88	0.88
time (sec)	N/A	0.158	0.000	0.021	0.195	0.252	0.020	0.283	0.022

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	194	63	57	0	387	53	212	174
N.S.	1	1.03	0.34	0.30	0.00	2.06	0.28	1.13	0.93
time (sec)	N/A	0.459	0.014	0.048	0.000	0.288	0.204	0.278	0.321

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	268	112	134	0	704	192	289	247
N.S.	1	1.09	0.46	0.55	0.00	2.87	0.78	1.18	1.01
time (sec)	N/A	0.486	0.044	0.085	0.000	0.275	0.664	0.321	10.171

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	342	149	277	0	1268	474	366	483
N.S.	1	1.12	0.49	0.91	0.00	4.16	1.55	1.20	1.58
time (sec)	N/A	0.548	0.058	0.146	0.000	0.283	1.390	0.278	10.188

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	653	842	461	727	1018	987	787
N.S.	1	1.00	1.81	2.33	1.28	2.01	2.82	2.73	2.18
time (sec)	N/A	0.985	0.160	0.053	0.212	0.251	0.087	0.286	9.629

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	241	188	180	269	345	346	270
N.S.	1	1.00	1.25	0.97	0.93	1.39	1.79	1.79	1.40
time (sec)	N/A	0.501	0.056	0.037	0.206	0.231	0.043	0.351	0.048

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	76	53	50	52	63	50	54
N.S.	1	1.00	1.36	0.95	0.89	0.93	1.12	0.89	0.96
time (sec)	N/A	0.187	0.000	0.027	0.201	0.238	0.019	0.262	0.026

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	80	87	112	112	0	131	106
N.S.	1	1.00	0.93	1.01	1.30	1.30	0.00	1.52	1.23
time (sec)	N/A	0.284	0.048	0.126	0.217	2.906	0.000	0.285	9.712

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	232	235	2096	0	0	1435	1940
N.S.	1	1.00	0.99	1.00	8.96	0.00	0.00	6.13	8.29
time (sec)	N/A	0.633	0.368	0.716	0.316	0.000	0.000	0.282	13.890

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	495	495	490	514	11005	0	0	7111	82532
N.S.	1	1.00	0.99	1.04	22.23	0.00	0.00	14.37	166.73
time (sec)	N/A	1.598	0.718	2.373	0.740	0.000	0.000	0.404	23.447

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	20	25
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00
time (sec)	N/A	0.179	0.006	0.056	0.286	0.313	0.057	0.278	0.030

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	25	25	24	26	25
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.77	0.84	0.81
time (sec)	N/A	0.197	0.006	0.063	0.288	0.332	0.065	0.321	0.034

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.155	0.001	0.016	0.200	0.329	0.019	0.284	0.019

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	26	28	26	19	30	25
N.S.	1	1.00	1.00	0.93	1.00	0.93	0.68	1.07	0.89
time (sec)	N/A	0.190	0.005	0.691	0.198	0.310	0.082	0.296	0.038

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	26	22	21	20	18	15	24	18
N.S.	1	1.18	1.00	0.95	0.91	0.82	0.68	1.09	0.82
time (sec)	N/A	0.168	0.005	0.683	0.193	0.311	0.098	0.276	9.264

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	63	61	61	0	211	564	62	213
N.S.	1	1.02	0.98	0.98	0.00	3.40	9.10	1.00	3.44
time (sec)	N/A	0.252	0.052	0.089	0.000	0.323	4.405	0.333	0.283

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	111	89	27	98	299	20	112	99
N.S.	1	0.97	0.77	0.23	0.85	2.60	0.17	0.97	0.86
time (sec)	N/A	0.280	0.021	0.680	0.282	0.320	0.071	0.277	0.143

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	17	16	29	16	16
N.S.	1	1.00	1.00	1.06	1.06	1.00	1.81	1.00	1.00
time (sec)	N/A	0.144	0.002	0.027	0.202	0.298	0.277	0.266	9.845

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	53	0	0	0	0	0	56
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	1.02
time (sec)	N/A	0.192	0.027	0.000	0.000	0.000	0.000	0.000	9.640

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	59	61	0	0	0	0	0	56
N.S.	1	1.11	1.15	0.00	0.00	0.00	0.00	0.00	1.06
time (sec)	N/A	0.197	0.030	0.000	0.000	0.000	0.000	0.000	10.146

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	129	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.339	0.138	0.000	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	35	44	203	0	0	0	34	0	41
N.S.	1	1.26	5.80	0.00	0.00	0.00	0.97	0.00	1.17
time (sec)	N/A	0.171	0.136	0.000	0.000	0.000	5.126	0.000	10.478

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	276	285	267	372	271	299	277	261
N.S.	1	1.02	1.06	0.99	1.38	1.00	1.11	1.03	0.97
time (sec)	N/A	0.591	0.032	0.050	0.196	0.271	0.048	0.279	10.568

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	189	171	163	205	163	180	166	160
N.S.	1	1.11	1.00	0.95	1.20	0.95	1.05	0.97	0.94
time (sec)	N/A	0.422	0.015	0.047	0.218	0.263	0.037	0.281	10.577

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	117	92	83	94	82	95	83	82
N.S.	1	1.27	1.00	0.90	1.02	0.89	1.03	0.90	0.89
time (sec)	N/A	0.312	0.010	0.038	0.190	0.284	0.028	0.269	0.023

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	28	28	31	28	28
N.S.	1	1.00	1.00	0.91	0.88	0.88	0.97	0.88	0.88
time (sec)	N/A	0.167	0.000	0.029	0.195	0.279	0.020	0.266	0.021

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	529	555	71	64	0	905	88	603	1551
N.S.	1	1.05	0.13	0.12	0.00	1.71	0.17	1.14	2.93
time (sec)	N/A	1.027	0.020	0.137	0.000	0.293	0.595	0.290	12.187

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	746	797	182	230	0	3222	0	1057	5844
N.S.	1	1.07	0.24	0.31	0.00	4.32	0.00	1.42	7.83
time (sec)	N/A	1.260	0.071	0.122	0.000	0.373	0.000	0.389	12.090

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	304	345	332	383	332	366	353	331
N.S.	1	1.03	1.17	1.13	1.30	1.13	1.24	1.20	1.12
time (sec)	N/A	0.594	0.045	0.059	0.195	0.286	0.051	0.295	10.542

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	215	207	200	214	198	218	205	201
N.S.	1	1.06	1.02	0.99	1.05	0.98	1.07	1.01	0.99
time (sec)	N/A	0.427	0.019	0.048	0.202	0.274	0.038	0.295	0.100

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	135	109	99	101	98	112	99	98
N.S.	1	1.26	1.02	0.93	0.94	0.92	1.05	0.93	0.92
time (sec)	N/A	0.314	0.011	0.038	0.198	0.272	0.026	0.293	10.174

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	33	33	36	33	33
N.S.	1	1.00	1.00	0.92	0.89	0.89	0.97	0.89	0.89
time (sec)	N/A	0.169	0.000	0.030	0.191	0.258	0.020	0.275	0.023

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	165	71	67	0	1115	122	577	1264
N.S.	1	1.08	0.46	0.44	0.00	7.29	0.80	3.77	8.26
time (sec)	N/A	0.361	0.018	0.261	0.000	0.310	0.991	0.284	11.302

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	377	234	288	0	4285	0	1115	10351
N.S.	1	1.10	0.68	0.84	0.00	12.53	0.00	3.26	30.27
time (sec)	N/A	0.582	0.096	0.133	0.000	0.482	0.000	0.317	13.831

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	85	84	84	94	84	84
N.S.	1	1.00	1.00	0.89	0.88	0.88	0.98	0.88	0.88
time (sec)	N/A	0.239	0.003	0.030	0.193	0.281	0.028	0.294	0.197

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	65	64	64	71	64	64
N.S.	1	1.00	1.00	0.88	0.86	0.86	0.96	0.86	0.86
time (sec)	N/A	0.221	0.001	0.028	0.187	0.289	0.027	0.279	0.066

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	49	44	44
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.199	0.002	0.022	0.187	0.308	0.022	0.336	0.020

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.150	0.000	0.022	0.194	0.281	0.026	0.291	0.017

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	321	45	41	0	1015	41	0	123
N.S.	1	1.20	0.17	0.15	0.00	3.79	0.15	0.00	0.46
time (sec)	N/A	0.656	0.008	0.047	0.000	1.086	0.540	0.000	10.581

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	432	113	83	0	1201	3834	0	176
N.S.	1	1.21	0.32	0.23	0.00	3.36	10.74	0.00	0.49
time (sec)	N/A	0.862	0.014	0.055	0.000	1.049	1.777	0.000	0.136

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	78	77	77	94	77	77
N.S.	1	1.00	1.00	0.80	0.79	0.79	0.97	0.79	0.79
time (sec)	N/A	0.244	0.002	0.029	0.199	0.256	0.029	0.303	0.118

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83
time (sec)	N/A	0.220	0.002	0.026	0.196	0.270	0.026	0.277	0.063

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	37	37	42	37	37
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.93	0.82	0.82
time (sec)	N/A	0.197	0.002	0.024	0.199	0.292	0.025	0.306	0.018

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	19	17	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.90	0.81	0.81
time (sec)	N/A	0.152	0.000	0.021	0.190	0.242	0.018	0.304	0.016

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	270	47	41	0	499	3432	265	87
N.S.	1	1.15	0.20	0.18	0.00	2.13	14.67	1.13	0.37
time (sec)	N/A	0.586	0.018	0.059	0.000	0.989	1.343	0.309	10.965

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	C	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	350	108	79	0	704	3834	315	174
N.S.	1	1.10	0.34	0.25	0.00	2.22	12.09	0.99	0.55
time (sec)	N/A	0.755	0.019	0.079	0.000	0.975	1.936	0.380	10.660

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	104	85	84	84	100	84	84
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.96	0.81	0.81
time (sec)	N/A	0.248	0.002	0.034	0.208	0.257	0.031	0.300	10.543

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	65	64	64	73	64	64
N.S.	1	1.00	1.00	0.86	0.84	0.84	0.96	0.84	0.84
time (sec)	N/A	0.223	0.002	0.029	0.196	0.259	0.027	0.293	0.065

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	45	44	44	49	44	44
N.S.	1	1.00	1.00	0.87	0.85	0.85	0.94	0.85	0.85
time (sec)	N/A	0.197	0.002	0.025	0.194	0.242	0.024	0.285	0.020

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	27	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.90	0.80	0.80
time (sec)	N/A	0.151	0.000	0.021	0.200	0.267	0.018	0.290	0.012

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	313	55	49	0	1297	41	0	123
N.S.	1	1.19	0.21	0.19	0.00	4.93	0.16	0.00	0.47
time (sec)	N/A	0.708	0.009	0.045	0.000	0.977	1.271	0.000	0.267

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	428	128	96	0	1540	3839	0	181
N.S.	1	1.17	0.35	0.26	0.00	4.21	10.49	0.00	0.49
time (sec)	N/A	0.888	0.014	0.055	0.000	1.099	2.232	0.000	0.150

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	164	592	163	185	163	163
N.S.	1	1.00	1.00	11.71	42.29	11.64	13.21	11.64	11.64
time (sec)	N/A	0.165	0.002	0.052	0.206	0.276	0.040	0.271	0.127

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	109	228	108	114	108	108
N.S.	1	1.00	1.00	7.79	16.29	7.71	8.14	7.71	7.71
time (sec)	N/A	0.167	0.002	0.040	0.207	0.300	0.031	0.279	0.055

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	61	13	53	53	60	53	53
N.S.	1	1.00	4.36	0.93	3.79	3.79	4.29	3.79	3.79
time (sec)	N/A	0.159	0.000	0.032	0.203	0.275	0.021	0.295	0.016

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	46	46	49	12	48
N.S.	1	1.00	1.00	0.93	3.29	3.29	3.50	0.86	3.43
time (sec)	N/A	0.162	0.004	0.058	0.196	0.290	0.134	0.294	0.033

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	101	101	109	12	103
N.S.	1	1.00	1.00	0.93	7.21	7.21	7.79	0.86	7.36
time (sec)	N/A	0.165	0.003	0.130	0.203	0.266	0.282	0.280	10.735

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	156	156	168	12	158
N.S.	1	1.00	1.00	0.93	11.14	11.14	12.00	0.86	11.29
time (sec)	N/A	0.161	0.003	0.244	0.219	0.282	0.426	0.281	11.831

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	30	30	29	31	36
N.S.	1	1.00	1.00	0.82	0.79	0.79	0.76	0.82	0.95
time (sec)	N/A	0.196	0.006	0.081	0.281	0.261	0.073	0.268	0.030

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	65	64	64	80	64	64
N.S.	1	1.00	1.00	0.77	0.76	0.76	0.95	0.76	0.76
time (sec)	N/A	0.852	0.003	0.039	0.197	0.257	0.033	0.272	0.077

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	50	49	49	60	49	49
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.95	0.78	0.78
time (sec)	N/A	0.890	0.003	0.033	0.193	0.245	0.028	0.281	0.053

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	35	34	34	41	34	34
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.93	0.77	0.77
time (sec)	N/A	0.608	0.001	0.028	0.190	0.272	0.026	0.276	0.015

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	22	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.88	0.76	0.76
time (sec)	N/A	0.157	0.000	0.046	0.189	0.254	0.019	0.287	0.020

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	62	42	54	56	63	62	27
N.S.	1	1.00	2.00	1.35	1.74	1.81	2.03	2.00	0.87
time (sec)	N/A	0.187	0.012	0.069	0.288	0.288	0.072	0.290	10.332

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	103	84	89	177	104	97	64
N.S.	1	1.00	1.16	0.94	1.00	1.99	1.17	1.09	0.72
time (sec)	N/A	0.245	0.039	0.117	0.273	0.264	0.817	0.296	0.047

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	137	108	119	282	134	112	93
N.S.	1	1.00	0.85	0.67	0.74	1.75	0.83	0.70	0.58
time (sec)	N/A	0.335	0.061	0.140	0.278	0.298	0.877	0.301	0.049

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	245	132	116	114	223	296	134	126
N.S.	1	2.69	1.45	1.27	1.25	2.45	3.25	1.47	1.38
time (sec)	N/A	0.443	0.065	0.131	0.286	0.277	0.806	0.298	10.083

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	74	73	111	81	198	209	77	87
N.S.	1	0.95	0.94	1.42	1.04	2.54	2.68	0.99	1.12
time (sec)	N/A	0.224	0.035	0.906	0.284	0.283	0.395	0.273	9.963

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	47	54	70	61	157	153	54	206
N.S.	1	0.94	1.08	1.40	1.22	3.14	3.06	1.08	4.12
time (sec)	N/A	0.199	0.021	0.882	0.278	0.285	0.257	0.285	0.057

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	39	38	54	50	136	124	43	46
N.S.	1	0.95	0.93	1.32	1.22	3.32	3.02	1.05	1.12
time (sec)	N/A	0.172	0.012	0.845	0.284	0.282	0.116	0.281	9.987

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	28	24	83	54	17	17
N.S.	1	1.00	1.00	1.33	1.14	3.95	2.57	0.81	0.81
time (sec)	N/A	0.144	0.007	0.826	0.277	0.272	0.085	0.291	0.026

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	57	48	74	68	154	738	62	173
N.S.	1	0.97	0.81	1.25	1.15	2.61	12.51	1.05	2.93
time (sec)	N/A	0.195	0.028	0.843	0.292	0.284	1.845	0.295	10.576

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	93	81	96	123	229	1620	117	425
N.S.	1	1.18	1.03	1.22	1.56	2.90	20.51	1.48	5.38
time (sec)	N/A	0.256	0.031	0.865	0.288	0.295	6.884	0.295	10.100

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	134	106	151	197	371	3284	195	573
N.S.	1	1.11	0.88	1.25	1.63	3.07	27.14	1.61	4.74
time (sec)	N/A	0.315	0.097	0.882	0.298	0.288	42.252	0.291	10.425

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	34	30	109	61	24	27
N.S.	1	1.00	1.00	1.10	0.97	3.52	1.97	0.77	0.87
time (sec)	N/A	0.156	0.009	1.112	0.281	0.270	0.089	0.301	0.037

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	61	60	86	75	253	117	65	76
N.S.	1	0.97	0.95	1.37	1.19	4.02	1.86	1.03	1.21
time (sec)	N/A	0.168	0.021	0.990	0.288	0.282	0.294	0.289	9.907

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	94	75	139	184	595	257	103	181
N.S.	1	1.03	0.82	1.53	2.02	6.54	2.82	1.13	1.99
time (sec)	N/A	0.183	0.044	0.963	0.298	0.282	0.619	0.305	10.264

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	198	42	66	279	92	30	31
N.S.	1	1.00	5.66	1.20	1.89	7.97	2.63	0.86	0.89
time (sec)	N/A	0.167	0.089	1.077	0.277	0.273	0.098	0.280	0.073

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	18	10	24	10	10
N.S.	1	1.00	1.00	1.10	1.80	1.00	2.40	1.00	1.00
time (sec)	N/A	0.138	0.005	0.981	0.273	0.250	0.078	0.294	0.025

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	35	31	43	51	55	56	41	42
N.S.	1	0.95	0.84	1.16	1.38	1.49	1.51	1.11	1.14
time (sec)	N/A	0.147	0.011	0.924	0.301	0.272	0.225	0.304	10.887

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	71	115	153	146	73	111
N.S.	1	1.00	0.87	1.18	1.92	2.55	2.43	1.22	1.85
time (sec)	N/A	0.159	0.014	0.935	0.282	0.253	0.480	0.282	0.077

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	32	23	25	22	22	27	10
N.S.	1	1.00	3.20	2.30	2.50	2.20	2.20	2.70	1.00
time (sec)	N/A	0.136	0.005	0.875	0.189	0.285	0.080	0.299	10.474

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	37	45	52	56	85	54	56	43
N.S.	1	0.95	1.15	1.33	1.44	2.18	1.38	1.44	1.10
time (sec)	N/A	0.148	0.019	0.822	0.198	0.288	0.225	0.278	10.358

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	65	78	122	220	141	88	114
N.S.	1	1.00	1.02	1.22	1.91	3.44	2.20	1.38	1.78
time (sec)	N/A	0.165	0.023	0.844	0.196	0.258	0.513	0.288	10.060

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	15	12	11	11	10	13	4
N.S.	1	1.00	3.75	3.00	2.75	2.75	2.50	3.25	1.00
time (sec)	N/A	0.128	0.003	0.708	0.202	0.264	0.044	0.297	0.110

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	25	39	24	27	23
N.S.	1	1.00	0.96	0.89	0.93	1.44	0.89	1.00	0.85
time (sec)	N/A	0.145	0.013	0.760	0.192	0.268	0.052	0.302	0.046

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	50	37	36	44	71	44	39	36
N.S.	1	1.11	0.82	0.80	0.98	1.58	0.98	0.87	0.80
time (sec)	N/A	0.157	0.014	0.731	0.188	0.254	0.067	0.284	10.257

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	65	64	61	54	54	58	62	55
N.S.	1	1.10	1.08	1.03	0.92	0.92	0.98	1.05	0.93
time (sec)	N/A	0.208	0.018	0.899	0.186	0.259	0.076	0.284	0.033

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	11	11	12	11	11	10	11	11
N.S.	1	1.10	1.10	1.20	1.10	1.10	1.00	1.10	1.10
time (sec)	N/A	0.163	0.007	0.892	0.187	0.266	0.038	0.285	10.372

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	41	55	25	36	35	26	23	0
N.S.	1	0.93	1.25	0.57	0.82	0.80	0.59	0.52	0.00
time (sec)	N/A	0.175	0.050	0.987	0.268	0.294	0.441	0.286	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	63	163	107	139	92	231	64	0
N.S.	1	0.94	2.43	1.60	2.07	1.37	3.45	0.96	0.00
time (sec)	N/A	0.195	0.299	1.005	0.276	0.295	0.751	0.307	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	60	86	87	135	70	228	86	0
N.S.	1	0.95	1.37	1.38	2.14	1.11	3.62	1.37	0.00
time (sec)	N/A	0.187	0.225	1.052	0.195	0.297	0.643	0.302	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	227	132	108	0	6315	238	0	374
N.S.	1	0.97	0.56	0.46	0.00	26.99	1.02	0.00	1.60
time (sec)	N/A	0.473	0.037	0.071	0.000	10.969	1.641	0.000	9.623

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	197	81	74	0	4759	158	0	437
N.S.	1	0.94	0.39	0.35	0.00	22.66	0.75	0.00	2.08
time (sec)	N/A	0.475	0.021	0.056	0.000	0.945	0.542	0.000	9.864

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	171	79	72	0	1950	83	0	145
N.S.	1	0.95	0.44	0.40	0.00	10.83	0.46	0.00	0.81
time (sec)	N/A	0.369	0.017	0.059	0.000	0.964	0.368	0.000	9.747

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	131	116	71	0	442	26	160	144
N.S.	1	0.94	0.83	0.51	0.00	3.16	0.19	1.14	1.03
time (sec)	N/A	0.278	0.024	0.047	0.000	0.285	0.118	0.302	10.075

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	241	119	105	0	4370	0	0	553
N.S.	1	1.08	0.53	0.47	0.00	19.51	0.00	0.00	2.47
time (sec)	N/A	0.520	0.037	0.092	0.000	0.950	0.000	0.000	0.075

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	315	173	143	0	8919	0	0	1588
N.S.	1	1.00	0.55	0.46	0.00	28.40	0.00	0.00	5.06
time (sec)	N/A	0.699	0.062	0.122	0.000	1.334	0.000	0.000	9.900

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	380	244	200	0	14765	0	0	1328
N.S.	1	0.97	0.62	0.51	0.00	37.57	0.00	0.00	3.38
time (sec)	N/A	0.824	0.103	0.151	0.000	3.883	0.000	0.000	9.265

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	342	106	97	0	0	374	0	1003
N.S.	1	0.96	0.30	0.27	0.00	0.00	1.05	0.00	2.82
time (sec)	N/A	0.497	0.028	0.097	0.000	0.000	2.238	0.000	9.617

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	307	106	97	0	61993	274	0	625
N.S.	1	0.97	0.33	0.31	0.00	194.95	0.86	0.00	1.97
time (sec)	N/A	0.427	0.024	0.078	0.000	58.699	1.600	0.000	9.271

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	250	104	95	0	40785	131	0	205
N.S.	1	0.96	0.40	0.36	0.00	156.26	0.50	0.00	0.79
time (sec)	N/A	0.377	0.019	0.074	0.000	11.051	0.478	0.000	9.128

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	228	161	94	0	152	26	103	60
N.S.	1	1.03	0.73	0.43	0.00	0.69	0.12	0.47	0.27
time (sec)	N/A	0.401	0.059	0.062	0.000	0.298	0.132	0.311	0.073

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	396	163	139	0	307773	0	0	882
N.S.	1	1.01	0.41	0.35	0.00	783.14	0.00	0.00	2.24
time (sec)	N/A	0.656	0.045	0.110	0.000	4.194	0.000	0.000	9.294

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	496	497	238	184	0	1128605	0	0	2440
N.S.	1	1.00	0.48	0.37	0.00	2275.41	0.00	0.00	4.92
time (sec)	N/A	0.957	0.088	0.142	0.000	78.189	0.000	0.000	9.527

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	125	195	179	192	179	199	219	175
N.S.	1	1.02	1.59	1.46	1.56	1.46	1.62	1.78	1.42
time (sec)	N/A	0.366	0.020	0.046	0.197	0.241	0.047	0.277	0.205

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	85	114	110	119	107	114	128	108
N.S.	1	0.71	0.95	0.92	0.99	0.89	0.95	1.07	0.90
time (sec)	N/A	0.298	0.012	0.033	0.199	0.253	0.031	0.296	0.081

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	52	66	60	65	56	65	65	61
N.S.	1	0.72	0.92	0.83	0.90	0.78	0.90	0.90	0.85
time (sec)	N/A	0.226	0.007	0.030	0.209	0.249	0.030	0.285	0.023

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	22	22	22	22
N.S.	1	1.00	1.00	0.88	0.85	0.85	0.85	0.85	0.85
time (sec)	N/A	0.153	0.000	0.023	0.204	0.248	0.020	0.294	0.011

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	57	51	0	457	66	2669	571
N.S.	1	1.00	0.64	0.57	0.00	5.13	0.74	29.99	6.42
time (sec)	N/A	0.231	0.018	0.067	0.000	0.249	0.543	2.518	0.323

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	165	150	158	0	1948	294	8503	4591
N.S.	1	0.98	0.89	0.93	0.00	11.53	1.74	50.31	27.17
time (sec)	N/A	0.313	0.047	0.069	0.000	0.269	3.339	6.370	11.032

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	254	254	400	0	3971	697	16632	8242
N.S.	1	1.01	1.01	1.59	0.00	15.76	2.77	66.00	32.71
time (sec)	N/A	0.442	0.091	0.112	0.000	0.310	8.015	11.962	12.115

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	204	182	182	182	212	222	178
N.S.	1	1.00	0.97	0.87	0.87	0.87	1.01	1.06	0.85
time (sec)	N/A	0.410	0.021	0.112	0.204	0.250	0.044	0.258	0.206

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	130	115	113	113	128	133	113
N.S.	1	1.00	0.97	0.86	0.84	0.84	0.96	0.99	0.84
time (sec)	N/A	0.316	0.013	0.109	0.206	0.237	0.033	0.272	9.638

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	75	63	59	59	70	68	64
N.S.	1	1.00	0.95	0.80	0.75	0.75	0.89	0.86	0.81
time (sec)	N/A	0.245	0.007	0.097	0.192	0.251	0.023	0.259	0.024

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	29	27	27
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.83	0.77	0.77
time (sec)	N/A	0.179	0.003	0.022	0.203	0.253	0.017	0.265	0.013

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	121	59	52	0	140500	155	0	275
N.S.	1	1.04	0.51	0.45	0.00	1211.21	1.34	0.00	2.37
time (sec)	N/A	0.324	0.016	0.043	0.000	1.452	2.478	0.000	9.580

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	B	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	166	158	0	0	539	0	1167
N.S.	1	1.00	0.72	0.68	0.00	0.00	2.33	0.00	5.05
time (sec)	N/A	0.450	0.049	0.060	0.000	0.000	16.396	0.000	10.332

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	B	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	359	284	409	0	0	1102	0	2200
N.S.	1	1.03	0.81	1.17	0.00	0.00	3.16	0.00	6.30
time (sec)	N/A	0.667	0.096	0.114	0.000	0.000	42.836	0.000	9.931

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	204	182	182	182	219	222	178
N.S.	1	1.00	0.97	0.87	0.87	0.87	1.04	1.06	0.85
time (sec)	N/A	0.405	0.020	0.122	0.193	0.239	0.042	0.290	9.273

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	132	115	113	113	134	133	113
N.S.	1	1.00	0.96	0.83	0.82	0.82	0.97	0.96	0.82
time (sec)	N/A	0.330	0.012	0.126	0.192	0.242	0.032	0.284	9.107

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	73	63	59	59	73	68	64
N.S.	1	1.00	0.92	0.80	0.75	0.75	0.92	0.86	0.81
time (sec)	N/A	0.256	0.007	0.107	0.189	0.239	0.024	0.293	0.026

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	29	27	27
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.83	0.77	0.77
time (sec)	N/A	0.184	0.002	0.022	0.189	0.238	0.018	0.277	0.012

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	105	61	54	0	1515766	172	0	878
N.S.	1	1.06	0.62	0.55	0.00	15310.77	1.74	0.00	8.87
time (sec)	N/A	0.357	0.016	0.040	0.000	17.658	4.082	0.000	0.399

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	B	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	220	182	160	0	0	561	0	1218
N.S.	1	0.98	0.81	0.71	0.00	0.00	2.49	0.00	5.41
time (sec)	N/A	0.465	0.051	0.064	0.000	0.000	18.357	0.000	9.444

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	545	530	99	93	0	0	0	0	1563
N.S.	1	0.97	0.18	0.17	0.00	0.00	0.00	0.00	2.87
time (sec)	N/A	1.280	0.040	0.078	0.000	0.000	0.000	0.000	9.522

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	472	99	93	0	0	0	0	1354
N.S.	1	0.97	0.20	0.19	0.00	0.00	0.00	0.00	2.78
time (sec)	N/A	0.889	0.031	0.078	0.000	0.000	0.000	0.000	9.569

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	319	97	93	0	27094	0	0	825
N.S.	1	0.96	0.29	0.28	0.00	81.12	0.00	0.00	2.47
time (sec)	N/A	0.626	0.026	0.082	0.000	2.428	0.000	0.000	9.515

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	469	454	95	91	0	0	0	0	1057
N.S.	1	0.97	0.20	0.19	0.00	0.00	0.00	0.00	2.25
time (sec)	N/A	0.857	0.027	0.081	0.000	0.000	0.000	0.000	9.612

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	507	99	90	0	0	0	0	1394
N.S.	1	0.97	0.19	0.17	0.00	0.00	0.00	0.00	2.67
time (sec)	N/A	1.049	0.036	0.072	0.000	0.000	0.000	0.000	9.501

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	563	548	157	134	0	0	0	0	4002
N.S.	1	0.97	0.28	0.24	0.00	0.00	0.00	0.00	7.11
time (sec)	N/A	1.262	0.071	0.109	0.000	0.000	0.000	0.000	9.073

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	645	625	163	133	0	0	0	0	2663
N.S.	1	0.97	0.25	0.21	0.00	0.00	0.00	0.00	4.13
time (sec)	N/A	1.524	0.076	0.111	0.000	0.000	0.000	0.000	9.683

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	402	61	56	0	0	70	0	427
N.S.	1	1.02	0.15	0.14	0.00	0.00	0.18	0.00	1.08
time (sec)	N/A	1.360	0.013	0.056	0.000	0.000	0.121	0.000	0.375

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	379	61	56	0	0	65	0	390
N.S.	1	1.01	0.16	0.15	0.00	0.00	0.17	0.00	1.03
time (sec)	N/A	1.075	0.010	0.045	0.000	0.000	0.135	0.000	9.234

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	354	61	56	0	0	61	0	276
N.S.	1	0.98	0.17	0.16	0.00	0.00	0.17	0.00	0.76
time (sec)	N/A	0.847	0.010	0.044	0.000	0.000	0.111	0.000	0.304

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	241	59	56	0	1277	48	0	247
N.S.	1	0.97	0.24	0.23	0.00	5.15	0.19	0.00	1.00
time (sec)	N/A	0.651	0.009	0.046	0.000	0.929	0.083	0.000	9.526

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	354	57	54	0	0	61	0	176
N.S.	1	0.98	0.16	0.15	0.00	0.00	0.17	0.00	0.49
time (sec)	N/A	0.901	0.010	0.045	0.000	0.000	0.112	0.000	9.097

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	370	62	53	0	0	65	0	306
N.S.	1	0.98	0.16	0.14	0.00	0.00	0.17	0.00	0.81
time (sec)	N/A	1.070	0.009	0.043	0.000	0.000	0.132	0.000	9.356

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	415	401	103	73	0	0	82	0	432
N.S.	1	0.97	0.25	0.18	0.00	0.00	0.20	0.00	1.04
time (sec)	N/A	1.268	0.013	0.066	0.000	0.000	0.561	0.000	9.438

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	453	109	69	0	0	70	0	340
N.S.	1	1.01	0.24	0.15	0.00	0.00	0.16	0.00	0.76
time (sec)	N/A	1.409	0.014	0.066	0.000	0.000	0.191	0.000	9.038

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1064	1012	167	122	0	0	112	0	388
N.S.	1	0.95	0.16	0.11	0.00	0.00	0.11	0.00	0.36
time (sec)	N/A	2.764	0.025	0.064	0.000	0.000	0.234	0.000	9.153

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1005	954	167	122	0	0	112	0	387
N.S.	1	0.95	0.17	0.12	0.00	0.00	0.11	0.00	0.39
time (sec)	N/A	2.738	0.019	0.065	0.000	0.000	0.281	0.000	9.322

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	677	638	167	122	0	0	112	0	388
N.S.	1	0.94	0.25	0.18	0.00	0.00	0.17	0.00	0.57
time (sec)	N/A	1.970	0.025	0.060	0.000	0.000	0.248	0.000	9.045

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	682	666	167	122	0	1445	104	0	299
N.S.	1	0.98	0.24	0.18	0.00	2.12	0.15	0.00	0.44
time (sec)	N/A	1.623	0.018	0.064	0.000	0.959	0.120	0.000	0.185

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	850	826	167	122	0	0	112	0	388
N.S.	1	0.97	0.20	0.14	0.00	0.00	0.13	0.00	0.46
time (sec)	N/A	2.009	0.024	0.063	0.000	0.000	0.246	0.000	9.104

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	873	855	167	122	0	0	112	0	387
N.S.	1	0.98	0.19	0.14	0.00	0.00	0.13	0.00	0.44
time (sec)	N/A	2.391	0.018	0.062	0.000	0.000	0.282	0.000	9.314

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	986	892	167	122	0	0	112	0	388
N.S.	1	0.90	0.17	0.12	0.00	0.00	0.11	0.00	0.39
time (sec)	N/A	2.463	0.027	0.059	0.000	0.000	0.235	0.000	9.090

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.215	0.003	0.713	0.181	0.238	0.026	0.276	0.021

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	79	96	105	105	88	365	106
N.S.	1	1.00	0.84	1.02	1.12	1.12	0.94	3.88	1.13
time (sec)	N/A	0.313	0.024	0.771	0.182	0.259	0.144	0.282	0.033

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	172	13	13	154	175	13	154
N.S.	1	1.00	11.47	0.87	0.87	10.27	11.67	0.87	10.27
time (sec)	N/A	0.158	0.004	0.764	0.188	0.267	0.054	0.306	8.985

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	182	15	156	156	182	156	156
N.S.	1	1.00	11.38	0.94	9.75	9.75	11.38	9.75	9.75
time (sec)	N/A	0.153	0.004	0.781	0.187	0.236	0.049	0.282	8.856

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	186	15	156	156	185	156	156
N.S.	1	1.00	11.62	0.94	9.75	9.75	11.56	9.75	9.75
time (sec)	N/A	0.159	0.005	0.808	0.200	0.226	0.052	0.286	9.110

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	229	262	0	189	229
N.S.	1	1.00	1.00	10.95	10.90	12.48	0.00	9.00	10.90
time (sec)	N/A	0.164	0.012	0.021	0.201	0.252	0.000	0.500	10.208

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	9	9	10	10	8	11	8
N.S.	1	1.00	0.90	0.90	1.00	1.00	0.80	1.10	0.80
time (sec)	N/A	0.142	0.004	0.724	0.192	0.261	0.056	0.276	9.154

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	17	15	14	13	13	12	18	13
N.S.	1	1.13	1.00	0.93	0.87	0.87	0.80	1.20	0.87
time (sec)	N/A	0.190	0.004	0.735	0.194	0.256	0.084	0.266	8.969

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	17	15	14	13	13	12	15	13
N.S.	1	1.13	1.00	0.93	0.87	0.87	0.80	1.00	0.87
time (sec)	N/A	0.193	0.005	0.733	0.207	0.249	0.093	0.268	8.925

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	17	19	18	47	23	29	0	0
N.S.	1	1.13	1.27	1.20	3.13	1.53	1.93	0.00	0.00
time (sec)	N/A	0.186	0.006	0.817	0.199	0.274	0.537	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	14	13	13	81	87	13	12
N.S.	1	1.00	0.93	0.87	0.87	5.40	5.80	0.87	0.80
time (sec)	N/A	0.141	0.015	0.717	0.199	0.260	0.459	0.268	10.133

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	81	81	87	15	14
N.S.	1	1.00	1.00	0.94	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.149	0.020	0.761	0.208	0.256	0.693	0.287	1.366

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	81	81	87	15	14
N.S.	1	1.00	1.00	0.94	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.155	0.024	0.856	0.217	0.265	0.926	0.299	12.023

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	203	612	143	0	0	0
N.S.	1	1.00	1.00	9.67	29.14	6.81	0.00	0.00	0.00
time (sec)	N/A	0.156	0.009	0.020	0.245	0.387	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	17	20	19	26	46	19	23
N.S.	1	1.00	0.89	1.05	1.00	1.37	2.42	1.00	1.21
time (sec)	N/A	0.143	0.006	0.726	0.203	0.253	0.272	0.263	9.105

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	97	31	35	32	80	54	45
N.S.	1	1.00	3.59	1.15	1.30	1.19	2.96	2.00	1.67
time (sec)	N/A	0.164	0.063	1.445	0.226	0.262	28.849	0.370	9.528

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	C	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	27	116	97	97	35	33	0	54	0
N.S.	1	4.30	3.59	3.59	1.30	1.22	0.00	2.00	0.00
time (sec)	N/A	0.251	0.002	4.743	0.236	0.271	0.000	0.311	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	99	33	35	34	0	58	49
N.S.	1	1.00	3.41	1.14	1.21	1.17	0.00	2.00	1.69
time (sec)	N/A	0.168	0.061	2.509	0.239	0.285	0.000	0.278	9.288

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	108	0	39	42	0	0	0
N.S.	1	1.00	3.00	0.00	1.08	1.17	0.00	0.00	0.00
time (sec)	N/A	0.179	0.172	0.000	0.276	0.296	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	26	29	26	26
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.206	0.002	0.688	0.196	0.269	0.028	0.286	0.027

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	14	12	11	10	10	8	10	10
N.S.	1	1.17	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.198	0.002	0.706	0.203	0.268	0.028	0.289	0.011

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	32	31	98	124	31	32
N.S.	1	1.00	1.00	0.76	0.74	2.33	2.95	0.74	0.76
time (sec)	N/A	0.225	0.014	0.784	0.284	0.257	0.134	0.265	9.211

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	25	38	0	25	54
N.S.	1	1.00	0.92	1.04	1.00	1.52	0.00	1.00	2.16
time (sec)	N/A	0.178	0.067	0.144	0.215	0.258	0.000	0.274	9.245

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	21	25	24	36	0	24	46
N.S.	1	1.00	0.88	1.04	1.00	1.50	0.00	1.00	1.92
time (sec)	N/A	0.169	0.247	0.114	0.199	0.263	0.000	0.290	9.315

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	26	39	35	0	65	51
N.S.	1	1.00	0.96	1.04	1.56	1.40	0.00	2.60	2.04
time (sec)	N/A	0.170	0.084	1.873	0.252	0.271	0.000	0.304	9.114

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	28	0	20	39
N.S.	1	1.00	1.00	1.05	1.00	1.40	0.00	1.00	1.95
time (sec)	N/A	0.160	0.028	0.099	0.193	0.254	0.000	0.263	9.124

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	106	20	19	26	63	19	25
N.S.	1	1.00	5.58	1.05	1.00	1.37	3.32	1.00	1.32
time (sec)	N/A	0.157	0.079	0.795	0.203	0.260	5.275	0.271	9.206

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	108	23	31	27	66	39	26
N.S.	1	1.00	4.91	1.05	1.41	1.23	3.00	1.77	1.18
time (sec)	N/A	0.141	0.029	1.453	0.259	0.255	23.671	0.443	9.022

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	22	32	0	22	43
N.S.	1	1.00	0.95	1.05	1.00	1.45	0.00	1.00	1.95
time (sec)	N/A	0.163	0.052	0.094	0.219	0.256	0.000	0.274	9.017

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	22	21	30	53	21	27
N.S.	1	1.00	0.90	1.05	1.00	1.43	2.52	1.00	1.29
time (sec)	N/A	0.161	0.041	0.754	0.194	0.259	0.431	0.262	9.135

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	28	32	31	56	51	28
N.S.	1	1.00	0.92	1.17	1.33	1.29	2.33	2.12	1.17
time (sec)	N/A	0.158	0.013	1.184	0.244	0.248	1.549	0.401	9.519

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	32	29	53	41	26
N.S.	1	1.00	1.00	1.05	1.45	1.32	2.41	1.86	1.18
time (sec)	N/A	0.140	0.011	1.158	0.229	0.257	1.528	0.272	9.138

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	32	32	0	22	43
N.S.	1	1.00	0.95	1.05	1.45	1.45	0.00	1.00	1.95
time (sec)	N/A	0.156	0.002	0.097	0.238	0.291	0.000	0.300	9.013

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	26	32	30	53	21	27
N.S.	1	1.00	0.90	1.24	1.52	1.43	2.52	1.00	1.29
time (sec)	N/A	0.152	0.002	0.761	0.227	0.242	0.423	0.287	9.175

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	143	20	19	1528	1771	160	1576
N.S.	1	1.00	6.81	0.95	0.90	72.76	84.33	7.62	75.05
time (sec)	N/A	0.294	0.093	0.158	0.195	0.258	0.183	0.304	9.903

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	18	18	441	469	18	418
N.S.	1	1.00	0.90	0.90	0.90	22.05	23.45	0.90	20.90
time (sec)	N/A	0.190	0.017	0.273	0.191	0.267	0.084	0.297	9.396

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	439	441	441	469	18	418
N.S.	1	1.00	0.95	23.11	23.21	23.21	24.68	0.95	22.00
time (sec)	N/A	0.233	0.007	0.964	0.194	0.250	0.075	0.281	9.143

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	127	15	14	456	483	120	438
N.S.	1	1.00	7.94	0.94	0.88	28.50	30.19	7.50	27.38
time (sec)	N/A	0.198	0.037	0.220	0.198	0.237	0.082	0.272	9.690

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	98	14	13	88	97	13	88
N.S.	1	1.00	6.53	0.93	0.87	5.87	6.47	0.87	5.87
time (sec)	N/A	0.154	0.004	0.749	0.199	0.237	0.042	0.277	0.031

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	98	89	88	88	97	13	88
N.S.	1	1.00	6.12	5.56	5.50	5.50	6.06	0.81	5.50
time (sec)	N/A	0.142	0.002	0.768	0.200	0.247	0.034	0.331	0.023

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	115	17	16	458	484	136	440
N.S.	1	1.00	6.39	0.94	0.89	25.44	26.89	7.56	24.44
time (sec)	N/A	0.191	0.037	0.099	0.202	0.250	0.078	0.291	9.575

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	98	13	15	88	97	15	88
N.S.	1	1.00	5.76	0.76	0.88	5.18	5.71	0.88	5.18
time (sec)	N/A	0.166	0.006	0.725	0.200	0.229	0.041	0.378	9.325

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	98	13	88	88	97	15	88
N.S.	1	1.00	7.00	0.93	6.29	6.29	6.93	1.07	6.29
time (sec)	N/A	0.141	0.002	0.694	0.180	0.227	0.035	0.280	0.026

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	98	88	88	88	97	15	88
N.S.	1	1.00	7.00	6.29	6.29	6.29	6.93	1.07	6.29
time (sec)	N/A	0.136	0.002	0.718	0.195	0.235	0.035	0.281	0.023

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	115	17	458	458	484	488	440
N.S.	1	1.00	6.39	0.94	25.44	25.44	26.89	27.11	24.44
time (sec)	N/A	0.225	0.007	0.068	0.185	0.246	0.074	0.290	0.434

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	98	13	88	88	97	88	88
N.S.	1	1.00	7.00	0.93	6.29	6.29	6.93	6.29	6.29
time (sec)	N/A	0.134	0.002	0.722	0.186	0.241	0.035	0.266	0.024

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	14	98	13	88	88	97	88	88
N.S.	1	0.78	5.44	0.72	4.89	4.89	5.39	4.89	4.89
time (sec)	N/A	0.133	0.002	0.734	0.190	0.256	0.036	0.294	0.025

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	88	88	97	88	88
N.S.	1	1.00	7.00	6.36	6.29	6.29	6.93	6.29	6.29
time (sec)	N/A	0.129	0.002	0.683	0.192	0.250	0.034	0.274	0.024

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	30	80	25	66	66	70	24	66
N.S.	1	1.07	2.86	0.89	2.36	2.36	2.50	0.86	2.36
time (sec)	N/A	0.196	0.004	0.724	0.186	0.244	0.034	0.293	0.030

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	32	108	27	187	187	194	88	180
N.S.	1	1.03	3.48	0.87	6.03	6.03	6.26	2.84	5.81
time (sec)	N/A	0.222	0.025	0.882	0.186	0.264	0.048	0.281	0.077

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	31	52	48	228	30	31
N.S.	1	1.00	1.00	0.91	1.53	1.41	6.71	0.88	0.91
time (sec)	N/A	0.163	0.340	0.818	0.300	0.262	18.289	0.267	9.074

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	36	73	33	54	52	0	32	58
N.S.	1	1.03	2.09	0.94	1.54	1.49	0.00	0.91	1.66
time (sec)	N/A	0.169	0.212	1.041	0.336	0.269	0.000	0.286	9.051

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	93	25	77	77	87	24	77
N.S.	1	1.00	3.10	0.83	2.57	2.57	2.90	0.80	2.57
time (sec)	N/A	0.210	0.006	0.801	0.191	0.248	0.036	0.276	9.206

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	32	140	27	280	280	314	105	266
N.S.	1	1.03	4.52	0.87	9.03	9.03	10.13	3.39	8.58
time (sec)	N/A	0.263	0.030	0.196	0.188	0.256	0.060	0.267	9.490

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	40	98	31	80	80	90	30	80
N.S.	1	1.18	2.88	0.91	2.35	2.35	2.65	0.88	2.35
time (sec)	N/A	0.206	0.006	0.732	0.195	0.266	0.038	0.281	0.040

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	42	146	33	289	289	321	126	273
N.S.	1	1.02	3.56	0.80	7.05	7.05	7.83	3.07	6.66
time (sec)	N/A	0.229	0.035	0.082	0.198	0.269	0.062	0.279	9.412

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	244	37	289	289	323	37	270
N.S.	1	1.00	5.30	0.80	6.28	6.28	7.02	0.80	5.87
time (sec)	N/A	0.244	0.045	0.356	0.188	0.383	0.068	0.283	9.318

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	48	248	39	773	773	930	153	753
N.S.	1	1.02	5.28	0.83	16.45	16.45	19.79	3.26	16.02
time (sec)	N/A	0.340	0.076	0.530	0.203	0.252	0.116	0.436	9.340

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	36	31	54	48	190	30	33
N.S.	1	1.00	1.06	0.91	1.59	1.41	5.59	0.88	0.97
time (sec)	N/A	0.180	0.195	0.803	0.303	0.274	40.139	0.288	9.163

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	42	37	71	57	189	36	37
N.S.	1	1.00	0.95	0.84	1.61	1.30	4.30	0.82	0.84
time (sec)	N/A	0.174	0.174	0.817	0.310	0.257	73.303	0.340	9.485

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	49	43	83	72	0	42	73
N.S.	1	1.00	0.98	0.86	1.66	1.44	0.00	0.84	1.46
time (sec)	N/A	0.177	0.267	0.162	0.316	0.259	0.000	0.271	9.204

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	33	18	17	29	29	30	29
N.S.	1	1.00	1.74	0.95	0.89	1.53	1.53	1.58	1.53
time (sec)	N/A	0.160	0.002	0.062	0.191	0.245	0.019	0.276	0.014

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	21	15	14	17	17	22	17
N.S.	1	1.00	1.31	0.94	0.88	1.06	1.06	1.38	1.06
time (sec)	N/A	0.151	0.002	0.106	0.189	0.239	0.018	0.273	0.019

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	96	96	29	86	86	94	28	86
N.S.	1	2.91	2.91	0.88	2.61	2.61	2.85	0.85	2.61
time (sec)	N/A	0.483	0.006	0.783	0.185	0.254	0.032	0.293	9.264

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	96	96	86	86	86	94	28	86
N.S.	1	2.91	2.91	2.61	2.61	2.61	2.85	0.85	2.61
time (sec)	N/A	0.420	0.004	0.786	0.191	0.255	0.035	0.313	0.192

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	57	56	12	12
N.S.	1	1.00	1.00	0.93	0.86	4.07	4.00	0.86	0.86
time (sec)	N/A	0.150	0.005	0.049	0.192	0.234	0.080	0.291	9.369

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	0.93	0.87
time (sec)	N/A	0.154	0.005	0.031	0.184	0.234	0.037	0.289	0.030

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	20	14	15	15	14	18	13
N.S.	1	1.00	1.18	0.82	0.88	0.88	0.82	1.06	0.76
time (sec)	N/A	0.154	0.005	0.035	0.187	0.231	0.041	0.311	0.043

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	23	24	23	23	22	23	23
N.S.	1	1.00	0.58	0.60	0.58	0.58	0.55	0.58	0.58
time (sec)	N/A	0.283	0.040	0.067	0.196	0.267	12.989	0.328	0.064

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	605	597	98	535	0	0	0	0	0
N.S.	1	0.99	0.16	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.852	0.040	0.157	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	55	53	0	83	58	58	75
N.S.	1	1.00	0.87	0.84	0.00	1.32	0.92	0.92	1.19
time (sec)	N/A	0.301	0.022	0.091	0.000	0.317	0.056	0.287	0.111

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	22	38	20	13	12
N.S.	1	1.00	1.00	0.93	1.57	2.71	1.43	0.93	0.86
time (sec)	N/A	0.206	0.006	0.039	0.191	0.305	0.036	0.301	0.023

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	24	22	32	46	29	23	21
N.S.	1	1.00	0.86	0.79	1.14	1.64	1.04	0.82	0.75
time (sec)	N/A	0.223	0.011	0.036	0.193	0.294	0.044	0.291	0.022

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	28	29	38	38	36	28	28
N.S.	1	1.00	0.47	0.49	0.64	0.64	0.61	0.47	0.47
time (sec)	N/A	0.456	0.009	0.050	0.215	0.323	0.061	0.569	0.029

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	11	11
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	1.00	1.00
time (sec)	N/A	0.156	0.005	0.054	0.188	0.300	0.043	0.285	9.457

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	245	132	116	114	223	272	134	124
N.S.	1	2.69	1.45	1.27	1.25	2.45	2.99	1.47	1.36
time (sec)	N/A	0.458	0.065	0.113	0.278	0.340	0.804	0.290	9.235

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	44	40	0	99	49
N.S.	1	1.00	0.92	1.04	1.76	1.60	0.00	3.96	1.96
time (sec)	N/A	0.189	1.358	4.232	0.244	0.509	0.000	0.543	9.146

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	39	39	0	89	39
N.S.	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.70
time (sec)	N/A	0.222	2.163	0.260	0.229	0.327	0.000	0.531	9.106

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	39	39	0	89	39
N.S.	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.70
time (sec)	N/A	0.206	1.065	0.218	0.224	0.294	0.000	0.394	9.269

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	22	37	37	0	87	37
N.S.	1	1.00	0.90	1.05	1.76	1.76	0.00	4.14	1.76
time (sec)	N/A	0.189	0.803	0.174	0.221	0.313	0.000	0.400	9.399

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	17	20	33	33	0	52	19
N.S.	1	1.00	0.89	1.05	1.74	1.74	0.00	2.74	1.00
time (sec)	N/A	0.183	0.062	0.151	0.230	0.306	0.000	0.270	9.108

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	23
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00
time (sec)	N/A	0.187	0.798	0.203	0.221	0.343	0.000	0.000	9.651

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	23
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00
time (sec)	N/A	0.192	0.853	0.205	0.266	0.341	0.000	0.000	10.067

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	23
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00
time (sec)	N/A	0.187	1.013	0.207	0.233	0.331	0.000	0.000	9.856

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	83	74	73	79	97	73	97
N.S.	1	1.00	0.86	0.76	0.75	0.81	1.00	0.75	1.00
time (sec)	N/A	0.311	0.025	0.092	0.272	0.332	0.117	0.299	0.120

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	78	69	68	74	92	68	92
N.S.	1	1.00	0.87	0.77	0.76	0.82	1.02	0.76	1.02
time (sec)	N/A	0.296	0.017	0.073	0.270	0.336	0.121	0.277	0.102

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	72	62	61	67	78	61	85
N.S.	1	1.00	0.94	0.81	0.79	0.87	1.01	0.79	1.10
time (sec)	N/A	0.278	0.021	0.072	0.273	0.274	0.113	0.308	9.259

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	69	57	56	62	75	56	80
N.S.	1	1.00	0.96	0.79	0.78	0.86	1.04	0.78	1.11
time (sec)	N/A	0.262	0.018	0.063	0.291	0.303	0.128	0.282	9.195

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	65	56	55	61	75	55	79
N.S.	1	1.00	0.92	0.79	0.77	0.86	1.06	0.77	1.11
time (sec)	N/A	0.250	0.013	0.059	0.275	0.274	0.116	0.307	0.084

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	69	60	59	65	78	60	83
N.S.	1	1.00	0.92	0.80	0.79	0.87	1.04	0.80	1.11
time (sec)	N/A	0.282	0.016	0.080	0.286	0.293	0.142	0.337	9.296

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	78	65	64	76	87	65	88
N.S.	1	1.00	0.93	0.77	0.76	0.90	1.04	0.77	1.05
time (sec)	N/A	0.291	0.023	0.091	0.283	0.298	0.160	0.291	9.087

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	82	70	69	89	94	70	92
N.S.	1	1.00	0.90	0.77	0.76	0.98	1.03	0.77	1.01
time (sec)	N/A	0.297	0.037	0.082	0.271	0.275	0.164	0.280	0.091

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	228	109	74	0	1202	61	0	128
N.S.	1	0.74	0.36	0.24	0.00	3.92	0.20	0.00	0.42
time (sec)	N/A	0.734	0.014	0.054	0.000	1.058	0.580	0.000	9.856

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	216	101	67	0	1145	3662	0	188
N.S.	1	0.80	0.38	0.25	0.00	4.26	13.61	0.00	0.70
time (sec)	N/A	0.581	0.012	0.046	0.000	1.024	1.587	0.000	0.078

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	217	94	62	0	1190	48	0	183
N.S.	1	0.94	0.41	0.27	0.00	5.17	0.21	0.00	0.80
time (sec)	N/A	0.506	0.013	0.041	0.000	0.996	0.519	0.000	9.478

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	214	90	58	0	1189	46	0	181
N.S.	1	1.08	0.45	0.29	0.00	6.01	0.23	0.00	0.91
time (sec)	N/A	0.510	0.010	0.037	0.000	0.998	0.529	0.000	9.827

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	214	101	51	0	1143	60	0	237
N.S.	1	0.87	0.41	0.21	0.00	4.67	0.24	0.00	0.97
time (sec)	N/A	0.571	0.013	0.058	0.000	1.022	8.492	0.000	9.698

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	226	109	58	0	1245	25507	0	242
N.S.	1	0.80	0.39	0.21	0.00	4.43	90.77	0.00	0.86
time (sec)	N/A	0.601	0.013	0.066	0.000	1.011	18.687	0.000	9.668

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	235	116	62	0	1274	70	0	246
N.S.	1	0.74	0.37	0.20	0.00	4.02	0.22	0.00	0.78
time (sec)	N/A	0.637	0.014	0.064	0.000	0.977	1.748	0.000	9.596

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	87	49	0	83	44	0	252
N.S.	1	1.00	4.58	2.58	0.00	4.37	2.32	0.00	13.26
time (sec)	N/A	0.263	0.031	0.265	0.000	0.258	0.609	0.000	9.547

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	48	57	33	33	47	37	34	38
N.S.	1	1.12	1.33	0.77	0.77	1.09	0.86	0.79	0.88
time (sec)	N/A	0.279	0.018	0.842	0.267	0.267	0.081	0.309	0.034

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	18	21
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	1.06	1.24
time (sec)	N/A	0.217	0.005	0.788	0.240	0.249	0.060	0.289	10.429

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	20	24	30
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.80	0.96	1.20
time (sec)	N/A	0.222	0.005	0.800	0.191	0.253	0.058	0.295	9.754

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	20	19	18	24	14	28	18
N.S.	1	1.00	0.91	0.86	0.82	1.09	0.64	1.27	0.82
time (sec)	N/A	0.167	0.007	0.790	0.183	0.247	0.048	0.318	0.024

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	38	22	21	21	26	22	25
N.S.	1	1.00	1.41	0.81	0.78	0.78	0.96	0.81	0.93
time (sec)	N/A	0.210	0.010	0.810	0.281	0.257	0.068	0.274	0.036

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	17	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.81	0.81
time (sec)	N/A	0.181	0.004	0.783	0.275	0.251	0.037	0.292	0.018

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	23	22	23	23
N.S.	1	1.00	1.00	0.89	0.85	0.85	0.81	0.85	0.85
time (sec)	N/A	0.225	0.006	1.069	0.268	0.244	0.052	0.313	9.439

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	26	25	25	26	29	25
N.S.	1	1.00	1.00	0.67	0.64	0.64	0.67	0.74	0.64
time (sec)	N/A	0.196	0.005	0.057	0.191	0.261	0.127	0.297	9.283

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	23	19	18	18	17	19	18
N.S.	1	1.00	1.05	0.86	0.82	0.82	0.77	0.86	0.82
time (sec)	N/A	0.175	0.003	0.738	0.195	0.254	0.030	0.282	0.015

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.202	0.005	0.793	0.269	0.245	0.037	0.278	9.350

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	24	21	20	26	17	22	10
N.S.	1	1.00	2.00	1.75	1.67	2.17	1.42	1.83	0.83
time (sec)	N/A	0.176	0.010	0.038	0.186	0.262	0.044	0.294	0.044

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	22	19	23	19
N.S.	1	1.00	1.00	0.88	0.84	0.88	0.76	0.92	0.76
time (sec)	N/A	0.218	0.006	0.834	0.189	0.263	0.047	0.296	9.328

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85
time (sec)	N/A	0.189	0.006	0.069	0.264	0.263	0.050	0.318	0.023

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	31	35	55	36	30	35
N.S.	1	1.00	1.00	0.89	1.00	1.57	1.03	0.86	1.00
time (sec)	N/A	0.218	0.012	0.817	0.284	0.251	0.061	0.303	9.521

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	20	17
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.87	0.74
time (sec)	N/A	0.215	0.005	0.046	0.190	0.251	0.085	0.290	9.588

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	19	20	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.83	0.87	0.83
time (sec)	N/A	0.237	0.007	0.046	0.197	0.238	0.051	0.273	0.035

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	28	25	24	23	32	20	23	25
N.S.	1	0.97	0.86	0.83	0.79	1.10	0.69	0.79	0.86
time (sec)	N/A	0.174	0.008	0.833	0.272	0.250	0.054	0.295	0.019

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	42	41	58	49	43	55
N.S.	1	1.00	1.00	0.95	0.93	1.32	1.11	0.98	1.25
time (sec)	N/A	0.438	0.019	0.798	0.278	0.265	0.097	0.294	0.077

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	38	51	38	88
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.11	0.83	1.91
time (sec)	N/A	0.328	0.016	1.439	0.275	0.260	0.102	0.274	0.099

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	34	44	72	41	44	56
N.S.	1	1.00	1.00	1.03	1.33	2.18	1.24	1.33	1.70
time (sec)	N/A	0.372	0.014	1.328	0.190	0.280	0.073	0.309	9.390

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	11	11	10	11	11
N.S.	1	1.00	1.00	0.71	0.65	0.65	0.59	0.65	0.65
time (sec)	N/A	0.143	0.005	0.809	0.262	0.277	0.064	0.301	0.019

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	20	20	34	20	20
N.S.	1	1.00	0.92	0.88	0.83	0.83	1.42	0.83	0.83
time (sec)	N/A	0.187	0.008	0.822	0.287	0.277	0.081	0.292	9.343

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	23	18	17	17	17	20	19
N.S.	1	1.00	1.53	1.20	1.13	1.13	1.13	1.33	1.27
time (sec)	N/A	0.225	0.008	0.788	0.268	0.265	0.069	0.283	0.037

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	17	17	19	17	17
N.S.	1	1.00	1.00	0.90	0.85	0.85	0.95	0.85	0.85
time (sec)	N/A	0.144	0.008	0.774	0.262	0.272	0.070	0.275	0.031

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	27	44	29	60	35
N.S.	1	1.00	0.89	0.76	0.73	1.19	0.78	1.62	0.95
time (sec)	N/A	0.216	0.018	0.838	0.280	0.276	0.077	0.280	9.419

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	21	21	26	21	21
N.S.	1	1.00	1.00	0.85	0.81	0.81	1.00	0.81	0.81
time (sec)	N/A	0.165	0.008	0.766	0.279	0.288	0.039	0.281	0.019

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	16	10	11	10
N.S.	1	1.00	1.00	0.92	0.83	1.33	0.83	0.92	0.83
time (sec)	N/A	0.203	0.004	0.790	0.177	0.274	0.037	0.284	9.458

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	20	21
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.95	1.00
time (sec)	N/A	0.196	0.005	0.043	0.182	0.271	0.066	0.271	0.032

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	34	17	20	20
N.S.	1	1.00	1.00	0.95	0.91	1.55	0.77	0.91	0.91
time (sec)	N/A	0.185	0.007	0.063	0.267	0.265	0.051	0.288	0.020

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	32	19	20	22
N.S.	1	1.00	1.00	0.88	0.83	1.33	0.79	0.83	0.92
time (sec)	N/A	0.184	0.008	0.057	0.265	0.259	0.049	0.268	0.017

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	38	36	34	33	33	39	33	56
N.S.	1	1.06	1.00	0.94	0.92	0.92	1.08	0.92	1.56
time (sec)	N/A	0.182	0.012	0.813	0.276	0.295	0.099	0.265	0.065

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	45	37	28	27	27	29	27	37
N.S.	1	1.22	1.00	0.76	0.73	0.73	0.78	0.73	1.00
time (sec)	N/A	0.179	0.007	0.819	0.272	0.267	0.084	0.290	9.555

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	21	22	23	21
N.S.	1	1.00	1.00	0.76	0.72	0.72	0.76	0.79	0.72
time (sec)	N/A	0.179	0.005	0.873	0.176	0.255	0.056	0.270	9.809

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	14	16	15
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.74	0.84	0.79
time (sec)	N/A	0.173	0.004	0.802	0.185	0.287	0.031	0.270	9.674

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	35	34	34	46	34	36
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.12	0.83	0.88
time (sec)	N/A	0.200	0.010	1.190	0.261	0.250	0.052	0.278	0.026

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	39	32	31	31	34	31	31
N.S.	1	1.00	0.95	0.78	0.76	0.76	0.83	0.76	0.76
time (sec)	N/A	0.204	0.006	1.073	0.267	0.245	0.060	0.269	9.570

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	24	21	20	20	24	23	20
N.S.	1	1.00	0.80	0.70	0.67	0.67	0.80	0.77	0.67
time (sec)	N/A	0.211	0.012	0.849	0.188	0.278	0.069	0.272	9.509

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	31	30	27
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.89	0.86	0.77
time (sec)	N/A	0.207	0.006	0.055	0.178	0.277	0.071	0.275	0.027

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	32	25	24	34	27	34	26
N.S.	1	1.00	0.94	0.74	0.71	1.00	0.79	1.00	0.76
time (sec)	N/A	0.219	0.010	0.787	0.182	0.246	0.065	0.258	9.382

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	43	42	35	33	44	36	33	39
N.S.	1	1.02	1.00	0.83	0.79	1.05	0.86	0.79	0.93
time (sec)	N/A	0.186	0.015	0.763	0.265	0.256	0.058	0.272	9.210

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	38	37	37	46	37	41
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.94	0.76	0.84
time (sec)	N/A	0.347	0.010	1.382	0.268	0.266	0.104	0.301	0.045

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	20	19	19	24	22	19
N.S.	1	1.00	1.00	0.69	0.66	0.66	0.83	0.76	0.66
time (sec)	N/A	0.215	0.007	0.852	0.190	0.255	0.064	0.266	9.623

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	52	43	34	33	33	41	34	50
N.S.	1	1.13	0.93	0.74	0.72	0.72	0.89	0.74	1.09
time (sec)	N/A	0.238	0.014	0.799	0.262	0.253	0.068	0.281	0.053

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	14	15	14	17	10	15	14
N.S.	1	1.00	0.88	0.94	0.88	1.06	0.62	0.94	0.88
time (sec)	N/A	0.193	0.007	0.042	0.204	0.259	0.041	0.328	9.329

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	18	14	12	17	17	14	12	12
N.S.	1	0.86	0.67	0.57	0.81	0.81	0.67	0.57	0.57
time (sec)	N/A	0.155	0.003	0.027	0.189	0.237	0.039	0.261	9.283

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	23	23	17	30	15
N.S.	1	1.00	1.00	1.07	1.53	1.53	1.13	2.00	1.00
time (sec)	N/A	0.195	0.009	0.785	0.185	0.248	0.048	0.287	9.220

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	28	28	3	29	57
N.S.	1	1.00	1.00	0.94	0.90	0.90	0.10	0.94	1.84
time (sec)	N/A	0.210	0.009	0.051	0.268	0.262	0.062	0.275	0.069

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	19	19	19	22	19
N.S.	1	1.00	1.00	0.72	0.76	0.76	0.76	0.88	0.76
time (sec)	N/A	0.224	0.005	0.045	0.188	0.283	0.065	0.277	0.038

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	29	25	24	36	20	26	22
N.S.	1	1.00	0.97	0.83	0.80	1.20	0.67	0.87	0.73
time (sec)	N/A	0.204	0.012	0.049	0.186	0.248	0.042	0.261	9.441

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	18	21
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.78	0.91
time (sec)	N/A	0.214	0.004	0.788	0.268	0.264	0.069	0.271	9.807

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	93	72	77	136	88	74	96
N.S.	1	1.00	0.90	0.70	0.75	1.32	0.85	0.72	0.93
time (sec)	N/A	0.603	0.033	1.092	0.274	0.298	0.292	0.282	9.342

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	34	30	27	25	36	24	25	32
N.S.	1	1.03	0.91	0.82	0.76	1.09	0.73	0.76	0.97
time (sec)	N/A	0.187	0.009	0.793	0.258	0.290	0.055	0.264	0.021

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	29	44	27	30	33
N.S.	1	1.00	1.00	0.85	0.88	1.33	0.82	0.91	1.00
time (sec)	N/A	0.220	0.013	0.822	0.261	0.295	0.065	0.283	9.594

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	23	19	17	26	19
N.S.	1	1.00	1.00	0.80	0.92	0.76	0.68	1.04	0.76
time (sec)	N/A	0.222	0.005	0.845	0.189	0.279	0.044	0.269	0.019

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	31	31	36	31	56
N.S.	1	1.00	1.00	0.89	0.86	0.86	1.00	0.86	1.56
time (sec)	N/A	0.307	0.011	0.793	0.262	0.291	0.094	0.259	0.062

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	33	22	21	23
N.S.	1	1.00	1.00	0.76	0.72	1.14	0.76	0.72	0.79
time (sec)	N/A	0.283	0.012	0.829	0.266	0.311	0.083	0.283	0.026

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	36	35	39	46	36	49
N.S.	1	1.00	1.00	0.78	0.76	0.85	1.00	0.78	1.07
time (sec)	N/A	0.247	0.018	0.063	0.270	0.325	0.083	0.269	9.426

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	26	26	19	18	18	17	20	14
N.S.	1	1.18	1.18	0.86	0.82	0.82	0.77	0.91	0.64
time (sec)	N/A	0.187	0.004	0.835	0.180	0.340	0.040	0.296	0.024

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	18	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	1.06	0.88
time (sec)	N/A	0.220	0.005	0.042	0.185	0.339	0.064	0.262	0.042

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	18	14	16	14
N.S.	1	1.00	1.00	1.07	1.00	1.29	1.00	1.14	1.00
time (sec)	N/A	0.188	0.004	0.773	0.193	0.325	0.052	0.289	9.487

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	25	18	17	17	17	20	17
N.S.	1	1.00	1.32	0.95	0.89	0.89	0.89	1.05	0.89
time (sec)	N/A	0.195	0.005	0.039	0.184	0.340	0.069	0.276	9.620

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	19	17	33
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.83	0.74	1.43
time (sec)	N/A	0.208	0.009	0.072	0.268	0.316	0.090	0.265	0.039

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	57	51	50	50	68	53	58
N.S.	1	1.00	0.90	0.81	0.79	0.79	1.08	0.84	0.92
time (sec)	N/A	0.266	0.020	0.082	0.263	0.327	0.216	0.336	0.093

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	86	67	54	59	103	65	59	71
N.S.	1	1.25	0.97	0.78	0.86	1.49	0.94	0.86	1.03
time (sec)	N/A	0.284	0.032	0.075	0.283	0.298	0.114	0.289	0.083

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	19	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	1.12	0.71	0.76	0.76
time (sec)	N/A	0.164	0.006	0.790	0.270	0.247	0.049	0.265	0.020

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	15	17	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	0.79	0.89	0.89
time (sec)	N/A	0.207	0.005	0.724	0.260	0.260	0.046	0.266	9.578

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	7	28	19
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.78	3.11	2.11
time (sec)	N/A	0.244	0.005	0.798	0.266	0.298	0.072	0.297	9.263

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	12	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.172	0.005	0.936	0.181	0.315	0.036	0.298	0.017

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	58	41	51	66	99	55	48
N.S.	1	1.00	0.89	0.63	0.78	1.02	1.52	0.85	0.74
time (sec)	N/A	0.261	0.025	0.875	0.323	0.309	0.245	0.266	0.063

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	46	24	28	28
N.S.	1	1.00	1.00	1.04	1.00	1.64	0.86	1.00	1.00
time (sec)	N/A	0.217	0.009	0.946	0.350	0.260	0.058	0.266	9.431

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	26	29	26	26
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.218	0.023	0.829	0.349	0.265	0.048	0.310	0.014

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	47	39	24	23	23	26	27	23
N.S.	1	1.52	1.26	0.77	0.74	0.74	0.84	0.87	0.74
time (sec)	N/A	0.281	0.006	0.052	0.226	0.291	0.106	0.300	0.029

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	26	19	23	30
N.S.	1	1.00	1.00	0.96	0.92	1.08	0.79	0.96	1.25
time (sec)	N/A	0.240	0.007	0.831	0.358	0.276	0.068	0.276	9.211

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	35	20	19	23
N.S.	1	1.00	1.00	0.83	1.04	1.52	0.87	0.83	1.00
time (sec)	N/A	0.200	0.009	0.796	0.313	0.246	0.056	0.348	9.253

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	35	20	19	23
N.S.	1	1.00	1.00	0.83	1.04	1.52	0.87	0.83	1.00
time (sec)	N/A	0.232	0.004	0.074	0.301	0.259	0.059	0.273	0.017

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	15	10	13	13
N.S.	1	1.00	1.00	1.08	1.00	1.15	0.77	1.00	1.00
time (sec)	N/A	0.225	0.005	0.037	0.216	0.265	0.042	0.284	9.257

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	193	68	199	4545	138	208	357
N.S.	1	1.00	0.94	0.33	0.97	22.06	0.67	1.01	1.73
time (sec)	N/A	0.480	0.071	0.777	0.318	0.943	0.732	0.272	9.359

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	54	45	38	0	47	44	38	42
N.S.	1	1.20	1.00	0.84	0.00	1.04	0.98	0.84	0.93
time (sec)	N/A	0.273	0.019	0.056	0.000	0.273	0.078	0.418	9.337

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	66	44	41	0	75	46	42	47
N.S.	1	1.12	0.75	0.69	0.00	1.27	0.78	0.71	0.80
time (sec)	N/A	0.280	0.017	0.117	0.000	0.248	0.085	0.739	0.028

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	234	48	0	578003	0	1714	3942
N.S.	1	1.00	1.12	0.23	0.00	2765.56	0.00	8.20	18.86
time (sec)	N/A	0.483	0.154	0.151	0.000	29.017	0.000	1.121	10.016

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	245	54	0	540080	0	1724	3046
N.S.	1	1.00	1.09	0.24	0.00	2411.07	0.00	7.70	13.60
time (sec)	N/A	0.473	0.155	0.720	0.000	80.465	0.000	1.190	0.649

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	57	60	65	190	62	61
N.S.	1	1.00	1.00	1.02	1.07	1.16	3.39	1.11	1.09
time (sec)	N/A	0.215	0.018	0.809	0.184	0.275	0.612	0.270	0.108

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	73	75	84	162	0	85	347
N.S.	1	1.00	0.76	0.78	0.88	1.69	0.00	0.89	3.61
time (sec)	N/A	0.260	0.025	0.849	0.278	0.287	0.000	0.279	0.652

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	228	177	279	5975	0	320	570
N.S.	1	1.00	0.86	0.67	1.06	22.63	0.00	1.21	2.16
time (sec)	N/A	0.668	0.059	0.845	0.275	1.052	0.000	0.290	9.578

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	417	370	226	349	259898	0	406	823
N.S.	1	1.00	0.89	0.54	0.84	623.26	0.00	0.97	1.97
time (sec)	N/A	0.776	0.152	0.841	0.274	23.563	0.000	0.285	10.072

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	24	21	20	26	19	21	12
N.S.	1	1.00	1.50	1.31	1.25	1.62	1.19	1.31	0.75
time (sec)	N/A	0.154	0.007	0.746	0.190	0.249	0.044	0.276	0.029

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	27	24	23	34	20	30	17
N.S.	1	1.00	1.42	1.26	1.21	1.79	1.05	1.58	0.89
time (sec)	N/A	0.150	0.009	0.826	0.179	0.249	0.054	0.262	9.908

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	117	85	72	75	106	92	77	103
N.S.	1	1.21	0.88	0.74	0.77	1.09	0.95	0.79	1.06
time (sec)	N/A	0.307	0.036	0.822	0.268	0.269	0.184	0.291	0.111

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	0.87	0.87
time (sec)	N/A	0.292	0.006	0.801	0.269	0.262	0.051	0.266	10.154

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85
time (sec)	N/A	0.278	0.006	0.785	0.263	0.273	0.054	0.285	0.022

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	25	24	24	29	24	51
N.S.	1	1.00	1.00	0.86	0.83	0.83	1.00	0.83	1.76
time (sec)	N/A	0.304	0.011	0.764	0.269	0.291	0.091	0.333	9.894

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	20	14	15	14	17	10	14	14
N.S.	1	1.43	1.00	1.07	1.00	1.21	0.71	1.00	1.00
time (sec)	N/A	0.180	0.007	0.912	0.264	0.293	0.059	0.298	0.025

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	12	7	11	10
N.S.	1	1.00	1.00	0.92	0.83	1.00	0.58	0.92	0.83
time (sec)	N/A	0.156	0.003	0.791	0.203	0.244	0.038	0.299	0.028

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	10	17
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.91	1.55
time (sec)	N/A	0.209	0.005	0.783	0.263	0.252	0.059	0.282	0.026

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	13	8	11	10
N.S.	1	1.00	1.00	0.92	0.83	1.08	0.67	0.92	0.83
time (sec)	N/A	0.182	0.003	0.779	0.183	0.271	0.035	0.306	0.028

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	12	14	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.00	1.17	1.00
time (sec)	N/A	0.175	0.004	0.719	0.182	0.264	0.052	0.283	0.029

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	16	23
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	0.94	1.35
time (sec)	N/A	0.211	0.007	0.783	0.308	0.290	0.060	0.309	9.919

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	10	14	19
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.77	1.08	1.46
time (sec)	N/A	0.197	0.020	0.810	0.264	0.271	0.063	0.274	10.457

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	22	21	20	26	20	22	20
N.S.	1	1.00	0.79	0.75	0.71	0.93	0.71	0.79	0.71
time (sec)	N/A	0.186	0.013	0.815	0.192	0.252	0.053	0.274	0.046

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	25	26	37	26	43	22
N.S.	1	1.00	1.00	0.78	0.81	1.16	0.81	1.34	0.69
time (sec)	N/A	0.180	0.013	0.784	0.179	0.254	0.061	0.264	9.963

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	28	20	19	19	19	20	25
N.S.	1	1.00	1.22	0.87	0.83	0.83	0.83	0.87	1.09
time (sec)	N/A	0.203	0.006	0.765	0.265	0.279	0.063	0.270	0.030

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	20	36	20	47	28
N.S.	1	1.00	0.92	0.88	0.83	1.50	0.83	1.96	1.17
time (sec)	N/A	0.206	0.010	0.807	0.290	0.268	0.075	0.385	9.875

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	40	39	39	44	39	41
N.S.	1	1.00	1.00	0.82	0.80	0.80	0.90	0.80	0.84
time (sec)	N/A	0.332	0.011	1.234	0.278	0.274	0.105	0.277	9.705

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	19	20	19	19	19	22	19
N.S.	1	1.00	0.76	0.80	0.76	0.76	0.76	0.88	0.76
time (sec)	N/A	0.212	0.007	0.813	0.192	0.282	0.064	0.307	0.029

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	48	47	60	63	60	61
N.S.	1	1.00	0.90	0.80	0.78	1.00	1.05	1.00	1.02
time (sec)	N/A	0.266	0.037	1.022	0.271	0.269	0.125	0.274	0.076

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	7	9	9	5	9	6
N.S.	1	1.00	1.00	0.64	0.82	0.82	0.45	0.82	0.55
time (sec)	N/A	0.147	0.001	0.913	0.185	0.247	0.021	0.283	0.009

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	21	22	23	21
N.S.	1	1.00	1.00	0.76	0.72	0.72	0.76	0.79	0.72
time (sec)	N/A	0.185	0.004	0.821	0.194	0.256	0.057	0.340	9.255

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	50	45	37	37	52	37	37	49
N.S.	1	1.11	1.00	0.82	0.82	1.16	0.82	0.82	1.09
time (sec)	N/A	0.223	0.009	1.072	0.271	0.287	0.062	0.288	0.026

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	28	52	29	29	44
N.S.	1	1.00	1.00	0.91	0.88	1.62	0.91	0.91	1.38
time (sec)	N/A	0.422	0.015	0.847	0.268	0.271	0.119	0.292	9.284

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	159	44	132	106	146	122	124
N.S.	1	1.00	1.07	0.30	0.89	0.72	0.99	0.82	0.84
time (sec)	N/A	0.344	0.053	0.127	0.272	0.260	0.213	0.276	9.105

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	101	54	99	515	61	92	170
N.S.	1	1.00	0.90	0.48	0.88	4.60	0.54	0.82	1.52
time (sec)	N/A	0.332	0.034	0.805	0.277	0.954	0.526	0.272	9.350

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	14	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	1.00	0.86
time (sec)	N/A	0.179	0.003	0.788	0.183	0.248	0.043	0.322	0.030

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	12	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	1.00	0.83
time (sec)	N/A	0.204	0.004	0.864	0.209	0.255	0.047	0.281	9.144

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	21	14	17	15
N.S.	1	1.00	1.00	0.94	0.88	1.24	0.82	1.00	0.88
time (sec)	N/A	0.209	0.003	0.798	0.186	0.230	0.047	0.266	0.016

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	17	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	1.00	0.88
time (sec)	N/A	0.208	0.004	0.791	0.214	0.247	0.044	0.275	0.019

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	24	18	17	16	25	15	23	16
N.S.	1	1.33	1.00	0.94	0.89	1.39	0.83	1.28	0.89
time (sec)	N/A	0.199	0.004	0.768	0.269	0.232	0.046	0.271	0.033

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	14	7	11	10
N.S.	1	1.00	1.00	1.10	1.00	1.40	0.70	1.10	1.00
time (sec)	N/A	0.191	0.004	0.034	0.195	0.264	0.035	0.284	9.397

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	31	30	30	36	33	30
N.S.	1	1.00	1.00	0.74	0.71	0.71	0.86	0.79	0.71
time (sec)	N/A	0.232	0.006	0.052	0.185	0.242	0.073	0.277	0.032

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	38	51	38	88
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.11	0.83	1.91
time (sec)	N/A	0.328	0.013	0.984	0.297	0.248	0.103	0.285	0.001

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	21	19	14	15	15	14	17	8
N.S.	1	1.11	1.00	0.74	0.79	0.79	0.74	0.89	0.42
time (sec)	N/A	0.259	0.003	0.762	0.185	0.240	0.048	0.260	9.434

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	42	40	27	30	30	34	32	26
N.S.	1	1.05	1.00	0.68	0.75	0.75	0.85	0.80	0.65
time (sec)	N/A	0.200	0.004	0.035	0.192	0.248	0.053	0.267	9.305

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	35	33	22	25	25	27	27	21
N.S.	1	1.06	1.00	0.67	0.76	0.76	0.82	0.82	0.64
time (sec)	N/A	0.191	0.003	0.037	0.180	0.252	0.054	0.293	0.019

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	28	26	17	20	20	20	22	16
N.S.	1	1.08	1.00	0.65	0.77	0.77	0.77	0.85	0.62
time (sec)	N/A	0.185	0.005	0.034	0.181	0.232	0.054	0.303	0.042

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	23	21	14	17	17	17	19	13
N.S.	1	1.10	1.00	0.67	0.81	0.81	0.81	0.90	0.62
time (sec)	N/A	0.178	0.003	0.033	0.188	0.234	0.049	0.268	9.420

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	23	21	14	17	17	15	19	8
N.S.	1	1.10	1.00	0.67	0.81	0.81	0.71	0.90	0.38
time (sec)	N/A	0.177	0.002	0.045	0.181	0.253	0.055	0.289	0.046

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	29	27	18	21	21	24	24	17
N.S.	1	1.07	1.00	0.67	0.78	0.78	0.89	0.89	0.63
time (sec)	N/A	0.184	0.003	0.045	0.186	0.275	0.073	0.255	0.057

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	36	34	27	26	30	31	29	22
N.S.	1	1.06	1.00	0.79	0.76	0.88	0.91	0.85	0.65
time (sec)	N/A	0.198	0.003	0.049	0.186	0.278	0.081	0.255	0.022

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	43	41	31	31	39	36	34	26
N.S.	1	1.05	1.00	0.76	0.76	0.95	0.88	0.83	0.63
time (sec)	N/A	0.200	0.004	0.053	0.181	0.285	0.089	0.285	9.327

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	50	48	36	36	44	41	39	32
N.S.	1	1.04	1.00	0.75	0.75	0.92	0.85	0.81	0.67
time (sec)	N/A	0.213	0.004	0.049	0.183	0.285	0.092	0.259	0.022

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	61	54	0	1506	41	0	144
N.S.	1	1.00	0.39	0.34	0.00	9.59	0.26	0.00	0.92
time (sec)	N/A	0.412	0.015	0.071	0.000	1.019	0.115	0.000	9.908

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	61	56	0	1546	41	0	142
N.S.	1	1.00	0.39	0.36	0.00	9.85	0.26	0.00	0.90
time (sec)	N/A	0.378	0.015	0.075	0.000	0.987	0.117	0.000	9.943

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	61	54	0	2271	39	0	142
N.S.	1	1.00	0.32	0.29	0.00	12.08	0.21	0.00	0.76
time (sec)	N/A	0.493	0.014	0.056	0.000	0.976	0.097	0.000	10.041

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	61	56	0	2259	39	0	142
N.S.	1	1.00	0.32	0.30	0.00	12.02	0.21	0.00	0.76
time (sec)	N/A	0.422	0.012	0.058	0.000	0.992	0.099	0.000	10.268

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	663	663	63	69	0	322185	133	0	328
N.S.	1	1.00	0.10	0.10	0.00	485.95	0.20	0.00	0.49
time (sec)	N/A	1.350	0.027	0.261	0.000	1.659	2.158	0.000	9.828

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	663	663	63	69	0	322185	133	0	328
N.S.	1	1.00	0.10	0.10	0.00	485.95	0.20	0.00	0.49
time (sec)	N/A	1.100	0.023	0.075	0.000	1.656	2.139	0.000	0.001

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	C	F	C	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	0	95	67	0	5653	42	0	504
N.S.	1	0.00	0.57	0.40	0.00	33.65	0.25	0.00	3.00
time (sec)	N/A	0.000	0.042	0.822	0.000	0.922	1.041	0.000	10.476

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	322	54	310	141845	384	314	894
N.S.	1	1.00	1.01	0.17	0.97	443.27	1.20	0.98	2.79
time (sec)	N/A	0.485	0.153	0.769	0.266	14.634	28.210	0.276	9.674

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	243	43	275	86139	277	287	556
N.S.	1	1.00	0.84	0.15	0.95	296.01	0.95	0.99	1.91
time (sec)	N/A	0.433	0.067	0.813	0.272	2.481	1.685	0.291	9.198

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	184	32	207	41851	124	213	160
N.S.	1	1.00	0.84	0.15	0.95	191.10	0.57	0.97	0.73
time (sec)	N/A	0.354	0.044	0.795	0.300	1.227	0.442	0.286	9.161

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	200	134	27	169	112	20	179	33
N.S.	1	1.08	0.72	0.15	0.91	0.61	0.11	0.97	0.18
time (sec)	N/A	0.376	0.012	0.797	0.273	0.267	0.076	0.267	0.048

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	404	211	345	352864	0	377	874
N.S.	1	1.00	0.97	0.51	0.83	848.23	0.00	0.91	2.10
time (sec)	N/A	0.681	0.102	0.892	0.272	97.112	0.000	0.293	9.348

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	552	552	524	354	561	0	0	668	2436
N.S.	1	1.00	0.95	0.64	1.02	0.00	0.00	1.21	4.41
time (sec)	N/A	1.016	0.400	0.869	0.281	0.000	0.000	0.550	9.345

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	680	680	738	446	817	0	0	941	1955
N.S.	1	1.00	1.09	0.66	1.20	0.00	0.00	1.38	2.88
time (sec)	N/A	1.217	0.580	0.968	0.287	0.000	0.000	0.367	9.892

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	354	347	100	332	91191	0	345	670
N.S.	1	1.01	0.99	0.29	0.95	261.29	0.00	0.99	1.92
time (sec)	N/A	0.553	0.251	0.849	0.271	12.487	0.000	0.278	0.261

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	328	321	90	318	90963	318	325	391
N.S.	1	1.02	1.00	0.28	0.99	282.49	0.99	1.01	1.21
time (sec)	N/A	0.517	0.219	0.819	0.268	2.742	3.727	0.304	9.255

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	246	224	66	238	43065	155	238	282
N.S.	1	1.02	0.93	0.27	0.99	178.69	0.64	0.99	1.17
time (sec)	N/A	0.405	0.137	0.866	0.272	1.356	0.610	0.300	0.157

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	225	183	46	189	183	39	194	58
N.S.	1	1.11	0.91	0.23	0.94	0.91	0.19	0.96	0.29
time (sec)	N/A	0.397	0.074	0.762	0.272	0.259	0.139	0.276	0.051

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	855	855	558	430	601	0	0	795	1591
N.S.	1	1.00	0.65	0.50	0.70	0.00	0.00	0.93	1.86
time (sec)	N/A	1.234	0.261	0.938	0.280	0.000	0.000	0.459	9.659

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1141	1141	807	534	961	0	0	1145	2246
N.S.	1	1.00	0.71	0.47	0.84	0.00	0.00	1.00	1.97
time (sec)	N/A	1.978	0.486	1.005	0.283	0.000	0.000	6.610	10.589

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1384	1384	996	680	1394	0	0	1557	3256
N.S.	1	1.00	0.72	0.49	1.01	0.00	0.00	1.12	2.35
time (sec)	N/A	2.357	0.732	1.044	0.317	0.000	0.000	0.387	10.849

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	408	388	141	392	95566	413	392	721
N.S.	1	1.04	0.98	0.36	0.99	242.55	1.05	0.99	1.83
time (sec)	N/A	0.652	0.249	0.810	0.282	13.227	131.883	0.319	0.275

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	374	358	126	364	91420	374	358	676
N.S.	1	1.04	0.99	0.35	1.01	253.94	1.04	0.99	1.88
time (sec)	N/A	0.607	0.225	0.838	0.275	7.152	14.652	0.282	0.268

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	280	249	86	269	43180	192	256	315
N.S.	1	1.05	0.94	0.32	1.01	162.33	0.72	0.96	1.18
time (sec)	N/A	0.478	0.135	0.841	0.291	2.106	1.193	0.280	0.175

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	250	200	57	212	255	63	204	80
N.S.	1	1.14	0.91	0.26	0.97	1.16	0.29	0.93	0.37
time (sec)	N/A	0.432	0.058	0.802	0.267	0.316	0.248	0.274	0.053

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1352	1352	835	677	1015	0	0	1311	2720
N.S.	1	1.00	0.62	0.50	0.75	0.00	0.00	0.97	2.01
time (sec)	N/A	1.844	0.448	1.019	0.310	0.000	0.000	0.345	10.551

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1830	1830	1115	829	1564	0	0	1809	3572
N.S.	1	1.00	0.61	0.45	0.85	0.00	0.00	0.99	1.95
time (sec)	N/A	3.134	0.804	1.100	0.338	0.000	0.000	46.908	11.176

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2204	2204	1338	1008	2198	0	0	2200	6280
N.S.	1	1.00	0.61	0.46	1.00	0.00	0.00	1.00	2.85
time (sec)	N/A	3.507	1.474	1.210	0.336	0.000	0.000	0.436	12.674

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	33	33	29	28	28	34	28	30
N.S.	1	1.03	1.03	0.91	0.88	0.88	1.06	0.88	0.94
time (sec)	N/A	0.182	0.007	1.277	0.280	0.319	0.051	0.277	0.025

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	33	33	29	28	28	34	28	30
N.S.	1	1.03	1.03	0.91	0.88	0.88	1.06	0.88	0.94
time (sec)	N/A	0.212	0.005	0.743	0.276	0.274	0.057	0.287	0.020

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	36	31	27	26	26	36	26	30
N.S.	1	1.12	0.97	0.84	0.81	0.81	1.12	0.81	0.94
time (sec)	N/A	0.182	0.008	1.253	0.296	0.297	0.052	0.274	9.175

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	36	31	27	26	26	36	26	30
N.S.	1	1.12	0.97	0.84	0.81	0.81	1.12	0.81	0.94
time (sec)	N/A	0.215	0.004	0.868	0.273	0.268	0.056	0.336	0.020

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	29	35	45	39	44	34
N.S.	1	1.00	0.93	0.64	0.78	1.00	0.87	0.98	0.76
time (sec)	N/A	0.209	0.015	0.887	0.282	0.307	0.048	0.280	0.084

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	29	35	45	39	44	34
N.S.	1	1.00	0.93	0.64	0.78	1.00	0.87	0.98	0.76
time (sec)	N/A	0.214	0.004	0.031	0.271	0.335	0.055	0.284	0.026

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	23	7	17	17	15	15	6
N.S.	1	1.00	3.83	1.17	2.83	2.83	2.50	2.50	1.00
time (sec)	N/A	0.127	0.003	0.770	0.175	0.334	0.046	0.297	9.091

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	23	14	17	17	15	19	6
N.S.	1	1.00	1.10	0.67	0.81	0.81	0.71	0.90	0.29
time (sec)	N/A	0.150	0.002	0.798	0.179	0.347	0.043	0.303	0.083

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	9	24	22	9	9
N.S.	1	1.00	0.85	0.77	0.69	1.85	1.69	0.69	0.69
time (sec)	N/A	0.133	0.002	0.797	0.188	0.258	0.044	0.267	9.355

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	81	11	13	57	24	22	57	9
N.S.	1	6.23	0.85	1.00	4.38	1.85	1.69	4.38	0.69
time (sec)	N/A	0.184	0.004	0.941	0.186	0.252	0.137	0.275	0.015

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	83	78	67	66	66	85	68	94
N.S.	1	1.20	1.13	0.97	0.96	0.96	1.23	0.99	1.36
time (sec)	N/A	0.255	0.012	0.826	0.261	0.259	0.126	0.296	0.057

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	81	78	63	66	66	85	68	94
N.S.	1	1.17	1.13	0.91	0.96	0.96	1.23	0.99	1.36
time (sec)	N/A	0.275	0.004	0.058	0.257	0.264	0.129	0.287	0.021

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	31	21	20	20	20	21	20
N.S.	1	1.00	1.29	0.88	0.83	0.83	0.83	0.88	0.83
time (sec)	N/A	0.181	0.005	0.783	0.179	0.268	0.031	0.275	0.022

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	21	20	20	19	21	20
N.S.	1	1.00	0.96	0.81	0.77	0.77	0.73	0.81	0.77
time (sec)	N/A	0.174	0.004	0.786	0.184	0.270	0.029	0.290	0.019

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	24	22	16	15	18	15	15	17
N.S.	1	1.41	1.29	0.94	0.88	1.06	0.88	0.88	1.00
time (sec)	N/A	0.137	0.000	0.014	0.183	0.235	0.018	0.266	0.014

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	16	20	18	15	20	17
N.S.	1	1.00	0.92	0.67	0.83	0.75	0.62	0.83	0.71
time (sec)	N/A	0.147	0.001	0.020	0.178	0.222	0.019	0.279	0.012

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	24	23	22	25	20	23	28
N.S.	1	1.00	1.09	1.05	1.00	1.14	0.91	1.05	1.27
time (sec)	N/A	0.166	0.005	0.773	0.273	0.262	0.064	0.295	0.025

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	27	16	15	15	15	18	15
N.S.	1	1.00	1.59	0.94	0.88	0.88	0.88	1.06	0.88
time (sec)	N/A	0.161	0.004	0.806	0.193	0.250	0.059	0.294	9.175

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	25	19	14	13	13	12	16	13
N.S.	1	1.32	1.00	0.74	0.68	0.68	0.63	0.84	0.68
time (sec)	N/A	0.182	0.005	0.806	0.192	0.253	0.040	0.284	0.045

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	17	11	10	10	8	13	10
N.S.	1	1.00	1.42	0.92	0.83	0.83	0.67	1.08	0.83
time (sec)	N/A	0.146	0.006	0.814	0.182	0.255	0.036	0.274	0.028

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	11	11	10	10	8	11	10
N.S.	1	1.00	1.10	1.10	1.00	1.00	0.80	1.10	1.00
time (sec)	N/A	0.151	0.006	0.791	0.188	0.257	0.063	0.280	0.033

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	18	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	1.06	0.88
time (sec)	N/A	0.174	0.005	0.810	0.194	0.251	0.058	0.279	9.603

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	18	21
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.78	0.91
time (sec)	N/A	0.173	0.005	0.798	0.261	0.256	0.061	0.269	9.448

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.87	0.87
time (sec)	N/A	0.147	0.004	0.031	0.186	0.236	0.038	0.271	0.025

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	20	20	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.95	0.95	0.81
time (sec)	N/A	0.178	0.006	0.043	0.181	0.240	0.062	0.271	9.553

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	9	11	9	9	7	9	10
N.S.	1	1.00	0.90	1.10	0.90	0.90	0.70	0.90	1.00
time (sec)	N/A	0.159	0.005	0.033	0.179	0.238	0.032	0.276	0.026

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	17	17	20	20	17
N.S.	1	1.00	1.00	0.72	0.68	0.68	0.80	0.80	0.68
time (sec)	N/A	0.179	0.006	0.041	0.184	0.235	0.062	0.290	0.070

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	12	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	0.86	1.00	1.00
time (sec)	N/A	0.171	0.004	0.806	0.264	0.229	0.047	0.299	0.019

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	14	13	11	16	17	11	20
N.S.	1	1.00	1.08	1.00	0.85	1.23	1.31	0.85	1.54
time (sec)	N/A	0.147	0.006	0.829	0.185	0.230	0.055	0.280	9.884

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	26	26	25	20	27	25
N.S.	1	1.00	0.96	1.00	1.00	0.96	0.77	1.04	0.96
time (sec)	N/A	0.196	0.006	0.812	0.184	0.252	0.082	0.275	9.334

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	4	5	4	4	3	5	4
N.S.	1	1.00	0.67	0.83	0.67	0.67	0.50	0.83	0.67
time (sec)	N/A	0.141	0.001	0.023	0.191	0.247	0.020	0.310	0.010

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	24	20	19	20	25	19	27	18
N.S.	1	1.20	1.00	0.95	1.00	1.25	0.95	1.35	0.90
time (sec)	N/A	0.161	0.005	0.810	0.197	0.271	0.045	0.289	0.022

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	43	40	28	27	27	31	28	44
N.S.	1	1.13	1.05	0.74	0.71	0.71	0.82	0.74	1.16
time (sec)	N/A	0.167	0.008	0.770	0.280	0.252	0.060	0.292	0.095

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	15	15	14	16	15
N.S.	1	1.00	1.06	0.94	0.88	0.88	0.82	0.94	0.88
time (sec)	N/A	0.151	0.003	0.808	0.190	0.234	0.029	0.263	0.018

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	36	36	22	21	21	22	22	25
N.S.	1	1.16	1.16	0.71	0.68	0.68	0.71	0.71	0.81
time (sec)	N/A	0.155	0.005	0.835	0.277	0.252	0.066	0.297	9.316

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	25	19	14	17	15	17	17	13
N.S.	1	1.32	1.00	0.74	0.89	0.79	0.89	0.89	0.68
time (sec)	N/A	0.167	0.006	0.835	0.192	0.244	0.050	0.291	0.061

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	9	9	9	8	11	9
N.S.	1	1.00	0.82	0.82	0.82	0.82	0.73	1.00	0.82
time (sec)	N/A	0.143	0.003	0.796	0.203	0.252	0.035	0.303	9.010

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	12	14	12	12
N.S.	1	1.00	1.00	0.72	0.67	0.67	0.78	0.67	0.67
time (sec)	N/A	0.162	0.005	0.078	0.281	0.260	0.070	0.293	8.844

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	39	46	83	46	52	45
N.S.	1	1.00	0.96	0.85	1.00	1.80	1.00	1.13	0.98
time (sec)	N/A	0.181	0.012	0.779	0.195	0.257	0.094	0.274	8.875

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	8	8	7	9	8
N.S.	1	1.00	0.83	0.75	0.67	0.67	0.58	0.75	0.67
time (sec)	N/A	0.131	0.002	0.793	0.190	0.237	0.032	0.304	9.268

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	27	23	23	34	20	25	17
N.S.	1	1.00	1.29	1.10	1.10	1.62	0.95	1.19	0.81
time (sec)	N/A	0.140	0.006	0.812	0.196	0.268	0.049	0.309	0.017

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	21	12	15	17
N.S.	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	0.89
time (sec)	N/A	0.135	0.006	0.839	0.285	0.256	0.052	0.268	0.016

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	8	7	9	6
N.S.	1	1.00	1.00	0.70	0.80	0.80	0.70	0.90	0.60
time (sec)	N/A	0.123	0.002	0.795	0.189	0.248	0.018	0.294	0.041

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	20	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	2.00	1.00	1.00
time (sec)	N/A	0.132	0.002	0.842	0.282	0.239	0.052	0.287	0.023

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.137	0.003	0.802	0.268	0.265	0.060	0.284	8.861

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	16
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84
time (sec)	N/A	0.145	0.004	1.238	0.275	0.241	0.049	0.275	8.792

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	17	16	17
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.77	0.73	0.77
time (sec)	N/A	0.152	0.001	0.823	0.188	0.232	0.016	0.281	0.019

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	15	16	17
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.68	0.73	0.77
time (sec)	N/A	0.154	0.001	0.806	0.187	0.239	0.016	0.292	0.017

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	10	14	15
N.S.	1	1.00	1.00	0.94	0.88	0.94	0.62	0.88	0.94
time (sec)	N/A	0.165	0.001	0.025	0.200	0.252	0.026	0.262	0.015

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	40	37	31	30	30	39	30	30
N.S.	1	1.08	1.00	0.84	0.81	0.81	1.05	0.81	0.81
time (sec)	N/A	0.182	0.008	1.357	0.273	0.263	0.053	0.275	0.026

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	12	13
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.93
time (sec)	N/A	0.154	0.000	0.790	0.196	0.240	0.017	0.272	0.013

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	7	9	9	7	9	6
N.S.	1	1.00	1.00	0.64	0.82	0.82	0.64	0.82	0.55
time (sec)	N/A	0.134	0.000	0.773	0.186	0.243	0.018	0.271	0.010

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	28	25	20	19	19	20	20	25
N.S.	1	1.12	1.00	0.80	0.76	0.76	0.80	0.80	1.00
time (sec)	N/A	0.159	0.007	0.807	0.280	0.263	0.067	0.286	9.095

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	26	25	20	19	19	19	20	25
N.S.	1	1.04	1.00	0.80	0.76	0.76	0.76	0.80	1.00
time (sec)	N/A	0.155	0.004	0.795	0.271	0.282	0.061	0.316	0.025

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	26	25	20	19	19	19	20	25
N.S.	1	1.04	1.00	0.80	0.76	0.76	0.76	0.80	1.00
time (sec)	N/A	0.154	0.005	0.811	0.276	0.248	0.055	0.287	8.869

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	7	7	8	7	12	5	8	7
N.S.	1	0.78	0.78	0.89	0.78	1.33	0.56	0.89	0.78
time (sec)	N/A	0.146	0.003	0.793	0.214	0.245	0.032	0.300	0.012

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.151	0.010	0.072	0.279	0.244	0.071	0.291	0.033

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	40	37	31	30	30	44	30	30
N.S.	1	1.08	1.00	0.84	0.81	0.81	1.19	0.81	0.81
time (sec)	N/A	0.180	0.011	1.368	0.284	0.235	0.058	0.287	0.027

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	15	19	14	16	13
N.S.	1	1.00	1.00	0.93	1.00	1.27	0.93	1.07	0.87
time (sec)	N/A	0.163	0.002	0.031	0.204	0.241	0.037	0.276	8.823

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	15	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	1.00	0.67	0.93	0.87
time (sec)	N/A	0.161	0.001	0.028	0.191	0.235	0.030	0.269	0.013

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	19	11	12	13	11	8	13	11
N.S.	1	1.73	1.00	1.09	1.18	1.00	0.73	1.18	1.00
time (sec)	N/A	0.161	0.003	0.838	0.197	0.250	0.040	0.270	0.033

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	27	19	18	17	17	17	17	17
N.S.	1	1.23	0.86	0.82	0.77	0.77	0.77	0.77	0.77
time (sec)	N/A	0.156	0.001	0.062	0.190	0.231	0.020	0.283	0.017

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	19	19	19	9	9
N.S.	1	1.00	1.00	0.91	1.73	1.73	1.73	0.82	0.82
time (sec)	N/A	0.128	0.001	0.800	0.185	0.234	0.018	0.261	0.081

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	16	12	13	13	12	14	11
N.S.	1	1.00	1.23	0.92	1.00	1.00	0.92	1.08	0.85
time (sec)	N/A	0.150	0.003	0.811	0.202	0.259	0.029	0.297	0.020

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	18	14	16	12
N.S.	1	1.00	1.00	0.94	0.88	1.12	0.88	1.00	0.75
time (sec)	N/A	0.156	0.003	0.827	0.192	0.267	0.043	0.282	0.021

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	31	26	24	25	39	24	27	23
N.S.	1	1.15	0.96	0.89	0.93	1.44	0.89	1.00	0.85
time (sec)	N/A	0.183	0.012	0.042	0.195	0.252	0.048	0.284	8.809

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	15	15	12	16	15
N.S.	1	1.00	1.06	0.94	0.88	0.88	0.71	0.94	0.88
time (sec)	N/A	0.151	0.003	0.816	0.186	0.276	0.029	0.267	0.015

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	20	15	17	16
N.S.	1	1.00	1.00	0.94	0.89	1.11	0.83	0.94	0.89
time (sec)	N/A	0.168	0.001	0.032	0.188	0.249	0.029	0.264	0.015

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	14	14	13
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.78	0.78	0.72
time (sec)	N/A	0.153	0.001	0.049	0.266	0.257	0.018	0.291	0.013

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	34	46	31	25	23
N.S.	1	1.00	1.00	0.96	1.48	2.00	1.35	1.09	1.00
time (sec)	N/A	0.181	0.007	0.789	0.192	0.275	0.048	0.277	0.019

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	12	13	12	15	10	32	12
N.S.	1	1.00	0.75	0.81	0.75	0.94	0.62	2.00	0.75
time (sec)	N/A	0.139	0.007	0.851	0.270	0.288	0.052	0.271	9.022

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	30	26	25	25	24	26	25
N.S.	1	1.00	1.03	0.90	0.86	0.86	0.83	0.90	0.86
time (sec)	N/A	0.184	0.007	0.804	0.192	0.249	0.030	0.281	0.015

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	12	13
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.81
time (sec)	N/A	0.160	0.000	0.793	0.197	0.243	0.020	0.279	0.012

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	13	17	25	14	12	17
N.S.	1	1.00	1.00	0.76	1.00	1.47	0.82	0.71	1.00
time (sec)	N/A	0.163	0.010	0.859	0.279	0.251	0.065	0.280	0.018

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	50	47	43	0	190	151	46	42
N.S.	1	1.06	1.00	0.91	0.00	4.04	3.21	0.98	0.89
time (sec)	N/A	0.200	0.019	1.286	0.000	0.272	0.163	0.280	0.055

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	60	61	61	0	240	294	60	82
N.S.	1	1.05	1.07	1.07	0.00	4.21	5.16	1.05	1.44
time (sec)	N/A	0.214	0.018	1.307	0.000	0.264	0.184	0.274	0.037

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	190	39	36	0	53	24	147	101
N.S.	1	1.01	0.21	0.19	0.00	0.28	0.13	0.78	0.54
time (sec)	N/A	0.415	0.032	0.108	0.000	0.288	0.304	0.653	0.057

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	64	27	28	65	65	60	30	27
N.S.	1	1.07	0.45	0.47	1.08	1.08	1.00	0.50	0.45
time (sec)	N/A	0.433	0.010	0.054	0.195	0.278	0.090	0.290	8.996

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	B	B	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	0	27	28	65	65	60	197	27
N.S.	1	0.00	1.00	1.04	2.41	2.41	2.22	7.30	1.00
time (sec)	N/A	0.000	0.006	0.110	0.232	0.273	0.149	0.285	0.025

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	B	B	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	0	27	28	65	65	60	111	27
N.S.	1	0.00	1.00	1.04	2.41	2.41	2.22	4.11	1.00
time (sec)	N/A	0.000	0.005	0.089	0.192	0.267	0.112	0.309	0.023

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [411] had the largest ratio of [1.1111100000000004]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.14	24	0.167
2	A	2	2	0.83	29	0.069
3	A	2	2	1.00	29	0.069
4	A	2	2	1.00	29	0.069
5	A	2	2	0.40	27	0.074
6	A	2	2	1.00	29	0.069
7	A	2	2	1.00	29	0.069
8	A	2	2	1.00	29	0.069
9	A	4	3	1.11	27	0.111
10	A	4	3	1.14	27	0.111
11	A	1	1	1.00	25	0.040
12	A	10	9	1.03	27	0.333
13	A	11	10	1.09	27	0.370
14	A	12	11	1.12	27	0.407
15	A	3	3	1.00	46	0.065
16	A	3	3	1.00	46	0.065
17	A	1	1	1.00	44	0.023
18	A	2	2	1.00	46	0.043
19	A	2	2	1.00	46	0.043
20	A	2	2	1.00	46	0.043
21	A	2	2	1.00	11	0.182
22	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	1	1	1.00	7	0.143
24	A	3	3	1.00	13	0.231
25	A	6	5	1.18	11	0.455
26	A	8	7	1.02	16	0.438
27	A	9	8	0.97	9	0.889
28	A	2	2	1.00	7	0.286
29	A	3	3	1.00	13	0.231
30	A	3	3	1.11	11	0.273
31	A	4	3	1.00	16	0.188
32	A	2	2	1.26	9	0.222
33	A	4	3	1.02	29	0.103
34	A	4	3	1.11	29	0.103
35	A	4	3	1.27	29	0.103
36	A	1	1	1.00	27	0.037
37	A	10	9	1.05	29	0.310
38	A	12	11	1.07	29	0.379
39	A	4	3	1.03	32	0.094
40	A	4	3	1.06	32	0.094
41	A	4	3	1.26	32	0.094
42	A	1	1	1.00	30	0.033
43	A	4	3	1.08	32	0.094
44	A	6	5	1.10	32	0.156
45	A	2	2	1.00	17	0.118
46	A	2	2	1.00	17	0.118
47	A	2	2	1.00	17	0.118
48	A	1	1	1.00	15	0.067
49	A	15	14	1.20	17	0.824
50	A	19	18	1.21	17	1.059
51	A	2	2	1.00	17	0.118
52	A	2	2	1.00	17	0.118
53	A	2	2	1.00	17	0.118
54	A	1	1	1.00	15	0.067
55	A	14	13	1.15	17	0.765
56	A	18	17	1.10	17	1.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	2	2	1.00	22	0.091
58	A	2	2	1.00	22	0.091
59	A	2	2	1.00	22	0.091
60	A	1	1	1.00	20	0.050
61	A	15	14	1.19	22	0.636
62	A	19	18	1.17	22	0.818
63	A	2	2	1.00	51	0.039
64	A	2	2	1.00	51	0.039
65	A	2	2	1.00	49	0.041
66	A	2	2	1.00	51	0.039
67	A	2	2	1.00	51	0.039
68	A	2	2	1.00	51	0.039
69	A	2	2	1.00	13	0.154
70	A	5	5	1.00	19	0.263
71	A	7	7	1.00	19	0.368
72	A	5	5	1.00	19	0.263
73	A	1	1	1.00	17	0.059
74	A	2	2	1.00	19	0.105
75	A	2	2	1.00	19	0.105
76	A	2	2	1.00	19	0.105
77	B	2	2	2.69	17	0.118
78	A	5	4	0.95	15	0.267
79	A	4	3	0.94	15	0.200
80	A	6	5	0.95	13	0.385
81	A	3	2	1.00	11	0.182
82	A	7	6	0.97	15	0.400
83	A	5	4	1.18	15	0.267
84	A	6	5	1.11	15	0.333
85	A	3	2	1.00	13	0.154
86	A	4	3	0.97	13	0.231
87	A	5	4	1.03	13	0.308
88	A	3	2	1.00	19	0.105
89	A	3	2	1.00	11	0.182
90	A	4	3	0.95	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	5	4	1.00	11	0.364
92	A	3	2	1.00	13	0.154
93	A	4	3	0.95	13	0.231
94	A	5	4	1.00	13	0.308
95	A	3	2	1.00	11	0.182
96	A	4	3	1.00	11	0.273
97	A	5	4	1.11	11	0.364
98	A	5	4	1.10	15	0.267
99	A	4	3	1.10	13	0.231
100	A	5	4	0.93	17	0.235
101	A	6	5	0.94	19	0.263
102	A	6	5	0.95	17	0.294
103	A	5	4	0.97	17	0.235
104	A	14	13	0.94	17	0.765
105	A	12	11	0.95	15	0.733
106	A	10	9	0.94	13	0.692
107	A	5	4	1.08	17	0.235
108	A	4	3	1.00	17	0.176
109	A	5	4	0.97	17	0.235
110	A	5	4	0.96	17	0.235
111	A	4	3	0.97	17	0.176
112	A	5	4	0.96	15	0.267
113	A	10	9	1.03	13	0.692
114	A	5	4	1.01	17	0.235
115	A	4	3	1.00	17	0.176
116	A	4	3	1.02	22	0.136
117	A	4	3	0.71	22	0.136
118	A	4	3	0.72	22	0.136
119	A	1	1	1.00	20	0.050
120	A	4	3	1.00	22	0.136
121	A	6	5	0.98	22	0.227
122	A	8	7	1.01	22	0.318
123	A	2	2	1.00	24	0.083
124	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	2	2	1.00	24	0.083
126	A	2	2	1.00	22	0.091
127	A	8	7	1.04	24	0.292
128	A	11	10	1.00	24	0.417
129	A	14	13	1.03	24	0.542
130	A	2	2	1.00	26	0.077
131	A	2	2	1.00	26	0.077
132	A	2	2	1.00	26	0.077
133	A	2	2	1.00	24	0.083
134	A	10	9	1.06	26	0.346
135	A	13	12	0.98	26	0.462
136	A	2	2	0.97	46	0.043
137	A	2	2	0.97	46	0.043
138	A	2	2	0.96	46	0.043
139	A	2	2	0.97	44	0.045
140	A	2	2	0.97	42	0.048
141	A	2	2	0.97	46	0.043
142	A	2	2	0.97	46	0.043
143	A	2	2	1.02	26	0.077
144	A	2	2	1.01	26	0.077
145	A	2	2	0.98	26	0.077
146	A	2	2	0.97	26	0.077
147	A	2	2	0.98	24	0.083
148	A	2	2	0.98	22	0.091
149	A	2	2	0.97	26	0.077
150	A	2	2	1.01	26	0.077
151	A	2	2	0.95	26	0.077
152	A	2	2	0.95	26	0.077
153	A	2	2	0.94	26	0.077
154	A	2	2	0.98	26	0.077
155	A	2	2	0.97	26	0.077
156	A	2	2	0.98	26	0.077
157	A	2	2	0.90	26	0.077
158	A	2	2	1.00	52	0.038

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	5	5	1.00	52	0.096
160	A	1	1	1.00	18	0.056
161	A	4	3	1.00	23	0.130
162	A	4	3	1.00	23	0.130
163	A	4	3	1.00	29	0.103
164	A	1	1	1.00	18	0.056
165	A	5	4	1.13	20	0.200
166	A	5	4	1.13	20	0.200
167	A	5	4	1.13	22	0.182
168	A	1	1	1.00	18	0.056
169	A	4	3	1.00	23	0.130
170	A	4	3	1.00	23	0.130
171	A	4	3	1.00	29	0.103
172	A	1	1	1.00	18	0.056
173	A	1	1	1.00	25	0.040
174	C	1	1	4.30	38	0.026
175	A	1	1	1.00	27	0.037
176	A	1	1	1.00	31	0.032
177	A	2	2	1.00	54	0.037
178	A	3	3	1.17	54	0.056
179	A	5	5	1.00	54	0.093
180	A	1	1	1.00	30	0.033
181	A	1	1	1.00	29	0.034
182	A	1	1	1.00	28	0.036
183	A	1	1	1.00	21	0.048
184	A	1	1	1.00	20	0.050
185	A	1	1	1.00	21	0.048
186	A	1	1	1.00	26	0.038
187	A	1	1	1.00	25	0.040
188	A	2	2	1.00	26	0.077
189	A	2	2	1.00	24	0.083
190	A	1	1	1.00	24	0.042
191	A	1	1	1.00	23	0.043
192	A	1	1	1.00	30	0.033

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	1	1	1.00	29	0.034
194	A	1	1	1.00	28	0.036
195	A	1	1	1.00	21	0.048
196	A	1	1	1.00	20	0.050
197	A	3	2	1.00	21	0.095
198	A	1	1	1.00	26	0.038
199	A	1	1	1.00	25	0.040
200	A	3	3	1.00	26	0.115
201	A	2	2	1.00	22	0.091
202	A	1	1	1.00	24	0.042
203	A	2	2	1.00	23	0.087
204	A	2	2	0.78	23	0.087
205	A	1	1	1.00	19	0.053
206	A	3	2	1.07	22	0.091
207	A	3	2	1.03	23	0.087
208	A	3	2	1.00	22	0.091
209	A	3	2	1.03	23	0.087
210	A	3	2	1.00	24	0.083
211	A	3	2	1.03	25	0.080
212	A	3	2	1.18	31	0.065
213	A	3	2	1.02	32	0.062
214	A	3	2	1.00	35	0.057
215	A	3	2	1.02	36	0.056
216	A	3	2	1.00	24	0.083
217	A	3	2	1.00	31	0.065
218	A	3	2	1.00	35	0.057
219	A	1	1	1.00	22	0.045
220	A	1	1	1.00	18	0.056
221	B	3	3	2.91	26	0.115
222	B	2	2	2.91	28	0.071
223	A	1	1	1.00	18	0.056
224	A	1	1	1.00	20	0.050
225	A	1	1	1.00	21	0.048
226	A	4	4	1.00	52	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	2	2	0.99	38	0.053
228	A	2	2	1.00	32	0.062
229	A	4	4	1.00	33	0.121
230	A	4	4	1.00	34	0.118
231	A	9	8	1.00	43	0.186
232	A	1	1	1.00	16	0.062
233	B	2	2	2.69	25	0.080
234	A	1	1	1.00	56	0.018
235	A	1	1	1.00	51	0.020
236	A	1	1	1.00	49	0.020
237	A	1	1	1.00	46	0.022
238	A	2	2	1.00	48	0.042
239	A	1	1	1.00	49	0.020
240	A	1	1	1.00	48	0.021
241	A	1	1	1.00	48	0.021
242	A	2	2	1.00	35	0.057
243	A	2	2	1.00	35	0.057
244	A	2	2	1.00	35	0.057
245	A	2	2	1.00	33	0.061
246	A	2	2	1.00	32	0.062
247	A	2	2	1.00	35	0.057
248	A	2	2	1.00	35	0.057
249	A	2	2	1.00	35	0.057
250	A	2	2	0.74	35	0.057
251	A	2	2	0.80	35	0.057
252	A	2	2	0.94	33	0.061
253	A	2	2	1.08	32	0.062
254	A	2	2	0.87	35	0.057
255	A	2	2	0.80	35	0.057
256	A	2	2	0.74	35	0.057
257	A	3	2	1.00	40	0.050
258	A	4	4	1.12	20	0.200
259	A	3	3	1.00	20	0.150
260	A	3	3	1.00	20	0.150

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	2	2	1.00	16	0.125
262	A	2	2	1.00	22	0.091
263	A	2	2	1.00	21	0.095
264	A	3	3	1.00	26	0.115
265	A	2	2	1.00	20	0.100
266	A	2	2	1.00	11	0.182
267	A	3	3	1.00	22	0.136
268	A	2	2	1.00	21	0.095
269	A	3	3	1.00	25	0.120
270	A	4	4	1.00	22	0.182
271	A	6	6	1.00	31	0.194
272	A	3	3	1.00	21	0.143
273	A	3	3	1.00	33	0.091
274	A	4	4	0.97	14	0.286
275	A	3	3	1.00	33	0.091
276	A	2	2	1.00	29	0.069
277	A	2	2	1.00	44	0.045
278	A	2	2	1.00	15	0.133
279	A	2	2	1.00	15	0.133
280	A	3	3	1.00	18	0.167
281	A	2	2	1.00	20	0.100
282	A	2	2	1.00	26	0.077
283	A	2	2	1.00	13	0.154
284	A	3	3	1.00	18	0.167
285	A	2	2	1.00	26	0.077
286	A	6	6	1.00	19	0.316
287	A	6	6	1.00	24	0.250
288	A	7	6	1.06	20	0.300
289	A	7	6	1.22	18	0.333
290	A	2	2	1.00	19	0.105
291	A	2	2	1.00	13	0.154
292	A	2	2	1.00	22	0.091
293	A	2	2	1.00	24	0.083
294	A	2	2	1.00	29	0.069

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
295	A	2	2	1.00	30	0.067
296	A	2	2	1.00	19	0.105
297	A	5	5	1.02	16	0.312
298	A	2	2	1.00	36	0.056
299	A	2	2	1.00	21	0.095
300	A	4	4	1.13	16	0.250
301	A	2	2	1.00	24	0.083
302	A	2	2	0.86	21	0.095
303	A	2	2	1.00	24	0.083
304	A	2	2	1.00	26	0.077
305	A	3	3	1.00	25	0.120
306	A	2	2	1.00	29	0.069
307	A	3	3	1.00	20	0.150
308	A	2	2	1.00	32	0.062
309	A	5	5	1.03	23	0.217
310	A	4	4	1.00	26	0.154
311	A	3	3	1.00	26	0.115
312	A	2	2	1.00	25	0.080
313	A	2	2	1.00	23	0.087
314	A	3	3	1.00	23	0.130
315	A	2	2	1.18	20	0.100
316	A	3	3	1.00	25	0.120
317	A	3	3	1.00	22	0.136
318	A	2	2	1.00	24	0.083
319	A	5	5	1.00	24	0.208
320	A	2	2	1.00	43	0.047
321	A	2	2	1.25	50	0.040
322	A	2	2	1.00	16	0.125
323	A	3	3	1.00	15	0.200
324	A	4	4	1.00	20	0.200
325	A	2	2	1.00	24	0.083
326	A	2	2	1.00	27	0.074
327	A	8	7	1.00	26	0.269
328	A	3	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
329	A	9	8	1.52	22	0.364
330	A	2	2	1.00	24	0.083
331	A	5	5	1.00	26	0.192
332	A	6	6	1.00	36	0.167
333	A	3	3	1.00	26	0.115
334	A	2	2	1.00	20	0.100
335	A	8	7	1.20	27	0.259
336	A	8	7	1.12	20	0.350
337	A	8	7	1.00	25	0.280
338	A	8	7	1.00	22	0.318
339	A	2	2	1.00	18	0.111
340	A	2	2	1.00	20	0.100
341	A	2	2	1.00	20	0.100
342	A	2	2	1.00	20	0.100
343	A	2	2	1.00	14	0.143
344	A	2	2	1.00	20	0.100
345	A	10	9	1.21	20	0.450
346	A	2	2	1.00	26	0.077
347	A	2	2	1.00	24	0.083
348	A	2	2	1.00	30	0.067
349	A	5	4	1.43	21	0.190
350	A	2	2	1.00	15	0.133
351	A	3	3	1.00	18	0.167
352	A	3	3	1.00	22	0.136
353	A	3	3	1.00	16	0.188
354	A	3	3	1.00	16	0.188
355	A	2	2	1.00	25	0.080
356	A	4	4	1.00	19	0.211
357	A	2	2	1.00	23	0.087
358	A	2	2	1.00	23	0.087
359	A	2	2	1.00	21	0.095
360	A	2	2	1.00	28	0.071
361	A	2	2	1.00	24	0.083
362	A	2	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
363	A	2	2	1.00	14	0.143
364	A	2	2	1.00	16	0.125
365	A	8	7	1.11	16	0.438
366	A	2	2	1.00	43	0.047
367	A	2	2	1.00	26	0.077
368	A	3	3	1.00	16	0.188
369	A	3	3	1.00	15	0.200
370	A	3	3	1.00	15	0.200
371	A	3	3	1.00	17	0.176
372	A	3	3	1.00	15	0.200
373	A	5	4	1.33	15	0.267
374	A	4	4	1.00	18	0.222
375	A	3	3	1.00	20	0.150
376	A	2	2	1.00	29	0.069
377	A	3	3	1.11	22	0.136
378	A	3	3	1.05	16	0.188
379	A	3	3	1.06	16	0.188
380	A	3	3	1.08	14	0.214
381	A	3	3	1.10	12	0.250
382	A	3	3	1.10	16	0.188
383	A	3	3	1.07	16	0.188
384	A	3	3	1.06	16	0.188
385	A	3	3	1.05	16	0.188
386	A	3	3	1.04	16	0.188
387	A	2	2	1.00	17	0.118
388	A	2	2	1.00	19	0.105
389	A	2	2	1.00	15	0.133
390	A	2	2	1.00	17	0.118
391	A	2	2	1.00	23	0.087
392	A	5	5	1.00	21	0.238
393	F	0	0	N/A	0.000	N/A
394	A	2	2	1.00	17	0.118
395	A	2	2	1.00	17	0.118

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
396	A	2	2	1.00	15	0.133
397	A	9	8	1.08	9	0.889
398	A	2	2	1.00	17	0.118
399	A	2	2	1.00	17	0.118
400	A	2	2	1.00	17	0.118
401	A	5	5	1.01	17	0.294
402	A	4	4	1.02	17	0.235
403	A	4	4	1.02	15	0.267
404	A	10	9	1.11	9	1.000
405	A	2	2	1.00	17	0.118
406	A	2	2	1.00	17	0.118
407	A	2	2	1.00	17	0.118
408	A	6	6	1.04	17	0.353
409	A	6	6	1.04	17	0.353
410	A	6	6	1.05	15	0.400
411	A	11	10	1.14	9	1.111
412	A	2	2	1.00	17	0.118
413	A	2	2	1.00	17	0.118
414	A	2	2	1.00	17	0.118
415	A	6	5	1.03	14	0.357
416	A	8	7	1.03	13	0.538
417	A	6	5	1.12	16	0.312
418	A	8	7	1.12	18	0.389
419	A	2	2	1.00	14	0.143
420	A	3	3	1.00	16	0.188
421	A	2	2	1.00	11	0.182
422	A	1	1	1.00	17	0.059
423	A	1	1	1.00	11	0.091
424	B	1	1	6.23	73	0.014
425	A	10	9	1.20	13	0.692
426	A	13	12	1.17	19	0.632
427	A	3	3	1.00	15	0.200
428	A	3	3	1.00	11	0.273
429	A	1	1	1.41	11	0.091

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
430	A	1	1	1.00	11	0.091
431	A	2	2	1.00	16	0.125
432	A	2	2	1.00	16	0.125
433	A	5	4	1.32	15	0.267
434	A	1	1	1.00	15	0.067
435	A	1	1	1.00	20	0.050
436	A	3	3	1.00	15	0.200
437	A	3	3	1.00	13	0.231
438	A	1	1	1.00	22	0.045
439	A	3	3	1.00	18	0.167
440	A	4	4	1.00	20	0.200
441	A	3	3	1.00	16	0.188
442	A	4	4	1.00	17	0.235
443	A	1	1	1.00	17	0.059
444	A	4	4	1.00	25	0.160
445	A	2	2	1.00	20	0.100
446	A	4	3	1.20	18	0.167
447	A	8	8	1.13	15	0.533
448	A	2	2	1.00	11	0.182
449	A	5	5	1.16	13	0.385
450	A	5	4	1.32	20	0.200
451	A	2	2	1.00	16	0.125
452	A	3	3	1.00	16	0.188
453	A	2	2	1.00	16	0.125
454	A	1	1	1.00	9	0.111
455	A	2	2	1.00	7	0.286
456	A	2	2	1.00	11	0.182
457	A	1	1	1.00	7	0.143
458	A	1	1	1.00	9	0.111
459	A	1	1	1.00	9	0.111
460	A	3	2	1.00	10	0.200
461	A	2	2	1.00	13	0.154
462	A	2	2	1.00	11	0.182
463	A	2	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
464	A	6	5	1.08	16	0.312
465	A	2	2	1.00	7	0.286
466	A	2	2	1.00	11	0.182
467	A	4	4	1.12	13	0.308
468	A	4	4	1.04	13	0.308
469	A	5	5	1.04	14	0.357
470	A	2	2	0.78	13	0.154
471	A	2	2	1.00	20	0.100
472	A	6	5	1.08	18	0.278
473	A	2	2	1.00	12	0.167
474	A	2	2	1.00	10	0.200
475	A	5	4	1.73	16	0.250
476	A	2	2	1.23	11	0.182
477	A	1	1	1.00	7	0.143
478	A	2	2	1.00	15	0.133
479	A	2	2	1.00	12	0.167
480	A	4	4	1.15	16	0.250
481	A	2	2	1.00	11	0.182
482	A	3	3	1.00	17	0.176
483	A	2	2	1.00	29	0.069
484	A	2	2	1.00	18	0.111
485	A	3	3	1.00	14	0.214
486	A	2	2	1.00	24	0.083
487	A	2	2	1.00	11	0.182
488	A	2	2	1.00	18	0.111
489	A	4	3	1.06	15	0.200
490	A	4	3	1.05	16	0.188
491	A	9	8	1.01	15	0.533
492	A	10	10	1.07	50	0.200
493	F	0	0	N/A	0.000	N/A
494	F	0	0	N/A	0.000	N/A

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{1}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx$

3.1.1 Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{1}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx=\frac{1}{3\sqrt{3}\sqrt{b}\left(\sqrt{3}\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}x\right)}{27b}$$

output
```
-1/27*ln(-x*3^(1/2)+b^(1/2))/b+1/27*ln(x*3^(1/2)+2*b^(1/2))/b+1/9*3^(1/2)/
b^(1/2)/(-3*x+3^(1/2)*b^(1/2))
```

3.1.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.86

$$\int \frac{1}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx=$$
$$-\frac{\left(-\sqrt{3}\sqrt{b}+3x\right)\left(2\sqrt{3}\sqrt{b}+3x\right)\left(3\sqrt{3}\sqrt{b}+\left(-\sqrt{3}\sqrt{b}+3x\right)\log\left(-\sqrt{3}\sqrt{b}+3x\right)+\left(\sqrt{3}\sqrt{b}-3x\right)\log\left(2\right.\right.}{81b\left(2\sqrt{3}b^{3/2}-9bx+9x^3\right)}$$

input
```
Integrate[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1),x]
```

output
```
-1/81*((-(Sqrt[3]*Sqrt[b]) + 3*x)*(2*Sqrt[3]*Sqrt[b] + 3*x)*(3*Sqrt[3]*Sqr
t[b] + (-(Sqrt[3]*Sqrt[b]) + 3*x)*Log[-(Sqrt[3]*Sqrt[b]) + 3*x] + (Sqrt[3]
*Sqrt[b] - 3*x)*Log[2*Sqrt[3]*Sqrt[b] + 3*x]))/(b*(2*Sqrt[3]*b^(3/2) - 9*b
*x + 9*x^3))
```

3.1.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2472, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2\sqrt{3}b^{3/2}-9bx+9x^3}dx$$

$$\Big\downarrow \ 2472$$

$$324b^3\int \frac{1}{108\sqrt{3}b^3\left(\sqrt{3}\sqrt{b}-3x\right)^2\left(\sqrt{3}x+2\sqrt{b}\right)}dx$$

$$\Big\downarrow \ 27$$

$$\sqrt{3}\int \frac{1}{\left(\sqrt{3}\sqrt{b}-3x\right)^2\left(\sqrt{3}x+2\sqrt{b}\right)}dx$$

$$\Big\downarrow \ 54$$

$$\sqrt{3}\int \left(\frac{1}{3\left(\sqrt{3}\sqrt{b}-3x\right)^2\sqrt{b}}+\frac{1}{27\left(\sqrt{b}-\sqrt{3}x\right)b}+\frac{1}{27\left(\sqrt{3}x+2\sqrt{b}\right)b}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\sqrt{3}\left(\frac{1}{9\sqrt{b}\left(\sqrt{3}\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}x\right)}{27\sqrt{3}b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}x\right)}{27\sqrt{3}b}\right)$$

input
```
Int[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1),x]
```

output
```
Sqrt[3]*(1/(9*Sqrt[b]*(Sqrt[3]*Sqrt[b] - 3*x)) - Log[Sqrt[b] - Sqrt[3]*x]/
(27*Sqrt[3]*b) + Log[2*Sqrt[b] + Sqrt[3]*x]/(27*Sqrt[3]*b))
```

3.1.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2472
```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Simp[1/(3^(3*p)*
a^(2*p))   Int[(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d}
, x] && EqQ[4*b^3 + 27*a^2*d, 0] && IntegerQ[p]
```

3.1.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(-9b\_Z+9\_Z^3+2b^{\frac{3}{2}}\sqrt{3}\right)}\frac{\ln\left(x-\_R\right)}{3\_R^2-b}}{9}$	43

input
```
int(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/9*sum(1/(3*_R^2-b)*ln(x-_R),_R=RootOf(-9*b*_Z+9*_Z^3+2*b^(3/2)*3^(1/2)))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{1}{2\sqrt{3}b^{3/2} - 9bx + 9x^3}\,dx = -\frac{3\,\sqrt{3}\sqrt{b}x - (3\,x^2 - b)\log\left(2\,\sqrt{3}\sqrt{b} + 3\,x\right) + (3\,x^2 - b)\log\left(-\sqrt{3}\sqrt{b} + 3\,x\right) + 3\,b}{27\,(3\,bx^2 - b^2)}$$

input
```
integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="fricas")
```

output
```
-1/27*(3*sqrt(3)*sqrt(b)*x - (3*x^2 - b)*log(2*sqrt(3)*sqrt(b) + 3*x) + (3
*x^2 - b)*log(-sqrt(3)*sqrt(b) + 3*x) + 3*b)/(3*b*x^2 - b^2)
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$\int \frac{1}{2\sqrt{3}b^{3/2} - 9bx + 9x^3}\,dx = -\frac{3\sqrt{3}}{81\sqrt{b}x - 27\sqrt{3}b} + \frac{-\frac{\log\left(-\frac{\sqrt{3}\sqrt{b}}{3}+x\right)}{27} + \frac{\log\left(\frac{2\sqrt{3}\sqrt{b}}{3}+x\right)}{27}}{b}$$

input
```
integrate(1/(-9*b*x+9*x**3+2*b**(3/2)*3**(1/2)),x)
```

output
```
-3*sqrt(3)/(81*sqrt(b)*x - 27*sqrt(3)*b) + (-log(-sqrt(3)*sqrt(b)/3 + x)/2
7 + log(2*sqrt(3)*sqrt(b)/3 + x)/27)/b
```

3.1.7 Maxima [F]

$$\int \frac{1}{2\sqrt{3}b^{3/2} - 9bx + 9x^3}\,dx = \int \frac{1}{9\,x^3 + 2\,\sqrt{3}b^{\frac{3}{2}} - 9\,bx}\,dx$$

input
```
integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(9*x^3 + 2*sqrt(3)*b^(3/2) - 9*b*x), x)
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{1}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx = \frac{\log\left(\left|9\sqrt{3}x+18\sqrt{b}\right|\right)}{27\,b} - \frac{\log\left(\left|-\sqrt{3}x+\sqrt{b}\right|\right)}{27\,b} - \frac{1}{9\left(\sqrt{3}x-\sqrt{b}\right)\sqrt{b}}$$

input
```
integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="giac")
```

output
```
1/27*log(abs(9*sqrt(3)*x + 18*sqrt(b)))/b - 1/27*log(abs(-sqrt(3)*x + sqrt
(b)))/b - 1/9/((sqrt(3)*x - sqrt(b))*sqrt(b))
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\int \frac{1}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx = \frac{2\sqrt{3}\sqrt{27}\,\mathrm{atanh}\left(\frac{\sqrt{3}\sqrt{27}}{27}+\frac{2\sqrt{27}\,x}{9\sqrt{b}}\right)}{243\,b} - \frac{\sqrt{3}}{27\sqrt{b}\left(x-\frac{\sqrt{3}\sqrt{b}}{3}\right)}$$

input
```
int(1/(2*3^(1/2)*b^(3/2) - 9*b*x + 9*x^3),x)
```

output
```
(2*3^(1/2)*27^(1/2)*atanh((3^(1/2)*27^(1/2))/27 + (2*27^(1/2)*x)/(9*b^(1/2
))))/(243*b) - 3^(1/2)/(27*b^(1/2)*(x - (3^(1/2)*b^(1/2))/3))
```

3.2 $\int (a^3+3a^2bx+3ab^2x^2+b^3x^3)^p\, dx$

3.2.1 Optimal result

Integrand size = 29, antiderivative size = 30

$$\int (a^3+3a^2bx+3ab^2x^2+b^3x^3)^p\, dx = \frac{\left(\frac{a}{b}+x\right)\left(b^3\left(\frac{a}{b}+x\right)^3\right)^p}{1+3p}$$

output
```
(a/b+x)*(b^3*(a/b+x)^3)^p/(1+3*p)
```

3.2.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int (a^3+3a^2bx+3ab^2x^2+b^3x^3)^p\, dx = \frac{(a+bx)\left((a+bx)^3\right)^p}{b(1+3p)}$$

input
```
Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]
```

output
```
((a + b*x)*((a + b*x)^3)^p)/(b*(1 + 3*p))
```

3.2.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2008, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx$$

$$\Big\downarrow \text{2008}$$

$$(a+bx)^{-3p}\left((a+bx)^3\right)^p\int (a+bx)^{3p}dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+bx)\left((a+bx)^3\right)^p}{b(3p+1)}$$

input
```
Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]
```

output
```
((a + b*x)*((a + b*x)^3)^p)/(b*(1 + 3*p))
```

3.2.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2008
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Simp[((a + b*x)^Exp
on[Px, x])^p/(a + b*x)^(Expon[Px, x]*p)   Int[u*(a + b*x)^(Expon[Px, x]*p),
 x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /;  !IntegerQ[p] && PolyQ[Px, x
] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

3.2.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

method	result	size
risch	$\frac{(bx+a)\left((bx+a)^3\right)^p}{b(1+3p)}$	26
gosper	$\frac{(bx+a)\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)^p}{b(1+3p)}$	46
parallelrisch	$\frac{x\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)^p ab+\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)^p a^2}{(1+3p)ab}$	82
norman	$\frac{x\,e^{p\ln\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)}}{1+3p}+\frac{a\,e^{p\ln\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)}}{b(1+3p)}$	85

input
```
int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)/b/(1+3*p)*((b*x+a)^3)^p
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx=\frac{(bx+a)(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)^p}{3\,bp+b}$$

input
```
integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="fricas")
```

output
```
(b*x + a)*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p/(3*b*p + b)
```

3.2.6 Sympy [F]

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx$$
$$=\begin{cases}\frac{x}{\sqrt[3]{a^3}} & \text{for } b=0\wedge p=-\frac{1}{3}\\ x\left(a^3\right)^p & \text{for } b=0\\ \int\frac{1}{\sqrt[3]{a^3+3a^2bx+3ab^2x^2+b^3x^3}}\,dx & \text{for } p=-\frac{1}{3}\\ \frac{a\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p}{3bp+b}+\frac{bx\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p}{3bp+b} & \text{otherwise}\end{cases}$$

input
```
integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**p,x)
```

output
```
Piecewise((x/(a**3)**(1/3), Eq(b, 0) & Eq(p, -1/3)), (x*(a**3)**p, Eq(b, 0
)), (Integral((a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**(-1/3), x),
 Eq(p, -1/3)), (a*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(3*b*
p + b) + b*x*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(3*b*p + b
), True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx=\frac{(bx+a)(bx+a)^{3\,p}}{b(3\,p+1)}$$

input
```
integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="maxima")
```

output
```
(b*x + a)*(b*x + a)^(3*p)/(b*(3*p + 1))
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(30) = 60.

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.43

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx$$
$$=\frac{(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)^pbx+(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)^pa}{3\,bp+b}$$

input
```
integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="giac")
```

output
```
((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*b*x + (b^3*x^3 + 3*a*b^2*x^2
+ 3*a^2*b*x + a^3)^p*a)/(3*b*p + b)
```

3.2.9 Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx=\left(\frac{x}{3\,p+1}+\frac{a}{b\,\left(3\,p+1\right)}\right)\,{\left(a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3\right)}^p$$

input
```
int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^p,x)
```

output
```
(x/(3*p + 1) + a/(b*(3*p + 1)))*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^
p
```

3.3 $\int (a^3+3a^2bx+3ab^2x^2+b^3x^3)^3\,dx$

3.3.1 Optimal result

Integrand size = 29, antiderivative size = 14

$$\int (a^3+3a^2bx+3ab^2x^2+b^3x^3)^3\,dx=\frac{(a+bx)^{10}}{10b}$$

output
```
1/10*(b*x+a)^10/b
```

3.3.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a^3+3a^2bx+3ab^2x^2+b^3x^3)^3\,dx=\frac{(a+bx)^{10}}{10b}$$

input
```
Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]
```

output
```
(a + b*x)^10/(10*b)
```

3.3.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2006, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3 \, dx$$
$$\Big\downarrow \; 2006$$
$$\int (a+bx)^9 dx$$
$$\Big\downarrow \; 17$$
$$\frac{(a+bx)^{10}}{10b}$$

input
```
Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]
```

output
```
(a + b*x)^10/(10*b)
```

3.3.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

3.3.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(12) = 24.

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 7.00

method	result
default	$\frac{1}{10}b^9x^{10}+a\,b^8x^9+\frac{9}{2}a^2b^7x^8+12a^3b^6x^7+21a^4b^5x^6+\frac{126}{5}a^5b^4x^5+21a^6b^3x^4+12a^7b^2x^3+\frac{9}{2}a$
norman	$\frac{1}{10}b^9x^{10}+a\,b^8x^9+\frac{9}{2}a^2b^7x^8+12a^3b^6x^7+21a^4b^5x^6+\frac{126}{5}a^5b^4x^5+21a^6b^3x^4+12a^7b^2x^3+\frac{9}{2}a$
risch	$\frac{1}{10}b^9x^{10}+a\,b^8x^9+\frac{9}{2}a^2b^7x^8+12a^3b^6x^7+21a^4b^5x^6+\frac{126}{5}a^5b^4x^5+21a^6b^3x^4+12a^7b^2x^3+\frac{9}{2}a$
parallelrisch	$\frac{1}{10}b^9x^{10}+a\,b^8x^9+\frac{9}{2}a^2b^7x^8+12a^3b^6x^7+21a^4b^5x^6+\frac{126}{5}a^5b^4x^5+21a^6b^3x^4+12a^7b^2x^3+\frac{9}{2}a$
gosper	$\frac{x\left(b^9x^9+10a\,b^8x^8+45a^2b^7x^7+120a^3b^6x^6+210a^4b^5x^5+252a^5b^4x^4+210a^6b^3x^3+120a^7b^2x^2+45a^8bx+10a^9\right)}{10}$

input
```
int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*b^9*x^10+a*b^8*x^9+9/2*a^2*b^7*x^8+12*a^3*b^6*x^7+21*a^4*b^5*x^6+126/
5*a^5*b^4*x^5+21*a^6*b^3*x^4+12*a^7*b^2*x^3+9/2*a^8*b*x^2+a^9*x
```

3.3.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(12) = 24.

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3\,dx=\frac{1}{10}\,b^9x^{10}+ab^8x^9+\frac{9}{2}\,a^2b^7x^8+12\,a^3b^6x^7+21\,a^4b^5x^6+\frac{126}{5}\,a^5b^4x^5+21\,a^6b^3x^4+12\,a^7b^2x^3+\frac{9}{2}\,a^8bx^2+a^9x$$

input
```
integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="fricas")
```

output
```
1/10*b^9*x^10 + a*b^8*x^9 + 9/2*a^2*b^7*x^8 + 12*a^3*b^6*x^7 + 21*a^4*b^5*
x^6 + 126/5*a^5*b^4*x^5 + 21*a^6*b^3*x^4 + 12*a^7*b^2*x^3 + 9/2*a^8*b*x^2
+ a^9*x
```

3.3.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(8) = 16.

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 7.64

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3\,dx = a^9x+\frac{9a^8bx^2}{2}+12a^7b^2x^3+21a^6b^3x^4+\frac{126a^5b^4x^5}{5}+21a^4b^5x^6+12a^3b^6x^7+\frac{9a^2b^7x^8}{2}+ab^8x^9+\frac{b^9x^{10}}{10}$$

input
```
integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)
```

output
```
a**9*x + 9*a**8*b*x**2/2 + 12*a**7*b**2*x**3 + 21*a**6*b**3*x**4 + 126*a**
5*b**4*x**5/5 + 21*a**4*b**5*x**6 + 12*a**3*b**6*x**7 + 9*a**2*b**7*x**8/2
 + a*b**8*x**9 + b**9*x**10/10
```

3.3.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(12) = 24.

Time = 0.20 (sec) , antiderivative size = 216, normalized size of antiderivative = 15.43

$$\begin{aligned}&\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3\,dx\\&=\frac{1}{10}b^9x^{10}+ab^8x^9+\frac{27}{8}a^2b^7x^8+\frac{27}{7}a^3b^6x^7+\frac{27}{4}a^6b^3x^4+a^9x\\&\quad+\frac{3}{4}\left(b^3x^4+4\,ab^2x^3+6\,a^2bx^2\right)a^6+\frac{9}{10}\left(5\,b^3x^6+18\,ab^2x^5\right)a^4b^2\\&\quad+\frac{3}{70}\left(10\,b^6x^7+70\,ab^5x^6+126\,a^2b^4x^5+210\,a^4b^2x^3+21\left(4\,b^3x^5+15\,ab^2x^4\right)a^2b\right)a^3\\&\quad+\frac{9}{56}\left(7\,b^6x^8+48\,ab^5x^7+84\,a^2b^4x^6\right)a^2b\end{aligned}$$

input
```
integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="maxima")
```

output
```
1/10*b^9*x^10 + a*b^8*x^9 + 27/8*a^2*b^7*x^8 + 27/7*a^3*b^6*x^7 + 27/4*a^6
*b^3*x^4 + a^9*x + 3/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2)*a^6 + 9/10*(5
*b^3*x^6 + 18*a*b^2*x^5)*a^4*b^2 + 3/70*(10*b^6*x^7 + 70*a*b^5*x^6 + 126*a
^2*b^4*x^5 + 210*a^4*b^2*x^3 + 21*(4*b^3*x^5 + 15*a*b^2*x^4)*a^2*b)*a^3 +
9/56*(7*b^6*x^8 + 48*a*b^5*x^7 + 84*a^2*b^4*x^6)*a^2*b
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(12) = 24.

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3\,dx=\frac{1}{10}\,b^9x^{10}+ab^8x^9+\frac{9}{2}\,a^2b^7x^8+12\,a^3b^6x^7+21\,a^4b^5x^6+\frac{126}{5}\,a^5b^4x^5+21\,a^6b^3x^4+12\,a^7b^2x^3+\frac{9}{2}\,a^8bx^2+a^9x$$

input
```
integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="giac")
```

output
```
1/10*b^9*x^10 + a*b^8*x^9 + 9/2*a^2*b^7*x^8 + 12*a^3*b^6*x^7 + 21*a^4*b^5*
x^6 + 126/5*a^5*b^4*x^5 + 21*a^6*b^3*x^4 + 12*a^7*b^2*x^3 + 9/2*a^8*b*x^2
+ a^9*x
```

3.3.9 Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3\,dx=a^9\,x+\frac{9\,a^8\,b\,x^2}{2}+12\,a^7\,b^2\,x^3+21\,a^6\,b^3\,x^4+\frac{126\,a^5\,b^4\,x^5}{5}+21\,a^4\,b^5\,x^6+12\,a^3\,b^6\,x^7+\frac{9\,a^2\,b^7\,x^8}{2}+a\,b^8\,x^9+\frac{b^9\,x^{10}}{10}$$

input
```
int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^3,x)
```

output
```
a^9*x + (b^9*x^10)/10 + (9*a^8*b*x^2)/2 + a*b^8*x^9 + 12*a^7*b^2*x^3 + 21*
a^6*b^3*x^4 + (126*a^5*b^4*x^5)/5 + 21*a^4*b^5*x^6 + 12*a^3*b^6*x^7 + (9*a
^2*b^7*x^8)/2
```

3.4 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2 \, dx$

3.4.1 Optimal result

Integrand size = 29, antiderivative size = 14

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2 \, dx=\frac{(a+bx)^7}{7b}$$

output
```
1/7*(b*x+a)^7/b
```

3.4.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2 \, dx=\frac{(a+bx)^7}{7b}$$

input
```
Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]
```

output
```
(a + b*x)^7/(7*b)
```

3.4.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2006, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx$$

$$\Big\downarrow \; 2006$$

$$\int (a+bx)^6dx$$

$$\Big\downarrow \; 17$$

$$\frac{(a+bx)^7}{7b}$$

input
```
Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]
```

output
```
(a + b*x)^7/(7*b)
```

3.4.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] && !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

3.4.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.64

method	result	size
default	$\frac{1}{7}b^6x^7 + a\,b^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5b\,x^2 + a^6x$	65
norman	$\frac{1}{7}b^6x^7 + a\,b^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5b\,x^2 + a^6x$	65
risch	$\frac{1}{7}b^6x^7 + a\,b^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5b\,x^2 + a^6x$	65
parallelrisch	$\frac{1}{7}b^6x^7 + a\,b^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5b\,x^2 + a^6x$	65
gosper	$\frac{x(b^6x^6+7a\,b^5x^5+21a^2b^4x^4+35a^3b^3x^3+35a^4b^2x^2+21a^5bx+7a^6)}{7}$	66

input
```
int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^6*x^7+a*b^5*x^6+3*a^2*b^4*x^5+5*a^3*b^3*x^4+5*a^4*b^2*x^3+3*a^5*b*x^
2+a^6*x
```

3.4.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 4.57

$$\int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^2 \, dx = \frac{1}{7}\,b^6x^7 + ab^5x^6 + 3\,a^2b^4x^5 + 5\,a^3b^3x^4 + 5\,a^4b^2x^3 + 3\,a^5bx^2 + a^6x$$

input
```
integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="fricas")
```

output
```
1/7*b^6*x^7 + a*b^5*x^6 + 3*a^2*b^4*x^5 + 5*a^3*b^3*x^4 + 5*a^4*b^2*x^3 +
3*a^5*b*x^2 + a^6*x
```

3.4.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(8) = 16.

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 4.71

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx = a^6x+3a^5bx^2+5a^4b^2x^3+5a^3b^3x^4+3a^2b^4x^5+ab^5x^6+\frac{b^6x^7}{7}$$

input
```
integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)
```

output
```
a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*a**2*b**4
*x**5 + a*b**5*x**6 + b**6*x**7/7
```

3.4.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(12) = 24.

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 7.07

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx = \frac{1}{7}\,b^6x^7+ab^5x^6+\frac{9}{5}\,a^2b^4x^5+3\,a^4b^2x^3+a^6x+\frac{1}{2}\left(b^3x^4+4\,ab^2x^3+6\,a^2bx^2\right)a^3+\frac{3}{10}\left(4\,b^3x^5+15\,ab^2x^4\right)a^2b$$

input
```
integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="maxima")
```

output
```
1/7*b^6*x^7 + a*b^5*x^6 + 9/5*a^2*b^4*x^5 + 3*a^4*b^2*x^3 + a^6*x + 1/2*(b
^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2)*a^3 + 3/10*(4*b^3*x^5 + 15*a*b^2*x^4)*
a^2*b
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(12) = 24$.

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 4.57

$$\int \left(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3\right)^2 \, dx = \frac{1}{7}\,b^6x^7 + ab^5x^6 + 3\,a^2b^4x^5 + 5\,a^3b^3x^4 + 5\,a^4b^2x^3 + 3\,a^5bx^2 + a^6x$$

input
```
integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="giac")
```

output
```
1/7*b^6*x^7 + a*b^5*x^6 + 3*a^2*b^4*x^5 + 5*a^3*b^3*x^4 + 5*a^4*b^2*x^3 +
3*a^5*b*x^2 + a^6*x
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 4.57

$$\int \left(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3\right)^2 \, dx = a^6\,x + 3\,a^5\,b\,x^2 + 5\,a^4\,b^2\,x^3 + 5\,a^3\,b^3\,x^4 + 3\,a^2\,b^4\,x^5 + a\,b^5\,x^6 + \frac{b^6\,x^7}{7}$$

input
```
int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^2,x)
```

output
```
a^6*x + (b^6*x^7)/7 + 3*a^5*b*x^2 + a*b^5*x^6 + 5*a^4*b^2*x^3 + 5*a^3*b^3*
x^4 + 3*a^2*b^4*x^5
```

3.5 $\int (a^3+3a^2bx+3ab^2x^2+b^3x^3)\,dx$

3.5.1 Optimal result

Integrand size = 27, antiderivative size = 35

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx=a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

output
```
a^3*x+3/2*a^2*b*x^2+a*b^2*x^3+1/4*b^3*x^4
```

3.5.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx=a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

input
```
Integrate[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3,x]
```

output
```
a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4
```

3.5.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.40, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2006, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx$$

$$\Big\downarrow \text{ 2006}$$

$$\int (a+bx)^3dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(a+bx)^4}{4b}$$

input
```
Int[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3,x]
```

output
```
(a + b*x)^4/(4*b)
```

3.5.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

3.5.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.37

method	result	size
default	$\frac{(bx+a)^4}{4b}$	13
norman	$a^3x+\frac{3}{2}a^2b\,x^2+a\,b^2x^3+\frac{1}{4}b^3x^4$	32
risch	$a^3x+\frac{3}{2}a^2b\,x^2+a\,b^2x^3+\frac{1}{4}b^3x^4$	32
parallelrisch	$a^3x+\frac{3}{2}a^2b\,x^2+a\,b^2x^3+\frac{1}{4}b^3x^4$	32
parts	$a^3x+\frac{3}{2}a^2b\,x^2+a\,b^2x^3+\frac{1}{4}b^3x^4$	32
gosper	$\frac{x(b^3x^3+4a\,b^2x^2+6a^2bx+4a^3)}{4}$	33

input
```
int(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b*x+a)^4/b
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int (a^3+3a^2bx+3ab^2x^2+b^3x^3)\ dx=\frac{1}{4}b^3x^4+ab^2x^3+\frac{3}{2}a^2bx^2+a^3x$$

input
```
integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="fricas")
```

output
```
1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int (a^3+3a^2bx+3ab^2x^2+b^3x^3)\ dx=a^3x+\frac{3a^2bx^2}{2}+ab^2x^3+\frac{b^3x^4}{4}$$

input
```
integrate(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3,x)
```

output
```
a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx=\frac{1}{4}\,b^3x^4+ab^2x^3+\frac{3}{2}\,a^2bx^2+a^3x$$

input
```
integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="maxima")
```

output
```
1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx=\frac{1}{4}\,b^3x^4+ab^2x^3+\frac{3}{2}\,a^2bx^2+a^3x$$

input
```
integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="giac")
```

output
```
1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx=a^3\,x+\frac{3\,a^2\,b\,x^2}{2}+a\,b^2\,x^3+\frac{b^3\,x^4}{4}$$

input
```
int(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x,x)
```

output
```
a^3*x + (b^3*x^4)/4 + (3*a^2*b*x^2)/2 + a*b^2*x^3
```

3.6 $\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3} \, dx$

3.6.1 Optimal result

Integrand size = 29, antiderivative size = 14

$$\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3} \, dx = -\frac{1}{2b(a+bx)^2}$$

output
```
-1/2/b/(b*x+a)^2
```

3.6.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3} \, dx = -\frac{1}{2b(a+bx)^2}$$

input
```
Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1),x]
```

output
```
-1/2*1/(b*(a + b*x)^2)
```

3.6.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2007, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}dx$$
$$\Big\downarrow \text{ 2007}$$
$$\int \frac{1}{(a+bx)^3}dx$$
$$\Big\downarrow \text{ 17}$$
$$-\frac{1}{2b(a+bx)^2}$$

input
```
Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1),x]
```

output
```
-1/2*1/(b*(a + b*x)^2)
```

3.6.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2007
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Ex
pon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; IntegerQ[p] && Pol
yQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

3.6.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{2b(bx+a)^2}$	13
norman	$-\frac{1}{2b(bx+a)^2}$	13
gosper	$-\frac{1}{2b(b^2x^2+2abx+a^2)}$	24
risch	$-\frac{1}{2b(b^2x^2+2abx+a^2)}$	24
parallelrisch	$-\frac{1}{2b(b^2x^2+2abx+a^2)}$	24

input
```
int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/(b*x+a)^2
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{1}{a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3}\, dx = -\frac{1}{2\,(b^3x^2 + 2\,ab^2x + a^2b)}$$

input
```
integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="fricas")
```

output
```
-1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

3.6.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{1}{a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3}\, dx = -\frac{1}{2a^2b + 4ab^2x + 2b^3x^2}$$

input
```
integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3),x)
```

output
```
-1/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx = -\frac{1}{2\left(b^3x^2+2\,ab^2x+a^2b\right)}$$

input
```
integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="maxima")
```

output
```
-1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx = -\frac{1}{2\left(bx+a\right)^2b}$$

input
```
integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="giac")
```

output
```
-1/2/((b*x + a)^2*b)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx = -\frac{1}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2}$$

input
```
int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x),x)
```

output
```
-1/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x)
```

3.7 $\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2}\,dx$

3.7.1 Optimal result

Integrand size = 29, antiderivative size = 14

$$\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2}\,dx=-\frac{1}{5b(a+bx)^5}$$

output
```
-1/5/b/(b*x+a)^5
```

3.7.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2}\,dx=-\frac{1}{5b(a+bx)^5}$$

input
```
Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2),x]
```

output
```
-1/5*1/(b*(a + b*x)^5)
```

3.7.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2007, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}dx$$
$$\Big\downarrow \; 2007$$
$$\int \frac{1}{(a+bx)^6}dx$$
$$\Big\downarrow \; 17$$
$$-\frac{1}{5b(a+bx)^5}$$

input
```
Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2),x]
```

output
```
-1/5*1/(b*(a + b*x)^5)
```

3.7.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2007
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Ex
pon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; IntegerQ[p] && Pol
yQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

3.7.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{5b(bx+a)^{5}}$	13
norman	$-\frac{1}{5b(bx+a)^{5}}$	13
risch	$-\frac{1}{5b(b^{2}x^{2}+2abx+a^{2})^{2}(bx+a)}$	31
gosper	$-\frac{1}{5(b^{2}x^{2}+2abx+a^{2})(b^{3}x^{3}+3a\,b^{2}x^{2}+3a^{2}bx+a^{3})b}$	53
parallelrisch	$-\frac{1}{5(b^{2}x^{2}+2abx+a^{2})(b^{3}x^{3}+3a\,b^{2}x^{2}+3a^{2}bx+a^{3})b}$	53

input
```
int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5/b/(b*x+a)^5
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(12) = 24.

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.07

$$\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}\,dx = -\frac{1}{5\,(b^6x^5+5\,ab^5x^4+10\,a^2b^4x^3+10\,a^3b^3x^2+5\,a^4b^2x+a^5b)}$$

input
```
integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="fricas")
```

output
```
-1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*
x + a^5*b)
```

3.7.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(12) = 24$.

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 4.36

$$\int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^2}\,dx = -\frac{1}{5a^5b + 25a^4b^2x + 50a^3b^3x^2 + 50a^2b^4x^3 + 25ab^5x^4 + 5b^6x^5}$$

input
```
integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)
```

output
```
-1/(5*a**5*b + 25*a**4*b**2*x + 50*a**3*b**3*x**2 + 50*a**2*b**4*x**3 + 25
*a*b**5*x**4 + 5*b**6*x**5)
```

3.7.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(12) = 24$.

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.07

$$\int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^2}\,dx = -\frac{1}{5\,(b^6x^5 + 5\,ab^5x^4 + 10\,a^2b^4x^3 + 10\,a^3b^3x^2 + 5\,a^4b^2x + a^5b)}$$

input
```
integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="maxima")
```

output
```
-1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*
x + a^5*b)
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}\,dx = -\frac{1}{5\,(bx+a)^5b}$$

input
```
integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="giac")
```

output
```
-1/5/((b*x + a)^5*b)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 4.21

$$\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}\,dx = -\frac{1}{5\,a^5\,b+25\,a^4\,b^2\,x+50\,a^3\,b^3\,x^2+50\,a^2\,b^4\,x^3+25\,a\,b^5\,x^4+5\,b^6\,x^5}$$

input
```
int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^2,x)
```

output
```
-1/(5*a^5*b + 5*b^6*x^5 + 25*a^4*b^2*x + 25*a*b^5*x^4 + 50*a^3*b^3*x^2 + 5
0*a^2*b^4*x^3)
```

3.8 $\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3}\,dx$

3.8.1 Optimal result

Integrand size = 29, antiderivative size = 14

$$\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3}\,dx=-\frac{1}{8b(a+bx)^8}$$

output
```
-1/8/b/(b*x+a)^8
```

3.8.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3}\,dx=-\frac{1}{8b(a+bx)^8}$$

input
```
Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3),x]
```

output
```
-1/8*1/(b*(a + b*x)^8)
```

3.8.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2007, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^3}\,dx$$

$$\Big\downarrow \text{2007}$$

$$\int \frac{1}{(a+bx)^9}dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{1}{8b(a+bx)^8}$$

input
```
Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3),x]
```

output
```
-1/8*1/(b*(a + b*x)^8)
```

3.8.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2007
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Ex
pon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; IntegerQ[p] && Pol
yQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

3.8.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{8b(bx+a)^{8}}$	13
norman	$-\frac{1}{8b(bx+a)^{8}}$	13
risch	$-\frac{1}{8b(b^{2}x^{2}+2abx+a^{2})^{3}(bx+a)^{2}}$	31
gosper	$-\frac{1}{8(b^{2}x^{2}+2abx+a^{2})(b^{3}x^{3}+3a\,b^{2}x^{2}+3a^{2}bx+a^{3})^{2}b}$	53
parallelrisch	$-\frac{1}{8(b^{2}x^{2}+2abx+a^{2})(b^{3}x^{3}+3a\,b^{2}x^{2}+3a^{2}bx+a^{3})^{2}b}$	53

input
```
int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8/b/(b*x+a)^8
```

3.8.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(12) = 24$.

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 6.43

$$\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^3}\,dx = -\frac{1}{8\,(b^9x^8+8\,ab^8x^7+28\,a^2b^7x^6+56\,a^3b^6x^5+70\,a^4b^5x^4+56\,a^5b^4x^3+28\,a^6b^3x^2+8\,a^7b^2x+a^8b)}$$

input
```
integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="fricas")
```

output
```
-1/8/(b^9*x^8 + 8*a*b^8*x^7 + 28*a^2*b^7*x^6 + 56*a^3*b^6*x^5 + 70*a^4*b^5
*x^4 + 56*a^5*b^4*x^3 + 28*a^6*b^3*x^2 + 8*a^7*b^2*x + a^8*b)
```

3.8.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3} dx = -\frac{1}{8a^8b + 64a^7b^2x + 224a^6b^3x^2 + 448a^5b^4x^3 + 560a^4b^5x^4 + 448a^3b^6x^5 + 224a^2b^7x^6 + 64ab^8x^7 + 8b^9x^8}$$

input
```
integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)
```

output
```
-1/(8*a**8*b + 64*a**7*b**2*x + 224*a**6*b**3*x**2 + 448*a**5*b**4*x**3 +
560*a**4*b**5*x**4 + 448*a**3*b**6*x**5 + 224*a**2*b**7*x**6 + 64*a*b**8*x
**7 + 8*b**9*x**8)
```

3.8.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(12) = 24$.

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 6.43

$$\int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3} dx = -\frac{1}{8 (b^9x^8 + 8 ab^8x^7 + 28 a^2b^7x^6 + 56 a^3b^6x^5 + 70 a^4b^5x^4 + 56 a^5b^4x^3 + 28 a^6b^3x^2 + 8 a^7b^2x + a^8b)}$$

input
```
integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="maxima")
```

output
```
-1/8/(b^9*x^8 + 8*a*b^8*x^7 + 28*a^2*b^7*x^6 + 56*a^3*b^6*x^5 + 70*a^4*b^5
*x^4 + 56*a^5*b^4*x^3 + 28*a^6*b^3*x^2 + 8*a^7*b^2*x + a^8*b)
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^3}\,dx = -\frac{1}{8\,(bx+a)^8b}$$

input
```
integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="giac")
```

output
```
-1/8/((b*x + a)^8*b)
```

3.8.9 Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 6.57

$$\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^3}\,dx =$$

$$-\frac{1}{8\,a^8\,b+64\,a^7\,b^2\,x+224\,a^6\,b^3\,x^2+448\,a^5\,b^4\,x^3+560\,a^4\,b^5\,x^4+448\,a^3\,b^6\,x^5+224\,a^2\,b^7\,x^6+64\,a\,b^8\,x^7+}$$

input
```
int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^3,x)
```

output
```
-1/(8*a^8*b + 8*b^9*x^8 + 64*a^7*b^2*x + 64*a*b^8*x^7 + 224*a^6*b^3*x^2 +
448*a^5*b^4*x^3 + 560*a^4*b^5*x^4 + 448*a^3*b^6*x^5 + 224*a^2*b^7*x^6)
```

3.9 $\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx$

3.9.1 Optimal result

Integrand size = 27, antiderivative size = 84

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx=-\frac{b^3(b^2-3ac)^3x}{c^3}+\frac{3b^2(b^2-3ac)^2(b+cx)^4}{4c^4}-\frac{3b(b^2-3ac)(b+cx)^7}{7c^4}+\frac{(b+cx)^{10}}{10c^4}$$

output
```
-b^3*(-3*a*c+b^2)^3*x/c^3+3/4*b^2*(-3*a*c+b^2)^2*(c*x+b)^4/c^4-3/7*b*(-3*a
*c+b^2)*(c*x+b)^7/c^4+1/10*(c*x+b)^10/c^4
```

3.9.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.89

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx=27a^3b^3x+\frac{81}{2}a^2b^4x^2+27ab^3\left(b^2+ac\right)x^3+\frac{27}{4}b^2\left(b^4+6ab^2c+a^2c^2\right)x^4+\frac{27}{5}b^3c\left(3b^2+5ac\right)x^5+9b^2c^2\left(2b^2+ac\right)x^6+\frac{9}{7}bc^3\left(9b^2+ac\right)x^7+\frac{9}{2}b^2c^4x^8+bc^5x^9+\frac{c^6x^{10}}{10}$$

input
```
Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

output
```
27*a^3*b^3*x + (81*a^2*b^4*x^2)/2 + 27*a*b^3*(b^2 + a*c)*x^3 + (27*b^2*(b^
4 + 6*a*b^2*c + a^2*c^2)*x^4)/4 + (27*b^3*c*(3*b^2 + 5*a*c)*x^5)/5 + 9*b^2
*c^2*(2*b^2 + a*c)*x^6 + (9*b*c^3*(9*b^2 + a*c)*x^7)/7 + (9*b^2*c^4*x^8)/2
 + b*c^5*x^9 + (c^6*x^10)/10
```

3.9.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2458, 747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3 dx$$

$$\Big\downarrow \text{2458}$$

$$\int \left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^3 d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{747}$$

$$\int \left(\frac{\left(3abc-b^3\right)^3}{c^3}+3\left(b^3-3abc\right)^2\left(\frac{b}{c}+x\right)^3-3bc^3\left(b^2-3ac\right)\left(\frac{b}{c}+x\right)^6+c^6\left(\frac{b}{c}+x\right)^9\right) d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{7}bc^3\left(b^2-3ac\right)\left(\frac{b}{c}+x\right)^7+\frac{3}{4}b^2\left(b^2-3ac\right)^2\left(\frac{b}{c}+x\right)^4-\frac{b^3\left(b^2-3ac\right)^3\left(\frac{b}{c}+x\right)}{c^3}+\frac{1}{10}c^6\left(\frac{b}{c}+x\right)^{10}$$

input
```
Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

output
```
-((b^3*(b^2 - 3*a*c)^3*(b/c + x))/c^3) + (3*b^2*(b^2 - 3*a*c)^2*(b/c + x)^
4)/4 - (3*b*c^3*(b^2 - 3*a*c)*(b/c + x)^7)/7 + (c^6*(b/c + x)^10)/10
```

3.9.3.1 Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.9.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(78) = 156$.

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.89

method	result
norman	$\frac{c^6x^{10}}{10}+b\,c^5x^9+\frac{9b^2c^4x^8}{2}+\left(\frac{9}{7}ab\,c^4+\frac{81}{7}b^3c^3\right)x^7+\left(9a\,b^2c^3+18b^4c^2\right)x^6+\left(27a\,b^3c^2+\frac{81}{5}b^5c\right)x$
gosper	$\frac{1}{10}c^6x^{10}+b\,c^5x^9+\frac{9}{2}b^2c^4x^8+\frac{9}{7}x^7ab\,c^4+\frac{81}{7}x^7b^3c^3+9a\,b^2c^3x^6+18b^4c^2x^6+27x^5a\,b^3c^2+\frac{81}{5}x$
risch	$\frac{1}{10}c^6x^{10}+b\,c^5x^9+\frac{9}{2}b^2c^4x^8+\frac{9}{7}x^7ab\,c^4+\frac{81}{7}x^7b^3c^3+9a\,b^2c^3x^6+18b^4c^2x^6+27x^5a\,b^3c^2+\frac{81}{5}x$
parallelrisch	$\frac{1}{10}c^6x^{10}+b\,c^5x^9+\frac{9}{2}b^2c^4x^8+\frac{9}{7}x^7ab\,c^4+\frac{81}{7}x^7b^3c^3+9a\,b^2c^3x^6+18b^4c^2x^6+27x^5a\,b^3c^2+\frac{81}{5}x$
default	$\frac{c^6x^{10}}{10}+b\,c^5x^9+\frac{9b^2c^4x^8}{2}+\frac{\left(3ab\,c^4+63b^3c^3+c^2\left(6ab\,c^2+18b^3c\right)\right)x^7}{7}+\frac{\left(18a\,b^2c^3+45b^4c^2+3bc\left(6ab\,c^2+18b^3c\right)+c^2\left(18a\right.\right.}{6}$

input
```
int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*c^6*x^10+b*c^5*x^9+9/2*b^2*c^4*x^8+(9/7*a*b*c^4+81/7*b^3*c^3)*x^7+(9*
a*b^2*c^3+18*b^4*c^2)*x^6+(27*a*b^3*c^2+81/5*b^5*c)*x^5+(27/4*a^2*b^2*c^2+
81/2*a*b^4*c+27/4*b^6)*x^4+(27*a^2*b^3*c+27*a*b^5)*x^3+81/2*a^2*b^4*x^2+27
*a^3*b^3*x
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.86

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx = \frac{1}{10}\,c^6x^{10}+bc^5x^9+\frac{9}{2}\,b^2c^4x^8+\frac{81}{2}\,a^2b^4x^2 + \frac{9}{7}\left(9\,b^3c^3+abc^4\right)x^7+27\,a^3b^3x + 9\left(2\,b^4c^2+ab^2c^3\right)x^6+\frac{27}{5}\left(3\,b^5c+5\,ab^3c^2\right)x^5 + \frac{27}{4}\left(b^6+6\,ab^4c+a^2b^2c^2\right)x^4+27\left(ab^5+a^2b^3c\right)x^3$$

input
```
integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="fricas")
```

output
```
1/10*c^6*x^10 + b*c^5*x^9 + 9/2*b^2*c^4*x^8 + 81/2*a^2*b^4*x^2 + 9/7*(9*b^
3*c^3 + a*b*c^4)*x^7 + 27*a^3*b^3*x + 9*(2*b^4*c^2 + a*b^2*c^3)*x^6 + 27/5
*(3*b^5*c + 5*a*b^3*c^2)*x^5 + 27/4*(b^6 + 6*a*b^4*c + a^2*b^2*c^2)*x^4 +
27*(a*b^5 + a^2*b^3*c)*x^3
```

3.9.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(80) = 160.

Time = 0.04 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.08

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx = 27a^3b^3x+\frac{81a^2b^4x^2}{2}+\frac{9b^2c^4x^8}{2}+bc^5x^9 + \frac{c^6x^{10}}{10}+x^7\cdot\left(\frac{9abc^4}{7}+\frac{81b^3c^3}{7}\right)+x^6 \cdot\left(9ab^2c^3+18b^4c^2\right)+x^5\cdot\left(27ab^3c^2+\frac{81b^5c}{5}\right)+x^4 \cdot\left(\frac{27a^2b^2c^2}{4}+\frac{81ab^4c}{2}+\frac{27b^6}{4}\right)+x^3\cdot\left(27a^2b^3c+27ab^5\right)$$

input
```
integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

output
```
27*a**3*b**3*x + 81*a**2*b**4*x**2/2 + 9*b**2*c**4*x**8/2 + b*c**5*x**9 +
c**6*x**10/10 + x**7*(9*a*b*c**4/7 + 81*b**3*c**3/7) + x**6*(9*a*b**2*c**3
 + 18*b**4*c**2) + x**5*(27*a*b**3*c**2 + 81*b**5*c/5) + x**4*(27*a**2*b**
2*c**2/4 + 81*a*b**4*c/2 + 27*b**6/4) + x**3*(27*a**2*b**3*c + 27*a*b**5)
```

3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(78) = 156$.

Time = 0.21 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.43

$$\begin{aligned}\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx &= \frac{1}{10}\,c^6x^{10}+bc^5x^9+\frac{27}{8}\,b^2c^4x^8+\frac{27}{7}\,b^3c^3x^7+\frac{27}{4}\,b^6x^4+27\,a^3b^3x\\ &+\frac{27}{4}\,\left(c^2x^4+4\,bcx^3+6\,b^2x^2\right)a^2b^2+\frac{9}{10}\,\left(5\,c^2x^6+18\,bcx^5\right)b^4\\ &+\frac{9}{70}\,\left(10\,c^4x^7+70\,bc^3x^6+126\,b^2c^2x^5+210\,b^4x^3+21\,\left(4\,c^2x^5+15\,bcx^4\right)b^2\right)ab\\ &+\frac{9}{56}\,\left(7\,c^4x^8+48\,bc^3x^7+84\,b^2c^2x^6\right)b^2\end{aligned}$$

input
```
integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="maxima")
```

output
```
1/10*c^6*x^10 + b*c^5*x^9 + 27/8*b^2*c^4*x^8 + 27/7*b^3*c^3*x^7 + 27/4*b^6
*x^4 + 27*a^3*b^3*x + 27/4*(c^2*x^4 + 4*b*c*x^3 + 6*b^2*x^2)*a^2*b^2 + 9/1
0*(5*c^2*x^6 + 18*b*c*x^5)*b^4 + 9/70*(10*c^4*x^7 + 70*b*c^3*x^6 + 126*b^2
*c^2*x^5 + 210*b^4*x^3 + 21*(4*c^2*x^5 + 15*b*c*x^4)*b^2)*a*b + 9/56*(7*c^
4*x^8 + 48*b*c^3*x^7 + 84*b^2*c^2*x^6)*b^2
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(78) = 156$.

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.98

$$\begin{aligned}\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx &= \frac{1}{10}\,c^6x^{10}+bc^5x^9+\frac{9}{2}\,b^2c^4x^8+\frac{81}{7}\,b^3c^3x^7\\ &+\frac{9}{7}\,abc^4x^7+18\,b^4c^2x^6+9\,ab^2c^3x^6+\frac{81}{5}\,b^5cx^5\\ &+27\,ab^3c^2x^5+\frac{27}{4}\,b^6x^4+\frac{81}{2}\,ab^4cx^4+\frac{27}{4}\,a^2b^2c^2x^4\\ &+27\,ab^5x^3+27\,a^2b^3cx^3+\frac{81}{2}\,a^2b^4x^2+27\,a^3b^3x\end{aligned}$$

input
```
integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="giac")
```

output
```
1/10*c^6*x^10 + b*c^5*x^9 + 9/2*b^2*c^4*x^8 + 81/7*b^3*c^3*x^7 + 9/7*a*b*c
^4*x^7 + 18*b^4*c^2*x^6 + 9*a*b^2*c^3*x^6 + 81/5*b^5*c*x^5 + 27*a*b^3*c^2*
x^5 + 27/4*b^6*x^4 + 81/2*a*b^4*c*x^4 + 27/4*a^2*b^2*c^2*x^4 + 27*a*b^5*x^
3 + 27*a^2*b^3*c*x^3 + 81/2*a^2*b^4*x^2 + 27*a^3*b^3*x
```

3.9.9 Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.77

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx = x^4\left(\frac{27\,a^2\,b^2\,c^2}{4}+\frac{81\,a\,b^4\,c}{2}+\frac{27\,b^6}{4}\right)+\frac{c^6\,x^{10}}{10}+27\,a^3\,b^3\,x+b\,c^5\,x^9+\frac{81\,a^2\,b^4\,x^2}{2}+\frac{9\,b^2\,c^4\,x^8}{2}+9\,b^2\,c^2\,x^6\left(2\,b^2+a\,c\right)+27\,a\,b^3\,x^3\left(b^2+a\,c\right)+\frac{27\,b^3\,c\,x^5\,(3\,b^2+5\,a\,c)}{5}+\frac{9\,b\,c^3\,x^7\,(9\,b^2+a\,c)}{7}$$

input
```
int((3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^3,x)
```

output
```
x^4*((27*b^6)/4 + (27*a^2*b^2*c^2)/4 + (81*a*b^4*c)/2) + (c^6*x^10)/10 + 2
7*a^3*b^3*x + b*c^5*x^9 + (81*a^2*b^4*x^2)/2 + (9*b^2*c^4*x^8)/2 + 9*b^2*c
^2*x^6*(a*c + 2*b^2) + 27*a*b^3*x^3*(a*c + b^2) + (27*b^3*c*x^5*(5*a*c + 3
*b^2))/5 + (9*b*c^3*x^7*(a*c + 9*b^2))/7
```

3.10 $\int (3ab+3b^2x+3bcx^2+c^2x^3)^2\,dx$

3.10.1 Optimal result

Integrand size = 27, antiderivative size = 56

$$\int (3ab+3b^2x+3bcx^2+c^2x^3)^2\,dx=\frac{b^2(b^2-3ac)^2x}{c^2}-\frac{b(b^2-3ac)(b+cx)^4}{2c^3}+\frac{(b+cx)^7}{7c^3}$$

output
```
b^2*(-3*a*c+b^2)^2*x/c^2-1/2*b*(-3*a*c+b^2)*(c*x+b)^4/c^3+1/7*(c*x+b)^7/c^
3
```

3.10.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.46

$$\int (3ab+3b^2x+3bcx^2+c^2x^3)^2\,dx=9a^2b^2x+9ab^3x^2+3b^2(b^2+2ac)x^3+\frac{3}{2}bc(3b^2+ac)x^4+3b^2c^2x^5+bc^3x^6+\frac{c^4x^7}{7}$$

input
```
Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

output
```
9*a^2*b^2*x + 9*a*b^3*x^2 + 3*b^2*(b^2 + 2*a*c)*x^3 + (3*b*c*(3*b^2 + a*c)
*x^4)/2 + 3*b^2*c^2*x^5 + b*c^3*x^6 + (c^4*x^7)/7
```

3.10.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2458, 747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2 dx$$

$$\Big\downarrow \text{2458}$$

$$\int \left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2 d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{747}$$

$$\int \left(\frac{\left(3abc-b^3\right)^2}{c^2}-2bc\left(b^2-3ac\right)\left(\frac{b}{c}+x\right)^3+c^4\left(\frac{b}{c}+x\right)^6\right) d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\left(b^2-3ac\right)^2\left(\frac{b}{c}+x\right)}{c^2}-\frac{1}{2}bc\left(b^2-3ac\right)\left(\frac{b}{c}+x\right)^4+\frac{1}{7}c^4\left(\frac{b}{c}+x\right)^7$$

input
```
Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

output
```
(b^2*(b^2 - 3*a*c)^2*(b/c + x))/c^2 - (b*c*(b^2 - 3*a*c)*(b/c + x)^4)/2 +
(c^4*(b/c + x)^7)/7
```

3.10.3.1 Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.10.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.46

method	result	size
norman	$\frac{c^4x^7}{7}+c^3x^6b+3b^2c^2x^5+\left(\frac{3}{2}ab\,c^2+\frac{9}{2}b^3c\right)x^4+\left(6a\,b^2c+3b^4\right)x^3+9a\,b^3x^2+9a^2b^2x$	82
gosper	$\frac{1}{7}c^4x^7+c^3x^6b+3b^2c^2x^5+\frac{3}{2}x^4ab\,c^2+\frac{9}{2}x^4b^3c+6a\,b^2c\,x^3+3b^4x^3+9a\,b^3x^2+9a^2b^2x$	84
default	$\frac{c^4x^7}{7}+c^3x^6b+3b^2c^2x^5+\frac{(6ab\,c^2+18b^3c)x^4}{4}+\frac{(18a\,b^2c+9b^4)x^3}{3}+9a\,b^3x^2+9a^2b^2x$	84
risch	$\frac{1}{7}c^4x^7+c^3x^6b+3b^2c^2x^5+\frac{3}{2}x^4ab\,c^2+\frac{9}{2}x^4b^3c+6a\,b^2c\,x^3+3b^4x^3+9a\,b^3x^2+9a^2b^2x$	84
parallelrisch	$\frac{1}{7}c^4x^7+c^3x^6b+3b^2c^2x^5+\frac{3}{2}x^4ab\,c^2+\frac{9}{2}x^4b^3c+6a\,b^2c\,x^3+3b^4x^3+9a\,b^3x^2+9a^2b^2x$	84

input
```
int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*c^4*x^7+c^3*x^6*b+3*b^2*c^2*x^5+(3/2*a*b*c^2+9/2*b^3*c)*x^4+(6*a*b^2*c
+3*b^4)*x^3+9*a*b^3*x^2+9*a^2*b^2*x
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.43

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx=\frac{1}{7}\,c^4x^7+bc^3x^6+3\,b^2c^2x^5+9\,ab^3x^2+9\,a^2b^2x+\frac{3}{2}\left(3\,b^3c+abc^2\right)x^4+3\left(b^4+2\,ab^2c\right)x^3$$

input
```
integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="fricas")
```

output
```
1/7*c^4*x^7 + b*c^3*x^6 + 3*b^2*c^2*x^5 + 9*a*b^3*x^2 + 9*a^2*b^2*x + 3/2*
(3*b^3*c + a*b*c^2)*x^4 + 3*(b^4 + 2*a*b^2*c)*x^3
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx = 9a^2b^2x+9ab^3x^2+3b^2c^2x^5+bc^3x^6+\frac{c^4x^7}{7}+x^4\cdot\left(\frac{3abc^2}{2}+\frac{9b^3c}{2}\right)+x^3\cdot\left(6ab^2c+3b^4\right)$$

input
```
integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

output
```
9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**2*c**2*x**5 + b*c**3*x**6 + c**4*x**7
/7 + x**4*(3*a*b*c**2/2 + 9*b**3*c/2) + x**3*(6*a*b**2*c + 3*b**4)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.66

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx = \frac{1}{7}\,c^4x^7+bc^3x^6+\frac{9}{5}\,b^2c^2x^5+3\,b^4x^3+9\,a^2b^2x+\frac{3}{2}\,\left(c^2x^4+4\,bcx^3+6\,b^2x^2\right)ab+\frac{3}{10}\,\left(4\,c^2x^5+15\,bcx^4\right)b^2$$

input
```
integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="maxima")
```

output
```
1/7*c^4*x^7 + b*c^3*x^6 + 9/5*b^2*c^2*x^5 + 3*b^4*x^3 + 9*a^2*b^2*x + 3/2*
(c^2*x^4 + 4*b*c*x^3 + 6*b^2*x^2)*a*b + 3/10*(4*c^2*x^5 + 15*b*c*x^4)*b^2
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.48

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx = \frac{1}{7}\,c^4x^7+bc^3x^6+3\,b^2c^2x^5+\frac{9}{2}\,b^3cx^4+\frac{3}{2}\,abc^2x^4+3\,b^4x^3+6\,ab^2cx^3+9\,ab^3x^2+9\,a^2b^2x$$

input
```
integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="giac")
```

output
```
1/7*c^4*x^7 + b*c^3*x^6 + 3*b^2*c^2*x^5 + 9/2*b^3*c*x^4 + 3/2*a*b*c^2*x^4
+ 3*b^4*x^3 + 6*a*b^2*c*x^3 + 9*a*b^3*x^2 + 9*a^2*b^2*x
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx = x^3\left(3\,b^4+6\,a\,c\,b^2\right)+\frac{c^4\,x^7}{7}+9\,a^2\,b^2\,x+9\,a\,b^3\,x^2+b\,c^3\,x^6+3\,b^2\,c^2\,x^5+\frac{3\,b\,c\,x^4\,\left(3\,b^2+a\,c\right)}{2}$$

input
```
int((3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^2,x)
```

output
```
x^3*(3*b^4 + 6*a*b^2*c) + (c^4*x^7)/7 + 9*a^2*b^2*x + 9*a*b^3*x^2 + b*c^3*
x^6 + 3*b^2*c^2*x^5 + (3*b*c*x^4*(a*c + 3*b^2))/2
```

3.11 $\int (3ab + 3b^2x + 3bcx^2 + c^2x^3) \, dx$

3.11.1 Optimal result

Integrand size = 25, antiderivative size = 32

$$\int (3ab + 3b^2x + 3bcx^2 + c^2x^3) \, dx = 3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

output
```
3*a*b*x+3/2*b^2*x^2+b*c*x^3+1/4*c^2*x^4
```

3.11.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (3ab + 3b^2x + 3bcx^2 + c^2x^3) \, dx = 3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

input
```
Integrate[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3,x]
```

output
```
3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4
```

3.11.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$3abx+\frac{3b^2x^2}{2}+bcx^3+\frac{c^2x^4}{4}$$

input
```
Int[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3,x]
```

output
```
3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4
```

3.11.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.11.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
gosper	$3abx+\frac{3}{2}b^2x^2+bc\,x^3+\frac{1}{4}c^2x^4$	29
default	$3abx+\frac{3}{2}b^2x^2+bc\,x^3+\frac{1}{4}c^2x^4$	29
norman	$3abx+\frac{3}{2}b^2x^2+bc\,x^3+\frac{1}{4}c^2x^4$	29
risch	$3abx+\frac{3}{2}b^2x^2+bc\,x^3+\frac{1}{4}c^2x^4$	29
parallelrisch	$3abx+\frac{3}{2}b^2x^2+bc\,x^3+\frac{1}{4}c^2x^4$	29
parts	$3abx+\frac{3}{2}b^2x^2+bc\,x^3+\frac{1}{4}c^2x^4$	29

input
```
int(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x,method=_RETURNVERBOSE)
```

output
```
3*a*b*x+3/2*b^2*x^2+b*c*x^3+1/4*c^2*x^4
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int (3ab + 3b^2x + 3bcx^2 + c^2x^3) \, dx = \frac{1}{4}\,c^2x^4 + bcx^3 + \frac{3}{2}\,b^2x^2 + 3\,abx$$

input
```
integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="fricas")
```

output
```
1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int (3ab + 3b^2x + 3bcx^2 + c^2x^3) \, dx = 3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

input
```
integrate(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b,x)
```

output
```
3*a*b*x + 3*b**2*x**2/2 + b*c*x**3 + c**2*x**4/4
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int (3ab + 3b^2x + 3bcx^2 + c^2x^3) \, dx = \frac{1}{4}\,c^2x^4 + bcx^3 + \frac{3}{2}\,b^2x^2 + 3\,abx$$

input
```
integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="maxima")
```

output
```
1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)\,dx=\frac{1}{4}\,c^2x^4+bcx^3+\frac{3}{2}\,b^2x^2+3\,abx$$

input
```
integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="giac")
```

output
```
1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)\,dx=\frac{3\,b^2\,x^2}{2}+b\,c\,x^3+3\,a\,b\,x+\frac{c^2\,x^4}{4}$$

input
```
int(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2,x)
```

output
```
(3*b^2*x^2)/2 + (c^2*x^4)/4 + 3*a*b*x + b*c*x^3
```

3.12 $\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx$

3.12.1 Optimal result

Integrand size = 27, antiderivative size = 188

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{b}+\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}b^{2/3}\left(b^2-3ac\right)^{2/3}}+\frac{\log\left(b-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{\log\left(b^{2/3}(b^2-3ac)^{2/3}+\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+c^2\left(\frac{b}{c}+x\right)^2\right)}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

output
```
1/3*ln(b-b^(1/3)*(-3*a*c+b^2)^(1/3)+c*x)/b^(2/3)/(-3*a*c+b^2)^(2/3)-1/6*ln
(b^(2/3)*(-3*a*c+b^2)^(2/3)+b^(1/3)*c*(-3*a*c+b^2)^(1/3)*(b/c+x)+c^2*(b/c+
x)^2)/b^(2/3)/(-3*a*c+b^2)^(2/3)-1/3*arctan(1/3*(b^(1/3)+2*(c*x+b)/(-3*a*c
+b^2)^(1/3))/b^(1/3)*3^(1/2))/b^(2/3)/(-3*a*c+b^2)^(2/3)*3^(1/2)
```

3.12.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.34

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = \frac{1}{3}\text{RootSum}\left[3ab+3b^2\#1+3bc\#1^2 + c^2\#1^3\&,\frac{\log(x-\#1)}{b^2+2bc\#1+c^2\#1^2}\&\right]$$

input
```
Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1),x]
```

output
```
RootSum[3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*
b*c*#1 + c^2*#1^2) & ]/3
```

3.12.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2458, 750, 16, 25, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3}d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{750}$$

$$\frac{c^{2/3}\int \frac{1}{c^{2/3}\left(\frac{b}{c}+x\right)-\frac{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}+\frac{c^{2/3}\int -\frac{c\left(\frac{b}{c}+x\right)+2\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}\left(c^{4/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}\right)}d\left(\frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

$$\Big\downarrow \; 16$$

$$\frac{c^{2/3}\int -\frac{c\left(\frac{b}{c}+x\right)+2\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}\left(c^{4/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}\right)}d\left(\frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}+\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

$$\Big\downarrow \; 25$$

$$\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c^{2/3}\int \frac{c\left(\frac{b}{c}+x\right)+2\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{c^{5/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c^{2/3}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

$$\Big\downarrow \; 1142$$

$$\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c^{2/3}\left(\frac{3}{2}\sqrt[3]{b}\sqrt[3]{b^2-3ac}\int \frac{1}{c^{5/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c^{2/3}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+x\right)+\frac{\int \frac{c^{2/3}\left(2c\left(\frac{b}{c}+x\right)+\sqrt[3]{b}\ldots\right.}{c^{5/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c^{2/3}\sqrt[3]{b^2-3\ldots}}}{2c\ldots}\right.}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c^{2/3}\left(\frac{3}{2}\sqrt[3]{b}\sqrt[3]{b^2-3ac}\int \frac{1}{c^{5/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c^{2/3}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+x\right)+\frac{1}{2}\int \frac{2c\left(\frac{b}{c}+x\right)+\sqrt[3]{\ldots}}{c^{5/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c^{2/3}\sqrt[3]{b^2-\ldots}}\right.}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

$$\Big\downarrow \; 1082$$

3.12. $\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx$

$$\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c^{2/3}\left(\frac{1}{2}\int \frac{2c\left(\frac{b}{c}+x\right)+\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{c^{5/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c^{2/3}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+x\right)-\frac{3\int \frac{1}{-\left(\frac{2c\left(\frac{b}{c}+x\right)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}+1\right)^2-3}d\left(\frac{2c\left(\frac{b}{c}+x\right)}{\sqrt[3]{b}\sqrt[3]{b^2-3\cdots}}\right.}{c^{2/3}}\cdots\right.}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

$\Big\downarrow$ 217

$$\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c^{2/3}\left(\frac{1}{2}\int \frac{2c\left(\frac{b}{c}+x\right)+\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{c^{5/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c^{2/3}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+x\right)+\frac{\sqrt{3}\arctan\left(\frac{\frac{2c\left(\frac{b}{c}+x\right)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}+1}{\sqrt{3}}\right)}{c^{2/3}}\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

$\Big\downarrow$ 1103

$$\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c^{2/3}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2c\left(\frac{b}{c}+x\right)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}+1}{\sqrt{3}}\right)}{c^{2/3}}+\frac{\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{2c^{2/3}}\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

input
```
Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1),x]
```

3.12. $\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx$

output
```
Log[b^(1/3)*(b^2 - 3*a*c)^(1/3) - c*(b/c + x)]/(3*b^(2/3)*(b^2 - 3*a*c)^(2
/3)) - (c^(2/3)*((Sqrt[3]*ArcTan[(1 + (2*c*(b/c + x))/(b^(1/3)*(b^2 - 3*a*
c)^(1/3)))/Sqrt[3]])/c^(2/3) + Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) + b^(1/3)*c
*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^2]/(2*c^(2/3))))/(3*b^(2/3)
*(b^2 - 3*a*c)^(2/3))
```

3.12.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.12.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.30

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c^{2}\_Z^{3}+3b\_Z^{2}c+3b^{2}\_Z+3ab\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}c^{2}+2\_Rbc+b^{2}}\right)}{3}$	57
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c^{2}\_Z^{3}+3b\_Z^{2}c+3b^{2}\_Z+3ab\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}c^{2}+2\_Rbc+b^{2}}\right)}{3}$	57

input
```
int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(x-_R),_R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3
*_Z*b^2+3*a*b))
```

3.12.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(151) = 302$.

Time = 0.29 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.06

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx =$$
$$-\frac{2\sqrt{3}(b^6-6\,ab^4c+9\,a^2b^2c^2)^{\frac{1}{6}}(b^3-3\,abc)\arctan\left(\frac{2\sqrt{3}(b^6-6\,ab^4c+9\,a^2b^2c^2)^{\frac{2}{3}}(cx+b)+\sqrt{3}(b^6-6\,ab^4c+9\,a^2b^2c^2)^{\frac{1}{3}}(b^3-3\,a}{3\,(b^6-6\,ab^4c+9\,a^2b^2c^2)^{\frac{5}{6}}}\right.}{}$$

input
```
integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/6)*(b^3 - 3*a*b*c)*ar
ctan(1/3*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sq
rt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*
b^4*c + 9*a^2*b^2*c^2)^(5/6)) + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*lo
g(-b^5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x
 - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) - (b^6 - 6*a*b^4*c +
9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c)) - 2*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2
)^(2/3)*log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c +
9*a^2*b^2*c^2)^(2/3)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.28

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = \operatorname{RootSum}\left(t^3\cdot\left(243a^2b^2c^2-162ab^4c+27b^6\right)-1,\left(t\mapsto t\log\left(x+\frac{9tabc-3tb^3+b}{c}\right)\right)\right)$$

input
```
integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b),x)
```

output
```
RootSum(_t**3*(243*a**2*b**2*c**2 - 162*a*b**4*c + 27*b**6) - 1, Lambda(_t
, _t*log(x + (9*_t*a*b*c - 3*_t*b**3 + b)/c)))
```

3.12.7 Maxima [F]

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = \int \frac{1}{c^2x^3+3\,bcx^2+3\,b^2x+3\,ab}\,dx$$

input
```
integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="maxima")
```

output
```
integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.13

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}(-b^3+3\,abc)^{\frac{1}{3}}}{cx+b+(-b^3+3\,abc)^{\frac{1}{3}}}\right)}{3\,(b^6-6\,ab^4c+9\,a^2b^2c^2)^{\frac{1}{3}}} - \frac{\log\left(4\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}(-b^3+3\,abc)^{\frac{1}{3}}\right)^2+4\left(cx+b+(-b^3+3\,abc)^{\frac{1}{3}}\right)^2\right)}{6\,(b^6-6\,ab^4c+9\,a^2b^2c^2)^{\frac{1}{3}}} + \frac{\log\left(\left|cx+b+(-b^3+3\,abc)^{\frac{1}{3}}\right|\right)}{3\,(b^6-6\,ab^4c+9\,a^2b^2c^2)^{\frac{1}{3}}}$$

input
```
integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan((sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/
3))/(c*x + b + (-b^3 + 3*a*b*c)^(1/3)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^
(1/3) - 1/6*log(4*(sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3
))^2 + 4*(c*x + b + (-b^3 + 3*a*b*c)^(1/3))^2)/(b^6 - 6*a*b^4*c + 9*a^2*b^
2*c^2)^(1/3) + 1/3*log(abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3)))/(b^6 - 6*a*b
^4*c + 9*a^2*b^2*c^2)^(1/3)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.93

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = \frac{\ln\left(b+b^{1/3}\,\left(3\,a\,c-b^2\right)^{1/3}+c\,x\right)}{3\,b^{2/3}\,\left(3\,a\,c-b^2\right)^{2/3}} + \frac{\ln\left(3\,b\,c^3+3\,c^4\,x+\frac{3\,b^{1/3}\,c^3\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(3\,a\,c-b^2\right)^{1/3}}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,b^{2/3}\,\left(3\,a\,c-b^2\right)^{2/3}} - \frac{\ln\left(3\,b\,c^3+3\,c^4\,x-\frac{3\,b^{1/3}\,c^3\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(3\,a\,c-b^2\right)^{1/3}}{2}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,b^{2/3}\,\left(3\,a\,c-b^2\right)^{2/3}}$$

input
```
int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2),x)
```

output
```
log(b + b^(1/3)*(3*a*c - b^2)^(1/3) + c*x)/(3*b^(2/3)*(3*a*c - b^2)^(2/3))
 + (log(3*b*c^3 + 3*c^4*x + (3*b^(1/3)*c^3*(3^(1/2)*1i - 1)*(3*a*c - b^2)^
(1/3))/2)*(3^(1/2)*1i - 1))/(6*b^(2/3)*(3*a*c - b^2)^(2/3)) - (log(3*b*c^3
 + 3*c^4*x - (3*b^(1/3)*c^3*(3^(1/2)*1i + 1)*(3*a*c - b^2)^(1/3))/2)*(3^(1
/2)*1i + 1))/(6*b^(2/3)*(3*a*c - b^2)^(2/3))
```

3.12. $\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx$

3.13 $\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2} dx$

3.13.1 Optimal result

Integrand size = 27, antiderivative size = 245

$$\begin{aligned}
&\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2} dx \\
&= -\frac{c\left(\frac{b}{c}+x\right)}{3b\,(b^2-3ac)\,(3ab+3b^2x+3bcx^2+c^2x^3)} \\
&\quad + \frac{2c\arctan\left(\frac{\sqrt[3]{b}+\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}b^{5/3}\,(b^2-3ac)^{5/3}} - \frac{2c\log\left(b-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+cx\right)}{9b^{5/3}\,(b^2-3ac)^{5/3}} \\
&\quad + \frac{c\log\left(b^{2/3}(b^2-3ac)^{2/3}+\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+c^2\left(\frac{b}{c}+x\right)^2\right)}{9b^{5/3}\,(b^2-3ac)^{5/3}}
\end{aligned}$$

output
```
-1/3*c*(b/c+x)/b/(-3*a*c+b^2)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)-2/9*c*ln(b
-b^(1/3)*(-3*a*c+b^2)^(1/3)+c*x)/b^(5/3)/(-3*a*c+b^2)^(5/3)+1/9*c*ln(b^(2/
3)*(-3*a*c+b^2)^(2/3)+b^(1/3)*c*(-3*a*c+b^2)^(1/3)*(b/c+x)+c^2*(b/c+x)^2)/
b^(5/3)/(-3*a*c+b^2)^(5/3)+2/9*c*arctan(1/3*(b^(1/3)+2*(c*x+b)/(-3*a*c+b^2
)^(1/3))/b^(1/3)*3^(1/2))/b^(5/3)/(-3*a*c+b^2)^(5/3)*3^(1/2)
```

3.13.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.46

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx$$
$$= -\frac{\frac{3(b+cx)}{3ab+x(3b^2+3bcx+c^2x^2)}+2c\text{RootSum}\left[3ab+3b^2\#1+3bc\#1^2+c^2\#1^3\&,\frac{\log(x-\#1)}{b^2+2bc\#1+c^2\#1^2}\&\right]}{9(b^3-3abc)}$$

input
```
Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2),x]
```

output
```
-1/9*((3*(b + c*x))/(3*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2)) + 2*c*RootSum[
3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b*c*#1 +
 c^2*#1^2) & ])/(b^3 - 3*a*b*c)
```

3.13.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2458, 749, 750, 16, 25, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx$$
$$\Big\downarrow \text{2458}$$
$$\int \frac{1}{\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}d\left(\frac{b}{c}+x\right)$$
$$\Big\downarrow \text{749}$$
$$-\frac{2c\int \frac{1}{c^2\left(\frac{b}{c}+x\right)^3+b\left(3a-\frac{b^2}{c}\right)}d\left(\frac{b}{c}+x\right)}{3b(b^2-3ac)}-\frac{c\left(\frac{b}{c}+x\right)}{3b(b^2-3ac)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}$$
$$\Big\downarrow \text{750}$$

$$-\frac{2c\left(\frac{c^{2/3}\int \frac{1}{c^{2/3}\left(\frac{b}{c}+x\right)-\frac{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}+\frac{c^{2/3}\int -\frac{c\left(\frac{b}{c}+x\right)+2\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}\left(c^{4/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}\right)}d\left(\frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}\right)}{3b\left(b^2-3ac\right)}$$

$$\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}$$

$\downarrow$ 16

$$-\frac{2c\left(\frac{c^{2/3}\int -\frac{c\left(\frac{b}{c}+x\right)+2\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}\left(c^{4/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}\right)}d\left(\frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}+\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}\right)}{3b\left(b^2-3ac\right)}-\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}$$

$\downarrow$ 25

$$-\frac{2c\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c^{2/3}\int \frac{c\left(\frac{b}{c}+x\right)+2\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{c^{5/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c^{2/3}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}\right)}{3b\left(b^2-3ac\right)}-\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}$$

$\downarrow$ 1142

3.13. $\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}\,dx$

$$-\frac{2c\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}(b^2-3ac)^{2/3}}-\frac{c^{2/3}\left(\frac{3}{2}\sqrt[3]{b}\sqrt[3]{b^2-3ac}\int\frac{1}{c^{5/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c^{2/3}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+x\right)\right.}{3b^{2/3}(b^2-3ac)^{2/3}}\right.}{3b\left(b^2-3ac\right)}$$

$$\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}$$

$$\Big\downarrow \; 27$$

$$-\frac{2c\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}(b^2-3ac)^{2/3}}-\frac{c^{2/3}\left(\frac{3}{2}\sqrt[3]{b}\sqrt[3]{b^2-3ac}\int\frac{1}{c^{5/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c^{2/3}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+x\right)\right.}{3b^{2/3}(b^2-3ac)^{2/3}}\right.}{3b\left(b^2-3ac\right)}$$

$$\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}$$

$$\Big\downarrow \; 1082$$

3.13. $\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx$

$$-\frac{2c\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}(b^2-3ac)^{2/3}}-\frac{c^{2/3}\left(\frac{1}{2}\int\frac{2c\left(\frac{b}{c}+x\right)+\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{c^{5/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c^{2/3}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+x\right)-\frac{3\int\cdots}{-\left(\frac{2c\left(\cdots\right.}{\sqrt[3]{b}\sqrt[3]{\cdots}}\right.}\cdots\right)}{3b^{2/3}(b^2-3ac)^{2/3}}\cdots\right)}{3b\left(b^2-3ac\right)\cdots}$$

$$\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}$$

$\downarrow$ 217

$$-\frac{2c\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}(b^2-3ac)^{2/3}}-\frac{c^{2/3}\left(\frac{1}{2}\int\frac{2c\left(\frac{b}{c}+x\right)+\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{c^{5/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c^{2/3}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+x\right)+\frac{\sqrt{3}\arctan\left(\frac{\cdots}{\sqrt[3]{b}\cdots}\right)}{\cdots}\cdots\right)}{3b^{2/3}(b^2-3ac)^{2/3}}\cdots\right)}{3b\left(b^2-3ac\right)\cdots}$$

$$\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}$$

$\downarrow$ 1103

3.13. $\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx$

$$-\frac{2c\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}(b^2-3ac)^{2/3}}-\frac{c^{2/3}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2c\left(\frac{b}{c}+x\right)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}+1}{\sqrt{3}}\right)}{c^{2/3}}+\frac{\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}\right.\right.}{2c^{2/3}}\right.}{3b^{2/3}(b^2-3ac)^{2/3}}\right.}{3b\,(b^2-3ac)}$$

$$-\frac{c\left(\frac{b}{c}+x\right)}{3b\,(b^2-3ac)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}$$

input
```
Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2),x]
```

output
```
-1/3*(c*(b/c + x))/(b*(b^2 - 3*a*c)*(b*(3*a - b^2/c) + c^2*(b/c + x)^3)) -
 (2*c*(Log[b^(1/3)*(b^2 - 3*a*c)^(1/3) - c*(b/c + x)]/(3*b^(2/3)*(b^2 - 3*
a*c)^(2/3)) - (c^(2/3)*((Sqrt[3]*ArcTan[(1 + (2*c*(b/c + x))/(b^(1/3)*(b^2
 - 3*a*c)^(1/3)))/Sqrt[3]])/c^(2/3) + Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) + b^
(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^2]/(2*c^(2/3))))/(3*
b^(2/3)*(b^2 - 3*a*c)^(2/3))))/(3*b*(b^2 - 3*a*c))
```

3.13.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.13. $\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx$

3.13.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\frac{cx}{9b\left(3ac-b^2\right)}+\frac{1}{27ac-9b^2}}{\frac{1}{3}c^2x^3+bc\,x^2+b^2x+ab}+\frac{2c\left(\sum_{\_R=\mathrm{RootOf}\left(c^2\_Z^3+3b\_Z^2c+3b^2\_Z+3ab\right)}\frac{\ln\left(x-\_R\right)}{\_R^2c^2+2\_Rbc+b^2}\right)}{9b(3ac-b^2)}$	134
risch	$\frac{\frac{cx}{9b\left(3ac-b^2\right)}+\frac{1}{27ac-9b^2}}{\frac{1}{3}c^2x^3+bc\,x^2+b^2x+ab}+\frac{2c\left(\sum_{\_R=\mathrm{RootOf}\left(c^2\_Z^3+3b\_Z^2c+3b^2\_Z+3ab\right)}\frac{\ln\left(x-\_R\right)}{\left(3ac-b^2\right)\left(\_R^2c^2+2\_Rbc+b^2\right)}\right)}{9b}$	134

input
```
int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/9*c/b/(3*a*c-b^2)*x+1/9/(3*a*c-b^2))/(1/3*c^2*x^3+b*c*x^2+b^2*x+a*b)+2/
9*c/b/(3*a*c-b^2)*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(x-_R),_R=RootOf(_Z^3*c^
2+3*_Z^2*b*c+3*_Z*b^2+3*a*b))
```

3.13.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 704 vs. 2(204) = 408.

Time = 0.28 (sec) , antiderivative size = 704, normalized size of antiderivative = 2.87

$$\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}\,dx =$$

$$-\frac{3\,b^7-18\,ab^5c+27\,a^2b^3c^2-2\,\sqrt{3}(b^6-6\,ab^4c+9\,a^2b^2c^2)^{\frac{1}{6}}(3\,ab^4c-9\,a^2b^2c^2+(b^3c^3-3\,abc^4)x^3+3\,(b^4c}{}$$

input
```
integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="fricas")
```

output
```
-1/9*(3*b^7 - 18*a*b^5*c + 27*a^2*b^3*c^2 - 2*sqrt(3)*(b^6 - 6*a*b^4*c + 9
*a^2*b^2*c^2)^(1/6)*(3*a*b^4*c - 9*a^2*b^2*c^2 + (b^3*c^3 - 3*a*b*c^4)*x^3
 + 3*(b^4*c^2 - 3*a*b^2*c^3)*x^2 + 3*(b^5*c - 3*a*b^3*c^2)*x)*arctan(1/3*(
2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt(3)*(b^6
 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4*c + 9*
a^2*b^2*c^2)^(5/6)) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c^3*x^3 + 3
*b*c^2*x^2 + 3*b^2*c*x + 3*a*b*c)*log(-b^5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*
c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(
2/3)*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))
+ 2*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2
*c*x + 3*a*b*c)*log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*
b^4*c + 9*a^2*b^2*c^2)^(2/3)) + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3)*x)
/(3*a*b^10 - 27*a^2*b^8*c + 81*a^3*b^6*c^2 - 81*a^4*b^4*c^3 + (b^9*c^2 - 9
*a*b^7*c^3 + 27*a^2*b^5*c^4 - 27*a^3*b^3*c^5)*x^3 + 3*(b^10*c - 9*a*b^8*c^
2 + 27*a^2*b^6*c^3 - 27*a^3*b^4*c^4)*x^2 + 3*(b^11 - 9*a*b^9*c + 27*a^2*b^
7*c^2 - 27*a^3*b^5*c^3)*x)
```

3.13.6 Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.78

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx$$
$$= \frac{b+cx}{27a^2b^2c-9ab^4+x^3\cdot(9abc^3-3b^3c^2)+x^2\cdot(27ab^2c^2-9b^4c)+x\,(27ab^3c-9b^5)}$$
$$+\operatorname{RootSum}\Big(t^3\cdot(177147a^5b^5c^5-295245a^4b^7c^4+196830a^3b^9c^3-65610a^2b^{11}c^2+10935ab^{13}c-729b^{15})$$

input
```
integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

output
```
(b + c*x)/(27*a**2*b**2*c - 9*a*b**4 + x**3*(9*a*b*c**3 - 3*b**3*c**2) + x
**2*(27*a*b**2*c**2 - 9*b**4*c) + x*(27*a*b**3*c - 9*b**5)) + RootSum(_t**
3*(177147*a**5*b**5*c**5 - 295245*a**4*b**7*c**4 + 196830*a**3*b**9*c**3 -
 65610*a**2*b**11*c**2 + 10935*a*b**13*c - 729*b**15) - 8*c**3, Lambda(_t,
 _t*log(x + (81*_t*a**2*b**2*c**2 - 54*_t*a*b**4*c + 9*_t*b**6 + 2*b*c)/(2
*c**2))))
```

3.13.7 Maxima [F]

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx = \int \frac{1}{(c^2x^3+3\,bcx^2+3\,b^2x+3\,ab)^2}\,dx$$

input
```
integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="maxima")
```

output
```
-2/3*c*integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(b^3 - 3*a*
b*c) - 1/3*(c*x + b)/(3*a*b^4 - 9*a^2*b^2*c + (b^3*c^2 - 3*a*b*c^3)*x^3 +
3*(b^4*c - 3*a*b^2*c^2)*x^2 + 3*(b^5 - 3*a*b^3*c)*x)
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.18

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx = -\frac{2\sqrt{3}\left(\frac{c^3}{b^6-6\,ab^4c+9\,a^2b^2c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}(-b^3+3\,abc)^{\frac{1}{3}}}{cx+b+(-b^3+3\,abc)^{\frac{1}{3}}}\right)-\left(\frac{c^3}{b^6-6\,ab^4c+9\,a^2b^2c^2}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}cx+\sqrt{3}b\right.\right.}{9} - \frac{cx+b}{3\,(c^2x^3+3\,bcx^2+3\,b^2x+3\,ab)(b^3-3\,abc)}$$

input
```
integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="giac")
```

output
```
-1/9*(2*sqrt(3)*(c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*arctan((sqrt
(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))/(c*x + b + (-b^3 + 3
*a*b*c)^(1/3))) - (c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(4*(sqr
t(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))^2 + 4*(c*x + b + (-
b^3 + 3*a*b*c)^(1/3))^2) + 2*(c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)
*log(abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3))))/(b^3 - 3*a*b*c) - 1/3*(c*x +
b)/((c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)*(b^3 - 3*a*b*c))
```

3.13.9 Mupad [B] (verification not implemented)

Time = 10.17 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.01

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx$$
$$= \frac{\frac{1}{3\,(3\,a\,c-b^2)}+\frac{c\,x}{3\,b\,(3\,a\,c-b^2)}}{3\,b^2\,x+3\,b\,c\,x^2+3\,a\,b+c^2\,x^3}+\frac{2\,c\,\ln\left(b+b^{1/3}\,(3\,a\,c-b^2)^{1/3}+c\,x\right)}{9\,b^{5/3}\,(3\,a\,c-b^2)^{5/3}}$$
$$-\frac{\ln\left(2\,b-b^{1/3}\,(3\,a\,c-b^2)^{1/3}+2\,c\,x-\sqrt{3}\,b^{1/3}\,(3\,a\,c-b^2)^{1/3}\,1\mathrm{i}\right)\,\left(c+\sqrt{3}\,c\,1\mathrm{i}\right)}{9\,b^{5/3}\,(3\,a\,c-b^2)^{5/3}}$$
$$-\frac{\ln\left(2\,b-b^{1/3}\,(3\,a\,c-b^2)^{1/3}+2\,c\,x+\sqrt{3}\,b^{1/3}\,(3\,a\,c-b^2)^{1/3}\,1\mathrm{i}\right)\,\left(c-\sqrt{3}\,c\,1\mathrm{i}\right)}{9\,b^{5/3}\,(3\,a\,c-b^2)^{5/3}}$$

input
```
int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^2,x)
```

output
```
(1/(3*(3*a*c - b^2)) + (c*x)/(3*b*(3*a*c - b^2)))/(3*a*b + 3*b^2*x + c^2*x
^3 + 3*b*c*x^2) + (2*c*log(b + b^(1/3)*(3*a*c - b^2)^(1/3) + c*x))/(9*b^(5
/3)*(3*a*c - b^2)^(5/3)) - (log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2*c*x
- 3^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*(c + 3^(1/2)*c*1i))/(9*b^(5/3)*(
3*a*c - b^2)^(5/3)) - (log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2*c*x + 3^(
1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*(c - 3^(1/2)*c*1i))/(9*b^(5/3)*(3*a*c
 - b^2)^(5/3))
```

3.14 $\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$

3.14.1 Optimal result

Integrand size = 27, antiderivative size = 305

$$\begin{aligned}\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx &= -\frac{c\left(\frac{b}{c}+x\right)}{6b(b^2-3ac)(3ab+3b^2x+3bcx^2+c^2x^3)^2} \\ &\quad+\frac{5c^2\left(\frac{b}{c}+x\right)}{18b^2(b^2-3ac)^2(3ab+3b^2x+3bcx^2+c^2x^3)} \\ &\quad-\frac{5c^2\arctan\left(\frac{\sqrt[3]{b}+\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}}{\sqrt{3}\sqrt[3]{b}}\right)}{9\sqrt{3}b^{8/3}(b^2-3ac)^{8/3}}+\frac{5c^2\log\left(b-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+cx\right)}{27b^{8/3}(b^2-3ac)^{8/3}} \\ &\quad-\frac{5c^2\log\left(b^{2/3}(b^2-3ac)^{2/3}+\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+c^2\left(\frac{b}{c}+x\right)^2\right)}{54b^{8/3}(b^2-3ac)^{8/3}}\end{aligned}$$

output
```
-1/6*c*(b/c+x)/b/(-3*a*c+b^2)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2+5/18*c^2
*(b/c+x)/b^2/(-3*a*c+b^2)^2/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)+5/27*c^2*ln(
b-b^(1/3)*(-3*a*c+b^2)^(1/3)+c*x)/b^(8/3)/(-3*a*c+b^2)^(8/3)-5/54*c^2*ln(b
^(2/3)*(-3*a*c+b^2)^(2/3)+b^(1/3)*c*(-3*a*c+b^2)^(1/3)*(b/c+x)+c^2*(b/c+x)
^2)/b^(8/3)/(-3*a*c+b^2)^(8/3)-5/27*c^2*arctan(1/3*(b^(1/3)+2*(c*x+b)/(-3*
a*c+b^2)^(1/3))/b^(1/3)*3^(1/2))/b^(8/3)/(-3*a*c+b^2)^(8/3)*3^(1/2)
```

3.14.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.49

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$$
$$= \frac{-\frac{3(b+cx)(3b^3-15b^2cx-5c^3x^3-3bc(8a+5cx^2))}{(3ab+x(3b^2+3bcx+c^2x^2))^2}+10c^2\text{RootSum}\left[3ab+3b^2\#1+3bc\#1^2+c^2\#1^3\&,\frac{\log(x-\#1)}{b^2+2bc\#1+c^2\#1^2}\&\right]}{54(b^3-3abc)^2}$$

input
```
Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3),x]
```

output
```
((-3*(b + c*x)*(3*b^3 - 15*b^2*c*x - 5*c^3*x^3 - 3*b*c*(8*a + 5*c*x^2)))/(
3*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2))^2 + 10*c^2*RootSum[3*a*b + 3*b^2*#1
 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b*c*#1 + c^2*#1^2) & ])/
(54*(b^3 - 3*a*b*c)^2)
```

3.14.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {2458, 749, 749, 750, 16, 25, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$$
$$\Big\downarrow \; 2458$$
$$\int \frac{1}{\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^3}d\left(\frac{b}{c}+x\right)$$
$$\Big\downarrow \; 749$$
$$-\frac{5c\int \frac{1}{\left(c^2\left(\frac{b}{c}+x\right)^3+b\left(3a-\frac{b^2}{c}\right)\right)^2}d\left(\frac{b}{c}+x\right)}{6b(b^2-3ac)}-\frac{c\left(\frac{b}{c}+x\right)}{6b(b^2-3ac)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}$$

$$\Big\downarrow \; 749$$

$$-\frac{5c\left(-\frac{2c\int \frac{1}{c^2\left(\frac{b}{c}+x\right)^3+b\left(3a-\frac{b^2}{c}\right)}d\left(\frac{b}{c}+x\right)}{3b(b^2-3ac)}-\frac{c\left(\frac{b}{c}+x\right)}{3b(b^2-3ac)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}\right)}{6b\left(b^2-3ac\right)}-\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}$$

$$\Big\downarrow \; 750$$

$$-\frac{5c\left(-\frac{2c\left(\frac{c^{2/3}\int \frac{1}{c^{2/3}\left(\frac{b}{c}+x\right)-\frac{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}+\frac{c^{2/3}\int -\frac{c\left(\frac{b}{c}+x\right)+2\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}\left(c^{4/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}\right)}d\left(\frac{b}{c}+\right.}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}\right.}{3b(b^2-3ac)}\right.}{6b\left(b^2-3ac\right)}$$

$$\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}$$

$$\Big\downarrow \; 16$$

3.14. $\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3}\,dx$

$$-\frac{5c\left(-\frac{2c\left(\frac{c^{2/3}\int -\frac{c\left(\frac{b}{c}+x\right)+2\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}\left(c^{4/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}\right)}d\left(\frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}+\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}\right)}{3b(b^2-3ac)}-\frac{}{3b(}\right.}{6b\left(b^2-3ac\right)}$$

$$\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}$$

$\downarrow$ 25

$$-\frac{5c\left(-\frac{2c\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c^{2/3}\int \frac{c\left(\frac{b}{c}+x\right)+2\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{c^{5/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c^{2/3}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}\right)}{3b(b^2-3ac)}-\frac{}{3b(b^2-3ac)(}\right.}{6b\left(b^2-3ac\right)}$$

$$\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}$$

$\downarrow$ 1142

3.14. $\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$

$$-\frac{5c\left(-\frac{2c\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c^{2/3}\left(\frac{3}{2}\sqrt[3]{b}\sqrt[3]{b^2-3ac}\int\frac{1}{c^{5/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c^{2/3}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+\right.\right.}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}}{3b(b^2-3ac)}}{6b\left(b^2-3ac\right)}$$

$$\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}$$

$\downarrow$ 27

3.14. $\int\frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3}\,dx$

$$-\frac{5c\left(-\frac{2c\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c^{2/3}\left(\frac{3}{2}\sqrt[3]{b}\sqrt[3]{b^2-3ac}\int\frac{1}{c^{5/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c^{2/3}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+\right.\right.}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}\right.}{3b(b^2-3ac)}\right.}{6b\left(b^2-3ac\right)}$$

$$\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}$$

$$\Big\downarrow \text{1082}$$

3.14. $\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$

$$-\frac{5c\left(-\frac{2c\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c^{2/3}\left(\frac{1}{2}\int\frac{2c\left(\frac{b}{c}+x\right)+\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{c^{5/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c^{2/3}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+x\right)-\frac{3\int\cdots}{-\left(\frac{2c\cdots}{\sqrt[3]{b}\sqrt[3]{\cdots}}\right)\cdots}\cdots\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}\cdots\right)}{3b(b^2-3ac)}\cdots\right)}{6b\left(b^2-3ac\right)}\cdots$$

$$\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}$$

$\downarrow$ 217

3.14. $\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$

$$-\frac{5c\left(-\frac{2c\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c^{2/3}\left(\frac{1}{2}\int\frac{2c\left(\frac{b}{c}+x\right)+\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{c^{5/3}\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c^{2/3}\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{\sqrt[3]{c}}}d\left(\frac{b}{c}+x\right)+\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{\cdots}}{\cdots}\right.}{\cdots}\right.}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}\right.}{3b(b^2-3ac)}\right.}{6b\left(b^2-3ac\right)}$$

$$\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}$$

$$\downarrow 1103$$

3.14. $\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$

$$-\frac{5c\left(-\frac{2c\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c^{2/3}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2c\left(\frac{b}{c}+x\right)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}+1}{\sqrt{3}}\right)}{c^{2/3}}+\frac{\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\right.}{2c^{2/3}}\right.}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}\right.}{3b(b^2-3ac)}\right.}{6b\left(b^2-3ac\right)}$$

$$\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}$$

input
```
Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3),x]
```

3.14. $\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3}\,dx$

output
```
-1/6*(c*(b/c + x))/(b*(b^2 - 3*a*c)*(b*(3*a - b^2/c) + c^2*(b/c + x)^3)^2)
 - (5*c*(-1/3*(c*(b/c + x))/(b*(b^2 - 3*a*c)*(b*(3*a - b^2/c) + c^2*(b/c +
 x)^3)) - (2*c*(Log[b^(1/3)*(b^2 - 3*a*c)^(1/3) - c*(b/c + x)]/(3*b^(2/3)*
(b^2 - 3*a*c)^(2/3)) - (c^(2/3)*((Sqrt[3]*ArcTan[(1 + (2*c*(b/c + x))/(b^(
1/3)*(b^2 - 3*a*c)^(1/3)))/Sqrt[3]])/c^(2/3) + Log[b^(2/3)*(b^2 - 3*a*c)^(
2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^2]/(2*c^(2/
3))))/(3*b^(2/3)*(b^2 - 3*a*c)^(2/3))))/(3*b*(b^2 - 3*a*c))))/(6*b*(b^2 -
3*a*c))
```

3.14.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.14.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.91

method	result
risch	$\frac{\frac{5c^4x^4}{18\left(9a^2c^2-6a\,b^2c+b^4\right)b^2}+\frac{10c^3x^3}{9b\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{5c^2x^2}{3\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{2\left(2ac+b^2\right)cx}{3b\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{9\left(8ac-b^2\right)}{486a^2c^2-324a\,b^2c+54b^4}}{\left(c^2x^3+3bc\,x^2+3b^2x+3ab\right)^2}+\frac{5c^2\Big(\underset{\_R=\mathrm{R}}{}$
default	$\frac{\frac{5c^4x^4}{18\left(9a^2c^2-6a\,b^2c+b^4\right)b^2}+\frac{10c^3x^3}{9b\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{5c^2x^2}{3\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{2\left(2ac+b^2\right)cx}{3b\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{9\left(8ac-b^2\right)}{486a^2c^2-324a\,b^2c+54b^4}}{\left(c^2x^3+3bc\,x^2+3b^2x+3ab\right)^2}+\frac{5c^2\Big(\underset{\_R=\mathrm{R}}{}$

input
```
int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x,method=_RETURNVERBOSE)
```

output
```
9*(5/162*c^4/(9*a^2*c^2-6*a*b^2*c+b^4)/b^2*x^4+10/81/b*c^3/(9*a^2*c^2-6*a*
b^2*c+b^4)*x^3+5/27*c^2/(9*a^2*c^2-6*a*b^2*c+b^4)*x^2+2/27*(2*a*c+b^2)*c/b
/(9*a^2*c^2-6*a*b^2*c+b^4)*x+1/54*(8*a*c-b^2)/(9*a^2*c^2-6*a*b^2*c+b^4))/(
c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2+5/27*c^2/b^2*sum(1/(9*a^2*c^2-6*a*b^2*c
+b^4)/(_R^2*c^2+2*_R*b*c+b^2)*ln(x-_R),_R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3*_Z*
b^2+3*a*b))
```

3.14.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. $2(262) = 524$.

Time = 0.28 (sec) , antiderivative size = 1268, normalized size of antiderivative = 4.16

$$\int \frac{1}{(3ab + 3b^2x + 3bcx^2 + c^2x^3)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="fricas")
```

output
```
-1/54*(9*b^10 - 126*a*b^8*c + 513*a^2*b^6*c^2 - 648*a^3*b^4*c^3 - 15*(b^6*
c^4 - 6*a*b^4*c^5 + 9*a^2*b^2*c^6)*x^4 - 60*(b^7*c^3 - 6*a*b^5*c^4 + 9*a^2
*b^3*c^5)*x^3 - 90*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*b^4*c^4)*x^2 + 10*sqrt(3
)*(9*a^2*b^5*c^2 - 27*a^3*b^3*c^3 + (b^3*c^6 - 3*a*b*c^7)*x^6 + 6*(b^4*c^5
 - 3*a*b^2*c^6)*x^5 + 15*(b^5*c^4 - 3*a*b^3*c^5)*x^4 + 6*(3*b^6*c^3 - 8*a*
b^4*c^4 - 3*a^2*b^2*c^5)*x^3 + 9*(b^7*c^2 - a*b^5*c^3 - 6*a^2*b^3*c^4)*x^2
 + 18*(a*b^6*c^2 - 3*a^2*b^4*c^3)*x)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/
6)*arctan(1/3*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b)
 + sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 -
 6*a*b^4*c + 9*a^2*b^2*c^2)^(5/6)) + 5*(c^6*x^6 + 6*b*c^5*x^5 + 15*b^2*c^4
*x^4 + 18*a*b^3*c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a*b*c^4)*x^3 + 9*(b
^4*c^2 + 2*a*b^2*c^3)*x^2)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b^
5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b
^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2
*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c)) - 10*(c^6*x^6 + 6*b*c^5*x^5 + 15*b^2*c^4*
x^4 + 18*a*b^3*c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a*b*c^4)*x^3 + 9*(b^
4*c^2 + 2*a*b^2*c^3)*x^2)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b^4
 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(
2/3)) - 36*(b^9*c - 4*a*b^7*c^2 - 3*a^2*b^5*c^3 + 18*a^3*b^3*c^4)*x)/(9*a^
2*b^14 - 108*a^3*b^12*c + 486*a^4*b^10*c^2 - 972*a^5*b^8*c^3 + 729*a^6*...
```

3.14.6 Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.55

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$$

$$= \frac{}{1458a^4b^4c^2-972a^3b^6c+162a^2b^8+x^6\cdot(162a^2b^2c^6-108ab^4c^5+18b^6c^4)+x^5\cdot(972a^2b^3c^5-648ab^5c^4+1}$$
$$+\operatorname{RootSum}\left(t^3\cdot(129140163a^8b^8c^8-344373768a^7b^{10}c^7+401769396a^6b^{12}c^6-267846264a^5b^{14}c^5+1116\right.$$

input
```
integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

output
```
(24*a*b**2*c - 3*b**4 + 30*b**2*c**2*x**2 + 20*b*c**3*x**3 + 5*c**4*x**4 +
 x*(24*a*b*c**2 + 12*b**3*c))/(1458*a**4*b**4*c**2 - 972*a**3*b**6*c + 162
*a**2*b**8 + x**6*(162*a**2*b**2*c**6 - 108*a*b**4*c**5 + 18*b**6*c**4) +
x**5*(972*a**2*b**3*c**5 - 648*a*b**5*c**4 + 108*b**7*c**3) + x**4*(2430*a
**2*b**4*c**4 - 1620*a*b**6*c**3 + 270*b**8*c**2) + x**3*(972*a**3*b**3*c*
*4 + 2268*a**2*b**5*c**3 - 1836*a*b**7*c**2 + 324*b**9*c) + x**2*(2916*a**
3*b**4*c**3 - 486*a**2*b**6*c**2 - 648*a*b**8*c + 162*b**10) + x*(2916*a**
3*b**5*c**2 - 1944*a**2*b**7*c + 324*a*b**9)) + RootSum(_t**3*(129140163*a
**8*b**8*c**8 - 344373768*a**7*b**10*c**7 + 401769396*a**6*b**12*c**6 - 26
7846264*a**5*b**14*c**5 + 111602610*a**4*b**16*c**4 - 29760696*a**3*b**18*
c**3 + 4960116*a**2*b**20*c**2 - 472392*a*b**22*c + 19683*b**24) - 125*c**
6, Lambda(_t, _t*log(x + (729*_t*a**3*b**3*c**3 - 729*_t*a**2*b**5*c**2 +
243*_t*a*b**7*c - 27*_t*b**9 + 5*b*c**2)/(5*c**3))))
```

3.14.7 Maxima [F]

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx = \int \frac{1}{(c^2x^3+3\,bcx^2+3\,b^2x+3\,ab)^3}\,dx$$

input
```
integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="maxima")
```

output
```
5/9*c^2*integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(b^6 - 6*a
*b^4*c + 9*a^2*b^2*c^2) + 1/18*(5*c^4*x^4 + 20*b*c^3*x^3 + 30*b^2*c^2*x^2
- 3*b^4 + 24*a*b^2*c + 12*(b^3*c + 2*a*b*c^2)*x)/(9*a^2*b^8 - 54*a^3*b^6*c
 + 81*a^4*b^4*c^2 + (b^6*c^4 - 6*a*b^4*c^5 + 9*a^2*b^2*c^6)*x^6 + 6*(b^7*c
^3 - 6*a*b^5*c^4 + 9*a^2*b^3*c^5)*x^5 + 15*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*
b^4*c^4)*x^4 + 6*(3*b^9*c - 17*a*b^7*c^2 + 21*a^2*b^5*c^3 + 9*a^3*b^3*c^4)
*x^3 + 9*(b^10 - 4*a*b^8*c - 3*a^2*b^6*c^2 + 18*a^3*b^4*c^3)*x^2 + 18*(a*b
^9 - 6*a^2*b^7*c + 9*a^3*b^5*c^2)*x)
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.20

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$$
$$= \frac{5\left(2\sqrt{3}\left(\frac{c^6}{b^6-6\,ab^4c+9\,a^2b^2c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}(-b^3+3\,abc)^{\frac{1}{3}}}{cx+b+(-b^3+3\,abc)^{\frac{1}{3}}}\right)-\left(\frac{c^6}{b^6-6\,ab^4c+9\,a^2b^2c^2}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}cx+\sqrt{3}b\right.\right.\right.}{54\,(b^6-}$$
$$+\frac{5\,c^4x^4+20\,bc^3x^3+30\,b^2c^2x^2+12\,b^3cx+24\,abc^2x-3\,b^4+24\,ab^2c}{18\,(b^6-6\,ab^4c+9\,a^2b^2c^2)(c^2x^3+3\,bcx^2+3\,b^2x+3\,ab)^2}$$

input
```
integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="giac")
```

output
```
5/54*(2*sqrt(3)*(c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*arctan((sqrt
(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))/(c*x + b + (-b^3 + 3
*a*b*c)^(1/3))) - (c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(4*(sqr
t(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))^2 + 4*(c*x + b + (-
b^3 + 3*a*b*c)^(1/3))^2) + 2*(c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)
*log(abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3))))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*
c^2) + 1/18*(5*c^4*x^4 + 20*b*c^3*x^3 + 30*b^2*c^2*x^2 + 12*b^3*c*x + 24*a
*b*c^2*x - 3*b^4 + 24*a*b^2*c)/((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)*(c^2*x^3
 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^2)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 10.19 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.58

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$$
$$= \frac{\frac{8\,a\,c-b^2}{6\,(9\,a^2\,c^2-6\,a\,b^2\,c+b^4)}+\frac{5\,c^2\,x^2}{3\,(9\,a^2\,c^2-6\,a\,b^2\,c+b^4)}+\frac{10\,c^3\,x^3}{9\,b\,(9\,a^2\,c^2-6\,a\,b^2\,c+b^4)}+\frac{5\,c^4\,x^4}{18\,b^2\,(9\,a^2\,c^2-6\,a\,b^2\,c+b^4)}+\frac{2\,c\,x\,(b^2+2\,a\,c)}{3\,b\,(9\,a^2\,c^2-6\,a\,b^2\,c+b^4)}}{x^2\,(9\,b^4+18\,a\,c\,b^2)+9\,a^2\,b^2+c^4\,x^6+x^3\,(18\,b^3\,c+6\,a\,b\,c^2)+6\,b\,c^3\,x^5+15\,b^2\,c^2\,x^4+18\,a\,b^3\,x}$$
$$+\frac{5\,c^2\,\ln\left(b\,(3\,a\,c-b^2)^{8/3}-b^{19/3}+c\,x\,(3\,a\,c-b^2)^{8/3}+27\,a^3\,b^{1/3}\,c^3-27\,a^2\,b^{7/3}\,c^2+9\,a\,b^{13/3}\,c\right)}{27\,b^{8/3}\,(3\,a\,c-b^2)^{8/3}}$$
$$-\frac{5\,c^2\,\ln\left(2\,b-b^{1/3}\,(3\,a\,c-b^2)^{1/3}+2\,c\,x-\sqrt{3}\,b^{1/3}\,(3\,a\,c-b^2)^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,b^{8/3}\,(3\,a\,c-b^2)^{8/3}}$$
$$+\frac{5\,c^2\,\ln\left(2\,b-b^{1/3}\,(3\,a\,c-b^2)^{1/3}+2\,c\,x+\sqrt{3}\,b^{1/3}\,(3\,a\,c-b^2)^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,b^{8/3}\,(3\,a\,c-b^2)^{8/3}}$$

input
```
int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^3,x)
```

output
```
((8*a*c - b^2)/(6*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) + (5*c^2*x^2)/(3*(b^4 + 9
*a^2*c^2 - 6*a*b^2*c)) + (10*c^3*x^3)/(9*b*(b^4 + 9*a^2*c^2 - 6*a*b^2*c))
+ (5*c^4*x^4)/(18*b^2*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) + (2*c*x*(2*a*c + b^2
))/(3*b*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)))/(x^2*(9*b^4 + 18*a*b^2*c) + 9*a^2*
b^2 + c^4*x^6 + x^3*(18*b^3*c + 6*a*b*c^2) + 6*b*c^3*x^5 + 15*b^2*c^2*x^4
+ 18*a*b^3*x) + (5*c^2*log(b*(3*a*c - b^2)^(8/3) - b^(19/3) + c*x*(3*a*c -
 b^2)^(8/3) + 27*a^3*b^(1/3)*c^3 - 27*a^2*b^(7/3)*c^2 + 9*a*b^(13/3)*c))/(
27*b^(8/3)*(3*a*c - b^2)^(8/3)) - (5*c^2*log(2*b - b^(1/3)*(3*a*c - b^2)^(
1/3) + 2*c*x - 3^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*((3^(1/2)*1i)/2 + 1
/2))/(27*b^(8/3)*(3*a*c - b^2)^(8/3)) + (5*c^2*log(2*b - b^(1/3)*(3*a*c -
b^2)^(1/3) + 2*c*x + 3^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*((3^(1/2)*1i)
/2 - 1/2))/(27*b^(8/3)*(3*a*c - b^2)^(8/3))
```

3.15 $\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 -$

3.15.1 Optimal result

Integrand size = 46, antiderivative size = 361

$$\begin{aligned}
&\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx \\
&=\frac{(bc-ad)^3(be-af)^3(a+bx)^4}{4b^7}+\frac{3(bc-ad)^2(be-af)^2(bde+bcf-2adf)(a+bx)^5}{5b^7} \\
&+\frac{(bc-ad)(be-af)\left(5a^2d^2f^2-5abdf(de+cf)+b^2(d^2e^2+3cdef+c^2f^2)\right)(a+bx)^6}{2b^7} \\
&+\frac{(bde+bcf-2adf)\left(10a^2d^2f^2-10abdf(de+cf)+b^2(d^2e^2+8cdef+c^2f^2)\right)(a+bx)^7}{7b^7} \\
&+\frac{3df\left(5a^2d^2f^2-5abdf(de+cf)+b^2(d^2e^2+3cdef+c^2f^2)\right)(a+bx)^8}{8b^7} \\
&+\frac{d^2f^2(bde+bcf-2adf)(a+bx)^9}{3b^7}+\frac{d^3f^3(a+bx)^{10}}{10b^7}
\end{aligned}$$

output
```
1/4*(-a*d+b*c)^3*(-a*f+b*e)^3*(b*x+a)^4/b^7+3/5*(-a*d+b*c)^2*(-a*f+b*e)^2*
(-2*a*d*f+b*c*f+b*d*e)*(b*x+a)^5/b^7+1/2*(-a*d+b*c)*(-a*f+b*e)*(5*a^2*d^2*
f^2-5*a*b*d*f*(c*f+d*e)+b^2*(c^2*f^2+3*c*d*e*f+d^2*e^2))*(b*x+a)^6/b^7+1/7
*(-2*a*d*f+b*c*f+b*d*e)*(10*a^2*d^2*f^2-10*a*b*d*f*(c*f+d*e)+b^2*(c^2*f^2+
8*c*d*e*f+d^2*e^2))*(b*x+a)^7/b^7+3/8*d*f*(5*a^2*d^2*f^2-5*a*b*d*f*(c*f+d*
e)+b^2*(c^2*f^2+3*c*d*e*f+d^2*e^2))*(b*x+a)^8/b^7+1/3*d^2*f^2*(-2*a*d*f+b*
c*f+b*d*e)*(b*x+a)^9/b^7+1/10*d^3*f^3*(b*x+a)^10/b^7
```

3.15.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.81

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3 dx$$
$$= a^3c^3e^3x+\frac{3}{2}a^2c^2e^2(bce+ade+acf)x^2$$
$$+ace\left(b^2c^2e^2+3abce(de+cf)+a^2\left(d^2e^2+3cdef+c^2f^2\right)\right)x^3+\frac{1}{4}\left(b^3c^3e^3+9ab^2c^2e^2(de+cf)\right.$$
$$\left.+9a^2bce\left(d^2e^2+3cdef+c^2f^2\right)+a^3\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^4$$
$$+\frac{3}{5}\left(b^3c^2e^2(de+cf)+3ab^2ce\left(d^2e^2+3cdef+c^2f^2\right)+a^3df\left(d^2e^2+3cdef+c^2f^2\right)\right.$$
$$\left.+a^2b\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^5$$
$$+\frac{1}{2}\left(a^3d^2f^2(de+cf)+b^3ce\left(d^2e^2+3cdef+c^2f^2\right)+3a^2bdf\left(d^2e^2+3cdef+c^2f^2\right)\right.$$
$$\left.+ab^2\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^6+\frac{1}{7}\left(a^3d^3f^3+9a^2bd^2f^2(de+cf)\right.$$
$$\left.+9ab^2df\left(d^2e^2+3cdef+c^2f^2\right)+b^3\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^7$$
$$+\frac{3}{8}bdf\left(a^2d^2f^2+3abdf(de+cf)+b^2\left(d^2e^2+3cdef+c^2f^2\right)\right)x^8$$
$$+\frac{1}{3}b^2d^2f^2(bde+bcf+adf)x^9+\frac{1}{10}b^3d^3f^3x^{10}$$

input
```
Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
 + b*d*f*x^3)^3,x]
```

output
```
a^3*c^3*e^3*x + (3*a^2*c^2*e^2*(b*c*e + a*d*e + a*c*f)*x^2)/2 + a*c*e*(b^2
*c^2*e^2 + 3*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^
3 + ((b^3*c^3*e^3 + 9*a*b^2*c^2*e^2*(d*e + c*f) + 9*a^2*b*c*e*(d^2*e^2 + 3
*c*d*e*f + c^2*f^2) + a^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f
^3))*x^4)/4 + (3*(b^3*c^2*e^2*(d*e + c*f) + 3*a*b^2*c*e*(d^2*e^2 + 3*c*d*e
*f + c^2*f^2) + a^3*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a^2*b*(d^3*e^3 +
 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^5)/5 + ((a^3*d^2*f^2*(d*e + c
*f) + b^3*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + 3*a^2*b*d*f*(d^2*e^2 + 3*c
*d*e*f + c^2*f^2) + a*b^2*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f
^3))*x^6)/2 + ((a^3*d^3*f^3 + 9*a^2*b*d^2*f^2*(d*e + c*f) + 9*a*b^2*d*f*(d
^2*e^2 + 3*c*d*e*f + c^2*f^2) + b^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f
^2 + c^3*f^3))*x^7)/7 + (3*b*d*f*(a^2*d^2*f^2 + 3*a*b*d*f*(d*e + c*f) + b^
2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^8)/8 + (b^2*d^2*f^2*(b*d*e + b*c*f +
a*d*f)*x^9)/3 + (b^3*d^3*f^3*x^10)/10
```

3.15.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2464, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^3 \, dx$$

$$\Big\downarrow \text{ 2464}$$

$$\int (a+bx)^3(c+dx)^3(e+fx)^3dx$$

$$\Big\downarrow \text{ 99}$$

$$\int \left(\frac{3df(a+bx)^7\left(5a^2d^2f^2-5abdf(cf+de)+b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)}{b^6}+\frac{(a+bx)^6(-2adf+bcf+bde)\left(10a^2\right.}{}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3df(a+bx)^8\left(5a^2d^2f^2-5abdf(cf+de)+b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)}{8b^7}+\frac{(a+bx)^7(-2adf+bcf+bde)\left(10a^2d^2f^2-10abdf(cf+de)+b^2\left(c^2f^2+8cdef+d^2e^2\right)\right)}{7b^7}+\frac{(a+bx)^6(bc-ad)(be-af)\left(5a^2d^2f^2-5abdf(cf+de)+b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)}{2b^7}+\frac{d^2f^2(a+bx)^9(-2adf+bcf+bde)}{3b^7}+\frac{3(a+bx)^5(bc-ad)^2(be-af)^2(-2adf+bcf+bde)}{5b^7}+\frac{(a+bx)^4(bc-ad)^3(be-af)^3}{4b^7}+\frac{d^3f^3(a+bx)^{10}}{10b^7}$$

input
```
Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d
*f*x^3)^3,x]
```

output
```
((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^4)/(4*b^7) + (3*(b*c - a*d)^2*(b*e
- a*f)^2*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^5)/(5*b^7) + ((b*c - a*d)*(b*
e - a*f)*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 3*c*d*e*f
 + c^2*f^2))*(a + b*x)^6)/(2*b^7) + ((b*d*e + b*c*f - 2*a*d*f)*(10*a^2*d^2
*f^2 - 10*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 8*c*d*e*f + c^2*f^2))*(a +
b*x)^7)/(7*b^7) + (3*d*f*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2
*e^2 + 3*c*d*e*f + c^2*f^2))*(a + b*x)^8)/(8*b^7) + (d^2*f^2*(b*d*e + b*c*
f - 2*a*d*f)*(a + b*x)^9)/(3*b^7) + (d^3*f^3*(a + b*x)^10)/(10*b^7)
```

3.15.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2464
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[u*Qx^p, x] /
;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ[Expon[Px, x], 2] &&
  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && IGtQ[p, 1]
```

3.15.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 841 vs. $2(347) = 694$.

Time = 0.05 (sec) , antiderivative size = 842, normalized size of antiderivative = 2.33

method	result
norman	$\frac{d^3 f^3 b^3 x^{10}}{10} + \left(\frac{1}{3} d^3 f^3 b^2 a + \frac{1}{3} b^3 c\, d^2 f^3 + \frac{1}{3} b^3 d^3 e\, f^2\right) x^9 + \left(\frac{3}{8} d^3 f^3 a^2 b + \frac{9}{8} a\, b^2 c\, d^2 f^3 + \frac{9}{8} a\, b^2 d^3 e\, f^2 \right.$
default	$\frac{d^3 f^3 b^3 x^{10}}{10} + \frac{(adf+fbc+bde) b^2 d^2 f^2 x^9}{3} + \frac{\left((acf+eda+ebc) b^2 d^2 f^2 + 2(adf+fbc+bde)^2 bdf + bdf\left(2(acf+eda+ebc)bdf+(adf\right.\right.}{8}$
gosper	$\frac{x\left(84d^3 f^3 b^3 x^9 + 280x^8 d^3 f^3 b^2 a + 280x^8 b^3 c\, d^2 f^3 + 280x^8 b^3 d^3 e\, f^2 + 315x^7 d^3 f^3 a^2 b + 945x^7 a\, b^2 c\, d^2 f^3 + 945x^7 a\, b^2 d^3 e\, f^2 + 315x^7 b^3\right.}{}$
risch	$\frac{9}{8} x^8 a\, b^2 c\, d^2 f^3 + \frac{9}{8} x^8 a\, b^2 d^3 e\, f^2 + \frac{9}{8} x^8 b^3 c\, d^2 e\, f^2 + \frac{9}{7} x^7 a^2 bc\, d^2 f^3 + \frac{9}{7} x^7 a^2 b\, d^3 e\, f^2 + \frac{9}{7} x^7 b^3 c\, d^2 e^2$
parallelrisch	$\frac{9}{8} x^8 a\, b^2 c\, d^2 f^3 + \frac{9}{8} x^8 a\, b^2 d^3 e\, f^2 + \frac{9}{8} x^8 b^3 c\, d^2 e\, f^2 + \frac{9}{7} x^7 a^2 bc\, d^2 f^3 + \frac{9}{7} x^7 a^2 b\, d^3 e\, f^2 + \frac{9}{7} x^7 b^3 c\, d^2 e^2$

input
```
int((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x,me
thod=_RETURNVERBOSE)
```

output
```
1/10*d^3*f^3*b^3*x^10+(1/3*d^3*f^3*b^2*a+1/3*b^3*c*d^2*f^3+1/3*b^3*d^3*e*f
^2)*x^9+(3/8*d^3*f^3*a^2*b+9/8*a*b^2*c*d^2*f^3+9/8*a*b^2*d^3*e*f^2+3/8*b^3
*c^2*d*f^3+9/8*b^3*c*d^2*e*f^2+3/8*b^3*d^3*e^2*f)*x^8+(1/7*d^3*f^3*a^3+9/7
*a^2*b*c*d^2*f^3+9/7*a^2*b*d^3*e*f^2+9/7*a*b^2*c^2*d*f^3+27/7*a*b^2*c*d^2*
e*f^2+9/7*a*b^2*d^3*e^2*f+1/7*b^3*c^3*f^3+9/7*b^3*c^2*d*e*f^2+9/7*b^3*c*d^
2*e^2*f+1/7*b^3*d^3*e^3)*x^7+(1/2*a^3*c*d^2*f^3+1/2*a^3*d^3*e*f^2+3/2*a^2*
b*c^2*d*f^3+9/2*a^2*b*c*d^2*e*f^2+3/2*a^2*b*d^3*e^2*f+1/2*a*b^2*c^3*f^3+9/
2*a*b^2*c^2*d*e*f^2+9/2*a*b^2*c*d^2*e^2*f+1/2*a*b^2*d^3*e^3+1/2*b^3*c^3*e*
f^2+3/2*b^3*c^2*d*e^2*f+1/2*b^3*c*d^2*e^3)*x^6+(3/5*a^3*c^2*d*f^3+9/5*a^3*
c*d^2*e*f^2+3/5*a^3*d^3*e^2*f+3/5*a^2*b*c^3*f^3+27/5*a^2*b*c^2*d*e*f^2+27/
5*a^2*b*c*d^2*e^2*f+3/5*a^2*b*d^3*e^3+9/5*a*b^2*c^3*e*f^2+27/5*a*b^2*c^2*d
*e^2*f+9/5*a*b^2*c*d^2*e^3+3/5*b^3*c^3*e^2*f+3/5*b^3*c^2*d*e^3)*x^5+(1/4*a
^3*c^3*f^3+9/4*a^3*c^2*d*e*f^2+9/4*a^3*c*d^2*e^2*f+1/4*a^3*d^3*e^3+9/4*a^2
*b*c^3*e*f^2+27/4*a^2*b*c^2*d*e^2*f+9/4*a^2*b*c*d^2*e^3+9/4*a*b^2*c^3*e^2*
f+9/4*a*b^2*c^2*d*e^3+1/4*c^3*e^3*b^3)*x^4+(a^3*c^3*e*f^2+3*a^3*c^2*d*e^2*
f+a^3*c*d^2*e^3+3*a^2*b*c^3*e^2*f+3*a^2*b*c^2*d*e^3+a*b^2*c^3*e^3)*x^3+(3/
2*a^3*c^3*e^2*f+3/2*a^3*c^2*d*e^3+3/2*c^3*e^3*a^2*b)*x^2+c^3*e^3*a^3*x
```

3.15.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(347) = 694$.

Time = 0.25 (sec) , antiderivative size = 727, normalized size of antiderivative = 2.01

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right)^3 \, dx$$
$$= \frac{1}{10} b^3 d^3 f^3 x^{10} + a^3 c^3 e^3 x + \frac{1}{3} \left(b^3 d^3 e f^2 + \left(b^3 c d^2 + a b^2 d^3\right) f^3\right) x^9$$
$$+ \frac{3}{8} \left(b^3 d^3 e^2 f + 3 \left(b^3 c d^2 + a b^2 d^3\right) e f^2 + \left(b^3 c^2 d + 3 a b^2 c d^2 + a^2 b d^3\right) f^3\right) x^8$$
$$+ \frac{1}{7} \left(b^3 d^3 e^3 + 9 \left(b^3 c d^2 + a b^2 d^3\right) e^2 f + 9 \left(b^3 c^2 d + 3 a b^2 c d^2 + a^2 b d^3\right) e f^2 + \left(b^3 c^3 + 9 a b^2 c^2 d + 9 a^2 b c d^2 + a^3 c\right.$$
$$+ \frac{1}{2} \left(\left(b^3 c d^2 + a b^2 d^3\right) e^3 + 3 \left(b^3 c^2 d + 3 a b^2 c d^2 + a^2 b d^3\right) e^2 f + \left(b^3 c^3 + 9 a b^2 c^2 d + 9 a^2 b c d^2 + a^3 d^3\right) e f^2 + (ab\right.$$
$$+ \frac{3}{5} \left(\left(b^3 c^2 d + 3 a b^2 c d^2 + a^2 b d^3\right) e^3 + \left(b^3 c^3 + 9 a b^2 c^2 d + 9 a^2 b c d^2 + a^3 d^3\right) e^2 f + 3 \left(a b^2 c^3 + 3 a^2 b c^2 d + a^3 c d^2\right.\right.$$
$$+ \frac{1}{4} \left(a^3 c^3 f^3 + \left(b^3 c^3 + 9 a b^2 c^2 d + 9 a^2 b c d^2 + a^3 d^3\right) e^3 + 9 \left(a b^2 c^3 + 3 a^2 b c^2 d + a^3 c d^2\right) e^2 f + 9 \left(a^2 b c^3 + a^3 c^2 \right.\right.$$
$$+ \left(a^3 c^3 e f^2 + \left(a b^2 c^3 + 3 a^2 b c^2 d + a^3 c d^2\right) e^3 + 3 \left(a^2 b c^3 + a^3 c^2 d\right) e^2 f\right) x^3$$
$$+ \frac{3}{2} \left(a^3 c^3 e^2 f + \left(a^2 b c^3 + a^3 c^2 d\right) e^3\right) x^2$$

input
```
integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^
3,x, algorithm="fricas")
```

output
```
1/10*b^3*d^3*f^3*x^10 + a^3*c^3*e^3*x + 1/3*(b^3*d^3*e*f^2 + (b^3*c*d^2 +
a*b^2*d^3)*f^3)*x^9 + 3/8*(b^3*d^3*e^2*f + 3*(b^3*c*d^2 + a*b^2*d^3)*e*f^2
 + (b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^3)*x^8 + 1/7*(b^3*d^3*e^3 + 9
*(b^3*c*d^2 + a*b^2*d^3)*e^2*f + 9*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)
*e*f^2 + (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3)*x^7 + 1/
2*((b^3*c*d^2 + a*b^2*d^3)*e^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)
*e^2*f + (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*f^2 + (a*b^
2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^3)*x^6 + 3/5*((b^3*c^2*d + 3*a*b^2*c*
d^2 + a^2*b*d^3)*e^3 + (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)
*e^2*f + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e*f^2 + (a^2*b*c^3 + a^
3*c^2*d)*f^3)*x^5 + 1/4*(a^3*c^3*f^3 + (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*
c*d^2 + a^3*d^3)*e^3 + 9*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e^2*f + 9
*(a^2*b*c^3 + a^3*c^2*d)*e*f^2)*x^4 + (a^3*c^3*e*f^2 + (a*b^2*c^3 + 3*a^2*
b*c^2*d + a^3*c*d^2)*e^3 + 3*(a^2*b*c^3 + a^3*c^2*d)*e^2*f)*x^3 + 3/2*(a^3
*c^3*e^2*f + (a^2*b*c^3 + a^3*c^2*d)*e^3)*x^2
```

3.15.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. $2(364) = 728$.

Time = 0.09 (sec) , antiderivative size = 1018, normalized size of antiderivative = 2.82

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx$$
$$= a^3c^3e^3x+\frac{b^3d^3f^3x^{10}}{10}+x^9\left(\frac{ab^2d^3f^3}{3}+\frac{b^3cd^2f^3}{3}+\frac{b^3d^3ef^2}{3}\right)+x^8 \cdot\left(\frac{3a^2bd^3f^3}{8}+\frac{9ab^2cd^2f^3}{8}+\frac{9ab^2d^3ef^2}{8}+\frac{3b^3c^2df^3}{8}+\frac{9b^3cd^2ef^2}{8}+\frac{3b^3d^3e^2f}{8}\right)$$
$$+x^7\left(\frac{a^3d^3f^3}{7}+\frac{9a^2bcd^2f^3}{7}+\frac{9a^2bd^3ef^2}{7}+\frac{9ab^2c^2df^3}{7}+\frac{27ab^2cd^2ef^2}{7}+\frac{9ab^2d^3e^2f}{7}+\frac{b^3c^3f^3}{7}+\frac{9b^3c^2def^2}{7}+\frac{9b^3cd^2e^2f}{7}+\frac{b^3d^3e^3}{7}\right)$$
$$+x^6\left(\frac{a^3cd^2f^3}{2}+\frac{a^3d^3ef^2}{2}+\frac{3a^2bc^2df^3}{2}+\frac{9a^2bcd^2ef^2}{2}+\frac{3a^2bd^3e^2f}{2}+\frac{ab^2c^3f^3}{2}+\frac{9ab^2c^2def^2}{2}+\frac{9ab^2cd^2e^2f}{2}+\frac{ab^2d^3e^3}{2}+\frac{b^3c^3ef^2}{2}+\frac{3b^3c^2de^2f}{2}+\frac{b^3cd^2e^3}{2}\right)+x^5$$
$$\cdot\left(\frac{3a^3c^2df^3}{5}+\frac{9a^3cd^2ef^2}{5}+\frac{3a^3d^3e^2f}{5}+\frac{3a^2bc^3f^3}{5}+\frac{27a^2bc^2def^2}{5}+\frac{27a^2bcd^2e^2f}{5}+\frac{3a^2bd^3e^3}{5}+\frac{9ab^2c^3ef^2}{5}+\frac{27ab^2c^2de^2f}{5}+\frac{9ab^2cd^2e^3}{5}+\frac{3b^3c^3e^2f}{5}+\frac{3b^3c^2de^3}{5}\right)$$
$$+x^4\left(\frac{a^3c^3f^3}{4}+\frac{9a^3c^2def^2}{4}+\frac{9a^3cd^2e^2f}{4}+\frac{a^3d^3e^3}{4}+\frac{9a^2bc^3ef^2}{4}+\frac{27a^2bc^2de^2f}{4}+\frac{9a^2bcd^2e^3}{4}+\frac{9ab^2c^3e^2f}{4}+\frac{9ab^2c^2de^3}{4}+\frac{b^3c^3e^3}{4}\right)$$
$$+x^3\left(a^3c^3ef^2+3a^3c^2de^2f+a^3cd^2e^3+3a^2bc^3e^2f+3a^2bc^2de^3+ab^2c^3e^3\right)$$
$$+x^2\cdot\left(\frac{3a^3c^3e^2f}{2}+\frac{3a^3c^2de^3}{2}+\frac{3a^2bc^3e^3}{2}\right)$$

input
```
integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3
)**3,x)
```

3.15. $\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx$

output
```
a**3*c**3*e**3*x + b**3*d**3*f**3*x**10/10 + x**9*(a*b**2*d**3*f**3/3 + b*
*3*c*d**2*f**3/3 + b**3*d**3*e*f**2/3) + x**8*(3*a**2*b*d**3*f**3/8 + 9*a*
b**2*c*d**2*f**3/8 + 9*a*b**2*d**3*e*f**2/8 + 3*b**3*c**2*d*f**3/8 + 9*b**
3*c*d**2*e*f**2/8 + 3*b**3*d**3*e**2*f/8) + x**7*(a**3*d**3*f**3/7 + 9*a**
2*b*c*d**2*f**3/7 + 9*a**2*b*d**3*e*f**2/7 + 9*a*b**2*c**2*d*f**3/7 + 27*a
*b**2*c*d**2*e*f**2/7 + 9*a*b**2*d**3*e**2*f/7 + b**3*c**3*f**3/7 + 9*b**3
*c**2*d*e*f**2/7 + 9*b**3*c*d**2*e**2*f/7 + b**3*d**3*e**3/7) + x**6*(a**3
*c*d**2*f**3/2 + a**3*d**3*e*f**2/2 + 3*a**2*b*c**2*d*f**3/2 + 9*a**2*b*c*
d**2*e*f**2/2 + 3*a**2*b*d**3*e**2*f/2 + a*b**2*c**3*f**3/2 + 9*a*b**2*c**
2*d*e*f**2/2 + 9*a*b**2*c*d**2*e**2*f/2 + a*b**2*d**3*e**3/2 + b**3*c**3*e
*f**2/2 + 3*b**3*c**2*d*e**2*f/2 + b**3*c*d**2*e**3/2) + x**5*(3*a**3*c**2
*d*f**3/5 + 9*a**3*c*d**2*e*f**2/5 + 3*a**3*d**3*e**2*f/5 + 3*a**2*b*c**3*
f**3/5 + 27*a**2*b*c**2*d*e*f**2/5 + 27*a**2*b*c*d**2*e**2*f/5 + 3*a**2*b*
d**3*e**3/5 + 9*a*b**2*c**3*e*f**2/5 + 27*a*b**2*c**2*d*e**2*f/5 + 9*a*b**
2*c*d**2*e**3/5 + 3*b**3*c**3*e**2*f/5 + 3*b**3*c**2*d*e**3/5) + x**4*(a**
3*c**3*f**3/4 + 9*a**3*c**2*d*e*f**2/4 + 9*a**3*c*d**2*e**2*f/4 + a**3*d**
3*e**3/4 + 9*a**2*b*c**3*e*f**2/4 + 27*a**2*b*c**2*d*e**2*f/4 + 9*a**2*b*c
*d**2*e**3/4 + 9*a*b**2*c**3*e**2*f/4 + 9*a*b**2*c**2*d*e**3/4 + b**3*c**3
*e**3/4) + x**3*(a**3*c**3*e*f**2 + 3*a**3*c**2*d*e**2*f + a**3*c*d**2*e**
3 + 3*a**2*b*c**3*e**2*f + 3*a**2*b*c**2*d*e**3 + a*b**2*c**3*e**3) + x...
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.28

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx$$
$$\begin{aligned}
&= \frac{1}{10}\,b^3d^3f^3x^{10} + \frac{1}{3}\,(bde+bcf+adf)b^2d^2f^2x^9 \\
&\quad + \frac{3}{8}\,(bde+bcf+adf)^2bdfx^8 + a^3c^3e^3x + \frac{1}{7}\,(bde+bcf+adf)^3x^7 \\
&\quad + \frac{1}{4}\,\left(3\,bdfx^4 + 4\,(bde+bcf+adf)x^3 + 6\,(bce+ade+acf)x^2\right)a^2c^2e^2 \\
&\quad + \frac{1}{4}\,(bce+ade+acf)^3x^4 \\
&\quad + \frac{1}{70}\,\left(30\,b^2d^2f^2x^7 + 70\,(bde+bcf+adf)bdfx^6 + 42\,(bde+bcf+adf)^2x^5 + 70\,(bce+ade+acf)^2x^3 + 2\right. \\
&\quad + \frac{1}{10}\,\left(5\,bdfx^6 + 6\,(bde+(bc+ad)f)x^5\right)(bce+ade+acf)^2 \\
&\quad + \frac{1}{56}\,\left(21\,b^2d^2f^2x^8 + 48\,\left(b^2d^2ef+\left(b^2cd+abd^2\right)f^2\right)x^7 + 28\,\left(b^2d^2e^2 + 2\,\left(b^2cd+abd^2\right)ef + \left(b^2c^2 + 2\,abcd\right.\right.\right.
\end{aligned}$$

input
```
integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^
3,x, algorithm="maxima")
```

output
```
1/10*b^3*d^3*f^3*x^10 + 1/3*(b*d*e + b*c*f + a*d*f)*b^2*d^2*f^2*x^9 + 3/8*
(b*d*e + b*c*f + a*d*f)^2*b*d*f*x^8 + a^3*c^3*e^3*x + 1/7*(b*d*e + b*c*f +
 a*d*f)^3*x^7 + 1/4*(3*b*d*f*x^4 + 4*(b*d*e + b*c*f + a*d*f)*x^3 + 6*(b*c*
e + a*d*e + a*c*f)*x^2)*a^2*c^2*e^2 + 1/4*(b*c*e + a*d*e + a*c*f)^3*x^4 +
1/70*(30*b^2*d^2*f^2*x^7 + 70*(b*d*e + b*c*f + a*d*f)*b*d*f*x^6 + 42*(b*d*
e + b*c*f + a*d*f)^2*x^5 + 70*(b*c*e + a*d*e + a*c*f)^2*x^3 + 21*(4*b*d*f*
x^5 + 5*(b*d*e + (b*c + a*d)*f)*x^4)*(b*c*e + a*d*e + a*c*f))*a*c*e + 1/10
*(5*b*d*f*x^6 + 6*(b*d*e + (b*c + a*d)*f)*x^5)*(b*c*e + a*d*e + a*c*f)^2 +
 1/56*(21*b^2*d^2*f^2*x^8 + 48*(b^2*d^2*e*f + (b^2*c*d + a*b*d^2)*f^2)*x^7
 + 28*(b^2*d^2*e^2 + 2*(b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 + 2*a*b*c*d + a^
2*d^2)*f^2)*x^6)*(b*c*e + a*d*e + a*c*f)
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 987 vs. $2(347) = 694$.

Time = 0.29 (sec) , antiderivative size = 987, normalized size of antiderivative = 2.73

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3 \, dx$$
$$= \frac{1}{10}b^3d^3f^3x^{10} + \frac{1}{3}b^3d^3ef^2x^9 + \frac{1}{3}b^3cd^2f^3x^9 + \frac{1}{3}ab^2d^3f^3x^9 + \frac{3}{8}b^3d^3e^2fx^8 + \frac{9}{8}b^3cd^2ef^2x^8 + \frac{9}{8}ab^2d^3ef^2x^8 + \frac{3}{8}b^3c^2df^3x^8 + \frac{9}{8}ab^2cd^2f^3x^8 + \frac{3}{8}a^2bd^3f^3x^8 + \frac{1}{7}b^3d^3e^3x^7 + \frac{9}{7}b^3cd^2e^2fx^7 + \frac{9}{7}ab^2d^3e^2fx^7 + \frac{9}{7}b^3c^2def^2x^7 + \frac{27}{7}ab^2cd^2ef^2x^7 + \frac{9}{7}a^2bd^3ef^2x^7 + \frac{1}{7}b^3c^3f^3x^7 + \frac{9}{7}ab^2c^2df^3x^7 + \frac{9}{7}a^2bcd^2f^3x^7 + \frac{1}{7}a^3d^3f^3x^7 + \frac{1}{2}b^3cd^2e^3x^6 + \frac{1}{2}ab^2d^3e^3x^6 + \frac{3}{2}b^3c^2de^2fx^6 + \frac{9}{2}ab^2cd^2e^2fx^6 + \frac{3}{2}a^2bd^3e^2fx^6 + \frac{1}{2}b^3c^3ef^2x^6 + \frac{9}{2}ab^2c^2def^2x^6 + \frac{9}{2}a^2bcd^2ef^2x^6 + \frac{1}{2}a^3d^3ef^2x^6 + \frac{1}{2}ab^2c^3f^3x^6 + \frac{3}{2}a^2bc^2df^3x^6 + \frac{1}{2}a^3cd^2f^3x^6 + \frac{3}{5}b^3c^2de^3x^5 + \frac{9}{5}ab^2cd^2e^3x^5 + \frac{3}{5}a^2bd^3e^3x^5 + \frac{3}{5}b^3c^3e^2fx^5 + \frac{27}{5}ab^2c^2de^2fx^5 + \frac{27}{5}a^2bcd^2e^2fx^5 + \frac{3}{5}a^3d^3e^2fx^5 + \frac{9}{5}ab^2c^3ef^2x^5 + \frac{27}{5}a^2bc^2def^2x^5 + \frac{9}{5}a^3cd^2ef^2x^5 + \frac{3}{5}a^2bc^3f^3x^5 + \frac{3}{5}a^3c^2df^3x^5 + \frac{1}{4}b^3c^3e^3x^4 + \frac{9}{4}ab^2c^2de^3x^4 + \frac{9}{4}a^2bcd^2e^3x^4 + \frac{1}{4}a^3d^3e^3x^4 + \frac{9}{4}ab^2c^3e^2fx^4 + \frac{27}{4}a^2bc^2de^2fx^4 + \frac{9}{4}a^3cd^2e^2fx^4 + \frac{9}{4}a^2bc^3ef^2x^4 + \frac{9}{4}a^3c^2def^2x^4 + \frac{1}{4}a^3c^3f^3x^4 + ab^2c^3e^3x^3 + 3a^2bc^2de^3x^3 + a^3cd^2e^3x^3 + 3a^2bc^3e^2fx^3 + 3a^3c^2de^2fx^3 + a^3c^3ef^2x^3 + \frac{3}{2}a^2bc^3e^3x^2 + \frac{3}{2}a^3c^2de^3x^2 + \frac{3}{2}a^3c^3e^2fx^2 + a^3c^3e^3x$$

input
```
integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^
3,x, algorithm="giac")
```

3.15. $\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^3 \, dx$

output
```
1/10*b^3*d^3*f^3*x^10 + 1/3*b^3*d^3*e*f^2*x^9 + 1/3*b^3*c*d^2*f^3*x^9 + 1/
3*a*b^2*d^3*f^3*x^9 + 3/8*b^3*d^3*e^2*f*x^8 + 9/8*b^3*c*d^2*e*f^2*x^8 + 9/
8*a*b^2*d^3*e*f^2*x^8 + 3/8*b^3*c^2*d*f^3*x^8 + 9/8*a*b^2*c*d^2*f^3*x^8 +
3/8*a^2*b*d^3*f^3*x^8 + 1/7*b^3*d^3*e^3*x^7 + 9/7*b^3*c*d^2*e^2*f*x^7 + 9/
7*a*b^2*d^3*e^2*f*x^7 + 9/7*b^3*c^2*d*e*f^2*x^7 + 27/7*a*b^2*c*d^2*e*f^2*x
^7 + 9/7*a^2*b*d^3*e*f^2*x^7 + 1/7*b^3*c^3*f^3*x^7 + 9/7*a*b^2*c^2*d*f^3*x
^7 + 9/7*a^2*b*c*d^2*f^3*x^7 + 1/7*a^3*d^3*f^3*x^7 + 1/2*b^3*c*d^2*e^3*x^6
 + 1/2*a*b^2*d^3*e^3*x^6 + 3/2*b^3*c^2*d*e^2*f*x^6 + 9/2*a*b^2*c*d^2*e^2*f
*x^6 + 3/2*a^2*b*d^3*e^2*f*x^6 + 1/2*b^3*c^3*e*f^2*x^6 + 9/2*a*b^2*c^2*d*e
*f^2*x^6 + 9/2*a^2*b*c*d^2*e*f^2*x^6 + 1/2*a^3*d^3*e*f^2*x^6 + 1/2*a*b^2*c
^3*f^3*x^6 + 3/2*a^2*b*c^2*d*f^3*x^6 + 1/2*a^3*c*d^2*f^3*x^6 + 3/5*b^3*c^2
*d*e^3*x^5 + 9/5*a*b^2*c*d^2*e^3*x^5 + 3/5*a^2*b*d^3*e^3*x^5 + 3/5*b^3*c^3
*e^2*f*x^5 + 27/5*a*b^2*c^2*d*e^2*f*x^5 + 27/5*a^2*b*c*d^2*e^2*f*x^5 + 3/5
*a^3*d^3*e^2*f*x^5 + 9/5*a*b^2*c^3*e*f^2*x^5 + 27/5*a^2*b*c^2*d*e*f^2*x^5
+ 9/5*a^3*c*d^2*e*f^2*x^5 + 3/5*a^2*b*c^3*f^3*x^5 + 3/5*a^3*c^2*d*f^3*x^5
+ 1/4*b^3*c^3*e^3*x^4 + 9/4*a*b^2*c^2*d*e^3*x^4 + 9/4*a^2*b*c*d^2*e^3*x^4
+ 1/4*a^3*d^3*e^3*x^4 + 9/4*a*b^2*c^3*e^2*f*x^4 + 27/4*a^2*b*c^2*d*e^2*f*x
^4 + 9/4*a^3*c*d^2*e^2*f*x^4 + 9/4*a^2*b*c^3*e*f^2*x^4 + 9/4*a^3*c^2*d*e*f
^2*x^4 + 1/4*a^3*c^3*f^3*x^4 + a*b^2*c^3*e^3*x^3 + 3*a^2*b*c^2*d*e^3*x^3 +
 a^3*c*d^2*e^3*x^3 + 3*a^2*b*c^3*e^2*f*x^3 + 3*a^3*c^2*d*e^2*f*x^3 + a^...
```

3.15. $\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3 \, dx$

3.15.9 Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 787, normalized size of antiderivative = 2.18

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx$$
$$= x^7\left(\frac{a^3\,d^3\,f^3}{7}+\frac{9\,a^2\,b\,c\,d^2\,f^3}{7}+\frac{9\,a^2\,b\,d^3\,e\,f^2}{7}+\frac{9\,a\,b^2\,c^2\,d\,f^3}{7}+\frac{27\,a\,b^2\,c\,d^2\,e\,f^2}{7}+\frac{9\,a\,b^2\,d^3\,e^2\,f}{7}+\frac{b^3\,c^3\,f^3}{7}+\frac{9\,b^3\,c^2\,d\,e\,f^2}{7}+\frac{9\,b^3\,c\,d^2\,e^2\,f}{7}+\frac{b^3\,d^3\,e^3}{7}\right)+x^5\left(\frac{3\,a^3\,c^2\,d\,f^3}{5}+\frac{9\,a^3\,c\,d^2\,e\,f^2}{5}+\frac{3\,a^3\,d^3\,e^2\,f}{5}+\frac{3\,a^2\,b\,c^3\,f^3}{5}+\frac{27\,a^2\,b\,c^2\,d\,e\,f^2}{5}+\frac{27\,a^2\,b\,c\,d^2\,e^2\,f}{5}+\frac{3\,a^2\,b\,d^3\,e^3}{5}+\frac{9\,a\,b^2\,c^3\,e\,f^2}{5}+\frac{27\,a\,b^2\,c^2\,d\,e^2\,f}{5}+\frac{9\,a\,b^2\,c\,d^2\,e^3}{5}+\frac{3\,b^3\,c^3\,e^2\,f}{5}+\frac{3\,b^3\,c^2\,d\,e^3}{5}\right)+x^6\left(\frac{a^3\,c\,d^2\,f^3}{2}+\frac{a^3\,d^3\,e\,f^2}{2}+\frac{3\,a^2\,b\,c^2\,d\,f^3}{2}+\frac{9\,a^2\,b\,c\,d^2\,e\,f^2}{2}+\frac{3\,a^2\,b\,d^3\,e^2\,f}{2}+\frac{a\,b^2\,c^3\,f^3}{2}+\frac{9\,a\,b^2\,c^2\,d\,e\,f^2}{2}+\frac{9\,a\,b^2\,c\,d^2\,e^2\,f}{2}+\frac{a\,b^2\,d^3\,e^3}{2}+\frac{b^3\,c^3\,e\,f^2}{2}+\frac{3\,b^3\,c^2\,d\,e^2\,f}{2}+\frac{b^3\,c\,d^2\,e^3}{2}\right)+x^4\left(\frac{a^3\,c^3\,f^3}{4}+\frac{9\,a^3\,c^2\,d\,e\,f^2}{4}+\frac{9\,a^3\,c\,d^2\,e^2\,f}{4}+\frac{a^3\,d^3\,e^3}{4}+\frac{9\,a^2\,b\,c^3\,e\,f^2}{4}+\frac{27\,a^2\,b\,c^2\,d\,e^2\,f}{4}+\frac{9\,a^2\,b\,c\,d^2\,e^3}{4}+\frac{9\,a\,b^2\,c^3\,e^2\,f}{4}+\frac{9\,a\,b^2\,c^2\,d\,e^3}{4}+\frac{b^3\,c^3\,e^3}{4}\right)+a^3\,c^3\,e^3\,x+\frac{b^3\,d^3\,f^3\,x^{10}}{10}+\frac{3\,a^2\,c^2\,e^2\,x^2\,(a\,c\,f+a\,d\,e+b\,c\,e)}{2}+\frac{b^2\,d^2\,f^2\,x^9\,(a\,d\,f+b\,c\,f+b\,d\,e)}{3}+a\,c\,e\,x^3\left(a^2\,c^2\,f^2+3\,a^2\,c\,d\,e\,f+a^2\,d^2\,e^2+3\,a\,b\,c^2\,e\,f+3\,a\,b\,c\,d\,e^2+b^2\,c^2\,e^2\right)+\frac{3\,b\,d\,f\,x^8\,\left(a^2\,d^2\,f^2+3\,a\,b\,c\,d\,f^2+3\,a\,b\,d^2\,e\,f+b^2\,c^2\,f^2+3\,b^2\,c\,d\,e\,f+b^2\,d^2\,e^2\right)}{8}$$

input
```
int((x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d
*f*x^3)^3,x)
```

output
```
x^7*((a^3*d^3*f^3)/7 + (b^3*c^3*f^3)/7 + (b^3*d^3*e^3)/7 + (9*a*b^2*c^2*d*
f^3)/7 + (9*a^2*b*c*d^2*f^3)/7 + (9*a*b^2*d^3*e^2*f)/7 + (9*a^2*b*d^3*e*f^
2)/7 + (9*b^3*c*d^2*e^2*f)/7 + (9*b^3*c^2*d*e*f^2)/7 + (27*a*b^2*c*d^2*e*f
^2)/7) + x^5*((3*a^2*b*c^3*f^3)/5 + (3*a^2*b*d^3*e^3)/5 + (3*a^3*c^2*d*f^3
)/5 + (3*b^3*c^2*d*e^3)/5 + (3*a^3*d^3*e^2*f)/5 + (3*b^3*c^3*e^2*f)/5 + (9
*a*b^2*c*d^2*e^3)/5 + (9*a*b^2*c^3*e*f^2)/5 + (9*a^3*c*d^2*e*f^2)/5 + (27*
a*b^2*c^2*d*e^2*f)/5 + (27*a^2*b*c*d^2*e^2*f)/5 + (27*a^2*b*c^2*d*e*f^2)/5
) + x^6*((a*b^2*c^3*f^3)/2 + (a*b^2*d^3*e^3)/2 + (a^3*c*d^2*f^3)/2 + (b^3*
c*d^2*e^3)/2 + (a^3*d^3*e*f^2)/2 + (b^3*c^3*e*f^2)/2 + (3*a^2*b*c^2*d*f^3)
/2 + (3*a^2*b*d^3*e^2*f)/2 + (3*b^3*c^2*d*e^2*f)/2 + (9*a*b^2*c*d^2*e^2*f)
/2 + (9*a*b^2*c^2*d*e*f^2)/2 + (9*a^2*b*c*d^2*e*f^2)/2) + x^4*((a^3*c^3*f^
3)/4 + (a^3*d^3*e^3)/4 + (b^3*c^3*e^3)/4 + (9*a*b^2*c^2*d*e^3)/4 + (9*a^2*
b*c*d^2*e^3)/4 + (9*a*b^2*c^3*e^2*f)/4 + (9*a^2*b*c^3*e*f^2)/4 + (9*a^3*c*
d^2*e^2*f)/4 + (9*a^3*c^2*d*e*f^2)/4 + (27*a^2*b*c^2*d*e^2*f)/4) + a^3*c^3
*e^3*x + (b^3*d^3*f^3*x^10)/10 + (3*a^2*c^2*e^2*x^2*(a*c*f + a*d*e + b*c*e
))/2 + (b^2*d^2*f^2*x^9*(a*d*f + b*c*f + b*d*e))/3 + a*c*e*x^3*(a^2*c^2*f^
2 + a^2*d^2*e^2 + b^2*c^2*e^2 + 3*a*b*c*d*e^2 + 3*a*b*c^2*e*f + 3*a^2*c*d*
e*f) + (3*b*d*f*x^8*(a^2*d^2*f^2 + b^2*c^2*f^2 + b^2*d^2*e^2 + 3*a*b*c*d*f
^2 + 3*a*b*d^2*e*f + 3*b^2*c*d*e*f))/8
```

3.15. $\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^3 \, dx$

3.16 $\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2$ -

3.16.1 Optimal result

Integrand size = 46, antiderivative size = 193

$$\begin{aligned}\int & \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2\,dx\\ &= \frac{(bc-ad)^2(be-af)^2(a+bx)^3}{3b^5}+\frac{(bc-ad)(be-af)(bde+bcf-2adf)(a+bx)^4}{2b^5}\\ &\quad+\frac{\left(6a^2d^2f^2-6abdf(de+cf)+b^2(d^2e^2+4cdef+c^2f^2)\right)(a+bx)^5}{5b^5}\\ &\quad+\frac{df(bde+bcf-2adf)(a+bx)^6}{3b^5}+\frac{d^2f^2(a+bx)^7}{7b^5}\end{aligned}$$

output
```
1/3*(-a*d+b*c)^2*(-a*f+b*e)^2*(b*x+a)^3/b^5+1/2*(-a*d+b*c)*(-a*f+b*e)*(-2*
a*d*f+b*c*f+b*d*e)*(b*x+a)^4/b^5+1/5*(6*a^2*d^2*f^2-6*a*b*d*f*(c*f+d*e)+b^
2*(c^2*f^2+4*c*d*e*f+d^2*e^2))*(b*x+a)^5/b^5+1/3*d*f*(-2*a*d*f+b*c*f+b*d*e
)*(b*x+a)^6/b^5+1/7*d^2*f^2*(b*x+a)^7/b^5
```

3.16.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.25

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2\,dx$$
$$=a^2c^2e^2x+ace(bce+ade+acf)x^2+\frac{1}{3}\left(b^2c^2e^2+4abce(de+cf)+a^2\left(d^2e^2+4cdef+c^2f^2\right)\right)x^3$$
$$+\frac{1}{2}\left(b^2ce(de+cf)+a^2df(de+cf)+ab\left(d^2e^2+4cdef+c^2f^2\right)\right)x^4$$
$$+\frac{1}{5}\left(a^2d^2f^2+4abdf(de+cf)+b^2\left(d^2e^2+4cdef+c^2f^2\right)\right)x^5$$
$$+\frac{1}{3}bdf(bde+bcf+adf)x^6+\frac{1}{7}b^2d^2f^2x^7$$

input
```
Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
 + b*d*f*x^3)^2,x]
```

output
```
a^2*c^2*e^2*x + a*c*e*(b*c*e + a*d*e + a*c*f)*x^2 + ((b^2*c^2*e^2 + 4*a*b*
c*e*(d*e + c*f) + a^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^3)/3 + ((b^2*c*e*
(d*e + c*f) + a^2*d*f*(d*e + c*f) + a*b*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x
^4)/2 + ((a^2*d^2*f^2 + 4*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f +
 c^2*f^2))*x^5)/5 + (b*d*f*(b*d*e + b*c*f + a*d*f)*x^6)/3 + (b^2*d^2*f^2*x
^7)/7
```

3.16.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2464, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^2\,dx$$
$$\Big\downarrow \text{ 2464}$$
$$\int (a+bx)^2(c+dx)^2(e+fx)^2dx$$
$$\Big\downarrow \text{ 99}$$

$$\int \left(\frac{(a+bx)^4 \left(6a^2d^2f^2 - 6abdf(cf+de) + b^2\left(c^2f^2 + 4cdef + d^2e^2\right)\right)}{b^4} + \frac{2df(a+bx)^5(-2adf+bcf+bde)}{b^4} + \frac{2(a$$

$$\Big\downarrow \ 2009$$

$$\frac{(a+bx)^5 \left(6a^2d^2f^2 - 6abdf(cf+de) + b^2\left(c^2f^2 + 4cdef + d^2e^2\right)\right)}{5b^5} + \frac{df(a+bx)^6(-2adf+bcf+bde)}{3b^5} + \frac{(a+bx)^4(bc-ad)(be-af)(-2adf+bcf+bde)}{2b^5} + \frac{(a+bx)^3(bc-ad)^2(be-af)^2}{3b^5} + \frac{d^2f^2(a+bx)^7}{7b^5}$$

input
```
Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d
*f*x^3)^2,x]
```

output
```
((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^3)/(3*b^5) + ((b*c - a*d)*(b*e - a*
f)*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^4)/(2*b^5) + ((6*a^2*d^2*f^2 - 6*a*
b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*(a + b*x)^5)/(5*b
^5) + (d*f*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^6)/(3*b^5) + (d^2*f^2*(a +
b*x)^7)/(7*b^5)
```

3.16.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2464
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[u*Qx^p, x] /
;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ[Expon[Px, x], 2] &&
  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && IGtQ[p, 1]
```

3.16.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.97

method	result
default	$\frac{d^{2} f^{2} b^{2} x^{7}}{7}+\frac{(adf+fbc+bde)bdf\,x^{6}}{3}+\frac{\left(2(acf+eda+ebc)bdf+(adf+fbc+bde)^{2}\right)x^{5}}{5}+\frac{(2acebdf+2(acf+eda+ebc)(adf+fb}{4}$
norman	$\frac{d^{2} f^{2} b^{2} x^{7}}{7}+\left(\frac{1}{3}d^{2} f^{2} ab+\frac{1}{3}b^{2} cd\,f^{2}+\frac{1}{3}b^{2} d^{2} ef\right)x^{6}+\left(\frac{1}{5}a^{2} d^{2} f^{2}+\frac{4}{5}abcd\,f^{2}+\frac{4}{5}ab\,d^{2} ef+\frac{1}{5}b^{2} c^{2} f^{2}\right.$
gosper	$\frac{x\left(30d^{2} f^{2} b^{2} x^{6}+70x^{5} d^{2} f^{2} ab+70x^{5} b^{2} cd\,f^{2}+70x^{5} b^{2} d^{2} ef+42x^{4} a^{2} d^{2} f^{2}+168x^{4} abcd\,f^{2}+168x^{4} ab\,d^{2} ef+42x^{4} b^{2} c^{2} f^{2}+168x^{4} b^{2} cd\right.}{}$
risch	$\frac{4}{5}x^{5} abcd\,f^{2}+\frac{4}{5}x^{5} ab\,d^{2} ef+\frac{4}{5}x^{5} b^{2} cdef+\frac{4}{3}x^{3} a^{2} cdef+\frac{4}{3}x^{3} ab\,c^{2} ef+\frac{4}{3}x^{3} abcd\,e^{2}+a^{2} c^{2} e^{2} x+$
parallelrisch	$\frac{4}{5}x^{5} abcd\,f^{2}+\frac{4}{5}x^{5} ab\,d^{2} ef+\frac{4}{5}x^{5} b^{2} cdef+\frac{4}{3}x^{3} a^{2} cdef+\frac{4}{3}x^{3} ab\,c^{2} ef+\frac{4}{3}x^{3} abcd\,e^{2}+a^{2} c^{2} e^{2} x+$

input
```
int((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x,me
thod=_RETURNVERBOSE)
```

output
```
1/7*d^2*f^2*b^2*x^7+1/3*(a*d*f+b*c*f+b*d*e)*b*d*f*x^6+1/5*(2*(a*c*f+a*d*e+
b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)*x^5+1/4*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+
b*c*e)*(a*d*f+b*c*f+b*d*e))*x^4+1/3*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*
d*e+b*c*e)^2)*x^3+a*c*e*(a*c*f+a*d*e+b*c*e)*x^2+a^2*c^2*e^2*x
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.39

$$
\begin{aligned}
&\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^{2}+bdfx^{3}\right)^{2}\,dx \\
&= \frac{1}{7}\,b^{2}d^{2}f^{2}x^{7}+a^{2}c^{2}e^{2}x+\frac{1}{3}\left(b^{2}d^{2}ef+\left(b^{2}cd+abd^{2}\right)f^{2}\right)x^{6} \\
&\quad+\frac{1}{5}\left(b^{2}d^{2}e^{2}+4\left(b^{2}cd+abd^{2}\right)ef+\left(b^{2}c^{2}+4\,abcd+a^{2}d^{2}\right)f^{2}\right)x^{5} \\
&\quad+\frac{1}{2}\left(\left(b^{2}cd+abd^{2}\right)e^{2}+\left(b^{2}c^{2}+4\,abcd+a^{2}d^{2}\right)ef+\left(abc^{2}+a^{2}cd\right)f^{2}\right)x^{4} \\
&\quad+\frac{1}{3}\left(a^{2}c^{2}f^{2}+\left(b^{2}c^{2}+4\,abcd+a^{2}d^{2}\right)e^{2}+4\left(abc^{2}+a^{2}cd\right)ef\right)x^{3} \\
&\quad+\left(a^{2}c^{2}ef+\left(abc^{2}+a^{2}cd\right)e^{2}\right)x^{2}
\end{aligned}
$$

input
```
integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^
2,x, algorithm="fricas")
```

output
```
1/7*b^2*d^2*f^2*x^7 + a^2*c^2*e^2*x + 1/3*(b^2*d^2*e*f + (b^2*c*d + a*b*d^
2)*f^2)*x^6 + 1/5*(b^2*d^2*e^2 + 4*(b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*f^2)*x^5 + 1/2*((b^2*c*d + a*b*d^2)*e^2 + (b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*e*f + (a*b*c^2 + a^2*c*d)*f^2)*x^4 + 1/3*(a^2*c^2*f^2 +
 (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2 + 4*(a*b*c^2 + a^2*c*d)*e*f)*x^3 + (a
^2*c^2*e*f + (a*b*c^2 + a^2*c*d)*e^2)*x^2
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.79

$$\begin{aligned}
&\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2\,dx \\
&= a^2c^2e^2x+\frac{b^2d^2f^2x^7}{7}+x^6\left(\frac{abd^2f^2}{3}+\frac{b^2cdf^2}{3}+\frac{b^2d^2ef}{3}\right) \\
&\quad +x^5\left(\frac{a^2d^2f^2}{5}+\frac{4abcdf^2}{5}+\frac{4abd^2ef}{5}+\frac{b^2c^2f^2}{5}+\frac{4b^2cdef}{5}+\frac{b^2d^2e^2}{5}\right) \\
&\quad +x^4\left(\frac{a^2cdf^2}{2}+\frac{a^2d^2ef}{2}+\frac{abc^2f^2}{2}+2abcdef+\frac{abd^2e^2}{2}+\frac{b^2c^2ef}{2}+\frac{b^2cde^2}{2}\right) \\
&\quad +x^3\left(\frac{a^2c^2f^2}{3}+\frac{4a^2cdef}{3}+\frac{a^2d^2e^2}{3}+\frac{4abc^2ef}{3}+\frac{4abcde^2}{3}+\frac{b^2c^2e^2}{3}\right) \\
&\quad +x^2\left(a^2c^2ef+a^2cde^2+abc^2e^2\right)
\end{aligned}$$

input
```
integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3
)**2,x)
```

output
```
a**2*c**2*e**2*x + b**2*d**2*f**2*x**7/7 + x**6*(a*b*d**2*f**2/3 + b**2*c*
d*f**2/3 + b**2*d**2*e*f/3) + x**5*(a**2*d**2*f**2/5 + 4*a*b*c*d*f**2/5 +
4*a*b*d**2*e*f/5 + b**2*c**2*f**2/5 + 4*b**2*c*d*e*f/5 + b**2*d**2*e**2/5)
 + x**4*(a**2*c*d*f**2/2 + a**2*d**2*e*f/2 + a*b*c**2*f**2/2 + 2*a*b*c*d*e
*f + a*b*d**2*e**2/2 + b**2*c**2*e*f/2 + b**2*c*d*e**2/2) + x**3*(a**2*c**
2*f**2/3 + 4*a**2*c*d*e*f/3 + a**2*d**2*e**2/3 + 4*a*b*c**2*e*f/3 + 4*a*b*
c*d*e**2/3 + b**2*c**2*e**2/3) + x**2*(a**2*c**2*e*f + a**2*c*d*e**2 + a*b
*c**2*e**2)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.93

$$\begin{aligned}&\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2\,dx\\&=\frac{1}{7}\,b^2d^2f^2x^7+\frac{1}{3}\,(bde+bcf+adf)bdfx^6+a^2c^2e^2x\\&\quad+\frac{1}{5}\,(bde+bcf+adf)^2x^5+\frac{1}{3}\,(bce+ade+acf)^2x^3\\&\quad+\frac{1}{6}\,\left(3\,bdfx^4+4\,(bde+bcf+adf)x^3+6\,(bce+ade+acf)x^2\right)ace\\&\quad+\frac{1}{10}\,\left(4\,bdfx^5+5\,(bde+(bc+ad)f)x^4\right)(bce+ade+acf)\end{aligned}$$

input
```
integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^
2,x, algorithm="maxima")
```

output
```
1/7*b^2*d^2*f^2*x^7 + 1/3*(b*d*e + b*c*f + a*d*f)*b*d*f*x^6 + a^2*c^2*e^2*
x + 1/5*(b*d*e + b*c*f + a*d*f)^2*x^5 + 1/3*(b*c*e + a*d*e + a*c*f)^2*x^3
+ 1/6*(3*b*d*f*x^4 + 4*(b*d*e + b*c*f + a*d*f)*x^3 + 6*(b*c*e + a*d*e + a*
c*f)*x^2)*a*c*e + 1/10*(4*b*d*f*x^5 + 5*(b*d*e + (b*c + a*d)*f)*x^4)*(b*c*
e + a*d*e + a*c*f)
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.79

$$\begin{aligned}&\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2\,dx\\&=\frac{1}{7}\,b^2d^2f^2x^7+\frac{1}{3}\,b^2d^2efx^6+\frac{1}{3}\,b^2cdf^2x^6+\frac{1}{3}\,abd^2f^2x^6+\frac{1}{5}\,b^2d^2e^2x^5+\frac{4}{5}\,b^2cdefx^5\\&\quad+\frac{4}{5}\,abd^2efx^5+\frac{1}{5}\,b^2c^2f^2x^5+\frac{4}{5}\,abcdf^2x^5+\frac{1}{5}\,a^2d^2f^2x^5+\frac{1}{2}\,b^2cde^2x^4\\&\quad+\frac{1}{2}\,abd^2e^2x^4+\frac{1}{2}\,b^2c^2efx^4+2\,abcdefx^4+\frac{1}{2}\,a^2d^2efx^4+\frac{1}{2}\,abc^2f^2x^4\\&\quad+\frac{1}{2}\,a^2cdf^2x^4+\frac{1}{3}\,b^2c^2e^2x^3+\frac{4}{3}\,abcde^2x^3+\frac{1}{3}\,a^2d^2e^2x^3+\frac{4}{3}\,abc^2efx^3\\&\quad+\frac{4}{3}\,a^2cdefx^3+\frac{1}{3}\,a^2c^2f^2x^3+abc^2e^2x^2+a^2cde^2x^2+a^2c^2efx^2+a^2c^2e^2x\end{aligned}$$

input
```
integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^
2,x, algorithm="giac")
```

output
```
1/7*b^2*d^2*f^2*x^7 + 1/3*b^2*d^2*e*f*x^6 + 1/3*b^2*c*d*f^2*x^6 + 1/3*a*b*
d^2*f^2*x^6 + 1/5*b^2*d^2*e^2*x^5 + 4/5*b^2*c*d*e*f*x^5 + 4/5*a*b*d^2*e*f*
x^5 + 1/5*b^2*c^2*f^2*x^5 + 4/5*a*b*c*d*f^2*x^5 + 1/5*a^2*d^2*f^2*x^5 + 1/
2*b^2*c*d*e^2*x^4 + 1/2*a*b*d^2*e^2*x^4 + 1/2*b^2*c^2*e*f*x^4 + 2*a*b*c*d*
e*f*x^4 + 1/2*a^2*d^2*e*f*x^4 + 1/2*a*b*c^2*f^2*x^4 + 1/2*a^2*c*d*f^2*x^4
+ 1/3*b^2*c^2*e^2*x^3 + 4/3*a*b*c*d*e^2*x^3 + 1/3*a^2*d^2*e^2*x^3 + 4/3*a*
b*c^2*e*f*x^3 + 4/3*a^2*c*d*e*f*x^3 + 1/3*a^2*c^2*f^2*x^3 + a*b*c^2*e^2*x^
2 + a^2*c*d*e^2*x^2 + a^2*c^2*e*f*x^2 + a^2*c^2*e^2*x
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.40

$$\begin{aligned}\int (ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2\,dx\\ &= x^4\left(\frac{a^2\,c\,d\,f^2}{2}+\frac{a^2\,d^2\,e\,f}{2}+\frac{a\,b\,c^2\,f^2}{2}+2\,a\,b\,c\,d\,e\,f+\frac{a\,b\,d^2\,e^2}{2}+\frac{b^2\,c^2\,e\,f}{2}+\frac{b^2\,c\,d\,e^2}{2}\right)\\ &\quad+x^3\left(\frac{a^2\,c^2\,f^2}{3}+\frac{4\,a^2\,c\,d\,e\,f}{3}+\frac{a^2\,d^2\,e^2}{3}+\frac{4\,a\,b\,c^2\,e\,f}{3}+\frac{4\,a\,b\,c\,d\,e^2}{3}+\frac{b^2\,c^2\,e^2}{3}\right)\\ &\quad+x^5\left(\frac{a^2\,d^2\,f^2}{5}+\frac{4\,a\,b\,c\,d\,f^2}{5}+\frac{4\,a\,b\,d^2\,e\,f}{5}+\frac{b^2\,c^2\,f^2}{5}+\frac{4\,b^2\,c\,d\,e\,f}{5}+\frac{b^2\,d^2\,e^2}{5}\right)\\ &\quad+a^2\,c^2\,e^2\,x+\frac{b^2\,d^2\,f^2\,x^7}{7}+a\,c\,e\,x^2\,(a\,c\,f+a\,d\,e+b\,c\,e)+\frac{b\,d\,f\,x^6\,(a\,d\,f+b\,c\,f+b\,d\,e)}{3}\end{aligned}$$

input
```
int((x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d
*f*x^3)^2,x)
```

output
```
x^4*((a*b*c^2*f^2)/2 + (a*b*d^2*e^2)/2 + (a^2*c*d*f^2)/2 + (b^2*c*d*e^2)/2
 + (a^2*d^2*e*f)/2 + (b^2*c^2*e*f)/2 + 2*a*b*c*d*e*f) + x^3*((a^2*c^2*f^2)
/3 + (a^2*d^2*e^2)/3 + (b^2*c^2*e^2)/3 + (4*a*b*c*d*e^2)/3 + (4*a*b*c^2*e*
f)/3 + (4*a^2*c*d*e*f)/3) + x^5*((a^2*d^2*f^2)/5 + (b^2*c^2*f^2)/5 + (b^2*
d^2*e^2)/5 + (4*a*b*c*d*f^2)/5 + (4*a*b*d^2*e*f)/5 + (4*b^2*c*d*e*f)/5) +
a^2*c^2*e^2*x + (b^2*d^2*f^2*x^7)/7 + a*c*e*x^2*(a*c*f + a*d*e + b*c*e) +
(b*d*f*x^6*(a*d*f + b*c*f + b*d*e))/3
```

3.17 $\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2$ -

3.17.1 Optimal result

Integrand size = 44, antiderivative size = 56

$$\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3) \ dx = acex + \frac{1}{2}(bce + ade + acf)x^2 + \frac{1}{3}(bde + bcf + adf)x^3 + \frac{1}{4}bdfx^4$$

output
```
a*c*e*x+1/2*(a*c*f+a*d*e+b*c*e)*x^2+1/3*(a*d*f+b*c*f+b*d*e)*x^3+1/4*b*d*f*
x^4
```

3.17.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36

$$\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3) \ dx = acex + \frac{1}{2}bcex^2 + \frac{1}{2}adex^2 + \frac{1}{2}acfx^2 + \frac{1}{3}bdex^3 + \frac{1}{3}bcfx^3 + \frac{1}{3}adfx^3 + \frac{1}{4}bdfx^4$$

input
```
Integrate[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3,x]
```

output
```
a*c*e*x + (b*c*e*x^2)/2 + (a*d*e*x^2)/2 + (a*c*f*x^2)/2 + (b*d*e*x^3)/3 +
(b*c*f*x^3)/3 + (a*d*f*x^3)/3 + (b*d*f*x^4)/4
```

3.17.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)\,dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}x^3(adf+bcf+bde)+\frac{1}{2}x^2(acf+ade+bce)+acex+\frac{1}{4}bdfx^4$$

input
```
Int[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3,x]
```

output
```
a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^2)/2 + ((b*d*e + b*c*f + a*d*f)*x^3)/
3 + (b*d*f*x^4)/4
```

3.17.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.17.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{bdf\,x^4}{4}+\frac{((cf+ed)b+adf)x^3}{3}+\frac{(ebc+(cf+ed)a)x^2}{2}+acex$	53
norman	$\frac{bdf\,x^4}{4}+\left(\frac{1}{3}adf+\frac{1}{3}fbc+\frac{1}{3}bde\right)x^3+\left(\frac{1}{2}acf+\frac{1}{2}eda+\frac{1}{2}ebc\right)x^2+acex$	55
gosper	$\frac{x\left(3bdf\,x^3+4adf\,x^2+4bcf\,x^2+4bde\,x^2+6acfx+6edax+6bcex+12ace\right)}{12}$	60
risch	$acex+\frac{1}{2}acf\,x^2+\frac{1}{2}ade\,x^2+\frac{1}{2}bce\,x^2+\frac{1}{3}adf\,x^3+\frac{1}{3}bcf\,x^3+\frac{1}{3}eb\,x^3d+\frac{1}{4}bdf\,x^4$	63
parallelrisch	$acex+\frac{1}{2}acf\,x^2+\frac{1}{2}ade\,x^2+\frac{1}{2}bce\,x^2+\frac{1}{3}adf\,x^3+\frac{1}{3}bcf\,x^3+\frac{1}{3}eb\,x^3d+\frac{1}{4}bdf\,x^4$	63
parts	$acex+\frac{1}{2}acf\,x^2+\frac{1}{2}ade\,x^2+\frac{1}{2}bce\,x^2+\frac{1}{3}adf\,x^3+\frac{1}{3}bcf\,x^3+\frac{1}{3}eb\,x^3d+\frac{1}{4}bdf\,x^4$	63

input
```
int(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,method
=_RETURNVERBOSE)
```

output
```
1/4*b*d*f*x^4+1/3*((c*f+d*e)*b+a*d*f)*x^3+1/2*(e*b*c+(c*f+d*e)*a)*x^2+a*c*
e*x
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3) \, dx$$
$$= \frac{1}{4}bdfx^4 + acex + \frac{1}{3}(bde + (bc + ad)f)x^3 + \frac{1}{2}(acf + (bc + ad)e)x^2$$

input
```
integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
 algorithm="fricas")
```

output
```
1/4*b*d*f*x^4 + a*c*e*x + 1/3*(b*d*e + (b*c + a*d)*f)*x^3 + 1/2*(a*c*f + (
b*c + a*d)*e)*x^2
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3) \, dx$$
$$= acex + \frac{bdfx^4}{4} + x^3\left(\frac{adf}{3} + \frac{bcf}{3} + \frac{bde}{3}\right) + x^2\left(\frac{acf}{2} + \frac{ade}{2} + \frac{bce}{2}\right)$$

input
```
integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3,
x)
```

output
```
a*c*e*x + b*d*f*x**4/4 + x**3*(a*d*f/3 + b*c*f/3 + b*d*e/3) + x**2*(a*c*f/
2 + a*d*e/2 + b*c*e/2)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)\,dx$$
$$=\frac{1}{4}\,bdfx^4+acex+\frac{1}{3}\,(bde+bcf+adf)x^3+\frac{1}{2}\,(bce+ade+acf)x^2$$

input
```
integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
 algorithm="maxima")
```

output
```
1/4*b*d*f*x^4 + a*c*e*x + 1/3*(b*d*e + b*c*f + a*d*f)*x^3 + 1/2*(b*c*e + a
*d*e + a*c*f)*x^2
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)\,dx$$
$$=\frac{1}{4}\,bdfx^4+acex+\frac{1}{3}\,(bde+bcf+adf)x^3+\frac{1}{2}\,(bce+ade+acf)x^2$$

input
```
integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
 algorithm="giac")
```

output
```
1/4*b*d*f*x^4 + a*c*e*x + 1/3*(b*d*e + b*c*f + a*d*f)*x^3 + 1/2*(b*c*e + a
*d*e + a*c*f)*x^2
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)\,dx$$
$$=\frac{b\,d\,f\,x^4}{4}+\left(\frac{a\,d\,f}{3}+\frac{b\,c\,f}{3}+\frac{b\,d\,e}{3}\right)\,x^3+\left(\frac{a\,c\,f}{2}+\frac{a\,d\,e}{2}+\frac{b\,c\,e}{2}\right)\,x^2+a\,c\,e\,x$$

input
```
int(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*
f*x^3,x)
```

output
```
x^2*((a*c*f)/2 + (a*d*e)/2 + (b*c*e)/2) + x^3*((a*d*f)/3 + (b*c*f)/3 + (b*
d*e)/3) + a*c*e*x + (b*d*f*x^4)/4
```

3.17. $\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)\ dx$

3.18 $\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3} dx$

3.18.1 Optimal result

Integrand size = 46, antiderivative size = 86

$$\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3} dx = \frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}+\frac{f\log(e+fx)}{(be-af)(de-cf)}$$

output
```
b*ln(b*x+a)/(-a*d+b*c)/(-a*f+b*e)-d*ln(d*x+c)/(-a*d+b*c)/(-c*f+d*e)+f*ln(f
*x+e)/(-a*f+b*e)/(-c*f+d*e)
```

3.18.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3} dx = \frac{b(-de+cf)\log(a+bx)+d(be-af)\log(c+dx)+(-bc+ad)f\log(e+fx)}{(bc-ad)(be-af)(-de+cf)}$$

input
```
Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
 + b*d*f*x^3)^(-1),x]
```

output
```
(b*(-(d*e) + c*f)*Log[a + b*x] + d*(b*e - a*f)*Log[c + d*x] + (-(b*c) + a*
d)*f*Log[e + f*x])/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f))
```

3.18.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{b^2}{(a+bx)(bc-ad)(be-af)}+\frac{d^2}{(c+dx)(bc-ad)(cf-de)}+\frac{f^2}{(e+fx)(be-af)(de-cf)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}+\frac{f\log(e+fx)}{(be-af)(de-cf)}$$

input
```
Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d
*f*x^3)^(-1),x]
```

output
```
(b*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)) - (d*Log[c + d*x])/((b*c - a*d)
*(d*e - c*f)) + (f*Log[e + f*x])/((b*e - a*f)*(d*e - c*f))
```

3.18.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.18.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{f\ln(fx+e)}{(cf-ed)(af-be)}-\frac{d\ln(dx+c)}{(cf-ed)(da-bc)}+\frac{b\ln(bx+a)}{(af-be)(da-bc)}$	87
norman	$\frac{f\ln(fx+e)}{ac\,f^2-adef-bcef+d\,e^2b}+\frac{b\ln(bx+a)}{(af-be)(da-bc)}-\frac{d\ln(dx+c)}{(cf-ed)(da-bc)}$	94
parallelrisch	$\frac{\ln(bx+a)bcf-\ln(bx+a)bde-\ln(dx+c)adf+\ln(dx+c)bde+\ln(fx+e)adf-\ln(fx+e)bcf}{(ac\,f^2-adef-bcef+d\,e^2b)(da-bc)}$	103
risch	$-\frac{d\ln(dx+c)}{acdf-a\,d^2e-b\,c^2f+bcde}+\frac{f\ln(-fx-e)}{ac\,f^2-adef-bcef+d\,e^2b}+\frac{b\ln(bx+a)}{a^2df-abcf-abde+b^2ce}$	111

input
```
int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3),x,me
thod=_RETURNVERBOSE)
```

output
```
f/(c*f-d*e)/(a*f-b*e)*ln(f*x+e)-d/(c*f-d*e)/(a*d-b*c)*ln(d*x+c)+b/(a*f-b*e
)/(a*d-b*c)*ln(b*x+a)
```

3.18.5 Fricas [A] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

$$\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx = \frac{(bc-ad)f\log(fx+e)+(bde-bcf)\log(bx+a)-(bde-adf)\log(dx+c)}{(b^2cd-abd^2)e^2-(b^2c^2-a^2d^2)ef+(abc^2-a^2cd)f^2}$$

input
```
integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3
),x, algorithm="fricas")
```

output
```
((b*c - a*d)*f*log(f*x + e) + (b*d*e - b*c*f)*log(b*x + a) - (b*d*e - a*d*
f)*log(d*x + c))/((b^2*c*d - a*b*d^2)*e^2 - (b^2*c^2 - a^2*d^2)*e*f + (a*b
*c^2 - a^2*c*d)*f^2)
```

3.18.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3}\, dx = \text{Timed out}$$

input
```
integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
*3),x)
```

output
```
Timed out
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

$$\int \frac{1}{ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3}\, dx = \frac{b\log(bx + a)}{(b^2c - abd)e - (abc - a^2d)f} - \frac{d\log(dx + c)}{(bcd - ad^2)e - (bc^2 - acd)f} + \frac{f\log(fx + e)}{bde^2 + acf^2 - (bc + ad)ef}$$

input
```
integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3
),x, algorithm="maxima")
```

output
```
b*log(b*x + a)/((b^2*c - a*b*d)*e - (a*b*c - a^2*d)*f) - d*log(d*x + c)/((
b*c*d - a*d^2)*e - (b*c^2 - a*c*d)*f) + f*log(f*x + e)/(b*d*e^2 + a*c*f^2
- (b*c + a*d)*e*f)
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.52

$$\int \frac{1}{ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3}\, dx = \frac{b^2\log(|bx + a|)}{b^3ce - ab^2de - ab^2cf + a^2bdf} - \frac{d^2\log(|dx + c|)}{bcd^2e - ad^3e - bc^2df + acd^2f} + \frac{f^2\log(|fx + e|)}{bde^2f - bcef^2 - adef^2 + acf^3}$$

input
```
integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3
),x, algorithm="giac")
```

output
```
b^2*log(abs(b*x + a))/(b^3*c*e - a*b^2*d*e - a*b^2*c*f + a^2*b*d*f) - d^2*
log(abs(d*x + c))/(b*c*d^2*e - a*d^3*e - b*c^2*d*f + a*c*d^2*f) + f^2*log(
abs(f*x + e))/(b*d*e^2*f - b*c*e*f^2 - a*d*e*f^2 + a*c*f^3)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.23

$$\int \frac{1}{ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3}\,dx = \frac{b\ln(a+b\,x)}{b^2\,c\,e + a^2\,d\,f - a\,b\,c\,f - a\,b\,d\,e} + \frac{d\ln(c+d\,x)}{a\,d^2\,e + b\,c^2\,f - a\,c\,d\,f - b\,c\,d\,e} + \frac{f\ln(e+f\,x)}{a\,c\,f^2 + b\,d\,e^2 - a\,d\,e\,f - b\,c\,e\,f}$$

input
```
int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b
*d*f*x^3),x)
```

output
```
(b*log(a + b*x))/(b^2*c*e + a^2*d*f - a*b*c*f - a*b*d*e) + (d*log(c + d*x)
)/(a*d^2*e + b*c^2*f - a*c*d*f - b*c*d*e) + (f*log(e + f*x))/(a*c*f^2 + b*
d*e^2 - a*d*e*f - b*c*e*f)
```

3.19 $\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2} dx$

3.19.1 Optimal result

Integrand size = 46, antiderivative size = 234

$$\begin{aligned}&\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2} dx\\&=-\frac{b^3}{(bc-ad)^2(be-af)^2(a+bx)}-\frac{d^3}{(bc-ad)^2(de-cf)^2(c+dx)}\\&\quad-\frac{f^3}{(be-af)^2(de-cf)^2(e+fx)}-\frac{2b^3(bde+bcf-2adf)\log(a+bx)}{(bc-ad)^3(be-af)^3}\\&\quad+\frac{2d^3(bde-2bcf+adf)\log(c+dx)}{(bc-ad)^3(de-cf)^3}+\frac{2f^3(2bde-bcf-adf)\log(e+fx)}{(be-af)^3(de-cf)^3}\end{aligned}$$

output
```
-b^3/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)-d^3/(-a*d+b*c)^2/(-c*f+d*e)^2/(d*x+
c)-f^3/(-a*f+b*e)^2/(-c*f+d*e)^2/(f*x+e)-2*b^3*(-2*a*d*f+b*c*f+b*d*e)*ln(b
*x+a)/(-a*d+b*c)^3/(-a*f+b*e)^3+2*d^3*(a*d*f-2*b*c*f+b*d*e)*ln(d*x+c)/(-a*
d+b*c)^3/(-c*f+d*e)^3+2*f^3*(-a*d*f-b*c*f+2*b*d*e)*ln(f*x+e)/(-a*f+b*e)^3/
(-c*f+d*e)^3
```

3.19.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.99

$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2}\,dx$$
$$= -\frac{b^3}{(bc-ad)^2(be-af)^2(a+bx)} - \frac{d^3}{(bc-ad)^2(de-cf)^2(c+dx)} - \frac{f^3}{(be-af)^2(de-cf)^2(e+fx)} - \frac{2b^3(bde+bcf-2adf)\log(a+bx)}{(bc-ad)^3(be-af)^3} - \frac{2d^3(bde-2bcf+adf)\log(c+dx)}{(bc-ad)^3(-de+cf)^3} - \frac{2f^3(-2bde+bcf+adf)\log(e+fx)}{(be-af)^3(de-cf)^3}$$

input
```
Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
 + b*d*f*x^3)^(-2),x]
```

output
```
-(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)^2*(d*e -
 c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (2*b^3*
(b*d*e + b*c*f - 2*a*d*f)*Log[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) - (2
*d^3*(b*d*e - 2*b*c*f + a*d*f)*Log[c + d*x])/((b*c - a*d)^3*(-(d*e) + c*f)
^3) - (2*f^3*(-2*b*d*e + b*c*f + a*d*f)*Log[e + f*x])/((b*e - a*f)^3*(d*e
- c*f)^3)
```

3.19.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3)^2}\,dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(-\frac{2b^4(-2adf+bcf+bde)}{(a+bx)(bc-ad)^3(be-af)^3} + \frac{b^4}{(a+bx)^2(bc-ad)^2(be-af)^2} - \frac{2d^4(adf-2bcf+bde)}{(c+dx)(bc-ad)^3(cf-de)^3} + \frac{}{(c+dx)^2}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2}-\frac{2b^3\log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3}-\frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2}+\frac{2d^3\log(c+dx)(adf-2bcf+bde)}{(bc-ad)^3(de-cf)^3}-\frac{f^3}{(e+fx)(be-af)^2(de-cf)^2}+\frac{2f^3\log(e+fx)(-adf-bcf+2bde)}{(be-af)^3(de-cf)^3}$$

input
```
Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d
*f*x^3)^(-2),x]
```

output
```
-(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)^2*(d*e -
 c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (2*b^3*
(b*d*e + b*c*f - 2*a*d*f)*Log[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) + (2
*d^3*(b*d*e - 2*b*c*f + a*d*f)*Log[c + d*x])/((b*c - a*d)^3*(d*e - c*f)^3)
 + (2*f^3*(2*b*d*e - b*c*f - a*d*f)*Log[e + f*x])/((b*e - a*f)^3*(d*e - c*
f)^3)
```

3.19.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.19.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00

method	result
default	$-\frac{f^3}{(cf-ed)^2(af-be)^2(fx+e)}-\frac{2f^3(adf+fbc-2bde)\ln(fx+e)}{(cf-ed)^3(af-be)^3}-\frac{d^3}{(cf-ed)^2(da-bc)^2(dx+c)}+\frac{2d^3(adf-2fbc+bde)\ln(dx}{(cf-ed)^3(da-bc)^3}$
norman	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x,
method=_RETURNVERBOSE)
```

output
```
-f^3/(c*f-d*e)^2/(a*f-b*e)^2/(f*x+e)-2*f^3*(a*d*f+b*c*f-2*b*d*e)/(c*f-d*e)
^3/(a*f-b*e)^3*ln(f*x+e)-d^3/(c*f-d*e)^2/(a*d-b*c)^2/(d*x+c)+2*d^3*(a*d*f-
2*b*c*f+b*d*e)/(c*f-d*e)^3/(a*d-b*c)^3*ln(d*x+c)-b^3/(a*f-b*e)^2/(a*d-b*c)
^2/(b*x+a)+2*b^3*(2*a*d*f-b*c*f-b*d*e)/(a*f-b*e)^3/(a*d-b*c)^3*ln(b*x+a)
```

3.19.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3
)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.19.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x*
*3)**2,x)
```

output
```
Timed out
```

3.19.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2096 vs. 2(234) = 468.

Time = 0.32 (sec) , antiderivative size = 2096, normalized size of antiderivative = 8.96

$$\int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2} dx = \text{Too large to display}$$

input
```
integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3
)^2,x, algorithm="maxima")
```

output
```
-2*(b^4*d*e + (b^4*c - 2*a*b^3*d)*f)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*
d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*e^3 - 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d +
3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*e^2*f + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d +
3*a^4*b^2*c*d^2 - a^5*b*d^3)*e*f^2 - (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^
5*b*c*d^2 - a^6*d^3)*f^3) + 2*(b*d^4*e - (2*b*c*d^3 - a*d^4)*f)*log(d*x +
c)/((b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6)*e^3 - 3*(b^3
*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^5)*e^2*f + 3*(b^3*c
^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4)*e*f^2 - (b^3*c^6 -
 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*f^3) + 2*(2*b*d*e*f^3 - (b
*c + a*d)*f^4)*log(f*x + e)/(b^3*d^3*e^6 + a^3*c^3*f^6 - 3*(b^3*c*d^2 + a*
b^2*d^3)*e^5*f + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^4*f^2 - (b^3*
c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^3*f^3 + 3*(a*b^2*c^3 + 3*
a^2*b*c^2*d + a^3*c*d^2)*e^2*f^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*e*f^5) - ((b^
3*c*d^2 + a*b^2*d^3)*e^3 - 2*(b^3*c^2*d + a^2*b*d^3)*e^2*f + (b^3*c^3 + a^
3*d^3)*e*f^2 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*f^3 + 2*(b^3*d^3*e^
2*f - (b^3*c*d^2 + a*b^2*d^3)*e*f^2 + (b^3*c^2*d - a*b^2*c*d^2 + a^2*b*d^3
)*f^3)*x^2 + (2*b^3*d^3*e^3 - (b^3*c*d^2 + a*b^2*d^3)*e^2*f - (b^3*c^2*d +
 a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + 2*a^3*d^3)*f^
3)*x)/((a*b^4*c^3*d^2 - 2*a^2*b^3*c^2*d^3 + a^3*b^2*c*d^4)*e^5 - 2*(a*b^4*
c^4*d - a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3 + a^4*b*c*d^4)*e^4*f + (a*b^4...
```

3.19.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1435 vs. 2(234) = 468.

Time = 0.28 (sec) , antiderivative size = 1435, normalized size of antiderivative = 6.13

$$\int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2} dx = \text{Too large to display}$$

input
```
integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3
)^2,x, algorithm="giac")
```

output
```
-2*(b^5*d*e + b^5*c*f - 2*a*b^4*d*f)*log(abs(b*x + a))/(b^7*c^3*e^3 - 3*a*
b^6*c^2*d*e^3 + 3*a^2*b^5*c*d^2*e^3 - a^3*b^4*d^3*e^3 - 3*a*b^6*c^3*e^2*f
+ 9*a^2*b^5*c^2*d*e^2*f - 9*a^3*b^4*c*d^2*e^2*f + 3*a^4*b^3*d^3*e^2*f + 3*
a^2*b^5*c^3*e*f^2 - 9*a^3*b^4*c^2*d*e*f^2 + 9*a^4*b^3*c*d^2*e*f^2 - 3*a^5*
b^2*d^3*e*f^2 - a^3*b^4*c^3*f^3 + 3*a^4*b^3*c^2*d*f^3 - 3*a^5*b^2*c*d^2*f^
3 + a^6*b*d^3*f^3) + 2*(b*d^5*e - 2*b*c*d^4*f + a*d^5*f)*log(abs(d*x + c))
/(b^3*c^3*d^4*e^3 - 3*a*b^2*c^2*d^5*e^3 + 3*a^2*b*c*d^6*e^3 - a^3*d^7*e^3
- 3*b^3*c^4*d^3*e^2*f + 9*a*b^2*c^3*d^4*e^2*f - 9*a^2*b*c^2*d^5*e^2*f + 3*
a^3*c*d^6*e^2*f + 3*b^3*c^5*d^2*e*f^2 - 9*a*b^2*c^4*d^3*e*f^2 + 9*a^2*b*c^
3*d^4*e*f^2 - 3*a^3*c^2*d^5*e*f^2 - b^3*c^6*d*f^3 + 3*a*b^2*c^5*d^2*f^3 -
3*a^2*b*c^4*d^3*f^3 + a^3*c^3*d^4*f^3) + 2*(2*b*d*e*f^4 - b*c*f^5 - a*d*f^
5)*log(abs(f*x + e))/(b^3*d^3*e^6*f - 3*b^3*c*d^2*e^5*f^2 - 3*a*b^2*d^3*e^
5*f^2 + 3*b^3*c^2*d*e^4*f^3 + 9*a*b^2*c*d^2*e^4*f^3 + 3*a^2*b*d^3*e^4*f^3
- b^3*c^3*e^3*f^4 - 9*a*b^2*c^2*d*e^3*f^4 - 9*a^2*b*c*d^2*e^3*f^4 - a^3*d^
3*e^3*f^4 + 3*a*b^2*c^3*e^2*f^5 + 9*a^2*b*c^2*d*e^2*f^5 + 3*a^3*c*d^2*e^2*
f^5 - 3*a^2*b*c^3*e*f^6 - 3*a^3*c^2*d*e*f^6 + a^3*c^3*f^7) - (2*b^3*d^3*e^
2*f*x^2 - 2*b^3*c*d^2*e*f^2*x^2 - 2*a*b^2*d^3*e*f^2*x^2 + 2*b^3*c^2*d*f^3*
x^2 - 2*a*b^2*c*d^2*f^3*x^2 + 2*a^2*b*d^3*f^3*x^2 + 2*b^3*d^3*e^3*x - b^3*
c*d^2*e^2*f*x - a*b^2*d^3*e^2*f*x - b^3*c^2*d*e*f^2*x - a^2*b*d^3*e*f^2*x
+ 2*b^3*c^3*f^3*x - a*b^2*c^2*d*f^3*x - a^2*b*c*d^2*f^3*x + 2*a^3*d^3*f...
```

3.19.9 Mupad [B] (verification not implemented)

Time = 13.89 (sec) , antiderivative size = 1940, normalized size of antiderivative = 8.29

$$\int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b
*d*f*x^3)^2,x)
```

output
```
- ((a*b^2*c^3*f^3 + a*b^2*d^3*e^3 + a^3*c*d^2*f^3 + b^3*c*d^2*e^3 + a^3*d^
3*e*f^2 + b^3*c^3*e*f^2 - 2*a^2*b*c^2*d*f^3 - 2*a^2*b*d^3*e^2*f - 2*b^3*c^
2*d*e^2*f)/(a^2*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*
d^2*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*
c^3*d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*
b^4*c^3*d*e^3*f + 2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*e^2*f^2 + 2*a^2*b^
2*c*d^3*e^3*f + 2*a^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2*f^2 + 2*a^3*b*c^
2*d^2*e*f^3 - 6*a^2*b^2*c^2*d^2*e^2*f^2) + (2*x^2*(a^2*b*d^3*f^3 + b^3*c^2
*d*f^3 + b^3*d^3*e^2*f - a*b^2*c*d^2*f^3 - a*b^2*d^3*e*f^2 - b^3*c*d^2*e*f
^2))/(a^2*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*d^2*e^
4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*c^3*d*
f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*b^4*c^
3*d*e^3*f + 2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*e^2*f^2 + 2*a^2*b^2*c*d^
3*e^3*f + 2*a^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2*f^2 + 2*a^3*b*c^2*d^2*
e*f^3 - 6*a^2*b^2*c^2*d^2*e^2*f^2) - (x*(a*b^2*c^2*d*f^3 - 2*b^3*c^3*f^3 -
 2*b^3*d^3*e^3 - 2*a^3*d^3*f^3 + a^2*b*c*d^2*f^3 + a*b^2*d^3*e^2*f + a^2*b
*d^3*e*f^2 + b^3*c*d^2*e^2*f + b^3*c^2*d*e*f^2))/(a^2*b^2*c^4*f^4 + a^2*b^
2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*d^2*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*
e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*c^3*d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^
3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*b^4*c^3*d*e^3*f + 2*a*b^3*c^2*d^2...
```

3.19. $\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2}\,dx$

3.20 $\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx$

3.20.1 Optimal result

Integrand size = 46, antiderivative size = 495

$$\begin{aligned}
&\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx \\
&= -\frac{b^5}{2(bc-ad)^3(be-af)^3(a+bx)^2}+\frac{3b^5(bde+bcf-2adf)}{(bc-ad)^4(be-af)^4(a+bx)} \\
&\quad+\frac{d^5}{2(bc-ad)^3(de-cf)^3(c+dx)^2}+\frac{3d^5(bde-2bcf+adf)}{(bc-ad)^4(de-cf)^4(c+dx)} \\
&\quad-\frac{f^5}{2(be-af)^3(de-cf)^3(e+fx)^2}-\frac{3f^5(2bde-bcf-adf)}{(be-af)^4(de-cf)^4(e+fx)} \\
&\quad+\frac{3b^5(7a^2d^2f^2-7abdf(de+cf)+b^2(2d^2e^2+3cdef+2c^2f^2))\log(a+bx)}{(bc-ad)^5(be-af)^5} \\
&\quad-\frac{3d^5(2a^2d^2f^2+abdf(3de-7cf)+b^2(2d^2e^2-7cdef+7c^2f^2))\log(c+dx)}{(bc-ad)^5(de-cf)^5} \\
&\quad+\frac{3f^5(2a^2d^2f^2-abdf(7de-3cf)+b^2(7d^2e^2-7cdef+2c^2f^2))\log(e+fx)}{(be-af)^5(de-cf)^5}
\end{aligned}$$

output
```
-1/2*b^5/(-a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)^2+3*b^5*(-2*a*d*f+b*c*f+b*d*e)/
(-a*d+b*c)^4/(-a*f+b*e)^4/(b*x+a)+1/2*d^5/(-a*d+b*c)^3/(-c*f+d*e)^3/(d*x+c
)^2+3*d^5*(a*d*f-2*b*c*f+b*d*e)/(-a*d+b*c)^4/(-c*f+d*e)^4/(d*x+c)-1/2*f^5/
(-a*f+b*e)^3/(-c*f+d*e)^3/(f*x+e)^2-3*f^5*(-a*d*f-b*c*f+2*b*d*e)/(-a*f+b*e
)^4/(-c*f+d*e)^4/(f*x+e)+3*b^5*(7*a^2*d^2*f^2-7*a*b*d*f*(c*f+d*e)+b^2*(2*c
^2*f^2+3*c*d*e*f+2*d^2*e^2))*ln(b*x+a)/(-a*d+b*c)^5/(-a*f+b*e)^5-3*d^5*(2*
a^2*d^2*f^2+a*b*d*f*(-7*c*f+3*d*e)+b^2*(7*c^2*f^2-7*c*d*e*f+2*d^2*e^2))*ln
(d*x+c)/(-a*d+b*c)^5/(-c*f+d*e)^5+3*f^5*(2*a^2*d^2*f^2-a*b*d*f*(-3*c*f+7*d
*e)+b^2*(2*c^2*f^2-7*c*d*e*f+7*d^2*e^2))*ln(f*x+e)/(-a*f+b*e)^5/(-c*f+d*e)
^5
```

3.20.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 490, normalized size of antiderivative = 0.99

$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx$$
$$=\frac{1}{2}\left(-\frac{b^5}{(bc-ad)^3(be-af)^3(a+bx)^2}+\frac{6b^5(bde+bcf-2adf)}{(bc-ad)^4(be-af)^4(a+bx)}-\frac{d^5}{(bc-ad)^3(-de+cf)^3(c+dx)^2}+\frac{6d^5(bde-2bcf+adf)}{(bc-ad)^4(de-cf)^4(c+dx)}-\frac{f^5}{(be-af)^3(de-cf)^3(e+fx)^2}+\frac{6f^5(-2bde+bcf+adf)}{(be-af)^4(de-cf)^4(e+fx)}+\frac{6b^5(7a^2d^2f^2-7abdf(de+cf)+b^2(2d^2e^2+3cdef+2c^2f^2))\log(a+bx)}{(bc-ad)^5(be-af)^5}+\frac{6d^5(2a^2d^2f^2+abdf(3de-7cf)+b^2(2d^2e^2-7cdef+7c^2f^2))\log(c+dx)}{(bc-ad)^5(-de+cf)^5}+\frac{6f^5(2a^2d^2f^2+abdf(-7de+3cf)+b^2(7d^2e^2-7cdef+2c^2f^2))\log(e+fx)}{(be-af)^5(de-cf)^5}\right)$$

input
```
Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
 + b*d*f*x^3)^(-3),x]
```

output
```
(-(b^5/((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2)) + (6*b^5*(b*d*e + b*c*f
- 2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) - d^5/((b*c - a*d)^3*(
-(d*e) + c*f)^3*(c + d*x)^2) + (6*d^5*(b*d*e - 2*b*c*f + a*d*f))/((b*c - a
*d)^4*(d*e - c*f)^4*(c + d*x)) - f^5/((b*e - a*f)^3*(d*e - c*f)^3*(e + f*x
)^2) + (6*f^5*(-2*b*d*e + b*c*f + a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e
+ f*x)) + (6*b^5*(7*a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 +
 3*c*d*e*f + 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) + (6*
d^5*(2*a^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*d*e*f
+ 7*c^2*f^2))*Log[c + d*x])/((b*c - a*d)^5*(-(d*e) + c*f)^5) + (6*f^5*(2*a
^2*d^2*f^2 + a*b*d*f*(-7*d*e + 3*c*f) + b^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2
*f^2))*Log[e + f*x])/((b*e - a*f)^5*(d*e - c*f)^5))/2
```

3.20.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^3}\,dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{3f^6\left(2a^2d^2f^2-abdf(7de-3cf)+b^2\left(2c^2f^2-7cdef+7d^2e^2\right)\right)}{(e+fx)(be-af)^5(de-cf)^5}+\frac{3d^6\left(-2a^2d^2f^2-abdf(3de-7cf)-\left(b^2(7c\right.\right.}{(c+dx)(bc-ad)^5(de-}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3f^5\log(e+fx)\left(2a^2d^2f^2-abdf(7de-3cf)+b^2\left(2c^2f^2-7cdef+7d^2e^2\right)\right)}{(be-af)^5(de-cf)^5}-\frac{3d^5\log(c+dx)\left(2a^2d^2f^2+abdf(3de-7cf)+b^2\left(7c^2f^2-7cdef+2d^2e^2\right)\right)}{(bc-ad)^5(de-cf)^5}+\frac{3b^5\log(a+bx)\left(7a^2d^2f^2-7abdf(cf+de)+b^2\left(2c^2f^2+3cdef+2d^2e^2\right)\right)}{(bc-ad)^5(be-af)^5}+\frac{3b^5(-2adf+bcf+bde)}{(a+bx)(bc-ad)^4(be-af)^4}-\frac{b^5}{2(a+bx)^2(bc-ad)^3(be-af)^3}+\frac{3d^5(adf-2bcf+bde)}{(c+dx)(bc-ad)^4(de-cf)^4}+\frac{d^5}{2(c+dx)^2(bc-ad)^3(de-cf)^3}-\frac{3f^5(-adf-bcf+2bde)}{(e+fx)(be-af)^4(de-cf)^4}-\frac{f^5}{2(e+fx)^2(be-af)^3(de-cf)^3}$$

input
```
Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d
*f*x^3)^(-3),x]
```

output
```
-1/2*b^5/((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2) + (3*b^5*(b*d*e + b*c*f
 - 2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) + d^5/(2*(b*c - a*d)^
3*(d*e - c*f)^3*(c + d*x)^2) + (3*d^5*(b*d*e - 2*b*c*f + a*d*f))/((b*c - a
*d)^4*(d*e - c*f)^4*(c + d*x)) - f^5/(2*(b*e - a*f)^3*(d*e - c*f)^3*(e + f
*x)^2) - (3*f^5*(2*b*d*e - b*c*f - a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e
 + f*x)) + (3*b^5*(7*a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2
+ 3*c*d*e*f + 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) - (3
*d^5*(2*a^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*d*e*f
 + 7*c^2*f^2))*Log[c + d*x])/((b*c - a*d)^5*(d*e - c*f)^5) + (3*f^5*(2*a^2
*d^2*f^2 - a*b*d*f*(7*d*e - 3*c*f) + b^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^
2))*Log[e + f*x])/((b*e - a*f)^5*(d*e - c*f)^5)
```

3.20.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.20.4 Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.04

method	result
default	$-\frac{f^5}{2(cf-ed)^3(af-be)^3(fx+e)^2}+\frac{3f^5(adf+fbc-2bde)}{(cf-ed)^4(af-be)^4(fx+e)}+\frac{3f^5(2a^2d^2f^2+3abcd\,f^2-7ab\,d^2ef+2b^2c^2f^2-7b^2cdef+7b}{(cf-ed)^5(af-be)^5}$
norman	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x,
method=_RETURNVERBOSE)
```

output
```
-1/2*f^5/(c*f-d*e)^3/(a*f-b*e)^3/(f*x+e)^2+3*f^5*(a*d*f+b*c*f-2*b*d*e)/(c*
f-d*e)^4/(a*f-b*e)^4/(f*x+e)+3*f^5*(2*a^2*d^2*f^2+3*a*b*c*d*f^2-7*a*b*d^2*
e*f+2*b^2*c^2*f^2-7*b^2*c*d*e*f+7*b^2*d^2*e^2)/(c*f-d*e)^5/(a*f-b*e)^5*ln(
f*x+e)+1/2*d^5/(c*f-d*e)^3/(a*d-b*c)^3/(d*x+c)^2+3*d^5*(a*d*f-2*b*c*f+b*d*
e)/(c*f-d*e)^4/(a*d-b*c)^4/(d*x+c)-3*d^5*(2*a^2*d^2*f^2-7*a*b*c*d*f^2+3*a*
b*d^2*e*f+7*b^2*c^2*f^2-7*b^2*c*d*e*f+2*b^2*d^2*e^2)/(c*f-d*e)^5/(a*d-b*c)
^5*ln(d*x+c)-1/2*b^5/(a*f-b*e)^3/(a*d-b*c)^3/(b*x+a)^2-3*b^5*(2*a*d*f-b*c*
f-b*d*e)/(a*f-b*e)^4/(a*d-b*c)^4/(b*x+a)+3*b^5*(7*a^2*d^2*f^2-7*a*b*c*d*f^
2-7*a*b*d^2*e*f+2*b^2*c^2*f^2+3*b^2*c*d*e*f+2*b^2*d^2*e^2)/(a*f-b*e)^5/(a*
d-b*c)^5*ln(b*x+a)
```

3.20.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3
)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.20.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x*
*3)**3,x)
```

output
```
Timed out
```

3.20.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11005 vs. $2(489) = 978$.

Time = 0.74 (sec) , antiderivative size = 11005, normalized size of antiderivative = 22.23

$$\int \frac{1}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3
)^3,x, algorithm="maxima")
```

output
```
3*(2*b^7*d^2*e^2 + (3*b^7*c*d - 7*a*b^6*d^2)*e*f + (2*b^7*c^2 - 7*a*b^6*c*
d + 7*a^2*b^5*d^2)*f^2)*log(b*x + a)/((b^10*c^5 - 5*a*b^9*c^4*d + 10*a^2*b
^8*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5*b^5*d^5)*e^5 - 5*(
a*b^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5*
a^5*b^5*c*d^4 - a^6*b^4*d^5)*e^4*f + 10*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 1
0*a^4*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*e^
3*f^2 - 10*(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^
4*c^2*d^3 + 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*e^2*f^3 + 5*(a^4*b^6*c^5 - 5*a^
5*b^5*c^4*d + 10*a^6*b^4*c^3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 -
a^9*b*d^5)*e*f^4 - (a^5*b^5*c^5 - 5*a^6*b^4*c^4*d + 10*a^7*b^3*c^3*d^2 - 1
0*a^8*b^2*c^2*d^3 + 5*a^9*b*c*d^4 - a^10*d^5)*f^5) - 3*(2*b^2*d^7*e^2 - (7
*b^2*c*d^6 - 3*a*b*d^7)*e*f + (7*b^2*c^2*d^5 - 7*a*b*c*d^6 + 2*a^2*d^7)*f^
2)*log(d*x + c)/((b^5*c^5*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*
a^3*b^2*c^2*d^8 + 5*a^4*b*c*d^9 - a^5*d^10)*e^5 - 5*(b^5*c^6*d^4 - 5*a*b^4
*c^5*d^5 + 10*a^2*b^3*c^4*d^6 - 10*a^3*b^2*c^3*d^7 + 5*a^4*b*c^2*d^8 - a^5
*c*d^9)*e^4*f + 10*(b^5*c^7*d^3 - 5*a*b^4*c^6*d^4 + 10*a^2*b^3*c^5*d^5 - 1
0*a^3*b^2*c^4*d^6 + 5*a^4*b*c^3*d^7 - a^5*c^2*d^8)*e^3*f^2 - 10*(b^5*c^8*d
^2 - 5*a*b^4*c^7*d^3 + 10*a^2*b^3*c^6*d^4 - 10*a^3*b^2*c^5*d^5 + 5*a^4*b*c
^4*d^6 - a^5*c^3*d^7)*e^2*f^3 + 5*(b^5*c^9*d - 5*a*b^4*c^8*d^2 + 10*a^2*b^
3*c^7*d^3 - 10*a^3*b^2*c^6*d^4 + 5*a^4*b*c^5*d^5 - a^5*c^4*d^6)*e*f^4 -...
```

3.20.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7111 vs. $2(489) = 978$.

Time = 0.40 (sec) , antiderivative size = 7111, normalized size of antiderivative = 14.37

$$\int \frac{1}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3
)^3,x, algorithm="giac")
```

output
```
3*(2*b^8*d^2*e^2 + 3*b^8*c*d*e*f - 7*a*b^7*d^2*e*f + 2*b^8*c^2*f^2 - 7*a*b
^7*c*d*f^2 + 7*a^2*b^6*d^2*f^2)*log(abs(b*x + a))/(b^11*c^5*e^5 - 5*a*b^10
*c^4*d*e^5 + 10*a^2*b^9*c^3*d^2*e^5 - 10*a^3*b^8*c^2*d^3*e^5 + 5*a^4*b^7*c
*d^4*e^5 - a^5*b^6*d^5*e^5 - 5*a*b^10*c^5*e^4*f + 25*a^2*b^9*c^4*d*e^4*f -
 50*a^3*b^8*c^3*d^2*e^4*f + 50*a^4*b^7*c^2*d^3*e^4*f - 25*a^5*b^6*c*d^4*e^
4*f + 5*a^6*b^5*d^5*e^4*f + 10*a^2*b^9*c^5*e^3*f^2 - 50*a^3*b^8*c^4*d*e^3*
f^2 + 100*a^4*b^7*c^3*d^2*e^3*f^2 - 100*a^5*b^6*c^2*d^3*e^3*f^2 + 50*a^6*b
^5*c*d^4*e^3*f^2 - 10*a^7*b^4*d^5*e^3*f^2 - 10*a^3*b^8*c^5*e^2*f^3 + 50*a^
4*b^7*c^4*d*e^2*f^3 - 100*a^5*b^6*c^3*d^2*e^2*f^3 + 100*a^6*b^5*c^2*d^3*e^
2*f^3 - 50*a^7*b^4*c*d^4*e^2*f^3 + 10*a^8*b^3*d^5*e^2*f^3 + 5*a^4*b^7*c^5*
e*f^4 - 25*a^5*b^6*c^4*d*e*f^4 + 50*a^6*b^5*c^3*d^2*e*f^4 - 50*a^7*b^4*c^2
*d^3*e*f^4 + 25*a^8*b^3*c*d^4*e*f^4 - 5*a^9*b^2*d^5*e*f^4 - a^5*b^6*c^5*f^
5 + 5*a^6*b^5*c^4*d*f^5 - 10*a^7*b^4*c^3*d^2*f^5 + 10*a^8*b^3*c^2*d^3*f^5
- 5*a^9*b^2*c*d^4*f^5 + a^10*b*d^5*f^5) - 3*(2*b^2*d^8*e^2 - 7*b^2*c*d^7*e
*f + 3*a*b*d^8*e*f + 7*b^2*c^2*d^6*f^2 - 7*a*b*c*d^7*f^2 + 2*a^2*d^8*f^2)*
log(abs(d*x + c))/(b^5*c^5*d^6*e^5 - 5*a*b^4*c^4*d^7*e^5 + 10*a^2*b^3*c^3*
d^8*e^5 - 10*a^3*b^2*c^2*d^9*e^5 + 5*a^4*b*c*d^10*e^5 - a^5*d^11*e^5 - 5*b
^5*c^6*d^5*e^4*f + 25*a*b^4*c^5*d^6*e^4*f - 50*a^2*b^3*c^4*d^7*e^4*f + 50*
a^3*b^2*c^3*d^8*e^4*f - 25*a^4*b*c^2*d^9*e^4*f + 5*a^5*c*d^10*e^4*f + 10*b
^5*c^7*d^4*e^3*f^2 - 50*a*b^4*c^6*d^5*e^3*f^2 + 100*a^2*b^3*c^5*d^6*e^3...
```

3.20.9 Mupad [B] (verification not implemented)

Time = 23.45 (sec) , antiderivative size = 82532, normalized size of antiderivative = 166.73

$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b
*d*f*x^3)^3,x)
```

output
```
symsum(log(root(756756*a^10*b^10*c^10*d^10*e^10*f^10*z^3 + 573300*a^12*b^8
*c^9*d^11*e^9*f^11*z^3 + 573300*a^11*b^9*c^11*d^9*e^8*f^12*z^3 + 573300*a^
11*b^9*c^8*d^12*e^11*f^9*z^3 + 573300*a^9*b^11*c^12*d^8*e^9*f^11*z^3 + 573
300*a^9*b^11*c^9*d^11*e^12*f^8*z^3 + 573300*a^8*b^12*c^11*d^9*e^11*f^9*z^3
 - 343980*a^11*b^9*c^10*d^10*e^9*f^11*z^3 - 343980*a^11*b^9*c^9*d^11*e^10*
f^10*z^3 - 343980*a^10*b^10*c^11*d^9*e^9*f^11*z^3 - 343980*a^10*b^10*c^9*d
^11*e^11*f^9*z^3 - 343980*a^9*b^11*c^11*d^9*e^10*f^10*z^3 - 343980*a^9*b^1
1*c^10*d^10*e^11*f^9*z^3 + 326340*a^13*b^7*c^10*d^10*e^7*f^13*z^3 + 326340
*a^13*b^7*c^7*d^13*e^10*f^10*z^3 + 326340*a^10*b^10*c^13*d^7*e^7*f^13*z^3
+ 326340*a^10*b^10*c^7*d^13*e^13*f^7*z^3 + 326340*a^7*b^13*c^13*d^7*e^10*f
^10*z^3 + 326340*a^7*b^13*c^10*d^10*e^13*f^7*z^3 - 267540*a^12*b^8*c^10*d^
10*e^8*f^12*z^3 - 267540*a^12*b^8*c^8*d^12*e^10*f^10*z^3 - 267540*a^10*b^1
0*c^12*d^8*e^8*f^12*z^3 - 267540*a^10*b^10*c^8*d^12*e^12*f^8*z^3 - 267540*
a^8*b^12*c^12*d^8*e^10*f^10*z^3 - 267540*a^8*b^12*c^10*d^10*e^12*f^8*z^3 +
 245700*a^14*b^6*c^8*d^12*e^8*f^12*z^3 + 245700*a^12*b^8*c^12*d^8*e^6*f^14
*z^3 + 245700*a^12*b^8*c^6*d^14*e^12*f^8*z^3 + 245700*a^8*b^12*c^14*d^6*e^
8*f^12*z^3 + 245700*a^8*b^12*c^8*d^12*e^14*f^6*z^3 + 245700*a^6*b^14*c^12*
d^8*e^12*f^8*z^3 - 191100*a^13*b^7*c^9*d^11*e^8*f^12*z^3 - 191100*a^13*b^7
*c^8*d^12*e^9*f^11*z^3 - 191100*a^12*b^8*c^11*d^9*e^7*f^13*z^3 - 191100*a^
12*b^8*c^7*d^13*e^11*f^9*z^3 - 191100*a^11*b^9*c^12*d^8*e^7*f^13*z^3 - ...
```

3.20. $\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx$

3.21 $\int \frac{1}{1+x+x^2+x^3} \, dx$

3.21.1 Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{1}{1+x+x^2+x^3} \, dx = \frac{\arctan(x)}{2} + \frac{1}{2}\log(1+x) - \frac{1}{4}\log\left(1+x^2\right)$$

output
```
1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

3.21.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x+x^2+x^3} \, dx = \frac{\arctan(x)}{2} + \frac{1}{2}\log(1+x) - \frac{1}{4}\log\left(1+x^2\right)$$

input
```
Integrate[(1 + x + x^2 + x^3)^(-1),x]
```

output
```
ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

3.21.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3+x^2+x+1}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{1-x}{2(x^2+1)}+\frac{1}{2(x+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan(x)}{2}-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)$$

input
```
Int[(1 + x + x^2 + x^3)^(-1),x]
```

output
```
ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

3.21.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.21.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{2}-\frac{\ln(x^2+1)}{4}$	20
risch	$\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{2}-\frac{\ln(x^2+1)}{4}$	20
parallelrisch	$\frac{\ln(x+1)}{2}-\frac{\ln(x-i)}{4}-\frac{i\ln(x-i)}{4}-\frac{\ln(x+i)}{4}+\frac{i\ln(x+i)}{4}$	38

input
```
int(1/(x^3+x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x)+1/2*ln(x+1)-1/4*ln(x^2+1)
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

input
```
integrate(1/(x^3+x^2+x+1),x, algorithm="fricas")
```

output
```
1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \frac{\log(x+1)}{2} - \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(1/(x**3+x**2+x+1),x)
```

output
```
log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

input
```
integrate(1/(x^3+x^2+x+1),x, algorithm="maxima")
```

output
```
1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(|x+1|)$$

input
```
integrate(1/(x^3+x^2+x+1),x, algorithm="giac")
```

output
```
1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \frac{\ln(x+1)}{2} + \ln(x-\mathrm{i})\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

input
```
int(1/(x + x^2 + x^3 + 1),x)
```

output
```
log(x + 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)
```

3.22 $\int \frac{1}{-1+4x-4x^2+16x^3}\,dx$

3.22.1 Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{1}{-1+4x-4x^2+16x^3}\,dx = -\frac{1}{10}\arctan(2x)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\log\left(1+4x^2\right)$$

output
```
-1/10*arctan(2*x)+1/5*ln(1-4*x)-1/10*ln(4*x^2+1)
```

3.22.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{-1+4x-4x^2+16x^3}\,dx = -\frac{1}{10}\arctan(2x)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\log\left(1+4x^2\right)$$

input
```
Integrate[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1),x]
```

output
```
-1/10*ArcTan[2*x] + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10
```

3.22.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{16x^3-4x^2+4x-1}dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(\frac{-4x-1}{5(4x^2+1)}+\frac{4}{5(4x-1)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{10}\arctan(2x)-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(1-4x)$$

input
```
Int[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1),x]
```

output
```
-1/10*ArcTan[2*x] + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10
```

3.22.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.22.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\ln(4x^2+1)}{10}-\frac{\arctan(2x)}{10}+\frac{\ln(-1+4x)}{5}$	26
risch	$-\frac{\ln(4x^2+1)}{10}-\frac{\arctan(2x)}{10}+\frac{\ln(-1+4x)}{5}$	26
parallelrisch	$-\frac{\ln(x-\frac{i}{2})}{10}+\frac{i\ln(x-\frac{i}{2})}{20}-\frac{\ln(x+\frac{i}{2})}{10}-\frac{i\ln(x+\frac{i}{2})}{20}+\frac{\ln(x-\frac{1}{4})}{5}$	38

input
```
int(1/(16*x^3-4*x^2+4*x-1),x,method=_RETURNVERBOSE)
```

output
```
-1/10*ln(4*x^2+1)-1/10*arctan(2*x)+1/5*ln(-1+4*x)
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{-1+4x-4x^2+16x^3}\,dx=-\frac{1}{10}\arctan(2x)-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(4x-1)$$

input
```
integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="fricas")
```

output
```
-1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{-1+4x-4x^2+16x^3}\,dx=\frac{\log\left(x-\frac{1}{4}\right)}{5}-\frac{\log\left(x^2+\frac{1}{4}\right)}{10}-\frac{\operatorname{atan}(2x)}{10}$$

input
```
integrate(1/(16*x**3-4*x**2+4*x-1),x)
```

output
```
log(x - 1/4)/5 - log(x**2 + 1/4)/10 - atan(2*x)/10
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{-1+4x-4x^2+16x^3}\,dx = -\frac{1}{10}\arctan(2x) - \frac{1}{10}\log(4x^2+1) + \frac{1}{5}\log(4x-1)$$

input
```
integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="maxima")
```

output
```
-1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{-1+4x-4x^2+16x^3}\,dx = -\frac{1}{10}\arctan(2x) - \frac{1}{10}\log(4x^2+1) + \frac{1}{5}\log(|4x-1|)$$

input
```
integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="giac")
```

output
```
-1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(abs(4*x - 1))
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{-1+4x-4x^2+16x^3}\,dx = \frac{\ln\left(x-\frac{1}{4}\right)}{5} + \ln\left(x-\frac{1}{2}\mathrm{i}\right)\left(-\frac{1}{10}+\frac{1}{20}\mathrm{i}\right) + \ln\left(x+\frac{1}{2}\mathrm{i}\right)\left(-\frac{1}{10}-\frac{1}{20}\mathrm{i}\right)$$

input
```
int(1/(4*x - 4*x^2 + 16*x^3 - 1),x)
```

output
```
log(x - 1/4)/5 - log(x - 1i/2)*(1/10 - 1i/20) - log(x + 1i/2)*(1/10 + 1i/2
0)
```

3.23 $\int \frac{1}{dx^3} \, dx$

3.23.1 Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{dx^3} \, dx = -\frac{1}{2dx^2}$$

output
```
-1/2/d/x^2
```

3.23.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{dx^3} \, dx = -\frac{1}{2dx^2}$$

input
```
Integrate[1/(d*x^3),x]
```

output
```
-1/2*1/(d*x^2)
```

3.23.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{dx^3}\,dx$$

$$\Big\downarrow \; 15$$

$$-\frac{1}{2dx^2}$$

input
```
Int[1/(d*x^3),x]
```

output
```
-1/2*1/(d*x^2)
```

3.23.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

3.23.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
gosper	$-\frac{1}{2 d \,x^{2}}$	9
default	$-\frac{1}{2 d \,x^{2}}$	9
norman	$-\frac{1}{2 d \,x^{2}}$	9
risch	$-\frac{1}{2 d \,x^{2}}$	9
parallelrisch	$-\frac{1}{2 d \,x^{2}}$	9

input
```
int(1/d/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/x^2
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{dx^3}\,dx = -\frac{1}{2\,dx^2}$$

input
```
integrate(1/d/x^3,x, algorithm="fricas")
```

output
```
-1/2/(d*x^2)
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{dx^3}\,dx = -\frac{1}{2dx^2}$$

input
```
integrate(1/d/x**3,x)
```

output
```
-1/(2*d*x**2)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{dx^3}\,dx = -\frac{1}{2\,dx^2}$$

input
```
integrate(1/d/x^3,x, algorithm="maxima")
```

output
```
-1/2/(d*x^2)
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{dx^3}\,dx = -\frac{1}{2\,dx^2}$$

input
```
integrate(1/d/x^3,x, algorithm="giac")
```

output
```
-1/2/(d*x^2)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{dx^3}\,dx = -\frac{1}{2\,d\,x^2}$$

input
```
int(1/(d*x^3),x)
```

output
```
-1/(2*d*x^2)
```

3.24 $\int \frac{1}{cx^2+dx^3}\,dx$

3.24.1 Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{cx^2+dx^3}\,dx = -\frac{1}{cx}-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}$$

output
```
-1/c/x-d*ln(x)/c^2+d*ln(d*x+c)/c^2
```

3.24.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{cx^2+dx^3}\,dx = -\frac{1}{cx}-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}$$

input
```
Integrate[(c*x^2 + d*x^3)^(-1),x]
```

output
```
-(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2
```

3.24.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2026, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{cx^2+dx^3}\,dx$$

$$\Big\downarrow \; 2026$$

$$\int \frac{1}{x^2(c+dx)}dx$$

$$\Big\downarrow \; 54$$

$$\int \left(\frac{d^2}{c^2(c+dx)}-\frac{d}{c^2x}+\frac{1}{cx^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}-\frac{1}{cx}$$

input
```
Int[(c*x^2 + d*x^3)^(-1),x]
```

output
```
-(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2
```

3.24.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.24.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$-\frac{d\ln(x)x-d\ln(dx+c)x+c}{c^2x}$	26
default	$-\frac{1}{cx}-\frac{d\ln(x)}{c^2}+\frac{d\ln(dx+c)}{c^2}$	29
norman	$-\frac{1}{cx}-\frac{d\ln(x)}{c^2}+\frac{d\ln(dx+c)}{c^2}$	29
risch	$-\frac{1}{cx}+\frac{d\ln(-dx-c)}{c^2}-\frac{d\ln(x)}{c^2}$	32

input
```
int(1/(d*x^3+c*x^2),x,method=_RETURNVERBOSE)
```

output
```
-(d*ln(x)*x-d*ln(d*x+c)*x+c)/c^2/x
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{cx^2+dx^3}\,dx=\frac{dx\log(dx+c)-dx\log(x)-c}{c^2x}$$

input
```
integrate(1/(d*x^3+c*x^2),x, algorithm="fricas")
```

output
```
(d*x*log(d*x + c) - d*x*log(x) - c)/(c^2*x)
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{cx^2+dx^3}\,dx=-\frac{1}{cx}+\frac{d\left(-\log(x)+\log\left(\frac{c}{d}+x\right)\right)}{c^2}$$

input
```
integrate(1/(d*x**3+c*x**2),x)
```

output
```
-1/(c*x) + d*(-log(x) + log(c/d + x))/c**2
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{cx^2 + dx^3} \, dx = \frac{d \log (dx + c)}{c^2} - \frac{d \log (x)}{c^2} - \frac{1}{cx}$$

input
```
integrate(1/(d*x^3+c*x^2),x, algorithm="maxima")
```

output
```
d*log(d*x + c)/c^2 - d*log(x)/c^2 - 1/(c*x)
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{cx^2 + dx^3} \, dx = \frac{d \log (|dx + c|)}{c^2} - \frac{d \log (|x|)}{c^2} - \frac{1}{cx}$$

input
```
integrate(1/(d*x^3+c*x^2),x, algorithm="giac")
```

output
```
d*log(abs(d*x + c))/c^2 - d*log(abs(x))/c^2 - 1/(c*x)
```

3.24.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{cx^2 + dx^3} \, dx = \frac{2 \, d \, \mathrm{atanh}\left(\frac{2 \, d \, x}{c} + 1\right)}{c^2} - \frac{1}{c \, x}$$

input
```
int(1/(c*x^2 + d*x^3),x)
```

output
```
(2*d*atanh((2*d*x)/c + 1))/c^2 - 1/(c*x)
```

3.25 $\int \frac{1}{bx+dx^3} dx$

3.25.1 Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{1}{bx+dx^3} dx = \frac{\log(x)}{b} - \frac{\log(b+dx^2)}{2b}$$

output
```
ln(x)/b-1/2*ln(d*x^2+b)/b
```

3.25.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{bx+dx^3} dx = \frac{\log(x)}{b} - \frac{\log(b+dx^2)}{2b}$$

input
```
Integrate[(b*x + d*x^3)^(-1),x]
```

output
```
Log[x]/b - Log[b + d*x^2]/(2*b)
```

3.25.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {2026, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{bx+dx^3}dx$$
$$\Big\downarrow \text{2026}$$
$$\int \frac{1}{x(b+dx^2)}dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2}\int \frac{1}{x^2(dx^2+b)}dx^2$$
$$\Big\downarrow \text{47}$$
$$\frac{1}{2}\left(\frac{\int \frac{1}{x^2}dx^2}{b}-\frac{d\int \frac{1}{dx^2+b}dx^2}{b}\right)$$
$$\Big\downarrow \text{14}$$
$$\frac{1}{2}\left(\frac{\log(x^2)}{b}-\frac{d\int \frac{1}{dx^2+b}dx^2}{b}\right)$$
$$\Big\downarrow \text{16}$$
$$\frac{1}{2}\left(\frac{\log(x^2)}{b}-\frac{\log(b+dx^2)}{b}\right)$$

input
```
Int[(b*x + d*x^3)^(-1),x]
```

output
```
(Log[x^2]/b - Log[b + d*x^2]/b)/2
```

3.25.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.25.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\ln(x)}{b}-\frac{\ln(d\,x^{2}+b)}{2b}$	21
norman	$\frac{\ln(x)}{b}-\frac{\ln(d\,x^{2}+b)}{2b}$	21
risch	$\frac{\ln(x)}{b}-\frac{\ln(d\,x^{2}+b)}{2b}$	21
parallelrisch	$\frac{2\ln(x)-\ln(d\,x^{2}+b)}{2b}$	21

input
```
int(1/(d*x^3+b*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/b-1/2*ln(d*x^2+b)/b
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{bx + dx^3} \, dx = -\frac{\log (dx^2 + b) - 2 \log (x)}{2 \, b}$$

input
```
integrate(1/(d*x^3+b*x),x, algorithm="fricas")
```

output
```
-1/2*(log(d*x^2 + b) - 2*log(x))/b
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{1}{bx + dx^3} \, dx = \frac{\log (x)}{b} - \frac{\log \left(\frac{b}{d} + x^2\right)}{2b}$$

input
```
integrate(1/(d*x**3+b*x),x)
```

output
```
log(x)/b - log(b/d + x**2)/(2*b)
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{bx + dx^3} \, dx = -\frac{\log (dx^2 + b)}{2 \, b} + \frac{\log (x)}{b}$$

input
```
integrate(1/(d*x^3+b*x),x, algorithm="maxima")
```

output
```
-1/2*log(d*x^2 + b)/b + log(x)/b
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{bx+dx^3}\,dx = \frac{\log(x^2)}{2\,b} - \frac{\log(|dx^2+b|)}{2\,b}$$

input
```
integrate(1/(d*x^3+b*x),x, algorithm="giac")
```

output
```
1/2*log(x^2)/b - 1/2*log(abs(d*x^2 + b))/b
```

3.25.9 Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{bx+dx^3}\,dx = -\frac{\ln(d\,x^2+b) - 2\,\ln(x)}{2\,b}$$

input
```
int(1/(b*x + d*x^3),x)
```

output
```
-(log(b + d*x^2) - 2*log(x))/(2*b)
```

3.26 $\int \frac{1}{bx+cx^2+dx^3}\,dx$

3.26.1 Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{1}{bx+cx^2+dx^3}\,dx=\frac{c\,\text{arctanh}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}}+\frac{\log(x)}{b}-\frac{\log\left(b+cx+dx^2\right)}{2b}$$

output
```
ln(x)/b-1/2*ln(d*x^2+c*x+b)/b+c*arctanh((2*d*x+c)/(-4*b*d+c^2)^(1/2))/b/(-
4*b*d+c^2)^(1/2)
```

3.26.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{1}{bx+cx^2+dx^3}\,dx=-\frac{\frac{2c\arctan\left(\frac{c+2dx}{\sqrt{-c^2+4bd}}\right)}{\sqrt{-c^2+4bd}}-2\log(x)+\log(b+x(c+dx))}{2b}$$

input
```
Integrate[(b*x + c*x^2 + d*x^3)^(-1),x]
```

output
```
-1/2*((2*c*ArcTan[(c + 2*d*x)/Sqrt[-c^2 + 4*b*d]])/Sqrt[-c^2 + 4*b*d] - 2*
Log[x] + Log[b + x*(c + d*x)])/b
```

3.26.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1949, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{bx+cx^2+dx^3}dx$$

$$\Big\downarrow \text{1949}$$

$$\int \frac{1}{x\left(b+cx+dx^2\right)}dx$$

$$\Big\downarrow \text{1144}$$

$$\frac{\int -\frac{c+dx}{dx^2+cx+b}dx}{b}+\frac{\log(x)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\log(x)}{b}-\frac{\int \frac{c+dx}{dx^2+cx+b}dx}{b}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\log(x)}{b}-\frac{\frac{1}{2}c\int \frac{1}{dx^2+cx+b}dx+\frac{1}{2}\int \frac{c+2dx}{dx^2+cx+b}dx}{b}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\log(x)}{b}-\frac{\frac{1}{2}\int \frac{c+2dx}{dx^2+cx+b}dx-c\int \frac{1}{c^2-(c+2dx)^2-4bd}d(c+2dx)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\log(x)}{b}-\frac{\frac{1}{2}\int \frac{c+2dx}{dx^2+cx+b}dx-\frac{c\,\text{arctanh}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{\sqrt{c^2-4bd}}}{b}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\log(x)}{b}-\frac{\frac{1}{2}\log\left(b+cx+dx^2\right)-\frac{c\,\text{arctanh}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{\sqrt{c^2-4bd}}}{b}$$

input
```
Int[(b*x + c*x^2 + d*x^3)^(-1),x]
```

output
```
Log[x]/b - (-((c*ArcTanh[(c + 2*d*x)/Sqrt[c^2 - 4*b*d]])/Sqrt[c^2 - 4*b*d]
) + Log[b + c*x + d*x^2]/2)/b
```

3.26.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1949
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b,
c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && IntegerQ[p]
```

3.26.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result
default	$\frac{\ln(x)}{b}+\frac{-\frac{\ln\left(d\,x^2+cx+b\right)}{2}-\frac{c\arctan\left(\frac{2dx+c}{\sqrt{4bd-c^2}}\right)}{\sqrt{4bd-c^2}}}{b}$
risch	$-\frac{2\ln\left(\left(8b\,c^2d-2c^4+6\sqrt{-c^2(4bd-c^2)}\,bd-2\sqrt{-c^2(4bd-c^2)}\,c^2\right)x+12b^2cd-3b\,c^3-\sqrt{-c^2(4bd-c^2)}\,bc\right)d}{4bd-c^2}+\frac{\ln\left(\left(8b\,c^2d-2c^4+6\sqrt{-c\right.\right.}{}$

input
```
int(1/(d*x^3+c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/b+1/b*(-1/2*ln(d*x^2+c*x+b)-c/(4*b*d-c^2)^(1/2)*arctan((2*d*x+c)/(4*
b*d-c^2)^(1/2)))
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.40

$$\int \frac{1}{bx+cx^2+dx^3}\,dx$$
$$=\left[\frac{\sqrt{c^2-4\,bd}c\log\left(\frac{2\,d^2x^2+2\,cdx+c^2-2\,bd+\sqrt{c^2-4\,bd}(2\,dx+c)}{dx^2+cx+b}\right)-(c^2-4\,bd)\log\left(dx^2+cx+b\right)+2\,(c^2-4\,bd)\log\left(x\right.}{2\,(bc^2-4\,b^2d)}\right.$$

input
```
integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(c^2 - 4*b*d)*c*log((2*d^2*x^2 + 2*c*d*x + c^2 - 2*b*d + sqrt(c^
2 - 4*b*d)*(2*d*x + c))/(d*x^2 + c*x + b)) - (c^2 - 4*b*d)*log(d*x^2 + c*x
 + b) + 2*(c^2 - 4*b*d)*log(x))/(b*c^2 - 4*b^2*d), 1/2*(2*sqrt(-c^2 + 4*b*
d)*c*arctan(-sqrt(-c^2 + 4*b*d)*(2*d*x + c)/(c^2 - 4*b*d)) - (c^2 - 4*b*d)
*log(d*x^2 + c*x + b) + 2*(c^2 - 4*b*d)*log(x))/(b*c^2 - 4*b^2*d)]
```

3.26.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(54) = 108.

Time = 4.40 (sec) , antiderivative size = 564, normalized size of antiderivative = 9.10

$$\int \frac{1}{bx+cx^2+dx^3}\,dx = \left(-\frac{c\sqrt{-4bd+c^2}}{2b\,(4bd-c^2)} - \frac{1}{2b}\right)\log\left(x + \frac{24b^4d^2\left(-\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2 - 14b^3c^2d\left(-\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2 - 12b^3d^2\left(-\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right) + \cdots}{9bcd^2-2c\cdots}\right.$$

$$+\left(\frac{c\sqrt{-4bd+c^2}}{2b\,(4bd-c^2)} - \frac{1}{2b}\right)\log\left(x + \frac{24b^4d^2\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2 - 14b^3c^2d\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2 - 12b^3d^2\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right) + 2b^2c^{4}\cdots}{9bcd^2-2c^3d}\right.$$

$$+\frac{\log(x)}{b}$$

input
```
integrate(1/(d*x**3+c*x**2+b*x),x)
```

output
```
(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))*log(x + (24*b**4*d
**2*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 14*b**3*c
**2*d*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 12*b**3
*d**2*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b)) + 2*b**2*c**
4*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 + 3*b**2*c**2
*d*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b)) - 12*b**2*d**2
+ 11*b*c**2*d - 2*c**4)/(9*b*c*d**2 - 2*c**3*d)) + (c*sqrt(-4*b*d + c**2)/
(2*b*(4*b*d - c**2)) - 1/(2*b))*log(x + (24*b**4*d**2*(c*sqrt(-4*b*d + c**
2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 14*b**3*c**2*d*(c*sqrt(-4*b*d + c*
*2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 12*b**3*d**2*(c*sqrt(-4*b*d + c**
2)/(2*b*(4*b*d - c**2)) - 1/(2*b)) + 2*b**2*c**4*(c*sqrt(-4*b*d + c**2)/(2
*b*(4*b*d - c**2)) - 1/(2*b))**2 + 3*b**2*c**2*d*(c*sqrt(-4*b*d + c**2)/(2
*b*(4*b*d - c**2)) - 1/(2*b)) - 12*b**2*d**2 + 11*b*c**2*d - 2*c**4)/(9*b*
c*d**2 - 2*c**3*d)) + log(x)/b
```

3.26.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{bx + cx^2 + dx^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b*d-c^2>0)', see `assume?` for
 more deta
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{1}{bx + cx^2 + dx^3} \, dx = -\frac{c \arctan\left(\frac{2\,dx+c}{\sqrt{-c^2+4\,bd}}\right)}{\sqrt{-c^2+4\,bd}b} - \frac{\log\left(dx^2+cx+b\right)}{2\,b} + \frac{\log\left(|x|\right)}{b}$$

input
```
integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="giac")
```

output
```
-c*arctan((2*d*x + c)/sqrt(-c^2 + 4*b*d))/(sqrt(-c^2 + 4*b*d)*b) - 1/2*log
(d*x^2 + c*x + b)/b + log(abs(x))/b
```

3.26.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.44

$$\int \frac{1}{bx + cx^2 + dx^3} \, dx = \frac{\ln(x)}{b} - \ln\left(\left(x\left(6\,b\,d^2 - 2\,c^2\,d\right) - b\,c\,d\right)\left(\frac{1}{2\,b} - \frac{c\sqrt{c^2-4\,b\,d}}{2\,\left(b\,c^2-4\,b^2\,d\right)}\right) - c\,d - 3\,d^2\,x\right)\left(\frac{1}{2\,b} - \frac{c\sqrt{c^2-4\,b\,d}}{2\,\left(b\,c^2-4\,b^2\,d\right)}\right) - \ln\left(\left(x\left(6\,b\,d^2 - 2\,c^2\,d\right) - b\,c\,d\right)\left(\frac{1}{2\,b} + \frac{c\sqrt{c^2-4\,b\,d}}{2\,\left(b\,c^2-4\,b^2\,d\right)}\right) - c\,d - 3\,d^2\,x\right)\left(\frac{1}{2\,b} + \frac{c\sqrt{c^2-4\,b\,d}}{2\,\left(b\,c^2-4\,b^2\,d\right)}\right)$$

input
```
int(1/(b*x + c*x^2 + d*x^3),x)
```

output
```
log(x)/b - log((x*(6*b*d^2 - 2*c^2*d) - b*c*d)*(1/(2*b) - (c*(c^2 - 4*b*d)
^(1/2))/(2*(b*c^2 - 4*b^2*d))) - c*d - 3*d^2*x)*(1/(2*b) - (c*(c^2 - 4*b*d
)^(1/2))/(2*(b*c^2 - 4*b^2*d))) - log((x*(6*b*d^2 - 2*c^2*d) - b*c*d)*(1/(
2*b) + (c*(c^2 - 4*b*d)^(1/2))/(2*(b*c^2 - 4*b^2*d))) - c*d - 3*d^2*x)*(1/
(2*b) + (c*(c^2 - 4*b*d)^(1/2))/(2*(b*c^2 - 4*b^2*d)))
```

3.26. $\int \frac{1}{bx+cx^2+dx^3}\,dx$

3.27 $\int \frac{1}{a+dx^3} \, dx$

3.27.1 Optimal result

Integrand size = 9, antiderivative size = 115

$$\int \frac{1}{a+dx^3} \, dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}$$

output
```
1/3*ln(a^(1/3)+d^(1/3)*x)/a^(2/3)/d^(1/3)-1/6*ln(a^(2/3)-a^(1/3)*d^(1/3)*x
+d^(2/3)*x^2)/a^(2/3)/d^(1/3)-1/3*arctan(1/3*(a^(1/3)-2*d^(1/3)*x)/a^(1/3)
*3^(1/2))/a^(2/3)/d^(1/3)*3^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{1}{a+dx^3} \, dx = -\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}$$

input
```
Integrate[(a + d*x^3)^(-1),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3
) + d^(1/3)*x] + Log[a^(2/3) - a^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(a^(2/3)*
d^(1/3))
```

3.27.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+dx^3} \, dx$$

$$\Big\downarrow \ 750$$

$$\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{d}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{a}}dx}{3a^{2/3}}$$

$$\Big\downarrow \ 16$$

$$\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{d}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}$$

$$\Big\downarrow \ 1142$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{d}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{a}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{d}x+a^{2/3}}dx}{2\sqrt[3]{d}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{d}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{d}\left(\sqrt[3]{a}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{d}x+a^{2/3}}dx}{2\sqrt[3]{d}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{d}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{d}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{d}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{d}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{d}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}}$$

input
```
Int[(a + d*x^3)^(-1),x]
```

output
```
Log[a^(1/3) + d^(1/3)*x]/(3*a^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*
d^(1/3)*x)/a^(1/3))/Sqrt[3]])/d^(1/3)) - Log[a^(2/3) - a^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2]/(2*d^(1/3)))/(3*a^(2/3))
```

3.27.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.27.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.23

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}}{3d}$	27
default	$\frac{\ln\left(x+\left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{a}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{d}\right)^{\frac{1}{3}}x+\left(\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{a}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{a}{d}\right)^{\frac{2}{3}}}$	91

input
```
int(1/(d*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+a))
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.60

$$\int \frac{1}{a+dx^3}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ad\sqrt{-\frac{(a^2d)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,adx^3-3\,(a^2d)^{\frac{1}{3}}\,ax-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,adx^2+(a^2d)^{\frac{2}{3}}\,x-(a^2d)^{\frac{1}{3}}\,a\right)\sqrt{-\frac{(a^2d)^{\frac{1}{3}}}{d}}}{dx^3+a}\right)-(a^2d)^{\frac{2}{3}}\log\left(adx^2-\right.}{6\,a^2d}\right.$$

input
```
integrate(1/(d*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*d*sqrt(-(a^2*d)^(1/3)/d)*log((2*a*d*x^3 - 3*(a^2*d)^(1
/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*d*x^2 + (a^2*d)^(2/3)*x - (a^2*d)^(1/3)*a
)*sqrt(-(a^2*d)^(1/3)/d))/(d*x^3 + a)) - (a^2*d)^(2/3)*log(a*d*x^2 - (a^2*
d)^(2/3)*x + (a^2*d)^(1/3)*a) + 2*(a^2*d)^(2/3)*log(a*d*x + (a^2*d)^(2/3))
)/(a^2*d), 1/6*(6*sqrt(1/3)*a*d*sqrt((a^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*
(a^2*d)^(2/3)*x - (a^2*d)^(1/3)*a)*sqrt((a^2*d)^(1/3)/d)/a^2) - (a^2*d)^(2
/3)*log(a*d*x^2 - (a^2*d)^(2/3)*x + (a^2*d)^(1/3)*a) + 2*(a^2*d)^(2/3)*log
(a*d*x + (a^2*d)^(2/3)))/(a^2*d)]
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.17

$$\int \frac{1}{a+dx^3}\,dx = \operatorname{RootSum}\left(27t^3a^2d-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

input
```
integrate(1/(d*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**2*d - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

$$\int \frac{1}{a+dx^3}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{d}\right)^{\frac{1}{3}}}\right)}{3\,d\left(\frac{a}{d}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{d}\right)^{\frac{1}{3}}+\left(\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6\,d\left(\frac{a}{d}\right)^{\frac{2}{3}}} + \frac{\log\left(x+\left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\,d\left(\frac{a}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/(d*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/d)^(1/3))/(a/d)^(1/3))/(d*(a/d)^(
2/3)) - 1/6*log(x^2 - x*(a/d)^(1/3) + (a/d)^(2/3))/(d*(a/d)^(2/3)) + 1/3*l
og(x + (a/d)^(1/3))/(d*(a/d)^(2/3))
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+dx^3}\,dx = -\frac{\left(-\frac{a}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,a} + \frac{\sqrt{3}(-ad^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{d}\right)^{\frac{1}{3}}}\right)}{3\,ad} + \frac{(-ad^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{d}\right)^{\frac{1}{3}}+\left(-\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6\,ad}$$

input
```
integrate(1/(d*x^3+a),x, algorithm="giac")
```

output
```
-1/3*(-a/d)^(1/3)*log(abs(x - (-a/d)^(1/3)))/a + 1/3*sqrt(3)*(-a*d^2)^(1/3
)*arctan(1/3*sqrt(3)*(2*x + (-a/d)^(1/3))/(-a/d)^(1/3))/(a*d) + 1/6*(-a*d^
2)^(1/3)*log(x^2 + x*(-a/d)^(1/3) + (-a/d)^(2/3))/(a*d)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+dx^3}\,dx = \frac{\ln\left(d^{1/3}\,x+a^{1/3}\right)}{3\,a^{2/3}\,d^{1/3}} + \frac{\ln\left(3\,d^2\,x+\frac{3\,a^{1/3}\,d^{5/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,d^{1/3}} - \frac{\ln\left(3\,d^2\,x-\frac{3\,a^{1/3}\,d^{5/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,d^{1/3}}$$

input
```
int(1/(a + d*x^3),x)
```

output
```
log(d^(1/3)*x + a^(1/3))/(3*a^(2/3)*d^(1/3)) + (log(3*d^2*x + (3*a^(1/3)*d
^(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(2/3)*d^(1/3)) - (log(3
*d^2*x - (3*a^(1/3)*d^(5/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2
/3)*d^(1/3))
```

3.28 $\int (dx^3)^n \, dx$

3.28.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int (dx^3)^n \, dx = \frac{x(dx^3)^n}{1+3n}$$

output
```
x*(d*x^3)^n/(1+3*n)
```

3.28.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx^3)^n \, dx = \frac{x(dx^3)^n}{1+3n}$$

input
```
Integrate[(d*x^3)^n,x]
```

output
```
(x*(d*x^3)^n)/(1 + 3*n)
```

3.28.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx^3)^n \, dx$$

$$\Big\downarrow \; 20$$

$$x^{-3n}(dx^3)^n \int x^{3n}dx$$

$$\Big\downarrow \; 15$$

$$\frac{x(dx^3)^n}{3n+1}$$

input
```
Int[(d*x^3)^n,x]
```

output
```
(x*(d*x^3)^n)/(1 + 3*n)
```

3.28.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

3.28.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{x\left(x^{3} d\right)^{n}}{1+3 n}$	17
risch	$\frac{x\left(x^{3} d\right)^{n}}{1+3 n}$	17
parallelrisch	$\frac{x\left(x^{3} d\right)^{n}}{1+3 n}$	17
norman	$\frac{x\,\mathrm{e}^{n \ln\left(x^{3} d\right)}}{1+3 n}$	19

input
```
int((x^3*d)^n,x,method=_RETURNVERBOSE)
```

output
```
x*(x^3*d)^n/(1+3*n)
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(dx^{3}\right)^{n}\,dx=\frac{\left(dx^{3}\right)^{n} x}{3\,n+1}$$

input
```
integrate((d*x^3)^n,x, algorithm="fricas")
```

output
```
(d*x^3)^n*x/(3*n + 1)
```

3.28.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \left(dx^{3}\right)^{n}\,dx=\begin{cases} \frac{x\left(dx^{3}\right)^{n}}{3n+1} & \text{for}\ n \neq -\frac{1}{3} \\ \frac{x\log\left(x\right)}{\sqrt[3]{dx^{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**3)**n,x)
```

output
```
Piecewise((x*(d*x**3)**n/(3*n + 1), Ne(n, -1/3)), (x*log(x)/(d*x**3)**(1/3
), True))
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \left(dx^3\right)^n\,dx=\frac{d^n x x^{3\,n}}{3\,n+1}$$

input
```
integrate((d*x^3)^n,x, algorithm="maxima")
```

output
```
d^n*x*x^(3*n)/(3*n + 1)
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(dx^3\right)^n\,dx=\frac{\left(dx^3\right)^n x}{3\,n+1}$$

input
```
integrate((d*x^3)^n,x, algorithm="giac")
```

output
```
(d*x^3)^n*x/(3*n + 1)
```

3.28.9 Mupad [B] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(dx^3\right)^n\,dx=\frac{x\left(d\,x^3\right)^n}{3\,n+1}$$

input
```
int((d*x^3)^n,x)
```

output
```
(x*(d*x^3)^n)/(3*n + 1)
```

3.29 $\int (cx^2 + dx^3)^n \, dx$

3.29.1 Optimal result

Integrand size = 13, antiderivative size = 55

$$\int (cx^2+dx^3)^n \, dx = \frac{x\left(1+\frac{dx}{c}\right)^{-n}(cx^2+dx^3)^n \text{Hypergeometric2F1}\left(-n,1+2n,2(1+n),-\frac{dx}{c}\right)}{1+2n}$$

output
```
x*(d*x^3+c*x^2)^n*hypergeom([-n, 1+2*n],[2+2*n],-d*x/c)/(1+2*n)/((1+d*x/c)
^n)
```

3.29.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int (cx^2+dx^3)^n \, dx = \frac{x(x^2(c+dx))^n\left(1+\frac{dx}{c}\right)^{-n} \text{Hypergeometric2F1}\left(-n,1+2n,2+2n,-\frac{dx}{c}\right)}{1+2n}$$

input
```
Integrate[(c*x^2 + d*x^3)^n,x]
```

output
```
(x*(x^2*(c + d*x))^n*Hypergeometric2F1[-n, 1 + 2*n, 2 + 2*n, -((d*x)/c)])/
((1 + 2*n)*(1 + (d*x)/c)^n)
```

3.29.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1917, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(cx^2+dx^3\right)^n \, dx$$

$$\Big\downarrow \text{1917}$$

$$x^{-2n}(c+dx)^{-n}\left(cx^2+dx^3\right)^n\int x^{2n}(c+dx)^ndx$$

$$\Big\downarrow \text{76}$$

$$x^{-2n}\left(\frac{dx}{c}+1\right)^{-n}\left(cx^2+dx^3\right)^n\int x^{2n}\left(\frac{dx}{c}+1\right)^ndx$$

$$\Big\downarrow \text{74}$$

$$\frac{x\left(\frac{dx}{c}+1\right)^{-n}\left(cx^2+dx^3\right)^n\text{Hypergeometric2F1}\left(-n,2n+1,2(n+1),-\frac{dx}{c}\right)}{2n+1}$$

input
```
Int[(c*x^2 + d*x^3)^n,x]
```

output
```
(x*(c*x^2 + d*x^3)^n*Hypergeometric2F1[-n, 1 + 2*n, 2*(1 + n), -((d*x)/c)]
)/((1 + 2*n)*(1 + (d*x)/c)^n)
```

3.29.3.1 Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.29.4 Maple [F]

$$\int \left(x^{3} d+c\,x^{2}\right)^{n}dx$$

input
```
int((d*x^3+c*x^2)^n,x)
```

output
```
int((d*x^3+c*x^2)^n,x)
```

3.29.5 Fricas [F]

$$\int \left(cx^2+dx^3\right)^n\,dx=\int \left(dx^{3}+cx^{2}\right)^{n}\,dx$$

input
```
integrate((d*x^3+c*x^2)^n,x, algorithm="fricas")
```

output
```
integral((d*x^3 + c*x^2)^n, x)
```

3.29.6 Sympy [F]

$$\int \left(cx^2 + dx^3\right)^n \, dx = \int \left(cx^2 + dx^3\right)^n \, dx$$

input
```
integrate((d*x**3+c*x**2)**n,x)
```

output
```
Integral((c*x**2 + d*x**3)**n, x)
```

3.29.7 Maxima [F]

$$\int \left(cx^2 + dx^3\right)^n \, dx = \int \left(dx^3 + cx^2\right)^n \, dx$$

input
```
integrate((d*x^3+c*x^2)^n,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c*x^2)^n, x)
```

3.29.8 Giac [F]

$$\int \left(cx^2 + dx^3\right)^n \, dx = \int \left(dx^3 + cx^2\right)^n \, dx$$

input
```
integrate((d*x^3+c*x^2)^n,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c*x^2)^n, x)
```

3.29.9 Mupad [B] (verification not implemented)

Time = 9.64 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \left(cx^2+dx^3\right)^n \,dx = \frac{x\,{\left(d\,x^3+c\,x^2\right)}^n\,{}_2\mathrm{F}_1\left(2\,n+1,-n;\ 2\,n+2;\ -\frac{d\,x}{c}\right)}{\left(2\,n+1\right)\,{\left(\frac{d\,x}{c}+1\right)}^n}$$

input
```
int((c*x^2 + d*x^3)^n,x)
```

output
```
(x*(c*x^2 + d*x^3)^n*hypergeom([2*n + 1, -n], 2*n + 2, -(d*x)/c))/((2*n +
1)*((d*x)/c + 1)^n)
```

3.30 $\int (bx + dx^3)^n \, dx$

3.30.1 Optimal result

Integrand size = 11, antiderivative size = 53

$$\int (bx + dx^3)^n \, dx = \frac{x(b + dx^2)(bx + dx^3)^n \text{Hypergeometric2F1}\left(1, \frac{3(1+n)}{2}, \frac{3+n}{2}, -\frac{dx^2}{b}\right)}{b(1+n)}$$

output
```
x*(d*x^2+b)*(d*x^3+b*x)^n*hypergeom([1, 3/2+3/2*n],[3/2+1/2*n],-d*x^2/b)/b
/(1+n)
```

3.30.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int (bx + dx^3)^n \, dx = \frac{x(x(b + dx^2))^n \left(1 + \frac{dx^2}{b}\right)^{-n} \text{Hypergeometric2F1}\left(-n, \frac{1+n}{2}, 1 + \frac{1+n}{2}, -\frac{dx^2}{b}\right)}{1+n}$$

input
```
Integrate[(b*x + d*x^3)^n,x]
```

output
```
(x*(x*(b + d*x^2))^n*Hypergeometric2F1[-n, (1 + n)/2, 1 + (1 + n)/2, -((d*
x^2)/b)])/((1 + n)*(1 + (d*x^2)/b)^n)
```

3.30.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1917, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+dx^3\right)^n dx$$

$$\Big\downarrow \text{1917}$$

$$x^{-n}\left(b+dx^2\right)^{-n}\left(bx+dx^3\right)^n\int x^n\left(dx^2+b\right)^n dx$$

$$\Big\downarrow \text{279}$$

$$x^{-n}\left(\frac{dx^2}{b}+1\right)^{-n}\left(bx+dx^3\right)^n\int x^n\left(\frac{dx^2}{b}+1\right)^n dx$$

$$\Big\downarrow \text{278}$$

$$\frac{x\left(\frac{dx^2}{b}+1\right)^{-n}\left(bx+dx^3\right)^n \text{Hypergeometric2F1}\left(-n,\frac{n+1}{2},\frac{n+3}{2},-\frac{dx^2}{b}\right)}{n+1}$$

input
```
Int[(b*x + d*x^3)^n,x]
```

output
```
(x*(b*x + d*x^3)^n*Hypergeometric2F1[-n, (1 + n)/2, (3 + n)/2, -((d*x^2)/b
)])/((1 + n)*(1 + (d*x^2)/b)^n)
```

3.30.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

3.30.4 Maple [F]

$$\int \left(x^3 d + bx\right)^n dx$$

input
```
int((d*x^3+b*x)^n,x)
```

output
```
int((d*x^3+b*x)^n,x)
```

3.30.5 Fricas [F]

$$\int \left(bx + dx^3\right)^n \, dx = \int \left(dx^3 + bx\right)^n \, dx$$

input
```
integrate((d*x^3+b*x)^n,x, algorithm="fricas")
```

output
```
integral((d*x^3 + b*x)^n, x)
```

3.30.6 Sympy [F]

$$\int \left(bx + dx^3\right)^n \, dx = \int \left(bx + dx^3\right)^n \, dx$$

input
```
integrate((d*x**3+b*x)**n,x)
```

output
```
Integral((b*x + d*x**3)**n, x)
```

3.30.7 Maxima [F]

$$\int (bx + dx^3)^n \, dx = \int (dx^3 + bx)^n \, dx$$

input
```
integrate((d*x^3+b*x)^n,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + b*x)^n, x)
```

3.30.8 Giac [F]

$$\int (bx + dx^3)^n \, dx = \int (dx^3 + bx)^n \, dx$$

input
```
integrate((d*x^3+b*x)^n,x, algorithm="giac")
```

output
```
integrate((d*x^3 + b*x)^n, x)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int (bx + dx^3)^n \, dx = \frac{x \, (d \, x^3 + b \, x)^n \, {}_2F_1\left(\frac{n}{2} + \frac{1}{2}, -n; \, \frac{n}{2} + \frac{3}{2}; \, -\frac{d \, x^2}{b}\right)}{\left(\frac{d \, x^2}{b} + 1\right)^n \, (n+1)}$$

input
```
int((b*x + d*x^3)^n,x)
```

output
```
(x*(b*x + d*x^3)^n*hypergeom([n/2 + 1/2, -n], n/2 + 3/2, -(d*x^2)/b))/(((d
*x^2)/b + 1)^n*(n + 1))
```

3.31 $\int (bx + cx^2 + dx^3)^n \, dx$

3.31.1 Optimal result

Integrand size = 16, antiderivative size = 132

$$\int (bx + cx^2 + dx^3)^n \, dx$$
$$= \frac{x\left(1+\frac{2dx}{c-\sqrt{c^2-4bd}}\right)^{-n}\left(1+\frac{2dx}{c+\sqrt{c^2-4bd}}\right)^{-n}(bx+cx^2+dx^3)^n \operatorname{AppellF1}\left(1+n,-n,-n,2+n,-\frac{2dx}{c-\sqrt{c^2-4bd}},-\right.}{1+n}$$

output
```
x*(d*x^3+c*x^2+b*x)^n*AppellF1(1+n,-n,-n,2+n,-2*d*x/(c-(-4*b*d+c^2)^(1/2))
,-2*d*x/(c+(-4*b*d+c^2)^(1/2)))/(1+n)/((1+2*d*x/(c-(-4*b*d+c^2)^(1/2)))^n)
/((1+2*d*x/(c+(-4*b*d+c^2)^(1/2)))^n)
```

3.31.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

$$\int (bx + cx^2 + dx^3)^n \, dx$$
$$= \frac{x\left(1+\frac{2dx}{c-\sqrt{c^2-4bd}}\right)^{-n}\left(1+\frac{2dx}{c+\sqrt{c^2-4bd}}\right)^{-n}(x(b+x(c+dx)))^n \operatorname{AppellF1}\left(1+n,-n,-n,2+n,-\frac{2dx}{c+\sqrt{c^2-4bd}},\right.}{1+n}$$

input
```
Integrate[(b*x + c*x^2 + d*x^3)^n,x]
```

output
```
(x*(x*(b + x*(c + d*x)))^n*AppellF1[1 + n, -n, -n, 2 + n, (-2*d*x)/(c + Sq
rt[c^2 - 4*b*d]), (2*d*x)/(-c + Sqrt[c^2 - 4*b*d])])/((1 + n)*(1 + (2*d*x)
/(c - Sqrt[c^2 - 4*b*d]))^n*(1 + (2*d*x)/(c + Sqrt[c^2 - 4*b*d]))^n)
```

3.31.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1955, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2+dx^3\right)^n\,dx$$

$$\Big\downarrow \; 1955$$

$$x^{-n}\left(b+cx+dx^2\right)^{-n}\left(bx+cx^2+dx^3\right)^n\int x^n\left(dx^2+cx+b\right)^n dx$$

$$\Big\downarrow \; 1179$$

$$x^{-n}\left(\frac{2dx}{c-\sqrt{c^2-4bd}}+1\right)^{-n}\left(\frac{2dx}{\sqrt{c^2-4bd}+c}+1\right)^{-n}\left(bx+cx^2+dx^3\right)^n\int x^n\left(\frac{2dx}{c-\sqrt{c^2-4bd}}+1\right)^n\left(\frac{2dx}{c+\sqrt{c^2-}\right.$$

$$\Big\downarrow \; 150$$

$$\frac{x\left(\frac{2dx}{c-\sqrt{c^2-4bd}}+1\right)^{-n}\left(\frac{2dx}{\sqrt{c^2-4bd}+c}+1\right)^{-n}\left(bx+cx^2+dx^3\right)^n \text{AppellF1}\left(n+1,-n,-n,n+2,-\frac{2dx}{c-\sqrt{c^2-4bd}},-\frac{2dx}{c+\sqrt{c^2}}\right.}{n+1}$$

input
```
Int[(b*x + c*x^2 + d*x^3)^n,x]
```

output
```
(x*(b*x + c*x^2 + d*x^3)^n*AppellF1[1 + n, -n, -n, 2 + n, (-2*d*x)/(c - Sq
rt[c^2 - 4*b*d]), (-2*d*x)/(c + Sqrt[c^2 - 4*b*d])])/((1 + n)*(1 + (2*d*x)
/(c - Sqrt[c^2 - 4*b*d]))^n*(1 + (2*d*x)/(c + Sqrt[c^2 - 4*b*d]))^n)
```

3.31.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1955
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Simp[(a*x^q + b*x^n + c*x^(2*n - q))^p/(x^(p*q)*(a + b*x^(n - q) + c*x
^(2*(n - q)))^p)   Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !In
tegerQ[p]
```

3.31.4 Maple [F]

$$\int \left(x^3 d + c\,x^2 + bx\right)^n dx$$

input
```
int((d*x^3+c*x^2+b*x)^n,x)
```

output
```
int((d*x^3+c*x^2+b*x)^n,x)
```

3.31.5 Fricas [F]

$$\int \left(bx + cx^2 + dx^3\right)^n \, dx = \int \left(dx^3 + cx^2 + bx\right)^n dx$$

input
```
integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="fricas")
```

output
```
integral((d*x^3 + c*x^2 + b*x)^n, x)
```

3.31.6 Sympy [F]

$$\int \left(bx+cx^2+dx^3\right)^n\,dx=\int \left(bx+cx^2+dx^3\right)^n\,dx$$

input
```
integrate((d*x**3+c*x**2+b*x)**n,x)
```

output
```
Integral((b*x + c*x**2 + d*x**3)**n, x)
```

3.31.7 Maxima [F]

$$\int \left(bx+cx^2+dx^3\right)^n\,dx=\int \left(dx^3+cx^2+bx\right)^n\,dx$$

input
```
integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c*x^2 + b*x)^n, x)
```

3.31.8 Giac [F]

$$\int \left(bx+cx^2+dx^3\right)^n\,dx=\int \left(dx^3+cx^2+bx\right)^n\,dx$$

input
```
integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c*x^2 + b*x)^n, x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \left(bx+cx^2+dx^3\right)^n\,dx=\int {\left(d\,x^3+c\,x^2+b\,x\right)}^n\,dx$$

input
```
int((b*x + c*x^2 + d*x^3)^n,x)
```

output
```
int((b*x + c*x^2 + d*x^3)^n, x)
```

3.32 $\int (a+dx^3)^n \, dx$

3.32.1 Optimal result

Integrand size = 9, antiderivative size = 35

$$\int (a+dx^3)^n \, dx = \frac{x(a+dx^3)^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{4}{3}+n,\frac{4}{3},-\frac{dx^3}{a}\right)}{a}$$

output
```
x*(d*x^3+a)^(1+n)*hypergeom([1, 4/3+n],[4/3],-d*x^3/a)/a
```

3.32.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 5.80

$$\int (a+dx^3)^n \, dx$$

$$= \frac{2^{-n}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{d}x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{d}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-n}\left(\frac{i\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}\right)^{-n}(a+dx^3)^n\,\text{AppellF1}\left(1+n,-n,-n,2\right.}{\sqrt[3]{d}(1+n)}$$

input
```
Integrate[(a + d*x^3)^n,x]
```

output
```
(((-1)^(2/3)*a^(1/3) + d^(1/3)*x)*(a + d*x^3)^n*AppellF1[1 + n, -n, -n, 2
+ n, -(((-1)^(2/3)*((-1)^(2/3)*a^(1/3) + d^(1/3)*x))/((1 + (-1)^(1/3))*a^(
1/3))), (I + Sqrt[3] - ((2*I)*d^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])])/(2^n*d
^(1/3)*(1 + n)*((a^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)
))^n*((I*(1 + (d^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3]))^n)
```

3.32.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+dx^3\right)^n dx$$

$$\Big\downarrow \text{779}$$

$$\left(a+dx^3\right)^n \left(\frac{dx^3}{a}+1\right)^{-n} \int \left(\frac{dx^3}{a}+1\right)^n dx$$

$$\Big\downarrow \text{778}$$

$$x\left(a+dx^3\right)^n \left(\frac{dx^3}{a}+1\right)^{-n} \text{Hypergeometric2F1}\left(\frac{1}{3},-n,\frac{4}{3},-\frac{dx^3}{a}\right)$$

input
```
Int[(a + d*x^3)^n,x]
```

output
```
(x*(a + d*x^3)^n*Hypergeometric2F1[1/3, -n, 4/3, -((d*x^3)/a)])/(1 + (d*x^
3)/a)^n
```

3.32.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

3.32.4 Maple [F]

$$\int \left(x^3d+a\right)^n dx$$

input
```
int((d*x^3+a)^n,x)
```

output
```
int((d*x^3+a)^n,x)
```

3.32.5 Fricas [F]

$$\int \left(a+dx^3\right)^n \, dx=\int \left(dx^3+a\right)^n dx$$

input
```
integrate((d*x^3+a)^n,x, algorithm="fricas")
```

output
```
integral((d*x^3 + a)^n, x)
```

3.32.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int (a+dx^3)^n\,dx = \frac{a^n x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},-n\\ \frac{4}{3}\end{matrix}\middle|\frac{dx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((d*x**3+a)**n,x)
```

output
```
a**n*x*gamma(1/3)*hyper((1/3, -n), (4/3,), d*x**3*exp_polar(I*pi)/a)/(3*ga
mma(4/3))
```

3.32.7 Maxima [F]

$$\int (a+dx^3)^n\,dx = \int (dx^3+a)^n\,dx$$

input
```
integrate((d*x^3+a)^n,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + a)^n, x)
```

3.32.8 Giac [F]

$$\int (a+dx^3)^n\,dx = \int (dx^3+a)^n\,dx$$

input
```
integrate((d*x^3+a)^n,x, algorithm="giac")
```

output
```
integrate((d*x^3 + a)^n, x)
```

3.32.9 Mupad [B] (verification not implemented)

Time = 10.48 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \left(a+dx^3\right)^n\,dx=\frac{x\,{\left(d\,x^3+a\right)}^n\,{}_2\mathrm{F}_1\left(\frac{1}{3},-n;\ \frac{4}{3};\ -\frac{d\,x^3}{a}\right)}{{\left(\frac{d\,x^3}{a}+1\right)}^n}$$

input
```
int((a + d*x^3)^n,x)
```

output
```
(x*(a + d*x^3)^n*hypergeom([1/3, -n], 4/3, -(d*x^3)/a))/((d*x^3)/a + 1)^n
```

3.33 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4 \, dx$

3.33.1 Optimal result

Integrand size = 29, antiderivative size = 270

$$\begin{aligned}\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4 \, dx = {} & \frac{c^4(c^3+4ad^2)^4 x}{d^8} - \frac{8c^5(c^3+4ad^2)^3 \left(\frac{c}{d}+x\right)^3}{3d^6} \\ & + \frac{4c^3(c^3+4ad^2)^2 (7c^3+4ad^2) \left(\frac{c}{d}+x\right)^5}{5d^4} \\ & - \frac{8c^4(c^3+4ad^2)(7c^3+12ad^2) \left(\frac{c}{d}+x\right)^7}{7d^2} \\ & + \frac{2}{9}c^2\left(35c^6+120ac^3d^2+48a^2d^4\right)\left(\frac{c}{d}+x\right)^9 \\ & - \frac{8}{11}c^3d^2\left(7c^3+12ad^2\right)\left(\frac{c}{d}+x\right)^{11} \\ & + \frac{4}{13}cd^4\left(7c^3+4ad^2\right)\left(\frac{c}{d}+x\right)^{13} \\ & - \frac{8}{15}c^2d^6\left(\frac{c}{d}+x\right)^{15} + \frac{1}{17}d^8\left(\frac{c}{d}+x\right)^{17}\end{aligned}$$

output
```
c^4*(4*a*d^2+c^3)^4*x/d^8-8/3*c^5*(4*a*d^2+c^3)^3*(c/d+x)^3/d^6+4/5*c^3*(4
*a*d^2+c^3)^2*(4*a*d^2+7*c^3)*(c/d+x)^5/d^4-8/7*c^4*(4*a*d^2+c^3)*(12*a*d^
2+7*c^3)*(c/d+x)^7/d^2+2/9*c^2*(48*a^2*d^4+120*a*c^3*d^2+35*c^6)*(c/d+x)^9
-8/11*c^3*d^2*(12*a*d^2+7*c^3)*(c/d+x)^11+4/13*c*d^4*(4*a*d^2+7*c^3)*(c/d+
x)^13-8/15*c^2*d^6*(c/d+x)^15+1/17*d^8*(c/d+x)^17
```

3.33.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.06

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx = 256a^4c^4x+\frac{1024}{3}a^3c^5x^3+256a^3c^4dx^4 + \frac{256}{5}a^2c^3\left(6c^3+ad^2\right)x^5+512a^2c^5dx^6 + \frac{256}{7}ac^4\left(4c^3+9ad^2\right)x^7+96ac^3d\left(4c^3+ad^2\right)x^8 + \frac{32}{9}c^2\left(8c^6+120ac^3d^2+3a^2d^4\right)x^9 + \frac{256}{5}c^4d\left(2c^3+5ad^2\right)x^{10}+\frac{64}{11}c^3d^2\left(28c^3+15ad^2\right)x^{11} + \frac{16}{3}c^2d^3\left(28c^3+3ad^2\right)x^{12}+\frac{16}{13}cd^4\left(70c^3+ad^2\right)x^{13} + 32c^3d^5x^{14}+\frac{112}{15}c^2d^6x^{15}+cd^7x^{16}+\frac{d^8x^{17}}{17}$$

input
```
Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]
```

output
```
256*a^4*c^4*x + (1024*a^3*c^5*x^3)/3 + 256*a^3*c^4*d*x^4 + (256*a^2*c^3*(6
*c^3 + a*d^2)*x^5)/5 + 512*a^2*c^5*d*x^6 + (256*a*c^4*(4*c^3 + 9*a*d^2)*x^
7)/7 + 96*a*c^3*d*(4*c^3 + a*d^2)*x^8 + (32*c^2*(8*c^6 + 120*a*c^3*d^2 + 3
*a^2*d^4)*x^9)/9 + (256*c^4*d*(2*c^3 + 5*a*d^2)*x^10)/5 + (64*c^3*d^2*(28*
c^3 + 15*a*d^2)*x^11)/11 + (16*c^2*d^3*(28*c^3 + 3*a*d^2)*x^12)/3 + (16*c*
d^4*(70*c^3 + a*d^2)*x^13)/13 + 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 +
c*d^7*x^16 + (d^8*x^17)/17
```

3.33.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx$$

$$\Big\downarrow \; 2458$$

$$\int \left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^4 d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{1403}$$

$$\int \left(16c^8\left(\frac{6a^2d^4}{c^6}+\frac{15ad^2}{c^3}+\frac{35}{8}\right)\left(\frac{c}{d}+x\right)^8+\frac{(4acd^2+c^4)^4}{d^8}-\frac{32c^7(4ad^2+c^3)\left(\frac{3ad^2}{c^3}+\frac{7}{4}\right)\left(\frac{c}{d}+x\right)^6}{d^2}-32c^6d^2\left(\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}c^2(48a^2d^4+120ac^3d^2+35c^6)\left(\frac{c}{d}+x\right)^9-\frac{8}{11}c^3d^2(12ad^2+7c^3)\left(\frac{c}{d}+x\right)^{11}+\frac{4}{13}cd^4(4ad^2+7c^3)\left(\frac{c}{d}+x\right)^{13}+\frac{4c^3(4ad^2+c^3)^2(4ad^2+7c^3)\left(\frac{c}{d}+x\right)^5}{5d^4}-\frac{8c^5(4ad^2+c^3)^3\left(\frac{c}{d}+x\right)^3}{3d^6}-\frac{8c^4(4ad^2+c^3)(12ad^2+7c^3)\left(\frac{c}{d}+x\right)^7}{7d^2}+\frac{c^4(4ad^2+c^3)^4\left(\frac{c}{d}+x\right)}{d^8}-\frac{8}{15}c^2d^6\left(\frac{c}{d}+x\right)^{15}+\frac{1}{17}d^8\left(\frac{c}{d}+x\right)^{17}$$

input
```
Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]
```

output
```
(c^4*(c^3 + 4*a*d^2)^4*(c/d + x))/d^8 - (8*c^5*(c^3 + 4*a*d^2)^3*(c/d + x)
^3)/(3*d^6) + (4*c^3*(c^3 + 4*a*d^2)^2*(7*c^3 + 4*a*d^2)*(c/d + x)^5)/(5*d
^4) - (8*c^4*(c^3 + 4*a*d^2)*(7*c^3 + 12*a*d^2)*(c/d + x)^7)/(7*d^2) + (2*
c^2*(35*c^6 + 120*a*c^3*d^2 + 48*a^2*d^4)*(c/d + x)^9)/9 - (8*c^3*d^2*(7*c
^3 + 12*a*d^2)*(c/d + x)^11)/11 + (4*c*d^4*(7*c^3 + 4*a*d^2)*(c/d + x)^13)
/13 - (8*c^2*d^6*(c/d + x)^15)/15 + (d^8*(c/d + x)^17)/17
```

3.33.3.1 Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.33.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.99

method	result
norman	$256a^4c^4x + \frac{1024a^3c^5x^3}{3} + 256a^3c^4d\,x^4 + \left(\frac{256}{5}a^3c^3d^2 + \frac{1536}{5}a^2c^6\right)x^5 + 512a^2c^5d\,x^6 + \left(\frac{2304}{7}a^2c^4d\right.$
gosper	$384a\,c^6d\,x^8 + 96a^2c^3d^3x^8 + \frac{960}{11}x^{11}a\,c^3d^4 + 16x^{12}a\,c^2d^5 + \frac{16}{13}x^{13}ac\,d^6 + \frac{32}{3}x^9a^2c^2d^4 + \frac{1280}{3}x^9a$
risch	$384a\,c^6d\,x^8 + 96a^2c^3d^3x^8 + \frac{960}{11}x^{11}a\,c^3d^4 + 16x^{12}a\,c^2d^5 + \frac{16}{13}x^{13}ac\,d^6 + \frac{32}{3}x^9a^2c^2d^4 + \frac{1280}{3}x^9a$
parallelrisch	$384a\,c^6d\,x^8 + 96a^2c^3d^3x^8 + \frac{960}{11}x^{11}a\,c^3d^4 + 16x^{12}a\,c^2d^5 + \frac{16}{13}x^{13}ac\,d^6 + \frac{32}{3}x^9a^2c^2d^4 + \frac{1280}{3}x^9a$
default	$\frac{d^8x^{17}}{17} + d^7c\,x^{16} + \frac{112c^2d^6x^{15}}{15} + 32c^3d^5x^{14} + \frac{(2(8d^2ac+16c^4)d^4+1088c^4d^4)x^{13}}{13} + \frac{(64a\,c^2d^5+16(8d^2ac+16c^4)c \cdots}{12}$

input
```
int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x,method=_RETURNVERBOSE)
```

output
```
256*a^4*c^4*x+1024/3*a^3*c^5*x^3+256*a^3*c^4*d*x^4+(256/5*a^3*c^3*d^2+1536
/5*a^2*c^6)*x^5+512*a^2*c^5*d*x^6+(2304/7*a^2*c^4*d^2+1024/7*a*c^7)*x^7+(9
6*a^2*c^3*d^3+384*a*c^6*d)*x^8+(32/3*a^2*c^2*d^4+1280/3*a*c^5*d^2+256/9*c^
8)*x^9+(256*a*c^4*d^3+512/5*c^7*d)*x^10+(960/11*a*c^3*d^4+1792/11*c^6*d^2)
*x^11+(16*a*c^2*d^5+448/3*c^5*d^3)*x^12+(16/13*a*c*d^6+1120/13*c^4*d^4)*x^
13+32*c^3*d^5*x^14+112/15*c^2*d^6*x^15+d^7*c*x^16+1/17*d^8*x^17
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4 \, dx = \frac{1}{17}\,d^8x^{17}+cd^7x^{16}+\frac{112}{15}\,c^2d^6x^{15}+32\,c^3d^5x^{14}+512\,a^2c^5dx^6+\frac{16}{13}\left(70\,c^4d^4+acd^6\right)x^{13}+\frac{16}{3}\left(28\,c^5d^3+3\,ac^2d^5\right)x^{12}+256\,a^3c^4dx^4+\frac{64}{11}\left(28\,c^6d^2+15\,ac^3d^4\right)x^{11}+\frac{1024}{3}\,a^3c^5x^3+\frac{256}{5}\left(2\,c^7d+5\,ac^4d^3\right)x^{10}+\frac{32}{9}\left(8\,c^8+120\,ac^5d^2+3\,a^2c^2d^4\right)x^9+256\,a^4c^4x+96\left(4\,ac^6d+a^2c^3d^3\right)x^8+\frac{256}{7}\left(4\,ac^7+9\,a^2c^4d^2\right)x^7+\frac{256}{5}\left(6\,a^2c^6+a^3c^3d^2\right)x^5$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="fricas")
```

output
```
1/17*d^8*x^17 + c*d^7*x^16 + 112/15*c^2*d^6*x^15 + 32*c^3*d^5*x^14 + 512*a
^2*c^5*d*x^6 + 16/13*(70*c^4*d^4 + a*c*d^6)*x^13 + 16/3*(28*c^5*d^3 + 3*a*
c^2*d^5)*x^12 + 256*a^3*c^4*d*x^4 + 64/11*(28*c^6*d^2 + 15*a*c^3*d^4)*x^11
 + 1024/3*a^3*c^5*x^3 + 256/5*(2*c^7*d + 5*a*c^4*d^3)*x^10 + 32/9*(8*c^8 +
 120*a*c^5*d^2 + 3*a^2*c^2*d^4)*x^9 + 256*a^4*c^4*x + 96*(4*a*c^6*d + a^2*
c^3*d^3)*x^8 + 256/7*(4*a*c^7 + 9*a^2*c^4*d^2)*x^7 + 256/5*(6*a^2*c^6 + a^
3*c^3*d^2)*x^5
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.11

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx = 256a^4c^4x+\frac{1024a^3c^5x^3}{3}+256a^3c^4dx^4+512a^2c^5dx^6 +32c^3d^5x^{14}+\frac{112c^2d^6x^{15}}{15}+cd^7x^{16}+\frac{d^8x^{17}}{17}+x^{13}\cdot\left(\frac{16acd^6}{13}+\frac{1120c^4d^4}{13}\right)+x^{12}\cdot\left(16ac^2d^5+\frac{448c^5d^3}{3}\right) +x^{11}\cdot\left(\frac{960ac^3d^4}{11}+\frac{1792c^6d^2}{11}\right) +x^{10}\cdot\left(256ac^4d^3+\frac{512c^7d}{5}\right)+x^9\cdot\left(\frac{32a^2c^2d^4}{3}+\frac{1280ac^5d^2}{3}+\frac{256c^8}{9}\right) +x^8\cdot\left(96a^2c^3d^3+384ac^6d\right) +x^7\cdot\left(\frac{2304a^2c^4d^2}{7}+\frac{1024ac^7}{7}\right) +x^5\cdot\left(\frac{256a^3c^3d^2}{5}+\frac{1536a^2c^6}{5}\right)$$

input
```
integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**4,x)
```

output
```
256*a**4*c**4*x + 1024*a**3*c**5*x**3/3 + 256*a**3*c**4*d*x**4 + 512*a**2*
c**5*d*x**6 + 32*c**3*d**5*x**14 + 112*c**2*d**6*x**15/15 + c*d**7*x**16 +
 d**8*x**17/17 + x**13*(16*a*c*d**6/13 + 1120*c**4*d**4/13) + x**12*(16*a*
c**2*d**5 + 448*c**5*d**3/3) + x**11*(960*a*c**3*d**4/11 + 1792*c**6*d**2/
11) + x**10*(256*a*c**4*d**3 + 512*c**7*d/5) + x**9*(32*a**2*c**2*d**4/3 +
 1280*a*c**5*d**2/3 + 256*c**8/9) + x**8*(96*a**2*c**3*d**3 + 384*a*c**6*d
) + x**7*(2304*a**2*c**4*d**2/7 + 1024*a*c**7/7) + x**5*(256*a**3*c**3*d**
2/5 + 1536*a**2*c**6/5)
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.38

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx$$
$$= \frac{1}{17}\,d^8x^{17} + cd^7x^{16} + \frac{32}{5}\,c^2d^6x^{15} + \frac{128}{7}\,c^3d^5x^{14} + \frac{256}{13}\,c^4d^4x^{13} + \frac{256}{9}\,c^8x^9$$
$$+ 256\,a^4c^4x + \frac{256}{15}\left(3\,d^2x^5 + 15\,cdx^4 + 20\,c^2x^3\right)a^3c^3 + \frac{256}{55}\left(5\,d^2x^{11} + 22\,cdx^{10}\right)c^6$$
$$+ \frac{32}{105}\left(35\,d^4x^9 + 315\,cd^3x^8 + 720\,c^2d^2x^7 + 1008\,c^4x^5 + 120\left(3\,d^2x^7 + 14\,cdx^6\right)c^2\right)a^2c^2$$
$$+ \frac{32}{143}\left(33\,d^4x^{13} + 286\,cd^3x^{12} + 624\,c^2d^2x^{11}\right)c^4$$
$$+ \frac{16}{15015}\left(1155\,d^6x^{13} + 15015\,cd^5x^{12} + 65520\,c^2d^4x^{11} + 96096\,c^3d^3x^{10} + 137280\,c^6x^7 + 40040\left(2\,d^2x^9 + 9\,c\right.\right.$$
$$+ \frac{16}{1365}\left(91\,d^6x^{15} + 1170\,cd^5x^{14} + 5040\,c^2d^4x^{13} + 7280\,c^3d^3x^{12}\right)c^2$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="maxima")
```

output
```
1/17*d^8*x^17 + c*d^7*x^16 + 32/5*c^2*d^6*x^15 + 128/7*c^3*d^5*x^14 + 256/
13*c^4*d^4*x^13 + 256/9*c^8*x^9 + 256*a^4*c^4*x + 256/15*(3*d^2*x^5 + 15*c
*d*x^4 + 20*c^2*x^3)*a^3*c^3 + 256/55*(5*d^2*x^11 + 22*c*d*x^10)*c^6 + 32/
105*(35*d^4*x^9 + 315*c*d^3*x^8 + 720*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*
d^2*x^7 + 14*c*d*x^6)*c^2)*a^2*c^2 + 32/143*(33*d^4*x^13 + 286*c*d^3*x^12
+ 624*c^2*d^2*x^11)*c^4 + 16/15015*(1155*d^6*x^13 + 15015*c*d^5*x^12 + 655
20*c^2*d^4*x^11 + 96096*c^3*d^3*x^10 + 137280*c^6*x^7 + 40040*(2*d^2*x^9 +
 9*c*d*x^8)*c^4 + 364*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2
)*a*c + 16/1365*(91*d^6*x^15 + 1170*c*d^5*x^14 + 5040*c^2*d^4*x^13 + 7280*
c^3*d^3*x^12)*c^2
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.03

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4 \, dx = \frac{1}{17}\,d^8x^{17}+cd^7x^{16}+\frac{112}{15}\,c^2d^6x^{15}+32\,c^3d^5x^{14}+\frac{1120}{13}\,c^4d^4x^{13}+\frac{16}{13}\,acd^6x^{13}+\frac{448}{3}\,c^5d^3x^{12}+16\,ac^2d^5x^{12}+\frac{1792}{11}\,c^6d^2x^{11}+\frac{960}{11}\,ac^3d^4x^{11}+\frac{512}{5}\,c^7dx^{10}+256\,ac^4d^3x^{10}+\frac{256}{9}\,c^8x^9+\frac{1280}{3}\,ac^5d^2x^9+\frac{32}{3}\,a^2c^2d^4x^9+384\,ac^6dx^8+96\,a^2c^3d^3x^8+\frac{1024}{7}\,ac^7x^7+\frac{2304}{7}\,a^2c^4d^2x^7+512\,a^2c^5dx^6+\frac{1536}{5}\,a^2c^6x^5+\frac{256}{5}\,a^3c^3d^2x^5+256\,a^3c^4dx^4+\frac{1024}{3}\,a^3c^5x^3+256\,a^4c^4x$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="giac")
```

output
```
1/17*d^8*x^17 + c*d^7*x^16 + 112/15*c^2*d^6*x^15 + 32*c^3*d^5*x^14 + 1120/
13*c^4*d^4*x^13 + 16/13*a*c*d^6*x^13 + 448/3*c^5*d^3*x^12 + 16*a*c^2*d^5*x
^12 + 1792/11*c^6*d^2*x^11 + 960/11*a*c^3*d^4*x^11 + 512/5*c^7*d*x^10 + 25
6*a*c^4*d^3*x^10 + 256/9*c^8*x^9 + 1280/3*a*c^5*d^2*x^9 + 32/3*a^2*c^2*d^4
*x^9 + 384*a*c^6*d*x^8 + 96*a^2*c^3*d^3*x^8 + 1024/7*a*c^7*x^7 + 2304/7*a^
2*c^4*d^2*x^7 + 512*a^2*c^5*d*x^6 + 1536/5*a^2*c^6*x^5 + 256/5*a^3*c^3*d^2
*x^5 + 256*a^3*c^4*d*x^4 + 1024/3*a^3*c^5*x^3 + 256*a^4*c^4*x
```

3.33.9 Mupad [B] (verification not implemented)

Time = 10.57 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.97

$$\int (4ac+4c^2x^2+4cdx^3+d^2x^4)^4\,dx = x^{10}\left(\frac{512\,c^7\,d}{5}+256\,a\,c^4\,d^3\right)+x^{13}\left(\frac{1120\,c^4\,d^4}{13}+\frac{16\,a\,c\,d^6}{13}\right)+x^9\left(\frac{32\,a^2\,c^2\,d^4}{3}+\frac{1280\,a\,c^5\,d^2}{3}+\frac{256\,c^8}{9}\right)+x^{12}\left(\frac{448\,c^5\,d^3}{3}+16\,a\,c^2\,d^5\right)+x^{11}\left(\frac{1792\,c^6\,d^2}{11}+\frac{960\,a\,c^3\,d^4}{11}\right)+\frac{d^8\,x^{17}}{17}+256\,a^4\,c^4\,x+c\,d^7\,x^{16}+\frac{1024\,a^3\,c^5\,x^3}{3}+32\,c^3\,d^5\,x^{14}+\frac{112\,c^2\,d^6\,x^{15}}{15}+256\,a^3\,c^4\,d\,x^4+512\,a^2\,c^5\,d\,x^6+\frac{256\,a\,c^4\,x^7\,(4\,c^3+9\,a\,d^2)}{7}+\frac{256\,a^2\,c^3\,x^5\,(6\,c^3+a\,d^2)}{5}+96\,a\,c^3\,d\,x^8\,\left(4\,c^3+a\,d^2\right)$$

input
```
int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^4,x)
```

output
```
x^10*((512*c^7*d)/5 + 256*a*c^4*d^3) + x^13*((1120*c^4*d^4)/13 + (16*a*c*d
^6)/13) + x^9*((256*c^8)/9 + (1280*a*c^5*d^2)/3 + (32*a^2*c^2*d^4)/3) + x^
12*((448*c^5*d^3)/3 + 16*a*c^2*d^5) + x^11*((1792*c^6*d^2)/11 + (960*a*c^3
*d^4)/11) + (d^8*x^17)/17 + 256*a^4*c^4*x + c*d^7*x^16 + (1024*a^3*c^5*x^3
)/3 + 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 + 256*a^3*c^4*d*x^4 + 512*a^
2*c^5*d*x^6 + (256*a*c^4*x^7*(9*a*d^2 + 4*c^3))/7 + (256*a^2*c^3*x^5*(a*d^
2 + 6*c^3))/5 + 96*a*c^3*d*x^8*(a*d^2 + 4*c^3)
```

3.34 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3 \, dx$

3.34.1 Optimal result

Integrand size = 29, antiderivative size = 171

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3 \, dx = 64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+\frac{48}{5}ac^2\left(4c^3+ad^2\right)x^5 + 64ac^4dx^6+\frac{32}{7}c^3\left(2c^3+9ad^2\right)x^7 + 12c^2d\left(2c^3+ad^2\right)x^8+\frac{4}{3}cd^2\left(20c^3+ad^2\right)x^9 + 16c^3d^3x^{10}+\frac{60}{11}c^2d^4x^{11}+cd^5x^{12}+\frac{d^6x^{13}}{13}$$

output
```
64*a^3*c^3*x+64*a^2*c^4*x^3+48*a^2*c^3*d*x^4+48/5*a*c^2*(a*d^2+4*c^3)*x^5+
64*a*c^4*d*x^6+32/7*c^3*(9*a*d^2+2*c^3)*x^7+12*c^2*d*(a*d^2+2*c^3)*x^8+4/3
*c*d^2*(a*d^2+20*c^3)*x^9+16*c^3*d^3*x^10+60/11*c^2*d^4*x^11+c*d^5*x^12+1/
13*d^6*x^13
```

3.34.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3 \, dx = 64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+\frac{48}{5}ac^2\left(4c^3+ad^2\right)x^5 + 64ac^4dx^6+\frac{32}{7}c^3\left(2c^3+9ad^2\right)x^7 + 12c^2d\left(2c^3+ad^2\right)x^8+\frac{4}{3}cd^2\left(20c^3+ad^2\right)x^9 + 16c^3d^3x^{10}+\frac{60}{11}c^2d^4x^{11}+cd^5x^{12}+\frac{d^6x^{13}}{13}$$

input
```
Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]
```

output
```
64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c^3 + a*d^
2)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x^7)/7 + 12*c^2*d*(
2*c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*x^9)/3 + 16*c^3*d^3*x^10 +
(60*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d^6*x^13)/13
```

3.34.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3 dx$$

$$\Big\downarrow \; 2458$$

$$\int \left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^3 d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \; 1403$$

$$\int \left(\frac{\left(4acd^2+c^4\right)^3}{d^6}-8c^6\left(\frac{6ad^2}{c^3}+\frac{5}{2}\right)\left(\frac{c}{d}+x\right)^6+\frac{12c^5\left(4ad^2+c^3\right)\left(\frac{ad^2}{c^3}+\frac{5}{4}\right)\left(\frac{c}{d}+x\right)^4}{d^2}+12c^4d^2\left(\frac{ad^2}{c^3}+\frac{5}{4}\right)\left(\frac{c}{d}\right.\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}cd^2\left(4ad^2+5c^3\right)\left(\frac{c}{d}+x\right)^9-\frac{4}{7}c^3\left(12ad^2+5c^3\right)\left(\frac{c}{d}+x\right)^7+\frac{c^3\left(4ad^2+c^3\right)^3\left(\frac{c}{d}+x\right)}{d^6}-\frac{2c^4\left(4ad^2+c^3\right)^2\left(\frac{c}{d}+x\right)^3}{d^4}+\frac{3c^2\left(4ad^2+c^3\right)\left(4ad^2+5c^3\right)\left(\frac{c}{d}+x\right)^5}{5d^2}-\frac{6}{11}c^2d^4\left(\frac{c}{d}+x\right)^{11}+\frac{1}{13}d^6\left(\frac{c}{d}+x\right)^{13}$$

input
```
Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]
```

output
```
(c^3*(c^3 + 4*a*d^2)^3*(c/d + x))/d^6 - (2*c^4*(c^3 + 4*a*d^2)^2*(c/d + x)
^3)/d^4 + (3*c^2*(c^3 + 4*a*d^2)*(5*c^3 + 4*a*d^2)*(c/d + x)^5)/(5*d^2) -
(4*c^3*(5*c^3 + 12*a*d^2)*(c/d + x)^7)/7 + (c*d^2*(5*c^3 + 4*a*d^2)*(c/d +
 x)^9)/3 - (6*c^2*d^4*(c/d + x)^11)/11 + (d^6*(c/d + x)^13)/13
```

3.34.3.1 Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.34.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95

method	result
norman	$\frac{d^6x^{13}}{13}+c\,d^5x^{12}+\frac{60c^2d^4x^{11}}{11}+16c^3d^3x^{10}+\left(\frac{4}{3}ac\,d^4+\frac{80}{3}d^2c^4\right)x^9+\left(12a\,c^2d^3+24c^5d\right)x^8+\left(\frac{28}{7}\right.$
gosper	$\frac{1}{13}d^6x^{13}+c\,d^5x^{12}+\frac{60}{11}c^2d^4x^{11}+16c^3d^3x^{10}+\frac{4}{3}x^9ac\,d^4+\frac{80}{3}x^9d^2c^4+12a\,c^2d^3x^8+24c^5d\,x^8+$
risch	$\frac{1}{13}d^6x^{13}+c\,d^5x^{12}+\frac{60}{11}c^2d^4x^{11}+16c^3d^3x^{10}+\frac{4}{3}x^9ac\,d^4+\frac{80}{3}x^9d^2c^4+12a\,c^2d^3x^8+24c^5d\,x^8+$
parallelrisch	$\frac{1}{13}d^6x^{13}+c\,d^5x^{12}+\frac{60}{11}c^2d^4x^{11}+16c^3d^3x^{10}+\frac{4}{3}x^9ac\,d^4+\frac{80}{3}x^9d^2c^4+12a\,c^2d^3x^8+24c^5d\,x^8+$
default	$\frac{d^6x^{13}}{13}+c\,d^5x^{12}+\frac{60c^2d^4x^{11}}{11}+16c^3d^3x^{10}+\frac{\left(4ac\,d^4+224d^2c^4+d^2\left(8d^2ac+16c^4\right)\right)x^9}{9}+\frac{\left(64a\,c^2d^3+128c^5d+4cd(8\right.}{8}$

input
```
int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*d^6*x^13+c*d^5*x^12+60/11*c^2*d^4*x^11+16*c^3*d^3*x^10+(4/3*a*c*d^4+8
0/3*d^2*c^4)*x^9+(12*a*c^2*d^3+24*c^5*d)*x^8+(288/7*a*c^3*d^2+64/7*c^6)*x^
7+64*a*c^4*d*x^6+(48/5*a^2*c^2*d^2+192/5*a*c^5)*x^5+48*a^2*c^3*d*x^4+64*a^
2*c^4*x^3+64*a^3*c^3*x
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx=\frac{1}{13}\,d^6x^{13}+cd^5x^{12}+\frac{60}{11}\,c^2d^4x^{11}+16\,c^3d^3x^{10}+64\,ac^4dx^6+48\,a^2c^3dx^4+\frac{4}{3}\,\left(20\,c^4d^2+acd^4\right)x^9+64\,a^2c^4x^3+12\,\left(2\,c^5d+ac^2d^3\right)x^8+\frac{32}{7}\,\left(2\,c^6+9\,ac^3d^2\right)x^7+64\,a^3c^3x+\frac{48}{5}\,\left(4\,ac^5+a^2c^2d^2\right)x^5$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="fricas")
```

output
```
1/13*d^6*x^13 + c*d^5*x^12 + 60/11*c^2*d^4*x^11 + 16*c^3*d^3*x^10 + 64*a*c
^4*d*x^6 + 48*a^2*c^3*d*x^4 + 4/3*(20*c^4*d^2 + a*c*d^4)*x^9 + 64*a^2*c^4*
x^3 + 12*(2*c^5*d + a*c^2*d^3)*x^8 + 32/7*(2*c^6 + 9*a*c^3*d^2)*x^7 + 64*a
^3*c^3*x + 48/5*(4*a*c^5 + a^2*c^2*d^2)*x^5
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.05

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx=64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+64ac^4dx^6+16c^3d^3x^{10}+\frac{60c^2d^4x^{11}}{11}+cd^5x^{12}+\frac{d^6x^{13}}{13}+x^9\cdot\left(\frac{4acd^4}{3}+\frac{80c^4d^2}{3}\right)+x^8\cdot\left(12ac^2d^3+24c^5d\right)+x^7\cdot\left(\frac{288ac^3d^2}{7}+\frac{64c^6}{7}\right)+x^5\cdot\left(\frac{48a^2c^2d^2}{5}+\frac{192ac^5}{5}\right)$$

input
```
integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**3,x)
```

output
```
64*a**3*c**3*x + 64*a**2*c**4*x**3 + 48*a**2*c**3*d*x**4 + 64*a*c**4*d*x**
6 + 16*c**3*d**3*x**10 + 60*c**2*d**4*x**11/11 + c*d**5*x**12 + d**6*x**13
/13 + x**9*(4*a*c*d**4/3 + 80*c**4*d**2/3) + x**8*(12*a*c**2*d**3 + 24*c**
5*d) + x**7*(288*a*c**3*d**2/7 + 64*c**6/7) + x**5*(48*a**2*c**2*d**2/5 +
192*a*c**5/5)
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.20

$$\begin{aligned}
&\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3 \, dx \\
&= \frac{1}{13}\,d^6x^{13}+cd^5x^{12}+\frac{48}{11}\,c^2d^4x^{11}+\frac{32}{5}\,c^3d^3x^{10}+\frac{64}{7}\,c^6x^7+64\,a^3c^3x \\
&\quad+\frac{16}{5}\,\left(3\,d^2x^5+15\,cdx^4+20\,c^2x^3\right)a^2c^2+\frac{8}{3}\,\left(2\,d^2x^9+9\,cdx^8\right)c^4 \\
&\quad+\frac{4}{105}\,\left(35\,d^4x^9+315\,cd^3x^8+720\,c^2d^2x^7+1008\,c^4x^5+120\,\left(3\,d^2x^7+14\,cdx^6\right)c^2\right)ac \\
&\quad+\frac{4}{165}\,\left(45\,d^4x^{11}+396\,cd^3x^{10}+880\,c^2d^2x^9\right)c^2
\end{aligned}$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="maxima")
```

output
```
1/13*d^6*x^13 + c*d^5*x^12 + 48/11*c^2*d^4*x^11 + 32/5*c^3*d^3*x^10 + 64/7
*c^6*x^7 + 64*a^3*c^3*x + 16/5*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a^2*c
^2 + 8/3*(2*d^2*x^9 + 9*c*d*x^8)*c^4 + 4/105*(35*d^4*x^9 + 315*c*d^3*x^8 +
 720*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*d^2*x^7 + 14*c*d*x^6)*c^2)*a*c +
4/165*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.97

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx = \frac{1}{13}\,d^6x^{13}+cd^5x^{12}+\frac{60}{11}\,c^2d^4x^{11}+16\,c^3d^3x^{10}+\frac{80}{3}\,c^4d^2x^9+\frac{4}{3}\,acd^4x^9+24\,c^5dx^8+12\,ac^2d^3x^8+\frac{64}{7}\,c^6x^7+\frac{288}{7}\,ac^3d^2x^7+64\,ac^4dx^6+\frac{192}{5}\,ac^5x^5+\frac{48}{5}\,a^2c^2d^2x^5+48\,a^2c^3dx^4+64\,a^2c^4x^3+64\,a^3c^3x$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="giac")
```

output
```
1/13*d^6*x^13 + c*d^5*x^12 + 60/11*c^2*d^4*x^11 + 16*c^3*d^3*x^10 + 80/3*c
^4*d^2*x^9 + 4/3*a*c*d^4*x^9 + 24*c^5*d*x^8 + 12*a*c^2*d^3*x^8 + 64/7*c^6*
x^7 + 288/7*a*c^3*d^2*x^7 + 64*a*c^4*d*x^6 + 192/5*a*c^5*x^5 + 48/5*a^2*c^
2*d^2*x^5 + 48*a^2*c^3*d*x^4 + 64*a^2*c^4*x^3 + 64*a^3*c^3*x
```

3.34.9 Mupad [B] (verification not implemented)

Time = 10.58 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.94

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx = x^8\left(24\,c^5\,d+12\,a\,c^2\,d^3\right)+x^9\left(\frac{80\,c^4\,d^2}{3}+\frac{4\,a\,c\,d^4}{3}\right)+\frac{d^6\,x^{13}}{13}+x^7\left(\frac{64\,c^6}{7}+\frac{288\,a\,c^3\,d^2}{7}\right)+64\,a^3\,c^3\,x+c\,d^5\,x^{12}+64\,a^2\,c^4\,x^3+16\,c^3\,d^3\,x^{10}+\frac{60\,c^2\,d^4\,x^{11}}{11}+48\,a^2\,c^3\,d\,x^4+\frac{48\,a\,c^2\,x^5\left(4\,c^3+a\,d^2\right)}{5}+64\,a\,c^4\,d\,x^6$$

input
```
int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^3,x)
```

output
```
x^8*(24*c^5*d + 12*a*c^2*d^3) + x^9*((80*c^4*d^2)/3 + (4*a*c*d^4)/3) + (d^
6*x^13)/13 + x^7*((64*c^6)/7 + (288*a*c^3*d^2)/7) + 64*a^3*c^3*x + c*d^5*x
^12 + 64*a^2*c^4*x^3 + 16*c^3*d^3*x^10 + (60*c^2*d^4*x^11)/11 + 48*a^2*c^3
*d*x^4 + (48*a*c^2*x^5*(a*d^2 + 4*c^3))/5 + 64*a*c^4*d*x^6
```

3.35 $\int (4ac+4c^2x^2+4cdx^3+d^2x^4)^2 \, dx$

3.35.1 Optimal result

Integrand size = 29, antiderivative size = 92

$$\int (4ac+4c^2x^2+4cdx^3+d^2x^4)^2 \, dx = 16a^2c^2x+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{8}{5}c(2c^3+ad^2)\,x^5 +\frac{16}{3}c^3dx^6+\frac{24}{7}c^2d^2x^7+cd^3x^8+\frac{d^4x^9}{9}$$

output
```
16*a^2*c^2*x+32/3*a*c^3*x^3+8*a*c^2*d*x^4+8/5*c*(a*d^2+2*c^3)*x^5+16/3*c^3
*d*x^6+24/7*c^2*d^2*x^7+c*d^3*x^8+1/9*d^4*x^9
```

3.35.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int (4ac+4c^2x^2+4cdx^3+d^2x^4)^2 \, dx = 16a^2c^2x+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{8}{5}c(2c^3+ad^2)\,x^5 +\frac{16}{3}c^3dx^6+\frac{24}{7}c^2d^2x^7+cd^3x^8+\frac{d^4x^9}{9}$$

input
```
Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]
```

output
```
16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a*d^2)*x^5
)/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8 + (d^4*x^9)/9
```

3.35.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2 dx$$

$$\Big\downarrow \text{ 2458}$$

$$\int \left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^2 d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{ 1403}$$

$$\int \left(\frac{\left(4acd^2+c^4\right)^2}{d^4}-\frac{4c^3\left(4ad^2+c^3\right)\left(\frac{c}{d}+x\right)^2}{d^2}+4c^4\left(\frac{2ad^2}{c^3}+\frac{3}{2}\right)\left(\frac{c}{d}+x\right)^4-4c^2d^2\left(\frac{c}{d}+x\right)^6+d^4\left(\frac{c}{d}+x\right)^8\right) d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{5}c\left(4ad^2+3c^3\right)\left(\frac{c}{d}+x\right)^5-\frac{4c^3\left(4ad^2+c^3\right)\left(\frac{c}{d}+x\right)^3}{3d^2}+\frac{c^2\left(4ad^2+c^3\right)^2\left(\frac{c}{d}+x\right)}{d^4}-\frac{4}{7}c^2d^2\left(\frac{c}{d}+x\right)^7+\frac{1}{9}d^4\left(\frac{c}{d}+x\right)^9$$

input
```
Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]
```

output
```
(c^2*(c^3 + 4*a*d^2)^2*(c/d + x))/d^4 - (4*c^3*(c^3 + 4*a*d^2)*(c/d + x)^3
)/(3*d^2) + (2*c*(3*c^3 + 4*a*d^2)*(c/d + x)^5)/5 - (4*c^2*d^2*(c/d + x)^7
)/7 + (d^4*(c/d + x)^9)/9
```

3.35.3.1 Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.35.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90

method	result
norman	$\frac{d^4x^9}{9}+c\,d^3x^8+\frac{24c^2d^2x^7}{7}+\frac{16c^3d\,x^6}{3}+\left(\frac{8}{5}d^2ac+\frac{16}{5}c^4\right)x^5+8a\,c^2d\,x^4+\frac{32a\,c^3x^3}{3}+16a^2c^2x$
gosper	$\frac{1}{9}d^4x^9+c\,d^3x^8+\frac{24}{7}c^2d^2x^7+\frac{16}{3}c^3d\,x^6+\frac{8}{5}x^5d^2ac+\frac{16}{5}x^5c^4+8a\,c^2d\,x^4+\frac{32}{3}a\,c^3x^3+16a^2c^2x$
default	$\frac{d^4x^9}{9}+c\,d^3x^8+\frac{24c^2d^2x^7}{7}+\frac{16c^3d\,x^6}{3}+\frac{(8d^2ac+16c^4)x^5}{5}+8a\,c^2d\,x^4+\frac{32a\,c^3x^3}{3}+16a^2c^2x$
risch	$\frac{1}{9}d^4x^9+c\,d^3x^8+\frac{24}{7}c^2d^2x^7+\frac{16}{3}c^3d\,x^6+\frac{8}{5}x^5d^2ac+\frac{16}{5}x^5c^4+8a\,c^2d\,x^4+\frac{32}{3}a\,c^3x^3+16a^2c^2x$
parallelrisch	$\frac{1}{9}d^4x^9+c\,d^3x^8+\frac{24}{7}c^2d^2x^7+\frac{16}{3}c^3d\,x^6+\frac{8}{5}x^5d^2ac+\frac{16}{5}x^5c^4+8a\,c^2d\,x^4+\frac{32}{3}a\,c^3x^3+16a^2c^2x$

input
```
int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*d^4*x^9+c*d^3*x^8+24/7*c^2*d^2*x^7+16/3*c^3*d*x^6+(8/5*d^2*a*c+16/5*c^
4)*x^5+8*a*c^2*d*x^4+32/3*a*c^3*x^3+16*a^2*c^2*x
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2\,dx=\frac{1}{9}\,d^4x^9+cd^3x^8+\frac{24}{7}\,c^2d^2x^7+\frac{16}{3}\,c^3dx^6+8\,ac^2dx^4+\frac{32}{3}\,ac^3x^3+\frac{8}{5}\,(2\,c^4+acd^2)x^5+16\,a^2c^2x$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")
```

output
```
1/9*d^4*x^9 + c*d^3*x^8 + 24/7*c^2*d^2*x^7 + 16/3*c^3*d*x^6 + 8*a*c^2*d*x^
4 + 32/3*a*c^3*x^3 + 8/5*(2*c^4 + a*c*d^2)*x^5 + 16*a^2*c^2*x
```

3.35.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2\,dx = 16a^2c^2x+\frac{32ac^3x^3}{3}+8ac^2dx^4+\frac{16c^3dx^6}{3}+\frac{24c^2d^2x^7}{7}+cd^3x^8+\frac{d^4x^9}{9}+x^5\cdot\left(\frac{8acd^2}{5}+\frac{16c^4}{5}\right)$$

input
```
integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

output
```
16*a**2*c**2*x + 32*a*c**3*x**3/3 + 8*a*c**2*d*x**4 + 16*c**3*d*x**6/3 + 2
4*c**2*d**2*x**7/7 + c*d**3*x**8 + d**4*x**9/9 + x**5*(8*a*c*d**2/5 + 16*c
**4/5)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.02

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2\,dx = \frac{1}{9}\,d^4x^9+cd^3x^8+\frac{16}{7}\,c^2d^2x^7+\frac{16}{5}\,c^4x^5+16\,a^2c^2x+\frac{8}{15}\left(3\,d^2x^5+15\,cdx^4+20\,c^2x^3\right)ac+\frac{8}{21}\left(3\,d^2x^7+14\,cdx^6\right)c^2$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")
```

output
```
1/9*d^4*x^9 + c*d^3*x^8 + 16/7*c^2*d^2*x^7 + 16/5*c^4*x^5 + 16*a^2*c^2*x +
 8/15*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a*c + 8/21*(3*d^2*x^7 + 14*c*d
*x^6)*c^2
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2\,dx=\frac{1}{9}\,d^4x^9+cd^3x^8+\frac{24}{7}\,c^2d^2x^7+\frac{16}{3}\,c^3dx^6+\frac{16}{5}\,c^4x^5+\frac{8}{5}\,acd^2x^5+8\,ac^2dx^4+\frac{32}{3}\,ac^3x^3+16\,a^2c^2x$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="giac")
```

output
```
1/9*d^4*x^9 + c*d^3*x^8 + 24/7*c^2*d^2*x^7 + 16/3*c^3*d*x^6 + 16/5*c^4*x^5
 + 8/5*a*c*d^2*x^5 + 8*a*c^2*d*x^4 + 32/3*a*c^3*x^3 + 16*a^2*c^2*x
```

3.35.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2\,dx=x^5\left(\frac{16\,c^4}{5}+\frac{8\,a\,c\,d^2}{5}\right)+\frac{d^4\,x^9}{9}+16\,a^2\,c^2\,x+\frac{32\,a\,c^3\,x^3}{3}+\frac{16\,c^3\,d\,x^6}{3}+c\,d^3\,x^8+\frac{24\,c^2\,d^2\,x^7}{7}+8\,a\,c^2\,d\,x^4$$

input
```
int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^2,x)
```

output
```
x^5*((16*c^4)/5 + (8*a*c*d^2)/5) + (d^4*x^9)/9 + 16*a^2*c^2*x + (32*a*c^3*
x^3)/3 + (16*c^3*d*x^6)/3 + c*d^3*x^8 + (24*c^2*d^2*x^7)/7 + 8*a*c^2*d*x^4
```

3.36 $\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)\,dx$

3.36.1 Optimal result

Integrand size = 27, antiderivative size = 32

$$\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)\,dx = 4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

output
```
4*a*c*x+4/3*c^2*x^3+c*d*x^4+1/5*d^2*x^5
```

3.36.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)\,dx = 4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

input
```
Integrate[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]
```

output
```
4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

3.36.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

input
```
Int[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]
```

output
```
4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

3.36.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.36.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
gosper	$4acx+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29
default	$4acx+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29
norman	$4acx+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29
risch	$4acx+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29
parallelrisch	$4acx+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29
parts	$4acx+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29

input
```
int(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x,method=_RETURNVERBOSE)
```

output
```
4*a*c*x+4/3*c^2*x^3+c*d*x^4+1/5*d^2*x^5
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4) \, dx = \frac{1}{5} d^2x^5 + cdx^4 + \frac{4}{3} c^2x^3 + 4\,acx$$

input
```
integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="fricas")
```

output
```
1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

3.36.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4) \, dx = 4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

input
```
integrate(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c,x)
```

output
```
4*a*c*x + 4*c**2*x**3/3 + c*d*x**4 + d**2*x**5/5
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4) \, dx = \frac{1}{5} d^2x^5 + cdx^4 + \frac{4}{3} c^2x^3 + 4\,acx$$

input
```
integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="maxima")
```

output
```
1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)\,dx=\frac{1}{5}\,d^2x^5+cdx^4+\frac{4}{3}\,c^2x^3+4\,acx$$

input
```
integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="giac")
```

output
```
1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

3.36.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)\,dx=\frac{4\,c^2\,x^3}{3}+c\,d\,x^4+4\,a\,c\,x+\frac{d^2\,x^5}{5}$$

input
```
int(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3,x)
```

output
```
(4*c^2*x^3)/3 + (d^2*x^5)/5 + 4*a*c*x + c*d*x^4
```

3.37 $\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

3.37.1 Optimal result

Integrand size = 29, antiderivative size = 529

$$\begin{aligned}\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx &= -\frac{d\operatorname{arctanh}\left(\frac{\sqrt{2}c+\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}+\sqrt{2}dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}(c+dx)}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\\ &\quad-\frac{d\log\left(\sqrt{c}\sqrt{c^3+4ad^2}-\sqrt{2}\sqrt[4]{c}d\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{4\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}\\ &\quad+\frac{d\log\left(\sqrt{c}\sqrt{c^3+4ad^2}+\sqrt{2}\sqrt[4]{c}d\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{4\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}\end{aligned}$$

output
```
-1/4*d*arctanh((c*2^(1/2)+d*x*2^(1/2)+c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1/2)
)^(1/2))/c^(1/4)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(4*a
*d^2+c^3)^(1/2)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2)+1/4*d*arctanh((-(d*x+c
)*2^(1/2)+c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(1/4)/(c^(3/2)-(4
*a*d^2+c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(4*a*d^2+c^3)^(1/2)/(c^(3/2)-(4*
a*d^2+c^3)^(1/2))^(1/2)-1/8*d*ln(d^2*(c/d+x)^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)
-c^(1/4)*d*(c/d+x)*2^(1/2)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(3/4)*2^
(1/2)/(4*a*d^2+c^3)^(1/2)/(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)+1/8*d*ln(d^2
*(c/d+x)^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)+c^(1/4)*d*(c/d+x)*2^(1/2)*(c^(3/2)+
(4*a*d^2+c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(4*a*d^2+c^3)^(1/2)/(c^(3/2)+(
4*a*d^2+c^3)^(1/2))^(1/2)
```

3.37.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.13

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \frac{1}{4}\text{RootSum}\left[4ac+4c^2\#1^2+4cd\#1^3+d^2\#1^4\&,\frac{\log(x-\#1)}{2c^2\#1+3cd\#1^2+d^2\#1^3}\&\right]$$

input
```
Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]
```

output
```
RootSum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , Log[x - #1]/(2*c^2*
#1 + 3*c*d*#1^2 + d^2*#1^3) & ]/4
```

3.37.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2458, 1407, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{1407}$$

$$\frac{d\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-d\left(\frac{c}{d}+x\right)}{d\left(\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}\right)}d\left(\frac{c}{d}+x\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{d\int \frac{d\left(\frac{c}{d}+x\right)+\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d\left(\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}\right)}d\left(\frac{c}{d}+x\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-d\left(\frac{c}{d}+x\right)}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{\int \frac{d\left(\frac{c}{d}+x\right)+\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\Big\downarrow 1142$$

$$\frac{\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\int \frac{1}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}-\frac{1}{2}d\int -\frac{\sqrt{2}\left(\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}\right.}{d\left(\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+\right.}{d}\right.}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\frac{\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\int \frac{1}{\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}+\frac{1}{2}d\int \frac{\sqrt{2}\left(\sqrt{2}d\left(\frac{c}{d}+x\right)+\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3}}\right.}{d\left(\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}\right.}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\Big\downarrow 25$$

$$\frac{\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\int \frac{1}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}+\frac{1}{2}d\int \frac{\sqrt{2}\left(\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}d\right.}{d\left(\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}\right.}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\frac{\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\int \frac{1}{\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}+\frac{1}{2}d\int \frac{\sqrt{2}\left(\sqrt{2}d\left(\frac{c}{d}+x\right)+\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3}}\right.}{d\left(\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}\right.}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\Big\downarrow 27$$

3.37. $\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

$$\frac{\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\int \frac{1}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}+\frac{\int \frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}d\left(\frac{c}{d}+x\right)}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\sqrt{c}\sqrt{c}\cdots}\cdots}{\sqrt{2}}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\frac{\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\int \frac{1}{\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}+\frac{\int \frac{\sqrt{2}d\left(\frac{c}{d}+x\right)+\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\sqrt{c}\sqrt{c}\cdots}\cdots}{\sqrt{2}}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{\int \frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}d\left(\frac{c}{d}+x\right)}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}-\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\int \frac{1}{\frac{2\sqrt{c}\left(c^{3/2}-\sqrt{c^3+4ad^2}\right)}{d^2}-\left(2\left(\frac{c}{d}+x\right)-\cdots\right.}\cdots}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\frac{\frac{\int \frac{\sqrt{2}d\left(\frac{c}{d}+x\right)+\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}-\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\int \frac{1}{\frac{2\sqrt{c}\left(c^{3/2}-\sqrt{c^3+4ad^2}\right)}{d^2}-\left(2\left(\frac{c}{d}+x\right)+\cdots\right.}\cdots}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\int \frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}d\left(\frac{c}{d}+x\right)}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}-\frac{d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\,\text{arctanh}\left(\frac{d\left(2\left(\frac{c}{d}+x\right)-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}{d}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\frac{\frac{\int \frac{\sqrt{2}d\left(\frac{c}{d}+x\right)+\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}-\frac{d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\,\text{arctanh}\left(\frac{d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}{d}+2\left(\frac{c}{d}+x\right)\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\Big\downarrow \; 1103$$

3.37. $\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

$$-\frac{-\frac{d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\operatorname{arctanh}\left(\frac{d\left(2\left(\frac{c}{d}+x\right)-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}{d}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}-\frac{1}{2}d\log\left(\sqrt{c}\sqrt{4ad^2+c^3}-\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c\ldots\right.}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\frac{\frac{1}{2}d\log\left(\sqrt{c}\sqrt{4ad^2+c^3}+\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)-\frac{d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\operatorname{arctanh}\left(\frac{d\left(\frac{\sqrt{2}\sqrt[4]{c}\ldots}{}\right.}{\sqrt{2}\ldots}\right.}{\sqrt{c^{3/2}-\sqrt{4ad^2\ldots}}}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

input
```
Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]
```

output
```
(-((d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*ArcTanh[(d*(-((Sqrt[2]*c^(1/4)*S
qrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]])/d) + 2*(c/d + x)))/(Sqrt[2]*c^(1/4)*Sq
rt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])
- (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] - Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sq
rt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/2)/(2*Sqrt[2]*c^(3/4)*Sqr
t[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]) + (-((d*Sqrt[c^(3/2)
 + Sqrt[c^3 + 4*a*d^2]]*ArcTanh[(d*((Sqrt[2]*c^(1/4)*Sqrt[c^(3/2) + Sqrt[c
^3 + 4*a*d^2]])/d + 2*(c/d + x)))/(Sqrt[2]*c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3
 + 4*a*d^2]])])/Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) + (d*Log[Sqrt[c]*Sqrt
[c^3 + 4*a*d^2] + Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c
/d + x) + d^2*(c/d + x)^2])/2)/(2*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt
[c^(3/2) + Sqrt[c^3 + 4*a*d^2]])
```

3.37.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.37.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.12

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(d^2\_Z^4+4cd\_Z^3+4c^2\_Z^2+4ac\right)}\frac{\ln\left(x-\_R\right)}{\_R^3d^2+3\_R^2cd+2\_Rc^2}\right)}{4}$	64
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(d^2\_Z^4+4cd\_Z^3+4c^2\_Z^2+4ac\right)}\frac{\ln\left(x-\_R\right)}{\_R^3d^2+3\_R^2cd+2\_Rc^2}\right)}{4}$	64

input
```
int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/(_R^3*d^2+3*_R^2*c*d+2*_R*c^2)*ln(x-_R),_R=RootOf(_Z^4*d^2+4*_Z^
3*c*d+4*_Z^2*c^2+4*a*c))
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 905 vs. $2(408) = 816$.

Time = 0.29 (sec) , antiderivative size = 905, normalized size of antiderivative = 1.71

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$$

$$= \frac{1}{8}\sqrt{-\frac{2\left(ac^3+4\,a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}+1}{ac^3+4\,a^2d^2}}\log\left(d^2x+cd\right.$$
$$+\left(2\,acd^2+\left(ac^7+4\,a^2c^4d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}\right)\sqrt{-\frac{2\left(ac^3+4\,a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}}{ac^3+4\,a^2d^2}}$$
$$-\frac{1}{8}\sqrt{-\frac{2\left(ac^3+4\,a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}+1}{ac^3+4\,a^2d^2}}\log\left(d^2x+cd\right.$$
$$-\left(2\,acd^2+\left(ac^7+4\,a^2c^4d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}\right)\sqrt{-\frac{2\left(ac^3+4\,a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}}{ac^3+4\,a^2d^2}}$$
$$+\frac{1}{8}\sqrt{\frac{2\left(ac^3+4\,a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}-1}{ac^3+4\,a^2d^2}}\log\left(d^2x+cd\right.$$
$$+\left(2\,acd^2-\left(ac^7+4\,a^2c^4d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}\right)\sqrt{\frac{2\left(ac^3+4\,a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}-}{ac^3+4\,a^2d^2}}$$
$$-\frac{1}{8}\sqrt{\frac{2\left(ac^3+4\,a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}-1}{ac^3+4\,a^2d^2}}\log\left(d^2x+cd\right.$$
$$-\left(2\,acd^2-\left(ac^7+4\,a^2c^4d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}\right)\sqrt{\frac{2\left(ac^3+4\,a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8\,a^2c^6d^2+16\,a^3c^3d^4}}-}{ac^3+4\,a^2d^2}}$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="fricas")
```

3.37. $\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

output
```
1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3
*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d + (2*a*c*d^2 + (a*c^7
 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqr
t(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^
4)) + 1)/(a*c^3 + 4*a^2*d^2))) - 1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^
2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(
d^2*x + c*d - (2*a*c*d^2 + (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^
2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^
9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))) + 1/8*sqrt
((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)
) - 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d + (2*a*c*d^2 - (a*c^7 + 4*a^2*
c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt((2*(a*c
^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(
a*c^3 + 4*a^2*d^2))) - 1/8*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 +
8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d
 - (2*a*c*d^2 - (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 +
 16*a^3*c^3*d^4)))*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^
6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2)))
```

3.37.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.17

$$\int \frac{1}{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \, dx$$
$$= \operatorname{RootSum}\left(t^4 \cdot \left(16384a^3c^3d^2 + 4096a^2c^6\right) + 128t^2ac^3 + 1, \left(t \mapsto t\log\left(x + \frac{-1024t^3a^2c^4d^2 - 256t^3ac^7 + 1}{d^2}\right.\right.\right.$$

input
```
integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c),x)
```

output
```
RootSum(_t**4*(16384*a**3*c**3*d**2 + 4096*a**2*c**6) + 128*_t**2*a*c**3 +
 1, Lambda(_t, _t*log(x + (-1024*_t**3*a**2*c**4*d**2 - 256*_t**3*a*c**7 +
 16*_t*a*c*d**2 - 4*_t*c**4 + c*d)/d**2)))
```

3.37.7 Maxima [F]

$$\int \frac{1}{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}\,dx = \int \frac{1}{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}\,dx$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="maxima")
```

output
```
integrate(1/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.14

$$\int \frac{1}{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}\,dx =$$
$$-\frac{\log\left(x+\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}{4\left(d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^3-3\,cd\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^2+2\,c^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)\right)}$$
$$+\frac{\log\left(x-\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}{4\left(d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^3+3\,cd\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^2+2\,c^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)\right)}$$
$$-\frac{\log\left(x+\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}{4\left(d^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^3-3\,cd\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^2+2\,c^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)\right)}$$
$$+\frac{\log\left(x-\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}{4\left(d^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^3+3\,cd\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^2+2\,c^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)\right)}$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="giac")
```

output
```
-1/4*log(x + sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2
*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*
c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/
d)) + 1/4*log(x - sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt
((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 + 2*sqr
t(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4)
 - c/d)) - 1/4*log(x + sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*
(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 -
2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)
/d^4) + c/d)) + 1/4*log(x - sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/
(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d
^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)
*d^3)/d^4) - c/d))
```

3.37.9 Mupad [B] (verification not implemented)

Time = 12.19 (sec) , antiderivative size = 1551, normalized size of antiderivative = 2.93

$$\int \frac{1}{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}\,dx = \text{Too large to display}$$

input
```
int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3),x)
```

output

```
atan(((-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/
2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(6
4*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) - 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2)
+ a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)*1i + (
-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((2
56*a*c^4*d^5 + 256*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*
c^6 + 4*a^3*c^3*d^2)))^(1/2) + 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3
)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)*1i)/((-(2*d*(
-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^
4*d^5 + 256*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4
*a^3*c^3*d^2)))^(1/2) - 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(
a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x) - (-(2*d*(-a^3*c^3)^
(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 25
6*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d
^2)))^(1/2) + 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 +
4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)))*(-(2*d*(-a^3*c^3)^(1/2) + a*
c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*2i + atan((((2*d*(-a^3*c^3)^(1/
2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 256*a
*c^3*d^6*x)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2))
)^(1/2) - 64*a*c*d^6)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*...
```

3.37. $\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

3.38 $\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx$

3.38.1 Optimal result

Integrand size = 29, antiderivative size = 746

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx = -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\,(c^3+4ad^2)\,(4ac+4c^2x^2+4cdx^3+d^2x^4)}$$
$$-\frac{d\left(c^3+12ad^2+c^{3/2}\sqrt{c^3+4ad^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}c+\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}+\sqrt{2}dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{32\sqrt{2}ac^{7/4}\,(c^3+4ad^2)^{3/2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}$$
$$+\frac{d\left(c^3+12ad^2+c^{3/2}\sqrt{c^3+4ad^2}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}(c+dx)}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{32\sqrt{2}ac^{7/4}\,(c^3+4ad^2)^{3/2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}$$
$$-\frac{d\left(c^3+12ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\log\left(\sqrt{c}\sqrt{c^3+4ad^2}-\sqrt{2}\sqrt[4]{c}d\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right.\right.}{64\sqrt{2}ac^{7/4}\,(c^3+4ad^2)^{3/2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$
$$+\frac{d\left(c^3+12ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\log\left(\sqrt{c}\sqrt{c^3+4ad^2}+\sqrt{2}\sqrt[4]{c}d\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right.\right.}{64\sqrt{2}ac^{7/4}\,(c^3+4ad^2)^{3/2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$

output
```
-1/16*(c/d+x)*(c^3-4*a*d^2-c*d^2*(c/d+x)^2)/a/c/(4*a*d^2+c^3)/(d^2*x^4+4*c
*d*x^3+4*c^2*x^2+4*a*c)-1/64*d*arctanh((c*2^(1/2)+d*x*2^(1/2)+c^(1/4)*(c^(
3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(1/4)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/
2))*(c^3+12*a*d^2+c^(3/2)*(4*a*d^2+c^3)^(1/2))/a/c^(7/4)/(4*a*d^2+c^3)^(3/
2)*2^(1/2)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2)+1/64*d*arctanh((-(d*x+c)*2^
(1/2)+c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(1/4)/(c^(3/2)-(4*a*d
^2+c^3)^(1/2))^(1/2))*(c^3+12*a*d^2+c^(3/2)*(4*a*d^2+c^3)^(1/2))/a/c^(7/4)
/(4*a*d^2+c^3)^(3/2)*2^(1/2)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2)-1/128*d*l
n(d^2*(c/d+x)^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)-c^(1/4)*d*(c/d+x)*2^(1/2)*(c^(
3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))*(c^3+12*a*d^2-c^(3/2)*(4*a*d^2+c^3)^(1/2)
)/a/c^(7/4)/(4*a*d^2+c^3)^(3/2)*2^(1/2)/(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2
)+1/128*d*ln(d^2*(c/d+x)^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)+c^(1/4)*d*(c/d+x)*2
^(1/2)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))*(c^3+12*a*d^2-c^(3/2)*(4*a*d^2
+c^3)^(1/2))/a/c^(7/4)/(4*a*d^2+c^3)^(3/2)*2^(1/2)/(c^(3/2)+(4*a*d^2+c^3)^
(1/2))^(1/2)
```

3.38.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.24

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx$$
$$= \frac{\frac{4(c+dx)(4ad+cx(2c+dx))}{4ac+x^2(2c+dx)^2} + \text{RootSum}\left[4ac+4c^2\#1^2+4cd\#1^3+d^2\#1^4\&, \frac{2c^3\log(x-\#1)+12ad^2\log(x-\#1)+2c^2d\log(x}{2c^2\#1+3cd\#1^2+c}\right.}{64ac\left(c^3+4ad^2\right)}$$

input
```
Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2),x]
```

output
```
((4*(c + d*x)*(4*a*d + c*x*(2*c + d*x)))/(4*a*c + x^2*(2*c + d*x)^2) + Roo
tSum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , (2*c^3*Log[x - #1] + 1
2*a*d^2*Log[x - #1] + 2*c^2*d*Log[x - #1]*#1 + c*d^2*Log[x - #1]*#1^2)/(2*
c^2*#1 + 3*c*d*#1^2 + d^2*#1^3) & ])/(64*a*c*(c^3 + 4*a*d^2))
```

3.38.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 797, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2458, 1405, 27, 1483, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}dx$$

$$\Big\downarrow \text{ 2458}$$

$$\int \frac{1}{\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^2}d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{\int \frac{2c\left(c^3+d^2\left(\frac{c}{d}+x\right)^2c+12ad^2\right)}{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}d\left(\frac{c}{d}+x\right)}{32ac^2(4ad^2+c^3)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac(4ad^2+c^3)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{c^3+d^2\left(\frac{c}{d}+x\right)^2c+12ad^2}{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}d\left(\frac{c}{d}+x\right)}{16ac(4ad^2+c^3)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac(4ad^2+c^3)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)}$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{\frac{d\int \frac{\sqrt{2}\sqrt[4]{c}(c^3+12ad^2)\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-d\left(c^3-\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\left(\frac{c}{d}+x\right)}{d\left(\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}\right)}d\left(\frac{c}{d}+x\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{d\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}(c^3+12ad^2)+d\left(c^3-\sqrt{c^3+4ad^2}\cdots\right.}{d\left(\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\cdots\right.}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+\cdots}}}{16ac(4ad^2+c^3)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac(4ad^2+c^3)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\left(c^3+12ad^2\right)\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-d\left(c^3-\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\left(\frac{c}{d}+x\right)}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(c^3+12ad^2\right)+d\left(c^3-\sqrt{c^3+4ad^2}c^{3/\cdots}\right.}{\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\sqrt{c}\sqrt{c\cdots}}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3\cdots}}}}{16ac\left(4ad^2+c^3\right)}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(4ad^2+c^3\right)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)}$$

$$\Big\downarrow \ 1142$$

$$\frac{\frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(c^3+\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\int \frac{1}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}-\frac{1}{2}d\left(c^3-\sqrt{c^3+4ad^2}c^{3/2}+12a\cdots\right.}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}\cdots$$

$$\frac{\left(\frac{c}{d}+x\right)\left(c^3-d^2\left(\frac{c}{d}+x\right)^2c-4ad^2\right)}{16ac\left(c^3+4ad^2\right)\left(d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(c^3+\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\int \frac{1}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}+\frac{1}{2}d\left(c^3-\sqrt{c^3+4ad^2}c^{3/2}+12a\cdots\right.}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}\cdots$$

$$\frac{\left(\frac{c}{d}+x\right)\left(c^3-d^2\left(\frac{c}{d}+x\right)^2c-4ad^2\right)}{16ac\left(c^3+4ad^2\right)\left(d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(c^3+\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\int \frac{1}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}+\frac{\left(c^3-\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\int\cdots}{\cdots}}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}\cdots$$

$$\frac{\left(\frac{c}{d}+x\right)\left(c^3-d^2\left(\frac{c}{d}+x\right)^2c-4ad^2\right)}{16ac\left(c^3+4ad^2\right)\left(d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)\right)}$$

$$\Big\downarrow \ 1083$$

3.38. $\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2}\,dx$

$$\frac{\frac{\left(c^3-\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\int \frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}d\left(\frac{c}{d}+x\right)}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}-\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(c^3+\sqrt{c^3+4ad^2}c^{3/2}+1\right.}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4a}}}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(c^3-d^2\left(\frac{c}{d}+x\right)^2c-4ad^2\right)}{16ac\left(c^3+4ad^2\right)\left(d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)\right)}$$

$\downarrow$ 219

$$\frac{\frac{\left(c^3-\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\int \frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}d\left(\frac{c}{d}+x\right)}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}-\frac{d\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(c^3+\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\text{arc}}{\sqrt{c^{3/2}-\sqrt{}}}}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(c^3-d^2\left(\frac{c}{d}+x\right)^2c-4ad^2\right)}{16ac\left(c^3+4ad^2\right)\left(d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)\right)}$$

$\downarrow$ 1103

$$\frac{-\frac{d\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(c^3+\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\text{arctanh}\left(\frac{d\left(2\left(\frac{c}{d}+x\right)-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}-\frac{1}{2}d\left(c^3-\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\log\left(d^2\left(\frac{c}{d}\right.\right.}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(c^3-d^2\left(\frac{c}{d}+x\right)^2c-4ad^2\right)}{16ac\left(c^3+4ad^2\right)\left(d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)\right)}$$

input
```
Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2),x]
```

3.38. $\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx$

output
```
-1/16*((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(a*c*(c^3 + 4*a*d^2)
*(c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4)) + ((-((d*Sqrt[
c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c^3 + 12*a*d^2 + c^(3/2)*Sqrt[c^3 + 4*a*d^
2])*ArcTanh[(d*(-((Sqrt[2]*c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]])/d)
 + 2*(c/d + x)))/(Sqrt[2]*c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/S
qrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*(c^3 + 12*a*d^2 - c^(3/2)*Sqrt[c^
3 + 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] - Sqrt[2]*c^(1/4)*d*Sqrt[c^(
3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/2)/(2*Sqrt[2]*c^
(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]) + (-((d*Sqr
t[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c^3 + 12*a*d^2 + c^(3/2)*Sqrt[c^3 + 4*a*
d^2])*ArcTanh[(d*((Sqrt[2]*c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]])/d
+ 2*(c/d + x)))/(Sqrt[2]*c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/Sq
rt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) + (d*(c^3 + 12*a*d^2 - c^(3/2)*Sqrt[c^3
 + 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt[2]*c^(1/4)*d*Sqrt[c^(3
/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/2)/(2*Sqrt[2]*c^(
3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]))/(16*a*c*(c^
3 + 4*a*d^2))
```

3.38.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.38.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.31

method	result
default	$\frac{\frac{d^{2} x^{3}}{64 a \left(4 a \,d^{2}+c^{3}\right)}+\frac{3 d c \,x^{2}}{64 a \left(4 a \,d^{2}+c^{3}\right)}+\frac{\left(2 a \,d^{2}+c^{3}\right) x}{32 c \left(4 a \,d^{2}+c^{3}\right) a}+\frac{d}{64 a \,d^{2}+16 c^{3}}}{\frac{1}{4} d^{2} x^{4}+c d \,x^{3}+c^{2} x^{2}+a c}+\frac{\sum_{\_R =\operatorname{RootOf}\left(d^{2} \_Z^{4}+4 c d \_Z^{3}+4 c^{2} \_Z^{2}+4 a c \right)} \frac{\left(\_R^{2} c \,d^{2}+2 \_R \,c^{2} \cdots\right.}{\_R^{3} d^{2}+3 \cdots}}{64 a c \left(4 a \,d^{2}+c^{3}\right)}$
risch	$\frac{\frac{d^{2} x^{3}}{64 a \left(4 a \,d^{2}+c^{3}\right)}+\frac{3 d c \,x^{2}}{64 a \left(4 a \,d^{2}+c^{3}\right)}+\frac{\left(2 a \,d^{2}+c^{3}\right) x}{32 c \left(4 a \,d^{2}+c^{3}\right) a}+\frac{d}{64 a \,d^{2}+16 c^{3}}}{\frac{1}{4} d^{2} x^{4}+c d \,x^{3}+c^{2} x^{2}+a c}+\frac{\sum_{\_R =\operatorname{RootOf}\left(d^{2} \_Z^{4}+4 c d \_Z^{3}+4 c^{2} \_Z^{2}+4 a c \right)} \frac{\left(\frac{d^{2} \_R^{2}}{4 a \,d^{2}+c^{3}}+\frac{2 c d \_R \cdots}{4 a \,d^{2}+c^{3}}\cdots\right.}{\_R^{3} d^{2}+3 \cdots}}{64 a}$

input
```
int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/64*d^2/a/(4*a*d^2+c^3)*x^3+3/64*d/a*c/(4*a*d^2+c^3)*x^2+1/32/c*(2*a*d^2
+c^3)/(4*a*d^2+c^3)/a*x+1/16*d/(4*a*d^2+c^3))/(1/4*d^2*x^4+c*d*x^3+c^2*x^2
+a*c)+1/64/a/c/(4*a*d^2+c^3)*sum((_R^2*c*d^2+2*_R*c^2*d+12*a*d^2+2*c^3)/(_
R^3*d^2+3*_R^2*c*d+2*_R*c^2)*ln(x-_R),_R=RootOf(_Z^4*d^2+4*_Z^3*c*d+4*_Z^2
*c^2+4*a*c))
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3222 vs. $2(608) = 1216$.

Time = 0.37 (sec) , antiderivative size = 3222, normalized size of antiderivative = 4.32

$$\int \frac{1}{(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")
```

output
```
1/64*(4*c*d^2*x^3 + 12*c^2*d*x^2 + 16*a*c*d + (4*a^2*c^5 + 16*a^3*c^2*d^2
+ (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c
^6 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c
^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6
+ 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^1
9*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*
a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d
^6))*log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*
d^6 + 324*a^2*d^8)*x + (5*a^2*c^8*d^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 +
 (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a
^7*c^7*d^8)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 +
 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^
8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 +
60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^
6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c
^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*
a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c
^5*d^4 + 64*a^6*c^2*d^6))) - (4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*
a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^
2)*x^2)*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c...
```

3.38.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

output
```
Timed out
```

3.38.7 Maxima [F]

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx = \int \frac{1}{(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac)^2}\,dx$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")
```

output
```
1/16*(c*d^2*x^3 + 3*c^2*d*x^2 + 4*a*c*d + 2*(c^3 + 2*a*d^2)*x)/(4*a^2*c^5
+ 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*
d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2) + 1/16*integrate((c*d^2*x^2 + 2*
c^2*d*x + 2*c^3 + 12*a*d^2)/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)/
(a*c^4 + 4*a^2*c*d^2)
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 1057, normalized size of antiderivative = 1.42

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx =$$

$$-\frac{\frac{\left(cd^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-acd^3}}{d^4}}+\frac{c}{d}\right)^2-2\,c^2d\left(\sqrt{\frac{c^2d^2+2\sqrt{-acd^3}}{d^4}}+\frac{c}{d}\right)+2\,c^3+12\,ad^2\right)\log\left(x+\sqrt{\frac{c^2d^2+2\sqrt{-acd^3}}{d^4}}+\frac{c}{d}\right)}{d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-acd^3}}{d^4}}+\frac{c}{d}\right)^3-3\,cd\left(\sqrt{\frac{c^2d^2+2\sqrt{-acd^3}}{d^4}}+\frac{c}{d}\right)^2+2\,c^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-acd^3}}{d^4}}+\frac{c}{d}\right)}-\frac{\left(cd^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-acd^3}}{d^4}}-\cdots\right.\right.}{d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-\cdots}}{d^4}}\right.}}{}$$

$$+\frac{cd^2x^3+3\,c^2dx^2+2\,c^3x+4\,ad^2x+4\,acd}{16\,(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac)(ac^4+4\,a^2cd^2)}$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="giac")
```

output
```
-1/64*((c*d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 - 2*c^2*d*(
sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d) + 2*c^3 + 12*a*d^2)*log(x +
sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt
(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4)
+ c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)) - (c*d^2*
(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*d*(sqrt((c^2*d^2
+ 2*sqrt(-a*c)*d^3)/d^4) - c/d) + 2*c^3 + 12*a*d^2)*log(x - sqrt((c^2*d^2
+ 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^
4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c
^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)) + (c*d^2*(sqrt((c^2*d^2
 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 - 2*c^2*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)
*d^3)/d^4) + c/d) + 2*c^3 + 12*a*d^2)*log(x + sqrt((c^2*d^2 - 2*sqrt(-a*c)
*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 -
3*c*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*
d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)) - (c*d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c
)*d^3)/d^4) - c/d)^2 + 2*c^2*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c
/d) + 2*c^3 + 12*a*d^2)*log(x - sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c
/d)/(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c
^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-
a*c)*d^3)/d^4) - c/d)))/(a*c^4 + 4*a^2*c*d^2) + 1/16*(c*d^2*x^3 + 3*c^2...
```

3.38.9 Mupad [B] (verification not implemented)

Time = 12.09 (sec) , antiderivative size = 5844, normalized size of antiderivative = 7.83

$$\int \frac{1}{(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^2,x)
```

output
```
(d/(4*(4*a*d^2 + c^3)) + (d^2*x^3)/(16*a*(4*a*d^2 + c^3)) + (x*(2*a*d^2 +
c^3))/(8*a*c*(4*a*d^2 + c^3)) + (3*c*d*x^2)/(16*a*(4*a*d^2 + c^3)))/(4*a*c
 + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3) - atan(((-(a^3*c^11 + 10*c^3*d^3*(-a^9
*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))
/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(
1/2)*((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/
(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6
+ 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 +
16*a^4*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d
^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*
c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*a^3*c^8*d^6 +
 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 +
 16*a^5*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*
d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7
*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 23
04*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^
2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*
c^5*d^2 + 16*a^4*c^2*d^4)))*1i + (-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2)
 + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6
*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*(((...
```

3.38. $\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2} \, dx$

3.39 $\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^4 \, dx$

3.39.1 Optimal result

Integrand size = 32, antiderivative size = 295

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^4 \, dx = \frac{(5d^4+256ae^3)^4 x}{1048576e^4} - \frac{d^2(5d^4+256ae^3)^3\left(\frac{d}{4e}+x\right)^3}{8192e^2} + \frac{(5d^4+256ae^3)^2(59d^4+256ae^3)\left(\frac{d}{4e}+x\right)^5}{5120} - \frac{9}{224}d^2e^2(5d^4+256ae^3)(17d^4+256ae^3)\left(\frac{d}{4e}+x\right)^7 + \frac{1}{24}e^4(601d^8+20992ad^4e^3+65536a^2e^6)\left(\frac{d}{4e}+x\right)^9 - \frac{72}{11}d^2e^6(17d^4+256ae^3)\left(\frac{d}{4e}+x\right)^{11} + \frac{64}{13}e^8(59d^4+256ae^3)\left(\frac{d}{4e}+x\right)^{13} - \frac{2048}{5}d^2e^{10}\left(\frac{d}{4e}+x\right)^{15} + \frac{4096}{17}e^{12}\left(\frac{d}{4e}+x\right)^{17}$$

output
```
1/1048576*(256*a*e^3+5*d^4)^4*x/e^4-1/8192*d^2*(256*a*e^3+5*d^4)^3*(1/4*d/
e+x)^3/e^2+1/5120*(256*a*e^3+5*d^4)^2*(256*a*e^3+59*d^4)*(1/4*d/e+x)^5-9/2
24*d^2*e^2*(256*a*e^3+5*d^4)*(256*a*e^3+17*d^4)*(1/4*d/e+x)^7+1/24*e^4*(65
536*a^2*e^6+20992*a*d^4*e^3+601*d^8)*(1/4*d/e+x)^9-72/11*d^2*e^6*(256*a*e^
3+17*d^4)*(1/4*d/e+x)^11+64/13*e^8*(256*a*e^3+59*d^4)*(1/4*d/e+x)^13-2048/
5*d^2*e^10*(1/4*d/e+x)^15+4096/17*e^12*(1/4*d/e+x)^17
```

3.39.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.17

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4 dx = 4096a^4e^8x-1024a^3d^3e^6x^2+128a^2d^6e^4x^3 + 8ade^2\left(-d^8+512a^2e^6\right)x^4 + \frac{1}{5}\left(d^{12}-6144a^2d^4e^6+16384a^3e^9\right)x^5 - 128ad^3e^4\left(-d^4+8ae^3\right)x^6 - \frac{32}{7}d^2e^2\left(d^8-24ad^4e^3-768a^2e^6\right)x^7 - 4de^3\left(d^8+192ad^4e^3-1536a^2e^6\right)x^8 + \frac{128}{3}e^4\left(d^8-32ad^4e^3+64a^2e^6\right)x^9 + \frac{128}{5}d^3e^5\left(3d^4+40ae^3\right)x^{10} + \frac{128}{11}d^2e^6\left(-13d^4+384ae^3\right)x^{11} - 512de^7\left(d^4-8ae^3\right)x^{12} + \frac{2048}{13}e^8\left(-d^4+8ae^3\right)x^{13} + 1024d^3e^9x^{14} + \frac{8192}{5}d^2e^{10}x^{15} + 1024de^{11}x^{16} + \frac{4096e^{12}x^{17}}{17}$$

input
```
Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]
```

output
```
4096*a^4*e^8*x - 1024*a^3*d^3*e^6*x^2 + 128*a^2*d^6*e^4*x^3 + 8*a*d*e^2*(-
d^8 + 512*a^2*e^6)*x^4 + ((d^12 - 6144*a^2*d^4*e^6 + 16384*a^3*e^9)*x^5)/5
 - 128*a*d^3*e^4*(-d^4 + 8*a*e^3)*x^6 - (32*d^2*e^2*(d^8 - 24*a*d^4*e^3 -
768*a^2*e^6)*x^7)/7 - 4*d*e^3*(d^8 + 192*a*d^4*e^3 - 1536*a^2*e^6)*x^8 + (
128*e^4*(d^8 - 32*a*d^4*e^3 + 64*a^2*e^6)*x^9)/3 + (128*d^3*e^5*(3*d^4 + 4
0*a*e^3)*x^10)/5 + (128*d^2*e^6*(-13*d^4 + 384*a*e^3)*x^11)/11 - 512*d*e^7
*(d^4 - 8*a*e^3)*x^12 + (2048*e^8*(-d^4 + 8*a*e^3)*x^13)/13 + 1024*d^3*e^9
*x^14 + (8192*d^2*e^10*x^15)/5 + 1024*d*e^11*x^16 + (4096*e^12*x^17)/17
```

3.39.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4 \, dx$$

$$\Big\downarrow \text{ 2458}$$

$$\int \left(\frac{1}{32}\left(256ae^2+\frac{5d^4}{e}\right)-3d^2e\left(\frac{d}{4e}+x\right)^2+8e^3\left(\frac{d}{4e}+x\right)^4\right)^4 d\left(\frac{d}{4e}+x\right)$$

$$\Big\downarrow \text{ 1403}$$

$$\int \left(\frac{27}{512}d^4\left(256ae^3+5d^4\right)^2\left(\frac{1}{54}\left(\frac{256ae^3}{d^4}+5\right)+1\right)\left(\frac{d}{4e}+x\right)^4+3456d^4e^8\left(\frac{1}{54}\left(\frac{256ae^3}{d^4}+5\right)+1\right)\left(\frac{d}{4e}+x\right)^1$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{24}e^4\left(65536a^2e^6+20992ad^4e^3+601d^8\right)\left(\frac{d}{4e}+x\right)^9+\frac{\left(256ae^3+5d^4\right)^2\left(256ae^3+59d^4\right)\left(\frac{d}{4e}+x\right)^5}{5120}+\frac{64}{13}e^8\left(256ae^3+59d^4\right)\left(\frac{d}{4e}+x\right)^{13}+\frac{\left(256ae^3+5d^4\right)^4\left(\frac{d}{4e}+x\right)}{1048576e^4}-\frac{72}{11}d^2e^6\left(256ae^3+17d^4\right)\left(\frac{d}{4e}+x\right)^{11}-\frac{9}{224}d^2e^2\left(256ae^3+5d^4\right)\left(256ae^3+17d^4\right)\left(\frac{d}{4e}+x\right)^7-\frac{d^2\left(256ae^3+5d^4\right)^3\left(\frac{d}{4e}+x\right)^3}{8192e^2}-\frac{2048}{5}d^2e^{10}\left(\frac{d}{4e}+x\right)^{15}+\frac{4096}{17}e^{12}\left(\frac{d}{4e}+x\right)^{17}$$

input
```
Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]
```

output
```
((5*d^4 + 256*a*e^3)^4*(d/(4*e) + x))/(1048576*e^4) - (d^2*(5*d^4 + 256*a*
e^3)^3*(d/(4*e) + x)^3)/(8192*e^2) + ((5*d^4 + 256*a*e^3)^2*(59*d^4 + 256*
a*e^3)*(d/(4*e) + x)^5)/5120 - (9*d^2*e^2*(5*d^4 + 256*a*e^3)*(17*d^4 + 25
6*a*e^3)*(d/(4*e) + x)^7)/224 + (e^4*(601*d^8 + 20992*a*d^4*e^3 + 65536*a^
2*e^6)*(d/(4*e) + x)^9)/24 - (72*d^2*e^6*(17*d^4 + 256*a*e^3)*(d/(4*e) + x
)^11)/11 + (64*e^8*(59*d^4 + 256*a*e^3)*(d/(4*e) + x)^13)/13 - (2048*d^2*e
^10*(d/(4*e) + x)^15)/5 + (4096*e^12*(d/(4*e) + x)^17)/17
```

3.39.3.1 Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.39.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.13

method	result
norman	$4096a^4e^8x-1024a^3e^6d^3x^2+128a^2e^4d^6x^3+\left(4096a^3e^8d-8a\,d^9e^2\right)x^4+\left(\frac{16384}{5}a^3e^9-\frac{6144}{5}a^2\right.$
gosper	$-768a\,d^5e^6x^8+4096ad\,e^{10}x^{12}-1024a^2d^3e^7x^6+128a\,d^7e^4x^6+6144a^2d\,e^9x^8-\frac{4096}{3}x^9a\,d^4e$
risch	$-768a\,d^5e^6x^8+4096ad\,e^{10}x^{12}-1024a^2d^3e^7x^6+128a\,d^7e^4x^6+6144a^2d\,e^9x^8-\frac{4096}{3}x^9a\,d^4e$
parallelrisch	$-768a\,d^5e^6x^8+4096ad\,e^{10}x^{12}-1024a^2d^3e^7x^6+128a\,d^7e^4x^6+6144a^2d\,e^9x^8-\frac{4096}{3}x^9a\,d^4e$
default	$\frac{4096e^{12}x^{17}}{17}+1024d\,e^{11}x^{16}+\frac{8192d^2e^{10}x^{15}}{5}+1024d^3e^9x^{14}+\frac{128\left(128a\,e^5-16d^4e^2\right)e^6x^{13}}{13}+\frac{\left(16384ad\,e^{10}+25\right.}{}$

input
```
int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x,method=_RETURNVERBOSE)
```

output
```
4096*a^4*e^8*x-1024*a^3*e^6*d^3*x^2+128*a^2*e^4*d^6*x^3+(4096*a^3*d*e^8-8*
a*d^9*e^2)*x^4+(16384/5*a^3*e^9-6144/5*a^2*d^4*e^6+1/5*d^12)*x^5+(-1024*a^
2*d^3*e^7+128*a*d^7*e^4)*x^6+(24576/7*a^2*d^2*e^8+768/7*a*d^6*e^5-32/7*d^1
0*e^2)*x^7+(6144*a^2*d*e^9-768*a*d^5*e^6-4*d^9*e^3)*x^8+(8192/3*a^2*e^10-4
096/3*a*d^4*e^7+128/3*d^8*e^4)*x^9+(1024*a*d^3*e^8+384/5*d^7*e^5)*x^10+(49
152/11*a*d^2*e^9-1664/11*d^6*e^6)*x^11+(4096*a*d*e^10-512*d^5*e^7)*x^12+(1
6384/13*a*e^11-2048/13*d^4*e^8)*x^13+1024*d^3*e^9*x^14+8192/5*d^2*e^10*x^1
5+1024*d*e^11*x^16+4096/17*e^12*x^17
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.13

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4\,dx = \frac{4096}{17}\,e^{12}x^{17}+1024\,de^{11}x^{16}+\frac{8192}{5}\,d^2e^{10}x^{15} + 1024\,d^3e^9x^{14}+128\,a^2d^6e^4x^3 - 1024\,a^3d^3e^6x^2-\frac{2048}{13}\,\left(d^4e^8-8\,ae^{11}\right)x^{13} + 4096\,a^4e^8x-512\,\left(d^5e^7-8\,ade^{10}\right)x^{12} - \frac{128}{11}\,\left(13\,d^6e^6-384\,ad^2e^9\right)x^{11} + \frac{128}{5}\,\left(3\,d^7e^5+40\,ad^3e^8\right)x^{10} + \frac{128}{3}\,\left(d^8e^4-32\,ad^4e^7+64\,a^2e^{10}\right)x^9 - 4\,\left(d^9e^3+192\,ad^5e^6-1536\,a^2de^9\right)x^8 - \frac{32}{7}\,\left(d^{10}e^2-24\,ad^6e^5-768\,a^2d^2e^8\right)x^7 + 128\,\left(ad^7e^4-8\,a^2d^3e^7\right)x^6 + \frac{1}{5}\,\left(d^{12}-6144\,a^2d^4e^6+16384\,a^3e^9\right)x^5 - 8\,\left(ad^9e^2-512\,a^3de^8\right)x^4$$

input

```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="fricas")
```

output

```
4096/17*e^12*x^17 + 1024*d*e^11*x^16 + 8192/5*d^2*e^10*x^15 + 1024*d^3*e^9
*x^14 + 128*a^2*d^6*e^4*x^3 - 1024*a^3*d^3*e^6*x^2 - 2048/13*(d^4*e^8 - 8*
a*e^11)*x^13 + 4096*a^4*e^8*x - 512*(d^5*e^7 - 8*a*d*e^10)*x^12 - 128/11*(
13*d^6*e^6 - 384*a*d^2*e^9)*x^11 + 128/5*(3*d^7*e^5 + 40*a*d^3*e^8)*x^10 +
 128/3*(d^8*e^4 - 32*a*d^4*e^7 + 64*a^2*e^10)*x^9 - 4*(d^9*e^3 + 192*a*d^5
*e^6 - 1536*a^2*d*e^9)*x^8 - 32/7*(d^10*e^2 - 24*a*d^6*e^5 - 768*a^2*d^2*e
^8)*x^7 + 128*(a*d^7*e^4 - 8*a^2*d^3*e^7)*x^6 + 1/5*(d^12 - 6144*a^2*d^4*e
^6 + 16384*a^3*e^9)*x^5 - 8*(a*d^9*e^2 - 512*a^3*d*e^8)*x^4
```

3.39.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.24

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4\,dx = 4096a^4e^8x - 1024a^3d^3e^6x^2 + 128a^2d^6e^4x^3 + 1024d^3e^9x^{14} + \frac{8192d^2e^{10}x^{15}}{5} + 1024de^{11}x^{16} + \frac{4096e^{12}x^{17}}{17} + x^{13}\cdot\left(\frac{16384ae^{11}}{13} - \frac{2048d^4e^8}{13}\right) + x^{12}\cdot\left(4096ade^{10} - 512d^5e^7\right) + x^{11}\cdot\left(\frac{49152ad^2e^9}{11} - \frac{1664d^6e^6}{11}\right) + x^{10}\cdot\left(1024ad^3e^8 + \frac{384d^7e^5}{5}\right) + x^9\cdot\left(\frac{8192a^2e^{10}}{3} - \frac{4096ad^4e^7}{3} + \frac{128d^8e^4}{3}\right) + x^8\cdot\left(6144a^2de^9 - 768ad^5e^6 - 4d^9e^3\right) + x^7\cdot\left(\frac{24576a^2d^2e^8}{7} + \frac{768ad^6e^5}{7} - \frac{32d^{10}e^2}{7}\right) + x^6\left(-1024a^2d^3e^7 + 128ad^7e^4\right) + x^5\cdot\left(\frac{16384a^3e^9}{5} - \frac{6144a^2d^4e^6}{5} + \frac{d^{12}}{5}\right) + x^4\cdot\left(4096a^3de^8 - 8ad^9e^2\right)$$

input

```
integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**4,x)
```

output

```
4096*a**4*e**8*x - 1024*a**3*d**3*e**6*x**2 + 128*a**2*d**6*e**4*x**3 + 10
24*d**3*e**9*x**14 + 8192*d**2*e**10*x**15/5 + 1024*d*e**11*x**16 + 4096*e
**12*x**17/17 + x**13*(16384*a*e**11/13 - 2048*d**4*e**8/13) + x**12*(4096
*a*d*e**10 - 512*d**5*e**7) + x**11*(49152*a*d**2*e**9/11 - 1664*d**6*e**6
/11) + x**10*(1024*a*d**3*e**8 + 384*d**7*e**5/5) + x**9*(8192*a**2*e**10/
3 - 4096*a*d**4*e**7/3 + 128*d**8*e**4/3) + x**8*(6144*a**2*d*e**9 - 768*a
*d**5*e**6 - 4*d**9*e**3) + x**7*(24576*a**2*d**2*e**8/7 + 768*a*d**6*e**5
/7 - 32*d**10*e**2/7) + x**6*(-1024*a**2*d**3*e**7 + 128*a*d**7*e**4) + x*
*5*(16384*a**3*e**9/5 - 6144*a**2*d**4*e**6/5 + d**12/5) + x**4*(4096*a**3
*d*e**8 - 8*a*d**9*e**2)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.30

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4\,dx = \frac{4096}{17}\,e^{12}x^{17}+1024\,de^{11}x^{16}+\frac{8192}{5}\,d^2e^{10}x^{15} + \frac{8192}{7}\,d^3e^9x^{14}+\frac{4096}{13}\,d^4e^8x^{13}+\frac{1}{5}\,d^{12}x^5+4096\,a^4e^8x-\frac{4}{7}\left(7\,e^3x^8+8\,de^2x^7\right)d^9 + \frac{1024}{5}\left(16\,e^3x^5+20\,de^2x^4-5\,d^3x^2\right)a^3e^6+\frac{128}{165}\left(45\,e^6x^{11}+99\,de^5x^{10}+55\,d^2e^4x^9\right)d^6 + \frac{128}{105}\left(2240\,e^6x^9+5040\,de^5x^8+2880\,d^2e^4x^7+105\,d^6x^3-168\left(5\,e^3x^6+6\,de^2x^5\right)d^3\right)a^2e^4 - \frac{512}{1001}\left(286\,e^9x^{14}+924\,de^8x^{13}+1001\,d^2e^7x^{12}+364\,d^3e^6x^{11}\right)d^3 + \frac{8}{15015}\left(2365440\,e^9x^{13}+7687680\,de^8x^{12}+8386560\,d^2e^7x^{11}+3075072\,d^3e^6x^{10}-15015\,d^9x^4+34320\,(6\right.$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="maxima")
```

output
```
4096/17*e^12*x^17 + 1024*d*e^11*x^16 + 8192/5*d^2*e^10*x^15 + 8192/7*d^3*e
^9*x^14 + 4096/13*d^4*e^8*x^13 + 1/5*d^12*x^5 + 4096*a^4*e^8*x - 4/7*(7*e^
3*x^8 + 8*d*e^2*x^7)*d^9 + 1024/5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*
a^3*e^6 + 128/165*(45*e^6*x^11 + 99*d*e^5*x^10 + 55*d^2*e^4*x^9)*d^6 + 128
/105*(2240*e^6*x^9 + 5040*d*e^5*x^8 + 2880*d^2*e^4*x^7 + 105*d^6*x^3 - 168
*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3)*a^2*e^4 - 512/1001*(286*e^9*x^14 + 924*d*e
^8*x^13 + 1001*d^2*e^7*x^12 + 364*d^3*e^6*x^11)*d^3 + 8/15015*(2365440*e^9
*x^13 + 7687680*d*e^8*x^12 + 8386560*d^2*e^7*x^11 + 3075072*d^3*e^6*x^10 -
 15015*d^9*x^4 + 34320*(6*e^3*x^7 + 7*d*e^2*x^6)*d^6 - 32032*(36*e^6*x^10
+ 80*d*e^5*x^9 + 45*d^2*e^4*x^8)*d^3)*a*e^2
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.20

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4\,dx = \frac{4096}{17}\,e^{12}x^{17}+1024\,de^{11}x^{16}+\frac{8192}{5}\,d^2e^{10}x^{15}+1024\,d^3e^9x^{14}-\frac{2048}{13}\,d^4e^8x^{13}+\frac{16384}{13}\,ae^{11}x^{13}-512\,d^5e^7x^{12}+4096\,ade^{10}x^{12}-\frac{1664}{11}\,d^6e^6x^{11}+\frac{49152}{11}\,ad^2e^9x^{11}+\frac{384}{5}\,d^7e^5x^{10}+1024\,ad^3e^8x^{10}+\frac{128}{3}\,d^8e^4x^9-\frac{4096}{3}\,ad^4e^7x^9+\frac{8192}{3}\,a^2e^{10}x^9-4\,d^9e^3x^8-768\,ad^5e^6x^8+6144\,a^2de^9x^8-\frac{32}{7}\,d^{10}e^2x^7+\frac{768}{7}\,ad^6e^5x^7+\frac{24576}{7}\,a^2d^2e^8x^7+128\,ad^7e^4x^6-1024\,a^2d^3e^7x^6+\frac{1}{5}\,d^{12}x^5-\frac{6144}{5}\,a^2d^4e^6x^5+\frac{16384}{5}\,a^3e^9x^5-8\,ad^9e^2x^4+4096\,a^3de^8x^4+128\,a^2d^6e^4x^3-1024\,a^3d^3e^6x^2+4096\,a^4e^8x$$

input

```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="giac")
```

output

```
4096/17*e^12*x^17 + 1024*d*e^11*x^16 + 8192/5*d^2*e^10*x^15 + 1024*d^3*e^9
*x^14 - 2048/13*d^4*e^8*x^13 + 16384/13*a*e^11*x^13 - 512*d^5*e^7*x^12 + 4
096*a*d*e^10*x^12 - 1664/11*d^6*e^6*x^11 + 49152/11*a*d^2*e^9*x^11 + 384/5
*d^7*e^5*x^10 + 1024*a*d^3*e^8*x^10 + 128/3*d^8*e^4*x^9 - 4096/3*a*d^4*e^7
*x^9 + 8192/3*a^2*e^10*x^9 - 4*d^9*e^3*x^8 - 768*a*d^5*e^6*x^8 + 6144*a^2*
d*e^9*x^8 - 32/7*d^10*e^2*x^7 + 768/7*a*d^6*e^5*x^7 + 24576/7*a^2*d^2*e^8*
x^7 + 128*a*d^7*e^4*x^6 - 1024*a^2*d^3*e^7*x^6 + 1/5*d^12*x^5 - 6144/5*a^2
*d^4*e^6*x^5 + 16384/5*a^3*e^9*x^5 - 8*a*d^9*e^2*x^4 + 4096*a^3*d*e^8*x^4
+ 128*a^2*d^6*e^4*x^3 - 1024*a^3*d^3*e^6*x^2 + 4096*a^4*e^8*x
```

3.39.9 Mupad [B] (verification not implemented)

Time = 10.54 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.12

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4\,dx = x^5\left(\frac{16384\,a^3\,e^9}{5}-\frac{6144\,a^2\,d^4\,e^6}{5}+\frac{d^{12}}{5}\right) + x^{10}\left(\frac{384\,d^7\,e^5}{5}+1024\,a\,d^3\,e^8\right) - x^{11}\left(\frac{1664\,d^6\,e^6}{11}-\frac{49152\,a\,d^2\,e^9}{11}\right) + \frac{4096\,e^{12}\,x^{17}}{17}+\frac{2048\,e^8\,x^{13}\,(8\,a\,e^3-d^4)}{13} + \frac{128\,e^4\,x^9\,(64\,a^2\,e^6-32\,a\,d^4\,e^3+d^8)}{3} + 4096\,a^4\,e^8\,x+1024\,d\,e^{11}\,x^{16}+1024\,d^3\,e^9\,x^{14} + \frac{8192\,d^2\,e^{10}\,x^{15}}{5}+512\,d\,e^7\,x^{12}\left(8\,a\,e^3-d^4\right) + \frac{32\,d^2\,e^2\,x^7\,(768\,a^2\,e^6+24\,a\,d^4\,e^3-d^8)}{7} - 1024\,a^3\,d^3\,e^6\,x^2+128\,a^2\,d^6\,e^4\,x^3 - 4\,d\,e^3\,x^8\left(-1536\,a^2\,e^6+192\,a\,d^4\,e^3+d^8\right) - 128\,a\,d^3\,e^4\,x^6\left(8\,a\,e^3-d^4\right) - 8\,a\,d\,e^2\,x^4\left(d^8-512\,a^2\,e^6\right)$$

input
```
int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^4,x)
```

output
```
x^5*(d^12/5 + (16384*a^3*e^9)/5 - (6144*a^2*d^4*e^6)/5) + x^10*((384*d^7*e
^5)/5 + 1024*a*d^3*e^8) - x^11*((1664*d^6*e^6)/11 - (49152*a*d^2*e^9)/11)
+ (4096*e^12*x^17)/17 + (2048*e^8*x^13*(8*a*e^3 - d^4))/13 + (128*e^4*x^9*
(d^8 + 64*a^2*e^6 - 32*a*d^4*e^3))/3 + 4096*a^4*e^8*x + 1024*d*e^11*x^16 +
 1024*d^3*e^9*x^14 + (8192*d^2*e^10*x^15)/5 + 512*d*e^7*x^12*(8*a*e^3 - d^
4) + (32*d^2*e^2*x^7*(768*a^2*e^6 - d^8 + 24*a*d^4*e^3))/7 - 1024*a^3*d^3*
e^6*x^2 + 128*a^2*d^6*e^4*x^3 - 4*d*e^3*x^8*(d^8 - 1536*a^2*e^6 + 192*a*d^
4*e^3) - 128*a*d^3*e^4*x^6*(8*a*e^3 - d^4) - 8*a*d*e^2*x^4*(d^8 - 512*a^2*
e^6)
```

3.40 $\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^3 \, dx$

3.40.1 Optimal result

Integrand size = 32, antiderivative size = 203

$$\begin{aligned}\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^3 \, dx = {} & 512a^3e^6x - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 \\ & - \frac{1}{4}d(d^8 - 1536a^2e^6)\,x^4 - \frac{384}{5}ae^4(d^4 - 4ae^3)\,x^5 \\ & + 4d^3e^2(d^4 - 16ae^3)\,x^6 + \frac{24}{7}d^2e^3(d^4 + 64ae^3)\,x^7 \\ & - 24de^4(d^4 - 16ae^3)\,x^8 - \frac{128}{3}e^5(d^4 - 4ae^3)\,x^9 \\ & + 32d^3e^6x^{10} + \frac{1536}{11}d^2e^7x^{11} + 128de^8x^{12} + \frac{512e^9x^{13}}{13}\end{aligned}$$

output
```
512*a^3*e^6*x-96*a^2*d^3*e^4*x^2+8*a*d^6*e^2*x^3-1/4*d*(-1536*a^2*e^6+d^8)
*x^4-384/5*a*e^4*(-4*a*e^3+d^4)*x^5+4*d^3*e^2*(-16*a*e^3+d^4)*x^6+24/7*d^2
*e^3*(64*a*e^3+d^4)*x^7-24*d*e^4*(-16*a*e^3+d^4)*x^8-128/3*e^5*(-4*a*e^3+d
^4)*x^9+32*d^3*e^6*x^10+1536/11*d^2*e^7*x^11+128*d*e^8*x^12+512/13*e^9*x^1
3
```

3.40.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.02

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3 dx = 512a^3e^6x-96a^2d^3e^4x^2+8ad^6e^2x^3 - \frac{1}{4}d\left(d^8-1536a^2e^6\right)x^4+\frac{384}{5}ae^4\left(-d^4+4ae^3\right)x^5 + 4d^3e^2\left(d^4-16ae^3\right)x^6+\frac{24}{7}d^2e^3\left(d^4+64ae^3\right)x^7 - 24de^4\left(d^4-16ae^3\right)x^8+\frac{128}{3}e^5\left(-d^4+4ae^3\right)x^9 + 32d^3e^6x^{10}+\frac{1536}{11}d^2e^7x^{11}+128de^8x^{12}+\frac{512e^9x^{13}}{13}$$

input
```
Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3,x]
```

output
```
512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 - 1536*a^2*
e^6)*x^4)/4 + (384*a*e^4*(-d^4 + 4*a*e^3)*x^5)/5 + 4*d^3*e^2*(d^4 - 16*a*e
^3)*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 - 24*d*e^4*(d^4 - 16*a*e^3)*
x^8 + (128*e^5*(-d^4 + 4*a*e^3)*x^9)/3 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x
^11)/11 + 128*d*e^8*x^12 + (512*e^9*x^13)/13
```

3.40.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3 dx$$

$$\Big\downarrow \text{2458}$$

$$\int \left(\frac{1}{32}\left(256ae^2+\frac{5d^4}{e}\right)-3d^2e\left(\frac{d}{4e}+x\right)^2+8e^3\left(\frac{d}{4e}+x\right)^4\right)^3 d\left(\frac{d}{4e}+x\right)$$

$$\Big\downarrow \text{1403}$$

$$\int \left(\frac{27}{32}d^4e\left(256ae^3+5d^4\right)\left(\frac{64ae^3}{9d^4}+\frac{41}{36}\right)\left(\frac{d}{4e}+x\right)^4+\frac{\left(256ae^3+5d^4\right)^3}{32768e^3}+216d^4e^5\left(\frac{64ae^3}{9d^4}+\frac{41}{36}\right)\left(\frac{d}{4e}+x\right)^8-\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{640}e\left(256ae^3+5d^4\right)\left(256ae^3+41d^4\right)\left(\frac{d}{4e}+x\right)^5+\frac{\left(256ae^3+5d^4\right)^3\left(\frac{d}{4e}+x\right)}{32768e^3}+\frac{2}{3}e^5\left(256ae^3+41d^4\right)\left(\frac{d}{4e}+x\right)^9-\frac{9}{14}d^2e^3\left(256ae^3+11d^4\right)\left(\frac{d}{4e}+x\right)^7-\frac{3d^2\left(256ae^3+5d^4\right)^2\left(\frac{d}{4e}+x\right)^3}{1024e}-\frac{576}{11}d^2e^7\left(\frac{d}{4e}+x\right)^{11}+\frac{512}{13}e^9\left(\frac{d}{4e}+x\right)^{13}$$

input
```
Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3,x]
```

output
```
((5*d^4 + 256*a*e^3)^3*(d/(4*e) + x))/(32768*e^3) - (3*d^2*(5*d^4 + 256*a*
e^3)^2*(d/(4*e) + x)^3)/(1024*e) + (3*e*(5*d^4 + 256*a*e^3)*(41*d^4 + 256*
a*e^3)*(d/(4*e) + x)^5)/640 - (9*d^2*e^3*(11*d^4 + 256*a*e^3)*(d/(4*e) + x
)^7)/14 + (2*e^5*(41*d^4 + 256*a*e^3)*(d/(4*e) + x)^9)/3 - (576*d^2*e^7*(d
/(4*e) + x)^11)/11 + (512*e^9*(d/(4*e) + x)^13)/13
```

3.40.3.1 Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.40.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.99

method	result
norman	$\frac{512e^9x^{13}}{13}+128d\,e^8x^{12}+\frac{1536d^2e^7x^{11}}{11}+32d^3e^6x^{10}+\left(\frac{512}{3}a\,e^8-\frac{128}{3}d^4e^5\right)x^9+\left(384a\,e^7d-24d^5e\right.$
gosper	$\frac{512}{13}e^9x^{13}+128d\,e^8x^{12}+\frac{1536}{11}d^2e^7x^{11}+32d^3e^6x^{10}+\frac{512}{3}x^9a\,e^8-\frac{128}{3}x^9d^4e^5+384ad\,e^7x^8-24$
risch	$\frac{512}{13}e^9x^{13}+128d\,e^8x^{12}+\frac{1536}{11}d^2e^7x^{11}+32d^3e^6x^{10}+\frac{512}{3}x^9a\,e^8-\frac{128}{3}x^9d^4e^5+384ad\,e^7x^8-24$
parallelrisch	$\frac{512}{13}e^9x^{13}+128d\,e^8x^{12}+\frac{1536}{11}d^2e^7x^{11}+32d^3e^6x^{10}+\frac{512}{3}x^9a\,e^8-\frac{128}{3}x^9d^4e^5+384ad\,e^7x^8-24$
default	$\frac{512e^9x^{13}}{13}+128d\,e^8x^{12}+\frac{1536d^2e^7x^{11}}{11}+32d^3e^6x^{10}+\frac{(512a\,e^8-256d^4e^5+8e^3(128a\,e^5-16d^4e^2))x^9}{9}+\frac{(2048a\,e}{}$

input
```
int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x,method=_RETURNVERBOSE)
```

output
```
512/13*e^9*x^13+128*d*e^8*x^12+1536/11*d^2*e^7*x^11+32*d^3*e^6*x^10+(512/3
*a*e^8-128/3*d^4*e^5)*x^9+(384*a*d*e^7-24*d^5*e^4)*x^8+(1536/7*a*e^6*d^2+2
4/7*d^6*e^3)*x^7+(-64*a*d^3*e^5+4*d^7*e^2)*x^6+(1536/5*a^2*e^7-384/5*a*d^4
*e^4)*x^5+(384*a^2*e^6*d-1/4*d^9)*x^4+8*a*d^6*e^2*x^3-96*a^2*d^3*e^4*x^2+5
12*a^3*e^6*x
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.98

$$\begin{aligned}\int\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3\,dx=&\ \frac{512}{13}\,e^9x^{13}+128\,de^8x^{12}+\frac{1536}{11}\,d^2e^7x^{11}\\&+32\,d^3e^6x^{10}+8\,ad^6e^2x^3-96\,a^2d^3e^4x^2+512\,a^3e^6x\\&-\frac{128}{3}\left(d^4e^5-4\,ae^8\right)x^9-24\left(d^5e^4-16\,ade^7\right)x^8\\&+\frac{24}{7}\left(d^6e^3+64\,ad^2e^6\right)x^7+4\left(d^7e^2-16\,ad^3e^5\right)x^6\\&-\frac{384}{5}\left(ad^4e^4-4\,a^2e^7\right)x^5-\frac{1}{4}\left(d^9-1536\,a^2de^6\right)x^4\end{aligned}$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="fricas")
```

output
```
512/13*e^9*x^13 + 128*d*e^8*x^12 + 1536/11*d^2*e^7*x^11 + 32*d^3*e^6*x^10
+ 8*a*d^6*e^2*x^3 - 96*a^2*d^3*e^4*x^2 + 512*a^3*e^6*x - 128/3*(d^4*e^5 -
4*a*e^8)*x^9 - 24*(d^5*e^4 - 16*a*d*e^7)*x^8 + 24/7*(d^6*e^3 + 64*a*d^2*e^
6)*x^7 + 4*(d^7*e^2 - 16*a*d^3*e^5)*x^6 - 384/5*(a*d^4*e^4 - 4*a^2*e^7)*x^
5 - 1/4*(d^9 - 1536*a^2*d*e^6)*x^4
```

3.40.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.07

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3\,dx=512a^3e^6x-96a^2d^3e^4x^2+8ad^6e^2x^3+32d^3e^6x^{10}+\frac{1536d^2e^7x^{11}}{11}+128de^8x^{12}+\frac{512e^9x^{13}}{13}+x^9\cdot\left(\frac{512ae^8}{3}-\frac{128d^4e^5}{3}\right)+x^8\cdot\left(384ade^7-24d^5e^4\right)+x^7\cdot\left(\frac{1536ad^2e^6}{7}+\frac{24d^6e^3}{7}\right)+x^6\left(-64ad^3e^5+4d^7e^2\right)+x^5\cdot\left(\frac{1536a^2e^7}{5}-\frac{384ad^4e^4}{5}\right)+x^4\cdot\left(384a^2de^6-\frac{d^9}{4}\right)$$

input
```
integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**3,x)
```

output
```
512*a**3*e**6*x - 96*a**2*d**3*e**4*x**2 + 8*a*d**6*e**2*x**3 + 32*d**3*e*
*6*x**10 + 1536*d**2*e**7*x**11/11 + 128*d*e**8*x**12 + 512*e**9*x**13/13
+ x**9*(512*a*e**8/3 - 128*d**4*e**5/3) + x**8*(384*a*d*e**7 - 24*d**5*e**
4) + x**7*(1536*a*d**2*e**6/7 + 24*d**6*e**3/7) + x**6*(-64*a*d**3*e**5 +
4*d**7*e**2) + x**5*(1536*a**2*e**7/5 - 384*a*d**4*e**4/5) + x**4*(384*a**
2*d*e**6 - d**9/4)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.05

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3\,dx = \frac{512}{13}\,e^9x^{13}+128\,de^8x^{12}+\frac{1536}{11}\,d^2e^7x^{11}+\frac{256}{5}\,d^3e^6x^{10} - \frac{1}{4}\,d^9x^4+512\,a^3e^6x+\frac{4}{7}\,\left(6\,e^3x^7+7\,de^2x^6\right)d^6 + \frac{96}{5}\,\left(16\,e^3x^5+20\,de^2x^4-5\,d^3x^2\right)a^2e^4-\frac{8}{15}\,\left(36\,e^6x^{10}+80\,de^5x^9+45\,d^2e^4x^8\right)d^3 + \frac{8}{105}\,\left(2240\,e^6x^9+5040\,de^5x^8+2880\,d^2e^4x^7+105\,d^6x^3-168\,\left(5\,e^3x^6+6\,de^2x^5\right)d^3\right)ae^2$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="maxima")
```

output
```
512/13*e^9*x^13 + 128*d*e^8*x^12 + 1536/11*d^2*e^7*x^11 + 256/5*d^3*e^6*x^
10 - 1/4*d^9*x^4 + 512*a^3*e^6*x + 4/7*(6*e^3*x^7 + 7*d*e^2*x^6)*d^6 + 96/
5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*a^2*e^4 - 8/15*(36*e^6*x^10 + 80
*d*e^5*x^9 + 45*d^2*e^4*x^8)*d^3 + 8/105*(2240*e^6*x^9 + 5040*d*e^5*x^8 +
2880*d^2*e^4*x^7 + 105*d^6*x^3 - 168*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3)*a*e^2
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.01

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3\,dx = \frac{512}{13}\,e^9x^{13}+128\,de^8x^{12}+\frac{1536}{11}\,d^2e^7x^{11}+32\,d^3e^6x^{10} - \frac{128}{3}\,d^4e^5x^9+\frac{512}{3}\,ae^8x^9-24\,d^5e^4x^8+384\,ade^7x^8 + \frac{24}{7}\,d^6e^3x^7+\frac{1536}{7}\,ad^2e^6x^7+4\,d^7e^2x^6 - 64\,ad^3e^5x^6-\frac{384}{5}\,ad^4e^4x^5+\frac{1536}{5}\,a^2e^7x^5-\frac{1}{4}\,d^9x^4 + 384\,a^2de^6x^4+8\,ad^6e^2x^3-96\,a^2d^3e^4x^2+512\,a^3e^6x$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="giac")
```

output
```
512/13*e^9*x^13 + 128*d*e^8*x^12 + 1536/11*d^2*e^7*x^11 + 32*d^3*e^6*x^10
- 128/3*d^4*e^5*x^9 + 512/3*a*e^8*x^9 - 24*d^5*e^4*x^8 + 384*a*d*e^7*x^8 +
 24/7*d^6*e^3*x^7 + 1536/7*a*d^2*e^6*x^7 + 4*d^7*e^2*x^6 - 64*a*d^3*e^5*x^
6 - 384/5*a*d^4*e^4*x^5 + 1536/5*a^2*e^7*x^5 - 1/4*d^9*x^4 + 384*a^2*d*e^6
*x^4 + 8*a*d^6*e^2*x^3 - 96*a^2*d^3*e^4*x^2 + 512*a^3*e^6*x
```

3.40.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.99

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^3 \, dx = \frac{512\,e^9\,x^{13}}{13} - x^4\left(\frac{d^9}{4} - 384\,a^2\,d\,e^6\right) + \frac{128\,e^5\,x^9\,(4\,a\,e^3 - d^4)}{3} + 512\,a^3\,e^6\,x + 128\,d\,e^8\,x^{12} + 32\,d^3\,e^6\,x^{10} + \frac{1536\,d^2\,e^7\,x^{11}}{11} + 8\,a\,d^6\,e^2\,x^3 + \frac{384\,a\,e^4\,x^5\,(4\,a\,e^3 - d^4)}{5} + 24\,d\,e^4\,x^8\left(16\,a\,e^3 - d^4\right) + \frac{24\,d^2\,e^3\,x^7\,(d^4 + 64\,a\,e^3)}{7} - 96\,a^2\,d^3\,e^4\,x^2 - 4\,d^3\,e^2\,x^6\left(16\,a\,e^3 - d^4\right)$$

input
```
int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^3,x)
```

output
```
(512*e^9*x^13)/13 - x^4*(d^9/4 - 384*a^2*d*e^6) + (128*e^5*x^9*(4*a*e^3 -
d^4))/3 + 512*a^3*e^6*x + 128*d*e^8*x^12 + 32*d^3*e^6*x^10 + (1536*d^2*e^7
*x^11)/11 + 8*a*d^6*e^2*x^3 + (384*a*e^4*x^5*(4*a*e^3 - d^4))/5 + 24*d*e^4
*x^8*(16*a*e^3 - d^4) + (24*d^2*e^3*x^7*(64*a*e^3 + d^4))/7 - 96*a^2*d^3*e
^4*x^2 - 4*d^3*e^2*x^6*(16*a*e^3 - d^4)
```

3.41 $\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2 \, dx$

3.41.1 Optimal result

Integrand size = 32, antiderivative size = 107

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2 \, dx = 64a^2e^4x - 8ad^3e^2x^2 + \frac{d^6x^3}{3} + 32ade^4x^4 - \frac{16}{5}e^2(d^4 - 8ae^3)x^5 - \frac{8}{3}d^3e^3x^6 + \frac{64}{7}d^2e^4x^7 + 16de^5x^8 + \frac{64e^6x^9}{9}$$

output
```
64*a^2*e^4*x-8*a*d^3*e^2*x^2+1/3*d^6*x^3+32*a*d*e^4*x^4-16/5*e^2*(-8*a*e^3
+d^4)*x^5-8/3*d^3*e^3*x^6+64/7*d^2*e^4*x^7+16*d*e^5*x^8+64/9*e^6*x^9
```

3.41.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.02

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2 \, dx = 64a^2e^4x - 8ad^3e^2x^2 + \frac{d^6x^3}{3} + 32ade^4x^4 + \frac{16}{5}e^2(-d^4 + 8ae^3)x^5 - \frac{8}{3}d^3e^3x^6 + \frac{64}{7}d^2e^4x^7 + 16de^5x^8 + \frac{64e^6x^9}{9}$$

input
```
Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]
```

output
```
64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 + (16*e^2*(-
d^4 + 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 + 16*d*e^5*
x^8 + (64*e^6*x^9)/9
```

3.41.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2 dx$$

$$\Big\downarrow \text{2458}$$

$$\int \left(\frac{1}{32}\left(256ae^2+\frac{5d^4}{e}\right)-3d^2e\left(\frac{d}{4e}+x\right)^2+8e^3\left(\frac{d}{4e}+x\right)^4\right)^2 d\left(\frac{d}{4e}+x\right)$$

$$\Big\downarrow \text{1403}$$

$$\int \left(9d^4e^2\left(\frac{128ae^3}{9d^4}+\frac{23}{18}\right)\left(\frac{d}{4e}+x\right)^4+\frac{\left(256ae^3+5d^4\right)^2}{1024e^2}-\frac{3}{16}d^2\left(256ae^3+5d^4\right)\left(\frac{d}{4e}+x\right)^2-48d^2e^4\left(\frac{d}{4e}+x\right)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}e^2\left(256ae^3+23d^4\right)\left(\frac{d}{4e}+x\right)^5+\frac{\left(256ae^3+5d^4\right)^2\left(\frac{d}{4e}+x\right)}{1024e^2}-\frac{1}{16}d^2\left(256ae^3+5d^4\right)\left(\frac{d}{4e}+x\right)^3-\frac{48}{7}d^2e^4\left(\frac{d}{4e}+x\right)^7+\frac{64}{9}e^6\left(\frac{d}{4e}+x\right)^9$$

input
```
Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]
```

output
```
((5*d^4 + 256*a*e^3)^2*(d/(4*e) + x))/(1024*e^2) - (d^2*(5*d^4 + 256*a*e^3
)*(d/(4*e) + x)^3)/16 + (e^2*(23*d^4 + 256*a*e^3)*(d/(4*e) + x)^5)/10 - (4
8*d^2*e^4*(d/(4*e) + x)^7)/7 + (64*e^6*(d/(4*e) + x)^9)/9
```

3.41.3.1 Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.41.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

method	result
norman	$\frac{64e^6x^9}{9}+16d\,e^5x^8+\frac{64d^2e^4x^7}{7}-\frac{8d^3e^3x^6}{3}+\left(\frac{128}{5}a\,e^5-\frac{16}{5}d^4e^2\right)x^5+32ad\,e^4x^4+\frac{d^6x^3}{3}-8a\,d^3e^2x$
gosper	$\frac{64}{9}e^6x^9+16d\,e^5x^8+\frac{64}{7}d^2e^4x^7-\frac{8}{3}d^3e^3x^6+\frac{128}{5}x^5a\,e^5-\frac{16}{5}x^5d^4e^2+32ad\,e^4x^4+\frac{1}{3}d^6x^3-8a\,c$
default	$\frac{64e^6x^9}{9}+16d\,e^5x^8+\frac{64d^2e^4x^7}{7}-\frac{8d^3e^3x^6}{3}+\frac{(128a\,e^5-16d^4e^2)x^5}{5}+32ad\,e^4x^4+\frac{d^6x^3}{3}-8a\,d^3e^2x^2+6$
risch	$\frac{64}{9}e^6x^9+16d\,e^5x^8+\frac{64}{7}d^2e^4x^7-\frac{8}{3}d^3e^3x^6+\frac{128}{5}x^5a\,e^5-\frac{16}{5}x^5d^4e^2+32ad\,e^4x^4+\frac{1}{3}d^6x^3-8a\,c$
parallelrisch	$\frac{64}{9}e^6x^9+16d\,e^5x^8+\frac{64}{7}d^2e^4x^7-\frac{8}{3}d^3e^3x^6+\frac{128}{5}x^5a\,e^5-\frac{16}{5}x^5d^4e^2+32ad\,e^4x^4+\frac{1}{3}d^6x^3-8a\,c$

input
```
int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x,method=_RETURNVERBOSE)
```

output
```
64/9*e^6*x^9+16*d*e^5*x^8+64/7*d^2*e^4*x^7-8/3*d^3*e^3*x^6+(128/5*a*e^5-16
/5*d^4*e^2)*x^5+32*a*d*e^4*x^4+1/3*d^6*x^3-8*a*d^3*e^2*x^2+64*a^2*e^4*x
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^2 \, dx = \frac{64}{9}\, e^6x^9 + 16\, de^5x^8 + \frac{64}{7}\, d^2e^4x^7 - \frac{8}{3}\, d^3e^3x^6 + 32\, ade^4x^4 + \frac{1}{3}\, d^6x^3 - 8\, ad^3e^2x^2 + 64\, a^2e^4x - \frac{16}{5}\, \left(d^4e^2 - 8\, ae^5\right)x^5$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")
```

output
```
64/9*e^6*x^9 + 16*d*e^5*x^8 + 64/7*d^2*e^4*x^7 - 8/3*d^3*e^3*x^6 + 32*a*d*
e^4*x^4 + 1/3*d^6*x^3 - 8*a*d^3*e^2*x^2 + 64*a^2*e^4*x - 16/5*(d^4*e^2 - 8
*a*e^5)*x^5
```

3.41.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^2 \, dx = 64a^2e^4x - 8ad^3e^2x^2 + 32ade^4x^4 + \frac{d^6x^3}{3} - \frac{8d^3e^3x^6}{3} + \frac{64d^2e^4x^7}{7} + 16de^5x^8 + \frac{64e^6x^9}{9} + x^5 \cdot \left(\frac{128ae^5}{5} - \frac{16d^4e^2}{5}\right)$$

input
```
integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
```

output
```
64*a**2*e**4*x - 8*a*d**3*e**2*x**2 + 32*a*d*e**4*x**4 + d**6*x**3/3 - 8*d
**3*e**3*x**6/3 + 64*d**2*e**4*x**7/7 + 16*d*e**5*x**8 + 64*e**6*x**9/9 +
x**5*(128*a*e**5/5 - 16*d**4*e**2/5)
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2 dx = \frac{64}{9}e^6x^9+16de^5x^8+\frac{64}{7}d^2e^4x^7+\frac{1}{3}d^6x^3+64a^2e^4x-\frac{8}{15}\left(5e^3x^6+6de^2x^5\right)d^3+\frac{8}{5}\left(16e^3x^5+20de^2x^4-5d^3x^2\right)ae^2$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")
```

output
```
64/9*e^6*x^9 + 16*d*e^5*x^8 + 64/7*d^2*e^4*x^7 + 1/3*d^6*x^3 + 64*a^2*e^4*
x - 8/15*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3 + 8/5*(16*e^3*x^5 + 20*d*e^2*x^4 -
5*d^3*x^2)*a*e^2
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2 dx = \frac{64}{9}e^6x^9+16de^5x^8+\frac{64}{7}d^2e^4x^7-\frac{8}{3}d^3e^3x^6-\frac{16}{5}d^4e^2x^5+\frac{128}{5}ae^5x^5+32ade^4x^4+\frac{1}{3}d^6x^3-8ad^3e^2x^2+64a^2e^4x$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")
```

output
```
64/9*e^6*x^9 + 16*d*e^5*x^8 + 64/7*d^2*e^4*x^7 - 8/3*d^3*e^3*x^6 - 16/5*d^
4*e^2*x^5 + 128/5*a*e^5*x^5 + 32*a*d*e^4*x^4 + 1/3*d^6*x^3 - 8*a*d^3*e^2*x
^2 + 64*a^2*e^4*x
```

3.41.9 Mupad [B] (verification not implemented)

Time = 10.17 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2\,dx = x^5\left(\frac{128\,a\,e^5}{5}-\frac{16\,d^4\,e^2}{5}\right)+\frac{d^6\,x^3}{3}+\frac{64\,e^6\,x^9}{9}+64\,a^2\,e^4\,x+16\,d\,e^5\,x^8-\frac{8\,d^3\,e^3\,x^6}{3}+\frac{64\,d^2\,e^4\,x^7}{7}-8\,a\,d^3\,e^2\,x^2+32\,a\,d\,e^4\,x^4$$

input
```
int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^2,x)
```

output
```
x^5*((128*a*e^5)/5 - (16*d^4*e^2)/5) + (d^6*x^3)/3 + (64*e^6*x^9)/9 + 64*a
^2*e^4*x + 16*d*e^5*x^8 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 - 8*a*d^3
*e^2*x^2 + 32*a*d*e^4*x^4
```

3.42 $\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4) \, dx$

3.42.1 Optimal result

Integrand size = 30, antiderivative size = 37

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4) \, dx = 8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

output
```
8*a*e^2*x-1/2*d^3*x^2+2*d*e^2*x^4+8/5*e^3*x^5
```

3.42.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4) \, dx = 8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

input
```
Integrate[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4,x]
```

output
```
8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

3.42.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right) dx$$
$$\Big\downarrow \text{ 2009}$$
$$8ae^2x-\frac{d^3x^2}{2}+2de^2x^4+\frac{8e^3x^5}{5}$$

input
```
Int[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4,x]
```

output
```
8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

3.42.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.42.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
gosper	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}e^3x^5$	34
default	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}e^3x^5$	34
norman	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}e^3x^5$	34
risch	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}e^3x^5$	34
parallelrisch	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}e^3x^5$	34
parts	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}e^3x^5$	34

input
```
int(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x,method=_RETURNVERBOSE)
```

output
```
8*a*e^2*x-1/2*d^3*x^2+2*d*e^2*x^4+8/5*e^3*x^5
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) dx = \frac{8}{5} e^3x^5 + 2 de^2x^4 - \frac{1}{2} d^3x^2 + 8 ae^2x$$

input
```
integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="fricas")
```

output
```
8/5*e^3*x^5 + 2*d*e^2*x^4 - 1/2*d^3*x^2 + 8*a*e^2*x
```

3.42.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) dx = 8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

input
```
integrate(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2,x)
```

output
```
8*a*e**2*x - d**3*x**2/2 + 2*d*e**2*x**4 + 8*e**3*x**5/5
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) dx = \frac{8}{5} e^3x^5 + 2 de^2x^4 - \frac{1}{2} d^3x^2 + 8 ae^2x$$

input
```
integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="maxima")
```

output
```
8/5*e^3*x^5 + 2*d*e^2*x^4 - 1/2*d^3*x^2 + 8*a*e^2*x
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) \, dx = \frac{8}{5}\, e^3x^5 + 2\, de^2x^4 - \frac{1}{2}\, d^3x^2 + 8\, ae^2x$$

input
```
integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="giac")
```

output
```
8/5*e^3*x^5 + 2*d*e^2*x^4 - 1/2*d^3*x^2 + 8*a*e^2*x
```

3.42.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) \, dx = -\frac{d^3\, x^2}{2} + 2\, d\, e^2\, x^4 + \frac{8\, e^3\, x^5}{5} + 8\, a\, e^2\, x$$

input
```
int(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3,x)
```

output
```
(8*e^3*x^5)/5 - (d^3*x^2)/2 + 2*d*e^2*x^4 + 8*a*e^2*x
```

3.43 $\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4} dx$

3.43.1 Optimal result

Integrand size = 32, antiderivative size = 153

$$\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx = \frac{2\operatorname{arctanh}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}} - \frac{2\operatorname{arctanh}\left(\frac{d+4ex}{\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}$$

output
```
2*arctanh((4*e*x+d)/(3*d^2-2*(-64*a*e^3+d^4)^(1/2))^(1/2))/(-64*a*e^3+d^4)
^(1/2)/(3*d^2-2*(-64*a*e^3+d^4)^(1/2))^(1/2)-2*arctanh((4*e*x+d)/(3*d^2+2*
(-64*a*e^3+d^4)^(1/2))^(1/2))/(-64*a*e^3+d^4)^(1/2)/(3*d^2+2*(-64*a*e^3+d^
4)^(1/2))^(1/2)
```

3.43.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

$$\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx = -\text{RootSum}\left[8ae^2-d^3\#1+8de^2\#1^3+8e^3\#1^4\&,\frac{\log(x-\#1)}{d^3-24de^2\#1^2-32e^3\#1^3}\&\right]$$

input
```
Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]
```

output
```
-RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 & , Log[x - #1]/(d^3
 - 24*d*e^2*#1^2 - 32*e^3*#1^3) & ]
```

3.43.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2458, 1406, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{\frac{1}{32}\left(256ae^2+\frac{5d^4}{e}\right)-3d^2e\left(\frac{d}{4e}+x\right)^2+8e^3\left(\frac{d}{4e}+x\right)^4}d\left(\frac{d}{4e}+x\right)$$

$$\Big\downarrow \text{1406}$$

$$\frac{4e^2\int \frac{1}{8e^3\left(\frac{d}{4e}+x\right)^2-\frac{1}{2}e\left(3d^2+2\sqrt{d^4-64ae^3}\right)}d\left(\frac{d}{4e}+x\right)}{\sqrt{d^4-64ae^3}}-\frac{4e^2\int \frac{1}{8e^3\left(\frac{d}{4e}+x\right)^2-\frac{1}{2}e\left(3d^2-2\sqrt{d^4-64ae^3}\right)}d\left(\frac{d}{4e}+x\right)}{\sqrt{d^4-64ae^3}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\text{arctanh}\left(\frac{4e\left(\frac{d}{4e}+x\right)}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}-\frac{2\text{arctanh}\left(\frac{4e\left(\frac{d}{4e}+x\right)}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

input
```
Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]
```

output
```
(2*ArcTanh[(4*e*(d/(4*e) + x))/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqr
t[d^4 - 64*a*e^3]*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) - (2*ArcTanh[(4*e*
(d/(4*e) + x))/Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 64*a*e^3
]*Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])
```

3.43.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.43.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.44

method	result	size
default	$\sum_{\_R=\text{RootOf}\left(8e^3\_Z^4+8d\,e^2\_Z^3-d^3\_Z+8a\,e^2\right)}\frac{\ln\left(x-\_R\right)}{32\_R^3e^3+24\_R^2d\,e^2-d^3}$	67
risch	$\sum_{\_R=\text{RootOf}\left(8e^3\_Z^4+8d\,e^2\_Z^3-d^3\_Z+8a\,e^2\right)}\frac{\ln\left(x-\_R\right)}{32\_R^3e^3+24\_R^2d\,e^2-d^3}$	67

input
```
int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x,method=_RETURNVERBOSE)
```

output
```
sum(1/(32*_R^3*e^3+24*_R^2*d*e^2-d^3)*ln(x-_R),_R=RootOf(8*_Z^4*e^3+8*_Z^3
*d*e^2-_Z*d^3+8*a*e^2))
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1115 vs. $2(133) = 266$.

Time = 0.31 (sec) , antiderivative size = 1115, normalized size of antiderivative = 7.29

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \, dx = \text{Too large to display}$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="fricas")
```

output
```
-sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960
*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 -
 16384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e^3 - 3*(5*d^10 - 64*a*d^6*e
^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 -
 4194304*a^3*e^9))*sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/
sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^
8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 + 2*(5*d^8 - 64*a*
d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6
- 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x - 2*
(2*d^4 - 128*a*e^3 - 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25
*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2
+ 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 -
98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^
6)) + 2*d) - sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(2
5*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64
*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e^3 + 3*(5*d^10
- 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a
^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 163
84*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3
*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 - 2...
```

3.43.6 Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.80

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \, dx$$
$$= \text{RootSum}\left(t^4 \cdot \left(1048576a^3e^9 - 12288a^2d^4e^6 - 384ad^8e^3 + 5d^{12}\right) + t^2 \cdot \left(384ad^2e^3 - 6d^6\right) + 1, \left(t \mapsto t \log\right.\right.$$

input
```
integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2),x)
```

output
```
RootSum(_t**4*(1048576*a**3*e**9 - 12288*a**2*d**4*e**6 - 384*a*d**8*e**3
+ 5*d**12) + _t**2*(384*a*d**2*e**3 - 6*d**6) + 1, Lambda(_t, _t*log(x + (
-49152*_t**3*a**2*d**2*e**6 - 192*_t**3*a*d**6*e**3 + 15*_t**3*d**10 + 256
*_t*a*e**3 - 13*_t*d**4 + 2*d)/(8*e))))
```

3.43.7 Maxima [F]

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx = \int \frac{1}{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}\,dx$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="maxima")
```

output
```
integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

3.43.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(133) = 266$.

Time = 0.28 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.77

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx$$
$$= -\frac{2\log\left(x+\frac{1}{4}\sqrt{\frac{3\,d^2e^2+2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}+\frac{d}{4\,e}\right)}{e^3\left(\sqrt{\frac{3\,d^2e^2+2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}+\frac{d}{e}\right)^3-3\,de^2\left(\sqrt{\frac{3\,d^2e^2+2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}+\frac{d}{e}\right)^2+2\,d^3}$$
$$+\frac{2\log\left(x-\frac{1}{4}\sqrt{\frac{3\,d^2e^2+2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}+\frac{d}{4\,e}\right)}{e^3\left(\sqrt{\frac{3\,d^2e^2+2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}-\frac{d}{e}\right)^3+3\,de^2\left(\sqrt{\frac{3\,d^2e^2+2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}-\frac{d}{e}\right)^2-2\,d^3}$$
$$-\frac{2\log\left(x+\frac{1}{4}\sqrt{\frac{3\,d^2e^2-2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}+\frac{d}{4\,e}\right)}{e^3\left(\sqrt{\frac{3\,d^2e^2-2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}+\frac{d}{e}\right)^3-3\,de^2\left(\sqrt{\frac{3\,d^2e^2-2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}+\frac{d}{e}\right)^2+2\,d^3}$$
$$+\frac{2\log\left(x-\frac{1}{4}\sqrt{\frac{3\,d^2e^2-2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}+\frac{d}{4\,e}\right)}{e^3\left(\sqrt{\frac{3\,d^2e^2-2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}-\frac{d}{e}\right)^3+3\,de^2\left(\sqrt{\frac{3\,d^2e^2-2\,\sqrt{d^4-64\,ae^3}e^2}{e^4}}-\frac{d}{e}\right)^2-2\,d^3}$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="giac")
```

output
```
-2*log(x + 1/4*sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + 1/4*d/
e)/(e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^3 - 3*d
*e^2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^2 + 2*d^3)
 + 2*log(x - 1/4*sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + 1/4*
d/e)/(e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^3 + 3
*d*e^2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^2 - 2*d^
3) - 2*log(x + 1/4*sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + 1/
4*d/e)/(e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^3 -
 3*d*e^2*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^2 + 2*
d^3) + 2*log(x - 1/4*sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) +
1/4*d/e)/(e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^3
 + 3*d*e^2*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^2 -
2*d^3)
```

3.43. $\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$

3.43.9 Mupad [B] (verification not implemented)

Time = 11.30 (sec) , antiderivative size = 1264, normalized size of antiderivative = 8.26

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx =$$

$$-\mathrm{atan}\left(\frac{d^3\sqrt{-262144\,a^3\,e^9 + 12288\,a^2\,d^4\,e^6 - 192\,a\,d^8\,e^3 + d^{12}}\,3\mathrm{i} + d^9\,2}{5\,d^{12}\sqrt{\frac{2\sqrt{-262144\,a^3\,e^9+12288\,a^2\,d^4\,e^6-192\,a\,d^8\,e^3+d^{12}}+3\,d^6-192\,a\,d^2\,e^3}{1048576\,a^3\,e^9-12288\,a^2\,d^4\,e^6-384\,a\,d^8\,e^3+5\,d^{12}}} + 1048576\,a^3\,e^9\sqrt{\frac{2\sqrt{-262144\,a^3\,e^9+12288}}{1048576\,a^3\,e^9-}}}\right.$$

$$+\mathrm{atan}\left(\frac{d^3\sqrt{-262144\,a^3\,e^9 + 12288\,a^2\,d^4\,e^6 - 192\,a\,d^8\,e^3 + d^{12}}\,3\mathrm{i} -}{5\,d^{12}\sqrt{-\frac{2\sqrt{-262144\,a^3\,e^9+12288\,a^2\,d^4\,e^6-192\,a\,d^8\,e^3+d^{12}}-3\,d^6+192\,a\,d^2\,e^3}{1048576\,a^3\,e^9-12288\,a^2\,d^4\,e^6-384\,a\,d^8\,e^3+5\,d^{12}}} + 1048576\,a^3\,e^9\sqrt{-\frac{2\sqrt{-262144\,a^3\,e^9+}}{1048576\,a}}}\right.$$

input
```
int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3),x)
```

output
```
atan((d^3*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2
)*3i - d^9*2i + a*d^5*e^3*256i - a^2*d*e^6*8192i - a^2*e^7*x*32768i - d^8*
e*x*8i + d^2*e*x*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^
6)^(1/2)*12i + a*d^4*e^4*x*1024i)/(5*d^12*(-(2*(d^12 - 262144*a^3*e^9 - 19
2*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 +
1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) + 1048576*a^3*
e^9*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)
 - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 1228
8*a^2*d^4*e^6))^(1/2) - 384*a*d^8*e^3*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*
d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048
576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) - 12288*a^2*d^4*e^
6*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) -
 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*
a^2*d^4*e^6))^(1/2)))*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*
a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 38
4*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2)*2i - atan((d^3*(d^12 - 262144*a^3*
e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)*3i + d^9*2i - a*d^5*e^3*256
i + a^2*d*e^6*8192i + a^2*e^7*x*32768i + d^8*e*x*8i + d^2*e*x*(d^12 - 2621
44*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)*12i - a*d^4*e^4*x*10
24i)/(5*d^12*((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4...
```

3.44 $\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^2}\,dx$

3.44.1 Optimal result

Integrand size = 32, antiderivative size = 342

$$\begin{aligned}\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^2}\,dx &= \frac{2e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(5d^8-64ad^4e^3-16384a^2e^6)\,(8ae^2-d^3x+8de^2x^3+8e^3x^4)} \\ &\quad - \frac{24e\left(d^4+128ae^3-d^2\sqrt{d^4-64ae^3}\right)\operatorname{arctanh}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{(d^4-64ae^3)^{3/2}\,(5d^4+256ae^3)\,\sqrt{3d^2-2\sqrt{d^4-64ae^3}}} \\ &\quad + \frac{24e\left(d^4+128ae^3+d^2\sqrt{d^4-64ae^3}\right)\operatorname{arctanh}\left(\frac{d+4ex}{\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}\right)}{(d^4-64ae^3)^{3/2}\,(5d^4+256ae^3)\,\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}\end{aligned}$$

output
```
2*e*(1/4*d/e+x)*(13*d^4-256*a*e^3-48*d^2*e^2*(1/4*d/e+x)^2)/(-16384*a^2*e^
6-64*a*d^4*e^3+5*d^8)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)-24*e*arctanh((
4*e*x+d)/(3*d^2-2*(-64*a*e^3+d^4)^(1/2))^(1/2))*(d^4+128*a*e^3-d^2*(-64*a*
e^3+d^4)^(1/2))/(-64*a*e^3+d^4)^(3/2)/(256*a*e^3+5*d^4)/(3*d^2-2*(-64*a*e^
3+d^4)^(1/2))^(1/2)+24*e*arctanh((4*e*x+d)/(3*d^2+2*(-64*a*e^3+d^4)^(1/2))
^(1/2))*(d^4+128*a*e^3+d^2*(-64*a*e^3+d^4)^(1/2))/(-64*a*e^3+d^4)^(3/2)/(2
56*a*e^3+5*d^4)/(3*d^2+2*(-64*a*e^3+d^4)^(1/2))^(1/2)
```

3.44.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.68

$$\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^2}\,dx = \frac{(d+4ex)\left(5d^4-128ae^3-12d^3ex-24d^2e^2x^2\right)}{(d^4-64ae^3)\left(5d^4+256ae^3\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)} + \frac{48e^2\text{RootSum}\left[8ae^2-d^3\#1+8de^2\#1^3+8e^3\#1^4\&,\frac{32ae^2\log(x-\#1)+d^3\log(x-\#1)\#1+2d^2e\log(x-\#1)\#1^2}{-d^3+24de^2\#1^2+32e^3\#1^3}\&\right]}{-5d^8+64ad^4e^3+16384a^2e^6}$$

input
```
Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]
```

output
```
((d + 4*e*x)*(5*d^4 - 128*a*e^3 - 12*d^3*e*x - 24*d^2*e^2*x^2))/((d^4 - 64
*a*e^3)*(5*d^4 + 256*a*e^3)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)) +
 (48*e^2*RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 & , (32*a*e^
2*Log[x - #1] + d^3*Log[x - #1]*#1 + 2*d^2*e*Log[x - #1]*#1^2)/(-d^3 + 24*
d*e^2*#1^2 + 32*e^3*#1^3) & ])/(-5*d^8 + 64*a*d^4*e^3 + 16384*a^2*e^6)
```

3.44.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2458, 1405, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^2}\,dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{\left(\frac{1}{32}\left(256ae^2+\frac{5d^4}{e}\right)-3d^2e\left(\frac{d}{4e}+x\right)^2+8e^3\left(\frac{d}{4e}+x\right)^4\right)^2}d\left(\frac{d}{4e}+x\right)$$

$$\Big\downarrow \text{1405}$$

$$\frac{64e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(-16384a^2e^6-64ad^4e^3+5d^8)\left(256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4\right)}-\frac{4\int -\frac{48e^2\left(d^4-16e^2\left(\frac{d}{4e}+x\right)^2d^2-256ae^3\right)}{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}d\left(\frac{d}{4e}+x\right)}{e\left(-16384a^2e^6-64ad^4e^3+5d^8\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{192e\int \frac{d^4-16e^2\left(\frac{d}{4e}+x\right)^2d^2-256ae^3}{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}d\left(\frac{d}{4e}+x\right)}{-16384a^2e^6-64ad^4e^3+5d^8}+\frac{64e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(-16384a^2e^6-64ad^4e^3+5d^8)\left(256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4\right)}$$

$$\Big\downarrow \; 1480$$

$$\frac{192e\left(\frac{8e^2\left(-d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\int \frac{1}{256e^3\left(\frac{d}{4e}+x\right)^2-16e\left(3d^2-2\sqrt{d^4-64ae^3}\right)}d\left(\frac{d}{4e}+x\right)}{\sqrt{d^4-64ae^3}}-\frac{8e^2\left(d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\int \frac{}{256e^3\left(\frac{d}{4e}+\right.}}{\sqrt{d^4-64ae}}\right.}{-16384a^2e^6-64ad^4e^3+5d^8}$$
$$\frac{64e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(-16384a^2e^6-64ad^4e^3+5d^8)\left(256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{192e\left(\frac{\left(d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\text{arctanh}\left(\frac{4e\left(\frac{d}{4e}+x\right)}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{8\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}-\frac{\left(-d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\text{arctanh}\left(\frac{4e\left(\frac{d}{4e}+x\right)}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{8\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{-16384a^2e^6-64ad^4e^3+5d^8}$$
$$\frac{64e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(-16384a^2e^6-64ad^4e^3+5d^8)\left(256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4\right)}$$

input
```
Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]
```

3.44. $\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^2}\,dx$

output
```
(64*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)^2))/((5
*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*((5*d^4)/e + 256*a*e^2 - 96*d^2*e*(d/
(4*e) + x)^2 + 256*e^3*(d/(4*e) + x)^4)) + (192*e*(-1/8*((d^4 + 128*a*e^3
- d^2*Sqrt[d^4 - 64*a*e^3])*ArcTanh[(4*e*(d/(4*e) + x))/Sqrt[3*d^2 - 2*Sqr
t[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 64*a*e^3]*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*
e^3]]) + ((d^4 + 128*a*e^3 + d^2*Sqrt[d^4 - 64*a*e^3])*ArcTanh[(4*e*(d/(4*
e) + x))/Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]]])/(8*Sqrt[d^4 - 64*a*e^3]*Sq
rt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^
6)
```

3.44.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.44.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.84

method	result
default	$\frac{\frac{12d^{2}e^{3}x^{3}}{\left(256a\,e^{3}+5d^{4}\right)\left(64a\,e^{3}-d^{4}\right)}+\frac{9d^{3}e^{2}x^{2}}{\left(256a\,e^{3}+5d^{4}\right)\left(64a\,e^{3}-d^{4}\right)}+\frac{ex}{256a\,e^{3}+5d^{4}}+\frac{d\left(128a\,e^{3}-5d^{4}\right)}{131072a^{2}e^{6}+512a\,d^{4}e^{3}-40d^{8}}}{e^{3}x^{4}+d\,e^{2}x^{3}-\frac{1}{8}d^{3}x+a\,e^{2}}+\frac{384e^{2}\left(\sum_{\_R=\mathrm{RootOf}\left(8e^{3}\_Z^{4}+\right.}\right.}{}$
risch	$\frac{\frac{12d^{2}e^{3}x^{3}}{\left(256a\,e^{3}+5d^{4}\right)\left(64a\,e^{3}-d^{4}\right)}+\frac{9d^{3}e^{2}x^{2}}{\left(256a\,e^{3}+5d^{4}\right)\left(64a\,e^{3}-d^{4}\right)}+\frac{ex}{256a\,e^{3}+5d^{4}}+\frac{d\left(128a\,e^{3}-5d^{4}\right)}{131072a^{2}e^{6}+512a\,d^{4}e^{3}-40d^{8}}}{e^{3}x^{4}+d\,e^{2}x^{3}-\frac{1}{8}d^{3}x+a\,e^{2}}+48e^{2}\left(\sum_{\_R=\mathrm{RootOf}\left(8e^{3}\_Z\right.}\right.$

input
```
int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(12*d^2*e^3/(256*a*e^3+5*d^4)/(64*a*e^3-d^4)*x^3+9*d^3*e^2/(256*a*e^3+5*d^
4)/(64*a*e^3-d^4)*x^2+e/(256*a*e^3+5*d^4)*x+1/8*d*(128*a*e^3-5*d^4)/(16384
*a^2*e^6+64*a*d^4*e^3-5*d^8))/(e^3*x^4+d*e^2*x^3-1/8*d^3*x+a*e^2)+384*e^2/
(2048*a*e^3+40*d^4)/(64*a*e^3-d^4)*sum((2*_R^2*d^2*e+_R*d^3+32*a*e^2)/(32*
_R^3*e^3+24*_R^2*d*e^2-d^3)*ln(x-_R),_R=RootOf(8*_Z^4*e^3+8*_Z^3*d*e^2-_Z*
d^3+8*a*e^2))
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4285 vs. 2(316) = 632.

Time = 0.48 (sec) , antiderivative size = 4285, normalized size of antiderivative = 12.53

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2}\, dx = \text{Too large to display}$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")
```

output
```
-(96*d^2*e^3*x^3 + 72*d^3*e^2*x^2 - 5*d^5 + 128*a*d*e^3 + 12*sqrt(2)*(40*a
*d^8*e^2 - 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6
- 16384*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3
- (5*d^11 - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^
6*e^5 + 40960*a^2*d^2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16
*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d
^4*e^15 - 4398046511104*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^
2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 211353
60000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*
e^15 + 2744381022928896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5
188146770730811392*a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^
24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825
205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*
log(884736*a*d^5*e^6 + 226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*
e^10)*x + 13824*sqrt(2)*(d^16*e^2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8
388608*a^3*d^4*e^11 - 268435456*a^4*e^14 - (125*d^30 + 59200*a*d^26*e^3 -
3624960*a^2*d^22*e^6 - 566493184*a^3*d^18*e^9 + 19797114880*a^4*d^14*e^12
+ 1906965479424*a^5*d^10*e^15 - 30786325577728*a^6*d^6*e^18 - 225179981368
5248*a^7*d^2*e^21)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*
d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^...
```

3.44.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
```

output
```
Timed out
```

3.44.7 Maxima [F]

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2} \, dx = \int \frac{1}{(8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2)^2} \, dx$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")
```

output
```
-48*e^2*integrate((2*d^2*e*x^2 + d^3*x + 32*a*e^2)/(8*e^3*x^4 + 8*d*e^2*x^
3 - d^3*x + 8*a*e^2), x)/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6) - (96*d^2*
e^3*x^3 + 72*d^3*e^2*x^2 - 5*d^5 + 128*a*d*e^3 - 8*(d^4*e - 64*a*e^4)*x)/(
40*a*d^8*e^2 - 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*
e^6 - 16384*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*
x^3 - (5*d^11 - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)
```

3.44.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1115 vs. $2(316) = 632$.

Time = 0.32 (sec) , antiderivative size = 1115, normalized size of antiderivative = 3.26

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")
```

output
```
12*((d^2*e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^2
- 2*d^3*e^2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e) + 2
56*a*e^4)*log(x + 1/4*sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) +
 1/4*d/e)/(e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^
3 - 3*d*e^2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^2 +
 2*d^3) - (d^2*e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d
/e)^2 + 2*d^3*e^2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/
e) + 256*a*e^4)*log(x - 1/4*sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/
e^4) + 1/4*d/e)/(e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) -
 d/e)^3 + 3*d*e^2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/
e)^2 - 2*d^3) + (d^2*e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^
4) + d/e)^2 - 2*d^3*e^2*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4
) + d/e) + 256*a*e^4)*log(x + 1/4*sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)
*e^2)/e^4) + 1/4*d/e)/(e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/
e^4) + d/e)^3 - 3*d*e^2*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4
) + d/e)^2 + 2*d^3) - (d^2*e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e
^2)/e^4) - d/e)^2 + 2*d^3*e^2*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^
2)/e^4) - d/e) + 256*a*e^4)*log(x - 1/4*sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*
a*e^3)*e^2)/e^4) + 1/4*d/e)/(e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)
*e^2)/e^4) - d/e)^3 + 3*d*e^2*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)...
```

3.44.9 Mupad [B] (verification not implemented)

Time = 13.83 (sec) , antiderivative size = 10351, normalized size of antiderivative = 30.27

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^2,x)
```

output
```
((8*e*x)/(256*a*e^3 + 5*d^4) - (5*d^5 - 128*a*d*e^3)/((64*a*e^3 - d^4)*(25
6*a*e^3 + 5*d^4)) + (72*d^3*e^2*x^2)/((64*a*e^3 - d^4)*(256*a*e^3 + 5*d^4)
) + (96*d^2*e^3*x^3)/((64*a*e^3 - d^4)*(256*a*e^3 + 5*d^4)))/(8*a*e^2 - d^
3*x + 8*e^3*x^4 + 8*d*e^2*x^3) + atan((((288*(d^22*e^2 + d^4*e^2*(-(64*a*e
^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10
*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(6
4*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800
*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*
a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 -
 791648371998720*a^7*d^8*e^21 - 4053239664633464*a^8*d^4*e^24))^(1/2)*(((
1536*(68719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d
^12*e^15 - 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 171
79869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d
^8*e^9 + 134217728*a^4*d^4*e^12) - ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11
+ 24576*a^2*d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 -
 25769803776*a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 171798
69184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*
e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 -
118784*a^2*d^14*e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 1
7179869184*a^5*d^2*e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^...
```

3.44. $\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^2}\,dx$

3.45 $\int (8+8x-x^3+8x^4)^4 \, dx$

3.45.1 Optimal result

Integrand size = 17, antiderivative size = 96

$$\int (8+8x-x^3+8x^4)^4 \, dx = 4096x+8192x^2+8192x^3+3584x^4+\frac{14336x^5}{5}+7168x^6+6784x^7+1376x^8+1408x^9+\frac{21488x^{10}}{5}+\frac{25312x^{11}}{11}-448x^{12}+\frac{10241x^{13}}{13}+1168x^{14}+\frac{128x^{15}}{5}-128x^{16}+\frac{4096x^{17}}{17}$$

output
```
4096*x+8192*x^2+8192*x^3+3584*x^4+14336/5*x^5+7168*x^6+6784*x^7+1376*x^8+1
408*x^9+21488/5*x^10+25312/11*x^11-448*x^12+10241/13*x^13+1168*x^14+128/5*
x^15-128*x^16+4096/17*x^17
```

3.45.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int (8+8x-x^3+8x^4)^4 \, dx = 4096x+8192x^2+8192x^3+3584x^4+\frac{14336x^5}{5}+7168x^6+6784x^7+1376x^8+1408x^9+\frac{21488x^{10}}{5}+\frac{25312x^{11}}{11}-448x^{12}+\frac{10241x^{13}}{13}+1168x^{14}+\frac{128x^{15}}{5}-128x^{16}+\frac{4096x^{17}}{17}$$

input
```
Integrate[(8 + 8*x - x^3 + 8*x^4)^4,x]
```

output
```
4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*
x^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 +
(10241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

3.45.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8x^4-x^3+8x+8\right)^4 dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(4096x^{16}-2048x^{15}+384x^{14}+16352x^{13}+10241x^{12}-5376x^{11}+25312x^{10}+42976x^9+12672x^8+11008x^7\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7168x^6+\frac{14336x^5}{5}+3584x^4+8192x^3+8192x^2+4096x$$

input
```
Int[(8 + 8*x - x^3 + 8*x^4)^4,x]
```

output
```
4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*
x^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 +
(10241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

3.45.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

3.45.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

method	result
gosper	$4096x + 8192x^2 + 8192x^3 + 3584x^4 + \frac{14336}{5}x^5 + 7168x^6 + 6784x^7 + 1376x^8 + 1408x^9 + \frac{214}{5}$
default	$4096x + 8192x^2 + 8192x^3 + 3584x^4 + \frac{14336}{5}x^5 + 7168x^6 + 6784x^7 + 1376x^8 + 1408x^9 + \frac{214}{5}$
norman	$4096x + 8192x^2 + 8192x^3 + 3584x^4 + \frac{14336}{5}x^5 + 7168x^6 + 6784x^7 + 1376x^8 + 1408x^9 + \frac{214}{5}$
risch	$4096x + 8192x^2 + 8192x^3 + 3584x^4 + \frac{14336}{5}x^5 + 7168x^6 + 6784x^7 + 1376x^8 + 1408x^9 + \frac{214}{5}$
parallelrisch	$4096x + 8192x^2 + 8192x^3 + 3584x^4 + \frac{14336}{5}x^5 + 7168x^6 + 6784x^7 + 1376x^8 + 1408x^9 + \frac{214}{5}$

input
```
int((8*x^4-x^3+8*x+8)^4,x,method=_RETURNVERBOSE)
```

output
```
4096*x+8192*x^2+8192*x^3+3584*x^4+14336/5*x^5+7168*x^6+6784*x^7+1376*x^8+1
408*x^9+21488/5*x^10+25312/11*x^11-448*x^12+10241/13*x^13+1168*x^14+128/5*
x^15-128*x^16+4096/17*x^17
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \left(8+8x-x^3+8x^4\right)^4 \, dx = \frac{4096}{17}\,x^{17} - 128\,x^{16} + \frac{128}{5}\,x^{15} + 1168\,x^{14} + \frac{10241}{13}\,x^{13} - 448\,x^{12} + \frac{25312}{11}\,x^{11} + \frac{21488}{5}\,x^{10} + 1408\,x^9 + 1376\,x^8 + 6784\,x^7 + 7168\,x^6 + \frac{14336}{5}\,x^5 + 3584\,x^4 + 8192\,x^3 + 8192\,x^2 + 4096\,x$$

input
```
integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="fricas")
```

output
```
4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^1
2 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x
^6 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

3.45.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

$$\int \left(8+8x-x^3+8x^4\right)^4 \, dx = \frac{4096x^{17}}{17} - 128x^{16} + \frac{128x^{15}}{5} + 1168x^{14} + \frac{10241x^{13}}{13} - 448x^{12} + \frac{25312x^{11}}{11} + \frac{21488x^{10}}{5} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + \frac{14336x^5}{5} + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

input
```
integrate((8*x**4-x**3+8*x+8)**4,x)
```

output
```
4096*x**17/17 - 128*x**16 + 128*x**15/5 + 1168*x**14 + 10241*x**13/13 - 44
8*x**12 + 25312*x**11/11 + 21488*x**10/5 + 1408*x**9 + 1376*x**8 + 6784*x*
*7 + 7168*x**6 + 14336*x**5/5 + 3584*x**4 + 8192*x**3 + 8192*x**2 + 4096*x
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \left(8+8x-x^3+8x^4\right)^4 \, dx = \frac{4096}{17}\,x^{17} - 128\,x^{16} + \frac{128}{5}\,x^{15} + 1168\,x^{14} + \frac{10241}{13}\,x^{13} - 448\,x^{12} + \frac{25312}{11}\,x^{11} + \frac{21488}{5}\,x^{10} + 1408\,x^9 + 1376\,x^8 + 6784\,x^7 + 7168\,x^6 + \frac{14336}{5}\,x^5 + 3584\,x^4 + 8192\,x^3 + 8192\,x^2 + 4096\,x$$

input
```
integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="maxima")
```

output
```
4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^1
2 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x
^6 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int (8+8x-x^3+8x^4)^4\,dx = \frac{4096}{17}\,x^{17}-128\,x^{16}+\frac{128}{5}\,x^{15}+1168\,x^{14}+\frac{10241}{13}\,x^{13}-448\,x^{12}+\frac{25312}{11}\,x^{11}+\frac{21488}{5}\,x^{10}+1408\,x^9+1376\,x^8+6784\,x^7+7168\,x^6+\frac{14336}{5}\,x^5+3584\,x^4+8192\,x^3+8192\,x^2+4096\,x$$

input
```
integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="giac")
```

output
```
4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^1
2 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x
^6 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int (8+8x-x^3+8x^4)^4\,dx = \frac{4096\,x^{17}}{17}-128\,x^{16}+\frac{128\,x^{15}}{5}+1168\,x^{14}+\frac{10241\,x^{13}}{13}-448\,x^{12}+\frac{25312\,x^{11}}{11}+\frac{21488\,x^{10}}{5}+1408\,x^9+1376\,x^8+6784\,x^7+7168\,x^6+\frac{14336\,x^5}{5}+3584\,x^4+8192\,x^3+8192\,x^2+4096\,x$$

input
```
int((8*x - x^3 + 8*x^4 + 8)^4,x)
```

output
```
4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*
x^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 +
(10241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

3.46 $\int (8+8x-x^3+8x^4)^3\,dx$

3.46.1 Optimal result

Integrand size = 17, antiderivative size = 74

$$\int (8+8x-x^3+8x^4)^3\,dx = 512x+768x^2+512x^3+80x^4+\frac{1152x^5}{5}+480x^6+\frac{1560x^7}{7} - 45x^8+128x^9+\frac{307x^{10}}{2}+\frac{24x^{11}}{11}-16x^{12}+\frac{512x^{13}}{13}$$

output
```
512*x+768*x^2+512*x^3+80*x^4+1152/5*x^5+480*x^6+1560/7*x^7-45*x^8+128*x^9+
307/2*x^10+24/11*x^11-16*x^12+512/13*x^13
```

3.46.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int (8+8x-x^3+8x^4)^3\,dx = 512x+768x^2+512x^3+80x^4+\frac{1152x^5}{5}+480x^6+\frac{1560x^7}{7} - 45x^8+128x^9+\frac{307x^{10}}{2}+\frac{24x^{11}}{11}-16x^{12}+\frac{512x^{13}}{13}$$

input
```
Integrate[(8 + 8*x - x^3 + 8*x^4)^3,x]
```

output
```
512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
 - 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/1
3
```

3.46.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8x^4-x^3+8x+8\right)^3 dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(512x^{12}-192x^{11}+24x^{10}+1535x^9+1152x^8-360x^7+1560x^6+2880x^5+1152x^4+320x^3+1536x^2+1536$$

$$\Big\downarrow \text{2009}$$

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

input
```
Int[(8 + 8*x - x^3 + 8*x^4)^3,x]
```

output
```
512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
 - 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/1
3
```

3.46.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

3.46.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

method	result
gosper	$512x+768x^2+512x^3+80x^4+\frac{1152}{5}x^5+480x^6+\frac{1560}{7}x^7-45x^8+128x^9+\frac{307}{2}x^{10}+\frac{24}{11}x^{11}$
default	$512x+768x^2+512x^3+80x^4+\frac{1152}{5}x^5+480x^6+\frac{1560}{7}x^7-45x^8+128x^9+\frac{307}{2}x^{10}+\frac{24}{11}x^{11}$
norman	$512x+768x^2+512x^3+80x^4+\frac{1152}{5}x^5+480x^6+\frac{1560}{7}x^7-45x^8+128x^9+\frac{307}{2}x^{10}+\frac{24}{11}x^{11}$
risch	$512x+768x^2+512x^3+80x^4+\frac{1152}{5}x^5+480x^6+\frac{1560}{7}x^7-45x^8+128x^9+\frac{307}{2}x^{10}+\frac{24}{11}x^{11}$
parallelrisch	$512x+768x^2+512x^3+80x^4+\frac{1152}{5}x^5+480x^6+\frac{1560}{7}x^7-45x^8+128x^9+\frac{307}{2}x^{10}+\frac{24}{11}x^{11}$

input
```
int((8*x^4-x^3+8*x+8)^3,x,method=_RETURNVERBOSE)
```

output
```
512*x+768*x^2+512*x^3+80*x^4+1152/5*x^5+480*x^6+1560/7*x^7-45*x^8+128*x^9+
307/2*x^10+24/11*x^11-16*x^12+512/13*x^13
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \left(8+8x-x^3+8x^4\right)^3\,dx=\frac{512}{13}\,x^{13}-16\,x^{12}+\frac{24}{11}\,x^{11}+\frac{307}{2}\,x^{10}+128\,x^9-45\,x^8+\frac{1560}{7}\,x^7+480\,x^6+\frac{1152}{5}\,x^5+80\,x^4+512\,x^3+768\,x^2+512\,x$$

input
```
integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="fricas")
```

output
```
512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/
7*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

3.46.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \left(8+8x-x^3+8x^4\right)^3 \, dx = \frac{512x^{13}}{13} - 16x^{12} + \frac{24x^{11}}{11} + \frac{307x^{10}}{2} + 128x^9 - 45x^8 + \frac{1560x^7}{7} + 480x^6 + \frac{1152x^5}{5} + 80x^4 + 512x^3 + 768x^2 + 512x$$

input
```
integrate((8*x**4-x**3+8*x+8)**3,x)
```

output
```
512*x**13/13 - 16*x**12 + 24*x**11/11 + 307*x**10/2 + 128*x**9 - 45*x**8 +
 1560*x**7/7 + 480*x**6 + 1152*x**5/5 + 80*x**4 + 512*x**3 + 768*x**2 + 51
2*x
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \left(8+8x-x^3+8x^4\right)^3 \, dx = \frac{512}{13}\,x^{13} - 16\,x^{12} + \frac{24}{11}\,x^{11} + \frac{307}{2}\,x^{10} + 128\,x^9 - 45\,x^8 + \frac{1560}{7}\,x^7 + 480\,x^6 + \frac{1152}{5}\,x^5 + 80\,x^4 + 512\,x^3 + 768\,x^2 + 512\,x$$

input
```
integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="maxima")
```

output
```
512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/
7*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \left(8+8x-x^3+8x^4\right)^3 \, dx = \frac{512}{13}\,x^{13} - 16\,x^{12} + \frac{24}{11}\,x^{11} + \frac{307}{2}\,x^{10} + 128\,x^9 - 45\,x^8 + \frac{1560}{7}\,x^7 + 480\,x^6 + \frac{1152}{5}\,x^5 + 80\,x^4 + 512\,x^3 + 768\,x^2 + 512\,x$$

input
```
integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="giac")
```

output
```
512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/
7*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

3.46.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int (8+8x-x^3+8x^4)^3 \,dx = \frac{512\,x^{13}}{13} - 16\,x^{12} + \frac{24\,x^{11}}{11} + \frac{307\,x^{10}}{2} + 128\,x^9 - 45\,x^8 + \frac{1560\,x^7}{7} + 480\,x^6 + \frac{1152\,x^5}{5} + 80\,x^4 + 512\,x^3 + 768\,x^2 + 512\,x$$

input
```
int((8*x - x^3 + 8*x^4 + 8)^3,x)
```

output
```
512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
 - 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/1
3
```

3.47 $\int (8+8x-x^3+8x^4)^2 \, dx$

3.47.1 Optimal result

Integrand size = 17, antiderivative size = 54

$$\int (8+8x-x^3+8x^4)^2 \, dx = 64x+64x^2+\frac{64x^3}{3}-4x^4+\frac{112x^5}{5}+\frac{64x^6}{3}+\frac{x^7}{7}-2x^8+\frac{64x^9}{9}$$

output
```
64*x+64*x^2+64/3*x^3-4*x^4+112/5*x^5+64/3*x^6+1/7*x^7-2*x^8+64/9*x^9
```

3.47.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int (8+8x-x^3+8x^4)^2 \, dx = 64x+64x^2+\frac{64x^3}{3}-4x^4+\frac{112x^5}{5}+\frac{64x^6}{3}+\frac{x^7}{7}-2x^8+\frac{64x^9}{9}$$

input
```
Integrate[(8 + 8*x - x^3 + 8*x^4)^2,x]
```

output
```
64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*
x^8 + (64*x^9)/9
```

3.47.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8x^4-x^3+8x+8\right)^2 dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(64x^8-16x^7+x^6+128x^5+112x^4-16x^3+64x^2+128x+64\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

input
```
Int[(8 + 8*x - x^3 + 8*x^4)^2,x]
```

output
```
64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*
x^8 + (64*x^9)/9
```

3.47.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

3.47.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
gosper	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45
default	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45
norman	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45
risch	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45
parallelrisch	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45

input
```
int((8*x^4-x^3+8*x+8)^2,x,method=_RETURNVERBOSE)
```

output
```
64*x+64*x^2+64/3*x^3-4*x^4+112/5*x^5+64/3*x^6+1/7*x^7-2*x^8+64/9*x^9
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(8+8x-x^3+8x^4\right)^2\,dx=\frac{64}{9}\,x^9-2\,x^8+\frac{1}{7}\,x^7+\frac{64}{3}\,x^6+\frac{112}{5}\,x^5-4\,x^4+\frac{64}{3}\,x^3+64\,x^2+64\,x$$

input
```
integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="fricas")
```

output
```
64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*
x^2 + 64*x
```

3.47.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \left(8+8x-x^3+8x^4\right)^2\,dx=\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

input
```
integrate((8*x**4-x**3+8*x+8)**2,x)
```

output
```
64*x**9/9 - 2*x**8 + x**7/7 + 64*x**6/3 + 112*x**5/5 - 4*x**4 + 64*x**3/3
+ 64*x**2 + 64*x
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int (8+8x-x^3+8x^4)^2 \, dx = \frac{64}{9}\,x^9 - 2\,x^8 + \frac{1}{7}\,x^7 + \frac{64}{3}\,x^6 + \frac{112}{5}\,x^5 - 4\,x^4 + \frac{64}{3}\,x^3 + 64\,x^2 + 64\,x$$

input
```
integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")
```

output
```
64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*
x^2 + 64*x
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int (8+8x-x^3+8x^4)^2 \, dx = \frac{64}{9}\,x^9 - 2\,x^8 + \frac{1}{7}\,x^7 + \frac{64}{3}\,x^6 + \frac{112}{5}\,x^5 - 4\,x^4 + \frac{64}{3}\,x^3 + 64\,x^2 + 64\,x$$

input
```
integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="giac")
```

output
```
64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*
x^2 + 64*x
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int (8+8x-x^3+8x^4)^2 \, dx = \frac{64\,x^9}{9} - 2\,x^8 + \frac{x^7}{7} + \frac{64\,x^6}{3} + \frac{112\,x^5}{5} - 4\,x^4 + \frac{64\,x^3}{3} + 64\,x^2 + 64\,x$$

input
```
int((8*x - x^3 + 8*x^4 + 8)^2,x)
```

output
```
64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*
x^8 + (64*x^9)/9
```

3.48 $\int (8+8x-x^3+8x^4)\,dx$

3.48.1 Optimal result

Integrand size = 15, antiderivative size = 23

$$\int (8+8x-x^3+8x^4)\,dx = 8x+4x^2-\frac{x^4}{4}+\frac{8x^5}{5}$$

output
```
8*x+4*x^2-1/4*x^4+8/5*x^5
```

3.48.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (8+8x-x^3+8x^4)\,dx = 8x+4x^2-\frac{x^4}{4}+\frac{8x^5}{5}$$

input
```
Integrate[8 + 8*x - x^3 + 8*x^4,x]
```

output
```
8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

3.48.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8x^4 - x^3 + 8x + 8\right) dx$$
$$\big\downarrow \text{ 2009}$$
$$\frac{8x^5}{5} - \frac{x^4}{4} + 4x^2 + 8x$$

input
```
Int[8 + 8*x - x^3 + 8*x^4,x]
```

output
```
8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

3.48.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.48.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
gosper	$8x + 4x^2 - \frac{1}{4}x^4 + \frac{8}{5}x^5$	20
default	$8x + 4x^2 - \frac{1}{4}x^4 + \frac{8}{5}x^5$	20
norman	$8x + 4x^2 - \frac{1}{4}x^4 + \frac{8}{5}x^5$	20
risch	$8x + 4x^2 - \frac{1}{4}x^4 + \frac{8}{5}x^5$	20
parallelrisch	$8x + 4x^2 - \frac{1}{4}x^4 + \frac{8}{5}x^5$	20
parts	$8x + 4x^2 - \frac{1}{4}x^4 + \frac{8}{5}x^5$	20

input
```
int(8*x^4-x^3+8*x+8,x,method=_RETURNVERBOSE)
```

output
```
8*x+4*x^2-1/4*x^4+8/5*x^5
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(8+8x-x^3+8x^4\right)\,dx=\frac{8}{5}\,x^{5}-\frac{1}{4}\,x^{4}+4\,x^{2}+8\,x$$

input
```
integrate(8*x^4-x^3+8*x+8,x, algorithm="fricas")
```

output
```
8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

3.48.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(8+8x-x^3+8x^4\right)\,dx=\frac{8x^{5}}{5}-\frac{x^{4}}{4}+4x^{2}+8x$$

input
```
integrate(8*x**4-x**3+8*x+8,x)
```

output
```
8*x**5/5 - x**4/4 + 4*x**2 + 8*x
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(8+8x-x^3+8x^4\right)\,dx=\frac{8}{5}\,x^{5}-\frac{1}{4}\,x^{4}+4\,x^{2}+8\,x$$

input
```
integrate(8*x^4-x^3+8*x+8,x, algorithm="maxima")
```

output
```
8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(8+8x-x^3+8x^4\right)\,dx=\frac{8}{5}\,x^5-\frac{1}{4}\,x^4+4\,x^2+8\,x$$

input
```
integrate(8*x^4-x^3+8*x+8,x, algorithm="giac")
```

output
```
8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

3.48.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(8+8x-x^3+8x^4\right)\,dx=\frac{8\,x^5}{5}-\frac{x^4}{4}+4\,x^2+8\,x$$

input
```
int(8*x - x^3 + 8*x^4 + 8,x)
```

output
```
8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

3.49 $\int \frac{1}{8+8x-x^3+8x^4} \, dx$

3.49.1 Optimal result

Integrand size = 17, antiderivative size = 268

$$\begin{aligned}\int \frac{1}{8+8x-x^3+8x^4} \, dx = &-\frac{\arctan\left(\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right)}{12\sqrt{7}} \\ &-\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\arctan\left(\frac{2-\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right) \\ &-\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\arctan\left(\frac{2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right) \\ &-\frac{1}{24}\sqrt{\frac{-109+67\sqrt{29}}{1218}}\log\left(3\sqrt{29}-\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right) \right. \\ &\left. +\left(1+\frac{4}{x}\right)^2\right)+\frac{1}{24}\sqrt{\frac{-109+67\sqrt{29}}{1218}}\log\left(3\sqrt{29} \right. \\ &\left. +\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right)\end{aligned}$$

output
```
-1/84*arctan(1/42*(3-(1+4/x)^2)*7^(1/2))*7^(1/2)-1/29232*ln((1+4/x)^2+3*29
^(1/2)-(1+4/x)*(6+6*29^(1/2))^(1/2))*(-132762+81606*29^(1/2))^(1/2)+1/2923
2*ln((1+4/x)^2+3*29^(1/2)+(1+4/x)*(6+6*29^(1/2))^(1/2))*(-132762+81606*29^
(1/2))^(1/2)-1/14616*arctan((2+8/x-(6+6*29^(1/2))^(1/2))/(-6+6*29^(1/2))^(
1/2))*(132762+81606*29^(1/2))^(1/2)-1/14616*arctan((2+8/x+(6+6*29^(1/2))^(
1/2))/(-6+6*29^(1/2))^(1/2))*(132762+81606*29^(1/2))^(1/2)
```

3.49.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.17

$$\int \frac{1}{8+8x-x^3+8x^4} \, dx = \text{RootSum}\left[8+8\#1-\#1^3+8\#1^4\&,\frac{\log(x-\#1)}{8-3\#1^2+32\#1^3}\&\right]$$

input
```
Integrate[(8 + 8*x - x^3 + 8*x^4)^(-1),x]
```

output
```
RootSum[8 + 8*#1 - #1^3 + 8*#1^4 & , Log[x - #1]/(8 - 3*#1^2 + 32*#1^3) &
]
```

3.49.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.20, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.824, Rules used = {2504, 27, 2202, 27, 1432, 1083, 217, 1483, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{8x^4-x^3+8x+8} \, dx$$

$$\Big\downarrow \text{2504}$$

$$-1024\int \frac{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^2}{512\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)}d\left(\frac{1}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$-2\int \frac{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^2}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \text{2202}$$

$$-2\left(\int \frac{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)+\int -\frac{8\left(\frac{1}{4}+\frac{1}{x}\right)}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$-2\left(\int \frac{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)-8\int \frac{\frac{1}{4}+\frac{1}{x}}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)\right)$$

$$\Big\downarrow \text{1432}$$

$$-2\left(\int \frac{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)-4\int \frac{1}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)^2\right)$$

$$\Big\downarrow \text{1083}$$

$$-2\left(8\int \frac{1}{-\left(\frac{1}{4}+\frac{1}{x}\right)^4-258048}d\left(512\left(\frac{1}{4}+\frac{1}{x}\right)^2-96\right)+\int \frac{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$-2\left(\int \frac{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)-\frac{\arctan\left(\frac{512\left(\frac{1}{x}+\frac{1}{4}\right)^2-96}{192\sqrt{7}}\right)}{24\sqrt{7}}\right)$$

$$\Big\downarrow \text{1483}$$

$$-2\left(\frac{\int \frac{8\left(\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}-2\left(1-3\sqrt{29}\right)\left(\frac{1}{4}+\frac{1}{x}\right)\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{24\sqrt{174\left(1+\sqrt{29}\right)}}+\frac{\int \frac{8\left(2\left(1-3\sqrt{29}\right)\left(\frac{1}{4}+\frac{1}{x}\right)+\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{24\sqrt{174\left(1+\sqrt{29}\right)}}-\frac{\arctan\left(\frac{512(\cdots}{\cdots}\right.}{24\cdots}\right.$$

$$\Big\downarrow \text{27}$$

3.49. $\int \frac{1}{8+8x-x^3+8x^4}\,dx$

$$-2\left(\frac{\int \frac{\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}-2\left(1-3\sqrt{29}\right)\left(\frac{1}{4}+\frac{1}{x}\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{3\sqrt{174\left(1+\sqrt{29}\right)}}+\frac{\int \frac{2\left(1-3\sqrt{29}\right)\left(\frac{1}{4}+\frac{1}{x}\right)+\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{3\sqrt{174\left(1+\sqrt{29}\right)}}-\frac{\arctan\left(\frac{512(\cdots}{\cdots}\right.}{24\cdots}\cdots\right.$$

$$\Big\downarrow \text{1142}$$

$$-2\left(\frac{\sqrt{\frac{3}{2}\left(109+67\sqrt{29}\right)}\int \frac{1}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)-\frac{1}{16}\left(1-3\sqrt{29}\right)\int -\frac{4\left(\sqrt{6\left(1+\sqrt{29}\right)}-8\left(\frac{1}{4}\cdots\right.\right.}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\cdots\right.}\cdots}{3\sqrt{174\left(1+\sqrt{29}\right)}}\cdots\right.$$

$$\Big\downarrow \text{27}$$

$$-2\left(\frac{\sqrt{\frac{3}{2}\left(109+67\sqrt{29}\right)}\int \frac{1}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)+\frac{1}{4}\left(1-3\sqrt{29}\right)\int \frac{\sqrt{6\left(1+\sqrt{29}\right)}-8\left(\frac{1}{4}+\frac{1}{x}\cdots\right.}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\cdots\right.}\cdots}{3\sqrt{174\left(1+\sqrt{29}\right)}}\cdots\right.$$

$$\Big\downarrow \text{1083}$$

$$-2\left(\frac{\frac{1}{4}\left(1-3\sqrt{29}\right)\int \frac{\sqrt{6\left(1+\sqrt{29}\right)}-8\left(\frac{1}{4}+\frac{1}{x}\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)-\sqrt{6\left(109+67\sqrt{29}\right)}\int \frac{1}{96\left(1-\sqrt{29}\right)-\left(32\left(\frac{1}{4}+\frac{1}{x}\right)-4\sqrt{6\cdots}\right.}\cdots}{3\sqrt{174\left(1+\sqrt{29}\right)}}\cdots\right.$$

$$\Big\downarrow \text{217}$$

$$-2\left(\frac{\frac{1}{4}\left(1-3\sqrt{29}\right)\int \frac{\sqrt{6\left(1+\sqrt{29}\right)}-8\left(\frac{1}{4}+\frac{1}{x}\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)+\frac{1}{4}\sqrt{\frac{109+67\sqrt{29}}{\sqrt{29}-1}}\arctan\left(\frac{32\left(\frac{1}{x}+\frac{1}{4}\right)-4\sqrt{6\left(1+\sqrt{29}\right)}}{4\sqrt{6\left(\sqrt{29}-1\right)}}\right)}{3\sqrt{174\left(1+\sqrt{29}\right)}}\cdots\right.$$

$$\Big\downarrow \text{1103}$$

3.49. $\int \frac{1}{8+8x-x^3+8x^4}\,dx$

$$-2\left(-\frac{\arctan\left(\frac{512\left(\frac{1}{x}+\frac{1}{4}\right)^2-96}{192\sqrt{7}}\right)}{24\sqrt{7}}+\frac{\frac{1}{4}\sqrt{\frac{109+67\sqrt{29}}{\sqrt{29}-1}}\arctan\left(\frac{32\left(\frac{1}{x}+\frac{1}{4}\right)-4\sqrt{6\left(1+\sqrt{29}\right)}}{4\sqrt{6\left(\sqrt{29}-1\right)}}\right)-\frac{1}{16}\left(1-3\sqrt{29}\right)\log\left(16\left(\frac{1}{x}+\frac{1}{4}\right)^2\right.}{3\sqrt{174\left(1+\sqrt{29}\right)}}\right.$$

input
```
Int[(8 + 8*x - x^3 + 8*x^4)^(-1),x]
```

output
```
-2*(-1/24*ArcTan[(-96 + 512*(1/4 + x^(-1))^2)/(192*Sqrt[7])]/Sqrt[7] + ((S
qrt[(109 + 67*Sqrt[29])/(-1 + Sqrt[29])]*ArcTan[(-4*Sqrt[6*(1 + Sqrt[29])]
 + 32*(1/4 + x^(-1)))/(4*Sqrt[6*(-1 + Sqrt[29])])])/4 - ((1 - 3*Sqrt[29])*
Log[3*Sqrt[29] - 4*Sqrt[6*(1 + Sqrt[29])]*(1/4 + x^(-1)) + 16*(1/4 + x^(-1
))^2])/16)/(3*Sqrt[174*(1 + Sqrt[29])]) + ((Sqrt[(109 + 67*Sqrt[29])/(-1 +
 Sqrt[29])]*ArcTan[(4*Sqrt[6*(1 + Sqrt[29])] + 32*(1/4 + x^(-1)))/(4*Sqrt[
6*(-1 + Sqrt[29])])])/4 + ((1 - 3*Sqrt[29])*Log[3*Sqrt[29] + 4*Sqrt[6*(1 +
 Sqrt[29])]*(1/4 + x^(-1)) + 16*(1/4 + x^(-1))^2])/16)/(3*Sqrt[174*(1 + Sq
rt[29])]))
```

3.49.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

3.49. $\int \frac{1}{8+8x-x^3+8x^4}\,dx$

3.49.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.15

method	result	size
default	$\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-\_Z^3+8\_Z+8\right)}\frac{\ln\left(x-\_R\right)}{32\_R^3-3\_R^2+8}$	41
risch	$\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-\_Z^3+8\_Z+8\right)}\frac{\ln\left(x-\_R\right)}{32\_R^3-3\_R^2+8}$	41

input
```
int(1/(8*x^4-x^3+8*x+8),x,method=_RETURNVERBOSE)
```

output
```
sum(1/(32*_R^3-3*_R^2+8)*ln(x-_R),_R=RootOf(8*_Z^4-_Z^3+8*_Z+8))
```

3.49.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 1015, normalized size of antiderivative = 3.79

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="fricas")
```

output
```
-1/168*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696))*log(28731419
5392*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^3 - 1203
8906880*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^2 + 1
6878104*x + 4897683*I*sqrt(7) - 411405372*sqrt(65/43848*I*sqrt(7) - 109/87
696) + 6055613) - 1/168*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/876
96))*log(-35914274424*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109
/87696))^3 + 16443*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I*sqrt(7) - 109/
87696))^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7) - 109/87696)
 - 91520) + 609*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 1
09/87696))^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/87696))
 + 2109763*x - 1911147/8*I*sqrt(7) + 40134087/2*sqrt(65/43848*I*sqrt(7) -
109/87696) - 1461344) + 1/1044*(sqrt(174)*sqrt(-4698*(1/168*I*sqrt(7) - 1/
2*sqrt(65/43848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*s
qrt(-65/43848*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/
43848*I*sqrt(7) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 1
09/87696)) - 7) + 261*sqrt(65/43848*I*sqrt(7) - 109/87696) + 261*sqrt(-65/
43848*I*sqrt(7) - 109/87696))*log(-16443/2*(-1/168*I*sqrt(7) - 1/2*sqrt(-6
5/43848*I*sqrt(7) - 109/87696))^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/4384
8*I*sqrt(7) - 109/87696) - 91520) - 609/2*(351027*(1/168*I*sqrt(7) - 1/2*s
qrt(65/43848*I*sqrt(7) - 109/87696))^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/...
```

3.49.6 Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.15

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx$$
$$= \text{RootSum}\left(66298176t^4+74088t^2+4095t+64, \left(t\mapsto t\log\left(\frac{35914274424t^3}{2109763}-\frac{1504863360t^2}{2109763}+\frac{1028513}{21097}\right.\right.\right.$$

input
```
integrate(1/(8*x**4-x**3+8*x+8),x)
```

output
```
RootSum(66298176*_t**4 + 74088*_t**2 + 4095*_t + 64, Lambda(_t, _t*log(359
14274424*_t**3/2109763 - 1504863360*_t**2/2109763 + 102851343*_t/2109763 +
 x + 6055613/16878104)))
```

3.49.7 Maxima [F]

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx = \int \frac{1}{8\,x^4-x^3+8\,x+8}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="maxima")
```

output
```
integrate(1/(8*x^4 - x^3 + 8*x + 8), x)
```

3.49.8 Giac [F]

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx = \int \frac{1}{8\,x^4-x^3+8\,x+8}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="giac")
```

output
```
integrate(1/(8*x^4 - x^3 + 8*x + 8), x)
```

3.49.9 Mupad [B] (verification not implemented)

Time = 10.58 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.46

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx$$
$$= \sum_{k=1}^{4} \ln\left(-\frac{\operatorname{root}\left(z^4+\frac{7\,z^2}{6264}+\frac{65\,z}{1052352}+\frac{1}{1035909},z,k\right)\,\left(8064\operatorname{root}\left(z^4+\frac{7\,z^2}{6264}+\frac{65\,z}{1052352}+\frac{1}{1035909},z,k\right)+256\,x\right.}{}\right.$$
$$\left. +\frac{7\,z^2}{6264}+\frac{65\,z}{1052352}+\frac{1}{1035909},z,k\right)$$

input
```
int(1/(8*x - x^3 + 8*x^4 + 8),x)
```

output
```
symsum(log(-(root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)*(
8064*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k) + 256*x +
 12285*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)*x + 148
176*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)^2*x + 1980
72*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)^2 - 8))/409
6)*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k), k, 1, 4)
```

3.49. $\int \frac{1}{8+8x-x^3+8x^4}\,dx$

3.50 $\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx$

3.50.1 Optimal result

Integrand size = 17, antiderivative size = 357

$$\begin{aligned}
&\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx \\
&= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}+\frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} \\
&\quad -\frac{17\arctan\left(\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right)}{1008\sqrt{7}}-\frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\arctan\left(\frac{2-\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right)}{87696} \\
&\quad -\frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\arctan\left(\frac{2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right)}{87696} \\
&\quad -\frac{\sqrt{\frac{-180983329+45923327\sqrt{29}}{1218}}\log\left(3\sqrt{29}-\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right)}{175392} \\
&\quad +\frac{\sqrt{\frac{-180983329+45923327\sqrt{29}}{1218}}\log\left(3\sqrt{29}+\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right)}{175392}
\end{aligned}$$

output
```
1/336*(-207-29*(1+4/x)^2)/(261-6*(1+4/x)^2+(1+4/x)^4)+5/87696*(5157+199*(1
+4/x)^2)*(1+4/x)/(261-6*(1+4/x)^2+(1+4/x)^4)-17/7056*arctan(1/42*(3-(1+4/x
)^2)*7^(1/2))*7^(1/2)-1/213627456*ln((1+4/x)^2+3*29^(1/2)-(1+4/x)*(6+6*29^
(1/2))^(1/2))*(-220437694722+55934612286*29^(1/2))^(1/2)+1/213627456*ln((1
+4/x)^2+3*29^(1/2)+(1+4/x)*(6+6*29^(1/2))^(1/2))*(-220437694722+5593461228
6*29^(1/2))^(1/2)-1/106813728*arctan((2+8/x-(6+6*29^(1/2))^(1/2))/(-6+6*29
^(1/2))^(1/2))*(220437694722+55934612286*29^(1/2))^(1/2)-1/106813728*arcta
n((2+8/x+(6+6*29^(1/2))^(1/2))/(-6+6*29^(1/2))^(1/2))*(220437694722+559346
12286*29^(1/2))^(1/2)
```

3.50.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.32

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx = \frac{544+1539x-1146x^2+784x^3}{43848\,(8+8x-x^3+8x^4)} + \frac{\text{RootSum}\left[8+8\#1-\#1^3+8\#1^4\&,\frac{2243\log(x-\#1)-1097\log(x-\#1)\#1+392\log(x-\#1)\#1^2}{8-3\#1^2+32\#1^3}\&\right]}{21924}$$

input
```
Integrate[(8 + 8*x - x^3 + 8*x^4)^(-2),x]
```

output
```
(544 + 1539*x - 1146*x^2 + 784*x^3)/(43848*(8 + 8*x - x^3 + 8*x^4)) + Root
Sum[8 + 8*#1 - #1^3 + 8*#1^4 & , (2243*Log[x - #1] - 1097*Log[x - #1]*#1 +
 392*Log[x - #1]*#1^2)/(8 - 3*#1^2 + 32*#1^3) & ]/21924
```

3.50.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.21, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.059$, Rules used = {2504, 27, 2202, 2194, 27, 2191, 27, 1083, 217, 2206, 27, 1483, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(8x^4-x^3+8x+8\right)^2}\,dx$$

$$\Big\downarrow \ 2504$$

$$-1024\int \frac{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^6}{4096\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^2}d\left(\frac{1}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{4}\int \frac{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^6}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^2}d\left(\frac{1}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \ 2202$$

$$\frac{1}{4}\left(-\int \frac{4096\left(\frac{1}{4}+\frac{1}{x}\right)^6+3840\left(\frac{1}{4}+\frac{1}{x}\right)^4+240\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^2}d\left(\frac{1}{4}+\frac{1}{x}\right)-\int \frac{\left(-6144\left(\frac{1}{4}+\frac{1}{x}\right)^4-1280\left(\frac{1}{4}+\frac{1}{x}\right)^2-2\right.}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+2\right.}\right.$$

$$\Big\downarrow \ 2194$$

$$\frac{1}{4}\left(-\frac{1}{2}\int -\frac{8\left(768\left(\frac{1}{4}+\frac{1}{x}\right)^4+160\left(\frac{1}{4}+\frac{1}{x}\right)^2+3\right)}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^2}d\left(\frac{1}{4}+\frac{1}{x}\right)^2-\int \frac{4096\left(\frac{1}{4}+\frac{1}{x}\right)^6+3840\left(\frac{1}{4}+\frac{1}{x}\right)^4+240\left(\frac{1}{4}+\frac{1}{x}\right)^2}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^2}\right.$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}\left(4\int \frac{768\left(\frac{1}{4}+\frac{1}{x}\right)^4+160\left(\frac{1}{4}+\frac{1}{x}\right)^2+3}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^2}d\left(\frac{1}{4}+\frac{1}{x}\right)^2-\int \frac{4096\left(\frac{1}{4}+\frac{1}{x}\right)^6+3840\left(\frac{1}{4}+\frac{1}{x}\right)^4+240\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^2}\right.$$

$$\Big\downarrow \ 2191$$

$$\frac{1}{4}\left(4\left(\frac{\int \frac{417792}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)^2}{258048}-\frac{464\left(\frac{1}{x}+\frac{1}{4}\right)^2+207}{336\left(256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261\right)}\right)-\int \frac{4096\left(\frac{1}{4}+\frac{1}{x}\right)^6+3}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right.\right.}\right.$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}\left(4\left(\frac{34}{21}\int \frac{1}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)^2-\frac{464\left(\frac{1}{x}+\frac{1}{4}\right)^2+207}{336\left(256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261\right)}\right)-\int \frac{40}{}\right.$$

$$\Big\downarrow \ 1083$$

3.50. $\int \frac{1}{\left(8+8x-x^3+8x^4\right)^2}\,dx$

$$\frac{1}{4}\left(4\left(-\frac{68}{21}\int \frac{1}{-\left(\frac{1}{4}+\frac{1}{x}\right)^4-258048}d\left(512\left(\frac{1}{4}+\frac{1}{x}\right)^2-96\right)-\frac{464\left(\frac{1}{x}+\frac{1}{4}\right)^2+207}{336\left(256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261\right)}\right)-\int \frac{4}{}\right.$$

$\downarrow$ 217

$$\frac{1}{4}\left(4\left(\frac{17\arctan\left(\frac{512\left(\frac{1}{x}+\frac{1}{4}\right)^2-96}{192\sqrt{7}}\right)}{1008\sqrt{7}}-\frac{464\left(\frac{1}{x}+\frac{1}{4}\right)^2+207}{336\left(256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261\right)}\right)-\int \frac{4096\left(\frac{1}{4}+\frac{1}{x}\right)^6+3840\left(\frac{1}{4}+\frac{1}{x}\right.}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}\right.\right.}\right.$$

$\downarrow$ 2206

$$\frac{1}{4}\left(-\frac{\int \frac{49152\left(35888\left(\frac{1}{4}+\frac{1}{x}\right)^2+12903\right)}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)}{134701056}+4\left(\frac{17\arctan\left(\frac{512\left(\frac{1}{x}+\frac{1}{4}\right)^2-96}{192\sqrt{7}}\right)}{1008\sqrt{7}}-\frac{464\left(\frac{1}{x}+\frac{1}{4}\right)^2+207}{336\left(256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+\right.}\right.\right.$$

$\downarrow$ 27

$$\frac{1}{4}\left(-\frac{2\int \frac{35888\left(\frac{1}{4}+\frac{1}{x}\right)^2+12903}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)}{5481}+4\left(\frac{17\arctan\left(\frac{512\left(\frac{1}{x}+\frac{1}{4}\right)^2-96}{192\sqrt{7}}\right)}{1008\sqrt{7}}-\frac{464\left(\frac{1}{x}+\frac{1}{4}\right)^2+207}{336\left(256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+\right.}\right.\right.$$

$\downarrow$ 1483

$$\frac{1}{4}\left(-\frac{2\left(\frac{\int \frac{24\left(4301\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}-2\left(4301-2243\sqrt{29}\right)\left(\frac{1}{4}+\frac{1}{x}\right)\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{24\sqrt{174\left(1+\sqrt{29}\right)}}+\frac{\int \frac{24\left(2\left(4301-2243\sqrt{29}\right)\left(\frac{1}{4}+\frac{1}{x}\right)+4301\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{24\sqrt{174\left(1+\sqrt{29}\right)}}\right)}{5481}+4\left(\frac{17}{}\right.\right.$$

$\downarrow$ 27

3.50. $\int \frac{1}{\left(8+8x-x^3+8x^4\right)^2}\,dx$

$$\frac{1}{4}\left(-\frac{2\left(\frac{\int \frac{4301\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}-2\left(4301-2243\sqrt{29}\right)\left(\frac{1}{4}+\frac{1}{x}\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d(\frac{1}{4}+\frac{1}{x})}{\sqrt{174\left(1+\sqrt{29}\right)}}+\frac{\int \frac{2\left(4301-2243\sqrt{29}\right)\left(\frac{1}{4}+\frac{1}{x}\right)+4301\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d(\frac{1}{4}+\frac{1}{x})}{\sqrt{174\left(1+\sqrt{29}\right)}}\right)}{5481}+4\left(\frac{17\arctan\ldots}{\ldots}\right.\right.$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{4}\left(-\frac{2\left(\frac{\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}\left(4301+2243\sqrt{29}\right)\int \frac{1}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d(\frac{1}{4}+\frac{1}{x})-\frac{1}{16}\left(4301-2243\sqrt{29}\right)\int -\frac{4\left(\sqrt{6\left(1+\sqrt{29}\right)}-8\ldots\right.}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\ldots}}{\sqrt{174\left(1+\sqrt{29}\right)}}\ldots\right.}{\ldots}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\left(-\frac{2\left(\frac{\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}\left(4301+2243\sqrt{29}\right)\int \frac{1}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d(\frac{1}{4}+\frac{1}{x})+\frac{1}{4}\left(4301-2243\sqrt{29}\right)\int \frac{\sqrt{6\left(1+\sqrt{29}\right)}-8\left(\frac{1}{4}\ldots\right.}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\ldots\right.}}{\sqrt{174\left(1+\sqrt{29}\right)}}\ldots\right.}{\ldots}\right.$$

$$\Big\downarrow \; 1083$$

3.50. $\int \frac{1}{\left(8+8x-x^3+8x^4\right)^2}\,dx$

$$\frac{1}{4}\left(-\frac{2\left(\frac{\frac{1}{4}\left(4301-2243\sqrt{29}\right)\int \frac{\sqrt{6\left(1+\sqrt{29}\right)}-8\left(\frac{1}{4}+\frac{1}{x}\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)-\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}\left(4301+2243\sqrt{29}\right)\int \frac{1}{96\left(1-\sqrt{29}\right)-\left(32\left(\frac{1}{4}+\frac{1}{x}\right)-4\sqrt{}\right.}}{\sqrt{174\left(1+\sqrt{29}\right)}}\right.}{}\right.$$

$$\downarrow 217$$

$$\frac{1}{4}\left(-\frac{2\left(\frac{\frac{1}{4}\left(4301-2243\sqrt{29}\right)\int \frac{\sqrt{6\left(1+\sqrt{29}\right)}-8\left(\frac{1}{4}+\frac{1}{x}\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)+\frac{1}{8}\sqrt{\frac{1+\sqrt{29}}{\sqrt{29}-1}}\left(4301+2243\sqrt{29}\right)\arctan\left(\frac{32\left(\frac{1}{x}+\frac{1}{4}\right)-4\sqrt{6\left(1+\sqrt{29}\right)}}{4\sqrt{6\left(\sqrt{29}-1\right)}}\right.}{\sqrt{174\left(1+\sqrt{29}\right)}}\right.}{}\right.$$

$$\downarrow 1103$$

$$\frac{1}{4}\left(4\left(\frac{17\arctan\left(\frac{512\left(\frac{1}{x}+\frac{1}{4}\right)^2-96}{192\sqrt{7}}\right)}{1008\sqrt{7}}-\frac{464\left(\frac{1}{x}+\frac{1}{4}\right)^2+207}{336\left(256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261\right)}\right)-\frac{2\left(\frac{\frac{1}{8}\sqrt{\frac{1+\sqrt{29}}{\sqrt{29}-1}}\left(4301+2243\sqrt{29}\right)\text{arcta}}{}\right.}{}\right.$$

input
```
Int[(8 + 8*x - x^3 + 8*x^4)^(-2),x]
```

3.50. $\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx$

output
```
((5*(5157 + 3184*(1/4 + x^(-1))^2)*(1/4 + x^(-1)))/(5481*(261 - 96*(1/4 +
x^(-1))^2 + 256*(1/4 + x^(-1))^4)) + 4*(-1/336*(207 + 464*(1/4 + x^(-1))^2
)/(261 - 96*(1/4 + x^(-1))^2 + 256*(1/4 + x^(-1))^4) + (17*ArcTan[(-96 + 5
12*(1/4 + x^(-1))^2)/(192*Sqrt[7])])/(1008*Sqrt[7])) - (2*(((Sqrt[(1 + Sqr
t[29])/(-1 + Sqrt[29])]*(4301 + 2243*Sqrt[29])*ArcTan[(-4*Sqrt[6*(1 + Sqrt
[29])] + 32*(1/4 + x^(-1)))/(4*Sqrt[6*(-1 + Sqrt[29])])])/8 - ((4301 - 224
3*Sqrt[29])*Log[3*Sqrt[29] - 4*Sqrt[6*(1 + Sqrt[29])]*(1/4 + x^(-1)) + 16*
(1/4 + x^(-1))^2])/16)/Sqrt[174*(1 + Sqrt[29])] + ((Sqrt[(1 + Sqrt[29])/(-
1 + Sqrt[29])]*(4301 + 2243*Sqrt[29])*ArcTan[(4*Sqrt[6*(1 + Sqrt[29])] + 3
2*(1/4 + x^(-1)))/(4*Sqrt[6*(-1 + Sqrt[29])])])/8 + ((4301 - 2243*Sqrt[29]
)*Log[3*Sqrt[29] + 4*Sqrt[6*(1 + Sqrt[29])]*(1/4 + x^(-1)) + 16*(1/4 + x^(
-1))^2])/16)/Sqrt[174*(1 + Sqrt[29])]))/5481)/4
```

3.50.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.50. $\int \frac{1}{(8+8x-x^3+8x^4)^2} \, dx$

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

3.50.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.23

method	result	size
default	$\frac{\frac{7}{3132}x^{3}-\frac{191}{58464}x^{2}+\frac{57}{12992}x+\frac{17}{10962}}{x^{4}-\frac{1}{8}x^{3}+x+1}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(8\_Z^{4}-\_Z^{3}+8\_Z+8\right)}\frac{\left(392\_R^{2}-1097\_R+2243\right)\ln\left(x-\_R\right)}{32\_R^{3}-3\_R^{2}+8}\right)}{21924}$	83
risch	$\frac{\frac{7}{3132}x^{3}-\frac{191}{58464}x^{2}+\frac{57}{12992}x+\frac{17}{10962}}{x^{4}-\frac{1}{8}x^{3}+x+1}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(8\_Z^{4}-\_Z^{3}+8\_Z+8\right)}\frac{\left(392\_R^{2}-1097\_R+2243\right)\ln\left(x-\_R\right)}{32\_R^{3}-3\_R^{2}+8}\right)}{21924}$	83

input
```
int(1/(8*x^4-x^3+8*x+8)^2,x,method=_RETURNVERBOSE)
```

output
```
(7/3132*x^3-191/58464*x^2+57/12992*x+17/10962)/(x^4-1/8*x^3+x+1)+1/21924*s
um((392*_R^2-1097*_R+2243)/(32*_R^3-3*_R^2+8)*ln(x-_R),_R=RootOf(8*_Z^4-_Z
^3+8*_Z+8))
```

3.50.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 1201, normalized size of antiderivative = 3.36

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="fricas")
```

output
```
1/213627456*(3819648*x^3 - 15138*(8*x^4 - x^3 + 8*x + 8)*(-17*I*sqrt(7) +
7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))*log(6
21785056787306565435997385932 8*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/3345
40596096*I*sqrt(7) - 180983329/4683568345344))^3 - 1002876724317971747863 2
775680*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180
983329/4683568345344))^2 + 67481665655469287031416*x + 320944207138750561 9
64778*I*sqrt(7) - 13321072503358964501314 5504*sqrt(4550065/334540596096*I*
sqrt(7) - 180983329/4683568345344) + 333979081113202533090737) - 15138*(8*
x^4 - x^3 + 8*x + 8)*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqr
t(7) - 180983329/4683568345344))*log(-777231320984133206794996732416*(17/1
4112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/46835
68345344))^3 + 878169064752*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/33454
0596096*I*sqrt(7) - 180983329/4683568345344))^2*(-1066184864424603*I*sqrt(
7) + 442529435492941104*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/46
83568345344) - 1427510892508480) + 7569*(727651150781043057307 2*(17/14112*
I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345
344))^2 - 23359423554371543)*(17*I*sqrt(7) + 7056*sqrt(-4550065/3345405960
96*I*sqrt(7) - 180983329/4683568345344)) + 843520820693366087892 7*x - 1484
4919514132868277263 3/4*I*sqrt(7) + 154037870723119880241720 36*sqrt(4550065
/334540596096*I*sqrt(7) - 180983329/4683568345344) - 473936066066965950...
```

3.50.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3834 vs. $2(274) = 548$.

Time = 1.78 (sec) , antiderivative size = 3834, normalized size of antiderivative = 10.74

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(8*x**4-x**3+8*x+8)**2,x)
```

output
```
(784*x**3 - 1146*x**2 + 1539*x + 544)/(350784*x**4 - 43848*x**3 + 350784*x
 + 350784) - sqrt(-180983329/37468546762752 + 1583563*sqrt(29)/12920188538
88)*log(x**2 + x*(-62716756730859*sqrt(1218)*sqrt(-180983329 + 45923327*sq
rt(29))*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-18098
3329 + 45923327*sqrt(29)) + 40699873480352667)/22700832326499868157368342 4
 - 267658292345340*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*
sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/8435208206933660
878927 - 215737452097035286682 3*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt
(29))/113504161632499340786841712 + 3881045239007430*sqrt(29)/532672726436
1229 + 435853770857118353330297/337408328277346435157 08 + 20905585576953*s
qrt(42)*sqrt(-180983329 + 45923327*sqrt(29))/85227636229779664) - 29428140
741014294150840305101822042500675569 53*sqrt(214095423017213*sqrt(29) + 471
06822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 406998734803526
67)/888496186751485201253966401139075287452416534006272 - 1425762563285631
483583114297276510260901053955935109 3/277655058359839125391864500355961027
32888016687696 - 751846315028188373888759000608813558 71*sqrt(1218)*sqrt(-1
80983329 + 45923327*sqrt(29))*sqrt(214095423017213*sqrt(29) + 47106822945*
sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/30637
79954315466211220573797031294094663505289 6768 - 963314181796141259748858 76
61065704878094062299*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/30...
```

3.50.7 Maxima [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx = \int \frac{1}{(8\,x^4-x^3+8\,x+8)^2}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")
```

output
```
1/43848*(784*x^3 - 1146*x^2 + 1539*x + 544)/(8*x^4 - x^3 + 8*x + 8) + 1/21
924*integrate((392*x^2 - 1097*x + 2243)/(8*x^4 - x^3 + 8*x + 8), x)
```

3.50.8 Giac [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx = \int \frac{1}{(8\,x^4-x^3+8\,x+8)^2}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="giac")
```

output
```
integrate((8*x^4 - x^3 + 8*x + 8)^(-2), x)
```

3.50.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.49

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx = \left(\sum_{k=1}^{4}\ln\left(\frac{2615257\,\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)}{72918171648}+\frac{4225\,x}{40375589184}-\frac{\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)\,x\,34885379}{72918171648}-\frac{\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^2\,x\,191555}{475136}-\frac{\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^3\,x\,9261}{256}-\frac{11205\,\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^2}{59392}-\frac{24759\,\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^3}{512}+\frac{10901}{107668237824}\right)\,\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)\right)+\frac{\frac{7\,x^3}{3132}-\frac{191\,x^2}{58464}+\frac{57\,x}{12992}+\frac{17}{10962}}{x^4-\frac{x^3}{8}+x+1}$$

input
```
int(1/(8*x - x^3 + 8*x^4 + 8)^2,x)
```

output
```
symsum(log((2615257*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/6
74433841729536 + 1114096/13723971258377709, z, k))/72918171648 + (4225*x)/
40375589184 - (34885379*root(z^4 + (6630191*z^2)/167270298048 + (77351105*
z)/674433841729536 + 1114096/13723971258377709, z, k)*x)/72918171648 - (19
1555*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536
+ 1114096/13723971258377709, z, k)^2*x)/475136 - (9261*root(z^4 + (6630191
*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377
709, z, k)^3*x)/256 - (11205*root(z^4 + (6630191*z^2)/167270298048 + (7735
1105*z)/674433841729536 + 1114096/13723971258377709, z, k)^2)/59392 - (247
59*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 +
1114096/13723971258377709, z, k)^3)/512 + 10901/107668237824)*root(z^4 + (
6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/1372397
1258377709, z, k), k, 1, 4) + ((57*x)/12992 - (191*x^2)/58464 + (7*x^3)/31
32 + 17/10962)/(x - x^3/8 + x^4 + 1)
```

3.50. $\int \frac{1}{(8+8x-x^3+8x^4)^2} \, dx$

3.51 $\int (1+4x+4x^2+4x^4)^4 \, dx$

3.51.1 Optimal result

Integrand size = 17, antiderivative size = 97

$$\int (1+4x+4x^2+4x^4)^4 \, dx = x+8x^2+\frac{112x^3}{3}+112x^4+\frac{1136x^5}{5}+\frac{992x^6}{3}+\frac{2752x^7}{7}+448x^8+\frac{4192x^9}{9}+384x^{10}+\frac{3328x^{11}}{11}+256x^{12}+\frac{1792x^{13}}{13}+\frac{512x^{14}}{7}+\frac{1024x^{15}}{15}+\frac{256x^{17}}{17}$$

output
```
x+8*x^2+112/3*x^3+112*x^4+1136/5*x^5+992/3*x^6+2752/7*x^7+448*x^8+4192/9*x
^9+384*x^10+3328/11*x^11+256*x^12+1792/13*x^13+512/7*x^14+1024/15*x^15+256
/17*x^17
```

3.51.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int (1+4x+4x^2+4x^4)^4 \, dx = x+8x^2+\frac{112x^3}{3}+112x^4+\frac{1136x^5}{5}+\frac{992x^6}{3}+\frac{2752x^7}{7}+448x^8+\frac{4192x^9}{9}+384x^{10}+\frac{3328x^{11}}{11}+256x^{12}+\frac{1792x^{13}}{13}+\frac{512x^{14}}{7}+\frac{1024x^{15}}{15}+\frac{256x^{17}}{17}$$

input
```
Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^4,x]
```

output
```
x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7
)/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (179
2*x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

3.51.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4x^4+4x^2+4x+1\right)^4 dx$$

$$\Big\downarrow \ 2465$$

$$\int (256x^{16}+1024x^{14}+1024x^{13}+1792x^{12}+3072x^{11}+3328x^{10}+3840x^9+4192x^8+3584x^7+2752x^6+1984x$$

$$\Big\downarrow \ 2009$$

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10}+\frac{4192x^9}{9}+448x^8+\frac{2752x^7}{7}+\frac{992x^6}{3}+\frac{1136x^5}{5}+112x^4+\frac{112x^3}{3}+8x^2+x$$

input
```
Int[(1 + 4*x + 4*x^2 + 4*x^4)^4,x]
```

output
```
x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7
)/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (179
2*x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

3.51.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

3.51.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

method	result
gosper	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}+$
default	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}+$
norman	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}+$
risch	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}+$
parallelrisch	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}+$

input
```
int((4*x^4+4*x^2+4*x+1)^4,x,method=_RETURNVERBOSE)
```

output
```
x+8*x^2+112/3*x^3+112*x^4+1136/5*x^5+992/3*x^6+2752/7*x^7+448*x^8+4192/9*x
^9+384*x^10+3328/11*x^11+256*x^12+1792/13*x^13+512/7*x^14+1024/15*x^15+256
/17*x^17
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int (1+4x+4x^2+4x^4)^4\,dx = \frac{256}{17}\,x^{17}+\frac{1024}{15}\,x^{15}+\frac{512}{7}\,x^{14}+\frac{1792}{13}\,x^{13}+256\,x^{12}+\frac{3328}{11}\,x^{11}+384\,x^{10}+\frac{4192}{9}\,x^9+448\,x^8+\frac{2752}{7}\,x^7+\frac{992}{3}\,x^6+\frac{1136}{5}\,x^5+112\,x^4+\frac{112}{3}\,x^3+8\,x^2+x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="fricas")
```

output
```
256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/1
1*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5
*x^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

3.51.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int (1+4x+4x^2+4x^4)^4 \, dx = \frac{256x^{17}}{17} + \frac{1024x^{15}}{15} + \frac{512x^{14}}{7} + \frac{1792x^{13}}{13} + 256x^{12} + \frac{3328x^{11}}{11} + 384x^{10} + \frac{4192x^9}{9} + 448x^8 + \frac{2752x^7}{7} + \frac{992x^6}{3} + \frac{1136x^5}{5} + 112x^4 + \frac{112x^3}{3} + 8x^2 + x$$

input
```
integrate((4*x**4+4*x**2+4*x+1)**4,x)
```

output
```
256*x**17/17 + 1024*x**15/15 + 512*x**14/7 + 1792*x**13/13 + 256*x**12 + 3
328*x**11/11 + 384*x**10 + 4192*x**9/9 + 448*x**8 + 2752*x**7/7 + 992*x**6
/3 + 1136*x**5/5 + 112*x**4 + 112*x**3/3 + 8*x**2 + x
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int (1+4x+4x^2+4x^4)^4 \, dx = \frac{256}{17}\,x^{17} + \frac{1024}{15}\,x^{15} + \frac{512}{7}\,x^{14} + \frac{1792}{13}\,x^{13} + 256\,x^{12} + \frac{3328}{11}\,x^{11} + 384\,x^{10} + \frac{4192}{9}\,x^9 + 448\,x^8 + \frac{2752}{7}\,x^7 + \frac{992}{3}\,x^6 + \frac{1136}{5}\,x^5 + 112\,x^4 + \frac{112}{3}\,x^3 + 8\,x^2 + x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="maxima")
```

output
```
256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/1
1*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5
*x^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int \left(1+4x+4x^2+4x^4\right)^4 \, dx = \frac{256}{17}\,x^{17}+\frac{1024}{15}\,x^{15}+\frac{512}{7}\,x^{14}+\frac{1792}{13}\,x^{13}+256\,x^{12}+\frac{3328}{11}\,x^{11}+384\,x^{10}+\frac{4192}{9}\,x^{9}+448\,x^{8}+\frac{2752}{7}\,x^{7}+\frac{992}{3}\,x^{6}+\frac{1136}{5}\,x^{5}+112\,x^{4}+\frac{112}{3}\,x^{3}+8\,x^{2}+x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="giac")
```

output
```
256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/1
1*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5
*x^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

3.51.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int \left(1+4x+4x^2+4x^4\right)^4 \, dx = \frac{256\,x^{17}}{17}+\frac{1024\,x^{15}}{15}+\frac{512\,x^{14}}{7}+\frac{1792\,x^{13}}{13}+256\,x^{12}+\frac{3328\,x^{11}}{11}+384\,x^{10}+\frac{4192\,x^{9}}{9}+448\,x^{8}+\frac{2752\,x^{7}}{7}+\frac{992\,x^{6}}{3}+\frac{1136\,x^{5}}{5}+112\,x^{4}+\frac{112\,x^{3}}{3}+8\,x^{2}+x$$

input
```
int((4*x + 4*x^2 + 4*x^4 + 1)^4,x)
```

output
```
x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7
)/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (179
2*x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

3.52 $\int (1 + 4x + 4x^2 + 4x^4)^3 \, dx$

3.52.1 Optimal result

Integrand size = 17, antiderivative size = 69

$$\int (1 + 4x + 4x^2 + 4x^4)^3 \, dx = x + 6x^2 + 20x^3 + 40x^4 + \frac{252x^5}{5} + 48x^6 + \frac{352x^7}{7} + 48x^8 + \frac{80x^9}{3} + \frac{96x^{10}}{5} + \frac{192x^{11}}{11} + \frac{64x^{13}}{13}$$

output
```
x+6*x^2+20*x^3+40*x^4+252/5*x^5+48*x^6+352/7*x^7+48*x^8+80/3*x^9+96/5*x^10
+192/11*x^11+64/13*x^13
```

3.52.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int (1 + 4x + 4x^2 + 4x^4)^3 \, dx = x + 6x^2 + 20x^3 + 40x^4 + \frac{252x^5}{5} + 48x^6 + \frac{352x^7}{7} + 48x^8 + \frac{80x^9}{3} + \frac{96x^{10}}{5} + \frac{192x^{11}}{11} + \frac{64x^{13}}{13}$$

input
```
Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^3,x]
```

output
```
x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8
+ (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

3.52.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4x^4+4x^2+4x+1\right)^3 dx$$

$$\Big\downarrow \; 2465$$

$$\int \left(64x^{12}+192x^{10}+192x^9+240x^8+384x^7+352x^6+288x^5+252x^4+160x^3+60x^2+12x+1\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

input
```
Int[(1 + 4*x + 4*x^2 + 4*x^4)^3,x]
```

output
```
x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8
+ (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

3.52.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

3.52.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	si
gosper	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$	5
default	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$	5
norman	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$	5
risch	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$	5
parallelrisch	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$	5

input
```
int((4*x^4+4*x^2+4*x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
x+6*x^2+20*x^3+40*x^4+252/5*x^5+48*x^6+352/7*x^7+48*x^8+80/3*x^9+96/5*x^10
+192/11*x^11+64/13*x^13
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \left(1+4x+4x^2+4x^4\right)^3\,dx=\frac{64}{13}\,x^{13}+\frac{192}{11}\,x^{11}+\frac{96}{5}\,x^{10}+\frac{80}{3}\,x^9+48\,x^8+\frac{352}{7}\,x^7+48\,x^6+\frac{252}{5}\,x^5+40\,x^4+20\,x^3+6\,x^2+x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="fricas")
```

output
```
64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*
x^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

3.52.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int (1+4x+4x^2+4x^4)^3\,dx = \frac{64x^{13}}{13} + \frac{192x^{11}}{11} + \frac{96x^{10}}{5} + \frac{80x^9}{3} + 48x^8 + \frac{352x^7}{7} + 48x^6 + \frac{252x^5}{5} + 40x^4 + 20x^3 + 6x^2 + x$$

input
```
integrate((4*x**4+4*x**2+4*x+1)**3,x)
```

output
```
64*x**13/13 + 192*x**11/11 + 96*x**10/5 + 80*x**9/3 + 48*x**8 + 352*x**7/7
 + 48*x**6 + 252*x**5/5 + 40*x**4 + 20*x**3 + 6*x**2 + x
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int (1+4x+4x^2+4x^4)^3\,dx = \frac{64}{13}\,x^{13} + \frac{192}{11}\,x^{11} + \frac{96}{5}\,x^{10} + \frac{80}{3}\,x^9 + 48\,x^8 + \frac{352}{7}\,x^7 + 48\,x^6 + \frac{252}{5}\,x^5 + 40\,x^4 + 20\,x^3 + 6\,x^2 + x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="maxima")
```

output
```
64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*
x^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int (1+4x+4x^2+4x^4)^3\,dx = \frac{64}{13}\,x^{13} + \frac{192}{11}\,x^{11} + \frac{96}{5}\,x^{10} + \frac{80}{3}\,x^9 + 48\,x^8 + \frac{352}{7}\,x^7 + 48\,x^6 + \frac{252}{5}\,x^5 + 40\,x^4 + 20\,x^3 + 6\,x^2 + x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="giac")
```

output
```
64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*
x^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

3.52.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int (1+4x+4x^2+4x^4)^3 \, dx = \frac{64\,x^{13}}{13} + \frac{192\,x^{11}}{11} + \frac{96\,x^{10}}{5} + \frac{80\,x^9}{3} + 48\,x^8 + \frac{352\,x^7}{7} + 48\,x^6 + \frac{252\,x^5}{5} + 40\,x^4 + 20\,x^3 + 6\,x^2 + x$$

input
```
int((4*x + 4*x^2 + 4*x^4 + 1)^3,x)
```

output
```
x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8
+ (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

3.53 $\int (1+4x+4x^2+4x^4)^2\,dx$

3.53.1 Optimal result

Integrand size = 17, antiderivative size = 45

$$\int (1+4x+4x^2+4x^4)^2\,dx = x+4x^2+8x^3+8x^4+\frac{24x^5}{5}+\frac{16x^6}{3}+\frac{32x^7}{7}+\frac{16x^9}{9}$$

output
```
x+4*x^2+8*x^3+8*x^4+24/5*x^5+16/3*x^6+32/7*x^7+16/9*x^9
```

3.53.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int (1+4x+4x^2+4x^4)^2\,dx = x+4x^2+8x^3+8x^4+\frac{24x^5}{5}+\frac{16x^6}{3}+\frac{32x^7}{7}+\frac{16x^9}{9}$$

input
```
Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]
```

output
```
x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9
)/9
```

3.53.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4x^4+4x^2+4x+1\right)^2 dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(16x^8+32x^6+32x^5+24x^4+32x^3+24x^2+8x+1\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

input
```
Int[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]
```

output
```
x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9
)/9
```

3.53.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

3.53.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

method	result	size
gosper	$x+4x^{2}+8x^{3}+8x^{4}+\frac{24}{5}x^{5}+\frac{16}{3}x^{6}+\frac{32}{7}x^{7}+\frac{16}{9}x^{9}$	38
default	$x+4x^{2}+8x^{3}+8x^{4}+\frac{24}{5}x^{5}+\frac{16}{3}x^{6}+\frac{32}{7}x^{7}+\frac{16}{9}x^{9}$	38
norman	$x+4x^{2}+8x^{3}+8x^{4}+\frac{24}{5}x^{5}+\frac{16}{3}x^{6}+\frac{32}{7}x^{7}+\frac{16}{9}x^{9}$	38
risch	$x+4x^{2}+8x^{3}+8x^{4}+\frac{24}{5}x^{5}+\frac{16}{3}x^{6}+\frac{32}{7}x^{7}+\frac{16}{9}x^{9}$	38
parallelrisch	$x+4x^{2}+8x^{3}+8x^{4}+\frac{24}{5}x^{5}+\frac{16}{3}x^{6}+\frac{32}{7}x^{7}+\frac{16}{9}x^{9}$	38

input
```
int((4*x^4+4*x^2+4*x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x+4*x^2+8*x^3+8*x^4+24/5*x^5+16/3*x^6+32/7*x^7+16/9*x^9
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \left(1+4x+4x^{2}+4x^{4}\right)^{2}\,dx=\frac{16}{9}\,x^{9}+\frac{32}{7}\,x^{7}+\frac{16}{3}\,x^{6}+\frac{24}{5}\,x^{5}+8\,x^{4}+8\,x^{3}+4\,x^{2}+x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")
```

output
```
16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

3.53.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \left(1+4x+4x^{2}+4x^{4}\right)^{2}\,dx=\frac{16x^{9}}{9}+\frac{32x^{7}}{7}+\frac{16x^{6}}{3}+\frac{24x^{5}}{5}+8x^{4}+8x^{3}+4x^{2}+x$$

input
```
integrate((4*x**4+4*x**2+4*x+1)**2,x)
```

output
```
16*x**9/9 + 32*x**7/7 + 16*x**6/3 + 24*x**5/5 + 8*x**4 + 8*x**3 + 4*x**2 +
 x
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \left(1+4x+4x^2+4x^4\right)^2 \, dx = \frac{16}{9}\,x^9 + \frac{32}{7}\,x^7 + \frac{16}{3}\,x^6 + \frac{24}{5}\,x^5 + 8\,x^4 + 8\,x^3 + 4\,x^2 + x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")
```

output
```
16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \left(1+4x+4x^2+4x^4\right)^2 \, dx = \frac{16}{9}\,x^9 + \frac{32}{7}\,x^7 + \frac{16}{3}\,x^6 + \frac{24}{5}\,x^5 + 8\,x^4 + 8\,x^3 + 4\,x^2 + x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")
```

output
```
16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

3.53.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \left(1+4x+4x^2+4x^4\right)^2 \, dx = \frac{16\,x^9}{9} + \frac{32\,x^7}{7} + \frac{16\,x^6}{3} + \frac{24\,x^5}{5} + 8\,x^4 + 8\,x^3 + 4\,x^2 + x$$

input
```
int((4*x + 4*x^2 + 4*x^4 + 1)^2,x)
```

output
```
x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9
)/9
```

3.54 $\int (1 + 4x + 4x^2 + 4x^4) \, dx$

3.54.1 Optimal result

Integrand size = 15, antiderivative size = 21

$$\int (1 + 4x + 4x^2 + 4x^4) \, dx = x + 2x^2 + \frac{4x^3}{3} + \frac{4x^5}{5}$$

output
```
x+2*x^2+4/3*x^3+4/5*x^5
```

3.54.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (1 + 4x + 4x^2 + 4x^4) \, dx = x + 2x^2 + \frac{4x^3}{3} + \frac{4x^5}{5}$$

input
```
Integrate[1 + 4*x + 4*x^2 + 4*x^4,x]
```

output
```
x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

3.54.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4x^4+4x^2+4x+1\right) \, dx$$
$$\Big\downarrow \; 2009$$
$$\frac{4x^5}{5}+\frac{4x^3}{3}+2x^2+x$$

input
```
Int[1 + 4*x + 4*x^2 + 4*x^4,x]
```

output
```
x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

3.54.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.54.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18
default	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18
norman	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18
risch	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18
parallelrisch	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18
parts	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18

input
```
int(4*x^4+4*x^2+4*x+1,x,method=_RETURNVERBOSE)
```

output
```
x+2*x^2+4/3*x^3+4/5*x^5
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int (1+4x+4x^2+4x^4)\,dx = \frac{4}{5}\,x^5+\frac{4}{3}\,x^3+2\,x^2+x$$

input
```
integrate(4*x^4+4*x^2+4*x+1,x, algorithm="fricas")
```

output
```
4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

3.54.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int (1+4x+4x^2+4x^4)\,dx = \frac{4x^5}{5}+\frac{4x^3}{3}+2x^2+x$$

input
```
integrate(4*x**4+4*x**2+4*x+1,x)
```

output
```
4*x**5/5 + 4*x**3/3 + 2*x**2 + x
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int (1+4x+4x^2+4x^4)\,dx = \frac{4}{5}\,x^5+\frac{4}{3}\,x^3+2\,x^2+x$$

input
```
integrate(4*x^4+4*x^2+4*x+1,x, algorithm="maxima")
```

output
```
4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(1+4x+4x^2+4x^4\right)\,dx = \frac{4}{5}\,x^5+\frac{4}{3}\,x^3+2\,x^2+x$$

input
```
integrate(4*x^4+4*x^2+4*x+1,x, algorithm="giac")
```

output
```
4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

3.54.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(1+4x+4x^2+4x^4\right)\,dx = \frac{4\,x^5}{5}+\frac{4\,x^3}{3}+2\,x^2+x$$

input
```
int(4*x + 4*x^2 + 4*x^4 + 1,x)
```

output
```
x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

3.55 $\int \frac{1}{1+4x+4x^2+4x^4} \, dx$

3.55.1 Optimal result

Integrand size = 17, antiderivative size = 234

$$\begin{aligned}\int \frac{1}{1+4x+4x^2+4x^4} \, dx = {} & \frac{1}{2}\arctan\left(\frac{1}{2}\left(-1+\left(1+\frac{1}{x}\right)^2\right)\right) \\ & -\frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\arctan\left(\frac{2-\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right) \\ & -\frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\arctan\left(\frac{2+\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right) \\ & -\frac{1}{4}\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\log\left(\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right) + \left(1+\frac{1}{x}\right)^2\right) + \frac{1}{4}\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\log\left(\sqrt{5} + \sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right)\end{aligned}$$

output
```
1/2*arctan(-1/2+1/2*(1+1/x)^2)-1/20*ln((1+1/x)^2+5^(1/2)-(1+1/x)*(2+2*5^(1
/2))^(1/2))*(-10+5*5^(1/2))^(1/2)+1/20*ln((1+1/x)^2+5^(1/2)+(1+1/x)*(2+2*5
^(1/2))^(1/2))*(-10+5*5^(1/2))^(1/2)-1/10*arctan((2+2/x-(2+2*5^(1/2))^(1/2
))/(-2+2*5^(1/2))^(1/2))*(10+5*5^(1/2))^(1/2)-1/10*arctan((2+2/x+(2+2*5^(1
/2))^(1/2))/(-2+2*5^(1/2))^(1/2))*(10+5*5^(1/2))^(1/2)
```

3.55.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.20

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = \frac{1}{4}\text{RootSum}\left[1+4\#1+4\#1^2+4\#1^4\&,\frac{\log(x-\#1)}{1+2\#1+4\#1^3}\&\right]$$

input
```
Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-1),x]
```

output
```
RootSum[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , Log[x - #1]/(1 + 2*#1 + 4*#1^3) & ]
/4
```

3.55.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {2504, 27, 2202, 27, 1432, 1083, 217, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4x^4+4x^2+4x+1}dx$$

$$\Big\downarrow \text{2504}$$

$$-16\int \frac{1}{16\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)x^2}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)x^2}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{2202}$$

$$-\int \frac{\left(1+\frac{1}{x}\right)^2+1}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)-\int -\frac{2\left(1+\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{1+\frac{1}{x}}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)-\int \frac{\left(1+\frac{1}{x}\right)^2+1}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{1}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)^2-\int \frac{\left(1+\frac{1}{x}\right)^2+1}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{-\left(1+\frac{1}{x}\right)^4-16}d\left(2\left(1+\frac{1}{x}\right)^2-2\right)-\int \frac{\left(1+\frac{1}{x}\right)^2+1}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-\int \frac{\left(1+\frac{1}{x}\right)^2+1}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{1483}$$

$$-\frac{\int \frac{\sqrt{2\left(1+\sqrt{5}\right)}-\left(1-\sqrt{5}\right)\left(1+\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\int \frac{\left(1-\sqrt{5}\right)\left(1+\frac{1}{x}\right)+\sqrt{2\left(1+\sqrt{5}\right)}}{\left(1+\frac{1}{x}\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{1}{2}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)$$

$$\Big\downarrow \text{1142}$$

$$-\frac{\frac{\left(1+\sqrt{5}\right)^{3/2}\int \frac{1}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{5}\right)\int -\frac{\sqrt{2\left(1+\sqrt{5}\right)}-2\left(1+\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\frac{\left(1+\sqrt{5}\right)^{3/2}\int \frac{1}{\left(1+\frac{1}{x}\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{2\left(1+\frac{1}{x}\right)+\sqrt{2\left(1+\sqrt{5}\right)}}{\left(1+\frac{1}{x}\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{1}{2}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)$$

$$\Big\downarrow \text{25}$$

3.55. $\int \frac{1}{1+4x+4x^2+4x^4}\,dx$

$$-\frac{\frac{\left(1+\sqrt{5}\right)^{3/2}\int \frac{1}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{\sqrt{2\left(1+\sqrt{5}\right)}-2\left(1+\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\frac{\left(1+\sqrt{5}\right)^{3/2}\int \frac{1}{\left(1+\frac{1}{x}\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{2\left(1+\frac{1}{x}\right)+\sqrt{2\left(1+\sqrt{5}\right)}}{\left(1+\frac{1}{x}\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{1}{2}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)$$

$\downarrow$ 1083

$$-\frac{\frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{\sqrt{2\left(1+\sqrt{5}\right)}-2\left(1+\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)-\sqrt{2}\left(1+\sqrt{5}\right)^{3/2}\int \frac{1}{2\left(1-\sqrt{5}\right)-\left(2\left(1+\frac{1}{x}\right)-\sqrt{2\left(1+\sqrt{5}\right)}\right)^2}d\left(2\left(1+\frac{1}{x}\right.\right.}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

$$\frac{\frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{2\left(1+\frac{1}{x}\right)+\sqrt{2\left(1+\sqrt{5}\right)}}{\left(1+\frac{1}{x}\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)-\sqrt{2}\left(1+\sqrt{5}\right)^{3/2}\int \frac{1}{2\left(1-\sqrt{5}\right)-\left(2\left(1+\frac{1}{x}\right)+\sqrt{2\left(1+\sqrt{5}\right)}\right)^2}d\left(2\left(1+\frac{1}{x}\right)\right.}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

$$\frac{1}{2}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)$$

$\downarrow$ 217

$$-\frac{\frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{\sqrt{2\left(1+\sqrt{5}\right)}-2\left(1+\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)+\frac{\left(1+\sqrt{5}\right)^{3/2}\arctan\left(\frac{2\left(\frac{1}{x}+1\right)-\sqrt{2\left(1+\sqrt{5}\right)}}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)}{\sqrt{\sqrt{5}-1}}}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{2\left(1+\frac{1}{x}\right)+\sqrt{2\left(1+\sqrt{5}\right)}}{\left(1+\frac{1}{x}\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)+\frac{\left(1+\sqrt{5}\right)^{3/2}\arctan\left(\frac{2\left(\frac{1}{x}+1\right)+\sqrt{2\left(1+\sqrt{5}\right)}}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)}{\sqrt{\sqrt{5}-1}}}{2\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{1}{2}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)$$

$\downarrow$ 1103

3.55. $\int \frac{1}{1+4x+4x^2+4x^4}\,dx$

$$\frac{1}{2}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-\frac{\frac{\left(1+\sqrt{5}\right)^{3/2}\arctan\left(\frac{2\left(\frac{1}{x}+1\right)-\sqrt{2\left(1+\sqrt{5}\right)}}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)}{\sqrt{\sqrt{5}-1}}-\frac{1}{2}(1-\sqrt{5})\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\frac{\left(1+\sqrt{5}\right)^{3/2}\arctan\left(\frac{2\left(\frac{1}{x}+1\right)+\sqrt{2\left(1+\sqrt{5}\right)}}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)}{\sqrt{\sqrt{5}-1}}+\frac{1}{2}(1-\sqrt{5})\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

input
```
Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-1),x]
```

output
```
ArcTan[(-2 + 2*(1 + x^(-1))^2)/4]/2 - (((1 + Sqrt[5])^(3/2)*ArcTan[(-Sqrt[
2*(1 + Sqrt[5])] + 2*(1 + x^(-1)))/Sqrt[2*(-1 + Sqrt[5])]])/Sqrt[-1 + Sqrt
[5]] - ((1 - Sqrt[5])*Log[Sqrt[5] - Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (
1 + x^(-1))^2])/2)/(2*Sqrt[10*(1 + Sqrt[5])]) - (((1 + Sqrt[5])^(3/2)*ArcT
an[(Sqrt[2*(1 + Sqrt[5])] + 2*(1 + x^(-1)))/Sqrt[2*(-1 + Sqrt[5])]])/Sqrt[
-1 + Sqrt[5]] + ((1 - Sqrt[5])*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5])]*(1 + x^
(-1)) + (1 + x^(-1))^2])/2)/(2*Sqrt[10*(1 + Sqrt[5])])
```

3.55.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

3.55. $\int \frac{1}{1+4x+4x^2+4x^4}\,dx$

3.55.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.18

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)}\frac{\ln\left(x-\_R\right)}{4\_R^3+2\_R+1}\right)}{4}$	41
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)}\frac{\ln\left(x-\_R\right)}{4\_R^3+2\_R+1}\right)}{4}$	41

input
```
int(1/(4*x^4+4*x^2+4*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/(4*_R^3+2*_R+1)*ln(x-_R),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1))
```

3.55.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.13

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="fricas")
```

output
```
-1/20*(sqrt(10)*sqrt(-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(2*sqrt(1/10
*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*I - 1/5) +
 I)^2 - 9) - 5*sqrt(1/10*I - 1/5) - 5*sqrt(-1/10*I - 1/5))*log(5/2*(2*sqrt
(1/10*I - 1/5) - I)^2*(12*sqrt(-1/10*I - 1/5) + 6*I - 1) + 15*(2*sqrt(1/10
*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)^2 - 5/2*(2*sqrt(-1/10*I - 1/5)
+ I)^2 + ((6*sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I) - sqrt(10))*(2*sqrt(1/10
*I - 1/5) - I) - sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I))*sqrt(-15/8*(2*sqrt(
1/10*I - 1/5) - I)^2 - 5/4*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/
5) + I) - 15/8*(2*sqrt(-1/10*I - 1/5) + I)^2 - 9) + 8*x + 3) + 1/20*(sqrt(
10)*sqrt(-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(2*sqrt(1/10*I - 1/5) -
I)*(2*sqrt(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*I - 1/5) + I)^2 - 9) +
 5*sqrt(1/10*I - 1/5) + 5*sqrt(-1/10*I - 1/5))*log(5/2*(2*sqrt(1/10*I - 1/
5) - I)^2*(12*sqrt(-1/10*I - 1/5) + 6*I - 1) + 15*(2*sqrt(1/10*I - 1/5) -
I)*(2*sqrt(-1/10*I - 1/5) + I)^2 - 5/2*(2*sqrt(-1/10*I - 1/5) + I)^2 - ((6
*sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I) - sqrt(10))*(2*sqrt(1/10*I - 1/5) -
I) - sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I))*sqrt(-15/8*(2*sqrt(1/10*I - 1/5
) - I)^2 - 5/4*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I) - 15
/8*(2*sqrt(-1/10*I - 1/5) + I)^2 - 9) + 8*x + 3) - 1/4*(2*sqrt(1/10*I - 1/
5) - I)*log(-5*(2*sqrt(1/10*I - 1/5) - I)^2*(12*sqrt(-1/10*I - 1/5) + 6*I
- 1) - 30*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)^2 - 30...
```

3.55.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3432 vs. $2(190) = 380$.

Time = 1.34 (sec) , antiderivative size = 3432, normalized size of antiderivative = 14.67

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(4*x**4+4*x**2+4*x+1),x)
```

output
```
sqrt(-1/40 + sqrt(5)/80)*log(x**2 + x*(-8 - 21*sqrt(5)*sqrt(-2 + sqrt(5))/
10 - sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/2 - sqrt(5)/2 + 12
*sqrt(-2 + sqrt(5)) + 9*sqrt(5)*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2
 + sqrt(5)) + sqrt(5) + 19)/5) - 841*sqrt(5)*sqrt(-2*sqrt(5)*sqrt(-2 + sqr
t(5)) + sqrt(5) + 19)/20 - 14351/40 - 441*sqrt(-2 + sqrt(5))/4 - 75*sqrt(5
)*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/8
- 3*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)
+ 301*sqrt(5)*sqrt(-2 + sqrt(5))/10 + 7407*sqrt(5)/40 + 3913*sqrt(-2*sqrt(
5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/40) - sqrt(-1/40 + sqrt(5)/80)*log(x
**2 + x*(-8 - 12*sqrt(-2 + sqrt(5)) - sqrt(5)/2 + 21*sqrt(5)*sqrt(-2 + sqr
t(5))/10 + sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/2 + 9*sqrt(5)
*sqrt(-2 + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/5) -
 3913*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/40 - 14351/40 - 75
*sqrt(5)*sqrt(-2 + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) +
19)/8 - 301*sqrt(5)*sqrt(-2 + sqrt(5))/10 - 3*sqrt(-2 + sqrt(5))*sqrt(2*sq
rt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 441*sqrt(-2 + sqrt(5))/4 + 7407
*sqrt(5)/40 + 841*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19
)/20) - 2*sqrt(3/80 + 3*sqrt(5)/80 + sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) +
sqrt(5) + 19)/40)*atan(-20*x/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*s
qrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt(5))*sqrt(...
```

3.55.7 Maxima [F]

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx=\int \frac{1}{4\,x^4+4\,x^2+4\,x+1}\,dx$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="maxima")
```

output
```
integrate(1/(4*x^4 + 4*x^2 + 4*x + 1), x)
```

3.55.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.13

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = -\frac{1}{20}\left((i+2)\sqrt{\sqrt{5}-2}\left(\frac{i}{\sqrt{5}-2}+1\right)+5i\right)\log\left((406i+174)\sqrt{5}x+(868i+372)x+29\sqrt{5}\sqrt{29\sqrt{5}+62}+(87i-203)\sqrt{5}+(19i+62)\sqrt{29\sqrt{5}+62}+186i-434\right) - \frac{1}{20}\left((i+2)\sqrt{\sqrt{5}-2}\left(-\frac{i}{\sqrt{5}-2}-1\right)+5i\right)\log\left((406i+174)\sqrt{5}x+(868i+372)x-29\sqrt{5}\sqrt{29\sqrt{5}+62}+(87i-203)\sqrt{5}-(19i+62)\sqrt{29\sqrt{5}+62}+186i-434\right) - \frac{1}{20}\left((2i+1)\sqrt{\sqrt{5}+2}\left(-\frac{i}{\sqrt{5}+2}-1\right)-5i\right)\log\left((26i+130)\sqrt{5}x-(44i+220)x+13\sqrt{5}\sqrt{13\sqrt{5}-22}-(65i-13)\sqrt{5}+(19i-22)\sqrt{13\sqrt{5}-22}+110i-22\right) - \frac{1}{20}\left((2i+1)\sqrt{\sqrt{5}+2}\left(\frac{i}{\sqrt{5}+2}+1\right)-5i\right)\log\left((26i+130)\sqrt{5}x-(44i+220)x-13\sqrt{5}\sqrt{13\sqrt{5}-22}-(65i-13)\sqrt{5}-(19i-22)\sqrt{13\sqrt{5}-22}+110i-22\right)$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="giac")
```

output
```
-1/20*((I + 2)*sqrt(sqrt(5) - 2)*(I/(sqrt(5) - 2) + 1) + 5*I)*log((406*I +
 174)*sqrt(5)*x + (868*I + 372)*x + 29*sqrt(5)*sqrt(29*sqrt(5) + 62) + (87
*I - 203)*sqrt(5) + (19*I + 62)*sqrt(29*sqrt(5) + 62) + 186*I - 434) - 1/2
0*((I + 2)*sqrt(sqrt(5) - 2)*(-I/(sqrt(5) - 2) - 1) + 5*I)*log((406*I + 17
4)*sqrt(5)*x + (868*I + 372)*x - 29*sqrt(5)*sqrt(29*sqrt(5) + 62) + (87*I
- 203)*sqrt(5) - (19*I + 62)*sqrt(29*sqrt(5) + 62) + 186*I - 434) - 1/20*(
(2*I + 1)*sqrt(sqrt(5) + 2)*(-I/(sqrt(5) + 2) - 1) - 5*I)*log((26*I + 130)
*sqrt(5)*x - (44*I + 220)*x + 13*sqrt(5)*sqrt(13*sqrt(5) - 22) - (65*I - 1
3)*sqrt(5) + (19*I - 22)*sqrt(13*sqrt(5) - 22) + 110*I - 22) - 1/20*((2*I
+ 1)*sqrt(sqrt(5) + 2)*(I/(sqrt(5) + 2) + 1) - 5*I)*log((26*I + 130)*sqrt(
5)*x - (44*I + 220)*x - 13*sqrt(5)*sqrt(13*sqrt(5) - 22) - (65*I - 13)*sqr
t(5) - (19*I - 22)*sqrt(13*sqrt(5) - 22) + 110*I - 22)
```

3.55.9 Mupad [B] (verification not implemented)

Time = 10.97 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.37

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = \sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)\left(\frac{x}{4}+\mathrm{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)\left(6\,x+\mathrm{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)(36\,x+16)\right)\right)\right)\mathrm{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)$$

input
```
int(1/(4*x + 4*x^2 + 4*x^4 + 1),x)
```

output
```
symsum(log(-root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k)*(x/4 + root(z^4 +
 (9*z^2)/40 + z/40 + 1/1280, z, k)*(6*x + root(z^4 + (9*z^2)/40 + z/40 + 1
/1280, z, k)*(36*x + 16))))*root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k),
k, 1, 4)
```

3.56 $\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx$

3.56.1 Optimal result

Integrand size = 17, antiderivative size = 317

$$
\begin{aligned}
\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx
&= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)}+\frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} \\
&\quad+\frac{7}{4}\arctan\left(\frac{1}{2}\left(-1+\left(1+\frac{1}{x}\right)^2\right)\right) \\
&\quad-\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\arctan\left(\frac{2-\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right) \\
&\quad-\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\arctan\left(\frac{2+\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right) \\
&\quad+\frac{1}{40}\sqrt{\frac{1}{10}\left(-5959+2665\sqrt{5}\right)}\log\left(\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right) \\
&\quad-\frac{1}{40}\sqrt{\frac{1}{10}\left(-5959+2665\sqrt{5}\right)}\log\left(\sqrt{5}+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right)
\end{aligned}
$$

output
```
1/2*(-17+(1+1/x)^2)/(5-2*(1+1/x)^2+(1+1/x)^4)+1/10*(59-17*(1+1/x)^2)*(1+1/
x)/(5-2*(1+1/x)^2+(1+1/x)^4)+7/4*arctan(-1/2+1/2*(1+1/x)^2)+1/400*ln((1+1/
x)^2+5^(1/2)-(1+1/x)*(2+2*5^(1/2))^(1/2))*(-59590+26650*5^(1/2))^(1/2)-1/4
00*ln((1+1/x)^2+5^(1/2)+(1+1/x)*(2+2*5^(1/2))^(1/2))*(-59590+26650*5^(1/2)
)^(1/2)-1/200*arctan((2+2/x-(2+2*5^(1/2))^(1/2))/(-2+2*5^(1/2))^(1/2))*(59
590+26650*5^(1/2))^(1/2)-1/200*arctan((2+2/x+(2+2*5^(1/2))^(1/2))/(-2+2*5^
(1/2))^(1/2))*(59590+26650*5^(1/2))^(1/2)
```

3.56.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.34

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx = \frac{1}{40}\left(\frac{38+84x-32x^2+72x^3}{1+4x+4x^2+4x^4}+\text{RootSum}\left[1+4\#1+4\#1^2+4\#1^4\&,\frac{27\log(x-\#1)-16\log(x-\#1)\#1+18\log(x-\#1)\#1^2}{1+2\#1+4\#1^3}\&\right]\right)$$

input
```
Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-2),x]
```

output
```
((38 + 84*x - 32*x^2 + 72*x^3)/(1 + 4*x + 4*x^2 + 4*x^4) + RootSum[1 + 4*#
1 + 4*#1^2 + 4*#1^4 & , (27*Log[x - #1] - 16*Log[x - #1]*#1 + 18*Log[x - #
1]*#1^2)/(1 + 2*#1 + 4*#1^3) & ])/40
```

3.56.3 Rubi [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.10, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2504, 27, 2202, 2194, 27, 2191, 27, 1083, 217, 2206, 27, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4x^4+4x^2+4x+1)^2}\,dx$$

$$\Big\downarrow 2504$$

$$-16\int \frac{1}{16\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2x^6}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow 27$$

$$-\int \frac{1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2x^6}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow 2202$$

$$-\int \frac{\left(1+\frac{1}{x}\right)^6+15\left(1+\frac{1}{x}\right)^4+15\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)-\int \frac{\left(-6\left(1+\frac{1}{x}\right)^4-20\left(1+\frac{1}{x}\right)^2-6\right)\left(1+\frac{1}{x}\right)}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow 2194$$

$$-\frac{1}{2}\int -\frac{2\left(3\left(1+\frac{1}{x}\right)^4+10\left(1+\frac{1}{x}\right)^2+3\right)}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)^2-\int \frac{\left(1+\frac{1}{x}\right)^6+15\left(1+\frac{1}{x}\right)^4+15\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow 27$$

$$\int \frac{3\left(1+\frac{1}{x}\right)^4+10\left(1+\frac{1}{x}\right)^2+3}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)^2-\int \frac{\left(1+\frac{1}{x}\right)^6+15\left(1+\frac{1}{x}\right)^4+15\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow 2191$$

$$\frac{1}{16}\int \frac{56}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)^2-\int \frac{\left(1+\frac{1}{x}\right)^6+15\left(1+\frac{1}{x}\right)^4+15\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$$\Big\downarrow 27$$

3.56. $\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx$

$$\frac{7}{2}\int \frac{1}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)^2-$$
$$\int \frac{\left(1+\frac{1}{x}\right)^6+15\left(1+\frac{1}{x}\right)^4+15\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$$\Big\downarrow \text{1083}$$

$$-7\int \frac{1}{-\left(1+\frac{1}{x}\right)^4-16}d\left(2\left(1+\frac{1}{x}\right)^2-2\right)-$$
$$\int \frac{\left(1+\frac{1}{x}\right)^6+15\left(1+\frac{1}{x}\right)^4+15\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$$\Big\downarrow \text{217}$$

$$-\int \frac{\left(1+\frac{1}{x}\right)^6+15\left(1+\frac{1}{x}\right)^4+15\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)+\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-$$
$$\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$$\Big\downarrow \text{2206}$$

$$-\frac{1}{160}\int \frac{16\left(27\left(1+\frac{1}{x}\right)^2+61\right)}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)+\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-$$
$$\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{10}\int \frac{27\left(1+\frac{1}{x}\right)^2+61}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)+\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-$$
$$\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$$\Big\downarrow \text{1483}$$

3.56. $\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx$

$$\frac{1}{10}\left(-\frac{\int \frac{61\sqrt{2\left(1+\sqrt{5}\right)}-\left(61-27\sqrt{5}\right)\left(1+\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\int \frac{\left(61-27\sqrt{5}\right)\left(1+\frac{1}{x}\right)+61\sqrt{2\left(1+\sqrt{5}\right)}}{\left(1+\frac{1}{x}\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}\right)+$$
$$\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+$$
$$\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$\big\downarrow$ 1142

$$\frac{1}{10}\left(-\frac{\sqrt{2\left(5959+2665\sqrt{5}\right)}\int \frac{1}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)-\frac{1}{2}\left(61-27\sqrt{5}\right)\int -\frac{\sqrt{2\left(1+\sqrt{5}\right)}-2\left(1+\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}}{2\sqrt{10\left(1+\sqrt{5}\right)}}\right.$$
$$\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+$$
$$\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$\big\downarrow$ 25

$$\frac{1}{10}\left(-\frac{\sqrt{2\left(5959+2665\sqrt{5}\right)}\int \frac{1}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)+\frac{1}{2}\left(61-27\sqrt{5}\right)\int \frac{\sqrt{2\left(1+\sqrt{5}\right)}-2\left(1+\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}}{2\sqrt{10\left(1+\sqrt{5}\right)}}\right.$$
$$\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+$$
$$\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$\big\downarrow$ 1083

3.56. $\int \frac{1}{\left(1+4x+4x^2+4x^4\right)^2}\,dx$

$$\frac{1}{10}\left(-\frac{\frac{1}{2}(61-27\sqrt{5})\int \frac{\sqrt{2\left(1+\sqrt{5}\right)}-2(1+\frac{1}{x})}{(1+\frac{1}{x})^2-\sqrt{2\left(1+\sqrt{5}\right)}(1+\frac{1}{x})+\sqrt{5}}d(1+\frac{1}{x})-2\sqrt{2\left(5959+2665\sqrt{5}\right)}\int \frac{1}{2\left(1-\sqrt{5}\right)-\left(2(1+\frac{1}{x})-\sqrt{2\left(1+\sqrt{5}\right)}\right.}}{2\sqrt{10\left(1+\sqrt{5}\right)}}\right.$$

$$\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17(\frac{1}{x}+1)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$\downarrow$ 217

$$\frac{1}{10}\left(-\frac{\frac{1}{2}(61-27\sqrt{5})\int \frac{\sqrt{2\left(1+\sqrt{5}\right)}-2(1+\frac{1}{x})}{(1+\frac{1}{x})^2-\sqrt{2\left(1+\sqrt{5}\right)}(1+\frac{1}{x})+\sqrt{5}}d(1+\frac{1}{x})+2\sqrt{\frac{5959+2665\sqrt{5}}{\sqrt{5}-1}}\arctan\left(\frac{2(\frac{1}{x}+1)-\sqrt{2\left(1+\sqrt{5}\right)}}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\frac{1}{2}(\cdots}{\cdots}\right.$$

$$\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17(\frac{1}{x}+1)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$\downarrow$ 1103

$$\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)+$$

$$\frac{1}{10}\left(-\frac{2\sqrt{\frac{5959+2665\sqrt{5}}{\sqrt{5}-1}}\arctan\left(\frac{2(\frac{1}{x}+1)-\sqrt{2\left(1+\sqrt{5}\right)}}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)-\frac{1}{2}(61-27\sqrt{5})\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}\right.$$

$$\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17(\frac{1}{x}+1)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

input

```
Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-2),x]
```

3.56. $\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx$

output
```
-1/2*(16 - x^(-1))/(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4) + ((59 - 17*(1
+ x^(-1))^2)*(1 + x^(-1)))/(10*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)) +
(7*ArcTan[(-2 + 2*(1 + x^(-1))^2)/4])/4 + (-1/2*(2*Sqrt[(5959 + 2665*Sqrt[
5])/(-1 + Sqrt[5])]*ArcTan[(-Sqrt[2*(1 + Sqrt[5])] + 2*(1 + x^(-1)))/Sqrt[
2*(-1 + Sqrt[5])]] - ((61 - 27*Sqrt[5])*Log[Sqrt[5] - Sqrt[2*(1 + Sqrt[5])
]*(1 + x^(-1)) + (1 + x^(-1))^2])/2)/Sqrt[10*(1 + Sqrt[5])] - (2*Sqrt[(595
9 + 2665*Sqrt[5])/(-1 + Sqrt[5])]*ArcTan[(Sqrt[2*(1 + Sqrt[5])] + 2*(1 + x
^(-1)))/Sqrt[2*(-1 + Sqrt[5])]] + ((61 - 27*Sqrt[5])*Log[Sqrt[5] + Sqrt[2*
(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(-1))^2])/2)/(2*Sqrt[10*(1 + Sqrt[5])
]))/10
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.56. $\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx$

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.56. $\int \frac{1}{(1+4x+4x^2+4x^4)^2} dx$

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

3.56.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.25

method	result	size
default	$\frac{\frac{9}{20}x^3-\frac{1}{5}x^2+\frac{21}{40}x+\frac{19}{80}}{x^4+x^2+x+\frac{1}{4}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)}\frac{\left(18\_R^2-16\_R+27\right)\ln\left(x-\_R\right)}{4\_R^3+2\_R+1}\right)}{40}$	79
risch	$\frac{\frac{9}{20}x^3-\frac{1}{5}x^2+\frac{21}{40}x+\frac{19}{80}}{x^4+x^2+x+\frac{1}{4}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)}\frac{\left(18\_R^2-16\_R+27\right)\ln\left(x-\_R\right)}{4\_R^3+2\_R+1}\right)}{40}$	79

input
```
int(1/(4*x^4+4*x^2+4*x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
(9/20*x^3-1/5*x^2+21/40*x+19/80)/(x^4+x^2+x+1/4)+1/40*sum((18*_R^2-16*_R+2
7)/(4*_R^3+2*_R+1)*ln(x-_R),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1))
```

3.56.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 704, normalized size of antiderivative = 2.22

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx=\text{Too large to display}$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")
```

output
```
1/400*(720*x^3 - 50*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(19/1000*I - 5959/200
0) + 7*I)*log(33368250*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^3 - 11755375/
4*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 541735337*x + 25784243612*sqrt
(19/1000*I - 5959/2000) + 45122426321*I - 71080995) - 50*(4*x^4 + 4*x^2 +
4*x + 1)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)*log(-33368250*(4*sqrt(19/1
000*I - 5959/2000) + 7*I)^3 - 125/4*(4271136*sqrt(19/1000*I - 5959/2000) +
 7474488*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 25*(1334730
*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 59
59/2000) - 7*I) + 541735337*x - 25806203712*sqrt(19/1000*I - 5959/2000) -
45160856496*I - 355111539) - 320*x^2 - (4*sqrt(10)*(4*x^4 + 4*x^2 + 4*x +
1)*sqrt(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(1
9/1000*I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/
32*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021) - 25*(4*x^4 + 4*x^2 +
4*x + 1)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) - 25*(4*x^4 + 4*x^2 + 4*x +
 1)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I))*log(125/8*(4271136*sqrt(19/100
0*I - 5959/2000) + 7474488*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*
I)^2 + 11755375/8*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 25/2*(1334730*
(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 595
9/2000) - 7*I) + 1/2*sqrt(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2
- 125/16*(4*sqrt(19/1000*I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 595...
```

3.56.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3834 vs. $2(257) = 514$.

Time = 1.94 (sec) , antiderivative size = 3834, normalized size of antiderivative = 12.09

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(4*x**4+4*x**2+4*x+1)**2,x)
```

output
```
(36*x**3 - 16*x**2 + 42*x + 19)/(80*x**4 + 80*x**2 + 80*x + 20) - sqrt(-59
59/16000 + 533*sqrt(5)/3200)*log(x**2 + x*(-1601676*sqrt(10)*sqrt(-5959 +
2665*sqrt(5))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(
5) + 36004639)/13543383425 - 1067784*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/10
16389 + 3131659367*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/13543383425 + 29168
9395/1083470674 + 470215*sqrt(5)/2032778 + 94043*sqrt(-665*sqrt(10)*sqrt(-
5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/541735337) - 40634464149
111451*sqrt(5)*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt
(5) + 36004639)/27530691871904650 - 2885835544225227917282997/146738587677
251784500 - 83803227754187*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/100111606806
926 - 50208805356*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))*sqrt(-665*sqrt(10)*sq
rt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/550613837438093 - 53
8485754891933*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(-665*sqrt(10)*sqrt(
-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/14673858767725178450 -
92532195509690141l*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/2934771753545035690
0 + 484304611938766076267*sqrt(5)/55061383743809300 + 220130360870147854O3
*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639
)/6669935803511444750) + sqrt(-5959/16000 + 533*sqrt(5)/3200)*log(x**2 + x
*(-94043*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 3
6004639)/541735337 - 1601676*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(6...
```

3.56.7 Maxima [F]

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx=\int \frac{1}{(4\,x^4+4\,x^2+4\,x+1)^2}\,dx$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")
```

output
```
1/20*(36*x^3 - 16*x^2 + 42*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1) + 1/10*integr
ate((18*x^2 - 16*x + 27)/(4*x^4 + 4*x^2 + 4*x + 1), x)
```

3.56.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.99

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx$$

$$= -\frac{1}{400}\left(-(i+3)\sqrt{2665\sqrt{5}-4790}\left(\frac{709i}{533\sqrt{5}-958}+1\right)-350i\right)\log\left((2534636224790i + 16853816172010)\sqrt{5}x-(3913528401620i+26022625108780)x + 5049076145\sqrt{5}\sqrt{1424281\sqrt{5}-2199118}-(8426908086005i-1267318112395)\sqrt{5} + (8166407345i-7795873310)\sqrt{1424281\sqrt{5}-2199118}+13011312554390i - 1956764200810\right) - \frac{1}{400}\left((i+3)\sqrt{2665\sqrt{5}-4790}\left(\frac{709i}{533\sqrt{5}-958}+1\right)-350i\right)\log\left((2534636224790i + 16853816172010)\sqrt{5}x-(3913528401620i+26022625108780)x - 5049076145\sqrt{5}\sqrt{1424281\sqrt{5}-2199118}-(8426908086005i-1267318112395)\sqrt{5} - (8166407345i-7795873310)\sqrt{1424281\sqrt{5}-2199118}+13011312554390i - 1956764200810\right) - \frac{1}{400}\left((3i+1)\sqrt{2665\sqrt{5}+4790}\left(\frac{709i}{533\sqrt{5}+958}+1\right)+350i\right)\log\left((16722951192450i + 2480822188910)\sqrt{5}x+(25712356272300i+3814385585140)x + 5021907265\sqrt{5}\sqrt{1416617\sqrt{5}+2178118}+(1240411094455i-8361475596225)\sqrt{5} + (81533361745i+7721428310)\sqrt{1416617\sqrt{5}+2178118}+1907192792570i - 12856178136150\right) - \frac{1}{400}\left(-(3i+1)\sqrt{2665\sqrt{5}+4790}\left(\frac{709i}{533\sqrt{5}+958}+1\right)+350i\right)\log\left((16722951192450i + 2480822188910)\sqrt{5}x+(25712356272300i+3814385585140)x - 5021907265\sqrt{5}\sqrt{1416617\sqrt{5}+2178118}+(1240411094455i-8361475596225)\sqrt{5} - (81533361745i+7721428310)\sqrt{1416617\sqrt{5}+2178118}+1907192792570i - 12856178136150\right) + \frac{36\,x^3-16\,x^2+42\,x+19}{20\,(4\,x^4+4\,x^2+4\,x+1)}$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")
```

output
```
-1/400*(-(I + 3)*sqrt(2665*sqrt(5) - 4790)*(709*I/(533*sqrt(5) - 958) + 1)
 - 350*I)*log((2534636224790*I + 16853816172010)*sqrt(5)*x - (391352840162
0*I + 26022625108780)*x + 5049076145*sqrt(5)*sqrt(1424281*sqrt(5) - 219911
8) - (8426908086005*I - 1267318112395)*sqrt(5) + (8166407345*I - 779587331
0)*sqrt(1424281*sqrt(5) - 2199118) + 13011312554390*I - 1956764200810) - 1
/400*((I + 3)*sqrt(2665*sqrt(5) - 4790)*(709*I/(533*sqrt(5) - 958) + 1) -
350*I)*log((2534636224790*I + 16853816172010)*sqrt(5)*x - (3913528401620*I
 + 26022625108780)*x - 5049076145*sqrt(5)*sqrt(1424281*sqrt(5) - 2199118)
- (8426908086005*I - 1267318112395)*sqrt(5) - (8166407345*I - 7795873310)*
sqrt(1424281*sqrt(5) - 2199118) + 13011312554390*I - 1956764200810) - 1/40
0*((3*I + 1)*sqrt(2665*sqrt(5) + 4790)*(709*I/(533*sqrt(5) + 958) + 1) + 3
50*I)*log((16722951192450*I + 2480822188910)*sqrt(5)*x + (25712356272300*I
 + 3814385585140)*x + 5021907265*sqrt(5)*sqrt(1416617*sqrt(5) + 2178118) +
 (1240411094455*I - 8361475596225)*sqrt(5) + (8153361745*I + 7721428310)*s
qrt(1416617*sqrt(5) + 2178118) + 1907192792570*I - 12856178136150) - 1/400
*(-(3*I + 1)*sqrt(2665*sqrt(5) + 4790)*(709*I/(533*sqrt(5) + 958) + 1) + 3
50*I)*log((16722951192450*I + 2480822188910)*sqrt(5)*x + (25712356272300*I
 + 3814385585140)*x - 5021907265*sqrt(5)*sqrt(1416617*sqrt(5) + 2178118) +
 (1240411094455*I - 8361475596225)*sqrt(5) - (8153361745*I + 7721428310)*s
qrt(1416617*sqrt(5) + 2178118) + 1907192792570*I - 12856178136150) + 1/...
```

3.56. $\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx$

3.56.9 Mupad [B] (verification not implemented)

Time = 10.66 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.55

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx = \left(\sum_{k=1}^{4}\ln\left(-\frac{169\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)}{100} + \frac{11\,x}{1600} + \frac{\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)\,x\,131}{100} - \frac{\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^2\,x\,72}{5} - \mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^3\,x\,36 + \frac{59\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^2}{20} - 16\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^3 + \frac{27}{1600}\right)\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)\right) + \frac{\frac{9\,x^3}{20}-\frac{x^2}{5}+\frac{21\,x}{40}+\frac{19}{80}}{x^4+x^2+x+\frac{1}{4}}$$

input
```
int(1/(4*x + 4*x^2 + 4*x^4 + 1)^2,x)
```

output
```
symsum(log((11*x)/1600 - (169*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 +
29/64000, z, k))/100 + (131*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29
/64000, z, k)*x)/100 - (72*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/
64000, z, k)^2*x)/5 - 36*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64
000, z, k)^3*x + (59*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000,
 z, k)^2)/20 - 16*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z,
 k)^3 + 27/1600)*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z,
k), k, 1, 4) + ((21*x)/40 - x^2/5 + (9*x^3)/20 + 19/80)/(x + x^2 + x^4 + 1
/4)
```

3.57 $\int (8+24x+8x^2-15x^3+8x^4)^4\,dx$

3.57.1 Optimal result

Integrand size = 22, antiderivative size = 104

$$\int (8+24x+8x^2-15x^3+8x^4)^4\,dx = 4096x+24576x^2+\frac{237568x^3}{3}+139776x^4+\frac{538624x^5}{5} -30720x^6-\frac{566912x^7}{7}+36384x^8+\frac{641152x^9}{9} -\frac{169584x^{10}}{5}-\frac{331040x^{11}}{11}+31128x^{12}-\frac{12095x^{13}}{13} -\frac{75504x^{14}}{7}+\frac{102784x^{15}}{15}-1920x^{16}+\frac{4096x^{17}}{17}$$

output
```
4096*x+24576*x^2+237568/3*x^3+139776*x^4+538624/5*x^5-30720*x^6-566912/7*x
^7+36384*x^8+641152/9*x^9-169584/5*x^10-331040/11*x^11+31128*x^12-12095/13
*x^13-75504/7*x^14+102784/15*x^15-1920*x^16+4096/17*x^17
```

3.57.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int (8+24x+8x^2-15x^3+8x^4)^4\,dx = 4096x+24576x^2+\frac{237568x^3}{3}+139776x^4+\frac{538624x^5}{5} -30720x^6-\frac{566912x^7}{7}+36384x^8+\frac{641152x^9}{9} -\frac{169584x^{10}}{5}-\frac{331040x^{11}}{11}+31128x^{12}-\frac{12095x^{13}}{13} -\frac{75504x^{14}}{7}+\frac{102784x^{15}}{15}-1920x^{16}+\frac{4096x^{17}}{17}$$

input
```
Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4,x]
```

output
```
4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*
x^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (331
040*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^1
5)/15 - 1920*x^16 + (4096*x^17)/17
```

3.57.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8x^4-15x^3+8x^2+24x+8\right)^4 dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(4096x^{16}-30720x^{15}+102784x^{14}-151008x^{13}-12095x^{12}+373536x^{11}-331040x^{10}-339168x^9+641152x^8\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9}+36384x^8-\frac{566912x^7}{7}-30720x^6+\frac{538624x^5}{5}+139776x^4+\frac{237568x^3}{3}+24576x^2+4096x$$

input
```
Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4,x]
```

output
```
4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*
x^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (331
040*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^1
5)/15 - 1920*x^16 + (4096*x^17)/17
```

3.57.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

3.57.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.82

method	result
gosper	$4096x + 24576x^2 + \frac{237568}{3}x^3 + 139776x^4 + \frac{538624}{5}x^5 - 30720x^6 - \frac{566912}{7}x^7 + 36384x^8 + \frac{64115}{9}$
default	$4096x + 24576x^2 + \frac{237568}{3}x^3 + 139776x^4 + \frac{538624}{5}x^5 - 30720x^6 - \frac{566912}{7}x^7 + 36384x^8 + \frac{64115}{9}$
norman	$4096x + 24576x^2 + \frac{237568}{3}x^3 + 139776x^4 + \frac{538624}{5}x^5 - 30720x^6 - \frac{566912}{7}x^7 + 36384x^8 + \frac{64115}{9}$
risch	$4096x + 24576x^2 + \frac{237568}{3}x^3 + 139776x^4 + \frac{538624}{5}x^5 - 30720x^6 - \frac{566912}{7}x^7 + 36384x^8 + \frac{64115}{9}$
parallelrisch	$4096x + 24576x^2 + \frac{237568}{3}x^3 + 139776x^4 + \frac{538624}{5}x^5 - 30720x^6 - \frac{566912}{7}x^7 + 36384x^8 + \frac{64115}{9}$

input
```
int((8*x^4-15*x^3+8*x^2+24*x+8)^4,x,method=_RETURNVERBOSE)
```

output
```
4096*x+24576*x^2+237568/3*x^3+139776*x^4+538624/5*x^5-30720*x^6-566912/7*x
^7+36384*x^8+641152/9*x^9-169584/5*x^10-331040/11*x^11+31128*x^12-12095/13
*x^13-75504/7*x^14+102784/15*x^15-1920*x^16+4096/17*x^17
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\begin{aligned}\int \left(8+24x+8x^2-15x^3+8x^4\right)^4 \, dx = {} & \frac{4096}{17}\,x^{17} - 1920\,x^{16} + \frac{102784}{15}\,x^{15} - \frac{75504}{7}\,x^{14} \\ & - \frac{12095}{13}\,x^{13} + 31128\,x^{12} - \frac{331040}{11}\,x^{11} \\ & - \frac{169584}{5}\,x^{10} + \frac{641152}{9}\,x^{9} + 36384\,x^{8} \\ & - \frac{566912}{7}\,x^{7} - 30720\,x^{6} + \frac{538624}{5}\,x^{5} \\ & + 139776\,x^{4} + \frac{237568}{3}\,x^{3} + 24576\,x^{2} + 4096\,x\end{aligned}$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="fricas")
```

output
```
4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
 31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 -
566912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 2457
6*x^2 + 4096*x
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int (8+24x+8x^2-15x^3+8x^4)^4\,dx = \frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9}+36384x^8-\frac{566912x^7}{7}-30720x^6+\frac{538624x^5}{5}+139776x^4+\frac{237568x^3}{3}+24576x^2+4096x$$

input
```
integrate((8*x**4-15*x**3+8*x**2+24*x+8)**4,x)
```

output
```
4096*x**17/17 - 1920*x**16 + 102784*x**15/15 - 75504*x**14/7 - 12095*x**13
/13 + 31128*x**12 - 331040*x**11/11 - 169584*x**10/5 + 641152*x**9/9 + 363
84*x**8 - 566912*x**7/7 - 30720*x**6 + 538624*x**5/5 + 139776*x**4 + 23756
8*x**3/3 + 24576*x**2 + 4096*x
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int (8+24x+8x^2-15x^3+8x^4)^4\,dx = \frac{4096}{17}\,x^{17}-1920\,x^{16}+\frac{102784}{15}\,x^{15}-\frac{75504}{7}\,x^{14}-\frac{12095}{13}\,x^{13}+31128\,x^{12}-\frac{331040}{11}\,x^{11}-\frac{169584}{5}\,x^{10}+\frac{641152}{9}\,x^9+36384\,x^8-\frac{566912}{7}\,x^7-30720\,x^6+\frac{538624}{5}\,x^5+139776\,x^4+\frac{237568}{3}\,x^3+24576\,x^2+4096\,x$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="maxima")
```

output
```
4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
 31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 -
566912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 2457
6*x^2 + 4096*x
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int (8+24x+8x^2-15x^3+8x^4)^4\,dx = \frac{4096}{17}\,x^{17}-1920\,x^{16}+\frac{102784}{15}\,x^{15}-\frac{75504}{7}\,x^{14} - \frac{12095}{13}\,x^{13}+31128\,x^{12}-\frac{331040}{11}\,x^{11} - \frac{169584}{5}\,x^{10}+\frac{641152}{9}\,x^{9}+36384\,x^{8} - \frac{566912}{7}\,x^{7}-30720\,x^{6}+\frac{538624}{5}\,x^{5} + 139776\,x^{4}+\frac{237568}{3}\,x^{3}+24576\,x^{2}+4096\,x$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="giac")
```

output
```
4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
 31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 -
566912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 2457
6*x^2 + 4096*x
```

3.57.9 Mupad [B] (verification not implemented)

Time = 10.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int (8+24x+8x^2-15x^3+8x^4)^4\,dx = \frac{4096\,x^{17}}{17} - 1920\,x^{16} + \frac{102784\,x^{15}}{15} - \frac{75504\,x^{14}}{7} - \frac{12095\,x^{13}}{13} + 31128\,x^{12} - \frac{331040\,x^{11}}{11} - \frac{169584\,x^{10}}{5} + \frac{641152\,x^9}{9} + 36384\,x^8 - \frac{566912\,x^7}{7} - 30720\,x^6 + \frac{538624\,x^5}{5} + 139776\,x^4 + \frac{237568\,x^3}{3} + 24576\,x^2 + 4096\,x$$

input
```
int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^4,x)
```

output
```
4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*
x^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (331
040*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^1
5)/15 - 1920*x^16 + (4096*x^17)/17
```

3.58 $\int (8+24x+8x^2-15x^3+8x^4)^3\,dx$

3.58.1 Optimal result

Integrand size = 22, antiderivative size = 76

$$\int (8+24x+8x^2-15x^3+8x^4)^3\,dx = 512x+2304x^2+5120x^3+5040x^4-\frac{384x^5}{5}-2976x^6+\frac{5528x^7}{7}+2097x^8-\frac{2936x^9}{3}-\frac{4527x^{10}}{10}+\frac{6936x^{11}}{11}-240x^{12}+\frac{512x^{13}}{13}$$

output
```
512*x+2304*x^2+5120*x^3+5040*x^4-384/5*x^5-2976*x^6+5528/7*x^7+2097*x^8-29
36/3*x^9-4527/10*x^10+6936/11*x^11-240*x^12+512/13*x^13
```

3.58.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int (8+24x+8x^2-15x^3+8x^4)^3\,dx = 512x+2304x^2+5120x^3+5040x^4-\frac{384x^5}{5}-2976x^6+\frac{5528x^7}{7}+2097x^8-\frac{2936x^9}{3}-\frac{4527x^{10}}{10}+\frac{6936x^{11}}{11}-240x^{12}+\frac{512x^{13}}{13}$$

input
```
Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3,x]
```

output
```
512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^
7)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^1
2 + (512*x^13)/13
```

3.58.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8x^4-15x^3+8x^2+24x+8\right)^3 dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(512x^{12}-2880x^{11}+6936x^{10}-4527x^9-8808x^8+16776x^7+5528x^6-17856x^5-384x^4+20160x^3+15360x$$

$$\Big\downarrow \text{2009}$$

$$\frac{512x^{13}}{13}-240x^{12}+\frac{6936x^{11}}{11}-\frac{4527x^{10}}{10}-\frac{2936x^9}{3}+2097x^8+\frac{5528x^7}{7}-2976x^6-\frac{384x^5}{5}+5040x^4+5120x^3+2304x^2+512x$$

input
```
Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3,x]
```

output
```
512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^
7)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^1
2 + (512*x^13)/13
```

3.58.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

3.58.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

method	result
gosper	$512x+2304x^2+5120x^3+5040x^4-\frac{384}{5}x^5-2976x^6+\frac{5528}{7}x^7+2097x^8-\frac{2936}{3}x^9-\frac{4527}{10}x^{10}$
default	$512x+2304x^2+5120x^3+5040x^4-\frac{384}{5}x^5-2976x^6+\frac{5528}{7}x^7+2097x^8-\frac{2936}{3}x^9-\frac{4527}{10}x^{10}$
norman	$512x+2304x^2+5120x^3+5040x^4-\frac{384}{5}x^5-2976x^6+\frac{5528}{7}x^7+2097x^8-\frac{2936}{3}x^9-\frac{4527}{10}x^{10}$
risch	$512x+2304x^2+5120x^3+5040x^4-\frac{384}{5}x^5-2976x^6+\frac{5528}{7}x^7+2097x^8-\frac{2936}{3}x^9-\frac{4527}{10}x^{10}$
parallelrisch	$512x+2304x^2+5120x^3+5040x^4-\frac{384}{5}x^5-2976x^6+\frac{5528}{7}x^7+2097x^8-\frac{2936}{3}x^9-\frac{4527}{10}x^{10}$

input
```
int((8*x^4-15*x^3+8*x^2+24*x+8)^3,x,method=_RETURNVERBOSE)
```

output
```
512*x+2304*x^2+5120*x^3+5040*x^4-384/5*x^5-2976*x^6+5528/7*x^7+2097*x^8-29
36/3*x^9-4527/10*x^10+6936/11*x^11-240*x^12+512/13*x^13
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\begin{aligned}\int (8+24x+8x^2-15x^3+8x^4)^3\,dx &= \frac{512}{13}\,x^{13}-240\,x^{12}+\frac{6936}{11}\,x^{11}-\frac{4527}{10}\,x^{10}\\&\quad-\frac{2936}{3}\,x^9+2097\,x^8+\frac{5528}{7}\,x^7-2976\,x^6\\&\quad-\frac{384}{5}\,x^5+5040\,x^4+5120\,x^3+2304\,x^2+512\,x\end{aligned}$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="fricas")
```

output
```
512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x
^8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 +
512*x
```

3.58.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int (8+24x+8x^2-15x^3+8x^4)^3\,dx = \frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^9}{3} + 2097x^8 + \frac{5528x^7}{7} - 2976x^6 - \frac{384x^5}{5} + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

input
```
integrate((8*x**4-15*x**3+8*x**2+24*x+8)**3,x)
```

output
```
512*x**13/13 - 240*x**12 + 6936*x**11/11 - 4527*x**10/10 - 2936*x**9/3 + 2
097*x**8 + 5528*x**7/7 - 2976*x**6 - 384*x**5/5 + 5040*x**4 + 5120*x**3 +
2304*x**2 + 512*x
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int (8+24x+8x^2-15x^3+8x^4)^3\,dx = \frac{512}{13}\,x^{13} - 240\,x^{12} + \frac{6936}{11}\,x^{11} - \frac{4527}{10}\,x^{10} - \frac{2936}{3}\,x^9 + 2097\,x^8 + \frac{5528}{7}\,x^7 - 2976\,x^6 - \frac{384}{5}\,x^5 + 5040\,x^4 + 5120\,x^3 + 2304\,x^2 + 512\,x$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="maxima")
```

output
```
512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x
^8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 +
512*x
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3 \, dx = \frac{512}{13}\,x^{13}-240\,x^{12}+\frac{6936}{11}\,x^{11}-\frac{4527}{10}\,x^{10}-\frac{2936}{3}\,x^9+2097\,x^8+\frac{5528}{7}\,x^7-2976\,x^6-\frac{384}{5}\,x^5+5040\,x^4+5120\,x^3+2304\,x^2+512\,x$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="giac")
```

output
```
512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x
^8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 +
512*x
```

3.58.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3 \, dx = \frac{512\,x^{13}}{13}-240\,x^{12}+\frac{6936\,x^{11}}{11}-\frac{4527\,x^{10}}{10}-\frac{2936\,x^9}{3}+2097\,x^8+\frac{5528\,x^7}{7}-2976\,x^6-\frac{384\,x^5}{5}+5040\,x^4+5120\,x^3+2304\,x^2+512\,x$$

input
```
int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^3,x)
```

output
```
512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^
7)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^1
2 + (512*x^13)/13
```

3.59 $\int (8+24x+8x^2-15x^3+8x^4)^2\,dx$

3.59.1 Optimal result

Integrand size = 22, antiderivative size = 52

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2\,dx=64x+192x^2+\frac{704x^3}{3}+36x^4-\frac{528x^5}{5}+24x^6+\frac{353x^7}{7}-30x^8+\frac{64x^9}{9}$$

output
```
64*x+192*x^2+704/3*x^3+36*x^4-528/5*x^5+24*x^6+353/7*x^7-30*x^8+64/9*x^9
```

3.59.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2\,dx=64x+192x^2+\frac{704x^3}{3}+36x^4-\frac{528x^5}{5}+24x^6+\frac{353x^7}{7}-30x^8+\frac{64x^9}{9}$$

input
```
Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]
```

output
```
64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
 - 30*x^8 + (64*x^9)/9
```

3.59.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8x^4-15x^3+8x^2+24x+8\right)^2 dx$$

$$\Big\downarrow \text{ 2465}$$

$$\int \left(64x^8-240x^7+353x^6+144x^5-528x^4+144x^3+704x^2+384x+64\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

input
```
Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]
```

output
```
64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
 - 30*x^8 + (64*x^9)/9
```

3.59.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

3.59.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

method	result	size
gosper	$64x+192x^2+\frac{704}{3}x^3+36x^4-\frac{528}{5}x^5+24x^6+\frac{353}{7}x^7-30x^8+\frac{64}{9}x^9$	45
default	$64x+192x^2+\frac{704}{3}x^3+36x^4-\frac{528}{5}x^5+24x^6+\frac{353}{7}x^7-30x^8+\frac{64}{9}x^9$	45
norman	$64x+192x^2+\frac{704}{3}x^3+36x^4-\frac{528}{5}x^5+24x^6+\frac{353}{7}x^7-30x^8+\frac{64}{9}x^9$	45
risch	$64x+192x^2+\frac{704}{3}x^3+36x^4-\frac{528}{5}x^5+24x^6+\frac{353}{7}x^7-30x^8+\frac{64}{9}x^9$	45
parallelrisch	$64x+192x^2+\frac{704}{3}x^3+36x^4-\frac{528}{5}x^5+24x^6+\frac{353}{7}x^7-30x^8+\frac{64}{9}x^9$	45

input
```
int((8*x^4-15*x^3+8*x^2+24*x+8)^2,x,method=_RETURNVERBOSE)
```

output
```
64*x+192*x^2+704/3*x^3+36*x^4-528/5*x^5+24*x^6+353/7*x^7-30*x^8+64/9*x^9
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2\,dx=\frac{64}{9}\,x^9-30\,x^8+\frac{353}{7}\,x^7+24\,x^6-\frac{528}{5}\,x^5+36\,x^4+\frac{704}{3}\,x^3+192\,x^2+64\,x$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")
```

output
```
64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 +
192*x^2 + 64*x
```

3.59.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2\,dx=\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

input
```
integrate((8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

output
```
64*x**9/9 - 30*x**8 + 353*x**7/7 + 24*x**6 - 528*x**5/5 + 36*x**4 + 704*x*
*3/3 + 192*x**2 + 64*x
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2\,dx = \frac{64}{9}\,x^9-30\,x^8+\frac{353}{7}\,x^7+24\,x^6-\frac{528}{5}\,x^5+36\,x^4+\frac{704}{3}\,x^3+192\,x^2+64\,x$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")
```

output
```
64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 +
192*x^2 + 64*x
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2\,dx = \frac{64}{9}\,x^9-30\,x^8+\frac{353}{7}\,x^7+24\,x^6-\frac{528}{5}\,x^5+36\,x^4+\frac{704}{3}\,x^3+192\,x^2+64\,x$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")
```

output
```
64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 +
192*x^2 + 64*x
```

3.59.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2 \, dx = \frac{64\,x^9}{9} - 30\,x^8 + \frac{353\,x^7}{7} + 24\,x^6 - \frac{528\,x^5}{5} + 36\,x^4 + \frac{704\,x^3}{3} + 192\,x^2 + 64\,x$$

input
```
int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^2,x)
```

output
```
64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
 - 30*x^8 + (64*x^9)/9
```

3.60 $\int (8+24x+8x^2-15x^3+8x^4)\ dx$

3.60.1 Optimal result

Integrand size = 20, antiderivative size = 30

$$\int \left(8+24x+8x^2-15x^3+8x^4\right) dx = 8x+12x^2+\frac{8x^3}{3}-\frac{15x^4}{4}+\frac{8x^5}{5}$$

output
```
8*x+12*x^2+8/3*x^3-15/4*x^4+8/5*x^5
```

3.60.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(8+24x+8x^2-15x^3+8x^4\right) dx = 8x+12x^2+\frac{8x^3}{3}-\frac{15x^4}{4}+\frac{8x^5}{5}$$

input
```
Integrate[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]
```

output
```
8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

3.60.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8x^4-15x^3+8x^2+24x+8\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{8x^5}{5}-\frac{15x^4}{4}+\frac{8x^3}{3}+12x^2+8x$$

input
```
Int[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]
```

output
```
8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

3.60.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.60.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25
default	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25
norman	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25
risch	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25
parallelrisch	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25
parts	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25

input
```
int(8*x^4-15*x^3+8*x^2+24*x+8,x,method=_RETURNVERBOSE)
```

output
```
8*x+12*x^2+8/3*x^3-15/4*x^4+8/5*x^5
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(8+24x+8x^2-15x^3+8x^4\right) \, dx = \frac{8}{5}\,x^5 - \frac{15}{4}\,x^4 + \frac{8}{3}\,x^3 + 12\,x^2 + 8\,x$$

input
```
integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="fricas")
```

output
```
8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

3.60.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \left(8+24x+8x^2-15x^3+8x^4\right) \, dx = \frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

input
```
integrate(8*x**4-15*x**3+8*x**2+24*x+8,x)
```

output
```
8*x**5/5 - 15*x**4/4 + 8*x**3/3 + 12*x**2 + 8*x
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(8+24x+8x^2-15x^3+8x^4\right) \, dx = \frac{8}{5}\,x^5 - \frac{15}{4}\,x^4 + \frac{8}{3}\,x^3 + 12\,x^2 + 8\,x$$

input
```
integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="maxima")
```

output
```
8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)\,dx=\frac{8}{5}\,x^5-\frac{15}{4}\,x^4+\frac{8}{3}\,x^3+12\,x^2+8\,x$$

input
```
integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="giac")
```

output
```
8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

3.60.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)\,dx=\frac{8\,x^5}{5}-\frac{15\,x^4}{4}+\frac{8\,x^3}{3}+12\,x^2+8\,x$$

input
```
int(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8,x)
```

output
```
8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

3.61 $\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$

3.61.1 Optimal result

Integrand size = 22, antiderivative size = 263

$$\begin{aligned}\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx &= -\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\arctan\left(\frac{6-\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}}{\sqrt{2\left(-19+\sqrt{517}\right)}}\right)\\ &\quad-\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\arctan\left(\frac{6+\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}}{\sqrt{2\left(-19+\sqrt{517}\right)}}\right)\\ &\quad+\frac{1}{4}\sqrt{\frac{3}{13}}\arctan\left(\frac{8+12x-5x^2}{\sqrt{39}x^2}\right)\\ &\quad-\frac{1}{8}\sqrt{\frac{-5167+235\sqrt{517}}{40326}}\log\left(\sqrt{517}-\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right)\\ &\quad+\frac{1}{8}\sqrt{\frac{-5167+235\sqrt{517}}{40326}}\log\left(\sqrt{517}+\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right)\end{aligned}$$

output
```
1/52*arctan(1/39*(-5*x^2+12*x+8)/x^2*39^(1/2))*39^(1/2)-1/322608*ln((3+4/x
)^2+517^(1/2)-(3+4/x)*(38+2*517^(1/2))^(1/2))*(-208364442+9476610*517^(1/2
))^(1/2)+1/322608*ln((3+4/x)^2+517^(1/2)+(3+4/x)*(38+2*517^(1/2))^(1/2))*(
-208364442+9476610*517^(1/2))^(1/2)-1/161304*arctan((6+8/x-(38+2*517^(1/2)
)^(1/2))/(-38+2*517^(1/2))^(1/2))*(208364442+9476610*517^(1/2))^(1/2)-1/16
1304*arctan((6+8/x+(38+2*517^(1/2))^(1/2))/(-38+2*517^(1/2))^(1/2))*(20836
4442+9476610*517^(1/2))^(1/2)
```

3.61.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.21

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx = \text{RootSum}\left[8+24\#1+8\#1^2-15\#1^3+8\#1^4\&,\frac{\log(x-\#1)}{24+16\#1-45\#1^2+32\#1^3}\&\right]$$

input
```
Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1),x]
```

output
```
RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , Log[x - #1]/(24 + 16*#1
- 45*#1^2 + 32*#1^3) & ]
```

3.61.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.19, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {2504, 27, 2202, 27, 1432, 1083, 217, 1483, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{8x^4-15x^3+8x^2+24x+8}dx$$

$$\Big\downarrow \ 2504$$

$$-1024\int \frac{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^2}{512\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)}d\left(\frac{3}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \ 27$$

$$-2\int \frac{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^2}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \ 2202$$

$$-2\left(\int \frac{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+9}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)+\int -\frac{24\left(\frac{3}{4}+\frac{1}{x}\right)}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)\right)$$

$$\Big\downarrow 27$$

$$-2\left(\int \frac{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+9}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)-24\int \frac{\frac{3}{4}+\frac{1}{x}}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)\right)$$

$$\Big\downarrow 1432$$

$$-2\left(\int \frac{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+9}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)-12\int \frac{1}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)^2\right)$$

$$\Big\downarrow 1083$$

$$-2\left(24\int \frac{1}{-\left(\frac{3}{4}+\frac{1}{x}\right)^4-159744}d\left(512\left(\frac{3}{4}+\frac{1}{x}\right)^2-608\right)+\int \frac{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+9}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)\right)$$

$$\Big\downarrow 217$$

$$-2\left(\int \frac{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+9}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)-\frac{1}{8}\sqrt{\frac{3}{13}}\arctan\left(\frac{512\left(\frac{1}{x}+\frac{3}{4}\right)^2-608}{64\sqrt{39}}\right)\right)$$

$$\Big\downarrow 1483$$

$$-2\left(\frac{\int \frac{8\left(9\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}-2\left(9-\sqrt{517}\right)\left(\frac{3}{4}+\frac{1}{x}\right)\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{8\sqrt{1034\left(19+\sqrt{517}\right)}}+\frac{\int \frac{8\left(2\left(9-\sqrt{517}\right)\left(\frac{3}{4}+\frac{1}{x}\right)+9\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{8\sqrt{1034\left(19+\sqrt{517}\right)}}-\frac{1}{8}\sqrt{\frac{3}{13}}\text{ar}$$

$$\Big\downarrow 27$$

$$-2\left(\frac{\int \frac{9\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}-2\left(9-\sqrt{517}\right)\left(\frac{3}{4}+\frac{1}{x}\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{\sqrt{1034\left(19+\sqrt{517}\right)}}+\frac{\int \frac{2\left(9-\sqrt{517}\right)\left(\frac{3}{4}+\frac{1}{x}\right)+9\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{\sqrt{1034\left(19+\sqrt{517}\right)}}-\frac{1}{8}\sqrt{\frac{3}{13}}\text{ar}$$

$$\Big\downarrow 1142$$

3.61. $\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$

$$-2\left(\frac{\sqrt{\frac{1}{2}\left(5167+235\sqrt{517}\right)}\int \frac{1}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)-\frac{1}{16}\left(9-\sqrt{517}\right)\int -\frac{4\left(\sqrt{2\left(19+\sqrt{5\right.}\right.}}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19-\right.}}}{\sqrt{1034\left(19+\sqrt{517}\right)}}\right.$$

$\downarrow$ 27

$$-2\left(\frac{\sqrt{\frac{1}{2}\left(5167+235\sqrt{517}\right)}\int \frac{1}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+\frac{1}{4}\left(9-\sqrt{517}\right)\int \frac{\sqrt{2\left(19+\sqrt{517}\right)}\cdot}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{5\right.}}}}{\sqrt{1034\left(19+\sqrt{517}\right)}}\right.$$

$\downarrow$ 1083

$$-2\left(\frac{\frac{1}{4}\left(9-\sqrt{517}\right)\int \frac{\sqrt{2\left(19+\sqrt{517}\right)}-8\left(\frac{3}{4}+\frac{1}{x}\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)-\sqrt{2\left(5167+235\sqrt{517}\right)}\int \frac{1}{32\left(19-\sqrt{517}\right)-\left(32\left(\frac{3}{4}+\right.\right.}}{\sqrt{1034\left(19+\sqrt{517}\right)}}\right.$$

$\downarrow$ 217

$$-2\left(\frac{\frac{1}{4}\left(9-\sqrt{517}\right)\int \frac{\sqrt{2\left(19+\sqrt{517}\right)}-8\left(\frac{3}{4}+\frac{1}{x}\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{\sqrt{517}-19}}\arctan\left(\frac{32\left(\frac{1}{x}+\frac{3}{4}\right)-4\sqrt{2\left(19+\sqrt{5\right.}}}{4\sqrt{2\left(\sqrt{517}-19\right)}}\right.}{\sqrt{1034\left(19+\sqrt{517}\right)}}\right.$$

$\downarrow$ 1103

$$-2\left(-\frac{1}{8}\sqrt{\frac{3}{13}}\arctan\left(\frac{512\left(\frac{1}{x}+\frac{3}{4}\right)^2-608}{64\sqrt{39}}\right)+\frac{\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{\sqrt{517}-19}}\arctan\left(\frac{32\left(\frac{1}{x}+\frac{3}{4}\right)-4\sqrt{2\left(19+\sqrt{517}\right)}}{4\sqrt{2\left(\sqrt{517}-19\right)}}\right)-\frac{1}{16}\left(9-\sqrt{517\right.}}{\sqrt{1034\left(19+\sqrt{\right.}}}\right.$$

3.61. $\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$

input
```
Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1),x]
```

output
```
-2*(-1/8*(Sqrt[3/13]*ArcTan[(-608 + 512*(3/4 + x^(-1))^2)/(64*Sqrt[39])])
+ ((Sqrt[(5167 + 235*Sqrt[517])/(-19 + Sqrt[517])]*ArcTan[(-4*Sqrt[2*(19 +
 Sqrt[517])] + 32*(3/4 + x^(-1)))/(4*Sqrt[2*(-19 + Sqrt[517])])])/4 - ((9
- Sqrt[517])*Log[Sqrt[517] - 4*Sqrt[2*(19 + Sqrt[517])]*(3/4 + x^(-1)) + 1
6*(3/4 + x^(-1))^2])/16)/Sqrt[1034*(19 + Sqrt[517])] + ((Sqrt[(5167 + 235*
Sqrt[517])/(-19 + Sqrt[517])]*ArcTan[(4*Sqrt[2*(19 + Sqrt[517])] + 32*(3/4
 + x^(-1)))/(4*Sqrt[2*(-19 + Sqrt[517])])])/4 + ((9 - Sqrt[517])*Log[Sqrt[
517] + 4*Sqrt[2*(19 + Sqrt[517])]*(3/4 + x^(-1)) + 16*(3/4 + x^(-1))^2])/1
6)/Sqrt[1034*(19 + Sqrt[517])])
```

3.61.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

3.61.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.19

method	result	size
default	$\sum_{\_R=\mathrm{RootOf}(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8)} \frac{\ln(x-\_R)}{32\_R^3-45\_R^2+16\_R+24}$	49
risch	$\sum_{\_R=\mathrm{RootOf}(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8)} \frac{\ln(x-\_R)}{32\_R^3-45\_R^2+16\_R+24}$	49

input
```
int(1/(8*x^4-15*x^3+8*x^2+24*x+8),x,method=_RETURNVERBOSE)
```

output
```
sum(1/(32*_R^3-45*_R^2+16*_R+24)*ln(x-_R),_R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+
24*_Z+8))
```

3.61.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 1297, normalized size of antiderivative = 4.93

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="fricas")
```

output
```
-1/104*(-I*sqrt(13)*sqrt(3) + 52*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167
/322608))*log(37895495846208*(1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(109/1613
04*I*sqrt(13)*sqrt(3) - 5167/322608))^3 - 537872704512*(1/104*I*sqrt(13)*s
qrt(3) - 1/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2 + 561402
7117*I*sqrt(13)*sqrt(3) + 1789133960*x - 291929410084*sqrt(109/161304*I*sq
rt(13)*sqrt(3) - 5167/322608) + 2270349121) - 1/104*(I*sqrt(13)*sqrt(3) +
52*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))*log(-4736936980776*
(1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/
322608))^3 + 20163*(-1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-109/161304*I*sqr
t(13)*sqrt(3) - 5167/322608))^2*(-2258963*I*sqrt(13)*sqrt(3) + 117466076*s
qrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608) - 3334528) + 517*(8809955
7*(1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 516
7/322608))^2 - 16507)*(I*sqrt(13)*sqrt(3) + 52*sqrt(-109/161304*I*sqrt(13)
*sqrt(3) - 5167/322608)) - 5545754165/8*I*sqrt(13)*sqrt(3) + 223641745*x +
 72094804145/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608) - 9165628
24) + 1/80652*(sqrt(40326)*sqrt(-120978*(1/104*I*sqrt(13)*sqrt(3) - 1/2*sq
rt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2 - 120978*(-1/104*I*sqrt
(13)*sqrt(3) - 1/2*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2 -
 1551/208*(I*sqrt(13)*sqrt(3) + 52*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5
167/322608))*(-I*sqrt(13)*sqrt(3) + 52*sqrt(109/161304*I*sqrt(13)*sqrt(...
```

3.61.6 Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.16

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$$
$$= \text{RootSum}\left(50326848t^4 + 765960t^2 + 12753t + 64, \left(t \mapsto t\log\left(\frac{100785893208t^3}{4758335} - \frac{1430512512t^2}{4758335} + \frac{7298}{223}\right.\right.\right.$$

input
```
integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8),x)
```

output
```
RootSum(50326848*_t**4 + 765960*_t**2 + 12753*_t + 64, Lambda(_t, _t*log(1
00785893208*_t**3/4758335 - 1430512512*_t**2/4758335 + 72982352521*_t/2236
41745 + x + 2270349121/1789133960)))
```

3.61.7 Maxima [F]

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx=\int \frac{1}{8\,x^4-15\,x^3+8\,x^2+24\,x+8}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="maxima")
```

output
```
integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

3.61.8 Giac [F]

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx=\int \frac{1}{8\,x^4-15\,x^3+8\,x^2+24\,x+8}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="giac")
```

output
```
integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.47

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$$
$$=\sum_{k=1}^{4}\ln\left(-\frac{\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)\left(2184\,\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)+256\,x\right.}{}\right.$$
$$\left.+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)$$

input
```
int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8),x)
```

output
```
symsum(log(-(root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z,
k)*(2184*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k) +
 256*x + 38259*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z
, k)*x + 1531920*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357,
 z, k)^2*x + 805896*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/7863
57, z, k)^2 - 120))/4096)*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 +
1/786357, z, k), k, 1, 4)
```

3.61. $\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$

3.62 $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx$

3.62.1 Optimal result

Integrand size = 22, antiderivative size = 366

$$\begin{aligned}
&\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx \\
&= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)}+\frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} \\
&-\frac{\sqrt{\frac{19+\sqrt{517}}{40326}}\left(1678181+74897\sqrt{517}\right)\arctan\left(\frac{6-\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}}{\sqrt{2\left(-19+\sqrt{517}\right)}}\right)}{645216} \\
&-\frac{\sqrt{\frac{19+\sqrt{517}}{40326}}\left(1678181+74897\sqrt{517}\right)\arctan\left(\frac{6+\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}}{\sqrt{2\left(-19+\sqrt{517}\right)}}\right)}{645216} \\
&+\frac{73}{208}\sqrt{\frac{3}{13}}\arctan\left(\frac{8+12x-5x^2}{\sqrt{39}x^2}\right) \\
&-\frac{\sqrt{\frac{-59644114671451+2623170438295\sqrt{517}}{40326}}\log\left(\sqrt{517}-\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right)}{645216} \\
&+\frac{\sqrt{\frac{-59644114671451+2623170438295\sqrt{517}}{40326}}\log\left(\sqrt{517}+\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right)}{645216}
\end{aligned}$$

output
```
-3/208*(3359-107*(3+4/x)^2)/(517-38*(3+4/x)^2+(3+4/x)^4)+1/322608*(3327931
-129631*(3+4/x)^2)*(3+4/x)/(517-38*(3+4/x)^2+(3+4/x)^4)+73/2704*arctan(1/3
9*(-5*x^2+12*x+8)/x^2*39^(1/2))*39^(1/2)-1/26018980416*arctan((6+8/x-(38+2
*517^(1/2))^(1/2))/(-38+2*517^(1/2))^(1/2))*(1678181+74897*517^(1/2))*(766
194+40326*517^(1/2))^(1/2)-1/26018980416*arctan((6+8/x+(38+2*517^(1/2))^(1
/2))/(-38+2*517^(1/2))^(1/2))*(1678181+74897*517^(1/2))*(766194+40326*517^
(1/2))^(1/2)-1/26018980416*ln((3+4/x)^2+517^(1/2)-(3+4/x)*(38+2*517^(1/2))
^(1/2))*(-2405208568240933026+10578197109468417O*517^(1/2))^(1/2)+1/260189
80416*ln((3+4/x)^2+517^(1/2)+(3+4/x)*(38+2*517^(1/2))^(1/2))*(-24052085682
40933026+105781971094684170*517^(1/2))^(1/2)
```

3.62.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.35

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx = \frac{72888+89033x-94314x^2+39280x^3}{161304\,(8+24x+8x^2-15x^3+8x^4)} + \frac{\text{RootSum}\left[8+24\#1+8\#1^2-15\#1^3+8\#1^4\&, \frac{74897\log(x-\#1)-57489\log(x-\#1)\#1+19640\log(x-\#1)\#1^2}{24+16\#1-45\#1^2+32\#1^3}\&\right]}{80652}$$

input
```
Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2),x]
```

output
```
(72888 + 89033*x - 94314*x^2 + 39280*x^3)/(161304*(8 + 24*x + 8*x^2 - 15*x
^3 + 8*x^4)) + RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , (74897*Lo
g[x - #1] - 57489*Log[x - #1]*#1 + 19640*Log[x - #1]*#1^2)/(24 + 16*#1 - 4
5*#1^2 + 32*#1^3) & ]/80652
```

3.62.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.17, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {2504, 27, 2202, 2194, 27, 2191, 27, 1083, 217, 2206, 27, 1483, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8x^4-15x^3+8x^2+24x+8)^2}\,dx$$

$$\Big\downarrow \; 2504$$

$$-1024\int \frac{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^6}{4096\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^2}d\left(\frac{3}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{4}\int \frac{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^6}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^2}d\left(\frac{3}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \; 2202$$

$$\frac{1}{4}\left(-\int \frac{4096\left(\frac{3}{4}+\frac{1}{x}\right)^6+34560\left(\frac{3}{4}+\frac{1}{x}\right)^4+19440\left(\frac{3}{4}+\frac{1}{x}\right)^2+729}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^2}d\left(\frac{3}{4}+\frac{1}{x}\right)-\int \frac{\left(-18432\left(\frac{3}{4}+\frac{1}{x}\right)^4-34560\left(\frac{3}{4}+\right.\right.}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\right.\right.}\right.$$

$$\Big\downarrow \; 2194$$

$$\frac{1}{4}\left(-\frac{1}{2}\int -\frac{72\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4+480\left(\frac{3}{4}+\frac{1}{x}\right)^2+81\right)}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^2}d\left(\frac{3}{4}+\frac{1}{x}\right)^2-\int \frac{4096\left(\frac{3}{4}+\frac{1}{x}\right)^6+34560\left(\frac{3}{4}+\frac{1}{x}\right)^4+19440\left(\frac{3}{4}\right.}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+51\right.}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\left(36\int \frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4+480\left(\frac{3}{4}+\frac{1}{x}\right)^2+81}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^2}d\left(\frac{3}{4}+\frac{1}{x}\right)^2-\int \frac{4096\left(\frac{3}{4}+\frac{1}{x}\right)^6+34560\left(\frac{3}{4}+\frac{1}{x}\right)^4+19440\left(\frac{3}{4}+\frac{1}{x}\right.}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^2}\right.$$

$$\Big\downarrow \; 2191$$

$$\frac{1}{4}\left(36\left(\frac{\int \frac{598016}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)^2}{159744}-\frac{3359-1712\left(\frac{1}{x}+\frac{3}{4}\right)^2}{624\left(256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517\right)}\right)-\int \frac{4096\left(\frac{3}{4}+\frac{1}{x}\right)^6}{\left(256\right.}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\left(36\left(\frac{146}{39}\int \frac{1}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)^2-\frac{3359-1712\left(\frac{1}{x}+\frac{3}{4}\right)^2}{624\left(256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517\right)}\right)-\right.$$

$$\Big\downarrow \; 1083$$

3.62. $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx$

$$\frac{1}{4}\left(36\left(-\frac{292}{39}\int \frac{1}{-\left(\frac{3}{4}+\frac{1}{x}\right)^4-159744}d\left(512\left(\frac{3}{4}+\frac{1}{x}\right)^2-608\right)-\frac{3359-1712\left(\frac{1}{x}+\frac{3}{4}\right)^2}{624\left(256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517\right)}\right)-\right.$$

$\downarrow$ 217

$$\frac{1}{4}\left(36\left(\frac{73\arctan\left(\frac{512\left(\frac{1}{x}+\frac{3}{4}\right)^2-608}{64\sqrt{39}}\right)}{624\sqrt{39}}-\frac{3359-1712\left(\frac{1}{x}+\frac{3}{4}\right)^2}{624\left(256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517\right)}\right)-\int \frac{4096\left(\frac{3}{4}+\frac{1}{x}\right)^6+34560\left(\frac{3}{4}\right.}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-\right.}\right.$$

$\downarrow$ 2206

$$\frac{1}{4}\left(-\frac{\int \frac{16384\left(1198352\left(\frac{3}{4}+\frac{1}{x}\right)^2+1678181\right)}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)}{165175296}+36\left(\frac{73\arctan\left(\frac{512\left(\frac{1}{x}+\frac{3}{4}\right)^2-608}{64\sqrt{39}}\right)}{624\sqrt{39}}-\frac{3359-1712\left(\frac{1}{x}+\frac{3}{4}\right.}{624\left(256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\right.\right.}\right.\right.$$

$\downarrow$ 27

$$\frac{1}{4}\left(-\frac{2\int \frac{1198352\left(\frac{3}{4}+\frac{1}{x}\right)^2+1678181}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)}{20163}+36\left(\frac{73\arctan\left(\frac{512\left(\frac{1}{x}+\frac{3}{4}\right)^2-608}{64\sqrt{39}}\right)}{624\sqrt{39}}-\frac{3359-1712\left(\frac{1}{x}+\frac{3}{4}\right)^2}{624\left(256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right.\right.}\right.\right.$$

$\downarrow$ 1483

$$\frac{1}{4}\left(-\frac{2\left(\frac{\int \frac{8\left(1678181\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}-2\left(1678181-74897\sqrt{517}\right)\left(\frac{3}{4}+\frac{1}{x}\right)\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{8\sqrt{1034\left(19+\sqrt{517}\right)}}+\frac{\int \frac{8\left(2\left(1678181-74897\sqrt{517}\right)\left(\frac{3}{4}+\frac{1}{x}\right)+1678181\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}\right.}{8\sqrt{1034\left(19+\sqrt{517}\right)}}\right.}{20163}\right.$$

$\downarrow$ 27

3.62. $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx$

$$\frac{1}{4}\left(-\frac{2\left(\frac{\int \frac{1678181\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}-2\left(1678181-74897\sqrt{517}\right)\left(\frac{3}{4}+\frac{1}{x}\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{\sqrt{1034\left(19+\sqrt{517}\right)}}+\frac{\int \frac{2\left(1678181-74897\sqrt{517}\right)\left(\frac{3}{4}+\frac{1}{x}\right)+1678181\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{\sqrt{1034\left(19+\sqrt{517}\right)}}\right)}{20163}\right.$$

$\downarrow$ 1142

$$\frac{1}{4}\left(-\frac{2\left(\frac{\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}\left(1678181+74897\sqrt{517}\right)\int \frac{1}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)-\frac{1}{16}\left(1678181-74897\sqrt{517}\right)\int -\frac{\cdots}{16\left(\frac{3}{4}+\cdots\right.}}{\sqrt{1034\left(19+\sqrt{517}\right)}}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 27

$$\frac{1}{4}\left(-\frac{2\left(\frac{\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}\left(1678181+74897\sqrt{517}\right)\int \frac{1}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+\frac{1}{4}\left(1678181-74897\sqrt{517}\right)\int \frac{\cdots}{16\left(\frac{3}{4}+\frac{1}{x}\right)\cdots}}{\sqrt{1034\left(19+\sqrt{517}\right)}}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 1083

3.62. $\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^2}\,dx$

$$\frac{1}{4}\left(-\frac{2\left(\frac{\frac{1}{4}\left(1678181-74897\sqrt{517}\right)\int \frac{\sqrt{2\left(19+\sqrt{517}\right)}-8\left(\frac{3}{4}+\frac{1}{x}\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)-\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}\left(1678181+74897\sqrt{517}\right)\int \frac{\cdots}{32\left(19-\sqrt{517}\cdots\right.}\cdots}{\sqrt{1034\left(19+\sqrt{517}\right)}}\cdots\right.}{\cdots}\cdots\right.$$

$\downarrow$ 217

$$\frac{1}{4}\left(-\frac{2\left(\frac{\frac{1}{4}\left(1678181-74897\sqrt{517}\right)\int \frac{\sqrt{2\left(19+\sqrt{517}\right)}-8\left(\frac{3}{4}+\frac{1}{x}\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+\frac{1}{8}\sqrt{\frac{19+\sqrt{517}}{\sqrt{517}-19}}\left(1678181+74897\sqrt{517}\right)\arctan\left(\frac{32\left(\frac{1}{x}+\frac{3}{4}\cdots\right.}{4\sqrt{\cdots}}\cdots\right.}{\sqrt{1034\left(19+\sqrt{517}\right)}}\cdots\right.}{\cdots}\cdots\right.$$

$\downarrow$ 1103

$$\frac{1}{4}\left(36\left(\frac{73\arctan\left(\frac{512\left(\frac{1}{x}+\frac{3}{4}\right)^2-608}{64\sqrt{39}}\right)}{624\sqrt{39}}-\frac{3359-1712\left(\frac{1}{x}+\frac{3}{4}\right)^2}{624\left(256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517\right)}\right)-\frac{2\left(\frac{\frac{1}{8}\sqrt{\frac{19+\sqrt{517}}{\sqrt{517}-19}}\left(1678181+74897\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

input

```
Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2),x]
```

3.62. $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx$

output
```
(((3327931 - 2074096*(3/4 + x^(-1))^2)*(3/4 + x^(-1)))/(20163*(517 - 608*(
3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4)) + 36*(-1/624*(3359 - 1712*(3/4 +
x^(-1))^2)/(517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4) + (73*ArcTa
n[(-608 + 512*(3/4 + x^(-1))^2)/(64*Sqrt[39])])/(624*Sqrt[39])) - (2*(((Sq
rt[(19 + Sqrt[517])/(-19 + Sqrt[517])]*(1678181 + 74897*Sqrt[517])*ArcTan[
(-4*Sqrt[2*(19 + Sqrt[517])] + 32*(3/4 + x^(-1)))/(4*Sqrt[2*(-19 + Sqrt[51
7])])])/8 - ((1678181 - 74897*Sqrt[517])*Log[Sqrt[517] - 4*Sqrt[2*(19 + Sq
rt[517])]*(3/4 + x^(-1)) + 16*(3/4 + x^(-1))^2])/16)/Sqrt[1034*(19 + Sqrt[
517])] + ((Sqrt[(19 + Sqrt[517])/(-19 + Sqrt[517])]*(1678181 + 74897*Sqrt[
517])*ArcTan[(4*Sqrt[2*(19 + Sqrt[517])] + 32*(3/4 + x^(-1)))/(4*Sqrt[2*(-
19 + Sqrt[517])])])/8 + ((1678181 - 74897*Sqrt[517])*Log[Sqrt[517] + 4*Sqr
t[2*(19 + Sqrt[517])]*(3/4 + x^(-1)) + 16*(3/4 + x^(-1))^2])/16)/Sqrt[1034
*(19 + Sqrt[517])]))/20163)/4
```

3.62.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.62. $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx$

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

3.62.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.26

method	result
default	$\frac{\frac{2455}{80652}x^3-\frac{1429}{19552}x^2+\frac{89033}{1290432}x+\frac{3037}{53768}}{x^4-\frac{15}{8}x^3+x^2+3x+1}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)}\frac{\left(19640\_R^2-57489\_R+74897\right)\ln\left(x-\_R\right)}{32\_R^3-45\_R^2+16\_R+24}\right.}{80652}$
risch	$\frac{\frac{2455}{80652}x^3-\frac{1429}{19552}x^2+\frac{89033}{1290432}x+\frac{3037}{53768}}{x^4-\frac{15}{8}x^3+x^2+3x+1}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)}\frac{\left(19640\_R^2-57489\_R+74897\right)\ln\left(x-\_R\right)}{32\_R^3-45\_R^2+16\_R+24}\right.}{80652}$

input
```
int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x,method=_RETURNVERBOSE)
```

output
```
(2455/80652*x^3-1429/19552*x^2+89033/1290432*x+3037/53768)/(x^4-15/8*x^3+x
^2+3*x+1)+1/80652*sum((19640*_R^2-57489*_R+74897)/(32*_R^3-45*_R^2+16*_R+2
4)*ln(x-_R),_R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8))
```

3.62.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 1540, normalized size of antiderivative = 4.21

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx=\text{Too large to display}$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")
```

output
```
1/26018980416*(6336021120*x^3 - 4811202*(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8
)*(-73*I*sqrt(13)*sqrt(3) + 2704*sqrt(537508757/26903625750144*I*sqrt(13)*
sqrt(3) - 59644114671451/2098482808511232))*log(-131155175531952*(21770028
8287626155772963*I*sqrt(13)*sqrt(3) + 806385725383207020835742 4*sqrt(-5375
08757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232)
 - 2904532176689925771712)*(73/5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt(53750875
7/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232))^2
+ 2115233227181899165359763637490696823296*(-73/5408*I*sqrt(13)*sqrt(3) -
1/2*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/209
8482808511232))^3 - 801867*(4877746614270480948333322 20336*(-73/5408*I*sqr
t(13)*sqrt(3) - 1/2*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59
644114671451/2098482808511232))^2 + 3281707530577268651899)*(-73*I*sqrt(13
)*sqrt(3) + 2704*sqrt(537508757/26903625750144*I*sqrt(13)*sqrt(3) - 596441
14671451/2098482808511232)) - 32462651961611566147760515527844884 93/4*I*sq
rt(13)*sqrt(3) + 1509305314029940798815339032152 65*x - 300613051041772859 1
217275136551115371 6*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59
644114671451/2098482808511232) + 1090507114917617311013907313810175 2) - 48
11202*(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)*(73*I*sqrt(13)*sqrt(3) + 2704*sq
rt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/209848280
8511232))*log(-16921865817455193322878109099925574586368*(-73/5408*I*sq...
```

3.62.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3839 vs. $2(292) = 584$.

Time = 2.23 (sec) , antiderivative size = 3839, normalized size of antiderivative = 10.49

$$\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

output
```
(39280*x**3 - 94314*x**2 + 89033*x + 72888)/(1290432*x**4 - 2419560*x**3 +
 1290432*x**2 + 3871296*x + 1290432) + sqrt(-59644114671451/16787862468089
856 + 5073830635*sqrt(517)/32471687559168)*log(x**2 + x*(-1123969950204685
0330693256748475546 3/6037221256119763195261356128610 60 - 29643869829812833
230907750777733957*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(5
17))/193641939879239446163785514191223839608 0 - 18153326104312036073 2*sqrt
(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(5
17)) + 626362156858715004293 5*sqrt(517) + 35575799716919912947693826 75)/15
093053140299407988153390321526 5 - 46926347979646613249222*sqrt(517)/297468
60362632912338339 + 994065243322493861977*sqrt(78)*sqrt(-59644114671451 +
2623170438295*sqrt(517))/142784929740637979224027 2 + 994065243322493861977
*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-7120427
417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62
636215685871500429 35*sqrt(517) + 35575799716919912947693826 75)/12909462658
61596307758570094608158930720) - 4597149706773066968921854791223560238809 1
89313591735176029*sqrt(517)*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644
114671451 + 2623170438295*sqrt(517)) + 626362156858715004293 5*sqrt(517) +
35575799716919912947693826 75)/18432767186998698626450604048374763890148748
05380627572841955840 - 10221327637202671758827804250636131310886019359583 0
387808115871094971545996741148644 7/3022018123806816906346315343858920 67...
```

3.62.7 Maxima [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^2}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")
```

output
```
1/161304*(39280*x^3 - 94314*x^2 + 89033*x + 72888)/(8*x^4 - 15*x^3 + 8*x^2
 + 24*x + 8) + 1/80652*integrate((19640*x^2 - 57489*x + 74897)/(8*x^4 - 15
*x^3 + 8*x^2 + 24*x + 8), x)
```

3.62.8 Giac [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^2}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")
```

output
```
integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-2), x)
```

3.62.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.49

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx = \frac{\frac{2455\,x^3}{80652}-\frac{1429\,x^2}{19552}+\frac{89033\,x}{1290432}+\frac{3037}{53768}}{x^4-\frac{15\,x^3}{8}+x^2+3\,x+1} + \left(\sum_{k=1}^{4}\ln\left(\frac{2146659825\,\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)}{2960381771776} + \frac{2222183\,x}{338246745408} + \frac{\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)x\,924124364159}{26643435945984} - \frac{\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)^2x\,72451101}{8470528} - \frac{\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)^3x\,95745}{256} + \frac{389551\,\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)^2}{264704} - \frac{100737\,\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)^3}{512} + \frac{271033}{624455529984}\right)\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)\right)$$

input
```
int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^2,x)
```

output
```
((89033*x)/1290432 - (1429*x^2)/19552 + (2455*x^3)/80652 + 3037/53768)/(3*
x + x^2 - (15*x^3)/8 + x^4 + 1) + symsum(log((2146659825*root(z^4 + (14911
625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 430234
40/44204510553294663, z, k))/2960381771776 + (2222183*x)/338246745408 + (9
24124364159*root(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261
*z)/3730636104019968 + 43023440/44204510553294663, z, k)*x)/26643435945984
 - (72451101*root(z^4 + (14911625619311*z^2)/524620702127808 + (3923813926
1*z)/3730636104019968 + 43023440/44204510553294663, z, k)^2*x)/8470528 - (
95745*root(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/37
30636104019968 + 43023440/44204510553294663, z, k)^3*x)/256 + (389551*root
(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/373063610401
9968 + 43023440/44204510553294663, z, k)^2)/264704 - (100737*root(z^4 + (1
4911625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43
023440/44204510553294663, z, k)^3)/512 + 271033/624455529984)*root(z^4 + (
14911625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 4
3023440/44204510553294663, z, k), k, 1, 4)
```

3.62. $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx$

3.63 $\int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5$

3.63.1 Optimal result

Integrand size = 51, antiderivative size = 14

$$\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right)^3 dx = \frac{(a+bx)^{16}}{16b}$$

output
```
1/16*(b*x+a)^16/b
```

3.63.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right)^3 dx = \frac{(a+bx)^{16}}{16b}$$

input
```
Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
 + b^5*x^5)^3,x]
```

output
```
(a + b*x)^16/(16*b)
```

3.63.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2006, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3 dx$$

$$\Big\downarrow \text{2006}$$

$$\int (a+bx)^{15}dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+bx)^{16}}{16b}$$

input
```
Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5
*x^5)^3,x]
```

output
```
(a + b*x)^16/(16*b)
```

3.63.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

3.63.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(12) = 24.

Time = 0.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 11.71

method	result
default	$a^{15}x+\frac{15}{2}a^{14}b\,x^{2}+35a^{13}b^{2}x^{3}+\frac{455}{4}a^{12}b^{3}x^{4}+273a^{11}b^{4}x^{5}+\frac{1001}{2}a^{10}b^{5}x^{6}+715a^{9}b^{6}x^{7}+\frac{6435}{8}a^{8}$
norman	$a^{15}x+\frac{15}{2}a^{14}b\,x^{2}+35a^{13}b^{2}x^{3}+\frac{455}{4}a^{12}b^{3}x^{4}+273a^{11}b^{4}x^{5}+\frac{1001}{2}a^{10}b^{5}x^{6}+715a^{9}b^{6}x^{7}+\frac{6435}{8}a^{8}$
risch	$a^{15}x+\frac{15}{2}a^{14}b\,x^{2}+35a^{13}b^{2}x^{3}+\frac{455}{4}a^{12}b^{3}x^{4}+273a^{11}b^{4}x^{5}+\frac{1001}{2}a^{10}b^{5}x^{6}+715a^{9}b^{6}x^{7}+\frac{6435}{8}a^{8}$
parallelrisch	$a^{15}x+\frac{15}{2}a^{14}b\,x^{2}+35a^{13}b^{2}x^{3}+\frac{455}{4}a^{12}b^{3}x^{4}+273a^{11}b^{4}x^{5}+\frac{1001}{2}a^{10}b^{5}x^{6}+715a^{9}b^{6}x^{7}+\frac{6435}{8}a^{8}$
gosper	$\frac{x(b^{15}x^{15}+16a\,b^{14}x^{14}+120a^{2}b^{13}x^{13}+560a^{3}b^{12}x^{12}+1820a^{4}b^{11}x^{11}+4368a^{5}b^{10}x^{10}+8008a^{6}b^{9}x^{9}+11440a^{7}b^{8}x^{8}+12870a^{8}b^{7}x^{7}}{16}$

input
```
int((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x,
method=_RETURNVERBOSE)
```

output
```
a^15*x+15/2*a^14*b*x^2+35*a^13*b^2*x^3+455/4*a^12*b^3*x^4+273*a^11*b^4*x^5
+1001/2*a^10*b^5*x^6+715*a^9*b^6*x^7+6435/8*a^8*b^7*x^8+715*a^7*b^8*x^9+10
01/2*a^6*b^9*x^10+273*a^5*b^10*x^11+455/4*a^4*b^11*x^12+35*a^3*b^12*x^13+1
5/2*a^2*b^13*x^14+a*b^14*x^15+1/16*b^15*x^16
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(12) = 24.

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 11.64

$$\begin{aligned}&\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3\,dx\\&=\frac{1}{16}\,b^{15}x^{16}+ab^{14}x^{15}+\frac{15}{2}\,a^2b^{13}x^{14}+35\,a^3b^{12}x^{13}+\frac{455}{4}\,a^4b^{11}x^{12}\\&\quad+273\,a^5b^{10}x^{11}+\frac{1001}{2}\,a^6b^9x^{10}+715\,a^7b^8x^9+\frac{6435}{8}\,a^8b^7x^8+715\,a^9b^6x^7\\&\quad+\frac{1001}{2}\,a^{10}b^5x^6+273\,a^{11}b^4x^5+\frac{455}{4}\,a^{12}b^3x^4+35\,a^{13}b^2x^3+\frac{15}{2}\,a^{14}bx^2+a^{15}x\end{aligned}$$

input
```
integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5
)^3,x, algorithm="fricas")
```

output
```
1/16*b^15*x^16 + a*b^14*x^15 + 15/2*a^2*b^13*x^14 + 35*a^3*b^12*x^13 + 455
/4*a^4*b^11*x^12 + 273*a^5*b^10*x^11 + 1001/2*a^6*b^9*x^10 + 715*a^7*b^8*x
^9 + 6435/8*a^8*b^7*x^8 + 715*a^9*b^6*x^7 + 1001/2*a^10*b^5*x^6 + 273*a^11
*b^4*x^5 + 455/4*a^12*b^3*x^4 + 35*a^13*b^2*x^3 + 15/2*a^14*b*x^2 + a^15*x
```

3.63.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(8) = 16$.

Time = 0.04 (sec) , antiderivative size = 185, normalized size of antiderivative = 13.21

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3\,dx = a^{15}x+\frac{15a^{14}bx^2}{2}+35a^{13}b^2x^3+\frac{455a^{12}b^3x^4}{4}+273a^{11}b^4x^5+\frac{1001a^{10}b^5x^6}{2}+715a^9b^6x^7+\frac{6435a^8b^7x^8}{8}+715a^7b^8x^9+\frac{1001a^6b^9x^{10}}{2}+273a^5b^{10}x^{11}+\frac{455a^4b^{11}x^{12}}{4}+35a^3b^{12}x^{13}+\frac{15a^2b^{13}x^{14}}{2}+ab^{14}x^{15}+\frac{b^{15}x^{16}}{16}$$

input
```
integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a
**4*b*x+a**5)**3,x)
```

output
```
a**15*x + 15*a**14*b*x**2/2 + 35*a**13*b**2*x**3 + 455*a**12*b**3*x**4/4 +
 273*a**11*b**4*x**5 + 1001*a**10*b**5*x**6/2 + 715*a**9*b**6*x**7 + 6435*
a**8*b**7*x**8/8 + 715*a**7*b**8*x**9 + 1001*a**6*b**9*x**10/2 + 273*a**5*
b**10*x**11 + 455*a**4*b**11*x**12/4 + 35*a**3*b**12*x**13 + 15*a**2*b**13
*x**14/2 + a*b**14*x**15 + b**15*x**16/16
```

3.63.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(12) = 24$.

Time = 0.21 (sec) , antiderivative size = 592, normalized size of antiderivative = 42.29

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3 \, dx$$
$$= \frac{1}{16}\,b^{15}x^{16}+ab^{14}x^{15}+\frac{75}{14}\,a^2b^{13}x^{14}+\frac{125}{13}\,a^3b^{12}x^{13}+100\,a^6b^9x^{10}+\frac{1000}{7}\,a^9b^6x^7$$
$$+\frac{125}{4}\,a^{12}b^3x^4+a^{15}x+\frac{1}{2}\left(b^5x^6+6\,ab^4x^5+15\,a^2b^3x^4+20\,a^3b^2x^3+15\,a^4bx^2\right)a^{10}$$
$$+\frac{25}{56}\left(21\,b^5x^8+120\,ab^4x^7+280\,a^2b^3x^6+336\,a^3b^2x^5\right)a^8b^2$$
$$+\frac{5}{3}\left(18\,b^5x^{10}+100\,ab^4x^9+225\,a^2b^3x^8\right)a^6b^4+\frac{25}{11}\left(11\,b^5x^{12}+60\,ab^4x^{11}\right)a^4b^6$$
$$+\frac{1}{462}\left(126\,b^{10}x^{11}+1386\,ab^9x^{10}+3850\,a^2b^8x^9+19800\,a^4b^6x^7+27720\,a^6b^4x^5+11550\,a^8b^2x^3+330\,(6\,b^5\right.$$
$$+\frac{5}{308}\left(77\,b^{10}x^{12}+840\,ab^9x^{11}+4158\,a^2b^8x^{10}+12320\,a^3b^7x^9+23100\,a^4b^6x^8+26400\,a^5b^5x^7+15400\,a^6b^4\right.$$
$$+\frac{5}{429}\left(198\,b^{10}x^{13}+2145\,ab^9x^{12}+10530\,a^2b^8x^{11}+25740\,a^3b^7x^{10}+28600\,a^4b^6x^9\right)a^3b^2$$
$$+\frac{5}{182}\left(78\,b^{10}x^{14}+840\,ab^9x^{13}+2275\,a^2b^8x^{12}\right)a^2b^3$$

input
```
integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5
)^3,x, algorithm="maxima")
```

output
```
1/16*b^15*x^16 + a*b^14*x^15 + 75/14*a^2*b^13*x^14 + 125/13*a^3*b^12*x^13
+ 100*a^6*b^9*x^10 + 1000/7*a^9*b^6*x^7 + 125/4*a^12*b^3*x^4 + a^15*x + 1/
2*(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2)
*a^10 + 25/56*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6 + 336*a^3*b^2*
x^5)*a^8*b^2 + 5/3*(18*b^5*x^10 + 100*a*b^4*x^9 + 225*a^2*b^3*x^8)*a^6*b^4
 + 25/11*(11*b^5*x^12 + 60*a*b^4*x^11)*a^4*b^6 + 1/462*(126*b^10*x^11 + 13
86*a*b^9*x^10 + 3850*a^2*b^8*x^9 + 19800*a^4*b^6*x^7 + 27720*a^6*b^4*x^5 +
 11550*a^8*b^2*x^3 + 330*(6*b^5*x^7 + 35*a*b^4*x^6 + 84*a^2*b^3*x^5 + 105*
a^3*b^2*x^4)*a^4*b + 165*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6)*a^
3*b^2 + 385*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*b^3)*a^5 + 5/308*(77*b^10*x^12
+ 840*a*b^9*x^11 + 4158*a^2*b^8*x^10 + 12320*a^3*b^7*x^9 + 23100*a^4*b^6*x
^8 + 26400*a^5*b^5*x^7 + 15400*a^6*b^4*x^6)*a^4*b + 5/429*(198*b^10*x^13 +
 2145*a*b^9*x^12 + 10530*a^2*b^8*x^11 + 25740*a^3*b^7*x^10 + 28600*a^4*b^6
*x^9)*a^3*b^2 + 5/182*(78*b^10*x^14 + 840*a*b^9*x^13 + 2275*a^2*b^8*x^12)*
a^2*b^3
```

3.63. $\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3 \, dx$

3.63.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 11.64

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3\,dx = \frac{1}{16}\,b^{15}x^{16}+ab^{14}x^{15}+\frac{15}{2}\,a^2b^{13}x^{14}+35\,a^3b^{12}x^{13}+\frac{455}{4}\,a^4b^{11}x^{12}+273\,a^5b^{10}x^{11}+\frac{1001}{2}\,a^6b^9x^{10}+715\,a^7b^8x^9+\frac{6435}{8}\,a^8b^7x^8+715\,a^9b^6x^7+\frac{1001}{2}\,a^{10}b^5x^6+273\,a^{11}b^4x^5+\frac{455}{4}\,a^{12}b^3x^4+35\,a^{13}b^2x^3+\frac{15}{2}\,a^{14}bx^2+a^{15}x$$

input
```
integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5
)^3,x, algorithm="giac")
```

output
```
1/16*b^15*x^16 + a*b^14*x^15 + 15/2*a^2*b^13*x^14 + 35*a^3*b^12*x^13 + 455
/4*a^4*b^11*x^12 + 273*a^5*b^10*x^11 + 1001/2*a^6*b^9*x^10 + 715*a^7*b^8*x
^9 + 6435/8*a^8*b^7*x^8 + 715*a^9*b^6*x^7 + 1001/2*a^10*b^5*x^6 + 273*a^11
*b^4*x^5 + 455/4*a^12*b^3*x^4 + 35*a^13*b^2*x^3 + 15/2*a^14*b*x^2 + a^15*x
```

3.63.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 11.64

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3\,dx = a^{15}\,x+\frac{15\,a^{14}\,b\,x^2}{2}+35\,a^{13}\,b^2\,x^3+\frac{455\,a^{12}\,b^3\,x^4}{4}+273\,a^{11}\,b^4\,x^5+\frac{1001\,a^{10}\,b^5\,x^6}{2}+715\,a^9\,b^6\,x^7+\frac{6435\,a^8\,b^7\,x^8}{8}+715\,a^7\,b^8\,x^9+\frac{1001\,a^6\,b^9\,x^{10}}{2}+273\,a^5\,b^{10}\,x^{11}+\frac{455\,a^4\,b^{11}\,x^{12}}{4}+35\,a^3\,b^{12}\,x^{13}+\frac{15\,a^2\,b^{13}\,x^{14}}{2}+a\,b^{14}\,x^{15}+\frac{b^{15}\,x^{16}}{16}$$

input
```
int((a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4
*b*x)^3,x)
```

output
```
a^15*x + (b^15*x^16)/16 + (15*a^14*b*x^2)/2 + a*b^14*x^15 + 35*a^13*b^2*x^
3 + (455*a^12*b^3*x^4)/4 + 273*a^11*b^4*x^5 + (1001*a^10*b^5*x^6)/2 + 715*
a^9*b^6*x^7 + (6435*a^8*b^7*x^8)/8 + 715*a^7*b^8*x^9 + (1001*a^6*b^9*x^10)
/2 + 273*a^5*b^10*x^11 + (455*a^4*b^11*x^12)/4 + 35*a^3*b^12*x^13 + (15*a^
2*b^13*x^14)/2
```

3.63. $\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right)^3 \, dx$

3.64 $\int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5$

3.64.1 Optimal result

Integrand size = 51, antiderivative size = 14

$$\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right)^2 \, dx = \frac{(a+bx)^{11}}{11b}$$

output
```
1/11*(b*x+a)^11/b
```

3.64.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right)^2 \, dx = \frac{(a+bx)^{11}}{11b}$$

input
```
Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
 + b^5*x^5)^2,x]
```

output
```
(a + b*x)^11/(11*b)
```

3.64.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2006, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2 dx$$

$$\Big\downarrow \text{2006}$$

$$\int (a+bx)^{10}dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+bx)^{11}}{11b}$$

input
```
Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5
*x^5)^2,x]
```

output
```
(a + b*x)^11/(11*b)
```

3.64.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

3.64.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(12) = 24.

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 7.79

method	result
default	$\frac{1}{11}b^{10}x^{11} + a\,b^9x^{10} + 5a^2b^8x^9 + 15a^3b^7x^8 + 30a^4b^6x^7 + 42a^5b^5x^6 + 42a^6b^4x^5 + 30a^7b^3x^4 + 15$
norman	$\frac{1}{11}b^{10}x^{11} + a\,b^9x^{10} + 5a^2b^8x^9 + 15a^3b^7x^8 + 30a^4b^6x^7 + 42a^5b^5x^6 + 42a^6b^4x^5 + 30a^7b^3x^4 + 15$
risch	$\frac{1}{11}b^{10}x^{11} + a\,b^9x^{10} + 5a^2b^8x^9 + 15a^3b^7x^8 + 30a^4b^6x^7 + 42a^5b^5x^6 + 42a^6b^4x^5 + 30a^7b^3x^4 + 15$
parallelrisch	$\frac{1}{11}b^{10}x^{11} + a\,b^9x^{10} + 5a^2b^8x^9 + 15a^3b^7x^8 + 30a^4b^6x^7 + 42a^5b^5x^6 + 42a^6b^4x^5 + 30a^7b^3x^4 + 15$
gosper	$\frac{x\left(b^{10}x^{10}+11a\,b^9x^9+55a^2b^8x^8+165a^3b^7x^7+330a^4b^6x^6+462a^5b^5x^5+462a^6b^4x^4+330a^7b^3x^3+165a^8b^2x^2+55a^9bx+11a^{10}\right)}{11}$

input
```
int((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x,
method=_RETURNVERBOSE)
```

output
```
1/11*b^10*x^11+a*b^9*x^10+5*a^2*b^8*x^9+15*a^3*b^7*x^8+30*a^4*b^6*x^7+42*a
^5*b^5*x^6+42*a^6*b^4*x^5+30*a^7*b^3*x^4+15*a^8*b^2*x^3+5*a^9*b*x^2+a^10*x
```

3.64.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(12) = 24.

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 7.71

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2\,dx$$
$$= \frac{1}{11}\,b^{10}x^{11} + ab^9x^{10} + 5\,a^2b^8x^9 + 15\,a^3b^7x^8 + 30\,a^4b^6x^7 + 42\,a^5b^5x^6 + 42\,a^6b^4x^5 + 30\,a^7b^3x^4 + 15\,a^8b^2x^3 + 5\,a^9bx^2 + a^{10}x$$

input
```
integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5
)^2,x, algorithm="fricas")
```

output
```
1/11*b^10*x^11 + a*b^9*x^10 + 5*a^2*b^8*x^9 + 15*a^3*b^7*x^8 + 30*a^4*b^6*
x^7 + 42*a^5*b^5*x^6 + 42*a^6*b^4*x^5 + 30*a^7*b^3*x^4 + 15*a^8*b^2*x^3 +
5*a^9*b*x^2 + a^10*x
```

3.64.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(8) = 16$.

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 8.14

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2\,dx = a^{10}x+5a^9bx^2+15a^8b^2x^3+30a^7b^3x^4+42a^6b^4x^5+42a^5b^5x^6 + 30a^4b^6x^7+15a^3b^7x^8+5a^2b^8x^9+ab^9x^{10}+\frac{b^{10}x^{11}}{11}$$

input
```
integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a
**4*b*x+a**5)**2,x)
```

output
```
a**10*x + 5*a**9*b*x**2 + 15*a**8*b**2*x**3 + 30*a**7*b**3*x**4 + 42*a**6*
b**4*x**5 + 42*a**5*b**5*x**6 + 30*a**4*b**6*x**7 + 15*a**3*b**7*x**8 + 5*
a**2*b**8*x**9 + a*b**9*x**10 + b**10*x**11/11
```

3.64.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(12) = 24$.

Time = 0.21 (sec) , antiderivative size = 228, normalized size of antiderivative = 16.29

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2\,dx = \frac{1}{11}\,b^{10}x^{11}+ab^9x^{10}+\frac{25}{9}\,a^2b^8x^9+\frac{100}{7}\,a^4b^6x^7+20\,a^6b^4x^5+\frac{25}{3}\,a^8b^2x^3 + a^{10}x+\frac{1}{3}\left(b^5x^6+6\,ab^4x^5+15\,a^2b^3x^4+20\,a^3b^2x^3+15\,a^4bx^2\right)a^5 + \frac{5}{21}\left(6\,b^5x^7+35\,ab^4x^6+84\,a^2b^3x^5+105\,a^3b^2x^4\right)a^4b + \frac{5}{42}\left(21\,b^5x^8+120\,ab^4x^7+280\,a^2b^3x^6\right)a^3b^2+\frac{5}{18}\left(8\,b^5x^9+45\,ab^4x^8\right)a^2b^3$$

input
```
integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5
)^2,x, algorithm="maxima")
```

output
```
1/11*b^10*x^11 + a*b^9*x^10 + 25/9*a^2*b^8*x^9 + 100/7*a^4*b^6*x^7 + 20*a^
6*b^4*x^5 + 25/3*a^8*b^2*x^3 + a^10*x + 1/3*(b^5*x^6 + 6*a*b^4*x^5 + 15*a^
2*b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2)*a^5 + 5/21*(6*b^5*x^7 + 35*a*b^
4*x^6 + 84*a^2*b^3*x^5 + 105*a^3*b^2*x^4)*a^4*b + 5/42*(21*b^5*x^8 + 120*a
*b^4*x^7 + 280*a^2*b^3*x^6)*a^3*b^2 + 5/18*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*
b^3
```

3.64.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 7.71

$$\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right)^2 dx$$
$$= \frac{1}{11} b^{10}x^{11} + ab^9x^{10} + 5\,a^2b^8x^9 + 15\,a^3b^7x^8 + 30\,a^4b^6x^7 + 42\,a^5b^5x^6 + 42\,a^6b^4x^5 + 30\,a^7b^3x^4 + 15\,a^8b^2x^3 + 5\,a^9bx^2 + a^{10}x$$

input
```
integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5
)^2,x, algorithm="giac")
```

output
```
1/11*b^10*x^11 + a*b^9*x^10 + 5*a^2*b^8*x^9 + 15*a^3*b^7*x^8 + 30*a^4*b^6*
x^7 + 42*a^5*b^5*x^6 + 42*a^6*b^4*x^5 + 30*a^7*b^3*x^4 + 15*a^8*b^2*x^3 +
5*a^9*b*x^2 + a^10*x
```

3.64.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 7.71

$$\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right)^2 dx$$
$$= a^{10}\,x + 5\,a^9\,b\,x^2 + 15\,a^8\,b^2\,x^3 + 30\,a^7\,b^3\,x^4 + 42\,a^6\,b^4\,x^5 + 42\,a^5\,b^5\,x^6 + 30\,a^4\,b^6\,x^7 + 15\,a^3\,b^7\,x^8 + 5\,a^2\,b^8\,x^9 + a\,b^9\,x^{10} + \frac{b^{10}\,x^{11}}{11}$$

input
```
int((a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4
*b*x)^2,x)
```

output
```
a^10*x + (b^10*x^11)/11 + 5*a^9*b*x^2 + a*b^9*x^10 + 15*a^8*b^2*x^3 + 30*a
^7*b^3*x^4 + 42*a^6*b^4*x^5 + 42*a^5*b^5*x^6 + 30*a^4*b^6*x^7 + 15*a^3*b^7
*x^8 + 5*a^2*b^8*x^9
```

3.64. $\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right)^2 \, dx$

3.65 $\int (a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5$

3.65.1 Optimal result

Integrand size = 49, antiderivative size = 14

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)\,dx=\frac{(a+bx)^6}{6b}$$

output
```
1/6*(b*x+a)^6/b
```

3.65.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 61 vs. 2(14) = 28.

Time = 0.00 (sec) , antiderivative size = 61, normalized size of antiderivative = 4.36

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)\,dx =a^5x+\frac{5}{2}a^4bx^2+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^2b^3x^4+ab^4x^5+\frac{b^5x^6}{6}$$

input
```
Integrate[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5,x]
```

output
```
a^5*x + (5*a^4*b*x^2)/2 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2 + a*b^4*x
^5 + (b^5*x^6)/6
```

3.65.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.041, Rules used = {2006, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right) dx$$

$$\Big\downarrow \text{2006}$$

$$\int (a+bx)^5dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+bx)^6}{6b}$$

input
```
Int[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5,x]
```

output
```
(a + b*x)^6/(6*b)
```

3.65.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

3.65.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{(bx+a)^6}{6b}$	13
norman	$\frac{1}{6}b^5x^6+a\,b^4x^5+\frac{5}{2}a^2b^3x^4+\frac{10}{3}a^3x^3b^2+\frac{5}{2}x^2a^4b+a^5x$	54
risch	$\frac{1}{6}b^5x^6+a\,b^4x^5+\frac{5}{2}a^2b^3x^4+\frac{10}{3}a^3x^3b^2+\frac{5}{2}x^2a^4b+a^5x$	54
parallelrisch	$\frac{1}{6}b^5x^6+a\,b^4x^5+\frac{5}{2}a^2b^3x^4+\frac{10}{3}a^3x^3b^2+\frac{5}{2}x^2a^4b+a^5x$	54
parts	$\frac{1}{6}b^5x^6+a\,b^4x^5+\frac{5}{2}a^2b^3x^4+\frac{10}{3}a^3x^3b^2+\frac{5}{2}x^2a^4b+a^5x$	54
gosper	$\frac{x\left(b^5x^5+6a\,b^4x^4+15a^2b^3x^3+20a^3b^2x^2+15x\,a^4b+6a^5\right)}{6}$	55

input
```
int(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x,meth
od=_RETURNVERBOSE)
```

output
```
1/6*(b*x+a)^6/b
```

3.65.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(12) = 24.

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)\,dx = \frac{1}{6}\,b^5x^6+ab^4x^5+\frac{5}{2}\,a^2b^3x^4+\frac{10}{3}\,a^3b^2x^3+\frac{5}{2}\,a^4bx^2+a^5x$$

input
```
integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,
x, algorithm="fricas")
```

output
```
1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x
^2 + a^5*x
```

3.65.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(8) = 16$.

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 4.29

$$\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right) dx$$
$$= a^5x + \frac{5a^4bx^2}{2} + \frac{10a^3b^2x^3}{3} + \frac{5a^2b^3x^4}{2} + ab^4x^5 + \frac{b^5x^6}{6}$$

input
```
integrate(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a*
*4*b*x+a**5,x)
```

output
```
a**5*x + 5*a**4*b*x**2/2 + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x**4/2 + a*b*
*4*x**5 + b**5*x**6/6
```

3.65.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right) dx$$
$$= \frac{1}{6}\,b^5x^6 + ab^4x^5 + \frac{5}{2}\,a^2b^3x^4 + \frac{10}{3}\,a^3b^2x^3 + \frac{5}{2}\,a^4bx^2 + a^5x$$

input
```
integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,
x, algorithm="maxima")
```

output
```
1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x
^2 + a^5*x
```

3.65.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)\,dx = \frac{1}{6}\,b^5x^6+ab^4x^5+\frac{5}{2}\,a^2b^3x^4+\frac{10}{3}\,a^3b^2x^3+\frac{5}{2}\,a^4bx^2+a^5x$$

input
```
integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,
x, algorithm="giac")
```

output
```
1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x
^2 + a^5*x
```

3.65.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)\,dx = a^5\,x+\frac{5\,a^4\,b\,x^2}{2}+\frac{10\,a^3\,b^2\,x^3}{3}+\frac{5\,a^2\,b^3\,x^4}{2}+a\,b^4\,x^5+\frac{b^5\,x^6}{6}$$

input
```
int(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*
b*x,x)
```

output
```
a^5*x + (b^5*x^6)/6 + (5*a^4*b*x^2)/2 + a*b^4*x^5 + (10*a^3*b^2*x^3)/3 + (
5*a^2*b^3*x^4)/2
```

3.66 $\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx$

3.66.1 Optimal result

Integrand size = 51, antiderivative size = 14

$$\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx=-\frac{1}{4b(a+bx)^4}$$

output
```
-1/4/b/(b*x+a)^4
```

3.66.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx=-\frac{1}{4b(a+bx)^4}$$

input
```
Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
 + b^5*x^5)^(-1),x]
```

output
```
-1/4*1/(b*(a + b*x)^4)
```

3.66.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2007, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}dx$$

$$\Big\downarrow \text{ 2007}$$

$$\int \frac{1}{(a+bx)^5}dx$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{1}{4b(a+bx)^4}$$

input
```
Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5
*x^5)^(-1),x]
```

output
```
-1/4*1/(b*(a + b*x)^4)
```

3.66.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2007
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Ex
pon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; IntegerQ[p] && Pol
yQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

3.66.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{4b(bx+a)^{4}}$	13
norman	$-\frac{1}{4b(bx+a)^{4}}$	13
gosper	$-\frac{1}{4b(b^{4}x^{4}+4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+4a^{3}bx+a^{4})}$	46
risch	$-\frac{1}{4b(b^{4}x^{4}+4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+4a^{3}bx+a^{4})}$	46
parallelrisch	$-\frac{1}{4b(b^{4}x^{4}+4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+4a^{3}bx+a^{4})}$	46

input
```
int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5),x,
method=_RETURNVERBOSE)
```

output
```
-1/4/b/(b*x+a)^4
```

3.66.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(12) = 24$.

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.29

$$\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx = -\frac{1}{4\left(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b\right)}$$

input
```
integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a
^5),x, algorithm="fricas")
```

output
```
-1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

3.66.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.50

$$\int \frac{1}{a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5}\,dx = -\frac{1}{4a^4b + 16a^3b^2x + 24a^2b^3x^2 + 16ab^4x^3 + 4b^5x^4}$$

input
```
integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5
*a**4*b*x+a**5),x)
```

output
```
-1/(4*a**4*b + 16*a**3*b**2*x + 24*a**2*b**3*x**2 + 16*a*b**4*x**3 + 4*b**
5*x**4)
```

3.66.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(12) = 24$.

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.29

$$\int \frac{1}{a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5}\,dx = -\frac{1}{4\,(b^5x^4 + 4\,ab^4x^3 + 6\,a^2b^3x^2 + 4\,a^3b^2x + a^4b)}$$

input
```
integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a
^5),x, algorithm="maxima")
```

output
```
-1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx = -\frac{1}{4\,(bx+a)^4b}$$

input
```
integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a
^5),x, algorithm="giac")
```

output
```
-1/4/((b*x + a)^4*b)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 3.43

$$\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx$$
$$= -\frac{1}{4\,a^4\,b+16\,a^3\,b^2\,x+24\,a^2\,b^3\,x^2+16\,a\,b^4\,x^3+4\,b^5\,x^4}$$

input
```
int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a
^4*b*x),x)
```

output
```
-1/(4*a^4*b + 4*b^5*x^4 + 16*a^3*b^2*x + 16*a*b^4*x^3 + 24*a^2*b^3*x^2)
```

3.67 $\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^2}\,dx$

3.67.1 Optimal result

Integrand size = 51, antiderivative size = 14

$$\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^2}\,dx=-\frac{1}{9b(a+bx)^9}$$

output
```
-1/9/b/(b*x+a)^9
```

3.67.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^2}\,dx=-\frac{1}{9b(a+bx)^9}$$

input
```
Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
 + b^5*x^5)^(-2),x]
```

output
```
-1/9*1/(b*(a + b*x)^9)
```

3.67.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2007, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^2}dx$$

$$\Big\downarrow \ 2007$$

$$\int \frac{1}{(a+bx)^{10}}dx$$

$$\Big\downarrow \ 17$$

$$-\frac{1}{9b(a+bx)^9}$$

input
```
Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5
*x^5)^(-2),x]
```

output
```
-1/9*1/(b*(a + b*x)^9)
```

3.67.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2007
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Ex
pon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; IntegerQ[p] && Pol
yQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

3.67.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{9b(bx+a)^9}$	13
norman	$-\frac{1}{9b(bx+a)^9}$	13
risch	$-\frac{1}{9b(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)^2(bx+a)}$	53
gosper	$-\frac{1}{9(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5x\,a^4b+a^5)b}$	97
parallelrisch	$-\frac{1}{9(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5x\,a^4b+a^5)b}$	97

input
```
int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,
x,method=_RETURNVERBOSE)
```

output
```
-1/9/b/(b*x+a)^9
```

3.67.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 7.21

$$\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^2}\,dx = -\frac{1}{9\,(b^{10}x^9+9\,ab^9x^8+36\,a^2b^8x^7+84\,a^3b^7x^6+126\,a^4b^6x^5+126\,a^5b^5x^4+84\,a^6b^4x^3+36\,a^7b^3x^2+9\,a^8b^2x}$$

input
```
integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a
^5)^2,x, algorithm="fricas")
```

output
```
-1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b
^6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x +
 a^9*b)
```

3.67.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 7.79

$$\int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^2}\,dx = -\frac{1}{9a^9b + 81a^8b^2x + 324a^7b^3x^2 + 756a^6b^4x^3 + 1134a^5b^5x^4 + 1134a^4b^6x^5 + 756a^3b^7x^6 + 324a^2b^8x^7 + 81ab}$$

input
```
integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5
*a**4*b*x+a**5)**2,x)
```

output
```
-1/(9*a**9*b + 81*a**8*b**2*x + 324*a**7*b**3*x**2 + 756*a**6*b**4*x**3 +
1134*a**5*b**5*x**4 + 1134*a**4*b**6*x**5 + 756*a**3*b**7*x**6 + 324*a**2*
b**8*x**7 + 81*a*b**9*x**8 + 9*b**10*x**9)
```

3.67.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(12) = 24$.

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 7.21

$$\int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^2}\,dx = -\frac{1}{9\,(b^{10}x^9 + 9\,ab^9x^8 + 36\,a^2b^8x^7 + 84\,a^3b^7x^6 + 126\,a^4b^6x^5 + 126\,a^5b^5x^4 + 84\,a^6b^4x^3 + 36\,a^7b^3x^2 + 9\,a^8b^2x}$$

input
```
integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a
^5)^2,x, algorithm="maxima")
```

output
```
-1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b
^6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x +
 a^9*b)
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2}\,dx=-\frac{1}{9\,(bx+a)^9b}$$

input
```
integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a
^5)^2,x, algorithm="giac")
```

output
```
-1/9/((b*x + a)^9*b)
```

3.67.9 Mupad [B] (verification not implemented)

Time = 10.73 (sec) , antiderivative size = 103, normalized size of antiderivative = 7.36

$$\int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2}\,dx=$$
$$-\frac{1}{9\,a^9\,b+81\,a^8\,b^2\,x+324\,a^7\,b^3\,x^2+756\,a^6\,b^4\,x^3+1134\,a^5\,b^5\,x^4+1134\,a^4\,b^6\,x^5+756\,a^3\,b^7\,x^6+324\,a^2\,b^8\,x}$$

input
```
int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a
^4*b*x)^2,x)
```

output
```
-1/(9*a^9*b + 9*b^10*x^9 + 81*a^8*b^2*x + 81*a*b^9*x^8 + 324*a^7*b^3*x^2 +
 756*a^6*b^4*x^3 + 1134*a^5*b^5*x^4 + 1134*a^4*b^6*x^5 + 756*a^3*b^7*x^6 +
 324*a^2*b^8*x^7)
```

3.68 $\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^3} dx$

3.68.1 Optimal result

Integrand size = 51, antiderivative size = 14

$$\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^3} dx = -\frac{1}{14b(a+bx)^{14}}$$

output
```
-1/14/b/(b*x+a)^14
```

3.68.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^3} dx = -\frac{1}{14b(a+bx)^{14}}$$

input
```
Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
 + b^5*x^5)^(-3),x]
```

output
```
-1/14*1/(b*(a + b*x)^14)
```

3.68.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2007, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^3}dx$$
$$\Big\downarrow \ 2007$$
$$\int \frac{1}{(a+bx)^{15}}dx$$
$$\Big\downarrow \ 17$$
$$-\frac{1}{14b(a+bx)^{14}}$$

input
```
Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5
*x^5)^(-3),x]
```

output
```
-1/14*1/(b*(a + b*x)^14)
```

3.68.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2007
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Ex
pon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; IntegerQ[p] && Pol
yQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

3.68.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{14b(bx+a)^{14}}$	13
norman	$-\frac{1}{14b(bx+a)^{14}}$	13
risch	$-\frac{1}{14b(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)^3(bx+a)^2}$	53
gosper	$-\frac{1}{14(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5x\,a^4b+a^5)^2b}$	97
parallelrisch	$-\frac{1}{14(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5x\,a^4b+a^5)^2b}$	97

input
```
int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,
x,method=_RETURNVERBOSE)
```

output
```
-1/14/b/(b*x+a)^14
```

3.68.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(12) = 24.

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 11.14

$$\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^3}\,dx =$$

$$-\frac{1}{14\,(b^{15}x^{14}+14\,ab^{14}x^{13}+91\,a^2b^{13}x^{12}+364\,a^3b^{12}x^{11}+1001\,a^4b^{11}x^{10}+2002\,a^5b^{10}x^9+3003\,a^6b^9x^8+34}$$

input
```
integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a
^5)^3,x, algorithm="fricas")
```

output
```
-1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^12*x^11 +
 1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x^8 + 3432*a^7*b^8*
x^7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 1001*a^10*b^5*x^4 + 364*a^11*b
^4*x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2*x + a^14*b)
```

3.68.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(12) = 24$.

Time = 0.43 (sec) , antiderivative size = 168, normalized size of antiderivative = 12.00

$$\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^3}\,dx =$$

$$-\frac{}{14a^{14}b+196a^{13}b^2x+1274a^{12}b^3x^2+5096a^{11}b^4x^3+14014a^{10}b^5x^4+28028a^9b^6x^5+42042a^8b^7x^6+480}$$

input
```
integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5
*a**4*b*x+a**5)**3,x)
```

output
```
-1/(14*a**14*b + 196*a**13*b**2*x + 1274*a**12*b**3*x**2 + 5096*a**11*b**4
*x**3 + 14014*a**10*b**5*x**4 + 28028*a**9*b**6*x**5 + 42042*a**8*b**7*x**
6 + 48048*a**7*b**8*x**7 + 42042*a**6*b**9*x**8 + 28028*a**5*b**10*x**9 +
14014*a**4*b**11*x**10 + 5096*a**3*b**12*x**11 + 1274*a**2*b**13*x**12 + 1
96*a*b**14*x**13 + 14*b**15*x**14)
```

3.68.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(12) = 24$.

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 11.14

$$\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^3}\,dx =$$

$$-\frac{1}{14\,(b^{15}x^{14}+14\,ab^{14}x^{13}+91\,a^2b^{13}x^{12}+364\,a^3b^{12}x^{11}+1001\,a^4b^{11}x^{10}+2002\,a^5b^{10}x^9+3003\,a^6b^9x^8+34}$$

input
```
integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a
^5)^3,x, algorithm="maxima")
```

output
```
-1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^12*x^11 +
 1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x^8 + 3432*a^7*b^8*
x^7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 1001*a^10*b^5*x^4 + 364*a^11*b
^4*x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2*x + a^14*b)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^3}\,dx = -\frac{1}{14\,(bx+a)^{14}b}$$

input
```
integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a
^5)^3,x, algorithm="giac")
```

output
```
-1/14/((b*x + a)^14*b)
```

3.68.9 Mupad [B] (verification not implemented)

Time = 11.83 (sec) , antiderivative size = 158, normalized size of antiderivative = 11.29

$$\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^3}\,dx =$$

$$-\frac{}{14\,a^{14}\,b+196\,a^{13}\,b^2\,x+1274\,a^{12}\,b^3\,x^2+5096\,a^{11}\,b^4\,x^3+14014\,a^{10}\,b^5\,x^4+28028\,a^9\,b^6\,x^5+42042\,a^8\,b^7\,x}$$

input
```
int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a
^4*b*x)^3,x)
```

output
```
-1/(14*a^14*b + 14*b^15*x^14 + 196*a^13*b^2*x + 196*a*b^14*x^13 + 1274*a^1
2*b^3*x^2 + 5096*a^11*b^4*x^3 + 14014*a^10*b^5*x^4 + 28028*a^9*b^6*x^5 + 4
2042*a^8*b^7*x^6 + 48048*a^7*b^8*x^7 + 42042*a^6*b^9*x^8 + 28028*a^5*b^10*
x^9 + 14014*a^4*b^11*x^10 + 5096*a^3*b^12*x^11 + 1274*a^2*b^13*x^12)
```

3.69 $\int \frac{1}{1+x^2+x^3+x^5} dx$

3.69.1 Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{1}{1+x^2+x^3+x^5} dx = \frac{\arctan(x)}{2} + \frac{1}{6}\log(1+x) + \frac{1}{4}\log\left(1+x^2\right) - \frac{1}{3}\log\left(1-x+x^2\right)$$

output
```
1/2*arctan(x)+1/6*ln(1+x)+1/4*ln(x^2+1)-1/3*ln(x^2-x+1)
```

3.69.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^2+x^3+x^5} dx = \frac{\arctan(x)}{2} + \frac{1}{6}\log(1+x) + \frac{1}{4}\log\left(1+x^2\right) - \frac{1}{3}\log\left(1-x+x^2\right)$$

input
```
Integrate[(1 + x^2 + x^3 + x^5)^(-1),x]
```

output
```
ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3
```

3.69.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5+x^3+x^2+1} \, dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{1-2x}{3\left(x^2-x+1\right)}+\frac{x+1}{2\left(x^2+1\right)}+\frac{1}{6(x+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan(x)}{2}+\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)$$

input
```
Int[(1 + x^2 + x^3 + x^5)^(-1),x]
```

output
```
ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3
```

3.69.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.69.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{6}+\frac{\ln(x^2+1)}{4}-\frac{\ln(x^2-x+1)}{3}$	31
risch	$\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{6}+\frac{\ln(x^2+1)}{4}-\frac{\ln(x^2-x+1)}{3}$	31
parallelrisch	$\frac{\ln(x+1)}{6}+\frac{\ln(x-i)}{4}-\frac{i\ln(x-i)}{4}+\frac{\ln(x+i)}{4}+\frac{i\ln(x+i)}{4}-\frac{\ln(x^2-x+1)}{3}$	49

input
```
int(1/(x^5+x^3+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x)+1/6*ln(x+1)+1/4*ln(x^2+1)-1/3*ln(x^2-x+1)
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^3+x^5}\,dx = \frac{1}{2}\arctan(x)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{6}\log(x+1)$$

input
```
integrate(1/(x^5+x^3+x^2+1),x, algorithm="fricas")
```

output
```
1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(x + 1)
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+x^2+x^3+x^5}\,dx = \frac{\log(x+1)}{6}+\frac{\log(x^2+1)}{4}-\frac{\log(x^2-x+1)}{3}+\frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(1/(x**5+x**3+x**2+1),x)
```

output
```
log(x + 1)/6 + log(x**2 + 1)/4 - log(x**2 - x + 1)/3 + atan(x)/2
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+x^2+x^3+x^5}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{3}\log(x^2-x+1) + \frac{1}{4}\log(x^2+1) + \frac{1}{6}\log(x+1)$$

input
```
integrate(1/(x^5+x^3+x^2+1),x, algorithm="maxima")
```

output
```
1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(x + 1)
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+x^2+x^3+x^5}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{3}\log(x^2-x+1) + \frac{1}{4}\log(x^2+1) + \frac{1}{6}\log(|x+1|)$$

input
```
integrate(1/(x^5+x^3+x^2+1),x, algorithm="giac")
```

output
```
1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(abs(x +
1))
```

3.69.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{1}{1+x^2+x^3+x^5}\,dx = \frac{\ln(x+1)}{6} - \frac{\ln(x^2-x+1)}{3} + \ln(x-\mathrm{i})\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

input
```
int(1/(x^2 + x^3 + x^5 + 1),x)
```

output
```
log(x + 1)/6 + log(x - 1i)*(1/4 - 1i/4) + log(x + 1i)*(1/4 + 1i/4) - log(x
^2 - x + 1)/3
```

3.70 $\int (3-19x^2+32x^4-16x^6)^4 \, dx$

3.70.1 Optimal result

Integrand size = 19, antiderivative size = 84

$$\int (3-19x^2+32x^4-16x^6)^4 \, dx = 81x-684x^3+4590x^5-\frac{149700x^7}{7}+\frac{634321x^9}{9}-\frac{1841600x^{11}}{11}+\frac{3764416x^{13}}{13}-\frac{1094656x^{15}}{3}+\frac{5633536x^{17}}{17}-\frac{4014080x^{19}}{19}+\frac{1884160x^{21}}{21}-\frac{524288x^{23}}{23}+\frac{65536x^{25}}{25}$$

output
```
81*x-684*x^3+4590*x^5-149700/7*x^7+634321/9*x^9-1841600/11*x^11+3764416/13
*x^13-1094656/3*x^15+5633536/17*x^17-4014080/19*x^19+1884160/21*x^21-52428
8/23*x^23+65536/25*x^25
```

3.70.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int (3-19x^2+32x^4-16x^6)^4 \, dx = 81x-684x^3+4590x^5-\frac{149700x^7}{7}+\frac{634321x^9}{9}-\frac{1841600x^{11}}{11}+\frac{3764416x^{13}}{13}-\frac{1094656x^{15}}{3}+\frac{5633536x^{17}}{17}-\frac{4014080x^{19}}{19}+\frac{1884160x^{21}}{21}-\frac{524288x^{23}}{23}+\frac{65536x^{25}}{25}$$

input
```
Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4,x]
```

output
```
81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^1
1)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (401408
0*x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```

3.70.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2464, 2036, 2036, 396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-16x^6+32x^4-19x^2+3\right)^4 dx$$

$$\Big\downarrow \text{ 2464}$$

$$\int (x-1)^4(x+1)^4(2x-1)^4(2x+1)^4\left(4x^2-3\right)^4 dx$$

$$\Big\downarrow \text{ 2036}$$

$$\int (2x-1)^4(2x+1)^4\left(x^2-1\right)^4\left(4x^2-3\right)^4 dx$$

$$\Big\downarrow \text{ 2036}$$

$$\int \left(x^2-1\right)^4\left(4x^2-3\right)^4\left(4x^2-1\right)^4 dx$$

$$\Big\downarrow \text{ 396}$$

$$\int \left(65536x^{24}-524288x^{22}+1884160x^{20}-4014080x^{18}+5633536x^{16}-5473280x^{14}+3764416x^{12}-1841600x^{10}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{634321x^9}{9}-\frac{149700x^7}{7}+4590x^5-684x^3+81x$$

input
```
Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4,x]
```

output
```
81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^1
1)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (401408
0*x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```

3.70.3.1 Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2036
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

rule 2464
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[u*Qx^p, x] /
;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ[Expon[Px, x], 2] &&
  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && IGtQ[p, 1]
```

3.70.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

method	result
default	$81x-684x^3+4590x^5-\frac{149700}{7}x^7+\frac{634321}{9}x^9-\frac{1841600}{11}x^{11}+\frac{3764416}{13}x^{13}-\frac{1094656}{3}x^{15}+\frac{5633536}{17}$
norman	$81x-684x^3+4590x^5-\frac{149700}{7}x^7+\frac{634321}{9}x^9-\frac{1841600}{11}x^{11}+\frac{3764416}{13}x^{13}-\frac{1094656}{3}x^{15}+\frac{5633536}{17}$
risch	$81x-684x^3+4590x^5-\frac{149700}{7}x^7+\frac{634321}{9}x^9-\frac{1841600}{11}x^{11}+\frac{3764416}{13}x^{13}-\frac{1094656}{3}x^{15}+\frac{5633536}{17}$
parallelrisch	$81x-684x^3+4590x^5-\frac{149700}{7}x^7+\frac{634321}{9}x^9-\frac{1841600}{11}x^{11}+\frac{3764416}{13}x^{13}-\frac{1094656}{3}x^{15}+\frac{5633536}{17}$
gosper	$x(4386184298496x^{24}-38140733030400x^{22}+150122379264000x^{20}-353491826688000x^{18}+554471344627200x^{16}-6105248717$

input
```
int((-16*x^6+32*x^4-19*x^2+3)^4,x,method=_RETURNVERBOSE)
```

output
```
81*x-684*x^3+4590*x^5-149700/7*x^7+634321/9*x^9-1841600/11*x^11+3764416/13
*x^13-1094656/3*x^15+5633536/17*x^17-4014080/19*x^19+1884160/21*x^21-52428
8/23*x^23+65536/25*x^25
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \left(3-19x^2+32x^4-16x^6\right)^4\,dx = \frac{65536}{25}\,x^{25} - \frac{524288}{23}\,x^{23} + \frac{1884160}{21}\,x^{21} - \frac{4014080}{19}\,x^{19} + \frac{5633536}{17}\,x^{17} - \frac{1094656}{3}\,x^{15} + \frac{3764416}{13}\,x^{13} - \frac{1841600}{11}\,x^{11} + \frac{634321}{9}\,x^{9} - \frac{149700}{7}\,x^{7} + 4590\,x^{5} - 684\,x^{3} + 81\,x$$

input
```
integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="fricas")
```

output
```
65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 56335
36/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9
*x^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \left(3-19x^2+32x^4-16x^6\right)^4\,dx = \frac{65536x^{25}}{25} - \frac{524288x^{23}}{23} + \frac{1884160x^{21}}{21} - \frac{4014080x^{19}}{19} + \frac{5633536x^{17}}{17} - \frac{1094656x^{15}}{3} + \frac{3764416x^{13}}{13} - \frac{1841600x^{11}}{11} + \frac{634321x^{9}}{9} - \frac{149700x^{7}}{7} + 4590x^{5} - 684x^{3} + 81x$$

input
```
integrate((-16*x**6+32*x**4-19*x**2+3)**4,x)
```

output
```
65536*x**25/25 - 524288*x**23/23 + 1884160*x**21/21 - 4014080*x**19/19 + 5
633536*x**17/17 - 1094656*x**15/3 + 3764416*x**13/13 - 1841600*x**11/11 +
634321*x**9/9 - 149700*x**7/7 + 4590*x**5 - 684*x**3 + 81*x
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \left(3-19x^2+32x^4-16x^6\right)^4\,dx = \frac{65536}{25}\,x^{25}-\frac{524288}{23}\,x^{23}+\frac{1884160}{21}\,x^{21}-\frac{4014080}{19}\,x^{19}+\frac{5633536}{17}\,x^{17}-\frac{1094656}{3}\,x^{15}+\frac{3764416}{13}\,x^{13}-\frac{1841600}{11}\,x^{11}+\frac{634321}{9}\,x^{9}-\frac{149700}{7}\,x^{7}+4590\,x^{5}-684\,x^{3}+81\,x$$

input
```
integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="maxima")
```

output
```
65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 56335
36/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9
*x^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \left(3-19x^2+32x^4-16x^6\right)^4\,dx = \frac{65536}{25}\,x^{25}-\frac{524288}{23}\,x^{23}+\frac{1884160}{21}\,x^{21}-\frac{4014080}{19}\,x^{19}+\frac{5633536}{17}\,x^{17}-\frac{1094656}{3}\,x^{15}+\frac{3764416}{13}\,x^{13}-\frac{1841600}{11}\,x^{11}+\frac{634321}{9}\,x^{9}-\frac{149700}{7}\,x^{7}+4590\,x^{5}-684\,x^{3}+81\,x$$

input
```
integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="giac")
```

output
```
65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 56335
36/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9
*x^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

3.70.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \left(3-19x^2+32x^4-16x^6\right)^4\,dx=\frac{65536\,x^{25}}{25}-\frac{524288\,x^{23}}{23}+\frac{1884160\,x^{21}}{21}-\frac{4014080\,x^{19}}{19}+\frac{5633536\,x^{17}}{17}-\frac{1094656\,x^{15}}{3}+\frac{3764416\,x^{13}}{13}-\frac{1841600\,x^{11}}{11}+\frac{634321\,x^9}{9}-\frac{149700\,x^7}{7}+4590\,x^5-684\,x^3+81\,x$$

input
```
int((19*x^2 - 32*x^4 + 16*x^6 - 3)^4,x)
```

output
```
81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^1
1)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (401408
0*x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```

3.71 $\int \left(3-19 x^2+32 x^4-16 x^6\right)^3 \, dx$

3.71.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \left(3-19 x^2+32 x^4-16 x^6\right)^3 \, dx=27 x-171 x^3+\frac{4113 x^5}{5}-2605 x^7+\frac{16448 x^9}{3}-\frac{84912 x^{11}}{11}+\frac{93440 x^{13}}{13}-\frac{21248 x^{15}}{5}+\frac{24576 x^{17}}{17}-\frac{4096 x^{19}}{19}$$

output
```
27*x-171*x^3+4113/5*x^5-2605*x^7+16448/3*x^9-84912/11*x^11+93440/13*x^13-2
1248/5*x^15+24576/17*x^17-4096/19*x^19
```

3.71.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \left(3-19 x^2+32 x^4-16 x^6\right)^3 \, dx=27 x-171 x^3+\frac{4113 x^5}{5}-2605 x^7+\frac{16448 x^9}{3}-\frac{84912 x^{11}}{11}+\frac{93440 x^{13}}{13}-\frac{21248 x^{15}}{5}+\frac{24576 x^{17}}{17}-\frac{4096 x^{19}}{19}$$

input
```
Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3,x]
```

output
```
27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
 + (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

3.71.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2464, 25, 25, 2036, 2036, 396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-16 x^6+32 x^4-19 x^2+3\right)^3 \, dx$$

$$\Big\downarrow \text{2464}$$

$$\int -(x-1)^3 (x+1)^3 (2 x-1)^3 (2 x+1)^3 \left(4 x^2-3\right)^3dx$$

$$\Big\downarrow \text{25}$$

$$-\int -(1-2 x)^3 (1-x)^3 (x+1)^3 (2 x+1)^3 \left(3-4 x^2\right)^3dx$$

$$\Big\downarrow \text{25}$$

$$\int (1-2 x)^3 (1-x)^3 (x+1)^3 (2 x+1)^3 \left(3-4 x^2\right)^3dx$$

$$\Big\downarrow \text{2036}$$

$$\int (1-x)^3 (x+1)^3 \left(1-4 x^2\right)^3 \left(3-4 x^2\right)^3dx$$

$$\Big\downarrow \text{2036}$$

$$\int \left(1-4 x^2\right)^3 \left(3-4 x^2\right)^3 \left(1-x^2\right)^3dx$$

$$\Big\downarrow \text{396}$$

$$\int \left(-4096 x^{18}+24576 x^{16}-63744 x^{14}+93440 x^{12}-84912 x^{10}+49344 x^8-18235 x^6+4113 x^4-513 x^2+27\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4096 x^{19}}{19}+\frac{24576 x^{17}}{17}-\frac{21248 x^{15}}{5}+\frac{93440 x^{13}}{13}-\frac{84912 x^{11}}{11}+\frac{16448 x^9}{3}-2605 x^7+\frac{4113 x^5}{5}-171 x^3+27 x$$

input
```
Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3,x]
```

output
```
27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
 + (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

3.71.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2036
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

rule 2464
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[u*Qx^p, x] /
;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ[Expon[Px, x], 2] &&
  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && IGtQ[p, 1]
```

3.71.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

method	result
default	$27x-171x^{3}+\frac{4113}{5}x^{5}-2605x^{7}+\frac{16448}{3}x^{9}-\frac{84912}{11}x^{11}+\frac{93440}{13}x^{13}-\frac{21248}{5}x^{15}+\frac{24576}{17}x^{17}-\frac{4096}{19}$
norman	$27x-171x^{3}+\frac{4113}{5}x^{5}-2605x^{7}+\frac{16448}{3}x^{9}-\frac{84912}{11}x^{11}+\frac{93440}{13}x^{13}-\frac{21248}{5}x^{15}+\frac{24576}{17}x^{17}-\frac{4096}{19}$
risch	$27x-171x^{3}+\frac{4113}{5}x^{5}-2605x^{7}+\frac{16448}{3}x^{9}-\frac{84912}{11}x^{11}+\frac{93440}{13}x^{13}-\frac{21248}{5}x^{15}+\frac{24576}{17}x^{17}-\frac{4096}{19}$
parallelrisch	$27x-171x^{3}+\frac{4113}{5}x^{5}-2605x^{7}+\frac{16448}{3}x^{9}-\frac{84912}{11}x^{11}+\frac{93440}{13}x^{13}-\frac{21248}{5}x^{15}+\frac{24576}{17}x^{17}-\frac{4096}{19}$
gosper	$-\frac{x(149360640x^{18}-1001594880x^{16}+2944271616x^{14}-4979884800x^{12}+5348182320x^{10}-3798583360x^{8}+1804835175x^{6}-569}{692835}$

input
```
int((-16*x^6+32*x^4-19*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
27*x-171*x^3+4113/5*x^5-2605*x^7+16448/3*x^9-84912/11*x^11+93440/13*x^13-2
1248/5*x^15+24576/17*x^17-4096/19*x^19
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \left(3-19x^2+32x^4-16x^6\right)^3 \, dx = -\frac{4096}{19}\,x^{19}+\frac{24576}{17}\,x^{17}-\frac{21248}{5}\,x^{15}+\frac{93440}{13}\,x^{13}-\frac{84912}{11}\,x^{11}+\frac{16448}{3}\,x^{9}-2605\,x^{7}+\frac{4113}{5}\,x^{5}-171\,x^{3}+27\,x$$

input
```
integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="fricas")
```

output
```
-4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^
11 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \left(3-19x^2+32x^4-16x^6\right)^3 \, dx = -\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^{9}}{3}-2605x^{7}+\frac{4113x^{5}}{5}-171x^{3}+27x$$

input
```
integrate((-16*x**6+32*x**4-19*x**2+3)**3,x)
```

output
```
-4096*x**19/19 + 24576*x**17/17 - 21248*x**15/5 + 93440*x**13/13 - 84912*x
**11/11 + 16448*x**9/3 - 2605*x**7 + 4113*x**5/5 - 171*x**3 + 27*x
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \left(3-19x^2+32x^4-16x^6\right)^3 \,dx=-\frac{4096}{19}\,x^{19}+\frac{24576}{17}\,x^{17}-\frac{21248}{5}\,x^{15}+\frac{93440}{13}\,x^{13}-\frac{84912}{11}\,x^{11}+\frac{16448}{3}\,x^9-2605\,x^7+\frac{4113}{5}\,x^5-171\,x^3+27\,x$$

input
```
integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="maxima")
```

output
```
-4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^
11 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \left(3-19x^2+32x^4-16x^6\right)^3 \,dx=-\frac{4096}{19}\,x^{19}+\frac{24576}{17}\,x^{17}-\frac{21248}{5}\,x^{15}+\frac{93440}{13}\,x^{13}-\frac{84912}{11}\,x^{11}+\frac{16448}{3}\,x^9-2605\,x^7+\frac{4113}{5}\,x^5-171\,x^3+27\,x$$

input
```
integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="giac")
```

output
```
-4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^
11 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x
```

3.71.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \left(3-19x^2+32x^4-16x^6\right)^3 \,dx=-\frac{4096\,x^{19}}{19}+\frac{24576\,x^{17}}{17}-\frac{21248\,x^{15}}{5}+\frac{93440\,x^{13}}{13}-\frac{84912\,x^{11}}{11}+\frac{16448\,x^9}{3}-2605\,x^7+\frac{4113\,x^5}{5}-171\,x^3+27\,x$$

input
```
int(-(19*x^2 - 32*x^4 + 16*x^6 - 3)^3,x)
```

output
```
27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
 + (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

3.71. $\int (3 - 19x^2 + 32x^4 - 16x^6)^3 \, dx$

3.72 $\int (3-19x^2+32x^4-16x^6)^2 \, dx$

3.72.1 Optimal result

Integrand size = 19, antiderivative size = 44

$$\int (3-19x^2+32x^4-16x^6)^2 \, dx = 9x-38x^3+\frac{553x^5}{5}-\frac{1312x^7}{7}+\frac{544x^9}{3}-\frac{1024x^{11}}{11}+\frac{256x^{13}}{13}$$

output
```
9*x-38*x^3+553/5*x^5-1312/7*x^7+544/3*x^9-1024/11*x^11+256/13*x^13
```

3.72.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (3-19x^2+32x^4-16x^6)^2 \, dx = 9x-38x^3+\frac{553x^5}{5}-\frac{1312x^7}{7}+\frac{544x^9}{3}-\frac{1024x^{11}}{11}+\frac{256x^{13}}{13}$$

input
```
Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]
```

output
```
9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
 (256*x^13)/13
```

3.72.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2464, 2036, 2036, 396, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-16x^6+32x^4-19x^2+3\right)^2 dx$$

$$\Big\downarrow \text{2464}$$

$$\int (x-1)^2(x+1)^2(2x-1)^2(2x+1)^2\left(4x^2-3\right)^2 dx$$

$$\Big\downarrow \text{2036}$$

$$\int (2x-1)^2(2x+1)^2\left(x^2-1\right)^2\left(4x^2-3\right)^2 dx$$

$$\Big\downarrow \text{2036}$$

$$\int \left(x^2-1\right)^2\left(4x^2-3\right)^2\left(4x^2-1\right)^2 dx$$

$$\Big\downarrow \text{396}$$

$$\int \left(256x^{12}-1024x^{10}+1632x^8-1312x^6+553x^4-114x^2+9\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^9}{3}-\frac{1312x^7}{7}+\frac{553x^5}{5}-38x^3+9x$$

input
```
Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]
```

output
```
9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
 (256*x^13)/13
```

3.72.3.1 Defintions of rubi rules used

rule 396
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q*
(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ
[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2036
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

rule 2464
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[u*Qx^p, x] /
;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ[Expon[Px, x], 2] &&
  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && IGtQ[p, 1]
```

3.72.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

method	result	size
default	$9x-38x^3+\frac{553}{5}x^5-\frac{1312}{7}x^7+\frac{544}{3}x^9-\frac{1024}{11}x^{11}+\frac{256}{13}x^{13}$	35
norman	$9x-38x^3+\frac{553}{5}x^5-\frac{1312}{7}x^7+\frac{544}{3}x^9-\frac{1024}{11}x^{11}+\frac{256}{13}x^{13}$	35
risch	$9x-38x^3+\frac{553}{5}x^5-\frac{1312}{7}x^7+\frac{544}{3}x^9-\frac{1024}{11}x^{11}+\frac{256}{13}x^{13}$	35
parallelrisch	$9x-38x^3+\frac{553}{5}x^5-\frac{1312}{7}x^7+\frac{544}{3}x^9-\frac{1024}{11}x^{11}+\frac{256}{13}x^{13}$	35
gosper	$\frac{x(295680x^{12}-1397760x^{10}+2722720x^8-2814240x^6+1660659x^4-570570x^2+135135)}{15015}$	36

input
```
int((-16*x^6+32*x^4-19*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
9*x-38*x^3+553/5*x^5-1312/7*x^7+544/3*x^9-1024/11*x^11+256/13*x^13
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(3-19x^2+32x^4-16x^6\right)^2 \, dx = \frac{256}{13}\,x^{13}-\frac{1024}{11}\,x^{11}+\frac{544}{3}\,x^9-\frac{1312}{7}\,x^7+\frac{553}{5}\,x^5-38\,x^3+9\,x$$

input
```
integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="fricas")
```

output
```
256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
 9*x
```

3.72.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \left(3-19x^2+32x^4-16x^6\right)^2 \, dx = \frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^9}{3}-\frac{1312x^7}{7}+\frac{553x^5}{5}-38x^3+9x$$

input
```
integrate((-16*x**6+32*x**4-19*x**2+3)**2,x)
```

output
```
256*x**13/13 - 1024*x**11/11 + 544*x**9/3 - 1312*x**7/7 + 553*x**5/5 - 38*
x**3 + 9*x
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(3-19x^2+32x^4-16x^6\right)^2 \, dx = \frac{256}{13}\,x^{13}-\frac{1024}{11}\,x^{11}+\frac{544}{3}\,x^9-\frac{1312}{7}\,x^7+\frac{553}{5}\,x^5-38\,x^3+9\,x$$

input
```
integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="maxima")
```

output
```
256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
 9*x
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(3-19x^2+32x^4-16x^6\right)^2 \, dx = \frac{256}{13}\,x^{13}-\frac{1024}{11}\,x^{11}+\frac{544}{3}\,x^9-\frac{1312}{7}\,x^7+\frac{553}{5}\,x^5-38\,x^3+9\,x$$

input
```
integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="giac")
```

output
```
256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
 9*x
```

3.72.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(3-19x^2+32x^4-16x^6\right)^2 \, dx = \frac{256\,x^{13}}{13}-\frac{1024\,x^{11}}{11}+\frac{544\,x^9}{3}-\frac{1312\,x^7}{7}+\frac{553\,x^5}{5}-38\,x^3+9\,x$$

input
```
int((19*x^2 - 32*x^4 + 16*x^6 - 3)^2,x)
```

output
```
9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
 (256*x^13)/13
```

3.73 $\int (3-19x^2+32x^4-16x^6) \, dx$

3.73.1 Optimal result

Integrand size = 17, antiderivative size = 25

$$\int (3-19x^2+32x^4-16x^6) \, dx = 3x-\frac{19x^3}{3}+\frac{32x^5}{5}-\frac{16x^7}{7}$$

output
```
3*x-19/3*x^3+32/5*x^5-16/7*x^7
```

3.73.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (3-19x^2+32x^4-16x^6) \, dx = 3x-\frac{19x^3}{3}+\frac{32x^5}{5}-\frac{16x^7}{7}$$

input
```
Integrate[3 - 19*x^2 + 32*x^4 - 16*x^6,x]
```

output
```
3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

3.73.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-16x^6+32x^4-19x^2+3\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

input
```
Int[3 - 19*x^2 + 32*x^4 - 16*x^6,x]
```

output
```
3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

3.73.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.73.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$3x-\frac{19}{3}x^3+\frac{32}{5}x^5-\frac{16}{7}x^7$	20
norman	$3x-\frac{19}{3}x^3+\frac{32}{5}x^5-\frac{16}{7}x^7$	20
risch	$3x-\frac{19}{3}x^3+\frac{32}{5}x^5-\frac{16}{7}x^7$	20
parallelrisch	$3x-\frac{19}{3}x^3+\frac{32}{5}x^5-\frac{16}{7}x^7$	20
parts	$3x-\frac{19}{3}x^3+\frac{32}{5}x^5-\frac{16}{7}x^7$	20
gosper	$-\frac{x(240x^6-672x^4+665x^2-315)}{105}$	21

input
```
int(-16*x^6+32*x^4-19*x^2+3,x,method=_RETURNVERBOSE)
```

output
```
3*x-19/3*x^3+32/5*x^5-16/7*x^7
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(3-19x^2+32x^4-16x^6\right)\,dx=-\frac{16}{7}\,x^7+\frac{32}{5}\,x^5-\frac{19}{3}\,x^3+3\,x$$

input
```
integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="fricas")
```

output
```
-16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

3.73.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \left(3-19x^2+32x^4-16x^6\right)\,dx=-\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

input
```
integrate(-16*x**6+32*x**4-19*x**2+3,x)
```

output
```
-16*x**7/7 + 32*x**5/5 - 19*x**3/3 + 3*x
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(3-19x^2+32x^4-16x^6\right)\,dx=-\frac{16}{7}\,x^7+\frac{32}{5}\,x^5-\frac{19}{3}\,x^3+3\,x$$

input
```
integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="maxima")
```

output
```
-16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(3-19x^2+32x^4-16x^6\right) \, dx = -\frac{16}{7}\,x^7+\frac{32}{5}\,x^5-\frac{19}{3}\,x^3+3\,x$$

input
```
integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="giac")
```

output
```
-16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

3.73.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(3-19x^2+32x^4-16x^6\right) \, dx = -\frac{16\,x^7}{7}+\frac{32\,x^5}{5}-\frac{19\,x^3}{3}+3\,x$$

input
```
int(32*x^4 - 19*x^2 - 16*x^6 + 3,x)
```

output
```
3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

3.74 $\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx$

3.74.1 Optimal result

Integrand size = 19, antiderivative size = 31

$$\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx = \frac{\operatorname{arctanh}(x)}{3}+\frac{1}{3}\operatorname{arctanh}(2x)-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
1/3*arctanh(x)+1/3*arctanh(2*x)-1/3*arctanh(2/3*x*3^(1/2))*3^(1/2)
```

3.74.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.00

$$\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx = \frac{1}{6}\Big(\sqrt{3}\log\left(\sqrt{3}-2x\right)-\sqrt{3}\log\left(\sqrt{3}+2x\right) - \log\left(1-3x+2x^2\right)+\log\left(1+3x+2x^2\right)\Big)$$

input
```
Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1),x]
```

output
```
(Sqrt[3]*Log[Sqrt[3] - 2*x] - Sqrt[3]*Log[Sqrt[3] + 2*x] - Log[1 - 3*x + 2
*x^2] + Log[1 + 3*x + 2*x^2])/6
```

3.74.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-16x^6+32x^4-19x^2+3}dx$$

$$\Big\downarrow \text{ 2460}$$

$$\int \left(\frac{2}{4x^2-3}-\frac{2}{3(4x^2-1)}-\frac{1}{3(x^2-1)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\text{arctanh}(x)}{3}+\frac{1}{3}\text{arctanh}(2x)-\frac{\text{arctanh}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1),x]
```

output
```
ArcTanh[x]/3 + ArcTanh[2*x]/3 - ArcTanh[(2*x)/Sqrt[3]]/Sqrt[3]
```

3.74.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

3.74.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{\ln(1+2x)}{6}-\frac{\ln(2x-1)}{6}+\frac{\ln(x+1)}{6}-\frac{\ln(x-1)}{6}-\frac{\operatorname{arctanh}\left(\frac{2x\sqrt{3}}{3}\right)\sqrt{3}}{3}$	42
risch	$\frac{\sqrt{3}\,\ln\left(2x-\sqrt{3}\right)}{6}-\frac{\sqrt{3}\,\ln\left(2x+\sqrt{3}\right)}{6}+\frac{\ln(2x^2+3x+1)}{6}-\frac{\ln(2x^2-3x+1)}{6}$	56

input
```
int(1/(-16*x^6+32*x^4-19*x^2+3),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(1+2*x)-1/6*ln(2*x-1)+1/6*ln(x+1)-1/6*ln(x-1)-1/3*arctanh(2/3*x*3^(1
/2))*3^(1/2)
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(23) = 46.

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx = \frac{1}{6}\,\sqrt{3}\log\left(\frac{4\,x^2-4\,\sqrt{3}x+3}{4\,x^2-3}\right) + \frac{1}{6}\,\log\left(2\,x^2+3\,x+1\right) - \frac{1}{6}\,\log\left(2\,x^2-3\,x+1\right)$$

input
```
integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log((4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) + 1/6*log(2*x^2 + 3
*x + 1) - 1/6*log(2*x^2 - 3*x + 1)
```

3.74.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(29) = 58$.

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03

$$\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx = \frac{\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{6} - \frac{\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}\right)}{6} - \frac{\log\left(x^2-\frac{3x}{2}+\frac{1}{2}\right)}{6} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{1}{2}\right)}{6}$$

input
```
integrate(1/(-16*x**6+32*x**4-19*x**2+3),x)
```

output
```
sqrt(3)*log(x - sqrt(3)/2)/6 - sqrt(3)*log(x + sqrt(3)/2)/6 - log(x**2 - 3
*x/2 + 1/2)/6 + log(x**2 + 3*x/2 + 1/2)/6
```

3.74.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(23) = 46$.

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{2x-\sqrt{3}}{2x+\sqrt{3}}\right) + \frac{1}{6}\log(2x+1) - \frac{1}{6}\log(2x-1) + \frac{1}{6}\log(x+1) - \frac{1}{6}\log(x-1)$$

input
```
integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) + 1/6*log(2*x + 1) - 1/6*
log(2*x - 1) + 1/6*log(x + 1) - 1/6*log(x - 1)
```

3.74.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(23) = 46$.

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.00

$$\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{|8\,x-4\,\sqrt{3}|}{|8\,x+4\,\sqrt{3}|}\right)+\frac{1}{6}\log\left(|2\,x+1|\right)-\frac{1}{6}\log\left(|2\,x-1|\right)+\frac{1}{6}\log\left(|x+1|\right)-\frac{1}{6}\log\left(|x-1|\right)$$

input
```
integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) + 1/6*log(abs(2
*x + 1)) - 1/6*log(abs(2*x - 1)) + 1/6*log(abs(x + 1)) - 1/6*log(abs(x - 1
))
```

3.74.9 Mupad [B] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx = \frac{\operatorname{atanh}\left(\frac{x}{4608\left(\frac{x^2}{6912}+\frac{1}{13824}\right)}\right)}{3}-\frac{\sqrt{3}\operatorname{atanh}\left(\frac{2\sqrt{3}\,x}{3}\right)}{3}$$

input
```
int(-1/(19*x^2 - 32*x^4 + 16*x^6 - 3),x)
```

output
```
atanh(x/(4608*(x^2/6912 + 1/13824)))/3 - (3^(1/2)*atanh((2*3^(1/2)*x)/3))/
3
```

3.75 $\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx$

3.75.1 Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx = \frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)}-\frac{1}{18(1+2x)}+\frac{2x}{3(3-4x^2)}+\frac{67\text{arctanh}(x)}{54}-\frac{7}{27}\text{arctanh}(2x)-\frac{5\text{arctanh}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

output
```
1/18/(1-2*x)+1/36/(1-x)-1/36/(1+x)-1/18/(1+2*x)+2/3*x/(-4*x^2+3)+67/54*arc
tanh(x)-7/27*arctanh(2*x)-5/9*arctanh(2/3*x*3^(1/2))*3^(1/2)
```

3.75.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx = \frac{1}{108}\left(-\frac{6x(27-104x^2+80x^4)}{-3+19x^2-32x^4+16x^6}+14\log(1-2x)+30\sqrt{3}\log\left(\sqrt{3}-2x\right)-67\log(1-x)+67\log(1+x)-14\log(1+2x)-30\sqrt{3}\log\left(\sqrt{3}+2x\right)\right)$$

input
```
Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2),x]
```

output
```
((-6*x*(27 - 104*x^2 + 80*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^6) + 14*Log[1
 - 2*x] + 30*Sqrt[3]*Log[Sqrt[3] - 2*x] - 67*Log[1 - x] + 67*Log[1 + x] -
14*Log[1 + 2*x] - 30*Sqrt[3]*Log[Sqrt[3] + 2*x])/108
```

3.75.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-16x^6+32x^4-19x^2+3)^2}dx$$

$$\Big\downarrow \text{2460}$$

$$\int \left(\frac{4}{4x^2-3}+\frac{14}{27(4x^2-1)}+\frac{4}{(4x^2-3)^2}-\frac{67}{54(x^2-1)}+\frac{1}{36(x-1)^2}+\frac{1}{36(x+1)^2}+\frac{1}{9(2x-1)^2}+\frac{1}{9(2x+1)^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{67\text{arctanh}(x)}{54}-\frac{7}{27}\text{arctanh}(2x)-\frac{5\text{arctanh}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x}{3(3-4x^2)}+\frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(x+1)}-\frac{1}{18(2x+1)}$$

input
```
Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2),x]
```

output
```
1/(18*(1 - 2*x)) + 1/(36*(1 - x)) - 1/(36*(1 + x)) - 1/(18*(1 + 2*x)) + (2
*x)/(3*(3 - 4*x^2)) + (67*ArcTanh[x])/54 - (7*ArcTanh[2*x])/27 - (5*ArcTan
h[(2*x)/Sqrt[3]])/(3*Sqrt[3])
```

3.75.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

3.75.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

method	result
default	$-\frac{1}{18(1+2x)}-\frac{7\ln(1+2x)}{54}-\frac{1}{18(2x-1)}+\frac{7\ln(2x-1)}{54}-\frac{1}{36(x+1)}+\frac{67\ln(x+1)}{108}-\frac{1}{36(x-1)}-\frac{67\ln(x-1)}{108}-\frac{x}{6\left(x^2-\frac{3}{4}\right)}-$
risch	$\frac{-\frac{5}{18}x^5+\frac{13}{36}x^3-\frac{3}{32}x}{x^6-2x^4+\frac{19}{16}x^2-\frac{3}{16}}+\frac{67\ln(x+1)}{108}-\frac{7\ln(1+2x)}{54}+\frac{7\ln(2x-1)}{54}+\frac{5\sqrt{3}\,\ln\left(2x-\sqrt{3}\right)}{18}-\frac{5\sqrt{3}\,\ln\left(2x+\sqrt{3}\right)}{18}-\frac{67\ln(x-1)}{108}$

input
```
int(1/(-16*x^6+32*x^4-19*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/18/(1+2*x)-7/54*ln(1+2*x)-1/18/(2*x-1)+7/54*ln(2*x-1)-1/36/(x+1)+67/108
*ln(x+1)-1/36/(x-1)-67/108*ln(x-1)-1/6*x/(x^2-3/4)-5/9*arctanh(2/3*x*3^(1/
2))*3^(1/2)
```

3.75.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(67) = 134.

Time = 0.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.99

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx =$$
$$-\frac{480\,x^5-624\,x^3-30\,\sqrt{3}(16\,x^6-32\,x^4+19\,x^2-3)\log\left(\frac{4\,x^2-4\,\sqrt{3}x+3}{4\,x^2-3}\right)+14\,(16\,x^6-32\,x^4+19\,x^2-3)}{}$$

input
```
integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="fricas")
```

output
```
-1/108*(480*x^5 - 624*x^3 - 30*sqrt(3)*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(
(4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) + 14*(16*x^6 - 32*x^4 + 19*x^2 - 3)
*log(2*x + 1) - 14*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(2*x - 1) - 67*(16*x^
6 - 32*x^4 + 19*x^2 - 3)*log(x + 1) + 67*(16*x^6 - 32*x^4 + 19*x^2 - 3)*lo
g(x - 1) + 162*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3)
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx = \frac{-80x^5+104x^3-27x}{288x^6-576x^4+342x^2-54} - \frac{67\log(x-1)}{108} + \frac{7\log\left(x-\frac{1}{2}\right)}{54} - \frac{7\log\left(x+\frac{1}{2}\right)}{54} + \frac{67\log(x+1)}{108} + \frac{5\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{18} - \frac{5\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}\right)}{18}$$

input
```
integrate(1/(-16*x**6+32*x**4-19*x**2+3)**2,x)
```

output
```
(-80*x**5 + 104*x**3 - 27*x)/(288*x**6 - 576*x**4 + 342*x**2 - 54) - 67*lo
g(x - 1)/108 + 7*log(x - 1/2)/54 - 7*log(x + 1/2)/54 + 67*log(x + 1)/108 +
 5*sqrt(3)*log(x - sqrt(3)/2)/18 - 5*sqrt(3)*log(x + sqrt(3)/2)/18
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx = \frac{5}{18}\sqrt{3}\log\left(\frac{2x-\sqrt{3}}{2x+\sqrt{3}}\right) - \frac{80x^5-104x^3+27x}{18(16x^6-32x^4+19x^2-3)} - \frac{7}{54}\log(2x+1) + \frac{7}{54}\log(2x-1) + \frac{67}{108}\log(x+1) - \frac{67}{108}\log(x-1)$$

input
```
integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="maxima")
```

output
```
5/18*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) - 1/18*(80*x^5 - 104*x^3
 + 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*log(2*x + 1) + 7/54*log(2*x
 - 1) + 67/108*log(x + 1) - 67/108*log(x - 1)
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx = \frac{5}{18}\sqrt{3}\log\left(\frac{|8\,x-4\,\sqrt{3}|}{|8\,x+4\,\sqrt{3}|}\right) - \frac{80\,x^5-104\,x^3+27\,x}{18\,(16\,x^6-32\,x^4+19\,x^2-3)} - \frac{7}{54}\log(|2\,x+1|) + \frac{7}{54}\log(|2\,x-1|) + \frac{67}{108}\log(|x+1|) - \frac{67}{108}\log(|x-1|)$$

input
```
integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="giac")
```

output
```
5/18*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) - 1/18*(80*x^5
 - 104*x^3 + 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*log(abs(2*x + 1))
 + 7/54*log(abs(2*x - 1)) + 67/108*log(abs(x + 1)) - 67/108*log(abs(x - 1)
)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx = -\frac{\operatorname{atan}(x\,1\mathrm{i})\,67\mathrm{i}}{54} + \frac{\operatorname{atan}(x\,2\mathrm{i})\,7\mathrm{i}}{27} - \frac{\frac{5\,x^5}{18}-\frac{13\,x^3}{36}+\frac{3\,x}{32}}{x^6-2\,x^4+\frac{19\,x^2}{16}-\frac{3}{16}} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\,x\,2\mathrm{i}}{3}\right)\,5\mathrm{i}}{9}$$

input
```
int(1/(19*x^2 - 32*x^4 + 16*x^6 - 3)^2,x)
```

output
```
(atan(x*2i)*7i)/27 - (atan(x*1i)*67i)/54 - ((3*x)/32 - (13*x^3)/36 + (5*x^
5)/18)/((19*x^2)/16 - 2*x^4 + x^6 - 3/16) + (3^(1/2)*atan((3^(1/2)*x*2i)/3
)*5i)/9
```

3.76 $\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx$

3.76.1 Optimal result

Integrand size = 19, antiderivative size = 161

$$\begin{aligned}\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx =& \frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2} \\ &+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{67}{432(1+x)} \\ &-\frac{1}{108(1+2x)^2}+\frac{7}{108(1+2x)}-\frac{2x}{3(3-4x^2)^2} \\ &+\frac{5x}{3(3-4x^2)}+\frac{3913\text{arctanh}(x)}{648}+\frac{67}{162}\text{arctanh}(2x) \\ &+\frac{5\text{arctanh}\left(\frac{2x}{\sqrt{3}}\right)}{6\sqrt{3}}-4\sqrt{3}\text{arctanh}\left(\frac{2x}{\sqrt{3}}\right)\end{aligned}$$

output
```
1/108/(1-2*x)^2-7/108/(1-2*x)+1/432/(1-x)^2+67/432/(1-x)-1/432/(1+x)^2-67/
432/(1+x)-1/108/(1+2*x)^2+7/108/(1+2*x)-2/3*x/(-4*x^2+3)^2+5/3*x/(-4*x^2+3
)+3913/648*arctanh(x)+67/162*arctanh(2*x)-67/18*arctanh(2/3*x*3^(1/2))*3^(
1/2)
```

3.76.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.85

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^3} dx$$
$$= \frac{\frac{36x(27-104x^2+80x^4)}{(3-19x^2+32x^4-16x^6)^2} - \frac{6x(345-2384x^2+2288x^4)}{-3+19x^2-32x^4+16x^6} - 268\log(1-2x) + 2412\sqrt{3}\log\left(\sqrt{3}-2x\right) - 3913\log(1-x) +}{1296}$$

input
```
Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3),x]
```

output
```
((36*x*(27 - 104*x^2 + 80*x^4))/(3 - 19*x^2 + 32*x^4 - 16*x^6)^2 - (6*x*(3
45 - 2384*x^2 + 2288*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^6) - 268*Log[1 - 2
*x] + 2412*Sqrt[3]*Log[Sqrt[3] - 2*x] - 3913*Log[1 - x] + 3913*Log[1 + x]
+ 268*Log[1 + 2*x] - 2412*Sqrt[3]*Log[Sqrt[3] + 2*x])/1296
```

3.76.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-16x^6+32x^4-19x^2+3)^3} dx$$
$$\Big\downarrow \text{2460}$$
$$\int \left(\frac{24}{4x^2-3} - \frac{67}{81(4x^2-1)} + \frac{12}{(4x^2-3)^2} + \frac{8}{(4x^2-3)^3} - \frac{3913}{648(x^2-1)} + \frac{67}{432(x-1)^2} + \frac{67}{432(x+1)^2} - \frac{7}{54(2x-}\right.$$
$$\Big\downarrow \text{2009}$$
$$\frac{3913\text{arctanh}(x)}{648} + \frac{67}{162}\text{arctanh}(2x) - 4\sqrt{3}\text{arctanh}\left(\frac{2x}{\sqrt{3}}\right) + \frac{5\text{arctanh}\left(\frac{2x}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{5x}{3(3-4x^2)} - \frac{2x}{3(3-4x^2)^2} - \frac{7}{108(1-2x)} + \frac{67}{432(1-x)} - \frac{67}{432(x+1)} + \frac{7}{108(2x+1)} + \frac{1}{108(1-2x)^2} + \frac{1}{432(1-x)^2} - \frac{1}{432(x+1)^2} - \frac{1}{108(2x+1)^2}$$

input
```
Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3),x]
```

output
```
1/(108*(1 - 2*x)^2) - 7/(108*(1 - 2*x)) + 1/(432*(1 - x)^2) + 67/(432*(1 -
 x)) - 1/(432*(1 + x)^2) - 67/(432*(1 + x)) - 1/(108*(1 + 2*x)^2) + 7/(108
*(1 + 2*x)) - (2*x)/(3*(3 - 4*x^2)^2) + (5*x)/(3*(3 - 4*x^2)) + (3913*ArcT
anh[x])/648 + (67*ArcTanh[2*x])/162 + (5*ArcTanh[(2*x)/Sqrt[3]])/(6*Sqrt[3
]) - 4*Sqrt[3]*ArcTanh[(2*x)/Sqrt[3]]
```

3.76.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

3.76.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.67

method	result
risch	$\frac{-\frac{4576}{27}x^{11}+\frac{4640}{9}x^{9}-580x^{7}+\frac{7960}{27}x^{5}-\frac{4777}{72}x^{3}+\frac{133}{24}x}{\left(16x^{6}-32x^{4}+19x^{2}-3\right)^{2}}+\frac{67\ln(1+2x)}{324}+\frac{3913\ln(x+1)}{1296}+\frac{67\sqrt{3}\,\ln\left(2x-\sqrt{3}\right)}{36}-\frac{67\sqrt{3}\,\ln\left(2x+\sqrt{3}\right)}{36}$
default	$-\frac{1}{108(1+2x)^{2}}+\frac{7}{108(1+2x)}+\frac{67\ln(1+2x)}{324}+\frac{1}{108(2x-1)^{2}}+\frac{7}{108(2x-1)}-\frac{67\ln(2x-1)}{324}-\frac{1}{432(x+1)^{2}}-\frac{67}{432(x+1)}+$

input
```
int(1/(-16*x^6+32*x^4-19*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
256*(-143/216*x^11+145/72*x^9-145/64*x^7+995/864*x^5-4777/18432*x^3+133/61
44*x)/(16*x^6-32*x^4+19*x^2-3)^2+67/324*ln(1+2*x)+3913/1296*ln(x+1)+67/36*
3^(1/2)*ln(2*x-3^(1/2))-67/36*3^(1/2)*ln(2*x+3^(1/2))-67/324*ln(2*x-1)-391
3/1296*ln(x-1)
```

3.76.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(111) = 222$.

Time = 0.30 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.75

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx =$$
$$-\frac{219648\,x^{11}-668160\,x^9+751680\,x^7-382080\,x^5+85986\,x^3-2412\,\sqrt{3}(256\,x^{12}-1024\,x^{10}+1632\,x^8 -}{}$$

input
```
integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="fricas")
```

output
```
-1/1296*(219648*x^11 - 668160*x^9 + 751680*x^7 - 382080*x^5 + 85986*x^3 -
2412*sqrt(3)*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x
^2 + 9)*log((4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) - 268*(256*x^12 - 1024*
x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(2*x + 1) + 268*(25
6*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(2*x
- 1) - 3913*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^
2 + 9)*log(x + 1) + 3913*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553
*x^4 - 114*x^2 + 9)*log(x - 1) - 7182*x)/(256*x^12 - 1024*x^10 + 1632*x^8
- 1312*x^6 + 553*x^4 - 114*x^2 + 9)
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.83

$$\begin{aligned}\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx &= -\frac{36608x^{11}-111360x^9+125280x^7-63680x^5+14331x^3-1197x}{55296x^{12}-221184x^{10}+352512x^8-283392x^6+119448x^4-24624x^2+1944} \\ &\quad -\frac{3913\log(x-1)}{1296}-\frac{67\log\left(x-\frac{1}{2}\right)}{324}+\frac{67\log\left(x+\frac{1}{2}\right)}{324} \\ &\quad +\frac{3913\log(x+1)}{1296}+\frac{67\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{36}-\frac{67\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}\right)}{36}\end{aligned}$$

input
```
integrate(1/(-16*x**6+32*x**4-19*x**2+3)**3,x)
```

output
```
-(36608*x**11 - 111360*x**9 + 125280*x**7 - 63680*x**5 + 14331*x**3 - 1197
*x)/(55296*x**12 - 221184*x**10 + 352512*x**8 - 283392*x**6 + 119448*x**4
- 24624*x**2 + 1944) - 3913*log(x - 1)/1296 - 67*log(x - 1/2)/324 + 67*log
(x + 1/2)/324 + 3913*log(x + 1)/1296 + 67*sqrt(3)*log(x - sqrt(3)/2)/36 -
67*sqrt(3)*log(x + sqrt(3)/2)/36
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.74

$$\begin{aligned}&\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx\\&=\frac{67}{36}\sqrt{3}\log\left(\frac{2x-\sqrt{3}}{2x+\sqrt{3}}\right)\\&\quad-\frac{36608x^{11}-111360x^9+125280x^7-63680x^5+14331x^3-1197x}{216\,(256x^{12}-1024x^{10}+1632x^8-1312x^6+553x^4-114x^2+9)}\\&\quad+\frac{67}{324}\log(2x+1)-\frac{67}{324}\log(2x-1)+\frac{3913}{1296}\log(x+1)-\frac{3913}{1296}\log(x-1)\end{aligned}$$

input
```
integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="maxima")
```

output
```
67/36*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) - 1/216*(36608*x^11 - 1
11360*x^9 + 125280*x^7 - 63680*x^5 + 14331*x^3 - 1197*x)/(256*x^12 - 1024*
x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9) + 67/324*log(2*x + 1)
- 67/324*log(2*x - 1) + 3913/1296*log(x + 1) - 3913/1296*log(x - 1)
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

$$\begin{aligned}&\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx\\&=\frac{67}{36}\sqrt{3}\log\left(\frac{|8x-4\sqrt{3}|}{|8x+4\sqrt{3}|}\right)\\&\quad-\frac{36608x^{11}-111360x^9+125280x^7-63680x^5+14331x^3-1197x}{216\,(16x^6-32x^4+19x^2-3)^2}\\&\quad+\frac{67}{324}\log(|2x+1|)-\frac{67}{324}\log(|2x-1|)+\frac{3913}{1296}\log(|x+1|)-\frac{3913}{1296}\log(|x-1|)\end{aligned}$$

input
```
integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="giac")
```

output
```
67/36*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) - 1/216*(3660
8*x^11 - 111360*x^9 + 125280*x^7 - 63680*x^5 + 14331*x^3 - 1197*x)/(16*x^6
 - 32*x^4 + 19*x^2 - 3)^2 + 67/324*log(abs(2*x + 1)) - 67/324*log(abs(2*x
- 1)) + 3913/1296*log(abs(x + 1)) - 3913/1296*log(abs(x - 1))
```

3.76.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.58

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx = \frac{-\frac{143\,x^{11}}{216}+\frac{145\,x^9}{72}-\frac{145\,x^7}{64}+\frac{995\,x^5}{864}-\frac{4777\,x^3}{18432}+\frac{133\,x}{6144}}{x^{12}-4\,x^{10}+\frac{51\,x^8}{8}-\frac{41\,x^6}{8}+\frac{553\,x^4}{256}-\frac{57\,x^2}{128}+\frac{9}{256}} - \frac{\mathrm{atan}(x\,2\mathrm{i})\;67\mathrm{i}}{162} - \frac{\mathrm{atan}(x\,1\mathrm{i})\;3913\mathrm{i}}{648} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x\,2\mathrm{i}}{3}\right)\;67\mathrm{i}}{18}$$

input
```
int(-1/(19*x^2 - 32*x^4 + 16*x^6 - 3)^3,x)
```

output
```
((133*x)/6144 - (4777*x^3)/18432 + (995*x^5)/864 - (145*x^7)/64 + (145*x^9
)/72 - (143*x^11)/216)/((553*x^4)/256 - (57*x^2)/128 - (41*x^6)/8 + (51*x^
8)/8 - 4*x^10 + x^12 + 9/256) - (atan(x*2i)*67i)/162 - (atan(x*1i)*3913i)/
648 + (3^(1/2)*atan((3^(1/2)*x*2i)/3)*67i)/18
```

3.77 $\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx$

3.77.1 Optimal result

Integrand size = 17, antiderivative size = 91

$$\begin{aligned}\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx &= \frac{x}{32(1-x^2)}+\frac{x(99-17x^2)}{128(1-6x^2+x^4)}+\frac{5\operatorname{arctanh}(x)}{32}\\ &\quad+\frac{1}{512}\left(-4+3\sqrt{2}\right)\operatorname{arctanh}\left(\left(-1+\sqrt{2}\right)x\right)\\ &\quad+\frac{1}{512}\left(4+3\sqrt{2}\right)\operatorname{arctanh}\left(\left(1+\sqrt{2}\right)x\right)\end{aligned}$$

output
```
1/32*x/(-x^2+1)+1/128*x*(-17*x^2+99)/(x^4-6*x^2+1)+5/32*arctanh(x)+1/512*a
rctanh(x*(2^(1/2)-1))*(-4+3*2^(1/2))+1/512*arctanh(x*(1+2^(1/2)))*(4+3*2^(
1/2))
```

3.77.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.45

$$\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx$$
$$= \frac{-\frac{8x(103-140x^2+21x^4)}{-1+7x^2-7x^4+x^6}-80\log(1-x)-\left(4+3\sqrt{2}\right)\log\left(-1+\sqrt{2}-x\right)+\left(4-3\sqrt{2}\right)\log\left(1+\sqrt{2}-x\right)+80}{1024}$$

input
```
Integrate[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2),x]
```

output
```
((-8*x*(103 - 140*x^2 + 21*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 80*Log[1 - x
] - (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (4 - 3*Sqrt[2])*Log[1 + Sqrt[2
] - x] + 80*Log[1 + x] + (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (-4 + 3*S
qrt[2])*Log[1 + Sqrt[2] + x])/1024
```

3.77.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 245 vs. $2(91) = 182$.

Time = 0.44 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.69, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{2460, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^6-7x^4+7x^2-1)^2}\,dx$$

$$\Big\downarrow \text{2460}$$

$$\int \left(\frac{29-12x}{64\,(x^2-2x-1)^2}-\frac{5}{32\,(x^2-1)}+\frac{x+6}{128\,(x^2-2x-1)}+\frac{6-x}{128\,(x^2+2x-1)}+\frac{12x+29}{64\,(x^2+2x-1)^2}+\frac{1}{64(x-1)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\text{arctanh}(x)}{32}-\frac{41-17x}{256\,(-x^2+2x+1)}+\frac{17x+41}{256\,(-x^2-2x+1)}+\frac{1}{64(1-x)}-\frac{1}{64(x+1)}+\frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(-x-\sqrt{2}+1\right)+\frac{17\log\left(-x-\sqrt{2}+1\right)}{512\sqrt{2}}+\frac{1}{512}\left(2+7\sqrt{2}\right)\log\left(-x+\sqrt{2}+1\right)-\frac{17\log\left(-x+\sqrt{2}+1\right)}{512\sqrt{2}}-\frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)-\frac{17\log\left(x-\sqrt{2}+1\right)}{512\sqrt{2}}-\frac{1}{512}\left(2+7\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)+\frac{17\log\left(x+\sqrt{2}+1\right)}{512\sqrt{2}}$$

input
```
Int[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2),x]
```

output
```
1/(64*(1 - x)) - 1/(64*(1 + x)) + (41 + 17*x)/(256*(1 - 2*x - x^2)) - (41
- 17*x)/(256*(1 + 2*x - x^2)) + (5*ArcTanh[x])/32 + (17*Log[1 - Sqrt[2] -
x])/(512*Sqrt[2]) + ((2 - 7*Sqrt[2])*Log[1 - Sqrt[2] - x])/512 - (17*Log[1
 + Sqrt[2] - x])/(512*Sqrt[2]) + ((2 + 7*Sqrt[2])*Log[1 + Sqrt[2] - x])/51
2 - (17*Log[1 - Sqrt[2] + x])/(512*Sqrt[2]) - ((2 - 7*Sqrt[2])*Log[1 - Sqr
t[2] + x])/512 + (17*Log[1 + Sqrt[2] + x])/(512*Sqrt[2]) - ((2 + 7*Sqrt[2]
)*Log[1 + Sqrt[2] + x])/512
```

3.77.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

3.77.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

method	result
default	$-\frac{1}{64(x+1)}+\frac{5\ln(x+1)}{64}-\frac{\frac{17x}{2}+\frac{41}{2}}{128(x^2+2x-1)}-\frac{\ln(x^2+2x-1)}{256}+\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{512}-\frac{1}{64(x-1)}-\frac{5\ln(x-1)}{64}+\frac{-}{128x^2}$
risch	$\frac{-\frac{21}{128}x^5+\frac{35}{32}x^3-\frac{103}{128}x}{x^6-7x^4+7x^2-1}+\frac{\ln\left(x-1+\sqrt{2}\right)}{256}+\frac{3\ln\left(x-1+\sqrt{2}\right)\sqrt{2}}{1024}+\frac{\ln\left(x-1-\sqrt{2}\right)}{256}-\frac{3\ln\left(x-1-\sqrt{2}\right)\sqrt{2}}{1024}-\frac{\ln\left(1+x+\sqrt{2}\right)}{256}+\frac{3\ln}{}$

input
```
int(1/(x^6-7*x^4+7*x^2-1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/64/(x+1)+5/64*ln(x+1)-1/128*(17/2*x+41/2)/(x^2+2*x-1)-1/256*ln(x^2+2*x-
1)+3/512*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))-1/64/(x-1)-5/64*ln(x-1)+1/12
8*(-17/2*x+41/2)/(x^2-2*x-1)+1/256*ln(x^2-2*x-1)+3/512*2^(1/2)*arctanh(1/4
*(2*x-2)*2^(1/2))
```

3.77.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(71) = 142$.

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.45

$$\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx =$$
$$-\frac{168\,x^5 - 1120\,x^3 - 3\,\sqrt{2}(x^6 - 7\,x^4 + 7\,x^2 - 1)\log\left(\frac{x^2+2\,\sqrt{2}(x+1)+2\,x+3}{x^2+2\,x-1}\right) - 3\,\sqrt{2}(x^6 - 7\,x^4 + 7\,x^2 - 1)\log}{}$$

input
```
integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="fricas")
```

output
```
-1/1024*(168*x^5 - 1120*x^3 - 3*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2
 + 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 3*sqrt(2)*(x^6 - 7*x^4
+ 7*x^2 - 1)*log((x^2 + 2*sqrt(2)*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) + 4*
(x^6 - 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x - 1) - 4*(x^6 - 7*x^4 + 7*x^2 - 1)
*log(x^2 - 2*x - 1) - 80*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 80*(x^6 -
7*x^4 + 7*x^2 - 1)*log(x - 1) + 824*x)/(x^6 - 7*x^4 + 7*x^2 - 1)
```

3.77.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(75) = 150$.

Time = 0.81 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.25

$$\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx$$
$$= \frac{-21x^5+140x^3-103x}{128x^6-896x^4+896x^2-128} - \frac{5\log(x-1)}{64} + \frac{5\log(x+1)}{64} + \left(-\frac{1}{256}\right.$$
$$\left.+\frac{3\sqrt{2}}{1024}\right)\log\left(x-\frac{8071264001}{202624020}-\frac{471550901878784\left(-\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^3}{2979765}+\frac{1299552375287054336\left(-\frac{1}{256}+\frac{3}{10}\right.}{50656005}\right.$$
$$+\left(-\frac{3\sqrt{2}}{1024}\right.$$
$$\left.-\frac{1}{256}\right)\log\left(x-\frac{8071264001\sqrt{2}}{270165360}-\frac{8071264001}{202624020}+\frac{1299552375287054336\left(-\frac{3\sqrt{2}}{1024}-\frac{1}{256}\right)^5}{50656005}-\frac{47155090187}{}\right.$$
$$+\left(\frac{1}{256}\right.$$
$$\left.-\frac{3\sqrt{2}}{1024}\right)\log\left(x-\frac{8071264001\sqrt{2}}{270165360}+\frac{1299552375287054336\left(\frac{1}{256}-\frac{3\sqrt{2}}{1024}\right)^5}{50656005}-\frac{471550901878784\left(\frac{1}{256}-\frac{3\sqrt{}}{10}\right.}{2979765}\right.$$
$$+\left(\frac{1}{256}\right.$$
$$\left.+\frac{3\sqrt{2}}{1024}\right)\log\left(x-\frac{471550901878784\left(\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^3}{2979765}+\frac{1299552375287054336\left(\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^5}{50656005}+\frac{807126400}{20262402}\right.$$

input
```
integrate(1/(x**6-7*x**4+7*x**2-1)**2,x)
```

3.77. $\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx$

output
```
(-21*x**5 + 140*x**3 - 103*x)/(128*x**6 - 896*x**4 + 896*x**2 - 128) - 5*l
og(x - 1)/64 + 5*log(x + 1)/64 + (-1/256 + 3*sqrt(2)/1024)*log(x - 8071264
001/202624020 - 471550901878784*(-1/256 + 3*sqrt(2)/1024)**3/2979765 + 129
9552375287054336*(-1/256 + 3*sqrt(2)/1024)**5/50656005 + 8071264001*sqrt(2
)/270165360) + (-3*sqrt(2)/1024 - 1/256)*log(x - 8071264001*sqrt(2)/270165
360 - 8071264001/202624020 + 1299552375287054336*(-3*sqrt(2)/1024 - 1/256)
**5/50656005 - 471550901878784*(-3*sqrt(2)/1024 - 1/256)**3/2979765) + (1/
256 - 3*sqrt(2)/1024)*log(x - 8071264001*sqrt(2)/270165360 + 1299552375287
054336*(1/256 - 3*sqrt(2)/1024)**5/50656005 - 471550901878784*(1/256 - 3*s
qrt(2)/1024)**3/2979765 + 8071264001/202624020) + (1/256 + 3*sqrt(2)/1024)
*log(x - 471550901878784*(1/256 + 3*sqrt(2)/1024)**3/2979765 + 12995523752
87054336*(1/256 + 3*sqrt(2)/1024)**5/50656005 + 8071264001/202624020 + 807
1264001*sqrt(2)/270165360)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.25

$$\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx = -\frac{3}{1024}\,\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right) - \frac{3}{1024}\,\sqrt{2}\log\left(\frac{x-\sqrt{2}-1}{x+\sqrt{2}-1}\right) - \frac{21\,x^5-140\,x^3+103\,x}{128\,(x^6-7\,x^4+7\,x^2-1)} - \frac{1}{256}\,\log\left(x^2+2\,x-1\right) + \frac{1}{256}\,\log\left(x^2-2\,x-1\right) + \frac{5}{64}\,\log\left(x+1\right) - \frac{5}{64}\,\log\left(x-1\right)$$

input
```
integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="maxima")
```

output
```
-3/1024*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) - 3/1024*sqrt(2)*
log((x - sqrt(2) - 1)/(x + sqrt(2) - 1)) - 1/128*(21*x^5 - 140*x^3 + 103*x
)/(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*log(x^2 + 2*x - 1) + 1/256*log(x^2 - 2
*x - 1) + 5/64*log(x + 1) - 5/64*log(x - 1)
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.47

$$\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx = -\frac{3}{1024}\sqrt{2}\log\left(\frac{|2\,x-2\,\sqrt{2}+2|}{|2\,x+2\,\sqrt{2}+2|}\right) - \frac{3}{1024}\sqrt{2}\log\left(\frac{|2\,x-2\,\sqrt{2}-2|}{|2\,x+2\,\sqrt{2}-2|}\right) - \frac{21\,x^5-140\,x^3+103\,x}{128\,(x^6-7\,x^4+7\,x^2-1)} - \frac{1}{256}\log\left(|x^2+2\,x-1|\right) + \frac{1}{256}\log\left(|x^2-2\,x-1|\right) + \frac{5}{64}\log\left(|x+1|\right) - \frac{5}{64}\log\left(|x-1|\right)$$

input
```
integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="giac")
```

output
```
-3/1024*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) - 3
/1024*sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2)/abs(2*x + 2*sqrt(2) - 2)) - 1/1
28*(21*x^5 - 140*x^3 + 103*x)/(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*log(abs(x^
2 + 2*x - 1)) + 1/256*log(abs(x^2 - 2*x - 1)) + 5/64*log(abs(x + 1)) - 5/6
4*log(abs(x - 1))
```

3.77.9 Mupad [B] (verification not implemented)

Time = 10.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.38

$$\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx = -\frac{\mathrm{atan}(x\,1\mathrm{i})\,5\mathrm{i}}{32} - \frac{\frac{21\,x^5}{128}-\frac{35\,x^3}{32}+\frac{103\,x}{128}}{x^6-7\,x^4+7\,x^2-1} - \mathrm{atan}\left(\frac{x\,940311\mathrm{i}}{134217728\,\left(\frac{275445\,\sqrt{2}}{134217728}-\frac{389421}{134217728}\right)} - \frac{\sqrt{2}\,x\,332433\mathrm{i}}{67108864\,\left(\frac{275445\,\sqrt{2}}{134217728}-\frac{389421}{134217728}\right)}\right)\left(\frac{\sqrt{2}\,3\mathrm{i}}{512}-\frac{1}{128}\mathrm{i}\right) - \mathrm{atan}\left(\frac{x\,940311\mathrm{i}}{134217728\,\left(\frac{275445\,\sqrt{2}}{134217728}+\frac{389421}{134217728}\right)} + \frac{\sqrt{2}\,x\,332433\mathrm{i}}{67108864\,\left(\frac{275445\,\sqrt{2}}{134217728}+\frac{389421}{134217728}\right)}\right)\left(\frac{\sqrt{2}\,3\mathrm{i}}{512}+\frac{1}{128}\mathrm{i}\right)$$

input
```
int(1/(7*x^2 - 7*x^4 + x^6 - 1)^2,x)
```

output
```
- (atan(x*1i)*5i)/32 - ((103*x)/128 - (35*x^3)/32 + (21*x^5)/128)/(7*x^2 -
 7*x^4 + x^6 - 1) - atan((x*940311i)/(134217728*((275445*2^(1/2))/13421772
8 - 389421/134217728)) - (2^(1/2)*x*332433i)/(67108864*((275445*2^(1/2))/1
34217728 - 389421/134217728)))*((2^(1/2)*3i)/512 - 1i/128) - atan((x*94031
1i)/(134217728*((275445*2^(1/2))/134217728 + 389421/134217728)) + (2^(1/2)
*x*332433i)/(67108864*((275445*2^(1/2))/134217728 + 389421/134217728)))*((
2^(1/2)*3i)/512 + 1i/128)
```

3.77. $\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx$

3.78 $\int \frac{x^3}{c+(a+bx)^2} dx$

3.78.1 Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{x^3}{c+(a+bx)^2} dx = -\frac{3ax}{b^3}+\frac{(a+bx)^2}{2b^4}-\frac{a(a^2-3c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{(3a^2-c)\log\left(c+(a+bx)^2\right)}{2b^4}$$

output
```
-3*a*x/b^3+1/2*(b*x+a)^2/b^4+1/2*(3*a^2-c)*ln(c+(b*x+a)^2)/b^4-a*(a^2-3*c)
*arctan((b*x+a)/c^(1/2))/b^4/c^(1/2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{c+(a+bx)^2} dx = \frac{bx(-4a+bx)-\frac{2(a^3-3ac)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+(3a^2-c)\log\left(a^2+c+2abx+b^2x^2\right)}{2b^4}$$

input
```
Integrate[x^3/(c + (a + b*x)^2),x]
```

output
```
(b*x*(-4*a + b*x) - (2*(a^3 - 3*a*c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] +
(3*a^2 - c)*Log[a^2 + c + 2*a*b*x + b^2*x^2])/(2*b^4)
```

3.78.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {896, 25, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx)^2+c}dx$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int \frac{b^3x^3}{(a+bx)^2+c}d(a+bx)}{b^4}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -\frac{b^3x^3}{(a+bx)^2+c}d(a+bx)}{b^4}$$

$$\Big\downarrow \text{ 478}$$

$$-\frac{\int \left(2a-bx+\frac{a^3-3ca-(3a^2-c)(a+bx)}{(a+bx)^2+c}\right)d(a+bx)}{b^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{a(a^2-3c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{1}{2}(3a^2-c)\log\left((a+bx)^2+c\right)+\frac{1}{2}(a+bx)^2-3a(a+bx)}{b^4}$$

input
```
Int[x^3/(c + (a + b*x)^2),x]
```

output
```
(-3*a*(a + b*x) + (a + b*x)^2/2 - (a*(a^2 - 3*c)*ArcTan[(a + b*x)/Sqrt[c]]
)/Sqrt[c] + ((3*a^2 - c)*Log[c + (a + b*x)^2])/2)/b^4
```

3.78.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.78.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.42

method	result
default	$-\frac{-\frac{1}{2}b\,x^2+2ax}{b^3}+\frac{\frac{\left(3a^2b-bc\right)\ln\left(b^2x^2+2abx+a^2+c\right)}{2b^2}+\frac{\left(2a^3+2ac-\frac{\left(3a^2b-bc\right)a}{b}\right)\arctan\left(\frac{2b^2x+2ab}{2\sqrt{c}\,b}\right)}{\sqrt{c}\,b}}{b^3}$
risch	$\frac{x^2}{2b^2}-\frac{2ax}{b^3}+\frac{3\ln\left(-a^3c-\sqrt{-c\,a^2(a^2-3c)^2}\,bx+3a\,c^2-\sqrt{-c\,a^2(a^2-3c)^2}\,a\right)a^2}{2b^4}-\frac{c\ln\left(-a^3c-\sqrt{-c\,a^2(a^2-3c)^2}\,bx+3a\,c^2-\sqrt{-c\right.}{2b^4}$

input
```
int(x^3/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^3*(-1/2*b*x^2+2*a*x)+1/b^3*(1/2*(3*a^2*b-b*c)/b^2*ln(b^2*x^2+2*a*b*x+
a^2+c)+(2*a^3+2*a*c-(3*a^2*b-b*c)*a/b)/c^(1/2)/b*arctan(1/2*(2*b^2*x+2*a*b
)/c^(1/2)/b))
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.54

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = \left[\frac{b^2cx^2 - 4\,abcx + (a^3 - 3\,ac)\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right) + (3\,a^2c - c^2)\log\left(b^2x^2 + 2\,abx + a^2 + \right.}{2\,b^4c}\right.$$

input
```
integrate(x^3/(c+(b*x+a)^2),x, algorithm="fricas")
```

output
```
[1/2*(b^2*c*x^2 - 4*a*b*c*x + (a^3 - 3*a*c)*sqrt(-c)*log((b^2*x^2 + 2*a*b*
x + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + (3*a^
2*c - c^2)*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c), 1/2*(b^2*c*x^2 - 4*a
*b*c*x - 2*(a^3 - 3*a*c)*sqrt(c)*arctan((b*x + a)/sqrt(c)) + (3*a^2*c - c^
2)*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c)]
```

3.78.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(71) = 142$.

Time = 0.40 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.68

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = -\frac{2ax}{b^3} + \left(-\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right)\log\left(x + \frac{a^4 - 2b^4c\left(-\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right) - c^2}{a^3b - 3abc}\right) + \left(\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right)\log\left(x + \frac{a^4 - 2b^4c\left(\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right) - c^2}{a^3b - 3abc}\right) + \frac{x^2}{2b^2}$$

input
```
integrate(x**3/(c+(b*x+a)**2),x)
```

output
```
-2*a*x/b**3 + (-a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4)
)*log(x + (a**4 - 2*b**4*c*(-a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2
- c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) + (a*sqrt(-c)*(a**2 - 3*c)/(2*b
**4*c) + (3*a**2 - c)/(2*b**4))*log(x + (a**4 - 2*b**4*c*(a*sqrt(-c)*(a**2
 - 3*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) +
x**2/(2*b**2)
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = \frac{bx^2-4\,ax}{2\,b^3} + \frac{(3\,a^2-c)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^4} - \frac{(a^3-3\,ac)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^4\sqrt{c}}$$

input
```
integrate(x^3/(c+(b*x+a)^2),x, algorithm="maxima")
```

output
```
1/2*(b*x^2 - 4*a*x)/b^3 + 1/2*(3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)
/b^4 - (a^3 - 3*a*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/(b^4*sqrt(c))
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = \frac{(3\,a^2-c)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^4} - \frac{(a^3-3\,ac)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^4\sqrt{c}} + \frac{b^2x^2-4\,abx}{2\,b^4}$$

input
```
integrate(x^3/(c+(b*x+a)^2),x, algorithm="giac")
```

output
```
1/2*(3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^4 - (a^3 - 3*a*c)*arcta
n((b*x + a)/sqrt(c))/(b^4*sqrt(c)) + 1/2*(b^2*x^2 - 4*a*b*x)/b^4
```

3.78.9 Mupad [B] (verification not implemented)

Time = 9.96 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = \frac{x^2}{2\,b^2}-\frac{2\,a\,x}{b^3}-\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+c\right)\,\left(4\,b^4\,c^2-12\,a^2\,b^4\,c\right)}{8\,b^8\,c}+\frac{a\,\mathrm{atan}\left(\frac{a+b\,x}{\sqrt{c}}\right)\,\left(3\,c-a^2\right)}{b^4\,\sqrt{c}}$$

input
```
int(x^3/(c + (a + b*x)^2),x)
```

output
```
x^2/(2*b^2) - (2*a*x)/b^3 - (log(c + a^2 + b^2*x^2 + 2*a*b*x)*(4*b^4*c^2 -
 12*a^2*b^4*c))/(8*b^8*c) + (a*atan((a + b*x)/c^(1/2))*(3*c - a^2))/(b^4*c
^(1/2))
```

3.79 $\int \frac{x^2}{c+(a+bx)^2} dx$

3.79.1 Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{x^2}{c+(a+bx)^2} dx = \frac{x}{b^2} + \frac{(a^2-c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}} - \frac{a\log\left(c+(a+bx)^2\right)}{b^3}$$

output
```
x/b^2-a*ln(c+(b*x+a)^2)/b^3+(a^2-c)*arctan((b*x+a)/c^(1/2))/b^3/c^(1/2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{c+(a+bx)^2} dx = \frac{bx + \frac{(a^2-c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}} - a\log\left(a^2+c+2abx+b^2x^2\right)}{b^3}$$

input
```
Integrate[x^2/(c + (a + b*x)^2),x]
```

output
```
(b*x + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - a*Log[a^2 + c + 2*a
*b*x + b^2*x^2])/b^3
```

3.79.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {896, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx)^2+c}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{b^2x^2}{(a+bx)^2+c}d(a+bx)}{b^3}$$

$$\Big\downarrow \text{478}$$

$$\frac{\int \left(\frac{a^2-2(a+bx)a-c}{(a+bx)^2+c}+1\right)d(a+bx)}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a^2-c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+a\left(-\log\left((a+bx)^2+c\right)\right)+a+bx}{b^3}$$

input
```
Int[x^2/(c + (a + b*x)^2),x]
```

output
```
(a + b*x + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - a*Log[c + (a +
b*x)^2])/b^3
```

3.79.3.1 Defintions of rubi rules used

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.79.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

method	result
default	$\frac{x}{b^2}+\frac{-\frac{a\ln\left(b^2x^2+2abx+a^2+c\right)}{b}+\frac{\left(a^2-c\right)\arctan\left(\frac{2b^2x+2ab}{2\sqrt{c}\,b}\right)}{\sqrt{c}\,b}}{b^2}$
risch	$\frac{x}{b^2}-\frac{\ln\left(-\sqrt{-c(a^2-c)^2}\,bx+c\,a^2-\sqrt{-c(a^2-c)^2}\,a-c^2\right)a}{b^3}+\frac{\ln\left(-\sqrt{-c(a^2-c)^2}\,bx+c\,a^2-\sqrt{-c(a^2-c)^2}\,a-c^2\right)\sqrt{-c(a^2-c)^2}}{2b^3c}-$

input
```
int(x^2/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
x/b^2+1/b^2*(-a/b*ln(b^2*x^2+2*a*b*x+a^2+c)+(a^2-c)/c^(1/2)/b*arctan(1/2*(
2*b^2*x+2*a*b)/c^(1/2)/b))
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.14

$$\int\frac{x^2}{c+(a+bx)^2}\,dx$$
$$=\left[\frac{2\,bcx-2\,ac\log\left(b^2x^2+2\,abx+a^2+c\right)+(a^2-c)\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)}{2\,b^3c},\ \frac{bcx-ac\log\left(b^2\right.}{}\right.$$

input
```
integrate(x^2/(c+(b*x+a)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*b*c*x - 2*a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) + (a^2 - c)*sqrt(-c
)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*
b*x + a^2 + c)))/(b^3*c), (b*c*x - a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) +
(a^2 - c)*sqrt(c)*arctan((b*x + a)/sqrt(c)))/(b^3*c)]
```

3.79.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(44) = 88$.

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.06

$$\int \frac{x^2}{c+(a+bx)^2}\,dx = \left(-\frac{a}{b^3}-\frac{\sqrt{-c}(a^2-c)}{2b^3c}\right)\log\left(x+\frac{a^3+ac+2b^3c\left(-\frac{a}{b^3}-\frac{\sqrt{-c}(a^2-c)}{2b^3c}\right)}{a^2b-bc}\right) + \left(-\frac{a}{b^3} + \frac{\sqrt{-c}(a^2-c)}{2b^3c}\right)\log\left(x+\frac{a^3+ac+2b^3c\left(-\frac{a}{b^3}+\frac{\sqrt{-c}(a^2-c)}{2b^3c}\right)}{a^2b-bc}\right)+\frac{x}{b^2}$$

input
```
integrate(x**2/(c+(b*x+a)**2),x)
```

output
```
(-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*
(-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + (-a/b**3 + s
qrt(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*(-a/b**3 + s
qrt(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + x/b**2
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{c+(a+bx)^2}\,dx = \frac{x}{b^2}-\frac{a\log\left(b^2x^2+2\,abx+a^2+c\right)}{b^3}+\frac{(a^2-c)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^3\sqrt{c}}$$

input
```
integrate(x^2/(c+(b*x+a)^2),x, algorithm="maxima")
```

output
```
x/b^2 - a*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan((b^2*x +
 a*b)/(b*sqrt(c)))/(b^3*sqrt(c))
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{c+(a+bx)^2}\,dx = \frac{x}{b^2} - \frac{a\log\left(b^2x^2+2\,abx+a^2+c\right)}{b^3} + \frac{\left(a^2-c\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^3\sqrt{c}}$$

input
```
integrate(x^2/(c+(b*x+a)^2),x, algorithm="giac")
```

output
```
x/b^2 - a*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan((b*x + a
)/sqrt(c))/(b^3*sqrt(c))
```

3.79.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 206, normalized size of antiderivative = 4.12

$$\begin{aligned}\int \frac{x^2}{c+(a+bx)^2}\,dx &= \frac{x}{b^2} - \frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+c\right)}{b^3} \\ &\quad + \frac{\sqrt{c}\,\mathrm{atan}\left(\frac{a^3}{\sqrt{c}\,(c-a^2)} - \frac{\sqrt{c}\,x}{\frac{c}{b}-\frac{a^2}{b}} - \frac{a\,\sqrt{c}}{c-a^2} + \frac{a^2\,x}{\sqrt{c}\,\left(\frac{c}{b}-\frac{a^2}{b}\right)}\right)}{b^3} \\ &\quad - \frac{a^2\,\mathrm{atan}\left(\frac{a^3}{\sqrt{c}\,(c-a^2)} - \frac{\sqrt{c}\,x}{\frac{c}{b}-\frac{a^2}{b}} - \frac{a\,\sqrt{c}}{c-a^2} + \frac{a^2\,x}{\sqrt{c}\,\left(\frac{c}{b}-\frac{a^2}{b}\right)}\right)}{b^3\,\sqrt{c}}\end{aligned}$$

input
```
int(x^2/(c + (a + b*x)^2),x)
```

output
```
x/b^2 - (a*log(c + a^2 + b^2*x^2 + 2*a*b*x))/b^3 + (c^(1/2)*atan(a^3/(c^(1
/2)*(c - a^2)) - (c^(1/2)*x)/(c/b - a^2/b) - (a*c^(1/2))/(c - a^2) + (a^2*
x)/(c^(1/2)*(c/b - a^2/b))))/b^3 - (a^2*atan(a^3/(c^(1/2)*(c - a^2)) - (c^
(1/2)*x)/(c/b - a^2/b) - (a*c^(1/2))/(c - a^2) + (a^2*x)/(c^(1/2)*(c/b - a
^2/b))))/(b^3*c^(1/2))
```

3.80 $\int \frac{x}{c+(a+bx)^2} dx$

3.80.1 Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{x}{c+(a+bx)^2} dx = -\frac{a \arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\log(c+(a+bx)^2)}{2b^2}$$

output
```
1/2*ln(c+(b*x+a)^2)/b^2-a*arctan((b*x+a)/c^(1/2))/b^2/c^(1/2)
```

3.80.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{x}{c+(a+bx)^2} dx = \frac{-\frac{2a \arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}} + \log(c+(a+bx)^2)}{2b^2}$$

input
```
Integrate[x/(c + (a + b*x)^2),x]
```

output
```
((-2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + Log[c + (a + b*x)^2])/(2*b^2)
```

3.80.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {896, 25, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx)^2+c}dx$$

$$\Big\downarrow \; 896$$

$$\frac{\int \frac{bx}{(a+bx)^2+c}d(a+bx)}{b^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{bx}{(a+bx)^2+c}d(a+bx)}{b^2}$$

$$\Big\downarrow \; 452$$

$$\frac{\int \frac{a+bx}{(a+bx)^2+c}d(a+bx)-a\int \frac{1}{(a+bx)^2+c}d(a+bx)}{b^2}$$

$$\Big\downarrow \; 216$$

$$\frac{\int \frac{a+bx}{(a+bx)^2+c}d(a+bx)-\frac{a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}}{b^2}$$

$$\Big\downarrow \; 240$$

$$\frac{\frac{1}{2}\log\left((a+bx)^2+c\right)-\frac{a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}}{b^2}$$

input
```
Int[x/(c + (a + b*x)^2),x]
```

output
```
(-((a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c]) + Log[c + (a + b*x)^2]/2)/b^2
```

3.80.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

3.80.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{\ln\left(b^{2} x^{2}+2 a b x+a^{2}+c\right)}{2 b^{2}}-\frac{a \arctan\left(\frac{2 b^{2} x+2 a b}{2 \sqrt{c}\, b}\right)}{b^{2} \sqrt{c}}$	54
risch	$\frac{\ln\left(-\sqrt{-c}\, b x-a \sqrt{-c}-c\right) a \sqrt{-c}}{2 c\, b^{2}}+\frac{\ln\left(-\sqrt{-c}\, b x-a \sqrt{-c}-c\right)}{2 b^{2}}-\frac{\ln\left(\sqrt{-c}\, b x+a \sqrt{-c}-c\right) a \sqrt{-c}}{2 c\, b^{2}}+\frac{\ln\left(\sqrt{-c}\, b x+a \sqrt{-c}-c\right)}{2 b^{2}}$	124

input
```
int(x/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*ln(b^2*x^2+2*a*b*x+a^2+c)-a/b^2/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)
/c^(1/2)/b)
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.32

$$\int \frac{x}{c+(a+bx)^2}\,dx = \left[-\frac{a\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)-c\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2c}, -\frac{2\,a\sqrt{c}\arctan\left(\frac{bx+a}{\sqrt{c}}\right)-c\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2c}\right]$$

input
```
integrate(x/(c+(b*x+a)^2),x, algorithm="fricas")
```

output
```
[-1/2*(a*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)
/(b^2*x^2 + 2*a*b*x + a^2 + c)) - c*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^2
*c), -1/2*(2*a*sqrt(c)*arctan((b*x + a)/sqrt(c)) - c*log(b^2*x^2 + 2*a*b*x
 + a^2 + c))/(b^2*c)]
```

3.80.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(36) = 72.

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.02

$$\int \frac{x}{c+(a+bx)^2}\,dx = \left(-\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)\log\left(x+\frac{a^2-2b^2c\left(-\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)+c}{ab}\right)+\left(\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)\log\left(x+\frac{a^2-2b^2c\left(\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)+c}{ab}\right)$$

input
```
integrate(x/(c+(b*x+a)**2),x)
```

output
```
(-a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2))*log(x + (a**2 - 2*b**2*c*(-a*sqrt(-c
)/(2*b**2*c) + 1/(2*b**2)) + c)/(a*b)) + (a*sqrt(-c)/(2*b**2*c) + 1/(2*b**
2))*log(x + (a**2 - 2*b**2*c*(a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2)) + c)/(a*
b))
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \frac{x}{c+(a+bx)^2}\,dx = -\frac{a\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2}$$

input
```
integrate(x/(c+(b*x+a)^2),x, algorithm="maxima")
```

output
```
-a*arctan((b^2*x + a*b)/(b*sqrt(c)))/(b^2*sqrt(c)) + 1/2*log(b^2*x^2 + 2*a
*b*x + a^2 + c)/b^2
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \frac{x}{c+(a+bx)^2}\,dx = -\frac{a\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2}$$

input
```
integrate(x/(c+(b*x+a)^2),x, algorithm="giac")
```

output
```
-a*arctan((b*x + a)/sqrt(c))/(b^2*sqrt(c)) + 1/2*log(b^2*x^2 + 2*a*b*x + a
^2 + c)/b^2
```

3.80.9 Mupad [B] (verification not implemented)

Time = 9.99 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{x}{c+(a+bx)^2}\,dx = \frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+c\right)}{2\,b^2} - \frac{a\,\mathrm{atan}\left(\frac{a}{\sqrt{c}}+\frac{b\,x}{\sqrt{c}}\right)}{b^2\,\sqrt{c}}$$

input
```
int(x/(c + (a + b*x)^2),x)
```

output
```
log(c + a^2 + b^2*x^2 + 2*a*b*x)/(2*b^2) - (a*atan(a/c^(1/2) + (b*x)/c^(1/
2)))/(b^2*c^(1/2))
```

3.81 $\int \frac{1}{c+(a+bx)^2} dx$

3.81.1 Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \frac{1}{c+(a+bx)^2} \, dx = \frac{\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

output
```
arctan((b*x+a)/c^(1/2))/b/c^(1/2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{c+(a+bx)^2} \, dx = \frac{\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

input
```
Integrate[(c + (a + b*x)^2)^(-1),x]
```

output
```
ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

3.81.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {239, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^2+c}\,dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \frac{1}{(a+bx)^2+c}d(a+bx)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

input
```
Int[(c + (a + b*x)^2)^(-1),x]
```

output
```
ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

3.81.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

3.81.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

method	result	size
default	$\frac{\arctan\left(\frac{2b^2x+2ab}{2\sqrt{c}\,b}\right)}{\sqrt{c}\,b}$	28
risch	$-\frac{\ln(bx+\sqrt{-c}+a)}{2\sqrt{-c}\,b}+\frac{\ln(-bx+\sqrt{-c}-a)}{2\sqrt{-c}\,b}$	47

input
```
int(1/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
1/c^(1/2)/b*arctan(1/2*(2*b^2*x+2*a*b)/c^(1/2)/b)
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.95

$$\int \frac{1}{c+(a+bx)^2}\,dx = \left[-\frac{\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)}{2\,bc}, \frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}}\right]$$

input
```
integrate(1/(c+(b*x+a)^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b
^2*x^2 + 2*a*b*x + a^2 + c))/(b*c), arctan((b*x + a)/sqrt(c))/(b*sqrt(c))]
```

3.81.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(17) = 34$.

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.57

$$\int \frac{1}{c+(a+bx)^2}\,dx = \frac{-\frac{\sqrt{-\frac{1}{c}}\log\left(x+\frac{a-c\sqrt{-\frac{1}{c}}}{b}\right)}{2}+\frac{\sqrt{-\frac{1}{c}}\log\left(x+\frac{a+c\sqrt{-\frac{1}{c}}}{b}\right)}{2}}{b}$$

input
```
integrate(1/(c+(b*x+a)**2),x)
```

output
```
(-sqrt(-1/c)*log(x + (a - c*sqrt(-1/c))/b)/2 + sqrt(-1/c)*log(x + (a + c*s
qrt(-1/c))/b)/2)/b
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{1}{c+(a+bx)^2}\,dx = \frac{\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b\sqrt{c}}$$

input
```
integrate(1/(c+(b*x+a)^2),x, algorithm="maxima")
```

output
```
arctan((b^2*x + a*b)/(b*sqrt(c)))/(b*sqrt(c))
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{c+(a+bx)^2}\,dx = \frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}}$$

input
```
integrate(1/(c+(b*x+a)^2),x, algorithm="giac")
```

output
```
arctan((b*x + a)/sqrt(c))/(b*sqrt(c))
```

3.81.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{c+(a+bx)^2}\,dx = \frac{\mathrm{atan}\left(\frac{a+b\,x}{\sqrt{c}}\right)}{b\,\sqrt{c}}$$

input
```
int(1/(c + (a + b*x)^2),x)
```

output
```
atan((a + b*x)/c^(1/2))/(b*c^(1/2))
```

3.82 $\int \frac{1}{x(c+(a+bx)^2)}\,dx$

3.82.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx = -\frac{a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2+c)}+\frac{\log(x)}{a^2+c}-\frac{\log\left(c+(a+bx)^2\right)}{2\,(a^2+c)}$$

output
```
ln(x)/(a^2+c)-1/2*ln(c+(b*x+a)^2)/(a^2+c)-a*arctan((b*x+a)/c^(1/2))/(a^2+c
)/c^(1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx = -\frac{\frac{2a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}-2\log(bx)+\log\left(c+(a+bx)^2\right)}{2\,(a^2+c)}$$

input
```
Integrate[1/(x*(c + (a + b*x)^2)),x]
```

output
```
-1/2*((2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - 2*Log[b*x] + Log[c + (a +
b*x)^2])/(a^2 + c)
```

3.82.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {896, 25, 479, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left((a+bx)^2+c\right)}dx$$

$$\Big\downarrow \ 896$$

$$\int \frac{1}{bx\left((a+bx)^2+c\right)}d(a+bx)$$

$$\Big\downarrow \ 25$$

$$-\int -\frac{1}{bx\left((a+bx)^2+c\right)}d(a+bx)$$

$$\Big\downarrow \ 479$$

$$\frac{\log(-bx)}{a^2+c}-\frac{\int \frac{2a+bx}{(a+bx)^2+c}d(a+bx)}{a^2+c}$$

$$\Big\downarrow \ 452$$

$$\frac{\log(-bx)}{a^2+c}-\frac{a\int \frac{1}{(a+bx)^2+c}d(a+bx)+\int \frac{a+bx}{(a+bx)^2+c}d(a+bx)}{a^2+c}$$

$$\Big\downarrow \ 216$$

$$\frac{\log(-bx)}{a^2+c}-\frac{\int \frac{a+bx}{(a+bx)^2+c}d(a+bx)+\frac{a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}}{a^2+c}$$

$$\Big\downarrow \ 240$$

$$\frac{\log(-bx)}{a^2+c}-\frac{\frac{a\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{1}{2}\log\left((a+bx)^2+c\right)}{a^2+c}$$

input
```
Int[1/(x*(c + (a + b*x)^2)),x]
```

output
```
Log[-(b*x)]/(a^2 + c) - ((a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + Log[c + (
a + b*x)^2]/2)/(a^2 + c)
```

3.82.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

3.82.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.25

method	result	size
default	$\frac{\ln(x)}{a^2+c}-\frac{b\left(\frac{\ln\left(b^2x^2+2abx+a^2+c\right)}{2b}+\frac{a\arctan\left(\frac{2b^2x+2ab}{2\sqrt{c}\,b}\right)}{\sqrt{c}\,b}\right)}{a^2+c}$	74
risch	$\frac{\ln(x)}{a^2+c}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(c\,a^2+c^2\right)\_Z^2+2c\_Z\right)}\_R\ln\left(\left(\left(-a^2b+3bc\right)\_R+3b\right)x+\left(-a^3-ac\right)\_R+2a\right)\right)}{2}$	75

input
```
int(1/x/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/(a^2+c)-b/(a^2+c)*(1/2/b*ln(b^2*x^2+2*a*b*x+a^2+c)+a/c^(1/2)/b*arcta
n(1/2*(2*b^2*x+2*a*b)/c^(1/2)/b))
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.61

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx = \left[-\frac{a\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)+c\log\left(b^2x^2+2\,abx+a^2+c\right)-2\,c\log\left(x\right)}{2\left(a^2c+c^2\right)}, -\frac{2\,a\sqrt{c}\arctan\left(\frac{bx+a}{\sqrt{c}}\right)+c\log\left(b^2x^2+2\,abx+a^2+c\right)-2\,c\log\left(x\right)}{2\left(a^2c+c^2\right)}\right]$$

input
```
integrate(1/x/(c+(b*x+a)^2),x, algorithm="fricas")
```

output
```
[-1/2*(a*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)
/(b^2*x^2 + 2*a*b*x + a^2 + c)) + c*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*c
*log(x))/(a^2*c + c^2), -1/2*(2*a*sqrt(c)*arctan((b*x + a)/sqrt(c)) + c*lo
g(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*c*log(x))/(a^2*c + c^2)]
```

3.82.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(49) = 98$.

Time = 1.85 (sec) , antiderivative size = 738, normalized size of antiderivative = 12.51

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx = \left(-\frac{a\sqrt{-c}}{2c\,(a^2+c)} - \frac{1}{2\,(a^2+c)}\right)\log\left(x + \frac{-4a^6c\left(-\frac{a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\right)^2 + 4a^4c^2\left(-\frac{a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\right)^2 - 6a^4c\left(-\frac{a\sqrt{-c}}{2c(a^2+c)} - \cdots\right.}{\cdots}\right) + \left(\frac{a\sqrt{-c}}{2c\,(a^2+c)} - \frac{1}{2\,(a^2+c)}\right)\log\left(x + \frac{-4a^6c\left(\frac{a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\right)^2 + 4a^4c^2\left(\frac{a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\right)^2 - 6a^4c\left(\frac{a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\cdots\right.}{\cdots}\right) + \frac{\log\left(x + \frac{-\frac{4a^6c}{(a^2+c)^2} + \frac{4a^4c^2}{(a^2+c)^2} - \frac{6a^4c}{a^2+c} + \frac{20a^2c^3}{(a^2+c)^2} - \frac{12a^2c^2}{a^2+c} + 10a^2c + \frac{12c^4}{(a^2+c)^2} - \frac{6c^3}{a^2+c} - 6c^2}{a^3b+9abc}\right)}{a^2+c}$$

input
```
integrate(1/x/(c+(b*x+a)**2),x)
```

output
```
(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*log(x + (-4*a**6*c*(-a*s
qrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 + 4*a**4*c**2*(-a*sqrt(-c)
/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*a**4*c*(-a*sqrt(-c)/(2*c*(a**
2 + c)) - 1/(2*(a**2 + c))) + 20*a**2*c**3*(-a*sqrt(-c)/(2*c*(a**2 + c)) -
 1/(2*(a**2 + c)))**2 - 12*a**2*c**2*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*
(a**2 + c))) + 10*a**2*c + 12*c**4*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a
**2 + c)))**2 - 6*c**3*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) -
 6*c**2)/(a**3*b + 9*a*b*c)) + (a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 +
 c)))*log(x + (-4*a**6*c*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*
*2 + 4*a**4*c**2*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*a
**4*c*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) + 20*a**2*c**3*(a*s
qrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 12*a**2*c**2*(a*sqrt(-c)
/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) + 10*a**2*c + 12*c**4*(a*sqrt(-c)/(2
*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*c**3*(a*sqrt(-c)/(2*c*(a**2 + c)
) - 1/(2*(a**2 + c))) - 6*c**2)/(a**3*b + 9*a*b*c)) + log(x + (-4*a**6*c/(
a**2 + c)**2 + 4*a**4*c**2/(a**2 + c)**2 - 6*a**4*c/(a**2 + c) + 20*a**2*c
**3/(a**2 + c)**2 - 12*a**2*c**2/(a**2 + c) + 10*a**2*c + 12*c**4/(a**2 +
c)**2 - 6*c**3/(a**2 + c) - 6*c**2)/(a**3*b + 9*a*b*c))/(a**2 + c)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx = -\frac{a\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{(a^2+c)\sqrt{c}} - \frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,(a^2+c)} + \frac{\log(x)}{a^2+c}$$

input
```
integrate(1/x/(c+(b*x+a)^2),x, algorithm="maxima")
```

output
```
-a*arctan((b^2*x + a*b)/(b*sqrt(c)))/((a^2 + c)*sqrt(c)) - 1/2*log(b^2*x^2
 + 2*a*b*x + a^2 + c)/(a^2 + c) + log(x)/(a^2 + c)
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx = -\frac{a\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{(a^2+c)\sqrt{c}} - \frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,(a^2+c)} + \frac{\log(|x|)}{a^2+c}$$

input
```
integrate(1/x/(c+(b*x+a)^2),x, algorithm="giac")
```

output
```
-a*arctan((b*x + a)/sqrt(c))/((a^2 + c)*sqrt(c)) - 1/2*log(b^2*x^2 + 2*a*b
*x + a^2 + c)/(a^2 + c) + log(abs(x))/(a^2 + c)
```

3.82.9 Mupad [B] (verification not implemented)

Time = 10.58 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.93

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx = \frac{\ln(x)}{a^2+c} - \frac{\ln\left(2\,a\,b^3+3\,b^4\,x+\frac{b^3\,\left(c+a\,\sqrt{-c}\right)\,\left(a^3+b\,x\,a^2+c\,a-3\,b\,c\,x\right)}{c\,(a^2+c)}\right)\,\left(c+a\,\sqrt{-c}\right)}{2\,\left(a^2\,c+c^2\right)} - \frac{\ln\left(2\,a\,b^3+3\,b^4\,x+\frac{b^3\,\left(c-a\,\sqrt{-c}\right)\,\left(a^3+b\,x\,a^2+c\,a-3\,b\,c\,x\right)}{c\,(a^2+c)}\right)\,\left(c-a\,\sqrt{-c}\right)}{2\,\left(a^2\,c+c^2\right)}$$

input
```
int(1/(x*(c + (a + b*x)^2)),x)
```

output
```
log(x)/(c + a^2) - (log(2*a*b^3 + 3*b^4*x + (b^3*(c + a*(-c)^(1/2))*(a*c +
 a^3 - 3*b*c*x + a^2*b*x))/(c*(c + a^2)))*(c + a*(-c)^(1/2)))/(2*(a^2*c +
c^2)) - (log(2*a*b^3 + 3*b^4*x + (b^3*(c - a*(-c)^(1/2))*(a*c + a^3 - 3*b*
c*x + a^2*b*x))/(c*(c + a^2)))*(c - a*(-c)^(1/2)))/(2*(a^2*c + c^2))
```

3.82. $\int \frac{1}{x(c+(a+bx)^2)}\,dx$

3.83 $\int \frac{1}{x^2(c+(a+bx)^2)}\,dx$

3.83.1 Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{1}{x^2\left(c+(a+bx)^2\right)}\,dx=-\frac{1}{(a^2+c)\,x}+\frac{b(a^2-c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2+c)^2}-\frac{2ab\log(x)}{(a^2+c)^2}+\frac{ab\log\left(c+(a+bx)^2\right)}{(a^2+c)^2}$$

output
```
-1/(a^2+c)/x-2*a*b*ln(x)/(a^2+c)^2+a*b*ln(c+(b*x+a)^2)/(a^2+c)^2+b*(a^2-c)
*arctan((b*x+a)/c^(1/2))/(a^2+c)^2/c^(1/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2\left(c+(a+bx)^2\right)}\,dx=\frac{b(a^2-c)\,x\arctan\left(\frac{a+bx}{\sqrt{c}}\right)-\sqrt{c}(a^2+c+2abx\log(x)-abx\log\left(a^2+c+2abx+b^2x^2\right))}{\sqrt{c}\,(a^2+c)^2\,x}$$

input
```
Integrate[1/(x^2*(c + (a + b*x)^2)),x]
```

output
```
(b*(a^2 - c)*x*ArcTan[(a + b*x)/Sqrt[c]] - Sqrt[c]*(a^2 + c + 2*a*b*x*Log[
x] - a*b*x*Log[a^2 + c + 2*a*b*x + b^2*x^2]))/(Sqrt[c]*(a^2 + c)^2*x)
```

3.83.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {896, 480, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left((a+bx)^2+c\right)}dx$$

$$\Big\downarrow \text{896}$$

$$b\int \frac{1}{b^2x^2\left((a+bx)^2+c\right)}d(a+bx)$$

$$\Big\downarrow \text{480}$$

$$b\left(\frac{\int -\frac{2a+bx}{bx((a+bx)^2+c)}d(a+bx)}{a^2+c}-\frac{1}{bx\left(a^2+c\right)}\right)$$

$$\Big\downarrow \text{657}$$

$$b\left(\frac{\int \left(\frac{a^2+2(a+bx)a-c}{(a^2+c)((a+bx)^2+c)}-\frac{2a}{b(a^2+c)x}\right)d(a+bx)}{a^2+c}-\frac{1}{bx\left(a^2+c\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$b\left(\frac{\frac{(a^2-c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)}-\frac{2a\log(-bx)}{a^2+c}+\frac{a\log((a+bx)^2+c)}{a^2+c}}{a^2+c}-\frac{1}{bx\left(a^2+c\right)}\right)$$

input
```
Int[1/(x^2*(c + (a + b*x)^2)),x]
```

output
```
b*(-(1/(b*(a^2 + c)*x)) + (((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*
(a^2 + c)) - (2*a*Log[-(b*x)])/(a^2 + c) + (a*Log[c + (a + b*x)^2])/(a^2 +
 c))/(a^2 + c))
```

3.83.3.1 Defintions of rubi rules used

rule 480
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.83.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.22

method	result
default	$-\frac{1}{(a^2+c)x}-\frac{2ab\ln(x)}{(a^2+c)^2}+\frac{b^2\left(\frac{a\ln(b^2x^2+2abx+a^2+c)}{b}+\frac{(a^2-c)\arctan\left(\frac{2b^2x+2ab}{2\sqrt{c}\,b}\right)}{\sqrt{c}\,b}\right)}{(a^2+c)^2}$
risch	$-\frac{1}{(a^2+c)x}-\frac{2ab\ln(x)}{a^4+2c\,a^2+c^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((c\,a^4+2a^2c^2+c^3)\_Z^2-4abc\_Z+b^2\right)}\_R\ln\left(\left((-a^6b+a^4bc+5a^2b\,c^2+3b\,c^3)\_R^2+(\right.\right.\right.}{}$

input
```
int(1/x^2/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a^2+c)/x-2*a*b*ln(x)/(a^2+c)^2+b^2/(a^2+c)^2*(a/b*ln(b^2*x^2+2*a*b*x+a
^2+c)+(a^2-c)/c^(1/2)/b*arctan(1/2*(2*b^2*x+2*a*b)/c^(1/2)/b))
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.90

$$\int \frac{1}{x^2 (c + (a+bx)^2)} \, dx$$
$$= \left[\frac{2\,abcx \log \left(b^2x^2 + 2\,abx + a^2 + c\right) - 4\,abcx \log \left(x\right) + \left(a^2b - bc\right)\sqrt{-c}x \log \left(\frac{b^2x^2 + 2\,abx + a^2 + 2\,(bx+a)\sqrt{-c} - c}{b^2x^2 + 2\,abx + a^2 + c}\right) - }{2\,(a^4c + 2\,a^2c^2 + c^3)x} \right.$$

input
```
integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 4*a*b*c*x*log(x) + (a^2
*b - b*c)*sqrt(-c)*x*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) -
 c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) - 2*a^2*c - 2*c^2)/((a^4*c + 2*a^2*c^2
+ c^3)*x), (a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*a*b*c*x*log(x) +
(a^2*b - b*c)*sqrt(c)*x*arctan((b*x + a)/sqrt(c)) - a^2*c - c^2)/((a^4*c +
 2*a^2*c^2 + c^3)*x)]
```

3.83.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1620 vs. $2(73) = 146$.

Time = 6.88 (sec) , antiderivative size = 1620, normalized size of antiderivative = 20.51

$$\int \frac{1}{x^2 (c + (a+bx)^2)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x**2/(c+(b*x+a)**2),x)
```

output
```
-2*a*b*log(x + (-16*a**13*b**2*c/(a**2 + c)**4 + 48*a**11*b**2*c**2/(a**2
+ c)**4 + 352*a**9*b**2*c**3/(a**2 + c)**4 - 20*a**9*b**2*c/(a**2 + c)**2
+ 608*a**7*b**2*c**4/(a**2 + c)**4 - 64*a**7*b**2*c**2/(a**2 + c)**2 + 432
*a**5*b**2*c**5/(a**2 + c)**4 - 72*a**5*b**2*c**3/(a**2 + c)**2 + 36*a**5*
b**2*c + 112*a**3*b**2*c**6/(a**2 + c)**4 - 32*a**3*b**2*c**4/(a**2 + c)**
2 - 88*a**3*b**2*c**2 - 4*a*b**2*c**5/(a**2 + c)**2 + 4*a*b**2*c**3)/(a**6
*b**3 + 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3))/(a**2 + c)**2 + (
a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*
log(x + (-4*a**11*c*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4
+ 2*a**2*c + c**2)))**2 + 12*a**9*c**2*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a*
*2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 10*a**8*b*c*(a*b/(a**2 + c)**
2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 88*a**7*c**3*(
a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*
*2 + 32*a**6*b*c**2*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4
+ 2*a**2*c + c**2))) + 36*a**5*b**2*c + 152*a**5*c**4*(a*b/(a**2 + c)**2 -
 b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 36*a**4*b*c**3
*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))
) - 88*a**3*b**2*c**2 + 108*a**3*c**5*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**
2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 16*a**2*b*c**4*(a*b/(a**2 + c)
**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 4*a*b**2*...
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.56

$$\int \frac{1}{x^2\left(c+(a+bx)^2\right)}\,dx = \frac{ab\log\left(b^2x^2+2\,abx+a^2+c\right)}{a^4+2\,a^2c+c^2} - \frac{2\,ab\log\left(x\right)}{a^4+2\,a^2c+c^2} + \frac{\left(a^2b^2-b^2c\right)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{(a^4+2\,a^2c+c^2)b\sqrt{c}} - \frac{1}{(a^2+c)x}$$

input
```
integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="maxima")
```

output
```
a*b*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b*log(x)/
(a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*arctan((b^2*x + a*b)/(b*sqrt(c))
)/((a^4 + 2*a^2*c + c^2)*b*sqrt(c)) - 1/((a^2 + c)*x)
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^2 (c + (a + bx)^2)} \, dx = \frac{ab \log (b^2x^2 + 2\,abx + a^2 + c)}{a^4 + 2\,a^2c + c^2} - \frac{2\,ab \log (|x|)}{a^4 + 2\,a^2c + c^2} + \frac{(a^2b^2 - b^2c) \arctan \left(\frac{bx+a}{\sqrt{c}}\right)}{(a^4 + 2\,a^2c + c^2)b\sqrt{c}} - \frac{1}{(a^2 + c)x}$$

input
```
integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="giac")
```

output
```
a*b*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b*log(abs
(x))/(a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*arctan((b*x + a)/sqrt(c))/(
(a^4 + 2*a^2*c + c^2)*b*sqrt(c)) - 1/((a^2 + c)*x)
```

3.83.9 Mupad [B] (verification not implemented)

Time = 10.10 (sec) , antiderivative size = 425, normalized size of antiderivative = 5.38

$$\int \frac{1}{x^2 (c + (a + bx)^2)} \, dx = \frac{\ln \left((-c)^{13/2} - 35\,a^2\,(-c)^{11/2} + 34\,a^4\,(-c)^{9/2} + 34\,a^6\,(-c)^{7/2} - 35\,a^8\,(-c)^{5/2} + a^{10}\,(-c)^{3/2} + a\,c^6 - a^{11}\,c + \right.}{} - \frac{1}{x\,(a^2 + c)} - \frac{\ln \left((-c)^{13/2} - 35\,a^2\,(-c)^{11/2} + 34\,a^4\,(-c)^{9/2} + 34\,a^6\,(-c)^{7/2} - 35\,a^8\,(-c)^{5/2} + a^{10}\,(-c)^{3/2} - a\,c^6 + a^{11}\right.}{} - \frac{2\,a\,b\,\ln (x)}{(a^2 + c)^2}$$

input
```
int(1/(x^2*(c + (a + b*x)^2)),x)
```

output
```
(log((-c)^(13/2) - 35*a^2*(-c)^(11/2) + 34*a^4*(-c)^(9/2) + 34*a^6*(-c)^(7
/2) - 35*a^8*(-c)^(5/2) + a^10*(-c)^(3/2) + a*c^6 - a^11*c + 35*a^3*c^5 +
34*a^5*c^4 - 34*a^7*c^3 - 35*a^9*c^2 + b*c^6*x - a^10*b*c*x + 35*a^2*b*c^5
*x + 34*a^4*b*c^4*x - 34*a^6*b*c^3*x - 35*a^8*b*c^2*x)*(b*(-c)^(3/2) + 2*a
*b*c + a^2*b*(-c)^(1/2)))/(2*(a^4*c + c^3 + 2*a^2*c^2)) - 1/(x*(c + a^2))
- (log((-c)^(13/2) - 35*a^2*(-c)^(11/2) + 34*a^4*(-c)^(9/2) + 34*a^6*(-c)^
(7/2) - 35*a^8*(-c)^(5/2) + a^10*(-c)^(3/2) - a*c^6 + a^11*c - 35*a^3*c^5
- 34*a^5*c^4 + 34*a^7*c^3 + 35*a^9*c^2 - b*c^6*x + a^10*b*c*x - 35*a^2*b*c
^5*x - 34*a^4*b*c^4*x + 34*a^6*b*c^3*x + 35*a^8*b*c^2*x)*(b*(-c)^(3/2) - 2
*a*b*c + a^2*b*(-c)^(1/2)))/(2*(a^4*c + c^3 + 2*a^2*c^2)) - (2*a*b*log(x))
/(c + a^2)^2
```

3.83. $\int \frac{1}{x^2(c+(a+bx)^2)}\,dx$

3.84 $\int \frac{1}{x^3(c+(a+bx)^2)}\,dx$

3.84.1 Optimal result

Integrand size = 15, antiderivative size = 121

$$\int \frac{1}{x^3(c+(a+bx)^2)}\,dx = -\frac{1}{2(a^2+c)x^2}+\frac{2ab}{(a^2+c)^2x}-\frac{ab^2(a^2-3c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)^3}+\frac{b^2(3a^2-c)\log(x)}{(a^2+c)^3}-\frac{b^2(3a^2-c)\log(c+(a+bx)^2)}{2(a^2+c)^3}$$

output
```
-1/2/(a^2+c)/x^2+2*a*b/(a^2+c)^2/x+b^2*(3*a^2-c)*ln(x)/(a^2+c)^3-1/2*b^2*(
3*a^2-c)*ln(c+(b*x+a)^2)/(a^2+c)^3-a*b^2*(a^2-3*c)*arctan((b*x+a)/c^(1/2))
/(a^2+c)^3/c^(1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3(c+(a+bx)^2)}\,dx = -\frac{\frac{(a^2+c)(a^2+c-4abx)}{x^2}+\frac{2ab^2(a^2-3c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+2b^2(-3a^2+c)\log(x)+b^2(3a^2-c)\log(a^2+c+2abx+b^2x}{2(a^2+c)^3}$$

input
```
Integrate[1/(x^3*(c + (a + b*x)^2)),x]
```

output
```
-1/2*(((a^2 + c)*(a^2 + c - 4*a*b*x))/x^2 + (2*a*b^2*(a^2 - 3*c)*ArcTan[(a
 + b*x)/Sqrt[c]])/Sqrt[c] + 2*b^2*(-3*a^2 + c)*Log[x] + b^2*(3*a^2 - c)*Lo
g[a^2 + c + 2*a*b*x + b^2*x^2])/(a^2 + c)^3
```

3.84.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {896, 25, 480, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left((a+bx)^2+c\right)}dx$$

$$\Big\downarrow \text{ 896}$$

$$b^2\int \frac{1}{b^3x^3\left((a+bx)^2+c\right)}d(a+bx)$$

$$\Big\downarrow \text{ 25}$$

$$-b^2\int -\frac{1}{b^3x^3\left((a+bx)^2+c\right)}d(a+bx)$$

$$\Big\downarrow \text{ 480}$$

$$b^2\left(-\frac{\int \frac{2a+bx}{b^2x^2((a+bx)^2+c)}d(a+bx)}{a^2+c}-\frac{1}{2b^2x^2\left(a^2+c\right)}\right)$$

$$\Big\downarrow \text{ 657}$$

$$b^2\left(-\frac{\int \left(\frac{2a}{b^2(a^2+c)x^2}-\frac{3a^2-c}{b(a^2+c)^2x}+\frac{a(a^2-3c)+(3a^2-c)(a+bx)}{(a^2+c)^2((a+bx)^2+c)}\right)d(a+bx)}{a^2+c}-\frac{1}{2b^2x^2\left(a^2+c\right)}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$b^2\left(-\frac{\frac{a(a^2-3c)\arctan\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)^2}-\frac{2a}{bx(a^2+c)}-\frac{(3a^2-c)\log(-bx)}{(a^2+c)^2}+\frac{(3a^2-c)\log((a+bx)^2+c)}{2(a^2+c)^2}}{a^2+c}-\frac{1}{2b^2x^2\left(a^2+c\right)}\right)$$

input
```
Int[1/(x^3*(c + (a + b*x)^2)),x]
```

output
```
b^2*(-1/2*1/(b^2*(a^2 + c)*x^2) - ((-2*a)/(b*(a^2 + c)*x) + (a*(a^2 - 3*c)
*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c)^2) - ((3*a^2 - c)*Log[-(b*x
)])/(a^2 + c)^2 + ((3*a^2 - c)*Log[c + (a + b*x)^2])/(2*(a^2 + c)^2))/(a^2
 + c))
```

3.84.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 480
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.84.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.25

method	result
default	$-\frac{1}{2(a^2+c)x^2}+\frac{b^2(3a^2-c)\ln(x)}{(a^2+c)^3}+\frac{2ab}{(a^2+c)^2x}-\frac{b^3\left(\frac{(3a^2b-bc)\ln(b^2x^2+2abx+a^2+c)}{2b^2}+\frac{\left(4a^3-4ac-\frac{(3a^2b-bc)a}{b}\right)\arctan\left(\frac{2b^2x+2ab}{2\sqrt{c}\,b}\right)}{\sqrt{c}\,b}\right)}{(a^2+c)^3}$
risch	$\frac{\frac{2abx}{a^4+2ca^2+c^2}-\frac{1}{2(a^2+c)}}{x^2}+\frac{3b^2\ln(x)a^2}{a^6+3ca^4+3a^2c^2+c^3}-\frac{b^2\ln(x)c}{a^6+3ca^4+3a^2c^2+c^3}+\left(\sum_{\_R=\mathrm{RootOf}\left((ca^6+3a^4c^2+3a^2c^3+c^4)\_Z^2+(6a^2b^2c-\cdots\right.}\right.$

input
```
int(1/x^3/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a^2+c)/x^2+b^2*(3*a^2-c)*ln(x)/(a^2+c)^3+2*a*b/(a^2+c)^2/x-b^3/(a^2+
c)^3*(1/2*(3*a^2*b-b*c)/b^2*ln(b^2*x^2+2*a*b*x+a^2+c)+(4*a^3-4*a*c-(3*a^2*
b-b*c)*a/b)/c^(1/2)/b*arctan(1/2*(2*b^2*x+2*a*b)/c^(1/2)/b))
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 371, normalized size of antiderivative = 3.07

$$\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx = \left[-\frac{a^4c-(a^3b^2-3\,ab^2c)\sqrt{-c}x^2\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)+2\,a^2c^2+(3\,a^2b^2c-b^2c^2)x^2\log\left(b^2x^2+\cdots\right.}{2\,(a^6c+3\,a^4c^2+3\,a^2c^3+c^4)x^2}\right.$$
$$-\frac{a^4c+2\,(a^3b^2-3\,ab^2c)\sqrt{c}x^2\arctan\left(\frac{bx+a}{\sqrt{c}}\right)+2\,a^2c^2+(3\,a^2b^2c-b^2c^2)x^2\log\left(b^2x^2+2\,abx+a^2+c\right)-\cdots}{2\,(a^6c+3\,a^4c^2+3\,a^2c^3+c^4)x^2}$$

input
```
integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="fricas")
```

output
```
[-1/2*(a^4*c - (a^3*b^2 - 3*a*b^2*c)*sqrt(-c)*x^2*log((b^2*x^2 + 2*a*b*x +
 a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + 2*a^2*c^
2 + (3*a^2*b^2*c - b^2*c^2)*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^
2*b^2*c - b^2*c^2)*x^2*log(x) + c^3 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3
*a^4*c^2 + 3*a^2*c^3 + c^4)*x^2), -1/2*(a^4*c + 2*(a^3*b^2 - 3*a*b^2*c)*sq
rt(c)*x^2*arctan((b*x + a)/sqrt(c)) + 2*a^2*c^2 + (3*a^2*b^2*c - b^2*c^2)*
x^2*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^2*b^2*c - b^2*c^2)*x^2*log(x
) + c^3 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3*a^4*c^2 + 3*a^2*c^3 + c^4)*
x^2)]
```

3.84.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3284 vs. $2(109) = 218$.

Time = 42.25 (sec) , antiderivative size = 3284, normalized size of antiderivative = 27.14

$$\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**3/(c+(b*x+a)**2),x)
```

output
```
b**2*(3*a**2 - c)*log(x + (-4*a**16*b**4*c*(3*a**2 - c)**2/(a**2 + c)**6 +
 24*a**14*b**4*c**2*(3*a**2 - c)**2/(a**2 + c)**6 + 216*a**12*b**4*c**3*(3
*a**2 - c)**2/(a**2 + c)**6 - 14*a**12*b**4*c*(3*a**2 - c)/(a**2 + c)**3 +
 568*a**10*b**4*c**4*(3*a**2 - c)**2/(a**2 + c)**6 - 44*a**10*b**4*c**2*(3
*a**2 - c)/(a**2 + c)**3 + 720*a**8*b**4*c**5*(3*a**2 - c)**2/(a**2 + c)**
6 - 42*a**8*b**4*c**3*(3*a**2 - c)/(a**2 + c)**3 + 78*a**8*b**4*c + 456*a*
*6*b**4*c**6*(3*a**2 - c)**2/(a**2 + c)**6 - 8*a**6*b**4*c**4*(3*a**2 - c)
/(a**2 + c)**3 - 464*a**6*b**4*c**2 + 104*a**4*b**4*c**7*(3*a**2 - c)**2/(
a**2 + c)**6 - 2*a**4*b**4*c**5*(3*a**2 - c)/(a**2 + c)**3 + 380*a**4*b**4
*c**3 - 24*a**2*b**4*c**8*(3*a**2 - c)**2/(a**2 + c)**6 - 12*a**2*b**4*c**
6*(3*a**2 - c)/(a**2 + c)**3 - 96*a**2*b**4*c**4 - 12*b**4*c**9*(3*a**2 -
c)**2/(a**2 + c)**6 - 6*b**4*c**7*(3*a**2 - c)/(a**2 + c)**3 + 6*b**4*c**5
)/(a**9*b**5 + 72*a**7*b**5*c - 270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 -
27*a*b**5*c**4))/(a**2 + c)**3 + (-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6
 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))*
log(x + (-4*a**16*c*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 24*a**14
*c**2*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 +
 c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 14*a**12*b**2*c*(-a*b*
*2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) -...
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.63

$$\int \frac{1}{x^3 (c+(a+bx)^2)}\,dx = -\frac{(3\,a^2b^2-b^2c)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,(a^6+3\,a^4c+3\,a^2c^2+c^3)} + \frac{(3\,a^2b^2-b^2c)\log(x)}{a^6+3\,a^4c+3\,a^2c^2+c^3} - \frac{(a^3b^3-3\,ab^3c)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{(a^6+3\,a^4c+3\,a^2c^2+c^3)b\sqrt{c}} + \frac{4\,abx-a^2-c}{2\,(a^4+2\,a^2c+c^2)x^2}$$

input
```
integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="maxima")
```

output
```
-1/2*(3*a^2*b^2 - b^2*c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3*a^4*c +
 3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*log(x)/(a^6 + 3*a^4*c + 3*a^2*c^2
+ c^3) - (a^3*b^3 - 3*a*b^3*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/((a^6 + 3
*a^4*c + 3*a^2*c^2 + c^3)*b*sqrt(c)) + 1/2*(4*a*b*x - a^2 - c)/((a^4 + 2*a
^2*c + c^2)*x^2)
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.61

$$\int \frac{1}{x^3 (c+(a+bx)^2)}\,dx = -\frac{(3\,a^2b^2-b^2c)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,(a^6+3\,a^4c+3\,a^2c^2+c^3)} + \frac{(3\,a^2b^2-b^2c)\log(|x|)}{a^6+3\,a^4c+3\,a^2c^2+c^3} - \frac{(a^3b^3-3\,ab^3c)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{(a^6+3\,a^4c+3\,a^2c^2+c^3)b\sqrt{c}} - \frac{a^4+2\,a^2c+c^2-4\,(a^3b+abc)x}{2\,(a^2+c)^3x^2}$$

input
```
integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="giac")
```

output
```
-1/2*(3*a^2*b^2 - b^2*c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3*a^4*c +
 3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*log(abs(x))/(a^6 + 3*a^4*c + 3*a^2
*c^2 + c^3) - (a^3*b^3 - 3*a*b^3*c)*arctan((b*x + a)/sqrt(c))/((a^6 + 3*a^
4*c + 3*a^2*c^2 + c^3)*b*sqrt(c)) - 1/2*(a^4 + 2*a^2*c + c^2 - 4*(a^3*b +
a*b*c)*x)/((a^2 + c)^3*x^2)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 10.43 (sec) , antiderivative size = 573, normalized size of antiderivative = 4.74

$$\int \frac{1}{x^3 (c+(a+bx)^2)} dx = \ln(x) \left(\frac{3 b^2}{(a^2+c)^2} - \frac{4 b^2 c}{(a^2+c)^3}\right) - \frac{\frac{1}{2(a^2+c)} - \frac{2abx}{(a^2+c)^2}}{x^2} - \frac{\ln\left(27(-c)^{15/2} + 90a^2(-c)^{13/2} + 9a^4(-c)^{11/2} - 324a^6(-c)^{9/2} + 125a^8(-c)^{7/2} + 74a^{10}(-c)^{5/2} - a^{12}\right.}{} + \frac{\ln\left(27(-c)^{15/2} + 90a^2(-c)^{13/2} + 9a^4(-c)^{11/2} - 324a^6(-c)^{9/2} + 125a^8(-c)^{7/2} + 74a^{10}(-c)^{5/2} - a^{12}\right.}{}$$

input
```
int(1/(x^3*(c + (a + b*x)^2)),x)
```

output
```
log(x)*((3*b^2)/(c + a^2)^2 - (4*b^2*c)/(c + a^2)^3) - (1/(2*(c + a^2)) -
(2*a*b*x)/(c + a^2)^2)/x^2 - (log(27*(-c)^(15/2) + 90*a^2*(-c)^(13/2) + 9*
a^4*(-c)^(11/2) - 324*a^6*(-c)^(9/2) + 125*a^8*(-c)^(7/2) + 74*a^10*(-c)^(
5/2) - a^12*(-c)^(3/2) - 27*a*c^7 + a^13*c + 90*a^3*c^6 - 9*a^5*c^5 - 324*
a^7*c^4 - 125*a^9*c^3 + 74*a^11*c^2 - 27*b*c^7*x + a^12*b*c*x + 90*a^2*b*c
^6*x - 9*a^4*b*c^5*x - 324*a^6*b*c^4*x - 125*a^8*b*c^3*x + 74*a^10*b*c^2*x
)*(a^3*b^2*(-c)^(1/2) - b^2*c^2 + 3*a^2*b^2*c + 3*a*b^2*(-c)^(3/2)))/(2*(a
^6*c + c^4 + 3*a^2*c^3 + 3*a^4*c^2)) + (log(27*(-c)^(15/2) + 90*a^2*(-c)^(
13/2) + 9*a^4*(-c)^(11/2) - 324*a^6*(-c)^(9/2) + 125*a^8*(-c)^(7/2) + 74*a
^10*(-c)^(5/2) - a^12*(-c)^(3/2) + 27*a*c^7 - a^13*c - 90*a^3*c^6 + 9*a^5*
c^5 + 324*a^7*c^4 + 125*a^9*c^3 - 74*a^11*c^2 + 27*b*c^7*x - a^12*b*c*x -
90*a^2*b*c^6*x + 9*a^4*b*c^5*x + 324*a^6*b*c^4*x + 125*a^8*b*c^3*x - 74*a^
10*b*c^2*x)*(b^2*c^2 + a^3*b^2*(-c)^(1/2) - 3*a^2*b^2*c + 3*a*b^2*(-c)^(3/
2)))/(2*(a^6*c + c^4 + 3*a^2*c^3 + 3*a^4*c^2))
```

3.85 $\int \frac{1}{a+b(c+dx)^2}\,dx$

3.85.1 Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{1}{a+b(c+dx)^2}\,dx=\frac{\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

output
```
arctan((d*x+c)*b^(1/2)/a^(1/2))/d/a^(1/2)/b^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b(c+dx)^2}\,dx=\frac{\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

input
```
Integrate[(a + b*(c + d*x)^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

3.85.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {239, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b(c+dx)^2} \, dx$$
$$\Big\downarrow \text{239}$$
$$\frac{\int \frac{1}{b(c+dx)^2+a}d(c+dx)}{d}$$
$$\Big\downarrow \text{218}$$
$$\frac{\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

input
```
Int[(a + b*(c + d*x)^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

3.85.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

3.85.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\arctan\left(\frac{2b\,d^2x+2bcd}{2d\sqrt{ab}}\right)}{d\sqrt{ab}}$	34
risch	$-\frac{\ln\left(bdx+bc+\sqrt{-ab}\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(-bdx-bc+\sqrt{-ab}\right)}{2\sqrt{-ab}\,d}$	56

input
```
int(1/(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(a*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(a*b)^(1/2))
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.52

$$\int \frac{1}{a+b(c+dx)^2}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{bd^2x^2+2\,bcdx+bc^2-2\,\sqrt{-ab}(dx+c)-a}{bd^2x^2+2\,bcdx+bc^2+a}\right)}{2\,abd}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}(dx+c)}{a}\right)}{abd}\right]$$

input
```
integrate(1/(a+b*(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x +
c) - a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(a*b*d), sqrt(a*b)*arctan(sqr
t(a*b)*(d*x + c)/a)/(a*b*d)]
```

3.85.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(27) = 54$.

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \frac{1}{a+b(c+dx)^2}\,dx = \frac{-\frac{\sqrt{-\frac{1}{ab}}\log\left(x+\frac{-a\sqrt{-\frac{1}{ab}}+c}{d}\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(x+\frac{a\sqrt{-\frac{1}{ab}}+c}{d}\right)}{2}}{d}$$

input
```
integrate(1/(a+b*(d*x+c)**2),x)
```

output
```
(-sqrt(-1/(a*b))*log(x + (-a*sqrt(-1/(a*b)) + c)/d)/2 + sqrt(-1/(a*b))*log
(x + (a*sqrt(-1/(a*b)) + c)/d)/2)/d
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+b(c+dx)^2}\,dx = \frac{\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}d}\right)}{\sqrt{ab}d}$$

input
```
integrate(1/(a+b*(d*x+c)^2),x, algorithm="maxima")
```

output
```
arctan((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*d)
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{a+b(c+dx)^2}\,dx = \frac{\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{\sqrt{ab}d}$$

input
```
integrate(1/(a+b*(d*x+c)^2),x, algorithm="giac")
```

output
```
arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*d)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{a+b(c+dx)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,c+\sqrt{b}\,d\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

input
```
int(1/(a + b*(c + d*x)^2),x)
```

output
```
atan((b^(1/2)*c + b^(1/2)*d*x)/a^(1/2))/(a^(1/2)*b^(1/2)*d)
```

3.86 $\int \frac{1}{(a+b(c+dx)^2)^2} dx$

3.86.1 Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{1}{(a+b(c+dx)^2)^2} dx = \frac{c+dx}{2ad(a+b(c+dx)^2)} + \frac{\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d}$$

output
```
1/2*(d*x+c)/a/d/(a+b*(d*x+c)^2)+1/2*arctan((d*x+c)*b^(1/2)/a^(1/2))/a^(3/2
)/d/b^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+b(c+dx)^2)^2} dx = \frac{\frac{\sqrt{a}(c+dx)}{a+b(c+dx)^2} + \frac{\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{2a^{3/2}d}$$

input
```
Integrate[(a + b*(c + d*x)^2)^(-2),x]
```

output
```
((Sqrt[a]*(c + d*x))/(a + b*(c + d*x)^2) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt
[a]]/Sqrt[b])/(2*a^(3/2)*d)
```

3.86.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {239, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b(c+dx)^2)^2}\,dx$$
$$\Big\downarrow \ 239$$
$$\frac{\int \frac{1}{(b(c+dx)^2+a)^2}d(c+dx)}{d}$$
$$\Big\downarrow \ 215$$
$$\frac{\frac{\int \frac{1}{b(c+dx)^2+a}d(c+dx)}{2a}+\frac{c+dx}{2a(a+b(c+dx)^2)}}{d}$$
$$\Big\downarrow \ 218$$
$$\frac{\frac{\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{c+dx}{2a(a+b(c+dx)^2)}}{d}$$

input
```
Int[(a + b*(c + d*x)^2)^(-2),x]
```

output
```
((c + d*x)/(2*a*(a + b*(c + d*x)^2)) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]
/(2*a^(3/2)*Sqrt[b]))/d
```

3.86.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

3.86.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

method	result	size
default	$\frac{2b\,d^{2}x+2bcd}{4ab\,d^{2}\left(b\,d^{2}x^{2}+2bcdx+b\,c^{2}+a\right)}+\frac{\arctan\left(\frac{2b\,d^{2}x+2bcd}{2d\sqrt{ab}}\right)}{2da\sqrt{ab}}$	86
risch	$\frac{\frac{x}{2a}+\frac{c}{2da}}{b\,d^{2}x^{2}+2bcdx+b\,c^{2}+a}-\frac{\ln\left(bdx+bc+\sqrt{-ab}\right)}{4\sqrt{-ab}\,da}+\frac{\ln\left(-bdx-bc+\sqrt{-ab}\right)}{4\sqrt{-ab}\,da}$	102

input
```
int(1/(a+b*(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*b*d^2*x+2*b*c*d)/a/b/d^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)+1/2/d/a/(a*b
)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(a*b)^(1/2))
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 4.02

$$\int \frac{1}{(a+b(c+dx)^2)^2}\,dx$$
$$= \left[\frac{2\,abdx+2\,abc-(bd^2x^2+2\,bcdx+bc^2+a)\sqrt{-ab}\log\left(\frac{bd^2x^2+2\,bcdx+bc^2-2\,\sqrt{-ab}(dx+c)-a}{bd^2x^2+2\,bcdx+bc^2+a}\right)}{4\,(a^2b^2d^3x^2+2\,a^2b^2cd^2x+(a^2b^2c^2+a^3b)d)},\frac{abdx+abc+(bd}{2\,(a^2b^2}\right.$$

input
```
integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a*b*d*x + 2*a*b*c - (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*sqrt(-a*b)
*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) - a)/(b*d^2*x
^2 + 2*b*c*d*x + b*c^2 + a)))/(a^2*b^2*d^3*x^2 + 2*a^2*b^2*c*d^2*x + (a^2*
b^2*c^2 + a^3*b)*d), 1/2*(a*b*d*x + a*b*c + (b*d^2*x^2 + 2*b*c*d*x + b*c^2
 + a)*sqrt(a*b)*arctan(sqrt(a*b)*(d*x + c)/a))/(a^2*b^2*d^3*x^2 + 2*a^2*b^
2*c*d^2*x + (a^2*b^2*c^2 + a^3*b)*d)]
```

3.86.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(51) = 102.

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a+b(c+dx)^2)^2}\,dx = \frac{c+dx}{2a^2d+2abc^2d+4abcd^2x+2abd^3x^2} + \frac{-\frac{\sqrt{-\frac{1}{a^3b}}\log\left(x+\frac{-a^2\sqrt{-\frac{1}{a^3b}}+c}{d}\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b}}\log\left(x+\frac{a^2\sqrt{-\frac{1}{a^3b}}+c}{d}\right)}{4}}{d}$$

input
```
integrate(1/(a+b*(d*x+c)**2)**2,x)
```

output
```
(c + d*x)/(2*a**2*d + 2*a*b*c**2*d + 4*a*b*c*d**2*x + 2*a*b*d**3*x**2) + (
-sqrt(-1/(a**3*b))*log(x + (-a**2*sqrt(-1/(a**3*b)) + c)/d)/4 + sqrt(-1/(a
**3*b))*log(x + (a**2*sqrt(-1/(a**3*b)) + c)/d)/4)/d
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int \frac{1}{(a+b(c+dx)^2)^2}\,dx = \frac{dx+c}{2\,(abd^3x^2+2\,abcd^2x+(abc^2+a^2)d)} + \frac{\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}d}\right)}{2\,\sqrt{ab}ad}$$

input
```
integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(d*x + c)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (a*b*c^2 + a^2)*d) + 1/2*arct
an((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*a*d)
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a+b(c+dx)^2)^2}\,dx = \frac{\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{2\sqrt{ab}ad} + \frac{dx+c}{2\,(bd^2x^2+2\,bcdx+bc^2+a)ad}$$

input
```
integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*a*d) + 1/2*(d*x + c)/((b*d^
2*x^2 + 2*b*c*d*x + b*c^2 + a)*a*d)
```

3.86.9 Mupad [B] (verification not implemented)

Time = 9.91 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+b(c+dx)^2)^2}\,dx = \frac{\frac{x}{2\,a}+\frac{c}{2\,a\,d}}{b\,c^2+2\,b\,c\,d\,x+b\,d^2\,x^2+a} + \frac{\mathrm{atan}\left(2\,a\,\left(\frac{\sqrt{b}\,c}{2\,a^{3/2}}+\frac{\sqrt{b}\,d\,x}{2\,a^{3/2}}\right)\right)}{2\,a^{3/2}\,\sqrt{b}\,d}$$

input
```
int(1/(a + b*(c + d*x)^2)^2,x)
```

output
```
(x/(2*a) + c/(2*a*d))/(a + b*c^2 + b*d^2*x^2 + 2*b*c*d*x) + atan(2*a*((b^(
1/2)*c)/(2*a^(3/2)) + (b^(1/2)*d*x)/(2*a^(3/2))))/(2*a^(3/2)*b^(1/2)*d)
```

3.87 $\int \frac{1}{(a+b(c+dx)^2)^3} dx$

3.87.1 Optimal result

Integrand size = 13, antiderivative size = 91

$$\int \frac{1}{(a+b(c+dx)^2)^3} dx = \frac{c+dx}{4ad(a+b(c+dx)^2)^2} + \frac{3(c+dx)}{8a^2d(a+b(c+dx)^2)} + \frac{3\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d}$$

output
```
1/4*(d*x+c)/a/d/(a+b*(d*x+c)^2)^2+3/8*(d*x+c)/a^2/d/(a+b*(d*x+c)^2)+3/8*ar
ctan((d*x+c)*b^(1/2)/a^(1/2))/a^(5/2)/d/b^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a+b(c+dx)^2)^3} dx = \frac{\frac{\sqrt{a}(c+dx)(5a+3b(c+dx)^2)}{(a+b(c+dx)^2)^2} + \frac{3\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{8a^{5/2}d}$$

input
```
Integrate[(a + b*(c + d*x)^2)^(-3),x]
```

output
```
((Sqrt[a]*(c + d*x)*(5*a + 3*b*(c + d*x)^2))/(a + b*(c + d*x)^2)^2 + (3*Ar
cTan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/Sqrt[b])/(8*a^(5/2)*d)
```

3.87.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {239, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b(c+dx)^2)^3}\,dx$$

$$\Big\downarrow \text{ 239}$$

$$\frac{\int \frac{1}{(b(c+dx)^2+a)^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\frac{3\int \frac{1}{\left(b(c+dx)^2+a\right)^2}d(c+dx)}{4a}+\frac{c+dx}{4a(a+b(c+dx)^2)^2}}{d}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\frac{3\left(\frac{\int \frac{1}{b(c+dx)^2+a}d(c+dx)}{2a}+\frac{c+dx}{2a\left(a+b(c+dx)^2\right)}\right)}{4a}+\frac{c+dx}{4a(a+b(c+dx)^2)^2}}{d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{c+dx}{2a\left(a+b(c+dx)^2\right)}\right)}{4a}+\frac{c+dx}{4a(a+b(c+dx)^2)^2}}{d}$$

input
```
Int[(a + b*(c + d*x)^2)^(-3),x]
```

output
```
((c + d*x)/(4*a*(a + b*(c + d*x)^2)^2) + (3*((c + d*x)/(2*a*(a + b*(c + d*
x)^2)) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a))/
d
```

3.87.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

3.87.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.53

method	result	size
default	$\frac{2b\,d^2x+2bcd}{8ab\,d^2(b\,d^2x^2+2bcdx+b\,c^2+a)^2}+\frac{\frac{3\left(2b\,d^2x+2bcd\right)}{16ab\,d^2\left(b\,d^2x^2+2bcdx+b\,c^2+a\right)}+\frac{3\arctan\left(\frac{2b\,d^2x+2bcd}{2d\sqrt{ab}}\right)}{8da\sqrt{ab}}}{a}$	139
risch	$\frac{\frac{3b\,d^2x^3}{8a^2}+\frac{9c\,x^2bd}{8a^2}+\frac{\left(9b\,c^2+5a\right)x}{8a^2}+\frac{c\left(3b\,c^2+5a\right)}{8d\,a^2}}{\left(b\,d^2x^2+2bcdx+b\,c^2+a\right)^2}-\frac{3\ln\left(bdx+bc+\sqrt{-ab}\right)}{16\sqrt{-ab}\,d\,a^2}+\frac{3\ln\left(-bdx-bc+\sqrt{-ab}\right)}{16\sqrt{-ab}\,d\,a^2}$	145

input
```
int(1/(a+b*(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*b*d^2*x+2*b*c*d)/a/b/d^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)^2+3/4/a*(1/4
*(2*b*d^2*x+2*b*c*d)/a/b/d^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)+1/2/d/a/(a*b)^(
1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(a*b)^(1/2)))
```

3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(77) = 154.

Time = 0.28 (sec) , antiderivative size = 595, normalized size of antiderivative = 6.54

$$\int \frac{1}{(a+b(c+dx)^2)^3}\,dx$$
$$= \left[\frac{6\,ab^2d^3x^3 + 18\,ab^2cd^2x^2 + 6\,ab^2c^3 + 10\,a^2bc + 2\,(9\,ab^2c^2 + 5\,a^2b)dx - 3\,(b^2d^4x^4 + 4\,b^2cd^3x^3 + b^2c^4 + 2\,(3}{16\,(a^3b^3d^5x^4 + 4\,a^3b^3cd^4x^3 + 2\,(3\,a^3b^3c^2 + a^4b^2)d^3x^2 + 4\,(a^3}\right.$$

input
```
integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(6*a*b^2*d^3*x^3 + 18*a*b^2*c*d^2*x^2 + 6*a*b^2*c^3 + 10*a^2*b*c + 2
*(9*a*b^2*c^2 + 5*a^2*b)*d*x - 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + b^2*c^4
+ 2*(3*b^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c^2 + 4*(b^2*c^3 + a*b*c)*d*x + a^2)
*sqrt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) -
a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)))/(a^3*b^3*d^5*x^4 + 4*a^3*b^3*c*d^
4*x^3 + 2*(3*a^3*b^3*c^2 + a^4*b^2)*d^3*x^2 + 4*(a^3*b^3*c^3 + a^4*b^2*c)*
d^2*x + (a^3*b^3*c^4 + 2*a^4*b^2*c^2 + a^5*b)*d), 1/8*(3*a*b^2*d^3*x^3 + 9
*a*b^2*c*d^2*x^2 + 3*a*b^2*c^3 + 5*a^2*b*c + (9*a*b^2*c^2 + 5*a^2*b)*d*x +
 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + b^2*c^4 + 2*(3*b^2*c^2 + a*b)*d^2*x^2
+ 2*a*b*c^2 + 4*(b^2*c^3 + a*b*c)*d*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)*(d
*x + c)/a))/(a^3*b^3*d^5*x^4 + 4*a^3*b^3*c*d^4*x^3 + 2*(3*a^3*b^3*c^2 + a^
4*b^2)*d^3*x^2 + 4*(a^3*b^3*c^3 + a^4*b^2*c)*d^2*x + (a^3*b^3*c^4 + 2*a^4*
b^2*c^2 + a^5*b)*d)]
```

3.87.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(82) = 164.

Time = 0.62 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.82

$$\int \frac{1}{(a+b(c+dx)^2)^3}\,dx$$
$$= \frac{5ac + 3bc^3 + 9bcd^2x^2 + 3bd^3x^3 + x(5ad + 9bc^2d)}{8a^4d + 16a^3bc^2d + 8a^2b^2c^4d + 32a^2b^2cd^4x^3 + 8a^2b^2d^5x^4 + x^2\cdot(16a^3bd^3 + 48a^2b^2c^2d^3) + x\,(32a^3bcd^2 + 32}$$
$$+ \frac{-\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(x+\frac{-3a^3\sqrt{-\frac{1}{a^5b}}+3c}{3d}\right)}{16} + \frac{3\sqrt{-\frac{1}{a^5b}}\log\left(x+\frac{3a^3\sqrt{-\frac{1}{a^5b}}+3c}{3d}\right)}{16}}{d}$$

input
```
integrate(1/(a+b*(d*x+c)**2)**3,x)
```

output
```
(5*a*c + 3*b*c**3 + 9*b*c*d**2*x**2 + 3*b*d**3*x**3 + x*(5*a*d + 9*b*c**2*
d))/(8*a**4*d + 16*a**3*b*c**2*d + 8*a**2*b**2*c**4*d + 32*a**2*b**2*c*d**
4*x**3 + 8*a**2*b**2*d**5*x**4 + x**2*(16*a**3*b*d**3 + 48*a**2*b**2*c**2*
d**3) + x*(32*a**3*b*c*d**2 + 32*a**2*b**2*c**3*d**2)) + (-3*sqrt(-1/(a**5
*b))*log(x + (-3*a**3*sqrt(-1/(a**5*b)) + 3*c)/(3*d))/16 + 3*sqrt(-1/(a**5
*b))*log(x + (3*a**3*sqrt(-1/(a**5*b)) + 3*c)/(3*d))/16)/d
```

3.87.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(77) = 154$.

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.02

$$\int \frac{1}{(a+b(c+dx)^2)^3}\,dx = \frac{3\,bd^3x^3 + 9\,bcd^2x^2 + 3\,bc^3 + (9\,bc^2 + 5\,a)dx + 5\,ac}{8\,(a^2b^2d^5x^4 + 4\,a^2b^2cd^4x^3 + 2\,(3\,a^2b^2c^2 + a^3b)d^3x^2 + 4\,(a^2b^2c^3 + a^3bc)d^2x + (a^2b^2c^4 + 2\,a^3bc^2 + a^4)d)} + \frac{3\,\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}d}\right)}{8\,\sqrt{ab}a^2d}$$

input
```
integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/8*(3*b*d^3*x^3 + 9*b*c*d^2*x^2 + 3*b*c^3 + (9*b*c^2 + 5*a)*d*x + 5*a*c)/
(a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + 2*(3*a^2*b^2*c^2 + a^3*b)*d^3*x^2
 + 4*(a^2*b^2*c^3 + a^3*b*c)*d^2*x + (a^2*b^2*c^4 + 2*a^3*b*c^2 + a^4)*d)
+ 3/8*arctan((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*a^2*d)
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13

$$\int \frac{1}{(a+b(c+dx)^2)^3}\,dx = \frac{3\,\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2d} + \frac{3\,bd^3x^3 + 9\,bcd^2x^2 + 9\,bc^2dx + 3\,bc^3 + 5\,adx + 5\,ac}{8\,(bd^2x^2 + 2\,bcdx + bc^2 + a)^2a^2d}$$

input
```
integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
3/8*arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*a^2*d) + 1/8*(3*b*d^3*x^3 +
 9*b*c*d^2*x^2 + 9*b*c^2*d*x + 3*b*c^3 + 5*a*d*x + 5*a*c)/((b*d^2*x^2 + 2*
b*c*d*x + b*c^2 + a)^2*a^2*d)
```

3.87.9 Mupad [B] (verification not implemented)

Time = 10.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.99

$$\int \frac{1}{(a+b(c+dx)^2)^3}\,dx = \frac{\frac{x\,(9\,b\,c^2+5\,a)}{8\,a^2}+\frac{3\,b\,c^3+5\,a\,c}{8\,a^2\,d}+\frac{3\,b\,d^2\,x^3}{8\,a^2}+\frac{9\,b\,c\,d\,x^2}{8\,a^2}}{x^2\,(6\,b^2\,c^2\,d^2+2\,a\,b\,d^2)+x\,(4\,d\,b^2\,c^3+4\,a\,d\,b\,c)+a^2+b^2\,c^4+b^2\,d^4\,x^4+2\,a\,b\,c^2+4\,b^2\,c\,d^3\,x^3} + \frac{3\,\mathrm{atan}\left(\frac{8\,a^2\,\left(\frac{3\,\sqrt{b}\,c}{8\,a^{5/2}}+\frac{3\,\sqrt{b}\,d\,x}{8\,a^{5/2}}\right)}{3}\right)}{8\,a^{5/2}\,\sqrt{b}\,d}$$

input
```
int(1/(a + b*(c + d*x)^2)^3,x)
```

output
```
((x*(5*a + 9*b*c^2))/(8*a^2) + (5*a*c + 3*b*c^3)/(8*a^2*d) + (3*b*d^2*x^3)
/(8*a^2) + (9*b*c*d*x^2)/(8*a^2))/(x^2*(6*b^2*c^2*d^2 + 2*a*b*d^2) + x*(4*
b^2*c^3*d + 4*a*b*c*d) + a^2 + b^2*c^4 + b^2*d^4*x^4 + 2*a*b*c^2 + 4*b^2*c
*d^3*x^3) + (3*atan((8*a^2*((3*b^(1/2)*c)/(8*a^(5/2)) + (3*b^(1/2)*d*x)/(8
*a^(5/2))))/3))/(8*a^(5/2)*b^(1/2)*d)
```

3.88 $\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx$

3.88.1 Optimal result

Integrand size = 19, antiderivative size = 35

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx=\frac{\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}d}$$

output
```
arctan((d*x+c)*b^(1/2)/(-a)^(1/4))/(-a)^(1/4)/d/b^(1/2)
```

3.88.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 198 vs. 2(35) = 70.

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 5.66

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx$$
$$=\frac{2\left(\sqrt{-a}-\sqrt{a}\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{b}(c+dx)}{\sqrt[4]{a}}\right)-2\left(\sqrt{-a}-\sqrt{a}\right)\arctan\left(1+\frac{\sqrt{2}\sqrt{b}(c+dx)}{\sqrt[4]{a}}\right)+\left(\sqrt{-a}+\sqrt{a}\right)\left(\log\right.}{4\sqrt{2}a^{3/4}\sqrt{b}d}$$

input
```
Integrate[(Sqrt[-a] + b*(c + d*x)^2)^(-1),x]
```

output
```
(2*(Sqrt[-a] - Sqrt[a])*ArcTan[1 - (Sqrt[2]*Sqrt[b]*(c + d*x))/a^(1/4)] -
2*(Sqrt[-a] - Sqrt[a])*ArcTan[1 + (Sqrt[2]*Sqrt[b]*(c + d*x))/a^(1/4)] + (
Sqrt[-a] + Sqrt[a])*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*Sqrt[b]*(c + d*x) + b*(
c + d*x)^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*Sqrt[b]*(c + d*x) + b*(c + d*x
)^2]))/(4*Sqrt[2]*a^(3/4)*Sqrt[b]*d)
```

3.88.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {239, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}dx$$

$$\Big\downarrow \text{ 239}$$

$$\frac{\int \frac{1}{b(c+dx)^2+\sqrt{-a}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}d}$$

input
```
Int[(Sqrt[-a] + b*(c + d*x)^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)
```

3.88.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

3.88.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

method	result	size
default	$\frac{\arctan\left(\frac{2b\,d^{2}x+2bcd}{2d\sqrt{b\sqrt{-a}}}\right)}{d\sqrt{b\sqrt{-a}}}$	42

input
```
int(1/(b*(d*x+c)^2+(-a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/d/(b*(-a)^(1/2))^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(b*(-a)^(1/2))^(
1/2))
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 279, normalized size of antiderivative = 7.97

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx$$
$$= \left[\frac{\sqrt{\frac{\sqrt{-a}}{ab}}\log\left(\frac{b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4-2\,(bd^2x^2+2\,bcdx+bc^2)\sqrt{-a}+2\,(abdx+abc+(b^2d^3x^3+3\,b^2cd^2x^2+3\,b^2c^2dx+b}{b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4+a}\right.}{2\,d}\right.$$

input
```
integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="fricas")
```

output
```
[1/2*sqrt(sqrt(-a)/(a*b))*log((b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d
^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(
-a) + 2*(a*b*d*x + a*b*c + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*sqrt(-a))*sqrt(sqrt(-a)/(a*b)) - a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*
x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + a))/d, sqrt(-sqrt(-a)/
(a*b))*arctan((b*d*x + b*c)*sqrt(-sqrt(-a)/(a*b)))/d]
```

3.88.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(31) = 62$.

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.63

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx = \frac{-\frac{\sqrt{-\frac{1}{b\sqrt{-a}}}\log\left(x+\frac{c-\sqrt{-a}\sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right)}{2}+\frac{\sqrt{-\frac{1}{b\sqrt{-a}}}\log\left(x+\frac{c+\sqrt{-a}\sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right)}{2}}{d}$$

input
```
integrate(1/(b*(d*x+c)**2+(-a)**(1/2)),x)
```

output
```
(-sqrt(-1/(b*sqrt(-a)))*log(x + (c - sqrt(-a)*sqrt(-1/(b*sqrt(-a))))/d)/2
+ sqrt(-1/(b*sqrt(-a)))*log(x + (c + sqrt(-a)*sqrt(-1/(b*sqrt(-a))))/d)/2)
/d
```

3.88.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(27) = 54$.

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.89

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx = \frac{\log\left(\frac{bd^2x+bcd-\sqrt{-\sqrt{-a}b}d}{bd^2x+bcd+\sqrt{-\sqrt{-a}b}d}\right)}{2\sqrt{-\sqrt{-a}b}d}$$

input
```
integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="maxima")
```

output
```
1/2*log((b*d^2*x + b*c*d - sqrt(-sqrt(-a)*b)*d)/(b*d^2*x + b*c*d + sqrt(-s
qrt(-a)*b)*d))/(sqrt(-sqrt(-a)*b)*d)
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx=\frac{\arctan\left(\frac{bdx+bc}{(-a)^{\frac{1}{4}}\sqrt{b}}\right)}{(-a)^{\frac{1}{4}}\sqrt{b}d}$$

input
```
integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="giac")
```

output
```
arctan((b*d*x + b*c)/((-a)^(1/4)*sqrt(b)))/((-a)^(1/4)*sqrt(b)*d)
```

3.88.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx=\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,c+\sqrt{b}\,d\,x}{(-a)^{1/4}}\right)}{(-a)^{1/4}\,\sqrt{b}\,d}$$

input
```
int(1/(b*(c + d*x)^2 + (-a)^(1/2)),x)
```

output
```
atan((b^(1/2)*c + b^(1/2)*d*x)/(-a)^(1/4))/((-a)^(1/4)*b^(1/2)*d)
```

3.89 $\int \frac{1}{1+(c+dx)^2}\,dx$

3.89.1 Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{1}{1+(c+dx)^2}\,dx = \frac{\arctan(c+dx)}{d}$$

output
```
arctan(d*x+c)/d
```

3.89.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+(c+dx)^2}\,dx = \frac{\arctan(c+dx)}{d}$$

input
```
Integrate[(1 + (c + d*x)^2)^(-1),x]
```

output
```
ArcTan[c + d*x]/d
```

3.89.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {239, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2+1}dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \frac{1}{(c+dx)^2+1}d(c+dx)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan(c+dx)}{d}$$

input
```
Int[(1 + (c + d*x)^2)^(-1),x]
```

output
```
ArcTan[c + d*x]/d
```

3.89.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

3.89.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\arctan(dx+c)}{d}$	11
risch	$\frac{\arctan(dx+c)}{d}$	11
parallelrisch	$-\frac{i\ln(dx+c-i)-i\ln(dx+c+i)}{2d}$	29

input
```
int(1/(1+(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(d*x+c)/d
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+(c+dx)^2}\,dx = \frac{\arctan(dx+c)}{d}$$

input
```
integrate(1/(1+(d*x+c)^2),x, algorithm="fricas")
```

output
```
arctan(d*x + c)/d
```

3.89.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.40

$$\int \frac{1}{1+(c+dx)^2}\,dx = \frac{-\frac{i\log\left(x+\frac{c-i}{d}\right)}{2}+\frac{i\log\left(x+\frac{c+i}{d}\right)}{2}}{d}$$

input
```
integrate(1/(1+(d*x+c)**2),x)
```

output
```
(-I*log(x + (c - I)/d)/2 + I*log(x + (c + I)/d)/2)/d
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{1}{1+(c+dx)^2}\,dx = \frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d}$$

input
```
integrate(1/(1+(d*x+c)^2),x, algorithm="maxima")
```

output
```
arctan((d^2*x + c*d)/d)/d
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+(c+dx)^2}\,dx = \frac{\arctan\left(dx+c\right)}{d}$$

input
```
integrate(1/(1+(d*x+c)^2),x, algorithm="giac")
```

output
```
arctan(d*x + c)/d
```

3.89.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+(c+dx)^2}\,dx = \frac{\mathrm{atan}(c+d\,x)}{d}$$

input
```
int(1/((c + d*x)^2 + 1),x)
```

output
```
atan(c + d*x)/d
```

3.90 $\int \frac{1}{\left(1+(c+dx)^2\right)^2} \, dx$

3.90.1 Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{1}{\left(1+(c+dx)^2\right)^2} \, dx = \frac{c+dx}{2d\left(1+(c+dx)^2\right)}+\frac{\arctan(c+dx)}{2d}$$

output
```
1/2*(d*x+c)/d/(1+(d*x+c)^2)+1/2*arctan(d*x+c)/d
```

3.90.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(1+(c+dx)^2\right)^2} \, dx = \frac{\frac{c+dx}{1+(c+dx)^2}+\arctan(c+dx)}{2d}$$

input
```
Integrate[(1 + (c + d*x)^2)^(-2),x]
```

output
```
((c + d*x)/(1 + (c + d*x)^2) + ArcTan[c + d*x])/(2*d)
```

3.90.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {239, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{((c+dx)^2+1)^2} \, dx$$
$$\Big\downarrow \; 239$$
$$\frac{\int \frac{1}{((c+dx)^2+1)^2}d(c+dx)}{d}$$
$$\Big\downarrow \; 215$$
$$\frac{\frac{1}{2}\int \frac{1}{(c+dx)^2+1}d(c+dx)+\frac{c+dx}{2((c+dx)^2+1)}}{d}$$
$$\Big\downarrow \; 216$$
$$\frac{\frac{1}{2}\arctan(c+dx)+\frac{c+dx}{2((c+dx)^2+1)}}{d}$$

input
```
Int[(1 + (c + d*x)^2)^(-2),x]
```

output
```
((c + d*x)/(2*(1 + (c + d*x)^2)) + ArcTan[c + d*x]/2)/d
```

3.90.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

3.90.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

method	result
risch	$\frac{\frac{x}{2}+\frac{c}{2d}}{d^2x^2+2cdx+c^2+1}+\frac{\arctan(dx+c)}{2d}$
default	$\frac{2d^2x+2cd}{4d^2(d^2x^2+2cdx+c^2+1)}+\frac{\arctan\left(\frac{2d^2x+2cd}{2d}\right)}{2d}$
parallelrisch	$-\frac{i\ln(dx+c-i)x^2d^3-i\ln(dx+c+i)x^2d^3+2i\ln(dx+c-i)xc\,d^2-2i\ln(dx+c+i)xc\,d^2+i\ln(dx+c-i)c^2d-i\ln(dx+c+i)c^2d+i\ln(}{4d^2(d^2x^2+2cdx+c^2+1)}$

input
```
int(1/(1+(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*x+1/2*c/d)/(d^2*x^2+2*c*d*x+c^2+1)+1/2*arctan(d*x+c)/d
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.49

$$\int \frac{1}{(1+(c+dx)^2)^2}\,dx = \frac{dx+(d^2x^2+2\,cdx+c^2+1)\arctan(dx+c)+c}{2\,(d^3x^2+2\,cd^2x+(c^2+1)d)}$$

input
```
integrate(1/(1+(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/2*(d*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c) + c)/(d^3*x^2 + 2
*c*d^2*x + (c^2 + 1)*d)
```

3.90.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int \frac{1}{\left(1+(c+dx)^2\right)^2}\,dx = \frac{c+dx}{2c^2d+4cd^2x+2d^3x^2+2d} + \frac{-\frac{i\log\left(x+\frac{c-i}{d}\right)}{4}+\frac{i\log\left(x+\frac{c+i}{d}\right)}{4}}{d}$$

input
```
integrate(1/(1+(d*x+c)**2)**2,x)
```

output
```
(c + d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 + 2*d) + (-I*log(x + (c - I
)/d)/4 + I*log(x + (c + I)/d)/4)/d
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(1+(c+dx)^2\right)^2}\,dx = \frac{dx+c}{2\,(d^3x^2+2\,cd^2x+(c^2+1)d)} + \frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{2\,d}$$

input
```
integrate(1/(1+(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 + 1)*d) + 1/2*arctan((d^2*x + c*
d)/d)/d
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(1+(c+dx)^2\right)^2}\,dx = \frac{\arctan\left(dx+c\right)}{2\,d} + \frac{dx+c}{2\,(d^2x^2+2\,cdx+c^2+1)d}$$

input
```
integrate(1/(1+(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*arctan(d*x + c)/d + 1/2*(d*x + c)/((d^2*x^2 + 2*c*d*x + c^2 + 1)*d)
```

3.90.9 Mupad [B] (verification not implemented)

Time = 10.89 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(1+(c+dx)^2\right)^2}\,dx=\frac{\frac{x}{2}+\frac{c}{2\,d}}{c^2+2\,c\,d\,x+d^2\,x^2+1}+\frac{\mathrm{atan}(c+d\,x)}{2\,d}$$

input
```
int(1/((c + d*x)^2 + 1)^2,x)
```

output
```
(x/2 + c/(2*d))/(c^2 + d^2*x^2 + 2*c*d*x + 1) + atan(c + d*x)/(2*d)
```

3.91 $\int \frac{1}{\left(1+(c+dx)^2\right)^3}\,dx$

3.91.1 Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{1}{\left(1+(c+dx)^2\right)^3}\,dx=\frac{c+dx}{4d\left(1+(c+dx)^2\right)^2}+\frac{3(c+dx)}{8d\left(1+(c+dx)^2\right)}+\frac{3\arctan(c+dx)}{8d}$$

output
```
1/4*(d*x+c)/d/(1+(d*x+c)^2)^2+3/8*(d*x+c)/d/(1+(d*x+c)^2)+3/8*arctan(d*x+c
)/d
```

3.91.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(1+(c+dx)^2\right)^3}\,dx=\frac{\frac{2(c+dx)}{\left(1+(c+dx)^2\right)^2}+\frac{3(c+dx)}{1+(c+dx)^2}+3\arctan(c+dx)}{8d}$$

input
```
Integrate[(1 + (c + d*x)^2)^(-3),x]
```

output
```
((2*(c + d*x))/(1 + (c + d*x)^2)^2 + (3*(c + d*x))/(1 + (c + d*x)^2) + 3*A
rcTan[c + d*x])/(8*d)
```

3.91.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {239, 215, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{((c+dx)^2+1)^3}dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \frac{1}{((c+dx)^2+1)^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3}{4}\int \frac{1}{((c+dx)^2+1)^2}d(c+dx)+\frac{c+dx}{4((c+dx)^2+1)^2}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{(c+dx)^2+1}d(c+dx)+\frac{c+dx}{2((c+dx)^2+1)}\right)+\frac{c+dx}{4((c+dx)^2+1)^2}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\arctan(c+dx)+\frac{c+dx}{2((c+dx)^2+1)}\right)+\frac{c+dx}{4((c+dx)^2+1)^2}}{d}$$

input
```
Int[(1 + (c + d*x)^2)^(-3),x]
```

output
```
((c + d*x)/(4*(1 + (c + d*x)^2)^2) + (3*((c + d*x)/(2*(1 + (c + d*x)^2)) +
 ArcTan[c + d*x]/2))/4)/d
```

3.91.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

3.91.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

method	result
risch	$\frac{\frac{3 d^{2} x^{3}}{8}+\frac{9 x^{2} c d}{8}+\left(\frac{9 c^{2}}{8}+\frac{5}{8}\right) x+\frac{c \left(3 c^{2}+5\right)}{8 d}}{\left(d^{2} x^{2}+2 c d x+c^{2}+1\right)^{2}}+\frac{3 \arctan \left(d x+c\right)}{8 d}$
default	$\frac{2 d^{2} x+2 c d}{8 d^{2} \left(d^{2} x^{2}+2 c d x+c^{2}+1\right)^{2}}+\frac{\frac{3}{8} d^{2} x+\frac{3}{8} c d}{d^{2} \left(d^{2} x^{2}+2 c d x+c^{2}+1\right)}+\frac{3 \arctan \left(\frac{2 d^{2} x+2 c d}{2 d}\right)}{8 d}$
parallelrisch	$-\frac{-3 i \ln \left(d x+c+i\right) c^{4} d^{3}+6 i \ln \left(d x+c-i\right) c^{2} d^{3}-6 i \ln \left(d x+c+i\right) c^{2} d^{3}+3 i \ln \left(d x+c-i\right) x^{4} d^{7}-3 i \ln \left(d x+c+i\right) x^{4} d^{7}+6 i \ln \left(d x+c-i\right) x^{2} d}{}$

input
```
int(1/(1+(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/8*d^2*x^3+9/8*x^2*c*d+(9/8*c^2+5/8)*x+1/8*c/d*(3*c^2+5))/(d^2*x^2+2*c*d
*x+c^2+1)^2+3/8*arctan(d*x+c)/d
```

3.91.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(54) = 108$.

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.55

$$\int \frac{1}{\left(1+(c+dx)^2\right)^3}\,dx = \frac{3\,d^3x^3 + 9\,cd^2x^2 + 3\,c^3 + (9\,c^2 + 5)dx + 3\,(d^4x^4 + 4\,cd^3x^3 + 2\,(3\,c^2 + 1)d^2x^2 + c^4 + 4\,(c^3 + c)dx + 2\,c^2 + }{8\,(d^5x^4 + 4\,cd^4x^3 + 2\,(3\,c^2 + 1)d^3x^2 + 4\,(c^3 + c)d^2x + (c^4 + 2\,c^2 + 1)d)}$$

input
```
integrate(1/(1+(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 + 5)*d*x + 3*(d^4*x^4 + 4*c*
d^3*x^3 + 2*(3*c^2 + 1)*d^2*x^2 + c^4 + 4*(c^3 + c)*d*x + 2*c^2 + 1)*arcta
n(d*x + c) + 5*c)/(d^5*x^4 + 4*c*d^4*x^3 + 2*(3*c^2 + 1)*d^3*x^2 + 4*(c^3
+ c)*d^2*x + (c^4 + 2*c^2 + 1)*d)
```

3.91.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.43

$$\int \frac{1}{\left(1+(c+dx)^2\right)^3}\,dx = \frac{3c^3 + 9cd^2x^2 + 5c + 3d^3x^3 + x(9c^2d + 5d)}{8c^4d + 16c^2d + 32cd^4x^3 + 8d^5x^4 + 8d + x^2\cdot(48c^2d^3 + 16d^3) + x\,(32c^3d^2 + 32cd^2)} + \frac{-\frac{3i\log\left(x+\frac{3c-3i}{3d}\right)}{16} + \frac{3i\log\left(x+\frac{3c+3i}{3d}\right)}{16}}{d}$$

input
```
integrate(1/(1+(d*x+c)**2)**3,x)
```

output
```
(3*c**3 + 9*c*d**2*x**2 + 5*c + 3*d**3*x**3 + x*(9*c**2*d + 5*d))/(8*c**4*
d + 16*c**2*d + 32*c*d**4*x**3 + 8*d**5*x**4 + 8*d + x**2*(48*c**2*d**3 +
16*d**3) + x*(32*c**3*d**2 + 32*c*d**2)) + (-3*I*log(x + (3*c - 3*I)/(3*d)
)/16 + 3*I*log(x + (3*c + 3*I)/(3*d))/16)/d
```

3.91.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(54) = 108$.

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.92

$$\int \frac{1}{(1+(c+dx)^2)^3}\,dx = \frac{3\,d^3x^3+9\,cd^2x^2+3\,c^3+(9\,c^2+5)dx+5\,c}{8\,(d^5x^4+4\,cd^4x^3+2\,(3\,c^2+1)d^3x^2+4\,(c^3+c)d^2x+(c^4+2\,c^2+1)d)} + \frac{3\,\arctan\left(\frac{d^2x+cd}{d}\right)}{8\,d}$$

input
```
integrate(1/(1+(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 + 5)*d*x + 5*c)/(d^5*x^4 + 4
*c*d^4*x^3 + 2*(3*c^2 + 1)*d^3*x^2 + 4*(c^3 + c)*d^2*x + (c^4 + 2*c^2 + 1)
*d) + 3/8*arctan((d^2*x + c*d)/d)/d
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \frac{1}{(1+(c+dx)^2)^3}\,dx = \frac{3\,\arctan(dx+c)}{8\,d} + \frac{3\,d^3x^3+9\,cd^2x^2+9\,c^2dx+3\,c^3+5\,dx+5\,c}{8\,(d^2x^2+2\,cdx+c^2+1)^2d}$$

input
```
integrate(1/(1+(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
3/8*arctan(d*x + c)/d + 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 9*c^2*d*x + 3*c^3 +
 5*d*x + 5*c)/((d^2*x^2 + 2*c*d*x + c^2 + 1)^2*d)
```

3.91.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.85

$$\int \frac{1}{\left(1+(c+dx)^2\right)^3}\,dx$$
$$=\frac{3\,\mathrm{atan}(c+d\,x)}{8\,d}+\frac{x\left(\frac{9\,c^2}{8}+\frac{5}{8}\right)+\frac{3\,c^3+5\,c}{8\,d}+\frac{3\,d^2\,x^3}{8}+\frac{9\,c\,d\,x^2}{8}}{x^2\,\left(6\,c^2\,d^2+2\,d^2\right)+2\,c^2+c^4+x\,\left(4\,d\,c^3+4\,d\,c\right)+d^4\,x^4+4\,c\,d^3\,x^3+1}$$

input
```
int(1/((c + d*x)^2 + 1)^3,x)
```

output
```
(3*atan(c + d*x))/(8*d) + (x*((9*c^2)/8 + 5/8) + (5*c + 3*c^3)/(8*d) + (3*
d^2*x^3)/8 + (9*c*d*x^2)/8)/(x^2*(2*d^2 + 6*c^2*d^2) + 2*c^2 + c^4 + x*(4*
c*d + 4*c^3*d) + d^4*x^4 + 4*c*d^3*x^3 + 1)
```

3.92 $\int \frac{1}{1-(c+dx)^2} \, dx$

3.92.1 Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{1}{1-(c+dx)^2} \, dx = \frac{\operatorname{arctanh}(c+dx)}{d}$$

output
```
arctanh(d*x+c)/d
```

3.92.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 32 vs. 2(10) = 20.

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 3.20

$$\int \frac{1}{1-(c+dx)^2} \, dx = -\frac{\log(1-c-dx)}{2d} + \frac{\log(1+c+dx)}{2d}$$

input
```
Integrate[(1 - (c + d*x)^2)^(-1),x]
```

output
```
-1/2*Log[1 - c - d*x]/d + Log[1 + c + d*x]/(2*d)
```

3.92.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {239, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-(c+dx)^2} \, dx$$

$$\Big\downarrow \text{ 239}$$

$$\frac{\int \frac{1}{1-(c+dx)^2}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}(c+dx)}{d}$$

input
```
Int[(1 - (c + d*x)^2)^(-1),x]
```

output
```
ArcTanh[c + d*x]/d
```

3.92.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

3.92.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20.

Time = 0.88 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

method	result	size
parallelrisch	$-\frac{\ln(dx+c-1)-\ln(dx+c+1)}{2d}$	23
default	$-\frac{\ln(dx+c-1)}{2d}+\frac{\ln(dx+c+1)}{2d}$	26
norman	$-\frac{\ln(dx+c-1)}{2d}+\frac{\ln(dx+c+1)}{2d}$	26
risch	$-\frac{\ln(dx+c-1)}{2d}+\frac{\ln(-dx-c-1)}{2d}$	29

input
```
int(1/(1-(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(ln(d*x+c-1)-ln(d*x+c+1))/d
```

3.92.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20.

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.20

$$\int \frac{1}{1-(c+dx)^2}\,dx = \frac{\log(dx+c+1)-\log(dx+c-1)}{2\,d}$$

input
```
integrate(1/(1-(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*(log(d*x + c + 1) - log(d*x + c - 1))/d
```

3.92.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(7) = 14.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.20

$$\int \frac{1}{1-(c+dx)^2}\,dx = -\frac{\frac{\log\left(x+\frac{c-1}{d}\right)}{2}-\frac{\log\left(x+\frac{c+1}{d}\right)}{2}}{d}$$

input
```
integrate(1/(1-(d*x+c)**2),x)
```

output
```
-(log(x + (c - 1)/d)/2 - log(x + (c + 1)/d)/2)/d
```

3.92.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.

Time $= 0.19$ (sec) , antiderivative size $= 25$, normalized size of antiderivative $= 2.50$

$$\int \frac{1}{1-(c+dx)^2}\,dx = \frac{\log(dx+c+1)}{2\,d} - \frac{\log(dx+c-1)}{2\,d}$$

input
```
integrate(1/(1-(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*log(d*x + c + 1)/d - 1/2*log(d*x + c - 1)/d
```

3.92.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time $= 0.30$ (sec) , antiderivative size $= 27$, normalized size of antiderivative $= 2.70$

$$\int \frac{1}{1-(c+dx)^2}\,dx = \frac{\log(|dx+c+1|)}{2\,d} - \frac{\log(|dx+c-1|)}{2\,d}$$

input
```
integrate(1/(1-(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*log(abs(d*x + c + 1))/d - 1/2*log(abs(d*x + c - 1))/d
```

3.92.9 Mupad [B] (verification not implemented)

Time = 10.47 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-(c+dx)^2} \, dx = \frac{\operatorname{atanh}(c+d\,x)}{d}$$

input
```
int(-1/((c + d*x)^2 - 1),x)
```

output
```
atanh(c + d*x)/d
```

3.93 $\int \frac{1}{\left(1-(c+dx)^2\right)^2}\,dx$

3.93.1 Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{1}{\left(1-(c+dx)^2\right)^2}\,dx=\frac{c+dx}{2d\left(1-(c+dx)^2\right)}+\frac{\operatorname{arctanh}(c+dx)}{2d}$$

output
```
1/2*(d*x+c)/d/(1-(d*x+c)^2)+1/2*arctanh(d*x+c)/d
```

3.93.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(1-(c+dx)^2\right)^2}\,dx=\frac{-\frac{2(c+dx)}{-1+(c+dx)^2}-\log(1-c-dx)+\log(1+c+dx)}{4d}$$

input
```
Integrate[(1 - (c + d*x)^2)^(-2),x]
```

output
```
((-2*(c + d*x))/(-1 + (c + d*x)^2) - Log[1 - c - d*x] + Log[1 + c + d*x])/
(4*d)
```

3.93.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {239, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-(c+dx)^2)^2} \, dx$$
$$\Big\downarrow \text{239}$$
$$\frac{\int \frac{1}{(1-(c+dx)^2)^2}d(c+dx)}{d}$$
$$\Big\downarrow \text{215}$$
$$\frac{\frac{1}{2}\int \frac{1}{1-(c+dx)^2}d(c+dx)+\frac{c+dx}{2(1-(c+dx)^2)}}{d}$$
$$\Big\downarrow \text{219}$$
$$\frac{\frac{1}{2}\text{arctanh}(c+dx)+\frac{c+dx}{2(1-(c+dx)^2)}}{d}$$

input
```
Int[(1 - (c + d*x)^2)^(-2),x]
```

output
```
((c + d*x)/(2*(1 - (c + d*x)^2)) + ArcTanh[c + d*x]/2)/d
```

3.93.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

3.93.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

method	result
default	$-\frac{1}{4d(dx+c-1)}-\frac{\ln(dx+c-1)}{4d}-\frac{1}{4d(dx+c+1)}+\frac{\ln(dx+c+1)}{4d}$
norman	$\frac{-\frac{c}{2d}-\frac{x}{2}}{d^2x^2+2cdx+c^2-1}-\frac{\ln(dx+c-1)}{4d}+\frac{\ln(dx+c+1)}{4d}$
risch	$\frac{-\frac{c}{2d}-\frac{x}{2}}{d^2x^2+2cdx+c^2-1}-\frac{\ln(dx+c-1)}{4d}+\frac{\ln(-dx-c-1)}{4d}$
parallelrisch	$-\frac{\ln(dx+c-1)x^2d^3-\ln(dx+c+1)x^2d^3+2\ln(dx+c-1)xc\,d^2-2\ln(dx+c+1)xc\,d^2+\ln(dx+c-1)c^2d-\ln(dx+c+1)c^2d+2d^2x-\ln(}{4d^2(d^2x^2+2cdx+c^2-1)}$

input
```
int(1/(1-(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/(d*x+c-1)-1/4/d*ln(d*x+c-1)-1/4/d/(d*x+c+1)+1/4/d*ln(d*x+c+1)
```

3.93.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(33) = 66$.

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.18

$$\int \frac{1}{\left(1-(c+dx)^2\right)^2}\,dx = -\frac{2\,dx-(d^2x^2+2\,cdx+c^2-1)\log(dx+c+1)+(d^2x^2+2\,cdx+c^2-1)\log(dx+c-1)+2\,c}{4\,(d^3x^2+2\,cd^2x+(c^2-1)d)}$$

input
```
integrate(1/(1-(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*d*x - (d^2*x^2 + 2*c*d*x + c^2 - 1)*log(d*x + c + 1) + (d^2*x^2 +
2*c*d*x + c^2 - 1)*log(d*x + c - 1) + 2*c)/(d^3*x^2 + 2*c*d^2*x + (c^2 - 1
)*d)
```

3.93.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int \frac{1}{(1-(c+dx)^2)^2}\,dx = \frac{-c-dx}{2c^2d+4cd^2x+2d^3x^2-2d} + \frac{-\frac{\log\left(x+\frac{c-1}{d}\right)}{4}+\frac{\log\left(x+\frac{c+1}{d}\right)}{4}}{d}$$

input
```
integrate(1/(1-(d*x+c)**2)**2,x)
```

output
```
(-c - d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 - 2*d) + (-log(x + (c - 1)
/d)/4 + log(x + (c + 1)/d)/4)/d
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int \frac{1}{(1-(c+dx)^2)^2}\,dx = -\frac{dx+c}{2\,(d^3x^2+2\,cd^2x+(c^2-1)d)} + \frac{\log(dx+c+1)}{4\,d} - \frac{\log(dx+c-1)}{4\,d}$$

input
```
integrate(1/(1-(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 - 1)*d) + 1/4*log(d*x + c + 1)/
d - 1/4*log(d*x + c - 1)/d
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int \frac{1}{(1-(c+dx)^2)^2}\,dx = \frac{\log(|dx+c+1|)}{4\,d} - \frac{\log(|dx+c-1|)}{4\,d} - \frac{dx+c}{2\,(d^2x^2+2\,cdx+c^2-1)d}$$

input
```
integrate(1/(1-(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/4*log(abs(d*x + c + 1))/d - 1/4*log(abs(d*x + c - 1))/d - 1/2*(d*x + c)/
((d^2*x^2 + 2*c*d*x + c^2 - 1)*d)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 10.36 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{1}{(1-(c+dx)^2)^2}\,dx = \frac{\operatorname{atanh}(c+d\,x)}{2\,d} - \frac{\frac{x}{2}+\frac{c}{2\,d}}{c^2+2\,c\,d\,x+d^2\,x^2-1}$$

input
```
int(1/((c + d*x)^2 - 1)^2,x)
```

output
```
atanh(c + d*x)/(2*d) - (x/2 + c/(2*d))/(c^2 + d^2*x^2 + 2*c*d*x - 1)
```

3.94 $\int \frac{1}{(1-(c+dx)^2)^3} dx$

3.94.1 Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{1}{(1-(c+dx)^2)^3} dx = \frac{c+dx}{4d(1-(c+dx)^2)^2} + \frac{3(c+dx)}{8d(1-(c+dx)^2)} + \frac{3\text{arctanh}(c+dx)}{8d}$$

output
```
1/4*(d*x+c)/d/(1-(d*x+c)^2)^2+3/8*(d*x+c)/d/(1-(d*x+c)^2)+3/8*arctanh(d*x+
c)/d
```

3.94.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{1}{(1-(c+dx)^2)^3} dx = \frac{\frac{4(c+dx)}{(-1+(c+dx)^2)^2} - \frac{6(c+dx)}{-1+(c+dx)^2} - 3\log(1-c-dx) + 3\log(1+c+dx)}{16d}$$

input
```
Integrate[(1 - (c + d*x)^2)^(-3),x]
```

output
```
((4*(c + d*x))/(-1 + (c + d*x)^2)^2 - (6*(c + d*x))/(-1 + (c + d*x)^2) - 3
*Log[1 - c - d*x] + 3*Log[1 + c + d*x])/(16*d)
```

3.94.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {239, 215, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-(c+dx)^2)^3}dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \frac{1}{(1-(c+dx)^2)^3}d(c+dx)}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3}{4}\int \frac{1}{(1-(c+dx)^2)^2}d(c+dx)+\frac{c+dx}{4(1-(c+dx)^2)^2}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-(c+dx)^2}d(c+dx)+\frac{c+dx}{2(1-(c+dx)^2)}\right)+\frac{c+dx}{4(1-(c+dx)^2)^2}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(c+dx)+\frac{c+dx}{2(1-(c+dx)^2)}\right)+\frac{c+dx}{4(1-(c+dx)^2)^2}}{d}$$

input
```
Int[(1 - (c + d*x)^2)^(-3),x]
```

output
```
((c + d*x)/(4*(1 - (c + d*x)^2)^2) + (3*((c + d*x)/(2*(1 - (c + d*x)^2)) +
 ArcTanh[c + d*x]/2))/4)/d
```

3.94.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

3.94.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

method	result
default	$\frac{1}{16d(dx+c-1)^{2}}-\frac{3}{16d(dx+c-1)}-\frac{3\ln(dx+c-1)}{16d}-\frac{1}{16d(dx+c+1)^{2}}-\frac{3}{16d(dx+c+1)}+\frac{3\ln(dx+c+1)}{16d}$
risch	$\frac{-\frac{3d^{2}x^{3}}{8}-\frac{9x^{2}cd}{8}+\left(-\frac{9c^{2}}{8}+\frac{5}{8}\right)x-\frac{c\left(3c^{2}-5\right)}{8d}}{\left(d^{2}x^{2}+2cdx+c^{2}-1\right)^{2}}-\frac{3\ln(dx+c-1)}{16d}+\frac{3\ln(-dx-c-1)}{16d}$
norman	$\frac{\frac{-3c^{3}d^{3}+5d^{3}c}{8d^{4}}+\frac{\left(-9c^{2}d^{3}+5d^{3}\right)x}{8d^{3}}-\frac{3d^{2}x^{3}}{8}-\frac{9x^{2}cd}{8}}{\left(d^{2}x^{2}+2cdx+c^{2}-1\right)^{2}}-\frac{3\ln(dx+c-1)}{16d}+\frac{3\ln(dx+c+1)}{16d}$
parallelrisch	$-\frac{3\ln(dx+c-1)c^{4}d^{3}+6d^{6}x^{3}+18x^{2}c\,d^{5}+18x\,c^{2}d^{4}-6\ln(dx+c-1)c^{2}d^{3}+6c^{3}d^{3}-10d^{3}c+12\ln(dx+c-1)x^{3}c\,d^{6}-12\ln(dx+c+1)x}{}$

input
```
int(1/(1-(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16/d/(d*x+c-1)^2-3/16/d/(d*x+c-1)-3/16/d*ln(d*x+c-1)-1/16/d/(d*x+c+1)^2-
3/16/d/(d*x+c+1)+3/16/d*ln(d*x+c+1)
```

3.94.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(54) = 108$.

Time = 0.26 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.44

$$\int \frac{1}{\left(1-(c+dx)^2\right)^3}\,dx = -\frac{6\,d^3x^3+18\,cd^2x^2+6\,c^3+2\,(9\,c^2-5)dx-3\,(d^4x^4+4\,cd^3x^3+2\,(3\,c^2-1)d^2x^2+c^4+4\,(c^3-c)dx-2}{16\,(d^5x^4+4\,cd^4x^3+2\,(3\,c^2-}$$

input
```
integrate(1/(1-(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/16*(6*d^3*x^3 + 18*c*d^2*x^2 + 6*c^3 + 2*(9*c^2 - 5)*d*x - 3*(d^4*x^4 +
 4*c*d^3*x^3 + 2*(3*c^2 - 1)*d^2*x^2 + c^4 + 4*(c^3 - c)*d*x - 2*c^2 + 1)*
log(d*x + c + 1) + 3*(d^4*x^4 + 4*c*d^3*x^3 + 2*(3*c^2 - 1)*d^2*x^2 + c^4
+ 4*(c^3 - c)*d*x - 2*c^2 + 1)*log(d*x + c - 1) - 10*c)/(d^5*x^4 + 4*c*d^4
*x^3 + 2*(3*c^2 - 1)*d^3*x^2 + 4*(c^3 - c)*d^2*x + (c^4 - 2*c^2 + 1)*d)
```

3.94.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(53) = 106$.

Time = 0.51 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

$$\int \frac{1}{\left(1-(c+dx)^2\right)^3}\,dx = -\frac{3c^3+9cd^2x^2-5c+3d^3x^3+x(9c^2d-5d)}{8c^4d-16c^2d+32cd^4x^3+8d^5x^4+8d+x^2\cdot(48c^2d^3-16d^3)+x\,(32c^3d^2-32cd^2)} - \frac{\frac{3\log\left(x+\frac{3c-3}{3d}\right)}{16}-\frac{3\log\left(x+\frac{3c+3}{3d}\right)}{16}}{d}$$

input
```
integrate(1/(1-(d*x+c)**2)**3,x)
```

output
```
-(3*c**3 + 9*c*d**2*x**2 - 5*c + 3*d**3*x**3 + x*(9*c**2*d - 5*d))/(8*c**4
*d - 16*c**2*d + 32*c*d**4*x**3 + 8*d**5*x**4 + 8*d + x**2*(48*c**2*d**3 -
 16*d**3) + x*(32*c**3*d**2 - 32*c*d**2)) - (3*log(x + (3*c - 3)/(3*d))/16
 - 3*log(x + (3*c + 3)/(3*d))/16)/d
```

3.94.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(54) = 108$.

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.91

$$\int \frac{1}{(1-(c+dx)^2)^3}\,dx = -\frac{3\,d^3x^3 + 9\,cd^2x^2 + 3\,c^3 + (9\,c^2-5)dx - 5\,c}{8\,(d^5x^4 + 4\,cd^4x^3 + 2\,(3\,c^2-1)d^3x^2 + 4\,(c^3-c)d^2x + (c^4-2\,c^2+1)d)} + \frac{3\,\log(dx+c+1)}{16\,d} - \frac{3\,\log(dx+c-1)}{16\,d}$$

input
```
integrate(1/(1-(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 - 5)*d*x - 5*c)/(d^5*x^4 +
4*c*d^4*x^3 + 2*(3*c^2 - 1)*d^3*x^2 + 4*(c^3 - c)*d^2*x + (c^4 - 2*c^2 + 1
)*d) + 3/16*log(d*x + c + 1)/d - 3/16*log(d*x + c - 1)/d
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.38

$$\int \frac{1}{(1-(c+dx)^2)^3}\,dx = \frac{3\,\log(|dx+c+1|)}{16\,d} - \frac{3\,\log(|dx+c-1|)}{16\,d} - \frac{3\,d^3x^3 + 9\,cd^2x^2 + 9\,c^2dx + 3\,c^3 - 5\,dx - 5\,c}{8\,(d^2x^2 + 2\,cdx + c^2 - 1)^2d}$$

input
```
integrate(1/(1-(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
3/16*log(abs(d*x + c + 1))/d - 3/16*log(abs(d*x + c - 1))/d - 1/8*(3*d^3*x
^3 + 9*c*d^2*x^2 + 9*c^2*d*x + 3*c^3 - 5*d*x - 5*c)/((d^2*x^2 + 2*c*d*x +
c^2 - 1)^2*d)
```

3.94.9 Mupad [B] (verification not implemented)

Time = 10.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.78

$$\int \frac{1}{\left(1-(c+dx)^2\right)^3}\,dx = \frac{3\,\mathrm{atanh}(c+d\,x)}{8\,d} - \frac{x\,\left(\frac{9\,c^2}{8}-\frac{5}{8}\right)-\frac{5\,c-3\,c^3}{8\,d}+\frac{3\,d^2\,x^3}{8}+\frac{9\,c\,d\,x^2}{8}}{c^4-2\,c^2-x^2\,\left(2\,d^2-6\,c^2\,d^2\right)-x\,\left(4\,c\,d-4\,c^3\,d\right)+d^4\,x^4+4\,c\,d^3\,x^3+1}$$

input
```
int(-1/((c + d*x)^2 - 1)^3,x)
```

output
```
(3*atanh(c + d*x))/(8*d) - (x*((9*c^2)/8 - 5/8) - (5*c - 3*c^3)/(8*d) + (3
*d^2*x^3)/8 + (9*c*d*x^2)/8)/(c^4 - 2*c^2 - x^2*(2*d^2 - 6*c^2*d^2) - x*(4
*c*d - 4*c^3*d) + d^4*x^4 + 4*c*d^3*x^3 + 1)
```

3.95 $\int \frac{1}{1-(1+x)^2} dx$

3.95.1 Optimal result

Integrand size = 11, antiderivative size = 4

$$\int \frac{1}{1-(1+x)^2} dx = \operatorname{arctanh}(1+x)$$

output
```
arctanh(1+x)
```

3.95.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 15 vs. 2(4) = 8.

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \frac{1}{1-(1+x)^2} dx = -\frac{\log(x)}{2} + \frac{1}{2}\log(2+x)$$

input
```
Integrate[(1 - (1 + x)^2)^(-1),x]
```

output
```
-1/2*Log[x] + Log[2 + x]/2
```

3.95.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {239, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-(x+1)^2} \, dx$$

$$\Big\downarrow \; 239$$

$$\int \frac{1}{1-(x+1)^2} d(x+1)$$

$$\Big\downarrow \; 219$$

$$\text{arctanh}(x+1)$$

input
```
Int[(1 - (1 + x)^2)^(-1),x]
```

output
```
ArcTanh[1 + x]
```

3.95.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

3.95.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(4) = 8$.

Time = 0.71 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

method	result	size
default	$-\frac{\ln(x)}{2}+\frac{\ln(x+2)}{2}$	12
norman	$-\frac{\ln(x)}{2}+\frac{\ln(x+2)}{2}$	12
risch	$-\frac{\ln(x)}{2}+\frac{\ln(x+2)}{2}$	12
parallelrisch	$-\frac{\ln(x)}{2}+\frac{\ln(x+2)}{2}$	12
meijerg	$\frac{\ln(1+\frac{x}{2})}{2}-\frac{\ln(x)}{2}+\frac{\ln(2)}{2}$	18

input
```
int(1/(1-(x+1)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(x)+1/2*ln(x+2)
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(4) = 8$.

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.75

$$\int \frac{1}{1-(1+x)^2}\,dx = \frac{1}{2}\log(x+2) - \frac{1}{2}\log(x)$$

input
```
integrate(1/(1-(1+x)^2),x, algorithm="fricas")
```

output
```
1/2*log(x + 2) - 1/2*log(x)
```

3.95.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. 2(3) = 6.

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.50

$$\int \frac{1}{1-(1+x)^2}\,dx = -\frac{\log(x)}{2} + \frac{\log(x+2)}{2}$$

input
```
integrate(1/(1-(1+x)**2),x)
```

output
```
-log(x)/2 + log(x + 2)/2
```

3.95.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(4) = 8.

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.75

$$\int \frac{1}{1-(1+x)^2}\,dx = \frac{1}{2}\log(x+2) - \frac{1}{2}\log(x)$$

input
```
integrate(1/(1-(1+x)^2),x, algorithm="maxima")
```

output
```
1/2*log(x + 2) - 1/2*log(x)
```

3.95.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(4) = 8.

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 3.25

$$\int \frac{1}{1-(1+x)^2}\,dx = \frac{1}{2}\log(|x+2|) - \frac{1}{2}\log(|x|)$$

input
```
integrate(1/(1-(1+x)^2),x, algorithm="giac")
```

output
```
1/2*log(abs(x + 2)) - 1/2*log(abs(x))
```

3.95.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-(1+x)^2}\,dx = \operatorname{atanh}(x+1)$$

input
```
int(-1/((x + 1)^2 - 1),x)
```

output
```
atanh(x + 1)
```

3.96 $\int \frac{1}{\left(1-(1+x)^2\right)^2} \, dx$

3.96.1 Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \frac{1}{\left(1-(1+x)^2\right)^2} \, dx=\frac{1+x}{2\left(1-(1+x)^2\right)}+\frac{1}{2}\text{arctanh}(1+x)$$

output
```
1/2*(1+x)/(1-(1+x)^2)+1/2*arctanh(1+x)
```

3.96.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(1-(1+x)^2\right)^2} \, dx=\frac{1}{4} \left(-\frac{2(1+x)}{x(2+x)}-\log(x)+\log(2+x)\right)$$

input
```
Integrate[(1 - (1 + x)^2)^(-2),x]
```

output
```
((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4
```

3.96.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {239, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-(x+1)^2)^2} dx$$
$$\Big\downarrow \text{239}$$
$$\int \frac{1}{(1-(x+1)^2)^2} d(x+1)$$
$$\Big\downarrow \text{215}$$
$$\frac{1}{2} \int \frac{1}{1-(x+1)^2} d(x+1) + \frac{x+1}{2(1-(x+1)^2)}$$
$$\Big\downarrow \text{219}$$
$$\frac{1}{2} \text{arctanh}(x+1) + \frac{x+1}{2(1-(x+1)^2)}$$

input
```
Int[(1 - (1 + x)^2)^(-2),x]
```

output
```
(1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2
```

3.96.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

3.96.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{1}{4x}-\frac{\ln(x)}{4}-\frac{1}{4(x+2)}+\frac{\ln(x+2)}{4}$	24
norman	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(x+2)}-\frac{\ln(x)}{4}+\frac{\ln(x+2)}{4}$	26
risch	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(x+2)}-\frac{\ln(x)}{4}+\frac{\ln(x+2)}{4}$	26
meijerg	$\frac{3x}{16\left(\frac{3x}{2}+3\right)}+\frac{\ln\left(1+\frac{x}{2}\right)}{4}-\frac{1}{8}-\frac{\ln(x)}{4}+\frac{\ln(2)}{4}-\frac{1}{4x}$	34
parallelrisch	$-\frac{\ln(x)x^2-\ln(x+2)x^2+2+2\ln(x)x-2\ln(x+2)x+2x}{4x(x+2)}$	43

input
```
int(1/(1-(x+1)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/x-1/4*ln(x)-1/4/(x+2)+1/4*ln(x+2)
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{1}{(1-(1+x)^2)^2}\,dx = \frac{(x^2+2\,x)\log(x+2)-(x^2+2\,x)\log(x)-2\,x-2}{4\,(x^2+2\,x)}$$

input
```
integrate(1/(1-(1+x)^2)^2,x, algorithm="fricas")
```

output
```
1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)
```

3.96.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{(1-(1+x)^2)^2}\,dx = \frac{-x-1}{2x^2+4x} - \frac{\log(x)}{4} + \frac{\log(x+2)}{4}$$

input
```
integrate(1/(1-(1+x)**2)**2,x)
```

output
```
(-x - 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{(1-(1+x)^2)^2}\,dx = -\frac{x+1}{2(x^2+2x)} + \frac{1}{4}\log(x+2) - \frac{1}{4}\log(x)$$

input
```
integrate(1/(1-(1+x)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-(1+x)^2)^2}\,dx = -\frac{x+1}{2(x^2+2x)} + \frac{1}{4}\log(|x+2|) - \frac{1}{4}\log(|x|)$$

input
```
integrate(1/(1-(1+x)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))
```

3.96.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(1-(1+x)^2\right)^2}\,dx = \frac{\operatorname{atanh}(x+1)}{2} - \frac{x+1}{2\left((x+1)^2-1\right)}$$

input
```
int(1/((x + 1)^2 - 1)^2,x)
```

output
```
atanh(x + 1)/2 - (x + 1)/(2*((x + 1)^2 - 1))
```

3.97 $\int \frac{1}{\left(1-(1+x)^2\right)^3}\,dx$

3.97.1 Optimal result

Integrand size = 11, antiderivative size = 45

$$\int \frac{1}{\left(1-(1+x)^2\right)^3}\,dx=\frac{1+x}{4\left(1-(1+x)^2\right)^2}+\frac{3(1+x)}{8\left(1-(1+x)^2\right)}+\frac{3}{8}\text{arctanh}(1+x)$$

output
```
1/4*(1+x)/(1-(1+x)^2)^2+3/8*(1+x)/(1-(1+x)^2)+3/8*arctanh(1+x)
```

3.97.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(1-(1+x)^2\right)^3}\,dx=\frac{1}{16}\left(\frac{1}{x^2}-\frac{3}{x}-\frac{1}{(2+x)^2}-\frac{3}{2+x}-3\log(x)+3\log(2+x)\right)$$

input
```
Integrate[(1 - (1 + x)^2)^(-3),x]
```

output
```
(x^(-2) - 3/x - (2 + x)^(-2) - 3/(2 + x) - 3*Log[x] + 3*Log[2 + x])/16
```

3.97.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {239, 215, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-(x+1)^2)^3}dx$$

$$\Big\downarrow \text{ 239}$$

$$\int \frac{1}{(1-(x+1)^2)^3}d(x+1)$$

$$\Big\downarrow \text{ 215}$$

$$\frac{3}{4}\int \frac{1}{(1-(x+1)^2)^2}d(x+1)+\frac{x+1}{4\left(1-(x+1)^2\right)^2}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-(x+1)^2}d(x+1)+\frac{x+1}{2\left(1-(x+1)^2\right)}\right)+\frac{x+1}{4\left(1-(x+1)^2\right)^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(x+1)+\frac{x+1}{2\left(1-(x+1)^2\right)}\right)+\frac{x+1}{4\left(1-(x+1)^2\right)^2}$$

input
```
Int[(1 - (1 + x)^2)^(-3),x]
```

output
```
(1 + x)/(4*(1 - (1 + x)^2)^2) + (3*((1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[
1 + x]/2))/4
```

3.97.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

3.97.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{1}{16x^2}-\frac{3}{16x}-\frac{3\ln(x)}{16}-\frac{1}{16(x+2)^2}-\frac{3}{16(x+2)}+\frac{3\ln(x+2)}{16}$	36
norman	$\frac{\frac{1}{4}-\frac{9}{8}x^2-\frac{3}{8}x^3-\frac{1}{2}x}{x^2(x+2)^2}-\frac{3\ln(x)}{16}+\frac{3\ln(x+2)}{16}$	36
risch	$\frac{\frac{1}{4}-\frac{9}{8}x^2-\frac{3}{8}x^3-\frac{1}{2}x}{x^2(x+2)^2}-\frac{3\ln(x)}{16}+\frac{3\ln(x+2)}{16}$	36
meijerg	$\frac{x(\frac{7x}{2}+8)}{128(1+\frac{x}{2})^2}+\frac{3\ln(1+\frac{x}{2})}{16}-\frac{7}{64}-\frac{3\ln(x)}{16}+\frac{3\ln(2)}{16}+\frac{1}{16x^2}-\frac{3}{16x}$	44
parallelrisch	$-\frac{3\ln(x)x^4-3\ln(x+2)x^4-4+12\ln(x)x^3-12\ln(x+2)x^3+12\ln(x)x^2-12\ln(x+2)x^2+6x^3+18x^2+8x}{16x^2(x+2)^2}$	74

input
```
int(1/(1-(x+1)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16/x^2-3/16/x-3/16*ln(x)-1/16/(x+2)^2-3/16/(x+2)+3/16*ln(x+2)
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(35) = 70.

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58

$$\int \frac{1}{\left(1-(1+x)^2\right)^3}\,dx = -\frac{6\,x^3+18\,x^2-3\,\left(x^4+4\,x^3+4\,x^2\right)\log\left(x+2\right)+3\,\left(x^4+4\,x^3+4\,x^2\right)\log\left(x\right)+8\,x-4}{16\,\left(x^4+4\,x^3+4\,x^2\right)}$$

input
```
integrate(1/(1-(1+x)^2)^3,x, algorithm="fricas")
```

output
```
-1/16*(6*x^3 + 18*x^2 - 3*(x^4 + 4*x^3 + 4*x^2)*log(x + 2) + 3*(x^4 + 4*x^
3 + 4*x^2)*log(x) + 8*x - 4)/(x^4 + 4*x^3 + 4*x^2)
```

3.97.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(1-(1+x)^2\right)^3}\,dx = -\frac{3\log\left(x\right)}{16}+\frac{3\log\left(x+2\right)}{16}-\frac{3x^3+9x^2+4x-2}{8x^4+32x^3+32x^2}$$

input
```
integrate(1/(1-(1+x)**2)**3,x)
```

output
```
-3*log(x)/16 + 3*log(x + 2)/16 - (3*x**3 + 9*x**2 + 4*x - 2)/(8*x**4 + 32*
x**3 + 32*x**2)
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(1-(1+x)^2\right)^3}\,dx = -\frac{3\,x^3+9\,x^2+4\,x-2}{8\,\left(x^4+4\,x^3+4\,x^2\right)}+\frac{3}{16}\,\log\left(x+2\right)-\frac{3}{16}\,\log\left(x\right)$$

input
```
integrate(1/(1-(1+x)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^4 + 4*x^3 + 4*x^2) + 3/16*log(x + 2) - 3
/16*log(x)
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{1}{(1-(1+x)^2)^3}\,dx = -\frac{3\,x^3+9\,x^2+4\,x-2}{8\,(x^2+2\,x)^2}+\frac{3}{16}\,\log\left(|x+2|\right)-\frac{3}{16}\,\log\left(|x|\right)$$

input
```
integrate(1/(1-(1+x)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^2 + 2*x)^2 + 3/16*log(abs(x + 2)) - 3/16
*log(abs(x))
```

3.97.9 Mupad [B] (verification not implemented)

Time = 10.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{1}{(1-(1+x)^2)^3}\,dx = \frac{3\,\mathrm{atanh}(x+1)}{8}+\frac{\frac{5\,x}{8}-\frac{3\,(x+1)^3}{8}+\frac{5}{8}}{(x+1)^4-2\,(x+1)^2+1}$$

input
```
int(-1/((x + 1)^2 - 1)^3,x)
```

output
```
(3*atanh(x + 1))/8 + ((5*x)/8 - (3*(x + 1)^3)/8 + 5/8)/((x + 1)^4 - 2*(x +
 1)^2 + 1)
```

3.98 $\int \frac{(1+(a+bx)^2)^2}{x} dx$

3.98.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{(1+(a+bx)^2)^2}{x} dx = a(2+a^2)\,bx + \frac{1}{2}(2+a^2)\,(a+bx)^2 + \frac{1}{3}a(a+bx)^3 + \frac{1}{4}(a+bx)^4 + (1+a^2)^2\log(x)$$

output
```
a*(a^2+2)*b*x+1/2*(a^2+2)*(b*x+a)^2+1/3*a*(b*x+a)^3+1/4*(b*x+a)^4+(a^2+1)^
2*ln(x)
```

3.98.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int \frac{(1+(a+bx)^2)^2}{x} dx = a(2+a^2)\,(a+bx) + \frac{1}{2}(2+a^2)\,(a+bx)^2 + \frac{1}{3}a(a+bx)^3 + \frac{1}{4}(a+bx)^4 + (1+a^2)^2\log(bx)$$

input
```
Integrate[(1 + (a + b*x)^2)^2/x,x]
```

output
```
a*(2 + a^2)*(a + b*x) + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a
 + b*x)^4/4 + (1 + a^2)^2*Log[b*x]
```

3.98.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {896, 25, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left((a+bx)^2+1\right)^2}{x}dx$$

$$\Big\downarrow \text{896}$$

$$\int \frac{\left((a+bx)^2+1\right)^2}{bx}d(a+bx)$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{\left((a+bx)^2+1\right)^2}{bx}d(a+bx)$$

$$\Big\downarrow \text{476}$$

$$-\int \left(-(a+bx)^3-a(a+bx)^2-\left(a^2+2\right)(a+bx)-a\left(a^2+2\right)-\frac{\left(a^2+1\right)^2}{bx}\right)d(a+bx)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(a^2+2\right)(a+bx)^2+a\left(a^2+2\right)(a+bx)+\left(a^2+1\right)^2\log(-bx)+\frac{1}{4}(a+bx)^4+\frac{1}{3}a(a+bx)^3$$

input
```
Int[(1 + (a + b*x)^2)^2/x,x]
```

output
```
a*(2 + a^2)*(a + b*x) + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a
 + b*x)^4/4 + (1 + a^2)^2*Log[-(b*x)]
```

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.98.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

method	result	size
norman	$\left(3a^{2}b^{2}+b^{2}\right)x^{2}+\left(4a^{3}b+4ab\right)x+\frac{b^{4}x^{4}}{4}+\frac{4a\,b^{3}x^{3}}{3}+\left(a^{4}+2a^{2}+1\right)\ln\left(x\right)$	61
default	$\frac{b^{4}x^{4}}{4}+\frac{4a\,b^{3}x^{3}}{3}+3a^{2}b^{2}x^{2}+4a^{3}bx+b^{2}x^{2}+4abx+\left(a^{4}+2a^{2}+1\right)\ln\left(x\right)$	62
risch	$\frac{b^{4}x^{4}}{4}+\frac{4a\,b^{3}x^{3}}{3}+3a^{2}b^{2}x^{2}+b^{2}x^{2}+4a^{3}bx+4abx+a^{4}\ln\left(x\right)+2a^{2}\ln\left(x\right)+\ln\left(x\right)$	64
parallelrisch	$\frac{b^{4}x^{4}}{4}+\frac{4a\,b^{3}x^{3}}{3}+3a^{2}b^{2}x^{2}+b^{2}x^{2}+4a^{3}bx+4abx+a^{4}\ln\left(x\right)+2a^{2}\ln\left(x\right)+\ln\left(x\right)$	64

input
```
int((1+(b*x+a)^2)^2/x,x,method=_RETURNVERBOSE)
```

output
```
(3*a^2*b^2+b^2)*x^2+(4*a^3*b+4*a*b)*x+1/4*b^4*x^4+4/3*a*b^3*x^3+(a^4+2*a^2
+1)*ln(x)
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{(1+(a+bx)^2)^2}{x}\,dx = \frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + \left(3\,a^2+1\right)b^2x^2 + 4\,\left(a^3+a\right)bx + \left(a^4+2\,a^2+1\right)\log\left(x\right)$$

input
```
integrate((1+(b*x+a)^2)^2/x,x, algorithm="fricas")
```

output
```
1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4
 + 2*a^2 + 1)*log(x)
```

3.98.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \frac{(1+(a+bx)^2)^2}{x}\,dx = \frac{4ab^3x^3}{3} + \frac{b^4x^4}{4} + x^2\cdot\left(3a^2b^2+b^2\right) + x\left(4a^3b+4ab\right) + \left(a^2+1\right)^2\log\left(x\right)$$

input
```
integrate((1+(b*x+a)**2)**2/x,x)
```

output
```
4*a*b**3*x**3/3 + b**4*x**4/4 + x**2*(3*a**2*b**2 + b**2) + x*(4*a**3*b +
4*a*b) + (a**2 + 1)**2*log(x)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{(1+(a+bx)^2)^2}{x}\,dx = \frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + \left(3\,a^2+1\right)b^2x^2 + 4\,\left(a^3+a\right)bx + \left(a^4+2\,a^2+1\right)\log\left(x\right)$$

input
```
integrate((1+(b*x+a)^2)^2/x,x, algorithm="maxima")
```

output
```
1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4
 + 2*a^2 + 1)*log(x)
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \frac{\left(1+(a+bx)^2\right)^2}{x}\,dx = \frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + 3\,a^2b^2x^2 + 4\,a^3bx + b^2x^2 + 4\,abx + \left(a^4 + 2\,a^2 + 1\right)\log\left(|x|\right)$$

input
```
integrate((1+(b*x+a)^2)^2/x,x, algorithm="giac")
```

output
```
1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + b^2*x^2 + 4*a*b*
x + (a^4 + 2*a^2 + 1)*log(abs(x))
```

3.98.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{\left(1+(a+bx)^2\right)^2}{x}\,dx = \ln\left(x\right)\,\left(a^4+2\,a^2+1\right) + \frac{b^4\,x^4}{4} + \frac{4\,a\,b^3\,x^3}{3} + b^2\,x^2\,\left(3\,a^2+1\right) + 4\,a\,b\,x\,\left(a^2+1\right)$$

input
```
int(((a + b*x)^2 + 1)^2/x,x)
```

output
```
log(x)*(2*a^2 + a^4 + 1) + (b^4*x^4)/4 + (4*a*b^3*x^3)/3 + b^2*x^2*(3*a^2
+ 1) + 4*a*b*x*(a^2 + 1)
```

3.99 $\int \frac{x^2}{1+(-1+x)^2} dx$

3.99.1 Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{x^2}{1+(-1+x)^2} dx = x+\log\left(1+(-1+x)^2\right)$$

output
```
x+ln(1+(-1+x)^2)
```

3.99.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{1+(-1+x)^2} dx = x+\log\left(2-2x+x^2\right)$$

input
```
Integrate[x^2/(1 + (-1 + x)^2),x]
```

output
```
x + Log[2 - 2*x + x^2]
```

3.99.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {896, 478, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x-1)^2+1} \, dx$$

$$\Big\downarrow \text{ 896}$$

$$\int \frac{x^2}{(x-1)^2+1} d(x-1)$$

$$\Big\downarrow \text{ 478}$$

$$\int \left(\frac{2(x-1)}{(x-1)^2+1}+1\right) d(x-1)$$

$$\Big\downarrow \text{ 2009}$$

$$x+\log \left((x-1)^2+1\right)-1$$

input
```
Int[x^2/(1 + (-1 + x)^2),x]
```

output
```
-1 + x + Log[1 + (-1 + x)^2]
```

3.99.3.1 Defintions of rubi rules used

rule 478
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ
[n, 1]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.99.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

method	result	size
default	$x+\ln \left(x^{2}-2x+2\right)$	12
norman	$x+\ln \left(x^{2}-2x+2\right)$	12
risch	$x+\ln \left(x^{2}-2x+2\right)$	12
parallelrisch	$x+\ln \left(x^{2}-2x+2\right)$	12

input
```
int(x^2/(1+(x-1)^2),x,method=_RETURNVERBOSE)
```

output
```
x+ln(x^2-2*x+2)
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{1+(-1+x)^2}\,dx = x+\log \left(x^{2}-2\,x+2\right)$$

input
```
integrate(x^2/(1+(-1+x)^2),x, algorithm="fricas")
```

output
```
x + log(x^2 - 2*x + 2)
```

3.99.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{1+(-1+x)^2}\,dx = x+\log \left(x^{2}-2x+2\right)$$

input
```
integrate(x**2/(1+(-1+x)**2),x)
```

output
```
x + log(x**2 - 2*x + 2)
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{1+(-1+x)^2}\,dx = x + \log\left(x^2-2\,x+2\right)$$

input
```
integrate(x^2/(1+(-1+x)^2),x, algorithm="maxima")
```

output
```
x + log(x^2 - 2*x + 2)
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{1+(-1+x)^2}\,dx = x + \log\left(x^2-2\,x+2\right)$$

input
```
integrate(x^2/(1+(-1+x)^2),x, algorithm="giac")
```

output
```
x + log(x^2 - 2*x + 2)
```

3.99.9 Mupad [B] (verification not implemented)

Time = 10.37 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{1+(-1+x)^2}\,dx = x + \ln\left(x^2-2\,x+2\right)$$

input
```
int(x^2/((x - 1)^2 + 1),x)
```

output
```
x + log(x^2 - 2*x + 2)
```

3.100 $\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx$

3.100.1 Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx = \frac{3}{2}\sqrt{1-(1+x)^2}-\frac{1}{2}x\sqrt{1-(1+x)^2}+\frac{3}{2}\arcsin(1+x)$$

output
```
3/2*arcsin(1+x)+3/2*(1-(1+x)^2)^(1/2)-1/2*x*(1-(1+x)^2)^(1/2)
```

3.100.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx = \frac{x(-6-x+x^2)-6\sqrt{x}\sqrt{2+x}\log\left(-\sqrt{x}+\sqrt{2+x}\right)}{2\sqrt{-x(2+x)}}$$

input
```
Integrate[x^2/Sqrt[1 - (1 + x)^2],x]
```

output
```
(x*(-6 - x + x^2) - 6*Sqrt[x]*Sqrt[2 + x]*Log[-Sqrt[x] + Sqrt[2 + x]])/(2*
Sqrt[-(x*(2 + x))])
```

3.100.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {896, 469, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{1-(x+1)^2}}dx$$

$$\Big\downarrow \text{ 896}$$

$$\int \frac{x^2}{\sqrt{1-(x+1)^2}}d(x+1)$$

$$\Big\downarrow \text{ 469}$$

$$\frac{3}{2}\int -\frac{x}{\sqrt{1-(x+1)^2}}d(x+1)-\frac{1}{2}x\sqrt{1-(x+1)^2}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{3}{2}\left(\int \frac{1}{\sqrt{1-(x+1)^2}}d(x+1)+\sqrt{1-(x+1)^2}\right)-\frac{1}{2}x\sqrt{1-(x+1)^2}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{3}{2}\left(\arcsin(x+1)+\sqrt{1-(x+1)^2}\right)-\frac{1}{2}x\sqrt{1-(x+1)^2}$$

input
```
Int[x^2/Sqrt[1 - (1 + x)^2],x]
```

output
```
-1/2*(x*Sqrt[1 - (1 + x)^2]) + (3*(Sqrt[1 - (1 + x)^2] + ArcSin[1 + x]))/2
```

3.100.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

3.100.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{(-3+x)x(x+2)}{2\sqrt{-x(x+2)}}+\frac{3\arcsin(x+1)}{2}$	25
pseudoelliptic	$-3\arctan\left(\frac{\sqrt{-x(x+2)}}{x}\right)+\frac{(3-x)\sqrt{-x(x+2)}}{2}$	32
default	$-\frac{x\sqrt{-x^2-2x}}{2}+\frac{3\sqrt{-x^2-2x}}{2}+\frac{3\arcsin(x+1)}{2}$	35
meijerg	$-\frac{4i\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-5x+15)\sqrt{1+\frac{x}{2}}}{40}+\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)}{4}\right)}{\sqrt{\pi}}$	45
trager	$\left(\frac{3}{2}-\frac{x}{2}\right)\sqrt{-x^2-2x}-\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2-2x}+\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{2}$	54

input
```
int(x^2/(1-(x+1)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-3+x)*x*(x+2)/(-x*(x+2))^(1/2)+3/2*arcsin(x+1)
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx = -\frac{1}{2}\sqrt{-x^2-2\,x}(x-3) - 3\,\arctan\left(\frac{\sqrt{-x^2-2\,x}}{x}\right)$$

input
```
integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-x^2 - 2*x)*(x - 3) - 3*arctan(sqrt(-x^2 - 2*x)/x)
```

3.100.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx = \left(\frac{3}{2}-\frac{x}{2}\right)\sqrt{-x^2-2x} + \frac{3\,\mathrm{asin}\,(x+1)}{2}$$

input
```
integrate(x**2/(1-(1+x)**2)**(1/2),x)
```

output
```
(3/2 - x/2)*sqrt(-x**2 - 2*x) + 3*asin(x + 1)/2
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx = -\frac{1}{2}\sqrt{-x^2-2\,x}x + \frac{3}{2}\sqrt{-x^2-2\,x} - \frac{3}{2}\,\arcsin\,(-x-1)$$

input
```
integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-x^2 - 2*x)*x + 3/2*sqrt(-x^2 - 2*x) - 3/2*arcsin(-x - 1)
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.52

$$\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx = -\frac{1}{2}\sqrt{-x^2-2\,x}(x-3)+\frac{3}{2}\,\arcsin\left(x+1\right)$$

input
```
integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-x^2 - 2*x)*(x - 3) + 3/2*arcsin(x + 1)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx = \int \frac{x^2}{\sqrt{1-(x+1)^2}}\,dx$$

input
```
int(x^2/(1 - (x + 1)^2)^(1/2),x)
```

output
```
int(x^2/(1 - (x + 1)^2)^(1/2), x)
```

3.101 $\int \frac{x^2}{\sqrt{1-(a+bx)^2}} \, dx$

3.101.1 Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}} \, dx = \frac{3a\sqrt{1-(a+bx)^2}}{2b^3} - \frac{x\sqrt{1-(a+bx)^2}}{2b^2} + \frac{(1+2a^2)\arcsin(a+bx)}{2b^3}$$

output
```
1/2*(2*a^2+1)*arcsin(b*x+a)/b^3+3/2*a*(1-(b*x+a)^2)^(1/2)/b^3-1/2*x*(1-(b*
x+a)^2)^(1/2)/b^2
```

3.101.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 163 vs. 2(67) = 134.

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.43

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}} \, dx = \frac{-2b(-3a+bx)\sqrt{1-a^2-2abx-b^2x^2} - 2(1+2a^2)\,b\arctan\left(\frac{-\sqrt{-b^2}x+\sqrt{1-a^2-2abx-b^2x^2}}{a}\right) + (1+2a^2)\sqrt{-b^2}}{4b^4}$$

input
```
Integrate[x^2/Sqrt[1 - (a + b*x)^2],x]
```

output
```
(-2*b*(-3*a + b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] - 2*(1 + 2*a^2)*b*Arc
Tan[(-(Sqrt[-b^2]*x) + Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/a] + (1 + 2*a^2)
*Sqrt[-b^2]*Log[-1 + 2*a*b*x + 2*b^2*x^2 + 2*Sqrt[-b^2]*x*Sqrt[1 - a^2 - 2
*a*b*x - b^2*x^2]])/(4*b^4)
```

3.101.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {896, 497, 25, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{b^2x^2}{\sqrt{1-(a+bx)^2}}d(a+bx)}{b^3}$$

$$\Big\downarrow \text{497}$$

$$\frac{-\frac{1}{2}\int -\frac{2a^2-3(a+bx)a+1}{\sqrt{1-(a+bx)^2}}d(a+bx)-\frac{1}{2}bx\sqrt{1-(a+bx)^2}}{b^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\int \frac{2a^2-3(a+bx)a+1}{\sqrt{1-(a+bx)^2}}d(a+bx)-\frac{1}{2}bx\sqrt{1-(a+bx)^2}}{b^3}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{1}{2}\left(\left(2a^2+1\right)\int \frac{1}{\sqrt{1-(a+bx)^2}}d(a+bx)+3a\sqrt{1-(a+bx)^2}\right)-\frac{1}{2}bx\sqrt{1-(a+bx)^2}}{b^3}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{1}{2}\left(\left(2a^2+1\right)\arcsin(a+bx)+3a\sqrt{1-(a+bx)^2}\right)-\frac{1}{2}bx\sqrt{1-(a+bx)^2}}{b^3}$$

input
```
Int[x^2/Sqrt[1 - (a + b*x)^2],x]
```

output
```
(-1/2*(b*x*Sqrt[1 - (a + b*x)^2]) + (3*a*Sqrt[1 - (a + b*x)^2] + (1 + 2*a^
2)*ArcSin[a + b*x])/2)/b^3
```

3.101.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

3.101. $\int \frac{x^2}{\sqrt{1-(a+bx)^2}} dx$

3.101.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{(-bx+3a)(b^2x^2+2abx+a^2-1)}{2b^3\sqrt{-b^2x^2-2abx-a^2+1}}+\frac{(2a^2+1)\arctan\left(\frac{\sqrt{b^2}\left(\frac{a}{b}+x\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{2b^2\sqrt{b^2}}$
default	$-\frac{x\sqrt{-b^2x^2-2abx-a^2+1}}{2b^2}-\frac{3a\left(-\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{b^2}-\frac{a\arctan\left(\frac{\sqrt{b^2}\left(\frac{a}{b}+x\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{b\sqrt{b^2}}\right)}{2b}+\frac{(-a^2+1)\arctan\left(\frac{\sqrt{b^2}\left(\frac{a}{b}+x\right)}{\sqrt{-b^2x^2-2abx-}}\right)}{2b^2\sqrt{b^2}}$

input
```
int(x^2/(1-(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-b*x+3*a)*(b^2*x^2+2*a*b*x+a^2-1)/b^3/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
+1/2/b^2*(2*a^2+1)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(a/b+x)/(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2))
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx = -\frac{(2a^2+1)\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}(bx+a)}{b^2x^2+2abx+a^2-1}\right)+\sqrt{-b^2x^2-2abx-a^2+1}(bx-3a)}{2b^3}$$

input
```
integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*((2*a^2 + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2
*x^2 + 2*a*b*x + a^2 - 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x - 3*a
))/b^3
```

3.101.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(54) = 108$.

Time = 0.75 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.45

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx = \begin{cases} \left(\frac{3a}{2b^3}-\frac{x}{2b^2}\right)\sqrt{-a^2-2abx-b^2x^2+1}+\frac{\left(\frac{3a^2}{2b^2}+\frac{1-a^2}{2b^2}\right)\log\left(-2ab-2b^2x+2\sqrt{-b^2}\sqrt{-a^2-2abx-b^2x^2+1}\right)}{\sqrt{-b^2}} & \text{for } b^2\neq 0 \\ -\frac{a^4\sqrt{-a^2-2abx+1}-2a^2\sqrt{-a^2-2abx+1}+\frac{(2a^2-2)(-a^2-2abx+1)^{\frac{3}{2}}}{3}+\frac{(-a^2-2abx+1)^{\frac{5}{2}}}{5}+\sqrt{-a^2-2abx+1}}{4a^3b^3} & \text{for } ab\neq 0 \\ \frac{x^3}{3\sqrt{1-a^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(1-(b*x+a)**2)**(1/2),x)
```

output
```
Piecewise(((3*a/(2*b**3) - x/(2*b**2))*sqrt(-a**2 - 2*a*b*x - b**2*x**2 +
1) + (3*a**2/(2*b**2) + (1 - a**2)/(2*b**2))*log(-2*a*b - 2*b**2*x + 2*sqr
t(-b**2)*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1))/sqrt(-b**2), Ne(b**2, 0)),
 (-(a**4*sqrt(-a**2 - 2*a*b*x + 1) - 2*a**2*sqrt(-a**2 - 2*a*b*x + 1) + (2
*a**2 - 2)*(-a**2 - 2*a*b*x + 1)**(3/2)/3 + (-a**2 - 2*a*b*x + 1)**(5/2)/5
 + sqrt(-a**2 - 2*a*b*x + 1))/(4*a**3*b**3), Ne(a*b, 0)), (x**3/(3*sqrt(1
- a**2)), True))
```

3.101.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(57) = 114$.

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx = -\frac{3\,a^2\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{2\,b^3}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}x}{2\,b^2}+\frac{(a^2-1)\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{2\,b^3}+\frac{3\sqrt{-b^2x^2-2\,abx-a^2+1}a}{2\,b^3}$$

input
```
integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-3/2*a^2*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 - 1/2*sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x/b^2 + 1/2*(a^2 - 1)*arcsin(-(b^2*x + a*
b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 + 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2
+ 1)*a/b^3
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx = -\frac{1}{2}\sqrt{-b^2x^2-2\,abx-a^2+1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right) - \frac{(2\,a^2+1)\arcsin\left(-bx-a\right)\operatorname{sgn}(b)}{2\,b^2|b|}$$

input
```
integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 + 1
)*arcsin(-b*x - a)*sgn(b)/(b^2*abs(b))
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx = \int \frac{x^2}{\sqrt{1-(a+b\,x)^2}}\,dx$$

input
```
int(x^2/(1 - (a + b*x)^2)^(1/2),x)
```

output
```
int(x^2/(1 - (a + b*x)^2)^(1/2), x)
```

3.102 $\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx$

3.102.1 Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = -\frac{3a\sqrt{1+(a+bx)^2}}{2b^3}+\frac{x\sqrt{1+(a+bx)^2}}{2b^2}-\frac{(1-2a^2)\operatorname{arcsinh}(a+bx)}{2b^3}$$

output
```
-1/2*(-2*a^2+1)*arcsinh(b*x+a)/b^3-3/2*a*(1+(b*x+a)^2)^(1/2)/b^3+1/2*x*(1+
(b*x+a)^2)^(1/2)/b^2
```

3.102.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = \frac{(-3a+bx)\sqrt{1+a^2+2abx+b^2x^2}}{2b^3} + \frac{(1-2a^2)\operatorname{arctanh}\left(\frac{bx}{\sqrt{1+a^2}-\sqrt{1+a^2+2abx+b^2x^2}}\right)}{b^3}$$

input
```
Integrate[x^2/Sqrt[1 + (a + b*x)^2],x]
```

output
```
((-3*a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/(2*b^3) + ((1 - 2*a^2)*Ar
cTanh[(b*x)/(Sqrt[1 + a^2] - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])])/b^3
```

3.102.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {896, 497, 25, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{(a+bx)^2+1}}\,dx$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int \frac{b^2x^2}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 497}$$

$$\frac{\frac{1}{2}\int -\frac{-2a^2+3(a+bx)a+1}{\sqrt{(a+bx)^2+1}}d(a+bx)+\frac{1}{2}bx\sqrt{(a+bx)^2+1}}{b^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{1}{2}bx\sqrt{(a+bx)^2+1}-\frac{1}{2}\int \frac{-2a^2+3(a+bx)a+1}{\sqrt{(a+bx)^2+1}}d(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{\frac{1}{2}\left(-\left(1-2a^2\right)\int \frac{1}{\sqrt{(a+bx)^2+1}}d(a+bx)-3a\sqrt{(a+bx)^2+1}\right)+\frac{1}{2}bx\sqrt{(a+bx)^2+1}}{b^3}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{\frac{1}{2}\left(-\left(1-2a^2\right)\text{arcsinh}(a+bx)-3a\sqrt{(a+bx)^2+1}\right)+\frac{1}{2}bx\sqrt{(a+bx)^2+1}}{b^3}$$

input
```
Int[x^2/Sqrt[1 + (a + b*x)^2],x]
```

output
```
((b*x*Sqrt[1 + (a + b*x)^2])/2 + (-3*a*Sqrt[1 + (a + b*x)^2] - (1 - 2*a^2)
*ArcSinh[a + b*x])/2)/b^3
```

3.102.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

3.102.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

method	result
risch	$-\frac{(-bx+3a)\sqrt{b^2x^2+2abx+a^2+1}}{2b^3}+\frac{(2a^2-1)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2b^2\sqrt{b^2}}$
default	$\frac{x\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{3a\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^2}-\frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{b\sqrt{b^2}}\right)}{2b}-\frac{(a^2+1)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2-}\right.}{2b^2\sqrt{b^2}}$

input
```
int(x^2/(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-b*x+3*a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^3+1/2/b^2*(2*a^2-1)*ln((b^
2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = -\frac{(2\,a^2-1)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\sqrt{b^2x^2+2\,abx+a^2+1}(bx-3\,a)}{2\,b^3}$$

input
```
integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*((2*a^2 - 1)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a))/b^3
```

3.102.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(54) = 108$.

Time = 0.64 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.62

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = \begin{cases} \left(-\frac{3a}{2b^3}+\frac{x}{2b^2}\right)\sqrt{a^2+2abx+b^2x^2+1}+\frac{\left(\frac{3a^2}{2b^2}-\frac{a^2+1}{2b^2}\right)\log\left(2ab+2b^2x+2\sqrt{a^2+2abx+b^2x^2+1}\sqrt{b^2}\right)}{\sqrt{b^2}} & \text{for}\ b^2\neq 0 \\ \frac{a^4\sqrt{a^2+2abx+1}+2a^2\sqrt{a^2+2abx+1}+\frac{(-2a^2-2)(a^2+2abx+1)^{\frac{3}{2}}}{3}+\frac{(a^2+2abx+1)^{\frac{5}{2}}}{5}+\sqrt{a^2+2abx+1}}{4a^3b^3} & \text{for}\ ab\neq 0 \\ \frac{x^3}{3\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(1+(b*x+a)**2)**(1/2),x)
```

output
```
Piecewise(((-3*a/(2*b**3) + x/(2*b**2))*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1) + (3*a**2/(2*b**2) - (a**2 + 1)/(2*b**2))*log(2*a*b + 2*b**2*x + 2*sqrt
(a**2 + 2*a*b*x + b**2*x**2 + 1)*sqrt(b**2))/sqrt(b**2), Ne(b**2, 0)), ((a
**4*sqrt(a**2 + 2*a*b*x + 1) + 2*a**2*sqrt(a**2 + 2*a*b*x + 1) + (-2*a**2
- 2)*(a**2 + 2*a*b*x + 1)**(3/2)/3 + (a**2 + 2*a*b*x + 1)**(5/2)/5 + sqrt(
a**2 + 2*a*b*x + 1))/(4*a**3*b**3), Ne(a*b, 0)), (x**3/(3*sqrt(a**2 + 1)),
 True))
```

3.102.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(53) = 106$.

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.14

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = \frac{3\,a^2 \operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^3} + \frac{\sqrt{b^2x^2+2\,abx+a^2+1}x}{2\,b^2} - \frac{(a^2+1)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^3} - \frac{3\sqrt{b^2x^2+2\,abx+a^2+1}a}{2\,b^3}$$

input
```
integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
3/2*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 +
1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 - 1/2*(a^2 + 1)*arcsinh(2*(b^2
*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 3/2*sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)*a/b^3
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = \frac{1}{2}\sqrt{b^2x^2+2\,abx+a^2+1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right) - \frac{(2\,a^2-1)\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{2\,b^2|b|}$$

input
```
integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 - 1)*
log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^2*abs
(b))
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = \int \frac{x^2}{\sqrt{{\left(a+b\,x\right)}^2+1}}\,dx$$

input
```
int(x^2/((a + b*x)^2 + 1)^(1/2),x)
```

output
```
int(x^2/((a + b*x)^2 + 1)^(1/2), x)
```

3.103 $\int \frac{x^3}{a+b(c+dx)^3}\,dx$

3.103.1 Optimal result

Integrand size = 17, antiderivative size = 234

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx = \frac{x}{bd^3}+\frac{\left(a-3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}d^4} - \frac{\left(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{4/3}d^4} + \frac{\left(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{4/3}d^4} - \frac{c\log\left(a+b(c+dx)^3\right)}{bd^4}$$

output
```
x/b/d^3-1/3*(a+3*a^(1/3)*b^(2/3)*c^2+b*c^3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^
(2/3)/b^(4/3)/d^4+1/6*(a+3*a^(1/3)*b^(2/3)*c^2+b*c^3)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/b^(4/3)/d^4-c*ln(a+b*(d*x+c)^3)/
b/d^4+1/3*(a-3*a^(1/3)*b^(2/3)*c^2+b*c^3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d
*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/b^(4/3)/d^4*3^(1/2)
```

3.103.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.56

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx = -\frac{-3bdx+\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&,\frac{a\log(x-\#1)+bc^3\log(x-\#1)+3bc^2d\log(x-\#1)}{c^2+2cd\#1+d^2\#1^2}\right.}{3b^2d^4}$$

input
```
Integrate[x^3/(a + b*(c + d*x)^3),x]
```

output
```
-1/3*(-3*b*d*x + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3
*#1^3 & , (a*Log[x - #1] + b*c^3*Log[x - #1] + 3*b*c^2*d*Log[x - #1]*#1 +
3*b*c*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(b^2*d^4)
```

3.103.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {896, 25, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx$$

$$\Big\downarrow \; 896$$

$$\frac{\int \frac{d^3x^3}{b(c+dx)^3+a}d(c+dx)}{d^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{d^3x^3}{b(c+dx)^3+a}d(c+dx)}{d^4}$$

$$\Big\downarrow \; 2426$$

$$-\frac{\int \left(\frac{bc^3-3b(c+dx)c^2+3b(c+dx)^2c+a}{b(b(c+dx)^3+a)}-\frac{1}{b}\right)d(c+dx)}{d^4}$$

$$\downarrow \text{2009}$$

$$\frac{\frac{\left(-3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}}-\frac{\left(3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{4/3}}+\frac{\left(3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\log\left(a^{2/3}-\right.}{6a^{2/3}b}}{d^4}$$

input
```
Int[x^3/(a + b*(c + d*x)^3),x]
```

output
```
((c + d*x)/b + ((a - 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*ArcTan[(a^(1/3) - 2*b^
(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(4/3)) - ((a + 3*a
^(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b
^(4/3)) + ((a + 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(4/3)) - (c*Log[a + b*(c
 + d*x)^3])/b)/d^4
```

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.103.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.46

method	result	size
default	$\frac{x}{b\,d^{3}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^{3}\_Z^{3}+3bc\,d^{2}\_Z^{2}+3b\,c^{2}d\_Z+b\,c^{3}+a\right)}\frac{\left(-3\_R^{2}bc\,d^{2}-3\_Rb\,c^{2}d-b\,c^{3}-a\right)\ln\left(x-\_R\right)}{d^{2}\_R^{2}+2cd\_R+c^{2}}}{3b^{2}d^{4}}$	108
risch	$\frac{x}{b\,d^{3}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^{3}\_Z^{3}+3bc\,d^{2}\_Z^{2}+3b\,c^{2}d\_Z+b\,c^{3}+a\right)}\frac{\left(-3\_R^{2}bc\,d^{2}-3\_Rb\,c^{2}d-b\,c^{3}-a\right)\ln\left(x-\_R\right)}{d^{2}\_R^{2}+2cd\_R+c^{2}}}{3b^{2}d^{4}}$	108

input
```
int(x^3/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
x/b/d^3+1/3/b^2/d^4*sum((-3*_R^2*b*c*d^2-3*_R*b*c^2*d-b*c^3-a)/(_R^2*d^2+2
*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c
^3+a))
```

3.103.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.97 (sec) , antiderivative size = 6315, normalized size of antiderivative = 26.99

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.103.6 Sympy [A] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx$$
$$=\mathrm{RootSum}\left(27t^3a^2b^4d^{12}+81t^2a^2b^3cd^8+t\left(54a^2b^2c^2d^4-27ab^3c^5d^4\right)+a^3+3a^2bc^3+3ab^2c^6+b^3c^9,\left(t\mapsto t\right.\right.$$
$$+\frac{x}{bd^3}$$

input
```
integrate(x**3/(a+b*(d*x+c)**3),x)
```

output
```
RootSum(27*_t**3*a**2*b**4*d**12 + 81*_t**2*a**2*b**3*c*d**8 + _t*(54*a**2
*b**2*c**2*d**4 - 27*a*b**3*c**5*d**4) + a**3 + 3*a**2*b*c**3 + 3*a*b**2*c
**6 + b**3*c**9, Lambda(_t, _t*log(x + (-27*_t**2*a**2*b**3*c**2*d**8 - 3*
_t*a**3*b*d**4 - 60*_t*a**2*b**2*c**3*d**4 - 3*_t*a*b**3*c**6*d**4 - 2*a**
3*c - 12*a**2*b*c**4 - 9*a*b**2*c**7 + b**3*c**10)/(a**3*d + 3*a**2*b*c**3
*d - 24*a*b**2*c**6*d + b**3*c**9*d)))) + x/(b*d**3)
```

3.103.7 Maxima [F]

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx=\int \frac{x^3}{(dx+c)^3b+a}\,dx$$

input
```
integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
x/(b*d^3) - integrate((3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b*d^3*x^3
 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(b*d^3)
```

3.103.8 Giac [F]

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx=\int \frac{x^3}{(dx+c)^3b+a}\,dx$$

input
```
integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(x^3/((d*x + c)^3*b + a), x)
```

3.103.9 Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.60

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx=\left(\sum_{k=1}^{3}\ln\left(\frac{3\,(b\,c^5+a\,c^2)}{d^2}\right.\right.$$
$$-\mathrm{root}\big(27\,a^2\,b^4\,d^{12}\,z^3+81\,a^2\,b^3\,c\,d^8\,z^2+54\,a^2\,b^2\,c^2\,d^4\,z-27\,a\,b^3\,c^5\,d^4\,z+3\,a\,b^2\,c^6+3\,a^2\,b\,c^3+b^3\,c^9+a^3,z,k\big)$$
$$\left.-\frac{3\,x\,(a\,c-2\,b\,c^4)}{d}\right)\mathrm{root}\big(27\,a^2\,b^4\,d^{12}\,z^3+81\,a^2\,b^3\,c\,d^8\,z^2+54\,a^2\,b^2\,c^2\,d^4\,z-27\,a\,b^3\,c^5\,d^4\,z$$
$$\left.+3\,a\,b^2\,c^6+3\,a^2\,b\,c^3+b^3\,c^9+a^3,z,k\big)\right)+\frac{x}{b\,d^3}$$

input
```
int(x^3/(a + b*(c + d*x)^3),x)
```

output
```
symsum(log((3*(a*c^2 + b*c^5))/d^2 - root(27*a^2*b^4*d^12*z^3 + 81*a^2*b^3
*c*d^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4*z + 3*a*b^2*c^6 + 3*a
^2*b*c^3 + b^3*c^9 + a^3, z, k)*((3*(b^2*c^4*d^4 - 5*a*b*c*d^4))/d^2 + (3*
x*(b^2*c^3*d^4 + a*b*d^4))/d - 9*root(27*a^2*b^4*d^12*z^3 + 81*a^2*b^3*c*d
^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4*z + 3*a*b^2*c^6 + 3*a^2*b
*c^3 + b^3*c^9 + a^3, z, k)*a*b^2*d^6) - (3*x*(a*c - 2*b*c^4))/d)*root(27*
a^2*b^4*d^12*z^3 + 81*a^2*b^3*c*d^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*
c^5*d^4*z + 3*a*b^2*c^6 + 3*a^2*b*c^3 + b^3*c^9 + a^3, z, k), k, 1, 3) + x
/(b*d^3)
```

3.104 $\int \frac{x^2}{a+b(c+dx)^3} \, dx$

3.104.1 Optimal result

Integrand size = 17, antiderivative size = 210

$$\int \frac{x^2}{a+b(c+dx)^3} \, dx = \frac{c\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^3} + \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^3} - \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^3} + \frac{\log\left(a+b(c+dx)^3\right)}{3bd^3}$$

output
```
1/3*c*(2*a^(1/3)+b^(1/3)*c)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(2/3)/d^
3-1/6*c*(2*a^(1/3)+b^(1/3)*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(
d*x+c)^2)/a^(2/3)/b^(2/3)/d^3+1/3*ln(a+b*(d*x+c)^3)/b/d^3+1/3*c*(2*a^(1/3)
-b^(1/3)*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3
)/b^(2/3)/d^3*3^(1/2)
```

3.104.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \frac{\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&,\frac{\log(x-\#1)\#1^2}{c^2+2cd\#1+d^2\#1^2}\&\right]}{3bd}$$

input
```
Integrate[x^2/(a + b*(c + d*x)^3),x]
```

output
```
RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (Log[x
 - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ]/(3*b*d)
```

3.104.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.94, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {896, 2410, 792, 2399, 16, 25, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int \frac{d^2x^2}{b(c+dx)^3+a}d(c+dx)}{d^3}$$

$$\Big\downarrow \text{ 2410}$$

$$\frac{\int \frac{c^2-2c(c+dx)}{b(c+dx)^3+a}d(c+dx)+\int \frac{(c+dx)^2}{b(c+dx)^3+a}d(c+dx)}{d^3}$$

$$\Big\downarrow \text{ 792}$$

$$\frac{\int \frac{c^2-2c(c+dx)}{b(c+dx)^3+a}d(c+dx)+\frac{\log\left(a+b(c+dx)^3\right)}{3b}}{d^3}$$

$$\Big\downarrow \; 2399$$

$$\frac{\frac{\int -\frac{c\left(2\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)+\sqrt[3]{b}\left(\sqrt[3]{b}c+2\sqrt[3]{a}\right)(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}+\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}+\frac{\log\left(a+b(c+dx)^3\right)}{3b}}{d^3}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{\int -\frac{c\left(2\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)+\sqrt[3]{b}\left(\sqrt[3]{b}c+2\sqrt[3]{a}\right)(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}+\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}}+\frac{\log\left(a+b(c+dx)^3\right)}{3b}}{d^3}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \frac{c\left(2\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)+\sqrt[3]{b}\left(\sqrt[3]{b}c+2\sqrt[3]{a}\right)(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}+\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}}+\frac{\log\left(a+b(c+dx)^3\right)}{3b}}{d^3}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{c\int \frac{2\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)+\sqrt[3]{b}\left(\sqrt[3]{b}c+2\sqrt[3]{a}\right)(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}+\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}}+\frac{\log\left(a+b(c+dx)^3\right)}{3b}}{d^3}$$

$$\Big\downarrow \; 1142$$

$$\frac{-\frac{c\left(\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\left(\frac{2\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{c\left(\cdots\right.}{\cdots}}{d^3}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{c\left(\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\left(\frac{2\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{c\left(2\cdots\right.}{\cdots}}{d^3}$$

$$\Big\downarrow \; 27$$

3.104. $\int \frac{x^2}{a+b(c+dx)^3}\,dx$

$$-\frac{\frac{c\left(\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\sqrt[3]{b}\left(\frac{2\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}+}{d^3}$$

$\downarrow$ 1082

$$-\frac{\frac{c\left(\frac{3\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\sqrt[3]{b}\left(\frac{2\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{c\left(2\sqrt[3]{\cdots}\right.}{\cdots}}{d^3}$$

$\downarrow$ 217

$$-\frac{\frac{c\left(-\frac{1}{2}\sqrt[3]{b}\left(\frac{2\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\cdots\right.}{3a^{2/3}b\cdots}}{d^3}$$

$\downarrow$ 1103

$$-\frac{\frac{c\left(\frac{1}{2}\left(\frac{2\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)-\frac{\sqrt{3}\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{3a^{2/3}b^{2/3}}}{d^3}$$

input
```
Int[x^2/(a + b*(c + d*x)^3),x]
```

3.104. $\int\frac{x^2}{a+b(c+dx)^3}\,dx$

output
```
((c*(2*a^(1/3) + b^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b
^(2/3)) - (c*(-((Sqrt[3]*(2*a^(1/3) - b^(1/3)*c)*ArcTan[(1 - (2*b^(1/3)*(c
 + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3)) + (((2*a^(1/3))/b^(1/3) + c)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/2))/(3*a^(2/3)*b^(
1/3)) + Log[a + b*(c + d*x)^3]/(3*b))/d^3
```

3.104.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

3.104.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,d^{3}\_Z^{3}+3bc\,d^{2}\_Z^{2}+3b\,c^{2}d\_Z+b\,c^{3}+a\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{d^{2}\_R^{2}+2cd\_R+c^{2}}}{3bd}$	74
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,d^{3}\_Z^{3}+3bc\,d^{2}\_Z^{2}+3b\,c^{2}d\_Z+b\,c^{3}+a\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{d^{2}\_R^{2}+2cd\_R+c^{2}}}{3bd}$	74

input
```
int(x^2/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3/b/d*sum(_R^2/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_
Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

3.104.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 4759, normalized size of antiderivative = 22.66

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b
^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) +
2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b
^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) -
 2/(b*d^3))*b*d^3*log(-1/2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/
(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 -
 a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*
(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2
*b^3*d^9))^(1/3) - 2/(b*d^3))^2*a^2*b^2*d^6 + b^2*c^6 - a*b*c^3 - 1/2*(a*b
^2*c^3 + 4*a^2*b)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^
6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^
3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) +
(1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3
- a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9)
)^(1/3) - 2/(b*d^3))*d^3 + (b^2*c^5 - 8*a*b*c^2)*d*x - 2*a^2) - ((2*(1/2)^
(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 -
 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2
*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) +...
```

3.104.6 Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx$$
$$= \text{RootSum}\left(27t^3a^2b^3d^9 - 27t^2a^2b^2d^6 + t\left(9a^2bd^3 - 18ab^2c^3d^3\right) - a^2 - 2abc^3 - b^2c^6, \left(t \mapsto t\log\left(x + \frac{18t^2a}{\ldots}\right.\right.\right.$$

input
```
integrate(x**2/(a+b*(d*x+c)**3),x)
```

output
```
RootSum(27*_t**3*a**2*b**3*d**9 - 27*_t**2*a**2*b**2*d**6 + _t*(9*a**2*b*d
**3 - 18*a*b**2*c**3*d**3) - a**2 - 2*a*b*c**3 - b**2*c**6, Lambda(_t, _t*
log(x + (18*_t**2*a**2*b**2*d**6 - 12*_t*a**2*b*d**3 - 3*_t*a*b**2*c**3*d*
*3 + 2*a**2 + a*b*c**3 - b**2*c**6)/(8*a*b*c**2*d - b**2*c**5*d))))
```

3.104.7 Maxima [F]

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \int \frac{x^2}{(dx+c)^3b+a}\,dx$$

input
```
integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(x^2/((d*x + c)^3*b + a), x)
```

3.104.8 Giac [F]

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \int \frac{x^2}{(dx+c)^3b+a}\,dx$$

input
```
integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(x^2/((d*x + c)^3*b + a), x)
```

3.104.9 Mupad [B] (verification not implemented)

Time = 9.86 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.08

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \sum_{k=1}^{3} \ln\Big(a+b\,c^3-\mathrm{root}\big(27\,a^2\,b^3\,d^9\,z^3-27\,a^2\,b^2\,d^6\,z^2-18\,a\,b^2\,c^3\,d^3\,z+9\,a^2\,b\,d^3\,z-2\,a\,b\,c^3-b^2\,c^6-a^2,z,k\big)\,a\,b\,d^3\,6+3\,b\,c^2\,d\,x+\mathrm{root}\big(27\,a^2\,b^3\,d^9\,z^3-27\,a^2\,b^2\,d^6\,z^2-18\,a\,b^2\,c^3\,d^3\,z+9\,a^2\,b\,d^3\,z-2\,a\,b\,c^3-b^2\,c^6-a^2,z,k\big)^2\,a\,b^2\,d^6\,9+\mathrm{root}\big(27\,a^2\,b^3\,d^9\,z^3-27\,a^2\,b^2\,d^6\,z^2-18\,a\,b^2\,c^3\,d^3\,z+9\,a^2\,b\,d^3\,z-2\,a\,b\,c^3-b^2\,c^6-a^2,z,k\big)\,b^2\,c^3\,d^3\,3+\mathrm{root}\big(27\,a^2\,b^3\,d^9\,z^3-27\,a^2\,b^2\,d^6\,z^2-18\,a\,b^2\,c^3\,d^3\,z+9\,a^2\,b\,d^3\,z-2\,a\,b\,c^3-b^2\,c^6-a^2,z,k\big)\,b^2\,c^2\,d^4\,x\,3\Big)\,\mathrm{root}\big(27\,a^2\,b^3\,d^9\,z^3-27\,a^2\,b^2\,d^6\,z^2-18\,a\,b^2\,c^3\,d^3\,z+9\,a^2\,b\,d^3\,z-2\,a\,b\,c^3-b^2\,c^6-a^2,z,k\big)$$

input
```
int(x^2/(a + b*(c + d*x)^3),x)
```

output
```
symsum(log(a + b*c^3 - 6*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18
*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)*a*b*d^
3 + 3*b*c^2*d*x + 9*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^
2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)^2*a*b^2*d^6
 + 3*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9
*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)*b^2*c^3*d^3 + 3*root(27*a^
2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z -
2*a*b*c^3 - b^2*c^6 - a^2, z, k)*b^2*c^2*d^4*x)*root(27*a^2*b^3*d^9*z^3 -
27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*
c^6 - a^2, z, k), k, 1, 3)
```

3.105 $\int \frac{x}{a+b(c+dx)^3} dx$

3.105.1 Optimal result

Integrand size = 15, antiderivative size = 180

$$\int \frac{x}{a+b(c+dx)^3} dx = -\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^2} - \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2} + \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^2}$$

output
```
-1/3*(a^(1/3)+b^(1/3)*c)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(2/3)/d^2+1
/6*(a^(1/3)+b^(1/3)*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^
2)/a^(2/3)/b^(2/3)/d^2-1/3*(a^(1/3)-b^(1/3)*c)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/b^(2/3)/d^2*3^(1/2)
```

3.105.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.44

$$\int \frac{x}{a+b(c+dx)^3} dx = \frac{\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&,\frac{\log(x-\#1)\#1}{c^2+2cd\#1+d^2\#1^2}\&\right]}{3bd}$$

input
```
Integrate[x/(a + b*(c + d*x)^3),x]
```

output
```
RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (Log[x
- #1]*#1)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ]/(3*b*d)
```

3.105.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {896, 25, 2399, 16, 25, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b(c+dx)^3}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{dx}{b(c+dx)^3+a}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{dx}{b(c+dx)^3+a}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{2399}$$

$$-\frac{\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-2\sqrt[3]{b}c\right)+\sqrt[3]{b}\left(\sqrt[3]{b}c+\sqrt[3]{a}\right)(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}}{d^2}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-2\sqrt[3]{b}c\right)+\sqrt[3]{b}\left(\sqrt[3]{b}c+\sqrt[3]{a}\right)(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}}}{d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\sqrt[3]{a}\left(\sqrt[3]{a}-2\sqrt[3]{b}c\right)+\sqrt[3]{b}\left(\sqrt[3]{b}c+\sqrt[3]{a}\right)(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}}}{d^2}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{}}{d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right.}{}}{d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\sqrt[3]{b}\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{}}{d^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{\frac{3\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\sqrt[3]{b}\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)}{}}{d^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{-\frac{1}{2}\sqrt[3]{b}\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(\right.}{3a^{2/3}b^{2/3}}}{d^2}$$

$$\Big\downarrow \text{1103}$$

3.105. $\int \frac{x}{a+b(c+dx)^3}\,dx$

$$\frac{\frac{\frac{1}{2}\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)-\frac{\sqrt{3}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}}}{d^2}$$

input
```
Int[x/(a + b*(c + d*x)^3),x]
```

output
```
(-1/3*((a^(1/3) + b^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(a^(2/3)*b^
(2/3)) + (-((Sqrt[3]*(a^(1/3) - b^(1/3)*c)*ArcTan[(1 - (2*b^(1/3)*(c + d*x
))/a^(1/3))/Sqrt[3]])/b^(1/3)) + ((a^(1/3)/b^(1/3) + c)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/2)/(3*a^(2/3)*b^(1/3)))/d^2
```

3.105.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

3.105.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.40

method	result	size
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\_R\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	72
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\_R\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	72

input
```
int(x/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3/b/d*sum(_R/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^
2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

3.105.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 1950, normalized size of antiderivative = 10.83

$$\int \frac{x}{a+b(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a
)/(a^2*b^2*d^6))^(1/3) - 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c
^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt
(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2
*b^2*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4)) + c*(-I*sqrt(3) + 1)/(a*b
*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1
/3)))*log(1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*
(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c
^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^
4 - 1/6*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3
- a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)
/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + 2*b
*c^4 + 2*(b*c^3 - a)*d*x - 4*a*c + 1/12*sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/
54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(
-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 -
a)/(a^2*b^2*d^6))^(1/3)))*a^2*b*d^4 + 6*a*b*c^2*d^2)*sqrt(-((9*(I*sqrt(3)
+ 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1
/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*
(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4))) + 1/36*(
9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(...
```

3.105.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.46

$$\int \frac{x}{a+b(c+dx)^3}\,dx$$
$$= \operatorname{RootSum}\left(27t^3a^2b^2d^6 - 9tabcd^2 + a + bc^3, \left(t \mapsto t\log\left(x + \frac{9t^2a^2bd^4 + 3tabc^2d^2 - ac - bc^4}{ad - bc^3d}\right)\right)\right)$$

input
```
integrate(x/(a+b*(d*x+c)**3),x)
```

output
```
RootSum(27*_t**3*a**2*b**2*d**6 - 9*_t*a*b*c*d**2 + a + b*c**3, Lambda(_t,
 _t*log(x + (9*_t**2*a**2*b*d**4 + 3*_t*a*b*c**2*d**2 - a*c - b*c**4)/(a*d
 - b*c**3*d))))
```

3.105.7 Maxima [F]

$$\int \frac{x}{a+b(c+dx)^3}\,dx = \int \frac{x}{(dx+c)^3b+a}\,dx$$

input
```
integrate(x/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(x/((d*x + c)^3*b + a), x)
```

3.105.8 Giac [F]

$$\int \frac{x}{a+b(c+dx)^3}\,dx = \int \frac{x}{(dx+c)^3b+a}\,dx$$

input
```
integrate(x/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(x/((d*x + c)^3*b + a), x)
```

3.105.9 Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.81

$$\int \frac{x}{a+b(c+dx)^3}\,dx = \sum_{k=1}^{3}\ln\left(-\mathrm{root}\left(27\,a^2\,b^2\,d^6\,z^3-9\,a\,b\,c\,d^2\,z+b\,c^3+a,z,k\right)\,\left(3\,b^2\,c^2\,d^4-\mathrm{root}\left(27\,a^2\,b^2\,d^6\,z^3-9\,a\,b\,c\,d^2\,z+b\,c^3+a,z,k\right)\,a\,b^2\,d^6\,9+3\,b^2\,c\,d^5\,x\right)+b\,d^3\,x\right)\,\mathrm{root}\left(27\,a^2\,b^2\,d^6\,z^3-9\,a\,b\,c\,d^2\,z+b\,c^3+a,z,k\right)$$

input
```
int(x/(a + b*(c + d*x)^3),x)
```

output
```
symsum(log(b*d^3*x - root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 + a,
z, k)*(3*b^2*c^2*d^4 - 9*root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 +
 a, z, k)*a*b^2*d^6 + 3*b^2*c*d^5*x))*root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^
2*z + b*c^3 + a, z, k), k, 1, 3)
```

3.105. $\int \frac{x}{a+b(c+dx)^3} \, dx$

3.106 $\int \frac{1}{a+b(c+dx)^3} dx$

3.106.1 Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{1}{a+b(c+dx)^3} dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d}$$

output
```
1/3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(1/3)/d-1/6*ln(a^(2/3)-a^(1/3)*b
^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/b^(1/3)/d-1/3*arctan(1/3*(a^(1/3
)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/b^(1/3)/d*3^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\int \frac{1}{a+b(c+dx)^3} dx = \frac{2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d}$$

input
```
Integrate[(a + b*(c + d*x)^3)^(-1),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] + 2*
Log[a^(1/3) + b^(1/3)*(c + d*x)] - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x)
 + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(1/3)*d)
```

3.106.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {239, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b(c+dx)^3}dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \frac{1}{b(c+dx)^3+a}d(c+dx)}{d}$$

$$\Big\downarrow \text{750}$$

$$\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}}}{d}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

input
```
Int[(a + b*(c + d*x)^3)^(-1),x]
```

output
```
(Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan
[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^
(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(2/3)))/d
```

3.106. $\int \frac{1}{a+b(c+dx)^3}\,dx$

3.106.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.106.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	71
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	71

input
```
int(1/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2
*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 442, normalized size of antiderivative = 3.16

$$\int \frac{1}{a+b(c+dx)^3}\,dx$$
$$= \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abd^3x^3+6\,abcd^2x^2+6\,abc^2dx+2\,abc^3-a^2+3\sqrt{\frac{1}{3}}\left(2\,abd^2x^2+4\,abcdx+2\,abc^2+\left(a^2b\right)^{\frac{2}{3}}(dx+c)-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-}}{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\right.}{}\right.$$

input
```
integrate(1/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*d^3*x^3 + 6*a*b*c*
d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3*sqrt(1/3)*(2*a*b*d^2*x^2 + 4
*a*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt(-
(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c))/(b*d^3*x^3 + 3*b*c*d^2*x
^2 + 3*b*c^2*d*x + b*c^3 + a)) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d
*x + a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3
)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d), 1/6*(6*sqrt(1/3)*a*b*sq
rt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(d*x + c) - (a^2*b)^
(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*
b*c*d*x + a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)
^(2/3)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d)]
```

3.106.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.19

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \frac{\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(x+\frac{3ta+c}{d}\right)\right)\right)}{d}$$

input
```
integrate(1/(a+b*(d*x+c)**3),x)
```

output
```
RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(x + (3*_t*a + c)/d)))/d
```

3.106.7 Maxima [F]

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \int \frac{1}{(dx+c)^3b+a}\,dx$$

input
```
integrate(1/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)^3*b + a), x)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \frac{1}{3}\sqrt{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+(ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}\,(ab^2)^{\frac{1}{3}}}\right) - \frac{1}{6}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}(ab^2)^{\frac{1}{3}}\right)^2 + 4\left(bdx+bc+(ab^2)^{\frac{1}{3}}\right)^2\right) + \frac{1}{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\log\left(\left|bdx+bc+(ab^2)^{\frac{1}{3}}\right|\right)$$

input
```
integrate(1/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))/(s
qrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - 1/6*(1/(a^2*b*d^3))
^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(
b*d*x + b*c + (a*b^2)^(1/3))^2) + 1/3*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x
+ b*c + (a*b^2)^(1/3)))
```

3.106.9 Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.03

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \frac{\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{3\,a^{2/3}\,b^{1/3}\,d} + \frac{\ln\left(3\,b^2\,c\,d^5+3\,b^2\,d^6\,x+\frac{3\,a^{1/3}\,b^{5/3}\,d^5\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d} - \frac{\ln\left(3\,b^2\,c\,d^5+3\,b^2\,d^6\,x-\frac{3\,a^{1/3}\,b^{5/3}\,d^5\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d}$$

input
```
int(1/(a + b*(c + d*x)^3),x)
```

output
```
log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x)/(3*a^(2/3)*b^(1/3)*d) + (log(3*b^2*
c*d^5 + 3*b^2*d^6*x + (3*a^(1/3)*b^(5/3)*d^5*(3^(1/2)*1i - 1))/2)*(3^(1/2)
*1i - 1))/(6*a^(2/3)*b^(1/3)*d) - (log(3*b^2*c*d^5 + 3*b^2*d^6*x - (3*a^(1
/3)*b^(5/3)*d^5*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)*b^(1/3)*
d)
```

3.107 $\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx$

3.107.1 Optimal result

Integrand size = 17, antiderivative size = 224

$$\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx=\frac{\sqrt[3]{b}c\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}c+b^{2/3}c^2\right)}+\frac{\log(x)}{a+bc^3}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)}-\frac{\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}c+b^{2/3}c^2\right)}$$

output
```
ln(x)/(b*c^3+a)-1/3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(a^(1/3)+b^(1/3)*c
)-1/6*(2*a^(1/3)-b^(1/3)*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*
x+c)^2)/a^(2/3)/(a^(2/3)-a^(1/3)*b^(1/3)*c+b^(2/3)*c^2)+1/3*b^(1/3)*c*arct
an(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(a^(2/3)-a^(1/
3)*b^(1/3)*c+b^(2/3)*c^2)*3^(1/2)
```

3.107.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.53

$$\int \frac{1}{x\,(a+b(c+dx)^3)}\,dx =$$
$$-\frac{-3\log(x)+\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&,\frac{3c^2\log(x-\#1)+3cd\log(x-\#1)\#1+d^2\log(a}{c^2+2cd\#1+d^2\#1^2}\right.}{3\,(a+bc^3)}$$

input
```
Integrate[1/(x*(a + b*(c + d*x)^3)),x]
```

output
```
-1/3*(-3*Log[x] + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^
3*#1^3 & , (3*c^2*Log[x - #1] + 3*c*d*Log[x - #1]*#1 + d^2*Log[x - #1]*#1^
2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(a + b*c^3)
```

3.107.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {896, 25, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\,(a+b(c+dx)^3)}\,dx$$
$$\Big\downarrow \text{896}$$
$$\int \frac{1}{dx\,(a+b(c+dx)^3)}d(c+dx)$$
$$\Big\downarrow \text{25}$$
$$-\int -\frac{1}{dx\,(b(c+dx)^3+a)}d(c+dx)$$
$$\Big\downarrow \text{7276}$$
$$-\int \left(\frac{b(c^2+(c+dx)c+(c+dx)^2)}{(bc^3+a)\,(b(c+dx)^3+a)}-\frac{1}{(bc^3+a)\,dx}\right)d(c+dx)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)}-\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)}+\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)}+\frac{\log(-dx)}{a+bc^3}-\frac{\log\left(a+b(c+dx)^3\right)}{3\left(a+bc^3\right)}$$

input
```
Int[1/(x*(a + b*(c + d*x)^3)),x]
```

output
```
(b^(1/3)*c*(a^(1/3) + b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(S
qrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)) + Log[-(d*x)]/(a + b*c^3)
+ (b^(1/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^
(2/3)*(a + b*c^3)) - (b^(1/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)) - Log[
a + b*(c + d*x)^3]/(3*(a + b*c^3))
```

3.107.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.107.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.47

method	result
default	$\frac{\ln(x)}{b\,c^3+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\left(d^2\_R^2+3cd\_R+3c^2\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3(b\,c^3+a)}$
risch	$\frac{\ln(-x)}{b\,c^3+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a^2b\,c^3+a^3\right)\_Z^3+3a^2\_Z^2+3a\_Z\right)}\_R\ln\left(\left((2ab\,c^3d-4d\,a^2)\_R^2+(b\,c^3d-8da)\_R-4d\right)x+(ab\,c^4+c\right.$ … $}{3}$

input
```
int(1/x/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/(b*c^3+a)-1/3*sum((_R^2*d^2+3*_R*c*d+3*c^2)/(_R^2*d^2+2*_R*c*d+c^2)*
ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/(b*c^3
+a)
```

3.107.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 4370, normalized size of antiderivative = 19.51

$$\int \frac{1}{x\,(a+b(c+dx)^3)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/12*(2*(b*c^3 + a)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1
/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b
*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3)
 + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2
)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))*log(b*c^2*d*x + b
*c^3 + 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 +
 a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3)
+ 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(
I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*
c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 - 1/2*
(a*b*c^3 - 2*a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(
b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c
^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) +
 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*
(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + a) - ((b*c^3 + a)
*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*
c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3
+ a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^
3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/
(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 3*sqrt(1/3)*(b*c^3 + a)*sqrt(-(...
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(a+b*(d*x+c)**3),x)
```

output
```
Timed out
```

3.107.7 Maxima [F]

$$\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx = \int \frac{1}{\left((dx+c)^3b+a\right)x}\,dx$$

input
```
integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-b*d*integrate((d^2*x^2 + 3*c*d*x + 3*c^2)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*
b*c^2*d*x + b*c^3 + a), x)/(b*c^3 + a) + log(x)/(b*c^3 + a)
```

3.107.8 Giac [F]

$$\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx = \int \frac{1}{\left((dx+c)^3b+a\right)x}\,dx$$

input
```
integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(1/(((d*x + c)^3*b + a)*x), x)
```

3.107.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 553, normalized size of antiderivative = 2.47

$$\int \frac{1}{x\,(a+b(c+dx)^3)}\,dx = \frac{\ln(x)}{b\,c^3+a} + \left(\sum_{k=1}^{3} \ln\left(\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^2 b^4\,c^4\,d^8\,3 - \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)\,b^3\,c\,d^8\,3 - \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)\,b^3\,d^9\,x\,4 - \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^2 a\,b^3\,c\,d^8\,6 - \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^2 a\,b^3\,d^9\,x\,24 + \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^3 a^2\,b^3\,c\,d^8\,9 + \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^3 a\,b^4\,c^4\,d^8\,9 - \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^3 a^2\,b^3\,d^9\,x\,36 + \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^2 b^4\,c^3\,d^9\,x\,3 + \mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^3 a\,b^4\,c^3\,d^9\,x\,18\right)\,\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)\right)$$

input
```
int(1/(x*(a + b*(c + d*x)^3)),x)
```

output
```
log(x)/(a + b*c^3) + symsum(log(3*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*
a^2*z^2 + 9*a*z + 1, z, k)^2*b^4*c^4*d^8 - 3*root(27*a^2*b*c^3*z^3 + 27*a^
3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)*b^3*c*d^8 - 4*root(27*a^2*b*c^3*z^3
+ 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)*b^3*d^9*x - 6*root(27*a^2*b*c
^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*a*b^3*c*d^8 - 24*roo
t(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*a*b^3*d^
9*x + 9*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)
^3*a^2*b^3*c*d^8 + 9*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a
*z + 1, z, k)^3*a*b^4*c^4*d^8 - 36*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27
*a^2*z^2 + 9*a*z + 1, z, k)^3*a^2*b^3*d^9*x + 3*root(27*a^2*b*c^3*z^3 + 27
*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*b^4*c^3*d^9*x + 18*root(27*a^2*
b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a*b^4*c^3*d^9*x)*
root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k), k, 1,
3)
```

3.108 $\int \frac{1}{x^2(a+b(c+dx)^3)}\,dx$

3.108.1 Optimal result

Integrand size = 17, antiderivative size = 314

$$\begin{aligned}\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx &= -\frac{1}{(a+bc^3)\,x}+\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3 d\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)^2}\\&\quad-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}+\frac{\sqrt[3]{b}\left(\sqrt[3]{a}(a-2bc^3)-\sqrt[3]{b}c(2a-bc^3)\right)d\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^2}\\&\quad-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}(a-2bc^3)-\sqrt[3]{b}c(2a-bc^3)\right)d\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^2}\\&\quad+\frac{bc^2d\log\left(a+b(c+dx)^3\right)}{\left(a+bc^3\right)^2}\end{aligned}$$

output
```
-1/(b*c^3+a)/x-3*b*c^2*d*ln(x)/(b*c^3+a)^2+1/3*b^(1/3)*(a^(1/3)*(-2*b*c^3+
a)-b^(1/3)*c*(-b*c^3+2*a))*d*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(b*c^3+a)
^2-1/6*b^(1/3)*(a^(1/3)*(-2*b*c^3+a)-b^(1/3)*c*(-b*c^3+2*a))*d*ln(a^(2/3)-
a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/(b*c^3+a)^2+b*c^2*d*ln(
a+b*(d*x+c)^3)/(b*c^3+a)^2+1/3*b^(1/3)*(a^(1/3)-b^(1/3)*c)*(a^(1/3)+b^(1/3
)*c)^3*d*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(
b*c^3+a)^2*3^(1/2)
```

3.108.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^2 (a+b(c+dx)^3)} \, dx = \frac{-3(a+bc^3+3bc^2dx\log(x))+dx\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&,\frac{-3ac\log(x-\#1)+6b}{}\right.}{3\left(a+bc^3\right)^2 x}$$

input
```
Integrate[1/(x^2*(a + b*(c + d*x)^3)),x]
```

output
```
(-3*(a + b*c^3 + 3*b*c^2*d*x*Log[x]) + d*x*RootSum[a + b*c^3 + 3*b*c^2*d*#
1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (-3*a*c*Log[x - #1] + 6*b*c^4*Log[x -
#1] - a*d*Log[x - #1]*#1 + 8*b*c^3*d*Log[x - #1]*#1 + 3*b*c^2*d^2*Log[x -
#1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(3*(a + b*c^3)^2*x)
```

3.108.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {896, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+b(c+dx)^3)} \, dx$$

$$\Big\downarrow \text{896}$$

$$d\int \frac{1}{d^2x^2 \left(b(c+dx)^3+a\right)}d(c+dx)$$

$$\Big\downarrow \text{7276}$$

$$d\int \left(-\frac{3bc^2}{\left(bc^3+a\right)^2 dx}+\frac{b\left(3bc^2(c+dx)^2-\left(a-2bc^3\right)(c+dx)-c\left(2a-bc^3\right)\right)}{\left(bc^3+a\right)^2\left(b(c+dx)^3+a\right)}+\frac{1}{\left(bc^3+a\right)d^2x^2}\right)d(c+dx)$$

$$\Big\downarrow \text{2009}$$

$$d\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)^2}+\frac{b^{2/3}\left(-\frac{\sqrt[3]{a}(a-2bc^3)}{\sqrt[3]{b}}+2ac-bc^4\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{}\right.}{6a^{2/3}\left(a+bc^3\right)^2}\right.$$

input
```
Int[1/(x^2*(a + b*(c + d*x)^3)),x]
```

output
```
d*(-(1/((a + b*c^3)*d*x)) + (b^(1/3)*(a^(1/3) - b^(1/3)*c)*(a^(1/3) + b^(1
/3)*c)^3*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[
3]*a^(2/3)*(a + b*c^3)^2) - (3*b*c^2*Log[-(d*x)])/(a + b*c^3)^2 + (b^(1/3)
*(a^(1/3)*(a - 2*b*c^3) - b^(1/3)*c*(2*a - b*c^3))*Log[a^(1/3) + b^(1/3)*(
c + d*x)])/(3*a^(2/3)*(a + b*c^3)^2) + (b^(2/3)*(2*a*c - b*c^4 - (a^(1/3)*
(a - 2*b*c^3))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*
(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)^2) + (b*c^2*Log[a + b*(c + d*x)^3])/(
a + b*c^3)^2)
```

3.108.3.1 Defintions of rubi rules used

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.108.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.46

method	result
default	$-\frac{1}{(b\,c^3+a)x}-\frac{3b\,c^2 d\ln(x)}{(b\,c^3+a)^2}-\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2 d\_Z+b\,c^3+a\right)}\frac{\left(-3\_R^2 b\,c^2 d^2-8\_R b\,c^3 d-6b\,c^4+\_R a d+\right.}{d^2\_R^2+2cd\_R+c^2}\right.}{3(b\,c^3+a)^2}$
risch	$-\frac{1}{(b\,c^3+a)x}-\frac{3b\,c^2 d\ln(x)}{c^6 b^2+2ab\,c^3+a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^2 b^2 c^6+2b\,c^3 a^3+a^4\right)\_Z^3-9a^2 b\,c^2 d\_Z^2+6abc\,d^2\_Z-b\,d^3\right)}\_R\ln\left(\left(\left(2a\,b^3 c^9 d-\right.\right.\right.\right.}{}$

input
```
int(1/x^2/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
-1/(b*c^3+a)/x-3*b*c^2*d*ln(x)/(b*c^3+a)^2-1/3*d*sum((-3*_R^2*b*c^2*d^2-8*
_R*b*c^3*d-6*b*c^4+_R*a*d+3*a*c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootO
f(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/(b*c^3+a)^2
```

3.108.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 8919, normalized size of antiderivative = 28.40

$$\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a+b(c+dx)^3\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(a+b*(d*x+c)**3),x)
```

output
```
Timed out
```

3.108.7 Maxima [F]

$$\int \frac{1}{x^2 \left(a+b(c+dx)^3\right)}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-3*b*c^2*d*log(x)/(b^2*c^6 + 2*a*b*c^3 + a^2) + b*d^2*integrate((3*b*c^2*d
^2*x^2 + 6*b*c^4 + (8*b*c^3 - a)*d*x - 3*a*c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 +
 3*b*c^2*d*x + b*c^3 + a), x)/(b^2*c^6 + 2*a*b*c^3 + a^2) - 1/((b*c^3 + a)
*x)
```

3.108.8 Giac [F]

$$\int \frac{1}{x^2 \left(a+b(c+dx)^3\right)}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(1/(((d*x + c)^3*b + a)*x^2), x)
```

3.108.9 Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 1588, normalized size of antiderivative = 5.06

$$\int \frac{1}{x^2 (a + b(c + dx)^3)} dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*(c + d*x)^3)),x)
```

output
```
symsum(log((b^4*d^12*x - 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27
*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^3*b^3*d^
9 - 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c
^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*b^6*c^9*d^9 - 9*root(27*a^2*b^2
*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d
^2*z - b*d^3, z, k)*b^5*c^5*d^10 + 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c
^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2
*a^2*b^4*c^3*d^9 + 27*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*
z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^3*b^4*c^4*d^8
 + 27*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c
^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^2*b^5*c^7*d^8 - 9*root(27*a^2
*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b
*c*d^2*z - b*d^3, z, k)*a*b^4*c^2*d^10 - 9*root(27*a^2*b^2*c^6*z^3 + 54*a^
3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z,
 k)*b^5*c^4*d^11*x + 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4
*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^4*b^3*c*d^8
+ 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^
2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a*b^5*c^6*d^9 + 9*root(27*a^2*b^
2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*
d^2*z - b*d^3, z, k)^3*a*b^6*c^10*d^8 - 36*root(27*a^2*b^2*c^6*z^3 + 54...
```

3.109 $\int \frac{1}{x^3(a+b(c+dx)^3)}\,dx$

3.109.1 Optimal result

Integrand size = 17, antiderivative size = 393

$$\begin{aligned}\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx &= -\frac{1}{2\left(a+bc^3\right)x^2}+\frac{3bc^2d}{\left(a+bc^3\right)^2x} \\ &+\frac{b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3\left(a-3a^{2/3}\sqrt[3]{b}c+bc^3\right)d^2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)^3} \\ &-\frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} \\ &-\frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2-7abc^3-3\sqrt[3]{a}b^{5/3}c^5+b^2c^6\right)d^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^3} \\ &+\frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2-7abc^3-3\sqrt[3]{a}b^{5/3}c^5+b^2c^6\right)d^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^3} \\ &+\frac{bc\left(a-2bc^3\right)d^2\log\left(a+b(c+dx)^3\right)}{\left(a+bc^3\right)^3}\end{aligned}$$

output
```
-1/2/(b*c^3+a)/x^2+3*b*c^2*d/(b*c^3+a)^2/x-3*b*c*(-2*b*c^3+a)*d^2*ln(x)/(b
*c^3+a)^3-1/3*b^(2/3)*(a^2+6*a^(4/3)*b^(2/3)*c^2-7*a*b*c^3-3*a^(1/3)*b^(5/
3)*c^5+b^2*c^6)*d^2*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(b*c^3+a)^3+1/6*b^
(2/3)*(a^2+6*a^(4/3)*b^(2/3)*c^2-7*a*b*c^3-3*a^(1/3)*b^(5/3)*c^5+b^2*c^6)*
d^2*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/(b*c^3+a
)^3+b*c*(-2*b*c^3+a)*d^2*ln(a+b*(d*x+c)^3)/(b*c^3+a)^3+1/3*b^(2/3)*(a^(1/3
)+b^(1/3)*c)^3*(a-3*a^(2/3)*b^(1/3)*c+b*c^3)*d^2*arctan(1/3*(a^(1/3)-2*b^(
1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(b*c^3+a)^3*3^(1/2)
```

3.109.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^3 (a + b(c+dx)^3)} dx =$$

$$-\frac{3(a+bc^3)(a+bc^2(c-6dx)) + 18bc(a-2bc^3)d^2x^2\log(x) + 2d^2x^2\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bc\right.}{}$$

input
```
Integrate[1/(x^3*(a + b*(c + d*x)^3)),x]
```

output
```
-1/6*(3*(a + b*c^3)*(a + b*c^2*(c - 6*d*x)) + 18*b*c*(a - 2*b*c^3)*d^2*x^2
*Log[x] + 2*d^2*x^2*RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*
d^3*#1^3 & , (a^2*Log[x - #1] - 16*a*b*c^3*Log[x - #1] + 10*b^2*c^6*Log[x
- #1] - 12*a*b*c^2*d*Log[x - #1]*#1 + 15*b^2*c^5*d*Log[x - #1]*#1 - 3*a*b*
c*d^2*Log[x - #1]*#1^2 + 6*b^2*c^4*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 +
 d^2*#1^2) & ])/((a + b*c^3)^3*x^2)
```

3.109.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {896, 25, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx$$
$$\Big\downarrow\ 896$$
$$d^2\int \frac{1}{d^3x^3\left(b(c+dx)^3+a\right)}d(c+dx)$$
$$\Big\downarrow\ 25$$
$$-d^2\int -\frac{1}{d^3x^3\left(b(c+dx)^3+a\right)}d(c+dx)$$
$$\Big\downarrow\ 7276$$
$$-d^2\int \left(\frac{3bc^2}{\left(bc^3+a\right)^2d^2x^2}-\frac{3b\left(2bc^3-a\right)c}{\left(bc^3+a\right)^3dx}+\frac{b\left(b^2c^6-7abc^3-3b\left(2a-bc^3\right)(c+dx)c^2-3b\left(a-2bc^3\right)(c+dx)^2c-\right.}{\left(bc^3+a\right)^3\left(b(c+dx)^3+a\right)}\right.$$
$$dx)$$
$$\Big\downarrow\ 2009$$
$$d^2\left(\frac{b^{2/3}\left(-3a^{2/3}\sqrt[3]{b}c+a+bc^3\right)\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)^3}-\frac{b^{2/3}\left(a^2+3\sqrt[3]{a}b^{2/3}c^2\left(2a-bc^3\right)-7ab\right.}{3a^{2/3}\left(a+\right.}\right.$$

input
```
Int[1/(x^3*(a + b*(c + d*x)^3)),x]
```

output
```
d^2*(-1/2*1/((a + b*c^3)*d^2*x^2) + (3*b*c^2)/((a + b*c^3)^2*d*x) + (b^(2/
3)*(a^(1/3) + b^(1/3)*c)^3*(a - 3*a^(2/3)*b^(1/3)*c + b*c^3)*ArcTan[(a^(1/
3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)
^3) - (3*b*c*(a - 2*b*c^3)*Log[-(d*x)])/(a + b*c^3)^3 - (b^(2/3)*(a^2 - 7*
a*b*c^3 + b^2*c^6 + 3*a^(1/3)*b^(2/3)*c^2*(2*a - b*c^3))*Log[a^(1/3) + b^(
1/3)*(c + d*x)])/(3*a^(2/3)*(a + b*c^3)^3) + (b^(2/3)*(a^2 - 7*a*b*c^3 + b
^2*c^6 + 3*a^(1/3)*b^(2/3)*c^2*(2*a - b*c^3))*Log[a^(2/3) - a^(1/3)*b^(1/3
)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)^3) + (b*c*(a -
2*b*c^3)*Log[a + b*(c + d*x)^3])/(a + b*c^3)^3)
```

3.109. $\int \frac{1}{x^3(a+b(c+dx)^3)}\,dx$

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.109.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.51

method	result
default	$-\frac{1}{2(b\,c^3+a)x^2}+\frac{3b\,c^2 d}{(b\,c^3+a)^2 x}-\frac{3bc(-2b\,c^3+a)d^2\ln(x)}{(b\,c^3+a)^3}-\frac{d^2\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2 d\_Z+b\,c^3+a\right)}\frac{\left(6\_R^2 b^2 c^4 \cdots\right.}{\cdots}\right.}{\cdots}$
risch	$\frac{\frac{3b\,c^2 dx}{c^6 b^2+2ab\,c^3+a^2}-\frac{1}{2\left(b\,c^3+a\right)}}{x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^2b^3c^9+3a^3b^2c^6+3a^4b\,c^3+a^5\right)\_Z^3+\left(18d^2c^4b^2a^2-9a^3bc\,d^2\right)\_Z^2+9a\,b^2c^2d^4\_Z+b^2d^6\right)}\cdots\right.}{\cdots}$

input
```
int(1/x^3/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(b*c^3+a)/x^2+3*b*c^2*d/(b*c^3+a)^2/x-3*b*c*(-2*b*c^3+a)*d^2*ln(x)/(b
*c^3+a)^3-1/3*d^2*sum((6*_R^2*b^2*c^4*d^2+15*_R*b^2*c^5*d+10*b^2*c^6-3*_R^
2*a*b*c*d^2-12*_R*a*b*c^2*d-16*a*b*c^3+a^2)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_
R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/(b*c^3+a)^3
```

3.109.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.88 (sec) , antiderivative size = 14765, normalized size of antiderivative = 37.57

$$\int \frac{1}{x^3 (a + b(c + dx)^3)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + b(c + dx)^3)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(a+b*(d*x+c)**3),x)
```

output
```
Timed out
```

3.109.7 Maxima [F]

$$\int \frac{1}{x^3 (a + b(c + dx)^3)} \, dx = \int \frac{1}{((dx + c)^3 b + a) x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-b*d^3*integrate((10*b^2*c^6 - 16*a*b*c^3 + 3*(2*b^2*c^4 - a*b*c)*d^2*x^2
+ 3*(5*b^2*c^5 - 4*a*b*c^2)*d*x + a^2)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^
2*d*x + b*c^3 + a), x)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) + 3*(2*
b^2*c^4 - a*b*c)*d^2*log(x)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) +
1/2*(6*b*c^2*d*x - b*c^3 - a)/((b^2*c^6 + 2*a*b*c^3 + a^2)*x^2)
```

3.109.8 Giac [F]

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx=\int \frac{1}{\left((dx+c)^3b+a\right)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(1/(((d*x + c)^3*b + a)*x^3), x)
```

3.109.9 Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 1328, normalized size of antiderivative = 3.38

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx=\left(\sum_{k=1}^{3}\ln\left(\frac{6\,b^6\,c^4\,d^{14}-3\,a\,b^5\,c\,d^{14}}{a^4+4\,a^3\,b\,c^3+6\,a^2\,b^2\,c^6+4\,a\,b^3\,c^9+b^4\,c^{12}}\right.\right.$$
$$-\mathrm{root}\big(81\,a^3\,b^2\,c^6\,z^3+27\,a^2\,b^3\,c^9\,z^3+81\,a^4\,b\,c^3\,z^3+27\,a^5\,z^3-81\,a^3\,b\,c\,d^2\,z^2+162\,a^2\,b^2\,c^4\,d^2\,z^2+27\,a\,b^2\,c^2\,d^4\,z$$
$$\left.-\frac{x\,(b^6\,c^3\,d^{15}+a\,b^5\,d^{15})}{a^4+4\,a^3\,b\,c^3+6\,a^2\,b^2\,c^6+4\,a\,b^3\,c^9+b^4\,c^{12}}\right)\mathrm{root}\big(81\,a^3\,b^2\,c^6\,z^3+27\,a^2\,b^3\,c^9\,z^3$$
$$\left.+81\,a^4\,b\,c^3\,z^3+27\,a^5\,z^3-81\,a^3\,b\,c\,d^2\,z^2+162\,a^2\,b^2\,c^4\,d^2\,z^2+27\,a\,b^2\,c^2\,d^4\,z+b^2\,d^6,z,k\big)\right)$$
$$-\frac{1}{2\,\left(b\,c^3\,x^2+a\,x^2\right)}+\frac{3\,b\,c^2\,d}{x\,a^2+2\,x\,a\,b\,c^3+x\,b^2\,c^6}$$
$$+\frac{6\,b^2\,c^4\,d^2\,\ln(x)}{a^3+3\,a^2\,b\,c^3+3\,a\,b^2\,c^6+b^3\,c^9}-\frac{3\,a\,b\,c\,d^2\,\ln(x)}{a^3+3\,a^2\,b\,c^3+3\,a\,b^2\,c^6+b^3\,c^9}$$

input
```
int(1/(x^3*(a + b*(c + d*x)^3)),x)
```

output
```
symsum(log((6*b^6*c^4*d^14 - 3*a*b^5*c*d^14)/(a^4 + b^4*c^12 + 4*a^3*b*c^3
 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9
*z^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^
4*d^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*((a^3*b^4*d^12 + 19*b^7*c^
9*d^12 + 12*a*b^6*c^6*d^12 - 6*a^2*b^5*c^3*d^12)/(a^4 + b^4*c^12 + 4*a^3*b
*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3
*c^9*z^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^
2*c^4*d^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*(root(81*a^3*b^2*c^6*z
^3 + 27*a^2*b^3*c^9*z^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z
^2 + 162*a^2*b^2*c^4*d^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*((9*a^6
*b^3*c*d^8 + 9*a*b^8*c^16*d^8 + 45*a^5*b^4*c^4*d^8 + 90*a^4*b^5*c^7*d^8 +
90*a^3*b^6*c^10*d^8 + 45*a^2*b^7*c^13*d^8)/(a^4 + b^4*c^12 + 4*a^3*b*c^3 +
 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - (x*(36*a^6*b^3*d^9 - 18*a*b^8*c^15*d^9 + 1
26*a^5*b^4*c^3*d^9 + 144*a^4*b^5*c^6*d^9 + 36*a^3*b^6*c^9*d^9 - 36*a^2*b^7
*c^12*d^9))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6))
+ (3*b^8*c^14*d^10 - 42*a*b^7*c^11*d^10 + 30*a^4*b^4*c^2*d^10 + 12*a^3*b^5
*c^5*d^10 - 63*a^2*b^6*c^8*d^10)/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c
^9 + 6*a^2*b^2*c^6) + (x*(3*b^8*c^13*d^11 + 66*a^4*b^4*c*d^11 - 87*a*b^7*c
^10*d^11 + 39*a^3*b^5*c^4*d^11 - 117*a^2*b^6*c^7*d^11))/(a^4 + b^4*c^12 +
4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6)) + (x*(18*b^7*c^8*d^13 + 90*...
```

3.109. $\int \frac{1}{x^3(a+b(c+dx)^3)}\,dx$

3.110 $\int \frac{x^3}{a+b(c+dx)^4} dx$

3.110.1 Optimal result

Integrand size = 17, antiderivative size = 356

$$\int \frac{x^3}{a+b(c+dx)^4} dx = \frac{3c^2 \arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^4} + \frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4} - \frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4} - \frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4} + \frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4} + \frac{\log\left(a+b(c+dx)^4\right)}{4bd^4}$$

output
```
1/4*ln(a+b*(d*x+c)^4)/b/d^4+3/2*c^2*arctan((d*x+c)^2*b^(1/2)/a^(1/2))/d^4/
a^(1/2)/b^(1/2)-1/8*c*ln(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^
2*b^(1/2))*(3*a^(1/2)-b^(1/2)*c^2)/a^(3/4)/b^(3/4)/d^4*2^(1/2)+1/8*c*ln(a^
(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))*(3*a^(1/2)-b^(1/2
)*c^2)/a^(3/4)/b^(3/4)/d^4*2^(1/2)-1/4*c*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)
/a^(1/4))*(3*a^(1/2)+b^(1/2)*c^2)/a^(3/4)/b^(3/4)/d^4*2^(1/2)-1/4*c*arctan
(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(3*a^(1/2)+b^(1/2)*c^2)/a^(3/4)/b^(3/4
)/d^4*2^(1/2)
```

3.110.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.30

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx = \frac{\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&,\frac{\log(x-\#1)\#1^3}{c^3+3c^2d\#1+3cd^2\#1^2+d^3\#1^3}\&\right]}{4bd}$$

input
```
Integrate[x^3/(a + b*(c + d*x)^4),x]
```

output
```
RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d
^4*#1^4 & , (Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3
) & ]/(4*b*d)
```

3.110.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {896, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{d^3x^3}{b(c+dx)^4+a}d(c+dx)}{d^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{d^3x^3}{b(c+dx)^4+a}d(c+dx)}{d^4}$$

$$\Big\downarrow \text{2415}$$

$$-\frac{\int \left(\frac{(c+dx)\left(-3c^2-(c+dx)^2\right)}{b(c+dx)^4+a}+\frac{c^3+3(c+dx)^2c}{b(c+dx)^4+a}\right)d(c+dx)}{d^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(\right.}{4\sqrt{2}a^{3/4}b^{3/4}}}{d^4}$$

input
```
Int[x^3/(a + b*(c + d*x)^4),x]
```

output
```
((3*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) + (c*(3
*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(
2*Sqrt[2]*a^(3/4)*b^(3/4)) - (c*(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - (c*(3*Sqrt[
a] - Sqrt[b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b
]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + (c*(3*Sqrt[a] - Sqrt[b]*c^2)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(
4*Sqrt[2]*a^(3/4)*b^(3/4)) + Log[a + b*(c + d*x)^4]/(4*b))/d^4
```

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.110.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.27

method	result	size
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,d^{4}\_Z^{4}+4bc\,d^{3}\_Z^{3}+6b\,c^{2}d^{2}\_Z^{2}+4b\,c^{3}d\_Z+b\,c^{4}+a\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{d^{3}\_R^{3}+3c\,d^{2}\_R^{2}+3c^{2}d\_R+c^{3}}}{4bd}$	97
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,d^{4}\_Z^{4}+4bc\,d^{3}\_Z^{3}+6b\,c^{2}d^{2}\_Z^{2}+4b\,c^{3}d\_Z+b\,c^{4}+a\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{d^{3}\_R^{3}+3c\,d^{2}\_R^{2}+3c^{2}d\_R+c^{3}}}{4bd}$	97

input
```
int(x^3/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/4/b/d*sum(_R^3/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf
(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

3.110.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx=\text{Timed out}$$

input
```
integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

output
```
Timed out
```

3.110.6 Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx$$
$$=\operatorname{RootSum}\Big(256t^4a^3b^4d^{16}-256t^3a^3b^3d^{12}+t^2\cdot\left(96a^3b^2d^8+480a^2b^3c^4d^8\right)+t\left(-16a^3bd^4+192a^2b^2c^4d^4-4\right.$$

input
```
integrate(x**3/(a+b*(d*x+c)**4),x)
```

output
```
RootSum(256*_t**4*a**3*b**4*d**16 - 256*_t**3*a**3*b**3*d**12 + _t**2*(96*
a**3*b**2*d**8 + 480*a**2*b**3*c**4*d**8) + _t*(-16*a**3*b*d**4 + 192*a**2
*b**2*c**4*d**4 - 48*a*b**3*c**8*d**4) + a**3 + 3*a**2*b*c**4 + 3*a*b**2*c
**8 + b**3*c**12, Lambda(_t, _t*log(x + (-1728*_t**3*a**4*b**3*d**12 - 960
*_t**3*a**3*b**4*c**4*d**12 + 1296*_t**2*a**4*b**2*d**8 + 2016*_t**2*a**3*
b**3*c**4*d**8 - 48*_t**2*a**2*b**4*c**8*d**8 - 324*_t*a**4*b*d**4 - 4716*
_t*a**3*b**2*c**4*d**4 - 1452*_t*a**2*b**3*c**8*d**4 - 4*_t*a*b**4*c**12*d
**4 + 27*a**4 - 390*a**3*b*c**4 - 444*a**2*b**2*c**8 - 26*a*b**3*c**12 + b
**4*c**16)/(729*a**3*b*c**3*d - 1053*a**2*b**2*c**7*d - 117*a*b**3*c**11*d
 + b**4*c**15*d))))
```

3.110.7 Maxima [F]

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx = \int \frac{x^3}{(dx+c)^4b+a}\,dx$$

input
```
integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

output
```
integrate(x^3/((d*x + c)^4*b + a), x)
```

3.110.8 Giac [F]

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx = \int \frac{x^3}{(dx+c)^4b+a}\,dx$$

input
```
integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="giac")
```

output
```
integrate(x^3/((d*x + c)^4*b + a), x)
```

3.110.9 Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 1003, normalized size of antiderivative = 2.82

$$\int \frac{x^3}{a+b(c+dx)^4} dx = \sum_{k=1}^{4} \ln \Big(b c^2 d \Big(2 a c + 2 b c^5 - 3 a d x + 5 b c^4 d x$$
$$-\text{root}\big(256 a^3 b^4 d^{16} z^4 - 256 a^3 b^3 d^{12} z^3 + 480 a^2 b^3 c^4 d^8 z^2 + 96 a^3 b^2 d^8 z^2 + 192 a^2 b^2 c^4 d^4 z - 48 a b^3 c^8 d^4 z -$$
$$+\text{root}\big(256 a^3 b^4 d^{16} z^4 - 256 a^3 b^3 d^{12} z^3 + 480 a^2 b^3 c^4 d^8 z^2 + 96 a^3 b^2 d^8 z^2 + 192 a^2 b^2 c^4 d^4 z - 48 a b^3 c^8 d$$
$$+\text{root}\big(256 a^3 b^4 d^{16} z^4 - 256 a^3 b^3 d^{12} z^3 + 480 a^2 b^3 c^4 d^8 z^2 + 96 a^3 b^2 d^8 z^2 + 192 a^2 b^2 c^4 d^4 z - 48 a b^3 c^8 d$$
$$-\text{root}\big(256 a^3 b^4 d^{16} z^4 - 256 a^3 b^3 d^{12} z^3 + 480 a^2 b^3 c^4 d^8 z^2 + 96 a^3 b^2 d^8 z^2 + 192 a^2 b^2 c^4 d^4 z - 48 a b^3 c^8 d^4 z -$$
$$+\text{root}\big(256 a^3 b^4 d^{16} z^4 - 256 a^3 b^3 d^{12} z^3 + 480 a^2 b^3 c^4 d^8 z^2 + 96 a^3 b^2 d^8 z^2 + 192 a^2 b^2 c^4 d^4 z - 48 a b^3 c^8 d^4 z -$$
$$+\text{root}\big(256 a^3 b^4 d^{16} z^4 - 256 a^3 b^3 d^{12} z^3 + 480 a^2 b^3 c^4 d^8 z^2 + 96 a^3 b^2 d^8 z^2 + 192 a^2 b^2 c^4 d^4 z - 48 a b^3 c^8 d^4 z -$$
$$- 256 a^3 b^3 d^{12} z^3 + 480 a^2 b^3 c^4 d^8 z^2 + 96 a^3 b^2 d^8 z^2 + 192 a^2 b^2 c^4 d^4 z - 48 a b^3 c^8 d^4 z$$
$$- 16 a^3 b d^4 z + 3 a b^2 c^8 + 3 a^2 b c^4 + b^3 c^{12} + a^3, z, k \big)$$

input
```
int(x^3/(a + b*(c + d*x)^4),x)
```

output
```
symsum(log(2*b*c^2*d*(2*a*c + 2*b*c^5 - 3*a*d*x + 5*b*c^4*d*x - 2*root(256
*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^
3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*
z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*b^2*c^5*d^4 + 32*roo
t(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 +
96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b
*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)^2*a*b^2*c*d^8 +
 24*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8
*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 1
6*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)^2*a*b^2*
d^9*x - 2*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c
^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4
*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*b^
2*c^4*d^5*x + 38*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^
2*b^3*c^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*
c^8*d^4*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z
, k)*a*b*c*d^4 + 6*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*
a^2*b^3*c^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^
3*c^8*d^4*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3,
 z, k)*a*b*d^5*x))*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 4...
```

3.111 $\int \frac{x^2}{a+b(c+dx)^4}\,dx$

3.111.1 Optimal result

Integrand size = 17, antiderivative size = 318

$$\begin{aligned}\int \frac{x^2}{a+b(c+dx)^4}\,dx =& -\frac{c\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3} - \frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3} \\ &+ \frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3} \\ &+ \frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} \\ &- \frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3}\end{aligned}$$

output
```
-c*arctan((d*x+c)^2*b^(1/2)/a^(1/2))/d^3/a^(1/2)/b^(1/2)+1/8*ln(-a^(1/4)*b
^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^
(3/4)/b^(3/4)/d^3*2^(1/2)-1/8*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(
d*x+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^(3/4)/b^(3/4)/d^3*2^(1/2)+1/4*ar
ctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(a^(1/2)+b^(1/2)*c^2)/a^(3/4)/b^(
3/4)/d^3*2^(1/2)+1/4*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(a^(1/2)+b^
(1/2)*c^2)/a^(3/4)/b^(3/4)/d^3*2^(1/2)
```

3.111.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.33

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx = \frac{\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&,\frac{\log(x-\#1)\#1^2}{c^3+3c^2d\#1+3cd^2\#1^2+d^3\#1^3}\&\right]}{4bd}$$

input
```
Integrate[x^2/(a + b*(c + d*x)^4),x]
```

output
```
RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d
^4*#1^4 & , (Log[x - #1]*#1^2)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3
) & ]/(4*b*d)
```

3.111.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {896, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int \frac{d^2x^2}{b(c+dx)^4+a}d(c+dx)}{d^3}$$

$$\Big\downarrow \text{ 2415}$$

$$\frac{\int \left(\frac{c^2+(c+dx)^2}{b(c+dx)^4+a}-\frac{2c(c+dx)}{b(c+dx)^4+a}\right)d(c+dx)}{d^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+d\right.}{4\sqrt{2}a^{3/4}b^{3/4}}$$
$$\overline{\qquad\qquad d^3\qquad\qquad}$$

input
```
Int[x^2/(a + b*(c + d*x)^4),x]
```

output
```
(-((c*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])) - ((Sqrt[a
] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*b^(3/4)) + ((Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[a] - Sqrt[b]*c^
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])
/(4*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b
^(3/4)))/d^3
```

3.111.3.1 Defintions of rubi rules used

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.111.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.31

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4bd}$	97
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4bd}$	97

input
```
int(x^2/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/4/b/d*sum(_R^2/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf
(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

3.111.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 58.70 (sec) , antiderivative size = 61993, normalized size of antiderivative = 194.95

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

output
```
Too large to include
```

3.111.6 Sympy [A] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx$$
$$=\operatorname{RootSum}\left(256t^4a^3b^3d^{12}+192t^2a^2b^2c^2d^6+t\left(-32a^2bcd^3+32ab^2c^5d^3\right)+a^2+2abc^4+b^2c^8,\left(t\mapsto t\log\left(x\right.\right.\right.$$

input
```
integrate(x**2/(a+b*(d*x+c)**4),x)
```

output
```
RootSum(256*_t**4*a**3*b**3*d**12 + 192*_t**2*a**2*b**2*c**2*d**6 + _t*(-3
2*a**2*b*c*d**3 + 32*a*b**2*c**5*d**3) + a**2 + 2*a*b*c**4 + b**2*c**8, La
mbda(_t, _t*log(x + (64*_t**3*a**4*b**2*d**9 + 448*_t**3*a**3*b**3*c**4*d*
*9 + 160*_t**2*a**3*b**2*c**3*d**6 - 32*_t**2*a**2*b**3*c**7*d**6 + 60*_t*
a**3*b*c**2*d**3 + 256*_t*a**2*b**2*c**6*d**3 + 4*_t*a*b**3*c**10*d**3 - 5
*a**3*c - 9*a**2*b*c**5 - 3*a*b**2*c**9 + b**3*c**13)/(a**3*d - 33*a**2*b*
c**4*d - 33*a*b**2*c**8*d + b**3*c**12*d))))
```

3.111.7 Maxima [F]

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx=\int \frac{x^2}{(dx+c)^4b+a}\,dx$$

input
```
integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

output
```
integrate(x^2/((d*x + c)^4*b + a), x)
```

3.111.8 Giac [F]

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx=\int \frac{x^2}{(dx+c)^4b+a}\,dx$$

input
```
integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="giac")
```

output
```
integrate(x^2/((d*x + c)^4*b + a), x)
```

3.111.9 Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.97

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx = \sum_{k=1}^{4} \ln\Big(-b\,d^4\,\Big(a+b\,c^4+4\,b\,c^3\,d\,x$$
$$+\mathrm{root}\big(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\big)\;b^2\,c^5\,d^3\,4$$
$$+\mathrm{root}\big(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\big)\;b^2\,c^4\,d^4\,x\,4$$
$$-\mathrm{root}\big(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\big)\;a\,b\,c\,d^3\,20$$
$$-\mathrm{root}\big(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\big)\;a\,b\,d^4\,x\,4$$
$$+\mathrm{root}\big(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\big)^2\,a\,b^2$$
$$+\mathrm{root}\big(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\big)^2\,a\,b^2$$
$$+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\big)$$

input
```
int(x^2/(a + b*(c + d*x)^4),x)
```

output
```
symsum(log(-b*d^4*(a + b*c^4 + 4*b*c^3*d*x + 4*root(256*a^3*b^3*d^12*z^4 +
 192*a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c
^4 + b^2*c^8 + a^2, z, k)*b^2*c^5*d^3 + 4*root(256*a^3*b^3*d^12*z^4 + 192*
a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 +
b^2*c^8 + a^2, z, k)*b^2*c^4*d^4*x - 20*root(256*a^3*b^3*d^12*z^4 + 192*a^
2*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^
2*c^8 + a^2, z, k)*a*b*c*d^3 - 4*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c
^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 +
 a^2, z, k)*a*b*d^4*x + 48*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6
*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2,
z, k)^2*a*b^2*c^2*d^6 + 32*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6
*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2,
z, k)^2*a*b^2*c*d^7*x))*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^
2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z,
k), k, 1, 4)
```

3.112 $\int \frac{x}{a+b(c+dx)^4} dx$

3.112.1 Optimal result

Integrand size = 15, antiderivative size = 261

$$\int \frac{x}{a+b(c+dx)^4} dx = \frac{\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} - \frac{c\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} + \frac{c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} - \frac{c\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}$$

output
```
-1/4*c*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/d^2*2^(1
/2)-1/4*c*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/d^2*2^
(1/2)+1/8*c*ln(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))
/a^(3/4)/b^(1/4)/d^2*2^(1/2)-1/8*c*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1
/2)+(d*x+c)^2*b^(1/2))/a^(3/4)/b^(1/4)/d^2*2^(1/2)+1/2*arctan((d*x+c)^2*b^
(1/2)/a^(1/2))/d^2/a^(1/2)/b^(1/2)
```

3.112.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.40

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \frac{\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&,\frac{\log(x-\#1)\#1}{c^3+3c^2d\#1+3cd^2\#1^2+d^3\#1^3}\&\right]}{4bd}$$

input
```
Integrate[x/(a + b*(c + d*x)^4),x]
```

output
```
RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d
^4*#1^4 & , (Log[x - #1]*#1)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3)
& ]/(4*b*d)
```

3.112.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {896, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b(c+dx)^4}\,dx$$
$$\Big\downarrow \ 896$$
$$\frac{\int \frac{dx}{b(c+dx)^4+a}d(c+dx)}{d^2}$$
$$\Big\downarrow \ 25$$
$$-\frac{\int -\frac{dx}{b(c+dx)^4+a}d(c+dx)}{d^2}$$
$$\Big\downarrow \ 2415$$
$$-\frac{\int \left(\frac{c}{b(c+dx)^4+a}-\frac{c+dx}{b(c+dx)^4+a}\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{}\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}}{d^2}$$

input
```
Int[x/(a + b*(c + d*x)^4),x]
```

output
```
(ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) + (c*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - (c*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4))
 + (c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^
2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)) - (c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)))/d^2
```

3.112.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.112.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.36

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\_R\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4bd}$	95
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\_R\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4bd}$	95

input
```
int(x/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/4/b/d*sum(_R/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_
Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

3.112.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.05 (sec) , antiderivative size = 40785, normalized size of antiderivative = 156.26

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

output
```
Too large to include
```

3.112.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.50

$$\int \frac{x}{a+b(c+dx)^4}\,dx$$
$$= \text{RootSum}\left(256t^4a^3b^2d^8+32t^2a^2bd^4-16tabc^2d^2+a+bc^4,\left(t\mapsto t\log\left(x+\frac{128t^3a^3bd^6+16t^2a^2bc^2d^4+\right.\right.\right.$$
$$\frac{\qquad}{4acd}$$

input
```
integrate(x/(a+b*(d*x+c)**4),x)
```

output
```
RootSum(256*_t**4*a**3*b**2*d**8 + 32*_t**2*a**2*b*d**4 - 16*_t*a*b*c**2*d
**2 + a + b*c**4, Lambda(_t, _t*log(x + (128*_t**3*a**3*b*d**6 + 16*_t**2*
a**2*b*c**2*d**4 + 8*_t*a**2*d**2 + 4*_t*a*b*c**4*d**2 - a*c**2 - b*c**6)/
(4*a*c*d - b*c**5*d)))
```

3.112.7 Maxima [F]

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \int \frac{x}{(dx+c)^4b+a}\,dx$$

input
```
integrate(x/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

output
```
integrate(x/((d*x + c)^4*b + a), x)
```

3.112.8 Giac [F]

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \int \frac{x}{(dx+c)^4b+a}\,dx$$

input
```
integrate(x/(a+b*(d*x+c)^4),x, algorithm="giac")
```

output
```
integrate(x/((d*x + c)^4*b + a), x)
```

3.112.9 Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.79

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(256\,a^3\,b^2\,d^8\,z^4+32\,a^2\,b\,d^4\,z^2-16\,a\,b\,c^2\,d^2\,z+b\,c^4+a,z,k\right)\left(-\mathrm{root}\left(256\,a^3\,b^2\,d^8\,z^4+32\,a^2\,b\,d^4\,z^2-16\,a\,b\,c^2\,d^2\,z+b\,c^4+a,z,k\right)\left(16\,a\,x\,b^3\,d^{12}+32\,a\,c\,b^3\,d^{11}\right)+4\,b^3\,c^3\,d^9+4\,b^3\,c^2\,d^{10}\,x\right)+b^2\,d^8\,x\right)\mathrm{root}\left(256\,a^3\,b^2\,d^8\,z^4+32\,a^2\,b\,d^4\,z^2-16\,a\,b\,c^2\,d^2\,z+b\,c^4+a,z,k\right)$$

input
```
int(x/(a + b*(c + d*x)^4),x)
```

output
```
symsum(log(b^2*d^8*x - root(256*a^3*b^2*d^8*z^4 + 32*a^2*b*d^4*z^2 - 16*a*
b*c^2*d^2*z + b*c^4 + a, z, k)*(4*b^3*c^3*d^9 - root(256*a^3*b^2*d^8*z^4 +
 32*a^2*b*d^4*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k)*(32*a*b^3*c*d^11 +
 16*a*b^3*d^12*x) + 4*b^3*c^2*d^10*x))*root(256*a^3*b^2*d^8*z^4 + 32*a^2*b
*d^4*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k), k, 1, 4)
```

3.113 $\int \frac{1}{a+b(c+dx)^4}\,dx$

3.113.1 Optimal result

Integrand size = 13, antiderivative size = 221

$$\int \frac{1}{a+b(c+dx)^4}\,dx=-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d}-\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

output
```
1/4*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/d*2^(1/2)+1
/4*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/d*2^(1/2)-1/8
*ln(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))/a^(3/4)/b^
(1/4)/d*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b
^(1/2))/a^(3/4)/b^(1/4)/d*2^(1/2)
```

3.113.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.73

$$\int \frac{1}{a+b(c+dx)^4}\,dx$$
$$=\frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

input
```
Integrate[(a + b*(c + d*x)^4)^(-1),x]
```

output
```
(-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2
]*b^(1/4)*(c + d*x))/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c +
 d*x) + Sqrt[b]*(c + d*x)^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c +
d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)
```

3.113.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {239, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b(c+dx)^4}\,dx$$
$$\Big\downarrow \; 239$$
$$\frac{\int \frac{1}{b(c+dx)^4+a}d(c+dx)}{d}$$
$$\Big\downarrow \; 755$$
$$\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}(c+dx)^2}{b(c+dx)^4+a}d(c+dx)}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}(c+dx)^2+\sqrt{a}}{b(c+dx)^4+a}d(c+dx)}{2\sqrt{a}}}{d}$$
$$\Big\downarrow \; 1476$$

$$\frac{\frac{\frac{\int \frac{1}{(c+dx)^2-\frac{\sqrt{2}\sqrt[4]{a}(c+dx)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d(c+dx)}{2\sqrt{b}}+\frac{\int \frac{1}{(c+dx)^2+\frac{\sqrt{2}\sqrt[4]{a}(c+dx)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d(c+dx)}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}(c+dx)^2}{b(c+dx)^4+a}d(c+dx)}{2\sqrt{a}}}{d}$$

$\downarrow$ 1082

$$\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}(c+dx)^2}{b(c+dx)^4+a}d(c+dx)}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{d}$$

$\downarrow$ 217

$$\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}(c+dx)^2}{b(c+dx)^4+a}d(c+dx)}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{d}$$

$\downarrow$ 1479

$$\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}\left((c+dx)^2-\frac{\sqrt{2}\sqrt[4]{a}(c+dx)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d(c+dx)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}(c+dx)+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left((c+dx)^2+\frac{\sqrt{2}\sqrt[4]{a}(c+dx)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d(c+dx)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{d}$$

$\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}\left((c+dx)^2-\frac{\sqrt{2}\sqrt[4]{a}(c+dx)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d(c+dx)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}(c+dx)+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left((c+dx)^2+\frac{\sqrt{2}\sqrt[4]{a}(c+dx)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d(c+dx)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}(c+dx)}{(c+dx)^2-\frac{\sqrt{2}\sqrt[4]{a}(c+dx)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d(c+dx)}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}(c+dx)+\sqrt[4]{a}}{(c+dx)^2+\frac{\sqrt{2}\sqrt[4]{a}(c+dx)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d(c+dx)}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{d}$$

3.113. $\int \frac{1}{a+b(c+dx)^4}\,dx$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{d}$$

input
```
Int[(a + b*(c + d*x)^4)^(-1),x]
```

output
```
((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/
4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^
(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x
) + Sqrt[b]*(c + d*x)^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2]/(2*Sqrt[2]*a^(1/4)*b^(1/
4)))/(2*Sqrt[a]))/d
```

3.113.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.113.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.43

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^{4}\_Z^{4}+4bc\,d^{3}\_Z^{3}+6b\,c^{2}d^{2}\_Z^{2}+4b\,c^{3}d\_Z+b\,c^{4}+a\right)}\frac{\ln\left(x-\_R\right)}{d^{3}\_R^{3}+3c\,d^{2}\_R^{2}+3c^{2}d\_R+c^{3}}}{4bd}$	94
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^{4}\_Z^{4}+4bc\,d^{3}\_Z^{3}+6b\,c^{2}d^{2}\_Z^{2}+4b\,c^{3}d\_Z+b\,c^{4}+a\right)}\frac{\ln\left(x-\_R\right)}{d^{3}\_R^{3}+3c\,d^{2}\_R^{2}+3c^{2}d\_R+c^{3}}}{4bd}$	94

input
```
int(1/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/4/b/d*sum(1/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_Z
^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

3.113.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.69

$$\begin{aligned}\int \frac{1}{a+b(c+dx)^4}\,dx = {} & \frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right)\\ & +\frac{1}{4}i\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(i\,ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right)\\ & -\frac{1}{4}i\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(-i\,ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right)\\ & -\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(-ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right)\end{aligned}$$

input
```
integrate(1/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/4*(-1/(a^3*b*d^4))^(1/4)*log(a*d*(-1/(a^3*b*d^4))^(1/4) + d*x + c) + 1/4
*I*(-1/(a^3*b*d^4))^(1/4)*log(I*a*d*(-1/(a^3*b*d^4))^(1/4) + d*x + c) - 1/
4*I*(-1/(a^3*b*d^4))^(1/4)*log(-I*a*d*(-1/(a^3*b*d^4))^(1/4) + d*x + c) -
1/4*(-1/(a^3*b*d^4))^(1/4)*log(-a*d*(-1/(a^3*b*d^4))^(1/4) + d*x + c)
```

3.113.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.12

$$\int \frac{1}{a+b(c+dx)^4}\,dx = \frac{\operatorname{RootSum}\left(256t^4a^3b+1,\left(t\mapsto t\log\left(x+\frac{4ta+c}{d}\right)\right)\right)}{d}$$

input
```
integrate(1/(a+b*(d*x+c)**4),x)
```

output
```
RootSum(256*_t**4*a**3*b + 1, Lambda(_t, _t*log(x + (4*_t*a + c)/d)))/d
```

3.113.7 Maxima [F]

$$\int \frac{1}{a+b(c+dx)^4}\,dx = \int \frac{1}{(dx+c)^4 b + a}\,dx$$

input
```
integrate(1/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)^4*b + a), x)
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.47

$$\begin{aligned}\int \frac{1}{a+b(c+dx)^4}\,dx = & -\frac{1}{2}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{bdx+bc}{(-ab^3)^{\frac{1}{4}}}\right) \\ & +\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\left|bdx+bc+\left(-ab^3\right)^{\frac{1}{4}}\right|\right) \\ & -\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\left|-bdx-bc+\left(-ab^3\right)^{\frac{1}{4}}\right|\right)\end{aligned}$$

input
```
integrate(1/(a+b*(d*x+c)^4),x, algorithm="giac")
```

output
```
-1/2*(-1/(a^3*b*d^4))^(1/4)*arctan(-(b*d*x + b*c)/(-a*b^3)^(1/4)) + 1/4*(-
1/(a^3*b*d^4))^(1/4)*log(abs(b*d*x + b*c + (-a*b^3)^(1/4))) - 1/4*(-1/(a^3
*b*d^4))^(1/4)*log(abs(-b*d*x - b*c + (-a*b^3)^(1/4)))
```

3.113.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.27

$$\int \frac{1}{a+b(c+dx)^4}\,dx = -\frac{\mathrm{atan}\left(\frac{b^{1/4}\,c}{(-a)^{1/4}}+\frac{b^{1/4}\,d\,x}{(-a)^{1/4}}\right)+\mathrm{atanh}\left(\frac{b^{1/4}\,c}{(-a)^{1/4}}+\frac{b^{1/4}\,d\,x}{(-a)^{1/4}}\right)}{2\,(-a)^{3/4}\,b^{1/4}\,d}$$

input
```
int(1/(a + b*(c + d*x)^4),x)
```

output
```
-(atan((b^(1/4)*c)/(-a)^(1/4) + (b^(1/4)*d*x)/(-a)^(1/4)) + atanh((b^(1/4)
*c)/(-a)^(1/4) + (b^(1/4)*d*x)/(-a)^(1/4)))/(2*(-a)^(3/4)*b^(1/4)*d)
```

3.113. $\int \frac{1}{a+b(c+dx)^4} dx$

3.114 $\int \frac{1}{x(a+b(c+dx)^4)}\,dx$

3.114.1 Optimal result

Integrand size = 17, antiderivative size = 393

$$
\begin{aligned}
\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx
&= -\frac{\sqrt{b}c^2\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(a+bc^4\right)}+\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)} \\
&\quad -\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)}+\frac{\log(x)}{a+bc^4} \\
&\quad -\frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)} \\
&\quad +\frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)} \\
&\quad -\frac{\log\left(a+b(c+dx)^4\right)}{4\left(a+bc^4\right)}
\end{aligned}
$$

output
```
ln(x)/(b*c^4+a)-1/4*ln(a+b*(d*x+c)^4)/(b*c^4+a)-1/2*c^2*arctan((d*x+c)^2*b
^(1/2)/a^(1/2))*b^(1/2)/(b*c^4+a)/a^(1/2)-1/8*b^(1/4)*c*ln(-a^(1/4)*b^(1/4
)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^(3/4)
/(b*c^4+a)*2^(1/2)+1/8*b^(1/4)*c*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2
)+(d*x+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^(3/4)/(b*c^4+a)*2^(1/2)-1/4*b
^(1/4)*c*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(a^(1/2)+b^(1/2)*c^2)/
a^(3/4)/(b*c^4+a)*2^(1/2)-1/4*b^(1/4)*c*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a
^(1/4))*(a^(1/2)+b^(1/2)*c^2)/a^(3/4)/(b*c^4+a)*2^(1/2)
```

3.114.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.41

$$\int \frac{1}{x(a+b(c+dx)^4)}\,dx = -\frac{-4\log(x)+\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&,\frac{4c^3\log(x-\#1)+6c^2d\log(x-}{c^3+}\right.}{4(a+bc^4)}$$

input
```
Integrate[1/(x*(a + b*(c + d*x)^4)),x]
```

output
```
-1/4*(-4*Log[x] + RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*
b*c*d^3*#1^3 + b*d^4*#1^4 & , (4*c^3*Log[x - #1] + 6*c^2*d*Log[x - #1]*#1
+ 4*c*d^2*Log[x - #1]*#1^2 + d^3*Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c
*d^2*#1^2 + d^3*#1^3) & ])/(a + b*c^4)
```

3.114.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {896, 25, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+b(c+dx)^4)}\,dx$$

$$\Big\downarrow \text{896}$$

$$\int \frac{1}{dx\,(a+b(c+dx)^4)}d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{1}{dx\,(b(c+dx)^4+a)}d(c+dx)$$

$$\Big\downarrow \text{7276}$$

$$-\int \left(\frac{b(c^3+(c+dx)c^2+(c+dx)^2c+(c+dx)^3)}{(bc^4+a)\,(b(c+dx)^4+a)}-\frac{1}{(bc^4+a)\,dx}\right)d(c+dx)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(a+bc^4)}-\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\,(a+bc^4)}-\frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\,(a+bc^4)}+\frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\,(a+bc^4)}-\frac{\sqrt{b}c^2\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(a+bc^4)}+\frac{\log(-dx)}{a+bc^4}-\frac{\log\left(a+b(c+dx)^4\right)}{4\,(a+bc^4)}$$

input
```
Int[1/(x*(a + b*(c + d*x)^4)),x]
```

output
```
-1/2*(Sqrt[b]*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*(a + b*c
^4)) + (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c +
 d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)) - (b^(1/4)*c*(Sqrt[a] + S
qrt[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^
(3/4)*(a + b*c^4)) + Log[-(d*x)]/(a + b*c^4) - (b^(1/4)*c*(Sqrt[a] - Sqrt[
b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x
)^2])/(4*Sqrt[2]*a^(3/4)*(a + b*c^4)) + (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(
4*Sqrt[2]*a^(3/4)*(a + b*c^4)) - Log[a + b*(c + d*x)^4]/(4*(a + b*c^4))
```

3.114. $\int \frac{1}{x(a+b(c+dx)^4)}\,dx$

3.114.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.114.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.35

method	result
default	$\frac{\ln(x)}{b c^4+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b d^4 \_Z^4+4bc d^3 \_Z^3+6b c^2 d^2 \_Z^2+4b c^3 d \_Z+b c^4+a\right)} \frac{\left(d^3 \_R^3+4c d^2 \_R^2+6c^2 d \_R+4c^3\right)\ln\left(x-\_R\right)}{d^3 \_R^3+3c d^2 \_R^2+3c^2 d \_R+c^3}}{4(b c^4+a)}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a^3 b c^4+a^4\right)\_Z^4+4\_Z^3 a^3+6a^2\_Z^2+4a\_Z\right)} \_R \ln\left(\left(\left(-3a^2 b c^4 d+5a^3 d\right)\_R^3+\left(-3ab c^4 d+15d a^2\right)\_R^2+\left(-b c^4 d \right.\right.\right.}{4}$

input
```
int(1/x/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/(b*c^4+a)-1/4/(b*c^4+a)*sum((_R^3*d^3+4*_R^2*c*d^2+6*_R*c^2*d+4*c^3)
/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_Z^4*b*d^4+4*_Z
^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

3.114.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.19 (sec) , antiderivative size = 307773, normalized size of antiderivative = 783.14

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

output
```
Too large to include
```

3.114.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(a+b*(d*x+c)**4),x)
```

output
```
Timed out
```

3.114.7 Maxima [F]

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx = \int \frac{1}{\left((dx+c)^4b+a\right)x}\,dx$$

input
```
integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

output
```
-b*d*integrate((d^3*x^3 + 4*c*d^2*x^2 + 6*c^2*d*x + 4*c^3)/(b*d^4*x^4 + 4*
b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a), x)/(b*c^4 + a) +
 log(x)/(b*c^4 + a)
```

3.114.8 Giac [F]

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx=\int \frac{1}{\left((dx+c)^4b+a\right)x}\,dx$$

input
```
integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="giac")
```

output
```
integrate(1/((((d*x + c)^4*b + a)*x), x)
```

3.114.9 Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 882, normalized size of antiderivative = 2.24

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx=\frac{\ln(x)}{b\,c^4+a}$$
$$+\left(\sum_{k=1}^{4}\ln\Big(-\mathrm{root}\left(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\right)^2 b^5\,c^5\,d^{15}\,4\right.$$
$$+\mathrm{root}\left(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\right)\,b^4\,c\,d^{15}\,4$$
$$+\mathrm{root}\left(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\right)\,b^4\,d^{16}\,x\,5$$
$$-\mathrm{root}\left(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\right)^4 a^2\,b^5\,c^5\,d^{15}\,64$$
$$+\mathrm{root}\left(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\right)^2 a\,b^4\,c\,d^{15}\,28$$
$$+\mathrm{root}\left(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\right)^2 a\,b^4\,d^{16}\,x\,60$$
$$+\mathrm{root}\left(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\right)^3 a^2\,b^4\,c\,d^{15}\,32$$
$$-\mathrm{root}\left(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\right)^4 a^3\,b^4\,c\,d^{15}\,64$$
$$-\mathrm{root}\left(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\right)^3 a\,b^5\,c^5\,d^{15}\,32$$
$$+\mathrm{root}\left(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\right)^3 a^2\,b^4\,d^{16}\,x\,240$$
$$+\mathrm{root}\left(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\right)^4 a^3\,b^4\,d^{16}\,x\,320$$
$$-\mathrm{root}\left(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\right)^2 b^5\,c^4\,d^{16}\,x\,4$$
$$-\mathrm{root}\left(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\right)^3 a\,b^5\,c^4\,d^{16}\,x\,48-\mathrm{root}\Big(256\,a^3\,b\,c^4$$
$$\left.+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\right)\Big)$$

input
```
int(1/(x*(a + b*(c + d*x)^4)),x)
```

output
```
log(x)/(a + b*c^4) + symsum(log(4*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 2
56*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)*b^4*c*d^15 - 4*root(256*a^3*b*
c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^2*b^5
*c^5*d^15 + 5*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*
z^2 + 16*a*z + 1, z, k)*b^4*d^16*x - 64*root(256*a^3*b*c^4*z^4 + 256*a^4*z
^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^2*b^5*c^5*d^15 + 28*
root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z +
 1, z, k)^2*a*b^4*c*d^15 + 60*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a
^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^2*a*b^4*d^16*x + 32*root(256*a^3*b
*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a^
2*b^4*c*d^15 - 64*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*
a^2*z^2 + 16*a*z + 1, z, k)^4*a^3*b^4*c*d^15 - 32*root(256*a^3*b*c^4*z^4 +
 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a*b^5*c^5*d^
15 + 240*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 +
 16*a*z + 1, z, k)^3*a^2*b^4*d^16*x + 320*root(256*a^3*b*c^4*z^4 + 256*a^4
*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^3*b^4*d^16*x - 4*r
oot(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z +
1, z, k)^2*b^5*c^4*d^16*x - 48*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*
a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a*b^5*c^4*d^16*x - 192*root(256
*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z...
```

3.114. $\int \frac{1}{x(a+b(c+dx)^4)}\,dx$

3.115 $\int \frac{1}{x^2(a+b(c+dx)^4)}\,dx$

3.115.1 Optimal result

Integrand size = 17, antiderivative size = 496

$$\begin{aligned}
&\int \frac{1}{x^2\left(a+b(c+dx)^4\right)}\,dx \\
&= -\frac{1}{(a+bc^4)\,x} - \frac{\sqrt{b}c(a-bc^4)\,d\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,(a+bc^4)^2} \\
&\quad + \frac{\sqrt[4]{b}\left(\sqrt{a}(a-3bc^4)+\sqrt{b}c^2(3a-bc^4)\right)d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(a+bc^4)^2} \\
&\quad - \frac{\sqrt[4]{b}\left(\sqrt{a}(a-3bc^4)+\sqrt{b}c^2(3a-bc^4)\right)d\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(a+bc^4)^2} - \frac{4bc^3d\log(x)}{(a+bc^4)^2} \\
&\quad - \frac{\sqrt[4]{b}\left(\sqrt{a}(a-3bc^4)-\sqrt{b}c^2(3a-bc^4)\right)d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\,(a+bc^4)^2} \\
&\quad + \frac{\sqrt[4]{b}\left(\sqrt{a}(a-3bc^4)-\sqrt{b}c^2(3a-bc^4)\right)d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\,(a+bc^4)^2} \\
&\quad + \frac{bc^3d\log\left(a+b(c+dx)^4\right)}{(a+bc^4)^2}
\end{aligned}$$

output
```
-1/(b*c^4+a)/x-4*b*c^3*d*ln(x)/(b*c^4+a)^2+b*c^3*d*ln(a+b*(d*x+c)^4)/(b*c^
4+a)^2-c*(-b*c^4+a)*d*arctan((d*x+c)^2*b^(1/2)/a^(1/2))*b^(1/2)/(b*c^4+a)^
2/a^(1/2)-1/8*b^(1/4)*d*ln(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c
)^2*b^(1/2))*((-3*b*c^4+a)*a^(1/2)-c^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^
4+a)^2*2^(1/2)+1/8*b^(1/4)*d*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d
*x+c)^2*b^(1/2))*((-3*b*c^4+a)*a^(1/2)-c^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(
b*c^4+a)^2*2^(1/2)-1/4*b^(1/4)*d*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4)
)*((-3*b*c^4+a)*a^(1/2)+c^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+a)^2*2^(1
/2)-1/4*b^(1/4)*d*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*((-3*b*c^4+a)*
a^(1/2)+c^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+a)^2*2^(1/2)
```

3.115.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^2 (a+b(c+dx)^4)}\,dx$$
$$= \frac{-4(a+bc^4+4bc^3dx\log(x))+dx\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&,\frac{-6a}{}\right.}{}$$

input
```
Integrate[1/(x^2*(a + b*(c + d*x)^4)),x]
```

output
```
(-4*(a + b*c^4 + 4*b*c^3*d*x*Log[x]) + d*x*RootSum[a + b*c^4 + 4*b*c^3*d*#
1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^4*#1^4 & , (-6*a*c^2*Log[x - #
1] + 10*b*c^6*Log[x - #1] - 4*a*c*d*Log[x - #1]*#1 + 20*b*c^5*d*Log[x - #1
]*#1 - a*d^2*Log[x - #1]*#1^2 + 15*b*c^4*d^2*Log[x - #1]*#1^2 + 4*b*c^3*d^
3*Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) & ])/(4*(
a + b*c^4)^2*x)
```

3.115.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {896, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+b(c+dx)^4\right)}dx$$

$$\Big\downarrow \text{ 896}$$

$$d\int \frac{1}{d^2x^2\left(b(c+dx)^4+a\right)}d(c+dx)$$

$$\Big\downarrow \text{ 7276}$$

$$d\int \left(-\frac{4bc^3}{\left(bc^4+a\right)^2dx}+\frac{b\left(4bc^3(c+dx)^3-\left(a-3bc^4\right)(c+dx)^2-2c\left(a-bc^4\right)(c+dx)-c^2\left(3a-bc^4\right)\right)}{\left(bc^4+a\right)^2\left(b(c+dx)^4+a\right)}+\frac{1}{\left(bc^4+\right.}\right.$$
$$\left.dx\right)$$

$$\Big\downarrow \text{ 2009}$$

$$d\left(\frac{\sqrt[4]{b}\left(\sqrt{a}\left(a-3bc^4\right)+\sqrt{b}c^2\left(3a-bc^4\right)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)^2}-\frac{\sqrt[4]{b}\left(\sqrt{a}\left(a-3bc^4\right)+\sqrt{b}c^2\left(3a-bc^4\right)\right)\text{arct}}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)^2}\right.$$

input
```
Int[1/(x^2*(a + b*(c + d*x)^4)),x]
```

output
```
d*(-(1/((a + b*c^4)*d*x)) - (Sqrt[b]*c*(a - b*c^4)*ArcTan[(Sqrt[b]*(c + d*
x)^2)/Sqrt[a]])/(Sqrt[a]*(a + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4)
+ Sqrt[b]*c^2*(3*a - b*c^4))*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4
)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) +
Sqrt[b]*c^2*(3*a - b*c^4))*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]
)/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (4*b*c^3*Log[-(d*x)])/(a + b*c^4)^2
- (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^
(3/4)*(a + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a
- b*c^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d
*x)^2])/(4*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) + (b*c^3*Log[a + b*(c + d*x)^4])
/(a + b*c^4)^2)
```

3.115.3.1 Defintions of rubi rules used

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.115.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.37

method	result
default	$-\frac{1}{(b c^4+a)x}-\frac{4 b c^3 d\ln(x)}{(b c^4+a)^2}-\frac{d\left(\sum_{\_R=\text{RootOf}\left(b d^4 \_Z^4+4 b c d^3 \_Z^3+6 b c^2 d^2 \_Z^2+4 b c^3 d \_Z+b c^4+a\right)}\frac{\left(-4 b d^3 c^3 \_R^3+d^2\left(-15 b c^4+a\right)\right.}{d^3 \_R}\right.}{4(b c^4+a)^2}$
risch	$-\frac{1}{(b c^4+a)x}-\frac{4 b c^3 d\ln(x)}{b^2 c^8+2 a b c^4+a^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(a^3 b^2 c^8+2 a^4 b c^4+a^5\right)\_Z^4-16 a^3 b c^3 d \_Z^3+20 a^2 b c^2 d^2 \_Z^2-8 a b c d^3 \_Z+b d^4\right)}\_R\right.}{}$

input
```
int(1/x^2/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/(b*c^4+a)/x-4*b*c^3*d*ln(x)/(b*c^4+a)^2-1/4*d/(b*c^4+a)^2*sum((-4*b*d^3
*c^3*_R^3+d^2*(-15*b*c^4+a)*_R^2+4*c*d*(-5*b*c^4+a)*_R-10*b*c^6+6*a*c^2)/(
_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_Z^4*b*d^4+4*_Z^3
*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

3.115.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 78.19 (sec) , antiderivative size = 1128605, normalized size of antiderivative = 2275.41

$$\int \frac{1}{x^2 (a+b(c+dx)^4)} dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

output
```
Too large to include
```

3.115.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+b(c+dx)^4)} dx = \text{Timed out}$$

input
```
integrate(1/x**2/(a+b*(d*x+c)**4),x)
```

output
```
Timed out
```

3.115.7 Maxima [F]

$$\int \frac{1}{x^2 (a+b(c+dx)^4)} dx = \int \frac{1}{\left((dx+c)^4 b+a\right)x^2} dx$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

output
```
-4*b*c^3*d*log(x)/(b^2*c^8 + 2*a*b*c^4 + a^2) + b*d^2*integrate((4*b*c^3*d
^3*x^3 + 10*b*c^6 + (15*b*c^4 - a)*d^2*x^2 - 6*a*c^2 + 4*(5*b*c^5 - a*c)*d
*x)/(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a
), x)/(b^2*c^8 + 2*a*b*c^4 + a^2) - 1/((b*c^4 + a)*x)
```

3.115.8 Giac [F]

$$\int \frac{1}{x^2\left(a+b(c+dx)^4\right)}\,dx=\int \frac{1}{\left((dx+c)^4b+a\right)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="giac")
```

output
```
integrate(1/((((d*x + c)^4*b + a)*x^2), x)
```

3.115.9 Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 2440, normalized size of antiderivative = 4.92

$$\int \frac{1}{x^2\left(a+b(c+dx)^4\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(x^2*(a + b*(c + d*x)^4)),x)
```

output
```
symsum(log(-(4*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4
- 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z
, k)^2*b^7*c^11*d^17 - 16*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 2
56*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z
 + b*d^4, z, k)^3*a^4*b^4*d^16 - b^5*d^20*x + 16*root(256*a^3*b^2*c^8*z^4
+ 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d
^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)*b^6*c^6*d^18 - 60*root(256*a^3*b^2*
c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2
*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a^2*b^5*c^3*d^17 + 176*ro
ot(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*
d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^3*b^5*c^
4*d^16 + 192*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 -
1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z,
k)^4*a^4*b^5*c^5*d^15 + 144*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 +
 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3
*z + b*d^4, z, k)^3*a^2*b^6*c^8*d^16 + 192*root(256*a^3*b^2*c^8*z^4 + 512*
a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2
 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^3*b^6*c^9*d^15 + 64*root(256*a^3*b^2*
c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2
*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^2*b^7*c^13*d^15 + 16...
```

3.116 $\int (a+8x-8x^2+4x^3-x^4)^4 \, dx$

3.116.1 Optimal result

Integrand size = 22, antiderivative size = 123

$$\int (a+8x-8x^2+4x^3-x^4)^4 \, dx = -\frac{8}{3}(3+a)^3(-1+x)^3+\frac{4}{5}(3-a)(3+a)^2(-1+x)^5 +\frac{8}{7}(3+a)(5+3a)(-1+x)^7-\frac{2}{9}(37+6a-3a^2)(-1+x)^9 -\frac{8}{11}(5+3a)(-1+x)^{11}+\frac{4}{13}(3-a)(-1+x)^{13} +\frac{8}{15}(-1+x)^{15}+\frac{1}{17}(-1+x)^{17}+(3+a)^4x$$

output
```
-8/3*(3+a)^3*(-1+x)^3+4/5*(3-a)*(3+a)^2*(-1+x)^5+8/7*(3+a)*(5+3*a)*(-1+x)^
7-2/9*(-3*a^2+6*a+37)*(-1+x)^9-8/11*(5+3*a)*(-1+x)^11+4/13*(3-a)*(-1+x)^13
+8/15*(-1+x)^15+1/17*(-1+x)^17+(3+a)^4*x
```

3.116.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.59

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^4 \, dx = a^4x+16a^3x^2-\frac{32}{3}(-12+a)a^2x^3+4a\left(128-48a+a^2\right)x^4 - \frac{4}{5}\left(-1024+1536a-192a^2+a^3\right)x^5 - \frac{16}{3}\left(512-288a+15a^2\right)x^6+\frac{64}{7}\left(512-140a+3a^2\right)x^7 - 6\left(896-128a+a^2\right)x^8+\frac{2}{9}\left(20480-1536a+3a^2\right)x^9 + \frac{16}{5}(-928+35a)x^{10}-\frac{32}{11}(-524+9a)x^{11} + \frac{4}{3}(-464+3a)x^{12}-\frac{4}{13}(-640+a)x^{13} - 48x^{14}+\frac{128x^{15}}{15}-x^{16}+\frac{x^{17}}{17}$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

output
```
a^4*x + 16*a^3*x^2 - (32*(-12 + a)*a^2*x^3)/3 + 4*a*(128 - 48*a + a^2)*x^4
 - (4*(-1024 + 1536*a - 192*a^2 + a^3)*x^5)/5 - (16*(512 - 288*a + 15*a^2)
*x^6)/3 + (64*(512 - 140*a + 3*a^2)*x^7)/7 - 6*(896 - 128*a + a^2)*x^8 + (
2*(20480 - 1536*a + 3*a^2)*x^9)/9 + (16*(-928 + 35*a)*x^10)/5 - (32*(-524
+ 9*a)*x^11)/11 + (4*(-464 + 3*a)*x^12)/3 - (4*(-640 + a)*x^13)/13 - 48*x^
14 + (128*x^15)/15 - x^16 + x^17/17
```

3.116.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-x^4+4x^3-8x^2+8x\right)^4 dx$$

$$\Big\downarrow \; 2458$$

$$\int \left(a-(x-1)^4-2(x-1)^2+3\right)^4 d(x-1)$$

$$\Big\downarrow \text{1403}$$

$$\int \left(108\left(1-\frac{1}{27}a(a^2+3a-9)\right)(x-1)^4+81\left(\frac{1}{81}a(a^3+12a^2+54a+108)+1\right)+12\left(1-\frac{a}{3}\right)(x-1)^{12}-40\left(\right.$$
1)

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{9}\left(-3a^2+6a+37\right)(x-1)^9+\frac{4}{13}(3-a)(x-1)^{13}-\frac{8}{11}(3a+5)(x-1)^{11}+\frac{8}{7}(a+3)(3a+5)(x-1)^7+\frac{4}{5}(3-a)(a+3)^2(x-1)^5-\frac{8}{3}(a+3)^3(x-1)^3+(a+3)^4(x-1)+\frac{1}{17}(x-1)^{17}+\frac{8}{15}(x-1)^{15}$$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

output
```
(3 + a)^4*(-1 + x) - (8*(3 + a)^3*(-1 + x)^3)/3 + (4*(3 - a)*(3 + a)^2*(-1
 + x)^5)/5 + (8*(3 + a)*(5 + 3*a)*(-1 + x)^7)/7 - (2*(37 + 6*a - 3*a^2)*(-
1 + x)^9)/9 - (8*(5 + 3*a)*(-1 + x)^11)/11 + (4*(3 - a)*(-1 + x)^13)/13 +
(8*(-1 + x)^15)/15 + (-1 + x)^17/17
```

3.116.3.1 Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.116.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.46

method	result
norman	$a^4x+16a^3x^2+\left(-\frac{32}{3}a^3+128a^2\right)x^3+\left(4a^3-192a^2+512a\right)x^4+\left(-\frac{4}{5}a^3+\frac{768}{5}a^2-\frac{6144}{5}a+\right.$
gosper	$-\frac{14848}{5}x^{10}+\frac{16768}{11}x^{11}-\frac{1856}{3}x^{12}+\frac{2560}{13}x^{13}+\frac{4096}{5}x^5+\frac{40960}{9}x^9+\frac{32768}{7}x^7+512a\,x^4-48x^{14}-$
risch	$-\frac{14848}{5}x^{10}+\frac{16768}{11}x^{11}-\frac{1856}{3}x^{12}+\frac{2560}{13}x^{13}+\frac{4096}{5}x^5+\frac{40960}{9}x^9+\frac{32768}{7}x^7+512a\,x^4-48x^{14}-$
parallelrisch	$-\frac{14848}{5}x^{10}+\frac{16768}{11}x^{11}-\frac{1856}{3}x^{12}+\frac{2560}{13}x^{13}+\frac{4096}{5}x^5+\frac{40960}{9}x^9+\frac{32768}{7}x^7+512a\,x^4-48x^{14}-$
default	$\frac{x^{17}}{17}-x^{16}+\frac{128x^{15}}{15}-48x^{14}+\frac{(-4a+2560)x^{13}}{13}+\frac{(48a-7424)x^{12}}{12}+\frac{(-288a+16768)x^{11}}{11}+\frac{(1120a-29696)x^{10}}{10}$

input
```
int((-x^4+4*x^3-8*x^2+a+8*x)^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*x+16*a^3*x^2+(-32/3*a^3+128*a^2)*x^3+(4*a^3-192*a^2+512*a)*x^4+(-4/5*a
^3+768/5*a^2-6144/5*a+4096/5)*x^5+(-80*a^2+1536*a-8192/3)*x^6+(192/7*a^2-1
280*a+32768/7)*x^7+(-6*a^2+768*a-5376)*x^8+(2/3*a^2-1024/3*a+40960/9)*x^9+
(112*a-14848/5)*x^10+(-288/11*a+16768/11)*x^11+(4*a-1856/3)*x^12+(-4/13*a+
2560/13)*x^13-48*x^14+128/15*x^15-x^16+1/17*x^17
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.46

$$\begin{aligned}\int \left(a+8x-8x^2+4x^3-x^4\right)^4\,dx =& \frac{1}{17}x^{17}-x^{16}+\frac{128}{15}x^{15}-\frac{4}{13}(a-640)x^{13}\\ &-48\,x^{14}+\frac{4}{3}(3\,a-464)x^{12}-\frac{32}{11}(9\,a-524)x^{11}\\ &+\frac{16}{5}(35\,a-928)x^{10}+\frac{2}{9}\left(3\,a^2-1536\,a+20480\right)x^9\\ &-6\left(a^2-128\,a+896\right)x^8+\frac{64}{7}\left(3\,a^2-140\,a+512\right)x^7\\ &-\frac{16}{3}\left(15\,a^2-288\,a+512\right)x^6\\ &-\frac{4}{5}\left(a^3-192\,a^2+1536\,a-1024\right)x^5+a^4x+16\,a^3x^2\\ &+4\left(a^3-48\,a^2+128\,a\right)x^4-\frac{32}{3}\left(a^3-12\,a^2\right)x^3\end{aligned}$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

output
```
1/17*x^17 - x^16 + 128/15*x^15 - 4/13*(a - 640)*x^13 - 48*x^14 + 4/3*(3*a
- 464)*x^12 - 32/11*(9*a - 524)*x^11 + 16/5*(35*a - 928)*x^10 + 2/9*(3*a^2
 - 1536*a + 20480)*x^9 - 6*(a^2 - 128*a + 896)*x^8 + 64/7*(3*a^2 - 140*a +
 512)*x^7 - 16/3*(15*a^2 - 288*a + 512)*x^6 - 4/5*(a^3 - 192*a^2 + 1536*a
- 1024)*x^5 + a^4*x + 16*a^3*x^2 + 4*(a^3 - 48*a^2 + 128*a)*x^4 - 32/3*(a^
3 - 12*a^2)*x^3
```

3.116.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.62

$$\int (a+8x-8x^2+4x^3-x^4)^4\,dx = a^4x+16a^3x^2+\frac{x^{17}}{17}-x^{16}+\frac{128x^{15}}{15}-48x^{14}+x^{13}\cdot\left(\frac{2560}{13}-\frac{4a}{13}\right)+x^{12}\cdot\left(4a-\frac{1856}{3}\right)+x^{11}\cdot\left(\frac{16768}{11}-\frac{288a}{11}\right)+x^{10}\cdot\left(112a-\frac{14848}{5}\right)+x^9\cdot\left(\frac{2a^2}{3}-\frac{1024a}{3}+\frac{40960}{9}\right)+x^8\left(-6a^2+768a-5376\right)+x^7\cdot\left(\frac{192a^2}{7}-1280a+\frac{32768}{7}\right)+x^6\left(-80a^2+1536a-\frac{8192}{3}\right)+x^5\left(-\frac{4a^3}{5}+\frac{768a^2}{5}-\frac{6144a}{5}+\frac{4096}{5}\right)+x^4\cdot\left(4a^3-192a^2+512a\right)+x^3\left(-\frac{32a^3}{3}+128a^2\right)$$

input
```
integrate((-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

output
```
a**4*x + 16*a**3*x**2 + x**17/17 - x**16 + 128*x**15/15 - 48*x**14 + x**13
*(2560/13 - 4*a/13) + x**12*(4*a - 1856/3) + x**11*(16768/11 - 288*a/11) +
 x**10*(112*a - 14848/5) + x**9*(2*a**2/3 - 1024*a/3 + 40960/9) + x**8*(-6
*a**2 + 768*a - 5376) + x**7*(192*a**2/7 - 1280*a + 32768/7) + x**6*(-80*a
**2 + 1536*a - 8192/3) + x**5*(-4*a**3/5 + 768*a**2/5 - 6144*a/5 + 4096/5)
 + x**4*(4*a**3 - 192*a**2 + 512*a) + x**3*(-32*a**3/3 + 128*a**2)
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.56

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^4 \, dx = \frac{1}{17}\,x^{17}-x^{16}+\frac{128}{15}\,x^{15}-48\,x^{14}+\frac{2560}{13}\,x^{13}-\frac{1856}{3}\,x^{12}+\frac{16768}{11}\,x^{11}-\frac{14848}{5}\,x^{10}+\frac{40960}{9}\,x^{9} - 5376\,x^8+\frac{32768}{7}\,x^7-\frac{8192}{3}\,x^6+a^4x+\frac{4096}{5}\,x^5-\frac{4}{15}\left(3\,x^5-15\,x^4+40\,x^3-60\,x^2\right)a^3 + \frac{2}{105}\left(35\,x^9-315\,x^8+1440\,x^7-4200\,x^6+8064\,x^5-10080\,x^4+6720\,x^3\right)a^2 - \frac{4}{2145}\left(165\,x^{13}-2145\,x^{12}+14040\,x^{11}-60060\,x^{10}+183040\,x^9-411840\,x^8+686400\,x^7-823680\,x^6+\right.$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

output
```
1/17*x^17 - x^16 + 128/15*x^15 - 48*x^14 + 2560/13*x^13 - 1856/3*x^12 + 16
768/11*x^11 - 14848/5*x^10 + 40960/9*x^9 - 5376*x^8 + 32768/7*x^7 - 8192/3
*x^6 + a^4*x + 4096/5*x^5 - 4/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a^3 +
2/105*(35*x^9 - 315*x^8 + 1440*x^7 - 4200*x^6 + 8064*x^5 - 10080*x^4 + 672
0*x^3)*a^2 - 4/2145*(165*x^13 - 2145*x^12 + 14040*x^11 - 60060*x^10 + 1830
40*x^9 - 411840*x^8 + 686400*x^7 - 823680*x^6 + 658944*x^5 - 274560*x^4)*a
```

3.116.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(103) = 206.

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.78

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^4 \, dx = \frac{1}{17}\,x^{17}-x^{16}+\frac{128}{15}\,x^{15}-\frac{4}{13}\,ax^{13}-48\,x^{14}+4\,ax^{12} + \frac{2560}{13}\,x^{13}-\frac{288}{11}\,ax^{11}-\frac{1856}{3}\,x^{12}+\frac{2}{3}\,a^2x^9+112\,ax^{10} + \frac{16768}{11}\,x^{11}-6\,a^2x^8-\frac{1024}{3}\,ax^9-\frac{14848}{5}\,x^{10} + \frac{192}{7}\,a^2x^7+768\,ax^8+\frac{40960}{9}\,x^9-\frac{4}{5}\,a^3x^5-80\,a^2x^6 - 1280\,ax^7-5376\,x^8+4\,a^3x^4+\frac{768}{5}\,a^2x^5+1536\,ax^6 + \frac{32768}{7}\,x^7-\frac{32}{3}\,a^3x^3-192\,a^2x^4-\frac{6144}{5}\,ax^5-\frac{8192}{3}\,x^6 + a^4x+16\,a^3x^2+128\,a^2x^3+512\,ax^4+\frac{4096}{5}\,x^5$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

output
```
1/17*x^17 - x^16 + 128/15*x^15 - 4/13*a*x^13 - 48*x^14 + 4*a*x^12 + 2560/1
3*x^13 - 288/11*a*x^11 - 1856/3*x^12 + 2/3*a^2*x^9 + 112*a*x^10 + 16768/11
*x^11 - 6*a^2*x^8 - 1024/3*a*x^9 - 14848/5*x^10 + 192/7*a^2*x^7 + 768*a*x^
8 + 40960/9*x^9 - 4/5*a^3*x^5 - 80*a^2*x^6 - 1280*a*x^7 - 5376*x^8 + 4*a^3
*x^4 + 768/5*a^2*x^5 + 1536*a*x^6 + 32768/7*x^7 - 32/3*a^3*x^3 - 192*a^2*x
^4 - 6144/5*a*x^5 - 8192/3*x^6 + a^4*x + 16*a^3*x^2 + 128*a^2*x^3 + 512*a*
x^4 + 4096/5*x^5
```

3.116.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.42

$$\begin{aligned}\int (a+8x-8x^2+4x^3-x^4)^4\,dx &= x^{12}\left(4a-\frac{1856}{3}\right)-x^{13}\left(\frac{4a}{13}-\frac{2560}{13}\right)\\ &\quad+x^{10}\left(112a-\frac{14848}{5}\right)-x^{11}\left(\frac{288a}{11}-\frac{16768}{11}\right)\\ &\quad-x^8\left(6a^2-768a+5376\right)-x^6\left(80a^2-1536a+\frac{8192}{3}\right)\\ &\quad+x^7\left(\frac{192a^2}{7}-1280a+\frac{32768}{7}\right)\\ &\quad+x^9\left(\frac{2a^2}{3}-\frac{1024a}{3}+\frac{40960}{9}\right)\\ &\quad-x^5\left(\frac{4a^3}{5}-\frac{768a^2}{5}+\frac{6144a}{5}-\frac{4096}{5}\right)+a^4x\\ &\quad-48x^{14}+\frac{128x^{15}}{15}-x^{16}+\frac{x^{17}}{17}+16a^3x^2\\ &\quad+4ax^4\left(a^2-48a+128\right)-\frac{32a^2x^3(a-12)}{3}\end{aligned}$$

input
```
int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

output
```
x^12*(4*a - 1856/3) - x^13*((4*a)/13 - 2560/13) + x^10*(112*a - 14848/5) -
 x^11*((288*a)/11 - 16768/11) - x^8*(6*a^2 - 768*a + 5376) - x^6*(80*a^2 -
 1536*a + 8192/3) + x^7*((192*a^2)/7 - 1280*a + 32768/7) + x^9*((2*a^2)/3
- (1024*a)/3 + 40960/9) - x^5*((6144*a)/5 - (768*a^2)/5 + (4*a^3)/5 - 4096
/5) + a^4*x - 48*x^14 + (128*x^15)/15 - x^16 + x^17/17 + 16*a^3*x^2 + 4*a*
x^4*(a^2 - 48*a + 128) - (32*a^2*x^3*(a - 12))/3
```

3.117 $\int (a+8x-8x^2+4x^3-x^4)^3 \, dx$

3.117.1 Optimal result

Integrand size = 22, antiderivative size = 120

$$\int (a+8x-8x^2+4x^3-x^4)^3 \, dx = a^3x+12a^2x^2+8(8-a)ax^3+(128-96a+3a^2)\,x^4 - \frac{3}{5}(512-128a+a^2)\,x^5+8(48-5a)x^6-\frac{32}{7}(70-3a)x^7 + 3(64-a)x^8-\frac{1}{3}(256-a)x^9+28x^{10}-\frac{72x^{11}}{11}+x^{12}-\frac{x^{13}}{13}$$

output
```
a^3*x+12*a^2*x^2+8*(8-a)*a*x^3+(3*a^2-96*a+128)*x^4-3/5*(a^2-128*a+512)*x^
5+8*(48-5*a)*x^6-32/7*(70-3*a)*x^7+3*(64-a)*x^8-1/3*(256-a)*x^9+28*x^10-72
/11*x^11+x^12-1/13*x^13
```

3.117.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.95

$$\int (a+8x-8x^2+4x^3-x^4)^3 \, dx = a^3x+12a^2x^2-8(-8+a)ax^3+(128-96a+3a^2)\,x^4 - \frac{3}{5}(512-128a+a^2)\,x^5-8(-48+5a)x^6 + \frac{32}{7}(-70+3a)x^7-3(-64+a)x^8 + \frac{1}{3}(-256+a)x^9+28x^{10}-\frac{72x^{11}}{11}+x^{12}-\frac{x^{13}}{13}$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

output
```
a^3*x + 12*a^2*x^2 - 8*(-8 + a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4 - (3*(512
 - 128*a + a^2)*x^5)/5 - 8*(-48 + 5*a)*x^6 + (32*(-70 + 3*a)*x^7)/7 - 3*(-
64 + a)*x^8 + ((-256 + a)*x^9)/3 + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13
```

3.117.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-x^4+4x^3-8x^2+8x\right)^3 dx$$

$$\Big\downarrow \text{2458}$$

$$\int \left(a-(x-1)^4-2(x-1)^2+3\right)^3 d(x-1)$$

$$\Big\downarrow \text{1403}$$

$$\int \left(-3(1-a)(x-1)^8+28\left(\frac{3a}{7}+1\right)(x-1)^6+9\left(1-\frac{1}{3}a(a+2)\right)(x-1)^4-54\left(\frac{1}{9}a(a+6)+1\right)(x-1)^2+27 \right. \\ 1)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}(1-a)(x-1)^9+\frac{4}{7}(3a+7)(x-1)^7+\frac{3}{5}(1-a)(a+3)(x-1)^5-2(a+3)^2(x-1)^3+(a+3)^3(x-1)-\frac{1}{13}(x-1)^{13}-\frac{6}{11}(x-1)^{11}$$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

output
```
(3 + a)^3*(-1 + x) - 2*(3 + a)^2*(-1 + x)^3 + (3*(1 - a)*(3 + a)*(-1 + x)^
5)/5 + (4*(7 + 3*a)*(-1 + x)^7)/7 - ((1 - a)*(-1 + x)^9)/3 - (6*(-1 + x)^1
1)/11 - (-1 + x)^13/13
```

3.117.3.1 Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.117.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

method	result
norman	$-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}+\left(\frac{a}{3}-\frac{256}{3}\right)x^{9}+\left(-3a+192\right)x^{8}+\left(\frac{96a}{7}-320\right)x^{7}+\left(-40a+3\right.$
gosper	$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+28x^{10}+\frac{1}{3}x^{9}a-\frac{256}{3}x^{9}-3a\,x^{8}+192x^{8}+\frac{96}{7}a\,x^{7}-320x^{7}-40a\,x^{6}-$
risch	$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+28x^{10}+\frac{1}{3}x^{9}a-\frac{256}{3}x^{9}-3a\,x^{8}+192x^{8}+\frac{96}{7}a\,x^{7}-320x^{7}-40a\,x^{6}-$
parallelrisch	$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+28x^{10}+\frac{1}{3}x^{9}a-\frac{256}{3}x^{9}-3a\,x^{8}+192x^{8}+\frac{96}{7}a\,x^{7}-320x^{7}-40a\,x^{6}-$
default	$-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}+\frac{(3a-768)x^{9}}{9}+\frac{(-24a+1536)x^{8}}{8}+\frac{(96a-2240)x^{7}}{7}+\frac{(-240a+2304)x^{6}}{6}+\frac{(a(-$

input
```
int((-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/13*x^13+x^12-72/11*x^11+28*x^10+(1/3*a-256/3)*x^9+(-3*a+192)*x^8+(96/7*
a-320)*x^7+(-40*a+384)*x^6+(-3/5*a^2+384/5*a-1536/5)*x^5+(3*a^2-96*a+128)*
x^4+(-8*a^2+64*a)*x^3+12*a^2*x^2+a^3*x
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.89

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^3 dx = -\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+\frac{1}{3}(a-256)x^9+28x^{10} -3(a-64)x^8+\frac{32}{7}(3a-70)x^7-8(5a-48)x^6 -\frac{3}{5}\left(a^2-128a+512\right)x^5+\left(3a^2-96a+128\right)x^4 +a^3x+12a^2x^2-8\left(a^2-8a\right)x^3$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

output
```
-1/13*x^13 + x^12 - 72/11*x^11 + 1/3*(a - 256)*x^9 + 28*x^10 - 3*(a - 64)*
x^8 + 32/7*(3*a - 70)*x^7 - 8*(5*a - 48)*x^6 - 3/5*(a^2 - 128*a + 512)*x^5
 + (3*a^2 - 96*a + 128)*x^4 + a^3*x + 12*a^2*x^2 - 8*(a^2 - 8*a)*x^3
```

3.117.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.95

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^3 dx = a^3x+12a^2x^2-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10} +x^9\left(\frac{a}{3}-\frac{256}{3}\right)+x^8\cdot(192-3a)+x^7\cdot\left(\frac{96a}{7}-320\right) +x^6\cdot(384-40a)+x^5\left(-\frac{3a^2}{5}+\frac{384a}{5}-\frac{1536}{5}\right) +x^4\cdot\left(3a^2-96a+128\right)+x^3\left(-8a^2+64a\right)$$

input
```
integrate((-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

output
```
a**3*x + 12*a**2*x**2 - x**13/13 + x**12 - 72*x**11/11 + 28*x**10 + x**9*(
a/3 - 256/3) + x**8*(192 - 3*a) + x**7*(96*a/7 - 320) + x**6*(384 - 40*a)
+ x**5*(-3*a**2/5 + 384*a/5 - 1536/5) + x**4*(3*a**2 - 96*a + 128) + x**3*
(-8*a**2 + 64*a)
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99

$$\int (a + 8x - 8x^2 + 4x^3 - x^4)^3 \, dx$$
$$= -\frac{1}{13}x^{13} + x^{12} - \frac{72}{11}x^{11} + 28x^{10} - \frac{256}{3}x^9 + 192x^8 - 320x^7 + 384x^6 - \frac{1536}{5}x^5 + a^3x + 128x^4 - \frac{1}{5}(3x^5 - 15x^4 + 40x^3 - 60x^2)a^2 + \frac{1}{105}(35x^9 - 315x^8 + 1440x^7 - 4200x^6 + 8064x^5 - 10080x^4 + 6720x^3)a$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

output
```
-1/13*x^13 + x^12 - 72/11*x^11 + 28*x^10 - 256/3*x^9 + 192*x^8 - 320*x^7 +
 384*x^6 - 1536/5*x^5 + a^3*x + 128*x^4 - 1/5*(3*x^5 - 15*x^4 + 40*x^3 - 6
0*x^2)*a^2 + 1/105*(35*x^9 - 315*x^8 + 1440*x^7 - 4200*x^6 + 8064*x^5 - 10
080*x^4 + 6720*x^3)*a
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.07

$$\int (a + 8x - 8x^2 + 4x^3 - x^4)^3 \, dx = -\frac{1}{13}x^{13} + x^{12} - \frac{72}{11}x^{11} + \frac{1}{3}ax^9 + 28x^{10} - 3ax^8 - \frac{256}{3}x^9 + \frac{96}{7}ax^7 + 192x^8 - \frac{3}{5}a^2x^5 - 40ax^6 - 320x^7 + 3a^2x^4 + \frac{384}{5}ax^5 + 384x^6 - 8a^2x^3 - 96ax^4 - \frac{1536}{5}x^5 + a^3x + 12a^2x^2 + 64ax^3 + 128x^4$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

output
```
-1/13*x^13 + x^12 - 72/11*x^11 + 1/3*a*x^9 + 28*x^10 - 3*a*x^8 - 256/3*x^9
 + 96/7*a*x^7 + 192*x^8 - 3/5*a^2*x^5 - 40*a*x^6 - 320*x^7 + 3*a^2*x^4 + 3
84/5*a*x^5 + 384*x^6 - 8*a^2*x^3 - 96*a*x^4 - 1536/5*x^5 + a^3*x + 12*a^2*
x^2 + 64*a*x^3 + 128*x^4
```

3.117.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.90

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^3\,dx = x^9\left(\frac{a}{3}-\frac{256}{3}\right)-x^8\,(3\,a-192)-x^6\,(40\,a-384)+x^7\left(\frac{96\,a}{7}-320\right)+x^4\,\left(3\,a^2-96\,a+128\right)-x^5\left(\frac{3\,a^2}{5}-\frac{384\,a}{5}+\frac{1536}{5}\right)+a^3\,x+28\,x^{10}-\frac{72\,x^{11}}{11}+x^{12}-\frac{x^{13}}{13}+12\,a^2\,x^2-8\,a\,x^3\,(a-8)$$

input
```
int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

output
```
x^9*(a/3 - 256/3) - x^8*(3*a - 192) - x^6*(40*a - 384) + x^7*((96*a)/7 - 3
20) + x^4*(3*a^2 - 96*a + 128) - x^5*((3*a^2)/5 - (384*a)/5 + 1536/5) + a^
3*x + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13 + 12*a^2*x^2 - 8*a*x^3*(a -
8)
```

3.118 $\int (a+8x-8x^2+4x^3-x^4)^2 \, dx$

3.118.1 Optimal result

Integrand size = 22, antiderivative size = 72

$$\int (a+8x-8x^2+4x^3-x^4)^2 \, dx = a^2x+8ax^2+\frac{16}{3}(4-a)x^3-2(16-a)x^4 +\frac{2}{5}(64-a)x^5-\frac{40x^6}{3}+\frac{32x^7}{7}-x^8+\frac{x^9}{9}$$

output
```
a^2*x+8*a*x^2+16/3*(4-a)*x^3-2*(16-a)*x^4+2/5*(64-a)*x^5-40/3*x^6+32/7*x^7
-x^8+1/9*x^9
```

3.118.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int (a+8x-8x^2+4x^3-x^4)^2 \, dx = a^2x+8ax^2-\frac{16}{3}(-4+a)x^3+2(-16+a)x^4 -\frac{2}{5}(-64+a)x^5-\frac{40x^6}{3}+\frac{32x^7}{7}-x^8+\frac{x^9}{9}$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
a^2*x + 8*a*x^2 - (16*(-4 + a)*x^3)/3 + 2*(-16 + a)*x^4 - (2*(-64 + a)*x^5
)/5 - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

3.118.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-x^4+4x^3-8x^2+8x\right)^2 dx$$

$$\Big\downarrow \text{2458}$$

$$\int \left(a-(x-1)^4-2(x-1)^2+3\right)^2 d(x-1)$$

$$\Big\downarrow \text{1403}$$

$$\int \left(-2(a+1)(x-1)^4-12\left(\frac{a}{3}+1\right)(x-1)^2+9\left(\frac{1}{9}a(a+6)+1\right)+(x-1)^8+4(x-1)^6\right)d(x-1)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{5}(a+1)(x-1)^5-\frac{4}{3}(a+3)(x-1)^3+(a+3)^2(x-1)+\frac{1}{9}(x-1)^9+\frac{4}{7}(x-1)^7$$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
(3 + a)^2*(-1 + x) - (4*(3 + a)*(-1 + x)^3)/3 - (2*(1 + a)*(-1 + x)^5)/5 +
 (4*(-1 + x)^7)/7 + (-1 + x)^9/9
```

3.118.3.1 Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.118.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

method	result	siz
norman	$\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}+\left(-\frac{2a}{5}+\frac{128}{5}\right)x^5+\left(2a-32\right)x^4+\left(-\frac{16a}{3}+\frac{64}{3}\right)x^3+8a\,x^2+a^2x$	60
default	$\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}+\frac{(-2a+128)x^5}{5}+\frac{(8a-128)x^4}{4}+\frac{(-16a+64)x^3}{3}+8a\,x^2+a^2x$	63
gosper	$\frac{1}{9}x^9-x^8+\frac{32}{7}x^7-\frac{40}{3}x^6-\frac{2}{5}a\,x^5+\frac{128}{5}x^5+2a\,x^4-32x^4-\frac{16}{3}a\,x^3+\frac{64}{3}x^3+8a\,x^2+a^2x$	66
risch	$\frac{1}{9}x^9-x^8+\frac{32}{7}x^7-\frac{40}{3}x^6-\frac{2}{5}a\,x^5+\frac{128}{5}x^5+2a\,x^4-32x^4-\frac{16}{3}a\,x^3+\frac{64}{3}x^3+8a\,x^2+a^2x$	66
parallelrisch	$\frac{1}{9}x^9-x^8+\frac{32}{7}x^7-\frac{40}{3}x^6-\frac{2}{5}a\,x^5+\frac{128}{5}x^5+2a\,x^4-32x^4-\frac{16}{3}a\,x^3+\frac{64}{3}x^3+8a\,x^2+a^2x$	66

input
```
int((-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*x^9-x^8+32/7*x^7-40/3*x^6+(-2/5*a+128/5)*x^5+(2*a-32)*x^4+(-16/3*a+64/
3)*x^3+8*a*x^2+a^2*x
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{1}{9}\,x^9-x^8+\frac{32}{7}\,x^7-\frac{2}{5}\,(a-64)x^5-\frac{40}{3}\,x^6+2\,(a-16)x^4-\frac{16}{3}\,(a-4)x^3+a^2x+8\,ax^2$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

output
```
1/9*x^9 - x^8 + 32/7*x^7 - 2/5*(a - 64)*x^5 - 40/3*x^6 + 2*(a - 16)*x^4 -
16/3*(a - 4)*x^3 + a^2*x + 8*a*x^2
```

3.118.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int (a+8x-8x^2+4x^3-x^4)^2\,dx = a^2x+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}+x^5\cdot\left(\frac{128}{5}-\frac{2a}{5}\right)+x^4\cdot(2a-32)+x^3\cdot\left(\frac{64}{3}-\frac{16a}{3}\right)$$

input
```
integrate((-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

output
```
a**2*x + 8*a*x**2 + x**9/9 - x**8 + 32*x**7/7 - 40*x**6/3 + x**5*(128/5 -
2*a/5) + x**4*(2*a - 32) + x**3*(64/3 - 16*a/3)
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int (a+8x-8x^2+4x^3-x^4)^2\,dx = \frac{1}{9}x^9-x^8+\frac{32}{7}x^7-\frac{40}{3}x^6+\frac{128}{5}x^5-32x^4+a^2x+\frac{64}{3}x^3-\frac{2}{15}(3x^5-15x^4+40x^3-60x^2)a$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

output
```
1/9*x^9 - x^8 + 32/7*x^7 - 40/3*x^6 + 128/5*x^5 - 32*x^4 + a^2*x + 64/3*x^
3 - 2/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int (a+8x-8x^2+4x^3-x^4)^2\,dx = \frac{1}{9}x^9-x^8+\frac{32}{7}x^7-\frac{2}{5}ax^5-\frac{40}{3}x^6+2ax^4+\frac{128}{5}x^5-\frac{16}{3}ax^3-32x^4+a^2x+8ax^2+\frac{64}{3}x^3$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

output
```
1/9*x^9 - x^8 + 32/7*x^7 - 2/5*a*x^5 - 40/3*x^6 + 2*a*x^4 + 128/5*x^5 - 16
/3*a*x^3 - 32*x^4 + a^2*x + 8*a*x^2 + 64/3*x^3
```

3.118.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int (a+8x-8x^2+4x^3-x^4)^2\,dx = x^4\,(2\,a-32)-x^3\left(\frac{16\,a}{3}-\frac{64}{3}\right)-x^5\left(\frac{2\,a}{5}-\frac{128}{5}\right)+8\,a\,x^2+a^2\,x-\frac{40\,x^6}{3}+\frac{32\,x^7}{7}-x^8+\frac{x^9}{9}$$

input
```
int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

output
```
x^4*(2*a - 32) - x^3*((16*a)/3 - 64/3) - x^5*((2*a)/5 - 128/5) + 8*a*x^2 +
 a^2*x - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

3.119 $\int (a+8x-8x^2+4x^3-x^4)\,dx$

3.119.1 Optimal result

Integrand size = 20, antiderivative size = 26

$$\int (a+8x-8x^2+4x^3-x^4)\,dx = ax+4x^2-\frac{8x^3}{3}+x^4-\frac{x^5}{5}$$

output
```
a*x+4*x^2-8/3*x^3+x^4-1/5*x^5
```

3.119.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (a+8x-8x^2+4x^3-x^4)\,dx = ax+4x^2-\frac{8x^3}{3}+x^4-\frac{x^5}{5}$$

input
```
Integrate[a + 8*x - 8*x^2 + 4*x^3 - x^4,x]
```

output
```
a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

3.119.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-x^4+4x^3-8x^2+8x\right) \, dx$$
$$\Big\downarrow \; 2009$$
$$ax-\frac{x^5}{5}+x^4-\frac{8x^3}{3}+4x^2$$

input
```
Int[a + 8*x - 8*x^2 + 4*x^3 - x^4,x]
```

output
```
a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

3.119.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.119.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
gosper	$ax+4x^2-\frac{8}{3}x^3+x^4-\frac{1}{5}x^5$	23
default	$ax+4x^2-\frac{8}{3}x^3+x^4-\frac{1}{5}x^5$	23
norman	$ax+4x^2-\frac{8}{3}x^3+x^4-\frac{1}{5}x^5$	23
risch	$ax+4x^2-\frac{8}{3}x^3+x^4-\frac{1}{5}x^5$	23
parallelrisch	$ax+4x^2-\frac{8}{3}x^3+x^4-\frac{1}{5}x^5$	23
parts	$ax+4x^2-\frac{8}{3}x^3+x^4-\frac{1}{5}x^5$	23

input
```
int(-x^4+4*x^3-8*x^2+a+8*x,x,method=_RETURNVERBOSE)
```

output
```
a*x+4*x^2-8/3*x^3+x^4-1/5*x^5
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int (a+8x-8x^2+4x^3-x^4)\,dx=-\frac{1}{5}\,x^5+x^4-\frac{8}{3}\,x^3+ax+4\,x^2$$

input
```
integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="fricas")
```

output
```
-1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

3.119.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int (a+8x-8x^2+4x^3-x^4)\,dx=ax-\frac{x^5}{5}+x^4-\frac{8x^3}{3}+4x^2$$

input
```
integrate(-x**4+4*x**3-8*x**2+a+8*x,x)
```

output
```
a*x - x**5/5 + x**4 - 8*x**3/3 + 4*x**2
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int (a+8x-8x^2+4x^3-x^4)\,dx=-\frac{1}{5}\,x^5+x^4-\frac{8}{3}\,x^3+ax+4\,x^2$$

input
```
integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="maxima")
```

output
```
-1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \left(a+8x-8x^2+4x^3-x^4\right)\,dx=-\frac{1}{5}\,x^5+x^4-\frac{8}{3}\,x^3+ax+4\,x^2$$

input
```
integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="giac")
```

output
```
-1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

3.119.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \left(a+8x-8x^2+4x^3-x^4\right)\,dx=-\frac{x^5}{5}+x^4-\frac{8\,x^3}{3}+4\,x^2+a\,x$$

input
```
int(a + 8*x - 8*x^2 + 4*x^3 - x^4,x)
```

output
```
a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

3.120 $\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx$

3.120.1 Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx=-\frac{\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}}+\frac{\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}}$$

output
```
-1/2*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))/(4+a)^(1/2)/(1-(4+a)^(1/2))^(1/2
)+1/2*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2))/(4+a)^(1/2)/(1+(4+a)^(1/2))^(1/
2)
```

3.120.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.64

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx=-\frac{1}{4}\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&,\frac{\log(x-\#1)}{-2+4\#1-3\#1^2+\#1^3}\&\right]$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1),x]
```

output
```
-1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , Log[x - #1]/(-2 + 4*#1
- 3*#1^2 + #1^3) & ]
```

3.120.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2458, 1406, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-x^4+4x^3-8x^2+8x}dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{a-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{1406}$$

$$\frac{\int \frac{1}{-(x-1)^2+\sqrt{a+4}-1}d(x-1)}{2\sqrt{a+4}}-\frac{\int \frac{1}{-(x-1)^2-\sqrt{a+4}-1}d(x-1)}{2\sqrt{a+4}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}-\frac{\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}$$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1),x]
```

output
```
-1/2*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(Sqrt[4 + a]*Sqrt[1 - Sqrt[4 +
 a]]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqr
t[4 + a]])
```

3.120.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.120.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-4\_Z^{3}+8\_Z^{2}-8\_Z-a\right)}\frac{\ln\left(x-\_R\right)}{-\_R^{3}+3\_R^{2}-4\_R+2}\right)}{4}$	51
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-4\_Z^{3}+8\_Z^{2}-8\_Z-a\right)}\frac{\ln\left(x-\_R\right)}{-\_R^{3}+3\_R^{2}-4\_R+2}\right)}{4}$	51

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z
-a))
```

3.120.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(65) = 130$.

Time = 0.25 (sec) , antiderivative size = 457, normalized size of antiderivative = 5.13

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx$$
$$= \frac{1}{4}\sqrt{\frac{\frac{a^2+7\,a+12}{\sqrt{a^3+10\,a^2+33\,a+36}}+1}{a^2+7\,a+12}}\log\left(\left(a-\frac{a^2+7\,a+12}{\sqrt{a^3+10\,a^2+33\,a+36}}+4\right)\sqrt{\frac{\frac{a^2+7\,a+12}{\sqrt{a^3+10\,a^2+33\,a+36}}+1}{a^2+7\,a+12}}+x-1\right)$$
$$-\frac{1}{4}\sqrt{\frac{\frac{a^2+7\,a+12}{\sqrt{a^3+10\,a^2+33\,a+36}}+1}{a^2+7\,a+12}}\log\left(-\left(a-\frac{a^2+7\,a+12}{\sqrt{a^3+10\,a^2+33\,a+36}}+4\right)\sqrt{\frac{\frac{a^2+7\,a+12}{\sqrt{a^3+10\,a^2+33\,a+36}}+1}{a^2+7\,a+12}}+x-1\right)$$
$$+\frac{1}{4}\sqrt{-\frac{\frac{a^2+7\,a+12}{\sqrt{a^3+10\,a^2+33\,a+36}}-1}{a^2+7\,a+12}}\log\left(\left(a+\frac{a^2+7\,a+12}{\sqrt{a^3+10\,a^2+33\,a+36}}+4\right)\sqrt{-\frac{\frac{a^2+7\,a+12}{\sqrt{a^3+10\,a^2+33\,a+36}}-1}{a^2+7\,a+12}}+x-1\right)$$
$$-\frac{1}{4}\sqrt{-\frac{\frac{a^2+7\,a+12}{\sqrt{a^3+10\,a^2+33\,a+36}}-1}{a^2+7\,a+12}}\log\left(-\left(a+\frac{a^2+7\,a+12}{\sqrt{a^3+10\,a^2+33\,a+36}}+4\right)\sqrt{-\frac{\frac{a^2+7\,a+12}{\sqrt{a^3+10\,a^2+33\,a+36}}-1}{a^2+7\,a+12}}+x-1\right)$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

output
```
1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a
+ 12))*log((a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*sqrt(
((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12)) +
x - 1) - 1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a
^2 + 7*a + 12))*log(-(a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36)
+ 4)*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a
 + 12)) + x - 1) + 1/4*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a +
36) - 1)/(a^2 + 7*a + 12))*log((a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 3
3*a + 36) + 4)*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)
/(a^2 + 7*a + 12)) + x - 1) - 1/4*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^
2 + 33*a + 36) - 1)/(a^2 + 7*a + 12))*log(-(a + (a^2 + 7*a + 12)/sqrt(a^3
+ 10*a^2 + 33*a + 36) + 4)*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*
a + 36) - 1)/(a^2 + 7*a + 12)) + x - 1)
```

3.120.6 Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.74

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx = -\operatorname{RootSum}\left(t^4\cdot\left(256a^3+2816a^2+10240a+12288\right)+t^2(-32a-128)-1,\left(t\mapsto t\log\left(64t^3a^2+448t^3a\right.\right.\right.$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

output
```
-RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a - 1
28) - 1, Lambda(_t, _t*log(64*_t**3*a**2 + 448*_t**3*a + 768*_t**3 - 4*_t*
a - 20*_t + x - 1)))
```

3.120.7 Maxima [F]

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx = \int -\frac{1}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

output
```
-integrate(1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

3.120.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2669 vs. $2(65) = 130$.

Time = 2.52 (sec) , antiderivative size = 2669, normalized size of antiderivative = 29.99

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

output
```
-1/4*sqrt(((a + 4)^(3/2) + a + 4)/(a^3 + 11*a^2 + 40*a + 48))*log(abs(sqrt
(a + 4)*a^5 + sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^4*x +
a^5 + sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^3*
x - sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^4 + 17*sqrt(a +
4)*a^4 + 14*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^3*x - sq
rt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^3 + 17*a^4
 + 10*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^2*
x - 14*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^3 + 111*sqrt(
a + 4)*a^3 + 69*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^2*x
- 10*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^2 +
 111*a^3 + 29*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a +
 4)*a*x - 69*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^2 + 351
*sqrt(a + 4)*a^2 + 144*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)
*a*x - 29*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*
a + 351*a^2 + 28*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(
a + 4)*x - 144*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a + 544
*sqrt(a + 4)*a + 112*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*x
 - 28*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4) + 54
4*a - 112*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12) + 336*sqrt(a
 + 4) + 336)) + 1/4*sqrt(((a + 4)^(3/2) + a + 4)/(a^3 + 11*a^2 + 40*a +...
```

3.120.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 571, normalized size of antiderivative = 6.42

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx =$$

$$-\mathrm{atan}\left(-\frac{a\,8\mathrm{i}-x\,16\mathrm{i}+x\,\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a\,x\,8\mathrm{i}-\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a^2}{44\,a^2\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+4\,a^3\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+160\,a\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+19}\right.$$

$$-\mathrm{atan}\left(-\frac{a\,8\mathrm{i}-x\,16\mathrm{i}-x\,\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a\,x\,8\mathrm{i}+\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a^2}{160\,a\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+192\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+44\,a^2\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+4\,a}\right.$$

input
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

output
```
- atan(-(a*8i - x*16i + x*(48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a*x*8i - (
48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a^2*x*1i + a^2*1i + 16i)/(44*a^2*((a
- (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^
(1/2) + 4*a^3*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2
 + 16*a^3 + 768))^(1/2) + 160*a*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4
)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 192*((a - (48*a + 12*a^2 + a^3
 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)))*((a - (48*a +
12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)*2i -
 atan(-(a*8i - x*16i - x*(48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a*x*8i + (4
8*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a^2*x*1i + a^2*1i + 16i)/(160*a*((a +
(48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1
/2) + 192*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 1
6*a^3 + 768))^(1/2) + 44*a^2*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(
640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 4*a^3*((a + (48*a + 12*a^2 + a^3
+ 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)))*((a + (48*a + 1
2*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)*2i
```

3.121 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

3.121.1 Optimal result

Integrand size = 22, antiderivative size = 169

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} - \frac{(10+3a+\sqrt{4+a})\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1-\sqrt{4+a}}} + \frac{(10+3a-\sqrt{4+a})\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1+\sqrt{4+a}}}$$

output
```
1/4*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)-1/8*arcta
n((-1+x)/(1-(4+a)^(1/2))^(1/2))*(10+3*a+(4+a)^(1/2))/(3+a)/(4+a)^(3/2)/(1-
(4+a)^(1/2))^(1/2)+1/8*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2))*(10+3*a-(4+a)^
(1/2))/(3+a)/(4+a)^(3/2)/(1+(4+a)^(1/2))^(1/2)
```

3.121.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \frac{(-1+x)(6+a-2x+x^2)}{4(3+a)(4+a)(a-x(-8+8x-4x^2+x^3))} - \frac{\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&, \frac{12\log(x-\#1)+3a\log(x-\#1)-2\log(x-\#1)\#1+\log(x-\#1)\#1^2}{-2+4\#1-3\#1^2+\#1^3}\&\right]}{16(12+7a+a^2)}$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]
```

output
```
((-1 + x)*(6 + a - 2*x + x^2))/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2
 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (12*Log[x - #1]
+ 3*a*Log[x - #1] - 2*Log[x - #1]*#1 + Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1
^2 + #1^3) & ]/(16*(12 + 7*a + a^2))
```

3.121.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2458, 1405, 27, 1480, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-x^4+4x^3-8x^2+8x\right)^2}dx$$

$$\Big\downarrow \ 2458$$

$$\int \frac{1}{\left(a-(x-1)^4-2(x-1)^2+3\right)^2}d(x-1)$$

$$\Big\downarrow \ 1405$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\int -\frac{2\left((x-1)^2+3a+11\right)}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{8\left(a^2+7a+12\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(x-1)^2+3a+11}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \ 1480$$

$$\frac{\frac{1}{2}\left(1-\frac{3a+10}{\sqrt{a+4}}\right)\int \frac{1}{-(x-1)^2-\sqrt{a+4}-1}d(x-1)+\frac{1}{2}\left(\frac{3a+10}{\sqrt{a+4}}+1\right)\int \frac{1}{-(x-1)^2+\sqrt{a+4}-1}d(x-1)}{4\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \ 217$$

$$\frac{-\frac{\left(\frac{3a+10}{\sqrt{a+4}}+1\right)\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\left(1-\frac{3a+10}{\sqrt{a+4}}\right)\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}}{4\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]
```

output
```
((5 + a + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2
- (-1 + x)^4)) + (-1/2*((1 + (10 + 3*a)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[
1 - Sqrt[4 + a]]])/Sqrt[1 - Sqrt[4 + a]] - ((1 - (10 + 3*a)/Sqrt[4 + a])*A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(2*Sqrt[1 + Sqrt[4 + a]]))/(4*(12 +
 7*a + a^2))
```

3.121.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.121. $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.121.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

method	result
default	$\frac{\frac{x^3}{4a^2+28a+48}-\frac{3x^2}{4(4+a)(3+a)}+\frac{(a+8)x}{4a^2+28a+48}-\frac{6+a}{4\left(a^2+7a+12\right)}}{-x^4+4x^3-8x^2+a+8x}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(\_R^2-2\_R+3a+12\right)\ln\left(x-\right.}{-\_R^3+3\_R^2-4\_R+}}{16(4+a)(3+a)}$
risch	$\frac{\frac{x^3}{4a^2+28a+48}-\frac{3x^2}{4(4+a)(3+a)}+\frac{(a+8)x}{4a^2+28a+48}-\frac{6+a}{4\left(a^2+7a+12\right)}}{-x^4+4x^3-8x^2+a+8x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(\frac{\_R^2}{a^2+7a+12}-\frac{2\_R}{a^2+7a+12}+\frac{3}{3+}\right.}{-\_R^3+3\_R^2-4}\right.}{16}$

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/(a^2+7*a+12)*x^3-3/4/(4+a)/(3+a)*x^2+1/4*(a+8)/(a^2+7*a+12)*x-1/4*(6+
a)/(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)+1/16/(4+a)/(3+a)*sum((_R^2-2*_R+
3*a+12)/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a
))
```

3.121.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1948 vs. 2(139) = 278.

Time = 0.27 (sec) , antiderivative size = 1948, normalized size of antiderivative = 11.53

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

output
```
-1/16*(4*x^3 - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a
^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 + (
a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2
+ 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4
+ 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^
5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 +
 567*a + 992)*x + (27*a^4 + 408*a^3 + 2309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a
^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 55
8*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 11
1105*a^3 + 156492*a^2 + 128304*a + 46656)) + 5800*a + 5456)*sqrt((15*a^2 +
 (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^
2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^
4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*
a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) + ((a^
2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2
- 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*
a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9
 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 1564
92*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a
^3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a + 992)*x ...
```

3.121.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(144) = 288$.

Time = 3.34 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.74

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$$
$$= \frac{a-x^3+3x^2+x(-a-8)+6}{-4a^3-28a^2-48a+x^4\cdot(4a^2+28a+48)+x^3\,(-16a^2-112a-192)+x^2\cdot(32a^2+224a+384)+x\,(-}$$
$$+\mathrm{RootSum}\Big(t^4\cdot(65536a^9+2162688a^8+31653888a^7+269680640a^6+1473773568a^5+5357174784a^4$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

output
```
(a - x**3 + 3*x**2 + x*(-a - 8) + 6)/(-4*a**3 - 28*a**2 - 48*a + x**4*(4*a
**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a +
 384) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*
a**8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4
+ 12952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**
2*(-7680*a**5 - 145920*a**4 - 1107968*a**3 - 4202496*a**2 - 7962624*a - 60
29312) - 81*a**2 - 576*a - 1024, Lambda(_t, _t*log(x + (-16384*_t**3*a**7
- 401408*_t**3*a**6 - 4202496*_t**3*a**5 - 24371200*_t**3*a**4 - 84549632*
_t**3*a**3 - 175472640*_t**3*a**2 - 201719808*_t**3*a - 99090432*_t**3 + 4
32*_t*a**4 + 7488*_t*a**3 + 47024*_t*a**2 + 128096*_t*a + 128512*_t - 81*a
**2 - 567*a - 992)/(81*a**2 + 567*a + 992))))
```

3.121.7 Maxima [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \int \frac{1}{(x^4-4\,x^3+8\,x^2-a-8\,x)^2}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

output
```
-1/4*(x^3 + (a + 8)*x - 3*x^2 - a - 6)/((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*
a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x
- 12*a) - 1/4*integrate((x^2 + 3*a - 2*x + 12)/(x^4 - 4*x^3 + 8*x^2 - a -
8*x), x)/(a^2 + 7*a + 12)
```

3.121.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8503 vs. $2(139) = 278$.

Time = 6.37 (sec) , antiderivative size = 8503, normalized size of antiderivative = 50.31

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

output
```
1/16*(sqrt((15*a^3 + 165*a^2 + (9*a^3 + 103*a^2 + 392*a + 496)*sqrt(a + 4)
 + 604*a + 736)/(a^3 + 11*a^2 + 40*a + 48))*log(abs(243*sqrt(a + 4)*a^10 +
 324*a^10 + 8640*sqrt(a + 4)*a^9 + 81*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (
9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^
8*x + 11466*a^9 + 81*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 +
 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^7*x -
 81*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a
 + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^8 + 138027*sqrt(a + 4)*a^8 + 2340*
sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1
488)*sqrt(a + 4) + 2548*a + 2208)*a^7*x - 81*sqrt(15*a^4 + 210*a^3 + 1099*
a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2
208)*sqrt(a + 4)*a^7 + 182314*a^8 + 2016*sqrt(15*a^4 + 210*a^3 + 1099*a^2
+ (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)
*sqrt(a + 4)*a^6*x - 2340*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*
a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^7 + 1304648*
sqrt(a + 4)*a^7 + 29518*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^
3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^6*x - 2016*sqr
t(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488
)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^6 + 1715172*a^7 + 21454*sqrt(
15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 148...
```

3.121.9 Mupad [B] (verification not implemented)

Time = 11.03 (sec) , antiderivative size = 4591, normalized size of antiderivative = 27.17

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

output
```
atan(-(((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2
 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 1976
32*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 11059
2)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 9011
2*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5
+ 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147
456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((15552*a - 9*a*((a + 4)^
9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 +
 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 +
 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) - (733184*a + 396288*
a^2 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 12
9*a^3 + 18*a^4 + a^5 + 576)))*((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a +
 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(27648
0*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7
 + 33*a^8 + a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(
64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 +
104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*1i + ((15552*a - 9*a*((a
+ 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*
a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*
a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640*...
```

3.121. $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

3.122 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx$

3.122.1 Optimal result

Integrand size = 22, antiderivative size = 252

$$\begin{aligned}
&\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx \\
&= \frac{(5+a+(-1+x)^2)\,(-1+x)}{8\,(12+7a+a^2)\,(3+a-2(-1+x)^2-(-1+x)^4)^2} \\
&\quad+ \frac{((6+a)(25+7a)+6(7+2a)(-1+x)^2)\,(-1+x)}{32(3+a)^2(4+a)^2\,(3+a-2(-1+x)^2-(-1+x)^4)} \\
&\quad- \frac{3\left(80+7a^2+14\sqrt{4+a}+a\left(47+4\sqrt{4+a}\right)\right)\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{64(3+a)^2(4+a)^{5/2}\sqrt{1-\sqrt{4+a}}} \\
&\quad- \frac{3\left(14+4a-\frac{80+47a+7a^2}{\sqrt{4+a}}\right)\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{64(3+a)^2(4+a)^2\sqrt{1+\sqrt{4+a}}}
\end{aligned}$$

output
```
1/8*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^2+1/32*((
6+a)*(25+7*a)+6*(7+2*a)*(-1+x)^2)*(-1+x)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-
1+x)^4)-3/64*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(80+7*a^2+14*(4+a)^(1/2)
+a*(47+4*(4+a)^(1/2)))/(3+a)^2/(4+a)^(5/2)/(1-(4+a)^(1/2))^(1/2)-3/64*arct
an((-1+x)/(1+(4+a)^(1/2))^(1/2))*(14+4*a+(-7*a^2-47*a-80)/(4+a)^(1/2))/(3+
a)^2/(4+a)^2/(1+(4+a)^(1/2))^(1/2)
```

3.122.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \frac{1}{128}\left(\frac{16(-1+x)\,(6+a-2x+x^2)}{(3+a)(4+a)\,(a-x\,(-8+8x-4x^2+x^3))^2}\right.$$
$$+\frac{4(-1+x)\,(7a^2+6(32-14x+7x^2)+a(79-24x+12x^2))}{(3+a)^2(4+a)^2\,(a-x\,(-8+8x-4x^2+x^3))}$$
$$-\frac{3\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&,\frac{108\log(x-\#1)+55a\log(x-\#1)+7a^2\log(x-\#1)-28\log(x-\#1)\#1-8}{-2+4\#1-3\#1^2}\right.}{(12+7a+a^2)^2}$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3),x]
```

output
```
((16*(-1 + x)*(6 + a - 2*x + x^2))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x
^2 + x^3))^2) + (4*(-1 + x)*(7*a^2 + 6*(32 - 14*x + 7*x^2) + a*(79 - 24*x
+ 12*x^2)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - (3*Ro
otSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (108*Log[x - #1] + 55*a*Log[x
- #1] + 7*a^2*Log[x - #1] - 28*Log[x - #1]*#1 - 8*a*Log[x - #1]*#1 + 14*Lo
g[x - #1]*#1^2 + 4*a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ])/(1
2 + 7*a + a^2)^2)/128
```

3.122.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2458, 1405, 27, 1492, 27, 1480, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-x^4+4x^3-8x^2+8x)^3}dx$$
$$\Big\downarrow \ 2458$$

$$\int \frac{1}{\left(a-(x-1)^4-2(x-1)^2+3\right)^3}d(x-1)$$

$$\Big\downarrow \; 1405$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}-\frac{\int -\frac{2\left(5(x-1)^2+7a+27\right)}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)}{16\left(a^2+7a+12\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{5(x-1)^2+7a+27}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)}{8\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$

$$\Big\downarrow \; 1492$$

$$\frac{\frac{(x-1)\left(6(2a+7)(x-1)^2+(a+6)(7a+25)\right)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}-\frac{\int -\frac{6\left(7a^2+51a+2(2a+7)(x-1)^2+94\right)}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{8(a^2+7a+12)}}{8\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{7a^2+51a+2(2a+7)(x-1)^2+94}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4(a^2+7a+12)}+\frac{(x-1)\left(6(2a+7)(x-1)^2+(a+6)(7a+25)\right)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}}{8\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{3\left(\frac{1}{2}\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\int \frac{1}{-(x-1)^2-\sqrt{a+4}-1}d(x-1)+\frac{1}{2}\left(\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\int \frac{1}{-(x-1)^2+\sqrt{a+4}-1}d(x-1)\right)}{4(a^2+7a+12)}+\frac{(x-1)(6(2a+7)(x-1)^2+(a}{4(a^2+7a+12)(a-(x-1)^4-}}{8\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{3\left(-\frac{\left(\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}\right)}{4(a^2+7a+12)}+\frac{(x-1)\left(6(2a+7)(x-1)^2+(a+6)(7a+25)\right)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}}{8\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$

3.122. $\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^3}\,dx$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3),x]
```

output
```
((5 + a + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2
- (-1 + x)^4)^2) + ((((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2)*(-1 + x
))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + (3*(-1/2*((1
4 + 4*a + (80 + 47*a + 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4
 + a]]])/Sqrt[1 - Sqrt[4 + a]] - ((14 + 4*a - (80 + 47*a + 7*a^2)/Sqrt[4 +
 a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(2*Sqrt[1 + Sqrt[4 + a]])))/(
4*(12 + 7*a + a^2)))/(8*(12 + 7*a + a^2))
```

3.122.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.122.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.59

method	result
default	$-\frac{\frac{3(7+2a)x^7}{16\left(a^4+14a^3+73a^2+168a+144\right)}-\frac{21(7+2a)x^6}{16\left(a^2+8a+16\right)\left(a^2+6a+9\right)}+\frac{\left(7a^2+343a+1116\right)x^5}{32a^4+448a^3+2336a^2+5376a+4608}-\frac{5\left(7a^2+175a+528\right)x^4}{32\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{}{16a^4+}}{(-x^4+4x^3-8x^2+a}$
risch	$\frac{-\frac{3(7+2a)x^7}{16\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{21(7+2a)x^6}{16\left(a^2+8a+16\right)\left(a^2+6a+9\right)}-\frac{\left(7a^2+343a+1116\right)x^5}{32\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{5\left(7a^2+175a+528\right)x^4}{32\left(a^4+14a^3+73a^2+168a+144\right)}-\frac{\left(34a^2\right.}{16\left(a^4+14\right.}}{(-x^4+4x^3-8x^2+a+}$

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-(3/16*(7+2*a)/(a^4+14*a^3+73*a^2+168*a+144)*x^7-21/16*(7+2*a)/(a^2+8*a+16
)/(a^2+6*a+9)*x^6+1/32*(7*a^2+343*a+1116)/(a^4+14*a^3+73*a^2+168*a+144)*x^
5-5/32*(7*a^2+175*a+528)/(a^4+14*a^3+73*a^2+168*a+144)*x^4+1/16*(34*a^2+67
9*a+1968)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(32*a^2+623*a+1800)/(a^4+
14*a^3+73*a^2+168*a+144)*x^2-1/32*(11*a^3+107*a^2-84*a-1152)/(a^4+14*a^3+7
3*a^2+168*a+144)*x+1/32*(11*a^3+131*a^2+408*a+288)/(3+a)/(a^3+11*a^2+40*a+
48))/(-x^4+4*x^3-8*x^2+a+8*x)^2-3/128/(a^3+10*a^2+33*a+36)/(4+a)*sum((-108
+2*(-2*a-7)*_R^2+4*(7+2*a)*_R-7*a^2-55*a)/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_
R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

3.122.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3971 vs. $2(220) = 440$.

Time = 0.31 (sec) , antiderivative size = 3971, normalized size of antiderivative = 15.76

$$\int \frac{1}{(a + 8x - 8x^2 + 4x^3 - x^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

output
```
-1/128*(24*(2*a + 7)*x^7 - 168*(2*a + 7)*x^6 + 4*(7*a^2 + 343*a + 1116)*x^
5 - 20*(7*a^2 + 175*a + 528)*x^4 + 8*(34*a^2 + 679*a + 1968)*x^3 + 44*a^3
- 8*(32*a^2 + 623*a + 1800)*x^2 - 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)
*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73
*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x
^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 +
 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a
^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16
*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7
749*a^2 + (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 3
73020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 +
33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 +
16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 943
20045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3
+ 592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a
^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3
 + 950400*a^2 + 725760*a + 248832))*log(-64827*a^4 - 907578*a^3 - 4780647*
a^2 + 27*(2401*a^4 + 33614*a^3 + 177061*a^2 + 415884*a + 367536)*x + 27*(3
43*a^7 + 8981*a^6 + 100811*a^5 + 628887*a^4 + 2354874*a^3 + 5293208*a^2 -
(11*a^12 + 462*a^11 + 8881*a^10 + 103320*a^9 + 810205*a^8 + 4511542*a^7...
```

3.122.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(230) = 460$.

Time = 8.02 (sec) , antiderivative size = 697, normalized size of antiderivative = 2.77

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx =$$

$$-\frac{\quad}{32a^6+448a^5+2336a^4+5376a^3+4608a^2+x^8\cdot(32a^4+448a^3+2336a^2+5376a+4608)+x^7\,(-256}$$

$$-\mathrm{RootSum}\left(t^4\cdot\left(268435456a^{15}+14763950080a^{14}+378493992960a^{13}+5999532441600a^{12}+65757291\right.\right.$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

output
```
-(11*a**3 + 131*a**2 + 408*a + x**7*(12*a + 42) + x**6*(-84*a - 294) + x**
5*(7*a**2 + 343*a + 1116) + x**4*(-35*a**2 - 875*a - 2640) + x**3*(68*a**2
 + 1358*a + 3936) + x**2*(-64*a**2 - 1246*a - 3600) + x*(-11*a**3 - 107*a*
*2 + 84*a + 1152) + 288)/(32*a**6 + 448*a**5 + 2336*a**4 + 5376*a**3 + 460
8*a**2 + x**8*(32*a**4 + 448*a**3 + 2336*a**2 + 5376*a + 4608) + x**7*(-25
6*a**4 - 3584*a**3 - 18688*a**2 - 43008*a - 36864) + x**6*(1024*a**4 + 143
36*a**3 + 74752*a**2 + 172032*a + 147456) + x**5*(-2560*a**4 - 35840*a**3
- 186880*a**2 - 430080*a - 368640) + x**4*(-64*a**5 + 3200*a**4 + 52672*a*
*3 + 288256*a**2 + 678912*a + 589824) + x**3*(256*a**5 - 512*a**4 - 38656*
a**3 - 256000*a**2 - 651264*a - 589824) + x**2*(-512*a**5 - 5120*a**4 - 87
04*a**3 + 63488*a**2 + 270336*a + 294912) + x*(512*a**5 + 7168*a**4 + 3737
6*a**3 + 86016*a**2 + 73728*a)) - RootSum(_t**4*(268435456*a**15 + 1476395
0080*a**14 + 378493992960*a**13 + 5999532441600*a**12 + 65757291479040*a**
11 + 527875908304896*a**10 + 3206246773555200*a**9 + 15003759578972160*a**
8 + 54537151127224320*a**7 + 153980418717122560*a**6 + 334927734494986240*
a**5 + 551152193655275520*a**4 + 664192984106926080*a**3 + 553362212027105
280*a**2 + 284993413919539200*a + 68398419340689408) + _t**2*(-30965760*a*
*9 - 1052835840*a**8 - 15910207488*a**7 - 140262506496*a**6 - 795007254528
*a**5 - 3004516270080*a**4 - 7571263979520*a**3 - 12268037210112*a**2 - 11
598827618304*a - 4875324751872) - 194481*a**4 - 2762424*a**3 - 14762736...
```

3.122.7 Maxima [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \int -\frac{1}{(x^4-4\,x^3+8\,x^2-a-8\,x)^3}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

output
```
-1/32*(6*(2*a + 7)*x^7 - 42*(2*a + 7)*x^6 + (7*a^2 + 343*a + 1116)*x^5 - 5
*(7*a^2 + 175*a + 528)*x^4 + 2*(34*a^2 + 679*a + 1968)*x^3 + 11*a^3 - 2*(3
2*a^2 + 623*a + 1800)*x^2 + 131*a^2 - (11*a^3 + 107*a^2 - 84*a - 1152)*x +
 408*a + 288)/((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3
 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^
6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 -
 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a
^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4
+ 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^
3 + 168*a^2 + 144*a)*x) - 3/32*integrate((2*(2*a + 7)*x^2 + 7*a^2 - 4*(2*a
 + 7)*x + 55*a + 108)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^4 + 14*a^3 +
73*a^2 + 168*a + 144)
```

3.122.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16632 vs. $2(220) = 440$.

Time = 11.96 (sec) , antiderivative size = 16632, normalized size of antiderivative = 66.00

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

output
```
-3/128*(sqrt((105*a^5 + 1890*a^4 + 13629*a^3 + 49224*a^2 + (49*a^5 + 926*a
^4 + 6997*a^3 + 26428*a^2 + 49904*a + 37696)*sqrt(a + 4) + 89056*a + 64576
)/(a^3 + 11*a^2 + 40*a + 48))*log(abs(16807*sqrt(a + 4)*a^15 + 26411*a^15
+ 908950*sqrt(a + 4)*a^14 + 1420804*a^14 + 22929088*sqrt(a + 4)*a^13 + 240
1*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 +
 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a
+ 4) + 331744*a + 193728)*a^12*x + 35650176*a^13 + 2401*sqrt(105*a^6 + 220
5*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4
 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 19
3728)*sqrt(a + 4)*a^11*x - 2401*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 9011
1*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^
2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^12 + 357887692*s
qrt(a + 4)*a^12 + 105154*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 +
 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187
408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^11*x - 2401*sqrt(105*a^
6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9
775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744
*a + 193728)*sqrt(a + 4)*a^11 + 553458148*a^12 + 95550*sqrt(105*a^6 + 2205
*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4
+ 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + ...
```

3.122.9 Mupad [B] (verification not implemented)

Time = 12.12 (sec) , antiderivative size = 8242, normalized size of antiderivative = 32.71

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

output
```
atan(((((52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a
^4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*
a^9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^
4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776))
 + ((4290672328704*a + 6001143054336*a^2 + 5025917042688*a^3 + 28005200035
84*a^4 + 1090200272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 82753617
92*a^8 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904)/(16
384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833
*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3510632448*a +
4020240384*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*a^5 + 6140723
2*a^6 + 7438336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(256*(48384*a
+ 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 +
20736)))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/
2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^
4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))
/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 +
 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*
a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15
 + 254803968)))^(1/2))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((
a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*...
```

3.122. $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx$

3.123 $\int x(a+8x-8x^2+4x^3-x^4)^4 \, dx$

3.123.1 Optimal result

Integrand size = 24, antiderivative size = 210

$$\begin{aligned}\int x\left(a+8x-8x^2+4x^3-x^4\right)^4 \, dx = {} & \frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+8(12-a)a^2x^4+\frac{16}{5}a\left(128-48a+a^2\right)x^5 \\ & +\frac{2}{3}\left(1024-1536a+192a^2-a^3\right)x^6 \\ & -\frac{32}{7}\left(512-288a+15a^2\right)x^7+8(128-3a)(4-a)x^8 \\ & -\frac{16}{3}\left(896-128a+a^2\right)x^9+\frac{1}{5}\left(20480-1536a+3a^2\right)x^{10} \\ & -\frac{32}{11}(928-35a)x^{11}+\frac{8}{3}(524-9a)x^{12}-\frac{16}{13}(464-3a)x^{13} \\ & +\frac{2}{7}(640-a)x^{14}-\frac{224x^{15}}{5}+8x^{16}-\frac{16x^{17}}{17}+\frac{x^{18}}{18}\end{aligned}$$

output
```
1/2*a^4*x^2+32/3*a^3*x^3+8*(12-a)*a^2*x^4+16/5*a*(a^2-48*a+128)*x^5+2/3*(-
a^3+192*a^2-1536*a+1024)*x^6-32/7*(15*a^2-288*a+512)*x^7+8*(128-3*a)*(4-a)
*x^8-16/3*(a^2-128*a+896)*x^9+1/5*(3*a^2-1536*a+20480)*x^10-32/11*(928-35*
a)*x^11+8/3*(524-9*a)*x^12-16/13*(464-3*a)*x^13+2/7*(640-a)*x^14-224/5*x^1
5+8*x^16-16/17*x^17+1/18*x^18
```

3.123.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.97

$$\int x(a+8x-8x^2+4x^3-x^4)^4\,dx = \frac{a^4x^2}{2}+\frac{32a^3x^3}{3}-8(-12+a)a^2x^4+\frac{16}{5}a(128-48a+a^2)\,x^5 - \frac{2}{3}(-1024+1536a-192a^2+a^3)\,x^6 - \frac{32}{7}(512-288a+15a^2)\,x^7 + 8(512-140a+3a^2)\,x^8-\frac{16}{3}(896-128a+a^2)\,x^9 + \frac{1}{5}(20480-1536a+3a^2)\,x^{10}+\frac{32}{11}(-928+35a)x^{11} - \frac{8}{3}(-524+9a)x^{12}+\frac{16}{13}(-464+3a)x^{13} - \frac{2}{7}(-640+a)x^{14}-\frac{224x^{15}}{5}+8x^{16}-\frac{16x^{17}}{17}+\frac{x^{18}}{18}$$

input
```
Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

output
```
(a^4*x^2)/2 + (32*a^3*x^3)/3 - 8*(-12 + a)*a^2*x^4 + (16*a*(128 - 48*a + a
^2)*x^5)/5 - (2*(-1024 + 1536*a - 192*a^2 + a^3)*x^6)/3 - (32*(512 - 288*a
 + 15*a^2)*x^7)/7 + 8*(512 - 140*a + 3*a^2)*x^8 - (16*(896 - 128*a + a^2)*
x^9)/3 + ((20480 - 1536*a + 3*a^2)*x^10)/5 + (32*(-928 + 35*a)*x^11)/11 -
(8*(-524 + 9*a)*x^12)/3 + (16*(-464 + 3*a)*x^13)/13 - (2*(-640 + a)*x^14)/
7 - (224*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18
```

3.123.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a-x^4+4x^3-8x^2+8x\right)^4\,dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(a^4x+32a^3x^2+2\left(3a^2-1536a+20480\right)x^9-48\left(a^2-128a+896\right)x^8-32\left(15a^2-288a+512\right)x^6+16a\left(a^2 -\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}-\frac{16}{3}\left(a^2-128a+896\right)x^9-\frac{32}{7}\left(15a^2-288a+512\right)x^7+\frac{16}{5}a\left(a^2-48a+128\right)x^5+8(12-a)a^2x^4+\frac{2}{3}\left(-a^3+192a^2-1536a+1024\right)x^6+\frac{2}{7}(640-a)x^{14}-\frac{16}{13}(464-3a)x^{13}+\frac{8}{3}(524-9a)x^{12}-\frac{32}{11}(928-35a)x^{11}+8(128-3a)(4-a)x^8+\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}$$

input
```
Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

output
```
(a^4*x^2)/2 + (32*a^3*x^3)/3 + 8*(12 - a)*a^2*x^4 + (16*a*(128 - 48*a + a^
2)*x^5)/5 + (2*(1024 - 1536*a + 192*a^2 - a^3)*x^6)/3 - (32*(512 - 288*a +
 15*a^2)*x^7)/7 + 8*(128 - 3*a)*(4 - a)*x^8 - (16*(896 - 128*a + a^2)*x^9)
/3 + ((20480 - 1536*a + 3*a^2)*x^10)/5 - (32*(928 - 35*a)*x^11)/11 + (8*(5
24 - 9*a)*x^12)/3 - (16*(464 - 3*a)*x^13)/13 + (2*(640 - a)*x^14)/7 - (224
*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18
```

3.123.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

3.123.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

method	result
norman	$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+(-8a^3+96a^2)\,x^4+\left(\frac{16}{5}a^3-\frac{768}{5}a^2+\frac{2048}{5}a\right)x^5+\left(-\frac{2}{3}a^3+128a^2-1024a+\right.$
gosper	$4096x^{10}-\frac{29696}{11}x^{11}+\frac{4192}{3}x^{12}-\frac{7424}{13}x^{13}-\frac{14336}{3}x^9-\frac{16384}{7}x^7-\frac{2}{7}x^{14}a+\frac{3}{5}x^{10}a^2-\frac{2}{3}x^6a^3+\frac{128}{7}$
risch	$4096x^{10}-\frac{29696}{11}x^{11}+\frac{4192}{3}x^{12}-\frac{7424}{13}x^{13}-\frac{14336}{3}x^9-\frac{16384}{7}x^7-\frac{2}{7}x^{14}a+\frac{3}{5}x^{10}a^2-\frac{2}{3}x^6a^3+\frac{128}{7}$
parallelrisch	$4096x^{10}-\frac{29696}{11}x^{11}+\frac{4192}{3}x^{12}-\frac{7424}{13}x^{13}-\frac{14336}{3}x^9-\frac{16384}{7}x^7-\frac{2}{7}x^{14}a+\frac{3}{5}x^{10}a^2-\frac{2}{3}x^6a^3+\frac{128}{7}$
default	$\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}+\frac{(-4a+2560)x^{14}}{14}+\frac{(48a-7424)x^{13}}{13}+\frac{(-288a+16768)x^{12}}{12}+\frac{(1120a-29696)x^{11}}{11}$

input
```
int(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^4*x^2+32/3*a^3*x^3+(-8*a^3+96*a^2)*x^4+(16/5*a^3-768/5*a^2+2048/5*a)
*x^5+(-2/3*a^3+128*a^2-1024*a+2048/3)*x^6+(-480/7*a^2+9216/7*a-16384/7)*x^
7+(24*a^2-1120*a+4096)*x^8+(-16/3*a^2+2048/3*a-14336/3)*x^9+(3/5*a^2-1536/
5*a+4096)*x^10+(1120/11*a-29696/11)*x^11+(-24*a+4192/3)*x^12+(48/13*a-7424
/13)*x^13+(-2/7*a+1280/7)*x^14-224/5*x^15+8*x^16-16/17*x^17+1/18*x^18
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

$$\begin{aligned}
&\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx \\
&= \frac{1}{18}x^{18}-\frac{16}{17}x^{17}+8x^{16}-\frac{2}{7}(a-640)x^{14}-\frac{224}{5}x^{15}+\frac{16}{13}(3a-464)x^{13} \\
&\quad-\frac{8}{3}(9a-524)x^{12}+\frac{32}{11}(35a-928)x^{11}+\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10} \\
&\quad-\frac{16}{3}\left(a^2-128a+896\right)x^9+8\left(3a^2-140a+512\right)x^8 \\
&\quad-\frac{32}{7}\left(15a^2-288a+512\right)x^7-\frac{2}{3}\left(a^3-192a^2+1536a-1024\right)x^6 \\
&\quad+\frac{1}{2}a^4x^2+\frac{32}{3}a^3x^3+\frac{16}{5}\left(a^3-48a^2+128a\right)x^5-8\left(a^3-12a^2\right)x^4
\end{aligned}$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

output
```
1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*(a - 640)*x^14 - 224/5*x^15 + 16/13*
(3*a - 464)*x^13 - 8/3*(9*a - 524)*x^12 + 32/11*(35*a - 928)*x^11 + 1/5*(3
*a^2 - 1536*a + 20480)*x^10 - 16/3*(a^2 - 128*a + 896)*x^9 + 8*(3*a^2 - 14
0*a + 512)*x^8 - 32/7*(15*a^2 - 288*a + 512)*x^7 - 2/3*(a^3 - 192*a^2 + 15
36*a - 1024)*x^6 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 16/5*(a^3 - 48*a^2 + 128*a
)*x^5 - 8*(a^3 - 12*a^2)*x^4
```

3.123.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\int x(a+8x-8x^2+4x^3-x^4)^4\,dx = \frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16} -\frac{224x^{15}}{5}+x^{14}\cdot\left(\frac{1280}{7}-\frac{2a}{7}\right)+x^{13} \cdot\left(\frac{48a}{13}-\frac{7424}{13}\right)+x^{12}\cdot\left(\frac{4192}{3}-24a\right)+x^{11} \cdot\left(\frac{1120a}{11}-\frac{29696}{11}\right)+x^{10}\cdot\left(\frac{3a^2}{5}-\frac{1536a}{5}+4096\right) +x^9\left(-\frac{16a^2}{3}+\frac{2048a}{3}-\frac{14336}{3}\right) +x^8\cdot(24a^2-1120a+4096) +x^7\left(-\frac{480a^2}{7}+\frac{9216a}{7}-\frac{16384}{7}\right) +x^6\left(-\frac{2a^3}{3}+128a^2-1024a+\frac{2048}{3}\right)+x^5 \cdot\left(\frac{16a^3}{5}-\frac{768a^2}{5}+\frac{2048a}{5}\right)+x^4(-8a^3+96a^2)$$

input
```
integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

output
```
a**4*x**2/2 + 32*a**3*x**3/3 + x**18/18 - 16*x**17/17 + 8*x**16 - 224*x**1
5/5 + x**14*(1280/7 - 2*a/7) + x**13*(48*a/13 - 7424/13) + x**12*(4192/3 -
 24*a) + x**11*(1120*a/11 - 29696/11) + x**10*(3*a**2/5 - 1536*a/5 + 4096)
 + x**9*(-16*a**2/3 + 2048*a/3 - 14336/3) + x**8*(24*a**2 - 1120*a + 4096)
 + x**7*(-480*a**2/7 + 9216*a/7 - 16384/7) + x**6*(-2*a**3/3 + 128*a**2 -
1024*a + 2048/3) + x**5*(16*a**3/5 - 768*a**2/5 + 2048*a/5) + x**4*(-8*a**
3 + 96*a**2)
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

$$\begin{aligned}\int x(a+8x-8x^2+4x^3-x^4)^4\,dx &= \frac{1}{18}x^{18}-\frac{16}{17}x^{17}+8x^{16}-\frac{2}{7}(a-640)x^{14}-\frac{224}{5}x^{15}+\frac{16}{13}(3a-464)x^{13}\\ &\quad-\frac{8}{3}(9a-524)x^{12}+\frac{32}{11}(35a-928)x^{11}+\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}\\ &\quad-\frac{16}{3}\left(a^2-128a+896\right)x^9+8\left(3a^2-140a+512\right)x^8\\ &\quad-\frac{32}{7}\left(15a^2-288a+512\right)x^7-\frac{2}{3}\left(a^3-192a^2+1536a-1024\right)x^6\\ &\quad+\frac{1}{2}a^4x^2+\frac{32}{3}a^3x^3+\frac{16}{5}\left(a^3-48a^2+128a\right)x^5-8\left(a^3-12a^2\right)x^4\end{aligned}$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

output
```
1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*(a - 640)*x^14 - 224/5*x^15 + 16/13*
(3*a - 464)*x^13 - 8/3*(9*a - 524)*x^12 + 32/11*(35*a - 928)*x^11 + 1/5*(3
*a^2 - 1536*a + 20480)*x^10 - 16/3*(a^2 - 128*a + 896)*x^9 + 8*(3*a^2 - 14
0*a + 512)*x^8 - 32/7*(15*a^2 - 288*a + 512)*x^7 - 2/3*(a^3 - 192*a^2 + 15
36*a - 1024)*x^6 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 16/5*(a^3 - 48*a^2 + 128*a
)*x^5 - 8*(a^3 - 12*a^2)*x^4
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.06

$$\begin{aligned}\int x(a+8x-8x^2+4x^3-x^4)^4\,dx &= \frac{1}{18}x^{18}-\frac{16}{17}x^{17}+8x^{16}-\frac{2}{7}ax^{14}-\frac{224}{5}x^{15}+\frac{48}{13}ax^{13}\\ &\quad+\frac{1280}{7}x^{14}-24ax^{12}-\frac{7424}{13}x^{13}+\frac{3}{5}a^2x^{10}+\frac{1120}{11}ax^{11}\\ &\quad+\frac{4192}{3}x^{12}-\frac{16}{3}a^2x^9-\frac{1536}{5}ax^{10}-\frac{29696}{11}x^{11}\\ &\quad+24a^2x^8+\frac{2048}{3}ax^9+4096x^{10}-\frac{2}{3}a^3x^6-\frac{480}{7}a^2x^7\\ &\quad-1120ax^8-\frac{14336}{3}x^9+\frac{16}{5}a^3x^5+128a^2x^6+\frac{9216}{7}ax^7\\ &\quad+4096x^8-8a^3x^4-\frac{768}{5}a^2x^5-1024ax^6-\frac{16384}{7}x^7\\ &\quad+\frac{1}{2}a^4x^2+\frac{32}{3}a^3x^3+96a^2x^4+\frac{2048}{5}ax^5+\frac{2048}{3}x^6\end{aligned}$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

output
```
1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*a*x^14 - 224/5*x^15 + 48/13*a*x^13 +
 1280/7*x^14 - 24*a*x^12 - 7424/13*x^13 + 3/5*a^2*x^10 + 1120/11*a*x^11 +
4192/3*x^12 - 16/3*a^2*x^9 - 1536/5*a*x^10 - 29696/11*x^11 + 24*a^2*x^8 +
2048/3*a*x^9 + 4096*x^10 - 2/3*a^3*x^6 - 480/7*a^2*x^7 - 1120*a*x^8 - 1433
6/3*x^9 + 16/5*a^3*x^5 + 128*a^2*x^6 + 9216/7*a*x^7 + 4096*x^8 - 8*a^3*x^4
 - 768/5*a^2*x^5 - 1024*a*x^6 - 16384/7*x^7 + 1/2*a^4*x^2 + 32/3*a^3*x^3 +
 96*a^2*x^4 + 2048/5*a*x^5 + 2048/3*x^6
```

3.123.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.85

$$\int x(a+8x-8x^2+4x^3-x^4)^4\,dx = x^{13}\left(\frac{48\,a}{13}-\frac{7424}{13}\right)-x^{12}\left(24\,a-\frac{4192}{3}\right) - x^{14}\left(\frac{2\,a}{7}-\frac{1280}{7}\right)+x^{11}\left(\frac{1120\,a}{11}-\frac{29696}{11}\right) + x^8\left(24\,a^2-1120\,a+4096\right) + x^{10}\left(\frac{3\,a^2}{5}-\frac{1536\,a}{5}+4096\right) - x^9\left(\frac{16\,a^2}{3}-\frac{2048\,a}{3}+\frac{14336}{3}\right) - x^7\left(\frac{480\,a^2}{7}-\frac{9216\,a}{7}+\frac{16384}{7}\right) - x^6\left(\frac{2\,a^3}{3}-128\,a^2+1024\,a-\frac{2048}{3}\right)-\frac{224\,x^{15}}{5} + 8\,x^{16}-\frac{16\,x^{17}}{17}+\frac{x^{18}}{18}+\frac{32\,a^3\,x^3}{3}+\frac{a^4\,x^2}{2} + \frac{16\,a\,x^5\,(a^2-48\,a+128)}{5}-8\,a^2\,x^4\,(a-12)$$

input
```
int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

output
```
x^13*((48*a)/13 - 7424/13) - x^12*(24*a - 4192/3) - x^14*((2*a)/7 - 1280/7
) + x^11*((1120*a)/11 - 29696/11) + x^8*(24*a^2 - 1120*a + 4096) + x^10*((
3*a^2)/5 - (1536*a)/5 + 4096) - x^9*((16*a^2)/3 - (2048*a)/3 + 14336/3) -
x^7*((480*a^2)/7 - (9216*a)/7 + 16384/7) - x^6*(1024*a - 128*a^2 + (2*a^3)
/3 - 2048/3) - (224*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18 + (32*a^3*x^
3)/3 + (a^4*x^2)/2 + (16*a*x^5*(a^2 - 48*a + 128))/5 - 8*a^2*x^4*(a - 12)
```

3.123. $\int x(a+8x-8x^2+4x^3-x^4)^4\, dx$

3.124 $\int x(a+8x-8x^2+4x^3-x^4)^3\,dx$

3.124.1 Optimal result

Integrand size = 24, antiderivative size = 134

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\frac{a^3x^2}{2}+8a^2x^3+6(8-a)ax^4+\frac{4}{5}\left(128-96a+3a^2\right)x^5-\frac{1}{2}\left(512-128a+a^2\right)x^6+\frac{48}{7}(48-5a)x^7-4(70-3a)x^8+\frac{8}{3}(64-a)x^9-\frac{3}{10}(256-a)x^{10}+\frac{280x^{11}}{11}-6x^{12}+\frac{12x^{13}}{13}-\frac{x^{14}}{14}$$

output
```
1/2*a^3*x^2+8*a^2*x^3+6*(8-a)*a*x^4+4/5*(3*a^2-96*a+128)*x^5-1/2*(a^2-128*
a+512)*x^6+48/7*(48-5*a)*x^7-4*(70-3*a)*x^8+8/3*(64-a)*x^9-3/10*(256-a)*x^
10+280/11*x^11-6*x^12+12/13*x^13-1/14*x^14
```

3.124.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.97

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\frac{a^3x^2}{2}+8a^2x^3-6(-8+a)ax^4+\frac{4}{5}\left(128-96a+3a^2\right)x^5+\frac{1}{2}\left(-512+128a-a^2\right)x^6-\frac{48}{7}(-48+5a)x^7+4(-70+3a)x^8-\frac{8}{3}(-64+a)x^9+\frac{3}{10}(-256+a)x^{10}+\frac{280x^{11}}{11}-6x^{12}+\frac{12x^{13}}{13}-\frac{x^{14}}{14}$$

input
```
Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

output
```
(a^3*x^2)/2 + 8*a^2*x^3 - 6*(-8 + a)*a*x^4 + (4*(128 - 96*a + 3*a^2)*x^5)/
5 + ((-512 + 128*a - a^2)*x^6)/2 - (48*(-48 + 5*a)*x^7)/7 + 4*(-70 + 3*a)*
x^8 - (8*(-64 + a)*x^9)/3 + (3*(-256 + a)*x^10)/10 + (280*x^11)/11 - 6*x^1
2 + (12*x^13)/13 - x^14/14
```

3.124.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a-x^4+4x^3-8x^2+8x\right)^3 dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(a^3x-3\left(a^2-128a+512\right)x^5+4\left(3a^2-96a+128\right)x^4+24a^2x^2-3(256-a)x^9+24(64-a)x^8-32(70-3a$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^2}{2}-\frac{1}{2}\left(a^2-128a+512\right)x^6+\frac{4}{5}\left(3a^2-96a+128\right)x^5+8a^2x^3-\frac{3}{10}(256-a)x^{10}+\frac{8}{3}(64-a)x^9-4(70-3a)x^8+\frac{48}{7}(48-5a)x^7+6(8-a)ax^4-\frac{x^{14}}{14}+\frac{12x^{13}}{13}-6x^{12}+\frac{280x^{11}}{11}$$

input
```
Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

output
```
(a^3*x^2)/2 + 8*a^2*x^3 + 6*(8 - a)*a*x^4 + (4*(128 - 96*a + 3*a^2)*x^5)/5
 - ((512 - 128*a + a^2)*x^6)/2 + (48*(48 - 5*a)*x^7)/7 - 4*(70 - 3*a)*x^8
+ (8*(64 - a)*x^9)/3 - (3*(256 - a)*x^10)/10 + (280*x^11)/11 - 6*x^12 + (1
2*x^13)/13 - x^14/14
```

3.124.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

3.124.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

method	result
norman	$\frac{a^3x^2}{2}+8a^2x^3+(-6a^2+48a)x^4+\left(\frac{12}{5}a^2-\frac{384}{5}a+\frac{512}{5}\right)x^5+\left(-\frac{1}{2}a^2+64a-256\right)x^6+\left(-\frac{24}{7}\right.$
gosper	$\frac{1}{2}a^3x^2+8a^2x^3-6a^2x^4+48a\,x^4+\frac{12}{5}a^2x^5-\frac{384}{5}a\,x^5+\frac{512}{5}x^5-\frac{1}{2}a^2x^6+64a\,x^6-256x^6-\frac{24}{7}$
risch	$\frac{1}{2}a^3x^2+8a^2x^3-6a^2x^4+48a\,x^4+\frac{12}{5}a^2x^5-\frac{384}{5}a\,x^5+\frac{512}{5}x^5-\frac{1}{2}a^2x^6+64a\,x^6-256x^6-\frac{24}{7}$
parallelrisch	$\frac{1}{2}a^3x^2+8a^2x^3-6a^2x^4+48a\,x^4+\frac{12}{5}a^2x^5-\frac{384}{5}a\,x^5+\frac{512}{5}x^5-\frac{1}{2}a^2x^6+64a\,x^6-256x^6-\frac{24}{7}$
default	$-\frac{x^{14}}{14}+\frac{12x^{13}}{13}-6x^{12}+\frac{280x^{11}}{11}+\frac{(3a-768)x^{10}}{10}+\frac{(-24a+1536)x^9}{9}+\frac{(96a-2240)x^8}{8}+\frac{(-240a+2304)x^7}{7}+\frac{(a}{}$

input
```
int(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^3*x^2+8*a^2*x^3+(-6*a^2+48*a)*x^4+(12/5*a^2-384/5*a+512/5)*x^5+(-1/2
*a^2+64*a-256)*x^6+(-240/7*a+2304/7)*x^7+(12*a-280)*x^8+(-8/3*a+512/3)*x^9
+(3/10*a-384/5)*x^10+280/11*x^11-6*x^12+12/13*x^13-1/14*x^14
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=-\frac{1}{14}\,x^{14}+\frac{12}{13}\,x^{13}-6\,x^{12}+\frac{3}{10}\,(a-256)x^{10}+\frac{280}{11}\,x^{11}-\frac{8}{3}\,(a-64)x^9+4\,(3\,a-70)x^8-\frac{48}{7}\,(5\,a-48)x^7-\frac{1}{2}\,\left(a^2-128\,a+512\right)x^6+\frac{4}{5}\,\left(3\,a^2-96\,a+128\right)x^5+\frac{1}{2}\,a^3x^2+8\,a^2x^3-6\,\left(a^2-8\,a\right)x^4$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

output
```
-1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*(a - 256)*x^10 + 280/11*x^11 - 8/3
*(a - 64)*x^9 + 4*(3*a - 70)*x^8 - 48/7*(5*a - 48)*x^7 - 1/2*(a^2 - 128*a
+ 512)*x^6 + 4/5*(3*a^2 - 96*a + 128)*x^5 + 1/2*a^3*x^2 + 8*a^2*x^3 - 6*(a
^2 - 8*a)*x^4
```

3.124.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

$$\int x(a+8x-8x^2+4x^3-x^4)^3\,dx = \frac{a^3x^2}{2} + 8a^2x^3 - \frac{x^{14}}{14} + \frac{12x^{13}}{13} - 6x^{12} + \frac{280x^{11}}{11} + x^{10} \cdot \left(\frac{3a}{10} - \frac{384}{5}\right) + x^9 \cdot \left(\frac{512}{3} - \frac{8a}{3}\right) + x^8 \cdot (12a - 280) + x^7 \cdot \left(\frac{2304}{7} - \frac{240a}{7}\right) + x^6\left(-\frac{a^2}{2} + 64a - 256\right) + x^5 \cdot \left(\frac{12a^2}{5} - \frac{384a}{5} + \frac{512}{5}\right) + x^4(-6a^2 + 48a)$$

input
```
integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

output
```
a**3*x**2/2 + 8*a**2*x**3 - x**14/14 + 12*x**13/13 - 6*x**12 + 280*x**11/1
1 + x**10*(3*a/10 - 384/5) + x**9*(512/3 - 8*a/3) + x**8*(12*a - 280) + x*
*7*(2304/7 - 240*a/7) + x**6*(-a**2/2 + 64*a - 256) + x**5*(12*a**2/5 - 38
4*a/5 + 512/5) + x**4*(-6*a**2 + 48*a)
```

3.124.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int x(a+8x-8x^2+4x^3-x^4)^3\,dx = -\frac{1}{14}x^{14} + \frac{12}{13}x^{13} - 6x^{12} + \frac{3}{10}(a-256)x^{10} + \frac{280}{11}x^{11} - \frac{8}{3}(a-64)x^9 + 4(3a-70)x^8 - \frac{48}{7}(5a-48)x^7 - \frac{1}{2}(a^2-128a+512)x^6 + \frac{4}{5}(3a^2-96a+128)x^5 + \frac{1}{2}a^3x^2 + 8a^2x^3 - 6(a^2-8a)x^4$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

output
```
-1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*(a - 256)*x^10 + 280/11*x^11 - 8/3
*(a - 64)*x^9 + 4*(3*a - 70)*x^8 - 48/7*(5*a - 48)*x^7 - 1/2*(a^2 - 128*a
+ 512)*x^6 + 4/5*(3*a^2 - 96*a + 128)*x^5 + 1/2*a^3*x^2 + 8*a^2*x^3 - 6*(a
^2 - 8*a)*x^4
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx = -\frac{1}{14}\,x^{14}+\frac{12}{13}\,x^{13}-6\,x^{12}+\frac{3}{10}\,ax^{10}+\frac{280}{11}\,x^{11}-\frac{8}{3}\,ax^9 - \frac{384}{5}\,x^{10}+12\,ax^8+\frac{512}{3}\,x^9-\frac{1}{2}\,a^2x^6-\frac{240}{7}\,ax^7 - 280\,x^8+\frac{12}{5}\,a^2x^5+64\,ax^6+\frac{2304}{7}\,x^7-6\,a^2x^4 - \frac{384}{5}\,ax^5-256\,x^6+\frac{1}{2}\,a^3x^2+8\,a^2x^3+48\,ax^4+\frac{512}{5}\,x^5$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

output
```
-1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*a*x^10 + 280/11*x^11 - 8/3*a*x^9 -
 384/5*x^10 + 12*a*x^8 + 512/3*x^9 - 1/2*a^2*x^6 - 240/7*a*x^7 - 280*x^8 +
 12/5*a^2*x^5 + 64*a*x^6 + 2304/7*x^7 - 6*a^2*x^4 - 384/5*a*x^5 - 256*x^6
+ 1/2*a^3*x^2 + 8*a^2*x^3 + 48*a*x^4 + 512/5*x^5
```

3.124.9 Mupad [B] (verification not implemented)

Time = 9.64 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx = x^8\,(12\,a-280)+x^{10}\left(\frac{3\,a}{10}-\frac{384}{5}\right)-x^9\left(\frac{8\,a}{3}-\frac{512}{3}\right) - x^7\left(\frac{240\,a}{7}-\frac{2304}{7}\right)-x^6\left(\frac{a^2}{2}-64\,a+256\right) + x^5\left(\frac{12\,a^2}{5}-\frac{384\,a}{5}+\frac{512}{5}\right)+\frac{280\,x^{11}}{11}-6\,x^{12} + \frac{12\,x^{13}}{13}-\frac{x^{14}}{14}+8\,a^2\,x^3+\frac{a^3\,x^2}{2}-6\,a\,x^4\,(a-8)$$

input
```
int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

output
```
x^8*(12*a - 280) + x^10*((3*a)/10 - 384/5) - x^9*((8*a)/3 - 512/3) - x^7*(
(240*a)/7 - 2304/7) - x^6*(a^2/2 - 64*a + 256) + x^5*((12*a^2)/5 - (384*a)
/5 + 512/5) + (280*x^11)/11 - 6*x^12 + (12*x^13)/13 - x^14/14 + 8*a^2*x^3
+ (a^3*x^2)/2 - 6*a*x^4*(a - 8)
```

3.124. $\int x(a+8x-8x^2+4x^3-x^4)^3\,dx$

3.125 $\int x(a+8x-8x^2+4x^3-x^4)^2\,dx$

3.125.1 Optimal result

Integrand size = 24, antiderivative size = 79

$$\int x(a+8x-8x^2+4x^3-x^4)^2\,dx = \frac{a^2x^2}{2}+\frac{16ax^3}{3}+4(4-a)x^4-\frac{8}{5}(16-a)x^5 +\frac{1}{3}(64-a)x^6-\frac{80x^7}{7}+4x^8-\frac{8x^9}{9}+\frac{x^{10}}{10}$$

output
```
1/2*a^2*x^2+16/3*a*x^3+4*(4-a)*x^4-8/5*(16-a)*x^5+1/3*(64-a)*x^6-80/7*x^7+
4*x^8-8/9*x^9+1/10*x^10
```

3.125.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int x(a+8x-8x^2+4x^3-x^4)^2\,dx = \frac{a^2x^2}{2}+\frac{16ax^3}{3}-4(-4+a)x^4+\frac{8}{5}(-16+a)x^5 +\frac{1}{3}(64-a)x^6-\frac{80x^7}{7}+4x^8-\frac{8x^9}{9}+\frac{x^{10}}{10}$$

input
```
Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
(a^2*x^2)/2 + (16*a*x^3)/3 - 4*(-4 + a)*x^4 + (8*(-16 + a)*x^5)/5 + ((64 -
 a)*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

3.125.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a-x^4+4x^3-8x^2+8x\right)^2 dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(a^2x+2(64-a)x^5-8(16-a)x^4+16(4-a)x^3+16ax^2+x^9-8x^8+32x^7-80x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^2}{2}+\frac{1}{3}(64-a)x^6-\frac{8}{5}(16-a)x^5+4(4-a)x^4+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}$$

input
```
Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
(a^2*x^2)/2 + (16*a*x^3)/3 + 4*(4 - a)*x^4 - (8*(16 - a)*x^5)/5 + ((64 - a
)*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

3.125.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

3.125.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

method	result
norman	$\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}+\left(\frac{64}{3}-\frac{a}{3}\right)x^6+\left(\frac{8a}{5}-\frac{128}{5}\right)x^5+\left(-4a+16\right)x^4+\frac{16a\,x^3}{3}+\frac{a^2x^2}{2}$
default	$\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}+\frac{(-2a+128)x^6}{6}+\frac{(8a-128)x^5}{5}+\frac{(-16a+64)x^4}{4}+\frac{16a\,x^3}{3}+\frac{a^2x^2}{2}$
gosper	$\frac{1}{10}x^{10}-\frac{8}{9}x^9+4x^8-\frac{80}{7}x^7+\frac{64}{3}x^6-\frac{1}{3}a\,x^6+\frac{8}{5}a\,x^5-\frac{128}{5}x^5-4a\,x^4+16x^4+\frac{16}{3}a\,x^3+\frac{1}{2}a^2x^2$
risch	$\frac{1}{10}x^{10}-\frac{8}{9}x^9+4x^8-\frac{80}{7}x^7+\frac{64}{3}x^6-\frac{1}{3}a\,x^6+\frac{8}{5}a\,x^5-\frac{128}{5}x^5-4a\,x^4+16x^4+\frac{16}{3}a\,x^3+\frac{1}{2}a^2x^2$
parallelrisch	$\frac{1}{10}x^{10}-\frac{8}{9}x^9+4x^8-\frac{80}{7}x^7+\frac{64}{3}x^6-\frac{1}{3}a\,x^6+\frac{8}{5}a\,x^5-\frac{128}{5}x^5-4a\,x^4+16x^4+\frac{16}{3}a\,x^3+\frac{1}{2}a^2x^2$

input
```
int(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*x^10-8/9*x^9+4*x^8-80/7*x^7+(64/3-1/3*a)*x^6+(8/5*a-128/5)*x^5+(-4*a+
16)*x^4+16/3*a*x^3+1/2*a^2*x^2
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{1}{10}\,x^{10}-\frac{8}{9}\,x^9+4\,x^8-\frac{1}{3}\,(a-64)x^6-\frac{80}{7}\,x^7+\frac{8}{5}\,(a-16)x^5-4\,(a-4)x^4+\frac{1}{2}\,a^2x^2+\frac{16}{3}\,ax^3$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

output
```
1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*(a - 64)*x^6 - 80/7*x^7 + 8/5*(a - 16)*x
^5 - 4*(a - 4)*x^4 + 1/2*a^2*x^2 + 16/3*a*x^3
```

3.125.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int x(a+8x-8x^2+4x^3-x^4)^2\,dx = \frac{a^2x^2}{2}+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}+x^6\cdot\left(\frac{64}{3}-\frac{a}{3}\right)+x^5\cdot\left(\frac{8a}{5}-\frac{128}{5}\right)+x^4\cdot(16-4a)$$

input
```
integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

output
```
a**2*x**2/2 + 16*a*x**3/3 + x**10/10 - 8*x**9/9 + 4*x**8 - 80*x**7/7 + x**
6*(64/3 - a/3) + x**5*(8*a/5 - 128/5) + x**4*(16 - 4*a)
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int x(a+8x-8x^2+4x^3-x^4)^2\,dx = \frac{1}{10}\,x^{10}-\frac{8}{9}\,x^9+4\,x^8-\frac{1}{3}\,(a-64)x^6-\frac{80}{7}\,x^7+\frac{8}{5}\,(a-16)x^5-4\,(a-4)x^4+\frac{1}{2}\,a^2x^2+\frac{16}{3}\,ax^3$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

output
```
1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*(a - 64)*x^6 - 80/7*x^7 + 8/5*(a - 16)*x
^5 - 4*(a - 4)*x^4 + 1/2*a^2*x^2 + 16/3*a*x^3
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int x(a+8x-8x^2+4x^3-x^4)^2\,dx = \frac{1}{10}\,x^{10}-\frac{8}{9}\,x^9+4\,x^8-\frac{1}{3}\,ax^6-\frac{80}{7}\,x^7+\frac{8}{5}\,ax^5+\frac{64}{3}\,x^6-4\,ax^4-\frac{128}{5}\,x^5+\frac{1}{2}\,a^2x^2+\frac{16}{3}\,ax^3+16\,x^4$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

output
```
1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*a*x^6 - 80/7*x^7 + 8/5*a*x^5 + 64/3*x^6
- 4*a*x^4 - 128/5*x^5 + 1/2*a^2*x^2 + 16/3*a*x^3 + 16*x^4
```

3.125.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

$$\int x(a+8x-8x^2+4x^3-x^4)^2\,dx = x^5\left(\frac{8\,a}{5}-\frac{128}{5}\right)-x^6\left(\frac{a}{3}-\frac{64}{3}\right)-x^4\,(4\,a-16)+\frac{16\,a\,x^3}{3}-\frac{80\,x^7}{7}+4\,x^8-\frac{8\,x^9}{9}+\frac{x^{10}}{10}+\frac{a^2\,x^2}{2}$$

input
```
int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

output
```
x^5*((8*a)/5 - 128/5) - x^6*(a/3 - 64/3) - x^4*(4*a - 16) + (16*a*x^3)/3 -
 (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10 + (a^2*x^2)/2
```

3.126 $\int x(a+8x-8x^2+4x^3-x^4)\,dx$

3.126.1 Optimal result

Integrand size = 22, antiderivative size = 35

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)\,dx=\frac{ax^2}{2}+\frac{8x^3}{3}-2x^4+\frac{4x^5}{5}-\frac{x^6}{6}$$

output
```
1/2*a*x^2+8/3*x^3-2*x^4+4/5*x^5-1/6*x^6
```

3.126.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)\,dx=\frac{ax^2}{2}+\frac{8x^3}{3}-2x^4+\frac{4x^5}{5}-\frac{x^6}{6}$$

input
```
Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

output
```
(a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

3.126.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a-x^4+4x^3-8x^2+8x\right)\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(ax-x^5+4x^4-8x^3+8x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

input
```
Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

output
```
(a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

3.126.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

3.126.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{1}{2}a\,x^2+\frac{8}{3}x^3-2x^4+\frac{4}{5}x^5-\frac{1}{6}x^6$	28
default	$\frac{1}{2}a\,x^2+\frac{8}{3}x^3-2x^4+\frac{4}{5}x^5-\frac{1}{6}x^6$	28
norman	$\frac{1}{2}a\,x^2+\frac{8}{3}x^3-2x^4+\frac{4}{5}x^5-\frac{1}{6}x^6$	28
risch	$\frac{1}{2}a\,x^2+\frac{8}{3}x^3-2x^4+\frac{4}{5}x^5-\frac{1}{6}x^6$	28
parallelrisch	$\frac{1}{2}a\,x^2+\frac{8}{3}x^3-2x^4+\frac{4}{5}x^5-\frac{1}{6}x^6$	28

input
```
int(x*(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+8/3*x^3-2*x^4+4/5*x^5-1/6*x^6
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x(a+8x-8x^2+4x^3-x^4)\;dx=-\frac{1}{6}\,x^6+\frac{4}{5}\,x^5-2\,x^4+\frac{1}{2}\,ax^2+\frac{8}{3}\,x^3$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

output
```
-1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

3.126.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x(a+8x-8x^2+4x^3-x^4)\;dx=\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

input
```
integrate(x*(-x**4+4*x**3-8*x**2+a+8*x),x)
```

output
```
a*x**2/2 - x**6/6 + 4*x**5/5 - 2*x**4 + 8*x**3/3
```

3.126.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x(a+8x-8x^2+4x^3-x^4)\,dx = -\frac{1}{6}\,x^6+\frac{4}{5}\,x^5-2\,x^4+\frac{1}{2}\,ax^2+\frac{8}{3}\,x^3$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

output
```
-1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x(a+8x-8x^2+4x^3-x^4)\,dx = -\frac{1}{6}\,x^6+\frac{4}{5}\,x^5-2\,x^4+\frac{1}{2}\,ax^2+\frac{8}{3}\,x^3$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

output
```
-1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

3.126.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x(a+8x-8x^2+4x^3-x^4)\,dx = -\frac{x^6}{6}+\frac{4\,x^5}{5}-2\,x^4+\frac{8\,x^3}{3}+\frac{a\,x^2}{2}$$

input
```
int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

output
```
(a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

3.127 $\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx$

3.127.1 Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = -\frac{\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} + \frac{\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}} + \frac{\operatorname{arctanh}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{2\sqrt{4+a}}$$

output
```
1/2*arctanh((1+(-1+x)^2)/(4+a)^(1/2))/(4+a)^(1/2)-1/2*arctan((-1+x)/(1-(4+
a)^(1/2))^(1/2))/(4+a)^(1/2)/(1-(4+a)^(1/2))^(1/2)+1/2*arctan((-1+x)/(1+(4
+a)^(1/2))^(1/2))/(4+a)^(1/2)/(1+(4+a)^(1/2))^(1/2)
```

3.127.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.51

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = -\frac{1}{4}\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3 - \#1^4\&,\frac{\log(x-\#1)\#1}{-2+4\#1-3\#1^2+\#1^3}\&\right]$$

input
```
Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

output
```
-1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (Log[x - #1]*#1)/(-2 +
4*#1 - 3*#1^2 + #1^3) & ]
```

3.127.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2459, 2202, 1406, 217, 1432, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a-x^4+4x^3-8x^2+8x}dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{x}{a-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\int \frac{x-1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)$$

$$\Big\downarrow \text{1406}$$

$$-\frac{\int \frac{1}{-(x-1)^2-\sqrt{a+4}-1}d(x-1)}{2\sqrt{a+4}}+\frac{\int \frac{1}{-(x-1)^2+\sqrt{a+4}-1}d(x-1)}{2\sqrt{a+4}}+\int \frac{x-1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)$$

$$\Big\downarrow \text{217}$$

$$\int \frac{x-1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)-\frac{\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}+\frac{\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \frac{1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)^2-\frac{\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}+\frac{\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}$$

$$\Big\downarrow \text{1083}$$

$$-\int \frac{1}{4(a+4)-(x-1)^4}d(-2(x-1)^2-2)-\frac{\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}+\frac{\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}$$

$$\Big\downarrow \ 219$$

$$-\frac{\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}+\frac{\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2\sqrt{a+4}}$$

input
```
Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

output
```
-1/2*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(Sqrt[4 + a]*Sqrt[1 - Sqrt[4 +
 a]]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqr
t[4 + a]]) - ArcTanh[(-2 - 2*(-1 + x)^2)/(2*Sqrt[4 + a])]/(2*Sqrt[4 + a])
```

3.127.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.127.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.45

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}-4\_Z^{3}+8\_Z^{2}-8\_Z-a\right)}\frac{\_R\ln\left(x-\_R\right)}{-\_R^{3}+3\_R^{2}-4\_R+2}\right)}{4}$	52
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}-4\_Z^{3}+8\_Z^{2}-8\_Z-a\right)}\frac{\_R\ln\left(x-\_R\right)}{-\_R^{3}+3\_R^{2}-4\_R+2}\right)}{4}$	52

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_
Z-a))
```

3.127.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.45 (sec) , antiderivative size = 140500, normalized size of antiderivative = 1211.21

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

output
```
Too large to include
```

3.127.6 Sympy [A] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.34

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = -\operatorname{RootSum}\left(t^4\cdot\left(256a^3+2816a^2+10240a+12288\right)+t^2\left(-32a^2-256a-512\right)+t(-16a-64)+a,\left(\right.\right.$$

input
```
integrate(x/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

output
```
-RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a**2
- 256*a - 512) + _t*(-16*a - 64) + a, Lambda(_t, _t*log(x + (-128*_t**3*a*
*4 - 1728*_t**3*a**3 - 8640*_t**3*a**2 - 18944*_t**3*a - 15360*_t**3 + 48*
_t**2*a**3 + 464*_t**2*a**2 + 1472*_t**2*a + 1536*_t**2 + 8*_t*a**3 + 88*_
t*a**2 + 312*_t*a + 352*_t - a**2 - 2*a)/(4*a**2 + 21*a + 28))))
```

3.127.7 Maxima [F]

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = \int -\frac{x}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

output
```
-integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

3.127.8 Giac [F]

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = \int -\frac{x}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

output
```
integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

3.127.9 Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.37

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = \sum_{k=1}^{4}\ln\Big(-x-\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+a,z,k\big)\,\big(\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+a\ldots-8\big)\Big)\,\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+a,z,k\big)$$

input
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

output
```
symsum(log(- x - root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4
 - 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z + 64*z + a, z, k)*(root(2816*
a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 -
 512*z^2 + 16*a*z + 64*z + a, z, k)*(32*a - root(2816*a^2*z^4 + 256*a^3*z^
4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z +
64*z + a, z, k)*(64*a - x*(64*a + 256) + 256) - x*(16*a + 64) + 128) - 8))
*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 -
256*a*z^2 - 512*z^2 + 16*a*z + 64*z + a, z, k), k, 1, 4)
```

3.128 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

3.128.1 Optimal result

Integrand size = 24, antiderivative size = 231

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \frac{1+(-1+x)^2}{4(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} - \frac{\left(10+3a+\sqrt{4+a}\right)\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1-\sqrt{4+a}}} + \frac{\left(10+3a-\sqrt{4+a}\right)\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1+\sqrt{4+a}}} + \frac{\text{arctanh}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{4(4+a)^{3/2}}$$

output
```
1/4*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/4*(5+a+(-1+x)^2)*(-1+x)
/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/4*arctanh((1+(-1+x)^2)/(4+a)^(1/
2))/(4+a)^(3/2)-1/8*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(10+3*a+(4+a)^(1/
2))/(3+a)/(4+a)^(3/2)/(1-(4+a)^(1/2))^(1/2)+1/8*arctan((-1+x)/(1+(4+a)^(1/
2))^(1/2))*(10+3*a-(4+a)^(1/2))/(3+a)/(4+a)^(3/2)/(1+(4+a)^(1/2))^(1/2)
```

3.128.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.72

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \frac{a+2x-ax+ax^2+x^3}{4(3+a)(4+a)\left(a-x\left(-8+8x-4x^2+x^3\right)\right)} - \frac{\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&, \frac{6\log(x-\#1)+a\log(x-\#1)+4\log(x-\#1)\#1+2a\log(x-\#1)\#1+\log(x}{-2+4\#1-3\#1^2+\#1^3}\right.}{16\left(12+7a+a^2\right)}$$

input
```
Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
(a + 2*x - a*x + a*x^2 + x^3)/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2
+ x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (6*Log[x - #1] +
a*Log[x - #1] + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + Log[x - #1]*#1^2)/
(-2 + 4*#1 - 3*#1^2 + #1^3) & ]/(16*(12 + 7*a + a^2))
```

3.128.3 Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2459, 2202, 1405, 27, 1432, 1086, 1083, 219, 1480, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a-x^4+4x^3-8x^2+8x)^2}dx$$

$$\Big\downarrow \text{ 2459}$$

$$\int \frac{x}{(a-(x-1)^4-2(x-1)^2+3)^2}d(x-1)$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{1}{(-(x-1)^4-2(x-1)^2+a+3)^2}d(x-1)+\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^2}d(x-1)$$

$$\Big\downarrow \text{ 1405}$$

$$-\frac{\int -\frac{2((x-1)^2+3a+11)}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{8(a^2+7a+12)}+\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^2}d(x-1)+\frac{(x-1)(a+(x-1)^2+5)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(x-1)^2+3a+11}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4(a^2+7a+12)}+\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^2}d(x-1)+\frac{(x-1)(a+(x-1)^2+5)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}$$

$$\Big\downarrow \text{1432}$$

$$\frac{\int \frac{(x-1)^2+3a+11}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4(a^2+7a+12)}+\frac{1}{2}\int \frac{1}{(-(x-1)^4-2(x-1)^2+a+3)^2}d(x-1)^2+\frac{(x-1)(a+(x-1)^2+5)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}$$

$$\Big\downarrow \text{1086}$$

$$\frac{\int \frac{(x-1)^2+3a+11}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4(a^2+7a+12)}+\frac{1}{2}\left(\frac{\int \frac{1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)^2}{2(a+4)}+\frac{x}{2(a+4)(a-(x-1)^4-2(x-1)^2+3)}\right)+\frac{(x-1)(a+(x-1)^2+5)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\int \frac{(x-1)^2+3a+11}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4(a^2+7a+12)}+\frac{1}{2}\left(\frac{x}{2(a+4)(a-(x-1)^4-2(x-1)^2+3)}-\frac{\int \frac{1}{4(a+4)-(x-1)^4}d(-2(x-1)^2-2)}{a+4}\right)+\frac{(x-1)(a+(x-1)^2+5)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\int \frac{(x-1)^2+3a+11}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4(a^2+7a+12)}+\frac{(x-1)(a+(x-1)^2+5)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}+\frac{1}{2}\left(\frac{x}{2(a+4)(a-(x-1)^4-2(x-1)^2+3)}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}\right)$$

$$\Big\downarrow \text{1480}$$

3.128. $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

$$\frac{\frac{1}{2}\left(1-\frac{3a+10}{\sqrt{a+4}}\right)\int \frac{1}{-(x-1)^2-\sqrt{a+4}-1}d(x-1)+\frac{1}{2}\left(\frac{3a+10}{\sqrt{a+4}}+1\right)\int \frac{1}{-(x-1)^2+\sqrt{a+4}-1}d(x-1)}{4\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{1}{2}\left(\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}\right)$$

$\Big\downarrow$ 217

$$\frac{-\frac{\left(\frac{3a+10}{\sqrt{a+4}}+1\right)\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\left(1-\frac{3a+10}{\sqrt{a+4}}\right)\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}}{4\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{1}{2}\left(\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}\right)$$

input
```
Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
((5 + a + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2
- (-1 + x)^4)) + (-1/2*((1 + (10 + 3*a)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[
1 - Sqrt[4 + a]]])/Sqrt[1 - Sqrt[4 + a]] - ((1 - (10 + 3*a)/Sqrt[4 + a])*A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(2*Sqrt[1 + Sqrt[4 + a]]))/(4*(12 +
 7*a + a^2)) + (x/(2*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - ArcTan
h[(-2 - 2*(-1 + x)^2)/(2*Sqrt[4 + a])]/(2*(4 + a)^(3/2)))/2
```

3.128.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.128. $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

3.128. $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.128.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.68

method	result
default	$\frac{\frac{x^3}{4a^2+28a+48}+\frac{a\,x^2}{4(4+a)(3+a)}-\frac{(a-2)x}{4(a^2+7a+12)}+\frac{a}{4a^2+28a+48}}{-x^4+4x^3-8x^2+a+8x}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a)}\frac{(6+\_R^2+2(a+2)\_R+a)\ln(\cdots}{-\_R^3+3\_R^2-4\_R\cdots}}{16a^2+112a+192}$ (truncated)
risch	$\frac{\frac{x^3}{4a^2+28a+48}+\frac{a\,x^2}{4(4+a)(3+a)}-\frac{(a-2)x}{4(a^2+7a+12)}+\frac{a}{4a^2+28a+48}}{-x^4+4x^3-8x^2+a+8x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a)}\frac{\left(\frac{\_R^2}{a^2+7a+12}+\frac{2(a+2)\_R}{a^2+7a+12}+\cdots\right.}{-\_R^3+3\_R^2\cdots}\right.}{16}$ (truncated)

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/(a^2+7*a+12)*x^3+1/4*a/(4+a)/(3+a)*x^2-1/4*(a-2)/(a^2+7*a+12)*x+1/4/(
a^2+7*a+12)*a)/(-x^4+4*x^3-8*x^2+a+8*x)+1/16/(a^2+7*a+12)*sum((6+_R^2+2*(a
+2)*_R+a)/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z
-a))
```

3.128.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx=\text{Timed out}$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.128.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(197) = 394.

Time = 16.40 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.33

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$$
$$= \frac{-ax^2-a-x^3+x(a-2)}{-4a^3-28a^2-48a+x^4\cdot(4a^2+28a+48)+x^3\left(-16a^2-112a-192\right)+x^2\cdot(32a^2+224a+384)+x\left(\right.}$$
$$+\mathrm{RootSum}\left(t^4\cdot\left(65536a^9+2162688a^8+31653888a^7+269680640a^6+1473773568a^5+5357174784a^4\right.\right.$$

input
```
integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

output
```
(-a*x**2 - a - x**3 + x*(a - 2))/(-4*a**3 - 28*a**2 - 48*a + x**4*(4*a**2
+ 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a + 384
) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*a**8
 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4 + 12
952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**2*(-
2048*a**6 - 50688*a**5 - 520704*a**4 - 2842624*a**3 - 8699904*a**2 - 14155
776*a - 9568256) + _t*(1152*a**4 + 17792*a**3 + 102912*a**2 + 264192*a + 2
53952) + 16*a**3 - 57*a**2 - 984*a - 2064, Lambda(_t, _t*log(x + (98304*_t
**3*a**12 + 3948544*_t**3*a**11 + 72196096*_t**3*a**10 + 793837568*_t**3*a
**9 + 5839372288*_t**3*a**8 + 30226464768*_t**3*a**7 + 112668450816*_t**3*
a**6 + 303864643584*_t**3*a**5 + 586157391872*_t**3*a**4 + 784017129472*_t
**3*a**3 + 683648483328*_t**3*a**2 + 343136010240*_t**3*a + 72477573120*_t
**3 + 30208*_t**2*a**10 + 986624*_t**2*a**9 + 14420992*_t**2*a**8 + 124156
928*_t**2*a**7 + 696815104*_t**2*a**6 + 2661758464*_t**2*a**5 + 7001485312
*_t**2*a**4 + 12506562560*_t**2*a**3 + 14494924800*_t**2*a**2 + 9820569600
*_t**2*a + 2944401408*_t**2 - 1536*_t*a**9 - 52048*_t*a**8 - 757040*_t*a**
7 - 6200656*_t*a**6 - 31380496*_t*a**5 - 100736416*_t*a**4 - 200813696*_t*
a**3 - 228144640*_t*a**2 - 114632704*_t*a - 2490368*_t + 248*a**7 + 6797*a
**6 + 71132*a**5 + 369745*a**4 + 987758*a**3 + 1128896*a**2 - 129568*a - 9
56416)/(576*a**7 + 10985*a**6 + 88746*a**5 + 396609*a**4 + 1076268*a**3...
```

3.128.7 Maxima [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \int \frac{x}{(x^4-4\,x^3+8\,x^2-a-8\,x)^2}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

output
```
-1/4*(a*x^2 + x^3 - (a - 2)*x + a)/((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a +
12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12
*a) - 1/4*integrate((2*(a + 2)*x + x^2 + a + 6)/(x^4 - 4*x^3 + 8*x^2 - a -
 8*x), x)/(a^2 + 7*a + 12)
```

3.128.8 Giac [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \int \frac{x}{(x^4-4\,x^3+8\,x^2-a-8\,x)^2}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

output
```
integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)
```

3.128.9 Mupad [B] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 1167, normalized size of antiderivative = 5.05

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

output
```
symsum(log((35*a + 4*a^2 + 68)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a
^5 + 576)) - root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4
 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^
5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*
a^2*z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2
- 14155776*a*z^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z +
 264192*a*z + 253952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*((12800*a +
 3600*a^2 + 336*a^3 + 15104)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5
 + 576)) + root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 +
 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*
z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^
2*z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 -
14155776*a*z^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 2
64192*a*z + 253952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*(root(1295201
0752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 181193
93280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^
4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 2842624*a^3*z^2
 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z^2 - 956825
6*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 253952*z -
984*a - 57*a^2 + 16*a^3 - 2064, z, k)*((15728640*a + 10878976*a^2 + 399...
```

3.128. $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2} \, dx$

3.129 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3} dx$

3.129.1 Optimal result

Integrand size = 24, antiderivative size = 349

$$\begin{aligned}
\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3} dx &= \frac{1+(-1+x)^2}{8(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^2} \\
&+ \frac{3(1+(-1+x)^2)}{16(4+a)^2\left(3+a-2(-1+x)^2-(-1+x)^4\right)} \\
&+ \frac{(5+a+(-1+x)^2)(-1+x)}{8(12+7a+a^2)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^2} \\
&+ \frac{((6+a)(25+7a)+6(7+2a)(-1+x)^2)(-1+x)}{32(3+a)^2(4+a)^2\left(3+a-2(-1+x)^2-(-1+x)^4\right)} \\
&- \frac{3\left(80+7a^2+14\sqrt{4+a}+a\left(47+4\sqrt{4+a}\right)\right)\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{64(3+a)^2(4+a)^{5/2}\sqrt{1-\sqrt{4+a}}} \\
&- \frac{3\left(14+4a-\frac{80+47a+7a^2}{\sqrt{4+a}}\right)\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{64(3+a)^2(4+a)^2\sqrt{1+\sqrt{4+a}}} + \frac{3\operatorname{arctanh}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{16(4+a)^{5/2}}
\end{aligned}$$

output
```
1/8*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^2+3/16*(1+(-1+x)^2)/(4+a)
^2/(3+a-2*(-1+x)^2-(-1+x)^4)+1/8*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2
*(-1+x)^2-(-1+x)^4)^2+1/32*((6+a)*(25+7*a)+6*(7+2*a)*(-1+x)^2)*(-1+x)/(a^2
+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)+3/16*arctanh((1+(-1+x)^2)/(4+a)^(1/2)
)/(4+a)^(5/2)-3/64*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(80+7*a^2+14*(4+a)
^(1/2)+a*(47+4*(4+a)^(1/2)))/(3+a)^2/(4+a)^(5/2)/(1-(4+a)^(1/2))^(1/2)-3/6
4*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2))*(14+4*a+(-7*a^2-47*a-80)/(4+a)^(1/2
))/(3+a)^2/(4+a)^2/(1+(4+a)^(1/2))^(1/2)
```

3.129.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.81

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \frac{1}{128}\left(\frac{16(a+2x-ax+ax^2+x^3)}{(3+a)(4+a)\left(a-x\left(-8+8x-4x^2+x^3\right)\right)^2}\right.$$
$$+\frac{4(a^2(5-5x+6x^2)+6(-14+28x-12x^2+7x^3)+a(-7+31x+12x^3))}{(3+a)^2(4+a)^2\left(a-x\left(-8+8x-4x^2+x^3\right)\right)}$$
$$-\frac{3\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&,\frac{72\log(x-\#1)+31a\log(x-\#1)+3a^2\log(x-\#1)+8\log(x-\#1)\#1+16\ldots}{-2+4\#\ldots}\right.}{(12+7a+a^2)^2}$$

input
```
Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

output
```
((16*(a + 2*x - a*x + a*x^2 + x^3))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*
x^2 + x^3))^2) + (4*(a^2*(5 - 5*x + 6*x^2) + 6*(-14 + 28*x - 12*x^2 + 7*x^
3) + a*(-7 + 31*x + 12*x^3)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^
2 + x^3))) - (3*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (72*Log[x -
#1] + 31*a*Log[x - #1] + 3*a^2*Log[x - #1] + 8*Log[x - #1]*#1 + 16*a*Log[x
 - #1]*#1 + 4*a^2*Log[x - #1]*#1 + 14*Log[x - #1]*#1^2 + 4*a*Log[x - #1]*#
1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ])/(12 + 7*a + a^2)^2)/128
```

3.129.3 Rubi [A] (warning: unable to verify)

Time = 0.67 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {2459, 2202, 1405, 27, 1432, 1086, 1086, 1083, 219, 1492, 27, 1480, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a-x^4+4x^3-8x^2+8x)^3}dx$$

$$\Big\downarrow \text{ 2459}$$

$$\int \frac{x}{(a-(x-1)^4-2(x-1)^2+3)^3}d(x-1)$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{1}{(-(x-1)^4-2(x-1)^2+a+3)^3}d(x-1)+\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^3}d(x-1)$$

$$\Big\downarrow \text{ 1405}$$

$$-\frac{\int -\frac{2(5(x-1)^2+7a+27)}{(-(x-1)^4-2(x-1)^2+a+3)^2}d(x-1)}{16(a^2+7a+12)}+\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^3}d(x-1)+\frac{(x-1)(a+(x-1)^2+5)}{8(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{5(x-1)^2+7a+27}{(-(x-1)^4-2(x-1)^2+a+3)^2}d(x-1)}{8(a^2+7a+12)}+\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^3}d(x-1)+\frac{(x-1)(a+(x-1)^2+5)}{8(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)^2}$$

$$\Big\downarrow \text{ 1432}$$

$$\frac{\int \frac{5(x-1)^2+7a+27}{(-(x-1)^4-2(x-1)^2+a+3)^2}d(x-1)}{8(a^2+7a+12)}+\frac{1}{2}\int \frac{1}{(-(x-1)^4-2(x-1)^2+a+3)^3}d(x-1)^2+\frac{(x-1)(a+(x-1)^2+5)}{8(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)^2}$$

$$\Big\downarrow \text{ 1086}$$

$$\frac{\int \frac{5(x-1)^2+7a+27}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)}{8\left(a^2+7a+12\right)}+\frac{1}{2}\left(\frac{3\int \frac{1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)^2}{4(a+4)}+\frac{x}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}\right)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$

$\Big\downarrow$ 1086

$$\frac{\int \frac{5(x-1)^2+7a+27}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)}{8\left(a^2+7a+12\right)}+\frac{1}{2}\left(\frac{3\left(\frac{\int \frac{1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)^2}{2(a+4)}+\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}\right)}{4(a+4)}+\frac{x}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}\right)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$

$\Big\downarrow$ 1083

$$\frac{\int \frac{5(x-1)^2+7a+27}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)}{8\left(a^2+7a+12\right)}+\frac{1}{2}\left(\frac{3\left(\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\int \frac{1}{4(a+4)-(x-1)^4}d\left(-2(x-1)^2-2\right)}{a+4}\right)}{4(a+4)}+\frac{x}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}\right)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$

$\Big\downarrow$ 219

$$\frac{\int \frac{5(x-1)^2+7a+27}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)}{8\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}+\frac{1}{2}\left(\frac{3\left(\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}\right)}{4(a+4)}+\frac{x}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}\right)$$

$\Big\downarrow$ 1492

3.129. $\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^3}\,dx$

$$\frac{\frac{(x-1)(6(2a+7)(x-1)^2+(a+6)(7a+25))}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}-\frac{\int -\frac{6\left(7a^2+51a+2(2a+7)(x-1)^2+94\right)}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{8(a^2+7a+12)}}{8\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}+\frac{1}{2}\left(\frac{3\left(\frac{x}{2(a+4)(a-(x-1)^4-2(x-1)^2+3)}-\frac{\operatorname{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}\right)}{4(a+4)}+\frac{x}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{7a^2+51a+2(2a+7)(x-1)^2+94}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4(a^2+7a+12)}+\frac{(x-1)(6(2a+7)(x-1)^2+(a+6)(7a+25))}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}}{8\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}+\frac{1}{2}\left(\frac{3\left(\frac{x}{2(a+4)(a-(x-1)^4-2(x-1)^2+3)}-\frac{\operatorname{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}\right)}{4(a+4)}+\frac{x}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}\right)$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{3\left(\frac{1}{2}\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\int \frac{1}{-(x-1)^2-\sqrt{a+4}-1}d(x-1)+\frac{1}{2}\left(\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\int \frac{1}{-(x-1)^2+\sqrt{a+4}-1}d(x-1)\right)}{4(a^2+7a+12)}+\frac{(x-1)(6(2a+7)(x-1)^2+(a\cdots}{4(a^2+7a+12)(a-(x-1)^4-\cdots}}{8\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}+\frac{1}{2}\left(\frac{3\left(\frac{x}{2(a+4)(a-(x-1)^4-2(x-1)^2+3)}-\frac{\operatorname{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}\right)}{4(a+4)}+\frac{x}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}\right)$$

$$\Big\downarrow \; 217$$

3.129. $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx$

$$\frac{\frac{3\left(-\frac{\left(\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}\right)}{4(a^2+7a+12)}+\frac{(x-1)(6(2a+7)(x-1)^2+(a+6)(7a+25))}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}}{8(a^2+7a+12)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8(a^2+7a+12)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}+\frac{1}{2}\left(\frac{3\left(\frac{x}{2(a+4)(a-(x-1)^4-2(x-1)^2+3)}-\frac{\operatorname{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}\right)}{4(a+4)}+\frac{x}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}\right)$$

input
```
Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

output
```
((5 + a + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2
- (-1 + x)^4)^2) + ((((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2)*(-1 + x
))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + (3*(-1/2*((1
4 + 4*a + (80 + 47*a + 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4
 + a]]])/Sqrt[1 - Sqrt[4 + a]] - ((14 + 4*a - (80 + 47*a + 7*a^2)/Sqrt[4 +
 a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(2*Sqrt[1 + Sqrt[4 + a]])))/(
4*(12 + 7*a + a^2)))/(8*(12 + 7*a + a^2)) + (x/(4*(4 + a)*(3 + a - 2*(-1 +
 x)^2 - (-1 + x)^4)^2) + (3*(x/(2*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)
^4)) - ArcTanh[(-2 - 2*(-1 + x)^2)/(2*Sqrt[4 + a])]/(2*(4 + a)^(3/2))))/(4
*(4 + a)))/2
```

3.129.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.129. $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

3.129. $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx$

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.129.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.17

method	result
default	$-\frac{\frac{3(7+2a)x^7}{16\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{3\left(a^2-8a-40\right)x^6}{16\left(a^4+14a^3+73a^2+168a+144\right)}-\frac{\left(29a^2-127a-792\right)x^5}{32\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{\left(73a^2-227a-1668\right)x^4}{32a^4+448a^3+2336a^2+5376a+4608}-\frac{}{16}}{(-x^4+4x^3-8x^2-}$
risch	$\frac{-\frac{3(7+2a)x^7}{16\left(a^4+14a^3+73a^2+168a+144\right)}-\frac{3\left(a^2-8a-40\right)x^6}{16\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{\left(29a^2-127a-792\right)x^5}{32a^4+448a^3+2336a^2+5376a+4608}-\frac{\left(73a^2-227a-1668\right)x^4}{32\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{}{16}}{(-x^4+4x^3-8x^2}$

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-(3/16*(7+2*a)/(a^4+14*a^3+73*a^2+168*a+144)*x^7+3/16*(a^2-8*a-40)/(a^4+14
*a^3+73*a^2+168*a+144)*x^6-1/32*(29*a^2-127*a-792)/(a^4+14*a^3+73*a^2+168*
a+144)*x^5+1/32*(73*a^2-227*a-1668)/(a^4+14*a^3+73*a^2+168*a+144)*x^4-1/16
*(62*a^2-103*a-1104)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(5*a^3-26*a^2+
140*a+1008)/(a^4+14*a^3+73*a^2+168*a+144)*x^2+3/32*(3*a^3-17*a^2-40*a+192)
/(a^4+14*a^3+73*a^2+168*a+144)*x-3/32*a*(3*a^2+7*a-12)/(a^4+14*a^3+73*a^2+
168*a+144))/(-x^4+4*x^3-8*x^2+a+8*x)^2-3/128/(a^4+14*a^3+73*a^2+168*a+144)
*sum((-72+2*(-2*a-7)*_R^2+4*(-a^2-4*a-2)*_R-3*a^2-31*a)/(-_R^3+3*_R^2-4*_R
+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

3.129.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.129.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1102 vs. 2(318) = 636.

Time = 42.84 (sec) , antiderivative size = 1102, normalized size of antiderivative = 3.16

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

output
```
-(-9*a**3 - 21*a**2 + 36*a + x**7*(12*a + 42) + x**6*(6*a**2 - 48*a - 240)
 + x**5*(-29*a**2 + 127*a + 792) + x**4*(73*a**2 - 227*a - 1668) + x**3*(-
124*a**2 + 206*a + 2208) + x**2*(-10*a**3 + 52*a**2 - 280*a - 2016) + x*(9
*a**3 - 51*a**2 - 120*a + 576))/(32*a**6 + 448*a**5 + 2336*a**4 + 5376*a**
3 + 4608*a**2 + x**8*(32*a**4 + 448*a**3 + 2336*a**2 + 5376*a + 4608) + x*
*7*(-256*a**4 - 3584*a**3 - 18688*a**2 - 43008*a - 36864) + x**6*(1024*a**
4 + 14336*a**3 + 74752*a**2 + 172032*a + 147456) + x**5*(-2560*a**4 - 3584
0*a**3 - 186880*a**2 - 430080*a - 368640) + x**4*(-64*a**5 + 3200*a**4 + 5
2672*a**3 + 288256*a**2 + 678912*a + 589824) + x**3*(256*a**5 - 512*a**4 -
 38656*a**3 - 256000*a**2 - 651264*a - 589824) + x**2*(-512*a**5 - 5120*a*
*4 - 8704*a**3 + 63488*a**2 + 270336*a + 294912) + x*(512*a**5 + 7168*a**4
 + 37376*a**3 + 86016*a**2 + 73728*a)) - RootSum(_t**4*(268435456*a**15 +
14763950080*a**14 + 378493992960*a**13 + 5999532441600*a**12 + 65757291479
040*a**11 + 527875908304896*a**10 + 3206246773555200*a**9 + 15003759578972
160*a**8 + 54537151127224320*a**7 + 153980418717122560*a**6 + 334927734494
986240*a**5 + 551152193655275520*a**4 + 664192984106926080*a**3 + 55336221
2027105280*a**2 + 284993413919539200*a + 68398419340689408) + _t**2*(-4718
592*a**10 - 196116480*a**9 - 3648061440*a**8 - 40022212608*a**7 - 28693993
8816*a**6 - 1405437345792*a**5 - 4764645457920*a**4 - 11043392716800*a**3
- 16752587046912*a**2 - 15023392948224*a - 6049461436416) + _t*(-270950...
```

3.129.7 Maxima [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \int -\frac{x}{(x^4-4\,x^3+8\,x^2-a-8\,x)^3}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

output
```
-1/32*(6*(2*a + 7)*x^7 + 6*(a^2 - 8*a - 40)*x^6 - (29*a^2 - 127*a - 792)*x
^5 + (73*a^2 - 227*a - 1668)*x^4 - 2*(62*a^2 - 103*a - 1104)*x^3 - 9*a^3 -
 2*(5*a^3 - 26*a^2 + 140*a + 1008)*x^2 - 21*a^2 + 3*(3*a^3 - 17*a^2 - 40*a
 + 192)*x + 36*a)/((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14
*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144
)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a
^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 -
 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*
a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 7
3*a^3 + 168*a^2 + 144*a)*x) - 3/32*integrate((2*(2*a + 7)*x^2 + 3*a^2 + 4*
(a^2 + 4*a + 2)*x + 31*a + 72)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^4 +
14*a^3 + 73*a^2 + 168*a + 144)
```

3.129.8 Giac [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \int -\frac{x}{(x^4-4\,x^3+8\,x^2-a-8\,x)^3}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

output
```
integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3, x)
```

3.129.9 Mupad [B] (verification not implemented)

Time = 9.93 (sec) , antiderivative size = 2200, normalized size of antiderivative = 6.30

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

output
```
symsum(log(root(15003759578972160*a^8*z^4 + 54537151127224320*a^7*z^4 + 15
3980418717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 551152193655275520
*a^4*z^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z^4 + 59995
32441600*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200*a*z^4 +
3206246773555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a^11*z^4
+ 378493992960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^4 - 471
8592*a^10*z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 15023392948224
*a*z^2 - 16752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608*a^7*
z^2 - 11043392716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z^2 -
 6049461436416*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a^2*
z + 72880128*a^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z +
 33351008256*z - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 20
736*a^5 - 68345856, z, k)*((242823168*a + 170044416*a^2 + 63509760*a^3 + 1
3340736*a^4 + 1494144*a^5 + 69696*a^6 + 144506880)/(16384*(940032*a + 1195
776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 58
2*a^8 + 36*a^9 + a^10 + 331776)) + root(15003759578972160*a^8*z^4 + 545371
51127224320*a^7*z^4 + 153980418717122560*a^6*z^4 + 334927734494986240*a^5*
z^4 + 551152193655275520*a^4*z^4 + 664192984106926080*a^3*z^4 + 5533622120
27105280*a^2*z^4 + 5999532441600*a^12*z^4 + 527875908304896*a^10*z^4 + 284
993413919539200*a*z^4 + 3206246773555200*a^9*z^4 + 14763950080*a^14*z^4...
```

3.129. $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3} \, dx$

3.130 $\int x^2(a+8x-8x^2+4x^3-x^4)^4\,dx$

3.130.1 Optimal result

Integrand size = 26, antiderivative size = 210

$$\begin{aligned}\int x^2(a+8x-8x^2+4x^3-x^4)^4\,dx = {} & \frac{a^4x^3}{3}+8a^3x^4+\frac{32}{5}(12-a)a^2x^5+\frac{8}{3}a\left(128-48a+a^2\right)x^6 \\ & +\frac{4}{7}\left(1024-1536a+192a^2-a^3\right)x^7 \\ & -4\left(512-288a+15a^2\right)x^8+\frac{64}{9}(128-3a)(4-a)x^9 \\ & -\frac{24}{5}\left(896-128a+a^2\right)x^{10} \\ & +\frac{2}{11}\left(20480-1536a+3a^2\right)x^{11}-\frac{8}{3}(928-35a)x^{12} \\ & +\frac{32}{13}(524-9a)x^{13}-\frac{8}{7}(464-3a)x^{14} \\ & +\frac{4}{15}(640-a)x^{15}-42x^{16}+\frac{128x^{17}}{17}-\frac{8x^{18}}{9}+\frac{x^{19}}{19}\end{aligned}$$

output
```
1/3*a^4*x^3+8*a^3*x^4+32/5*(12-a)*a^2*x^5+8/3*a*(a^2-48*a+128)*x^6+4/7*(-a
^3+192*a^2-1536*a+1024)*x^7-4*(15*a^2-288*a+512)*x^8+64/9*(128-3*a)*(4-a)*
x^9-24/5*(a^2-128*a+896)*x^10+2/11*(3*a^2-1536*a+20480)*x^11-8/3*(928-35*a
)*x^12+32/13*(524-9*a)*x^13-8/7*(464-3*a)*x^14+4/15*(640-a)*x^15-42*x^16+1
28/17*x^17-8/9*x^18+1/19*x^19
```

3.130.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.97

$$\int x^2(a+8x-8x^2+4x^3-x^4)^4\,dx=\frac{a^4x^3}{3}+8a^3x^4-\frac{32}{5}(-12+a)a^2x^5+\frac{8}{3}a(128-48a+a^2)\,x^6 -\frac{4}{7}(-1024+1536a-192a^2+a^3)\,x^7 -4(512-288a+15a^2)\,x^8+\frac{64}{9}(512-140a+3a^2)\,x^9 -\frac{24}{5}(896-128a+a^2)\,x^{10} +\frac{2}{11}(20480-1536a+3a^2)\,x^{11}+\frac{8}{3}(-928+35a)x^{12} -\frac{32}{13}(-524+9a)x^{13}+\frac{8}{7}(-464+3a)x^{14} -\frac{4}{15}(-640+a)x^{15}-42x^{16}+\frac{128x^{17}}{17}-\frac{8x^{18}}{9}+\frac{x^{19}}{19}$$

input
```
Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

output
```
(a^4*x^3)/3 + 8*a^3*x^4 - (32*(-12 + a)*a^2*x^5)/5 + (8*a*(128 - 48*a + a^
2)*x^6)/3 - (4*(-1024 + 1536*a - 192*a^2 + a^3)*x^7)/7 - 4*(512 - 288*a +
15*a^2)*x^8 + (64*(512 - 140*a + 3*a^2)*x^9)/9 - (24*(896 - 128*a + a^2)*x
^10)/5 + (2*(20480 - 1536*a + 3*a^2)*x^11)/11 + (8*(-928 + 35*a)*x^12)/3 -
 (32*(-524 + 9*a)*x^13)/13 + (8*(-464 + 3*a)*x^14)/7 - (4*(-640 + a)*x^15)
/15 - 42*x^16 + (128*x^17)/17 - (8*x^18)/9 + x^19/19
```

3.130.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a-x^4+4x^3-8x^2+8x)^4\,dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(a^4x^2+32a^3x^3+2\left(3a^2-1536a+20480\right)x^{10}-48\left(a^2-128a+896\right)x^9-32\left(15a^2-288a+512\right)x^7+16a(a^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4x^3}{3}+8a^3x^4+\frac{2}{11}\left(3a^2-1536a+20480\right)x^{11}-\frac{24}{5}\left(a^2-128a+896\right)x^{10}-4\left(15a^2-288a+512\right)x^8+\frac{8}{3}a\left(a^2-48a+128\right)x^6+\frac{32}{5}(12-a)a^2x^5+\frac{4}{7}\left(-a^3+192a^2-1536a+1024\right)x^7+\frac{4}{15}(640-a)x^{15}-\frac{8}{7}(464-3a)x^{14}+\frac{32}{13}(524-9a)x^{13}-\frac{8}{3}(928-35a)x^{12}+\frac{64}{9}(128-3a)(4-a)x^9+\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}$$

input
```
Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

output
```
(a^4*x^3)/3 + 8*a^3*x^4 + (32*(12 - a)*a^2*x^5)/5 + (8*a*(128 - 48*a + a^2
)*x^6)/3 + (4*(1024 - 1536*a + 192*a^2 - a^3)*x^7)/7 - 4*(512 - 288*a + 15
*a^2)*x^8 + (64*(128 - 3*a)*(4 - a)*x^9)/9 - (24*(896 - 128*a + a^2)*x^10)
/5 + (2*(20480 - 1536*a + 3*a^2)*x^11)/11 - (8*(928 - 35*a)*x^12)/3 + (32*
(524 - 9*a)*x^13)/13 - (8*(464 - 3*a)*x^14)/7 + (4*(640 - a)*x^15)/15 - 42
*x^16 + (128*x^17)/17 - (8*x^18)/9 + x^19/19
```

3.130.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

3.130.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

method	result
norman	$\frac{a^4x^3}{3}+8a^3x^4+\left(-\frac{32}{5}a^3+\frac{384}{5}a^2\right)x^5+\left(\frac{8}{3}a^3-128a^2+\frac{1024}{3}a\right)x^6+\left(-\frac{4}{7}a^3+\frac{768}{7}a^2-\frac{6144}{7}a+\right.$
gosper	$-\frac{21504}{5}x^{10}+\frac{40960}{11}x^{11}-\frac{7424}{3}x^{12}+\frac{16768}{13}x^{13}+\frac{32768}{9}x^9+\frac{4096}{7}x^7+\frac{24}{7}x^{14}a-\frac{24}{5}x^{10}a^2+\frac{8}{3}x^6a^3-$
risch	$-\frac{21504}{5}x^{10}+\frac{40960}{11}x^{11}-\frac{7424}{3}x^{12}+\frac{16768}{13}x^{13}+\frac{32768}{9}x^9+\frac{4096}{7}x^7+\frac{24}{7}x^{14}a-\frac{24}{5}x^{10}a^2+\frac{8}{3}x^6a^3-$
parallelrisch	$-\frac{21504}{5}x^{10}+\frac{40960}{11}x^{11}-\frac{7424}{3}x^{12}+\frac{16768}{13}x^{13}+\frac{32768}{9}x^9+\frac{4096}{7}x^7+\frac{24}{7}x^{14}a-\frac{24}{5}x^{10}a^2+\frac{8}{3}x^6a^3-$
default	$\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}+\frac{(-4a+2560)x^{15}}{15}+\frac{(48a-7424)x^{14}}{14}+\frac{(-288a+16768)x^{13}}{13}+\frac{(1120a-29696)x^{12}}{12}$

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^4*x^3+8*a^3*x^4+(-32/5*a^3+384/5*a^2)*x^5+(8/3*a^3-128*a^2+1024/3*a)
*x^6+(-4/7*a^3+768/7*a^2-6144/7*a+4096/7)*x^7+(-60*a^2+1152*a-2048)*x^8+(6
4/3*a^2-8960/9*a+32768/9)*x^9+(-24/5*a^2+3072/5*a-21504/5)*x^10+(6/11*a^2-
3072/11*a+40960/11)*x^11+(280/3*a-7424/3)*x^12+(-288/13*a+16768/13)*x^13+(
24/7*a-3712/7)*x^14+(-4/15*a+512/3)*x^15-42*x^16+128/17*x^17-8/9*x^18+1/19
*x^19
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

$$\begin{aligned}\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx=&\frac{1}{19}x^{19}-\frac{8}{9}x^{18}+\frac{128}{17}x^{17}-\frac{4}{15}(a-640)x^{15}\\&-42x^{16}+\frac{8}{7}(3a-464)x^{14}-\frac{32}{13}(9a-524)x^{13}\\&+\frac{8}{3}(35a-928)x^{12}+\frac{2}{11}\left(3a^2-1536a+20480\right)x^{11}\\&-\frac{24}{5}\left(a^2-128a+896\right)x^{10}\\&+\frac{64}{9}\left(3a^2-140a+512\right)x^9\\&-4\left(15a^2-288a+512\right)x^8\\&-\frac{4}{7}\left(a^3-192a^2+1536a-1024\right)x^7+\frac{1}{3}a^4x^3\\&+8a^3x^4+\frac{8}{3}\left(a^3-48a^2+128a\right)x^6\\&-\frac{32}{5}\left(a^3-12a^2\right)x^5\end{aligned}$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

output
```
1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*(a - 640)*x^15 - 42*x^16 + 8/7*(
3*a - 464)*x^14 - 32/13*(9*a - 524)*x^13 + 8/3*(35*a - 928)*x^12 + 2/11*(3
*a^2 - 1536*a + 20480)*x^11 - 24/5*(a^2 - 128*a + 896)*x^10 + 64/9*(3*a^2
- 140*a + 512)*x^9 - 4*(15*a^2 - 288*a + 512)*x^8 - 4/7*(a^3 - 192*a^2 + 1
536*a - 1024)*x^7 + 1/3*a^4*x^3 + 8*a^3*x^4 + 8/3*(a^3 - 48*a^2 + 128*a)*x
^6 - 32/5*(a^3 - 12*a^2)*x^5
```

3.130.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.04

$$\int x^2(a+8x-8x^2+4x^3-x^4)^4\,dx = \frac{a^4x^3}{3}+8a^3x^4+\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}+x^{15}\cdot\left(\frac{512}{3}-\frac{4a}{15}\right)+x^{14}\cdot\left(\frac{24a}{7}-\frac{3712}{7}\right)+x^{13}\cdot\left(\frac{16768}{13}-\frac{288a}{13}\right)+x^{12}\cdot\left(\frac{280a}{3}-\frac{7424}{3}\right)+x^{11}\cdot\left(\frac{6a^2}{11}-\frac{3072a}{11}+\frac{40960}{11}\right)+x^{10}\left(-\frac{24a^2}{5}+\frac{3072a}{5}-\frac{21504}{5}\right)+x^9\cdot\left(\frac{64a^2}{3}-\frac{8960a}{9}+\frac{32768}{9}\right)+x^8(-60a^2+1152a-2048)+x^7\left(-\frac{4a^3}{7}+\frac{768a^2}{7}-\frac{6144a}{7}+\frac{4096}{7}\right)+x^6\cdot\left(\frac{8a^3}{3}-128a^2+\frac{1024a}{3}\right)+x^5\left(-\frac{32a^3}{5}+\frac{384a^2}{5}\right)$$

input
```
integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

output
```
a**4*x**3/3 + 8*a**3*x**4 + x**19/19 - 8*x**18/9 + 128*x**17/17 - 42*x**16
 + x**15*(512/3 - 4*a/15) + x**14*(24*a/7 - 3712/7) + x**13*(16768/13 - 28
8*a/13) + x**12*(280*a/3 - 7424/3) + x**11*(6*a**2/11 - 3072*a/11 + 40960/
11) + x**10*(-24*a**2/5 + 3072*a/5 - 21504/5) + x**9*(64*a**2/3 - 8960*a/9
 + 32768/9) + x**8*(-60*a**2 + 1152*a - 2048) + x**7*(-4*a**3/7 + 768*a**2
/7 - 6144*a/7 + 4096/7) + x**6*(8*a**3/3 - 128*a**2 + 1024*a/3) + x**5*(-3
2*a**3/5 + 384*a**2/5)
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

$$\begin{aligned}\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx &= \frac{1}{19}\,x^{19}-\frac{8}{9}\,x^{18}+\frac{128}{17}\,x^{17}-\frac{4}{15}\,(a-640)x^{15}\\ &-42\,x^{16}+\frac{8}{7}\,(3\,a-464)x^{14}-\frac{32}{13}\,(9\,a-524)x^{13}\\ &+\frac{8}{3}\,(35\,a-928)x^{12}+\frac{2}{11}\,\left(3\,a^2-1536\,a+20480\right)x^{11}\\ &-\frac{24}{5}\,\left(a^2-128\,a+896\right)x^{10}\\ &+\frac{64}{9}\,\left(3\,a^2-140\,a+512\right)x^9\\ &-4\,\left(15\,a^2-288\,a+512\right)x^8\\ &-\frac{4}{7}\,\left(a^3-192\,a^2+1536\,a-1024\right)x^7+\frac{1}{3}\,a^4x^3\\ &+8\,a^3x^4+\frac{8}{3}\,\left(a^3-48\,a^2+128\,a\right)x^6\\ &-\frac{32}{5}\,\left(a^3-12\,a^2\right)x^5\end{aligned}$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

output
```
1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*(a - 640)*x^15 - 42*x^16 + 8/7*(
3*a - 464)*x^14 - 32/13*(9*a - 524)*x^13 + 8/3*(35*a - 928)*x^12 + 2/11*(3
*a^2 - 1536*a + 20480)*x^11 - 24/5*(a^2 - 128*a + 896)*x^10 + 64/9*(3*a^2
- 140*a + 512)*x^9 - 4*(15*a^2 - 288*a + 512)*x^8 - 4/7*(a^3 - 192*a^2 + 1
536*a - 1024)*x^7 + 1/3*a^4*x^3 + 8*a^3*x^4 + 8/3*(a^3 - 48*a^2 + 128*a)*x
^6 - 32/5*(a^3 - 12*a^2)*x^5
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.06

$$\int x^2(a+8x-8x^2+4x^3-x^4)^4\,dx = \frac{1}{19}x^{19}-\frac{8}{9}x^{18}+\frac{128}{17}x^{17}-\frac{4}{15}ax^{15}-42x^{16}+\frac{24}{7}ax^{14}+\frac{512}{3}x^{15}-\frac{288}{13}ax^{13}-\frac{3712}{7}x^{14}+\frac{6}{11}a^2x^{11}+\frac{280}{3}ax^{12}+\frac{16768}{13}x^{13}-\frac{24}{5}a^2x^{10}-\frac{3072}{11}ax^{11}-\frac{7424}{3}x^{12}+\frac{64}{3}a^2x^9+\frac{3072}{5}ax^{10}+\frac{40960}{11}x^{11}-\frac{4}{7}a^3x^7-60a^2x^8-\frac{8960}{9}ax^9-\frac{21504}{5}x^{10}+\frac{8}{3}a^3x^6+\frac{768}{7}a^2x^7+1152ax^8+\frac{32768}{9}x^9-\frac{32}{5}a^3x^5-128a^2x^6-\frac{6144}{7}ax^7-2048x^8+\frac{1}{3}a^4x^3+8a^3x^4+\frac{384}{5}a^2x^5+\frac{1024}{3}ax^6+\frac{4096}{7}x^7$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

output
```
1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*a*x^15 - 42*x^16 + 24/7*a*x^14 +
 512/3*x^15 - 288/13*a*x^13 - 3712/7*x^14 + 6/11*a^2*x^11 + 280/3*a*x^12 +
 16768/13*x^13 - 24/5*a^2*x^10 - 3072/11*a*x^11 - 7424/3*x^12 + 64/3*a^2*x
^9 + 3072/5*a*x^10 + 40960/11*x^11 - 4/7*a^3*x^7 - 60*a^2*x^8 - 8960/9*a*x
^9 - 21504/5*x^10 + 8/3*a^3*x^6 + 768/7*a^2*x^7 + 1152*a*x^8 + 32768/9*x^9
 - 32/5*a^3*x^5 - 128*a^2*x^6 - 6144/7*a*x^7 - 2048*x^8 + 1/3*a^4*x^3 + 8*
a^3*x^4 + 384/5*a^2*x^5 + 1024/3*a*x^6 + 4096/7*x^7
```

3.130.9 Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.85

$$\int x^2(a+8x-8x^2+4x^3-x^4)^4\,dx = x^{14}\left(\frac{24\,a}{7}-\frac{3712}{7}\right)-x^{15}\left(\frac{4\,a}{15}-\frac{512}{3}\right)+x^{12}\left(\frac{280\,a}{3}-\frac{7424}{3}\right)-x^{13}\left(\frac{288\,a}{13}-\frac{16768}{13}\right)-x^8\left(60\,a^2-1152\,a+2048\right)-x^{10}\left(\frac{24\,a^2}{5}-\frac{3072\,a}{5}+\frac{21504}{5}\right)+x^9\left(\frac{64\,a^2}{3}-\frac{8960\,a}{9}+\frac{32768}{9}\right)+x^{11}\left(\frac{6\,a^2}{11}-\frac{3072\,a}{11}+\frac{40960}{11}\right)-x^7\left(\frac{4\,a^3}{7}-\frac{768\,a^2}{7}+\frac{6144\,a}{7}-\frac{4096}{7}\right)-42\,x^{16}+\frac{128\,x^{17}}{17}-\frac{8\,x^{18}}{9}+\frac{x^{19}}{19}+8\,a^3\,x^4+\frac{a^4\,x^3}{3}+\frac{8\,a\,x^6\,(a^2-48\,a+128)}{3}-\frac{32\,a^2\,x^5\,(a-12)}{5}$$

input
```
int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

output
```
x^14*((24*a)/7 - 3712/7) - x^15*((4*a)/15 - 512/3) + x^12*((280*a)/3 - 742
4/3) - x^13*((288*a)/13 - 16768/13) - x^8*(60*a^2 - 1152*a + 2048) - x^10*
((24*a^2)/5 - (3072*a)/5 + 21504/5) + x^9*((64*a^2)/3 - (8960*a)/9 + 32768
/9) + x^11*((6*a^2)/11 - (3072*a)/11 + 40960/11) - x^7*((6144*a)/7 - (768*
a^2)/7 + (4*a^3)/7 - 4096/7) - 42*x^16 + (128*x^17)/17 - (8*x^18)/9 + x^19
/19 + 8*a^3*x^4 + (a^4*x^3)/3 + (8*a*x^6*(a^2 - 48*a + 128))/3 - (32*a^2*x
^5*(a - 12))/5
```

3.131 $\int x^2(a+8x-8x^2+4x^3-x^4)^3\,dx$

3.131.1 Optimal result

Integrand size = 26, antiderivative size = 138

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\frac{a^3x^3}{3}+6a^2x^4+\frac{24}{5}(8-a)ax^5+\frac{2}{3}\left(128-96a+3a^2\right)x^6-\frac{3}{7}\left(512-128a+a^2\right)x^7+6(48-5a)x^8-\frac{32}{9}(70-3a)x^9+\frac{12}{5}(64-a)x^{10}-\frac{3}{11}(256-a)x^{11}+\frac{70x^{12}}{3}-\frac{72x^{13}}{13}+\frac{6x^{14}}{7}-\frac{x^{15}}{15}$$

output
```
1/3*a^3*x^3+6*a^2*x^4+24/5*(8-a)*a*x^5+2/3*(3*a^2-96*a+128)*x^6-3/7*(a^2-1
28*a+512)*x^7+6*(48-5*a)*x^8-32/9*(70-3*a)*x^9+12/5*(64-a)*x^10-3/11*(256-
a)*x^11+70/3*x^12-72/13*x^13+6/7*x^14-1/15*x^15
```

3.131.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\frac{a^3x^3}{3}+6a^2x^4-\frac{24}{5}(-8+a)ax^5+\frac{2}{3}\left(128-96a+3a^2\right)x^6-\frac{3}{7}\left(512-128a+a^2\right)x^7-6(-48+5a)x^8+\frac{32}{9}(-70+3a)x^9-\frac{12}{5}(-64+a)x^{10}+\frac{3}{11}(-256+a)x^{11}+\frac{70x^{12}}{3}-\frac{72x^{13}}{13}+\frac{6x^{14}}{7}-\frac{x^{15}}{15}$$

input
```
Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

output
```
(a^3*x^3)/3 + 6*a^2*x^4 - (24*(-8 + a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*
x^6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 - 6*(-48 + 5*a)*x^8 + (32*(-70 + 3*
a)*x^9)/9 - (12*(-64 + a)*x^10)/5 + (3*(-256 + a)*x^11)/11 + (70*x^12)/3 -
 (72*x^13)/13 + (6*x^14)/7 - x^15/15
```

3.131.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a-x^4+4 x^3-8 x^2+8 x\right)^3 \, dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(a^3x^2-3\left(a^2-128a+512\right)x^6+4\left(3a^2-96a+128\right)x^5+24a^2x^3-3(256-a)x^{10}+24(64-a)x^9-32(70-3\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3}{3}-\frac{3}{7}\left(a^2-128a+512\right)x^7+\frac{2}{3}\left(3a^2-96a+128\right)x^6+6a^2x^4-\frac{3}{11}(256-a)x^{11}+\frac{12}{5}(64-a)x^{10}-\frac{32}{9}(70-3a)x^9+6(48-5a)x^8+\frac{24}{5}(8-a)ax^5-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}$$

input
```
Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

output
```
(a^3*x^3)/3 + 6*a^2*x^4 + (24*(8 - a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*x
^6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 + 6*(48 - 5*a)*x^8 - (32*(70 - 3*a)*
x^9)/9 + (12*(64 - a)*x^10)/5 - (3*(256 - a)*x^11)/11 + (70*x^12)/3 - (72*
x^13)/13 + (6*x^14)/7 - x^15/15
```

3.131.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

3.131.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

method	result
norman	$\frac{a^3x^3}{3}+6a^2x^4+\left(-\frac{24}{5}a^2+\frac{192}{5}a\right)x^5+\left(2a^2-64a+\frac{256}{3}\right)x^6+\left(-\frac{3}{7}a^2+\frac{384}{7}a-\frac{1536}{7}\right)x^7+(-3$
gosper	$\frac{1}{3}a^3x^3+6a^2x^4-\frac{24}{5}a^2x^5+\frac{192}{5}a\,x^5+2a^2x^6-64a\,x^6+\frac{256}{3}x^6-\frac{3}{7}a^2x^7+\frac{384}{7}a\,x^7-\frac{1536}{7}x^7-3$
risch	$\frac{1}{3}a^3x^3+6a^2x^4-\frac{24}{5}a^2x^5+\frac{192}{5}a\,x^5+2a^2x^6-64a\,x^6+\frac{256}{3}x^6-\frac{3}{7}a^2x^7+\frac{384}{7}a\,x^7-\frac{1536}{7}x^7-3$
parallelrisch	$\frac{1}{3}a^3x^3+6a^2x^4-\frac{24}{5}a^2x^5+\frac{192}{5}a\,x^5+2a^2x^6-64a\,x^6+\frac{256}{3}x^6-\frac{3}{7}a^2x^7+\frac{384}{7}a\,x^7-\frac{1536}{7}x^7-3$
default	$-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}+\frac{(3a-768)x^{11}}{11}+\frac{(-24a+1536)x^{10}}{10}+\frac{(96a-2240)x^9}{9}+\frac{(-240a+2304)x^8}{8}+\frac{(a($

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^3*x^3+6*a^2*x^4+(-24/5*a^2+192/5*a)*x^5+(2*a^2-64*a+256/3)*x^6+(-3/7
*a^2+384/7*a-1536/7)*x^7+(-30*a+288)*x^8+(32/3*a-2240/9)*x^9+(-12/5*a+768/
5)*x^10+(3/11*a-768/11)*x^11+70/3*x^12-72/13*x^13+6/7*x^14-1/15*x^15
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\begin{aligned}\int x^2(a+8x-8x^2+4x^3-x^4)^3\,dx = &-\frac{1}{15}x^{15}+\frac{6}{7}x^{14}-\frac{72}{13}x^{13}+\frac{3}{11}(a-256)x^{11}+\frac{70}{3}x^{12}\\ &-\frac{12}{5}(a-64)x^{10}+\frac{32}{9}(3a-70)x^9-6(5a-48)x^8\\ &-\frac{3}{7}(a^2-128a+512)x^7+\frac{2}{3}(3a^2-96a+128)x^6\\ &+\frac{1}{3}a^3x^3+6a^2x^4-\frac{24}{5}(a^2-8a)x^5\end{aligned}$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

output
```
-1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*(a - 256)*x^11 + 70/3*x^12 - 12/
5*(a - 64)*x^10 + 32/9*(3*a - 70)*x^9 - 6*(5*a - 48)*x^8 - 3/7*(a^2 - 128*
a + 512)*x^7 + 2/3*(3*a^2 - 96*a + 128)*x^6 + 1/3*a^3*x^3 + 6*a^2*x^4 - 24
/5*(a^2 - 8*a)*x^5
```

3.131.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.97

$$\int x^2(a+8x-8x^2+4x^3-x^4)^3\,dx = \frac{a^3x^3}{3}+6a^2x^4-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}+x^{11}\cdot\left(\frac{3a}{11}-\frac{768}{11}\right)+x^{10}\cdot\left(\frac{768}{5}-\frac{12a}{5}\right)+x^9\cdot\left(\frac{32a}{3}-\frac{2240}{9}\right)+x^8\cdot(288-30a)+x^7\left(-\frac{3a^2}{7}+\frac{384a}{7}-\frac{1536}{7}\right)+x^6\cdot\left(2a^2-64a+\frac{256}{3}\right)+x^5\left(-\frac{24a^2}{5}+\frac{192a}{5}\right)$$

input
```
integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

output
```
a**3*x**3/3 + 6*a**2*x**4 - x**15/15 + 6*x**14/7 - 72*x**13/13 + 70*x**12/
3 + x**11*(3*a/11 - 768/11) + x**10*(768/5 - 12*a/5) + x**9*(32*a/3 - 2240
/9) + x**8*(288 - 30*a) + x**7*(-3*a**2/7 + 384*a/7 - 1536/7) + x**6*(2*a*
*2 - 64*a + 256/3) + x**5*(-24*a**2/5 + 192*a/5)
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int x^2(a+8x-8x^2+4x^3-x^4)^3\,dx = -\frac{1}{15}x^{15}+\frac{6}{7}x^{14}-\frac{72}{13}x^{13}+\frac{3}{11}(a-256)x^{11}+\frac{70}{3}x^{12}-\frac{12}{5}(a-64)x^{10}+\frac{32}{9}(3a-70)x^9-6(5a-48)x^8-\frac{3}{7}(a^2-128a+512)x^7+\frac{2}{3}(3a^2-96a+128)x^6+\frac{1}{3}a^3x^3+6a^2x^4-\frac{24}{5}(a^2-8a)x^5$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

output
```
-1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*(a - 256)*x^11 + 70/3*x^12 - 12/
5*(a - 64)*x^10 + 32/9*(3*a - 70)*x^9 - 6*(5*a - 48)*x^8 - 3/7*(a^2 - 128*
a + 512)*x^7 + 2/3*(3*a^2 - 96*a + 128)*x^6 + 1/3*a^3*x^3 + 6*a^2*x^4 - 24
/5*(a^2 - 8*a)*x^5
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx = -\frac{1}{15}x^{15}+\frac{6}{7}x^{14}-\frac{72}{13}x^{13}+\frac{3}{11}ax^{11}+\frac{70}{3}x^{12} -\frac{12}{5}ax^{10}-\frac{768}{11}x^{11}+\frac{32}{3}ax^9+\frac{768}{5}x^{10} -\frac{3}{7}a^2x^7-30\,ax^8-\frac{2240}{9}x^9+2\,a^2x^6+\frac{384}{7}ax^7 +288\,x^8-\frac{24}{5}a^2x^5-64\,ax^6-\frac{1536}{7}x^7 +\frac{1}{3}a^3x^3+6\,a^2x^4+\frac{192}{5}ax^5+\frac{256}{3}x^6$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

output
```
-1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*a*x^11 + 70/3*x^12 - 12/5*a*x^10
 - 768/11*x^11 + 32/3*a*x^9 + 768/5*x^10 - 3/7*a^2*x^7 - 30*a*x^8 - 2240/9
*x^9 + 2*a^2*x^6 + 384/7*a*x^7 + 288*x^8 - 24/5*a^2*x^5 - 64*a*x^6 - 1536/
7*x^7 + 1/3*a^3*x^3 + 6*a^2*x^4 + 192/5*a*x^5 + 256/3*x^6
```

3.131.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx = x^{11}\left(\frac{3\,a}{11}-\frac{768}{11}\right)-x^{10}\left(\frac{12\,a}{5}-\frac{768}{5}\right)-x^8\,(30\,a-288) +x^9\left(\frac{32\,a}{3}-\frac{2240}{9}\right)+x^6\left(2\,a^2-64\,a+\frac{256}{3}\right) -x^7\left(\frac{3\,a^2}{7}-\frac{384\,a}{7}+\frac{1536}{7}\right)+\frac{70\,x^{12}}{3}-\frac{72\,x^{13}}{13} +\frac{6\,x^{14}}{7}-\frac{x^{15}}{15}+6\,a^2\,x^4+\frac{a^3\,x^3}{3}-\frac{24\,a\,x^5\,(a-8)}{5}$$

input
```
int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

output
```
x^11*((3*a)/11 - 768/11) - x^10*((12*a)/5 - 768/5) - x^8*(30*a - 288) + x^
9*((32*a)/3 - 2240/9) + x^6*(2*a^2 - 64*a + 256/3) - x^7*((3*a^2)/7 - (384
*a)/7 + 1536/7) + (70*x^12)/3 - (72*x^13)/13 + (6*x^14)/7 - x^15/15 + 6*a^
2*x^4 + (a^3*x^3)/3 - (24*a*x^5*(a - 8))/5
```

3.131. $\int x^2(a+8x-8x^2+4x^3-x^4)^3\,dx$

3.132 $\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx$

3.132.1 Optimal result

Integrand size = 26, antiderivative size = 79

$$\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx=\frac{a^2x^3}{3}+4ax^4+\frac{16}{5}(4-a)x^5-\frac{4}{3}(16-a)x^6+\frac{2}{7}(64-a)x^7-10x^8+\frac{32x^9}{9}-\frac{4x^{10}}{5}+\frac{x^{11}}{11}$$

output
```
1/3*a^2*x^3+4*a*x^4+16/5*(4-a)*x^5-4/3*(16-a)*x^6+2/7*(64-a)*x^7-10*x^8+32
/9*x^9-4/5*x^10+1/11*x^11
```

3.132.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx=\frac{a^2x^3}{3}+4ax^4-\frac{16}{5}(-4+a)x^5+\frac{4}{3}(-16+a)x^6-\frac{2}{7}(-64+a)x^7-10x^8+\frac{32x^9}{9}-\frac{4x^{10}}{5}+\frac{x^{11}}{11}$$

input
```
Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
(a^2*x^3)/3 + 4*a*x^4 - (16*(-4 + a)*x^5)/5 + (4*(-16 + a)*x^6)/3 - (2*(-6
4 + a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11
```

3.132.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a - x^4 + 4x^3 - 8x^2 + 8x\right)^2 \, dx$$

$$\Big\downarrow \ 2465$$

$$\int \left(a^2x^2 + 2(64-a)x^6 - 8(16-a)x^5 + 16(4-a)x^4 + 16ax^3 + x^{10} - 8x^9 + 32x^8 - 80x^7\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^2x^3}{3} + \frac{2}{7}(64-a)x^7 - \frac{4}{3}(16-a)x^6 + \frac{16}{5}(4-a)x^5 + 4ax^4 + \frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8$$

input
```
Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
(a^2*x^3)/3 + 4*a*x^4 + (16*(4 - a)*x^5)/5 - (4*(16 - a)*x^6)/3 + (2*(64 -
 a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11
```

3.132.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

3.132.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

method	result
norman	$\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^{9}}{9}-10x^{8}+\left(-\frac{2a}{7}+\frac{128}{7}\right)x^{7}+\left(\frac{4a}{3}-\frac{64}{3}\right)x^{6}+\left(-\frac{16a}{5}+\frac{64}{5}\right)x^{5}+4a\,x^{4}+\frac{a^{2}x^{3}}{3}$
default	$\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^{9}}{9}-10x^{8}+\frac{(-2a+128)x^{7}}{7}+\frac{(8a-128)x^{6}}{6}+\frac{(-16a+64)x^{5}}{5}+4a\,x^{4}+\frac{a^{2}x^{3}}{3}$
gosper	$\frac{1}{11}x^{11}-\frac{4}{5}x^{10}+\frac{32}{9}x^{9}-10x^{8}-\frac{2}{7}a\,x^{7}+\frac{128}{7}x^{7}+\frac{4}{3}a\,x^{6}-\frac{64}{3}x^{6}-\frac{16}{5}a\,x^{5}+\frac{64}{5}x^{5}+4a\,x^{4}+\frac{1}{3}a^{2}$
risch	$\frac{1}{11}x^{11}-\frac{4}{5}x^{10}+\frac{32}{9}x^{9}-10x^{8}-\frac{2}{7}a\,x^{7}+\frac{128}{7}x^{7}+\frac{4}{3}a\,x^{6}-\frac{64}{3}x^{6}-\frac{16}{5}a\,x^{5}+\frac{64}{5}x^{5}+4a\,x^{4}+\frac{1}{3}a^{2}$
parallelrisch	$\frac{1}{11}x^{11}-\frac{4}{5}x^{10}+\frac{32}{9}x^{9}-10x^{8}-\frac{2}{7}a\,x^{7}+\frac{128}{7}x^{7}+\frac{4}{3}a\,x^{6}-\frac{64}{3}x^{6}-\frac{16}{5}a\,x^{5}+\frac{64}{5}x^{5}+4a\,x^{4}+\frac{1}{3}a^{2}$

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*x^11-4/5*x^10+32/9*x^9-10*x^8+(-2/7*a+128/7)*x^7+(4/3*a-64/3)*x^6+(-1
6/5*a+64/5)*x^5+4*a*x^4+1/3*a^2*x^3
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx=\frac{1}{11}x^{11}-\frac{4}{5}x^{10}+\frac{32}{9}x^9-\frac{2}{7}(a-64)x^7-10x^8+\frac{4}{3}(a-16)x^6-\frac{16}{5}(a-4)x^5+\frac{1}{3}a^2x^3+4ax^4$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

output
```
1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*(a - 64)*x^7 - 10*x^8 + 4/3*(a - 16)
*x^6 - 16/5*(a - 4)*x^5 + 1/3*a^2*x^3 + 4*a*x^4
```

3.132.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx = \frac{a^2x^3}{3}+4ax^4+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^9}{9}-10x^8+x^7\cdot\left(\frac{128}{7}-\frac{2a}{7}\right)+x^6\cdot\left(\frac{4a}{3}-\frac{64}{3}\right)+x^5\cdot\left(\frac{64}{5}-\frac{16a}{5}\right)$$

input
```
integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

output
```
a**2*x**3/3 + 4*a*x**4 + x**11/11 - 4*x**10/5 + 32*x**9/9 - 10*x**8 + x**7
*(128/7 - 2*a/7) + x**6*(4*a/3 - 64/3) + x**5*(64/5 - 16*a/5)
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx = \frac{1}{11}\,x^{11}-\frac{4}{5}\,x^{10}+\frac{32}{9}\,x^9-\frac{2}{7}\,(a-64)x^7-10\,x^8+\frac{4}{3}\,(a-16)x^6-\frac{16}{5}\,(a-4)x^5+\frac{1}{3}\,a^2x^3+4\,ax^4$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

output
```
1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*(a - 64)*x^7 - 10*x^8 + 4/3*(a - 16)
*x^6 - 16/5*(a - 4)*x^5 + 1/3*a^2*x^3 + 4*a*x^4
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx = \frac{1}{11}\,x^{11}-\frac{4}{5}\,x^{10}+\frac{32}{9}\,x^9-\frac{2}{7}\,ax^7-10\,x^8+\frac{4}{3}\,ax^6+\frac{128}{7}\,x^7-\frac{16}{5}\,ax^5-\frac{64}{3}\,x^6+\frac{1}{3}\,a^2x^3+4\,ax^4+\frac{64}{5}\,x^5$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

output
```
1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*a*x^7 - 10*x^8 + 4/3*a*x^6 + 128/7*x
^7 - 16/5*a*x^5 - 64/3*x^6 + 1/3*a^2*x^3 + 4*a*x^4 + 64/5*x^5
```

3.132.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=x^6\left(\frac{4\,a}{3}-\frac{64}{3}\right)-x^5\left(\frac{16\,a}{5}-\frac{64}{5}\right)-x^7\left(\frac{2\,a}{7}-\frac{128}{7}\right)+4\,a\,x^4-10\,x^8+\frac{32\,x^9}{9}-\frac{4\,x^{10}}{5}+\frac{x^{11}}{11}+\frac{a^2\,x^3}{3}$$

input
```
int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

output
```
x^6*((4*a)/3 - 64/3) - x^5*((16*a)/5 - 64/5) - x^7*((2*a)/7 - 128/7) + 4*a
*x^4 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11 + (a^2*x^3)/3
```

3.133 $\int x^2(a+8x-8x^2+4x^3-x^4)\,dx$

3.133.1 Optimal result

Integrand size = 24, antiderivative size = 35

$$\int x^2(a+8x-8x^2+4x^3-x^4)\,dx=\frac{ax^3}{3}+2x^4-\frac{8x^5}{5}+\frac{2x^6}{3}-\frac{x^7}{7}$$

output
```
1/3*a*x^3+2*x^4-8/5*x^5+2/3*x^6-1/7*x^7
```

3.133.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int x^2(a+8x-8x^2+4x^3-x^4)\,dx=\frac{ax^3}{3}+2x^4-\frac{8x^5}{5}+\frac{2x^6}{3}-\frac{x^7}{7}$$

input
```
Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

output
```
(a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

3.133.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a-x^4+4x^3-8x^2+8x)\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int (ax^2-x^6+4x^5-8x^4+8x^3)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^3}{3}-\frac{x^7}{7}+\frac{2x^6}{3}-\frac{8x^5}{5}+2x^4$$

input
```
Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

output
```
(a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

3.133.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

3.133.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28
default	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28
norman	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28
risch	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28
parallelrisch	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+2*x^4-8/5*x^5+2/3*x^6-1/7*x^7
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)\,dx=-\frac{1}{7}\,x^7+\frac{2}{3}\,x^6-\frac{8}{5}\,x^5+\frac{1}{3}\,ax^3+2\,x^4$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

output
```
-1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

3.133.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)\,dx=\frac{ax^3}{3}-\frac{x^7}{7}+\frac{2x^6}{3}-\frac{8x^5}{5}+2x^4$$

input
```
integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x),x)
```

output
```
a*x**3/3 - x**7/7 + 2*x**6/3 - 8*x**5/5 + 2*x**4
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)\,dx=-\frac{1}{7}\,x^7+\frac{2}{3}\,x^6-\frac{8}{5}\,x^5+\frac{1}{3}\,ax^3+2\,x^4$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

output
```
-1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)\,dx=-\frac{1}{7}\,x^7+\frac{2}{3}\,x^6-\frac{8}{5}\,x^5+\frac{1}{3}\,ax^3+2\,x^4$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

output
```
-1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

3.133.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)\,dx=-\frac{x^7}{7}+\frac{2\,x^6}{3}-\frac{8\,x^5}{5}+2\,x^4+\frac{a\,x^3}{3}$$

input
```
int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

output
```
(a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

3.134 $\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx$

3.134.1 Optimal result

Integrand size = 26, antiderivative size = 99

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx=-\frac{\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}}-\frac{\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}}+\frac{\operatorname{arctanh}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{\sqrt{4+a}}$$

output
```
arctanh((1+(-1+x)^2)/(4+a)^(1/2))/(4+a)^(1/2)-1/2*arctan((-1+x)/(1-(4+a)^(
1/2))^(1/2))/(1-(4+a)^(1/2))^(1/2)-1/2*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2)
)/(1+(4+a)^(1/2))^(1/2)
```

3.134.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx=-\frac{1}{4}\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&,\frac{\log(x-\#1)\#1^2}{-2+4\#1-3\#1^2+\#1^3}\&\right]$$

input
```
Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

output
```
-1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (Log[x - #1]*#1^2)/(-2
+ 4*#1 - 3*#1^2 + #1^3) & ]
```

3.134.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {2459, 2006, 2202, 27, 1432, 1083, 219, 1480, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a-x^4+4x^3-8x^2+8x}dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{(x-1)^2+2(x-1)+1}{a-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{2006}$$

$$\int \frac{x^2}{a-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{(x-1)^2+1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\int \frac{2(x-1)}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(x-1)^2+1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+2\int \frac{x-1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)^2+\int \frac{(x-1)^2+1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{(x-1)^2+1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)-2\int \frac{1}{4(a+4)-(x-1)^4}d\left(-2(x-1)^2-2\right)$$

$$\Big\downarrow \text{219}$$

$$\int \frac{(x-1)^2+1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)-\frac{\operatorname{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\int \frac{1}{-(x-1)^2-\sqrt{a+4}-1}d(x-1)+\frac{1}{2}\int \frac{1}{-(x-1)^2+\sqrt{a+4}-1}d(x-1)-\frac{\operatorname{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}-\frac{\operatorname{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

input
```
Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

output
```
-1/2*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/Sqrt[1 - Sqrt[4 + a]] - ArcTan
[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[1 + Sqrt[4 + a]]) - ArcTanh[(-2 -
 2*(-1 + x)^2)/(2*Sqrt[4 + a])]/Sqrt[4 + a]
```

3.134.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.134.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}\right)}{4}$	54
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}\right)}{4}$	54

input
```
int(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^2/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8
*_Z-a))
```

3.134.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.66 (sec) , antiderivative size = 1515766, normalized size of antiderivative = 15310.77

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

output
```
Too large to include
```

3.134.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(82) = 164.

Time = 4.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.74

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx =$$
$$-\operatorname{RootSum}\left(t^4\cdot\left(256a^3+2816a^2+10240a+12288\right)+t^2\left(-160a^2-1152a-2048\right)+t\left(-32a^2-256a\right.\right.$$

input
```
integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

output
```
-RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-160*a**2
 - 1152*a - 2048) + _t*(-32*a**2 - 256*a - 512) - a**2, Lambda(_t, _t*log(
x + (-64*_t**3*a**4 - 448*_t**3*a**3 - 256*_t**3*a**2 + 3584*_t**3*a + 614
4*_t**3 - 224*_t**2*a**3 - 2208*_t**2*a**2 - 7168*_t**2*a - 7680*_t**2 + 5
6*_t*a**3 + 400*_t*a**2 + 864*_t*a + 512*_t + 5*a**3 + 34*a**2 + 56*a)/(a*
*3 + 60*a**2 + 320*a + 448)))
```

3.134.7 Maxima [F]

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = \int -\frac{x^2}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

output
```
-integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

3.134.8 Giac [F]

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = \int -\frac{x^2}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

output
```
integrate(-x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

3.134.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 878, normalized size of antiderivative = 8.87

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = \sum_{k=1}^{4} \ln\Big(64\,\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4$$
$$+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a\,z+512\,z-a^2,z,k\big)-a$$
$$-8\,x+\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2$$
$$-2048\,z^2+32\,a^2\,z+256\,a\,z+512\,z-a^2,z,k\big)\ a\,20$$
$$-\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a$$
$$+\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a$$
$$+\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a$$
$$-\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a$$
$$-192\,\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+2$$
$$+256\,\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+2$$
$$-\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2$$
$$+32\,a^2\,z+256\,a\,z+512\,z-a^2,z,k\big)\ a\,x\,4$$
$$+\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a$$
$$-\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a$$
$$+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a\,z$$
$$+512\,z-a^2,z,k\big)$$

input
```
int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

output
```
symsum(log(64*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 -
160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z,
 k) - a - 8*x + 20*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z
^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^
2, z, k)*a - 48*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4
- 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2,
z, k)^2*a + 64*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 -
 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z
, k)^3*a + 128*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 -
 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z
, k)^2*x - 256*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 -
 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z
, k)^3*x - 192*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 -
 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z
, k)^2 + 256*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 1
60*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z,
k)^3 - 4*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a
^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)*a
*x + 32*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^
2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)...
```

3.134. $\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx$

3.135 $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

3.135.1 Optimal result

Integrand size = 26, antiderivative size = 225

$$\begin{aligned}\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx &= \frac{1+(-1+x)^2}{2(4+a)\,(3+a-2(-1+x)^2-(-1+x)^4)} \\ &+ \frac{(4+a)\,(2+(-1+x)^2)\,(-1+x)}{4\,(12+7a+a^2)\,(3+a-2(-1+x)^2-(-1+x)^4)} \\ &- \frac{\left(4+a+\sqrt{4+a}\right)\arctan\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{8(3+a)(4+a)\sqrt{1-\sqrt{4+a}}} \\ &- \frac{\left(4+a-\sqrt{4+a}\right)\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{8(3+a)(4+a)\sqrt{1+\sqrt{4+a}}} \\ &+ \frac{\operatorname{arctanh}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{2(4+a)^{3/2}}\end{aligned}$$

output
```
1/2*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/4*(4+a)*(2+(-1+x)^2)*(-
1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/2*arctanh((1+(-1+x)^2)/(4+a)
^(1/2))/(4+a)^(3/2)-1/8*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(4+a+(4+a)^(1
/2))/(3+a)/(4+a)/(1-(4+a)^(1/2))^(1/2)-1/8*arctan((-1+x)/(1+(4+a)^(1/2))^(
1/2))*(4+a-(4+a)^(1/2))/(3+a)/(4+a)/(1+(4+a)^(1/2))^(1/2)
```

3.135.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \frac{2x(4-3x+2x^2)+a(1+x-x^2+x^3)}{4(3+a)(4+a)\left(a-x\left(-8+8x-4x^2+x^3\right)\right)} - \frac{\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&, \frac{-a\log(x-\#1)+4\log(x-\#1)\#1+2a\log(x-\#1)\#1+4\log(x-\#1)\#1^2}{-2+4\#1-3\#1^2+\#1^3}\right.}{16\left(12+7a+a^2\right)}$$

input
```
Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
(2*x*(4 - 3*x + 2*x^2) + a*(1 + x - x^2 + x^3))/(4*(3 + a)*(4 + a)*(a - x*
(-8 + 8*x - 4*x^2 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & ,
 (-(a*Log[x - #1]) + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + 4*Log[x - #1]
*#1^2 + a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ]/(16*(12 + 7*a
+ a^2))
```

3.135.3 Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2459, 2006, 2202, 27, 1432, 1086, 1083, 219, 1492, 27, 1480, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-x^4+4x^3-8x^2+8x)^2}dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{(x-1)^2+2(x-1)+1}{(a-(x-1)^4-2(x-1)^2+3)^2}d(x-1)$$

$$\Big\downarrow \text{2006}$$

$$\int \frac{x^2}{(a-(x-1)^4-2(x-1)^2+3)^2}d(x-1)$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)+\int \frac{2(x-1)}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)+2\int \frac{x-1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)^2+\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)$$

$$\Big\downarrow \text{1086}$$

$$\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)+\frac{\int \frac{1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)^2}{2(a+4)}+\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\int \frac{1}{4(a+4)-(x-1)^4}d\left(-2(x-1)^2-2\right)}{a+4}+\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)+\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \text{219}$$

$$\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)-\frac{\operatorname{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}+\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \text{1492}$$

$$-\frac{\int -\frac{2(a+4)\left((x-1)^2+2\right)}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{8\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\operatorname{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}+\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+4)\int \frac{(x-1)^2+2}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\operatorname{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}+\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \text{1480}$$

3.135. $\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx$

$$\frac{(a+4)\left(\frac{1}{2}\left(1-\frac{1}{\sqrt{a+4}}\right)\int \frac{1}{-(x-1)^2-\sqrt{a+4}-1}d(x-1)+\frac{1}{2}\left(\frac{1}{\sqrt{a+4}}+1\right)\int \frac{1}{-(x-1)^2+\sqrt{a+4}-1}d(x-1)\right)}{4(a^2+7a+12)}+$$
$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{4(a^2+7a+12)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\operatorname{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}+$$
$$\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{(a+4)\left(-\frac{\left(\frac{1}{\sqrt{a+4}}+1\right)\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\left(1-\frac{1}{\sqrt{a+4}}\right)\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}\right)}{4(a^2+7a+12)}+$$
$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{4(a^2+7a+12)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\operatorname{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}+$$
$$\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

input
```
Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
((4 + a)*(2 + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x
)^2 - (-1 + x)^4)) + x/(2*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + (
(4 + a)*(-1/2*((1 + 1/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])
/Sqrt[1 - Sqrt[4 + a]] - ((1 - 1/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqr
t[4 + a]]])/(2*Sqrt[1 + Sqrt[4 + a]])))/(4*(12 + 7*a + a^2)) - ArcTanh[(-2
 - 2*(-1 + x)^2)/(2*Sqrt[4 + a])]/(2*(4 + a)^(3/2))
```

3.135.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

3.135. $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2} \, dx$

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.135.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.71

method	result
default	$\frac{\frac{x^3}{4a+12}-\frac{(6+a)x^2}{4(3+a)(4+a)}+\frac{(a+8)x}{4(3+a)(4+a)}+\frac{a}{4(4+a)(3+a)}}{-x^4+4x^3-8x^2+a+8x}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(\_R^2(4+a)+2(a+2)\_R-a\right)\ln\left(x-\_\right.}{-\_R^3+3\_R^2-4\_R+2}}{16(4+a)(3+a)}$
risch	$\frac{\frac{x^3}{4a+12}-\frac{(6+a)x^2}{4(3+a)(4+a)}+\frac{(a+8)x}{4(3+a)(4+a)}+\frac{a}{4(4+a)(3+a)}}{-x^4+4x^3-8x^2+a+8x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(\frac{\_R^2}{3+a}+\frac{2(a+2)\_R}{(3+a)(4+a)}-\frac{a}{(4+a)(3+a)}\right.}{-\_R^3+3\_R^2-4\_R}\right.}{16}$

input
```
int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/(3+a)*x^3-1/4*(6+a)/(3+a)/(4+a)*x^2+1/4*(a+8)/(3+a)/(4+a)*x+1/4*a/(4+
a)/(3+a))/(-x^4+4*x^3-8*x^2+a+8*x)+1/16/(4+a)/(3+a)*sum((_R^2*(4+a)+2*(a+2
)*_R-a)/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a
))
```

3.135.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.135.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(185) = 370.

Time = 18.36 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.49

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$$
$$= \frac{-a+x^3(-a-4)+x^2(a+6)+x(-a-8)}{-4a^3-28a^2-48a+x^4\cdot(4a^2+28a+48)+x^3(-16a^2-112a-192)+x^2\cdot(32a^2+224a+384)+x(-}$$
$$+\operatorname{RootSum}\Big(t^4\cdot(65536a^9+2162688a^8+31653888a^7+269680640a^6+1473773568a^5+5357174784a^4$$

input
```
integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

output
```
(-a + x**3*(-a - 4) + x**2*(a + 6) + x*(-a - 8))/(-4*a**3 - 28*a**2 - 48*a
 + x**4*(4*a**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a*
*2 + 224*a + 384) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**
9 + 2162688*a**8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357
174784*a**4 + 12952010752*a**3 + 20082327552*a**2 + 18119393280*a + 724775
7312) + _t**2*(-9728*a**6 - 209408*a**5 - 1878016*a**4 - 8986624*a**3 - 24
215552*a**2 - 34865152*a - 20971520) + _t*(256*a**5 + 5888*a**4 + 53248*a*
*3 + 237568*a**2 + 524288*a + 458752) - a**4 + 144*a**3 + 1024*a**2 + 1792
*a, Lambda(_t, _t*log(x + (4096*_t**3*a**12 - 61440*_t**3*a**11 - 5480448*
_t**3*a**10 - 111403008*_t**3*a**9 - 1227173888*_t**3*a**8 - 8682876928*_t
**3*a**7 - 42187440128*_t**3*a**6 - 144630284288*_t**3*a**5 - 350972280832
*_t**3*a**4 - 591750234112*_t**3*a**3 - 660716126208*_t**3*a**2 - 43984827
1872*_t**3*a - 132271570944*_t**3 - 28672*_t**2*a**10 - 993280*_t**2*a**9
- 15400960*_t**2*a**8 - 140742656*_t**2*a**7 - 839462912*_t**2*a**6 - 3414
427648*_t**2*a**5 - 9590087680*_t**2*a**4 - 18363547648*_t**2*a**3 - 22938
255360*_t**2*a**2 - 16873684992*_t**2*a - 5549064192*_t**2 - 848*_t*a**9 -
 6096*_t*a**8 + 174608*_t*a**7 + 3323792*_t*a**6 + 26276224*_t*a**5 + 1190
09280*_t*a**4 + 332017664*_t*a**3 + 566497280*_t*a**2 + 544112640*_t*a + 2
25837056*_t + 11*a**8 + 958*a**7 + 17419*a**6 + 142964*a**5 + 632632*a**4
+ 1567552*a**3 + 2049792*a**2 + 1100800*a)/(a**8 + 870*a**7 + 18289*a**...
```

3.135.7 Maxima [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \int \frac{x^2}{(x^4-4\,x^3+8\,x^2-a-8\,x)^2}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

output
```
-1/4*((a + 4)*x^3 - (a + 6)*x^2 + (a + 8)*x + a)/((a^2 + 7*a + 12)*x^4 - 4
*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*
a + 12)*x - 12*a) - 1/4*integrate(((a + 4)*x^2 + 2*(a + 2)*x - a)/(x^4 - 4
*x^3 + 8*x^2 - a - 8*x), x)/(a^2 + 7*a + 12)
```

3.135.8 Giac [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \int \frac{x^2}{(x^4-4\,x^3+8\,x^2-a-8\,x)^2}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)
```

3.135.9 Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 1218, normalized size of antiderivative = 5.41

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

output
```
symsum(log((x*(40*a + 7*a^2 + 56))/(8*(816*a + 460*a^2 + 129*a^3 + 18*a^4
+ a^5 + 576)) - (48*a + 12*a^2 - a^3)/(64*(816*a + 460*a^2 + 129*a^3 + 18*
a^4 + a^5 + 576)) - root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*
a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 147377
3568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 2
4215552*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a^4*z^2 - 209408*a^5*z^2 - 972
8*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2 + 237568*a^2*z + 53248*a^3*z + 5
888*a^4*z + 256*a^5*z + 524288*a*z + 458752*z + 1792*a + 1024*a^2 + 144*a^
3 - a^4, z, k)*((28160*a + 11328*a^2 + 2064*a^3 + 144*a^4 + 26624)/(64*(81
6*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + root(12952010752*a^3*z^4
+ 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 +
 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640
*a^6*z^4 + 7247757312*z^4 - 24215552*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a
^4*z^2 - 209408*a^5*z^2 - 9728*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2 + 2
37568*a^2*z + 53248*a^3*z + 5888*a^4*z + 256*a^5*z + 524288*a*z + 458752*z
 + 1792*a + 1024*a^2 + 144*a^3 - a^4, z, k)*(root(12952010752*a^3*z^4 + 31
653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 200
82327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6
*z^4 + 7247757312*z^4 - 24215552*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a^4*z
^2 - 209408*a^5*z^2 - 9728*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2 + 23...
```

3.136 $\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6} dx$

3.136.1 Optimal result

Integrand size = 46, antiderivative size = 545

$$\begin{aligned}
&\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx \\
&= -\frac{\sqrt[3]{-1}\left(2\sqrt[3]{-1}b+3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{5/6}b^2\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}c^{2/3}} \\
&\quad -\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}a^{5/6}b^2\sqrt{4b-3\sqrt[3]{a}c^{2/3}}c^{2/3}} \\
&\quad -\frac{(-1)^{2/3}\left(2b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{5/6}b^2\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}c^{2/3}} \\
&\quad -\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}+\frac{\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{6\left(1+\sqrt[3]{-1}\right)^2a^{2/3}b^2\sqrt[3]{c}} \\
&\quad +\frac{\sqrt[3]{-1}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}
\end{aligned}$$

output
```
-1/18*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(2/3)/b^2/c^(1/3)+1/6*ln(3*a-3*(
-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(2/3)/b^2/c^(1/3)+1/
18*(-1)^(1/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(2/3)/b^2/c^(
1/3)-1/27*(2*b-3*a^(1/3)*c^(2/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(
1/2)/a^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2))/a^(5/6)/b^2/c^(2/3)*3^(1/2)/(4
*b-3*a^(1/3)*c^(2/3))^(1/2)-1/9*(-1)^(2/3)*(2*b+3*(-1)^(1/3)*a^(1/3)*c^(2/
3))*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3
*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(5/6
)/b^2/c^(2/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)-1/9*(-1)^(1
/3)*(2*(-1)^(1/3)*b+3*a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^
(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+
(-1)^(1/3))^2/a^(5/6)/b^2/c^(2/3)*3^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3
))^(1/2)
```

3.136.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.18

$$\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4 + b^3\#1^6\&,\frac{\log(x-\#1)\#1^3}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\right]$$

input
```
Integrate[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^
6),x]
```

output
```
RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
 , (Log[x - #1]*#1^3)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4
) & ]/3
```

3.136.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 530, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$

$$\Big\downarrow \text{2466}$$

$$19683a^6\int \left(-\frac{\sqrt[3]{c}x+\sqrt[3]{a}}{177147a^{20/3}bc^{2/3}\left(bx^2+3a^{2/3}\sqrt[3]{c}x+3a\right)}+\frac{\sqrt[3]{c}x+(-1)^{2/3}\sqrt[3]{a}}{59049\left(1+\sqrt[3]{-1}\right)^2a^{20/3}bc^{2/3}\left(bx^2-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$19683a^6\left(-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}c^{2/3}+2\sqrt[3]{-1}b\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{59049\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{41/6}b^2c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{}}}\right.}{177147\sqrt{3}a^{41/6}b^2c^{2/3}\sqrt{4b-3\sqrt[3]{a}}}\right.$$

input
```
Int[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

output
```
19683*a^6*(-1/59049*((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTa
n[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1
)^(2/3)*a^(1/3)*c^(2/3)])])/(Sqrt[3]*(1 + (-1)^(1/3))^2*a^(41/6)*b^2*Sqrt[
4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*
ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c
^(2/3)])])/(177147*Sqrt[3]*a^(41/6)*b^2*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2
/3)) - ((-1)^(2/3)*(2*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/
3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/
3)*c^(2/3)])])/(177147*Sqrt[3]*a^(41/6)*b^2*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3
)*c^(2/3)]*c^(2/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(354294*a^(20
/3)*b^2*c^(1/3)) + Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(1180
98*(1 + (-1)^(1/3))^2*a^(20/3)*b^2*c^(1/3)) + ((-1)^(1/3)*Log[3*a + 3*(-1)
^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(354294*a^(20/3)*b^2*c^(1/3)))
```

3.136.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.136.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.17

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(b^{3}\_Z^{6}+9b^{2}a\_Z^{4}+27c\,a^{2}\_Z^{3}+27a^{2}b\_Z^{2}+27a^{3}\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{5}b^{3}+12\_R^{3}a\,b^{2}+27\_R^{2}a^{2}c+18a^{2}b\_R}\right)}{3}$	93
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(b^{3}\_Z^{6}+9b^{2}a\_Z^{4}+27c\,a^{2}\_Z^{3}+27a^{2}b\_Z^{2}+27a^{3}\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{5}b^{3}+12\_R^{3}a\,b^{2}+27\_R^{2}a^{2}c+18a^{2}b\_R}\right)}{3}$	93

input
```
int(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_R
ETURNVERBOSE)
```

output
```
1/3*sum(_R^4/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R)
,_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

3.136.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} \, dx = \text{Timed out}$$

input
```
integrate(x**4/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a
**3),x)
```

output
```
Timed out
```

3.136.7 Maxima [F]

$$\int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} \, dx = \int \frac{x^4}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3} \, dx$$

input
```
integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="maxima")
```

output
```
integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^
3), x)
```

3.136.8 Giac [F]

$$\int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \int \frac{x^4}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

input
```
integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="giac")
```

output
```
integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^
3), x)
```

3.136.9 Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 1563, normalized size of antiderivative = 2.87

$$\int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \text{Too large to display}$$

input
```
int(x^4/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

output
```
symsum(log(-19683*a^8*b^3*(c*x - b + 6561*root(918330048*a^5*b^9*c^4*z^6 -
 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3
*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)
^2*a^3*c^4 + 2*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6
+ 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 +
 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)*b^4*x - 198*root(918330048
*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1
023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*
a*b*c*z + 1, z, k)*a*b^2*c - 8991*root(918330048*a^5*b^9*c^4*z^6 - 3874204
89*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 5
31441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*a^2*b
^3*c^2 - 19683*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6
+ 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 +
 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^3*a^3*b^4*c^3 + 104976*roo
t(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*
c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2
*z^2 + 324*a*b*c*z + 1, z, k)^4*a^3*b^8*c^2 - 8503056*root(918330048*a^5*b
^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516
*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*
z + 1, z, k)^5*a^4*b^9*c^3 + 4782969*root(918330048*a^5*b^9*c^4*z^6 - 3...
```

3.136. $\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

3.137 $\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

3.137.1 Optimal result

Integrand size = 46, antiderivative size = 487

$$\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = -\frac{\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{7/6}b\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} - \frac{\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}a^{7/6}b\sqrt{4b-3\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} + \frac{\sqrt[3]{-1}\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{7/6}b\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} + \frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54a^{4/3}bc^{2/3}} - \frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{18\left(1+\sqrt[3]{-1}\right)^2a^{4/3}bc^{2/3}} + \frac{(-1)^{2/3}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54a^{4/3}bc^{2/3}}$$

output
```
1/54*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(4/3)/b/c^(2/3)-1/18*(-1)^(2/3)*l
n(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(4/3)/b/c^(
2/3)+1/54*(-1)^(2/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(4/3)/
b/c^(2/3)-1/27*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3
*a^(1/3)*c^(2/3))^(1/2))/a^(7/6)/b/c^(1/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))
^(1/2)+1/9*(-1)^(1/3)*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1
/2)/a^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-
1)^(1/3))^2/a^(7/6)/b/c^(1/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(
1/2)-1/9*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(
4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(7/6)/b/c^(1/3
)*3^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

3.137.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.20

$$\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4 + b^3\#1^6\&, \frac{\log(x-\#1)\#1^2}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\right]$$

input
```
Integrate[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^
6),x]
```

output
```
RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
 , (Log[x - #1]*#1^2)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4
) & ]/3
```

3.137.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 472, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$

$$\Big\downarrow \text{2466}$$

$$19683a^6\int \left(\frac{x}{531441a^{22/3}c^{2/3}\left(bx^2+3a^{2/3}\sqrt[3]{c}x+3a\right)}-\frac{(-1)^{2/3}x}{177147\left(1+\sqrt[3]{-1}\right)^2a^{22/3}c^{2/3}\left(bx^2-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$19683a^6\left(-\frac{\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{59049\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{43/6}b\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{177147\sqrt{3}a^{43/6}b\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}+\frac{\sqrt[3]{-1}}{1771}\right.$$

input
```
Int[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

output
```
19683*a^6*(-1/59049*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]
*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(Sqrt[3]*(1 + (-1)^(1/
3))^2*a^(43/6)*b*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTa
n[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3
)])]/(177147*Sqrt[3]*a^(43/6)*b*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + (
(-1)^(1/3)*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*
Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(177147*Sqrt[3]*a^(43/6)*b*Sqr
t[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[3*a + 3*a^(2/3)*c^(1/
3)*x + b*x^2]/(1062882*a^(22/3)*b*c^(2/3)) - ((-1)^(2/3)*Log[3*a - 3*(-1)^
(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(354294*(1 + (-1)^(1/3))^2*a^(22/3)*b*c^
(2/3)) + ((-1)^(2/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(1
062882*a^(22/3)*b*c^(2/3)))
```

3.137.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.137.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.19

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{3}\_Z^{6}+9b^{2}a\_Z^{4}+27c\,a^{2}\_Z^{3}+27a^{2}b\_Z^{2}+27a^{3}\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{2\_R^{5}b^{3}+12\_R^{3}a\,b^{2}+27\_R^{2}a^{2}c+18a^{2}b\_R}\right)}{3}$	93
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{3}\_Z^{6}+9b^{2}a\_Z^{4}+27c\,a^{2}\_Z^{3}+27a^{2}b\_Z^{2}+27a^{3}\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{2\_R^{5}b^{3}+12\_R^{3}a\,b^{2}+27\_R^{2}a^{2}c+18a^{2}b\_R}\right)}{3}$	93

input
```
int(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_R
ETURNVERBOSE)
```

output
```
1/3*sum(_R^3/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R)
,_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

3.137.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

3.137.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a
**3),x)
```

output
```
Timed out
```

3.137.7 Maxima [F]

$$\int \frac{x^3}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \int \frac{x^3}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

input
```
integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="maxima")
```

output
```
integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^
3), x)
```

3.137.8 Giac [F]

$$\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \int \frac{x^3}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx$$

input
```
integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="giac")
```

output
```
integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^
3), x)
```

3.137.9 Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 1354, normalized size of antiderivative = 2.78

$$\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \text{Too large to display}$$

input
```
int(x^3/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

output
```
symsum(log(4782969*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*
c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c
^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^2*a^9*b^6*c^3 - 729*a^5*b^7*x + 12
9140163*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 1
4348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19
683*a^3*b*c^2*z^2 - 1, z, k)^3*a^10*b^8*c^3 + 1549681956*root(10460353203*
a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 +
 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z,
 k)^4*a^11*b^10*c^3 + 167365651248*root(10460353203*a^9*b^3*c^6*z^6 - 2479
4911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^
3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^5*a^12*b^12*c^3 -
94143178827*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6
 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3
- 19683*a^3*b*c^2*z^2 - 1, z, k)^5*a^13*b^9*c^5 + 98415*root(10460353203*a
^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 +
314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z,
k)*a^7*b^7*c + 4374*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6
*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*
c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)*a^6*b^9*x - 2125764*root(10460353
203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^...
```

3.137. $\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

3.138 $\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

3.138.1 Optimal result

Integrand size = 46, antiderivative size = 334

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \frac{2(-1)^{2/3}\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{11/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}c^{2/3}}+\frac{2\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{11/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}c^{2/3}}+\frac{2(-1)^{2/3}\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{11/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}c^{2/3}}$$

output
```
2/81*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c
^(2/3))^(1/2))/a^(11/6)/c^(2/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)+2/27
*(-1)^(2/3)*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2
)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^
2/a^(11/6)/c^(2/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)+2/27*(
-1)^(2/3)*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/
(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(11/6)/c^(2/3
)*3^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

3.138.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.29

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4 + b^3\#1^6\&,\frac{\log(x-\#1)\#1}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\right]$$

input
```
Integrate[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^
6),x]
```

output
```
RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
 , (Log[x - #1]*#1)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4)
& ]/3
```

3.138.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$

$$\Big\downarrow \ 2466$$

$$19683a^6\int \left(\frac{1}{531441a^{22/3}\left(bx^2+3a^{2/3}\sqrt[3]{c}x+3a\right)c^{2/3}}-\frac{(-1)^{2/3}}{177147\left(1+\sqrt[3]{-1}\right)^2a^{22/3}\left(bx^2-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a\right)c^{2/}\right.$$

$$\Big\downarrow \ 2009$$

$$19683a^6\left(\frac{2(-1)^{2/3}\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{177147\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{47/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}+\frac{2\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{531441\sqrt{3}a^{47/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}+\frac{2(-1)}{5314}\right.$$

input
```
Int[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

output
```
19683*a^6*((2*(-1)^(2/3)*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sq
rt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(177147*Sqrt[3]*
(1 + (-1)^(1/3))^2*a^(47/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/
3)) + (2*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*
a^(1/3)*c^(2/3)])])/(531441*Sqrt[3]*a^(47/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]
*c^(2/3)) + (2*(-1)^(2/3)*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(S
qrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(531441*Sqrt[3]
*a^(47/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3)))
```

3.138.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.138.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.28

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27c\,a^2\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	93
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27c\,a^2\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	93

input
```
int(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_R
ETURNVERBOSE)
```

output
```
1/3*sum(_R^2/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R)
,_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

3.138.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.43 (sec) , antiderivative size = 27094, normalized size of antiderivative = 81.12

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="fricas")
```

output
```
Too large to include
```

3.138.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a
**3),x)
```

output
```
Timed out
```

3.138.7 Maxima [F]

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \int \frac{x^2}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx$$

input
```
integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="maxima")
```

output
```
integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^
3), x)
```

3.138.8 Giac [F]

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \int \frac{x^2}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx$$

input
```
integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="giac")
```

output
```
integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^
3), x)
```

3.138.9 Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 825, normalized size of antiderivative = 2.47

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$
$$=\sum_{k=1}^{6}\ln\Big(-a^3\,b^9\,\Big(-\mathrm{root}\big(669462604992\,a^{11}\,b^3\,c^4\,z^6-282429536481\,a^{12}\,c^6\,z^6+129140163\,a^8\,c^4\,z^4-19683\,a$$
$$-\mathrm{root}\big(669462604992\,a^{11}\,b^3\,c^4\,z^6-282429536481\,a^{12}\,c^6\,z^6+129140163\,a^8\,c^4\,z^4-19683\,a^4\,c^2\,z^2+1,z,k\big)$$
$$+\mathrm{root}\big(669462604992\,a^{11}\,b^3\,c^4\,z^6-282429536481\,a^{12}\,c^6\,z^6+129140163\,a^8\,c^4\,z^4-19683\,a^4\,c^2\,z^2+1,z,k\big)$$
$$+\mathrm{root}\big(669462604992\,a^{11}\,b^3\,c^4\,z^6-282429536481\,a^{12}\,c^6\,z^6+129140163\,a^8\,c^4\,z^4-19683\,a^4\,c^2\,z^2+1,z,k\big)$$
$$+\mathrm{root}\big(669462604992\,a^{11}\,b^3\,c^4\,z^6-282429536481\,a^{12}\,c^6\,z^6+129140163\,a^8\,c^4\,z^4-19683\,a^4\,c^2\,z^2+1,z,k\big)\,a^2$$
$$+\mathrm{root}\big(669462604992\,a^{11}\,b^3\,c^4\,z^6-282429536481\,a^{12}\,c^6\,z^6+129140163\,a^8\,c^4\,z^4-19683\,a^4\,c^2\,z^2+1,z,k\big)\,a\,b$$
$$-\mathrm{root}\big(669462604992\,a^{11}\,b^3\,c^4\,z^6-282429536481\,a^{12}\,c^6\,z^6+129140163\,a^8\,c^4\,z^4-19683\,a^4\,c^2\,z^2+1,z,k\big)$$
$$-\mathrm{root}\big(669462604992\,a^{11}\,b^3\,c^4\,z^6-282429536481\,a^{12}\,c^6\,z^6+129140163\,a^8\,c^4\,z^4-19683\,a^4\,c^2\,z^2+1,z,k\big)$$
$$+\mathrm{root}\big(669462604992\,a^{11}\,b^3\,c^4\,z^6-282429536481\,a^{12}\,c^6\,z^6+129140163\,a^8\,c^4\,z^4-19683\,a^4\,c^2\,z^2+1,z,k\big)$$
$$+\mathrm{root}\big(669462604992\,a^{11}\,b^3\,c^4\,z^6-282429536481\,a^{12}\,c^6\,z^6+129140163\,a^8\,c^4\,z^4-19683\,a^4\,c^2\,z^2+1,z,k\big)$$
$$+\mathrm{root}\big(669462604992\,a^{11}\,b^3\,c^4\,z^6-282429536481\,a^{12}\,c^6\,z^6+129140163\,a^8\,c^4\,z^4-19683\,a^4\,c^2\,z^2+1,z,k\big)$$
$$+\mathrm{root}\big(669462604992\,a^{11}\,b^3\,c^4\,z^6-282429536481\,a^{12}\,c^6\,z^6+129140163\,a^8\,c^4\,z^4-19683\,a^4\,c^2\,z^2+1,z,k\big)$$
$$+1\Big)\,27\Big)\,\mathrm{root}\big(669462604992\,a^{11}\,b^3\,c^4\,z^6-282429536481\,a^{12}\,c^6\,z^6+129140163\,a^8\,c^4\,z^4$$
$$-19683\,a^4\,c^2\,z^2+1,z,k\big)$$

input
```
int(x^2/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

output
```
symsum(log(-27*a^3*b^9*(43046721*root(669462604992*a^11*b^3*c^4*z^6 - 2824
29536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z,
k)^4*a^8*c^4 - 1062882*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a
^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^3*a^6*c
^3 - 13122*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6
+ 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^2*a^4*c^2 + 3486784
401*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 12914
0163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^10*c^5 + 81*root(66946
2604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z
^4 - 19683*a^4*c^2*z^2 + 1, z, k)*a^2*c + 18*root(669462604992*a^11*b^3*c^
4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*
z^2 + 1, z, k)*a*b^2*x - 25509168*root(669462604992*a^11*b^3*c^4*z^6 - 282
429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z,
 k)^4*a^7*b^3*c^2 - 6198727824*root(669462604992*a^11*b^3*c^4*z^6 - 282429
536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)
^5*a^9*b^3*c^3 + 5832*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^
12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^2*a^3*b^
2*c*x + 708588*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*
z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^3*a^5*b^2*c^2*x
 + 38263752*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*...
```

3.138. $\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

3.139 $\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

3.139.1 Optimal result

Integrand size = 44, antiderivative size = 469

$$\begin{aligned}
&\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx \\
&= -\frac{\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{13/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}}-\frac{\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{13/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} \\
&\quad+\frac{\sqrt[3]{-1}\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{13/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} \\
&\quad-\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{7/3}c^{2/3}}+\frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{7/3}c^{2/3}} \\
&\quad-\frac{(-1)^{2/3}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{7/3}c^{2/3}}
\end{aligned}$$

output
```
-1/162*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(7/3)/c^(2/3)+1/54*(-1)^(2/3)*l
n(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(7/3)/c^(2/
3)-1/162*(-1)^(2/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(7/3)/c
^(2/3)-1/81*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^
(1/3)*c^(2/3))^(1/2))/a^(13/6)/c^(1/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/
2)+1/27*(-1)^(1/3)*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)
/a^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^
(1/3))^2/a^(13/6)/c^(1/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)
-1/27*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b
-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(13/6)/c^(1/3)*3^
(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

3.139.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.20

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \frac{1}{3}\text{RootSum}\Big[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4 + b^3\#1^6\&, \frac{\log(x-\#1)}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\Big]$$

input
```
Integrate[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)
,x]
```

output
```
RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
 , Log[x - #1]/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) & ]/3
```

3.139.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$

$\Big\downarrow$ 2466

$$19683a^6\int \left(-\frac{bx+3a^{2/3}\sqrt[3]{c}}{1594323a^{25/3}c^{2/3}\left(bx^2+3a^{2/3}\sqrt[3]{c}x+3a\right)}+\frac{(-1)^{2/3}bx+3a^{2/3}\sqrt[3]{c}}{531441\left(1+\sqrt[3]{-1}\right)^2a^{25/3}c^{2/3}\left(bx^2-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+\right.}\right.$$

$\Big\downarrow$ 2009

$$19683a^6\left(-\frac{\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{177147\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{49/6}\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{531441\sqrt{3}a^{49/6}\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}+\frac{\sqrt[3]{-1}\,\ldots}{53144\ldots}\right.$$

input
```
Int[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

output
```
19683*a^6*(-1/177147*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3
]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(Sqrt[3]*(1 + (-1)^(1
/3))^2*a^(49/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTan
[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)
])]/(531441*Sqrt[3]*a^(49/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1
)^(1/3)*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqr
t[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(531441*Sqrt[3]*a^(49/6)*Sqrt[4*b
 + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)*x
+ b*x^2]/(3188646*a^(25/3)*c^(2/3)) + ((-1)^(2/3)*Log[3*a - 3*(-1)^(1/3)*a
^(2/3)*c^(1/3)*x + b*x^2])/(1062882*(1 + (-1)^(1/3))^2*a^(25/3)*c^(2/3)) -
 ((-1)^(2/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(3188646*a
^(25/3)*c^(2/3)))
```

3.139. $\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

3.139.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.139.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.19

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{3}\_Z^{6}+9b^{2}a\_Z^{4}+27c\,a^{2}\_Z^{3}+27a^{2}b\_Z^{2}+27a^{3}\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^{5}b^{3}+12\_R^{3}a\,b^{2}+27\_R^{2}a^{2}c+18a^{2}b\_R}\right)}{3}$	91
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{3}\_Z^{6}+9b^{2}a\_Z^{4}+27c\,a^{2}\_Z^{3}+27a^{2}b\_Z^{2}+27a^{3}\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^{5}b^{3}+12\_R^{3}a\,b^{2}+27\_R^{2}a^{2}c+18a^{2}b\_R}\right)}{3}$	91

input
```
int(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_RET
URNVERBOSE)
```

output
```
1/3*sum(_R/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_
R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

3.139.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algo
rithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

3.139.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \text{Timed out}$$

input
```
integrate(x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3
),x)
```

output
```
Timed out
```

3.139.7 Maxima [F]

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \int \frac{x}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx$$

input
```
integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algo
rithm="maxima")
```

output
```
integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
, x)
```

3.139.8 Giac [F]

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \int \frac{x}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx$$

input
```
integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algo
rithm="giac")
```

output
```
integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
, x)
```

3.139.9 Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 1057, normalized size of antiderivative = 2.25

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \sum_{k=1}^{6}\ln\Big(b^{12}\,x$$

$$+\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$

$$+\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$

$$-\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$

$$+\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2\,z^3-14348907\,a^8\,c^4\,z^3+177147\,a^5\,b^2\,c^2\,z^2+b^3,z,k\big)\;a^2\,b^{13}\,x\,54$$

$$+\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$

$$+\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$

$$-\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$

$$+\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$

$$+\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$

$$-\mathrm{root}\big(18075490334784\,a^{14}\,b^3\,c^4\,z^6-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2$$

$$-7625597484987\,a^{15}\,c^6\,z^6+1162261467\,a^{10}\,b\,c^4\,z^4+8503056\,a^7\,b^3\,c^2\,z^3-14348907\,a^8\,c^4\,z^3+177147\,a^5\,b^2\,c^2\,z^2+b^3,z,k\big)$$

input
```
int(x/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

output
```
symsum(log(b^12*x + 1033121304*root(18075490334784*a^14*b^3*c^4*z^6 - 7625
597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z
^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^4*a^10*b^1
1*c^3 + 167365651248*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*
a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348
907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^5*a^12*b^12*c^3 - 94
143178827*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^
6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4
*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^5*a^13*b^9*c^5 + 54*root(180754
90334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b
*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2
*c^2*z^2 + b^3, z, k)*a^2*b^13*x + 177147*root(18075490334784*a^14*b^3*c^4
*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^
7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)
^2*a^5*b^11*c^2*x + 17006112*root(18075490334784*a^14*b^3*c^4*z^6 - 762559
7484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3
 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^3*a^7*b^12*c
^2*x - 14348907*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*
c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a
^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^3*a^8*b^9*c^4*x + 2295...
```

3.139. $\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

3.140 $\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

3.140.1 Optimal result

Integrand size = 42, antiderivative size = 522

$$\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$
$$= -\frac{\sqrt[3]{-1}\left(2\sqrt[3]{-1}b+3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{17/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}c^{2/3}}$$
$$-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}a^{17/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}c^{2/3}}$$
$$-\frac{\left(2(-1)^{2/3}b-3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{17/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}c^{2/3}}$$
$$+\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}-\frac{\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{8/3}\sqrt[3]{c}}$$
$$-\frac{\sqrt[3]{-1}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}$$

output
```
1/162*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(8/3)/c^(1/3)-1/54*ln(3*a-3*(-1)
^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(8/3)/c^(1/3)-1/162*(-1
)^(1/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(8/3)/c^(1/3)-1/243
*(2*b-3*a^(1/3)*c^(2/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1
/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2))/a^(17/6)/c^(2/3)*3^(1/2)/(4*b-3*a^(1/3)
*c^(2/3))^(1/2)-1/81*(2*(-1)^(2/3)*b-3*a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)
^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^
(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(17/6)/c^(2/3)*3^(1/2)/(4*
b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)-1/81*(-1)^(1/3)*(2*(-1)^(1/3)*b+3*a^
(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(
1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(17/6)/c
^(2/3)*3^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

3.140.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.19

$$\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4 + b^3\#1^6\&,\frac{\log(x-\#1)}{18a^2b\#1+27a^2c\#1^2+12ab^2\#1^3+2b^3\#1^5}\&\right]$$

input
```
Integrate[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(
-1),x]
```

output
```
RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
 , Log[x - #1]/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12*a*b^2*#1^3 + 2*b^3*#1^5)
& ]/3
```

3.140.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 507, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx$$

$$\Big\downarrow \text{2466}$$

$$19683a^6 \int \left(-\frac{\sqrt[3]{a}\left(b - 3\sqrt[3]{a}c^{2/3}\right) - b\sqrt[3]{c}x}{1594323a^{26/3}c^{2/3}\left(bx^2 + 3a^{2/3}\sqrt[3]{c}x + 3a\right)} + \frac{\sqrt[3]{-1}\sqrt[3]{a}\left(\sqrt[3]{-1}b + 3\sqrt[3]{a}c^{2/3}\right) - b\sqrt[3]{c}x}{531441\left(1 + \sqrt[3]{-1}\right)^2 a^{26/3}c^{2/3}\left(bx^2 - 3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + \right.}$$

$$\Big\downarrow \text{2009}$$

$$19683a^6 \left(-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}c^{2/3} + 2\sqrt[3]{-1}b\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c} - 2bx}{\sqrt{3}\sqrt{a}\sqrt{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{531441\sqrt{3}\left(1 + \sqrt[3]{-1}\right)^2 a^{53/6}c^{2/3}\sqrt{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}} - \frac{\left(2b - 3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c} +}{\sqrt{3}\sqrt{a}\sqrt{4b - 3}}\right.}{1594323\sqrt{3}a^{53/6}c^{2/3}\sqrt{4b - 3\sqrt[3]{a}}}\right.$$

input
```
Int[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-1),x]
```

output
```
19683*a^6*(-1/531441*((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcT
an[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-
1)^(2/3)*a^(1/3)*c^(2/3)])])/(Sqrt[3]*(1 + (-1)^(1/3))^2*a^(53/6)*Sqrt[4*b
 - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*Arc
Tan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2
/3)])])/(1594323*Sqrt[3]*a^(53/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) -
 ((2*(-1)^(2/3)*b - 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3
) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(1
594323*Sqrt[3]*a^(53/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3))
+ Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(3188646*a^(26/3)*c^(1/3)) - Log[
3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(1062882*(1 + (-1)^(1/3))^2*
a^(26/3)*c^(1/3)) - ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x +
 b*x^2])/(3188646*a^(26/3)*c^(1/3)))
```

3.140.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.140.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.17

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27c\,a^2\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	90
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27c\,a^2\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	90

input
```
int(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_RET
URNVERBOSE)
```

output
```
1/3*sum(1/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R
=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

3.140.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algo
rithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

3.140.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\, dx = \text{Timed out}$$

input
```
integrate(1/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3
),x)
```

output
```
Timed out
```

3.140.7 Maxima [F]

$$\int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\, dx = \int \frac{1}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\, dx$$

input
```
integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algo
rithm="maxima")
```

output
```
integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
, x)
```

3.140.8 Giac [F]

$$\int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \int \frac{1}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

input
```
integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algo
rithm="giac")
```

output
```
integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
, x)
```

3.140.9 Mupad [B] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 1394, normalized size of antiderivative = 2.67

$$\int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \text{Too large to display}$$

input
```
int(1/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

output
```
symsum(log(6561*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^
18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 38
7420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z
, k)^2*a^4*b^12*c^2 - 6*root(488038239039168*a^17*b^3*c^4*z^6 - 2058911320
94649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*
z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z
+ b^6, z, k)*b^15*x - 4782969*root(488038239039168*a^17*b^3*c^4*z^6 - 2058
91132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^
3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^
5*c*z + b^6, z, k)^3*a^7*b^11*c^3 - 229582512*root(488038239039168*a^17*b^
3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 -
746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z
^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^9*b^13*c^2 - 387420489*root(4880382
39039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^1
2*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 26572
05*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^10*b^10*c^4 + 16736
5651248*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z
^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*
a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a
^12*b^12*c^3 - 94143178827*root(488038239039168*a^17*b^3*c^4*z^6 - 2058...
```

3.140. $\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

3.141 $\int \frac{1}{x(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx$

3.141.1 Optimal result

Integrand size = 46, antiderivative size = 563

$$\begin{aligned}
&\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx \\
&= \frac{\left(b-(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} \\
&+ \frac{\left(b-\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{19/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} \\
&+ \frac{(-1)^{2/3}\left((-1)^{2/3}b-\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} \\
&+ \frac{\log(x)}{27a^3} - \frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{486a^{10/3}} \\
&- \frac{\left(b+i\sqrt{3}b+6\sqrt[3]{a}c^{2/3}\right)\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{972a^{10/3}c^{2/3}} \\
&- \frac{\left(3\sqrt[3]{a}-\frac{(-1)^{2/3}b}{c^{2/3}}\right)\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{486a^{10/3}}
\end{aligned}$$

output
```
1/27*ln(x)/a^3-1/486*(3*a^(1/3)-b/c^(2/3))*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^
2)/a^(10/3)-1/486*(3*a^(1/3)-(-1)^(2/3)*b/c^(2/3))*ln(3*a+3*(-1)^(2/3)*a^(
2/3)*c^(1/3)*x+b*x^2)/a^(10/3)-1/972*ln(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x
+b*x^2)*(b+6*a^(1/3)*c^(2/3)+I*b*3^(1/2))/a^(10/3)/c^(2/3)+1/81*(b-a^(1/3)
*c^(2/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1
/3)*c^(2/3))^(1/2))/a^(19/6)/c^(1/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)
+1/27*(-1)^(2/3)*((-1)^(2/3)*b-a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(2/3)*a
^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(
1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(19/6)/c^(1/3)*3^(1/2)/(4*b+3*(-1)
^(1/3)*a^(1/3)*c^(2/3))^(1/2)+1/27*(b-(-1)^(2/3)*a^(1/3)*c^(2/3))*arctan(1
/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(2/3)*
a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(19/6)/c^(1/3)*3^(1/2)/(4*b-3*(
-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

3.141.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.28

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx =$$
$$-\frac{-3\log(x)+\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\&,\frac{27a^2b\log(x-\#1)+27a^2c\log(x-\#}{18a^2b+27a}\right.}{81a^3}$$

input
```
Integrate[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x
^6)),x]
```

output
```
-1/81*(-3*Log[x] + RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^
2*#1^4 + b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*
a*b^2*Log[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b + 27*a^2*c*#1 + 1
2*a*b^2*#1^2 + 2*b^3*#1^4) & ])/a^3
```

3.141.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 548, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}dx$$

$$\Big\downarrow \text{2466}$$

$$19683a^6\int\left(\frac{3a^{2/3}\sqrt[3]{c}\left(2b-3\sqrt[3]{a}c^{2/3}\right)+b\left(b-3\sqrt[3]{a}c^{2/3}\right)x}{4782969a^{28/3}c^{2/3}\left(bx^2+3a^{2/3}\sqrt[3]{c}x+3a\right)}+\frac{1}{531441a^9x}-\frac{3a^{2/3}\sqrt[3]{c}\left(2b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)+\sqrt[3]{-}}{1594323\left(1+\sqrt[3]{-1}\right)^2a^{28/3}c^{2/3}\left(bx^2-\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$19683a^6\left(\frac{\left(b-(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{177147\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{55/6}\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}+\frac{\left(b-\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{531441\sqrt{3}a^{55/6}\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right.$$

input
```
Int[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x
]
```

output
```
19683*a^6*(((b - (-1)^(2/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*
c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]
)])/(177147*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(55/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^
(1/3)*c^(2/3)]*c^(1/3)) + ((b - a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3)
 + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(531441*Sqrt[3
]*a^(55/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(2/3)*((-1)^(2/3
)*b - a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt
[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(531441*Sqrt[3]*a^
(55/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[x]/(531441*
a^9) - ((3*a^(1/3) - b/c^(2/3))*Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2])/(9
565938*a^(28/3)) - ((b + I*Sqrt[3]*b + 6*a^(1/3)*c^(2/3))*Log[3*a - 3*(-1)
^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(19131876*a^(28/3)*c^(2/3)) - ((3*a^(1/
3) - ((-1)^(2/3)*b)/c^(2/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*
x^2])/(9565938*a^(28/3)))
```

3.141.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.141.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.24

method	result
default	$\frac{\ln(x)}{27a^3} - \frac{\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27c\,a^2\_Z^3+27a^2b\_Z^2+27a^3\right)} \frac{\left(\_R^5b^3+9\_R^3ab^2+27\_R^2a^2c+27a^2b\_R\right)\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3ab^2+27\_R^2a^2c+18a^2b\_R}}{81a^3}$
risch	$\frac{\ln(-x)}{27a^3} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(27a^{21}c^6-64a^{20}b^3c^4\right)\_Z^6+\left(243a^{18}c^6-576a^{17}b^3c^4\right)\_Z^5+\left(729a^{15}c^6-1755a^{14}c^4b^3\right)\_Z^4+\left(729a^{12}c^6-1917a^{11}c^4\right.\right.}\right.}{}$

input
```
int(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_R
ETURNVERBOSE)
```

output
```
1/27*ln(x)/a^3-1/81/a^3*sum((_R^5*b^3+9*_R^3*a*b^2+27*_R^2*a^2*c+27*_R*a^2
*b)/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R=RootO
f(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

3.141.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

3.141.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

output
```
Timed out
```

3.141.7 Maxima [F]

$$\int \frac{1}{x(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx = \int \frac{1}{(b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3)x}\,dx$$

input
```
integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="maxima")
```

output
```
-1/27*integrate((b^3*x^5 + 9*a*b^2*x^3 + 27*a^2*c*x^2 + 27*a^2*b*x)/(b^3*x
^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)/a^3 + 1/27*lo
g(x)/a^3
```

3.141.8 Giac [F]

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \int \frac{1}{(b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3)x}\,dx$$

input
```
integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="giac")
```

output
```
integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
)*x), x)
```

3.141.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 4002, normalized size of antiderivative = 7.11

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3)),x
)
```

output
```
log(x)/(27*a^3) + symsum(log(7*root(13177032454057536*a^20*b^3*c^4*z^6 - 5
559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 20589113
2094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15
*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 104
60353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^
2 - 6561*a^4*b^6*c^2*z + b^9, z, k)*b^18*x - 162*root(13177032454057536*a^
20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*
c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 -
2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10
*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097
379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^2*a^3*b^18*x + 86093
442*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^
6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119
306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11
*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 143
48907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^
9, z, k)^3*a^8*b^13*c^3 + 34867844010*root(13177032454057536*a^20*b^3*c^4*
z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 2
05891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 25418658283
29*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2...
```

3.141. $\int \frac{1}{x(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx$

3.142 $\int \frac{1}{x^2(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx$

3.142.1 Optimal result

Integrand size = 46, antiderivative size = 645

$$
\begin{aligned}
&\int \frac{1}{x^2\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx \\
&= -\frac{1}{27a^3x}+\frac{\left(2(-1)^{2/3}b^2+12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3}+9a^{2/3}c^{4/3}\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{23/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\,c^{2/3}} \\
&\quad+\frac{\left(2b^2-12\sqrt[3]{a}bc^{2/3}+9a^{2/3}c^{4/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{243\sqrt{3}a^{23/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}\,c^{2/3}} \\
&\quad+\frac{(-1)^{2/3}\left(2b^2+12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3}+9(-1)^{2/3}a^{2/3}c^{4/3}\right)\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{23/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}\,c^{2/3}} \\
&\quad-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{486a^{11/3}\sqrt[3]{c}} \\
&\quad+\frac{\left(2b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162\left(1+\sqrt[3]{-1}\right)^2a^{11/3}\sqrt[3]{c}} \\
&\quad+\frac{\sqrt[3]{-1}\left(2b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{486a^{11/3}\sqrt[3]{c}}
\end{aligned}
$$

output
```
-1/27/a^3/x-1/486*(2*b-3*a^(1/3)*c^(2/3))*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2
)/a^(11/3)/c^(1/3)+1/162*(2*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))*ln(3*a-3*(-1)^
(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(11/3)/c^(1/3)+1/486*(-1
)^(1/3)*(2*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(
1/3)*x+b*x^2)/a^(11/3)/c^(1/3)+1/729*(2*b^2-12*a^(1/3)*b*c^(2/3)+9*a^(2/3)
*c^(4/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1
/3)*c^(2/3))^(1/2))/a^(23/6)/c^(2/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)
+1/243*(-1)^(2/3)*(2*b^2+12*(-1)^(1/3)*a^(1/3)*b*c^(2/3)+9*(-1)^(2/3)*a^(2
/3)*c^(4/3))*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/
2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))
^2/a^(23/6)/c^(2/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)+1/243
*(2*(-1)^(2/3)*b^2+12*(-1)^(1/3)*a^(1/3)*b*c^(2/3)+9*a^(2/3)*c^(4/3))*arct
an(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(2
/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(23/6)/c^(2/3)*3^(1/2)/(4*b
-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

3.142.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^2\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx =$$
$$-\frac{3+x\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\&,\frac{27a^2b\log(x-\#1)+27a^2c\log(x-\#1)\#1+9\cdots}{18a^2b\#1+27a^2c\#1^2\cdots}\right.}{81a^3x}$$

input
```
Integrate[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3
*x^6)),x]
```

output
```
-1/81*(3 + x*RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4
 + b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*a*b^2*
Log[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12
*a*b^2*#1^3 + 2*b^3*#1^5) & ])/(a^3*x)
```

3.142.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 625, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} \, dx$$

$$\Big\downarrow \text{2466}$$

$$19683a^6 \int \left(\frac{\sqrt[3]{a}\left(b^2 - 9\sqrt[3]{a}c^{2/3}b + 9a^{2/3}c^{4/3}\right) - b\left(2b - 3\sqrt[3]{a}c^{2/3}\right)\sqrt[3]{c}x}{4782969a^{29/3}c^{2/3}\left(bx^2 + 3a^{2/3}\sqrt[3]{c}x + 3a\right)} - \frac{\sqrt[3]{a}\left((-1)^{2/3}b^2 + 9\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}b + 9a^{2/3}c\right.}{1594323\left(1 + \sqrt[3]{-1}\right)^2 a^{29/3}c^{2/3}\left(b\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$19683a^6 \left(\frac{\left(9a^{2/3}c^{4/3} + 12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3} + 2(-1)^{2/3}b^2\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c} - 2bx}{\sqrt{3}\sqrt{a}\sqrt{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{1594323\sqrt{3}\left(1 + \sqrt[3]{-1}\right)^2 a^{59/6}c^{2/3}\sqrt{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}} + \frac{\left(9a^{2/3}c^{4/3} - 12\sqrt[3]{a}bc\right.}{4782969\sqrt{}}\right.$$

input
```
Int[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6))
,x]
```

output
```
19683*a^6*(-1/531441*1/(a^9*x) + ((2*(-1)^(2/3)*b^2 + 12*(-1)^(1/3)*a^(1/3
)*b*c^(2/3) + 9*a^(2/3)*c^(4/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*
b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(1594323
*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(59/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/
3)]*c^(2/3)) + ((2*b^2 - 12*a^(1/3)*b*c^(2/3) + 9*a^(2/3)*c^(4/3))*ArcTan[
(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]
)])/(4782969*Sqrt[3]*a^(59/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + ((2
*(-1)^(2/3)*b^2 - 12*a^(1/3)*b*c^(2/3) - 9*(-1)^(1/3)*a^(2/3)*c^(4/3))*Arc
Tan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(
-1)^(1/3)*a^(1/3)*c^(2/3)])])/(4782969*Sqrt[3]*a^(59/6)*Sqrt[4*b + 3*(-1)^
(1/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*Log[3*a + 3*a
^(2/3)*c^(1/3)*x + b*x^2])/(9565938*a^(29/3)*c^(1/3)) + ((2*b - 3*(-1)^(2/
3)*a^(1/3)*c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(31
88646*(1 + (-1)^(1/3))^2*a^(29/3)*c^(1/3)) + ((-1)^(1/3)*(2*b + 3*(-1)^(1/
3)*a^(1/3)*c^(2/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(95
65938*a^(29/3)*c^(1/3)))
```

3.142.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.142.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.21

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27c\,a^2\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\left(-\_R^4b^3-9\_R^2a\,b^2-27\_R\,a^2c-27a^2b\right)\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}}{81a^3}-\frac{1}{27a^3x}$
risch	$-\frac{1}{27a^3x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^{24}c^6-1728a^{23}b^3c^4\right)\_Z^6+\left(13122a^{17}b\,c^6-31347a^{16}b^4c^4\right)\_Z^4+\left(-19683c^7a^{14}+52488c^5b^3a^{13}-14472c^3b^6a^{1}\right.\right.}\right.}{}$

input
```
int(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=
_RETURNVERBOSE)
```

output
```
1/81/a^3*sum((-_R^4*b^3-9*_R^2*a*b^2-27*_R*a^2*c-27*a^2*b)/(2*_R^5*b^3+12*
_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R=RootOf(_Z^6*b^3+9*_Z^4*a
*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))-1/27/a^3/x
```

3.142.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,
algorithm="fricas")
```

output
```
Timed out
```

3.142.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27
*a**3),x)
```

output
```
Timed out
```

3.142.7 Maxima [F]

$$\int \frac{1}{x^2\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \int \frac{1}{(b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3)x^2}\,dx$$

input
```
integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,
algorithm="maxima")
```

output
```
-1/27*integrate((b^3*x^4 + 9*a*b^2*x^2 + 27*a^2*c*x + 27*a^2*b)/(b^3*x^6 +
 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)/a^3 - 1/27/(a^3*x
)
```

3.142.8 Giac [F]

$$\int \frac{1}{x^2\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \int \frac{1}{(b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3)x^2}\,dx$$

input
```
integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,
algorithm="giac")
```

output
```
integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
)*x^2), x)
```

3.142.9 Mupad [B] (verification not implemented)

Time = 9.68 (sec) , antiderivative size = 2663, normalized size of antiderivative = 4.13

$$\int \frac{1}{x^2 (27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx = \text{Too large to display}$$

input
```
int(1/(x^2*(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3))
,x)
```

output
```
symsum(log(-282429536481*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094
635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 10930023061814
7*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6
*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 1004423
49*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^23*b^9*(2*b^10*x +
 2541865828329*root(355779876259553472*a^23*b^3*c^4*z^6 - 1500946352969991
21*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4
*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 +
 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8
*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^4*a^17*c^5 - 45*a*b^8*c + 3874
20489*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c
^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4
- 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 28242953
6481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2
+ 17496*a^4*b^10*c*z + b^12, z, k)^2*a^10*c^6*x - 401769396*root(355779876
259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909
922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*
b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 +
258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z
 + b^12, z, k)^2*a^9*b^4*c^3 - 2066242608*root(355779876259553472*a^23*...
```

3.143 $\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6} \, dx$

3.143.1 Optimal result

Integrand size = 26, antiderivative size = 395

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6} \, dx$$

$$= -\frac{\sqrt[3]{-2}\left(1+\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}$$

$$+\frac{\sqrt[6]{\frac{3}{2}}\left(1-(-3)^{2/3}\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$

$$-\frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$

$$+\frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)+\frac{1}{108}\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x\right.$$

output
```
1/108*(18-(-2)^(2/3)*3^(1/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/108*(18-2
^(2/3)*3^(1/3))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/216*ln(6-3*(-3)^(1/3)*2^(2
/3)*x+x^2)*(36+2^(2/3)*3^(1/3)*(1+I*3^(1/2)))+1/2*3^(1/6)*2^(5/6)*(1-(-3)^
(2/3)*2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^
(1/3))^(1/2))/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/6*(1-2^(1/
3)*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^
(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)-1/3*(-2)^(1/3)*(1+(-2)
^(1/3)*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3
))^(1/2))*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(1/2)
```

3.143.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.15

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)\#1^4}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

input
```
Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^4)/
(36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

3.143.3 Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

$$\Big\downarrow \ 2466$$

$$1259712\int\left(\frac{(-1)^{2/3}\left(\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)x+3\sqrt[3]{-3}2^{2/3}\right)}{3779136\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{(-1)^{2/3}\left(3(-2)^{2/3}\sqrt[3]{3}-\left(1+3\sqrt[3]{-2}3^{2/3}\right)x\right)}{11337408\sqrt[3]{2}3^{2/3}\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}\right.$$

$\big\downarrow$ 2009

$$1259712\left(\frac{(-1)^{2/3}\left((-1)^{2/3}-\sqrt[3]{2}3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1259712\sqrt[6]{2}3^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{\left(2(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{\cdots}\right.}{\sqrt{3\left(4-3(-3)^{2/3}\cdots\right.}}\right.}{419904\ 6^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\cdots}}\right.$$

input
```
Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
1259712*(((-1)^(2/3)*((-1)^(2/3) - 2^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3
^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(1259712*2^(1/6)*3^(5/6
)*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) - ((2*(-3)^(2/3) - 2^(2/3))*ArcTan[(2^(1
/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(4199
04*6^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 - 2^(1
/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1
/3)*3^(2/3))]])/(1259712*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + (
(36 + 2^(2/3)*3^(1/3) + I*2^(2/3)*3^(5/6))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x
+ x^2])/272097792 + ((18 - (-2)^(2/3)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3
)*x + x^2])/136048896 + ((18 - 2^(2/3)*3^(1/3))*Log[6 + 3*2^(2/3)*3^(1/3)*
x + x^2])/136048896)
```

3.143.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.143. $\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx$

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.143.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R^{5}\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	56
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R^{5}\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	56

input
```
int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(_R^5/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4
+324*_Z^3+108*_Z^2+216))
```

3.143.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx=\text{Timed out}$$

input
```
integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
Timed out
```

3.143.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.18

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \mathrm{RootSum}\Big(72662865048t^6 - 72662865048t^5 + 24163559388t^4 - 2646786132t^3 - 6626610t^2 - 4374t - 1,$$

input
```
integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(72662865048*_t**6 - 72662865048*_t**5 + 24163559388*_t**4 - 264678
6132*_t**3 - 6626610*_t**2 - 4374*_t - 1, Lambda(_t, _t*log(-8923641713104
7376*_t**5/833243797 + 8930194953299812 8*_t**4/833243797 - 297405602818058
52*_t**3/833243797 + 192466080408420*_t**2/49014341 + 5867255361684*_t/833
243797 + x + 5365044886/2499731391)))
```

3.143.7 Maxima [F]

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x^5}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.143.8 Giac [F]

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x^5}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.143.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$=\sum_{k=1}^{6}\ln\left(\frac{362797056\left(19236852\,x\,\mathrm{root}(z^6+4374\,z^5+6626610\,z^4+2646786132\,z^3-24163559388\,z^2+72\right.}{}\right.$$
$$\left.-z^5+\frac{421\,z^4}{1266}-\frac{100853\,z^3}{2768742}-\frac{505\,z^2}{5537484}-\frac{z}{16612452}-\frac{1}{72662865048},z,k\right)$$

input
```
int(x^5/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

output
```
symsum(log((362797056*(19236852*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646
786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k) - 191318
76*x - 6482268*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 2416
3559388*z^2 + 72662865048*z - 72662865048, z, k)^2 + 742851*x*root(z^6 + 4
374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z -
 72662865048, z, k)^3 - 4130*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786
132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^4 + x*root(
z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865
048*z - 72662865048, z, k)^5 - 154944576*root(z^6 + 4374*z^5 + 6626610*z^4
 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^2
 + 17047422*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 241635593
88*z^2 + 72662865048*z - 72662865048, z, k)^3 + 27054*root(z^6 + 4374*z^5
+ 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 7266286
5048, z, k)^4 + 9*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 241
63559388*z^2 + 72662865048*z - 72662865048, z, k)^5 + 465542316*root(z^6 +
 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z
 - 72662865048, z, k) - 465542316))/root(z^6 + 4374*z^5 + 6626610*z^4 + 26
46786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^5)*roo
t(z^6 - z^5 + (421*z^4)/1266 - (100853*z^3)/2768742 - (505*z^2)/5537484 -
z/16612452 - 1/72662865048, z, k), k, 1, 6)
```

3.144 $\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx$

3.144.1 Optimal result

Integrand size = 26, antiderivative size = 377

$$\begin{aligned}
&\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx \\
&= \frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{9\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} \\
&\quad+\frac{\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{27\sqrt{3\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)}} \\
&\quad-\frac{\left(9-2^{2/3}\sqrt[3]{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{27\sqrt{6\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}+\frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{6\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2} \\
&\quad+\frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{18\ 2^{2/3}}-\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{18\ 2^{2/3}\sqrt[3]{3}}
\end{aligned}$$

output
```
1/36*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(2/3)/(1+(-1)^(1/3))^2+1/1
08*(-1)^(1/3)*3^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(1/3)-1/108*ln(6+
3*2^(2/3)*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)+1/27*(-1)^(2/3)*(3*(-3)^(2/3)-2^(
2/3))*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*
3^(5/6)/(1+(-1)^(1/3))^2/(8-6*(-3)^(2/3)*2^(1/3))^(1/2)-1/27*(9-2^(2/3)*3^
(1/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2)
)/(-24+18*2^(1/3)*3^(2/3))^(1/2)+1/27*(9-(-2)^(2/3)*3^(1/3))*arctan((3*(-2
)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(24+27*I*2^(1/3)*3^
(1/6)+9*2^(1/3)*3^(2/3))^(1/2)
```

3.144.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.16

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)\#1^3}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

input
```
Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^3)/
(36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

3.144.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1259712 \int \left(\frac{(-1)^{2/3} \left(2-\sqrt[3]{-3}2^{2/3}x\right)}{7558272\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)} - \frac{(-1)^{2/3}\left((-2)^{2/3}\sqrt[3]{3}x+2\right)}{22674816\sqrt[3]{2}3^{2/3}\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)} - \right.$$

$$\Big\downarrow \text{2009}$$

$$1259712 \left(-\frac{\left(2(-1)^{2/3}\sqrt[6]{2}3^{5/6}-9\sqrt{6}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{204073344\sqrt{4+3\sqrt[3]{-2}3^{2/3}}} + \frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{\right.}}{\sqrt{3\left(4-3\right.}}\right)}{11337408\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-\right.}} \right.$$

input
```
Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
1259712*(-1/204073344*((2*(-1)^(2/3)*2^(1/6)*3^(5/6) - 9*Sqrt[6])*ArcTan[(
3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/Sqrt[4 +
3*(-2)^(1/3)*3^(2/3)] + ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(2^(1/
6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(11337
408*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) - ((9 -
 2^(2/3)*3^(1/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3
*2^(1/3)*3^(2/3))]])/(34012224*Sqrt[6*(-4 + 3*2^(1/3)*3^(2/3))]) + Log[6 -
 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(7558272*2^(2/3)*3^(1/3)*(1 + (-1)^(1/3))^2
) + ((-1/3)^(1/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(22674816*2^(2/3)
) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(22674816*2^(2/3)*3^(1/3)))
```

3.144.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.144.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.15

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	56
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	56

input
```
int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(_R^4/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4
+324*_Z^3+108*_Z^2+216))
```

3.144.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
Timed out
```

3.144.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.17

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \mathrm{RootSum}\left(15695178850368t^6 - 2066242608t^4 + 1845163152t^3 - 1180980t^2 - 1944t - 1, \left(t \mapsto t\log\left(\frac{61}{}\right.\right.\right.$$

input
```
integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(15695178850368*_t**6 - 2066242608*_t**4 + 1845163152*_t**3 - 11809
80*_t**2 - 1944*_t - 1, Lambda(_t, _t*log(614714526178551746208*_t**5/5712
1295165 - 1270857362386176*_t**4/57121295165 - 80483053187684376*_t**3/571
21295165 + 72431318325103884*_t**2/57121295165 - 45358602689088*_t/5712129
5165 + x - 44532180783/57121295165)))
```

3.144.7 Maxima [F]

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx=\int \frac{x^4}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.144.8 Giac [F]

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx=\int \frac{x^4}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.144.9 Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.03

$$\int \frac{x^4}{216 + 108x^2 + 324x^3 + 18x^4 + x^6}\,dx$$
$$= \sum_{k=1}^{6} \ln\left(-\frac{5038848\,\big(1377495072\,x + 17006112\,x\,\mathrm{root}(z^6 + 1944\,z^5 + 1180980\,z^4 - 1845163152\,z^3 + 2066\right.$$
$$\left. - \frac{z^4}{7596} + \frac{217\,z^3}{1845828} - \frac{5\,z^2}{66449808} - \frac{z}{8073651672} - \frac{1}{15695178850368}, z, k\right)$$

input
```
int(x^4/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

output
```
symsum(log(-(5038848*(1377495072*x + 17006112*x*root(z^6 + 1944*z^5 + 1180
980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k) - 104976
*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 1
5695178850368, z, k)^2 + 158112*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845
163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^3 + 1946*x*root(z^6 +
1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368,
 z, k)^4 + 3*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 206624
2608*z^2 - 15695178850368, z, k)^5 - 4251528*root(z^6 + 1944*z^5 + 1180980
*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^2 + 3927852
*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 156
95178850368, z, k)^3 - 1188*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152
*z^3 + 2066242608*z^2 - 15695178850368, z, k)^4 - root(z^6 + 1944*z^5 + 11
80980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^5 + 75
58272*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2
- 15695178850368, z, k) + 33519046752))/root(z^6 + 1944*z^5 + 1180980*z^4
- 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^5)*root(z^6 - z^
4/7596 + (217*z^3)/1845828 - (5*z^2)/66449808 - z/8073651672 - 1/156951788
50368, z, k), k, 1, 6)
```

3.145 $\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx$

3.145.1 Optimal result

Integrand size = 26, antiderivative size = 361

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx = -\frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{6\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{\sqrt[3]{-1}\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{9\,2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{18\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}} - \frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{36\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} + \frac{(-1)^{2/3}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{108\sqrt[3]{2}3^{2/3}} + \frac{\log\left(6+3\,2^{2/3}\sqrt[3]{3}x+x^2\right)}{108\sqrt[3]{2}3^{2/3}}$$

output
```
-1/216*(-1)^(2/3)*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)
^(1/3))^2+1/648*(-1)^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3
)+1/648*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/36*arctan((3*(-3)^
(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(1/6)/(1+(-
1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)+1/108*arctanh(2^(1/6)*(3*3^(1/3
)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*
3^(2/3))^(1/2)+1/54*(-1)^(1/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-
2)^(1/3)*3^(2/3))^(1/2))*2^(1/3)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*
3^(2/3))^(1/2)
```

3.145.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.17

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)\#1^2}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

input
```
Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^2)/
(36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

3.145.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1259712\int\left(-\frac{(-1)^{2/3}x}{22674816\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{(-1)^{2/3}x}{68024448\sqrt[3]{2}3^{2/3}\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}\right.$$

$\big\downarrow$ 2009

$$1259712\left(\frac{\sqrt[3]{-1}\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{22674816\sqrt[6]{2}3^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{7558272\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\text{arctanh}\left(\frac{\sqrt[6]{2}}{\sqrt{3}}\right.}{22674816\sqrt[6]{2}3^{5}}\right.$$

input
```
Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
1259712*(((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2
)^(1/3)*3^(2/3))]])/(22674816*2^(1/6)*3^(5/6)*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3
)]) - ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)
*2^(1/3))]]/(7558272*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2
/3)*2^(1/3)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2
^(1/3)*3^(2/3))]]/(22674816*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])
- ((-1)^(2/3)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(45349632*2^(1/3)*3^(
2/3)*(1 + (-1)^(1/3))^2) + ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^
2])/(136048896*2^(1/3)*3^(2/3)) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(1360
48896*2^(1/3)*3^(2/3)))
```

3.145.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.145.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.16

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	56
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	56

input
```
int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(_R^3/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4
+324*_Z^3+108*_Z^2+216))
```

3.145.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx=\text{Timed out}$$

input
```
integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
Timed out
```

3.145.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.17

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$=\operatorname{RootSum}\left(3390158631679488t^6-74384733888t^4-1332145440t^3-1417176t^2-1,\left(t\mapsto t\log\left(-\frac{84823}{\ldots}\right.\right.\right.$$

input
```
integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(3390158631679488*_t**6 - 74384733888*_t**4 - 1332145440*_t**3 - 14
17176*_t**2 - 1, Lambda(_t, _t*log(-8482372214243328*_t**5/415817 + 221605
5910930560*_t**4/415817 - 2062546612992*_t**3/415817 - 57027208896*_t**2/4
15817 - 416583756*_t/415817 + x - 89938/415817)))
```

3.145.7 Maxima [F]

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x^3}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.145.8 Giac [F]

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x^3}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.145.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \sum_{k=1}^{6} \ln\left(-\frac{23328\left(297538935552\,x - 7992872640\,x\,\mathrm{root}(z^6+1417176\,z^4+1332145440\,z^3+74384733888\right.}{}\right.$$
$$\left. -\frac{z^4}{45576}-\frac{235\,z^3}{598048272}-\frac{z^2}{2392193088}-\frac{1}{3390158631679488}, z, k\right)$$

input
```
int(x^3/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

output
```
symsum(log(-(23328*(297538935552*x - 7992872640*x*root(z^6 + 1417176*z^4 +
 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k) + 52488*x*root
(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488,
z, k)^3 + 2904*x*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2
 - 3390158631679488, z, k)^4 + x*root(z^6 + 1417176*z^4 + 1332145440*z^3 +
 74384733888*z^2 - 3390158631679488, z, k)^5 - 153055008*root(z^6 + 141717
6*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k)^2 - 276
4368*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 339015863
1679488, z, k)^3 - 1620*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733
888*z^2 - 3390158631679488, z, k)^4 - 3673320192*root(z^6 + 1417176*z^4 +
1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k) + 7240114098432
))/root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 33901586316
79488, z, k)^5)*root(z^6 - z^4/45576 - (235*z^3)/598048272 - z^2/239219308
8 - 1/3390158631679488, z, k), k, 1, 6)
```

3.145. $\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx$

3.146 $\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx$

3.146.1 Optimal result

Integrand size = 26, antiderivative size = 248

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{(-1)^{2/3}\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\,2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{(-1)^{2/3}\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{81\sqrt[3]{2}\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{81\,2^{5/6}\sqrt[6]{3}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$

output
```
1/162*(-1)^(2/3)*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/
3))^(1/2))*2^(1/6)*3^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)
-1/486*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2)
)*2^(1/6)*3^(5/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)+1/486*(-1)^(2/3)*arctan((3*
(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(2/3)*3^(5/6)/
(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(1/2)
```

3.146.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)\#1}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

input
```
Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1)/(3
6 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

3.146.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1259712\int \left(\frac{(-1)^{2/3}}{68024448\sqrt[3]{2}3^{2/3}\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}+\frac{1}{68024448\sqrt[3]{2}3^{2/3}\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}-\frac{}{22674816\sqrt[3]{}}\right.$$

$$\Big\downarrow \text{2009}$$

$$1259712\left(\frac{(-1)^{2/3}\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{102036672\ 2^{5/6}\sqrt[6]{3}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}+\frac{(-1)^{2/3}\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{34012224\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\text{arctanh}}{102036672}\right.$$

input
```
Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
1259712*(((-1)^(2/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2
)^(1/3)*3^(2/3))]])/(102036672*2^(5/6)*3^(1/6)*Sqrt[4 + 3*(-2)^(1/3)*3^(2/
3)]) + ((-1)^(2/3)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 -
 3*(-3)^(2/3)*2^(1/3))]])/(34012224*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^2*Sqr
t[4 - 3*(-3)^(2/3)*2^(1/3)]) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/S
qrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(102036672*2^(5/6)*3^(1/6)*Sqrt[-4 + 3*2^
(1/3)*3^(2/3)]))
```

3.146.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.146.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.23

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	56
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	56

input
```
int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(_R^2/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4
+324*_Z^3+108*_Z^2+216))
```

3.146.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1277 vs. 2(162) = 324.

Time = 0.93 (sec) , antiderivative size = 1277, normalized size of antiderivative = 5.15

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
1/324*sqrt(1/633)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(1/211*sqrt(1/633)
*(3*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 248
67)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) +
81)^2 + 2*x + 729/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/324*sqrt
(1/633)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(-1/211*sqrt(1/633)*(3*(6*18
^(2/3) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 24867)*sqrt(
6*18^(2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 2
*x + 729/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/136728*sqrt(1266)
*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^
(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(
1/3) + 81)^2 + 18*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3
) + 48*18^(1/3) + 371)*(6*18^(2/3) + 8*18^(1/3) + 81) + 1/211*(6*sqrt(1266
)*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3)
 + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*
18^(1/3) + 81) - 211*sqrt(1266)) - 1247*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3
) + 81) + 51273*sqrt(1266))*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) +
8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18)
 + 3376*x - 2916*18^(2/3) - 3888*18^(1/3) - 16578) + 1/136728*sqrt(1266)*s
qrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2
/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^...
```

3.146.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.19

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \text{RootSum}\left(732274264442769408t^6 - 2677850419968t^4 + 2834352t^2 - 1, \left(t \mapsto t\log\left(1017047589503846\right.\right.\right.$$

input
```
integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(732274264442769408*_t**6 - 2677850419968*_t**4 + 2834352*_t**2 - 1
, Lambda(_t, _t*log(10170475895038464*_t**5 - 5231726283456*_t**4 - 318099
32496*_t**3 + 19131876*_t**2 + 19683*_t + x - 27/2)))
```

3.146.7 Maxima [F]

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x^2}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.146.8 Giac [F]

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x^2}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.146.9 Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \sum_{k=1}^{6} \ln\left(-\frac{216\left(32134205039616\,x-1836660096\,\mathrm{root}(z^6-2834352\,z^4+2677850419968\,z^2-7322742644\right.}{}\right.$$
$$\left. -\frac{z^4}{273456}+\frac{z^2}{258356853504}-\frac{1}{732274264442769408},z,k\right)$$

input
```
int(x^2/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

output

```
symsum(log(-(216*(32134205039616*x - 1836660096*root(z^6 - 2834352*z^4 + 2
677850419968*z^2 - 732274264442769408, z, k)^2 - 1889568*root(z^6 - 283435
2*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^3 + 972*root(z^6 - 2
834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^4 + root(z^6 -
2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^5 + 1322395269
12*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k
) + 204073344*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 7322742644427
69408, z, k)^2 + 139968*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732
274264442769408, z, k)^3 + 36*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2
 - 732274264442769408, z, k)^4 + 863230245120*root(z^6 - 2834352*z^4 + 267
7850419968*z^2 - 732274264442769408, z, k) + 781932322630656))/root(z^6 -
2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^5)*root(z^6 -
z^4/273456 + z^2/258356853504 - 1/732274264442769408, z, k), k, 1, 6)
```

3.146. $\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6} \, dx$

3.147 $\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx$

3.147.1 Optimal result

Integrand size = 24, antiderivative size = 361

$$\begin{aligned}\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx = &-\frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{36\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}\\
&+\frac{\sqrt[3]{-1}\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{54\ 2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}\\
&+\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{108\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}\\
&+\frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{216\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}\\
&-\frac{(-1)^{2/3}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\sqrt[3]{2}3^{2/3}}\\
&-\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{648\sqrt[3]{2}3^{2/3}}\end{aligned}$$

output
```
1/1296*(-1)^(2/3)*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)
^(1/3))^2-1/3888*(-1)^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/
3)-1/3888*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/216*arctan((3*(-
3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(1/6)/(1
+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)+1/648*arctanh(2^(1/6)*(3*3^(
1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/
3)*3^(2/3))^(1/2)+1/324*(-1)^(1/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+1
8*(-2)^(1/3)*3^(2/3))^(1/2))*2^(1/3)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1
/3)*3^(2/3))^(1/2)
```

3.147.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.16

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

input
```
Integrate[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36 + 1
62*#1 + 12*#1^2 + #1^4) & ]/6
```

3.147.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1259712\int \left(-\frac{(-1)^{2/3}\left(3\sqrt[3]{-3}2^{2/3}-x\right)}{136048896\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}-\frac{(-1)^{2/3}\left(x+3(-2)^{2/3}\sqrt[3]{3}\right)}{408146688\sqrt[3]{2}3^{2/3}\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$1259712\left(\frac{\sqrt[3]{-1}\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{136048896\sqrt[6]{2}3^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{45349632\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\text{arctanh}\left(\right.}{136048896\sqrt[6]{2}}\right.$$

input
```
Int[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
1259712*(((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2
)^(1/3)*3^(2/3))]])/(136048896*2^(1/6)*3^(5/6)*Sqrt[4 + 3*(-2)^(1/3)*3^(2/
3)]) - ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3
)*2^(1/3))]]/(45349632*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^
(2/3)*2^(1/3)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3
*2^(1/3)*3^(2/3))]]/(136048896*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)
]) + ((-1)^(2/3)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(272097792*2^(1/3)
*3^(2/3)*(1 + (-1)^(1/3))^2) - ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x
+ x^2])/(816293376*2^(1/3)*3^(2/3)) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(
816293376*2^(1/3)*3^(2/3)))
```

3.147.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.147.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.15

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	54
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	54

input
```
int(x/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(_R/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+3
24*_Z^3+108*_Z^2+216))
```

3.147.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx=\text{Timed out}$$

input
```
integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
Timed out
```

3.147.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.17

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$=\mathrm{RootSum}\left(158171241119638192128t^6-964026151188848t^4+287743415040t^3-51018336t^2-1,\left(t\mapsto t\right.\right.$$

input
```
integrate(x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(15817124111963819212 8*_t**6 - 96402615118848*_t**4 + 287743415040*
_t**3 - 51018336*_t**2 - 1, Lambda(_t, _t*log(65418399445721140961280*_t**
5/415817 + 2480926457425102848*_t**4/415817 - 39451802929737984*_t**3/4158
17 + 118071997444800*_t**2/415817 - 16745884920*_t/415817 + x - 268790/415
817)))
```

3.147.7 Maxima [F]

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx=\int \frac{x}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.147.8 Giac [F]

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx=\int \frac{x}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.147.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.49

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx = \sum_{k=1}^{6} \ln\left(x+\operatorname{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{1581712411119638192128},z,k\right)\left(216\,x+\operatorname{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{1581712411119638192128},z,k\right)\left(51018336\,x-\right.\right.\right.$$
$$-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{1581712411119638192128},z,k\Big)$$

input
```
int(x/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

output
```
symsum(log(x + root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/31002
82242048 - 1/1581712411119638192128, z, k)*(216*x + root(z^6 - z^4/1640736
+ (235*z^3)/129178426752 - z^2/3100282242048 - 1/1581712411119638192128, z,
 k)*(51018336*x - root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/31
00282242048 - 1/1581712411119638192128, z, k)*(277947894528*x - root(z^6 -
z^4/1640736 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/1581712411196
38192128, z, k)*(33192121254912*x - root(z^6 - z^4/1640736 + (235*z^3)/129
178426752 - z^2/3100282242048 - 1/1581712411119638192128, z, k)*(6940988288
557056*x + 168897381688221696) + 28563737812992)))))*root(z^6 - z^4/164073
6 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/1581712411119638192128,
z, k), k, 1, 6)
```

3.148 $\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx$

3.148.1 Optimal result

Integrand size = 22, antiderivative size = 377

$$
\begin{aligned}
&\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx \\
&= \frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{324\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} \\
&\quad+\frac{\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{972\sqrt{3\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)}} \\
&\quad-\frac{\left(9-2^{2/3}\sqrt[3]{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{972\sqrt{6\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}-\frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{216\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2} \\
&\quad-\frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\ 2^{2/3}}+\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{648\ 2^{2/3}\sqrt[3]{3}}
\end{aligned}
$$

output
```
-1/1296*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(2/3)/(1+(-1)^(1/3))^2-
1/3888*(-1)^(1/3)*3^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(1/3)+1/3888*
ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)+1/972*(-1)^(2/3)*(3*(-3)^(2/
3)-2^(2/3))*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(
1/2))*3^(5/6)/(1+(-1)^(1/3))^2/(8-6*(-3)^(2/3)*2^(1/3))^(1/2)-1/972*(9-2^(
2/3)*3^(1/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3)
)^(1/2))/(-24+18*2^(1/3)*3^(2/3))^(1/2)+1/972*(9-(-2)^(2/3)*3^(1/3))*arcta
n((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(24+27*I*2^
(1/3)*3^(1/6)+9*2^(1/3)*3^(2/3))^(1/2)
```

3.148.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.16

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)}{36\#1+162\#1^2+12\#1^3+\#1^5}\&\right]$$

input
```
Integrate[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```

output
```
RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36*#1
+ 162*#1^2 + 12*#1^3 + #1^5) & ]/6
```

3.148.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6+18x^4+324x^3+108x^2+216}\,dx$$
$$\Big\downarrow \; 2466$$

$$1259712 \int \left(\frac{\left(-\frac{1}{3}\right)^{2/3} \left(\sqrt[3]{-6}x + 2^{2/3}\left(1 - 3(-3)^{2/3}\sqrt[3]{2}\right)\right)}{272097792 \left(1 + \sqrt[3]{-1}\right)^2 \left(x^2 - 3\sqrt[3]{-3}2^{2/3}x + 6\right)} - \frac{\sqrt[3]{-6}x + 2^{2/3}\left((-1)^{2/3} - 3\sqrt[3]{2}3^{2/3}\right)}{816293376\ 3^{2/3}\left(x^2 + 3(-2)^{2/3}\sqrt[3]{3}x + 6\right)} + \frac{}{24488\cdots}\right.$$

$\downarrow$ 2009

$$1259712 \left(\frac{\left(9 - (-2)^{2/3}\sqrt[3]{3}\right) \arctan\left(\frac{2x + 3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4 + 3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1224440064\sqrt{6\left(4 + 3\sqrt[3]{-2}3^{2/3}\right)}} + \frac{(-1)^{2/3}\left(3(-3)^{2/3} - 2^{2/3}\right) \arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3} - \sqrt[3]{2}x\right)}{\sqrt{3\left(4 - 3(-3)^{2/3}\sqrt[3]{2}\cdots\right.}}\right.}{408146688\sqrt[6]{3}\left(1 + \sqrt[3]{-1}\right)^2\sqrt{2\left(4 - 3(-3)^{2/3}\sqrt[3]{2}\right)\cdots}}\right.$$

input
```
Int[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```

output
```
1259712*(((9 - (-2)^(2/3)*3^(1/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqr
t[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(1224440064*Sqrt[6*(4 + 3*(-2)^(1/3)*3^(
2/3))]) + ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/
3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(408146688*3^(1/6)*(
1 + (-1)^(1/3))^2*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) - ((9 - 2^(2/3)*3^(1
/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2
/3))]])/(1224440064*Sqrt[6*(-4 + 3*2^(1/3)*3^(2/3))]) - Log[6 - 3*(-3)^(1/
3)*2^(2/3)*x + x^2]/(272097792*2^(2/3)*3^(1/3)*(1 + (-1)^(1/3))^2) - ((-1/
3)^(1/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(816293376*2^(2/3)) + Log[
6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(816293376*2^(2/3)*3^(1/3)))
```

3.148.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.148. $\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx$

3.148.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	53
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	53

input
```
int(1/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(1/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+32
4*_Z^3+108*_Z^2+216))
```

3.148.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = \text{Timed out}$$

input
```
integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
Timed out
```

3.148.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.17

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \text{RootSum}\Big(34164988081841849499648t^6 - 3470494144278528t^4 - 86087932019712t^3 - 1530550080t^2 +$$

input
```
integrate(1/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(341649880818418494996648*_t**6 - 3470494144278528*_t**4 - 860879320
19712*_t**3 - 1530550080*_t**2 + 69984*_t - 1, Lambda(_t, _t*log(185904446
699109611410573787136*_t**5/57121295165 + 637730125326791738276659 2*_t**4/
57121295165 - 18904636002388564311552*_t**3/57121295165 - 4690805529151817
23968*_t**2/57121295165 - 24358640509989936*_t/57121295165 + x + 152427895
956/57121295165)))
```

3.148.7 Maxima [F]

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{1}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.148.8 Giac [F]

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{1}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.148.9 Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.81

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = \sum_{k=1}^{6}\ln\left(-\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{341649880818418494996 48},z,k\right)x\,6\right.$$
$$+\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{3416498808184 18\ldots}\right.$$
$$-\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{341649880818418\ldots}\right.$$
$$-\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{341649880818418\ldots}\right.$$
$$-\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{341649880818418\ldots}\right.$$
$$+944784\,\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{\ldots}{341649880\ldots}\right.$$
$$-16529940864\,\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{\ldots}{3416\ldots}\right.$$
$$-33192121254912\,\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\ldots\right.$$
$$-168897381688221696\,\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{48818284296184627\ldots}\right.$$
$$-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}$$
$$\left.-\frac{1}{34164988081841849499648},z,k\right)$$

input
```
int(1/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

output
```
symsum(log(349920*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)
/111610160713728 + z/488182842961846272 - 1/341649880818418494996648, z, k)
^2*x - 6*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161016
0713728 + z/488182842961846272 - 1/341649880818418494996648, z, k)*x - 6122
200320*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/1116101607
13728 + z/488182842961846272 - 1/341649880818418494996648, z, k)^3*x - 2582
63796059136*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161
0160713728 + z/488182842961846272 - 1/341649880818418494996648, z, k)^4*x -
 6940988288557056*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)
/111610160713728 + z/488182842961846272 - 1/341649880818418494996648, z, k)
^5*x + 944784*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111
610160713728 + z/488182842961846272 - 1/341649880818418494996648, z, k)^2 -
 16529940864*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/1116
10160713728 + z/488182842961846272 - 1/341649880818418494996648, z, k)^3 -
33192121254912*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11
1610160713728 + z/488182842961846272 - 1/341649880818418494996648, z, k)^4
- 168897381688221696*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z
^2)/111610160713728 + z/488182842961846272 - 1/341649880818418494996648, z,
 k)^5)*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/1116101607
13728 + z/488182842961846272 - 1/341649880818418494996648, z, k), k, 1, ...
```

3.148. $\int \frac{1}{216+108x^2+324x^3+18x^4+x^6} \, dx$

3.149 $\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx$

3.149.1 Optimal result

Integrand size = 26, antiderivative size = 415

$$\begin{aligned}
\int \frac{1}{x\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx
&= \frac{(-1)^{2/3}\left((-2)^{2/3}-2\ 3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{216\sqrt[3]{2}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} \\
&\quad - \frac{(-1)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{216\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} \\
&\quad - \frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{216\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}} + \frac{\log(x)}{216} \\
&\quad - \frac{\left(36+2^{2/3}\sqrt[3]{3}(1+i\sqrt{3})\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{46656} \\
&\quad - \frac{\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{23328} \\
&\quad - \frac{\left(18-2^{2/3}\sqrt[3]{3}\right)\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{23328}
\end{aligned}$$

output
```
1/216*ln(x)-1/23328*(18-(-2)^(2/3)*3^(1/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^
2)-1/23328*(18-2^(2/3)*3^(1/3))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)-1/46656*ln(6
-3*(-3)^(1/3)*2^(2/3)*x+x^2)*(36+2^(2/3)*3^(1/3)*(1+I*3^(1/2)))-1/1296*(-1
)^(2/3)*((-3)^(1/3)+3*2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12
-9*(-3)^(2/3)*2^(1/3))^(1/2))*6^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(
1/3))^(1/2)-1/1296*(1-2^(1/3)*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*
x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(
1/2)+1/1296*(-1)^(2/3)*((-2)^(2/3)-2*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)
+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(2/3)*3^(1/6)/(8+9*I*2^(1/3)*3^(
1/6)+3*2^(1/3)*3^(2/3))^(1/2)
```

3.149.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.25

$$\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx = \frac{\log(x)}{216} - \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{108\log(x-\#1)+324\log(x-\#1)\#1+18\log(x-\#1)\#1^2+\log}{36+162\#1+12\#1^2+\#1^4}\right.}{1296}$$

input
```
Integrate[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

output
```
Log[x]/216 - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*L
og[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/
(36 + 162*#1 + 12*#1^2 + #1^4) & ]/1296
```

3.149.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x^6+18x^4+324x^3+108x^2+216)}\,dx$$

$$\Big\downarrow \; 2466$$

$$1259712\int\left(\frac{(-1)^{2/3}\left(6\left(9+\sqrt[3]{-3}2^{2/3}\right)-\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)x\right)}{816293376\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{1}{272097792x}-\frac{(-1)^{2/3}\left(6\left(9-(-2)^{2/3}\sqrt[3]{3}\right.\right.}{2448880128\sqrt[3]{2}3^{2/3}\left(x^2\right.}\right.$$

$$\Big\downarrow \; 2009$$

$$1259712\left(\frac{(-1)^{2/3}\left((-2)^{2/3}-2\ 3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{272097792\ 6^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{(-1)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}\right.}{\sqrt{3\left(4-3(-3)\right.}}\right.}{272097792\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)}}\right.$$

input
```
Int[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

output
```
1259712*(((-1)^(2/3)*((-2)^(2/3) - 2*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3)
 + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(272097792*6^(5/6)*Sqrt[4 + 3
*(-2)^(1/3)*3^(2/3)]) - ((-1)^(2/3)*((-3)^(1/3) + 3*2^(1/3))*ArcTan[(2^(1/
6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(27209
7792*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 - 2^
(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^
(1/3)*3^(2/3))]])/(272097792*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])
 + Log[x]/272097792 - ((36 + 2^(2/3)*3^(1/3) + I*2^(2/3)*3^(5/6))*Log[6 -
3*(-3)^(1/3)*2^(2/3)*x + x^2])/58773123072 - ((18 - (-2)^(2/3)*3^(1/3))*Lo
g[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/29386561536 - ((18 - 2^(2/3)*3^(1/3))
*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/29386561536)
```

3.149. $\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx$

3.149.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.149.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.18

method	result
risch	$\frac{\ln(x)}{216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(136728\_Z^6+1230552\_Z^5+3682908\_Z^4+3630708\_Z^3-81810\_Z^2+486\_Z-1\right)}\_R\ln\left(-23672342955240\_R^5-\right.\right.}{}$
default	$\frac{\ln(x)}{216}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(\_R^5+18\_R^3+324\_R^2+108\_R\right)\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{1296}$

input
```
int(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/216*ln(x)+1/1944*sum(_R*ln(-23672342955240*_R^5-213056277916248*_R^4-637
689647288592*_R^3-628763677061560*_R^2+14004611129596*_R+2499731391*x-5513
3083786),_R=RootOf(136728*_Z^6+1230552*_Z^5+3682908*_Z^4+3630708*_Z^3-8181
0*_Z^2+486*_Z-1))
```

3.149.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
Timed out
```

3.149.6 Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.20

$$\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx = \frac{\log(x)}{216} + \text{RootSum}\Big(737963742567783949192396 8t^6 + 34164988081841849499648t^5 + 525988092506853703 68t^4$$

input
```
integrate(1/x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
log(x)/216 + RootSum(7379637425677839491923968*_t**6 + 3416498808184184949
9648*_t**5 + 52598809250685370368*_t**4 + 26673506015311872*_t**3 - 309171
116160*_t**2 + 944784*_t - 1, Lambda(_t, _t*log(814557009966881793678336 21
1511929736056012 8*_t**6/143425799309052440063 + 97706876677080638108735825
7564745728*_t**5/143425799309052440063 - 116529526608851264288400971539061
538816*_t**4/143425799309052440063 - 23935979498524220254250144071076659 2*
_t**3/143425799309052440063 - 13667831263813709443988734141824 0*_t**2/1434
25799309052440063 + 156311556906766379573541369 6*_t/143425799309052440063
+ x - 316444631507523619004 4/143425799309052440063))
```

3.149.7 Maxima [F]

$$\int \frac{1}{x\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx = \int \frac{1}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)x}\,dx$$

input
```
integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
-1/216*integrate((x^5 + 18*x^3 + 324*x^2 + 108*x)/(x^6 + 18*x^4 + 324*x^3
+ 108*x^2 + 216), x) + 1/216*log(x)
```

3.149.8 Giac [F]

$$\int \frac{1}{x\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx = \int \frac{1}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)x}\,dx$$

input
```
integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x), x)
```

3.149.9 Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.04

$$\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx = \frac{\ln(x)}{216}$$

$$+\left(\sum_{k=1}^{6} \ln\left(\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{7810925487389540352}\right.\right.\right.$$

$$-\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{7810925487389540352}-\frac{}{73}\right.$$

$$+\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{7810925487389540352}-\frac{}{73}\right.$$

$$-\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{7810925487389540352}-\frac{}{73}\right.$$

$$-\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{7810925487389540352}-\frac{}{73}\right.$$

$$-\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{7810925487389540352}-\frac{}{73}\right.$$

$$+839808\,\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{781092548738954035}\right.$$

$$+594896472576\,\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{781092548738}\right.$$

$$-8483430130458624\,\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{}{78109254}\right.$$

$$-3831425535283494912\,\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{}{7810}\right.$$

$$+1217393817906599165952\,\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{}{78}\right.$$

$$+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{7810925487389540352}$$

$$\left.\left.-\frac{1}{737963742567783949192396 8},z,k\right)\right)\right)$$

input
```
int(1/(x*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)
```

output
```
log(x)/216 + symsum(log(7*root(z^6 + z^5/216 + (421*z^4)/59066496 + (10085
3*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/781092548738954035
2 - 1/7379637425677839491923968, z, k)*x - 5670000*root(z^6 + z^5/216 + (4
21*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/120538973570826
24 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^2*x + 1546
875947520*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/279025401
78432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637425
677839491923968, z, k)^3*x - 106961147905609728*root(z^6 + z^5/216 + (421*
z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624
+ z/7810925487389540352 - 1/7379637425677839491923968, z, k)^4*x - 1405119
95854134018048*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/2790
2540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/73796
37425677839491923968, z, k)^5*x - 45607290567387619000320*root(z^6 + z^5/2
16 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897
357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^6*x
 + 839808*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/279025401
78432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637425
677839491923968, z, k)^2 + 594896472576*root(z^6 + z^5/216 + (421*z^4)/590
66496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810
925487389540352 - 1/7379637425677839491923968, z, k)^3 - 84834301304586...
```

3.149. $\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx$

3.150 $\int \frac{1}{x^2(216+108x^2+324x^3+18x^4+x^6)}\,dx$

3.150.1 Optimal result

Integrand size = 26, antiderivative size = 448

$$
\begin{aligned}
&\int \frac{1}{x^2\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx \\
&= -\frac{1}{216x} - \frac{\left(27\sqrt[3]{-6}-(-2)^{2/3}+12\ 3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{5832\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} \\
&\quad - \frac{(-1)^{2/3}\left(6(-6)^{2/3}+27\sqrt[3]{-3}-\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{1944\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} \\
&\quad - \frac{\left(\sqrt[3]{2}+27\sqrt[3]{3}-6\ 6^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{5832\sqrt[6]{6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}} \\
&\quad - \frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}2^{2/3}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} \\
&\quad + \frac{\left(3(-6)^{2/3}+2\sqrt[3]{-2}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{7776\sqrt[3]{3}} \\
&\quad - \frac{\left(2^{2/3}-3\ 3^{2/3}\right)\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{3888\sqrt[3]{6}}
\end{aligned}
$$

output
```
-1/216/x-1/7776*(-1)^(2/3)*(9+(-3)^(1/3)*2^(2/3))*ln(6-3*(-3)^(1/3)*2^(2/3
)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^(1/3))^2+1/23328*(3*(-6)^(2/3)+2*(-2)^(1/
3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*3^(2/3)-1/23328*(2^(2/3)-3*3^(2/3))*l
n(6+3*2^(2/3)*3^(1/3)*x+x^2)*6^(2/3)-1/11664*(-1)^(2/3)*(6*(-6)^(2/3)+27*(
-3)^(1/3)-2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3
)*2^(1/3))^(1/2))*6^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-
1/34992*(2^(1/3)+27*3^(1/3)-6*6^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*
x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*6^(5/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)-1/1
7496*(27*(-6)^(1/3)-(-2)^(2/3)+12*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*
x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1
/3)*3^(2/3))^(1/2)
```

3.150.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^2\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx = -\frac{1}{216x} - \frac{\text{RootSum}\Big[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{108\log(x-\#1)+324\log(x-\#1)\#1+18\log(x-\#1)\#1^2+\log}{36\#1+162\#1^2+12\#1^3+\#1^5}}{1296}$$

input
```
Integrate[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

output
```
-1/216*1/x - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*L
og[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/
(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/1296
```

3.150.3 Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(x^6+18x^4+324x^3+108x^2+216\right)}\,dx$$

$\Big\downarrow$ 2466

$$1259712\int\left(-\frac{(-1)^{2/3}\left(\left(9+\sqrt[3]{-3}2^{2/3}\right)x-27\sqrt[3]{-3}2^{2/3}-9(-3)^{2/3}\sqrt[3]{2}+1\right)}{816293376\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{(-1)^{2/3}\left(\left(9-(-2)^{2/3}\sqrt[3]{3}\right)x+9\right.}{2448880128\sqrt[3]{2}3^{2/3}\left(x^2\right.}\right.$$

$\Big\downarrow$ 2009

$$1259712\left(\frac{(-1)^{2/3}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{7346640384\ 2^{5/6}\sqrt[6]{3}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{(-1)^{2/3}\left(6(-6)^{2/3}+27\sqrt[3]{-3}\right.}{2448880128\sqrt[6]{6}\left(1\right.}\right.$$

input
```
Int[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

output
```
1259712*(-1/272097792*1/x + ((-1)^(2/3)*(2 + 27*(-2)^(2/3)*3^(1/3) + 12*(-
2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(
1/3)*3^(2/3))]])/(7346640384*2^(5/6)*3^(1/6)*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)
]) - ((-1)^(2/3)*(6*(-6)^(2/3) + 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/6)*
(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(24488801
28*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((2^(1/3)
+ 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3
*(-4 + 3*2^(1/3)*3^(2/3))]])/(7346640384*6^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/
3)]) - ((-1)^(2/3)*(9 + (-3)^(1/3)*2^(2/3))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x
 + x^2])/(1632586752*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2) - ((-1)^(2/3)*((-
2)^(2/3) - 3*3^(2/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(4897760256*6
^(1/3)) - ((2^(2/3) - 3*3^(2/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/(4897
760256*6^(1/3)))
```

3.150. $\int \frac{1}{x^2(216+108x^2+324x^3+18x^4+x^6)}\,dx$

3.150.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.150.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.15

method	result
risch	$-\frac{1}{216x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(633\_Z^6+204849\_Z^4-5446980\_Z^3-80433\_Z^2-72\_Z-1\right)}\_R\ln\left(-4620404398013514843 93\_R^5+136423\right.\right.}{}$
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-\_R^4-18\_R^2-324\_R-108\right)\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{1296}-\frac{1}{216x}$

input
```
int(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
-1/216/x+1/11664*sum(_R*ln(-462040439801351484393*_R^5+1364231865933925308
*_R^4-14952374096957448341 7612*_R^3+397631047190316263673 6042*_R^2+4696745
4543463546461111*_R+247008995694079835 90*x-25597852658707816584),_R=RootOf
(633*_Z^6+204849*_Z^4-5446980*_Z^3-80433*_Z^2-72*_Z-1))
```

3.150.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (216 + 108x^2 + 324x^3 + 18x^4 + x^6)} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
Timed out
```

3.150.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^2 (216 + 108x^2 + 324x^3 + 18x^4 + x^6)} \, dx$$
$$= \text{RootSum}\Big(1594001683946413330255577088t^6 + 3791612026460331638784t^4 - 8643672699589509120t^3$$
$$- \frac{1}{216x}$$

input
```
integrate(1/x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(1594001683946413330255577088*_t**6 + 37916120264603316387784*_t**4
- 8643672699589509120*_t**3 - 10942820851968*_t**2 - 839808*_t - 1, Lambda
(_t, _t*log(-49875532761902496003293561236914468028416*_t**5/1235044978470
3991795 + 12625489872431620388005975200497664*_t**4/12350449784703991795 -
 118637692607573771238550798852644864*_t**3/12350449784703991795 + 2704863
24927832147818193778754816*_t**2/12350449784703991795 + 273914194897479402
961199352*_t/12350449784703991795 + x - 12798926329353908292/1235044978470
3991795))) - 1/(216*x)
```

3.150.7 Maxima [F]

$$\int \frac{1}{x^2 (216 + 108x^2 + 324x^3 + 18x^4 + x^6)} \, dx = \int \frac{1}{(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216)x^2} \, dx$$

input
```
integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
-1/216/x - 1/216*integrate((x^4 + 18*x^2 + 324*x + 108)/(x^6 + 18*x^4 + 32
4*x^3 + 108*x^2 + 216), x)
```

3.150.8 Giac [F]

$$\int \frac{1}{x^2 (216 + 108x^2 + 324x^3 + 18x^4 + x^6)} \, dx = \int \frac{1}{(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216)x^2} \, dx$$

input
```
integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x^2), x)
```

3.150.9 Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2 (216 + 108x^2 + 324x^3 + 18x^4 + x^6)} \, dx$$

$$= \left(\sum_{k=1}^{6} \ln \left(\frac{5\,\mathrm{root}\left(z^6 + \frac{281\,z^4}{118132992} - \frac{50435\,z^3}{9300846726144} - \frac{331\,z^2}{48215589428330496} - \frac{z}{1898054893435658305536} - \frac{1}{1594001683946413330\cdots}\right.}{8} \right.\right.$$

$$- \frac{\mathrm{root}\left(z^6 + \frac{281\,z^4}{118132992} - \frac{50435\,z^3}{9300846726144} - \frac{331\,z^2}{48215589428330496} - \frac{z}{1898054893435658305536} - \frac{1}{1594001683946413330255577088}, \cdots\right.}{216}$$

$$-\mathrm{root}\left(z^6 + \frac{281\,z^4}{118132992} - \frac{50435\,z^3}{9300846726144} - \frac{331\,z^2}{48215589428330496} - \frac{z}{1898054893435658305536} - \frac{\cdots}{1594\cdots}\right.$$

$$-\mathrm{root}\left(z^6 + \frac{281\,z^4}{118132992} - \frac{50435\,z^3}{9300846726144} - \frac{331\,z^2}{48215589428330496} - \frac{z}{1898054893435658305536} - \frac{\cdots}{1594\cdots}\right.$$

$$+\mathrm{root}\left(z^6 + \frac{281\,z^4}{118132992} - \frac{50435\,z^3}{9300846726144} - \frac{331\,z^2}{48215589428330496} - \frac{z}{1898054893435658305536} - \frac{\cdots}{1594\cdots}\right.$$

$$-\mathrm{root}\left(z^6 + \frac{281\,z^4}{118132992} - \frac{50435\,z^3}{9300846726144} - \frac{331\,z^2}{48215589428330496} - \frac{z}{1898054893435658305536} - \frac{\cdots}{1594\cdots}\right.$$

$$+2344464\,\mathrm{root}\left(z^6 + \frac{281\,z^4}{118132992} - \frac{50435\,z^3}{9300846726144} - \frac{331\,z^2}{48215589428330496} - \frac{z}{1898054893435658305536}\cdots\right.$$

$$-210297580992\,\mathrm{root}\left(z^6 + \frac{281\,z^4}{118132992} - \frac{50435\,z^3}{9300846726144} - \frac{331\,z^2}{48215589428330496} - \frac{z}{189805489343565830\cdots}\right.$$

$$-10535082310656\,\mathrm{root}\left(z^6 + \frac{281\,z^4}{118132992} - \frac{50435\,z^3}{9300846726144} - \frac{331\,z^2}{48215589428330496} - \frac{z}{18980548934356\cdots}\right.$$

$$-168897381688221696\,\mathrm{root}\left(z^6 + \frac{281\,z^4}{118132992} - \frac{50435\,z^3}{9300846726144} - \frac{331\,z^2}{48215589428330496} - \frac{z}{18980548934\cdots}\right.$$

$$+ \frac{281\,z^4}{118132992} - \frac{50435\,z^3}{9300846726144} - \frac{331\,z^2}{48215589428330496} - \frac{z}{1898054893435658305536}$$

$$\left.\left.\left. - \frac{1}{1594001683946413330255577088}, z, k\right)\right)\right) - \frac{1}{216\,x}$$

input
```
int(1/(x^2*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)
```

output
```
symsum(log((5*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 -
 (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/15940016839464
13330255577088, z, k))/8 - (root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9
300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/
1594001683946413330255577088, z, k)*x)/216 - 396252*root(z^6 + (281*z^4)/1
18132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/189
8054893435658305536 - 1/1594001683946413330255577088, z, k)^2*x - 59822967
0528*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2
)/48215589428330496 - z/1898054893435658305536 - 1/15940016839464133302555
77088, z, k)^3*x + 82120746212352*root(z^6 + (281*z^4)/118132992 - (50435*
z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/189805489343565830553
6 - 1/1594001683946413330255577088, z, k)^4*x - 6940988288557056*root(z^6
+ (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/482155894283
30496 - z/1898054893435658305536 - 1/1594001683946413330255577088, z, k)^5
*x + 2344464*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 -
(331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/159400168394641
3330255577088, z, k)^2 - 210297580992*root(z^6 + (281*z^4)/118132992 - (50
435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/18980548934356583
05536 - 1/1594001683946413330255577088, z, k)^3 - 10535082310656*root(z^6
+ (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/482155894...
```

3.150. $\int \frac{1}{x^2(216+108x^2+324x^3+18x^4+x^6)}\,dx$

3.151 $\int \frac{x^8}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

3.151.1 Optimal result

Integrand size = 26, antiderivative size = 1064

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}2^{2/3}\right)+\left(2-2^{2/3}\left(6(-6)^{2/3}+27\sqrt[3]{-3}\right)\right)x\right)}{162\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$

$$-\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6-(-2)^{2/3}\sqrt[3]{3}\right)+\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)x\right)}{729\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{9\left(6-2^{2/3}\sqrt[3]{3}\right)+\left(2+2^{2/3}\left(27\sqrt[3]{3}-6\ 6^{2/3}\right)\right)x}{1458\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$-\frac{i\left((-2)^{2/3}+6\ 3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{162\ 2^{5/6}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$

$$-\frac{\sqrt[3]{-1}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{162\sqrt[6]{2}3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$

$$-\frac{\sqrt[3]{-1}\left(6(-6)^{2/3}+27\sqrt[3]{-3}-\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{81\sqrt{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}$$

$$+\frac{\left(i2^{2/3}-9\sqrt[6]{3}-3i3^{2/3}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{162\ 2^{5/6}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$

$$-\frac{\left(1+3\sqrt[3]{2}3^{2/3}\right)\text{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{1458\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$

$$+\frac{\left(\sqrt[3]{2}+27\sqrt[3]{3}-6\ 6^{2/3}\right)\text{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{81\sqrt{2}3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}-\frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{972\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^4}$$

output
```
-1/972*(-1)^(1/3)*3^(2/3)*(54+9*(-3)^(1/3)*2^(2/3)+(2-2^(2/3)*(6*(-6)^(2/3
)+27*(-3)^(1/3)))*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-
3*(-3)^(1/3)*2^(2/3)*x+x^2)-1/4374*(-1)^(1/3)*3^(2/3)*(54-9*(-2)^(2/3)*3^(
1/3)+(2+27*(-2)^(2/3)*3^(1/3)+12*(-2)^(1/3)*3^(2/3))*x)*2^(1/3)/(8+9*I*2^(
1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/8748*(54-
9*2^(2/3)*3^(1/3)+(2+2^(2/3)*(27*3^(1/3)-6*6^(2/3)))*x)*2^(1/3)*3^(2/3)/(4
-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)-1/972*(-1)^(1/3)*(2+27*(-2
)^(2/3)*3^(1/3)+12*(-2)^(1/3)*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(
24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^
(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3))^(3/2)-1/5832*ln(6-3*(-3)^(1/3)*2^(2/3)*x
+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^(1/3))^4+1/5832*I*ln(6+3*(-2)^(2/3)*3^(1/3)*
x+x^2)*2^(2/3)*3^(5/6)/(1+(-1)^(1/3))^5-1/52488*ln(6+3*2^(2/3)*3^(1/3)*x+x
^2)*2^(2/3)*3^(1/3)-1/486*(-1)^(1/3)*(6*(-6)^(2/3)+27*(-3)^(1/3)-2^(1/3))*
arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*3
^(1/6)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(3/2)*2^(1/2)+1/486*(2^(1
/3)+27*3^(1/3)-6*6^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(
1/3)*3^(2/3))^(1/2))*3^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2^(1/
3)*3^(2/3))^(3/2)*2^(1/2)+1/972*(I*2^(2/3)-9*3^(1/6)-3*I*3^(2/3))*arctan(2
^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/6)*3
^(2/3)/(1+(-1)^(1/3))^5/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/972*I*((-2)^(2...
```

3.151.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.16

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \frac{-7884+324x-3990x^2-11610x^3-203x^4-9x^5}{34182\,(216+108x^2+324x^3+18x^4+x^6)}$$
$$-\frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{324\log(x-\#1)-96\log(x-\#1)\#1+324\log(x-\#1)\#1^2+40\ldots}{36\#1+162\#1^2+12\#1^3+\ldots}\right.}{205092}$$

input
```
Integrate[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
(-7884 + 324*x - 3990*x^2 - 11610*x^3 - 203*x^4 - 9*x^5)/(34182*(216 + 108
*x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#
1^4 + #1^6 & , (324*Log[x - #1] - 96*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2
 + 406*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3
+ #1^5) & ]/205092
```

3.151.3 Rubi [A] (verified)

Time = 2.76 (sec) , antiderivative size = 1012, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(x^6+18x^4+324x^3+108x^2+216)^2}\,dx$$

$$\Big\downarrow \; 2466$$

$$1586874322944\int \left(-\frac{i(27-x)}{771220920950784\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}+\frac{27-}{6940988288557056\sqrt[3]{2}3^{2/}}\right.$$

$$\Big\downarrow \; 2009$$

$$1586874322944\left(-\frac{\sqrt[3]{-\frac{1}{3}}\left(\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)x+9\left(6-(-2)^{2/3}\sqrt[3]{3}\right)\right)}{2313662762852352\;2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}-\frac{\sqrt[3]{-1}\left(2+27(-2)^{2/3}\sqrt[3]{3}\right.}{23136627628}\right.$$

input
```
Int[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
1586874322944*(-1/257073640316928*((-1/3)^(1/3)*(9*(6 + (-3)^(1/3)*2^(2/3)
) + (2 - 3*2^(2/3)*(2*(-6)^(2/3) + 9*(-3)^(1/3)))*x))/(2^(2/3)*(1 + (-1)^(
1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - (
(-1/3)^(1/3)*(9*(6 - (-2)^(2/3)*3^(1/3)) + (2 + 27*(-2)^(2/3)*3^(1/3) + 12
*(-2)^(1/3)*3^(2/3))*x))/(231366276285235 2*2^(2/3)*(4 + 3*(-2)^(1/3)*3^(2/
3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (9*(6 - 2^(2/3)*3^(1/3)) + (2 +
2^(2/3)*(27*3^(1/3) - 6*6^(2/3)))*x)/(231366276285235 2*2^(2/3)*3^(1/3)*(4
- 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ((I/25707364031692
8)*((-2)^(2/3) + 6*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4
+ 3*(-2)^(1/3)*3^(2/3))]])/(2^(5/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*
(-2)^(1/3)*3^(2/3)]) - ((-1)^(1/3)*(2 + 27*(-2)^(2/3)*3^(1/3) + 12*(-2)^(1
/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*
3^(2/3))]])/(231366276285235 2*2^(1/6)*3^(5/6)*(4 + 3*(-2)^(1/3)*3^(2/3))^(
3/2)) - ((-1)^(1/3)*(6*(-6)^(2/3) + 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/
6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(12853
6820158464*Sqrt[2]*3^(5/6)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))^(
3/2)) + ((I*2^(2/3) - 9*3^(1/6) - (3*I)*3^(2/3))*ArcTan[(2^(1/6)*(3*(-3)^(
1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(257073640316928*2
^(5/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 +
3*2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4...
```

3.151.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.151. $\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

3.151.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.11

method	result
default	$\frac{-\frac{1}{3798}x^5-\frac{203}{34182}x^4-\frac{215}{633}x^3-\frac{665}{5697}x^2+\frac{2}{211}x-\frac{146}{633}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4-406\_R^3-324\_\right.}{\_R^5+12\_R^3}\right.}{205092}$
risch	$\frac{-\frac{1}{3798}x^5-\frac{203}{34182}x^4-\frac{215}{633}x^3-\frac{665}{5697}x^2+\frac{2}{211}x-\frac{146}{633}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4-406\_R^3-324\_\right.}{\_R^5+12\_R^3}\right.}{205092}$

input
```
int(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/3798*x^5-203/34182*x^4-215/633*x^3-665/5697*x^2+2/211*x-146/633)/(x^6+
18*x^4+324*x^3+108*x^2+216)+1/205092*sum((-9*_R^4-406*_R^3-324*_R^2+96*_R-
324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^
3+108*_Z^2+216))
```

3.151.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.151.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.11

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \text{RootSum}\Big(8525601705296418741512336066457 6t^6 + 501051915333854345687 04t^4 + 4888574805127748$$
$$+ \frac{-9x^5-203x^4-11610x^3-3990x^2+324x-7884}{34182x^6+615276x^4+11074968x^3+3691656x^2+7383312}$$

input
```
integrate(x**8/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

output
```
RootSum(85256017052964187415123360664576*_t**6 + 50105191533385434568704*_
t**4 + 4888574805127748601 6*_t**3 + 865447782603408*_t**2 + 3220532460*_t
+ 4513, Lambda(_t, _t*log(354920362040841744041191931354834874665907640 32*
_t**5/3569006970707929484758 45 - 19474160067218837086826809631017022308224
*_t**4/713801394141585896951 69 + 20779963076545132233894582764903396544*_t
**3/356900697070792948475845 + 20265219154367004972162198012037344*_t**2/3
569006970707929 48475845 + 27519246894921053204907514 5372*_t/35690069707079
2948475845 + x + 12902851912921772896220 12/10707020912123788454 27535))) +
(-9*x**5 - 203*x**4 - 11610*x**3 - 3990*x**2 + 324*x - 7884)/(34182*x**6 +
 615276*x**4 + 11074968*x**3 + 3691656*x**2 + 7383312)
```

3.151.7 Maxima [F]

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^8}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

output
```
-1/34182*(9*x^5 + 203*x^4 + 11610*x^3 + 3990*x^2 - 324*x + 7884)/(x^6 + 18
*x^4 + 324*x^3 + 108*x^2 + 216) - 1/34182*integrate((9*x^4 + 406*x^3 + 324
*x^2 - 96*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.151.8 Giac [F]

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^8}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

output
```
integrate(x^8/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.151.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.36

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

input
```
int(x^8/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

output
```
symsum(log((239491904*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14
149992416343982992 + (5171*z^2)/5093997269883833877l2 + (505*z)/1336868643
5083133627113728 + 4513/8525601705296418741512336O664576, z, k)*x)/8763068
43 - (275536*x)/638827688547 - (3848128*root(z^6 + (326*z^4)/554702231619
+ (8113597*z^3)/14149992416343982992 + (5171*z^2)/50939972698838338771 2 +
(505*z)/13368686435083133627113728 + 4513/85256017052964187415123360664576
, z, k))/3606201 - (152363520*root(z^6 + (326*z^4)/554702231619 + (8113597
*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/13
368686435083133627113728 + 4513/85256017052964187415123360664576, z, k)^2*
x)/44521 - (698075283456*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)
/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/1336868
6435083133627113728 + 4513/85256017052964187415123360664576, z, k)^3*x)/44
521 + (130789789876224*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/1
4149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/133686864
35083133627113728 + 4513/85256017052964187415123360664576, z, k)^4*x)/211
- 6940988288557056*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149
992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/1336868643508
3133627113728 + 4513/85256017052964187415123360664576, z, k)^5*x - (426422
0928*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/1414999241634398299
2 + (5171*z^2)/509399726988383387712 + (505*z)/1336868643508313362711137...
```

3.152 $\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2} \, dx$

3.152.1 Optimal result

Integrand size = 26, antiderivative size = 1005

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$

$$= -\frac{2\left(2\sqrt[3]{-1}3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)x}{972\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$

$$-\frac{\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)-9\left(1+\sqrt[3]{-2}3^{2/3}\right)x}{4374\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{2\left(2-3\sqrt[3]{2}3^{2/3}\right)-3\left(6-2^{2/3}\sqrt[3]{3}\right)x}{2916\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{\left(9i+\sqrt[3]{3}\left(2i2^{2/3}-9\sqrt[6]{3}+2\ 2^{2/3}\sqrt{3}\right)\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{5832\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}$$

$$+\frac{\left(1+\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{54\sqrt{6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$

$$+\frac{\left(9\sqrt[6]{3}+i\left(4\ 2^{2/3}-3\ 3^{2/3}\right)\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1944\ 3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}$$

$$-\frac{\sqrt[3]{-1}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{54\sqrt{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}$$

$$+\frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{54\sqrt{6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$

$$+\frac{\left(2\ 2^{2/3}+3\ 3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{26244\sqrt[6]{3}\sqrt{2\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}+\frac{i\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{648\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}$$

output
```
1/1944*(-4*(-1)^(1/3)*3^(2/3)-18*6^(1/3)+9*((-2)^(2/3)+2*(-1)^(1/3)*3^(2/3
))*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^
(2/3)*x+x^2)+1/4374*(-(-6)^(1/3)*(9*(-2)^(1/3)+2*3^(1/3))+9*(1+(-2)^(1/3)*
3^(2/3))*x)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1
/3)*x+x^2)+1/17496*(4-6*2^(1/3)*3^(2/3)-3*(6-2^(2/3)*3^(1/3))*x)*2^(1/3)*3
^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/3888*I*ln(6-3*(
-3)^(1/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5-1/104976*ln(6+3*
2^(2/3)*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)-1/324*(-1)^(1/3)*((-3)^(1/3)+3*2^(1
/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/
2))*3^(1/6)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(3/2)*2^(1/2)-1/7776
*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*(3^(1/2)+I)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3
))^5+1/324*(1+(-2)^(1/3)*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18
*(-2)^(1/3)*3^(2/3))^(1/2))/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1
/3)*3^(2/3))^(3/2)*6^(1/2)+1/324*(1-2^(1/3)*3^(2/3))*arctanh(2^(1/6)*(3*3^
(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))/(1-(-1)^(1/3))^2/(1+(-1)^(
1/3))^4/(-4+3*2^(1/3)*3^(2/3))^(3/2)*6^(1/2)+1/5832*arctan((3*(-3)^(1/3)*2
^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*(9*I+3^(1/3)*(2*I*2^(2/3)-9*
3^(1/6)+2*2^(2/3)*3^(1/2)))/(1+(-1)^(1/3))^5/(8-6*(-3)^(2/3)*2^(1/3))^(1/2
)+1/5832*(9*3^(1/6)+I*(4*2^(2/3)-3*3^(2/3)))*arctan((3*(-2)^(2/3)*3^(1/3)+
2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*3^(1/3)/(1+(-1)^(1/3))^5/(8+6*(-...
```

3.152.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.17

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \frac{648-96x+432x^2+908x^3-18x^4+73x^5}{68364\,(216+108x^2+324x^3+18x^4+x^6)} + \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{96\log(x-\#1)-216\log(x-\#1)\#1+96\log(x-\#1)\#1^2-36\,\mathrm{lo}\cdots}{36\#1+162\#1^2+12\#1^3+\cdots}\right.}{410184}$$

input
```
Integrate[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
(648 - 96*x + 432*x^2 + 908*x^3 - 18*x^4 + 73*x^5)/(68364*(216 + 108*x^2 +
 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 +
#1^6 & , (96*Log[x - #1] - 216*Log[x - #1]*#1 + 96*Log[x - #1]*#1^2 - 36*L
og[x - #1]*#1^3 + 73*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5)
 & ]/410184
```

3.152.3 Rubi [A] (verified)

Time = 2.74 (sec) , antiderivative size = 954, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(x^6+18x^4+324x^3+108x^2+216)^2}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1586874322944\int \left(\frac{3i\sqrt[3]{2}\sqrt[6]{3}x-i2^{2/3}3^{5/6}-9i\sqrt{3}-3\ 2^{2/3}\sqrt[3]{3}+27}{9254651051409408\left(1+\sqrt[3]{-1}\right)^5\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{2^{2/3}\left(27-9i\sqrt{3}+2i2^{2/3}3^{5/}\right.}{9254651051409408\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)}\right.$$

$$\Big\downarrow \text{2009}$$

$$1586874322944\left(\frac{2\left(2-3\sqrt[3]{2}3^{2/3}\right)-3\left(6-2^{2/3}\sqrt[3]{3}\right)x}{4627325525704704\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}-\frac{\left(9i-\sqrt[3]{3}\left(4i2^{2/3}+9\sqrt[6]{3}\right)\right)}{9254651051409408\left(1+\right.}\right.$$

input
```
Int[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
1586874322944*(-1/1542441841901568*(2*(2*(-1)^(1/3)*3^(2/3) + 9*6^(1/3)) -
 9*((-2)^(2/3) + 2*(-1)^(1/3)*3^(2/3))*x)/(2^(2/3)*(1 + (-1)^(1/3))^4*(4 -
 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3)*(
9*(-2)^(1/3) + 2*3^(1/3)) - 9*(1 + (-2)^(1/3)*3^(2/3))*x)/(13881976577114
12*(4 + 3*(-2)^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (2*(2
- 3*2^(1/3)*3^(2/3)) - 3*(6 - 2^(2/3)*3^(1/3))*x)/(4627325525704704*2^(2/3
)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((1 +
 (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2
)^(1/3)*3^(2/3))]])/(771220920950784*Sqrt[6]*(4 + 3*(-2)^(1/3)*3^(2/3))^(3
/2)) - ((9*I - 3^(1/3)*((4*I)*2^(2/3) + 9*3^(1/6)))*ArcTan[(3*(-2)^(2/3)*3
^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(9254651051409408*(1 +
(-1)^(1/3))^5*Sqrt[2*(4 + 3*(-2)^(1/3)*3^(2/3))]) - ((-1)^(1/3)*((-3)^(1/3
) + 3*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(
-3)^(2/3)*2^(1/3))]])/(85691213438976*Sqrt[2]*3^(5/6)*(1 + (-1)^(1/3))^4*(
4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) + ((9*I + 3^(1/3)*((2*I)*2^(2/3) - 9*3^(1
/6) + 2*2^(2/3)*Sqrt[3]))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt
[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(9254651051409408*(1 + (-1)^(1/3))^5*Sqrt
[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) + ((1 - 2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*
(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(7712209209507
84*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + ((2*2^(2/3) + 3*3^(2/3))*A...
```

3.152.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.152.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.12

method	result
default	$\frac{\frac{73}{68364}x^5-\frac{1}{3798}x^4+\frac{227}{17091}x^3+\frac{4}{633}x^2-\frac{8}{5697}x+\frac{2}{211}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(73\_R^4-36\_R^3+96\_R \cdots\right.}{\_R^5+12\_R^3 \cdots}\right.}{410184}$
risch	$\frac{\frac{73}{68364}x^5-\frac{1}{3798}x^4+\frac{227}{17091}x^3+\frac{4}{633}x^2-\frac{8}{5697}x+\frac{2}{211}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(73\_R^4-36\_R^3+96\_R \cdots\right.}{\_R^5+12\_R^3 \cdots}\right.}{410184}$

input
```
int(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

output
```
(73/68364*x^5-1/3798*x^4+227/17091*x^3+4/633*x^2-8/5697*x+2/211)/(x^6+18*x
^4+324*x^3+108*x^2+216)+1/410184*sum((73*_R^4-36*_R^3+96*_R^2-216*_R+96)/(
_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*
_Z^2+216))
```

3.152.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\text{Timed out}$$

input
```
integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.152.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.11

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \mathrm{RootSum}\Big(5892895898700884634133326689135493 12t^6 - 539640290266075248405737472t^4 + 92182638$$
$$+ \frac{73x^5-18x^4+908x^3+432x^2-96x+648}{68364x^6+1230552x^4+22149936x^3+7383312x^2+14766624}$$

input
```
integrate(x**7/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

output
```
RootSum(5892895898700884634133326689135493 12*_t**6 - 539640290266075248405
737472*_t**4 + 92182638168509682392064*_t**3 - 553241442069170496*_t**2 -
3759837842016*_t - 7197829, Lambda(_t, _t*log(429960276397274477140037433 0
51607461110184385010259993231 36*_t**5/15420600979105204449069438030323752 1
 - 425847662595081946846897154744222514051572098358476 80*_t**4/15420600979
105204449069438030323752 1 - 375124461288495881501083694493237540783173410 8
2112*_t**3/15420600979105204449069438030323752 1 + 715203759402167526763889
0715531672481920222144*_t**2/154206009791052044490694380303237521 - 442275
469988352977238302917949743105240 32*_t/15420600979105204449069438030323752
1 + x - 1745733490366760477341325695830248 55/15420600979105204449069438030
3237521))) + (73*x**5 - 18*x**4 + 908*x**3 + 432*x**2 - 96*x + 648)/(68364
*x**6 + 1230552*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)
```

3.152.7 Maxima [F]

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^7}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

output
```
1/68364*(73*x^5 - 18*x^4 + 908*x^3 + 432*x^2 - 96*x + 648)/(x^6 + 18*x^4 +
 324*x^3 + 108*x^2 + 216) + 1/68364*integrate((73*x^4 - 36*x^3 + 96*x^2 -
216*x + 96)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.152.8 Giac [F]

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\int \frac{x^7}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

output
```
integrate(x^7/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.152.9 Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.39

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\text{Too large to display}$$

input
```
int(x^7/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

output
```
symsum(log((8336932*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^
3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (19897
87*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668
913549312, z, k))/97367427 - (480227*x)/851770251396 - (759164282*root(z^6
 - (292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2
479189*z^2)/2640728184707779481899008 - (1989787*z)/3118647171576193412533
09046784 - 7197829/589289589870088463413332668913549312, z, k)*x)/78867615
87 - (207565888*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/7
5466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z
)/311864717157619341253309046784 - 7197829/5892895898700884634133326689135
49312, z, k)^2*x)/400689 - (108430970112*root(z^6 - (292589*z^4)/319508485
412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707
779481899008 - (1989787*z)/311864717157619341253309046784 - 7197829/589289
589870088463413332668913549312, z, k)^3*x)/44521 - (147138513610752*root(z
^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 -
(2479189*z^2)/2640728184707779481899008 - (1989787*z)/31186471715761934125
3309046784 - 7197829/589289589870088463413332668913549312, z, k)^4*x)/211
- 6940988288557056*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3
)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (198978
7*z)/311864717157619341253309046784 - 7197829/589289589870088463413332 6...
```

3.153 $\int \frac{x^6}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

3.153.1 Optimal result

Integrand size = 26, antiderivative size = 677

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$
$$+\frac{9\ 2^{2/3}+\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x}{13122\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$+\frac{3\ 2^{2/3}\sqrt[3]{3}-\left(2-3\sqrt[3]{2}3^{2/3}\right)x}{8748\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$+\frac{\sqrt[3]{-1}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{486\ 6^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}$$
$$+\frac{\left(3(-3)^{2/3}+\sqrt[3]{-1}2^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{486\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$
$$-\frac{\left(2^{2/3}-3\ 3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{486\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$
$$+\frac{\sqrt[6]{-\frac{1}{3}}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{5832\sqrt[3]{2}\left(1+\sqrt[3]{-1}\right)^5}$$
$$-\frac{i\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{5832\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}+\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{52488\sqrt[3]{2}3^{2/3}}$$

output
```
1/5832*(9*(-2)^(2/3)+6^(1/3)*(9+(-3)^(1/3)*2^(2/3))*x)*2^(1/3)/(1+(-1)^(1/
3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/26244*(9*2
^(2/3)+(-1)^(1/3)*3^(2/3)*(2+3*(-2)^(1/3)*3^(2/3))*x)*2^(1/3)/(8+9*I*2^(1/
3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/52488*(3*2^
(2/3)*3^(1/3)-(2-3*2^(1/3)*3^(2/3))*x)*2^(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3
))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/2916*(-1)^(1/3)*(3*(-3)^(2/3)-2^(2/3))*ar
ctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*6^(1/6)/
(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(3/2)+1/2916*(3*(-3)^(2/3)+(-1)^
(1/3)*2^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3)
)^(1/2))*6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3)
)^(3/2)-1/2916*(2^(2/3)-3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(
-12+9*2^(1/3)*3^(2/3))^(1/2))*6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-
4+3*2^(1/3)*3^(2/3))^(3/2)+1/34992*(-1)^(1/6)*3^(5/6)*ln(6-3*(-3)^(1/3)*2^
(2/3)*x+x^2)*2^(2/3)/(1+(-1)^(1/3))^5-1/34992*I*ln(6+3*(-2)^(2/3)*3^(1/3)*
x+x^2)*2^(2/3)*3^(5/6)/(1+(-1)^(1/3))^5+1/314928*ln(6+3*2^(2/3)*3^(1/3)*x+
x^2)*2^(2/3)*3^(1/3)
```

3.153.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.25

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \frac{-96+108x-64x^2-72x^3+73x^4-3x^5}{68364\,(216+108x^2+324x^3+18x^4+x^6)} - \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{108\log(x-\#1)-32\log(x-\#1)\#1+108\log(x-\#1)\#1^2-14}{36\#1+162\#1^2+12\#1^3+}\right.}{410184}$$

input
```
Integrate[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
(-96 + 108*x - 64*x^2 - 72*x^3 + 73*x^4 - 3*x^5)/(68364*(216 + 108*x^2 + 3
24*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1
^6 & , (108*Log[x - #1] - 32*Log[x - #1]*#1 + 108*Log[x - #1]*#1^2 - 146*L
og[x - #1]*#1^3 + 3*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5)
& ]/410184
```

3.153.3 Rubi [A] (verified)

Time = 1.97 (sec) , antiderivative size = 638, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(x^6+18x^4+324x^3+108x^2+216)^2}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1586874322944\int \left(\frac{3^{5/6}(1+\sqrt[3]{-1})-i\sqrt[3]{2}x}{4627325525704704\ 2^{2/3}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}-\frac{3i3^{5/6}}{4627325525704704\ 6^{2/3}}\right.$$

$$\Big\downarrow \text{2009}$$

$$1586874322944\left(\frac{\left(3(-3)^{2/3}+\sqrt[3]{-1}2^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{6940988288557056\ 6^{5/6}\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}-\frac{\left(\sqrt[3]{-1}2^{2/3}+3\ 3^{2/3}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{}\right.}{\sqrt{3\left(4-3\right.}}\right)}{771220920950784\ 6^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-\right.}\right.$$

input
```
Int[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
1586874322944*(((-1)^(2/3)*(6*3^(1/3) - (-2)^(1/3)*(2*(-1)^(1/3) + 3*2^(1/
3)*3^(2/3))*x))/(3084883683803136*3^(1/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(
2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) + (3*2^(2/3)*3^(1/3) + (
-1)^(1/3)*(2 + 3*(-2)^(1/3)*3^(2/3))*x)/(13881976577114112*2^(2/3)*3^(1/3)
*(4 + 3*(-2)^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (6*3^(1/
3) - (2*2^(1/3) - 3*6^(2/3))*x)/(27763953154228224*3^(1/3)*(4 - 3*2^(1/3)*
3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^
(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3
))]])/(6940988288557056*6^(5/6)*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - (((-1)
^(1/3)*2^(2/3) + 3*3^(2/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sq
rt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(771220920950784*6^(5/6)*(1 + (-1)^(1/3
))^4*(4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) - ((2^(2/3) - 3*3^(2/3))*ArcTanh[(2
^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(694098
8288557056*6^(5/6)*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + ((-1/3)^(1/6)*Log[6 -
 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(9254651051409408*2^(1/3)*(1 + (-1)^(1/3))
^5) - ((I/9254651051409408)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3
)*3^(1/6)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(832918
59462684672*2^(1/3)*3^(2/3)))
```

3.153.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.153. $\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

3.153.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.18

method	result
default	$\frac{-\frac{1}{22788}x^5+\frac{73}{68364}x^4-\frac{2}{1899}x^3-\frac{16}{17091}x^2+\frac{1}{633}x-\frac{8}{5697}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-3\_R^4+146\_R^3-1\right.}{\_R^5+12\_}\right.}{410184}$
risch	$\frac{-\frac{1}{22788}x^5+\frac{73}{68364}x^4-\frac{2}{1899}x^3-\frac{16}{17091}x^2+\frac{1}{633}x-\frac{8}{5697}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-3\_R^4+146\_R^3-1\right.}{\_R^5+12\_}\right.}{410184}$

input
```
int(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/22788*x^5+73/68364*x^4-2/1899*x^3-16/17091*x^2+1/633*x-8/5697)/(x^6+18
*x^4+324*x^3+108*x^2+216)+1/410184*sum((-3*_R^4+146*_R^3-108*_R^2+32*_R-10
8)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+
108*_Z^2+216))
```

3.153.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.153.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.17

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \text{RootSum}\left(397770473162309712803999551516645785 6t^6 - 1010314319415295961050951680t^4 - 201682\right.$$
$$+ \frac{-3x^5+73x^4-72x^3-64x^2+108x-96}{68364x^6+1230552x^4+22149936x^3+7383312x^2+14766624}$$

input
```
integrate(x**6/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

output
```
RootSum(3977704731623097128039995515166457856*_t**6 - 10103143194152959610
50951680*_t**4 - 2016822447709395715l232*_t**3 - 112582856818899648*_t**2
- 50648453064*_t - 880007, Lambda(_t, _t*log(-2736555670900189915706499414
1439556098619968804064460 8*_t**5/4979785539613990026757339569 5 + 118370084
7019604608530864623076435429280504457011 2*_t**4/497978553961399002675733 95
695 - 1057058190044671726637407748287331504778 7008*_t**3/497978553961399 00
267573395695 - 1552547411569469872387563218792789323392*_t**2/497978553961
39900267573395695 - 1254292379115914082690900325029592 8*_t/497978553961399
00267573395695 + x - 230665338703203224108343482 96/49797855396139900267573
395695))) + (-3*x**5 + 73*x**4 - 72*x**3 - 64*x**2 + 108*x - 96)/(68364*x*
*6 + 1230552*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)
```

3.153.7 Maxima [F]

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^6}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

output
```
-1/68364*(3*x^5 - 73*x^4 + 72*x^3 + 64*x^2 - 108*x + 96)/(x^6 + 18*x^4 + 3
24*x^3 + 108*x^2 + 216) - 1/68364*integrate((3*x^4 - 146*x^3 + 108*x^2 - 3
2*x + 108)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.153.8 Giac [F]

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^6}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

output
```
integrate(x^6/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.153.9 Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.57

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

input
```
int(x^6/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

output
```
symsum(log((7028852*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3
)/305639836193030032627 2 - (168169*z^2)/59416384155925038342727 68 - (3971*
z)/311864717157619341253309046784 - 880007/397770473162309712803999515166
457856, z, k))/2628920529 - (1980083*x)/310470256633842 - (235710556*root(
z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/305639836193030032627 2
- (168169*z^2)/59416384155925038342727 68 - (3971*z)/311864717157619341253 3
09046784 - 880007/397770473162309712803999515166457856, z, k)*x)/70980854
283 - (6628544*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/305
639836193030032627 2 - (168169*z^2)/59416384155925038342727 68 - (3971*z)/31
186471715761934125330904678 4 - 880007/39777047316230971280399951516645785
6, z, k)^2*x)/44521 - (141776759808*root(z^6 - (60865*z^4)/239631364059408
 - (15496909*z^3)/305639836193030032627 2 - (168169*z^2)/594163841559250383
4272768 - (3971*z)/311864717157619341253309046784 - 880007/397770473162309
712803999515166457856, z, k)^3*x)/44521 + (183701926508544*root(z^6 - (60
865*z^4)/239631364059408 - (15496909*z^3)/305639836193030032627 2 - (168169
*z^2)/59416384155925038342727 68 - (3971*z)/311864717157619341253309046784
- 880007/397770473162309712803999515166457856, z, k)^4*x)/211 - 694098828
8557056*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361
930300326272 - (168169*z^2)/59416384155925038342727 68 - (3971*z)/311864717
157619341253309046784 - 880007/397770473162309712803999515166457856, z...
```

3.154 $\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

3.154.1 Optimal result

Integrand size = 26, antiderivative size = 682

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \frac{\sqrt[3]{-\frac{1}{3}}(4-\sqrt[3]{-3}2^{2/3}x)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)(6-3\sqrt[3]{-3}2^{2/3}x+x^2)}$$
$$+ \frac{\sqrt[3]{-\frac{1}{3}}\left(4+(-2)^{2/3}\sqrt[3]{3}x\right)}{8748\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$- \frac{4+2^{2/3}\sqrt[3]{3}x}{17496\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$- \frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^4\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\sqrt{3}\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$
$$- \frac{i\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1458\ 2^{5/6}3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}} - \frac{\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{4374\sqrt{3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$
$$- \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{8748\sqrt{6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{39366\ 2^{5/6}\sqrt[6]{3}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$

output
```
1/11664*(-1)^(1/3)*3^(2/3)*(4-(-3)^(1/3)*2^(2/3)*x)*2^(1/3)/(1+(-1)^(1/3))
^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/52488*(-1)^(1
/3)*3^(2/3)*(4+(-2)^(2/3)*3^(1/3)*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1
/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/104976*(-4-2^(2/3)*3^(1/3)*x
)*2^(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/1312
2*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))/(8-9
*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)*3^(1/2)-1/13122*arctan((3*(-2)
^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(8+9*I*2^(1/3)*3^(1/
6)+3*2^(1/3)*3^(2/3))^(3/2)*3^(1/2)-1/52488*arctanh(2^(1/6)*(3*3^(1/3)+2^(
1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))/(-4+3*2^(1/3)*3^(2/3))^(3/2)*6^(1/2
)-1/26244*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/
2))*2^(1/6)*3^(5/6)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/8748
*I*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(
1/6)*3^(1/3)/(1+(-1)^(1/3))^5/(4+3*(-2)^(1/3)*3^(2/3))^(1/2)-1/236196*arct
anh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3
^(5/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)
```

3.154.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.24

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \frac{972-144x+648x^2+729x^3-27x^4+4x^5}{615276\,(216+108x^2+324x^3+18x^4+x^6)} + \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{144\log(x-\#1)-324\log(x-\#1)\#1+2043\log(x-\#1)\#1^2-\cdots}{36\#1+162\#1^2+12\#1^3+\cdots}\right.}{3691656}$$

input
```
Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
(972 - 144*x + 648*x^2 + 729*x^3 - 27*x^4 + 4*x^5)/(615276*(216 + 108*x^2
+ 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 +
 #1^6 & , (144*Log[x - #1] - 324*Log[x - #1]*#1 + 2043*Log[x - #1]*#1^2 -
54*Log[x - #1]*#1^3 + 4*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1
^5) & ]/3691656
```

3.154.3 Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 666, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(x^6+18x^4+324x^3+108x^2+216)^2}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1586874322944\int \left(-\frac{\sqrt[3]{-\frac{1}{3}}x}{1542441841901568\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)^2}-\frac{\sqrt[3]{-\ldots}}{13881976577114112\ 2^{2/3}\left(\ldots\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$1586874322944\left(-\frac{i\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{2313662762852352\ 2^{5/6}3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{13881976577114112\sqrt{6}\left(4+3\sqrt[3]{-\ldots}\right.}\right.$$

input
```
Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
1586874322944*(((-1/3)^(1/3)*(4 - (-3)^(1/3)*2^(2/3)*x))/(3084883683803136
*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2
^(2/3)*x + x^2)) + ((-1/3)^(1/3)*(4 + (-2)^(2/3)*3^(1/3)*x))/(277639531542
28224*2^(2/3)*(4 + 3*(-2)^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2
)) - (4 + 2^(2/3)*3^(1/3)*x)/(27763953154228224*2^(2/3)*3^(1/3)*(4 - 3*2^(
1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ArcTan[(3*(-2)^(2/3)*3^(1
/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]]/(13881976577114112*Sqrt[6]*
(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((I/2313662762852352)*ArcTan[(3*(-2)^(
2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2^(5/6)*3^(2/3)*
(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) - ArcTan[(2^(1/6)*(3*(-
3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(69409882885570
56*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^4*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + Ar
cTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3)
)]]/(6940988288557056*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/
3))^(3/2)) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1
/3)*3^(2/3))]]/(13881976577114112*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2))
- ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3)
)]]/(62468894597013504*2^(5/6)*3^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]))
```

3.154.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.154. $\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2} dx$

3.154.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.18

method	result
default	$\frac{\frac{1}{153819}x^5-\frac{1}{22788}x^4+\frac{1}{844}x^3+\frac{2}{1899}x^2-\frac{4}{17091}x+\frac{1}{633}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(4\_R^4-54\_R^3+2043\_\right.}{\_R^5+12\_R}\right.}{3691656}$
risch	$\frac{\frac{1}{153819}x^5-\frac{1}{22788}x^4+\frac{1}{844}x^3+\frac{2}{1899}x^2-\frac{4}{17091}x+\frac{1}{633}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(4\_R^4-54\_R^3+2043\_\right.}{\_R^5+12\_R}\right.}{3691656}$

input
```
int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/153819*x^5-1/22788*x^4+1/844*x^3+2/1899*x^2-4/17091*x+1/633)/(x^6+18*x^
4+324*x^3+108*x^2+216)+1/3691656*sum((4*_R^4-54*_R^3+2043*_R^2-324*_R+144)
/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+10
8*_Z^2+216))
```

3.154.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1445 vs. 2(463) = 926.

Time = 0.96 (sec) , antiderivative size = 1445, normalized size of antiderivative = 2.12

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

output
```
1/28041818976*(182304*x^5 - 1230552*x^4 + 33224904*x^3 + 422*sqrt(1/633)*(
x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(5034474*18^(2/3) + 9367856*18
^(1/3) + 44687457)*log(2/1982119441*sqrt(1/633)*(7238020557*(5034474*18^(2
/3) + 9367856*18^(1/3) + 44687457)^2 - 447902374840040617697967 3*18^(2/3)
- 833430652250766125864511 2*18^(1/3) - 268625598114228853471204 77)*sqrt(50
34474*18^(2/3) + 9367856*18^(1/3) + 44687457) - 7383041510/9393931*(503447
4*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 247458158879850620*x + 51322
55454960803463351330/9393931*18^(2/3) + 95498020363770460407535 20/9393931*
18^(1/3) + 27278928233033940032425830/9393931) - 422*sqrt(1/633)*(x^6 + 18
*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(5034474*18^(2/3) + 9367856*18^(1/3) +
 44687457)*log(-2/1982119441*sqrt(1/633)*(7238020557*(5034474*18^(2/3) + 9
367856*18^(1/3) + 44687457)^2 - 4479023748400406176979673*18^(2/3) - 83343
06522507661258645112*18^(1/3) - 26862559811422885347120477)*sqrt(5034474*1
8^(2/3) + 9367856*18^(1/3) + 44687457) - 7383041510/9393931*(5034474*18^(2
/3) + 9367856*18^(1/3) + 44687457)^2 + 247458158879850620*x + 513225545496
0803463351330/9393931*18^(2/3) + 9549802036377046040753520/9393931*18^(1/3
) + 27278928233033940032425830/9393931) - 9*sqrt(422)*(x^6 + 18*x^4 + 324*
x^3 + 108*x^2 + 216)*sqrt(-20718*18^(2/3) + sqrt(-1/19683*(5034474*18^(2/3
) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/
81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 367798)*log(147660...
```

3.154.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.15

$$\int \frac{x^5}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2}\,dx$$
$$= \text{RootSum}\Big(274938951049788473490124490008305567006 72t^6 - 131871818922695008886298319257 6t^4 +$$
$$+ \frac{4x^5 - 27x^4 + 729x^3 + 648x^2 - 144x + 972}{615276x^6 + 11074968x^4 + 199349424x^3 + 66449808x^2 + 132899616}$$

input
```
integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

output
```
RootSum(27493895104978847349012449000830556700672*_t**6 - 1318718189226950
088862983192576*_t**4 + 12120917704776776448*_t**2 - 39753025, Lambda(_t,
_t*log(947842259001288723909832054550209950242045952*_t**5/618645397199626
55 - 243458646817775607639654889480814592*_t**4/9811980923071 - 4168255647
5067500431787310779667456*_t**3/61864539719962655 + 1202687744266432861646
2272*_t**2/9811980923071 + 216142618488859793668428*_t/61864539719962655 +
 x - 308574300024117/39247923692284))) + (4*x**5 - 27*x**4 + 729*x**3 + 64
8*x**2 - 144*x + 972)/(615276*x**6 + 11074968*x**4 + 199349424*x**3 + 6644
9808*x**2 + 132899616)
```

3.154.7 Maxima [F]

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\int \frac{x^5}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

output
```
1/615276*(4*x^5 - 27*x^4 + 729*x^3 + 648*x^2 - 144*x + 972)/(x^6 + 18*x^4
+ 324*x^3 + 108*x^2 + 216) + 1/615276*integrate((4*x^4 - 54*x^3 + 2043*x^2
 - 324*x + 144)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.154.8 Giac [F]

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\int \frac{x^5}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

output
```
integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.154.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.44

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$

$$= \left(\sum_{k=1}^{6} \ln\left(-\frac{4477969\,\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{274938951049788473490124490008305567}\right.}{189282278088}\right.\right.$$

$$+\frac{6305\,x}{4967524106141472}$$

$$-\frac{\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{27493895104978847349012449000830556700672},z,k\right)\,x\,1634}{5110621508376}$$

$$-\frac{\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{27493895104978847349012449000830556700672},z,k\right)^2\,x\,4334}{10818603}$$

$$-\frac{\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{27493895104978847349012449000830556700672},z,k\right)^3\,x\,6533}{44521}$$

$$-\frac{\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{27493895104978847349012449000830556700672},z,k\right)^4\,x\,4002}{211}$$

$$-\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{274938951049788473490124490008}\right.$$

$$+\frac{5943884\,\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{27493895104978847349012449000830556700672},z,k\right.}{400689}$$

$$+\frac{224442467136\,\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{2749389510497884734901244900083055670067}\right.}{44521}$$

$$-\frac{137087493272064\,\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{274938951049788473490124490008305567}\right.}{211}$$

$$-168897381688221696\,\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{}{274938951049}\right.$$

$$\left.-\frac{13082875}{178830867821092992}\right)\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}\right.$$

$$\left.\left.-\frac{39753025}{27493895104978847349012449000830556700672},z,k\right)\right)$$

$$+\frac{\frac{x^5}{153819}-\frac{x^4}{22788}+\frac{x^3}{844}+\frac{2\,x^2}{1899}-\frac{4\,x}{17091}+\frac{1}{633}}{x^6+18\,x^4+324\,x^3+108\,x^2+216}$$

input
```
int(x^5/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

output
```
symsum(log((6305*x)/4967524106141472 - (4477969*root(z^6 - (183899*z^4)/38
34101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/27493895
104978847349012449000830556700672, z, k))/189282278088 - (16340881*root(z^
6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376
- 39753025/27493895104978847349012449000830556700672, z, k)*x)/51106215083
76 - (43348696*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083
883651774823903461376 - 39753025/27493895104978847349012449000830556700672
, z, k)^2*x)/10818603 - (65333687616*root(z^6 - (183899*z^4)/3834101824950
528 + (6209*z^2)/14083883651774823903461376 - 39753025/2749389510497884734
9012449000830556700672, z, k)^3*x)/44521 - (40024496812032*root(z^6 - (183
899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 397530
25/27493895104978847349012449000830556700672, z, k)^4*x)/211 - 69409882885
57056*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774
823903461376 - 39753025/27493895104978847349012449000830556700672, z, k)^5
*x + (5943884*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/140838
83651774823903461376 - 39753025/27493895104978847349012449000830556700672,
 z, k)^2)/400689 + (224442467136*root(z^6 - (183899*z^4)/3834101824950528
+ (6209*z^2)/14083883651774823903461376 - 39753025/27493895104978847349012
449000830556700672, z, k)^3)/44521 - (137087493272064*root(z^6 - (183899*z
^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025...
```

3.154. $\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

3.155 $\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

3.155.1 Optimal result

Integrand size = 26, antiderivative size = 850

$$\int \frac{x^4}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}2^{2/3}-2x\right)}{5832\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$
$$-\frac{\sqrt[3]{-\frac{1}{3}}\left(3(-2)^{2/3}\sqrt[3]{3}+2x\right)}{26244\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$-\frac{3\sqrt[3]{3}+\sqrt[3]{2}x}{52488\left(9\sqrt[3]{2}-4\sqrt[3]{3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$+\frac{\sqrt[3]{-1}\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{729\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$
$$-\frac{\sqrt[3]{-1}\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{2916\sqrt[6]{2}3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$
$$-\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{11664\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$
$$-\frac{i\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{5832\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{26244\sqrt[6]{2}3^{5/6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$
$$+\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{52488\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}-\frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{34992\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^4}$$
$$+\frac{i\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{34992\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}-\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{314928\sqrt[3]{2}3^{2/3}}$$

output
```
1/34992*(-1)^(1/3)*3^(2/3)*(3*(-3)^(1/3)*2^(2/3)-2*x)*2^(1/3)/(1+(-1)^(1/3
))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)-1/157464*(-1)
^(1/3)*3^(2/3)*(3*(-2)^(2/3)*3^(1/3)+2*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3
*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/52488*(-3*3^(1/3)-2^(1/
3)*x)/(9*2^(1/3)-4*3^(1/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/4374*(-1)^(1/3)*
arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/3
)*3^(1/6)/(1+(-1)^(1/3))^4/(8-9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)
-1/17496*(-1)^(1/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^
(2/3))^(1/2))*2^(5/6)*3^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^
(1/3)*3^(2/3))^(3/2)+1/157464*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9
*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(3/2)-1/20
9952*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^(1/3))^4+1/2
09952*I*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(5/6)/(1+(-1)^(1/3))^5-
1/1889568*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/34992*I*arctan(2
^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3
^(2/3)/(1+(-1)^(1/3))^5/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/69984*arctan((3*(
-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*(3^(1/2)+I)*2^(5/
6)*3^(2/3)/(1+(-1)^(1/3))^5/(4+3*(-2)^(1/3)*3^(2/3))^(1/2)+1/314928*arctan
h(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(
1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)
```

3.155.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.20

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \frac{-288+324x-1458x^2-216x^3+8x^4-9x^5}{1230552\,(216+108x^2+324x^3+18x^4+x^6)} - \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{324\log(x-\#1)-2628\log(x-\#1)\#1+324\log(x-\#1)\#1^2-}{36\#1+162\#1^2+12\#1^3+}\right.}{7383312}$$

input
```
Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
(-288 + 324*x - 1458*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(1230552*(216 + 108*x^
2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4
 + #1^6 & , (324*Log[x - #1] - 2628*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2
- 16*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 +
#1^5) & ]/7383312
```

3.155.3 Rubi [A] (warning: unable to verify)

Time = 2.01 (sec) , antiderivative size = 826, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(x^6+18x^4+324x^3+108x^2+216)^2}dx$$

$$\Big\downarrow \text{2466}$$

$$1586874322944\int \left(-\frac{x+3\ 2^{2/3}\sqrt[3]{3}}{249875578388054016\sqrt[3]{2}3^{2/3}\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}+\frac{6i3^{5/6}-\left(\sqrt[3]{-2}+\right.}{27763953154228224\ 6^{2/3}\left(1+\sqrt[3]{-1}\right)}\right.$$

$$\Big\downarrow \text{2009}$$

$$1586874322944\left(-\frac{\sqrt[3]{-\frac{1}{3}}\left(2x+3(-2)^{2/3}\sqrt[3]{3}\right)}{83291859462684672\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}-\frac{(1-i\sqrt{3})\arctan}{83291859462684672\sqrt[6]{2}}\right.$$

input
```
Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
1586874322944*(((-1/3)^(1/3)*(3*(-3)^(1/3) - 2^(1/3)*x))/(9254651051409408
*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x
 + x^2)) - ((-1/3)^(1/3)*(3*(-2)^(2/3)*3^(1/3) + 2*x))/(83291859462684672*
2^(2/3)*(4 + 3*(-2)^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (
3*3^(1/3) + 2^(1/3)*x)/(83291859462684672*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*
(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3
) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(41645929731342336*2^(1/6)*3
^(5/6)*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((1 - I*Sqrt[3])*ArcTan[(3*(-2)
^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(83291859462684
672*2^(1/6)*3^(1/3)*(3*I + Sqrt[3])*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) - ((I/
9254651051409408)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 -
3*(-3)^(2/3)*2^(1/3))]])/(2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-
3)^(2/3)*2^(1/3)]) + ((-1)^(1/3)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x
))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(1156831381426176*2^(2/3)*3^(5/6)*
(1 + (-1)^(1/3))^4*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2))
+ ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3)
)]]/(41645929731342336*2^(1/6)*3^(5/6)*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + A
rcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]
/(83291859462684672*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - Log[6
- 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(55527906308456448*2^(1/3)*3^(2/3)*(1 +...
```

3.155.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.155. $\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

3.155.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.14

method	result
default	$\frac{-\frac{1}{136728}x^5+\frac{1}{153819}x^4-\frac{1}{5697}x^3-\frac{1}{844}x^2+\frac{1}{3798}x-\frac{4}{17091}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4+16\_R^3-\right.}{\_R^5+1}\right.}{7383312}$
risch	$\frac{-\frac{1}{136728}x^5+\frac{1}{153819}x^4-\frac{1}{5697}x^3-\frac{1}{844}x^2+\frac{1}{3798}x-\frac{4}{17091}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4+16\_R^3-\right.}{\_R^5+1}\right.}{7383312}$

input
```
int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/136728*x^5+1/153819*x^4-1/5697*x^3-1/844*x^2+1/3798*x-4/17091)/(x^6+18
*x^4+324*x^3+108*x^2+216)+1/7383312*sum((-9*_R^4+16*_R^3-324*_R^2+2628*_R-
324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^
3+108*_Z^2+216))
```

3.155.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\text{Timed out}$$

input
```
integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.155.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.13

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \text{RootSum}\left(185583791958607219605834030755606257729536t^6 - 1309367357962223565522033377280t^4\right.$$
$$+ \frac{-9x^5+8x^4-216x^3-1458x^2+324x-288}{1230552x^6+22149936x^4+398698848x^3+132899616x^2+265799232}$$

input
```
integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

output
```
RootSum(185583791958607219605834030755606257729536*_t**6 - 130936735796222
3565522033377280*_t**4 + 4356336487052294744666112*_t**3 - 405298284548038
7328*_t**2 + 303890718384*_t - 880007, Lambda(_t, _t*log(39083462657955593
476841044707333565976412952759280634691584*_t**5/4979785539613990026757339
5695 + 8836979346223785538912817601414711102396804462575616*_t**4/49797855
396139900267573395695 - 264930581348308532588844249597134695706805067776*_
t**3/49797855396139900267573395695 + 8861353335473631852015151098261583762
50624*_t**2/49797855396139900267573395695 - 682321479574909906511394635855
601936*_t/49797855396139900267573395695 + x - 2137556077084648622429151956
8/49797855396139900267573395695))) + (-9*x**5 + 8*x**4 - 216*x**3 - 1458*x
**2 + 324*x - 288)/(1230552*x**6 + 22149936*x**4 + 398698848*x**3 + 132899
616*x**2 + 265799232)
```

3.155.7 Maxima [F]

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^4}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

output
```
-1/1230552*(9*x^5 - 8*x^4 + 216*x^3 + 1458*x^2 - 324*x + 288)/(x^6 + 18*x^
4 + 324*x^3 + 108*x^2 + 216) - 1/1230552*integrate((9*x^4 - 16*x^3 + 324*x
^2 - 2628*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.155.8 Giac [F]

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^4}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

output
```
integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.155.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.46

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

input
```
int(x^4/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

output
```
symsum(log((24389*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)
/6601820461769448704747752 - (168169*z^2)/7700363386607884969217507328 + (3
971*z)/2425060040617647997585731147792384 - 880007/18558379195860721960583
4030755606257729536, z, k))/851770251396 + (288041*x)/804738905194918464 -
 (1090723*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/6601820
46176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2
425060040617647997585731147792384 - 880007/1855837919586072196058340307556
06257729536, z, k)*x)/22997796787692 + (5850124*root(z^6 - (60865*z^4)/862
6729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/770
0363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 -
880007/185583791958607219605834030755606257729536, z, k)^2*x)/3606201 - (6
4554687936*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182
046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/
2425060040617647997585731147792384 - 880007/185583791958607219605834030755
606257729536, z, k)^3*x)/44521 + (31535589897216*root(z^6 - (60865*z^4)/86
26729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/77
00363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 -
 880007/185583791958607219605834030755606257729536, z, k)^4*x)/211 - 69409
88288557056*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/66018
2046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971...
```

3.156 $\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2} \, dx$

3.156.1 Optimal result

Integrand size = 26, antiderivative size = 873

$$
\begin{aligned}
&\int \frac{x^3}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx \\
&= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)} \\
&\quad-\frac{\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)+3x}{157464\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)} \\
&\quad-\frac{2\sqrt[3]{2}-3\ 6^{2/3}-\sqrt[3]{3}x}{104976\left(9\sqrt[3]{2}-4\sqrt[3]{3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)} \\
&\quad+\frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{26244\sqrt{3}\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} \\
&\quad-\frac{\left(9i-\sqrt[3]{3}\left(2i2^{2/3}+9\sqrt[6]{3}+2\ 2^{2/3}\sqrt{3}\right)\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{209952\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} \\
&\quad-\frac{\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{26244\sqrt{3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} \\
&\quad+\frac{\left(9i+\sqrt[3]{3}\left(4i2^{2/3}-9\sqrt[6]{3}\right)\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{209952\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4+3\sqrt[3]{-2}3^{2/3}\right)}} \\
&\quad-\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{52488\sqrt{6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}} \\
&\quad+\frac{\left(2\ 2^{2/3}-3\ 3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{944784\sqrt[6]{3}\sqrt{2\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}-\frac{i\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{23328\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^5}
\end{aligned}
$$

output
```
1/157464*((-6)^(1/3)*(2*(-3)^(1/3)+9*2^(1/3))-3*x)/(8-9*I*2^(1/3)*3^(1/6)+
3*2^(1/3)*3^(2/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/157464*(-(-6)^(1/3)*(9
*(-2)^(1/3)+2*3^(1/3))-3*x)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3
*(-2)^(2/3)*3^(1/3)*x+x^2)+1/104976*(-2*2^(1/3)+3*6^(2/3)+3^(1/3)*x)/(9*2^
(1/3)-4*3^(1/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)-1/139968*I*ln(6-3*(-3)^(1/3)*
2^(2/3)*x+x^2)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5+1/3779136*ln(6+3*2^(2/3)*3
^(1/3)*x+x^2)*2^(1/3)*3^(2/3)+1/78732*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(2
4-18*(-3)^(2/3)*2^(1/3))^(1/2))/(8-9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^
(3/2)*3^(1/2)-1/78732*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*
3^(2/3))^(1/2))/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)*3^(1/2)+1/
279936*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*(3^(1/2)+I)*2^(1/3)*3^(1/6)/(1+(-1
)^(1/3))^5-1/314928*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3
^(2/3))^(1/2))/(-4+3*2^(1/3)*3^(2/3))^(3/2)*6^(1/2)-1/209952*arctan((3*(-3
)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*(9*I-3^(1/3)*(2*I*2
^(2/3)+9*3^(1/6)+2*2^(2/3)*3^(1/2)))/(1+(-1)^(1/3))^5/(8-6*(-3)^(2/3)*2^(1
/3))^(1/2)+1/209952*(9*I+3^(1/3)*(4*I*2^(2/3)-9*3^(1/6)))*arctan((3*(-2)^(
2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(1+(-1)^(1/3))^5/(8+6*
(-2)^(1/3)*3^(2/3))^(1/2)+1/2834352*(2*2^(2/3)-3*3^(2/3))*arctanh(2^(1/6)*
(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(-8+6*2^(1/3)
*3^(2/3))^(1/2)
```

3.156.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.19

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \frac{972-3942x+648x^2+96x^3-27x^4+4x^5}{3691656\,(216+108x^2+324x^3+18x^4+x^6)} + \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{1971\log(x-\#1)-162\log(x-\#1)\#1+72\log(x-\#1)\#1^2-2}{36\#1+162\#1^2+12\#1^3+}\right.}{11074968}$$

input
```
Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
(972 - 3942*x + 648*x^2 + 96*x^3 - 27*x^4 + 4*x^5)/(3691656*(216 + 108*x^2
 + 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4
+ #1^6 & , (1971*Log[x - #1] - 162*Log[x - #1]*#1 + 72*Log[x - #1]*#1^2 -
27*Log[x - #1]*#1^3 + 2*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1
^5) & ]/11074968
```

3.156.3 Rubi [A] (warning: unable to verify)

Time = 2.39 (sec) , antiderivative size = 855, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(x^6+18x^4+324x^3+108x^2+216)^2}\,dx$$

$$\Big\downarrow \ 2466$$

$$1586874322944\int \left(-\frac{x+3\ 2^{2/3}\sqrt[3]{3}}{83291859462684672\ 2^{2/3}\sqrt[3]{3}\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)^2}-\frac{3i\sqrt[3]{2}\sqrt[6]{3}x+i2^{2/3}3^{5/6}-9i\sqrt{3}}{333167437850738688\left(1+\sqrt[3]{-1}\right)^5(}\right.$$

$$\Big\downarrow \ 2009$$

$$1586874322944\left(-\frac{3x+\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)}{499751156776108032\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}+\frac{\left(9i+\sqrt[3]{3}\left(4i2^{2/3}-9\sqrt[6]{3}\right)\right)}{333167437850738688\,(1+}\right.$$

input
```
Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
1586874322944*(-1/55527906308456448*(2*(2*(-1)^(1/3)*3^(2/3) + 9*6^(1/3))
+ 3*(-2)^(2/3)*x)/(2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(
6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3)*(9*(-2)^(1/3) + 2*3^(1/3)
) + 3*x)/(49975115677610803 2*(4 + 3*(-2)^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*
3^(1/3)*x + x^2)) + (2*2^(1/3) - 3*6^(2/3) - 3^(1/3)*x)/(1665837189253693
4*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ArcTa
n[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]]/(832918
59462684672*Sqrt[6]*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) + ((9*I + 3^(1/3)*((
4*I)*2^(2/3) - 9*3^(1/6)))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 +
 3*(-2)^(1/3)*3^(2/3))]])/(333167437850738688*(1 + (-1)^(1/3))^5*Sqrt[2*(4
 + 3*(-2)^(1/3)*3^(2/3))]) + ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/S
qrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(41645929731342336*Sqrt[3]*(8 - (9*I)*2
^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - ((9*I - 3^(1/3)*((2*I)*2^(2/3
) + 9*3^(1/6) + 2*2^(2/3)*Sqrt[3]))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3
)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(333167437850738688*(1 + (-1)^(
1/3))^5*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) - ArcTanh[(2^(1/6)*(3*3^(1/3)
+ 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(83291859462684672*Sqrt[6]
*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - (Sqrt[-4 + 3*2^(1/3)*3^(2/3)]*ArcTanh[(
2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(14992
53470328324096*2^(5/6)*3^(1/6)) - ((I/37018604205637632)*Log[6 - 3*(-3)...
```

3.156.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.156. $\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

3.156.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.14

method	result
default	$\frac{\frac{1}{922914}x^5-\frac{1}{136728}x^4+\frac{4}{153819}x^3+\frac{1}{5697}x^2-\frac{73}{68364}x+\frac{1}{3798}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(2\_R^4-27\_R^3+\right.}{\_R^5+1}\right.}{11074968}$
risch	$\frac{\frac{1}{922914}x^5-\frac{1}{136728}x^4+\frac{4}{153819}x^3+\frac{1}{5697}x^2-\frac{73}{68364}x+\frac{1}{3798}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(2\_R^4-27\_R^3+\right.}{\_R^5+1}\right.}{11074968}$

input
```
int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/922914*x^5-1/136728*x^4+4/153819*x^3+1/5697*x^2-73/68364*x+1/3798)/(x^6
+18*x^4+324*x^3+108*x^2+216)+1/11074968*sum((2*_R^4-27*_R^3+72*_R^2-162*_R
+1971)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_
Z^3+108*_Z^2+216))
```

3.156.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.156.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.13

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \text{RootSum}\Big(128275517001789310191552482058275045342655283 2t^6 - 9063884657755442444262511497 7$$
$$+ \frac{4x^5-27x^4+96x^3+648x^2-3942x+972}{3691656x^6+66449808x^4+1196096544x^3+398698848x^2+797397696}$$

input
```
integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

output
```
RootSum(1282755170017893101915524820582750453426552832*_t**6 - 90638846577
554424444262511497707 52*_t**4 - 4300873166389987741684137984*_t**3 - 717000
908921644962816*_t**2 + 135354162312576*_t - 7197829, Lambda(_t, _t*log(17
257935592810449901409556597891882995604001083339368041361480613888*_t**5/1
542060097910520444906943803032375 21 + 238960740062098552437635885357265220
718195632456058768405299 2*_t**4/154206009791052044490694380303237521 - 122
860721608832839307117159488782600789969921934883880 96*_t**3/1542060097910 5
204449069438030323752 1 - 594905535739591731611254960135279097541565584107 5
2*_t**2/154206009791052044490694380303237521 - 175201496798366911123670641
97713753004827200*_t/154206009791052044490694380303237521 + x + 7664229887
072296150558552870408873 32/154206009791052044490694380303237521))) + (4*x*
*5 - 27*x**4 + 96*x**3 + 648*x**2 - 3942*x + 972)/(3691656*x**6 + 66449808
*x**4 + 1196096544*x**3 + 398698848*x**2 + 797397696)
```

3.156.7 Maxima [F]

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^3}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

output
```
1/3691656*(4*x^5 - 27*x^4 + 96*x^3 + 648*x^2 - 3942*x + 972)/(x^6 + 18*x^4
 + 324*x^3 + 108*x^2 + 216) + 1/1845828*integrate((2*x^4 - 27*x^3 + 72*x^2
 - 162*x + 1971)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.156.8 Giac [F]

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\int \frac{x^3}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

output
```
integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.156.9 Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.44

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\text{Too large to display}$$

input
```
int(x^3/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

output
```
symsum(log((11*x)/603554178896188848 - (14059*root(z^6 - (292589*z^4)/4140
82997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/
4435409310686141742269284220928 + (1989787*z)/1885726687584283082922664540
5233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)
)/30663729050256 - (5658601*root(z^6 - (292589*z^4)/414082997094657024 - (
11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/443540931068614174
2269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 71978
29/1282755170017893101915524820582750453426552832, z, k)*x)/66233654748552
96 + (6603523*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/
3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928
+ (1989787*z)/18857266875842830829226645405233577984 - 7197829/12827551700
17893101915524820582750453426552832, z, k)^2*x)/584204562 - (1762321104*ro
ot(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705
975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18
857266875842830829226645405233577984 - 7197829/128275517001789310191552482
0582750453426552832, z, k)^3*x)/44521 - (59633904436992*root(z^6 - (292589
*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (247
9189*z^2)/4435409310686141742269284220928 + (1989787*z)/188572668758428308
29226645405233577984 - 7197829/1282755170017893101915524820582750453426552
832, z, k)^4*x)/211 - 6940988288557056*root(z^6 - (292589*z^4)/41408299...
```

3.157 $\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

3.157.1 Optimal result

Integrand size = 26, antiderivative size = 986

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$

$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{104976\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$

$$-\frac{27\ 2^{2/3}\left(1+\sqrt[3]{-2}3^{2/3}\right)-\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x}{472392\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$+\frac{9\left(6-2^{2/3}\sqrt[3]{3}\right)-\left(2-3\sqrt[3]{2}3^{2/3}\right)x}{314928\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$

$$-\frac{\left(1+i\sqrt{3}+3\sqrt[3]{2}3^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{8748\ 2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$

$$+\frac{\left(3(-3)^{2/3}+\sqrt[3]{-1}2^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{17496\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$

$$+\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{34992\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$

$$+\frac{i\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{17496\sqrt[6]{2}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$

$$-\frac{\left(2^{2/3}-3\ 3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{17496\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{157464\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}+\frac{\left(i+\sqrt{3}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{419904\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}$$

$$-\frac{i\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{\cdots}+\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{\cdots}$$

output
```
1/209952*(-27*(-2)^(2/3)-54*(-1)^(1/3)*3^(2/3)+6^(1/3)*(9+(-3)^(1/3)*2^(2/
3))*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2
^(2/3)*x+x^2)+1/944784*(-27*2^(2/3)-54*(-1)^(1/3)*3^(2/3)+(-1)^(1/3)*3^(2/
3)*(2+3*(-2)^(1/3)*3^(2/3))*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^
(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/1889568*(54-9*2^(2/3)*3^(1/3)-(2-3
*2^(1/3)*3^(2/3))*x)*2^(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^
(1/3)*x+x^2)+1/104976*(3*(-3)^(2/3)+(-1)^(1/3)*2^(2/3))*arctan((3*(-2)^(2/
3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*6^(1/6)/(1-(-1)^(1/3))^2
/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3))^(3/2)-1/104976*(2^(2/3)-3*3^(2/
3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*6
^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2^(1/3)*3^(2/3))^(3/2)-1/12
59712*I*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(5/6)/(1+(-1)^(1/3))^5+
1/11337408*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/52488*arctan((3
*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*(1+3*2^(1/3)*3^
(2/3)+I*3^(1/2))*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^4/(8-9*I*2^(1/3)*3^(1/6)+3
*2^(1/3)*3^(2/3))^(3/2)+1/2519424*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*(3^(1/2
)+I)*2^(2/3)*3^(5/6)/(1+(-1)^(1/3))^5+1/104976*I*arctan(2^(1/6)*(3*(-3)^(1
/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(2/3)/(1+(-1)^(1
/3))^5/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)+1/209952*arctan((3*(-2)^(2/3)*3^(1/3
)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*(3^(1/2)+I)*2^(5/6)*3^(2/3)/(1...
```

3.157.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.17

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \frac{-7884+324x-2724x^2-216x^3+8x^4-9x^5}{7383312\,(216+108x^2+324x^3+18x^4+x^6)} - \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\ \frac{324\log(x-\#1)+2436\log(x-\#1)\#1+324\log(x-\#1)\#1^2-}{36\#1+162\#1^2+12\#1^3+}\right.}{44299872}$$

input
```
Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
(-7884 + 324*x - 2724*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(7383312*(216 + 108*x
^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^
4 + #1^6 & , (324*Log[x - #1] + 2436*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2
 - 16*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 +
 #1^5) & ]/44299872
```

3.157.3 Rubi [A] (verified)

Time = 2.46 (sec) , antiderivative size = 892, normalized size of antiderivative = 0.90, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^6+18x^4+324x^3+108x^2+216)^2}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1586874322944\int \left(\frac{3x+\sqrt[3]{-6}\left(9\sqrt[3]{-2}+\sqrt[3]{3}\right)}{499751156776108032\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)^2}-\frac{i\left(18\sqrt[3]{3}-\sqrt[3]{2}(1-i\sqrt{3}\right.}{333167437850738688\ 2^{2/3}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^{5}}\right.$$

$$\Big\downarrow \text{2009}$$

$$1586874322944\left(-\frac{9\left((-6)^{2/3}+6\sqrt[3]{-3}\right)-\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)x}{55527906308456448\ 6^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{(i}{555279063}\right.$$

input
```
Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
1586874322944*(-1/55527906308456448*(9*((-6)^(2/3) + 6*(-3)^(1/3)) - (2*(-
3)^(1/3) + 9*2^(1/3))*x)/(6^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(
1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - (54*(1 + (-2)^(1/3)*3^(2/3)) +
 (-6)^(2/3)*((-2)^(2/3) - 3*3^(2/3))*x)/(2998506940656648192*(4 + 3*(-2)^(
1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (18*(3*2^(1/3) - 3^(1/
3)) - (2*2^(1/3) - 3*6^(2/3))*x)/(999502313552216064*3^(1/3)*(4 - 3*2^(1/3
)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((-1)^(1/3)*(2 + 3*(-2)^(1/3
)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^
(2/3))]])/(499751156776108032*2^(1/6)*3^(5/6)*(4 + 3*(-2)^(1/3)*3^(2/3))^(
3/2)) + ((I + Sqrt[3])*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(
-2)^(1/3)*3^(2/3))]])/(55527906308456448*2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^
5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) + ((I/27763953154228224)*ArcTan[(2^(1/6)
*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(2^(1/6)
*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((9 + (-3)^(
1/3)*2^(2/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-
3)^(2/3)*2^(1/3))]])/(83291859462684672*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^4
*(4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) - ((2^(2/3) - 3*3^(2/3))*ArcTanh[(2^(1/
6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(2498755783
88054016*6^(5/6)*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - ArcTanh[(2^(1/6)*(3*3^(
1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(24987557838805401...
```

3.157.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

3.157. $\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

3.157.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.12

method	result
default	$\frac{-\frac{1}{820368}x^5+\frac{1}{922914}x^4-\frac{1}{34182}x^3-\frac{227}{615276}x^2+\frac{1}{22788}x-\frac{73}{68364}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4+16\_\right.}{\_}\right.}{44299872}$
risch	$\frac{-\frac{1}{820368}x^5+\frac{1}{922914}x^4-\frac{1}{34182}x^3-\frac{227}{615276}x^2+\frac{1}{22788}x-\frac{73}{68364}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4+16\_\right.}{\_}\right.}{44299872}$

input
```
int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/820368*x^5+1/922914*x^4-1/34182*x^3-227/615276*x^2+1/22788*x-73/68364)
/(x^6+18*x^4+324*x^3+108*x^2+216)+1/44299872*sum((-9*_R^4+16*_R^3-324*_R^2
-2436*_R-324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^
4+324*_Z^3+108*_Z^2+216))
```

3.157.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.157.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.11

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \text{RootSum}\Big(865859739762077843792979253893356556062923161 6t^6 + 109068095871770168248838645612$$
$$+ \frac{-9x^5+8x^4-216x^3-2724x^2+324x-7884}{7383312x^6+132899616x^4+2392193088x^3+797397696x^2+1594795392}$$

input
```
integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

output
```
RootSum(865859739762077843792979253893356556062923161 6*_t**6 + 10906809587
177016824883864561254 4*_t**4 - 49265570759336691571349913 6*_t**3 + 4037833
1745144603648*_t**2 - 695635011360*_t + 4513, Lambda(_t, _t*log(1014425315
618041811131612870398593498518816196536317121658 88*_t**5/3569006970707929 4
8475845 - 1497965500823593351127094349719750889670502100500 48*_t**4/356900
697070792948475845 + 1222409754458272818505898777686707836172369 92*_t**3/
356900697070792948475845 - 5775055524251595723022901938558261453824*_t**2/
356900697070792948475845 + 96165242200260265765603930470432*_t/71380139414
158589695169 + x - 170591523411296981205455 84/107070209121237884542753 5))
 + (-9*x**5 + 8*x**4 - 216*x**3 - 2724*x**2 + 324*x - 7884)/(7383312*x**6
+ 132899616*x**4 + 2392193088*x**3 + 797397696*x**2 + 1594795392)
```

3.157.7 Maxima [F]

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^2}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

output
```
-1/7383312*(9*x^5 - 8*x^4 + 216*x^3 + 2724*x^2 - 324*x + 7884)/(x^6 + 18*x
^4 + 324*x^3 + 108*x^2 + 216) - 1/7383312*integrate((9*x^4 - 16*x^3 + 324*
x^2 + 2436*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

3.157.8 Giac [F]

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^2}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

output
```
integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

3.157.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

input
```
int(x^2/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

output
```
symsum(log((4897*x)/18772949180387057928192 - (8147*root(z^6 + (163*z^4)/1
2940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2)/
1108852327671535435567321055232 - (505*z)/62857556252809436097422151350778
59328 + 4513/8658597397620778437929792538933565560629231616, z, k))/110389
4245809216 - (1197643*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^
3)/142599321974220092022546432 + (5171*z^2)/110885232767153543556732105523
2 - (505*z)/6285755625280943609742215135077859328 + 4513/86585973976207784
37929792538933565560629231616, z, k)*x)/29805144636848832 + (452809*root(z
^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/1425993219742200920225464
32 + (5171*z^2)/1108852327671535435567321055232 - (505*z)/6285755625280943
609742215135077859328 + 4513/865859739762077843792979253893356556062923161
6, z, k)^2*x)/194734854 - (1241776944*root(z^6 + (163*z^4)/129400936592080
32 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2)/11088523276715
35435567321055232 - (505*z)/6285755625280943609742215135077859328 + 4513/8
658597397620778437929792538933565560629231616, z, k)^3*x)/44521 + (4524079
28832*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/1425993219742
20092022546432 + (5171*z^2)/1108852327671535435567321055232 - (505*z)/6285
755625280943609742215135077859328 + 4513/865859739762077843792979253893356
5560629231616, z, k)^4*x)/211 - 6940988288557056*root(z^6 + (163*z^4)/1294
0093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2)/...
```

3.158 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx}\,dx$

3.158.1 Optimal result

Integrand size = 52, antiderivative size = 25

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx}\,dx=a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

output
```
a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

3.158.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx}\,dx=a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

input
```
Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d
*x^5)/(c + d*x),x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

3.158.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2019, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx} \, dx$$

$$\Big\downarrow \text{2019}$$

$$\int \left(a^2+2abx^2+b^2x^4\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

input
```
Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/
(c + d*x),x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

3.158.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.158.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$a^2x + \frac{2}{3}ab\,x^3 + \frac{1}{5}b^2x^5$	22
norman	$a^2x + \frac{2}{3}ab\,x^3 + \frac{1}{5}b^2x^5$	22
risch	$a^2x + \frac{2}{3}ab\,x^3 + \frac{1}{5}b^2x^5$	22
parallelrisch	$a^2x + \frac{2}{3}ab\,x^3 + \frac{1}{5}b^2x^5$	22
parts	$a^2x + \frac{2}{3}ab\,x^3 + \frac{1}{5}b^2x^5$	22
gosper	$\frac{x(3b^2x^4+10ab\,x^2+15a^2)}{15}$	25

input
```
int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c),x,
method=_RETURNVERBOSE)
```

output
```
a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx}\,dx = \frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x
+c),x, algorithm="fricas")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

3.158.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx}\,dx = a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

input
```
integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2
*c)/(d*x+c),x)
```

output
```
a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx}\,dx = \frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x
+c),x, algorithm="maxima")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx}\,dx = \frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x
+c),x, algorithm="giac")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

3.158.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx}\,dx = a^2\,x + \frac{2\,a\,b\,x^3}{3} + \frac{b^2\,x^5}{5}$$

input
```
int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/
(c + d*x),x)
```

output
```
a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

3.159 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx$

3.159.1 Optimal result

Integrand size = 52, antiderivative size = 94

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx = -\frac{bc(bc^2+2ad^2)\,x}{d^4}+\frac{b(bc^2+2ad^2)\,x^2}{2d^3}-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}+\frac{(bc^2+ad^2)^2\log(c+dx)}{d^5}$$

output
```
-b*c*(2*a*d^2+b*c^2)*x/d^4+1/2*b*(2*a*d^2+b*c^2)*x^2/d^3-1/3*b^2*c*x^3/d^2
+1/4*b^2*x^4/d+(a*d^2+b*c^2)^2*ln(d*x+c)/d^5
```

3.159.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.84

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx = \frac{bdx(12ad^2(-2c+dx)+b(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3))+12(bc^2+ad^2)^2\log(c+dx)}{12d^5}$$

input
```
Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d
*x^5)/(c + d*x)^2,x]
```

output
```
(b*d*x*(12*a*d^2*(-2*c + d*x) + b*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d
^3*x^3)) + 12*(b*c^2 + a*d^2)^2*Log[c + d*x])/(12*d^5)
```

3.159.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.096, Rules used = {2019, 1380, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx$$

$$\Big\downarrow \ 2019$$

$$\int \frac{a^2+2abx^2+b^2x^4}{c+dx}dx$$

$$\Big\downarrow \ 1380$$

$$\frac{\int \frac{b^2(bx^2+a)^2}{c+dx}dx}{b^2}$$

$$\Big\downarrow \ 27$$

$$\int \frac{(a+bx^2)^2}{c+dx}dx$$

$$\Big\downarrow \ 476$$

$$\int \left(\frac{(ad^2+bc^2)^2}{d^4(c+dx)}-\frac{bc(2ad^2+bc^2)}{d^4}+\frac{bx(2ad^2+bc^2)}{d^3}-\frac{b^2cx^2}{d^2}+\frac{b^2x^3}{d}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{(ad^2+bc^2)^2\log(c+dx)}{d^5}-\frac{bcx(2ad^2+bc^2)}{d^4}+\frac{bx^2(2ad^2+bc^2)}{2d^3}-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}$$

input
```
Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/
(c + d*x)^2,x]
```

output
```
-((b*c*(b*c^2 + 2*a*d^2)*x)/d^4) + (b*(b*c^2 + 2*a*d^2)*x^2)/(2*d^3) - (b^
2*c*x^3)/(3*d^2) + (b^2*x^4)/(4*d) + ((b*c^2 + a*d^2)^2*Log[c + d*x])/d^5
```

3.159.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.159.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

method	result
default	$-\frac{b\left(-\frac{b\,x^{4} d^{3}}{4}+\frac{bc\,x^{3} d^{2}}{3}-\frac{\left(2a\,d^{2}+b\,c^{2}\right) x^{2} d}{2}+c\left(2a\,d^{2}+b\,c^{2}\right) x\right)}{d^{4}}+\frac{\left(a^{2} d^{4}+2b\,c^{2} d^{2} a+b^{2} c^{4}\right) \ln\left(dx+c\right)}{d^{5}}$
risch	$\frac{b^{2} x^{4}}{4d}-\frac{b^{2} c\,x^{3}}{3d^{2}}+\frac{ba\,x^{2}}{d}+\frac{b^{2} c^{2} x^{2}}{2d^{3}}-\frac{2bacx}{d^{2}}-\frac{b^{2} c^{3} x}{d^{4}}+\frac{\ln\left(dx+c\right) a^{2}}{d}+\frac{2\ln\left(dx+c\right) b\,c^{2} a}{d^{3}}+\frac{\ln\left(dx+c\right) b^{2} c^{4}}{d^{5}}$
parallelrisch	$\frac{3x^{4} b^{2} d^{4}-4b^{2} c\,x^{3} d^{3}+12x^{2} ab\,d^{4}+6x^{2} b^{2} c^{2} d^{2}+12\ln\left(dx+c\right) a^{2} d^{4}+24\ln\left(dx+c\right) ab\,c^{2} d^{2}+12\ln\left(dx+c\right) b^{2} c^{4}-24xabc\,d^{3}-12x\,b^{2} c^{3} d}{12d^{5}}$
norman	$\frac{\frac{c\left(2b\,c^{2} d^{2} a+b^{2} c^{4}\right)}{d^{5}}+\frac{b^{2} x^{5}}{4}+\frac{b\left(6a\,d^{2}+b\,c^{2}\right) x^{3}}{6d^{2}}-\frac{b^{2} c\,x^{4}}{12d}-\frac{bc\left(2a\,d^{2}+b\,c^{2}\right) x^{2}}{2d^{3}}}{dx+c}+\frac{\left(a^{2} d^{4}+2b\,c^{2} d^{2} a+b^{2} c^{4}\right) \ln\left(dx+c\right)}{d^{5}}$

input
```
int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c)^2,
x,method=_RETURNVERBOSE)
```

output
```
-b/d^4*(-1/4*b*x^4*d^3+1/3*b*c*x^3*d^2-1/2*(2*a*d^2+b*c^2)*x^2*d+c*(2*a*d^
2+b*c^2)*x)+(a^2*d^4+2*a*b*c^2*d^2+b^2*c^4)/d^5*ln(d*x+c)
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(c+dx)^2}\,dx$$
$$= \frac{3\,b^2d^4x^4 - 4\,b^2cd^3x^3 + 6\,(b^2c^2d^2 + 2\,abd^4)x^2 - 12\,(b^2c^3d + 2\,abcd^3)x + 12\,(b^2c^4 + 2\,abc^2d^2 + a^2d^4)\log{(dx}}{12\,d^5}$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x
+c)^2,x, algorithm="fricas")
```

output
```
1/12*(3*b^2*d^4*x^4 - 4*b^2*c*d^3*x^3 + 6*(b^2*c^2*d^2 + 2*a*b*d^4)*x^2 -
12*(b^2*c^3*d + 2*a*b*c*d^3)*x + 12*(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*lo
g(d*x + c))/d^5
```

3.159.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(c+dx)^2}\,dx$$
$$= -\frac{b^2cx^3}{3d^2} + \frac{b^2x^4}{4d} + x^2\left(\frac{ab}{d} + \frac{b^2c^2}{2d^3}\right) + x\left(-\frac{2abc}{d^2} - \frac{b^2c^3}{d^4}\right) + \frac{(ad^2 + bc^2)^2\log{(c+dx)}}{d^5}$$

input
```
integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2
*c)/(d*x+c)**2,x)
```

output
```
-b**2*c*x**3/(3*d**2) + b**2*x**4/(4*d) + x**2*(a*b/d + b**2*c**2/(2*d**3)
) + x*(-2*a*b*c/d**2 - b**2*c**3/d**4) + (a*d**2 + b*c**2)**2*log(c + d*x)
/d**5
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx = \frac{3\,b^2d^3x^4-4\,b^2cd^2x^3+6\,(b^2c^2d+2\,abd^3)x^2-12\,(b^2c^3+2\,abcd^2)x}{12\,d^4} + \frac{(b^2c^4+2\,abc^2d^2+a^2d^4)\log{(dx+c)}}{d^5}$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x
+c)^2,x, algorithm="maxima")
```

output
```
1/12*(3*b^2*d^3*x^4 - 4*b^2*c*d^2*x^3 + 6*(b^2*c^2*d + 2*a*b*d^3)*x^2 - 12
*(b^2*c^3 + 2*a*b*c*d^2)*x)/d^4 + (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*log(
d*x + c)/d^5
```

3.159.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(88) = 176.

Time = 0.28 (sec) , antiderivative size = 365, normalized size of antiderivative = 3.88

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx = -\frac{1}{12}\,b^2d\left(\frac{(dx+c)^4\left(\frac{20\,c}{dx+c}-\frac{60\,c^2}{(dx+c)^2}+\frac{120\,c^3}{(dx+c)^3}-3\right)}{d^6}+\frac{60\,c^4\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^6}-\frac{12\,c^5}{(dx+c)d^6}\right) - \frac{1}{3}\,b^2c\left(\frac{(dx+c)^3\left(\frac{6\,c}{dx+c}-\frac{18\,c^2}{(dx+c)^2}-1\right)}{d^5}-\frac{12\,c^3\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^5}+\frac{3\,c^4}{(dx+c)d^5}\right) - abd\left(\frac{(dx+c)^2\left(\frac{6\,c}{dx+c}-1\right)}{d^4}+\frac{6\,c^2\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^4}-\frac{2\,c^3}{(dx+c)d^4}\right) + 2\,abc\left(\frac{2\,c\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^3}+\frac{dx+c}{d^3}-\frac{c^2}{(dx+c)d^3}\right) - a^2\left(\frac{\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d}-\frac{c}{(dx+c)d}\right) - \frac{a^2c}{(dx+c)d}$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x
+c)^2,x, algorithm="giac")
```

output
```
-1/12*b^2*d*((d*x + c)^4*(20*c/(d*x + c) - 60*c^2/(d*x + c)^2 + 120*c^3/(d
*x + c)^3 - 3)/d^6 + 60*c^4*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^6 - 1
2*c^5/((d*x + c)*d^6)) - 1/3*b^2*c*((d*x + c)^3*(6*c/(d*x + c) - 18*c^2/(d
*x + c)^2 - 1)/d^5 - 12*c^3*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^5 + 3
*c^4/((d*x + c)*d^5)) - a*b*d*((d*x + c)^2*(6*c/(d*x + c) - 1)/d^4 + 6*c^2
*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^4 - 2*c^3/((d*x + c)*d^4)) + 2*a
*b*c*(2*c*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^3 + (d*x + c)/d^3 - c^2
/((d*x + c)*d^3)) - a^2*(log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d - c/((d*
x + c)*d)) - a^2*c/((d*x + c)*d)
```

3.159.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx = x^2\left(\frac{b^2\,c^2}{2\,d^3}+\frac{a\,b}{d}\right)+\frac{\ln\left(c+d\,x\right)\,\left(a^2\,d^4+2\,a\,b\,c^2\,d^2+b^2\,c^4\right)}{d^5} + \frac{b^2\,x^4}{4\,d}-\frac{b^2\,c\,x^3}{3\,d^2}-\frac{c\,x\,\left(\frac{b^2\,c^2}{d^3}+\frac{2\,a\,b}{d}\right)}{d}$$

input
```
int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/
(c + d*x)^2,x)
```

output
```
x^2*((b^2*c^2)/(2*d^3) + (a*b)/d) + (log(c + d*x)*(a^2*d^4 + b^2*c^4 + 2*a
*b*c^2*d^2))/d^5 + (b^2*x^4)/(4*d) - (b^2*c*x^3)/(3*d^2) - (c*x*((b^2*c^2)
/d^3 + (2*a*b)/d))/d
```

3.160 $\int (b+2cx) \left(bx+cx^2\right)^{13} \, dx$

3.160.1 Optimal result

Integrand size = 18, antiderivative size = 15

$$\int (b+2cx) \left(bx+cx^2\right)^{13} \, dx = \frac{1}{14} \left(bx+cx^2\right)^{14}$$

output
```
1/14*(c*x^2+b*x)^14
```

3.160.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(15) = 30.

Time = 0.00 (sec) , antiderivative size = 172, normalized size of antiderivative = 11.47

$$\begin{aligned}\int (b+2cx) \left(bx+cx^2\right)^{13} \, dx = {} & \frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17} \\ & +\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20} \\ & +\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24} \\ & +26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}\end{aligned}$$

input
```
Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

output
```
(b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (
143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7
*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^2
4)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)
/14
```

3.160.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)\left(bx+cx^2\right)^{13}dx$$

$$\Big\downarrow \text{1104}$$

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

input
```
Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

output
```
(b*x + c*x^2)^14/14
```

3.160.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

3.160.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result
gosper	$\frac{(cx+b)^{14} x^{14}}{14}$
default	$\frac{(c\,x^2+bx)^{14}}{14}$
norman	$26b^3c^{11}x^{25} + \frac{13}{2}x^{26}b^2c^{12} + b\,c^{13}x^{27} + \frac{1}{14}c^{14}x^{28} + \frac{1}{14}x^{14}b^{14} + b^{13}c\,x^{15} + \frac{13}{2}x^{16}b^{12}c^2 + 26b^{11}c^3x^1$
risch	$26b^3c^{11}x^{25} + \frac{13}{2}x^{26}b^2c^{12} + b\,c^{13}x^{27} + \frac{1}{14}c^{14}x^{28} + \frac{1}{14}x^{14}b^{14} + b^{13}c\,x^{15} + \frac{13}{2}x^{16}b^{12}c^2 + 26b^{11}c^3x^1$
parallelrisch	$26b^3c^{11}x^{25} + \frac{13}{2}x^{26}b^2c^{12} + b\,c^{13}x^{27} + \frac{1}{14}c^{14}x^{28} + \frac{1}{14}x^{14}b^{14} + b^{13}c\,x^{15} + \frac{13}{2}x^{16}b^{12}c^2 + 26b^{11}c^3x^1$

input
```
int((2*c*x+b)*(c*x^2+b*x)^13,x,method=_RETURNVERBOSE)
```

output
```
1/14*(c*x+b)^14*x^14
```

3.160.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 10.27

$$\begin{aligned}\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx =& \frac{1}{14}\,c^{14}x^{28} + bc^{13}x^{27} + \frac{13}{2}\,b^2c^{12}x^{26} + 26\,b^3c^{11}x^{25} \\ &+ \frac{143}{2}\,b^4c^{10}x^{24} + 143\,b^5c^9x^{23} + \frac{429}{2}\,b^6c^8x^{22} \\ &+ \frac{1716}{7}\,b^7c^7x^{21} + \frac{429}{2}\,b^8c^6x^{20} + 143\,b^9c^5x^{19} + \frac{143}{2}\,b^{10}c^4x^{18} \\ &+ 26\,b^{11}c^3x^{17} + \frac{13}{2}\,b^{12}c^2x^{16} + b^{13}cx^{15} + \frac{1}{14}\,b^{14}x^{14}\end{aligned}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="fricas")
```

output
```
1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143
/2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*
x^21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^
11*c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

3.160.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 175, normalized size of antiderivative = 11.67

$$\begin{aligned}\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx &= \frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13b^{12}c^2x^{16}}{2}+26b^{11}c^3x^{17}\\&\quad+\frac{143b^{10}c^4x^{18}}{2}+143b^9c^5x^{19}+\frac{429b^8c^6x^{20}}{2}\\&\quad+\frac{1716b^7c^7x^{21}}{7}+\frac{429b^6c^8x^{22}}{2}+143b^5c^9x^{23}+\frac{143b^4c^{10}x^{24}}{2}\\&\quad+26b^3c^{11}x^{25}+\frac{13b^2c^{12}x^{26}}{2}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}\end{aligned}$$

input
```
integrate((2*c*x+b)*(c*x**2+b*x)**13,x)
```

output
```
b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**
17 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2
+ 1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 1
43*b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c*
*13*x**27 + c**14*x**28/14
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx = \frac{1}{14}\left(cx^2+bx\right)^{14}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="maxima")
```

output
```
1/14*(c*x^2 + b*x)^14
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(cx^2+bx\right)^{14}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="giac")
```

output
```
1/14*(c*x^2 + b*x)^14
```

3.160.9 Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 154, normalized size of antiderivative = 10.27

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{b^{14}\,x^{14}}{14}+b^{13}\,c\,x^{15}+\frac{13\,b^{12}\,c^2\,x^{16}}{2}+26\,b^{11}\,c^3\,x^{17}+\frac{143\,b^{10}\,c^4\,x^{18}}{2}+143\,b^9\,c^5\,x^{19}+\frac{429\,b^8\,c^6\,x^{20}}{2}+\frac{1716\,b^7\,c^7\,x^{21}}{7}+\frac{429\,b^6\,c^8\,x^{22}}{2}+143\,b^5\,c^9\,x^{23}+\frac{143\,b^4\,c^{10}\,x^{24}}{2}+26\,b^3\,c^{11}\,x^{25}+\frac{13\,b^2\,c^{12}\,x^{26}}{2}+b\,c^{13}\,x^{27}+\frac{c^{14}\,x^{28}}{14}$$

input
```
int((b*x + c*x^2)^13*(b + 2*c*x),x)
```

output
```
(b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2
*x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (
429*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b
^5*c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26
)/2
```

3.161 $\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx$

3.161.1 Optimal result

Integrand size = 23, antiderivative size = 16

$$\int x^{14}\left(b+2cx^2\right)\left(bx+cx^3\right)^{13}\,dx=\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

output
```
1/28*x^28*(c*x^2+b)^14
```

3.161.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 182, normalized size of antiderivative = 11.38

$$\begin{aligned}\int x^{14}\left(b+2cx^2\right)\left(bx+cx^3\right)^{13}\,dx&=\frac{b^{14}x^{28}}{28}+\frac{1}{2}b^{13}cx^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}\\&+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}\\&+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}+\frac{143}{2}b^5c^9x^{46}+\frac{143}{4}b^4c^{10}x^{48}\\&+13b^3c^{11}x^{50}+\frac{13}{4}b^2c^{12}x^{52}+\frac{1}{2}bc^{13}x^{54}+\frac{c^{14}x^{56}}{28}\end{aligned}$$

input
```
Integrate[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]
```

output
```
(b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (
858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b
^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/
2 + (c^14*x^56)/28
```

3.161.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {9, 354, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{14}\left(b+2cx^2\right)\left(bx+cx^3\right)^{13}dx$$

$$\Big\downarrow \; 9$$

$$\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int x^{26}\left(cx^2+b\right)^{13}\left(2cx^2+b\right)dx^2$$

$$\Big\downarrow \; 83$$

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

input
```
Int[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]
```

output
```
(x^28*(b + c*x^2)^14)/28
```

3.161.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.161.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{x^{28}(c\,x^2+b)^{14}}{28}$
default	$\frac{1}{2}b\,c^{13}x^{54}+\frac{1}{28}c^{14}x^{56}+\frac{143}{2}x^{46}b^5c^9+\frac{143}{4}x^{48}b^4c^{10}+13x^{50}b^3c^{11}+\frac{13}{4}x^{52}b^2c^{12}+\frac{429}{4}x^{44}b^6c^8+\frac{858}{7}$
risch	$\frac{1}{2}b\,c^{13}x^{54}+\frac{1}{28}c^{14}x^{56}+\frac{143}{2}x^{46}b^5c^9+\frac{143}{4}x^{48}b^4c^{10}+13x^{50}b^3c^{11}+\frac{13}{4}x^{52}b^2c^{12}+\frac{429}{4}x^{44}b^6c^8+\frac{858}{7}$
parallelrisch	$\frac{1}{2}b\,c^{13}x^{54}+\frac{1}{28}c^{14}x^{56}+\frac{143}{2}x^{46}b^5c^9+\frac{143}{4}x^{48}b^4c^{10}+13x^{50}b^3c^{11}+\frac{13}{4}x^{52}b^2c^{12}+\frac{429}{4}x^{44}b^6c^8+\frac{858}{7}$

input
```
int(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x,method=_RETURNVERBOSE)
```

output
```
1/28*x^28*(c*x^2+b)^14
```

3.161.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(14) = 28.

Time = 0.24 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx=\frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^2c^{12}x^{52}+13\,b^3c^{11}x^{50}+\frac{143}{4}\,b^4c^{10}x^{48}+\frac{143}{2}\,b^5c^9x^{46}+\frac{429}{4}\,b^6c^8x^{44}+\frac{858}{7}\,b^7c^7x^{42}+\frac{429}{4}\,b^8c^6x^{40}+\frac{143}{2}\,b^9c^5x^{38}+\frac{143}{4}\,b^{10}c^4x^{36}+13\,b^{11}c^3x^{34}+\frac{13}{4}\,b^{12}c^2x^{32}+\frac{1}{2}\,b^{13}cx^{30}+\frac{1}{28}\,b^{14}x^{28}$$

input
```
integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="fricas")
```

output
```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7
*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36
+ 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

3.161.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(12) = 24.

Time = 0.05 (sec) , antiderivative size = 182, normalized size of antiderivative = 11.38

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx=\frac{b^{14}x^{28}}{28}+\frac{b^{13}cx^{30}}{2}+\frac{13b^{12}c^2x^{32}}{4}+13b^{11}c^3x^{34}+\frac{143b^{10}c^4x^{36}}{4}+\frac{143b^9c^5x^{38}}{2}+\frac{429b^8c^6x^{40}}{4}+\frac{858b^7c^7x^{42}}{7}+\frac{429b^6c^8x^{44}}{4}+\frac{143b^5c^9x^{46}}{2}+\frac{143b^4c^{10}x^{48}}{4}+13b^3c^{11}x^{50}+\frac{13b^2c^{12}x^{52}}{4}+\frac{bc^{13}x^{54}}{2}+\frac{c^{14}x^{56}}{28}$$

input
```
integrate(x**14*(2*c*x**2+b)*(c*x**3+b*x)**13,x)
```

output
```
b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x
**34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**4
0/4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/
2 + 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 +
 b*c**13*x**54/2 + c**14*x**56/28
```

3.161.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(14) = 28.

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx = \frac{1}{28}\,c^{14}x^{56} + \frac{1}{2}\,bc^{13}x^{54} + \frac{13}{4}\,b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}\,b^4c^{10}x^{48} + \frac{143}{2}\,b^5c^9x^{46} + \frac{429}{4}\,b^6c^8x^{44} + \frac{858}{7}\,b^7c^7x^{42} + \frac{429}{4}\,b^8c^6x^{40} + \frac{143}{2}\,b^9c^5x^{38} + \frac{143}{4}\,b^{10}c^4x^{36} + 13\,b^{11}c^3x^{34} + \frac{13}{4}\,b^{12}c^2x^{32} + \frac{1}{2}\,b^{13}cx^{30} + \frac{1}{28}\,b^{14}x^{28}$$

input
```
integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="maxima")
```

output
```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7
*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36
+ 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

3.161.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(14) = 28.

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx = \frac{1}{28}\,c^{14}x^{56} + \frac{1}{2}\,bc^{13}x^{54} + \frac{13}{4}\,b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}\,b^4c^{10}x^{48} + \frac{143}{2}\,b^5c^9x^{46} + \frac{429}{4}\,b^6c^8x^{44} + \frac{858}{7}\,b^7c^7x^{42} + \frac{429}{4}\,b^8c^6x^{40} + \frac{143}{2}\,b^9c^5x^{38} + \frac{143}{4}\,b^{10}c^4x^{36} + 13\,b^{11}c^3x^{34} + \frac{13}{4}\,b^{12}c^2x^{32} + \frac{1}{2}\,b^{13}cx^{30} + \frac{1}{28}\,b^{14}x^{28}$$

input
```
integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="giac")
```

output
```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7
*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36
+ 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

3.161.9 Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx = \frac{b^{14}\,x^{28}}{28}+\frac{b^{13}\,c\,x^{30}}{2}+\frac{13\,b^{12}\,c^2\,x^{32}}{4}+13\,b^{11}\,c^3\,x^{34}+\frac{143\,b^{10}\,c^4\,x^{36}}{4}+\frac{143\,b^9\,c^5\,x^{38}}{2}+\frac{429\,b^8\,c^6\,x^{40}}{4}+\frac{858\,b^7\,c^7\,x^{42}}{7}+\frac{429\,b^6\,c^8\,x^{44}}{4}+\frac{143\,b^5\,c^9\,x^{46}}{2}+\frac{143\,b^4\,c^{10}\,x^{48}}{4}+13\,b^3\,c^{11}\,x^{50}+\frac{13\,b^2\,c^{12}\,x^{52}}{4}+\frac{b\,c^{13}\,x^{54}}{2}+\frac{c^{14}\,x^{56}}{28}$$

input
```
int(x^14*(b*x + c*x^3)^13*(b + 2*c*x^2),x)
```

output
```
(b^14*x^28)/28 + (c^14*x^56)/28 + (b^13*c*x^30)/2 + (b*c^13*x^54)/2 + (13*
b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5
*x^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44
)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (1
3*b^2*c^12*x^52)/4
```

3.162 $\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13}\,dx$

3.162.1 Optimal result

Integrand size = 23, antiderivative size = 16

$$\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13}\,dx=\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

output
```
1/42*x^42*(c*x^3+b)^14
```

3.162.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 186 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 186, normalized size of antiderivative = 11.62

$$\begin{aligned}\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13}\,dx=&\ \frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}\\&+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}\\&+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{6}b^4c^{10}x^{72}\\&+\frac{26}{3}b^3c^{11}x^{75}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}bc^{13}x^{81}+\frac{c^{14}x^{84}}{42}\end{aligned}$$

input
```
Integrate[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]
```

output
```
(b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^5
1)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2
 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (1
43*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^1
3*x^81)/3 + (c^14*x^84)/42
```

3.162.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {9, 948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{28}\left(b+2cx^3\right)\left(bx+cx^4\right)^{13}dx$$

$$\Big\downarrow \; 9$$

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{3}\int x^{39}\left(cx^3+b\right)^{13}\left(2cx^3+b\right)dx^3$$

$$\Big\downarrow \; 83$$

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

input
```
Int[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]
```

output
```
(x^42*(b + c*x^3)^14)/42
```

3.162.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.162.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{x^{42}(c\,x^3+b)^{14}}{42}$
default	$\frac{1}{42}c^{14}x^{84}+\frac{13}{6}x^{78}b^2c^{12}+\frac{1}{3}b\,c^{13}x^{81}+\frac{143}{3}x^{69}b^5c^9+\frac{143}{6}x^{72}b^4c^{10}+\frac{26}{3}x^{75}b^3c^{11}+\frac{572}{7}x^{63}b^7c^7+\frac{143}{2}$
risch	$\frac{1}{42}c^{14}x^{84}+\frac{13}{6}x^{78}b^2c^{12}+\frac{1}{3}b\,c^{13}x^{81}+\frac{143}{3}x^{69}b^5c^9+\frac{143}{6}x^{72}b^4c^{10}+\frac{26}{3}x^{75}b^3c^{11}+\frac{572}{7}x^{63}b^7c^7+\frac{143}{2}$
parallelrisch	$\frac{1}{42}c^{14}x^{84}+\frac{13}{6}x^{78}b^2c^{12}+\frac{1}{3}b\,c^{13}x^{81}+\frac{143}{3}x^{69}b^5c^9+\frac{143}{6}x^{72}b^4c^{10}+\frac{26}{3}x^{75}b^3c^{11}+\frac{572}{7}x^{63}b^7c^7+\frac{143}{2}$

input
```
int(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x,method=_RETURNVERBOSE)
```

output
```
1/42*x^42*(c*x^3+b)^14
```

3.162.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13}\,dx = \frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^2c^{12}x^{78}+\frac{26}{3}b^3c^{11}x^{75}+\frac{143}{6}b^4c^{10}x^{72}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{2}b^6c^8x^{66}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^8c^6x^{60}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{6}b^{10}c^4x^{54}+\frac{26}{3}b^{11}c^3x^{51}+\frac{13}{6}b^{12}c^2x^{48}+\frac{1}{3}b^{13}cx^{45}+\frac{1}{42}b^{14}x^{42}$$

input
```
integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="fricas")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b
^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^5
4 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*
x^42
```

3.162.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(12) = 24$.

Time = 0.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 11.56

$$\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13}\,dx = \frac{b^{14}x^{42}}{42}+\frac{b^{13}cx^{45}}{3}+\frac{13b^{12}c^2x^{48}}{6}+\frac{26b^{11}c^3x^{51}}{3}+\frac{143b^{10}c^4x^{54}}{6}+\frac{143b^9c^5x^{57}}{3}+\frac{143b^8c^6x^{60}}{2}+\frac{572b^7c^7x^{63}}{7}+\frac{143b^6c^8x^{66}}{2}+\frac{143b^5c^9x^{69}}{3}+\frac{143b^4c^{10}x^{72}}{6}+\frac{26b^3c^{11}x^{75}}{3}+\frac{13b^2c^{12}x^{78}}{6}+\frac{bc^{13}x^{81}}{3}+\frac{c^{14}x^{84}}{42}$$

input
```
integrate(x**28*(2*c*x**3+b)*(c*x**4+b*x)**13,x)
```

output
```
b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x
**51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x*
*60/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**6
9/3 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78
/6 + b*c**13*x**81/3 + c**14*x**84/42
```

3.162.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13}\,dx = \frac{1}{42}\,c^{14}x^{84} + \frac{1}{3}\,bc^{13}x^{81} + \frac{13}{6}\,b^2c^{12}x^{78} + \frac{26}{3}\,b^3c^{11}x^{75} + \frac{143}{6}\,b^4c^{10}x^{72} + \frac{143}{3}\,b^5c^9x^{69} + \frac{143}{2}\,b^6c^8x^{66} + \frac{572}{7}\,b^7c^7x^{63} + \frac{143}{2}\,b^8c^6x^{60} + \frac{143}{3}\,b^9c^5x^{57} + \frac{143}{6}\,b^{10}c^4x^{54} + \frac{26}{3}\,b^{11}c^3x^{51} + \frac{13}{6}\,b^{12}c^2x^{48} + \frac{1}{3}\,b^{13}cx^{45} + \frac{1}{42}\,b^{14}x^{42}$$

input
```
integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="maxima")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b
^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^5
4 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*
x^42
```

3.162.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13}\,dx = \frac{1}{42}\,c^{14}x^{84} + \frac{1}{3}\,bc^{13}x^{81} + \frac{13}{6}\,b^2c^{12}x^{78} + \frac{26}{3}\,b^3c^{11}x^{75} + \frac{143}{6}\,b^4c^{10}x^{72} + \frac{143}{3}\,b^5c^9x^{69} + \frac{143}{2}\,b^6c^8x^{66} + \frac{572}{7}\,b^7c^7x^{63} + \frac{143}{2}\,b^8c^6x^{60} + \frac{143}{3}\,b^9c^5x^{57} + \frac{143}{6}\,b^{10}c^4x^{54} + \frac{26}{3}\,b^{11}c^3x^{51} + \frac{13}{6}\,b^{12}c^2x^{48} + \frac{1}{3}\,b^{13}cx^{45} + \frac{1}{42}\,b^{14}x^{42}$$

input
```
integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="giac")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b
^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^5
4 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*
x^42
```

3.162.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{28}\left(b+2cx^3\right)\left(bx+cx^4\right)^{13}\,dx=\frac{b^{14}\,x^{42}}{42}+\frac{b^{13}\,c\,x^{45}}{3}+\frac{13\,b^{12}\,c^2\,x^{48}}{6}+\frac{26\,b^{11}\,c^3\,x^{51}}{3}+\frac{143\,b^{10}\,c^4\,x^{54}}{6}+\frac{143\,b^9\,c^5\,x^{57}}{3}+\frac{143\,b^8\,c^6\,x^{60}}{2}+\frac{572\,b^7\,c^7\,x^{63}}{7}+\frac{143\,b^6\,c^8\,x^{66}}{2}+\frac{143\,b^5\,c^9\,x^{69}}{3}+\frac{143\,b^4\,c^{10}\,x^{72}}{6}+\frac{26\,b^3\,c^{11}\,x^{75}}{3}+\frac{13\,b^2\,c^{12}\,x^{78}}{6}+\frac{b\,c^{13}\,x^{81}}{3}+\frac{c^{14}\,x^{84}}{42}$$

input
```
int(x^28*(b*x + c*x^4)^13*(b + 2*c*x^3),x)
```

output
```
(b^14*x^42)/42 + (c^14*x^84)/42 + (b^13*c*x^45)/3 + (b*c^13*x^81)/3 + (13*
b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9
*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*
x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75
)/3 + (13*b^2*c^12*x^78)/6
```

3.163 $\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx$

3.163.1 Optimal result

Integrand size = 29, antiderivative size = 21

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

output
```
1/14*x^(14*n)*(b+c*x^n)^14/n
```

3.163.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

input
```
Integrate[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]
```

output
```
(x^(14*n)*(b + c*x^n)^14)/(14*n)
```

3.163.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {10, 948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{14(n-1)}(b+2cx^n)\left(bx+cx^{n+1}\right)^{13}dx$$
$$\Big\downarrow \text{10}$$
$$\int x^{14n-1}(b+cx^n)^{13}(b+2cx^n)dx$$
$$\Big\downarrow \text{948}$$
$$\frac{\int x^{13n}(cx^n+b)^{13}(2cx^n+b)dx^n}{n}$$
$$\Big\downarrow \text{83}$$
$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

input
```
Int[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]
```

output
```
(x^(14*n)*(b + c*x^n)^14)/(14*n)
```

3.163.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.163.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(19) = 38.

Time = 0.02 (sec) , antiderivative size = 230, normalized size of antiderivative = 10.95

$$\frac{c^{14}x^{28n}}{14n}+\frac{b\,c^{13}x^{27n}}{n}+\frac{13c^{12}x^{26n}b^2}{2n}+\frac{26c^{11}b^3x^{25n}}{n}+\frac{143c^{10}x^{24n}b^4}{2n}+\frac{143c^9b^5x^{23n}}{n}+\frac{429c^8x^{22n}b^6}{2n}+\frac{1716b^7c^7x^{21n}}{7n}+\frac{42}{}$$

input
```
int(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x)
```

output
```
1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*c^12/n*(x^n)^26*b^2+26*c^11*b^
3/n*(x^n)^25+143/2*c^10/n*(x^n)^24*b^4+143*c^9*b^5/n*(x^n)^23+429/2*c^8/n*
(x^n)^22*b^6+1716/7*b^7*c^7/n*(x^n)^21+429/2*c^6/n*(x^n)^20*b^8+143*b^9*c^
5/n*(x^n)^19+143/2*c^4/n*(x^n)^18*b^10+26*b^11*c^3/n*(x^n)^17+13/2*c^2/n*(
x^n)^16*b^12+b^13*c/n*(x^n)^15+1/14/n*(x^n)^14*b^14
```

3.163.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(19) = 38.

Time = 0.25 (sec) , antiderivative size = 262, normalized size of antiderivative = 12.48

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx$$
$$=\frac{b^{14}x^{14}x^{14n+14}+14\,b^{13}cx^{13}x^{15n+15}+91\,b^{12}c^2x^{12}x^{16n+16}+364\,b^{11}c^3x^{11}x^{17n+17}+1001\,b^{10}c^4x^{10}x^{18n+18}+20}{}$$

input
```
integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="fricas
")
```

output
```
1/14*(b^14*x^14*x^(14*n + 14) + 14*b^13*c*x^13*x^(15*n + 15) + 91*b^12*c^2
*x^12*x^(16*n + 16) + 364*b^11*c^3*x^11*x^(17*n + 17) + 1001*b^10*c^4*x^10
*x^(18*n + 18) + 2002*b^9*c^5*x^9*x^(19*n + 19) + 3003*b^8*c^6*x^8*x^(20*n
 + 20) + 3432*b^7*c^7*x^7*x^(21*n + 21) + 3003*b^6*c^8*x^6*x^(22*n + 22) +
 2002*b^5*c^9*x^5*x^(23*n + 23) + 1001*b^4*c^10*x^4*x^(24*n + 24) + 364*b^
3*c^11*x^3*x^(25*n + 25) + 91*b^2*c^12*x^2*x^(26*n + 26) + 14*b*c^13*x*x^(
27*n + 27) + c^14*x^(28*n + 28))/(n*x^28)
```

3.163.6 Sympy [F(-1)]

Timed out.

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx=\text{Timed out}$$

input
```
integrate(x**(-14+14*n)*(b+2*c*x**n)*(b*x+c*x**(1+n))**13,x)
```

output
```
Timed out
```

3.163.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(19) = 38.

Time = 0.20 (sec) , antiderivative size = 229, normalized size of antiderivative = 10.90

$$\begin{aligned}\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx=&\ \frac{c^{14}x^{28n}}{14n}+\frac{bc^{13}x^{27n}}{n}+\frac{13b^2c^{12}x^{26n}}{2n}\\&+\frac{26b^3c^{11}x^{25n}}{n}+\frac{143b^4c^{10}x^{24n}}{2n}+\frac{143b^5c^9x^{23n}}{n}\\&+\frac{429b^6c^8x^{22n}}{2n}+\frac{1716b^7c^7x^{21n}}{7n}+\frac{429b^8c^6x^{20n}}{2n}\\&+\frac{143b^9c^5x^{19n}}{n}+\frac{143b^{10}c^4x^{18n}}{2n}+\frac{26b^{11}c^3x^{17n}}{n}\\&+\frac{13b^{12}c^2x^{16n}}{2n}+\frac{b^{13}cx^{15n}}{n}+\frac{b^{14}x^{14n}}{14n}\end{aligned}$$

input
```
integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="maxima
")
```

output
```
1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b
^3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n +
429/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20
*n)/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x
^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14
*n)/n
```

3.163.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(19) = 38$.

Time = 0.50 (sec) , antiderivative size = 189, normalized size of antiderivative = 9.00

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx$$
$$= \frac{c^{14}x^{28\,n} + 14\,bc^{13}x^{27\,n} + 91\,b^2c^{12}x^{26\,n} + 364\,b^3c^{11}x^{25\,n} + 1001\,b^4c^{10}x^{24\,n} + 2002\,b^5c^9x^{23\,n} + 3003\,b^6c^8x^{22\,n} + \cdots}{}$$

input
```
integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="giac")
```

output
```
1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*
c^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*
c^8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^
5*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*
x^(16*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

3.163.9 Mupad [B] (verification not implemented)

Time = 10.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 10.90

$$\begin{aligned}\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx &= \frac{b^{14}\,x^{14\,n}}{14\,n} + \frac{c^{14}\,x^{28\,n}}{14\,n} + \frac{13\,b^{12}\,c^2\,x^{16\,n}}{2\,n} \\ &+ \frac{26\,b^{11}\,c^3\,x^{17\,n}}{n} + \frac{143\,b^{10}\,c^4\,x^{18\,n}}{2\,n} + \frac{143\,b^9\,c^5\,x^{19\,n}}{n} \\ &+ \frac{429\,b^8\,c^6\,x^{20\,n}}{2\,n} + \frac{1716\,b^7\,c^7\,x^{21\,n}}{7\,n} + \frac{429\,b^6\,c^8\,x^{22\,n}}{2\,n} \\ &+ \frac{143\,b^5\,c^9\,x^{23\,n}}{n} + \frac{143\,b^4\,c^{10}\,x^{24\,n}}{2\,n} + \frac{26\,b^3\,c^{11}\,x^{25\,n}}{n} \\ &+ \frac{13\,b^2\,c^{12}\,x^{26\,n}}{2\,n} + \frac{b^{13}\,c\,x^{15\,n}}{n} + \frac{b\,c^{13}\,x^{27\,n}}{n}\end{aligned}$$

input
```
int(x^(14*n - 14)*(b*x + c*x^(n + 1))^13*(b + 2*c*x^n),x)
```

output
```
(b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(
2*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9
*c^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/
(7*n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4
*c^10*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/
(2*n) + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

3.163. $\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx$

3.164 $\int \frac{b+2cx}{bx+cx^2}\,dx$

3.164.1 Optimal result

Integrand size = 18, antiderivative size = 10

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(bx+cx^2\right)$$

output
```
ln(c*x^2+b*x)
```

3.164.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log(x)+\log(b+cx)$$

input
```
Integrate[(b + 2*c*x)/(b*x + c*x^2),x]
```

output
```
Log[x] + Log[b + c*x]
```

3.164.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx}{bx+cx^2} \, dx$$

$$\Big\downarrow \text{1103}$$

$$\log \left(bx+cx^2\right)$$

input
```
Int[(b + 2*c*x)/(b*x + c*x^2),x]
```

output
```
Log[b*x + c*x^2]
```

3.164.3.1 Defintions of rubi rules used

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.164.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
default	$\ln\left(x(cx+b)\right)$	9
norman	$\ln\left(x\right)+\ln\left(cx+b\right)$	10
parallelrisch	$\ln\left(x\right)+\ln\left(cx+b\right)$	10
derivativedivides	$\ln\left(c\,x^2+bx\right)$	11
risch	$\ln\left(c\,x^2+bx\right)$	11

input
```
int((2*c*x+b)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x*(c*x+b))
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(cx^2+bx\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
log(c*x^2 + b*x)
```

3.164.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(bx+cx^2\right)$$

input
```
integrate((2*c*x+b)/(c*x**2+b*x),x)
```

output
```
log(b*x + c*x**2)
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(cx^2+bx\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
log(c*x^2 + b*x)
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(\left|cx^2+bx\right|\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
log(abs(c*x^2 + b*x))
```

3.164.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \ln\left(x\,(b+c\,x)\right)$$

input
```
int((b + 2*c*x)/(b*x + c*x^2),x)
```

output
```
log(x*(b + c*x))
```

3.165 $\int \frac{b+2cx^2}{bx+cx^3} \, dx$

3.165.1 Optimal result

Integrand size = 20, antiderivative size = 15

$$\int \frac{b+2cx^2}{bx+cx^3} \, dx = \log(x) + \frac{1}{2}\log\left(b+cx^2\right)$$

output
```
ln(x)+1/2*ln(c*x^2+b)
```

3.165.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^2}{bx+cx^3} \, dx = \log(x) + \frac{1}{2}\log\left(b+cx^2\right)$$

input
```
Integrate[(b + 2*c*x^2)/(b*x + c*x^3),x]
```

output
```
Log[x] + Log[b + c*x^2]/2
```

3.165.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2026, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^2}{bx+cx^3} \, dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{b+2cx^2}{x(b+cx^2)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{2cx^2+b}{x^2(cx^2+b)}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{c}{cx^2+b}+\frac{1}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\log\left(b+cx^2\right)+\log\left(x^2\right)\right)$$

input
```
Int[(b + 2*c*x^2)/(b*x + c*x^3),x]
```

output
```
(Log[x^2] + Log[b + c*x^2])/2
```

3.165.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.165.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\ln(x)+\frac{\ln(c\,x^2+b)}{2}$	14
norman	$\ln(x)+\frac{\ln(c\,x^2+b)}{2}$	14
risch	$\ln(x)+\frac{\ln(c\,x^2+b)}{2}$	14
parallelrisch	$\ln(x)+\frac{\ln(c\,x^2+b)}{2}$	14

input
```
int((2*c*x^2+b)/(c*x^3+b*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+1/2*ln(c*x^2+b)
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx = \frac{1}{2}\log\left(cx^2+b\right)+\log(x)$$

input
```
integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="fricas")
```

output
```
1/2*log(c*x^2 + b) + log(x)
```

3.165.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{b + 2cx^2}{bx + cx^3}\, dx = \log(x) + \frac{\log\left(\frac{b}{c} + x^2\right)}{2}$$

input
```
integrate((2*c*x**2+b)/(c*x**3+b*x),x)
```

output
```
log(x) + log(b/c + x**2)/2
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b + 2cx^2}{bx + cx^3}\, dx = \frac{1}{2}\log\left(cx^2 + b\right) + \log(x)$$

input
```
integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="maxima")
```

output
```
1/2*log(c*x^2 + b) + log(x)
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{b + 2cx^2}{bx + cx^3}\, dx = \frac{1}{2}\log\left(x^2\right) + \frac{1}{2}\log\left(\left|cx^2 + b\right|\right)$$

input
```
integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="giac")
```

output
```
1/2*log(x^2) + 1/2*log(abs(c*x^2 + b))
```

3.165.9 Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx = \frac{\ln\left(c\,x^2+b\right)}{2}+\ln\left(x\right)$$

input
```
int((b + 2*c*x^2)/(b*x + c*x^3),x)
```

output
```
log(b + c*x^2)/2 + log(x)
```

3.166 $\int \frac{b+2cx^3}{bx+cx^4} \, dx$

3.166.1 Optimal result

Integrand size = 20, antiderivative size = 15

$$\int \frac{b+2cx^3}{bx+cx^4} \, dx = \log(x) + \frac{1}{3} \log\left(b+cx^3\right)$$

output
```
ln(x)+1/3*ln(c*x^3+b)
```

3.166.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^3}{bx+cx^4} \, dx = \log(x) + \frac{1}{3} \log\left(b+cx^3\right)$$

input
```
Integrate[(b + 2*c*x^3)/(b*x + c*x^4),x]
```

output
```
Log[x] + Log[b + c*x^3]/3
```

3.166.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2026, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx$$
$$\Big\downarrow \text{2026}$$
$$\int \frac{b+2cx^3}{x\left(b+cx^3\right)}dx$$
$$\Big\downarrow \text{948}$$
$$\frac{1}{3}\int \frac{2cx^3+b}{x^3\left(cx^3+b\right)}dx^3$$
$$\Big\downarrow \text{86}$$
$$\frac{1}{3}\int \left(\frac{c}{cx^3+b}+\frac{1}{x^3}\right)dx^3$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3}\left(\log\left(b+cx^3\right)+\log\left(x^3\right)\right)$$

input
```
Int[(b + 2*c*x^3)/(b*x + c*x^4),x]
```

output
```
(Log[x^3] + Log[b + c*x^3])/3
```

3.166.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.166.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\ln(x)+\frac{\ln(c\,x^3+b)}{3}$	14
norman	$\ln(x)+\frac{\ln(c\,x^3+b)}{3}$	14
risch	$\ln(x)+\frac{\ln(c\,x^3+b)}{3}$	14
parallelrisch	$\ln(x)+\frac{\ln(c\,x^3+b)}{3}$	14

input
```
int((2*c*x^3+b)/(c*x^4+b*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+1/3*ln(c*x^3+b)
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx=\frac{1}{3}\log\left(cx^3+b\right)+\log(x)$$

input
```
integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="fricas")
```

output
```
1/3*log(c*x^3 + b) + log(x)
```

3.166.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx = \log{(x)} + \frac{\log{\left(\frac{b}{c}+x^3\right)}}{3}$$

input
```
integrate((2*c*x**3+b)/(c*x**4+b*x),x)
```

output
```
log(x) + log(b/c + x**3)/3
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx = \frac{1}{3}\log\left(cx^3+b\right) + \log\left(x\right)$$

input
```
integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="maxima")
```

output
```
1/3*log(c*x^3 + b) + log(x)
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx = \frac{1}{3}\log\left(\left|cx^3+b\right|\right) + \log\left(\left|x\right|\right)$$

input
```
integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="giac")
```

output
```
1/3*log(abs(c*x^3 + b)) + log(abs(x))
```

3.166.9 Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^3}{bx+cx^4}\,dx=\frac{\ln\left(c\,x^3+b\right)}{3}+\ln\left(x\right)$$

input
```
int((b + 2*c*x^3)/(b*x + c*x^4),x)
```

output
```
log(b + c*x^3)/3 + log(x)
```

3.167 $\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx$

3.167.1 Optimal result

Integrand size = 22, antiderivative size = 15

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx=\log(x)+\frac{\log(b+cx^n)}{n}$$

output
```
ln(x)+ln(b+c*x^n)/n
```

3.167.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx=\frac{\log(x^n)+\log(n(b+cx^n))}{n}$$

input
```
Integrate[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]
```

output
```
(Log[x^n] + Log[n*(b + c*x^n)])/n
```

3.167.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^n}{bx+cx^{n+1}}\,dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{b+2cx^n}{x\left(b+cx^n\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{-n}\left(2cx^n+b\right)}{cx^n+b}dx^n}{n}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(x^{-n}+\frac{c}{cx^n+b}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log\left(b+cx^n\right)+\log\left(x^n\right)}{n}$$

input
```
Int[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]
```

output
```
(Log[x^n] + Log[b + c*x^n])/n
```

3.167.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.167.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

method	result	size
norman	$\ln(x)+\frac{\ln\left(c\,{\mathrm e}^{n\ln(x)}+b\right)}{n}$	18
risch	$\ln(x)+\frac{\ln\left(x^{n}+\frac{b}{c}\right)}{n}$	18

input
```
int((b+2*c*x^n)/(b*x+c*x^(1+n)),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+1/n*ln(c*exp(n*ln(x))+b)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx=\frac{(n-1)\log(x)+\log\left(bx+cx^{n+1}\right)}{n}$$

input
```
integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="fricas")
```

output
```
((n - 1)*log(x) + log(b*x + c*x^(n + 1)))/n
```

3.167.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx = \begin{cases} \log(x) & \text{for } c=0 \wedge (c=0 \vee n=0) \\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n=0 \\ \log(x)+\frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{otherwise} \end{cases}$$

input
```
integrate((b+2*c*x**n)/(b*x+c*x**(1+n)),x)
```

output
```
Piecewise((log(x), Eq(c, 0) & (Eq(c, 0) | Eq(n, 0))), ((b + 2*c)*log(x)/(b
 + c), Eq(n, 0)), (log(x) + log(b/c + x**n)/n, True))
```

3.167.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(15) = 30.

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.13

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx = b\left(\frac{\log(x)}{b}-\frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right)+\frac{2\log\left(\frac{cx^n+b}{c}\right)}{n}$$

input
```
integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="maxima")
```

output
```
b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

3.167.8 Giac [F]

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx = \int \frac{2\,cx^n+b}{bx+cx^{n+1}}\,dx$$

input
```
integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="giac")
```

output
```
integrate((2*c*x^n + b)/(b*x + c*x^(n + 1)), x)
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx=\int \frac{b+2\,c\,x^n}{b\,x+c\,x^{n+1}}\,dx$$

input
```
int((b + 2*c*x^n)/(b*x + c*x^(n + 1)),x)
```

output
```
int((b + 2*c*x^n)/(b*x + c*x^(n + 1)), x)
```

3.168 $\int \frac{b+2cx}{(bx+cx^2)^8} dx$

3.168.1 Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{b+2cx}{(bx+cx^2)^8} dx = -\frac{1}{7(bx+cx^2)^7}$$

output
```
-1/7/(c*x^2+b*x)^7
```

3.168.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{b+2cx}{(bx+cx^2)^8} dx = -\frac{1}{7x^7(b+cx)^7}$$

input
```
Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

output
```
-1/7*1/(x^7*(b + c*x)^7)
```

3.168.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx$$

$$\Big\downarrow \ 1104$$

$$-\frac{1}{7\,(bx+cx^2)^7}$$

input
```
Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

output
```
-1/7*1/(b*x + c*x^2)^7
```

3.168.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

3.168.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result
gosper	$-\frac{1}{7x^7(cx+b)^7}$
norman	$-\frac{1}{7x^7(cx+b)^7}$
risch	$-\frac{1}{7x^7(cx+b)^7}$
parallelrisch	$-\frac{1}{7x^7(cx+b)^7}$
derivativedivides	$-\frac{1}{7(c\,x^2+bx)^7}$
default	$-\frac{1}{7b^7x^7}-\frac{132c^6}{b^{13}x}+\frac{66c^5}{b^{12}x^2}-\frac{30c^4}{b^{11}x^3}+\frac{12c^3}{b^{10}x^4}-\frac{4c^2}{b^9x^5}+\frac{c}{b^8x^6}+\frac{132c^7}{b^{13}(cx+b)}+\frac{66c^7}{b^{12}(cx+b)^2}+\frac{30c^7}{b^{11}(cx+b)^3}+$

input
```
int((2*c*x+b)/(c*x^2+b*x)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7/x^7/(c*x+b)^7
```

3.168.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(13) = 26$.

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.40

$$\int \frac{b + 2cx}{(bx + cx^2)^8} \, dx = -\frac{1}{7\left(c^7x^{14} + 7\,bc^6x^{13} + 21\,b^2c^5x^{12} + 35\,b^3c^4x^{11} + 35\,b^4c^3x^{10} + 21\,b^5c^2x^9 + 7\,b^6cx^8 + b^7x^7\right)}$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="fricas")
```

output
```
-1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4
*c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

3.168.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(14) = 28$.

Time = 0.46 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.80

$$\int \frac{b + 2cx}{(bx + cx^2)^8} \, dx = -\frac{1}{7b^7x^7 + 49b^6cx^8 + 147b^5c^2x^9 + 245b^4c^3x^{10} + 245b^3c^4x^{11} + 147b^2c^5x^{12} + 49bc^6x^{13} + 7c^7x^{14}}$$

input
```
integrate((2*c*x+b)/(c*x**2+b*x)**8,x)
```

output
```
-1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**1
0 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x
**14)
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7(cx^2+bx)^7}$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="maxima")
```

output
```
-1/7/(c*x^2 + b*x)^7
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7(cx^2+bx)^7}$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="giac")
```

output
```
-1/7/(c*x^2 + b*x)^7
```

3.168.9 Mupad [B] (verification not implemented)

Time = 10.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7\,x^7\,(b+c\,x)^7}$$

input
```
int((b + 2*c*x)/(b*x + c*x^2)^8,x)
```

output
```
-1/(7*x^7*(b + c*x)^7)
```

3.169 $\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx$

3.169.1 Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx=-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

output
```
-1/14/x^14/(c*x^2+b)^7
```

3.169.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx=-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

input
```
Integrate[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]
```

output
```
-1/14*1/(x^14*(b + c*x^2)^7)
```

3.169.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {9, 354, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^2}{x^7 \left(bx+cx^3\right)^8} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{b+2cx^2}{x^{15} \left(b+cx^2\right)^8}dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2} \int \frac{2cx^2+b}{x^{16} \left(cx^2+b\right)^8}dx^2$$

$$\Big\downarrow \; 83$$

$$-\frac{1}{14x^{14} \left(b+cx^2\right)^7}$$

input
```
Int[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]
```

output
```
-1/14*1/(x^14*(b + c*x^2)^7)
```

3.169.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.169.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
norman	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
risch	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
parallelrisch	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
default	$-\frac{1}{14b^7x^{14}}-\frac{66c^6}{b^{13}x^2}+\frac{33c^5}{b^{12}x^4}-\frac{15c^4}{b^{11}x^6}+\frac{6c^3}{b^{10}x^8}-\frac{2c^2}{b^9x^{10}}+\frac{c}{2b^8x^{12}}-\frac{c^8\left(-\frac{12b^3}{c\left(c\,x^2+b\right)^4}-\frac{b^5}{c\left(c\,x^2+b\right)^6}-\frac{66b}{c\left(c\,x^2+b\right)^2}-\cdots\right)}{2\cdots}$

input
```
int((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/14/x^14/(c*x^2+b)^7
```

3.169.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(14) = 28.

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx =$$
$$-\frac{1}{14\left(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14}\right)}$$

input
```
integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="fricas")
```

output
```
-1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^
4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

3.169.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.

Time = 0.69 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx = -\frac{1}{14b^7x^{14}+98b^6cx^{16}+294b^5c^2x^{18}+490b^4c^3x^{20}+490b^3c^4x^{22}+294b^2c^5x^{24}+98bc^6x^{26}+14c^7x^{28}}$$

input
```
integrate((2*c*x**2+b)/x**7/(c*x**3+b*x)**8,x)
```

output
```
-1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*
x**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c
**7*x**28)
```

3.169.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx = -\frac{1}{14\left(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14}\right)}$$

input
```
integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="maxima")
```

output
```
-1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^
4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx=-\frac{1}{14\left(cx^4+bx^2\right)^7}$$

input
```
integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="giac")
```

output
```
-1/14/(c*x^4 + b*x^2)^7
```

3.169.9 Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx=-\frac{1}{14\,x^{14}\left(c\,x^2+b\right)^7}$$

input
```
int((b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x)
```

output
```
-1/(14*x^14*(b + c*x^2)^7)
```

3.170 $\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx$

3.170.1 Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx=-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

output
```
-1/21/x^21/(c*x^3+b)^7
```

3.170.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx=-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

input
```
Integrate[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]
```

output
```
-1/21*1/(x^21*(b + c*x^3)^7)
```

3.170.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {9, 948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{2cx^3+b}{x^{24}\left(cx^3+b\right)^8}dx^3$$

$$\Big\downarrow \text{ 83}$$

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

input
```
Int[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]
```

output
```
-1/21*1/(x^21*(b + c*x^3)^7)
```

3.170.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.170.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{1}{21x^{21}(c\,x^3+b)^7}$
risch	$-\frac{1}{21x^{21}(c\,x^3+b)^7}$
parallelrisch	$-\frac{1}{21x^{21}(c\,x^3+b)^7}$
default	$-\frac{1}{21b^7x^{21}}-\frac{44c^6}{b^{13}x^3}+\frac{22c^5}{b^{12}x^6}-\frac{10c^4}{b^{11}x^9}+\frac{4c^3}{b^{10}x^{12}}-\frac{4c^2}{3b^9x^{15}}+\frac{c}{3b^8x^{18}}-\frac{c^8\left(-\frac{66b}{c\left(c\,x^3+b\right)^2}-\frac{4b^4}{c\left(c\,x^3+b\right)^5}-\frac{132}{c\left(c\,x^3+b\right)}-\cdots\right)}{\cdots}$

input
```
int((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/21/x^21/(c*x^3+b)^7
```

3.170.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx =$$
$$-\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

input
```
integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="fricas")
```

output
```
-1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^
4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

3.170.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.

Time = 0.93 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{b+2cx^3}{x^{14}(bx+cx^4)^8}\,dx = -\frac{1}{21b^7x^{21}+147b^6cx^{24}+441b^5c^2x^{27}+735b^4c^3x^{30}+735b^3c^4x^{33}+441b^2c^5x^{36}+147bc^6x^{39}+21c^7x^{42}}$$

input
```
integrate((2*c*x**3+b)/x**14/(c*x**4+b*x)**8,x)
```

output
```
-1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3
*x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21
*c**7*x**42)
```

3.170.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^3}{x^{14}(bx+cx^4)^8}\,dx = -\frac{1}{21\,(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21})}$$

input
```
integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="maxima")
```

output
```
-1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^
4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx = -\frac{1}{21\left(cx^6+bx^3\right)^7}$$

input
```
integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="giac")
```

output
```
-1/21/(c*x^6 + b*x^3)^7
```

3.170.9 Mupad [B] (verification not implemented)

Time = 12.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx = -\frac{1}{21\,x^{21}\left(c\,x^3+b\right)^7}$$

input
```
int((b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x)
```

output
```
-1/(21*x^21*(b + c*x^3)^7)
```

3.171 $\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx$

3.171.1 Optimal result

Integrand size = 29, antiderivative size = 21

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx = -\frac{x^{-7n}}{7n(b+cx^n)^7}$$

output
```
-1/7/n/(x^(7*n))/(b+c*x^n)^7
```

3.171.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx = -\frac{x^{-7n}}{7n(b+cx^n)^7}$$

input
```
Integrate[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8),x]
```

output
```
-1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

3.171.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {10, 948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-7(n-1)}(b+2cx^n)}{(bx+cx^{n+1})^8}dx$$

$$\Big\downarrow \text{ 10}$$

$$\int \frac{x^{-7n-1}(b+2cx^n)}{(b+cx^n)^8}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{\int \frac{x^{-8n}(2cx^n+b)}{(cx^n+b)^8}dx^n}{n}$$

$$\Big\downarrow \text{ 83}$$

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

input
```
Int[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8),x]
```

output
```
-1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

3.171.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.171.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(21) = 42.

Time = 0.02 (sec) , antiderivative size = 203, normalized size of antiderivative = 9.67

$$-\frac{132c^6x^{-n}}{b^{13}n}+\frac{66c^5x^{-2n}}{b^{12}n}-\frac{30c^4x^{-3n}}{b^{11}n}+\frac{12c^3x^{-4n}}{b^{10}n}-\frac{4c^2x^{-5n}}{b^9n}+\frac{c\,x^{-6n}}{b^8n}-\frac{x^{-7n}}{7b^7n}+\frac{c^7(924x^{6n}c^6+6006b\,c^5x^{5n}+1638\cdots}{\cdots}$$

input
```
int((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x)
```

output
```
-132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*
c^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*
c^7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^
3*(x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^
7
```

3.171.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(21) = 42.

Time = 0.39 (sec) , antiderivative size = 143, normalized size of antiderivative = 6.81

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx =$$

$$-\frac{x^{14}}{7\left(b^7nx^7x^{7n+7}+7\,b^6cnx^6x^{8n+8}+21\,b^5c^2nx^5x^{9n+9}+35\,b^4c^3nx^4x^{10n+10}+35\,b^3c^4nx^3x^{11n+11}+21\,b^2c^5n\cdots\right)}$$

input
```
integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="fricas"
)
```

output
```
-1/7*x^14/(b^7*n*x^7*x^(7*n + 7) + 7*b^6*c*n*x^6*x^(8*n + 8) + 21*b^5*c^2*
n*x^5*x^(9*n + 9) + 35*b^4*c^3*n*x^4*x^(10*n + 10) + 35*b^3*c^4*n*x^3*x^(1
1*n + 11) + 21*b^2*c^5*n*x^2*x^(12*n + 12) + 7*b*c^6*n*x*x^(13*n + 13) + c
^7*n*x^(14*n + 14))
```

3.171.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx = \text{Timed out}$$

input
```
integrate((b+2*c*x**n)/(x**(-7+7*n))/(b*x+c*x**(1+n))**8,x)
```

output
```
Timed out
```

3.171.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(21) = 42.

Time = 0.24 (sec) , antiderivative size = 612, normalized size of antiderivative = 29.14

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx =$$
$$-\frac{1}{105}b\left(\frac{360360\,c^{13}x^{13\,n}+2342340\,bc^{12}x^{12\,n}+6426420\,b^2c^{11}x^{11\,n}+9579570\,b^3c^{10}x^{10\,n}+8270262\,b^4c^9x^{9\,n}}{b^{14}c^7nx^{14\,n}+7\,b^{15}c^6nx^{13\,n}+21\,b^{16}c^5n}\right.$$
$$+\frac{1}{105}c\left(\frac{360360\,c^{12}x^{12\,n}+2342340\,bc^{11}x^{11\,n}+6426420\,b^2c^{10}x^{10\,n}+9579570\,b^3c^9x^{9\,n}+8270262\,b^4c^8x^{8\,n}+}{b^{13}c^7nx^{13\,n}+7\,b^{14}c^6nx^{12\,n}+21\,b^{15}c^5nx^{11\,n}+}\right.$$

input
```
integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="maxima"
)
```

output
```
-1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^
11*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 401801
4*b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*
b^8*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^
2*x^(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x
^(13*n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*
x^(10*n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) +
360360*c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c
*((360360*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10
*n) + 9579570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*
x^(7*n) + 934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^
(4*n) + 1001*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b
^12)/(b^13*c^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n)
 + 35*b^16*c^4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n)
+ 7*b^19*c*n*x^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c
^6*log((c*x^n + b)/c)/(b^14*n))
```

3.171.8 Giac [F]

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx = \int \frac{2\,cx^n+b}{(bx+cx^{n+1})^8x^{7\,n-7}}\,dx$$

input
```
integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="giac")
```

output
```
integrate((2*c*x^n + b)/((b*x + c*x^(n + 1))^8*x^(7*n - 7)), x)
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx = \int \frac{x^{7-7\,n}\,(b+2\,c\,x^n)}{(b\,x+c\,x^{n+1})^8}\,dx$$

input
```
int((x^(7 - 7*n)*(b + 2*c*x^n))/(b*x + c*x^(n + 1))^8,x)
```

output
```
int((x^(7 - 7*n)*(b + 2*c*x^n))/(b*x + c*x^(n + 1))^8, x)
```

3.172 $\int (b+2cx) (bx+cx^2)^p \, dx$

3.172.1 Optimal result

Integrand size = 18, antiderivative size = 19

$$\int (b+2cx) (bx+cx^2)^p \, dx = \frac{(bx+cx^2)^{1+p}}{1+p}$$

output
```
(c*x^2+b*x)^(p+1)/(p+1)
```

3.172.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int (b+2cx) (bx+cx^2)^p \, dx = \frac{(x(b+cx))^{1+p}}{1+p}$$

input
```
Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

output
```
(x*(b + c*x))^(1 + p)/(1 + p)
```

3.172.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx$$

$$\Big\downarrow \text{ 1104}$$

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

input
```
Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

output
```
(b*x + c*x^2)^(1 + p)/(1 + p)
```

3.172.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

3.172.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{(c x^2+bx)^{1+p}}{1+p}$	20
default	$\frac{(c x^2+bx)^{1+p}}{1+p}$	20
risch	$\frac{x(cx+b)(x(cx+b))^p}{1+p}$	22
gosper	$\frac{x(cx+b)(c x^2+bx)^p}{1+p}$	24
parallelrisch	$\frac{x^2(x(cx+b))^p bc+x(x(cx+b))^p b^2}{b(1+p)}$	40
norman	$\frac{bx\,e^{p\ln\left(c x^2+bx\right)}}{1+p}+\frac{c x^2 e^{p\ln\left(c x^2+bx\right)}}{1+p}$	46

input
```
int((2*c*x+b)*(c*x^2+b*x)^p,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+b*x)^(1+p)/(1+p)
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{(cx^2+bx)(cx^2+bx)^p}{p+1}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="fricas")
```

output
```
(c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)
```

3.172.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\begin{cases}\frac{bx(bx+cx^2)^p}{p+1}+\frac{cx^2(bx+cx^2)^p}{p+1} & \text{for } p\neq -1\\ \log(x)+\log\left(\frac{b}{c}+x\right) & \text{otherwise}\end{cases}$$

input
```
integrate((2*c*x+b)*(c*x**2+b*x)**p,x)
```

output
```
Piecewise((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)**p/(p + 1
), Ne(p, -1)), (log(x) + log(b/c + x), True))
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{(cx^2+bx)^{p+1}}{p+1}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="maxima")
```

output
```
(c*x^2 + b*x)^(p + 1)/(p + 1)
```

3.172.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{(cx^2+bx)^{p+1}}{p+1}$$

input
```
integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="giac")
```

output
```
(c*x^2 + b*x)^(p + 1)/(p + 1)
```

3.172.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{x\,{\left(c\,x^2+b\,x\right)}^p\,\left(b+c\,x\right)}{p+1}$$

input
```
int((b*x + c*x^2)^p*(b + 2*c*x),x)
```

output
```
(x*(b*x + c*x^2)^p*(b + c*x))/(p + 1)
```

3.173 $\int x^{1+p}(b+2cx^2)\left(bx+cx^3\right)^p\,dx$

3.173.1 Optimal result

Integrand size = 25, antiderivative size = 27

$$\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx=\frac{x^{1+p}(bx+cx^3)^{1+p}}{2(1+p)}$$

output
```
1/2*x^(p+1)*(c*x^3+b*x)^(p+1)/(p+1)
```

3.173.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx$$
$$=\frac{x^{2+p}(x(b+cx^2))^p\left(1+\frac{cx^2}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^2}{b}\right)+2c(1+p)x^2\,\text{Hyp}\right.}{2(1+p)(2+p)}$$

input
```
Integrate[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]
```

output
```
(x^(2 + p)*(x*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p
, -((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c
*x^2)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

3.173.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1942}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{p+1}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx$$

$$\Big\downarrow \ 1942$$

$$\frac{x^{p+1}\left(bx+cx^3\right)^{p+1}}{2(p+1)}$$

input
```
Int[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]
```

output
```
(x^(1 + p)*(b*x + c*x^3)^(1 + p))/(2*(1 + p))
```

3.173.3.1 Defintions of rubi rules used

rule 1942
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] /; FreeQ[{a, b, c, d, e, j, m
, n, p}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && EqQ[
a*d*(m + j*p + 1) - b*c*(m + n + p*(j + n) + 1), 0] && (GtQ[e, 0] || Intege
rsQ[j]) && NeQ[m + j*p + 1, 0]
```

3.173.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

method	result	siz
gosper	$\frac{x^{2+p} \left(c\,x^{2}+b\right) \left(c\,x^{3}+b x \right)^{p}}{2+2 p}$	31
parallelrisch	$\frac{x^{3} x^{1+p} \left(x \left(c\,x^{2}+b \right)\right)^{p} b c +x\,x^{1+p} \left(x \left(c\,x^{2}+b \right)\right)^{p} b^{2}}{2 b \left(1+p \right)}$	55
risch	$\frac{\left(c\,x^{2}+b \right) x\,x^{1+p} \left(c\,x^{2}+b \right)^{p} x^{p} {\mathrm e}^{-\frac{i \operatorname{csgn}\left(i x \left(c\,x^{2}+b \right)\right) \pi p \left(-\operatorname{csgn}\left(i x \left(c\,x^{2}+b \right)\right)+\operatorname{csgn}\left(i \left(c\,x^{2}+b \right)\right)\right) \left(-\operatorname{csgn}\left(i x \left(c\,x^{2}+b \right)\right)+\operatorname{csgn}\left(i x \right)\right)}{2}}}{2+2 p}$	97

input
```
int(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(2+p)/(1+p)*(c*x^2+b)*(c*x^3+b*x)^p
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x^{1+p}(b+2cx^2)\left(bx+cx^3\right)^p\,dx=\frac{(cx^3+bx)(cx^3+bx)^px^{p+1}}{2\,(p+1)}$$

input
```
integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="fricas")
```

output
```
1/2*(c*x^3 + b*x)*(c*x^3 + b*x)^p*x^(p + 1)/(p + 1)
```

3.173.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(20) = 40.

Time = 28.85 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.96

$$\int x^{1+p}(b+2cx^2)\left(bx+cx^3\right)^p\,dx=\begin{cases}\frac{bxx^{p+1}(bx+cx^3)^p}{2p+2}+\frac{cx^3x^{p+1}(bx+cx^3)^p}{2p+2} & \text{for } p\neq -1\\ \log(x)+\frac{\log\left(x-\sqrt{-\frac{b}{c}}\right)}{2}+\frac{\log\left(x+\sqrt{-\frac{b}{c}}\right)}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(1+p)*(2*c*x**2+b)*(c*x**3+b*x)**p,x)
```

output
```
Piecewise((b*x*x**(p + 1)*(b*x + c*x**3)**p/(2*p + 2) + c*x**3*x**(p + 1)*
(b*x + c*x**3)**p/(2*p + 2), Ne(p, -1)), (log(x) + log(x - sqrt(-b/c))/2 +
 log(x + sqrt(-b/c))/2, True))
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int x^{1+p}(b+2cx^2)\,(bx+cx^3)^p\,dx=\frac{(cx^4+bx^2)e^{(p\log(cx^2+b)+2\,p\log(x))}}{2\,(p+1)}$$

input
```
integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="maxima")
```

output
```
1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

3.173.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int x^{1+p}(b+2cx^2)\,(bx+cx^3)^p\,dx$$
$$=\frac{cx^3e^{(p\log(cx^2+b)+2\,p\log(x)+\log(x))}+bxe^{(p\log(cx^2+b)+2\,p\log(x)+\log(x))}}{2\,(p+1)}$$

input
```
integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="giac")
```

output
```
1/2*(c*x^3*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)) + b*x*e^(p*log(c*x^2
 + b) + 2*p*log(x) + log(x)))/(p + 1)
```

3.173.9 Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int x^{1+p}(b+2cx^2)\,(bx+cx^3)^p\,dx=(c\,x^3+b\,x)^p\left(\frac{b\,x\,x^{p+1}}{2\,p+2}+\frac{c\,x^{p+1}\,x^3}{2\,p+2}\right)$$

input
```
int(x^(p + 1)*(b*x + c*x^3)^p*(b + 2*c*x^2),x)
```

output
```
(b*x + c*x^3)^p*((b*x*x^(p + 1))/(2*p + 2) + (c*x^(p + 1)*x^3)/(2*p + 2))
```

3.174 $\int \left(bx^{1+p}(bx+cx^3)^p+2cx^{3+p}(bx+cx^3)^p\right) dx$

3.174.1 Optimal result

Integrand size = 38, antiderivative size = 27

$$\int \left(bx^{1+p}(bx+cx^3)^p+2cx^{3+p}(bx+cx^3)^p\right) dx=\frac{x^{1+p}(bx+cx^3)^{1+p}}{2(1+p)}$$

output
```
1/2*x^(p+1)*(c*x^3+b*x)^(p+1)/(p+1)
```

3.174.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int \left(bx^{1+p}(bx+cx^3)^p+2cx^{3+p}(bx+cx^3)^p\right) dx$$
$$=\frac{x^{2+p}(x(b+cx^2))^p\left(1+\frac{cx^2}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^2}{b}\right)+2c(1+p)x^2\,\text{Hyp}\right.}{2(1+p)(2+p)}$$

input
```
Integrate[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]
```

output
```
(x^(2 + p)*(x*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p
, -((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c
*x^2)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

3.174.3 Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.30, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx^{p+1}(bx+cx^3)^p+2cx^{p+3}(bx+cx^3)^p\right)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bx^{p+2}(bx+cx^3)^p\left(\frac{cx^2}{b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-p,p+1,p+2,-\frac{cx^2}{b}\right)}{2(p+1)}+\frac{cx^{p+4}(bx+cx^3)^p\left(\frac{cx^2}{b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-p,p+2,p+3,-\frac{cx^2}{b}\right)}{p+2}$$

input
```
Int[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]
```

output
```
(b*x^(2 + p)*(b*x + c*x^3)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x^2)
/b)])/(2*(1 + p)*(1 + (c*x^2)/b)^p) + (c*x^(4 + p)*(b*x + c*x^3)^p*Hyperge
ometric2F1[-p, 2 + p, 3 + p, -((c*x^2)/b)])/((2 + p)*(1 + (c*x^2)/b)^p)
```

3.174.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.174.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.74 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

method	result	size
risch	$\frac{(c\,x^2+b)x\,x^{1+p}(c\,x^2+b)^p x^p \mathrm{e}^{-\frac{i\,\mathrm{csgn}\left(ix\left(c\,x^2+b\right)\right)\pi p\left(-\mathrm{csgn}\left(ix\left(c\,x^2+b\right)\right)+\mathrm{csgn}\left(i\left(c\,x^2+b\right)\right)\right)\left(-\mathrm{csgn}\left(ix\left(c\,x^2+b\right)\right)+\mathrm{csgn}(ix)\right)}{2}}}{2+2p}$	97

input
```
int(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x,method=_RETURNVERB
OSE)
```

output
```
1/2*(c*x^2+b)*x*x^(1+p)/(1+p)*(c*x^2+b)^p*x^p*exp(-1/2*I*csgn(I*x*(c*x^2+b
))*Pi*p*(-csgn(I*x*(c*x^2+b))+csgn(I*(c*x^2+b)))*(-csgn(I*x*(c*x^2+b))+csg
n(I*x)))
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx=\frac{(cx^2+b)(cx^3+bx)^px^{p+3}}{2\,(p+1)x}$$

input
```
integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="
fricas")
```

output
```
1/2*(c*x^2 + b)*(c*x^3 + b*x)^p*x^(p + 3)/((p + 1)*x)
```

3.174.6 Sympy [F]

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx=\int \left(x\left(b+cx^2\right)\right)^p\left(bx^{p+1}+2cx^{p+3}\right)\,dx$$

input
```
integrate(b*x**(1+p)*(c*x**3+b*x)**p+2*c*x**(3+p)*(c*x**3+b*x)**p,x)
```

output
```
Integral((x*(b + c*x**2))**p*(b*x**(p + 1) + 2*c*x**(p + 3)), x)
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx=\frac{(cx^4+bx^2)e^{(p\log(cx^2+b)+2\,p\log(x))}}{2\,(p+1)}$$

input
```
integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="
maxima")
```

output
```
1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

3.174.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx$$
$$=\frac{cx^3e^{(p\log(cx^2+b)+2\,p\log(x)+\log(x))}+bxe^{(p\log(cx^2+b)+2\,p\log(x)+\log(x))}}{2\,(p+1)}$$

input
```
integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="
giac")
```

output
```
1/2*(c*x^3*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)) + b*x*e^(p*log(c*x^2
 + b) + 2*p*log(x) + log(x)))/(p + 1)
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx$$
$$=\int b\,x^{p+1}\,\left(c\,x^3+b\,x\right)^p+2\,c\,x^{p+3}\,\left(c\,x^3+b\,x\right)^p\,dx$$

input
```
int(b*x^(p + 1)*(b*x + c*x^3)^p + 2*c*x^(p + 3)*(b*x + c*x^3)^p,x)
```

output
```
int(b*x^(p + 1)*(b*x + c*x^3)^p + 2*c*x^(p + 3)*(b*x + c*x^3)^p, x)
```

3.175 $\int x^{2(1+p)}(b+2cx^3)\left(bx+cx^4\right)^p\,dx$

3.175.1 Optimal result

Integrand size = 27, antiderivative size = 29

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\frac{x^{2(1+p)}\left(bx+cx^4\right)^{1+p}}{3(1+p)}$$

output
```
1/3*x^(2+2*p)*(c*x^4+b*x)^(p+1)/(p+1)
```

3.175.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.41

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx$$
$$=\frac{x^{3+2p}(x(b+cx^3))^p\left(1+\frac{cx^3}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^3}{b}\right)+2c(1+p)x^3\,\text{Hy}\right.}{3(1+p)(2+p)}$$

input
```
Integrate[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]
```

output
```
(x^(3 + 2*p)*(x*(b + c*x^3))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 +
 p, -((c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -(
(c*x^3)/b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)
```

3.175.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1942}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2(p+1)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx$$

$$\Big\downarrow \; 1942$$

$$\frac{x^{2(p+1)}\left(bx+cx^4\right)^{p+1}}{3(p+1)}$$

input
```
Int[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]
```

output
```
(x^(2*(1 + p))*(b*x + c*x^4)^(1 + p))/(3*(1 + p))
```

3.175.3.1 Defintions of rubi rules used

rule 1942
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] /; FreeQ[{a, b, c, d, e, j, m
, n, p}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && EqQ[
a*d*(m + j*p + 1) - b*c*(m + n + p*(j + n) + 1), 0] && (GtQ[e, 0] || Intege
rsQ[j]) && NeQ[m + j*p + 1, 0]
```

3.175.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

method	result	si
gosper	$\frac{x^{3+2p}\left(c\,x^{3}+b\right)\left(c\,x^{4}+bx\right)^{p}}{3+3p}$	3
parallelrisch	$\frac{x^{4}x^{2+2p}\left(x\left(c\,x^{3}+b\right)\right)^{p}c^{2}+x\,x^{2+2p}\left(x\left(c\,x^{3}+b\right)\right)^{p}bc}{3c(1+p)}$	5
risch	$\frac{\left(c\,x^{3}+b\right)x\,x^{2+2p}\left(c\,x^{3}+b\right)^{p}x^{p}\mathrm{e}^{-\frac{i\pi\,\mathrm{csgn}\left(ix\left(c\,x^{3}+b\right)\right)p\left(-\mathrm{csgn}\left(ix\left(c\,x^{3}+b\right)\right)+\mathrm{csgn}\left(i\left(c\,x^{3}+b\right)\right)\right)\left(-\mathrm{csgn}\left(ix\left(c\,x^{3}+b\right)\right)+\mathrm{csgn}(ix)\right)}{2}}}{3+3p}$	9

input
```
int(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(3+2*p)/(1+p)*(c*x^3+b)*(c*x^4+b*x)^p
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\frac{(cx^4+bx)(cx^4+bx)^p x^{2p+2}}{3\,(p+1)}$$

input
```
integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="fricas")
```

output
```
1/3*(c*x^4 + b*x)*(c*x^4 + b*x)^p*x^(2*p + 2)/(p + 1)
```

3.175.6 Sympy [F(-1)]

Timed out.

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**(2+2*p)*(2*c*x**3+b)*(c*x**4+b*x)**p,x)
```

output
```
Timed out
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\frac{(cx^6+bx^3)e^{\left(p\log\left(cx^3+b\right)+3\,p\log\left(x\right)\right)}}{3\,(p+1)}$$

input
```
integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="maxima")
```

output
```
1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

3.175.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(27) = 54.

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.00

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx$$
$$=\frac{cx^4e^{\left(p\log\left(cx^3+b\right)+3\,p\log(x)+2\,\log(x)\right)}+bxe^{\left(p\log\left(cx^3+b\right)+3\,p\log(x)+2\,\log(x)\right)}}{3\,(p+1)}$$

input
```
integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="giac")
```

output
```
1/3*(c*x^4*e^(p*log(c*x^3 + b) + 3*p*log(x) + 2*log(x)) + b*x*e^(p*log(c*x
^3 + b) + 3*p*log(x) + 2*log(x)))/(p + 1)
```

3.175.9 Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\left(c\,x^4+b\,x\right)^p\left(\frac{c\,x^{2\,p+2}\,x^4}{3\,p+3}+\frac{b\,x\,x^{2\,p+2}}{3\,p+3}\right)$$

input
```
int(x^(2*p + 2)*(b*x + c*x^4)^p*(b + 2*c*x^3),x)
```

output
```
(b*x + c*x^4)^p*((c*x^(2*p + 2)*x^4)/(3*p + 3) + (b*x*x^(2*p + 2))/(3*p +
3))
```

3.176 $\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx$

3.176.1 Optimal result

Integrand size = 31, antiderivative size = 36

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\frac{x^{-((1-n)(1+p))}\left(bx+cx^{1+n}\right)^{1+p}}{n(1+p)}$$

output
```
(b*x+c*x^(1+n))^(p+1)/n/(p+1)/(x^((1-n)*(p+1)))
```

3.176.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.00

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx$$
$$=\frac{x^{-p}(x(b+cx^n))^p\left(1+\frac{cx^n}{b}\right)^{-p}\left(b(2+p)x^{n(1+p)}\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^n}{b}\right)+2c(1+p)x^n\right.}{n(1+p)(2+p)}$$

input
```
Integrate[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]
```

output
```
((x*(b + c*x^n))^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p, 2
 + p, -((c*x^n)/b)] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 +
p, 3 + p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*x^p*(1 + (c*x^n)/b)^p)
```

3.176.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1942}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{(n-1)(p+1)}(b+2cx^n)\left(bx+cx^{n+1}\right)^p\,dx$$

$$\Big\downarrow \; 1942$$

$$\frac{x^{-((1-n)(p+1))}\left(bx+cx^{n+1}\right)^{p+1}}{n(p+1)}$$

input
```
Int[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]
```

output
```
(b*x + c*x^(1 + n))^(1 + p)/(n*(1 + p)*x^((1 - n)*(1 + p)))
```

3.176.3.1 Defintions of rubi rules used

rule 1942
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] /; FreeQ[{a, b, c, d, e, j, m
, n, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && EqQ[
a*d*(m + j*p + 1) - b*c*(m + n + p*(j + n) + 1), 0] && (GtQ[e, 0] || Intege
rsQ[j]) && NeQ[m + j*p + 1, 0]
```

3.176.4 Maple [F]

$$\int x^{(-1+n)(1+p)}(b+2c\,x^n)\left(bx+c\,x^{1+n}\right)^p dx$$

input
```
int(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)
```

output
```
int(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\frac{(bx+cx^{n+1})(bx+cx^{n+1})^px^{(n-1)p+n-1}}{np+n}$$

input
```
integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="fri
cas")
```

output
```
(b*x + c*x^(n + 1))*(b*x + c*x^(n + 1))^p*x^((n - 1)*p + n - 1)/(n*p + n)
```

3.176.6 Sympy [F(-1)]

Timed out.

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**((-1+n)*(1+p))*(b+2*c*x**n)*(b*x+c*x**(1+n))**p,x)
```

output
```
Timed out
```

3.176.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\frac{(cx^{2n}+bx^n)e^{(np\log(x)+p\log(cx^n+b))}}{n(p+1)}$$

input
```
integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="max
ima")
```

output
```
(c*x^(2*n) + b*x^n)*e^(n*p*log(x) + p*log(c*x^n + b))/(n*(p + 1))
```

3.176.8 Giac [F]

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\int (2\,cx^n+b)\left(bx+cx^{n+1}\right)^p x^{(n-1)(p+1)}\,dx$$

input
```
integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="gia
c")
```

output
```
integrate((2*c*x^n + b)*(b*x + c*x^(n + 1))^p*x^((n - 1)*(p + 1)), x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\int x^{(n-1)\,(p+1)}\,\left(b\,x+c\,x^{n+1}\right)^p\,(b+2\,c\,x^n)\,dx$$

input
```
int(x^((n - 1)*(p + 1))*(b*x + c*x^(n + 1))^p*(b + 2*c*x^n),x)
```

output
```
int(x^((n - 1)*(p + 1))*(b*x + c*x^(n + 1))^p*(b + 2*c*x^n), x)
```

3.177 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2} dx$

3.177.1 Optimal result

Integrand size = 54, antiderivative size = 32

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2} dx = acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

output
```
a*c*x+1/2*a*d*x^2+1/3*b*c*x^3+1/4*b*d*x^4
```

3.177.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2} dx = acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

input
```
Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d
*x^5)/(a + b*x^2),x]
```

output
```
a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

3.177.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2019, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2}\,dx$$
$$\Big\downarrow \text{2019}$$
$$\int \left(ac+adx+bcx^2+bdx^3\right)dx$$
$$\Big\downarrow \text{2009}$$
$$acx+\frac{1}{2}adx^2+\frac{1}{3}bcx^3+\frac{1}{4}bdx^4$$

input
```
Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/
(a + b*x^2),x]
```

output
```
a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

3.177.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.177.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
default	$acx+\frac{1}{2}ad\,x^{2}+\frac{1}{3}bc\,x^{3}+\frac{1}{4}bd\,x^{4}$	27
norman	$acx+\frac{1}{2}ad\,x^{2}+\frac{1}{3}bc\,x^{3}+\frac{1}{4}bd\,x^{4}$	27
risch	$acx+\frac{1}{2}ad\,x^{2}+\frac{1}{3}bc\,x^{3}+\frac{1}{4}bd\,x^{4}$	27
parallelrisch	$acx+\frac{1}{2}ad\,x^{2}+\frac{1}{3}bc\,x^{3}+\frac{1}{4}bd\,x^{4}$	27
parts	$acx+\frac{1}{2}ad\,x^{2}+\frac{1}{3}bc\,x^{3}+\frac{1}{4}bd\,x^{4}$	27
gosper	$\frac{x\left(3x^{3}bd+4bc\,x^{2}+6adx+12ac\right)}{12}$	28

input
```
int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a),
x,method=_RETURNVERBOSE)
```

output
```
a*c*x+1/2*a*d*x^2+1/3*b*c*x^3+1/4*b*d*x^4
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2}\,dx=\frac{1}{4}\,bdx^4+\frac{1}{3}\,bcx^3+\frac{1}{2}\,adx^2+acx$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a),x, algorithm="fricas")
```

output
```
1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

3.177.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2}\,dx = acx + \frac{adx^2}{2} + \frac{bcx^3}{3} + \frac{bdx^4}{4}$$

input
```
integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2
*c)/(b*x**2+a),x)
```

output
```
a*c*x + a*d*x**2/2 + b*c*x**3/3 + b*d*x**4/4
```

3.177.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2}\,dx = \frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,adx^2 + acx$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a),x, algorithm="maxima")
```

output
```
1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

3.177.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2}\,dx = \frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,adx^2 + acx$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a),x, algorithm="giac")
```

output
```
1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

3.177.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2} \, dx = \frac{b\,d\,x^4}{4} + \frac{b\,c\,x^3}{3} + \frac{a\,d\,x^2}{2} + a\,c\,x$$

input
```
int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/
(a + b*x^2),x)
```

output
```
a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

3.178 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx$

3.178.1 Optimal result

Integrand size = 54, antiderivative size = 12

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx = cx+\frac{dx^2}{2}$$

output
```
c*x+1/2*d*x^2
```

3.178.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx = cx+\frac{dx^2}{2}$$

input
```
Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d
*x^5)/(a + b*x^2)^2,x]
```

output
```
c*x + (d*x^2)/2
```

3.178.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2019, 2019, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{ac+adx+bcx^2+bdx^3}{a+bx^2}dx$$

$$\Big\downarrow \text{2019}$$

$$\int (c+dx)dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(c+dx)^2}{2d}$$

input
```
Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/
(a + b*x^2)^2,x]
```

output
```
(c + d*x)^2/(2*d)
```

3.178.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.178.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$\frac{x(dx+2c)}{2}$	11
default	$cx+\frac{1}{2}d\,x^{2}$	11
risch	$cx+\frac{1}{2}d\,x^{2}$	11
parallelrisch	$cx+\frac{1}{2}d\,x^{2}$	11
parts	$cx+\frac{1}{2}d\,x^{2}$	11
norman	$\frac{acx+bc\,x^{3}-\frac{a^{2}d}{2b}+\frac{bd\,x^{4}}{2}}{b\,x^{2}+a}$	38

input
```
int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^
2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(d*x+2*c)
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx=\frac{1}{2}\,dx^2+cx$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a)^2,x, algorithm="fricas")
```

output
```
1/2*d*x^2 + c*x
```

3.178.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx=cx+\frac{dx^2}{2}$$

input
```
integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2
*c)/(b*x**2+a)**2,x)
```

output
```
c*x + d*x**2/2
```

3.178.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx=\frac{1}{2}\,dx^2+cx$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*d*x^2 + c*x
```

3.178.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx=\frac{1}{2}\,dx^2+cx$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a)^2,x, algorithm="giac")
```

output
```
1/2*d*x^2 + c*x
```

3.178.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^2}\,dx = \frac{d\,x^2}{2} + c\,x$$

input
```
int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/
(a + b*x^2)^2,x)
```

output
```
c*x + (d*x^2)/2
```

3.179 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3}\,dx$

3.179.1 Optimal result

Integrand size = 54, antiderivative size = 42

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3}\,dx=\frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{d\log(a+bx^2)}{2b}$$

output
```
1/2*d*ln(b*x^2+a)/b+c*arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

3.179.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3}\,dx=\frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{d\log(a+bx^2)}{2b}$$

input
```
Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d
*x^5)/(a + b*x^2)^3,x]
```

output
```
(c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*
b)
```

3.179.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {2019, 2019, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{\left(a+bx^2\right)^3}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{ac+adx+bcx^2+bdx^3}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{c+dx}{a+bx^2}dx$$

$$\Big\downarrow \text{452}$$

$$c\int \frac{1}{bx^2+a}dx+d\int \frac{x}{bx^2+a}dx$$

$$\Big\downarrow \text{218}$$

$$d\int \frac{x}{bx^2+a}dx+\frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

$$\Big\downarrow \text{240}$$

$$\frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}$$

input
```
Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/
(a + b*x^2)^3,x]
```

output
```
(c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*
b)
```

3.179.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.179.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{d\ln\left(b\,x^{2}+a\right)}{2b}+\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	32
risch	$\frac{\ln\left(-\sqrt{-ab}\,x+a\right)c\sqrt{-ab}}{2ab}+\frac{\ln\left(-\sqrt{-ab}\,x+a\right)d}{2b}-\frac{\ln\left(\sqrt{-ab}\,x+a\right)c\sqrt{-ab}}{2ab}+\frac{\ln\left(\sqrt{-ab}\,x+a\right)d}{2b}$	90

input
```
int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^
3,x,method=_RETURNVERBOSE)
```

output
```
1/2*d*ln(b*x^2+a)/b+c/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.33

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^3}\, dx$$
$$= \left[\frac{ad\log\left(bx^2 + a\right) - \sqrt{-ab}c\log\left(\frac{bx^2 - 2\sqrt{-ab}x - a}{bx^2 + a}\right)}{2\,ab}, \frac{ad\log\left(bx^2 + a\right) + 2\sqrt{ab}c\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\,ab}\right]$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a)^3,x, algorithm="fricas")
```

output
```
[1/2*(a*d*log(b*x^2 + a) - sqrt(-a*b)*c*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b), 1/2*(a*d*log(b*x^2 + a) + 2*sqrt(a*b)*c*arctan(sqrt(a*
b)*x/a))/(a*b)]
```

3.179.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(37) = 74.

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.95

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^3}\, dx$$
$$= \left(\frac{d}{2b} - \frac{c\sqrt{-ab^3}}{2ab^2}\right)\log\left(x + \frac{2ab\left(\frac{d}{2b} - \frac{c\sqrt{-ab^3}}{2ab^2}\right) - ad}{bc}\right)$$
$$+ \left(\frac{d}{2b} + \frac{c\sqrt{-ab^3}}{2ab^2}\right)\log\left(x + \frac{2ab\left(\frac{d}{2b} + \frac{c\sqrt{-ab^3}}{2ab^2}\right) - ad}{bc}\right)$$

input
```
integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2
*c)/(b*x**2+a)**3,x)
```

output
```
(d/(2*b) - c*sqrt(-a*b**3)/(2*a*b**2))*log(x + (2*a*b*(d/(2*b) - c*sqrt(-a
*b**3)/(2*a*b**2)) - a*d)/(b*c)) + (d/(2*b) + c*sqrt(-a*b**3)/(2*a*b**2))*
log(x + (2*a*b*(d/(2*b) + c*sqrt(-a*b**3)/(2*a*b**2)) - a*d)/(b*c))
```

3.179.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^3}\,dx = \frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{d\log\left(bx^2 + a\right)}{2\,b}$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a)^3,x, algorithm="maxima")
```

output
```
c*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*d*log(b*x^2 + a)/b
```

3.179.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^3}\,dx = \frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{d\log\left(bx^2 + a\right)}{2\,b}$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a)^3,x, algorithm="giac")
```

output
```
c*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*d*log(b*x^2 + a)/b
```

3.179.9 Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^3}\,dx = \frac{d\,\ln\left(b\,x^2 + a\right)}{2\,b} + \frac{c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/
(a + b*x^2)^3,x)
```

output
```
(d*log(a + b*x^2))/(2*b) + (c*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2))
```

3.180 $\int (b+2cx+3dx^2)(a+bx+cx^2+dx^3)^n\,dx$

3.180.1 Optimal result

Integrand size = 30, antiderivative size = 25

$$\int (b+2cx+3dx^2)\left(a+bx+cx^2+dx^3\right)^n\,dx=\frac{(a+bx+cx^2+dx^3)^{1+n}}{1+n}$$

output
```
(d*x^3+c*x^2+b*x+a)^(1+n)/(1+n)
```

3.180.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (b+2cx+3dx^2)\left(a+bx+cx^2+dx^3\right)^n\,dx=\frac{(a+x(b+x(c+dx)))^{1+n}}{1+n}$$

input
```
Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]
```

output
```
(a + x*(b + x*(c + d*x)))^(1 + n)/(1 + n)
```

3.180.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx+3dx^2)(a+bx+cx^2+dx^3)^n\,dx$$

$$\Big\downarrow \text{2021}$$

$$\frac{(a+bx+cx^2+dx^3)^{n+1}}{n+1}$$

input
```
Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]
```

output
```
(a + b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

3.180.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.180.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	si
gosper	$\frac{(x^3d+c\,x^2+bx+a)^{1+n}}{1+n}$	26
derivativedivides	$\frac{(x^3d+c\,x^2+bx+a)^{1+n}}{1+n}$	26
default	$\frac{(x^3d+c\,x^2+bx+a)^{1+n}}{1+n}$	26
risch	$\frac{(x^3d+c\,x^2+bx+a)\,(x^3d+c\,x^2+bx+a)^n}{1+n}$	39
parallelrisch	$\frac{x^3\,(x^3d+c\,x^2+bx+a)^n cd+x^2\,(x^3d+c\,x^2+bx+a)^n c^2+x\,(x^3d+c\,x^2+bx+a)^n bc+(x^3d+c\,x^2+bx+a)^n ac}{c(1+n)}$	99
norman	$\frac{a\,\mathrm{e}^{n\ln\left(x^3d+c\,x^2+bx+a\right)}}{1+n}+\frac{bx\,\mathrm{e}^{n\ln\left(x^3d+c\,x^2+bx+a\right)}}{1+n}+\frac{c\,x^2\mathrm{e}^{n\ln\left(x^3d+c\,x^2+bx+a\right)}}{1+n}+\frac{d\,x^3\mathrm{e}^{n\ln\left(x^3d+c\,x^2+bx+a\right)}}{1+n}$	11

input
```
int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x,method=_RETURNVERBOSE)
```

output
```
(d*x^3+c*x^2+b*x+a)^(1+n)/(1+n)
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^n\,dx=\frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^n}{n+1}$$

input
```
integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^n/(n + 1)
```

3.180.6 Sympy [F(-1)]

Timed out.

$$\int (b + 2cx + 3dx^2) (a + bx + cx^2 + dx^3)^n \, dx = \text{Timed out}$$

input
```
integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**n,x)
```

output
```
Timed out
```

3.180.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (b + 2cx + 3dx^2) (a + bx + cx^2 + dx^3)^n \, dx = \frac{(dx^3 + cx^2 + bx + a)^{n+1}}{n+1}$$

input
```
integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^(n + 1)/(n + 1)
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (b + 2cx + 3dx^2) (a + bx + cx^2 + dx^3)^n \, dx = \frac{(dx^3 + cx^2 + bx + a)^{n+1}}{n+1}$$

input
```
integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^(n + 1)/(n + 1)
```

3.180.9 Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int (b+2cx+3dx^2) (a+bx+cx^2+dx^3)^n \, dx$$
$$= (d\,x^3+c\,x^2+b\,x+a)^n \left(\frac{a}{n+1}+\frac{b\,x}{n+1}+\frac{c\,x^2}{n+1}+\frac{d\,x^3}{n+1}\right)$$

input
```
int((b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^n*(a/(n + 1) + (b*x)/(n + 1) + (c*x^2)/(n + 1) +
 (d*x^3)/(n + 1))
```

3.181 $\int (b+2cx+3dx^2)(bx+cx^2+dx^3)^n\,dx$

3.181.1 Optimal result

Integrand size = 29, antiderivative size = 24

$$\int (b+2cx+3dx^2)(bx+cx^2+dx^3)^n\,dx=\frac{(bx+cx^2+dx^3)^{1+n}}{1+n}$$

output
```
(d*x^3+c*x^2+b*x)^(1+n)/(1+n)
```

3.181.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int (b+2cx+3dx^2)(bx+cx^2+dx^3)^n\,dx=\frac{(x(b+x(c+dx)))^{1+n}}{1+n}$$

input
```
Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]
```

output
```
(x*(b + x*(c + d*x)))^(1 + n)/(1 + n)
```

3.181.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^n\,dx$$

$$\Big\downarrow \; 2021$$

$$\frac{\left(bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

input
```
Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]
```

output
```
(b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

3.181.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.181.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\left(x^{3} d+c\,x^{2}+b x\right)^{1+n}}{1+n}$	25
default	$\frac{\left(x^{3} d+c\,x^{2}+b x\right)^{1+n}}{1+n}$	25
risch	$\frac{x\left(d\,x^{2}+c x+b\right)\left(x\left(d\,x^{2}+c x+b\right)\right)^{n}}{1+n}$	32
gosper	$\frac{x\left(d\,x^{2}+c x+b\right)\left(x^{3} d+c\,x^{2}+b x\right)^{n}}{1+n}$	34
parallelrisch	$\frac{x^{3}\left(x\left(d\,x^{2}+c x+b\right)\right)^{n} d^{2}+x^{2}\left(x\left(d\,x^{2}+c x+b\right)\right)^{n} c d+x\left(x\left(d\,x^{2}+c x+b\right)\right)^{n} b d}{d(1+n)}$	70
norman	$\frac{b x\,{\mathrm e}^{n \ln\left(x^{3} d+c\,x^{2}+b x\right)}}{1+n}+\frac{c\,x^{2} {\mathrm e}^{n \ln\left(x^{3} d+c\,x^{2}+b x\right)}}{1+n}+\frac{d\,x^{3} {\mathrm e}^{n \ln\left(x^{3} d+c\,x^{2}+b x\right)}}{1+n}$	84

input
```
int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x,method=_RETURNVERBOSE)
```

output
```
(d*x^3+c*x^2+b*x)^(1+n)/(1+n)
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^n\,dx=\frac{(dx^3+cx^2+bx)(dx^3+cx^2+bx)^n}{n+1}$$

input
```
integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2 + b*x)*(d*x^3 + c*x^2 + b*x)^n/(n + 1)
```

3.181.6 Sympy [F(-1)]

Timed out.

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^n\,dx=\text{Timed out}$$

input
```
integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**n,x)
```

output
```
Timed out
```

3.181.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^n\,dx=\frac{\left(dx^3+cx^2+bx\right)^{n+1}}{n+1}$$

input
```
integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2 + b*x)^(n + 1)/(n + 1)
```

3.181.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^n\,dx=\frac{\left(dx^3+cx^2+bx\right)^{n+1}}{n+1}$$

input
```
integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + b*x)^(n + 1)/(n + 1)
```

3.181.9 Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int (b+2cx+3dx^2) (bx+cx^2+dx^3)^n \, dx = \left(\frac{b\,x}{n+1}+\frac{c\,x^2}{n+1}+\frac{d\,x^3}{n+1}\right) \left(d\,x^3+c\,x^2+b\,x\right)^n$$

input
```
int((b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x)
```

output
```
((b*x)/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(b*x + c*x^2 + d*x^3)^
n
```

3.182 $\int x^n(b+cx+dx^2)^n (b+2cx+3dx^2)\, dx$

3.182.1 Optimal result

Integrand size = 28, antiderivative size = 25

$$\int x^n(b+cx+dx^2)^n (b+2cx+3dx^2)\, dx = \frac{x^{1+n}(b+cx+dx^2)^{1+n}}{1+n}$$

output
```
x^(1+n)*(d*x^2+c*x+b)^(1+n)/(1+n)
```

3.182.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int x^n(b+cx+dx^2)^n (b+2cx+3dx^2)\, dx = \frac{x^{1+n}(b+x(c+dx))^{1+n}}{1+n}$$

input
```
Integrate[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]
```

output
```
(x^(1 + n)*(b + x*(c + d*x))^(1 + n))/(1 + n)
```

3.182.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^n (b + 2cx + 3dx^2) (b + cx + dx^2)^n \, dx$$

$$\Big\downarrow \; 2023$$

$$\frac{x^{n+1}(b + cx + dx^2)^{n+1}}{n+1}$$

input
```
Int[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]
```

output
```
(x^(1 + n)*(b + c*x + d*x^2)^(1 + n))/(1 + n)
```

3.182.3.1 Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

3.182.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{x^{1+n} \left(d\,x^{2}+c x +b \right)^{1+n}}{1+n}$	26
risch	$\frac{x \left(d\,x^{2}+c x +b \right) x^{n} \left(d\,x^{2}+c x +b \right)^{n}}{1+n}$	33
parallelrisch	$\frac{x^{3} x^{n} \left(d\,x^{2}+c x +b \right)^{n} d^{2}+x^{2} x^{n} \left(d\,x^{2}+c x +b \right)^{n} c d +x\,x^{n} \left(d\,x^{2}+c x +b \right)^{n} b d}{d \left(1+n \right)}$	73

input
```
int(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x,method=_RETURNVERBOSE)
```

output
```
x^(1+n)*(d*x^2+c*x+b)^(1+n)/(1+n)
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int x^n \left(b+cx+dx^2\right)^n \left(b+2cx+3dx^2\right)\,dx=\frac{(dx^3+cx^2+bx)(dx^2+cx+b)^n x^n}{n+1}$$

input
```
integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2 + b*x)*(d*x^2 + c*x + b)^n*x^n/(n + 1)
```

3.182.6 Sympy [F(-1)]

Timed out.

$$\int x^n \left(b+cx+dx^2\right)^n \left(b+2cx+3dx^2\right)\,dx=\text{Timed out}$$

input
```
integrate(x**n*(d*x**2+c*x+b)**n*(3*d*x**2+2*c*x+b),x)
```

output
```
Timed out
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int x^n \left(b+cx+dx^2\right)^n \left(b+2cx+3dx^2\right)\,dx=\frac{(dx^3+cx^2+bx)e^{\left(n\log\left(dx^2+cx+b\right)+n\log\left(x\right)\right)}}{n+1}$$

input
```
integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2 + b*x)*e^(n*log(d*x^2 + c*x + b) + n*log(x))/(n + 1)
```

3.182.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(25) = 50$.

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.60

$$\int x^n (b + cx + dx^2)^n (b + 2cx + 3dx^2) \, dx = \frac{(dx^2 + cx + b)^n dx^3 x^n + (dx^2 + cx + b)^n cx^2 x^n + (dx^2 + cx + b)^n bx x^n}{n + 1}$$

input
```
integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="giac")
```

output
```
((d*x^2 + c*x + b)^n*d*x^3*x^n + (d*x^2 + c*x + b)^n*c*x^2*x^n + (d*x^2 +
c*x + b)^n*b*x*x^n)/(n + 1)
```

3.182.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int x^n (b + cx + dx^2)^n (b + 2cx + 3dx^2) \, dx = \left(\frac{c\,x^n\,x^2}{n+1} + \frac{d\,x^n\,x^3}{n+1} + \frac{b\,x\,x^n}{n+1}\right) \left(d\,x^2 + c\,x + b\right)^n$$

input
```
int(x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x)
```

output
```
((c*x^n*x^2)/(n + 1) + (d*x^n*x^3)/(n + 1) + (b*x*x^n)/(n + 1))*(b + c*x +
 d*x^2)^n
```

3.183 $\int (b+3dx^2)(a+bx+dx^3)^n\,dx$

3.183.1 Optimal result

Integrand size = 21, antiderivative size = 20

$$\int (b+3dx^2)(a+bx+dx^3)^n\,dx=\frac{(a+bx+dx^3)^{1+n}}{1+n}$$

output
```
(d*x^3+b*x+a)^(1+n)/(1+n)
```

3.183.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (b+3dx^2)(a+bx+dx^3)^n\,dx=\frac{(a+bx+dx^3)^{1+n}}{1+n}$$

input
```
Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]
```

output
```
(a + b*x + d*x^3)^(1 + n)/(1 + n)
```

3.183.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b + 3dx^2) (a + bx + dx^3)^n \, dx$$
$$\Big\downarrow \text{2021}$$
$$\frac{(a + bx + dx^3)^{n+1}}{n + 1}$$

input
```
Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]
```

output
```
(a + b*x + d*x^3)^(1 + n)/(1 + n)
```

3.183.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.183.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{\left(x^{3} d+b x+a\right)^{1+n}}{1+n}$	21
derivativedivides	$\frac{\left(x^{3} d+b x+a\right)^{1+n}}{1+n}$	21
default	$\frac{\left(x^{3} d+b x+a\right)^{1+n}}{1+n}$	21
risch	$\frac{\left(x^{3} d+b x+a\right)\left(x^{3} d+b x+a\right)^{n}}{1+n}$	29
parallelrisch	$\frac{x^{3}\left(x^{3} d+b x+a\right)^{n} d^{2}+x\left(x^{3} d+b x+a\right)^{n} b d+\left(x^{3} d+b x+a\right)^{n} a d}{d(1+n)}$	61
norman	$\frac{a\,{\mathrm e}^{n \ln\left(x^{3} d+b x+a\right)}}{1+n}+\frac{b x\,{\mathrm e}^{n \ln\left(x^{3} d+b x+a\right)}}{1+n}+\frac{d\,x^{3} {\mathrm e}^{n \ln\left(x^{3} d+b x+a\right)}}{1+n}$	69

input
```
int((3*d*x^2+b)*(d*x^3+b*x+a)^n,x,method=_RETURNVERBOSE)
```

output
```
(d*x^3+b*x+a)^(1+n)/(1+n)
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \left(b+3 d x^2\right) \left(a+b x+d x^3\right)^n \, dx=\frac{(dx^3+bx+a)(dx^3+bx+a)^n}{n+1}$$

input
```
integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="fricas")
```

output
```
(d*x^3 + b*x + a)*(d*x^3 + b*x + a)^n/(n + 1)
```

3.183.6 Sympy [F(-1)]

Timed out.

$$\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^n\,dx=\text{Timed out}$$

input
```
integrate((3*d*x**2+b)*(d*x**3+b*x+a)**n,x)
```

output
```
Timed out
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^n\,dx=\frac{\left(dx^3+bx+a\right)^{n+1}}{n+1}$$

input
```
integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="maxima")
```

output
```
(d*x^3 + b*x + a)^(n + 1)/(n + 1)
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^n\,dx=\frac{\left(dx^3+bx+a\right)^{n+1}}{n+1}$$

input
```
integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="giac")
```

output
```
(d*x^3 + b*x + a)^(n + 1)/(n + 1)
```

3.183.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^n\,dx=\left(\frac{a}{n+1}+\frac{b\,x}{n+1}+\frac{d\,x^3}{n+1}\right)\left(d\,x^3+b\,x+a\right)^n$$

input
```
int((b + 3*d*x^2)*(a + b*x + d*x^3)^n,x)
```

output
```
(a/(n + 1) + (b*x)/(n + 1) + (d*x^3)/(n + 1))*(a + b*x + d*x^3)^n
```

3.184 $\int (b+3dx^2)(bx+dx^3)^n\,dx$

3.184.1 Optimal result

Integrand size = 20, antiderivative size = 19

$$\int (b+3dx^2)(bx+dx^3)^n\,dx=\frac{(bx+dx^3)^{1+n}}{1+n}$$

output
```
(d*x^3+b*x)^(1+n)/(1+n)
```

3.184.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 5.58

$$\int (b+3dx^2)(bx+dx^3)^n\,dx$$
$$=\frac{x(x(b+dx^2))^n\left(1+\frac{dx^2}{b}\right)^{-n}\left(b(3+n)\,\text{Hypergeometric2F1}\left(-n,\frac{1+n}{2},\frac{3+n}{2},-\frac{dx^2}{b}\right)+3d(1+n)x^2\,\text{Hyperge}\right.}{(1+n)(3+n)}$$

input
```
Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]
```

output
```
(x*(x*(b + d*x^2))^n*(b*(3 + n)*Hypergeometric2F1[-n, (1 + n)/2, (3 + n)/2
, -((d*x^2)/b)] + 3*d*(1 + n)*x^2*Hypergeometric2F1[-n, (3 + n)/2, (5 + n)
/2, -((d*x^2)/b)]))/((1 + n)*(3 + n)*(1 + (d*x^2)/b)^n)
```

3.184.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+3dx^2)\left(bx+dx^3\right)^n\,dx$$

$$\Big\downarrow \; 2021$$

$$\frac{\left(bx+dx^3\right)^{n+1}}{n+1}$$

input
```
Int[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]
```

output
```
(b*x + d*x^3)^(1 + n)/(1 + n)
```

3.184.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.184.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{\left(x^{3} d+b x\right)^{1+n}}{1+n}$	20
default	$\frac{\left(x^{3} d+b x\right)^{1+n}}{1+n}$	20
gosper	$\frac{x\left(d\,x^{2}+b\right)\left(x^{3} d+b x\right)^{n}}{1+n}$	26
risch	$\frac{x\left(d\,x^{2}+b\right)\left(x\left(d\,x^{2}+b\right)\right)^{n}}{1+n}$	26
parallelrisch	$\frac{x^{3}\left(x\left(d\,x^{2}+b\right)\right)^{n} b d+x\left(x\left(d\,x^{2}+b\right)\right)^{n} b^{2}}{b(1+n)}$	44
norman	$\frac{b x\,\mathrm{e}^{n \ln\left(x^{3} d+b x\right)}}{1+n}+\frac{d\,x^{3} \mathrm{e}^{n \ln\left(x^{3} d+b x\right)}}{1+n}$	46

input
```
int((3*d*x^2+b)*(d*x^3+b*x)^n,x,method=_RETURNVERBOSE)
```

output
```
(d*x^3+b*x)^(1+n)/(1+n)
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \left(b+3dx^2\right)\left(bx+dx^3\right)^n\,dx=\frac{(dx^3+bx)(dx^3+bx)^n}{n+1}$$

input
```
integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="fricas")
```

output
```
(d*x^3 + b*x)*(d*x^3 + b*x)^n/(n + 1)
```

3.184.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(14) = 28.

Time = 5.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.32

$$\int (b+3dx^2)(bx+dx^3)^n\,dx = \begin{cases} \frac{bx(bx+dx^3)^n}{n+1} + \frac{dx^3(bx+dx^3)^n}{n+1} & \text{for } n \neq -1 \\ \log(x) + \log\left(x - \sqrt{-\frac{b}{d}}\right) + \log\left(x + \sqrt{-\frac{b}{d}}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((3*d*x**2+b)*(d*x**3+b*x)**n,x)
```

output
```
Piecewise((b*x*(b*x + d*x**3)**n/(n + 1) + d*x**3*(b*x + d*x**3)**n/(n + 1
), Ne(n, -1)), (log(x) + log(x - sqrt(-b/d)) + log(x + sqrt(-b/d)), True))
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (b+3dx^2)(bx+dx^3)^n\,dx = \frac{(dx^3+bx)^{n+1}}{n+1}$$

input
```
integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="maxima")
```

output
```
(d*x^3 + b*x)^(n + 1)/(n + 1)
```

3.184.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (b+3dx^2)(bx+dx^3)^n\,dx = \frac{(dx^3+bx)^{n+1}}{n+1}$$

input
```
integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="giac")
```

output
```
(d*x^3 + b*x)^(n + 1)/(n + 1)
```

3.184.9 Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \left(b+3dx^2\right)\left(bx+dx^3\right)^n\,dx=\frac{x\,{\left(d\,x^3+b\,x\right)}^n\,\left(d\,x^2+b\right)}{n+1}$$

input
```
int((b*x + d*x^3)^n*(b + 3*d*x^2),x)
```

output
```
(x*(b*x + d*x^3)^n*(b + d*x^2))/(n + 1)
```

3.185 $\int x^n(b+dx^2)^n(b+3dx^2)\,dx$

3.185.1 Optimal result

Integrand size = 21, antiderivative size = 22

$$\int x^n\left(b+dx^2\right)^n\left(b+3dx^2\right)\,dx=\frac{x^{1+n}(b+dx^2)^{1+n}}{1+n}$$

output
```
x^(1+n)*(d*x^2+b)^(1+n)/(1+n)
```

3.185.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.91

$$\int x^n\left(b+dx^2\right)^n\left(b+3dx^2\right)\,dx$$
$$=\frac{x^{1+n}(b+dx^2)^n\left(1+\frac{dx^2}{b}\right)^{-n}\left(b(3+n)\,\text{Hypergeometric2F1}\left(-n,\frac{1+n}{2},\frac{3+n}{2},-\frac{dx^2}{b}\right)+3d(1+n)x^2\,\text{Hyperge}\right.}{(1+n)(3+n)}$$

input
```
Integrate[x^n*(b + d*x^2)^n*(b + 3*d*x^2),x]
```

output
```
(x^(1 + n)*(b + d*x^2)^n*(b*(3 + n)*Hypergeometric2F1[-n, (1 + n)/2, (3 +
n)/2, -((d*x^2)/b)] + 3*d*(1 + n)*x^2*Hypergeometric2F1[-n, (3 + n)/2, (5
+ n)/2, -((d*x^2)/b)]))/((1 + n)*(3 + n)*(1 + (d*x^2)/b)^n)
```

3.185.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {356}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^n (b + 3dx^2) (b + dx^2)^n \, dx$$

$$\Big\downarrow \text{356}$$

$$\frac{x^{n+1} (b + dx^2)^{n+1}}{n + 1}$$

input
```
Int[x^n*(b + d*x^2)^n*(b + 3*d*x^2),x]
```

output
```
(x^(1 + n)*(b + d*x^2)^(1 + n))/(1 + n)
```

3.185.3.1 Defintions of rubi rules used

rule 356
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] /;
 FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) -
b*c*(m + 2*p + 3), 0] && NeQ[m, -1]
```

3.185.4 Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{x^{1+n} (d\,x^{2}+b)^{1+n}}{1+n}$	23
risch	$\frac{x (d\,x^{2}+b) x^{n} (d\,x^{2}+b)^{n}}{1+n}$	27
parallelrisch	$\frac{x^{3} x^{n} (d\,x^{2}+b)^{n} b d+x\,x^{n} (d\,x^{2}+b)^{n} b^{2}}{b(1+n)}$	46

input
```
int(x^n*(d*x^2+b)^n*(3*d*x^2+b),x,method=_RETURNVERBOSE)
```

output
```
x^(1+n)*(d*x^2+b)^(1+n)/(1+n)
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int x^n \left(b+dx^2\right)^n \left(b+3dx^2\right)\,dx=\frac{(dx^3+bx)(dx^2+b)^n x^n}{n+1}$$

input
```
integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="fricas")
```

output
```
(d*x^3 + b*x)*(d*x^2 + b)^n*x^n/(n + 1)
```

3.185.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(17) = 34.

Time = 23.67 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.00

$$\int x^n \left(b+dx^2\right)^n \left(b+3dx^2\right)\,dx=\begin{cases}\frac{bxx^n(b+dx^2)^n}{n+1}+\frac{dx^3x^n(b+dx^2)^n}{n+1} & \text{for } n\neq -1\\ \log(x)+\log\left(x-\sqrt{-\frac{b}{d}}\right)+\log\left(x+\sqrt{-\frac{b}{d}}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(x**n*(d*x**2+b)**n*(3*d*x**2+b),x)
```

output
```
Piecewise((b*x*x**n*(b + d*x**2)**n/(n + 1) + d*x**3*x**n*(b + d*x**2)**n/
(n + 1), Ne(n, -1)), (log(x) + log(x - sqrt(-b/d)) + log(x + sqrt(-b/d)),
True))
```

3.185.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int x^n (b+dx^2)^n (b+3dx^2) \, dx = \frac{(dx^3+bx)e^{(n\log(dx^2+b)+n\log(x))}}{n+1}$$

input
```
integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="maxima")
```

output
```
(d*x^3 + b*x)*e^(n*log(d*x^2 + b) + n*log(x))/(n + 1)
```

3.185.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int x^n (b+dx^2)^n (b+3dx^2) \, dx = \frac{(dx^2+b)^n dx^3 x^n + (dx^2+b)^n bxx^n}{n+1}$$

input
```
integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="giac")
```

output
```
((d*x^2 + b)^n*d*x^3*x^n + (d*x^2 + b)^n*b*x*x^n)/(n + 1)
```

3.185.9 Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int x^n (b+dx^2)^n (b+3dx^2) \, dx = \frac{x\,x^n\,(d\,x^2+b)^n\,(d\,x^2+b)}{n+1}$$

input
```
int(x^n*(b + d*x^2)^n*(b + 3*d*x^2),x)
```

output
```
(x*x^n*(b + d*x^2)^n*(b + d*x^2))/(n + 1)
```

3.186 $\int (2cx + 3dx^2)(a + cx^2 + dx^3)^n \, dx$

3.186.1 Optimal result

Integrand size = 26, antiderivative size = 22

$$\int (2cx + 3dx^2)(a + cx^2 + dx^3)^n \, dx = \frac{(a + cx^2 + dx^3)^{1+n}}{1+n}$$

output
```
(d*x^3+c*x^2+a)^(1+n)/(1+n)
```

3.186.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int (2cx + 3dx^2)(a + cx^2 + dx^3)^n \, dx = \frac{(a + x^2(c + dx))^{1+n}}{1+n}$$

input
```
Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]
```

output
```
(a + x^2*(c + d*x))^(1 + n)/(1 + n)
```

3.186.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2cx+3dx^2)\left(a+cx^2+dx^3\right)^n\,dx$$

$$\Big\downarrow \text{ 2021}$$

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

input
```
Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]
```

output
```
(a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

3.186.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.186.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{(x^3 d+c x^2+a)^{1+n}}{1+n}$	23
derivativedivides	$\frac{(x^3 d+c x^2+a)^{1+n}}{1+n}$	23
default	$\frac{(x^3 d+c x^2+a)^{1+n}}{1+n}$	23
risch	$\frac{(x^3 d+c x^2+a)^n (x^3 d+c x^2+a)}{1+n}$	33
parallelrisch	$\frac{x^3 (x^3 d+c x^2+a)^n d^2+x^2 (x^3 d+c x^2+a)^n cd+(x^3 d+c x^2+a)^n ad}{d(1+n)}$	69
norman	$\frac{a\,\mathrm{e}^{n\ln(x^3 d+c x^2+a)}}{1+n}+\frac{c x^2 \mathrm{e}^{n\ln(x^3 d+c x^2+a)}}{1+n}+\frac{d x^3 \mathrm{e}^{n\ln(x^3 d+c x^2+a)}}{1+n}$	77

input
```
int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x,method=_RETURNVERBOSE)
```

output
```
(d*x^3+c*x^2+a)^(1+n)/(1+n)
```

3.186.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int (2cx+3dx^2)(a+cx^2+dx^3)^n\,dx=\frac{(dx^3+cx^2+a)(dx^3+cx^2+a)^n}{n+1}$$

input
```
integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

3.186.6 Sympy [F(-1)]

Timed out.

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx=\text{Timed out}$$

input
```
integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**n,x)
```

output
```
Timed out
```

3.186.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(dx^3+cx^2+a\right)^{n+1}}{n+1}$$

input
```
integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(dx^3+cx^2+a\right)^{n+1}}{n+1}$$

input
```
integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)
```

3.186.9 Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.95

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx=\left(\frac{a}{n+1}+\frac{c\,x^2}{n+1}+\frac{d\,x^3}{n+1}\right)\,\left(d\,x^3+c\,x^2+a\right)^n$$

input
```
int((2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x)
```

output
```
(a/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(a + c*x^2 + d*x^3)^n
```

3.187 $\int (2cx+3dx^2)(cx^2+dx^3)^n\,dx$

3.187.1 Optimal result

Integrand size = 25, antiderivative size = 21

$$\int (2cx+3dx^2)\left(cx^2+dx^3\right)^n\,dx=\frac{(cx^2+dx^3)^{1+n}}{1+n}$$

output
```
(d*x^3+c*x^2)^(1+n)/(1+n)
```

3.187.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int (2cx+3dx^2)\left(cx^2+dx^3\right)^n\,dx=\frac{(x^2(c+dx))^{1+n}}{1+n}$$

input
```
Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]
```

output
```
(x^2*(c + d*x))^(1 + n)/(1 + n)
```

3.187.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2cx+3dx^2)(cx^2+dx^3)^n\,dx$$

$$\Big\downarrow \text{2021}$$

$$\frac{(cx^2+dx^3)^{n+1}}{n+1}$$

input
```
Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]
```

output
```
(c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

3.187.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.187.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{\left(x^{3} d+c\,x^{2}\right)^{1+n}}{1+n}$	22
default	$\frac{\left(x^{3} d+c\,x^{2}\right)^{1+n}}{1+n}$	22
risch	$\frac{x^{2} (dx+c) \left(x^{2} (dx+c)\right)^{n}}{1+n}$	26
gosper	$\frac{\left(x^{3} d+c\,x^{2}\right)^{n} x^{2} (dx+c)}{1+n}$	28
parallelrisch	$\frac{x^{3} \left(x^{2} (dx+c)\right)^{n} c d+x^{2} \left(x^{2} (dx+c)\right)^{n} c^{2}}{c (1+n)}$	46
norman	$\frac{c\,x^{2} {\mathrm e}^{n \ln \left(x^{3} d+c\,x^{2}\right)}}{1+n}+\frac{d\,x^{3} {\mathrm e}^{n \ln \left(x^{3} d+c\,x^{2}\right)}}{1+n}$	52

input
```
int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x,method=_RETURNVERBOSE)
```

output
```
(d*x^3+c*x^2)^(1+n)/(1+n)
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int \left(2cx+3dx^2\right) \left(cx^2+dx^3\right)^n \, dx = \frac{(dx^3+cx^2)(dx^3+cx^2)^n}{n+1}$$

input
```
integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)
```

3.187.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.

Time = 0.43 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.52

$$\int (2cx + 3dx^2) (cx^2 + dx^3)^n \, dx = \begin{cases} \frac{cx^2(cx^2+dx^3)^n}{n+1} + \frac{dx^3(cx^2+dx^3)^n}{n+1} & \text{for } n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

input
```
integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**n,x)
```

output
```
Piecewise((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 + d*x**3)*
*n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

3.187.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (2cx + 3dx^2) (cx^2 + dx^3)^n \, dx = \frac{(dx^3 + cx^2)^{n+1}}{n+1}$$

input
```
integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2)^(n + 1)/(n + 1)
```

3.187.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (2cx + 3dx^2) (cx^2 + dx^3)^n \, dx = \frac{(dx^3 + cx^2)^{n+1}}{n+1}$$

input
```
integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2)^(n + 1)/(n + 1)
```

3.187.9 Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \left(2cx+3dx^2\right)\left(cx^2+dx^3\right)^n\,dx=\frac{x^2\,{\left(d\,x^3+c\,x^2\right)}^n\,\left(c+d\,x\right)}{n+1}$$

input
```
int((2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x)
```

output
```
(x^2*(c*x^2 + d*x^3)^n*(c + d*x))/(n + 1)
```

3.188 $\int x^n(cx+dx^2)^n\,(2cx+3dx^2)\;dx$

3.188.1 Optimal result

Integrand size = 26, antiderivative size = 24

$$\int x^n\left(cx+dx^2\right)^n\left(2cx+3dx^2\right)\,dx=\frac{x^{1+n}\left(cx+dx^2\right)^{1+n}}{1+n}$$

output
```
x^(1+n)*(d*x^2+c*x)^(1+n)/(1+n)
```

3.188.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^n\left(cx+dx^2\right)^n\left(2cx+3dx^2\right)\,dx=\frac{x^{1+n}(x(c+dx))^{1+n}}{1+n}$$

input
```
Integrate[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x]
```

output
```
(x^(1 + n)*(x*(c + d*x))^(1 + n))/(1 + n)
```

3.188.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {9, 1217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^n \left(2cx+3dx^2\right) \left(cx+dx^2\right)^n \, dx$$

$$\Big\downarrow \; 9$$

$$\int x^{n+1}(2c+3dx) \left(cx+dx^2\right)^n dx$$

$$\Big\downarrow \; 1217$$

$$\frac{x^{n+1}\left(cx+dx^2\right)^{n+1}}{n+1}$$

input
```
Int[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x]
```

output
```
(x^(1 + n)*(c*x + d*x^2)^(1 + n))/(1 + n)
```

3.188.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1217
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*
d^2 - b*d*e + a*e^2, 0] && EqQ[c*e*f*(m + 2*p + 2) + g*(c*d*m - b*e*(m + p
+ 1)), 0]
```

3.188.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

method	result	size
gosper	$\frac{x^{2+n}(dx+c)\left(d\,x^{2}+cx\right)^{n}}{1+n}$	28
parallelrisch	$\frac{x^{3} x^{n} \left(\left(dx+c\right)x\right)^{n} cd+x^{2} x^{n} \left(\left(dx+c\right)x\right)^{n} c^{2}}{c(1+n)}$	48
risch	$\frac{(dx+c)x^{2} x^{2n} (dx+c)^{n} \mathrm{e}^{-\frac{i\,\mathrm{csgn}(ix(dx+c))\pi n(-\mathrm{csgn}(ix(dx+c))+\mathrm{csgn}(i(dx+c)))(-\mathrm{csgn}(ix(dx+c))+\mathrm{csgn}(ix))}{2}}}{1+n}$	83

input
```
int(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x,method=_RETURNVERBOSE)
```

output
```
x^(2+n)/(1+n)*(d*x+c)*(d*x^2+c*x)^n
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int x^n \left(cx+dx^2\right)^n \left(2cx+3dx^2\right)\,dx = \frac{(dx^3+cx^2)(dx^2+cx)^n x^n}{n+1}$$

input
```
integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2)*(d*x^2 + c*x)^n*x^n/(n + 1)
```

3.188.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(19) = 38.

Time = 1.55 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int x^n \left(cx+dx^2\right)^n \left(2cx+3dx^2\right)\,dx = \begin{cases} \frac{cx^2x^n\left(cx+dx^2\right)^n}{n+1} + \frac{dx^3x^n\left(cx+dx^2\right)^n}{n+1} & \text{for } n \neq -1 \\ 2\log\left(x\right) + \log\left(\frac{c}{d}+x\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**n*(d*x**2+c*x)**n*(3*d*x**2+2*c*x),x)
```

output
```
Piecewise(((c*x**2*x**n*(c*x + d*x**2)**n/(n + 1) + d*x**3*x**n*(c*x + d*x*
*2)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

3.188.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int x^n (cx + dx^2)^n (2cx + 3dx^2) \, dx = \frac{(dx^3 + cx^2)e^{(n \log(dx+c)+2n\log(x))}}{n+1}$$

input
```
integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

3.188.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(24) = 48.

Time = 0.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int x^n (cx + dx^2)^n (2cx + 3dx^2) \, dx = \frac{dx^3 x^n e^{(n \log(dx+c)+n\log(x))} + cx^2 x^n e^{(n \log(dx+c)+n\log(x))}}{n+1}$$

input
```
integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="giac")
```

output
```
(d*x^3*x^n*e^(n*log(d*x + c) + n*log(x)) + c*x^2*x^n*e^(n*log(d*x + c) + n
*log(x)))/(n + 1)
```

3.188.9 Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int x^n (cx + dx^2)^n (2cx + 3dx^2) \, dx = \frac{x^n x^2 (d x^2 + c x)^n (c + d x)}{n+1}$$

input
```
int(x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x)
```

output
```
(x^n*x^2*(c*x + d*x^2)^n*(c + d*x))/(n + 1)
```

3.189 $\int x^{2n}(c+dx)^n\left(2cx+3dx^2\right)\,dx$

3.189.1 Optimal result

Integrand size = 24, antiderivative size = 22

$$\int x^{2n}(c+dx)^n\left(2cx+3dx^2\right)\,dx=\frac{x^{2(1+n)}(c+dx)^{1+n}}{1+n}$$

output
```
x^(2+2*n)*(d*x+c)^(1+n)/(1+n)
```

3.189.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^{2n}(c+dx)^n\left(2cx+3dx^2\right)\,dx=\frac{x^{2+2n}(c+dx)^{1+n}}{1+n}$$

input
```
Integrate[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2),x]
```

output
```
(x^(2 + 2*n)*(c + d*x)^(1 + n))/(1 + n)
```

3.189.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {9, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n}\left(2cx+3dx^2\right)(c+dx)^n\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^{2n+1}(2c+3dx)(c+dx)^n dx$$

$$\Big\downarrow \; 83$$

$$\frac{x^{2(n+1)}(c+dx)^{n+1}}{n+1}$$

input
```
Int[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2),x]
```

output
```
(x^(2*(1 + n))*(c + d*x)^(1 + n))/(1 + n)
```

3.189.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

3.189.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{x^{2+2n}(dx+c)^{1+n}}{1+n}$	23
risch	$\frac{(dx+c)^n x^{2n} x^2 (dx+c)}{1+n}$	27
parallelrisch	$\frac{x^3 x^{2n}(dx+c)^n cd+x^2 x^{2n}(dx+c)^n c^2}{c(1+n)}$	48

input
```
int(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x,method=_RETURNVERBOSE)
```

output
```
x^(2+2*n)*(d*x+c)^(1+n)/(1+n)
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int x^{2n}(c+dx)^n \left(2cx+3dx^2\right) \, dx = \frac{(dx^3+cx^2)(dx+c)^n x^{2\,n}}{n+1}$$

input
```
integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2)*(d*x + c)^n*x^(2*n)/(n + 1)
```

3.189.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(17) = 34.

Time = 1.53 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int x^{2n}(c+dx)^n \left(2cx+3dx^2\right) \, dx = \begin{cases} \frac{cx^2x^{2n}(c+dx)^n}{n+1} + \frac{dx^3x^{2n}(c+dx)^n}{n+1} & \text{for } n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d}+x\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(2*n)*(d*x+c)**n*(3*d*x**2+2*c*x),x)
```

output
```
Piecewise(((c*x**2*x**(2*n)*(c + d*x)**n/(n + 1) + d*x**3*x**(2*n)*(c + d*x
)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

3.189.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int x^{2n}(c+dx)^n\left(2cx+3dx^2\right)\,dx=\frac{(dx^3+cx^2)e^{(n\log(dx+c)+2\,n\log(x))}}{n+1}$$

input
```
integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int x^{2n}(c+dx)^n\left(2cx+3dx^2\right)\,dx=\frac{(dx+c)^n dx^3x^{2\,n}+(dx+c)^n cx^2x^{2\,n}}{n+1}$$

input
```
integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="giac")
```

output
```
((d*x + c)^n*d*x^3*x^(2*n) + (d*x + c)^n*c*x^2*x^(2*n))/(n + 1)
```

3.189.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int x^{2n}(c+dx)^n\left(2cx+3dx^2\right)\,dx=\frac{x^{2\,n}\,x^2\,(c+d\,x)^n\,(c+d\,x)}{n+1}$$

input
```
int(x^(2*n)*(2*c*x + 3*d*x^2)*(c + d*x)^n,x)
```

output
```
(x^(2*n)*x^2*(c + d*x)^n*(c + d*x))/(n + 1)
```

3.190 $\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx$

3.190.1 Optimal result

Integrand size = 24, antiderivative size = 22

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(a+cx^2+dx^3\right)^{1+n}}{1+n}$$

output
```
(d*x^3+c*x^2+a)^(1+n)/(1+n)
```

3.190.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(a+x^2(c+dx)\right)^{1+n}}{1+n}$$

input
```
Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]
```

output
```
(a + x^2*(c + d*x))^(1 + n)/(1 + n)
```

3.190.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx$$

$$\Big\downarrow \; 2021$$

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

input
```
Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]
```

output
```
(a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

3.190.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.190.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{\left(x^{3} d+c\,x^{2}+a\right)^{1+n}}{1+n}$	23
risch	$\frac{\left(x^{3} d+c\,x^{2}+a\right)^{n} \left(x^{3} d+c\,x^{2}+a\right)}{1+n}$	33
parallelrisch	$\frac{x^{3} \left(x^{3} d+c\,x^{2}+a\right)^{n} d^{2}+x^{2} \left(x^{3} d+c\,x^{2}+a\right)^{n} c d+\left(x^{3} d+c\,x^{2}+a\right)^{n} a d}{d \left(1+n\right)}$	69
norman	$\frac{a\,{\mathrm e}^{n \ln \left(x^{3} d+c\,x^{2}+a\right)}}{1+n}+\frac{c\,x^{2} {\mathrm e}^{n \ln \left(x^{3} d+c\,x^{2}+a\right)}}{1+n}+\frac{d\,x^{3} {\mathrm e}^{n \ln \left(x^{3} d+c\,x^{2}+a\right)}}{1+n}$	77

input
```
int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x,method=_RETURNVERBOSE)
```

output
```
(d*x^3+c*x^2+a)^(1+n)/(1+n)
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\frac{(dx^3+cx^2+a)(dx^3+cx^2+a)^n}{n+1}$$

input
```
integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

3.190.6 Sympy [F(-1)]

Timed out.

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\text{Timed out}$$

input
```
integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**n,x)
```

output
```
Timed out
```

3.190.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\frac{(dx^3+cx^2+a)(dx^3+cx^2+a)^n}{n+1}$$

input
```
integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(dx^3+cx^2+a\right)^{n+1}}{n+1}$$

input
```
integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)
```

3.190.9 Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.95

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\left(\frac{a}{n+1}+\frac{c\,x^2}{n+1}+\frac{d\,x^3}{n+1}\right)\left(d\,x^3+c\,x^2+a\right)^n$$

input
```
int(x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x)
```

output
```
(a/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(a + c*x^2 + d*x^3)^n
```

3.191 $\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx$

3.191.1 Optimal result

Integrand size = 23, antiderivative size = 21

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\frac{\left(cx^2+dx^3\right)^{1+n}}{1+n}$$

output
```
(d*x^3+c*x^2)^(1+n)/(1+n)
```

3.191.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\frac{\left(x^2(c+dx)\right)^{1+n}}{1+n}$$

input
```
Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]
```

output
```
(x^2*(c + d*x))^(1 + n)/(1 + n)
```

3.191.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1942}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx$$

$$\Big\downarrow \; 1942$$

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

input
```
Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]
```

output
```
(c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

3.191.3.1 Defintions of rubi rules used

rule 1942
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] /; FreeQ[{a, b, c, d, e, j, m
, n, p}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && EqQ[
a*d*(m + j*p + 1) - b*c*(m + n + p*(j + n) + 1), 0] && (GtQ[e, 0] || Intege
rsQ[j]) && NeQ[m + j*p + 1, 0]
```

3.191.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

method	result	size
risch	$\frac{x^{2} \left(d x +c \right) \left(x^{2} \left(d x +c \right)\right)^{n}}{1+n}$	26
gosper	$\frac{\left(x^{3} d +c \,x^{2}\right)^{n} x^{2} \left(d x +c \right)}{1+n}$	28
parallelrisch	$\frac{x^{3} \left(x^{2} \left(d x +c \right)\right)^{n} c d +x^{2} \left(x^{2} \left(d x +c \right)\right)^{n} c^{2}}{c \left(1+n \right)}$	46
norman	$\frac{c \,x^{2} {\mathrm e}^{n \ln \left(x^{3} d +c \,x^{2}\right)}}{1+n}+\frac{d \,x^{3} {\mathrm e}^{n \ln \left(x^{3} d +c \,x^{2}\right)}}{1+n}$	52

input
```
int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x,method=_RETURNVERBOSE)
```

output
```
x^2*(d*x+c)/(1+n)*(x^2*(d*x+c))^n
```

3.191.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\frac{(dx^3+cx^2)(dx^3+cx^2)^n}{n+1}$$

input
```
integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)
```

3.191.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.

Time = 0.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.52

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\begin{cases}\frac{cx^2(cx^2+dx^3)^n}{n+1}+\frac{dx^3(cx^2+dx^3)^n}{n+1} & \text{for } n\neq -1\\ 2\log(x)+\log\left(\frac{c}{d}+x\right) & \text{otherwise}\end{cases}$$

input
```
integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**n,x)
```

output
```
Piecewise(((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 + d*x**3)*
*n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

3.191.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.52

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\frac{(dx^3+cx^2)e^{(n\log(dx+c)+2\,n\log(x))}}{n+1}$$

input
```
integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

3.191.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\frac{\left(dx^3+cx^2\right)^{n+1}}{n+1}$$

input
```
integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2)^(n + 1)/(n + 1)
```

3.191.9 Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\frac{x^2\left(d\,x^3+c\,x^2\right)^n(c+d\,x)}{n+1}$$

input
```
int(x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x)
```

output
```
(x^2*(c*x^2 + d*x^3)^n*(c + d*x))/(n + 1)
```

3.192 $\int (b+2cx+3dx^2)(a+bx+cx^2+dx^3)^7 \, dx$

3.192.1 Optimal result

Integrand size = 30, antiderivative size = 21

$$\int (b+2cx+3dx^2)(a+bx+cx^2+dx^3)^7 \, dx = \frac{1}{8}(a+bx+cx^2+dx^3)^8$$

output
```
1/8*(d*x^3+c*x^2+b*x+a)^8
```

3.192.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 143 vs. 2(21) = 42.

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 6.81

$$\begin{aligned}&\int (b+2cx+3dx^2)(a+bx+cx^2+dx^3)^7 \, dx \\ &= \frac{1}{8}x(b+x(c+dx))\left(8a^7+28a^6x(b+x(c+dx))+56a^5x^2(b+x(c+dx))^2\right. \\ &\quad +70a^4x^3(b+x(c+dx))^3+56a^3x^4(b+x(c+dx))^4+28a^2x^5(b+x(c+dx))^5 \\ &\quad \left.+8ax^6(b+x(c+dx))^6+x^7(b+x(c+dx))^7\right)\end{aligned}$$

input
```
Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]
```

output
```
(x*(b + x*(c + d*x))*(8*a^7 + 28*a^6*x*(b + x*(c + d*x)) + 56*a^5*x^2*(b +
 x*(c + d*x))^2 + 70*a^4*x^3*(b + x*(c + d*x))^3 + 56*a^3*x^4*(b + x*(c +
d*x))^4 + 28*a^2*x^5*(b + x*(c + d*x))^5 + 8*a*x^6*(b + x*(c + d*x))^6 + x
^7*(b + x*(c + d*x))^7))/8
```

3.192.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx+3dx^2)\left(a+bx+cx^2+dx^3\right)^7 dx$$

$$\Big\downarrow \; 2021$$

$$\frac{1}{8}\left(a+bx+cx^2+dx^3\right)^8$$

input
```
Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]
```

output
```
(a + b*x + c*x^2 + d*x^3)^8/8
```

3.192.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.192.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\left(x^3 d+c\,x^2+b x+a\right)^8}{8}$	20
norman	Expression too large to display	1579
gosper	Expression too large to display	1957
parallelrisch	Expression too large to display	1957
risch	Expression too large to display	1962

input
```
int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/8*(d*x^3+c*x^2+b*x+a)^8
```

3.192.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1528 vs. $2(19) = 38$.

Time = 0.26 (sec) , antiderivative size = 1528, normalized size of antiderivative = 72.76

$$\int (b + 2cx + 3dx^2) (a + bx + cx^2 + dx^3)^7 \, dx = \text{Too large to display}$$

input
```
integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="fricas")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 1/2*(7*c^2*d^6 + 2*b*d^7)*x^22 + (7*c^3*d^5 +
7*b*c*d^6 + a*d^7)*x^21 + 7/4*(5*c^4*d^4 + 12*b*c^2*d^5 + 2*(b^2 + 2*a*c)*
d^6)*x^20 + 7*(c^5*d^3 + 5*b*c^3*d^4 + a*b*d^6 + 3*(b^2*c + a*c^2)*d^5)*x^
19 + 7/2*(c^6*d^2 + 10*b*c^4*d^3 + a^2*d^6 + 2*(b^3 + 6*a*b*c)*d^5 + 5*(3*
b^2*c^2 + 2*a*c^3)*d^4)*x^18 + (c^7*d + 21*b*c^5*d^2 + 21*(a*b^2 + a^2*c)*
d^5 + 35*(b^3*c + 3*a*b*c^2)*d^4 + 35*(2*b^2*c^3 + a*c^4)*d^3)*x^17 + 1/8*
(c^8 + 56*b*c^6*d + 168*a^2*b*d^5 + 70*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4
+ 560*(b^3*c^2 + 2*a*b*c^3)*d^3 + 84*(5*b^2*c^4 + 2*a*c^5)*d^2)*x^16 + (b*
c^7 + 7*a^3*d^5 + 35*(a*b^3 + 3*a^2*b*c)*d^4 + 35*(b^4*c + 6*a*b^2*c^2 + 2
*a^2*c^3)*d^3 + 35*(2*b^3*c^3 + 3*a*b*c^4)*d^2 + 7*(3*b^2*c^5 + a*c^6)*d)*
x^15 + 1/2*(7*b^2*c^6 + 2*a*c^7 + 35*(3*a^2*b^2 + 2*a^3*c)*d^4 + 14*(b^5 +
 20*a*b^3*c + 30*a^2*b*c^2)*d^3 + 105*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^
2 + 14*(5*b^3*c^4 + 6*a*b*c^5)*d)*x^14 + 7*(b^3*c^5 + a*b*c^6 + 5*a^3*b*d^
4 + 5*(a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^3 + 3*(b^5*c + 10*a*b^3*c^2 + 10
*a^2*b*c^3)*d^2 + (5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*d)*x^13 + 7/4*(5*
b^4*c^4 + 12*a*b^2*c^5 + 2*a^2*c^6 + 5*a^4*d^4 + 40*(a^2*b^3 + 2*a^3*b*c)*
d^3 + 2*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d^2 + 4*(3*b^5*c^
2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*d)*x^12 + 7*(b^5*c^3 + 5*a*b^3*c^4 + 3*a^
2*b*c^5 + 5*(2*a^3*b^2 + a^4*c)*d^3 + 3*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c
^2)*d^2 + (b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3*c^4)*d)*x^11 +...
```

3.192.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1771 vs. $2(17) = 34$.

Time = 0.18 (sec) , antiderivative size = 1771, normalized size of antiderivative = 84.33

$$\int (b+2cx+3dx^2)\left(a+bx+cx^2+dx^3\right)^7\,dx = \text{Too large to display}$$

input
```
integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**7,x)
```

output
```
a**7*b*x + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7*c**2*d**6/2) +
x**21*(a*d**7 + 7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 7*b**2*d**
6/2 + 21*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(7*a*b*d**6 + 21*a*c**2*d**
5 + 21*b**2*c*d**5 + 35*b*c**3*d**4 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2
+ 42*a*b*c*d**5 + 35*a*c**3*d**4 + 7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35
*b*c**4*d**3 + 7*c**6*d**2/2) + x**17*(21*a**2*c*d**5 + 21*a*b**2*d**5 + 1
05*a*b*c**2*d**4 + 35*a*c**4*d**3 + 35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 2
1*b*c**5*d**2 + c**7*d) + x**16*(21*a**2*b*d**5 + 105*a**2*c**2*d**4/2 + 1
05*a*b**2*c*d**4 + 140*a*b*c**3*d**3 + 21*a*c**5*d**2 + 35*b**4*d**4/4 + 7
0*b**3*c**2*d**3 + 105*b**2*c**4*d**2/2 + 7*b*c**6*d + c**8/8) + x**15*(7*
a**3*d**5 + 105*a**2*b*c*d**4 + 70*a**2*c**3*d**3 + 35*a*b**3*d**4 + 210*a
*b**2*c**2*d**3 + 105*a*b*c**4*d**2 + 7*a*c**6*d + 35*b**4*c*d**3 + 70*b**
3*c**3*d**2 + 21*b**2*c**5*d + b*c**7) + x**14*(35*a**3*c*d**4 + 105*a**2*
b**2*d**4/2 + 210*a**2*b*c**2*d**3 + 105*a**2*c**4*d**2/2 + 140*a*b**3*c*d
**3 + 210*a*b**2*c**3*d**2 + 42*a*b*c**5*d + a*c**7 + 7*b**5*d**3 + 105*b*
*4*c**2*d**2/2 + 35*b**3*c**4*d + 7*b**2*c**6/2) + x**13*(35*a**3*b*d**4 +
 70*a**3*c**2*d**3 + 210*a**2*b**2*c*d**3 + 210*a**2*b*c**3*d**2 + 21*a**2
*c**5*d + 35*a*b**4*d**3 + 210*a*b**3*c**2*d**2 + 105*a*b**2*c**4*d + 7*a*
b*c**6 + 21*b**5*c*d**2 + 35*b**4*c**3*d + 7*b**3*c**5) + x**12*(35*a**4*d
**4/4 + 140*a**3*b*c*d**3 + 70*a**3*c**3*d**2 + 70*a**2*b**3*d**3 + 315...
```

3.192.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int (b+2cx+3dx^2)\left(a+bx+cx^2+dx^3\right)^7\,dx = \frac{1}{8}\left(dx^3+cx^2+bx+a\right)^8$$

input
```
integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="maxima")
```

output
```
1/8*(d*x^3 + c*x^2 + b*x + a)^8
```

3.192.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(19) = 38.

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 7.62

$$\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^7\,dx$$
$$=\frac{1}{8}\left(dx^3+cx^2+bx\right)^8+\left(dx^3+cx^2+bx\right)^7a+\frac{7}{2}\left(dx^3+cx^2+bx\right)^6a^2$$
$$+7\left(dx^3+cx^2+bx\right)^5a^3+\frac{35}{4}\left(dx^3+cx^2+bx\right)^4a^4$$
$$+7\left(dx^3+cx^2+bx\right)^3a^5+\frac{7}{2}\left(dx^3+cx^2+bx\right)^2a^6+\left(dx^3+cx^2+bx\right)a^7$$

input
```
integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="giac")
```

output
```
1/8*(d*x^3 + c*x^2 + b*x)^8 + (d*x^3 + c*x^2 + b*x)^7*a + 7/2*(d*x^3 + c*x
^2 + b*x)^6*a^2 + 7*(d*x^3 + c*x^2 + b*x)^5*a^3 + 35/4*(d*x^3 + c*x^2 + b*
x)^4*a^4 + 7*(d*x^3 + c*x^2 + b*x)^3*a^5 + 7/2*(d*x^3 + c*x^2 + b*x)^2*a^6
 + (d*x^3 + c*x^2 + b*x)*a^7
```

3.192.9 Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 1576, normalized size of antiderivative = 75.05

$$\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^7\,dx=\text{Too large to display}$$

input
```
int((b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x)
```

output
```
x^12*((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + (35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21
*a*b^2*c^5 + 21*b^5*c^2*d + 70*a^2*b^3*d^3 + 70*a^3*c^3*d^2 + 315*a^2*b^2*
c^2*d^2 + 140*a*b^3*c^3*d + 105*a*b^4*c*d^2 + 105*a^2*b*c^4*d + 140*a^3*b*
c*d^3) + x^11*(7*b^5*c^3 + 35*a*b^3*c^4 + 21*a^2*b*c^5 + 21*a*b^5*d^2 + 35
*a^3*c^4*d + 35*a^4*c*d^3 + 70*a^3*b^2*d^3 + 7*b^6*c*d + 210*a^2*b^2*c^3*d
 + 210*a^2*b^3*c*d^2 + 210*a^3*b*c^2*d^2 + 105*a*b^4*c^2*d) + x^13*(7*b^3*
c^5 + 35*a*b^4*d^3 + 35*a^3*b*d^4 + 21*a^2*c^5*d + 35*b^4*c^3*d + 21*b^5*c
*d^2 + 70*a^3*c^2*d^3 + 7*a*b*c^6 + 210*a*b^3*c^2*d^2 + 210*a^2*b*c^3*d^2
+ 210*a^2*b^2*c*d^3 + 105*a*b^2*c^4*d) + x^5*(7*a^3*b^5 + 35*a^4*b^3*c + 2
1*a^5*b*c^2 + 21*a^5*b^2*d + 7*a^6*c*d) + x^19*(7*c^5*d^3 + 21*a*c^2*d^5 +
 35*b*c^3*d^4 + 21*b^2*c*d^5 + 7*a*b*d^6) + x^8*(b^8/8 + (35*a^4*c^4)/4 +
21*a^2*b^5*d + 21*a^5*c*d^2 + (105*a^2*b^4*c^2)/2 + 70*a^3*b^2*c^3 + (105*
a^4*b^2*d^2)/2 + 7*a*b^6*c + 140*a^3*b^3*c*d + 105*a^4*b*c^2*d) + x^9*(b^7
*c + 7*a^5*d^3 + 21*a*b^5*c^2 + 35*a^3*b*c^4 + 35*a^4*c^3*d + 70*a^2*b^3*c
^3 + 70*a^3*b^3*d^2 + 7*a*b^6*d + 210*a^3*b^2*c^2*d + 105*a^2*b^4*c*d + 10
5*a^4*b*c*d^2) + x^16*(c^8/8 + (35*b^4*d^4)/4 + 21*a^2*b*d^5 + 21*a*c^5*d^
2 + (105*a^2*c^2*d^4)/2 + (105*b^2*c^4*d^2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d
 + 140*a*b*c^3*d^3 + 105*a*b^2*c*d^4) + x^10*(b^7*d + 7*a^3*c^5 + (7*b^6*c
^2)/2 + 35*a*b^4*c^3 + 35*a^4*b*d^3 + (105*a^2*b^2*c^4)/2 + (105*a^2*b^4*d
^2)/2 + (105*a^4*c^2*d^2)/2 + 210*a^2*b^3*c^2*d + 210*a^3*b^2*c*d^2 + 4...
```

3.192. $\int (b+2cx+3dx^2)\left(a+bx+cx^2+dx^3\right)^7 \, dx$

3.193 $\int (b+2cx+3dx^2) (bx+cx^2+dx^3)^7 \, dx$

3.193.1 Optimal result

Integrand size = 29, antiderivative size = 20

$$\int (b+2cx+3dx^2) (bx+cx^2+dx^3)^7 \, dx = \frac{1}{8}(bx+cx^2+dx^3)^8$$

output
```
1/8*(d*x^3+c*x^2+b*x)^8
```

3.193.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (b+2cx+3dx^2) (bx+cx^2+dx^3)^7 \, dx = \frac{1}{8}x^8(b+x(c+dx))^8$$

input
```
Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]
```

output
```
(x^8*(b + x*(c + d*x))^8)/8
```

3.193.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^7 dx$$

$$\Big\downarrow \text{2021}$$

$$\frac{1}{8}\left(bx+cx^2+dx^3\right)^8$$

input
```
Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]
```

output
```
(b*x + c*x^2 + d*x^3)^8/8
```

3.193.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.193.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result
gosper	$\frac{x^8(dx^2+cx+b)^8}{8}$
default	$\frac{(x^3d+cx^2+bx)^8}{8}$
norman	$\frac{d^8x^{24}}{8}+c\,d^7x^{23}+\left(b\,d^7+\frac{7}{2}c^2d^6\right)x^{22}+\left(\frac{7}{2}b^2d^6+21b\,c^2d^5+\frac{35}{4}c^4d^4\right)x^{20}+\left(7bc\,d^6+7c^3d^5\right)x^{21}$
risch	$\frac{7}{2}x^{12}b^6d^2+\frac{35}{4}x^{12}b^4c^4+c\,d^7x^{23}+b\,c^7x^{15}+21b^2c\,d^5x^{19}+35b\,c^3d^4x^{19}+35b^3c\,d^4x^{17}+70b^2c^3$
parallelrisch	$\frac{7}{2}x^{12}b^6d^2+\frac{35}{4}x^{12}b^4c^4+c\,d^7x^{23}+b\,c^7x^{15}+21b^2c\,d^5x^{19}+35b\,c^3d^4x^{19}+35b^3c\,d^4x^{17}+70b^2c^3$

input
```
int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x,method=_RETURNVERBOSE)
```

output
```
1/8*x^8*(d*x^2+c*x+b)^8
```

3.193.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(18) = 36.

Time = 0.27 (sec) , antiderivative size = 441, normalized size of antiderivative = 22.05

$$\begin{aligned}
&\int (b+2cx+3dx^2)\left(bx+cx^2+dx^3\right)^7 \, dx \\
&= \frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{1}{2}\left(7c^2d^6 + 2bd^7\right)x^{22} + 7\left(c^3d^5 + bcd^6\right)x^{21} \\
&\quad + \frac{7}{4}\left(5c^4d^4 + 12bc^2d^5 + 2b^2d^6\right)x^{20} + 7\left(c^5d^3 + 5bc^3d^4 + 3b^2cd^5\right)x^{19} \\
&\quad + \frac{7}{2}\left(c^6d^2 + 10bc^4d^3 + 15b^2c^2d^4 + 2b^3d^5\right)x^{18} \\
&\quad + \left(c^7d + 21bc^5d^2 + 70b^2c^3d^3 + 35b^3cd^4\right)x^{17} + b^7cx^9 \\
&\quad + \frac{1}{8}\left(c^8 + 56bc^6d + 420b^2c^4d^2 + 560b^3c^2d^3 + 70b^4d^4\right)x^{16} \\
&\quad + \frac{1}{8}b^8x^8 + \left(bc^7 + 21b^2c^5d + 70b^3c^3d^2 + 35b^4cd^3\right)x^{15} \\
&\quad + \frac{7}{2}\left(b^2c^6 + 10b^3c^4d + 15b^4c^2d^2 + 2b^5d^3\right)x^{14} + 7\left(b^3c^5 + 5b^4c^3d + 3b^5cd^2\right)x^{13} \\
&\quad + \frac{7}{4}\left(5b^4c^4 + 12b^5c^2d + 2b^6d^2\right)x^{12} + 7\left(b^5c^3 + b^6cd\right)x^{11} + \frac{1}{2}\left(7b^6c^2 + 2b^7d\right)x^{10}
\end{aligned}$$

input
```
integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="fricas")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 1/2*(7*c^2*d^6 + 2*b*d^7)*x^22 + 7*(c^3*d^5 +
b*c*d^6)*x^21 + 7/4*(5*c^4*d^4 + 12*b*c^2*d^5 + 2*b^2*d^6)*x^20 + 7*(c^5*d
^3 + 5*b*c^3*d^4 + 3*b^2*c*d^5)*x^19 + 7/2*(c^6*d^2 + 10*b*c^4*d^3 + 15*b^
2*c^2*d^4 + 2*b^3*d^5)*x^18 + (c^7*d + 21*b*c^5*d^2 + 70*b^2*c^3*d^3 + 35*
b^3*c*d^4)*x^17 + b^7*c*x^9 + 1/8*(c^8 + 56*b*c^6*d + 420*b^2*c^4*d^2 + 56
0*b^3*c^2*d^3 + 70*b^4*d^4)*x^16 + 1/8*b^8*x^8 + (b*c^7 + 21*b^2*c^5*d + 7
0*b^3*c^3*d^2 + 35*b^4*c*d^3)*x^15 + 7/2*(b^2*c^6 + 10*b^3*c^4*d + 15*b^4*
c^2*d^2 + 2*b^5*d^3)*x^14 + 7*(b^3*c^5 + 5*b^4*c^3*d + 3*b^5*c*d^2)*x^13 +
 7/4*(5*b^4*c^4 + 12*b^5*c^2*d + 2*b^6*d^2)*x^12 + 7*(b^5*c^3 + b^6*c*d)*x
^11 + 1/2*(7*b^6*c^2 + 2*b^7*d)*x^10
```

3.193.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(15) = 30.

Time = 0.08 (sec) , antiderivative size = 469, normalized size of antiderivative = 23.45

$$\int (b+2cx+3dx^2)(bx+cx^2+dx^3)^7\,dx = \frac{b^8x^8}{8}+b^7cx^9+cd^7x^{23}+\frac{d^8x^{24}}{8}+x^{22}\left(bd^7+\frac{7c^2d^6}{2}\right)+x^{21}\cdot(7bcd^6+7c^3d^5)+x^{20}\cdot\left(\frac{7b^2d^6}{2}+21bc^2d^5+\frac{35c^4d^4}{4}\right)+x^{19}\cdot(21b^2cd^5+35bc^3d^4+7c^5d^3)+x^{18}\cdot\left(7b^3d^5+\frac{105b^2c^2d^4}{2}+35bc^4d^3+\frac{7c^6d^2}{2}\right)+x^{17}\cdot(35b^3cd^4+70b^2c^3d^3+21bc^5d^2+c^7d)+x^{16}\cdot\left(\frac{35b^4d^4}{4}+70b^3c^2d^3+\frac{105b^2c^4d^2}{2}+7bc^6d+\frac{c^8}{8}\right)+x^{15}\cdot(35b^4cd^3+70b^3c^3d^2+21b^2c^5d+bc^7)+x^{14}\cdot\left(7b^5d^3+\frac{105b^4c^2d^2}{2}+35b^3c^4d+\frac{7b^2c^6}{2}\right)+x^{13}\cdot(21b^5cd^2+35b^4c^3d+7b^3c^5)+x^{12}\cdot\left(\frac{7b^6d^2}{2}+21b^5c^2d+\frac{35b^4c^4}{4}\right)+x^{11}\cdot(7b^6cd+7b^5c^3)+x^{10}\left(b^7d+\frac{7b^6c^2}{2}\right)$$

input
```
integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**7,x)
```

output
```
b**8*x**8/8 + b**7*c*x**9 + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 +
7*c**2*d**6/2) + x**21*(7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*b**2*d**6/2 +
 21*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(21*b**2*c*d**5 + 35*b*c**3*d**4
 + 7*c**5*d**3) + x**18*(7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c**4*d*
*3 + 7*c**6*d**2/2) + x**17*(35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**
5*d**2 + c**7*d) + x**16*(35*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c*
*4*d**2/2 + 7*b*c**6*d + c**8/8) + x**15*(35*b**4*c*d**3 + 70*b**3*c**3*d*
*2 + 21*b**2*c**5*d + b*c**7) + x**14*(7*b**5*d**3 + 105*b**4*c**2*d**2/2
+ 35*b**3*c**4*d + 7*b**2*c**6/2) + x**13*(21*b**5*c*d**2 + 35*b**4*c**3*d
 + 7*b**3*c**5) + x**12*(7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4)
+ x**11*(7*b**6*c*d + 7*b**5*c**3) + x**10*(b**7*d + 7*b**6*c**2/2)
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (b+2cx+3dx^2)\left(bx+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(dx^3+cx^2+bx\right)^8$$

input
```
integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="maxima")
```

output
```
1/8*(d*x^3 + c*x^2 + b*x)^8
```

3.193.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (b+2cx+3dx^2)\left(bx+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(dx^3+cx^2+bx\right)^8$$

input
```
integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="giac")
```

output
```
1/8*(d*x^3 + c*x^2 + b*x)^8
```

3.193.9 Mupad [B] (verification not implemented)

Time = 9.40 (sec) , antiderivative size = 418, normalized size of antiderivative = 20.90

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^7\,dx = x^{14}\left(7\,b^5\,d^3+\frac{105\,b^4\,c^2\,d^2}{2}+35\,b^3\,c^4\,d+\frac{7\,b^2\,c^6}{2}\right)+x^{18}\left(7\,b^3\,d^5+\frac{105\,b^2\,c^2\,d^4}{2}+35\,b\,c^4\,d^3+\frac{7\,c^6\,d^2}{2}\right)+x^{12}\left(\frac{7\,b^6\,d^2}{2}+21\,b^5\,c^2\,d+\frac{35\,b^4\,c^4}{4}\right)+x^{20}\left(\frac{7\,b^2\,d^6}{2}+21\,b\,c^2\,d^5+\frac{35\,c^4\,d^4}{4}\right)+x^{16}\left(\frac{35\,b^4\,d^4}{4}+70\,b^3\,c^2\,d^3+\frac{105\,b^2\,c^4\,d^2}{2}+7\,b\,c^6\,d+\frac{c^8}{8}\right)+\frac{b^8\,x^8}{8}+\frac{d^8\,x^{24}}{8}+x^{10}\left(d\,b^7+\frac{7\,b^6\,c^2}{2}\right)+b^7\,c\,x^9+c\,d^7\,x^{23}+\frac{d^6\,x^{22}\,\left(7\,c^2+2\,b\,d\right)}{2}+7\,b^3\,c\,x^{13}\left(3\,b^2\,d^2+5\,b\,c^2\,d+c^4\right)+7\,c\,d^3\,x^{19}\left(3\,b^2\,d^2+5\,b\,c^2\,d+c^4\right)+b\,c\,x^{15}\left(35\,b^3\,d^3+70\,b^2\,c^2\,d^2+21\,b\,c^4\,d+c^6\right)+c\,d\,x^{17}\left(35\,b^3\,d^3+70\,b^2\,c^2\,d^2+21\,b\,c^4\,d+c^6\right)+7\,b^5\,c\,x^{11}\left(c^2+b\,d\right)+7\,c\,d^5\,x^{21}\left(c^2+b\,d\right)$$

input
```
int((b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x)
```

output
```
x^14*((7*b^2*c^6)/2 + 7*b^5*d^3 + 35*b^3*c^4*d + (105*b^4*c^2*d^2)/2) + x^
18*(7*b^3*d^5 + (7*c^6*d^2)/2 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2) + x^12
*((35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*b^5*c^2*d) + x^20*((7*b^2*d^6)/2 + (
35*c^4*d^4)/4 + 21*b*c^2*d^5) + x^16*(c^8/8 + (35*b^4*d^4)/4 + (105*b^2*c^
4*d^2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d) + (b^8*x^8)/8 + (d^8*x^24)/8 + x^10
*(b^7*d + (7*b^6*c^2)/2) + b^7*c*x^9 + c*d^7*x^23 + (d^6*x^22*(2*b*d + 7*c
^2))/2 + 7*b^3*c*x^13*(c^4 + 3*b^2*d^2 + 5*b*c^2*d) + 7*c*d^3*x^19*(c^4 +
3*b^2*d^2 + 5*b*c^2*d) + b*c*x^15*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*
b*c^4*d) + c*d*x^17*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d) + 7*b
^5*c*x^11*(b*d + c^2) + 7*c*d^5*x^21*(b*d + c^2)
```

3.194 $\int x^7(b+cx+dx^2)^7(b+2cx+3dx^2)\,dx$

3.194.1 Optimal result

Integrand size = 28, antiderivative size = 19

$$\int x^7(b+cx+dx^2)^7(b+2cx+3dx^2)\,dx=\frac{1}{8}x^8(b+cx+dx^2)^8$$

output
```
1/8*x^8*(d*x^2+c*x+b)^8
```

3.194.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int x^7(b+cx+dx^2)^7(b+2cx+3dx^2)\,dx=\frac{1}{8}x^8(b+x(c+dx))^8$$

input
```
Integrate[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x]
```

output
```
(x^8*(b + x*(c + d*x))^8)/8
```

3.194.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(b+cx+dx^2\right)^7 \left(b+2cx+3dx^2\right) \, dx$$

$$\Big\downarrow \text{2021}$$

$$\frac{1}{8} x^8 \left(b+cx+dx^2\right)^8$$

input
```
Int[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x]
```

output
```
(x^8*(b + c*x + d*x^2)^8)/8
```

3.194.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.194.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(17) = 34.

Time = 0.96 (sec) , antiderivative size = 439, normalized size of antiderivative = 23.11

method	result
norman	$\frac{d^8x^{24}}{8}+c\,d^7x^{23}+\left(b\,d^7+\frac{7}{2}c^2d^6\right)x^{22}+\left(\frac{7}{2}b^2d^6+21b\,c^2d^5+\frac{35}{4}c^4d^4\right)x^{20}+\left(7bc\,d^6+7c^3d^5\right)x^{21}$
gosper	$\frac{7}{2}x^{12}b^6d^2+\frac{35}{4}x^{12}b^4c^4+c\,d^7x^{23}+b\,c^7x^{15}+21b^2c\,d^5x^{19}+35b\,c^3d^4x^{19}+35b^3c\,d^4x^{17}+70b^2c^3$
risch	$\frac{7}{2}x^{12}b^6d^2+\frac{35}{4}x^{12}b^4c^4+c\,d^7x^{23}+b\,c^7x^{15}+21b^2c\,d^5x^{19}+35b\,c^3d^4x^{19}+35b^3c\,d^4x^{17}+70b^2c^3$
parallelrisch	$\frac{7}{2}x^{12}b^6d^2+\frac{35}{4}x^{12}b^4c^4+c\,d^7x^{23}+b\,c^7x^{15}+21b^2c\,d^5x^{19}+35b\,c^3d^4x^{19}+35b^3c\,d^4x^{17}+70b^2c^3$
default	Expression too large to display

input
```
int(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x,method=_RETURNVERBOSE)
```

output
```
1/8*d^8*x^24+c*d^7*x^23+(b*d^7+7/2*c^2*d^6)*x^22+(7/2*b^2*d^6+21*b*c^2*d^5
+35/4*c^4*d^4)*x^20+(7*b*c*d^6+7*c^3*d^5)*x^21+(35*b^4*c*d^3+70*b^3*c^3*d^
2+21*b^2*c^5*d+b*c^7)*x^15+(35/4*b^4*d^4+70*b^3*c^2*d^3+105/2*b^2*c^4*d^2+
7*b*c^6*d+1/8*c^8)*x^16+(35*b^3*c*d^4+70*b^2*c^3*d^3+21*b*c^5*d^2+c^7*d)*x
^17+(7*b^3*d^5+105/2*b^2*c^2*d^4+35*b*c^4*d^3+7/2*c^6*d^2)*x^18+(21*b^2*c*
d^5+35*b*c^3*d^4+7*c^5*d^3)*x^19+(7/2*b^6*d^2+21*b^5*c^2*d+35/4*b^4*c^4)*x
^12+(21*b^5*c*d^2+35*b^4*c^3*d+7*b^3*c^5)*x^13+(7*b^5*d^3+105/2*b^4*c^2*d^
2+35*b^3*c^4*d+7/2*c^6*b^2)*x^14+b^7*c*x^9+(b^7*d+7/2*c^2*b^6)*x^10+(7*b^6
*c*d+7*b^5*c^3)*x^11+1/8*x^8*b^8
```

3.194.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 441, normalized size of antiderivative = 23.21

$$\begin{aligned}\int x^7\left(b+cx+dx^2\right)^7\left(b+2cx+3dx^2\right)\,dx &= \frac{1}{8}d^8x^{24}+cd^7x^{23}+\frac{1}{2}\left(7\,c^2d^6+2\,bd^7\right)x^{22}+7\left(c^3d^5+bcd^6\right)x^{21}\\&+\frac{7}{4}\left(5\,c^4d^4+12\,bc^2d^5+2\,b^2d^6\right)x^{20}+7\left(c^5d^3+5\,bc^3d^4+3\,b^2cd^5\right)x^{19}\\&+\frac{7}{2}\left(c^6d^2+10\,bc^4d^3+15\,b^2c^2d^4+2\,b^3d^5\right)x^{18}\\&+\left(c^7d+21\,bc^5d^2+70\,b^2c^3d^3+35\,b^3cd^4\right)x^{17}+b^7cx^9\\&+\frac{1}{8}\left(c^8+56\,bc^6d+420\,b^2c^4d^2+560\,b^3c^2d^3+70\,b^4d^4\right)x^{16}\\&+\frac{1}{8}b^8x^8+\left(bc^7+21\,b^2c^5d+70\,b^3c^3d^2+35\,b^4cd^3\right)x^{15}\\&+\frac{7}{2}\left(b^2c^6+10\,b^3c^4d+15\,b^4c^2d^2+2\,b^5d^3\right)x^{14}+7\left(b^3c^5+5\,b^4c^3d+3\,b^5cd^2\right)x^{13}\\&+\frac{7}{4}\left(5\,b^4c^4+12\,b^5c^2d+2\,b^6d^2\right)x^{12}+7\left(b^5c^3+b^6cd\right)x^{11}+\frac{1}{2}\left(7\,b^6c^2+2\,b^7d\right)x^{10}\end{aligned}$$

input
```
integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="fricas")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 1/2*(7*c^2*d^6 + 2*b*d^7)*x^22 + 7*(c^3*d^5 +
b*c*d^6)*x^21 + 7/4*(5*c^4*d^4 + 12*b*c^2*d^5 + 2*b^2*d^6)*x^20 + 7*(c^5*d
^3 + 5*b*c^3*d^4 + 3*b^2*c*d^5)*x^19 + 7/2*(c^6*d^2 + 10*b*c^4*d^3 + 15*b^
2*c^2*d^4 + 2*b^3*d^5)*x^18 + (c^7*d + 21*b*c^5*d^2 + 70*b^2*c^3*d^3 + 35*
b^3*c*d^4)*x^17 + b^7*c*x^9 + 1/8*(c^8 + 56*b*c^6*d + 420*b^2*c^4*d^2 + 56
0*b^3*c^2*d^3 + 70*b^4*d^4)*x^16 + 1/8*b^8*x^8 + (b*c^7 + 21*b^2*c^5*d + 7
0*b^3*c^3*d^2 + 35*b^4*c*d^3)*x^15 + 7/2*(b^2*c^6 + 10*b^3*c^4*d + 15*b^4*
c^2*d^2 + 2*b^5*d^3)*x^14 + 7*(b^3*c^5 + 5*b^4*c^3*d + 3*b^5*c*d^2)*x^13 +
 7/4*(5*b^4*c^4 + 12*b^5*c^2*d + 2*b^6*d^2)*x^12 + 7*(b^5*c^3 + b^6*c*d)*x
^11 + 1/2*(7*b^6*c^2 + 2*b^7*d)*x^10
```

3.194.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(15) = 30$.

Time = 0.08 (sec) , antiderivative size = 469, normalized size of antiderivative = 24.68

$$\int x^7(b+cx+dx^2)^7(b+2cx+3dx^2)\,dx = \frac{b^8x^8}{8} + b^7cx^9 + cd^7x^{23} + \frac{d^8x^{24}}{8} + x^{22}\left(bd^7 + \frac{7c^2d^6}{2}\right) + x^{21}\cdot\left(7bcd^6 + 7c^3d^5\right) + x^{20}\cdot\left(\frac{7b^2d^6}{2} + 21bc^2d^5 + \frac{35c^4d^4}{4}\right) + x^{19}\cdot\left(21b^2cd^5 + 35bc^3d^4 + 7c^5d^3\right) + x^{18}\cdot\left(7b^3d^5 + \frac{105b^2c^2d^4}{2} + 35bc^4d^3 + \frac{7c^6d^2}{2}\right) + x^{17}\cdot\left(35b^3cd^4 + 70b^2c^3d^3 + 21bc^5d^2 + c^7d\right) + x^{16}\cdot\left(\frac{35b^4d^4}{4} + 70b^3c^2d^3 + \frac{105b^2c^4d^2}{2} + 7bc^6d + \frac{c^8}{8}\right) + x^{15}\cdot\left(35b^4cd^3 + 70b^3c^3d^2 + 21b^2c^5d + bc^7\right) + x^{14}\cdot\left(7b^5d^3 + \frac{105b^4c^2d^2}{2} + 35b^3c^4d + \frac{7b^2c^6}{2}\right) + x^{13}\cdot\left(21b^5cd^2 + 35b^4c^3d + 7b^3c^5\right) + x^{12}\cdot\left(\frac{7b^6d^2}{2} + 21b^5c^2d + \frac{35b^4c^4}{4}\right) + x^{11}\cdot\left(7b^6cd + 7b^5c^3\right) + x^{10}\left(b^7d + \frac{7b^6c^2}{2}\right)$$

input
```
integrate(x**7*(d*x**2+c*x+b)**7*(3*d*x**2+2*c*x+b),x)
```

output
```
b**8*x**8/8 + b**7*c*x**9 + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 +
7*c**2*d**6/2) + x**21*(7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*b**2*d**6/2 +
 21*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(21*b**2*c*d**5 + 35*b*c**3*d**4
 + 7*c**5*d**3) + x**18*(7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c**4*d*
*3 + 7*c**6*d**2/2) + x**17*(35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**
5*d**2 + c**7*d) + x**16*(35*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c*
*4*d**2/2 + 7*b*c**6*d + c**8/8) + x**15*(35*b**4*c*d**3 + 70*b**3*c**3*d*
*2 + 21*b**2*c**5*d + b*c**7) + x**14*(7*b**5*d**3 + 105*b**4*c**2*d**2/2
+ 35*b**3*c**4*d + 7*b**2*c**6/2) + x**13*(21*b**5*c*d**2 + 35*b**4*c**3*d
 + 7*b**3*c**5) + x**12*(7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4)
+ x**11*(7*b**6*c*d + 7*b**5*c**3) + x**10*(b**7*d + 7*b**6*c**2/2)
```

3.194.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 441, normalized size of antiderivative = 23.21

$$\begin{aligned}
&\int x^7(b+cx+dx^2)^7\left(b+2cx+3dx^2\right)\,dx \\
&= \frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{1}{2}\left(7\,c^2d^6 + 2\,bd^7\right)x^{22} + 7\left(c^3d^5 + bcd^6\right)x^{21} \\
&\quad + \frac{7}{4}\left(5\,c^4d^4 + 12\,bc^2d^5 + 2\,b^2d^6\right)x^{20} + 7\left(c^5d^3 + 5\,bc^3d^4 + 3\,b^2cd^5\right)x^{19} \\
&\quad + \frac{7}{2}\left(c^6d^2 + 10\,bc^4d^3 + 15\,b^2c^2d^4 + 2\,b^3d^5\right)x^{18} \\
&\quad + \left(c^7d + 21\,bc^5d^2 + 70\,b^2c^3d^3 + 35\,b^3cd^4\right)x^{17} + b^7cx^9 \\
&\quad + \frac{1}{8}\left(c^8 + 56\,bc^6d + 420\,b^2c^4d^2 + 560\,b^3c^2d^3 + 70\,b^4d^4\right)x^{16} \\
&\quad + \frac{1}{8}b^8x^8 + \left(bc^7 + 21\,b^2c^5d + 70\,b^3c^3d^2 + 35\,b^4cd^3\right)x^{15} \\
&\quad + \frac{7}{2}\left(b^2c^6 + 10\,b^3c^4d + 15\,b^4c^2d^2 + 2\,b^5d^3\right)x^{14} + 7\left(b^3c^5 + 5\,b^4c^3d + 3\,b^5cd^2\right)x^{13} \\
&\quad + \frac{7}{4}\left(5\,b^4c^4 + 12\,b^5c^2d + 2\,b^6d^2\right)x^{12} + 7\left(b^5c^3 + b^6cd\right)x^{11} + \frac{1}{2}\left(7\,b^6c^2 + 2\,b^7d\right)x^{10}
\end{aligned}$$

input
```
integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="maxima")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 1/2*(7*c^2*d^6 + 2*b*d^7)*x^22 + 7*(c^3*d^5 +
b*c*d^6)*x^21 + 7/4*(5*c^4*d^4 + 12*b*c^2*d^5 + 2*b^2*d^6)*x^20 + 7*(c^5*d
^3 + 5*b*c^3*d^4 + 3*b^2*c*d^5)*x^19 + 7/2*(c^6*d^2 + 10*b*c^4*d^3 + 15*b^
2*c^2*d^4 + 2*b^3*d^5)*x^18 + (c^7*d + 21*b*c^5*d^2 + 70*b^2*c^3*d^3 + 35*
b^3*c*d^4)*x^17 + b^7*c*x^9 + 1/8*(c^8 + 56*b*c^6*d + 420*b^2*c^4*d^2 + 56
0*b^3*c^2*d^3 + 70*b^4*d^4)*x^16 + 1/8*b^8*x^8 + (b*c^7 + 21*b^2*c^5*d + 7
0*b^3*c^3*d^2 + 35*b^4*c*d^3)*x^15 + 7/2*(b^2*c^6 + 10*b^3*c^4*d + 15*b^4*
c^2*d^2 + 2*b^5*d^3)*x^14 + 7*(b^3*c^5 + 5*b^4*c^3*d + 3*b^5*c*d^2)*x^13 +
 7/4*(5*b^4*c^4 + 12*b^5*c^2*d + 2*b^6*d^2)*x^12 + 7*(b^5*c^3 + b^6*c*d)*x
^11 + 1/2*(7*b^6*c^2 + 2*b^7*d)*x^10
```

3.194.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int x^7 \left(b+cx+dx^2\right)^7 \left(b+2cx+3dx^2\right)\,dx=\frac{1}{8}\left(dx^3+cx^2+bx\right)^8$$

input
```
integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="giac")
```

output
```
1/8*(d*x^3 + c*x^2 + b*x)^8
```

3.194.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 418, normalized size of antiderivative = 22.00

$$\int x^7 (b+cx+dx^2)^7 (b+2cx+3dx^2)\,dx = x^{14}\left(7\,b^5\,d^3+\frac{105\,b^4\,c^2\,d^2}{2}+35\,b^3\,c^4\,d+\frac{7\,b^2\,c^6}{2}\right) + x^{18}\left(7\,b^3\,d^5+\frac{105\,b^2\,c^2\,d^4}{2}+35\,b\,c^4\,d^3 + \frac{7\,c^6\,d^2}{2}\right)+x^{12}\left(\frac{7\,b^6\,d^2}{2}+21\,b^5\,c^2\,d+\frac{35\,b^4\,c^4}{4}\right) + x^{20}\left(\frac{7\,b^2\,d^6}{2}+21\,b\,c^2\,d^5+\frac{35\,c^4\,d^4}{4}\right) + x^{16}\left(\frac{35\,b^4\,d^4}{4}+70\,b^3\,c^2\,d^3+\frac{105\,b^2\,c^4\,d^2}{2} + 7\,b\,c^6\,d+\frac{c^8}{8}\right) + \frac{b^8\,x^8}{8}+\frac{d^8\,x^{24}}{8}+x^{10}\left(d\,b^7+\frac{7\,b^6\,c^2}{2}\right) + b^7\,c\,x^9+c\,d^7\,x^{23}+\frac{d^6\,x^{22}\,(7\,c^2+2\,b\,d)}{2} + 7\,b^3\,c\,x^{13}\,(3\,b^2\,d^2+5\,b\,c^2\,d+c^4) + 7\,c\,d^3\,x^{19}\,(3\,b^2\,d^2+5\,b\,c^2\,d+c^4) + b\,c\,x^{15}\,(35\,b^3\,d^3+70\,b^2\,c^2\,d^2+21\,b\,c^4\,d+c^6) + c\,d\,x^{17}\,(35\,b^3\,d^3+70\,b^2\,c^2\,d^2+21\,b\,c^4\,d+c^6) + 7\,b^5\,c\,x^{11}\,(c^2+b\,d)+7\,c\,d^5\,x^{21}\,(c^2+b\,d)$$

input
```
int(x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x)
```

output
```
x^14*((7*b^2*c^6)/2 + 7*b^5*d^3 + 35*b^3*c^4*d + (105*b^4*c^2*d^2)/2) + x^
18*(7*b^3*d^5 + (7*c^6*d^2)/2 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2) + x^12
*((35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*b^5*c^2*d) + x^20*((7*b^2*d^6)/2 + (
35*c^4*d^4)/4 + 21*b*c^2*d^5) + x^16*(c^8/8 + (35*b^4*d^4)/4 + (105*b^2*c^
4*d^2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d) + (b^8*x^8)/8 + (d^8*x^24)/8 + x^10
*(b^7*d + (7*b^6*c^2)/2) + b^7*c*x^9 + c*d^7*x^23 + (d^6*x^22*(2*b*d + 7*c
^2))/2 + 7*b^3*c*x^13*(c^4 + 3*b^2*d^2 + 5*b*c^2*d) + 7*c*d^3*x^19*(c^4 +
3*b^2*d^2 + 5*b*c^2*d) + b*c*x^15*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*
b*c^4*d) + c*d*x^17*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d) + 7*b
^5*c*x^11*(b*d + c^2) + 7*c*d^5*x^21*(b*d + c^2)
```

3.195 $\int (b+3dx^2)(a+bx+dx^3)^7\,dx$

3.195.1 Optimal result

Integrand size = 21, antiderivative size = 16

$$\int (b+3dx^2)\left(a+bx+dx^3\right)^7\,dx=\frac{1}{8}\left(a+bx+dx^3\right)^8$$

output
```
1/8*(d*x^3+b*x+a)^8
```

3.195.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 127 vs. 2(16) = 32.

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 7.94

$$\int (b+3dx^2)\left(a+bx+dx^3\right)^7\,dx=\frac{1}{8}x(b+dx^2)\left(8a^7+28a^6x(b+dx^2)+56a^5x^2(b+dx^2)^2+70a^4x^3(b+dx^2)^3+56a^3x^4(b+dx^2)^4+28a^2x^5(b+dx^2)^5+8ax^6(b+dx^2)^6+x^7(b+dx^2)^7\right)$$

input
```
Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]
```

output
```
(x*(b + d*x^2)*(8*a^7 + 28*a^6*x*(b + d*x^2) + 56*a^5*x^2*(b + d*x^2)^2 +
70*a^4*x^3*(b + d*x^2)^3 + 56*a^3*x^4*(b + d*x^2)^4 + 28*a^2*x^5*(b + d*x^
2)^5 + 8*a*x^6*(b + d*x^2)^6 + x^7*(b + d*x^2)^7))/8
```

3.195.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^7\,dx$$

$$\Big\downarrow \; 2021$$

$$\frac{1}{8}\left(a+bx+dx^3\right)^8$$

input
```
Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]
```

output
```
(a + b*x + d*x^3)^8/8
```

3.195.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.195.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(x^3d+bx+a\right)^8}{8}$
norman	$\frac{7x^{20}b^2d^6}{2}+a\,d^7x^{21}+x^{22}b\,d^7+\frac{d^8x^{24}}{8}+7ab\,d^6x^{19}+\left(7a^3d^5+35b^3a\,d^4\right)x^{15}+\left(21a^2b\,d^5+\frac{35}{4}b^4\right.$
gosper	$\frac{7}{2}x^{12}b^6d^2+\frac{35}{4}x^4a^4b^4+\frac{7}{2}x^6a^6d^2+\frac{7}{2}x^6a^2b^6+\frac{7}{2}x^2b^2a^6+70a^3b^3d^2x^9+7a\,b^6d\,x^9+70a^3b^2d^3x$
parallelrisch	$\frac{7}{2}x^{12}b^6d^2+\frac{35}{4}x^4a^4b^4+\frac{7}{2}x^6a^6d^2+\frac{7}{2}x^6a^2b^6+\frac{7}{2}x^2b^2a^6+70a^3b^3d^2x^9+7a\,b^6d\,x^9+70a^3b^2d^3x$
risch	$\frac{7}{2}x^{12}b^6d^2+\frac{35}{4}x^4a^4b^4+\frac{7}{2}x^6a^6d^2+\frac{7}{2}x^6a^2b^6+\frac{7}{2}x^2b^2a^6+70a^3b^3d^2x^9+7a\,b^6d\,x^9+70a^3b^2d^3x$

input
```
int((3*d*x^2+b)*(d*x^3+b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/8*(d*x^3+b*x+a)^8
```

3.195.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 456, normalized size of antiderivative = 28.50

$$\begin{aligned}\int (b+3dx^2)\,(a+bx+dx^3)^7\,dx &= \frac{1}{8}\,d^8x^{24}+bd^7x^{22}+ad^7x^{21}+\frac{7}{2}\,b^2d^6x^{20}\\&\quad+7\,abd^6x^{19}+21\,ab^2d^5x^{17}+\frac{7}{2}\,(2\,b^3d^5+a^2d^6)x^{18}\\&\quad+\frac{7}{4}\,(5\,b^4d^4+12\,a^2bd^5)x^{16}+7\,(5\,ab^3d^4+a^3d^5)x^{15}\\&\quad+\frac{7}{2}\,(2\,b^5d^3+15\,a^2b^2d^4)x^{14}+35\,(ab^4d^3+a^3bd^4)x^{13}\\&\quad+\frac{7}{4}\,(2\,b^6d^2+40\,a^2b^3d^3+5\,a^4d^4)x^{12}\\&\quad+7\,(3\,ab^5d^2+10\,a^3b^2d^3)x^{11}\\&\quad+\frac{1}{2}\,(2\,b^7d+105\,a^2b^4d^2+70\,a^4bd^3)x^{10}\\&\quad+\frac{7}{2}\,a^6b^2x^2+7\,(ab^6d+10\,a^3b^3d^2+a^5d^3)x^9\\&\quad+a^7bx+\frac{1}{8}\,(b^8+168\,a^2b^5d+420\,a^4b^2d^2)x^8\\&\quad+(ab^7+35\,a^3b^4d+21\,a^5bd^2)x^7\\&\quad+\frac{7}{2}\,(a^2b^6+10\,a^4b^3d+a^6d^2)x^6+7\,(a^3b^5+3\,a^5b^2d)x^5\\&\quad+\frac{7}{4}\,(5\,a^4b^4+4\,a^6bd)x^4+(7\,a^5b^3+a^7d)x^3\end{aligned}$$

input
```
integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="fricas")
```

output
```
1/8*d^8*x^24 + b*d^7*x^22 + a*d^7*x^21 + 7/2*b^2*d^6*x^20 + 7*a*b*d^6*x^19
 + 21*a*b^2*d^5*x^17 + 7/2*(2*b^3*d^5 + a^2*d^6)*x^18 + 7/4*(5*b^4*d^4 + 1
2*a^2*b*d^5)*x^16 + 7*(5*a*b^3*d^4 + a^3*d^5)*x^15 + 7/2*(2*b^5*d^3 + 15*a
^2*b^2*d^4)*x^14 + 35*(a*b^4*d^3 + a^3*b*d^4)*x^13 + 7/4*(2*b^6*d^2 + 40*a
^2*b^3*d^3 + 5*a^4*d^4)*x^12 + 7*(3*a*b^5*d^2 + 10*a^3*b^2*d^3)*x^11 + 1/2
*(2*b^7*d + 105*a^2*b^4*d^2 + 70*a^4*b*d^3)*x^10 + 7/2*a^6*b^2*x^2 + 7*(a*
b^6*d + 10*a^3*b^3*d^2 + a^5*d^3)*x^9 + a^7*b*x + 1/8*(b^8 + 168*a^2*b^5*d
 + 420*a^4*b^2*d^2)*x^8 + (a*b^7 + 35*a^3*b^4*d + 21*a^5*b*d^2)*x^7 + 7/2*
(a^2*b^6 + 10*a^4*b^3*d + a^6*d^2)*x^6 + 7*(a^3*b^5 + 3*a^5*b^2*d)*x^5 + 7
/4*(5*a^4*b^4 + 4*a^6*b*d)*x^4 + (7*a^5*b^3 + a^7*d)*x^3
```

3.195.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(12) = 24$.

Time = 0.08 (sec) , antiderivative size = 483, normalized size of antiderivative = 30.19

$$\int (b+3dx^2)\left(a+bx+dx^3\right)^7\,dx = a^7bx + \frac{7a^6b^2x^2}{2} + 21ab^2d^5x^{17} + 7abd^6x^{19} + ad^7x^{21} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8} + x^{18}\cdot\left(\frac{7a^2d^6}{2} + 7b^3d^5\right) + x^{16}\cdot\left(21a^2bd^5 + \frac{35b^4d^4}{4}\right) + x^{15}\cdot\left(7a^3d^5 + 35ab^3d^4\right) + x^{14}\cdot\left(\frac{105a^2b^2d^4}{2} + 7b^5d^3\right) + x^{13}\cdot\left(35a^3bd^4 + 35ab^4d^3\right) + x^{12}\cdot\left(\frac{35a^4d^4}{4} + 70a^2b^3d^3 + \frac{7b^6d^2}{2}\right) + x^{11}\cdot\left(70a^3b^2d^3 + 21ab^5d^2\right) + x^{10}\cdot\left(35a^4bd^3 + \frac{105a^2b^4d^2}{2} + b^7d\right) + x^9\cdot\left(7a^5d^3 + 70a^3b^3d^2 + 7ab^6d\right) + x^8\cdot\left(\frac{105a^4b^2d^2}{2} + 21a^2b^5d + \frac{b^8}{8}\right) + x^7\cdot\left(21a^5bd^2 + 35a^3b^4d + ab^7\right) + x^6\cdot\left(\frac{7a^6d^2}{2} + 35a^4b^3d + \frac{7a^2b^6}{2}\right) + x^5\cdot\left(21a^5b^2d + 7a^3b^5\right) + x^4\cdot\left(7a^6bd + \frac{35a^4b^4}{4}\right) + x^3\left(a^7d + 7a^5b^3\right)$$

input
```
integrate((3*d*x**2+b)*(d*x**3+b*x+a)**7,x)
```

output
```
a**7*b*x + 7*a**6*b**2*x**2/2 + 21*a*b**2*d**5*x**17 + 7*a*b*d**6*x**19 +
a*d**7*x**21 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 + d**8*x**24/8 + x**18*(
7*a**2*d**6/2 + 7*b**3*d**5) + x**16*(21*a**2*b*d**5 + 35*b**4*d**4/4) + x
**15*(7*a**3*d**5 + 35*a*b**3*d**4) + x**14*(105*a**2*b**2*d**4/2 + 7*b**5
*d**3) + x**13*(35*a**3*b*d**4 + 35*a*b**4*d**3) + x**12*(35*a**4*d**4/4 +
 70*a**2*b**3*d**3 + 7*b**6*d**2/2) + x**11*(70*a**3*b**2*d**3 + 21*a*b**5
*d**2) + x**10*(35*a**4*b*d**3 + 105*a**2*b**4*d**2/2 + b**7*d) + x**9*(7*
a**5*d**3 + 70*a**3*b**3*d**2 + 7*a*b**6*d) + x**8*(105*a**4*b**2*d**2/2 +
 21*a**2*b**5*d + b**8/8) + x**7*(21*a**5*b*d**2 + 35*a**3*b**4*d + a*b**7
) + x**6*(7*a**6*d**2/2 + 35*a**4*b**3*d + 7*a**2*b**6/2) + x**5*(21*a**5*
b**2*d + 7*a**3*b**5) + x**4*(7*a**6*b*d + 35*a**4*b**4/4) + x**3*(a**7*d
+ 7*a**5*b**3)
```

3.195.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int (b+3dx^2)\,(a+bx+dx^3)^7\,dx=\frac{1}{8}\,(dx^3+bx+a)^8$$

input
```
integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="maxima")
```

output
```
1/8*(d*x^3 + b*x + a)^8
```

3.195.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(14) = 28.

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 7.50

$$\begin{aligned}\int (b+3dx^2)\,(a+bx+dx^3)^7\,dx &= \frac{1}{8}\,(dx^3+bx)^8+(dx^3+bx)^7a+\frac{7}{2}\,(dx^3+bx)^6a^2\\&\quad+7\,(dx^3+bx)^5a^3+\frac{35}{4}\,(dx^3+bx)^4a^4\\&\quad+7\,(dx^3+bx)^3a^5+\frac{7}{2}\,(dx^3+bx)^2a^6+(dx^3+bx)a^7\end{aligned}$$

input
```
integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="giac")
```

output
```
1/8*(d*x^3 + b*x)^8 + (d*x^3 + b*x)^7*a + 7/2*(d*x^3 + b*x)^6*a^2 + 7*(d*x
^3 + b*x)^5*a^3 + 35/4*(d*x^3 + b*x)^4*a^4 + 7*(d*x^3 + b*x)^3*a^5 + 7/2*(
d*x^3 + b*x)^2*a^6 + (d*x^3 + b*x)*a^7
```

3.195.9 Mupad [B] (verification not implemented)

Time = 9.69 (sec) , antiderivative size = 438, normalized size of antiderivative = 27.38

$$\begin{aligned}\int (b+3dx^2)\left(a+bx+dx^3\right)^7\,dx = {} & x^{12}\left(\frac{35\,a^4\,d^4}{4}+70\,a^2\,b^3\,d^3+\frac{7\,b^6\,d^2}{2}\right) \\ & + x^4\left(7\,d\,a^6\,b+\frac{35\,a^4\,b^4}{4}\right)+x^{18}\left(\frac{7\,a^2\,d^6}{2}+7\,b^3\,d^5\right) \\ & + x^6\left(\frac{7\,a^6\,d^2}{2}+35\,a^4\,b^3\,d+\frac{7\,a^2\,b^6}{2}\right) \\ & + x^8\left(\frac{105\,a^4\,b^2\,d^2}{2}+21\,a^2\,b^5\,d+\frac{b^8}{8}\right) \\ & + \frac{d^8\,x^{24}}{8}+x^3\left(d\,a^7+7\,a^5\,b^3\right)+a\,d^7\,x^{21} \\ & + b\,d^7\,x^{22}+\frac{7\,a^6\,b^2\,x^2}{2}+\frac{7\,b^2\,d^6\,x^{20}}{2}+a^7\,b\,x \\ & + 21\,a\,b^2\,d^5\,x^{17}+a\,b\,x^7\left(21\,a^4\,d^2+35\,a^2\,b^3\,d+b^6\right) \\ & + 7\,a\,d\,x^9\left(a^4\,d^2+10\,a^2\,b^3\,d+b^6\right)+7\,a^3\,b^2\,x^5\left(3\,d\,a^2+b^3\right) \\ & + 7\,a\,d^4\,x^{15}\left(d\,a^2+5\,b^3\right)+\frac{7\,b\,d^4\,x^{16}\,(12\,d\,a^2+5\,b^3)}{4} \\ & + \frac{b\,d\,x^{10}\,(70\,a^4\,d^2+105\,a^2\,b^3\,d+2\,b^6)}{2} \\ & + 7\,a\,b\,d^6\,x^{19}+\frac{7\,b^2\,d^3\,x^{14}\,(15\,d\,a^2+2\,b^3)}{2} \\ & + 7\,a\,b^2\,d^2\,x^{11}\left(10\,d\,a^2+3\,b^3\right)+35\,a\,b\,d^3\,x^{13}\left(d\,a^2+b^3\right)\end{aligned}$$

input
```
int((b + 3*d*x^2)*(a + b*x + d*x^3)^7,x)
```

output
```
x^12*((35*a^4*d^4)/4 + (7*b^6*d^2)/2 + 70*a^2*b^3*d^3) + x^4*((35*a^4*b^4)
/4 + 7*a^6*b*d) + x^18*((7*a^2*d^6)/2 + 7*b^3*d^5) + x^6*((7*a^2*b^6)/2 +
(7*a^6*d^2)/2 + 35*a^4*b^3*d) + x^8*(b^8/8 + 21*a^2*b^5*d + (105*a^4*b^2*d
^2)/2) + (d^8*x^24)/8 + x^3*(a^7*d + 7*a^5*b^3) + a*d^7*x^21 + b*d^7*x^22
+ (7*a^6*b^2*x^2)/2 + (7*b^2*d^6*x^20)/2 + a^7*b*x + 21*a*b^2*d^5*x^17 + a
*b*x^7*(b^6 + 21*a^4*d^2 + 35*a^2*b^3*d) + 7*a*d*x^9*(b^6 + a^4*d^2 + 10*a
^2*b^3*d) + 7*a^3*b^2*x^5*(3*a^2*d + b^3) + 7*a*d^4*x^15*(a^2*d + 5*b^3) +
 (7*b*d^4*x^16*(12*a^2*d + 5*b^3))/4 + (b*d*x^10*(2*b^6 + 70*a^4*d^2 + 105
*a^2*b^3*d))/2 + 7*a*b*d^6*x^19 + (7*b^2*d^3*x^14*(15*a^2*d + 2*b^3))/2 +
7*a*b^2*d^2*x^11*(10*a^2*d + 3*b^3) + 35*a*b*d^3*x^13*(a^2*d + b^3)
```

3.195. $\int (b+3dx^2)(a+bx+dx^3)^7 \, dx$

3.196 $\int (b+3dx^2)(bx+dx^3)^7\,dx$

3.196.1 Optimal result

Integrand size = 20, antiderivative size = 15

$$\int (b+3dx^2)(bx+dx^3)^7\,dx=\frac{1}{8}(bx+dx^3)^8$$

output
```
1/8*(d*x^3+b*x)^8
```

3.196.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(15) = 30.

Time = 0.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 6.53

$$\int (b+3dx^2)(bx+dx^3)^7\,dx=\frac{b^8x^8}{8}+b^7dx^{10}+\frac{7}{2}b^6d^2x^{12}+7b^5d^3x^{14}+\frac{35}{4}b^4d^4x^{16}+7b^3d^5x^{18}+\frac{7}{2}b^2d^6x^{20}+bd^7x^{22}+\frac{d^8x^{24}}{8}$$

input
```
Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]
```

output
```
(b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d
^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7*x^22 + (d^8*x^24)
/8
```

3.196.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b+3dx^2\right)\left(bx+dx^3\right)^7\,dx$$

$$\Big\downarrow \text{ 2021}$$

$$\frac{1}{8}\left(bx+dx^3\right)^8$$

input
```
Int[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]
```

output
```
(b*x + d*x^3)^8/8
```

3.196.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.196.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result
default	$\frac{\left(x^3 d+b x\right)^8}{8}$
gosper	$\frac{x^8\left(d\,x^2+b\right)^8}{8}$
norman	$\frac{1}{8}d^8x^{24}+x^{22}b\,d^7+7x^{18}b^3d^5+\frac{7}{2}x^{20}b^2d^6+7x^{14}b^5d^3+\frac{35}{4}x^{16}b^4d^4+\frac{1}{8}x^8b^8+x^{10}b^7d+\frac{7}{2}x^{12}b^6d$
risch	$\frac{1}{8}d^8x^{24}+x^{22}b\,d^7+7x^{18}b^3d^5+\frac{7}{2}x^{20}b^2d^6+7x^{14}b^5d^3+\frac{35}{4}x^{16}b^4d^4+\frac{1}{8}x^8b^8+x^{10}b^7d+\frac{7}{2}x^{12}b^6d$
parallelrisch	$\frac{1}{8}d^8x^{24}+x^{22}b\,d^7+7x^{18}b^3d^5+\frac{7}{2}x^{20}b^2d^6+7x^{14}b^5d^3+\frac{35}{4}x^{16}b^4d^4+\frac{1}{8}x^8b^8+x^{10}b^7d+\frac{7}{2}x^{12}b^6d$

input
```
int((3*d*x^2+b)*(d*x^3+b*x)^7,x,method=_RETURNVERBOSE)
```

output
```
1/8*(d*x^3+b*x)^8
```

3.196.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(13) = 26$.

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.87

$$\int (b+3dx^2)(bx+dx^3)^7\,dx = \frac{1}{8}d^8x^{24} + bd^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8}b^8x^8$$

input
```
integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="fricas")
```

output
```
1/8*d^8*x^24 + b*d^7*x^22 + 7/2*b^2*d^6*x^20 + 7*b^3*d^5*x^18 + 35/4*b^4*d
^4*x^16 + 7*b^5*d^3*x^14 + 7/2*b^6*d^2*x^12 + b^7*d*x^10 + 1/8*b^8*x^8
```

3.196.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.47

$$\int (b+3dx^2)(bx+dx^3)^7\,dx = \frac{b^8x^8}{8} + b^7dx^{10} + \frac{7b^6d^2x^{12}}{2} + 7b^5d^3x^{14} + \frac{35b^4d^4x^{16}}{4} + 7b^3d^5x^{18} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8}$$

input
```
integrate((3*d*x**2+b)*(d*x**3+b*x)**7,x)
```

output
```
b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x**14 + 35*
b**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**20/2 + b*d**7*x**22
 + d**8*x**24/8
```

3.196.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int (b+3dx^2)\left(bx+dx^3\right)^7\,dx=\frac{1}{8}\left(dx^3+bx\right)^8$$

input
```
integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="maxima")
```

output
```
1/8*(d*x^3 + b*x)^8
```

3.196.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int (b+3dx^2)\left(bx+dx^3\right)^7\,dx=\frac{1}{8}\left(dx^3+bx\right)^8$$

input
```
integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="giac")
```

output
```
1/8*(d*x^3 + b*x)^8
```

3.196.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.87

$$\int (b+3dx^2)\left(bx+dx^3\right)^7\,dx=\frac{b^8\,x^8}{8}+b^7\,d\,x^{10}+\frac{7\,b^6\,d^2\,x^{12}}{2}+7\,b^5\,d^3\,x^{14}+\frac{35\,b^4\,d^4\,x^{16}}{4}+7\,b^3\,d^5\,x^{18}+\frac{7\,b^2\,d^6\,x^{20}}{2}+b\,d^7\,x^{22}+\frac{d^8\,x^{24}}{8}$$

input
```
int((b*x + d*x^3)^7*(b + 3*d*x^2),x)
```

output
```
(b^8*x^8)/8 + (d^8*x^24)/8 + b^7*d*x^10 + b*d^7*x^22 + (7*b^6*d^2*x^12)/2
+ 7*b^5*d^3*x^14 + (35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)
/2
```

3.197 $\int x^7(b+dx^2)^7(b+3dx^2)\,dx$

3.197.1 Optimal result

Integrand size = 21, antiderivative size = 16

$$\int x^7\left(b+dx^2\right)^7\left(b+3dx^2\right)\,dx=\frac{1}{8}x^8\left(b+dx^2\right)^8$$

output
```
1/8*x^8*(d*x^2+b)^8
```

3.197.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 6.12

$$\int x^7\left(b+dx^2\right)^7\left(b+3dx^2\right)\,dx=\frac{b^8x^8}{8}+b^7dx^{10}+\frac{7}{2}b^6d^2x^{12}+7b^5d^3x^{14}+\frac{35}{4}b^4d^4x^{16}+7b^3d^5x^{18}+\frac{7}{2}b^2d^6x^{20}+bd^7x^{22}+\frac{d^8x^{24}}{8}$$

input
```
Integrate[x^7*(b + d*x^2)^7*(b + 3*d*x^2),x]
```

output
```
(b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d
^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7*x^22 + (d^8*x^24)
/8
```

3.197.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {354, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(b+dx^2\right)^7 \left(b+3dx^2\right) \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int x^6 \left(dx^2+b\right)^7 \left(3dx^2+b\right) dx^2$$

$$\Big\downarrow \text{83}$$

$$\frac{1}{8} x^8 \left(b+dx^2\right)^8$$

input
```
Int[x^7*(b + d*x^2)^7*(b + 3*d*x^2),x]
```

output
```
(x^8*(b + d*x^2)^8)/8
```

3.197.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.197.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(14) = 28$.

Time = 0.77 (sec) , antiderivative size = 89, normalized size of antiderivative = 5.56

method	result
gosper	$\frac{1}{8}d^8x^{24} + x^{22}b\,d^7 + 7x^{18}b^3d^5 + \frac{7}{2}x^{20}b^2d^6 + 7x^{14}b^5d^3 + \frac{35}{4}x^{16}b^4d^4 + \frac{1}{8}x^8b^8 + x^{10}b^7d + \frac{7}{2}x^{12}b^6d$
default	$\frac{1}{8}d^8x^{24} + x^{22}b\,d^7 + 7x^{18}b^3d^5 + \frac{7}{2}x^{20}b^2d^6 + 7x^{14}b^5d^3 + \frac{35}{4}x^{16}b^4d^4 + \frac{1}{8}x^8b^8 + x^{10}b^7d + \frac{7}{2}x^{12}b^6d$
norman	$\frac{1}{8}d^8x^{24} + x^{22}b\,d^7 + 7x^{18}b^3d^5 + \frac{7}{2}x^{20}b^2d^6 + 7x^{14}b^5d^3 + \frac{35}{4}x^{16}b^4d^4 + \frac{1}{8}x^8b^8 + x^{10}b^7d + \frac{7}{2}x^{12}b^6d$
risch	$\frac{1}{8}d^8x^{24} + x^{22}b\,d^7 + 7x^{18}b^3d^5 + \frac{7}{2}x^{20}b^2d^6 + 7x^{14}b^5d^3 + \frac{35}{4}x^{16}b^4d^4 + \frac{1}{8}x^8b^8 + x^{10}b^7d + \frac{7}{2}x^{12}b^6d$
parallelrisch	$\frac{1}{8}d^8x^{24} + x^{22}b\,d^7 + 7x^{18}b^3d^5 + \frac{7}{2}x^{20}b^2d^6 + 7x^{14}b^5d^3 + \frac{35}{4}x^{16}b^4d^4 + \frac{1}{8}x^8b^8 + x^{10}b^7d + \frac{7}{2}x^{12}b^6d$

input
```
int(x^7*(d*x^2+b)^7*(3*d*x^2+b),x,method=_RETURNVERBOSE)
```

output
```
1/8*d^8*x^24+x^22*b*d^7+7*x^18*b^3*d^5+7/2*x^20*b^2*d^6+7*x^14*b^5*d^3+35/
4*x^16*b^4*d^4+1/8*x^8*b^8+x^10*b^7*d+7/2*x^12*b^6*d^2
```

3.197.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.50

$$\int x^7(b+dx^2)^7(b+3dx^2)\,dx = \frac{1}{8}d^8x^{24} + bd^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8}b^8x^8$$

input
```
integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="fricas")
```

output
```
1/8*d^8*x^24 + b*d^7*x^22 + 7/2*b^2*d^6*x^20 + 7*b^3*d^5*x^18 + 35/4*b^4*d
^4*x^16 + 7*b^5*d^3*x^14 + 7/2*b^6*d^2*x^12 + b^7*d*x^10 + 1/8*b^8*x^8
```

3.197.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(12) = 24.

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.06

$$\int x^7(b+dx^2)^7(b+3dx^2)\,dx = \frac{b^8x^8}{8}+b^7dx^{10}+\frac{7b^6d^2x^{12}}{2}+7b^5d^3x^{14}+\frac{35b^4d^4x^{16}}{4}+7b^3d^5x^{18}+\frac{7b^2d^6x^{20}}{2}+bd^7x^{22}+\frac{d^8x^{24}}{8}$$

input
```
integrate(x**7*(d*x**2+b)**7*(3*d*x**2+b),x)
```

output
```
b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x**14 + 35*
b**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**20/2 + b*d**7*x**22
 + d**8*x**24/8
```

3.197.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(14) = 28.

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.50

$$\int x^7(b+dx^2)^7(b+3dx^2)\,dx = \frac{1}{8}\,d^8x^{24}+bd^7x^{22}+\frac{7}{2}\,b^2d^6x^{20}+7\,b^3d^5x^{18}+\frac{35}{4}\,b^4d^4x^{16}+7\,b^5d^3x^{14}+\frac{7}{2}\,b^6d^2x^{12}+b^7dx^{10}+\frac{1}{8}\,b^8x^8$$

input
```
integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="maxima")
```

output
```
1/8*d^8*x^24 + b*d^7*x^22 + 7/2*b^2*d^6*x^20 + 7*b^3*d^5*x^18 + 35/4*b^4*d
^4*x^16 + 7*b^5*d^3*x^14 + 7/2*b^6*d^2*x^12 + b^7*d*x^10 + 1/8*b^8*x^8
```

3.197.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int x^7 \left(b+dx^2\right)^7 \left(b+3dx^2\right) \, dx = \frac{1}{8} \left(dx^3+bx\right)^8$$

input
```
integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="giac")
```

output
```
1/8*(d*x^3 + b*x)^8
```

3.197.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.50

$$\int x^7 \left(b+dx^2\right)^7 \left(b+3dx^2\right) \, dx = \frac{b^8\,x^8}{8}+b^7\,d\,x^{10}+\frac{7\,b^6\,d^2\,x^{12}}{2}+7\,b^5\,d^3\,x^{14}+\frac{35\,b^4\,d^4\,x^{16}}{4} +7\,b^3\,d^5\,x^{18}+\frac{7\,b^2\,d^6\,x^{20}}{2}+b\,d^7\,x^{22}+\frac{d^8\,x^{24}}{8}$$

input
```
int(x^7*(b + d*x^2)^7*(b + 3*d*x^2),x)
```

output
```
(b^8*x^8)/8 + (d^8*x^24)/8 + b^7*d*x^10 + b*d^7*x^22 + (7*b^6*d^2*x^12)/2
+ 7*b^5*d^3*x^14 + (35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)
/2
```

3.198 $\int (2cx+3dx^2)(a+cx^2+dx^3)^7\,dx$

3.198.1 Optimal result

Integrand size = 26, antiderivative size = 18

$$\int (2cx+3dx^2)\left(a+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

output
```
1/8*(d*x^3+c*x^2+a)^8
```

3.198.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 115 vs. 2(18) = 36.

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 6.39

$$\int (2cx+3dx^2)\left(a+cx^2+dx^3\right)^7\,dx=\frac{1}{8}x^2(c+dx)\left(8a^7+28a^6x^2(c+dx)+56a^5x^4(c+dx)^2+70a^4x^6(c+dx)^3+56a^3x^8(c+dx)^4+28a^2x^{10}(c+dx)^5+8ax^{12}(c+dx)^6+x^{14}(c+dx)^7\right)$$

input
```
Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]
```

output
```
(x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*x)^2 + 70
*a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x^10*(c + d*x)^5 +
8*a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8
```

3.198.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^7\,dx$$
$$\Big\downarrow \text{2021}$$
$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

input
```
Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]
```

output
```
(a + c*x^2 + d*x^3)^8/8
```

3.198.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.198.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(x^3 d+c\,x^2+a\right)^8}{8}$
norman	$\frac{7x^{22}c^2d^6}{2}+c\,d^7x^{23}+\frac{d^8x^{24}}{8}+\left(7ac\,d^6+\frac{35}{4}c^4d^4\right)x^{20}+\left(d^7a+7c^3d^5\right)x^{21}+\left(21a\,c^2d^5+7c^5d^3\right)x$
parallelrisch	$\frac{7}{2}x^4a^6c^2+\frac{7}{2}x^6a^6d^2+7x^6a^5c^3+x^2c\,a^7+35a^4c^3d\,x^9+35a^4c\,d^3x^{11}+35a^3c^4d\,x^{11}+c\,d^7x^{23}+$
gosper	$\frac{x^2\left(d^8x^{22}+8c\,d^7x^{21}+28x^{20}c^2d^6+8a\,d^7x^{19}+56c^3d^5x^{19}+56x^{18}ac\,d^6+70x^{18}c^4d^4+168a\,c^2d^5x^{17}+56c^5d^3x^{17}+28x^{16}a^2d^6+280x\right.}{}$
risch	$\frac{7}{2}x^4a^6c^2+\frac{7}{2}x^6a^6d^2+7x^6a^5c^3+x^2c\,a^7+35a^4c^3d\,x^9+35a^4c\,d^3x^{11}+35a^3c^4d\,x^{11}+c\,d^7x^{23}+$

input
```
int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/8*(d*x^3+c*x^2+a)^8
```

3.198.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(16) = 32$.

Time = 0.25 (sec) , antiderivative size = 458, normalized size of antiderivative = 25.44

$$\int (2cx+3dx^2)(a+cx^2+dx^3)^7\,dx = \frac{1}{8}d^8x^{24}+cd^7x^{23}+\frac{7}{2}c^2d^6x^{22}+(7c^3d^5+ad^7)x^{21} + \frac{7}{4}(5c^4d^4+4acd^6)x^{20}+7(c^5d^3+3ac^2d^5)x^{19} + \frac{7}{2}(c^6d^2+10ac^3d^4+a^2d^6)x^{18} + (c^7d+35ac^4d^3+21a^2cd^5)x^{17} + \frac{1}{8}(c^8+168ac^5d^2+420a^2c^2d^4)x^{16} + 7(ac^6d+10a^2c^3d^3+a^3d^5)x^{15}+21a^5c^2dx^7 + \frac{1}{2}(2ac^7+105a^2c^4d^2+70a^3cd^4)x^{14} + 7(3a^2c^5d+10a^3c^2d^3)x^{13}+7a^6cdx^5 + \frac{7}{4}(2a^2c^6+40a^3c^3d^2+5a^4d^4)x^{12} + \frac{7}{2}a^6c^2x^4+35(a^3c^4d+a^4cd^3)x^{11} + a^7dx^3+\frac{7}{2}(2a^3c^5+15a^4c^2d^2)x^{10} + a^7cx^2+7(5a^4c^3d+a^5d^3)x^9 + \frac{7}{4}(5a^4c^4+12a^5cd^2)x^8+\frac{7}{2}(2a^5c^3+a^6d^2)x^6$$

input
```
integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="fricas")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + (7*c^3*d^5 + a*d^7)*x^21 +
7/4*(5*c^4*d^4 + 4*a*c*d^6)*x^20 + 7*(c^5*d^3 + 3*a*c^2*d^5)*x^19 + 7/2*(c
^6*d^2 + 10*a*c^3*d^4 + a^2*d^6)*x^18 + (c^7*d + 35*a*c^4*d^3 + 21*a^2*c*d
^5)*x^17 + 1/8*(c^8 + 168*a*c^5*d^2 + 420*a^2*c^2*d^4)*x^16 + 7*(a*c^6*d +
 10*a^2*c^3*d^3 + a^3*d^5)*x^15 + 21*a^5*c^2*d*x^7 + 1/2*(2*a*c^7 + 105*a^
2*c^4*d^2 + 70*a^3*c*d^4)*x^14 + 7*(3*a^2*c^5*d + 10*a^3*c^2*d^3)*x^13 + 7
*a^6*c*d*x^5 + 7/4*(2*a^2*c^6 + 40*a^3*c^3*d^2 + 5*a^4*d^4)*x^12 + 7/2*a^6
*c^2*x^4 + 35*(a^3*c^4*d + a^4*c*d^3)*x^11 + a^7*d*x^3 + 7/2*(2*a^3*c^5 +
15*a^4*c^2*d^2)*x^10 + a^7*c*x^2 + 7*(5*a^4*c^3*d + a^5*d^3)*x^9 + 7/4*(5*
a^4*c^4 + 12*a^5*c*d^2)*x^8 + 7/2*(2*a^5*c^3 + a^6*d^2)*x^6
```

3.198.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 484, normalized size of antiderivative = 26.89

$$\int (2cx + 3dx^2) (a + cx^2 + dx^3)^7 \, dx = a^7cx^2 + a^7dx^3 + \frac{7a^6c^2x^4}{2} + 7a^6cdx^5 + 21a^5c^2dx^7 + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8} + x^{21}(ad^7 + 7c^3d^5) + x^{20} \cdot \left(7acd^6 + \frac{35c^4d^4}{4}\right) + x^{19} \cdot (21ac^2d^5 + 7c^5d^3) + x^{18} \cdot \left(\frac{7a^2d^6}{2} + 35ac^3d^4 + \frac{7c^6d^2}{2}\right) + x^{17} \cdot (21a^2cd^5 + 35ac^4d^3 + c^7d) + x^{16} \cdot \left(\frac{105a^2c^2d^4}{2} + 21ac^5d^2 + \frac{c^8}{8}\right) + x^{15} \cdot (7a^3d^5 + 70a^2c^3d^3 + 7ac^6d) + x^{14} \cdot \left(35a^3cd^4 + \frac{105a^2c^4d^2}{2} + ac^7\right) + x^{13} \cdot (70a^3c^2d^3 + 21a^2c^5d) + x^{12} \cdot \left(\frac{35a^4d^4}{4} + 70a^3c^3d^2 + \frac{7a^2c^6}{2}\right) + x^{11} \cdot (35a^4cd^3 + 35a^3c^4d) + x^{10} \cdot \left(\frac{105a^4c^2d^2}{2} + 7a^3c^5\right) + x^9 \cdot (7a^5d^3 + 35a^4c^3d) + x^8 \cdot \left(21a^5cd^2 + \frac{35a^4c^4}{4}\right) + x^6 \cdot \left(\frac{7a^6d^2}{2} + 7a^5c^3\right)$$

input
```
integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**7,x)
```

output
```
a**7*c*x**2 + a**7*d*x**3 + 7*a**6*c**2*x**4/2 + 7*a**6*c*d*x**5 + 21*a**5
*c**2*d*x**7 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8 + x**21*(
a*d**7 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 35*c**4*d**4/4) + x**19*(21*a*
c**2*d**5 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 35*a*c**3*d**4 + 7*c**6*
d**2/2) + x**17*(21*a**2*c*d**5 + 35*a*c**4*d**3 + c**7*d) + x**16*(105*a*
*2*c**2*d**4/2 + 21*a*c**5*d**2 + c**8/8) + x**15*(7*a**3*d**5 + 70*a**2*c
**3*d**3 + 7*a*c**6*d) + x**14*(35*a**3*c*d**4 + 105*a**2*c**4*d**2/2 + a*
c**7) + x**13*(70*a**3*c**2*d**3 + 21*a**2*c**5*d) + x**12*(35*a**4*d**4/4
 + 70*a**3*c**3*d**2 + 7*a**2*c**6/2) + x**11*(35*a**4*c*d**3 + 35*a**3*c*
*4*d) + x**10*(105*a**4*c**2*d**2/2 + 7*a**3*c**5) + x**9*(7*a**5*d**3 + 3
5*a**4*c**3*d) + x**8*(21*a**5*c*d**2 + 35*a**4*c**4/4) + x**6*(7*a**6*d**
2/2 + 7*a**5*c**3)
```

3.198.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(dx^3+cx^2+a\right)^8$$

input
```
integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="maxima")
```

output
```
1/8*(d*x^3 + c*x^2 + a)^8
```

3.198.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(16) = 32.

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 7.56

$$\begin{aligned}\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^7\,dx=&\ \frac{1}{8}\left(dx^3+cx^2\right)^8+\left(dx^3+cx^2\right)^7a\\&+\frac{7}{2}\left(dx^3+cx^2\right)^6a^2+7\left(dx^3+cx^2\right)^5a^3\\&+\frac{35}{4}\left(dx^3+cx^2\right)^4a^4+7\left(dx^3+cx^2\right)^3a^5\\&+\frac{7}{2}\left(dx^3+cx^2\right)^2a^6+\left(dx^3+cx^2\right)a^7\end{aligned}$$

input
```
integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="giac")
```

output
```
1/8*(d*x^3 + c*x^2)^8 + (d*x^3 + c*x^2)^7*a + 7/2*(d*x^3 + c*x^2)^6*a^2 +
7*(d*x^3 + c*x^2)^5*a^3 + 35/4*(d*x^3 + c*x^2)^4*a^4 + 7*(d*x^3 + c*x^2)^3
*a^5 + 7/2*(d*x^3 + c*x^2)^2*a^6 + (d*x^3 + c*x^2)*a^7
```

3.198.9 Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 440, normalized size of antiderivative = 24.44

$$\begin{aligned}\int (2cx+3dx^2)(a+cx^2+dx^3)^7\,dx &= x^{12}\left(\frac{35\,a^4\,d^4}{4}+70\,a^3\,c^3\,d^2+\frac{7\,a^2\,c^6}{2}\right)\\&+x^6\left(\frac{7\,a^6\,d^2}{2}+7\,a^5\,c^3\right)+x^{20}\left(\frac{35\,c^4\,d^4}{4}+7\,a\,c\,d^6\right)\\&+x^{16}\left(\frac{105\,a^2\,c^2\,d^4}{2}+21\,a\,c^5\,d^2+\frac{c^8}{8}\right)\\&+x^{18}\left(\frac{7\,a^2\,d^6}{2}+35\,a\,c^3\,d^4+\frac{7\,c^6\,d^2}{2}\right)\\&+\frac{d^8\,x^{24}}{8}+x^{21}\left(7\,c^3\,d^5+a\,d^7\right)+a^7\,c\,x^2\\&+a^7\,d\,x^3+c\,d^7\,x^{23}+\frac{7\,a^6\,c^2\,x^4}{2}+\frac{7\,c^2\,d^6\,x^{22}}{2}\\&+21\,a^5\,c^2\,d\,x^7+7\,a\,d\,x^{15}\left(a^2\,d^4+10\,a\,c^3\,d^2+c^6\right)\\&+c\,d\,x^{17}\left(21\,a^2\,d^4+35\,a\,c^3\,d^2+c^6\right)\\&+\frac{7\,a^4\,c\,x^8\,(5\,c^3+12\,a\,d^2)}{4}\\&+7\,a^4\,d\,x^9\left(5\,c^3+a\,d^2\right)+7\,c^2\,d^3\,x^{19}\left(c^3+3\,a\,d^2\right)\\&+\frac{a\,c\,x^{14}\,(70\,a^2\,d^4+105\,a\,c^3\,d^2+2\,c^6)}{2}\\&+7\,a^6\,c\,d\,x^5+\frac{7\,a^3\,c^2\,x^{10}\,(2\,c^3+15\,a\,d^2)}{2}\\&+7\,a^2\,c^2\,d\,x^{13}\left(3\,c^3+10\,a\,d^2\right)+35\,a^3\,c\,d\,x^{11}\left(c^3+a\,d^2\right)\end{aligned}$$

input
```
int((2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x)
```

output
```
x^12*((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + 70*a^3*c^3*d^2) + x^6*(7*a^5*c^3 +
(7*a^6*d^2)/2) + x^20*((35*c^4*d^4)/4 + 7*a*c*d^6) + x^16*(c^8/8 + 21*a*c^
5*d^2 + (105*a^2*c^2*d^4)/2) + x^18*((7*a^2*d^6)/2 + (7*c^6*d^2)/2 + 35*a*
c^3*d^4) + (d^8*x^24)/8 + x^21*(a*d^7 + 7*c^3*d^5) + a^7*c*x^2 + a^7*d*x^3
 + c*d^7*x^23 + (7*a^6*c^2*x^4)/2 + (7*c^2*d^6*x^22)/2 + 21*a^5*c^2*d*x^7
+ 7*a*d*x^15*(c^6 + a^2*d^4 + 10*a*c^3*d^2) + c*d*x^17*(c^6 + 21*a^2*d^4 +
 35*a*c^3*d^2) + (7*a^4*c*x^8*(12*a*d^2 + 5*c^3))/4 + 7*a^4*d*x^9*(a*d^2 +
 5*c^3) + 7*c^2*d^3*x^19*(3*a*d^2 + c^3) + (a*c*x^14*(2*c^6 + 70*a^2*d^4 +
 105*a*c^3*d^2))/2 + 7*a^6*c*d*x^5 + (7*a^3*c^2*x^10*(15*a*d^2 + 2*c^3))/2
 + 7*a^2*c^2*d*x^13*(10*a*d^2 + 3*c^3) + 35*a^3*c*d*x^11*(a*d^2 + c^3)
```

3.198. $\int (2cx + 3dx^2)(a + cx^2 + dx^3)^7 \, dx$

3.199 $\int (2cx+3dx^2)(cx^2+dx^3)^7 \, dx$

3.199.1 Optimal result

Integrand size = 25, antiderivative size = 17

$$\int (2cx+3dx^2)\left(cx^2+dx^3\right)^7 \, dx = \frac{1}{8}\left(cx^2+dx^3\right)^8$$

output
```
1/8*(d*x^3+c*x^2)^8
```

3.199.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(17) = 34.

Time = 0.01 (sec) , antiderivative size = 98, normalized size of antiderivative = 5.76

$$\int (2cx+3dx^2)\left(cx^2+dx^3\right)^7 \, dx = \frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20} +7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

input
```
Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]
```

output
```
(c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*
d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24
)/8
```

3.199.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2cx+3dx^2)(cx^2+dx^3)^7\,dx$$

$$\Big\downarrow \text{2021}$$

$$\frac{1}{8}(cx^2+dx^3)^8$$

input
```
Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]
```

output
```
(c*x^2 + d*x^3)^8/8
```

3.199.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.199.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result
gosper	$\frac{x^{16}(dx+c)^8}{8}$
default	$\frac{(x^3d+cx^2)^8}{8}$
norman	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x$
risch	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x$
parallelrisch	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x$

input
```
int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x,method=_RETURNVERBOSE)
```

output
```
1/8*x^16*(d*x+c)^8
```

3.199.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(15) = 30.

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.18

$$\int (2cx + 3dx^2) (cx^2 + dx^3)^7 \, dx = \frac{1}{8} d^8x^{24} + cd^7x^{23} + \frac{7}{2} c^2d^6x^{22} + 7 c^3d^5x^{21} + \frac{35}{4} c^4d^4x^{20} + 7 c^5d^3x^{19} + \frac{7}{2} c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8} c^8x^{16}$$

input
```
integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="fricas")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d
^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.199.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(12) = 24.

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 5.71

$$\int (2cx + 3dx^2) (cx^2 + dx^3)^7 \, dx = \frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

input
```
integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**7,x)
```

output
```
c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35
*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**2
3 + d**8*x**24/8
```

3.199.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int (2cx + 3dx^2) (cx^2 + dx^3)^7 \, dx = \frac{1}{8} (dx^3 + cx^2)^8$$

input
```
integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="maxima")
```

output
```
1/8*(d*x^3 + c*x^2)^8
```

3.199.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int (2cx + 3dx^2) (cx^2 + dx^3)^7 \, dx = \frac{1}{8} (dx^3 + cx^2)^8$$

input
```
integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="giac")
```

output
```
1/8*(d*x^3 + c*x^2)^8
```

3.199.9 Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.18

$$\int (2cx + 3dx^2) (cx^2 + dx^3)^7 \, dx = \frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7 c^6 d^2 x^{18}}{2} + 7 c^5 d^3 x^{19} + \frac{35 c^4 d^4 x^{20}}{4} + 7 c^3 d^5 x^{21} + \frac{7 c^2 d^6 x^{22}}{2} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

input
```
int((2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x)
```

output
```
(c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22
)/2
```

3.200 $\int x^7(cx+dx^2)^7(2cx+3dx^2)\,dx$

3.200.1 Optimal result

Integrand size = 26, antiderivative size = 14

$$\int x^7(cx+dx^2)^7(2cx+3dx^2)\,dx=\frac{1}{8}x^{16}(c+dx)^8$$

output
```
1/8*x^16*(d*x+c)^8
```

3.200.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(14) = 28.

Time = 0.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 7.00

$$\int x^7(cx+dx^2)^7(2cx+3dx^2)\,dx=\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

input
```
Integrate[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x]
```

output
```
(c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*
d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24
)/8
```

3.200.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {9, 9, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(cx+dx^2\right)^7 \left(2cx+3dx^2\right) \, dx$$

$$\Big\downarrow \; 9$$

$$\int x^{14}(c+dx)^7 \left(2cx+3dx^2\right) dx$$

$$\Big\downarrow \; 9$$

$$\int x^{15}(c+dx)^7(2c+3dx)dx$$

$$\Big\downarrow \; 83$$

$$\frac{1}{8}x^{16}(c+dx)^8$$

input
```
Int[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x]
```

output
```
(x^16*(c + d*x)^8)/8
```

3.200.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

3.200.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result
gosper	$\frac{x^{16}(dx+c)^8}{8}$
default	$\frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24} + \frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x$
norman	$\frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24} + \frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x$
risch	$\frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24} + \frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x$
parallelrisch	$\frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24} + \frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x$

input
```
int(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x,method=_RETURNVERBOSE)
```

output
```
1/8*x^16*(d*x+c)^8
```

3.200.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(12) = 24.

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^7 (cx + dx^2)^7 (2cx + 3dx^2) \, dx = \frac{1}{8} d^8x^{24} + cd^7x^{23} + \frac{7}{2} c^2d^6x^{22} + 7 c^3d^5x^{21} + \frac{35}{4} c^4d^4x^{20} + 7 c^5d^3x^{19} + \frac{7}{2} c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8} c^8x^{16}$$

input
```
integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="fricas")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d
^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.200.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(10) = 20.

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int x^7(cx+dx^2)^7(2cx+3dx^2)\,dx = \frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

input
```
integrate(x**7*(d*x**2+c*x)**7*(3*d*x**2+2*c*x),x)
```

output
```
c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35
*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**2
3 + d**8*x**24/8
```

3.200.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(12) = 24.

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^7(cx+dx^2)^7(2cx+3dx^2)\,dx = \frac{1}{8}d^8x^{24}+cd^7x^{23}+\frac{7}{2}c^2d^6x^{22}+7c^3d^5x^{21}+\frac{35}{4}c^4d^4x^{20}+7c^5d^3x^{19}+\frac{7}{2}c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}c^8x^{16}$$

input
```
integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d
^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.200.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int x^7 (cx + dx^2)^7 (2cx + 3dx^2) \, dx = \frac{1}{8} (dx^3 + cx^2)^8$$

input
```
integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="giac")
```

output
```
1/8*(d*x^3 + c*x^2)^8
```

3.200.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^7 (cx + dx^2)^7 (2cx + 3dx^2) \, dx = \frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7 c^6 d^2 x^{18}}{2} + 7 c^5 d^3 x^{19} + \frac{35 c^4 d^4 x^{20}}{4} + 7 c^3 d^5 x^{21} + \frac{7 c^2 d^6 x^{22}}{2} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

input
```
int(x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x)
```

output
```
(c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22
)/2
```

3.201 $\int x^{14}(c+dx)^7 \left(2cx+3dx^2\right) \, dx$

3.201.1 Optimal result

Integrand size = 22, antiderivative size = 14

$$\int x^{14}(c+dx)^7 \left(2cx+3dx^2\right) \, dx = \frac{1}{8}x^{16}(c+dx)^8$$

output
```
1/8*x^16*(d*x+c)^8
```

3.201.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(14) = 28.

Time = 0.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 7.00

$$\int x^{14}(c+dx)^7 \left(2cx+3dx^2\right) \, dx = \frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7}{2}c^6d^2x^{18} + 7c^5d^3x^{19} + \frac{35}{4}c^4d^4x^{20} + 7c^3d^5x^{21} + \frac{7}{2}c^2d^6x^{22} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

input
```
Integrate[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2),x]
```

output
```
(c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*
d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24
)/8
```

3.201.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {9, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{15}(c+dx)^7(2c+3dx)dx$$

$$\Big\downarrow \text{ 83}$$

$$\frac{1}{8}x^{16}(c+dx)^8$$

input
```
Int[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2),x]
```

output
```
(x^16*(c + d*x)^8)/8
```

3.201.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

3.201.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(12) = 24.

Time = 0.72 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

method	result
gosper	$\frac{x^{16}\left(d^{8} x^{8}+8 c\, d^{7} x^{7}+28 x^{6} c^{2} d^{6}+56 c^{3} d^{5} x^{5}+70 x^{4} c^{4} d^{4}+56 c^{5} d^{3} x^{3}+28 x^{2} c^{6} d^{2}+8 c^{7} d x+c^{8}\right)}{8}$
default	$\frac{7}{2} x^{22} c^{2} d^{6}+c\, d^{7} x^{23}+\frac{1}{8} d^{8} x^{24}+\frac{1}{8} x^{16} c^{8}+c^{7} d\, x^{17}+\frac{7}{2} x^{18} c^{6} d^{2}+7 c^{5} d^{3} x^{19}+\frac{35}{4} x^{20} c^{4} d^{4}+7 c^{3} d^{5} x$
norman	$\frac{7}{2} x^{22} c^{2} d^{6}+c\, d^{7} x^{23}+\frac{1}{8} d^{8} x^{24}+\frac{1}{8} x^{16} c^{8}+c^{7} d\, x^{17}+\frac{7}{2} x^{18} c^{6} d^{2}+7 c^{5} d^{3} x^{19}+\frac{35}{4} x^{20} c^{4} d^{4}+7 c^{3} d^{5} x$
risch	$\frac{7}{2} x^{22} c^{2} d^{6}+c\, d^{7} x^{23}+\frac{1}{8} d^{8} x^{24}+\frac{1}{8} x^{16} c^{8}+c^{7} d\, x^{17}+\frac{7}{2} x^{18} c^{6} d^{2}+7 c^{5} d^{3} x^{19}+\frac{35}{4} x^{20} c^{4} d^{4}+7 c^{3} d^{5} x$
parallelrisch	$\frac{7}{2} x^{22} c^{2} d^{6}+c\, d^{7} x^{23}+\frac{1}{8} d^{8} x^{24}+\frac{1}{8} x^{16} c^{8}+c^{7} d\, x^{17}+\frac{7}{2} x^{18} c^{6} d^{2}+7 c^{5} d^{3} x^{19}+\frac{35}{4} x^{20} c^{4} d^{4}+7 c^{3} d^{5} x$

input
```
int(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x,method=_RETURNVERBOSE)
```

output
```
1/8*x^16*(d^8*x^8+8*c*d^7*x^7+28*c^2*d^6*x^6+56*c^3*d^5*x^5+70*c^4*d^4*x^4
+56*c^5*d^3*x^3+28*c^6*d^2*x^2+8*c^7*d*x+c^8)
```

3.201.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(12) = 24.

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx=\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

input
```
integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="fricas")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d
^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.201.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(10) = 20$.

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx=\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

input
```
integrate(x**14*(d*x+c)**7*(3*d*x**2+2*c*x),x)
```

output
```
c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35
*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**2
3 + d**8*x**24/8
```

3.201.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx=\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

input
```
integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d
^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.201.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx=\frac{1}{8}\left(dx^3+cx^2\right)^8$$

input
```
integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="giac")
```

output
```
1/8*(d*x^3 + c*x^2)^8
```

3.201.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx=\frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7\,c^6\,d^2\,x^{18}}{2}+7\,c^5\,d^3\,x^{19}+\frac{35\,c^4\,d^4\,x^{20}}{4}+7\,c^3\,d^5\,x^{21}+\frac{7\,c^2\,d^6\,x^{22}}{2}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

input
```
int(x^14*(2*c*x + 3*d*x^2)*(c + d*x)^7,x)
```

output
```
(c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22
)/2
```

3.202 $\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx$

3.202.1 Optimal result

Integrand size = 24, antiderivative size = 18

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

output
```
1/8*(d*x^3+c*x^2+a)^8
```

3.202.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 115 vs. 2(18) = 36.

Time = 0.01 (sec) , antiderivative size = 115, normalized size of antiderivative = 6.39

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx=\frac{1}{8}x^2(c+dx)\left(8a^7+28a^6x^2(c+dx)+56a^5x^4(c+dx)^2+70a^4x^6(c+dx)^3+56a^3x^8(c+dx)^4+28a^2x^{10}(c+dx)^5+8ax^{12}(c+dx)^6+x^{14}(c+dx)^7\right)$$

input
```
Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]
```

output
```
(x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*x)^2 + 70
*a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x^10*(c + d*x)^5 +
8*a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8
```

3.202.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx$$

$$\Big\downarrow \text{ 2021}$$

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

input
```
Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]
```

output
```
(a + c*x^2 + d*x^3)^8/8
```

3.202.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.202.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(x^3d+c\,x^2+a\right)^8}{8}$
norman	$\left(d^7a+7c^3d^5\right)x^{21}+\frac{7x^{22}c^2d^6}{2}+c\,d^7x^{23}+\frac{d^8x^{24}}{8}+\left(35a^3c\,d^4+\frac{105}{2}a^2c^4d^2+a\,c^7\right)x^{14}+(7a^3d^5+$
gosper	$\frac{7}{2}x^4a^6c^2+\frac{7}{2}x^6a^6d^2+7x^6a^5c^3+x^2c\,a^7+35a^4c^3d\,x^9+35a^4c\,d^3x^{11}+35a^3c^4d\,x^{11}+c\,d^7x^{23}+$
parallelrisch	$\frac{7}{2}x^4a^6c^2+\frac{7}{2}x^6a^6d^2+7x^6a^5c^3+x^2c\,a^7+35a^4c^3d\,x^9+35a^4c\,d^3x^{11}+35a^3c^4d\,x^{11}+c\,d^7x^{23}+$
risch	$\frac{7}{2}x^4a^6c^2+\frac{7}{2}x^6a^6d^2+7x^6a^5c^3+x^2c\,a^7+35a^4c^3d\,x^9+35a^4c\,d^3x^{11}+35a^3c^4d\,x^{11}+c\,d^7x^{23}+$

input
```
int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/8*(d*x^3+c*x^2+a)^8
```

3.202.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(16) = 32$.

Time = 0.25 (sec) , antiderivative size = 458, normalized size of antiderivative = 25.44

$$\begin{aligned}\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx &= \frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + \left(7\,c^3d^5 + ad^7\right)x^{21} \\ &+ \frac{7}{4}\left(5\,c^4d^4 + 4\,acd^6\right)x^{20} + 7\left(c^5d^3 + 3\,ac^2d^5\right)x^{19} \\ &+ \frac{7}{2}\left(c^6d^2 + 10\,ac^3d^4 + a^2d^6\right)x^{18} \\ &+ \left(c^7d + 35\,ac^4d^3 + 21\,a^2cd^5\right)x^{17} \\ &+ \frac{1}{8}\left(c^8 + 168\,ac^5d^2 + 420\,a^2c^2d^4\right)x^{16} \\ &+ 7\left(ac^6d + 10\,a^2c^3d^3 + a^3d^5\right)x^{15} + 21\,a^5c^2dx^7 \\ &+ \frac{1}{2}\left(2\,ac^7 + 105\,a^2c^4d^2 + 70\,a^3cd^4\right)x^{14} \\ &+ 7\left(3\,a^2c^5d + 10\,a^3c^2d^3\right)x^{13} + 7\,a^6cdx^5 \\ &+ \frac{7}{4}\left(2\,a^2c^6 + 40\,a^3c^3d^2 + 5\,a^4d^4\right)x^{12} \\ &+ \frac{7}{2}\,a^6c^2x^4 + 35\left(a^3c^4d + a^4cd^3\right)x^{11} \\ &+ a^7dx^3 + \frac{7}{2}\left(2\,a^3c^5 + 15\,a^4c^2d^2\right)x^{10} \\ &+ a^7cx^2 + 7\left(5\,a^4c^3d + a^5d^3\right)x^9 \\ &+ \frac{7}{4}\left(5\,a^4c^4 + 12\,a^5cd^2\right)x^8 + \frac{7}{2}\left(2\,a^5c^3 + a^6d^2\right)x^6\end{aligned}$$

input
```
integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="fricas")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + (7*c^3*d^5 + a*d^7)*x^21 +
7/4*(5*c^4*d^4 + 4*a*c*d^6)*x^20 + 7*(c^5*d^3 + 3*a*c^2*d^5)*x^19 + 7/2*(c
^6*d^2 + 10*a*c^3*d^4 + a^2*d^6)*x^18 + (c^7*d + 35*a*c^4*d^3 + 21*a^2*c*d
^5)*x^17 + 1/8*(c^8 + 168*a*c^5*d^2 + 420*a^2*c^2*d^4)*x^16 + 7*(a*c^6*d +
 10*a^2*c^3*d^3 + a^3*d^5)*x^15 + 21*a^5*c^2*d*x^7 + 1/2*(2*a*c^7 + 105*a^
2*c^4*d^2 + 70*a^3*c*d^4)*x^14 + 7*(3*a^2*c^5*d + 10*a^3*c^2*d^3)*x^13 + 7
*a^6*c*d*x^5 + 7/4*(2*a^2*c^6 + 40*a^3*c^3*d^2 + 5*a^4*d^4)*x^12 + 7/2*a^6
*c^2*x^4 + 35*(a^3*c^4*d + a^4*c*d^3)*x^11 + a^7*d*x^3 + 7/2*(2*a^3*c^5 +
15*a^4*c^2*d^2)*x^10 + a^7*c*x^2 + 7*(5*a^4*c^3*d + a^5*d^3)*x^9 + 7/4*(5*
a^4*c^4 + 12*a^5*c*d^2)*x^8 + 7/2*(2*a^5*c^3 + a^6*d^2)*x^6
```

3.202.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 484, normalized size of antiderivative = 26.89

$$\begin{aligned}\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx = {} & a^7cx^2+a^7dx^3+\frac{7a^6c^2x^4}{2}+7a^6cdx^5+21a^5c^2dx^7 \\ & +\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}+x^{21}\left(ad^7+7c^3d^5\right) \\ & +x^{20}\cdot\left(7acd^6+\frac{35c^4d^4}{4}\right)+x^{19}\cdot\left(21ac^2d^5+7c^5d^3\right) \\ & +x^{18}\cdot\left(\frac{7a^2d^6}{2}+35ac^3d^4+\frac{7c^6d^2}{2}\right) \\ & +x^{17}\cdot\left(21a^2cd^5+35ac^4d^3+c^7d\right) \\ & +x^{16}\cdot\left(\frac{105a^2c^2d^4}{2}+21ac^5d^2+\frac{c^8}{8}\right) \\ & +x^{15}\cdot\left(7a^3d^5+70a^2c^3d^3+7ac^6d\right) \\ & +x^{14}\cdot\left(35a^3cd^4+\frac{105a^2c^4d^2}{2}+ac^7\right) \\ & +x^{13}\cdot\left(70a^3c^2d^3+21a^2c^5d\right)+x^{12} \\ & \cdot\left(\frac{35a^4d^4}{4}+70a^3c^3d^2+\frac{7a^2c^6}{2}\right) \\ & +x^{11}\cdot\left(35a^4cd^3+35a^3c^4d\right)+x^{10} \\ & \cdot\left(\frac{105a^4c^2d^2}{2}+7a^3c^5\right)+x^9\cdot\left(7a^5d^3+35a^4c^3d\right) \\ & +x^8\cdot\left(21a^5cd^2+\frac{35a^4c^4}{4}\right)+x^6\cdot\left(\frac{7a^6d^2}{2}+7a^5c^3\right)\end{aligned}$$

input
```
integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**7,x)
```

output
```
a**7*c*x**2 + a**7*d*x**3 + 7*a**6*c**2*x**4/2 + 7*a**6*c*d*x**5 + 21*a**5
*c**2*d*x**7 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8 + x**21*(
a*d**7 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 35*c**4*d**4/4) + x**19*(21*a*
c**2*d**5 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 35*a*c**3*d**4 + 7*c**6*
d**2/2) + x**17*(21*a**2*c*d**5 + 35*a*c**4*d**3 + c**7*d) + x**16*(105*a*
*2*c**2*d**4/2 + 21*a*c**5*d**2 + c**8/8) + x**15*(7*a**3*d**5 + 70*a**2*c
**3*d**3 + 7*a*c**6*d) + x**14*(35*a**3*c*d**4 + 105*a**2*c**4*d**2/2 + a*
c**7) + x**13*(70*a**3*c**2*d**3 + 21*a**2*c**5*d) + x**12*(35*a**4*d**4/4
 + 70*a**3*c**3*d**2 + 7*a**2*c**6/2) + x**11*(35*a**4*c*d**3 + 35*a**3*c*
*4*d) + x**10*(105*a**4*c**2*d**2/2 + 7*a**3*c**5) + x**9*(7*a**5*d**3 + 3
5*a**4*c**3*d) + x**8*(21*a**5*c*d**2 + 35*a**4*c**4/4) + x**6*(7*a**6*d**
2/2 + 7*a**5*c**3)
```

3.202.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(16) = 32$.

Time = 0.19 (sec) , antiderivative size = 458, normalized size of antiderivative = 25.44

$$\begin{aligned}\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx &= \frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+\left(7\,c^3d^5+ad^7\right)x^{21}\\&+\frac{7}{4}\left(5\,c^4d^4+4\,acd^6\right)x^{20}+7\left(c^5d^3+3\,ac^2d^5\right)x^{19}\\&+\frac{7}{2}\left(c^6d^2+10\,ac^3d^4+a^2d^6\right)x^{18}\\&+\left(c^7d+35\,ac^4d^3+21\,a^2cd^5\right)x^{17}\\&+\frac{1}{8}\left(c^8+168\,ac^5d^2+420\,a^2c^2d^4\right)x^{16}\\&+7\left(ac^6d+10\,a^2c^3d^3+a^3d^5\right)x^{15}+21\,a^5c^2dx^7\\&+\frac{1}{2}\left(2\,ac^7+105\,a^2c^4d^2+70\,a^3cd^4\right)x^{14}\\&+7\left(3\,a^2c^5d+10\,a^3c^2d^3\right)x^{13}+7\,a^6cdx^5\\&+\frac{7}{4}\left(2\,a^2c^6+40\,a^3c^3d^2+5\,a^4d^4\right)x^{12}\\&+\frac{7}{2}\,a^6c^2x^4+35\left(a^3c^4d+a^4cd^3\right)x^{11}\\&+a^7dx^3+\frac{7}{2}\left(2\,a^3c^5+15\,a^4c^2d^2\right)x^{10}\\&+a^7cx^2+7\left(5\,a^4c^3d+a^5d^3\right)x^9\\&+\frac{7}{4}\left(5\,a^4c^4+12\,a^5cd^2\right)x^8+\frac{7}{2}\left(2\,a^5c^3+a^6d^2\right)x^6\end{aligned}$$

input
```
integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="maxima")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + (7*c^3*d^5 + a*d^7)*x^21 +
7/4*(5*c^4*d^4 + 4*a*c*d^6)*x^20 + 7*(c^5*d^3 + 3*a*c^2*d^5)*x^19 + 7/2*(c
^6*d^2 + 10*a*c^3*d^4 + a^2*d^6)*x^18 + (c^7*d + 35*a*c^4*d^3 + 21*a^2*c*d
^5)*x^17 + 1/8*(c^8 + 168*a*c^5*d^2 + 420*a^2*c^2*d^4)*x^16 + 7*(a*c^6*d +
 10*a^2*c^3*d^3 + a^3*d^5)*x^15 + 21*a^5*c^2*d*x^7 + 1/2*(2*a*c^7 + 105*a^
2*c^4*d^2 + 70*a^3*c*d^4)*x^14 + 7*(3*a^2*c^5*d + 10*a^3*c^2*d^3)*x^13 + 7
*a^6*c*d*x^5 + 7/4*(2*a^2*c^6 + 40*a^3*c^3*d^2 + 5*a^4*d^4)*x^12 + 7/2*a^6
*c^2*x^4 + 35*(a^3*c^4*d + a^4*c*d^3)*x^11 + a^7*d*x^3 + 7/2*(2*a^3*c^5 +
15*a^4*c^2*d^2)*x^10 + a^7*c*x^2 + 7*(5*a^4*c^3*d + a^5*d^3)*x^9 + 7/4*(5*
a^4*c^4 + 12*a^5*c*d^2)*x^8 + 7/2*(2*a^5*c^3 + a^6*d^2)*x^6
```

3.202.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 488, normalized size of antiderivative = 27.11

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx = \frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + ad^7x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,acd^6x^{20} + 7\,c^5d^3x^{19} + 21\,ac^2d^5x^{19} + \frac{7}{2}\,c^6d^2x^{18} + 35\,ac^3d^4x^{18} + \frac{7}{2}\,a^2d^6x^{18} + c^7dx^{17} + 35\,ac^4d^3x^{17} + 21\,a^2cd^5x^{17} + \frac{1}{8}\,c^8x^{16} + 21\,ac^5d^2x^{16} + \frac{105}{2}\,a^2c^2d^4x^{16} + 7\,ac^6dx^{15} + 70\,a^2c^3d^3x^{15} + 7\,a^3d^5x^{15} + ac^7x^{14} + \frac{105}{2}\,a^2c^4d^2x^{14} + 35\,a^3cd^4x^{14} + 21\,a^2c^5dx^{13} + 70\,a^3c^2d^3x^{13} + \frac{7}{2}\,a^2c^6x^{12} + 70\,a^3c^3d^2x^{12} + \frac{35}{4}\,a^4d^4x^{12} + 35\,a^3c^4dx^{11} + 35\,a^4cd^3x^{11} + 7\,a^3c^5x^{10} + \frac{105}{2}\,a^4c^2d^2x^{10} + 35\,a^4c^3dx^9 + 7\,a^5d^3x^9 + \frac{35}{4}\,a^4c^4x^8 + 21\,a^5cd^2x^8 + 21\,a^5c^2dx^7 + 7\,a^5c^3x^6 + \frac{7}{2}\,a^6d^2x^6 + 7\,a^6cdx^5 + \frac{7}{2}\,a^6c^2x^4 + a^7dx^3 + a^7cx^2$$

input
```
integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="giac")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + a*d^7*x^21
 + 35/4*c^4*d^4*x^20 + 7*a*c*d^6*x^20 + 7*c^5*d^3*x^19 + 21*a*c^2*d^5*x^19
 + 7/2*c^6*d^2*x^18 + 35*a*c^3*d^4*x^18 + 7/2*a^2*d^6*x^18 + c^7*d*x^17 +
35*a*c^4*d^3*x^17 + 21*a^2*c*d^5*x^17 + 1/8*c^8*x^16 + 21*a*c^5*d^2*x^16 +
 105/2*a^2*c^2*d^4*x^16 + 7*a*c^6*d*x^15 + 70*a^2*c^3*d^3*x^15 + 7*a^3*d^5
*x^15 + a*c^7*x^14 + 105/2*a^2*c^4*d^2*x^14 + 35*a^3*c*d^4*x^14 + 21*a^2*c
^5*d*x^13 + 70*a^3*c^2*d^3*x^13 + 7/2*a^2*c^6*x^12 + 70*a^3*c^3*d^2*x^12 +
 35/4*a^4*d^4*x^12 + 35*a^3*c^4*d*x^11 + 35*a^4*c*d^3*x^11 + 7*a^3*c^5*x^1
0 + 105/2*a^4*c^2*d^2*x^10 + 35*a^4*c^3*d*x^9 + 7*a^5*d^3*x^9 + 35/4*a^4*c
^4*x^8 + 21*a^5*c*d^2*x^8 + 21*a^5*c^2*d*x^7 + 7*a^5*c^3*x^6 + 7/2*a^6*d^2
*x^6 + 7*a^6*c*d*x^5 + 7/2*a^6*c^2*x^4 + a^7*d*x^3 + a^7*c*x^2
```

3.202.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 440, normalized size of antiderivative = 24.44

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx = x^{12}\left(\frac{35\,a^4\,d^4}{4}+70\,a^3\,c^3\,d^2+\frac{7\,a^2\,c^6}{2}\right) + x^6\left(\frac{7\,a^6\,d^2}{2}+7\,a^5\,c^3\right)+x^{20}\left(\frac{35\,c^4\,d^4}{4}+7\,a\,c\,d^6\right) + x^{16}\left(\frac{105\,a^2\,c^2\,d^4}{2}+21\,a\,c^5\,d^2+\frac{c^8}{8}\right) + x^{18}\left(\frac{7\,a^2\,d^6}{2}+35\,a\,c^3\,d^4+\frac{7\,c^6\,d^2}{2}\right) + \frac{d^8\,x^{24}}{8}+x^{21}\left(7\,c^3\,d^5+a\,d^7\right)+a^7\,c\,x^2 + a^7\,d\,x^3+c\,d^7\,x^{23}+\frac{7\,a^6\,c^2\,x^4}{2}+\frac{7\,c^2\,d^6\,x^{22}}{2} + 21\,a^5\,c^2\,d\,x^7+7\,a\,d\,x^{15}\left(a^2\,d^4+10\,a\,c^3\,d^2+c^6\right) + c\,d\,x^{17}\left(21\,a^2\,d^4+35\,a\,c^3\,d^2+c^6\right) + \frac{7\,a^4\,c\,x^8\left(5\,c^3+12\,a\,d^2\right)}{4} + 7\,a^4\,d\,x^9\left(5\,c^3+a\,d^2\right)+7\,c^2\,d^3\,x^{19}\left(c^3+3\,a\,d^2\right) + \frac{a\,c\,x^{14}\left(70\,a^2\,d^4+105\,a\,c^3\,d^2+2\,c^6\right)}{2} + 7\,a^6\,c\,d\,x^5+\frac{7\,a^3\,c^2\,x^{10}\left(2\,c^3+15\,a\,d^2\right)}{2} + 7\,a^2\,c^2\,d\,x^{13}\left(3\,c^3+10\,a\,d^2\right)+35\,a^3\,c\,d\,x^{11}\left(c^3+a\,d^2\right)$$

input
```
int(x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x)
```

output
```
x^12*((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + 70*a^3*c^3*d^2) + x^6*(7*a^5*c^3 +
(7*a^6*d^2)/2) + x^20*((35*c^4*d^4)/4 + 7*a*c*d^6) + x^16*(c^8/8 + 21*a*c^
5*d^2 + (105*a^2*c^2*d^4)/2) + x^18*((7*a^2*d^6)/2 + (7*c^6*d^2)/2 + 35*a*
c^3*d^4) + (d^8*x^24)/8 + x^21*(a*d^7 + 7*c^3*d^5) + a^7*c*x^2 + a^7*d*x^3
 + c*d^7*x^23 + (7*a^6*c^2*x^4)/2 + (7*c^2*d^6*x^22)/2 + 21*a^5*c^2*d*x^7
+ 7*a*d*x^15*(c^6 + a^2*d^4 + 10*a*c^3*d^2) + c*d*x^17*(c^6 + 21*a^2*d^4 +
 35*a*c^3*d^2) + (7*a^4*c*x^8*(12*a*d^2 + 5*c^3))/4 + 7*a^4*d*x^9*(a*d^2 +
 5*c^3) + 7*c^2*d^3*x^19*(3*a*d^2 + c^3) + (a*c*x^14*(2*c^6 + 70*a^2*d^4 +
 105*a*c^3*d^2))/2 + 7*a^6*c*d*x^5 + (7*a^3*c^2*x^10*(15*a*d^2 + 2*c^3))/2
 + 7*a^2*c^2*d*x^13*(10*a*d^2 + 3*c^3) + 35*a^3*c*d*x^11*(a*d^2 + c^3)
```

3.202. $\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx$

3.203 $\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx$

3.203.1 Optimal result

Integrand size = 23, antiderivative size = 14

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx=\frac{1}{8}x^{16}(c+dx)^8$$

output
```
1/8*x^16*(d*x+c)^8
```

3.203.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(14) = 28.

Time = 0.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 7.00

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx=\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

input
```
Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]
```

output
```
(c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*
d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24
)/8
```

3.203.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {9, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^{15}(c+dx)^7(2c+3dx)dx$$

$$\Big\downarrow \; 83$$

$$\frac{1}{8}x^{16}(c+dx)^8$$

input
```
Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]
```

output
```
(x^16*(c + d*x)^8)/8
```

3.203.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

3.203.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result
gosper	$\frac{x^{16}(dx+c)^8}{8}$
default	$\frac{(x^3d+c\,x^2)^8}{8}$
norman	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x$
risch	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x$
parallelrisch	$\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x$

input
```
int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x,method=_RETURNVERBOSE)
```

output
```
1/8*x^16*(d*x+c)^8
```

3.203.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(12) = 24.

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx=\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

input
```
integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="fricas")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d
^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.203.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx = \frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

input
```
integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**7,x)
```

output
```
c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35
*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**2
3 + d**8*x**24/8
```

3.203.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx = \frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

input
```
integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="maxima")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d
^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.203.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx = \frac{1}{8}d^8x^{24}+cd^7x^{23}+\frac{7}{2}c^2d^6x^{22}+7c^3d^5x^{21}+\frac{35}{4}c^4d^4x^{20}+7c^5d^3x^{19}+\frac{7}{2}c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}c^8x^{16}$$

input
```
integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="giac")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d
^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.203.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx = \frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7\,c^6\,d^2\,x^{18}}{2}+7\,c^5\,d^3\,x^{19}+\frac{35\,c^4\,d^4\,x^{20}}{4}+7\,c^3\,d^5\,x^{21}+\frac{7\,c^2\,d^6\,x^{22}}{2}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

input
```
int(x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x)
```

output
```
(c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22
)/2
```

3.204 $\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx$

3.204.1 Optimal result

Integrand size = 23, antiderivative size = 18

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx=\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

output
```
1/8*x^8*(d*x^2+c*x)^8
```

3.204.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(18) = 36.

Time = 0.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 5.44

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx=\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

input
```
Integrate[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]
```

output
```
(c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*
d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24
)/8
```

3.204.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {9, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7 dx$$

$$\Big\downarrow \; 9$$

$$\int x^{15}(c+dx)^7(2c+3dx)dx$$

$$\Big\downarrow \; 83$$

$$\frac{1}{8}x^{16}(c+dx)^8$$

input
```
Int[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]
```

output
```
(x^16*(c + d*x)^8)/8
```

3.204.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

3.204.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

method	result
gosper	$\frac{x^{16}(dx+c)^{8}}{8}$
default	$\frac{7}{2}x^{22}c^{2}d^{6}+c\,d^{7}x^{23}+\frac{1}{8}d^{8}x^{24}+\frac{1}{8}x^{16}c^{8}+c^{7}d\,x^{17}+\frac{7}{2}x^{18}c^{6}d^{2}+7c^{5}d^{3}x^{19}+\frac{35}{4}x^{20}c^{4}d^{4}+7c^{3}d^{5}x$
norman	$\frac{7}{2}x^{22}c^{2}d^{6}+c\,d^{7}x^{23}+\frac{1}{8}d^{8}x^{24}+\frac{1}{8}x^{16}c^{8}+c^{7}d\,x^{17}+\frac{7}{2}x^{18}c^{6}d^{2}+7c^{5}d^{3}x^{19}+\frac{35}{4}x^{20}c^{4}d^{4}+7c^{3}d^{5}x$
risch	$\frac{7}{2}x^{22}c^{2}d^{6}+c\,d^{7}x^{23}+\frac{1}{8}d^{8}x^{24}+\frac{1}{8}x^{16}c^{8}+c^{7}d\,x^{17}+\frac{7}{2}x^{18}c^{6}d^{2}+7c^{5}d^{3}x^{19}+\frac{35}{4}x^{20}c^{4}d^{4}+7c^{3}d^{5}x$
parallelrisch	$\frac{7}{2}x^{22}c^{2}d^{6}+c\,d^{7}x^{23}+\frac{1}{8}d^{8}x^{24}+\frac{1}{8}x^{16}c^{8}+c^{7}d\,x^{17}+\frac{7}{2}x^{18}c^{6}d^{2}+7c^{5}d^{3}x^{19}+\frac{35}{4}x^{20}c^{4}d^{4}+7c^{3}d^{5}x$

input
```
int(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x,method=_RETURNVERBOSE)
```

output
```
1/8*x^16*(d*x+c)^8
```

3.204.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(16) = 32.

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 4.89

$$\int x^{8}(2c+3dx)\left(cx+dx^{2}\right)^{7}\,dx=\frac{1}{8}\,d^{8}x^{24}+cd^{7}x^{23}+\frac{7}{2}\,c^{2}d^{6}x^{22}+7\,c^{3}d^{5}x^{21}+\frac{35}{4}\,c^{4}d^{4}x^{20}+7\,c^{5}d^{3}x^{19}+\frac{7}{2}\,c^{6}d^{2}x^{18}+c^{7}dx^{17}+\frac{1}{8}\,c^{8}x^{16}$$

input
```
integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="fricas")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d
^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.204.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(14) = 28$.

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 5.39

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx = \frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

input
```
integrate(x**8*(3*d*x+2*c)*(d*x**2+c*x)**7,x)
```

output
```
c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35
*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**2
3 + d**8*x**24/8
```

3.204.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(16) = 32$.

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 4.89

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx = \frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

input
```
integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="maxima")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d
^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.204.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 4.89

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx = \frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

input
```
integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="giac")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d
^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.204.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 4.89

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx = \frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7\,c^6\,d^2\,x^{18}}{2}+7\,c^5\,d^3\,x^{19}+\frac{35\,c^4\,d^4\,x^{20}}{4}+7\,c^3\,d^5\,x^{21}+\frac{7\,c^2\,d^6\,x^{22}}{2}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

input
```
int(x^8*(c*x + d*x^2)^7*(2*c + 3*d*x),x)
```

output
```
(c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22
)/2
```

3.205 $\int x^{15}(c+dx)^7(2c+3dx)\,dx$

3.205.1 Optimal result

Integrand size = 19, antiderivative size = 14

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx=\frac{1}{8}x^{16}(c+dx)^8$$

output
```
1/8*x^16*(d*x+c)^8
```

3.205.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(14) = 28.

Time = 0.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 7.00

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx=\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

input
```
Integrate[x^15*(c + d*x)^7*(2*c + 3*d*x),x]
```

output
```
(c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*
d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24
)/8
```

3.205.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx$$
$$\Big\downarrow \; 83$$
$$\frac{1}{8}x^{16}(c+dx)^8$$

input
```
Int[x^15*(c + d*x)^7*(2*c + 3*d*x),x]
```

output
```
(x^16*(c + d*x)^8)/8
```

3.205.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

3.205.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(12) = 24.

Time = 0.68 (sec) , antiderivative size = 89, normalized size of antiderivative = 6.36

method	result
gosper	$\frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24} + \frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x$
default	$\frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24} + \frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x$
norman	$\frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24} + \frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x$
risch	$\frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24} + \frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x$
parallelrisch	$\frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24} + \frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x$

input
```
int(x^15*(d*x+c)^7*(3*d*x+2*c),x,method=_RETURNVERBOSE)
```

output
```
7/2*x^22*c^2*d^6+c*d^7*x^23+1/8*d^8*x^24+1/8*x^16*c^8+c^7*d*x^17+7/2*x^18*
c^6*d^2+7*c^5*d^3*x^19+35/4*x^20*c^4*d^4+7*c^3*d^5*x^21
```

3.205.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx = \frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

input
```
integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="fricas")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d
^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.205.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(10) = 20$.

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 6.93

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx = \frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

input
```
integrate(x**15*(d*x+c)**7*(3*d*x+2*c),x)
```

output
```
c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35
*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**2
3 + d**8*x**24/8
```

3.205.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(12) = 24.

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx=\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

input
```
integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="maxima")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d
^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.205.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx=\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

input
```
integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="giac")
```

output
```
1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d
^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

3.205.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.29

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx=\frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7\,c^6\,d^2\,x^{18}}{2}+7\,c^5\,d^3\,x^{19}+\frac{35\,c^4\,d^4\,x^{20}}{4}+7\,c^3\,d^5\,x^{21}+\frac{7\,c^2\,d^6\,x^{22}}{2}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

input
```
int(x^15*(2*c + 3*d*x)*(c + d*x)^7,x)
```

output
```
(c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22
)/2
```

3.205. $\int x^{15}(c+dx)^7(2c+3dx)\,dx$

3.206 $\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx$

3.206.1 Optimal result

Integrand size = 22, antiderivative size = 28

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx=ax+\frac{bx^2}{2}+\frac{1}{160}x^5(2a+bx)^5$$

output
```
a*x+1/2*b*x^2+1/160*x^5*(b*x+2*a)^5
```

3.206.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 80 vs. 2(28) = 56.

Time = 0.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.86

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx=ax+\frac{bx^2}{2}+\frac{a^5x^5}{5}+\frac{1}{2}a^4bx^6+\frac{1}{2}a^3b^2x^7+\frac{1}{4}a^2b^3x^8+\frac{1}{16}ab^4x^9+\frac{b^5x^{10}}{160}$$

input
```
Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]
```

output
```
a*x + (b*x^2)/2 + (a^5*x^5)/5 + (a^4*b*x^6)/2 + (a^3*b^2*x^7)/2 + (a^2*b^3
*x^8)/4 + (a*b^4*x^9)/16 + (b^5*x^10)/160
```

3.206.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\left(\left(ax+\frac{bx^2}{2}\right)^4+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(ax+\frac{bx^2}{2}\right)^4+1\right)d\left(ax+\frac{bx^2}{2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\left(ax+\frac{bx^2}{2}\right)^5+ax+\frac{bx^2}{2}$$

input
```
Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]
```

output
```
a*x + (b*x^2)/2 + (a*x + (b*x^2)/2)^5/5
```

3.206.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.206.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\left(a x+\frac{1}{2} b \,x^{2}\right)^{5}}{5}+a x +\frac{b \,x^{2}}{2}$	25
gosper	$\frac{x \left(x^{9} b^{5}+10 a \,x^{8} b^{4}+40 a^{2} b^{3} x^{7}+80 a^{3} b^{2} x^{6}+80 a^{4} b \,x^{5}+32 a^{5} x^{4}+80 b x +160 a \right)}{160}$	67
norman	$a x +\frac{1}{5} a^{5} x^{5}+\frac{1}{2} b \,x^{2}+\frac{1}{160} x^{10} b^{5}+\frac{1}{4} a^{2} b^{3} x^{8}+\frac{1}{2} a^{3} b^{2} x^{7}+\frac{1}{2} a^{4} b \,x^{6}+\frac{1}{16} b^{4} a \,x^{9}$	67
risch	$a x +\frac{1}{5} a^{5} x^{5}+\frac{1}{2} b \,x^{2}+\frac{1}{160} x^{10} b^{5}+\frac{1}{4} a^{2} b^{3} x^{8}+\frac{1}{2} a^{3} b^{2} x^{7}+\frac{1}{2} a^{4} b \,x^{6}+\frac{1}{16} b^{4} a \,x^{9}$	67
parallelrisch	$a x +\frac{1}{5} a^{5} x^{5}+\frac{1}{2} b \,x^{2}+\frac{1}{160} x^{10} b^{5}+\frac{1}{4} a^{2} b^{3} x^{8}+\frac{1}{2} a^{3} b^{2} x^{7}+\frac{1}{2} a^{4} b \,x^{6}+\frac{1}{16} b^{4} a \,x^{9}$	67

input
```
int((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x,method=_RETURNVERBOSE)
```

output
```
1/5*(a*x+1/2*b*x^2)^5+a*x+1/2*b*x^2
```

3.206.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx=\frac{1}{160}\,b^5x^{10}+\frac{1}{16}\,ab^4x^9+\frac{1}{4}\,a^2b^3x^8+\frac{1}{2}\,a^3b^2x^7+\frac{1}{2}\,a^4bx^6+\frac{1}{5}\,a^5x^5+\frac{1}{2}\,bx^2+ax$$

input
```
integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="fricas")
```

output
```
1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/2*a^3*b^2*x^7 + 1/2*
a^4*b*x^6 + 1/5*a^5*x^5 + 1/2*b*x^2 + a*x
```

3.206.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(22) = 44$.

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.50

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)dx = \frac{a^5x^5}{5}+\frac{a^4bx^6}{2}+\frac{a^3b^2x^7}{2}+\frac{a^2b^3x^8}{4}+\frac{ab^4x^9}{16}+ax+\frac{b^5x^{10}}{160}+\frac{bx^2}{2}$$

input
```
integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**4),x)
```

output
```
a**5*x**5/5 + a**4*b*x**6/2 + a**3*b**2*x**7/2 + a**2*b**3*x**8/4 + a*b**4
*x**9/16 + a*x + b**5*x**10/160 + b*x**2/2
```

3.206.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)dx = \frac{1}{160}b^5x^{10}+\frac{1}{16}ab^4x^9+\frac{1}{4}a^2b^3x^8+\frac{1}{2}a^3b^2x^7+\frac{1}{2}a^4bx^6+\frac{1}{5}a^5x^5+\frac{1}{2}bx^2+ax$$

input
```
integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="maxima")
```

output
```
1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/2*a^3*b^2*x^7 + 1/2*
a^4*b*x^6 + 1/5*a^5*x^5 + 1/2*b*x^2 + a*x
```

3.206.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx=\frac{1}{160}\left(bx^2+2\,ax\right)^5+\frac{1}{2}\,bx^2+ax$$

input
```
integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="giac")
```

output
```
1/160*(b*x^2 + 2*a*x)^5 + 1/2*b*x^2 + a*x
```

3.206.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx=\frac{a^5\,x^5}{5}+\frac{a^4\,b\,x^6}{2}+\frac{a^3\,b^2\,x^7}{2}+\frac{a^2\,b^3\,x^8}{4}+\frac{a\,b^4\,x^9}{16}+a\,x+\frac{b^5\,x^{10}}{160}+\frac{b\,x^2}{2}$$

input
```
int(((a*x + (b*x^2)/2)^4 + 1)*(a + b*x),x)
```

output
```
a*x + (b*x^2)/2 + (a^5*x^5)/5 + (b^5*x^10)/160 + (a^4*b*x^6)/2 + (a*b^4*x^
9)/16 + (a^3*b^2*x^7)/2 + (a^2*b^3*x^8)/4
```

3.207 $\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx$

3.207.1 Optimal result

Integrand size = 23, antiderivative size = 31

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx = ax+\frac{bx^2}{2}+\frac{1}{5}\left(c+ax+\frac{bx^2}{2}\right)^5$$

output
```
a*x+1/2*b*x^2+1/5*(c+a*x+1/2*b*x^2)^5
```

3.207.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(31) = 62.

Time = 0.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.48

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx = \frac{1}{160}x(2a+bx)\left(80+80c^4+16a^4x^4+32a^3bx^5+24a^2b^2x^6+8ab^3x^7+b^4x^8+80c^3x(2a+bx)+40c^2x^2(2a+bx)^2+10cx^3(2a+bx)^3\right)$$

input
```
Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4),x]
```

output
```
(x*(2*a + b*x)*(80 + 80*c^4 + 16*a^4*x^4 + 32*a^3*b*x^5 + 24*a^2*b^2*x^6 +
 8*a*b^3*x^7 + b^4*x^8 + 80*c^3*x*(2*a + b*x) + 40*c^2*x^2*(2*a + b*x)^2 +
 10*c*x^3*(2*a + b*x)^3))/160
```

3.207.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\left(\left(ax+\frac{bx^2}{2}+c\right)^4+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(ax+\frac{bx^2}{2}+c\right)^4+1\right)d\left(ax+\frac{bx^2}{2}+c\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\left(ax+\frac{bx^2}{2}+c\right)^5+ax+\frac{bx^2}{2}+c$$

input
```
Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4),x]
```

output
```
c + a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^5/5
```

3.207.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.207.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result
default	$\frac{\left(c+ax+\frac{1}{2}b\,x^{2}\right)^{5}}{5}+c+ax+\frac{b\,x^{2}}{2}$
norman	$\left(\frac{1}{4}a^{2}b^{3}+\frac{1}{16}b^{4}c\right)x^{8}+\left(\frac{1}{2}a^{3}b^{2}+\frac{1}{2}a\,b^{3}c\right)x^{7}+\left(\frac{1}{5}a^{5}+2a^{3}bc+\frac{3}{2}b^{2}a\,c^{2}\right)x^{5}+\left(2a^{2}c^{3}+\frac{1}{2}b\,c^{4}+\frac{1}{2}\right.$
gosper	$\frac{x\left(x^{9}b^{5}+10a\,x^{8}b^{4}+40a^{2}b^{3}x^{7}+10b^{4}c\,x^{7}+80a^{3}b^{2}x^{6}+80a\,b^{3}c\,x^{6}+80a^{4}b\,x^{5}+240a^{2}b^{2}c\,x^{5}+40b^{3}c^{2}x^{5}+32a^{5}x^{4}+320x^{4}a^{3}bc+240x\right.}{160}$
risch	$\frac{1}{160}x^{10}b^{5}+\frac{1}{16}b^{4}a\,x^{9}+\frac{1}{4}a^{2}b^{3}x^{8}+\frac{1}{16}x^{8}b^{4}c+\frac{1}{2}a^{3}b^{2}x^{7}+\frac{1}{2}x^{7}a\,b^{3}c+\frac{1}{2}a^{4}b\,x^{6}+\frac{3}{2}x^{6}a^{2}b^{2}c+\frac{1}{4}x^{6}b$
parallelrisch	$\frac{1}{160}x^{10}b^{5}+\frac{1}{16}b^{4}a\,x^{9}+\frac{1}{4}a^{2}b^{3}x^{8}+\frac{1}{16}x^{8}b^{4}c+\frac{1}{2}a^{3}b^{2}x^{7}+\frac{1}{2}x^{7}a\,b^{3}c+\frac{1}{2}a^{4}b\,x^{6}+\frac{3}{2}x^{6}a^{2}b^{2}c+\frac{1}{4}x^{6}b$

input
```
int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x,method=_RETURNVERBOSE)
```

output
```
1/5*(c+a*x+1/2*b*x^2)^5+c+a*x+1/2*b*x^2
```

3.207.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(27) = 54.

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 6.03

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)dx = \frac{1}{160}b^5x^{10}+\frac{1}{16}ab^4x^9 + \frac{1}{16}\left(4a^2b^3+b^4c\right)x^8+\frac{1}{2}\left(a^3b^2+ab^3c\right)x^7 + \frac{1}{4}\left(2a^4b+6a^2b^2c+b^3c^2\right)x^6 + \frac{1}{10}\left(2a^5+20a^3bc+15ab^2c^2\right)x^5 + \frac{1}{2}\left(2a^4c+6a^2bc^2+b^2c^3\right)x^4+2\left(a^3c^2+abc^3\right)x^3 + \frac{1}{2}\left(4a^2c^3+bc^4+b\right)x^2+\left(ac^4+a\right)x$$

input
```
integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="fricas")
```

output
```
1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/16*(4*a^2*b^3 + b^4*c)*x^8 + 1/2*(a^3*
b^2 + a*b^3*c)*x^7 + 1/4*(2*a^4*b + 6*a^2*b^2*c + b^3*c^2)*x^6 + 1/10*(2*a
^5 + 20*a^3*b*c + 15*a*b^2*c^2)*x^5 + 1/2*(2*a^4*c + 6*a^2*b*c^2 + b^2*c^3
)*x^4 + 2*(a^3*c^2 + a*b*c^3)*x^3 + 1/2*(4*a^2*c^3 + b*c^4 + b)*x^2 + (a*c
^4 + a)*x
```

3.207.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(24) = 48$.

Time = 0.05 (sec) , antiderivative size = 194, normalized size of antiderivative = 6.26

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)dx = \frac{ab^4x^9}{16}+\frac{b^5x^{10}}{160}+x^8\left(\frac{a^2b^3}{4}+\frac{b^4c}{16}\right)+x^7\left(\frac{a^3b^2}{2}+\frac{ab^3c}{2}\right)+x^6\left(\frac{a^4b}{2}+\frac{3a^2b^2c}{2}+\frac{b^3c^2}{4}\right)+x^5\left(\frac{a^5}{5}+2a^3bc+\frac{3ab^2c^2}{2}\right)+x^4\left(a^4c+3a^2bc^2+\frac{b^2c^3}{2}\right)+x^3\cdot\left(2a^3c^2+2abc^3\right)+x^2\cdot\left(2a^2c^3+\frac{bc^4}{2}+\frac{b}{2}\right)+x\left(ac^4+a\right)$$

input
```
integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**4),x)
```

output
```
a*b**4*x**9/16 + b**5*x**10/160 + x**8*(a**2*b**3/4 + b**4*c/16) + x**7*(a
**3*b**2/2 + a*b**3*c/2) + x**6*(a**4*b/2 + 3*a**2*b**2*c/2 + b**3*c**2/4)
 + x**5*(a**5/5 + 2*a**3*b*c + 3*a*b**2*c**2/2) + x**4*(a**4*c + 3*a**2*b*
c**2 + b**2*c**3/2) + x**3*(2*a**3*c**2 + 2*a*b*c**3) + x**2*(2*a**2*c**3
+ b*c**4/2 + b/2) + x*(a*c**4 + a)
```

3.207.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(27) = 54.

Time = 0.19 (sec) , antiderivative size = 187, normalized size of antiderivative = 6.03

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx = \frac{1}{160}\,b^5x^{10}+\frac{1}{16}\,ab^4x^9+\frac{1}{16}\,\left(4\,a^2b^3+b^4c\right)x^8+\frac{1}{2}\,\left(a^3b^2+ab^3c\right)x^7+\frac{1}{4}\,\left(2\,a^4b+6\,a^2b^2c+b^3c^2\right)x^6+\frac{1}{10}\,\left(2\,a^5+20\,a^3bc+15\,ab^2c^2\right)x^5+\frac{1}{2}\,\left(2\,a^4c+6\,a^2bc^2+b^2c^3\right)x^4+2\,\left(a^3c^2+abc^3\right)x^3+\frac{1}{2}\,\left(4\,a^2c^3+bc^4+b\right)x^2+\left(ac^4+a\right)x$$

input
```
integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="maxima")
```

output
```
1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/16*(4*a^2*b^3 + b^4*c)*x^8 + 1/2*(a^3*
b^2 + a*b^3*c)*x^7 + 1/4*(2*a^4*b + 6*a^2*b^2*c + b^3*c^2)*x^6 + 1/10*(2*a
^5 + 20*a^3*b*c + 15*a*b^2*c^2)*x^5 + 1/2*(2*a^4*c + 6*a^2*b*c^2 + b^2*c^3
)*x^4 + 2*(a^3*c^2 + a*b*c^3)*x^3 + 1/2*(4*a^2*c^3 + b*c^4 + b)*x^2 + (a*c
^4 + a)*x
```

3.207.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(27) = 54.

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.84

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx = \frac{1}{160}\,\left(bx^2+2\,ax\right)^5+\frac{1}{16}\,\left(bx^2+2\,ax\right)^4c+\frac{1}{4}\,\left(bx^2+2\,ax\right)^3c^2+\frac{1}{2}\,\left(bx^2+2\,ax\right)^2c^3+\frac{1}{2}\,\left(bx^2+2\,ax\right)c^4+\frac{1}{2}\,bx^2+ax$$

input
```
integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="giac")
```

output
```
1/160*(b*x^2 + 2*a*x)^5 + 1/16*(b*x^2 + 2*a*x)^4*c + 1/4*(b*x^2 + 2*a*x)^3
*c^2 + 1/2*(b*x^2 + 2*a*x)^2*c^3 + 1/2*(b*x^2 + 2*a*x)*c^4 + 1/2*b*x^2 + a
*x
```

3.207.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 5.81

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx = x^6\left(\frac{a^4\,b}{2}+\frac{3\,a^2\,b^2\,c}{2}+\frac{b^3\,c^2}{4}\right) + x^4\left(a^4\,c+3\,a^2\,b\,c^2+\frac{b^2\,c^3}{2}\right) + x^2\left(2\,a^2\,c^3+\frac{b\,c^4}{2}+\frac{b}{2}\right) + x^5\left(\frac{a^5}{5}+2\,a^3\,b\,c+\frac{3\,a\,b^2\,c^2}{2}\right)+\frac{b^5\,x^{10}}{160} + x^8\left(\frac{a^2\,b^3}{4}+\frac{c\,b^4}{16}\right)+\frac{a\,b^4\,x^9}{16}+a\,x\,\left(c^4+1\right) + \frac{a\,b^2\,x^7\,\left(a^2+b\,c\right)}{2}+2\,a\,c^2\,x^3\,\left(a^2+b\,c\right)$$

input
```
int(((c + a*x + (b*x^2)/2)^4 + 1)*(a + b*x),x)
```

output
```
x^6*((a^4*b)/2 + (b^3*c^2)/4 + (3*a^2*b^2*c)/2) + x^4*(a^4*c + (b^2*c^3)/2
 + 3*a^2*b*c^2) + x^2*(b/2 + (b*c^4)/2 + 2*a^2*c^3) + x^5*(a^5/5 + (3*a*b^
2*c^2)/2 + 2*a^3*b*c) + (b^5*x^10)/160 + x^8*((b^4*c)/16 + (a^2*b^3)/4) +
(a*b^4*x^9)/16 + a*x*(c^4 + 1) + (a*b^2*x^7*(b*c + a^2))/2 + 2*a*c^2*x^3*(
b*c + a^2)
```

3.208 $\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx$

3.208.1 Optimal result

Integrand size = 22, antiderivative size = 34

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx=ax+\frac{bx^2}{2}+\frac{\left(ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}$$

output
```
a*x+1/2*b*x^2+(a*x+1/2*b*x^2)^(1+n)/(1+n)
```

3.208.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx=\frac{x(2a+bx)\left(1+n+\left(ax+\frac{bx^2}{2}\right)^n\right)}{2(1+n)}$$

input
```
Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n),x]
```

output
```
(x*(2*a + b*x)*(1 + n + (a*x + (b*x^2)/2)^n))/(2*(1 + n))
```

3.208.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\left(\left(ax+\frac{bx^2}{2}\right)^n+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(ax+\frac{bx^2}{2}\right)^n+1\right)d\left(ax+\frac{bx^2}{2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(ax+\frac{bx^2}{2}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

input
```
Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n),x]
```

output
```
a*x + (b*x^2)/2 + (a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

3.208.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.208.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$ax+\frac{b\,x^{2}}{2}+\frac{\left(ax+\frac{1}{2}b\,x^{2}\right)^{1+n}}{1+n}$	31
default	$ax+\frac{b\,x^{2}}{2}+\frac{\left(ax+\frac{1}{2}b\,x^{2}\right)^{1+n}}{1+n}$	31
risch	$ax+\frac{b\,x^{2}}{2}+\frac{x(bx+2a)(x(bx+2a))^{n}\left(\frac{1}{2}\right)^{n}}{2+2n}$	40
norman	$ax+\frac{ax\,\mathrm{e}^{n\ln\left(ax+\frac{1}{2}b\,x^{2}\right)}}{1+n}+\frac{b\,x^{2}}{2}+\frac{b\,x^{2}\mathrm{e}^{n\ln\left(ax+\frac{1}{2}b\,x^{2}\right)}}{2+2n}$	58
parallelrisch	$\frac{x^{2}\left(\frac{x(bx+2a)}{2}\right)^{n}b^{2}+x^{2}b^{2}n+b^{2}x^{2}+2x\left(\frac{x(bx+2a)}{2}\right)^{n}ab+2abnx+2abx-4a^{2}n-4a^{2}}{2b(1+n)}$	85

input
```
int((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x,method=_RETURNVERBOSE)
```

output
```
a*x+1/2*b*x^2+(a*x+1/2*b*x^2)^(1+n)/(1+n)
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx=\frac{(bn+b)x^2+(bx^2+2\,ax)\left(\frac{1}{2}\,bx^2+ax\right)^n+2\,(an+a)x}{2\,(n+1)}$$

input
```
integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="fricas")
```

output
```
1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x)*(1/2*b*x^2 + a*x)^n + 2*(a*n + a)*x)/
(n + 1)
```

3.208.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(26) = 52.

Time = 18.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 6.71

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx$$
$$= \begin{cases} a\left(x+\frac{\log(x)}{a}\right) & \text{for } b=0\wedge n=- \\ a\left(\frac{nx}{n+1}+\frac{x(ax)^n}{n+1}+\frac{x}{n+1}\right) & \text{for } b=0 \\ ax+\frac{bx^2}{2}+\log(x)+\log\left(\frac{2a}{b}+x\right) & \text{for } n=-1 \\ \frac{2\cdot 2^n abnx}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2\cdot 2^n abx}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2^n b^2nx^2}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2^n b^2x^2}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2abx(2ax+bx^2)^n}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{b^2x^2(2ax+bx^2)^n}{2\cdot 2^n bn+2\cdot 2^n b} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**n),x)
```

output
```
Piecewise((a*(x + log(x)/a), Eq(b, 0) & Eq(n, -1)), (a*(n*x/(n + 1) + x*(a
*x)**n/(n + 1) + x/(n + 1)), Eq(b, 0)), (a*x + b*x**2/2 + log(x) + log(2*a
/b + x), Eq(n, -1)), (2*2**n*a*b*n*x/(2*2**n*b*n + 2*2**n*b) + 2*2**n*a*b*
x/(2*2**n*b*n + 2*2**n*b) + 2**n*b**2*n*x**2/(2*2**n*b*n + 2*2**n*b) + 2**
n*b**2*x**2/(2*2**n*b*n + 2*2**n*b) + 2*a*b*x*(2*a*x + b*x**2)**n/(2*2**n*
b*n + 2*2**n*b) + b**2*x**2*(2*a*x + b*x**2)**n/(2*2**n*b*n + 2*2**n*b), T
rue))
```

3.208.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.53

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx = \frac{1}{2}\,bx^2+ax+\frac{(bx^2+2\,ax)e^{(n\log(bx+2\,a)+n\log(x))}}{2^{n+1}n+2^{n+1}}$$

input
```
integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="maxima")
```

output
```
1/2*b*x^2 + a*x + (b*x^2 + 2*a*x)*e^(n*log(b*x + 2*a) + n*log(x))/(2^(n +
1)*n + 2^(n + 1))
```

3.208.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx=\frac{1}{2}\,bx^2+ax+\frac{\left(\frac{1}{2}\,bx^2+ax\right)^{n+1}}{n+1}$$

input
```
integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="giac")
```

output
```
1/2*b*x^2 + a*x + (1/2*b*x^2 + a*x)^(n + 1)/(n + 1)
```

3.208.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx=\frac{x\,(2\,a+b\,x)\,\left(n+\left(\frac{b\,x^2}{2}+a\,x\right)^n+1\right)}{2\,(n+1)}$$

input
```
int(((a*x + (b*x^2)/2)^n + 1)*(a + b*x),x)
```

output
```
(x*(2*a + b*x)*(n + (a*x + (b*x^2)/2)^n + 1))/(2*(n + 1))
```

3.209 $\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right) dx$

3.209.1 Optimal result

Integrand size = 23, antiderivative size = 35

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right) dx = ax+\frac{bx^2}{2}+\frac{\left(c+ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}$$

output
```
a*x+1/2*b*x^2+(c+a*x+1/2*b*x^2)^(1+n)/(1+n)
```

3.209.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right) dx = \frac{2c\left(c+ax+\frac{bx^2}{2}\right)^n+2ax\left(1+n+\left(c+ax+\frac{bx^2}{2}\right)^n\right)+bx^2\left(1+n+\left(c+ax+\frac{bx^2}{2}\right)^n\right)}{2(1+n)}$$

input
```
Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n),x]
```

output
```
(2*c*(c + a*x + (b*x^2)/2)^n + 2*a*x*(1 + n + (c + a*x + (b*x^2)/2)^n) + b
*x^2*(1 + n + (c + a*x + (b*x^2)/2)^n))/(2*(1 + n))
```

3.209.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\left(\left(ax+\frac{bx^2}{2}+c\right)^n+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(ax+\frac{bx^2}{2}+c\right)^n+1\right)d\left(ax+\frac{bx^2}{2}+c\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(ax+\frac{bx^2}{2}+c\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}+c$$

input
```
Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n),x]
```

output
```
c + a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

3.209.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.209.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

method	result
derivativedivides	$c+ax+\frac{b\,x^2}{2}+\frac{\left(c+ax+\frac{1}{2}b\,x^2\right)^{1+n}}{1+n}$
default	$c+ax+\frac{b\,x^2}{2}+\frac{\left(c+ax+\frac{1}{2}b\,x^2\right)^{1+n}}{1+n}$
risch	$ax+\frac{b\,x^2}{2}+\frac{(b\,x^2+2ax+2c)\,(b\,x^2+2ax+2c)^n\,\left(\frac{1}{2}\right)^n}{2+2n}$
norman	$ax+\frac{c\,\mathrm{e}^{n\ln\left(c+ax+\frac{1}{2}b\,x^2\right)}}{1+n}+\frac{ax\,\mathrm{e}^{n\ln\left(c+ax+\frac{1}{2}b\,x^2\right)}}{1+n}+\frac{b\,x^2}{2}+\frac{b\,x^2\mathrm{e}^{n\ln\left(c+ax+\frac{1}{2}b\,x^2\right)}}{2+2n}$
parallelrisch	$\frac{\left(c+ax+\frac{1}{2}b\,x^2\right)^n b^2x^2+x^2b^2n+b^2x^2+2\left(c+ax+\frac{1}{2}b\,x^2\right)^n abx+2abnx+2abx+2\left(c+ax+\frac{1}{2}b\,x^2\right)^n bc-4a^2n-2bcn-4a^2-2bc}{2b(1+n)}$

input
```
int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x,method=_RETURNVERBOSE)
```

output
```
c+a*x+1/2*b*x^2+(c+a*x+1/2*b*x^2)^(1+n)/(1+n)
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.49

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx$$
$$=\frac{(bn+b)x^2+(bx^2+2\,ax+2\,c)\left(\frac{1}{2}\,bx^2+ax+c\right)^n+2\,(an+a)x}{2\,(n+1)}$$

input
```
integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="fricas")
```

output
```
1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x + 2*c)*(1/2*b*x^2 + a*x + c)^n + 2*(a*
n + a)*x)/(n + 1)
```

3.209.6 Sympy [F(-1)]

Timed out.

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx=\text{Timed out}$$

input
```
integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**n),x)
```

output
```
Timed out
```

3.209.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx=\frac{1}{2}\,bx^2+ax+\frac{(bx^2+2\,ax+2\,c)(bx^2+2\,ax+2\,c)^n}{2^{n+1}n+2^{n+1}}$$

input
```
integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="maxima")
```

output
```
1/2*b*x^2 + a*x + (b*x^2 + 2*a*x + 2*c)*(b*x^2 + 2*a*x + 2*c)^n/(2^(n + 1)
*n + 2^(n + 1))
```

3.209.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx=\frac{1}{2}\,bx^2+ax+c+\frac{\left(\frac{1}{2}\,bx^2+ax+c\right)^{n+1}}{n+1}$$

input
```
integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="giac")
```

output
```
1/2*b*x^2 + a*x + c + (1/2*b*x^2 + a*x + c)^(n + 1)/(n + 1)
```

3.209.9 Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx=a\,x+\left(\frac{b\,x^2}{2}+a\,x+c\right)^n\left(\frac{2\,c}{2\,n+2}+\frac{b\,x^2}{2\,n+2}+\frac{2\,a\,x}{2\,n+2}\right)+\frac{b\,x^2}{2}$$

input
```
int(((c + a*x + (b*x^2)/2)^n + 1)*(a + b*x),x)
```

output
```
a*x + (c + a*x + (b*x^2)/2)^n*((2*c)/(2*n + 2) + (b*x^2)/(2*n + 2) + (2*a*
x)/(2*n + 2)) + (b*x^2)/2
```

3.210 $\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx$

3.210.1 Optimal result

Integrand size = 24, antiderivative size = 30

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx=ax+\frac{cx^3}{3}+\frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6$$

output
```
a*x+1/3*c*x^3+1/6*(a*x+1/3*c*x^3)^6
```

3.210.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 93 vs. 2(30) = 60.

Time = 0.01 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.10

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx=ax+\frac{cx^3}{3}+\frac{a^6x^6}{6}+\frac{1}{3}a^5cx^8+\frac{5}{18}a^4c^2x^{10}+\frac{10}{81}a^3c^3x^{12}+\frac{5}{162}a^2c^4x^{14}+\frac{1}{243}ac^5x^{16}+\frac{c^6x^{18}}{4374}$$

input
```
Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]
```

output
```
a*x + (c*x^3)/3 + (a^6*x^6)/6 + (a^5*c*x^8)/3 + (5*a^4*c^2*x^10)/18 + (10*
a^3*c^3*x^12)/81 + (5*a^2*c^4*x^14)/162 + (a*c^5*x^16)/243 + (c^6*x^18)/43
74
```

3.210.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)\left(\left(ax+\frac{cx^3}{3}\right)^5+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(ax+\frac{cx^3}{3}\right)^5+1\right)d\left(ax+\frac{cx^3}{3}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6+ax+\frac{cx^3}{3}$$

input
```
Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]
```

output
```
a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^6/6
```

3.210.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.210.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result
default	$ax+\frac{c\,x^{3}}{3}+\frac{\left(ax+\frac{1}{3}c\,x^{3}\right)^{6}}{6}$
norman	$ax+\frac{1}{6}a^{6}x^{6}+\frac{1}{3}c\,x^{3}+\frac{1}{4374}c^{6}x^{18}+\frac{1}{243}a\,c^{5}x^{16}+\frac{5}{162}a^{2}c^{4}x^{14}+\frac{5}{18}a^{4}c^{2}x^{10}+\frac{1}{3}c\,a^{5}x^{8}+\frac{10}{81}c^{3}a^{3}x^{1}$
risch	$ax+\frac{1}{6}a^{6}x^{6}+\frac{1}{3}c\,x^{3}+\frac{1}{4374}c^{6}x^{18}+\frac{1}{243}a\,c^{5}x^{16}+\frac{5}{162}a^{2}c^{4}x^{14}+\frac{5}{18}a^{4}c^{2}x^{10}+\frac{1}{3}c\,a^{5}x^{8}+\frac{10}{81}c^{3}a^{3}x^{1}$
parallelrisch	$ax+\frac{1}{6}a^{6}x^{6}+\frac{1}{3}c\,x^{3}+\frac{1}{4374}c^{6}x^{18}+\frac{1}{243}a\,c^{5}x^{16}+\frac{5}{162}a^{2}c^{4}x^{14}+\frac{5}{18}a^{4}c^{2}x^{10}+\frac{1}{3}c\,a^{5}x^{8}+\frac{10}{81}c^{3}a^{3}x^{1}$
gosper	$\frac{x\left(c^{6}x^{17}+18a\,c^{5}x^{15}+135a^{2}c^{4}x^{13}+540c^{3}a^{3}x^{11}+1215a^{4}c^{2}x^{9}+1458c\,a^{5}x^{7}+729a^{6}x^{5}+1458c\,x^{2}+4374a\right)}{4374}$

input
```
int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
```

output
```
a*x+1/3*c*x^3+1/6*(a*x+1/3*c*x^3)^6
```

3.210.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(24) = 48.

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.57

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)dx = \frac{1}{4374}c^6x^{18}+\frac{1}{243}ac^5x^{16}+\frac{5}{162}a^2c^4x^{14}+\frac{10}{81}a^3c^3x^{12}+\frac{5}{18}a^4c^2x^{10}+\frac{1}{3}a^5cx^8+\frac{1}{6}a^6x^6+\frac{1}{3}cx^3+ax$$

input
```
integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="fricas")
```

output
```
1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 5/162*a^2*c^4*x^14 + 10/81*a^3*c^3*x^
12 + 5/18*a^4*c^2*x^10 + 1/3*a^5*c*x^8 + 1/6*a^6*x^6 + 1/3*c*x^3 + a*x
```

3.210.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(22) = 44.

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.90

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)dx = \frac{a^6x^6}{6}+\frac{a^5cx^8}{3}+\frac{5a^4c^2x^{10}}{18}+\frac{10a^3c^3x^{12}}{81}+\frac{5a^2c^4x^{14}}{162}+\frac{ac^5x^{16}}{243}+ax+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}$$

input
```
integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**5),x)
```

output
```
a**6*x**6/6 + a**5*c*x**8/3 + 5*a**4*c**2*x**10/18 + 10*a**3*c**3*x**12/81
 + 5*a**2*c**4*x**14/162 + a*c**5*x**16/243 + a*x + c**6*x**18/4374 + c*x*
*3/3
```

3.210.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(24) = 48.

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.57

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)dx = \frac{1}{4374}c^6x^{18}+\frac{1}{243}ac^5x^{16}+\frac{5}{162}a^2c^4x^{14}+\frac{10}{81}a^3c^3x^{12}+\frac{5}{18}a^4c^2x^{10}+\frac{1}{3}a^5cx^8+\frac{1}{6}a^6x^6+\frac{1}{3}cx^3+ax$$

input
```
integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="maxima")
```

output
```
1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 5/162*a^2*c^4*x^14 + 10/81*a^3*c^3*x^
12 + 5/18*a^4*c^2*x^10 + 1/3*a^5*c*x^8 + 1/6*a^6*x^6 + 1/3*c*x^3 + a*x
```

3.210.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)dx=\frac{1}{4374}\left(cx^3+3\,ax\right)^6+\frac{1}{3}\,cx^3+ax$$

input
```
integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="giac")
```

output
```
1/4374*(c*x^3 + 3*a*x)^6 + 1/3*c*x^3 + a*x
```

3.210.9 Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.57

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)dx=\frac{a^6\,x^6}{6}+\frac{a^5\,c\,x^8}{3}+\frac{5\,a^4\,c^2\,x^{10}}{18}+\frac{10\,a^3\,c^3\,x^{12}}{81}+\frac{5\,a^2\,c^4\,x^{14}}{162}+\frac{a\,c^5\,x^{16}}{243}+a\,x+\frac{c^6\,x^{18}}{4374}+\frac{c\,x^3}{3}$$

input
```
int((a + c*x^2)*((a*x + (c*x^3)/3)^5 + 1),x)
```

output
```
a*x + (c*x^3)/3 + (a^6*x^6)/6 + (c^6*x^18)/4374 + (a^5*c*x^8)/3 + (a*c^5*x
^16)/243 + (5*a^4*c^2*x^10)/18 + (10*a^3*c^3*x^12)/81 + (5*a^2*c^4*x^14)/1
62
```

3.211 $\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)\,dx$

3.211.1 Optimal result

Integrand size = 25, antiderivative size = 31

$$\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)dx = ax+\frac{cx^3}{3}+\frac{1}{6}\left(d+ax+\frac{cx^3}{3}\right)^6$$

output
```
a*x+1/3*c*x^3+1/6*(d+a*x+1/3*c*x^3)^6
```

3.211.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(31) = 62.

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 4.52

$$\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)dx$$
$$=\frac{x(3a+cx^2)\left(1458+1458d^5+243a^5x^5+405a^4cx^7+270a^3c^2x^9+90a^2c^3x^{11}+15ac^4x^{13}+c^5x^{15}+1215d\right.}{4374}$$

input
```
Integrate[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]
```

output
```
(x*(3*a + c*x^2)*(1458 + 1458*d^5 + 243*a^5*x^5 + 405*a^4*c*x^7 + 270*a^3*
c^2*x^9 + 90*a^2*c^3*x^11 + 15*a*c^4*x^13 + c^5*x^15 + 1215*d^4*(3*a*x + c
*x^3) + 540*d^3*(3*a*x + c*x^3)^2 + 135*d^2*(3*a*x + c*x^3)^3 + 18*d*(3*a*
x + c*x^3)^4))/4374
```

3.211.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)\left(\left(ax+\frac{cx^3}{3}+d\right)^5+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(ax+\frac{cx^3}{3}+d\right)^5+1\right)d\left(ax+\frac{cx^3}{3}+d\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}+d\right)^6+ax+\frac{cx^3}{3}+d$$

input
```
Int[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]
```

output
```
d + a*x + (c*x^3)/3 + (d + a*x + (c*x^3)/3)^6/6
```

3.211.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.211.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result
default	$\frac{\left(d+ax+\frac{1}{3}c\,x^{3}\right)^{6}}{6}+d+ax+\frac{c\,x^{3}}{3}$
norman	$\left(\frac{5}{18}a^{4}c^{2}+\frac{10}{27}a\,c^{3}d^{2}\right)x^{10}+\left(\frac{5}{2}a^{4}d^{2}+\frac{5}{3}ac\,d^{4}\right)x^{4}+\left(\frac{1}{3}c\,a^{5}+\frac{5}{3}a^{2}c^{2}d^{2}\right)x^{8}+\left(\frac{10}{81}c^{3}a^{3}+\frac{5}{162}d^{2}c^{4}\right)$
risch	$\frac{5}{2}a^{2}d^{4}x^{2}+\frac{10}{3}x^{6}a^{3}c\,d^{2}+\frac{10}{3}x^{5}a^{2}c\,d^{3}+\frac{5}{3}x^{4}ac\,d^{4}+\frac{5}{81}a\,c^{4}d\,x^{13}+\frac{1}{6}a^{6}x^{6}+\frac{1}{3}c\,x^{3}+ax+\frac{5}{162}x^{12}$
parallelrisch	$\frac{5}{2}a^{2}d^{4}x^{2}+\frac{10}{3}x^{6}a^{3}c\,d^{2}+\frac{10}{3}x^{5}a^{2}c\,d^{3}+\frac{5}{3}x^{4}ac\,d^{4}+\frac{5}{81}a\,c^{4}d\,x^{13}+\frac{1}{6}a^{6}x^{6}+\frac{1}{3}c\,x^{3}+ax+\frac{5}{162}x^{12}$
gosper	$x\left(c^{6}x^{17}+18a\,c^{5}x^{15}+18c^{5}d\,x^{14}+135a^{2}c^{4}x^{13}+270a\,c^{4}d\,x^{12}+540c^{3}a^{3}x^{11}+135x^{11}d^{2}c^{4}+1620a^{2}c^{3}d\,x^{10}+1215a^{4}c^{2}x^{9}+1620x^{9}a\right.$

input
```
int((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
```

output
```
1/6*(d+a*x+1/3*c*x^3)^6+d+a*x+1/3*c*x^3
```

3.211.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(27) = 54.

Time = 0.26 (sec) , antiderivative size = 280, normalized size of antiderivative = 9.03

$$\int \left(a+cx^2\right)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)\,dx = \frac{1}{4374}\,c^6x^{18}+\frac{1}{243}\,ac^5x^{16}+\frac{1}{243}\,c^5dx^{15}+\frac{5}{162}\,a^2c^4x^{14}+\frac{5}{81}\,ac^4dx^{13}+\frac{10}{27}\,a^2c^3dx^{11}+\frac{5}{162}\,\left(4\,a^3c^3+c^4d^2\right)x^{12}+\frac{5}{54}\,\left(3\,a^4c^2+4\,ac^3d^2\right)x^{10}+\frac{10}{81}\,\left(9\,a^3c^2d+c^3d^3\right)x^9+\frac{1}{3}\,\left(a^5c+5\,a^2c^2d^2\right)x^8+\frac{5}{2}\,a^2d^4x^2+\frac{5}{9}\,\left(3\,a^4cd+2\,ac^2d^3\right)x^7+\frac{1}{18}\,\left(3\,a^6+60\,a^3cd^2+5\,c^2d^4\right)x^6+\frac{1}{3}\,\left(3\,a^5d+10\,a^2cd^3\right)x^5+\frac{5}{6}\,\left(3\,a^4d^2+2\,acd^4\right)x^4+\frac{1}{3}\,\left(10\,a^3d^3+cd^5+c\right)x^3+\left(ad^5+a\right)x$$

input
```
integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="fricas")
```

output
```
1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 1/243*c^5*d*x^15 + 5/162*a^2*c^4*x^14
 + 5/81*a*c^4*d*x^13 + 10/27*a^2*c^3*d*x^11 + 5/162*(4*a^3*c^3 + c^4*d^2)*
x^12 + 5/54*(3*a^4*c^2 + 4*a*c^3*d^2)*x^10 + 10/81*(9*a^3*c^2*d + c^3*d^3)
*x^9 + 1/3*(a^5*c + 5*a^2*c^2*d^2)*x^8 + 5/2*a^2*d^4*x^2 + 5/9*(3*a^4*c*d
+ 2*a*c^2*d^3)*x^7 + 1/18*(3*a^6 + 60*a^3*c*d^2 + 5*c^2*d^4)*x^6 + 1/3*(3*
a^5*d + 10*a^2*c*d^3)*x^5 + 5/6*(3*a^4*d^2 + 2*a*c*d^4)*x^4 + 1/3*(10*a^3*
d^3 + c*d^5 + c)*x^3 + (a*d^5 + a)*x
```

3.211. $\int \left(a+cx^2\right)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)\,dx$

3.211.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(24) = 48$.

Time = 0.06 (sec) , antiderivative size = 314, normalized size of antiderivative = 10.13

$$\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)dx = \frac{5a^2c^4x^{14}}{162}+\frac{10a^2c^3dx^{11}}{27}+\frac{5a^2d^4x^2}{2}+\frac{ac^5x^{16}}{243}+\frac{5ac^4dx^{13}}{81}+\frac{c^6x^{18}}{4374}+\frac{c^5dx^{15}}{243}+x^{12}\cdot\left(\frac{10a^3c^3}{81}+\frac{5c^4d^2}{162}\right)+x^{10}\cdot\left(\frac{5a^4c^2}{18}+\frac{10ac^3d^2}{27}\right)+x^9\cdot\left(\frac{10a^3c^2d}{9}+\frac{10c^3d^3}{81}\right)+x^8\left(\frac{a^5c}{3}+\frac{5a^2c^2d^2}{3}\right)+x^7\cdot\left(\frac{5a^4cd}{3}+\frac{10ac^2d^3}{9}\right)+x^6\left(\frac{a^6}{6}+\frac{10a^3cd^2}{3}+\frac{5c^2d^4}{18}\right)+x^5\left(a^5d+\frac{10a^2cd^3}{3}\right)+x^4\cdot\left(\frac{5a^4d^2}{2}+\frac{5acd^4}{3}\right)+x^3\cdot\left(\frac{10a^3d^3}{3}+\frac{cd^5}{3}+\frac{c}{3}\right)+x(ad^5+a)$$

input
```
integrate((c*x**2+a)*(1+(d+a*x+1/3*c*x**3)**5),x)
```

output
```
5*a**2*c**4*x**14/162 + 10*a**2*c**3*d*x**11/27 + 5*a**2*d**4*x**2/2 + a*c
**5*x**16/243 + 5*a*c**4*d*x**13/81 + c**6*x**18/4374 + c**5*d*x**15/243 +
 x**12*(10*a**3*c**3/81 + 5*c**4*d**2/162) + x**10*(5*a**4*c**2/18 + 10*a*
c**3*d**2/27) + x**9*(10*a**3*c**2*d/9 + 10*c**3*d**3/81) + x**8*(a**5*c/3
 + 5*a**2*c**2*d**2/3) + x**7*(5*a**4*c*d/3 + 10*a*c**2*d**3/9) + x**6*(a*
*6/6 + 10*a**3*c*d**2/3 + 5*c**2*d**4/18) + x**5*(a**5*d + 10*a**2*c*d**3/
3) + x**4*(5*a**4*d**2/2 + 5*a*c*d**4/3) + x**3*(10*a**3*d**3/3 + c*d**5/3
 + c/3) + x*(a*d**5 + a)
```

3.211.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(27) = 54$.

Time = 0.19 (sec) , antiderivative size = 280, normalized size of antiderivative = 9.03

$$\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)dx = \frac{1}{4374}c^6x^{18}+\frac{1}{243}ac^5x^{16}+\frac{1}{243}c^5dx^{15}+\frac{5}{162}a^2c^4x^{14}+\frac{5}{81}ac^4dx^{13}+\frac{10}{27}a^2c^3dx^{11}+\frac{5}{162}\left(4a^3c^3+c^4d^2\right)x^{12}+\frac{5}{54}\left(3a^4c^2+4ac^3d^2\right)x^{10}+\frac{10}{81}\left(9a^3c^2d+c^3d^3\right)x^9+\frac{1}{3}\left(a^5c+5a^2c^2d^2\right)x^8+\frac{5}{2}a^2d^4x^2+\frac{5}{9}\left(3a^4cd+2ac^2d^3\right)x^7+\frac{1}{18}\left(3a^6+60a^3cd^2+5c^2d^4\right)x^6+\frac{1}{3}\left(3a^5d+10a^2cd^3\right)x^5+\frac{5}{6}\left(3a^4d^2+2acd^4\right)x^4+\frac{1}{3}\left(10a^3d^3+cd^5+c\right)x^3+\left(ad^5+a\right)x$$

input
```
integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="maxima")
```

output
```
1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 1/243*c^5*d*x^15 + 5/162*a^2*c^4*x^14
 + 5/81*a*c^4*d*x^13 + 10/27*a^2*c^3*d*x^11 + 5/162*(4*a^3*c^3 + c^4*d^2)*
x^12 + 5/54*(3*a^4*c^2 + 4*a*c^3*d^2)*x^10 + 10/81*(9*a^3*c^2*d + c^3*d^3)
*x^9 + 1/3*(a^5*c + 5*a^2*c^2*d^2)*x^8 + 5/2*a^2*d^4*x^2 + 5/9*(3*a^4*c*d
+ 2*a*c^2*d^3)*x^7 + 1/18*(3*a^6 + 60*a^3*c*d^2 + 5*c^2*d^4)*x^6 + 1/3*(3*
a^5*d + 10*a^2*c*d^3)*x^5 + 5/6*(3*a^4*d^2 + 2*a*c*d^4)*x^4 + 1/3*(10*a^3*
d^3 + c*d^5 + c)*x^3 + (a*d^5 + a)*x
```

3.211.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(27) = 54.

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.39

$$\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)dx = \frac{1}{4374}\left(cx^3+3\,ax\right)^6+\frac{1}{243}\left(cx^3+3\,ax\right)^5d+\frac{5}{162}\left(cx^3+3\,ax\right)^4d^2+\frac{10}{81}\left(cx^3+3\,ax\right)^3d^3+\frac{5}{18}\left(cx^3+3\,ax\right)^2d^4+\frac{1}{3}\left(cx^3+3\,ax\right)d^5+\frac{1}{3}\,cx^3+ax$$

input
```
integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="giac")
```

output
```
1/4374*(c*x^3 + 3*a*x)^6 + 1/243*(c*x^3 + 3*a*x)^5*d + 5/162*(c*x^3 + 3*a*
x)^4*d^2 + 10/81*(c*x^3 + 3*a*x)^3*d^3 + 5/18*(c*x^3 + 3*a*x)^2*d^4 + 1/3*
(c*x^3 + 3*a*x)*d^5 + 1/3*c*x^3 + a*x
```

3.211.9 Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 266, normalized size of antiderivative = 8.58

$$\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)dx = x^5\left(a^5\,d+\frac{10\,c\,a^2\,d^3}{3}\right)+x^4\left(\frac{5\,a^4\,d^2}{2}+\frac{5\,c\,a\,d^4}{3}\right)+x^3\left(\frac{10\,a^3\,d^3}{3}+\frac{c\,d^5}{3}+\frac{c}{3}\right)+x^6\left(\frac{a^6}{6}+\frac{10\,a^3\,c\,d^2}{3}+\frac{5\,c^2\,d^4}{18}\right)+\frac{c^6\,x^{18}}{4374}+\frac{a\,c^5\,x^{16}}{243}+a\,x\,\left(d^5+1\right)+\frac{c^5\,d\,x^{15}}{243}+\frac{5\,a^2\,c^4\,x^{14}}{162}+\frac{5\,a^2\,d^4\,x^2}{2}+\frac{5\,c^3\,x^{12}\,\left(4\,a^3+c\,d^2\right)}{162}+\frac{a^2\,c\,x^8\,\left(a^3+5\,c\,d^2\right)}{3}+\frac{10\,a^2\,c^3\,d\,x^{11}}{27}+\frac{5\,a\,c^2\,x^{10}\,\left(3\,a^3+4\,c\,d^2\right)}{54}+\frac{10\,c^2\,d\,x^9\,\left(9\,a^3+c\,d^2\right)}{81}+\frac{5\,a\,c^4\,d\,x^{13}}{81}+\frac{5\,a\,c\,d\,x^7\,\left(3\,a^3+2\,c\,d^2\right)}{9}$$

input
```
int(((d + a*x + (c*x^3)/3)^5 + 1)*(a + c*x^2),x)
```

output
```
x^5*(a^5*d + (10*a^2*c*d^3)/3) + x^4*((5*a^4*d^2)/2 + (5*a*c*d^4)/3) + x^3
*(c/3 + (c*d^5)/3 + (10*a^3*d^3)/3) + x^6*(a^6/6 + (5*c^2*d^4)/18 + (10*a^
3*c*d^2)/3) + (c^6*x^18)/4374 + (a*c^5*x^16)/243 + a*x*(d^5 + 1) + (c^5*d*
x^15)/243 + (5*a^2*c^4*x^14)/162 + (5*a^2*d^4*x^2)/2 + (5*c^3*x^12*(c*d^2
+ 4*a^3))/162 + (a^2*c*x^8*(5*c*d^2 + a^3))/3 + (10*a^2*c^3*d*x^11)/27 + (
5*a*c^2*x^10*(4*c*d^2 + 3*a^3))/54 + (10*c^2*d*x^9*(c*d^2 + 9*a^3))/81 + (
5*a*c^4*d*x^13)/81 + (5*a*c*d*x^7*(2*c*d^2 + 3*a^3))/9
```

3.211. $\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)\,dx$

3.212 $\int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx$

3.212.1 Optimal result

Integrand size = 31, antiderivative size = 34

$$\int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx = \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{x^{12}(3b + 2cx)^6}{279936}$$

output
```
1/2*b*x^2+1/3*c*x^3+1/279936*x^12*(2*c*x+3*b)^6
```

3.212.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(34) = 68.

Time = 0.01 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.88

$$\int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx = \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{b^6x^{12}}{384} + \frac{1}{96}b^5cx^{13} + \frac{5}{288}b^4c^2x^{14} + \frac{5}{324}b^3c^3x^{15} + \frac{5}{648}b^2c^4x^{16} + \frac{1}{486}bc^5x^{17} + \frac{c^6x^{18}}{4374}$$

input
```
Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5),x]
```

output
```
(b*x^2)/2 + (c*x^3)/3 + (b^6*x^12)/384 + (b^5*c*x^13)/96 + (5*b^4*c^2*x^14
)/288 + (5*b^3*c^3*x^15)/324 + (5*b^2*c^4*x^16)/648 + (b*c^5*x^17)/486 + (
c^6*x^18)/4374
```

3.212.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx+cx^2)\left(\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5+1\right)d\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5),x]
```

output
```
(b*x^2)/2 + (c*x^3)/3 + ((b*x^2)/2 + (c*x^3)/3)^6/6
```

3.212.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.212.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result
default	$\frac{\left(\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)^{6}}{6}+\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}$
gosper	$\frac{x^{2}\left(64c^{6}x^{16}+576b\,c^{5}x^{15}+2160b^{2}c^{4}x^{14}+4320b^{3}c^{3}x^{13}+4860b^{4}c^{2}x^{12}+2916b^{5}c\,x^{11}+729b^{6}x^{10}+93312cx+139968b\right)}{279936}$
norman	$\frac{1}{2}b\,x^{2}+\frac{1}{384}b^{6}x^{12}+\frac{1}{3}c\,x^{3}+\frac{1}{4374}c^{6}x^{18}+\frac{1}{486}b\,c^{5}x^{17}+\frac{5}{648}b^{2}c^{4}x^{16}+\frac{5}{324}b^{3}c^{3}x^{15}+\frac{5}{288}b^{4}c^{2}x^{14}+$
risch	$\frac{1}{2}b\,x^{2}+\frac{1}{384}b^{6}x^{12}+\frac{1}{3}c\,x^{3}+\frac{1}{4374}c^{6}x^{18}+\frac{1}{486}b\,c^{5}x^{17}+\frac{5}{648}b^{2}c^{4}x^{16}+\frac{5}{324}b^{3}c^{3}x^{15}+\frac{5}{288}b^{4}c^{2}x^{14}+$
parallelrisch	$\frac{1}{2}b\,x^{2}+\frac{1}{384}b^{6}x^{12}+\frac{1}{3}c\,x^{3}+\frac{1}{4374}c^{6}x^{18}+\frac{1}{486}b\,c^{5}x^{17}+\frac{5}{648}b^{2}c^{4}x^{16}+\frac{5}{324}b^{3}c^{3}x^{15}+\frac{5}{288}b^{4}c^{2}x^{14}+$

input
```
int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
```

output
```
1/6*(1/2*b*x^2+1/3*c*x^3)^6+1/2*b*x^2+1/3*c*x^3
```

3.212.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(28) = 56.

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{1}{4374}\,c^6x^{18}+\frac{1}{486}\,bc^5x^{17}+\frac{5}{648}\,b^2c^4x^{16}+\frac{5}{324}\,b^3c^3x^{15}+\frac{5}{288}\,b^4c^2x^{14}+\frac{1}{96}\,b^5cx^{13}+\frac{1}{384}\,b^6x^{12}+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2$$

input
```
integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fricas")
```

output
```
1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 5/324*b^3*c^3*x^
15 + 5/288*b^4*c^2*x^14 + 1/96*b^5*c*x^13 + 1/384*b^6*x^12 + 1/3*c*x^3 + 1
/2*b*x^2
```

3.212.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(27) = 54$.

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx = \frac{b^6x^{12}}{384}+\frac{b^5cx^{13}}{96}+\frac{5b^4c^2x^{14}}{288}+\frac{5b^3c^3x^{15}}{324}+\frac{5b^2c^4x^{16}}{648}+\frac{bc^5x^{17}}{486}+\frac{bx^2}{2}+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}$$

input
```
integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**5),x)
```

output
```
b**6*x**12/384 + b**5*c*x**13/96 + 5*b**4*c**2*x**14/288 + 5*b**3*c**3*x**
15/324 + 5*b**2*c**4*x**16/648 + b*c**5*x**17/486 + b*x**2/2 + c**6*x**18/
4374 + c*x**3/3
```

3.212.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(28) = 56$.

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx = \frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{5}{648}b^2c^4x^{16}+\frac{5}{324}b^3c^3x^{15}+\frac{5}{288}b^4c^2x^{14}+\frac{1}{96}b^5cx^{13}+\frac{1}{384}b^6x^{12}+\frac{1}{3}cx^3+\frac{1}{2}bx^2$$

input
```
integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")
```

output
```
1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 5/324*b^3*c^3*x^
15 + 5/288*b^4*c^2*x^14 + 1/96*b^5*c*x^13 + 1/384*b^6*x^12 + 1/3*c*x^3 + 1
/2*b*x^2
```

3.212.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{1}{279936}\left(2\,cx^3+3\,bx^2\right)^6+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2$$

input
```
integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac")
```

output
```
1/279936*(2*c*x^3 + 3*b*x^2)^6 + 1/3*c*x^3 + 1/2*b*x^2
```

3.212.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{b^6\,x^{12}}{384}+\frac{b^5\,c\,x^{13}}{96}+\frac{5\,b^4\,c^2\,x^{14}}{288}+\frac{5\,b^3\,c^3\,x^{15}}{324}+\frac{5\,b^2\,c^4\,x^{16}}{648}+\frac{b\,c^5\,x^{17}}{486}+\frac{b\,x^2}{2}+\frac{c^6\,x^{18}}{4374}+\frac{c\,x^3}{3}$$

input
```
int((b*x + c*x^2)*(((b*x^2)/2 + (c*x^3)/3)^5 + 1),x)
```

output
```
(b*x^2)/2 + (c*x^3)/3 + (b^6*x^12)/384 + (c^6*x^18)/4374 + (b^5*c*x^13)/96
 + (b*c^5*x^17)/486 + (5*b^4*c^2*x^14)/288 + (5*b^3*c^3*x^15)/324 + (5*b^2
*c^4*x^16)/648
```

3.213 $\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

3.213.1 Optimal result

Integrand size = 32, antiderivative size = 41

$$\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6$$

output
```
1/2*b*x^2+1/3*c*x^3+1/6*(d+1/2*b*x^2+1/3*c*x^3)^6
```

3.213.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 146 vs. 2(41) = 82.

Time = 0.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.56

$$\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$$
$$=\frac{x^2(3b+2cx)\,(46656+46656d^5+243b^5x^{10}+810b^4cx^{11}+1080b^3c^2x^{12}+720b^2c^3x^{13}+240bc^4x^{14}+32c^5x^{1}}{279936}$$

input
```
Integrate[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

output
```
(x^2*(3*b + 2*c*x)*(46656 + 46656*d^5 + 243*b^5*x^10 + 810*b^4*c*x^11 + 10
80*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14 + 32*c^5*x^15 + 19440*
d^4*x^2*(3*b + 2*c*x) + 4320*d^3*x^4*(3*b + 2*c*x)^2 + 540*d^2*x^6*(3*b +
2*c*x)^3 + 36*d*x^8*(3*b + 2*c*x)^4))/279936
```

3.213.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx+cx^2)\left(\left(\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^5+1\right)dx$$

$$\Big\downarrow \text{ 2024}$$

$$\int \left(\left(\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^5+1\right)d\left(\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6}\left(\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+\frac{bx^2}{2}+\frac{cx^3}{3}+d$$

input
```
Int[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

output
```
d + (b*x^2)/2 + (c*x^3)/3 + (d + (b*x^2)/2 + (c*x^3)/3)^6/6
```

3.213.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.213.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result
default	$\frac{\left(d+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)^{6}}{6}+d+\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}$
norman	$\left(\frac{1}{2}b\,d^{5}+\frac{1}{2}b\right)x^{2}+\left(\frac{5}{324}b^{3}c^{3}+\frac{1}{243}c^{5}d\right)x^{15}+\left(\frac{5}{12}b^{3}d^{3}+\frac{5}{18}c^{2}d^{4}\right)x^{6}+\left(\frac{5}{288}b^{4}c^{2}+\frac{5}{162}b\,c^{4}d\right)x^{14}+$
gosper	$\frac{x^{2}\left(64c^{6}x^{16}+576b\,c^{5}x^{15}+2160b^{2}c^{4}x^{14}+4320b^{3}c^{3}x^{13}+1152c^{5}d\,x^{13}+4860b^{4}c^{2}x^{12}+8640b\,c^{4}d\,x^{12}+2916b^{5}c\,x^{11}+25920b^{2}c^{3}d\,x^{1}\right.}{}$
risch	$\frac{5}{162}x^{14}b\,c^{4}d+\frac{5}{54}x^{13}b^{2}c^{3}d+\frac{5}{12}x^{6}b^{3}d^{3}+\frac{1}{2}b\,d^{5}x^{2}+\frac{1}{384}b^{6}x^{12}+\frac{1}{3}c\,x^{3}+\frac{1}{2}b\,x^{2}+\frac{5}{162}x^{12}d^{2}c^{4}+\frac{10}{81}$
parallelrisch	$\frac{5}{162}x^{14}b\,c^{4}d+\frac{5}{54}x^{13}b^{2}c^{3}d+\frac{5}{12}x^{6}b^{3}d^{3}+\frac{1}{2}b\,d^{5}x^{2}+\frac{1}{384}b^{6}x^{12}+\frac{1}{3}c\,x^{3}+\frac{1}{2}b\,x^{2}+\frac{5}{162}x^{12}d^{2}c^{4}+\frac{10}{81}$

input
```
int((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
```

output
```
1/6*(d+1/2*b*x^2+1/3*c*x^3)^6+d+1/2*b*x^2+1/3*c*x^3
```

3.213.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(33) = 66.

Time = 0.27 (sec) , antiderivative size = 289, normalized size of antiderivative = 7.05

$$\begin{aligned}&\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx\\&=\frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{5}{648}b^2c^4x^{16}+\frac{1}{972}\left(15\,b^3c^3+4\,c^5d\right)x^{15}\\&+\frac{5}{2592}\left(9\,b^4c^2+16\,bc^4d\right)x^{14}+\frac{1}{864}\left(9\,b^5c+80\,b^2c^3d\right)x^{13}+\frac{5}{6}b^2cd^3x^7\\&+\frac{1}{10368}\left(27\,b^6+1440\,b^3c^2d+320\,c^4d^2\right)x^{12}+\frac{5}{432}\left(9\,b^4cd+16\,bc^3d^2\right)x^{11}\\&+\frac{5}{6}bcd^4x^5+\frac{1}{96}\left(3\,b^5d+40\,b^2c^2d^2\right)x^{10}+\frac{5}{8}b^2d^4x^4+\frac{5}{324}\left(27\,b^3cd^2+8\,c^3d^3\right)x^9\\&+\frac{5}{288}\left(9\,b^4d^2+32\,bc^2d^3\right)x^8+\frac{5}{36}\left(3\,b^3d^3+2\,c^2d^4\right)x^6+\frac{1}{3}\left(cd^5+c\right)x^3+\frac{1}{2}\left(bd^5+b\right)x^2\end{aligned}$$

input
```
integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fricas")
```

output
```
1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/972*(15*b^3*c^
3 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 16*b*c^4*d)*x^14 + 1/864*(9*b^5*c
+ 80*b^2*c^3*d)*x^13 + 5/6*b^2*c*d^3*x^7 + 1/10368*(27*b^6 + 1440*b^3*c^2*
d + 320*c^4*d^2)*x^12 + 5/432*(9*b^4*c*d + 16*b*c^3*d^2)*x^11 + 5/6*b*c*d^
4*x^5 + 1/96*(3*b^5*d + 40*b^2*c^2*d^2)*x^10 + 5/8*b^2*d^4*x^4 + 5/324*(27
*b^3*c*d^2 + 8*c^3*d^3)*x^9 + 5/288*(9*b^4*d^2 + 32*b*c^2*d^3)*x^8 + 5/36*
(3*b^3*d^3 + 2*c^2*d^4)*x^6 + 1/3*(c*d^5 + c)*x^3 + 1/2*(b*d^5 + b)*x^2
```

3.213.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(31) = 62$.

Time = 0.06 (sec) , antiderivative size = 321, normalized size of antiderivative = 7.83

$$\int (bx + cx^2)\left(1 + \left(d + \frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx = \frac{5b^2c^4x^{16}}{648} + \frac{5b^2cd^3x^7}{6} + \frac{5b^2d^4x^4}{8} + \frac{bc^5x^{17}}{486} + \frac{5bcd^4x^5}{6} + \frac{c^6x^{18}}{4374} + x^{15}\cdot\left(\frac{5b^3c^3}{324} + \frac{c^5d}{243}\right) + x^{14}\cdot\left(\frac{5b^4c^2}{288} + \frac{5bc^4d}{162}\right) + x^{13}\left(\frac{b^5c}{96} + \frac{5b^2c^3d}{54}\right) + x^{12}\left(\frac{b^6}{384} + \frac{5b^3c^2d}{36} + \frac{5c^4d^2}{162}\right) + x^{11}\cdot\left(\frac{5b^4cd}{48} + \frac{5bc^3d^2}{27}\right) + x^{10}\left(\frac{b^5d}{32} + \frac{5b^2c^2d^2}{12}\right) + x^9\cdot\left(\frac{5b^3cd^2}{12} + \frac{10c^3d^3}{81}\right) + x^8\cdot\left(\frac{5b^4d^2}{32} + \frac{5bc^2d^3}{9}\right) + x^6\cdot\left(\frac{5b^3d^3}{12} + \frac{5c^2d^4}{18}\right) + x^3\left(\frac{cd^5}{3} + \frac{c}{3}\right) + x^2\left(\frac{bd^5}{2} + \frac{b}{2}\right)$$

input
```
integrate((c*x**2+b*x)*(1+(d+1/2*b*x**2+1/3*c*x**3)**5),x)
```

output
```
5*b**2*c**4*x**16/648 + 5*b**2*c*d**3*x**7/6 + 5*b**2*d**4*x**4/8 + b*c**5
*x**17/486 + 5*b*c*d**4*x**5/6 + c**6*x**18/4374 + x**15*(5*b**3*c**3/324
+ c**5*d/243) + x**14*(5*b**4*c**2/288 + 5*b*c**4*d/162) + x**13*(b**5*c/9
6 + 5*b**2*c**3*d/54) + x**12*(b**6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/1
62) + x**11*(5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(b**5*d/32 + 5*b**2
*c**2*d**2/12) + x**9*(5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(5*b**4*
d**2/32 + 5*b*c**2*d**3/9) + x**6*(5*b**3*d**3/12 + 5*c**2*d**4/18) + x**3
*(c*d**5/3 + c/3) + x**2*(b*d**5/2 + b/2)
```

3.213.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(33) = 66$.

Time = 0.20 (sec) , antiderivative size = 289, normalized size of antiderivative = 7.05

$$\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$= \frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{1}{972}(15b^3c^3+4c^5d)x^{15} + \frac{5}{2592}(9b^4c^2+16bc^4d)x^{14} + \frac{1}{864}(9b^5c+80b^2c^3d)x^{13} + \frac{5}{6}b^2cd^3x^7 + \frac{1}{10368}(27b^6+1440b^3c^2d+320c^4d^2)x^{12} + \frac{5}{432}(9b^4cd+16bc^3d^2)x^{11} + \frac{5}{6}bcd^4x^5 + \frac{1}{96}(3b^5d+40b^2c^2d^2)x^{10} + \frac{5}{8}b^2d^4x^4 + \frac{5}{324}(27b^3cd^2+8c^3d^3)x^9 + \frac{5}{288}(9b^4d^2+32bc^2d^3)x^8 + \frac{5}{36}(3b^3d^3+2c^2d^4)x^6 + \frac{1}{3}(cd^5+c)x^3 + \frac{1}{2}(bd^5+b)x^2$$

input
```
integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")
```

output
```
1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/972*(15*b^3*c^
3 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 16*b*c^4*d)*x^14 + 1/864*(9*b^5*c
+ 80*b^2*c^3*d)*x^13 + 5/6*b^2*c*d^3*x^7 + 1/10368*(27*b^6 + 1440*b^3*c^2*
d + 320*c^4*d^2)*x^12 + 5/432*(9*b^4*c*d + 16*b*c^3*d^2)*x^11 + 5/6*b*c*d^
4*x^5 + 1/96*(3*b^5*d + 40*b^2*c^2*d^2)*x^10 + 5/8*b^2*d^4*x^4 + 5/324*(27
*b^3*c*d^2 + 8*c^3*d^3)*x^9 + 5/288*(9*b^4*d^2 + 32*b*c^2*d^3)*x^8 + 5/36*
(3*b^3*d^3 + 2*c^2*d^4)*x^6 + 1/3*(c*d^5 + c)*x^3 + 1/2*(b*d^5 + b)*x^2
```

3.213.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(33) = 66.

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.07

$$\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$=\frac{1}{279936}\left(2\,cx^3+3\,bx^2\right)^6+\frac{1}{7776}\left(2\,cx^3+3\,bx^2\right)^5d+\frac{5}{2592}\left(2\,cx^3+3\,bx^2\right)^4d^2+\frac{5}{324}\left(2\,cx^3+3\,bx^2\right)^3d^3+\frac{5}{72}\left(2\,cx^3+3\,bx^2\right)^2d^4+\frac{1}{6}\left(2\,cx^3+3\,bx^2\right)d^5+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2$$

input
```
integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac")
```

output
```
1/279936*(2*c*x^3 + 3*b*x^2)^6 + 1/7776*(2*c*x^3 + 3*b*x^2)^5*d + 5/2592*(
2*c*x^3 + 3*b*x^2)^4*d^2 + 5/324*(2*c*x^3 + 3*b*x^2)^3*d^3 + 5/72*(2*c*x^3
 + 3*b*x^2)^2*d^4 + 1/6*(2*c*x^3 + 3*b*x^2)*d^5 + 1/3*c*x^3 + 1/2*b*x^2
```

3.213.9 Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 273, normalized size of antiderivative = 6.66

$$\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx=x^{13}\left(\frac{b^5\,c}{96}+\frac{5\,d\,b^2\,c^3}{54}\right)+x^{14}\left(\frac{5\,b^4\,c^2}{288}+\frac{5\,d\,b\,c^4}{162}\right)+x^{12}\left(\frac{b^6}{384}+\frac{5\,b^3\,c^2\,d}{36}+\frac{5\,c^4\,d^2}{162}\right)+\frac{c^6\,x^{18}}{4374}+x^{15}\left(\frac{5\,b^3\,c^3}{324}+\frac{d\,c^5}{243}\right)+\frac{5\,d^3\,x^6\,(3\,b^3+2\,d\,c^2)}{36}+\frac{b\,c^5\,x^{17}}{486}+\frac{5\,b^2\,c^4\,x^{16}}{648}+\frac{b\,x^2\,(d^5+1)}{2}+\frac{5\,b^2\,d^4\,x^4}{8}+\frac{c\,x^3\,(d^5+1)}{3}+\frac{5\,b^2\,c\,d^3\,x^7}{6}+\frac{5\,b\,d^2\,x^8\,(9\,b^3+32\,d\,c^2)}{288}+\frac{b^2\,d\,x^{10}\,(3\,b^3+40\,d\,c^2)}{96}+\frac{5\,c\,d^2\,x^9\,(27\,b^3+8\,d\,c^2)}{324}+\frac{5\,b\,c\,d^4\,x^5}{6}+\frac{5\,b\,c\,d\,x^{11}\,(9\,b^3+16\,d\,c^2)}{432}$$

input
```
int((b*x + c*x^2)*((d + (b*x^2)/2 + (c*x^3)/3)^5 + 1),x)
```

output
```
x^13*((b^5*c)/96 + (5*b^2*c^3*d)/54) + x^14*((5*b^4*c^2)/288 + (5*b*c^4*d)
/162) + x^12*(b^6/384 + (5*c^4*d^2)/162 + (5*b^3*c^2*d)/36) + (c^6*x^18)/4
374 + x^15*((c^5*d)/243 + (5*b^3*c^3)/324) + (5*d^3*x^6*(2*c^2*d + 3*b^3))
/36 + (b*c^5*x^17)/486 + (5*b^2*c^4*x^16)/648 + (b*x^2*(d^5 + 1))/2 + (5*b
^2*d^4*x^4)/8 + (c*x^3*(d^5 + 1))/3 + (5*b^2*c*d^3*x^7)/6 + (5*b*d^2*x^8*(
32*c^2*d + 9*b^3))/288 + (b^2*d*x^10*(40*c^2*d + 3*b^3))/96 + (5*c*d^2*x^9
*(8*c^2*d + 27*b^3))/324 + (5*b*c*d^4*x^5)/6 + (5*b*c*d*x^11*(16*c^2*d + 9
*b^3))/432
```

3.213. $\int (bx + cx^2) \left(1 + \left(d + \frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) \, dx$

3.214 $\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

3.214.1 Optimal result

Integrand size = 35, antiderivative size = 46

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx=ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6$$

output
```
a*x+1/2*b*x^2+1/3*c*x^3+1/6*(a*x+1/2*b*x^2+1/3*c*x^3)^6
```

3.214.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 244 vs. 2(46) = 92.

Time = 0.05 (sec) , antiderivative size = 244, normalized size of antiderivative = 5.30

$$\begin{aligned}&\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx\\&=\frac{a^6x^6}{6}+\frac{1}{6}a^5x^7(3b+2cx)+\frac{5}{72}a^4x^8(3b+2cx)^2+\frac{5}{324}a^3x^9(3b+2cx)^3+\frac{5a^2x^{10}(3b+2cx)^4}{2592}\\&\quad+a\left(x+\frac{b^5x^{11}}{32}+\frac{5}{48}b^4cx^{12}+\frac{5}{36}b^3c^2x^{13}+\frac{5}{54}b^2c^3x^{14}+\frac{5}{162}bc^4x^{15}+\frac{c^5x^{16}}{243}\right)\\&\quad+\frac{x^2(729b^6x^{10}+2916b^5cx^{11}+4860b^4c^2x^{12}+4320b^3c^3x^{13}+2160b^2c^4x^{14}+576b(243+c^5x^{15})+64cx(145}{279936}\end{aligned}$$

input
```
Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

output
```
(a^6*x^6)/6 + (a^5*x^7*(3*b + 2*c*x))/6 + (5*a^4*x^8*(3*b + 2*c*x)^2)/72 +
 (5*a^3*x^9*(3*b + 2*c*x)^3)/324 + (5*a^2*x^10*(3*b + 2*c*x)^4)/2592 + a*(
x + (b^5*x^11)/32 + (5*b^4*c*x^12)/48 + (5*b^3*c^2*x^13)/36 + (5*b^2*c^3*x
^14)/54 + (5*b*c^4*x^15)/162 + (c^5*x^16)/243) + (x^2*(729*b^6*x^10 + 2916
*b^5*c*x^11 + 4860*b^4*c^2*x^12 + 4320*b^3*c^3*x^13 + 2160*b^2*c^4*x^14 +
576*b*(243 + c^5*x^15) + 64*c*x*(1458 + c^5*x^15)))/279936
```

3.214.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx+cx^2)\left(\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5+1\right)dx$$

$$\Big\downarrow \text{ 2024}$$

$$\int \left(\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5+1\right)d\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^6/6
```

3.214.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.214.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result
default	$ax+\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{\left(ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)^{6}}{6}$
norman	$ax+\left(\frac{1}{243}a\,c^{5}+\frac{5}{648}b^{2}c^{4}\right)x^{16}+\left(\frac{1}{3}c\,a^{5}+\frac{5}{8}b^{2}a^{4}\right)x^{8}+\left(\frac{5}{162}ab\,c^{4}+\frac{5}{324}b^{3}c^{3}\right)x^{15}+\left(\frac{5}{6}a^{4}bc+\frac{5}{12}a\right.$
risch	$\frac{1}{2}a^{5}b\,x^{7}+\frac{5}{12}a^{3}b^{3}x^{9}+\frac{1}{32}a\,b^{5}x^{11}+\frac{1}{384}b^{6}x^{12}+\frac{1}{6}a^{6}x^{6}+\frac{1}{3}c\,x^{3}+ax+\frac{1}{2}b\,x^{2}+\frac{1}{96}b^{5}c\,x^{13}+\frac{5}{288}b^{4}$
parallelrisch	$\frac{1}{2}a^{5}b\,x^{7}+\frac{5}{12}a^{3}b^{3}x^{9}+\frac{1}{32}a\,b^{5}x^{11}+\frac{1}{384}b^{6}x^{12}+\frac{1}{6}a^{6}x^{6}+\frac{1}{3}c\,x^{3}+ax+\frac{1}{2}b\,x^{2}+\frac{1}{96}b^{5}c\,x^{13}+\frac{5}{288}b^{4}$
gosper	$\frac{x\left(64c^{6}x^{17}+576b\,c^{5}x^{16}+1152a\,c^{5}x^{15}+2160b^{2}c^{4}x^{15}+8640ab\,c^{4}x^{14}+4320b^{3}c^{3}x^{14}+8640a^{2}c^{4}x^{13}+25920a\,b^{2}c^{3}x^{13}+4860b^{4}c^{2}x^{1}\right.}{}$

input
```
int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
```

output
```
a*x+1/2*b*x^2+1/3*c*x^3+1/6*(a*x+1/2*b*x^2+1/3*c*x^3)^6
```

3.214.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(37) = 74.

Time = 0.38 (sec) , antiderivative size = 289, normalized size of antiderivative = 6.28

$$
\begin{aligned}
&\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx \\
&= \frac{1}{4374}\,c^6x^{18}+\frac{1}{486}\,bc^5x^{17}+\frac{1}{1944}\,(15\,b^2c^4+8\,ac^5)x^{16}+\frac{5}{324}\,(b^3c^3+2\,abc^4)x^{15} \\
&\quad+\frac{5}{2592}\,(9\,b^4c^2+48\,ab^2c^3+16\,a^2c^4)x^{14}+\frac{1}{864}\,(9\,b^5c+120\,ab^3c^2+160\,a^2bc^3)x^{13} \\
&\quad+\frac{1}{2}\,a^5bx^7+\frac{1}{10368}\,(27\,b^6+1080\,ab^4c+4320\,a^2b^2c^2+1280\,a^3c^3)x^{12}+\frac{1}{6}\,a^6x^6 \\
&\quad+\frac{1}{288}\,(9\,ab^5+120\,a^2b^3c+160\,a^3bc^2)x^{11}+\frac{5}{288}\,(9\,a^2b^4+48\,a^3b^2c+16\,a^4c^2)x^{10} \\
&\quad+\frac{5}{12}\,(a^3b^3+2\,a^4bc)x^9+\frac{1}{24}\,(15\,a^4b^2+8\,a^5c)x^8+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax
\end{aligned}
$$

input
```
integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fric
as")
```

output
```
1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 +
5/324*(b^3*c^3 + 2*a*b*c^4)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^2*c^3 + 16*a
^2*c^4)*x^14 + 1/864*(9*b^5*c + 120*a*b^3*c^2 + 160*a^2*b*c^3)*x^13 + 1/2*
a^5*b*x^7 + 1/10368*(27*b^6 + 1080*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c
^3)*x^12 + 1/6*a^6*x^6 + 1/288*(9*a*b^5 + 120*a^2*b^3*c + 160*a^3*b*c^2)*x
^11 + 5/288*(9*a^2*b^4 + 48*a^3*b^2*c + 16*a^4*c^2)*x^10 + 5/12*(a^3*b^3 +
 2*a^4*b*c)*x^9 + 1/24*(15*a^4*b^2 + 8*a^5*c)*x^8 + 1/3*c*x^3 + 1/2*b*x^2
+ a*x
```

3.214.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(36) = 72.

Time = 0.07 (sec) , antiderivative size = 323, normalized size of antiderivative = 7.02

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$$
$$=\frac{a^6x^6}{6}+\frac{a^5bx^7}{2}+ax+\frac{bc^5x^{17}}{486}+\frac{bx^2}{2}+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}+x^{16}\left(\frac{ac^5}{243}+\frac{5b^2c^4}{648}\right)+x^{15}$$
$$\cdot\left(\frac{5abc^4}{162}+\frac{5b^3c^3}{324}\right)+x^{14}\cdot\left(\frac{5a^2c^4}{162}+\frac{5ab^2c^3}{54}+\frac{5b^4c^2}{288}\right)+x^{13}\cdot\left(\frac{5a^2bc^3}{27}+\frac{5ab^3c^2}{36}+\frac{b^5c}{96}\right)$$
$$+x^{12}\cdot\left(\frac{10a^3c^3}{81}+\frac{5a^2b^2c^2}{12}+\frac{5ab^4c}{48}+\frac{b^6}{384}\right)+x^{11}\cdot\left(\frac{5a^3bc^2}{9}+\frac{5a^2b^3c}{12}+\frac{ab^5}{32}\right)$$
$$+x^{10}\cdot\left(\frac{5a^4c^2}{18}+\frac{5a^3b^2c}{6}+\frac{5a^2b^4}{32}\right)+x^9\cdot\left(\frac{5a^4bc}{6}+\frac{5a^3b^3}{12}\right)+x^8\left(\frac{a^5c}{3}+\frac{5a^4b^2}{8}\right)$$

input
```
integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

output
```
a**6*x**6/6 + a**5*b*x**7/2 + a*x + b*c**5*x**17/486 + b*x**2/2 + c**6*x**
18/4374 + c*x**3/3 + x**16*(a*c**5/243 + 5*b**2*c**4/648) + x**15*(5*a*b*c
**4/162 + 5*b**3*c**3/324) + x**14*(5*a**2*c**4/162 + 5*a*b**2*c**3/54 + 5
*b**4*c**2/288) + x**13*(5*a**2*b*c**3/27 + 5*a*b**3*c**2/36 + b**5*c/96)
+ x**12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + b**6/384)
 + x**11*(5*a**3*b*c**2/9 + 5*a**2*b**3*c/12 + a*b**5/32) + x**10*(5*a**4*
c**2/18 + 5*a**3*b**2*c/6 + 5*a**2*b**4/32) + x**9*(5*a**4*b*c/6 + 5*a**3*
b**3/12) + x**8*(a**5*c/3 + 5*a**4*b**2/8)
```

3.214.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(37) = 74.

Time = 0.19 (sec) , antiderivative size = 289, normalized size of antiderivative = 6.28

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$$
$$=\frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{1}{1944}\left(15\,b^2c^4+8\,ac^5\right)x^{16}+\frac{5}{324}\left(b^3c^3+2\,abc^4\right)x^{15}$$
$$+\frac{5}{2592}\left(9\,b^4c^2+48\,ab^2c^3+16\,a^2c^4\right)x^{14}+\frac{1}{864}\left(9\,b^5c+120\,ab^3c^2+160\,a^2bc^3\right)x^{13}$$
$$+\frac{1}{2}a^5bx^7+\frac{1}{10368}\left(27\,b^6+1080\,ab^4c+4320\,a^2b^2c^2+1280\,a^3c^3\right)x^{12}+\frac{1}{6}a^6x^6$$
$$+\frac{1}{288}\left(9\,ab^5+120\,a^2b^3c+160\,a^3bc^2\right)x^{11}+\frac{5}{288}\left(9\,a^2b^4+48\,a^3b^2c+16\,a^4c^2\right)x^{10}$$
$$+\frac{5}{12}\left(a^3b^3+2\,a^4bc\right)x^9+\frac{1}{24}\left(15\,a^4b^2+8\,a^5c\right)x^8+\frac{1}{3}cx^3+\frac{1}{2}bx^2+ax$$

input
```
integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxi
ma")
```

output
```
1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 +
5/324*(b^3*c^3 + 2*a*b*c^4)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^2*c^3 + 16*a
^2*c^4)*x^14 + 1/864*(9*b^5*c + 120*a*b^3*c^2 + 160*a^2*b*c^3)*x^13 + 1/2*
a^5*b*x^7 + 1/10368*(27*b^6 + 1080*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c
^3)*x^12 + 1/6*a^6*x^6 + 1/288*(9*a*b^5 + 120*a^2*b^3*c + 160*a^3*b*c^2)*x
^11 + 5/288*(9*a^2*b^4 + 48*a^3*b^2*c + 16*a^4*c^2)*x^10 + 5/12*(a^3*b^3 +
 2*a^4*b*c)*x^9 + 1/24*(15*a^4*b^2 + 8*a^5*c)*x^8 + 1/3*c*x^3 + 1/2*b*x^2
+ a*x
```

3.214.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$=\frac{1}{279936}\left(2\,cx^3+3\,bx^2+6\,ax\right)^6+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

input
```
integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac
")
```

output
```
1/279936*(2*c*x^3 + 3*b*x^2 + 6*a*x)^6 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

3.214.9 Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 270, normalized size of antiderivative = 5.87

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$$
$$= x^{12}\left(\frac{10\,a^3\,c^3}{81}+\frac{5\,a^2\,b^2\,c^2}{12}+\frac{5\,a\,b^4\,c}{48}+\frac{b^6}{384}\right)+a\,x+\frac{b\,x^2}{2}+\frac{c\,x^3}{3}+\frac{a^6\,x^6}{6}+\frac{c^6\,x^{18}}{4374}+\frac{5\,a^2\,x^{10}\,(16\,a^2\,c^2+48\,a\,b^2\,c+9\,b^4)}{288}+\frac{5\,c^2\,x^{14}\,(16\,a^2\,c^2+48\,a\,b^2\,c+9\,b^4)}{2592}+\frac{a^5\,b\,x^7}{2}+\frac{b\,c^5\,x^{17}}{486}+\frac{a^4\,x^8\,(15\,b^2+8\,a\,c)}{24}+\frac{c^4\,x^{16}\,(15\,b^2+8\,a\,c)}{1944}+\frac{a\,b\,x^{11}\,(160\,a^2\,c^2+120\,a\,b^2\,c+9\,b^4)}{288}+\frac{b\,c\,x^{13}\,(160\,a^2\,c^2+120\,a\,b^2\,c+9\,b^4)}{864}+\frac{5\,a^3\,b\,x^9\,(b^2+2\,a\,c)}{12}+\frac{5\,b\,c^3\,x^{15}\,(b^2+2\,a\,c)}{324}$$

input
```
int(((a*x + (b*x^2)/2 + (c*x^3)/3)^5 + 1)*(a + b*x + c*x^2),x)
```

output
```
x^12*(b^6/384 + (10*a^3*c^3)/81 + (5*a^2*b^2*c^2)/12 + (5*a*b^4*c)/48) + a
*x + (b*x^2)/2 + (c*x^3)/3 + (a^6*x^6)/6 + (c^6*x^18)/4374 + (5*a^2*x^10*(
9*b^4 + 16*a^2*c^2 + 48*a*b^2*c))/288 + (5*c^2*x^14*(9*b^4 + 16*a^2*c^2 +
48*a*b^2*c))/2592 + (a^5*b*x^7)/2 + (b*c^5*x^17)/486 + (a^4*x^8*(8*a*c + 1
5*b^2))/24 + (c^4*x^16*(8*a*c + 15*b^2))/1944 + (a*b*x^11*(9*b^4 + 160*a^2
*c^2 + 120*a*b^2*c))/288 + (b*c*x^13*(9*b^4 + 160*a^2*c^2 + 120*a*b^2*c))/
864 + (5*a^3*b*x^9*(2*a*c + b^2))/12 + (5*b*c^3*x^15*(2*a*c + b^2))/324
```

3.215 $\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

3.215.1 Optimal result

Integrand size = 36, antiderivative size = 47

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6$$

output
```
a*x+1/2*b*x^2+1/3*c*x^3+1/6*(d+a*x+1/2*b*x^2+1/3*c*x^3)^6
```

3.215.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 248 vs. 2(47) = 94.

Time = 0.08 (sec) , antiderivative size = 248, normalized size of antiderivative = 5.28

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx = \frac{x(6a+x(3b+2cx))\,(46656+46656d^5+7776a^5x^5+243b^5x^{10}+810b^4cx^{11}+1080b^3c^2x^{12}+720b^2c^3x^{13}+\cdots}{}$$

input
```
Integrate[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

output
```
(x*(6*a + x*(3*b + 2*c*x))*(46656 + 46656*d^5 + 7776*a^5*x^5 + 243*b^5*x^1
0 + 810*b^4*c*x^11 + 1080*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14
 + 32*c^5*x^15 + 6480*a^4*x^6*(3*b + 2*c*x) + 2160*a^3*x^7*(3*b + 2*c*x)^2
 + 360*a^2*x^8*(3*b + 2*c*x)^3 + 30*a*x^9*(3*b + 2*c*x)^4 + 19440*d^4*x*(6
*a + x*(3*b + 2*c*x)) + 4320*d^3*x^2*(6*a + x*(3*b + 2*c*x))^2 + 540*d^2*x
^3*(6*a + x*(3*b + 2*c*x))^3 + 36*d*x^4*(6*a + x*(3*b + 2*c*x))^4))/279936
```

3.215.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx+cx^2)\left(\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^5+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^5+1\right)d\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d$$

input
```
Int[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

output
```
d + a*x + (b*x^2)/2 + (c*x^3)/3 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^6/6
```

3.215.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.215.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

method	result
default	$\frac{\left(d+ax+\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3\right)^6}{6}+d+ax+\frac{b\,x^2}{2}+\frac{c\,x^3}{3}$
norman	$\left(\frac{1}{243}a\,c^5+\frac{5}{648}b^2c^4\right)x^{16}+\left(\frac{5}{2}a^2d^4+\frac{1}{2}b\,d^5+\frac{1}{2}b\right)x^2+\left(\frac{5}{162}ab\,c^4+\frac{5}{324}b^3c^3+\frac{1}{243}c^5d\right)x^{15}+\left(\frac{5}{162}\right.$
gosper	$\frac{x\left(64c^6x^{17}+576b\,c^5x^{16}+1152a\,c^5x^{15}+2160b^2c^4x^{15}+8640ab\,c^4x^{14}+4320b^3c^3x^{14}+1152c^5d\,x^{14}+8640a^2c^4x^{13}+25920a\,b^2c^3x^{13}\right.}{}$
risch	$\frac{5}{162}x^{14}b\,c^4d+\frac{5}{54}x^{13}b^2c^3d+\frac{5}{2}a^2d^4x^2+\frac{5}{12}x^6b^3d^3+\frac{1}{2}b\,d^5x^2+\frac{10}{3}x^6a^3c\,d^2+\frac{10}{3}x^5a^2c\,d^3+\frac{5}{3}x^4a$
parallelrisch	$\frac{5}{162}x^{14}b\,c^4d+\frac{5}{54}x^{13}b^2c^3d+\frac{5}{2}a^2d^4x^2+\frac{5}{12}x^6b^3d^3+\frac{1}{2}b\,d^5x^2+\frac{10}{3}x^6a^3c\,d^2+\frac{10}{3}x^5a^2c\,d^3+\frac{5}{3}x^4a$

input
```
int((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x,method=_RETURNVERBOS
E)
```

output
```
1/6*(d+a*x+1/2*b*x^2+1/3*c*x^3)^6+d+a*x+1/2*b*x^2+1/3*c*x^3
```

3.215.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(40) = 80$.

Time = 0.25 (sec) , antiderivative size = 773, normalized size of antiderivative = 16.45

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$\begin{aligned}
&= \frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{1}{1944}(15b^2c^4+8ac^5)x^{16} \\
&+\frac{1}{972}(15b^3c^3+30abc^4+4c^5d)x^{15}+\frac{5}{2592}(9b^4c^2+48ab^2c^3+16a^2c^4+16bc^4d)x^{14} \\
&+\frac{1}{2592}(27b^5c+360ab^3c^2+480a^2bc^3+80(3b^2c^3+2ac^4)d)x^{13} \\
&+\frac{1}{10368}(27b^6+1080ab^4c+4320a^2b^2c^2+1280a^3c^3+320c^4d^2+480(3b^3c^2+8abc^3)d)x^{12} \\
&+\frac{1}{864}(27ab^5+360a^2b^3c+480a^3bc^2+160bc^3d^2+10(9b^4c+72ab^2c^2+32a^2c^3)d)x^{11} \\
&+\frac{1}{864}(135a^2b^4+720a^3b^2c+240a^4c^2+40(9b^2c^2+8ac^3)d^2+9(3b^5+80ab^3c+160a^2bc^2)d)x^{10} \\
&+\frac{5}{1296}(108a^3b^3+216a^4bc+32c^3d^3+108(b^3c+4abc^2)d^2+9(9ab^4+72a^2b^2c+32a^3c^2)d)x^9 \\
&+\frac{1}{288}(180a^4b^2+96a^5c+160bc^2d^3+15(3b^4+48ab^2c+32a^2c^2)d^2+120(3a^2b^3+8a^3bc)d)x^8 \\
&+\frac{1}{36}(18a^5b+10(3b^2c+4ac^2)d^3+45(ab^3+4a^2bc)d^2+30(3a^3b^2+2a^4c)d)x^7 \\
&+\frac{1}{36}(6a^6+90a^4bd+10c^2d^4+15(b^3+8abc)d^3+15(9a^2b^2+8a^3c)d^2)x^6 \\
&+\frac{1}{6}(6a^5d+30a^3bd^2+5bcd^4+5(3ab^2+4a^2c)d^3)x^5 \\
&+\frac{5}{24}(12a^4d^2+24a^2bd^3+(3b^2+8ac)d^4)x^4 \\
&+\frac{1}{6}(20a^3d^3+15abd^4+2cd^5+2c)x^3+\frac{1}{2}(5a^2d^4+bd^5+b)x^2+(ad^5+a)x
\end{aligned}$$

input
```
integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fr
icas")
```

output
```
1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 +
1/972*(15*b^3*c^3 + 30*a*b*c^4 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*
b^2*c^3 + 16*a^2*c^4 + 16*b*c^4*d)*x^14 + 1/2592*(27*b^5*c + 360*a*b^3*c^2
 + 480*a^2*b*c^3 + 80*(3*b^2*c^3 + 2*a*c^4)*d)*x^13 + 1/10368*(27*b^6 + 10
80*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3 + 320*c^4*d^2 + 480*(3*b^3*c^
2 + 8*a*b*c^3)*d)*x^12 + 1/864*(27*a*b^5 + 360*a^2*b^3*c + 480*a^3*b*c^2 +
 160*b*c^3*d^2 + 10*(9*b^4*c + 72*a*b^2*c^2 + 32*a^2*c^3)*d)*x^11 + 1/864*
(135*a^2*b^4 + 720*a^3*b^2*c + 240*a^4*c^2 + 40*(9*b^2*c^2 + 8*a*c^3)*d^2
+ 9*(3*b^5 + 80*a*b^3*c + 160*a^2*b*c^2)*d)*x^10 + 5/1296*(108*a^3*b^3 + 2
16*a^4*b*c + 32*c^3*d^3 + 108*(b^3*c + 4*a*b*c^2)*d^2 + 9*(9*a*b^4 + 72*a^
2*b^2*c + 32*a^3*c^2)*d)*x^9 + 1/288*(180*a^4*b^2 + 96*a^5*c + 160*b*c^2*d
^3 + 15*(3*b^4 + 48*a*b^2*c + 32*a^2*c^2)*d^2 + 120*(3*a^2*b^3 + 8*a^3*b*c
)*d)*x^8 + 1/36*(18*a^5*b + 10*(3*b^2*c + 4*a*c^2)*d^3 + 45*(a*b^3 + 4*a^2
*b*c)*d^2 + 30*(3*a^3*b^2 + 2*a^4*c)*d)*x^7 + 1/36*(6*a^6 + 90*a^4*b*d + 1
0*c^2*d^4 + 15*(b^3 + 8*a*b*c)*d^3 + 15*(9*a^2*b^2 + 8*a^3*c)*d^2)*x^6 + 1
/6*(6*a^5*d + 30*a^3*b*d^2 + 5*b*c*d^4 + 5*(3*a*b^2 + 4*a^2*c)*d^3)*x^5 +
5/24*(12*a^4*d^2 + 24*a^2*b*d^3 + (3*b^2 + 8*a*c)*d^4)*x^4 + 1/6*(20*a^3*d
^3 + 15*a*b*d^4 + 2*c*d^5 + 2*c)*x^3 + 1/2*(5*a^2*d^4 + b*d^5 + b)*x^2 + (
a*d^5 + a)*x
```

3.215.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 930 vs. $2(37) = 74$.

3.215. $\int (a + bx + cx^2) \left(1 + \left(d + ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx$

Time = 0.12 (sec) , antiderivative size = 930, normalized size of antiderivative = 19.79

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$$
$$= \frac{bc^5x^{17}}{486}+\frac{c^6x^{18}}{4374}+x^{16}\left(\frac{ac^5}{243}+\frac{5b^2c^4}{648}\right)+x^{15}\cdot\left(\frac{5abc^4}{162}+\frac{5b^3c^3}{324}+\frac{c^5d}{243}\right)+x^{14}$$
$$\cdot\left(\frac{5a^2c^4}{162}+\frac{5ab^2c^3}{54}+\frac{5b^4c^2}{288}+\frac{5bc^4d}{162}\right)+x^{13}\cdot\left(\frac{5a^2bc^3}{27}+\frac{5ab^3c^2}{36}+\frac{5ac^4d}{81}+\frac{b^5c}{96}+\frac{5b^2c^3d}{54}\right)$$
$$+x^{12}\cdot\left(\frac{10a^3c^3}{81}+\frac{5a^2b^2c^2}{12}+\frac{5ab^4c}{48}+\frac{10abc^3d}{27}+\frac{b^6}{384}+\frac{5b^3c^2d}{36}+\frac{5c^4d^2}{162}\right)+x^{11}$$
$$\cdot\left(\frac{5a^3bc^2}{9}+\frac{5a^2b^3c}{12}+\frac{10a^2c^3d}{27}+\frac{ab^5}{32}+\frac{5ab^2c^2d}{6}+\frac{5b^4cd}{48}+\frac{5bc^3d^2}{27}\right)+x^{10}$$
$$\cdot\left(\frac{5a^4c^2}{18}+\frac{5a^3b^2c}{6}+\frac{5a^2b^4}{32}+\frac{5a^2bc^2d}{3}+\frac{5ab^3cd}{6}+\frac{10ac^3d^2}{27}+\frac{b^5d}{32}+\frac{5b^2c^2d^2}{12}\right)+x^9$$
$$\cdot\left(\frac{5a^4bc}{6}+\frac{5a^3b^3}{12}+\frac{10a^3c^2d}{9}+\frac{5a^2b^2cd}{2}+\frac{5ab^4d}{16}+\frac{5abc^2d^2}{3}+\frac{5b^3cd^2}{12}+\frac{10c^3d^3}{81}\right)$$
$$+x^8\left(\frac{a^5c}{3}+\frac{5a^4b^2}{8}+\frac{10a^3bcd}{3}+\frac{5a^2b^3d}{4}+\frac{5a^2c^2d^2}{3}+\frac{5ab^2cd^2}{2}+\frac{5b^4d^2}{32}+\frac{5bc^2d^3}{9}\right)$$
$$+x^7\left(\frac{a^5b}{2}+\frac{5a^4cd}{3}+\frac{5a^3b^2d}{2}+5a^2bcd^2+\frac{5ab^3d^2}{4}+\frac{10ac^2d^3}{9}+\frac{5b^2cd^3}{6}\right)$$
$$+x^6\left(\frac{a^6}{6}+\frac{5a^4bd}{2}+\frac{10a^3cd^2}{3}+\frac{15a^2b^2d^2}{4}+\frac{10abcd^3}{3}+\frac{5b^3d^3}{12}+\frac{5c^2d^4}{18}\right)$$
$$+x^5\left(a^5d+5a^3bd^2+\frac{10a^2cd^3}{3}+\frac{5ab^2d^3}{2}+\frac{5bcd^4}{6}\right)+x^4\cdot\left(\frac{5a^4d^2}{2}+5a^2bd^3+\frac{5acd^4}{3}+\frac{5b^2d^4}{8}\right)$$
$$+x^3\cdot\left(\frac{10a^3d^3}{3}+\frac{5abd^4}{2}+\frac{cd^5}{3}+\frac{c}{3}\right)+x^2\cdot\left(\frac{5a^2d^4}{2}+\frac{bd^5}{2}+\frac{b}{2}\right)+x\left(ad^5+a\right)$$

input
```
integrate((c*x**2+b*x+a)*(1+(d+a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

3.215. $\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

output
```
b*c**5*x**17/486 + c**6*x**18/4374 + x**16*(a*c**5/243 + 5*b**2*c**4/648)
+ x**15*(5*a*b*c**4/162 + 5*b**3*c**3/324 + c**5*d/243) + x**14*(5*a**2*c*
*4/162 + 5*a*b**2*c**3/54 + 5*b**4*c**2/288 + 5*b*c**4*d/162) + x**13*(5*a
**2*b*c**3/27 + 5*a*b**3*c**2/36 + 5*a*c**4*d/81 + b**5*c/96 + 5*b**2*c**3
*d/54) + x**12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + 10
*a*b*c**3*d/27 + b**6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/162) + x**11*(5
*a**3*b*c**2/9 + 5*a**2*b**3*c/12 + 10*a**2*c**3*d/27 + a*b**5/32 + 5*a*b*
*2*c**2*d/6 + 5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(5*a**4*c**2/18 +
5*a**3*b**2*c/6 + 5*a**2*b**4/32 + 5*a**2*b*c**2*d/3 + 5*a*b**3*c*d/6 + 10
*a*c**3*d**2/27 + b**5*d/32 + 5*b**2*c**2*d**2/12) + x**9*(5*a**4*b*c/6 +
5*a**3*b**3/12 + 10*a**3*c**2*d/9 + 5*a**2*b**2*c*d/2 + 5*a*b**4*d/16 + 5*
a*b*c**2*d**2/3 + 5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5
*a**4*b**2/8 + 10*a**3*b*c*d/3 + 5*a**2*b**3*d/4 + 5*a**2*c**2*d**2/3 + 5*
a*b**2*c*d**2/2 + 5*b**4*d**2/32 + 5*b*c**2*d**3/9) + x**7*(a**5*b/2 + 5*a
**4*c*d/3 + 5*a**3*b**2*d/2 + 5*a**2*b*c*d**2 + 5*a*b**3*d**2/4 + 10*a*c**
2*d**3/9 + 5*b**2*c*d**3/6) + x**6*(a**6/6 + 5*a**4*b*d/2 + 10*a**3*c*d**2
/3 + 15*a**2*b**2*d**2/4 + 10*a*b*c*d**3/3 + 5*b**3*d**3/12 + 5*c**2*d**4/
18) + x**5*(a**5*d + 5*a**3*b*d**2 + 10*a**2*c*d**3/3 + 5*a*b**2*d**3/2 +
5*b*c*d**4/6) + x**4*(5*a**4*d**2/2 + 5*a**2*b*d**3 + 5*a*c*d**4/3 + 5*b**
2*d**4/8) + x**3*(10*a**3*d**3/3 + 5*a*b*d**4/2 + c*d**5/3 + c/3) + x**...
```

3.215.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(40) = 80$.

3.215. $\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Time = 0.20 (sec) , antiderivative size = 773, normalized size of antiderivative = 16.45

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$\begin{aligned}
&= \frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{1}{1944}\left(15\,b^2c^4+8\,ac^5\right)x^{16} \\
&+\frac{1}{972}\left(15\,b^3c^3+30\,abc^4+4\,c^5d\right)x^{15}+\frac{5}{2592}\left(9\,b^4c^2+48\,ab^2c^3+16\,a^2c^4+16\,bc^4d\right)x^{14} \\
&+\frac{1}{2592}\left(27\,b^5c+360\,ab^3c^2+480\,a^2bc^3+80\left(3\,b^2c^3+2\,ac^4\right)d\right)x^{13} \\
&+\frac{1}{10368}\left(27\,b^6+1080\,ab^4c+4320\,a^2b^2c^2+1280\,a^3c^3+320\,c^4d^2+480\left(3\,b^3c^2+8\,abc^3\right)d\right)x^{12} \\
&+\frac{1}{864}\left(27\,ab^5+360\,a^2b^3c+480\,a^3bc^2+160\,bc^3d^2+10\left(9\,b^4c+72\,ab^2c^2+32\,a^2c^3\right)d\right)x^{11} \\
&+\frac{1}{864}\left(135\,a^2b^4+720\,a^3b^2c+240\,a^4c^2+40\left(9\,b^2c^2+8\,ac^3\right)d^2+9\left(3\,b^5+80\,ab^3c+160\,a^2bc^2\right)d\right)x^{10} \\
&+\frac{5}{1296}\left(108\,a^3b^3+216\,a^4bc+32\,c^3d^3+108\left(b^3c+4\,abc^2\right)d^2+9\left(9\,ab^4+72\,a^2b^2c+32\,a^3c^2\right)d\right)x^9 \\
&+\frac{1}{288}\left(180\,a^4b^2+96\,a^5c+160\,bc^2d^3+15\left(3\,b^4+48\,ab^2c+32\,a^2c^2\right)d^2+120\left(3\,a^2b^3+8\,a^3bc\right)d\right)x^8 \\
&+\frac{1}{36}\left(18\,a^5b+10\left(3\,b^2c+4\,ac^2\right)d^3+45\left(ab^3+4\,a^2bc\right)d^2+30\left(3\,a^3b^2+2\,a^4c\right)d\right)x^7 \\
&+\frac{1}{36}\left(6\,a^6+90\,a^4bd+10\,c^2d^4+15\left(b^3+8\,abc\right)d^3+15\left(9\,a^2b^2+8\,a^3c\right)d^2\right)x^6 \\
&+\frac{1}{6}\left(6\,a^5d+30\,a^3bd^2+5\,bcd^4+5\left(3\,ab^2+4\,a^2c\right)d^3\right)x^5 \\
&+\frac{5}{24}\left(12\,a^4d^2+24\,a^2bd^3+\left(3\,b^2+8\,ac\right)d^4\right)x^4 \\
&+\frac{1}{6}\left(20\,a^3d^3+15\,abd^4+2\,cd^5+2\,c\right)x^3+\frac{1}{2}\left(5\,a^2d^4+bd^5+b\right)x^2+\left(ad^5+a\right)x
\end{aligned}$$

input
```
integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="ma
xima")
```

3.215. $\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$

output
```
1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 +
1/972*(15*b^3*c^3 + 30*a*b*c^4 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*
b^2*c^3 + 16*a^2*c^4 + 16*b*c^4*d)*x^14 + 1/2592*(27*b^5*c + 360*a*b^3*c^2
 + 480*a^2*b*c^3 + 80*(3*b^2*c^3 + 2*a*c^4)*d)*x^13 + 1/10368*(27*b^6 + 10
80*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3 + 320*c^4*d^2 + 480*(3*b^3*c^
2 + 8*a*b*c^3)*d)*x^12 + 1/864*(27*a*b^5 + 360*a^2*b^3*c + 480*a^3*b*c^2 +
 160*b*c^3*d^2 + 10*(9*b^4*c + 72*a*b^2*c^2 + 32*a^2*c^3)*d)*x^11 + 1/864*
(135*a^2*b^4 + 720*a^3*b^2*c + 240*a^4*c^2 + 40*(9*b^2*c^2 + 8*a*c^3)*d^2
+ 9*(3*b^5 + 80*a*b^3*c + 160*a^2*b*c^2)*d)*x^10 + 5/1296*(108*a^3*b^3 + 2
16*a^4*b*c + 32*c^3*d^3 + 108*(b^3*c + 4*a*b*c^2)*d^2 + 9*(9*a*b^4 + 72*a^
2*b^2*c + 32*a^3*c^2)*d)*x^9 + 1/288*(180*a^4*b^2 + 96*a^5*c + 160*b*c^2*d
^3 + 15*(3*b^4 + 48*a*b^2*c + 32*a^2*c^2)*d^2 + 120*(3*a^2*b^3 + 8*a^3*b*c
)*d)*x^8 + 1/36*(18*a^5*b + 10*(3*b^2*c + 4*a*c^2)*d^3 + 45*(a*b^3 + 4*a^2
*b*c)*d^2 + 30*(3*a^3*b^2 + 2*a^4*c)*d)*x^7 + 1/36*(6*a^6 + 90*a^4*b*d + 1
0*c^2*d^4 + 15*(b^3 + 8*a*b*c)*d^3 + 15*(9*a^2*b^2 + 8*a^3*c)*d^2)*x^6 + 1
/6*(6*a^5*d + 30*a^3*b*d^2 + 5*b*c*d^4 + 5*(3*a*b^2 + 4*a^2*c)*d^3)*x^5 +
5/24*(12*a^4*d^2 + 24*a^2*b*d^3 + (3*b^2 + 8*a*c)*d^4)*x^4 + 1/6*(20*a^3*d
^3 + 15*a*b*d^4 + 2*c*d^5 + 2*c)*x^3 + 1/2*(5*a^2*d^4 + b*d^5 + b)*x^2 + (
a*d^5 + a)*x
```

3.215.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(40) = 80$.

Time = 0.44 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.26

$$\int (a + bx + cx^2)\left(1 + \left(d + ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx$$
$$= \frac{1}{279936}\left(2\,cx^3 + 3\,bx^2 + 6\,ax\right)^6 + \frac{1}{7776}\left(2\,cx^3 + 3\,bx^2 + 6\,ax\right)^5 d + \frac{5}{2592}\left(2\,cx^3 + 3\,bx^2 + 6\,ax\right)^4 d^2 + \frac{5}{324}\left(2\,cx^3 + 3\,bx^2 + 6\,ax\right)^3 d^3 + \frac{5}{72}\left(2\,cx^3 + 3\,bx^2 + 6\,ax\right)^2 d^4 + \frac{1}{6}\left(2\,cx^3 + 3\,bx^2 + 6\,ax\right) d^5 + \frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2 + ax$$

input
```
integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="gi
ac")
```

output
```
1/279936*(2*c*x^3 + 3*b*x^2 + 6*a*x)^6 + 1/7776*(2*c*x^3 + 3*b*x^2 + 6*a*x
)^5*d + 5/2592*(2*c*x^3 + 3*b*x^2 + 6*a*x)^4*d^2 + 5/324*(2*c*x^3 + 3*b*x^
2 + 6*a*x)^3*d^3 + 5/72*(2*c*x^3 + 3*b*x^2 + 6*a*x)^2*d^4 + 1/6*(2*c*x^3 +
 3*b*x^2 + 6*a*x)*d^5 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

3.215.9 Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 753, normalized size of antiderivative = 16.02

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$= x^{10}\left(\frac{5a^4c^2}{18}+\frac{5a^3b^2c}{6}+\frac{5a^2b^4}{32}+\frac{5a^2bc^2d}{3}+\frac{5ab^3cd}{6}+\frac{10ac^3d^2}{27}+\frac{b^5d}{32}+\frac{5b^2c^2d^2}{12}\right)$$
$$+x^8\left(\frac{a^5c}{3}+\frac{5a^4b^2}{8}+\frac{10a^3bcd}{3}+\frac{5a^2b^3d}{4}+\frac{5a^2c^2d^2}{3}+\frac{5ab^2cd^2}{2}+\frac{5b^4d^2}{32}+\frac{5bc^2d^3}{9}\right)$$
$$+x^9\left(\frac{5a^4bc}{6}+\frac{5a^3b^3}{12}+\frac{10a^3c^2d}{9}+\frac{5a^2b^2cd}{2}+\frac{5ab^4d}{16}+\frac{5abc^2d^2}{3}+\frac{5b^3cd^2}{12}+\frac{10c^3d^3}{81}\right)$$
$$+x^{14}\left(\frac{5a^2c^4}{162}+\frac{5ab^2c^3}{54}+\frac{5b^4c^2}{288}+\frac{5dbc^4}{162}\right)$$
$$+x^{12}\left(\frac{10a^3c^3}{81}+\frac{5a^2b^2c^2}{12}+\frac{5ab^4c}{48}+\frac{10abc^3d}{27}+\frac{b^6}{384}+\frac{5b^3c^2d}{36}+\frac{5c^4d^2}{162}\right)$$
$$+x^6\left(\frac{a^6}{6}+\frac{5a^4bd}{2}+\frac{10a^3cd^2}{3}+\frac{15a^2b^2d^2}{4}+\frac{10abcd^3}{3}+\frac{5b^3d^3}{12}+\frac{5c^2d^4}{18}\right)$$
$$+x^3\left(\frac{10a^3d^3}{3}+\frac{5bad^4}{2}+\frac{cd^5}{3}+\frac{c}{3}\right)$$
$$+x^{11}\left(\frac{5a^3bc^2}{9}+\frac{5a^2b^3c}{12}+\frac{10a^2c^3d}{27}+\frac{ab^5}{32}+\frac{5ab^2c^2d}{6}+\frac{5b^4cd}{48}+\frac{5bc^3d^2}{27}\right)$$
$$+x^7\left(\frac{a^5b}{2}+\frac{5a^4cd}{3}+\frac{5a^3b^2d}{2}+5a^2bcd^2+\frac{5ab^3d^2}{4}+\frac{10ac^2d^3}{9}+\frac{5b^2cd^3}{6}\right)$$
$$+x^2\left(\frac{5a^2d^4}{2}+\frac{bd^5}{2}+\frac{b}{2}\right)+x^{13}\left(\frac{5a^2bc^3}{27}+\frac{5ab^3c^2}{36}+\frac{5dac^4}{81}+\frac{b^5c}{96}+\frac{5db^2c^3}{54}\right)$$
$$+x^5\left(a^5d+5a^3bd^2+\frac{10ca^2d^3}{3}+\frac{5ab^2d^3}{2}+\frac{5cbd^4}{6}\right)+\frac{c^6x^{18}}{4374}$$
$$+\frac{5d^2x^4(12a^4+24a^2bd+8cad^2+3b^2d^2)}{24}+ax(d^5+1)$$
$$+\frac{bc^5x^{17}}{486}+\frac{c^3x^{15}(15b^3+30abc+4dc^2)}{972}+\frac{c^4x^{16}(15b^2+8ac)}{1944}$$

input
```
int(((d + a*x + (b*x^2)/2 + (c*x^3)/3)^5 + 1)*(a + b*x + c*x^2),x)
```

output
```
x^10*((b^5*d)/32 + (5*a^2*b^4)/32 + (5*a^4*c^2)/18 + (5*a^3*b^2*c)/6 + (10
*a*c^3*d^2)/27 + (5*b^2*c^2*d^2)/12 + (5*a*b^3*c*d)/6 + (5*a^2*b*c^2*d)/3)
 + x^8*((a^5*c)/3 + (5*a^4*b^2)/8 + (5*b^4*d^2)/32 + (5*a^2*b^3*d)/4 + (5*
b*c^2*d^3)/9 + (5*a^2*c^2*d^2)/3 + (10*a^3*b*c*d)/3 + (5*a*b^2*c*d^2)/2) +
 x^9*((5*a^3*b^3)/12 + (10*c^3*d^3)/81 + (10*a^3*c^2*d)/9 + (5*b^3*c*d^2)/
12 + (5*a^4*b*c)/6 + (5*a*b^4*d)/16 + (5*a*b*c^2*d^2)/3 + (5*a^2*b^2*c*d)/
2) + x^14*((5*a^2*c^4)/162 + (5*b^4*c^2)/288 + (5*a*b^2*c^3)/54 + (5*b*c^4
*d)/162) + x^12*(b^6/384 + (10*a^3*c^3)/81 + (5*c^4*d^2)/162 + (5*b^3*c^2*
d)/36 + (5*a^2*b^2*c^2)/12 + (5*a*b^4*c)/48 + (10*a*b*c^3*d)/27) + x^6*(a^
6/6 + (5*b^3*d^3)/12 + (5*c^2*d^4)/18 + (10*a^3*c*d^2)/3 + (15*a^2*b^2*d^2
)/4 + (5*a^4*b*d)/2 + (10*a*b*c*d^3)/3) + x^3*(c/3 + (c*d^5)/3 + (10*a^3*d
^3)/3 + (5*a*b*d^4)/2) + x^11*((a*b^5)/32 + (5*a^2*b^3*c)/12 + (5*a^3*b*c^
2)/9 + (10*a^2*c^3*d)/27 + (5*b*c^3*d^2)/27 + (5*b^4*c*d)/48 + (5*a*b^2*c^
2*d)/6) + x^7*((a^5*b)/2 + (5*a*b^3*d^2)/4 + (5*a^3*b^2*d)/2 + (10*a*c^2*d
^3)/9 + (5*b^2*c*d^3)/6 + (5*a^4*c*d)/3 + 5*a^2*b*c*d^2) + x^2*(b/2 + (b*d
^5)/2 + (5*a^2*d^4)/2) + x^13*((b^5*c)/96 + (5*a*b^3*c^2)/36 + (5*a^2*b*c^
3)/27 + (5*b^2*c^3*d)/54 + (5*a*c^4*d)/81) + x^5*(a^5*d + (5*a*b^2*d^3)/2
+ 5*a^3*b*d^2 + (10*a^2*c*d^3)/3 + (5*b*c*d^4)/6) + (c^6*x^18)/4374 + (5*d
^2*x^4*(12*a^4 + 3*b^2*d^2 + 24*a^2*b*d + 8*a*c*d^2))/24 + a*x*(d^5 + 1) +
 (b*c^5*x^17)/486 + (c^3*x^15*(4*c^2*d + 15*b^3 + 30*a*b*c))/972 + (c^4...
```

3.215. $\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

3.216 $\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx$

3.216.1 Optimal result

Integrand size = 24, antiderivative size = 34

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx = ax+\frac{cx^3}{3}+\frac{\left(ax+\frac{cx^3}{3}\right)^{1+n}}{1+n}$$

output
```
a*x+1/3*c*x^3+(a*x+1/3*c*x^3)^(1+n)/(1+n)
```

3.216.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx = \frac{x(3a+cx^2)\left(1+n+\left(ax+\frac{cx^3}{3}\right)^n\right)}{3(1+n)}$$

input
```
Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]
```

output
```
(x*(3*a + c*x^2)*(1 + n + (a*x + (c*x^3)/3)^n))/(3*(1 + n))
```

3.216.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)\left(\left(ax+\frac{cx^3}{3}\right)^n+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(ax+\frac{cx^3}{3}\right)^n+1\right)d\left(ax+\frac{cx^3}{3}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(ax+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{cx^3}{3}$$

input
```
Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]
```

output
```
a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^(1 + n)/(1 + n)
```

3.216.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.216.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$ax+\frac{c\,x^{3}}{3}+\frac{\left(ax+\frac{1}{3}c\,x^{3}\right)^{1+n}}{1+n}$	31
default	$ax+\frac{c\,x^{3}}{3}+\frac{\left(ax+\frac{1}{3}c\,x^{3}\right)^{1+n}}{1+n}$	31
risch	$ax+\frac{c\,x^{3}}{3}+\frac{x\left(c\,x^{2}+3a\right)\left(\frac{1}{3}\right)^{n}\left(x\left(c\,x^{2}+3a\right)\right)^{n}}{3+3n}$	44
norman	$ax+\frac{ax\,\mathrm{e}^{n\ln\left(ax+\frac{1}{3}c\,x^{3}\right)}}{1+n}+\frac{c\,x^{3}}{3}+\frac{c\,x^{3}\mathrm{e}^{n\ln\left(ax+\frac{1}{3}c\,x^{3}\right)}}{3+3n}$	58
parallelrisch	$\frac{x^{3}\left(\frac{x\left(c\,x^{2}+3a\right)}{3}\right)^{n}c^{2}+x^{3}c^{2}n+c^{2}x^{3}+3x\left(\frac{x\left(c\,x^{2}+3a\right)}{3}\right)^{n}ac+3xacn+3acx}{3c(1+n)}$	78

input
```
int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x,method=_RETURNVERBOSE)
```

output
```
a*x+1/3*c*x^3+(a*x+1/3*c*x^3)^(1+n)/(1+n)
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx=\frac{(cn+c)x^3+(cx^3+3\,ax)\left(\frac{1}{3}\,cx^3+ax\right)^n+3\,(an+a)x}{3\,(n+1)}$$

input
```
integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="fricas")
```

output
```
1/3*((c*n + c)*x^3 + (c*x^3 + 3*a*x)*(1/3*c*x^3 + a*x)^n + 3*(a*n + a)*x)/
(n + 1)
```

3.216.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(26) = 52.

Time = 40.14 (sec) , antiderivative size = 190, normalized size of antiderivative = 5.59

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)dx$$
$$=\begin{cases}\frac{3\cdot 3^n anx}{3\cdot 3^n n+3\cdot 3^n}+\frac{3\cdot 3^n ax}{3\cdot 3^n n+3\cdot 3^n}+\frac{3^n cnx^3}{3\cdot 3^n n+3\cdot 3^n}+\frac{3^n cx^3}{3\cdot 3^n n+3\cdot 3^n}+\frac{3ax(3ax+cx^3)^n}{3\cdot 3^n n+3\cdot 3^n}+\frac{cx^3(3ax+cx^3)^n}{3\cdot 3^n n+3\cdot 3^n} & \text{for } n\neq -1\\ ax+\frac{cx^3}{3}+\log(x)+\log\left(x-\sqrt{3}\sqrt{-\frac{a}{c}}\right)+\log\left(x+\sqrt{3}\sqrt{-\frac{a}{c}}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**n),x)
```

output
```
Piecewise((3*3**n*a*n*x/(3*3**n*n + 3*3**n) + 3*3**n*a*x/(3*3**n*n + 3*3**
n) + 3**n*c*n*x**3/(3*3**n*n + 3*3**n) + 3**n*c*x**3/(3*3**n*n + 3*3**n) +
 3*a*x*(3*a*x + c*x**3)**n/(3*3**n*n + 3*3**n) + c*x**3*(3*a*x + c*x**3)**
n/(3*3**n*n + 3*3**n), Ne(n, -1)), (a*x + c*x**3/3 + log(x) + log(x - sqrt
(3)*sqrt(-a/c)) + log(x + sqrt(3)*sqrt(-a/c)), True))
```

3.216.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)dx=\frac{1}{3}cx^3+ax+\frac{(cx^3+3ax)e^{(n\log(cx^2+3a)+n\log(x))}}{3^{n+1}n+3^{n+1}}$$

input
```
integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="maxima")
```

output
```
1/3*c*x^3 + a*x + (c*x^3 + 3*a*x)*e^(n*log(c*x^2 + 3*a) + n*log(x))/(3^(n
+ 1)*n + 3^(n + 1))
```

3.216.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx=\frac{1}{3}\,cx^3+ax+\frac{\left(\frac{1}{3}\,cx^3+ax\right)^{n+1}}{n+1}$$

input
```
integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="giac")
```

output
```
1/3*c*x^3 + a*x + (1/3*c*x^3 + a*x)^(n + 1)/(n + 1)
```

3.216.9 Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx=\frac{x\,(c\,x^2+3\,a)\,\left(n+\left(\frac{c\,x^3}{3}+a\,x\right)^n+1\right)}{3\,(n+1)}$$

input
```
int((a + c*x^2)*((a*x + (c*x^3)/3)^n + 1),x)
```

output
```
(x*(3*a + c*x^2)*(n + (a*x + (c*x^3)/3)^n + 1))/(3*(n + 1))
```

3.217 $\int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^n\right) dx$

3.217.1 Optimal result

Integrand size = 31, antiderivative size = 44

$$\int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^n\right) dx = \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{\left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^{1+n}}{1+n}$$

output
```
1/2*b*x^2+1/3*c*x^3+(1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

3.217.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^n\right) dx = \frac{x^2(3b + 2cx) \left(1 + n + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^n\right)}{6(1+n)}$$

input
```
Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n),x]
```

output
```
(x^2*(3*b + 2*c*x)*(1 + n + ((b*x^2)/2 + (c*x^3)/3)^n))/(6*(1 + n))
```

3.217.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)\left(\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n+1\right)d\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n),x]
```

output
```
(b*x^2)/2 + (c*x^3)/3 + ((b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)
```

3.217.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.217.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{\left(\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)^{1+n}}{1+n}$	37
default	$\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{\left(\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)^{1+n}}{1+n}$	37
risch	$\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{x^{2}(2cx+3b)\left(\frac{1}{3}\right)^{n}\left(\frac{1}{2}\right)^{n}\left(x^{2}(2cx+3b)\right)^{n}}{6n+6}$	52
norman	$\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{b\,x^{2}{\mathrm e}^{n\ln\left(\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)}}{2+2n}+\frac{c\,x^{3}{\mathrm e}^{n\ln\left(\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)}}{3+3n}$	70
parallelrisch	$\frac{2x^{3}\left(\frac{x^{2}(2cx+3b)}{6}\right)^{n}c^{2}+2x^{3}c^{2}n+2c^{2}x^{3}+3x^{2}\left(\frac{x^{2}(2cx+3b)}{6}\right)^{n}bc+3bcn\,x^{2}+3bc\,x^{2}}{6c(1+n)}$	89

input
```
int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*x^2+1/3*c*x^3+(1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)dx$$
$$=\frac{2\,(cn+c)x^3+3\,(bn+b)x^2+\left(2\,cx^3+3\,bx^2\right)\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2\right)^n}{6\,(n+1)}$$

input
```
integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="fricas")
```

output
```
1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2)*(1/3*c*x^3 +
1/2*b*x^2)^n)/(n + 1)
```

3.217.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(32) = 64$.

Time = 73.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.30

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx = \begin{cases} \frac{3\cdot 6^n bnx^2}{6\cdot 6^n n+6\cdot 6^n}+\frac{3\cdot 6^n bx^2}{6\cdot 6^n n+6\cdot 6^n}+\frac{2\cdot 6^n cnx^3}{6\cdot 6^n n+6\cdot 6^n}+\frac{2\cdot 6^n cx^3}{6\cdot 6^n n+6\cdot 6^n}+\frac{3bx^2(3bx^2+2cx^3)^n}{6\cdot 6^n n+6\cdot 6^n}+\frac{2cx^3(3bx^2+2cx^3)^n}{6\cdot 6^n n+6\cdot 6^n} & \text{for } n\neq -1 \\ \frac{bx^2}{2}+\frac{cx^3}{3}+2\log(x)+\log\left(\frac{3b}{2c}+x\right) & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**n),x)
```

output
```
Piecewise((3*6**n*b*n*x**2/(6*6**n*n + 6*6**n) + 3*6**n*b*x**2/(6*6**n*n +
 6*6**n) + 2*6**n*c*n*x**3/(6*6**n*n + 6*6**n) + 2*6**n*c*x**3/(6*6**n*n +
 6*6**n) + 3*b*x**2*(3*b*x**2 + 2*c*x**3)**n/(6*6**n*n + 6*6**n) + 2*c*x**
3*(3*b*x**2 + 2*c*x**3)**n/(6*6**n*n + 6*6**n), Ne(n, -1)), (b*x**2/2 + c*
x**3/3 + 2*log(x) + log(3*b/(2*c) + x), True))
```

3.217.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx = \frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+\frac{(2\,cx^3+3\,bx^2)e^{(n\log(2\,cx+3\,b)+2\,n\log(x))}}{3^{n+1}2^{n+1}n+3^{n+1}2^{n+1}}$$

input
```
integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxima")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + (2*c*x^3 + 3*b*x^2)*e^(n*log(2*c*x + 3*b) + 2*n*lo
g(x))/(3^(n + 1)*2^(n + 1)*n + 3^(n + 1)*2^(n + 1))
```

3.217.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx=\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+\frac{\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2\right)^{n+1}}{n+1}$$

input
```
integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="giac")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + (1/3*c*x^3 + 1/2*b*x^2)^(n + 1)/(n + 1)
```

3.217.9 Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx=\frac{x^2\,(3\,b+2\,c\,x)\,\left(n+\left(\frac{c\,x^3}{3}+\frac{b\,x^2}{2}\right)^n+1\right)}{6\,(n+1)}$$

input
```
int((b*x + c*x^2)*(((b*x^2)/2 + (c*x^3)/3)^n + 1),x)
```

output
```
(x^2*(3*b + 2*c*x)*(n + ((b*x^2)/2 + (c*x^3)/3)^n + 1))/(6*(n + 1))
```

3.218 $\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx$

3.218.1 Optimal result

Integrand size = 35, antiderivative size = 50

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx=ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}}{1+n}$$

output
```
a*x+1/2*b*x^2+1/3*c*x^3+(a*x+1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

3.218.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx =\frac{x(6a+x(3b+2cx))\left(1+n+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)}{6(1+n)}$$

input
```
Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n),x]
```

output
```
(x*(6*a + x*(3*b + 2*c*x))*(1 + n + (a*x + (b*x^2)/2 + (c*x^3)/3)^n))/(6*(
1 + n))
```

3.218.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx+cx^2)\left(\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n+1\right)dx$$
$$\Big\downarrow \text{2024}$$
$$\int \left(\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n+1\right)d\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)$$
$$\Big\downarrow \text{2009}$$
$$\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n),x]
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n
)
```

3.218.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.218.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

method	result
derivativedivides	$ax+\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{\left(ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)^{1+n}}{1+n}$
default	$ax+\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{\left(ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)^{1+n}}{1+n}$
risch	$ax+\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{x\left(2c\,x^{2}+3bx+6a\right)\left(\frac{1}{3}\right)^{n}\left(\frac{1}{2}\right)^{n}\left(x\left(2c\,x^{2}+3bx+6a\right)\right)^{n}}{6n+6}$
norman	$ax+\frac{ax\,\mathrm{e}^{n\ln\left(ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)}}{1+n}+\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{b\,x^{2}\mathrm{e}^{n\ln\left(ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)}}{2+2n}+\frac{c\,x^{3}\mathrm{e}^{n\ln\left(ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)}}{3+3n}$
parallelrisch	$\frac{2x^{3}\left(\frac{x\left(2c\,x^{2}+3bx+6a\right)}{6}\right)^{n}c^{2}+2x^{3}c^{2}n+2c^{2}x^{3}+3x^{2}\left(\frac{x\left(2c\,x^{2}+3bx+6a\right)}{6}\right)^{n}bc+3bcn\,x^{2}+3bc\,x^{2}+6x\left(\frac{x\left(2c\,x^{2}+3bx+6a\right)}{6}\right)^{n}c}{6c(1+n)}$

input
```
int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x,method=_RETURNVERBOSE)
```

output
```
a*x+1/2*b*x^2+1/3*c*x^3+(a*x+1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.44

$$\int(a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)dx$$
$$=\frac{2\,(cn+c)x^3+3\,(bn+b)x^2+\left(2\,cx^3+3\,bx^2+6\,ax\right)\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax\right)^n+6\,(an+a)x}{6\,(n+1)}$$

input
```
integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="fric
as")
```

output
```
1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2 + 6*a*x)*(1/3*
c*x^3 + 1/2*b*x^2 + a*x)^n + 6*(a*n + a)*x)/(n + 1)
```

3.218.6 Sympy [F(-1)]

Timed out.

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**n),x)
```

output
```
Timed out
```

3.218.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.66

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx = \frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax+\frac{(2\,cx^3+3\,bx^2+6\,ax)e^{(n\log(2\,cx^2+3\,bx+6\,a)+n\log(x))}}{3^{n+1}2^{n+1}n+3^{n+1}2^{n+1}}$$

input
```
integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxi
ma")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + a*x + (2*c*x^3 + 3*b*x^2 + 6*a*x)*e^(n*log(2*c*x^2
 + 3*b*x + 6*a) + n*log(x))/(3^(n + 1)*2^(n + 1)*n + 3^(n + 1)*2^(n + 1))
```

3.218.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx = \frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax+\frac{\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax\right)^{n+1}}{n+1}$$

input
```
integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="giac
")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + a*x + (1/3*c*x^3 + 1/2*b*x^2 + a*x)^(n + 1)/(n + 1
)
```

3.218.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.46

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx$$
$$=a\,x+\left(\frac{3\,b\,x^2}{6\,n+6}+\frac{2\,c\,x^3}{6\,n+6}+\frac{6\,a\,x}{6\,n+6}\right)\left(\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+a\,x\right)^n+\frac{b\,x^2}{2}+\frac{c\,x^3}{3}$$

input
```
int(((a*x + (b*x^2)/2 + (c*x^3)/3)^n + 1)*(a + b*x + c*x^2),x)
```

output
```
a*x + ((3*b*x^2)/(6*n + 6) + (2*c*x^3)/(6*n + 6) + (6*a*x)/(6*n + 6))*(a*x
 + (b*x^2)/2 + (c*x^3)/3)^n + (b*x^2)/2 + (c*x^3)/3
```

3.219 $\int (-4+4x+x^2)(5-12x+6x^2+x^3)\,dx$

3.219.1 Optimal result

Integrand size = 22, antiderivative size = 19

$$\int (-4+4x+x^2)(5-12x+6x^2+x^3)\,dx=\frac{1}{6}(5-12x+6x^2+x^3)^2$$

output
```
1/6*(x^3+6*x^2-12*x+5)^2
```

3.219.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int (-4+4x+x^2)(5-12x+6x^2+x^3)\,dx=-20x+34x^2-\frac{67x^3}{3}+2x^4+2x^5+\frac{x^6}{6}$$

input
```
Integrate[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]
```

output
```
-20*x + 34*x^2 - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6
```

3.219.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^2+4x-4)(x^3+6x^2-12x+5)\,dx$$

$$\Big\downarrow \; 2021$$

$$\frac{1}{6}(x^3+6x^2-12x+5)^2$$

input
```
Int[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]
```

output
```
(5 - 12*x + 6*x^2 + x^3)^2/6
```

3.219.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.219.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{(x^3+6x^2-12x+5)^2}{6}$	18
gosper	$\frac{x(x^5+12x^4+12x^3-134x^2+204x-120)}{6}$	27
norman	$\frac{1}{6}x^6+2x^5+2x^4-\frac{67}{3}x^3+34x^2-20x$	30
parallelrisch	$\frac{1}{6}x^6+2x^5+2x^4-\frac{67}{3}x^3+34x^2-20x$	30
risch	$\frac{1}{6}x^6+2x^5+2x^4-\frac{67}{3}x^3+34x^2-20x+\frac{25}{6}$	31

input
```
int((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x,method=_RETURNVERBOSE)
```

output
```
1/6*(x^3+6*x^2-12*x+5)^2
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int (-4+4x+x^2)(5-12x+6x^2+x^3)\,dx = \frac{1}{6}\,x^6+2\,x^5+2\,x^4-\frac{67}{3}\,x^3+34\,x^2-20\,x$$

input
```
integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="fricas")
```

output
```
1/6*x^6 + 2*x^5 + 2*x^4 - 67/3*x^3 + 34*x^2 - 20*x
```

3.219.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int (-4+4x+x^2)(5-12x+6x^2+x^3)\,dx = \frac{x^6}{6}+2x^5+2x^4-\frac{67x^3}{3}+34x^2-20x$$

input
```
integrate((x**2+4*x-4)*(x**3+6*x**2-12*x+5),x)
```

output
```
x**6/6 + 2*x**5 + 2*x**4 - 67*x**3/3 + 34*x**2 - 20*x
```

3.219.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int (-4+4x+x^2)(5-12x+6x^2+x^3)\,dx = \frac{1}{6}\,(x^3+6\,x^2-12\,x+5)^2$$

input
```
integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="maxima")
```

output
```
1/6*(x^3 + 6*x^2 - 12*x + 5)^2
```

3.219.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int (-4+4x+x^2) (5-12x+6x^2+x^3) \, dx = \frac{5}{3}\,x^3 + \frac{1}{6}\left(x^3+6\,x^2-12\,x\right)^2 + 10\,x^2 - 20\,x$$

input
```
integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="giac")
```

output
```
5/3*x^3 + 1/6*(x^3 + 6*x^2 - 12*x)^2 + 10*x^2 - 20*x
```

3.219.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int (-4+4x+x^2) (5-12x+6x^2+x^3) \, dx = \frac{x^6}{6} + 2\,x^5 + 2\,x^4 - \frac{67\,x^3}{3} + 34\,x^2 - 20\,x$$

input
```
int((4*x + x^2 - 4)*(6*x^2 - 12*x + x^3 + 5),x)
```

output
```
34*x^2 - 20*x - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6
```

3.220 $\int (2x + x^3) (1 + 4x^2 + x^4) \, dx$

3.220.1 Optimal result

Integrand size = 18, antiderivative size = 16

$$\int (2x + x^3) (1 + 4x^2 + x^4) \, dx = \frac{1}{8} (1 + 4x^2 + x^4)^2$$

output
```
1/8*(x^4+4*x^2+1)^2
```

3.220.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int (2x + x^3) (1 + 4x^2 + x^4) \, dx = x^2 + \frac{9x^4}{4} + x^6 + \frac{x^8}{8}$$

input
```
Integrate[(2*x + x^3)*(1 + 4*x^2 + x^4),x]
```

output
```
x^2 + (9*x^4)/4 + x^6 + x^8/8
```

3.220.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^3+2x)\left(x^4+4x^2+1\right) dx$$

$$\Big\downarrow \text{ 2021}$$

$$\frac{1}{8}\left(x^4+4x^2+1\right)^2$$

input
```
Int[(2*x + x^3)*(1 + 4*x^2 + x^4),x]
```

output
```
(1 + 4*x^2 + x^4)^2/8
```

3.220.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.220.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\left(x^4+4x^2+1\right)^2}{8}$	15
norman	$\frac{1}{8}x^8+x^6+\frac{9}{4}x^4+x^2$	18
parallelrisch	$\frac{1}{8}x^8+x^6+\frac{9}{4}x^4+x^2$	18
risch	$\frac{1}{8}x^8+x^6+\frac{9}{4}x^4+x^2+\frac{1}{8}$	19
gosper	$\frac{x^2\left(x^6+8x^4+18x^2+8\right)}{8}$	21

input
```
int((x^3+2*x)*(x^4+4*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*(x^4+4*x^2+1)^2
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (2x+x^3)\left(1+4x^2+x^4\right)\,dx=\frac{1}{8}\,x^8+x^6+\frac{9}{4}\,x^4+x^2$$

input
```
integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="fricas")
```

output
```
1/8*x^8 + x^6 + 9/4*x^4 + x^2
```

3.220.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (2x+x^3)\left(1+4x^2+x^4\right)\,dx=\frac{x^8}{8}+x^6+\frac{9x^4}{4}+x^2$$

input
```
integrate((x**3+2*x)*(x**4+4*x**2+1),x)
```

output
```
x**8/8 + x**6 + 9*x**4/4 + x**2
```

3.220.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int (2x+x^3)\left(1+4x^2+x^4\right)\,dx=\frac{1}{8}\left(x^4+4\,x^2+1\right)^2$$

input
```
integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="maxima")
```

output
```
1/8*(x^4 + 4*x^2 + 1)^2
```

3.220.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (2x+x^3)(1+4x^2+x^4)\,dx=\frac{1}{4}\,x^4+\frac{1}{8}\,(x^4+4\,x^2)^2+x^2$$

input
```
integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="giac")
```

output
```
1/4*x^4 + 1/8*(x^4 + 4*x^2)^2 + x^2
```

3.220.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (2x+x^3)(1+4x^2+x^4)\,dx=\frac{x^8}{8}+x^6+\frac{9\,x^4}{4}+x^2$$

input
```
int((2*x + x^3)*(4*x^2 + x^4 + 1),x)
```

output
```
x^2 + (9*x^4)/4 + x^6 + x^8/8
```

3.221 $\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx$

3.221.1 Optimal result

Integrand size = 26, antiderivative size = 33

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx=81x^4(1+x)^4-36x^7(1+x)^7+\frac{49}{10}x^{10}(1+x)^{10}$$

output
```
81*x^4*(1+x)^4-36*x^7*(1+x)^7+49/10*x^10*(1+x)^10
```

3.221.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(33) = 66.

Time = 0.01 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.91

$$\begin{aligned}&\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx\\&=81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-1211x^{11}-\frac{1071x^{12}}{2}\\&\quad+336x^{13}+993x^{14}+\frac{6174x^{15}}{5}+1029x^{16}+588x^{17}+\frac{441x^{18}}{2}+49x^{19}+\frac{49x^{20}}{10}\end{aligned}$$

input
```
Integrate[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]
```

output
```
81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*
x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

3.221.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. $2(33) = 66$.

Time = 0.48 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2027, 2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)\left(x^2+x\right)^3\left(7\left(x^2+x\right)^3-18\right)^2 dx$$

$$\Big\downarrow \text{2027}$$

$$\int x^3(x+1)^3(2x+1)\left(7\left(x^2+x\right)^3-18\right)^2 dx$$

$$\Big\downarrow \text{2115}$$

$$\int \left(98x^{19}+931x^{18}+3969x^{17}+9996x^{16}+16464x^{15}+18522x^{14}+13902x^{13}+4368x^{12}-6426x^{11}-13321x^{10}-\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

input
```
Int[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]
```

output
```
81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*
x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

3.221.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

3.221.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result
default	$\frac{49(x^2+x)^{10}}{10}-36(x^2+x)^7+81(x^2+x)^4$
gosper	$\frac{(x+1)^3(49x^{13}+343x^{12}+1029x^{11}+1715x^{10}+1715x^9+1029x^8-17x^7-1391x^6-2160x^5-1440x^4-360x^3+810x+810)x^4}{10}$
norman	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}+$
risch	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}+$
parallelrisch	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}+$

input
```
int((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
49/10*(x^2+x)^10-36*(x^2+x)^7+81*(x^2+x)^4
```

3.221.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(31) = 62$.

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx$$
$$= \frac{49}{10}\,x^{20} + 49\,x^{19} + \frac{441}{2}\,x^{18} + 588\,x^{17} + 1029\,x^{16} + \frac{6174}{5}\,x^{15} + 993\,x^{14} + 336\,x^{13} - \frac{1071}{2}\,x^{12} - 1211\,x^{11} - \frac{12551}{10}\,x^{10} - 756\,x^9 - 171\,x^8 + 288\,x^7 + 486\,x^6 + 324\,x^5 + 81\,x^4$$

input
```
integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="fricas")
```

output
```
49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 9
93*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 1
71*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

3.221.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(31) = 62$.

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.85

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx$$
$$= \frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

input
```
integrate((1+2*x)*(x**2+x)**3*(-18+7*(x**2+x)**3)**2,x)
```

output
```
49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6174*x**15
/5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 - 12551*x**10/10 -
756*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324*x**5 + 81*x**4
```

3.221.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(31) = 62.

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx = \frac{49}{10}\,x^{20}+49\,x^{19}+\frac{441}{2}\,x^{18}+588\,x^{17}+1029\,x^{16}+\frac{6174}{5}\,x^{15}+993\,x^{14}+336\,x^{13} - \frac{1071}{2}\,x^{12}-1211\,x^{11}-\frac{12551}{10}\,x^{10}-756\,x^9-171\,x^8+288\,x^7+486\,x^6+324\,x^5+81\,x^4$$

input
```
integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="maxima")
```

output
```
49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 9
93*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 1
71*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

3.221.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx = \frac{49}{10}\left(x^2+x\right)^{10}-36\left(x^2+x\right)^7+81\left(x^2+x\right)^4$$

input
```
integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="giac")
```

output
```
49/10*(x^2 + x)^10 - 36*(x^2 + x)^7 + 81*(x^2 + x)^4
```

3.221.9 Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx = \frac{49\,x^{20}}{10}+49\,x^{19}+\frac{441\,x^{18}}{2}+588\,x^{17}+1029\,x^{16}+\frac{6174\,x^{15}}{5}+993\,x^{14}+336\,x^{13} - \frac{1071\,x^{12}}{2}-1211\,x^{11}-\frac{12551\,x^{10}}{10}-756\,x^9-171\,x^8+288\,x^7+486\,x^6+324\,x^5+81\,x^4$$

input
```
int((2*x + 1)*(x + x^2)^3*(7*(x + x^2)^3 - 18)^2,x)
```

output
```
81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*
x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

3.221. $\int (1+2x)\left(x+x^2\right)^3\left(-18+7(x+x^2)^3\right)^2\,dx$

3.222 $\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx$

3.222.1 Optimal result

Integrand size = 28, antiderivative size = 33

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx = 81x^4(1+x)^4-36x^7(1+x)^7+\frac{49}{10}x^{10}(1+x)^{10}$$

output
```
81*x^4*(1+x)^4-36*x^7*(1+x)^7+49/10*x^10*(1+x)^10
```

3.222.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(33) = 66.

Time = 0.00 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.91

$$\begin{aligned}\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx &= 81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-1211x^{11}-\frac{1071x^{12}}{2}\\ &\quad+336x^{13}+993x^{14}+\frac{6174x^{15}}{5}+1029x^{16}+588x^{17}+\frac{441x^{18}}{2}+49x^{19}+\frac{49x^{20}}{10}\end{aligned}$$

input
```
Integrate[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2,x]
```

output
```
81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*
x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

3.222.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. $2(33) = 66$.

Time = 0.42 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(x+1)^3(2x+1)\left(7x^3(x+1)^3-18\right)^2 dx$$

$$\Big\downarrow \text{2115}$$

$$\int (98x^{19}+931x^{18}+3969x^{17}+9996x^{16}+16464x^{15}+18522x^{14}+13902x^{13}+4368x^{12}-6426x^{11}-13321x^{10}-$$

$$\Big\downarrow \text{2009}$$

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

input
```
Int[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2,x]
```

output
```
81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*
x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

3.222.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

3.222.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(31) = 62$.

Time = 0.79 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

method	result
gosper	$\frac{x^4(49x^{16}+490x^{15}+2205x^{14}+5880x^{13}+10290x^{12}+12348x^{11}+9930x^{10}+3360x^9-5355x^8-12110x^7-12551x^6-7560x^5-1710x}{10}$
default	$81x^4 + 324x^5 + 486x^6 + 288x^7 - 171x^8 - 756x^9 - \frac{12551}{10}x^{10} - 1211x^{11} - \frac{1071}{2}x^{12} + 336x^{13} +$
norman	$81x^4 + 324x^5 + 486x^6 + 288x^7 - 171x^8 - 756x^9 - \frac{12551}{10}x^{10} - 1211x^{11} - \frac{1071}{2}x^{12} + 336x^{13} +$
risch	$81x^4 + 324x^5 + 486x^6 + 288x^7 - 171x^8 - 756x^9 - \frac{12551}{10}x^{10} - 1211x^{11} - \frac{1071}{2}x^{12} + 336x^{13} +$
parallelrisch	$81x^4 + 324x^5 + 486x^6 + 288x^7 - 171x^8 - 756x^9 - \frac{12551}{10}x^{10} - 1211x^{11} - \frac{1071}{2}x^{12} + 336x^{13} +$

input
```
int(x^3*(x+1)^3*(1+2*x)*(-18+7*x^3*(x+1)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*x^4*(49*x^16+490*x^15+2205*x^14+5880*x^13+10290*x^12+12348*x^11+9930*
x^10+3360*x^9-5355*x^8-12110*x^7-12551*x^6-7560*x^5-1710*x^4+2880*x^3+4860
*x^2+3240*x+810)
```

3.222.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(31) = 62$.

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx$$
$$= \frac{49}{10}\,x^{20}+49\,x^{19}+\frac{441}{2}\,x^{18}+588\,x^{17}+1029\,x^{16}+\frac{6174}{5}\,x^{15}+993\,x^{14}+336\,x^{13} - \frac{1071}{2}\,x^{12}-1211\,x^{11}-\frac{12551}{10}\,x^{10}-756\,x^9-171\,x^8+288\,x^7+486\,x^6+324\,x^5+81\,x^4$$

input
```
integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="fricas")
```

output
```
49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 9
93*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 1
71*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

3.222.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(31) = 62$.

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.85

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx$$
$$= \frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13} - \frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

input
```
integrate(x**3*(1+x)**3*(1+2*x)*(-18+7*x**3*(1+x)**3)**2,x)
```

output
```
49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6174*x**15
/5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 - 12551*x**10/10 -
756*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324*x**5 + 81*x**4
```

3.222.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(31) = 62.

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx = \frac{49}{10}\,x^{20}+49\,x^{19}+\frac{441}{2}\,x^{18}+588\,x^{17}+1029\,x^{16}+\frac{6174}{5}\,x^{15}+993\,x^{14}+336\,x^{13}-\frac{1071}{2}\,x^{12}-1211\,x^{11}-\frac{12551}{10}\,x^{10}-756\,x^{9}-171\,x^{8}+288\,x^{7}+486\,x^{6}+324\,x^{5}+81\,x^{4}$$

input
```
integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="maxima")
```

output
```
49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 9
93*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 1
71*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

3.222.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx = \frac{49}{10}\left(x^2+x\right)^{10}-36\left(x^2+x\right)^{7}+81\left(x^2+x\right)^{4}$$

input
```
integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="giac")
```

output
```
49/10*(x^2 + x)^10 - 36*(x^2 + x)^7 + 81*(x^2 + x)^4
```

3.222.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx = \frac{49\,x^{20}}{10}+49\,x^{19}+\frac{441\,x^{18}}{2}+588\,x^{17}+1029\,x^{16}+\frac{6174\,x^{15}}{5}+993\,x^{14}+336\,x^{13}-\frac{1071\,x^{12}}{2}-1211\,x^{11}-\frac{12551\,x^{10}}{10}-756\,x^{9}-171\,x^{8}+288\,x^{7}+486\,x^{6}+324\,x^{5}+81\,x^{4}$$

input
```
int(x^3*(2*x + 1)*(7*x^3*(x + 1)^3 - 18)^2*(x + 1)^3,x)
```

output
```
81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*
x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

3.222. $\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx$

3.223 $\int \frac{2-x^2}{(1-6x+x^3)^5} dx$

3.223.1 Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \frac{2-x^2}{(1-6x+x^3)^5} \, dx = \frac{1}{12(1-6x+x^3)^4}$$

output
```
1/12/(x^3-6*x+1)^4
```

3.223.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{2-x^2}{(1-6x+x^3)^5} \, dx = \frac{1}{12(1-6x+x^3)^4}$$

input
```
Integrate[(2 - x^2)/(1 - 6*x + x^3)^5,x]
```

output
```
1/(12*(1 - 6*x + x^3)^4)
```

3.223.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-x^2}{(x^3-6x+1)^5}\,dx$$

$$\Big\downarrow \text{ 2021}$$

$$\frac{1}{12\,(x^3-6x+1)^4}$$

input
```
Int[(2 - x^2)/(1 - 6*x + x^3)^5,x]
```

output
```
1/(12*(1 - 6*x + x^3)^4)
```

3.223.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.223.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{1}{12(x^3-6x+1)^4}$	13
default	$\frac{1}{12(x^3-6x+1)^4}$	13
norman	$\frac{1}{12(x^3-6x+1)^4}$	13
risch	$\frac{1}{12(x^3-6x+1)^4}$	13
parallelrisch	$\frac{1}{12(x^3-6x+1)^4}$	13

input
```
int((-x^2+2)/(x^3-6*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/12/(x^3-6*x+1)^4
```

3.223.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(12) = 24.

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.07

$$\int \frac{2-x^2}{(1-6x+x^3)^5}\,dx$$
$$=\frac{1}{12\,(x^{12}-24\,x^{10}+4\,x^9+216\,x^8-72\,x^7-858\,x^6+432\,x^5+1224\,x^4-860\,x^3+216\,x^2-24\,x+1)}$$

input
```
integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="fricas")
```

output
```
1/12/(x^12 - 24*x^10 + 4*x^9 + 216*x^8 - 72*x^7 - 858*x^6 + 432*x^5 + 1224
*x^4 - 860*x^3 + 216*x^2 - 24*x + 1)
```

3.223.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 4.00

$$\int \frac{2-x^2}{(1-6x+x^3)^5}\,dx$$
$$=\frac{1}{12x^{12}-288x^{10}+48x^9+2592x^8-864x^7-10296x^6+5184x^5+14688x^4-10320x^3+2592x^2-288x+}$$

input
```
integrate((-x**2+2)/(x**3-6*x+1)**5,x)
```

output
```
1/(12*x**12 - 288*x**10 + 48*x**9 + 2592*x**8 - 864*x**7 - 10296*x**6 + 51
84*x**5 + 14688*x**4 - 10320*x**3 + 2592*x**2 - 288*x + 12)
```

3.223.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2-x^2}{(1-6x+x^3)^5}\,dx = \frac{1}{12\,(x^3-6\,x+1)^4}$$

input
```
integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="maxima")
```

output
```
1/12/(x^3 - 6*x + 1)^4
```

3.223.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2-x^2}{(1-6x+x^3)^5}\,dx = \frac{1}{12\,(x^3-6\,x+1)^4}$$

input
```
integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="giac")
```

output
```
1/12/(x^3 - 6*x + 1)^4
```

3.223.9 Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2-x^2}{(1-6x+x^3)^5}\,dx = \frac{1}{12\,(x^3-6\,x+1)^4}$$

input
```
int(-(x^2 - 2)/(x^3 - 6*x + 1)^5,x)
```

output
```
1/(12*(x^3 - 6*x + 1)^4)
```

3.224 $\int \frac{2x+x^2}{4+3x^2+x^3}\,dx$

3.224.1 Optimal result

Integrand size = 20, antiderivative size = 15

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx=\frac{1}{3}\log\left(4+3x^2+x^3\right)$$

output
```
1/3*ln(x^3+3*x^2+4)
```

3.224.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx=\frac{1}{3}\log\left(4+3x^2+x^3\right)$$

input
```
Integrate[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

output
```
Log[4 + 3*x^2 + x^3]/3
```

3.224.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+2x}{x^3+3x^2+4}\,dx$$

$$\Big\downarrow \; 2020$$

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

input
```
Int[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

output
```
Log[4 + 3*x^2 + x^3]/3
```

3.224.3.1 Defintions of rubi rules used

rule 2020
```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

3.224.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln\left(x^3+3x^2+4\right)}{3}$	14
norman	$\frac{\ln\left(x^3+3x^2+4\right)}{3}$	14
risch	$\frac{\ln\left(x^3+3x^2+4\right)}{3}$	14
parallelrisch	$\frac{\ln\left(x^3+3x^2+4\right)}{3}$	14

input
```
int((x^2+2*x)/(x^3+3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(x^3+3*x^2+4)
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx = \frac{1}{3}\,\log\left(x^3+3\,x^2+4\right)$$

input
```
integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="fricas")
```

output
```
1/3*log(x^3 + 3*x^2 + 4)
```

3.224.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx = \frac{\log\left(x^3+3x^2+4\right)}{3}$$

input
```
integrate((x**2+2*x)/(x**3+3*x**2+4),x)
```

output
```
log(x**3 + 3*x**2 + 4)/3
```

3.224.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx = \frac{1}{3}\,\log\left(x^3+3\,x^2+4\right)$$

input
```
integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="maxima")
```

output
```
1/3*log(x^3 + 3*x^2 + 4)
```

3.224.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx = \frac{1}{3}\log\left(\left|x^3+3\,x^2+4\right|\right)$$

input
```
integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="giac")
```

output
```
1/3*log(abs(x^3 + 3*x^2 + 4))
```

3.224.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx = \frac{\ln\left(x^3+3\,x^2+4\right)}{3}$$

input
```
int((2*x + x^2)/(3*x^2 + x^3 + 4),x)
```

output
```
log(3*x^2 + x^3 + 4)/3
```

3.225 $\int \frac{1+x+x^3}{4x+2x^2+x^4} dx$

3.225.1 Optimal result

Integrand size = 21, antiderivative size = 17

$$\int \frac{1+x+x^3}{4x+2x^2+x^4} \, dx = \frac{1}{4}\log\left(4x+2x^2+x^4\right)$$

output
```
1/4*ln(x^4+2*x^2+4*x)
```

3.225.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \frac{1+x+x^3}{4x+2x^2+x^4} \, dx = \frac{\log(x)}{4} + \frac{1}{4}\log\left(4+2x+x^3\right)$$

input
```
Integrate[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

output
```
Log[x]/4 + Log[4 + 2*x + x^3]/4
```

3.225.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x+1}{x^4+2x^2+4x}\,dx$$
$$\Big\downarrow \text{ 2020}$$
$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

input
```
Int[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

output
```
Log[4*x + 2*x^2 + x^4]/4
```

3.225.3.1 Defintions of rubi rules used

rule 2020
```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

3.225.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\ln\left(x\left(x^{3}+2x+4\right)\right)}{4}$	14
risch	$\frac{\ln\left(x^{4}+2x^{2}+4x\right)}{4}$	16
norman	$\frac{\ln\left(x\right)}{4}+\frac{\ln\left(x^{3}+2x+4\right)}{4}$	17
parallelrisch	$\frac{\ln\left(x\right)}{4}+\frac{\ln\left(x^{3}+2x+4\right)}{4}$	17

input
```
int((x^3+x+1)/(x^4+2*x^2+4*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x*(x^3+2*x+4))
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx = \frac{1}{4}\,\log\left(x^4+2\,x^2+4\,x\right)$$

input
```
integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="fricas")
```

output
```
1/4*log(x^4 + 2*x^2 + 4*x)
```

3.225.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx = \frac{\log\left(x^4+2x^2+4x\right)}{4}$$

input
```
integrate((x**3+x+1)/(x**4+2*x**2+4*x),x)
```

output
```
log(x**4 + 2*x**2 + 4*x)/4
```

3.225.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx = \frac{1}{4}\,\log\left(x^4+2\,x^2+4\,x\right)$$

input
```
integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="maxima")
```

output
```
1/4*log(x^4 + 2*x^2 + 4*x)
```

3.225.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx = \frac{1}{4}\log\left(4\left|\frac{1}{4}\,x^4+\frac{1}{2}\,x^2+x\right|\right)$$

input
```
integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="giac")
```

output
```
1/4*log(4*abs(1/4*x^4 + 1/2*x^2 + x))
```

3.225.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx = \frac{\ln\left(x\left(x^3+2\,x+4\right)\right)}{4}$$

input
```
int((x + x^3 + 1)/(4*x + 2*x^2 + x^4),x)
```

output
```
log(x*(2*x + x^3 + 4))/4
```

3.226 $\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2} dx$

3.226.1 Optimal result

Integrand size = 52, antiderivative size = 40

$$\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2} dx = \frac{a}{c+dx+ex^2+fx^3} + \frac{bx}{c+dx+ex^2+fx^3}$$

output
```
a/(f*x^3+e*x^2+d*x+c)+b*x/(f*x^3+e*x^2+d*x+c)
```

3.226.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2} dx = \frac{a+bx}{c+dx+ex^2+fx^3}$$

input
```
Integrate[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x
 + e*x^2 + f*x^3)^2,x]
```

output
```
(a + b*x)/(c + d*x + e*x^2 + f*x^3)
```

3.226.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6, 2527, 27, 2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-ad-2aex-3afx^2+bc-bex^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}dx$$

$$\Big\downarrow \ 6$$

$$\int \frac{x^2(-3af-be)-ad-2aex+bc-2bfx^3}{(c+dx+ex^2+fx^3)^2}dx$$

$$\Big\downarrow \ 2527$$

$$\frac{bx}{c+dx+ex^2+fx^3}-\frac{\int \frac{2(3af^2x^2+2aefx+adf)}{(fx^3+ex^2+dx+c)^2}dx}{2f}$$

$$\Big\downarrow \ 27$$

$$\frac{bx}{c+dx+ex^2+fx^3}-\frac{\int \frac{3af^2x^2+2aefx+adf}{(fx^3+ex^2+dx+c)^2}dx}{f}$$

$$\Big\downarrow \ 2021$$

$$\frac{a}{c+dx+ex^2+fx^3}+\frac{bx}{c+dx+ex^2+fx^3}$$

input
```
Int[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x + e*x
^2 + f*x^3)^2,x]
```

output
```
a/(c + d*x + e*x^2 + f*x^3) + (b*x)/(c + d*x + e*x^2 + f*x^3)
```

3.226.3.1 Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

rule 2527
```
Int[(Pm_)*(Qn_)^(p_.), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x
]}, Simp[Coeff[Pm, x, m]*x^(m - n + 1)*(Qn^(p + 1)/((m + n*p + 1)*Coeff[Qn,
 x, n])), x] + Simp[1/((m + n*p + 1)*Coeff[Qn, x, n])   Int[ExpandToSum[(m
+ n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn +
 (p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 <
0] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]
```

3.226.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{bx+a}{f\,x^{3}+e\,x^{2}+dx+c}$	24
norman	$\frac{bx+a}{f\,x^{3}+e\,x^{2}+dx+c}$	24
risch	$\frac{bx+a}{f\,x^{3}+e\,x^{2}+dx+c}$	24
default	$-\frac{-bx-a}{f\,x^{3}+e\,x^{2}+dx+c}$	28
parallelrisch	$\frac{bex+ae}{e(f\,x^{3}+e\,x^{2}+dx+c)}$	30

input
```
int((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c)^2,x
,method=_RETURNVERBOSE)
```

output
```
(b*x+a)/(f*x^3+e*x^2+d*x+c)
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx = \frac{bx+a}{fx^3+ex^2+dx+c}$$

input
```
integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+
c)^2,x, algorithm="fricas")
```

output
```
(b*x + a)/(f*x^3 + e*x^2 + d*x + c)
```

3.226.6 Sympy [A] (verification not implemented)

Time = 12.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.55

$$\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx = -\frac{-a-bx}{c+dx+ex^2+fx^3}$$

input
```
integrate((-2*b*f*x**3-3*a*f*x**2-b*e*x**2-2*a*e*x-a*d+b*c)/(f*x**3+e*x**2
+d*x+c)**2,x)
```

output
```
-(-a - b*x)/(c + d*x + e*x**2 + f*x**3)
```

3.226.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx = \frac{bx+a}{fx^3+ex^2+dx+c}$$

input
```
integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+
c)^2,x, algorithm="maxima")
```

output
```
(b*x + a)/(f*x^3 + e*x^2 + d*x + c)
```

3.226.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{bc - ad - 2aex - bex^2 - 3afx^2 - 2bfx^3}{(c + dx + ex^2 + fx^3)^2} dx = \frac{bx + a}{fx^3 + ex^2 + dx + c}$$

input
```
integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+
c)^2,x, algorithm="giac")
```

output
```
(b*x + a)/(f*x^3 + e*x^2 + d*x + c)
```

3.226.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{bc - ad - 2aex - bex^2 - 3afx^2 - 2bfx^3}{(c + dx + ex^2 + fx^3)^2} dx = \frac{a + b\,x}{f\,x^3 + e\,x^2 + d\,x + c}$$

input
```
int(-(a*d - b*c + 2*a*e*x + 3*a*f*x^2 + b*e*x^2 + 2*b*f*x^3)/(c + d*x + e*
x^2 + f*x^3)^2,x)
```

output
```
(a + b*x)/(c + d*x + e*x^2 + f*x^3)
```

3.227 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4} dx$

3.227.1 Optimal result

Integrand size = 38, antiderivative size = 605

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4} dx$$

$$= \frac{\left(4a^2B+b\left(b-\sqrt{8a^2+b^2-4ac}\right)D-a\left(A\left(b-\sqrt{8a^2+b^2-4ac}\right)+bC-\sqrt{8a^2+b^2-4ac}C+2cD\right)\right) \mathrm{arc}}{\sqrt{2}a\sqrt{8a^2+b^2-4ac}\sqrt{4a^2+2ac-b\left(b-\sqrt{8a^2+b^2-4ac}\right)}}$$

$$- \frac{\left(4a^2B+b\left(b+\sqrt{8a^2+b^2-4ac}\right)D-a\left(A\left(b+\sqrt{8a^2+b^2-4ac}\right)+bC+\sqrt{8a^2+b^2-4ac}C+2cD\right)\right) \mathrm{a}}{\sqrt{2}a\sqrt{8a^2+b^2-4ac}\sqrt{4a^2+2ac-b\left(b+\sqrt{8a^2+b^2-4ac}\right)}}$$

$$- \frac{\left(2a(A-C)+\left(b-\sqrt{8a^2+b^2-4ac}\right)D\right)\log\left(2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)x+2ax^2\right)}{4a\sqrt{8a^2+b^2-4ac}}$$

$$+ \frac{\left(2a(A-C)+\left(b+\sqrt{8a^2+b^2-4ac}\right)D\right)\log\left(2a+\left(b+\sqrt{8a^2+b^2-4ac}\right)x+2ax^2\right)}{4a\sqrt{8a^2+b^2-4ac}}$$

output
```
-1/4*ln(2*a+2*a*x^2+x*(b-(8*a^2-4*a*c+b^2)^(1/2)))*(2*a*(A-C)+D*(b-(8*a^2-
4*a*c+b^2)^(1/2)))/a/(8*a^2-4*a*c+b^2)^(1/2)+1/4*ln(2*a+2*a*x^2+x*(b+(8*a^
2-4*a*c+b^2)^(1/2)))*(2*a*(A-C)+D*(b+(8*a^2-4*a*c+b^2)^(1/2)))/a/(8*a^2-4*
a*c+b^2)^(1/2)+1/2*arctan(1/2*(b+4*a*x-(8*a^2-4*a*c+b^2)^(1/2))*2^(1/2)/(4
*a^2+2*a*c-b*(b-(8*a^2-4*a*c+b^2)^(1/2)))^(1/2))*(4*a^2*B+b*D*(b-(8*a^2-4*
a*c+b^2)^(1/2))-a*(b*C+2*c*D+A*(b-(8*a^2-4*a*c+b^2)^(1/2))-C*(8*a^2-4*a*c+
b^2)^(1/2)))/a*2^(1/2)/(8*a^2-4*a*c+b^2)^(1/2)/(4*a^2+2*a*c-b*(b-(8*a^2-4*
a*c+b^2)^(1/2)))^(1/2)-1/2*arctan(1/2*(b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))*2^
(1/2)/(4*a^2+2*a*c-b*(b+(8*a^2-4*a*c+b^2)^(1/2)))^(1/2))*(4*a^2*B+b*D*(b+(
8*a^2-4*a*c+b^2)^(1/2))-a*(b*C+2*c*D+C*(8*a^2-4*a*c+b^2)^(1/2)+A*(b+(8*a^2
-4*a*c+b^2)^(1/2))))/a*2^(1/2)/(8*a^2-4*a*c+b^2)^(1/2)/(4*a^2+2*a*c-b*(b+(
8*a^2-4*a*c+b^2)^(1/2)))^(1/2)
```

3.227.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.16

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = \text{RootSum}\Big[a+b\#1+c\#1^2+b\#1^3 +a\#1^4\&, \frac{A\log(x-\#1)+B\log(x-\#1)\#1+C\log(x-\#1)\#1^2+D\log(x-\#1)\#1^3}{b+2c\#1+3b\#1^2+4a\#1^3}\&\Big]$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4),x]
```

output
```
RootSum[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , (A*Log[x - #1] + B*Log[x -
 #1]*#1 + C*Log[x - #1]*#1^2 + D*Log[x - #1]*#1^3)/(b + 2*c*#1 + 3*b*#1^2
+ 4*a*#1^3) & ]
```

3.227.3 Rubi [A] (verified)

Time = 3.85 (sec) , antiderivative size = 597, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{ax^4+a+bx^3+bx+cx^2}\,dx$$

$$\Big\downarrow \; 2492$$

$$\frac{\int \left(\frac{a\left(A\left(b+\sqrt{8a^2-4ca+b^2}\right)-2a(B-D)+\left(2a(A-C)+\left(b+\sqrt{8a^2-4ca+b^2}\right)D\right)x\right)}{\sqrt{8a^2-4ca+b^2}\left(2ax^2+\left(b+\sqrt{8a^2-4ca+b^2}\right)x+2a\right)}-\frac{a\left(A\left(b-\sqrt{8a^2-4ca+b^2}\right)-2a(B-D)+\left(2aA-2aC+bD-\sqrt{\ldots}\right.\right.}{\sqrt{8a^2-4ca+b^2}\left(2ax^2+\left(b-\sqrt{8a^2-4ca+b^2}\right)x\ldots\right.}\right.}{a}$$

$$\Big\downarrow \; 2009$$

$$\frac{\arctan\left(\frac{-\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}\right)\left(-a\left(A\left(b-\sqrt{8a^2-4ac+b^2}\right)-C\sqrt{8a^2-4ac+b^2}+bC+2cD\right)+bD\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2B\right)}{\sqrt{2}\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}-\frac{\text{arc}\ldots}{\ldots}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4),x]
```

output
```
(((4*a^2*B + b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])*D - a*(A*(b - Sqrt[8*a^2 +
b^2 - 4*a*c]) + b*C - Sqrt[8*a^2 + b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b - Sq
rt[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b - Sqrt
[8*a^2 + b^2 - 4*a*c])]])])/(Sqrt[2]*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 +
 2*a*c - b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])]) - ((4*a^2*B + b*(b + Sqrt[8*a
^2 + b^2 - 4*a*c])*D - a*(A*(b + Sqrt[8*a^2 + b^2 - 4*a*c]) + b*C + Sqrt[8
*a^2 + b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b + Sqrt[8*a^2 + b^2 - 4*a*c] + 4*
a*x)/(Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])]])])/(
Sqrt[2]*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 +
 b^2 - 4*a*c])]) - ((2*a*(A - C) + (b - Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[
2*a + (b - Sqrt[8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*Sqrt[8*a^2 + b^2 -
4*a*c]) + ((2*a*(A - C) + (b + Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (b
+ Sqrt[8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*Sqrt[8*a^2 + b^2 - 4*a*c]))/
a
```

3.227. $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx$

3.227.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

3.227.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 535, normalized size of antiderivative = 0.88

method	result
default	$4a\left(\frac{-\frac{\left(2Aa-2Ca-\sqrt{8a^{2}-4ac+b^{2}}\,D+Db\right)\ln\left(-2a\,x^{2}+\sqrt{8a^{2}-4ac+b^{2}}\,x-bx-2a\right)}{4a}+\frac{2\left(\frac{\left(2Aa-2Ca-\sqrt{8a^{2}-4ac+b^{2}}\,D+Db\right)\left(\sqrt{8a^{2}-4ac+b^{2}}-b\right)}{4a}\right.}{\sqrt{8a^{2}}}}{4a\sqrt{8a^{2}-4ac+b^{2}}}\right.$

input
```
int((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
4*a*(1/4/a/(8*a^2-4*a*c+b^2)^(1/2)*(-1/4*(2*A*a-2*C*a-(8*a^2-4*a*c+b^2)^(1
/2)*D+D*b)/a*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)+2*(1/4*(2*A*a-
2*C*a-(8*a^2-4*a*c+b^2)^(1/2)*D+D*b)/a*((8*a^2-4*a*c+b^2)^(1/2)-b)-(8*a^2-
4*a*c+b^2)^(1/2)*A+A*b-2*B*a+2*D*a)/(8*a^2+4*a*c-2*b^2+2*(8*a^2-4*a*c+b^2)
^(1/2)*b)^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b
^2+2*(8*a^2-4*a*c+b^2)^(1/2)*b)^(1/2)))+1/4/a/(8*a^2-4*a*c+b^2)^(1/2)*(1/4
*(2*A*a-2*C*a+(8*a^2-4*a*c+b^2)^(1/2)*D+D*b)/a*ln(2*a*x^2+(8*a^2-4*a*c+b^2
)^(1/2)*x+b*x+2*a)+2*(-1/4*(2*A*a-2*C*a+(8*a^2-4*a*c+b^2)^(1/2)*D+D*b)/a*(
b+(8*a^2-4*a*c+b^2)^(1/2))+(8*a^2-4*a*c+b^2)^(1/2)*A+A*b-2*B*a+2*D*a)/(8*a
^2+4*a*c-2*b^2-2*(8*a^2-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan((b+4*a*x+(8*a^2-4
*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*(8*a^2-4*a*c+b^2)^(1/2)*b)^(1/2))))
```

3.227.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = \text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="fric
as")
```

output
```
Timed out
```

3.227.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(a*x**4+b*x**3+c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.227.7 Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{ax^4+bx^3+cx^2+bx+a}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="maxi
ma")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(a*x^4 + b*x^3 + c*x^2 + b*x + a), x)
```

3.227.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument
Value
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{a\,x^4+b\,x^3+c\,x^2+b\,x+a}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x + a*x^4 + b*x^3 + c*x^2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x + a*x^4 + b*x^3 + c*x^2), x)
```

3.228 $\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} \, dx$

3.228.1 Optimal result

Integrand size = 32, antiderivative size = 63

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} \, dx = -\frac{2\log\left(2-\left(1-\sqrt{5}\right)x+2x^2\right)}{1-\sqrt{5}} - \frac{2\log\left(2-\left(1+\sqrt{5}\right)x+2x^2\right)}{1+\sqrt{5}}$$

output
```
-2*ln(2+2*x^2-x*(-5^(1/2)+1))/(-5^(1/2)+1)-2*ln(2+2*x^2-x*(5^(1/2)+1))/(5^
(1/2)+1)
```

3.228.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} \, dx = \frac{1}{2}\left(-\left(\left(-1+\sqrt{5}\right)\log\left(-2+x+\sqrt{5}x-2x^2\right)\right) + \left(1+\sqrt{5}\right)\log\left(2+\left(-1+\sqrt{5}\right)x+2x^2\right)\right)$$

input
```
Integrate[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4),x]
```

output
```
(-((-1 + Sqrt[5])*Log[-2 + x + Sqrt[5]*x - 2*x^2]) + (1 + Sqrt[5])*Log[2 +
 (-1 + Sqrt[5])*x + 2*x^2])/2
```

3.228.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3-4x^2+x+2}{x^4-x^3+x^2-x+1}\,dx$$

$$\Big\downarrow \text{2492}$$

$$\int \left(\frac{2\left(\left(1-\sqrt{5}\right)x+1\right)}{2x^2-\left(1+\sqrt{5}\right)x+2}+\frac{2\left(\left(1+\sqrt{5}\right)x+1\right)}{2x^2-\left(1-\sqrt{5}\right)x+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}{1-\sqrt{5}}-\frac{2\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}{1+\sqrt{5}}$$

input
```
Int[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4),x]
```

output
```
(-2*Log[2 - (1 - Sqrt[5])*x + 2*x^2])/(1 - Sqrt[5]) - (2*Log[2 - (1 + Sqrt
[5])*x + 2*x^2])/(1 + Sqrt[5])
```

3.228.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

3.228.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

method	result	size
default	$2\left(\frac{\sqrt{5}}{4}+\frac{1}{4}\right)\ln\left(x\sqrt{5}+2x^{2}-x+2\right)-2\left(\frac{\sqrt{5}}{4}-\frac{1}{4}\right)\ln\left(-x\sqrt{5}+2x^{2}-x+2\right)$	53
risch	$\frac{\ln\left(2+2x^{2}+\left(\sqrt{5}-1\right)x\right)}{2}+\frac{\ln\left(2+2x^{2}+\left(\sqrt{5}-1\right)x\right)\sqrt{5}}{2}+\frac{\ln\left(2+2x^{2}+\left(-\sqrt{5}-1\right)x\right)}{2}-\frac{\ln\left(2+2x^{2}+\left(-\sqrt{5}-1\right)x\right)\sqrt{5}}{2}$	80

input
```
int((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x,method=_RETURNVERBOSE)
```

output
```
2*(1/4*5^(1/2)+1/4)*ln(x*5^(1/2)+2*x^2-x+2)-2*(1/4*5^(1/2)-1/4)*ln(-x*5^(1
/2)+2*x^2-x+2)
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = \frac{1}{2}\sqrt{5}\log\left(\frac{2\,x^{4}-2\,x^{3}+7\,x^{2}+\sqrt{5}(2\,x^{3}-x^{2}+2\,x)-2\,x+2}{x^{4}-x^{3}+x^{2}-x+1}\right)+\frac{1}{2}\log\left(x^{4}-x^{3}+x^{2}-x+1\right)$$

input
```
integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(5)*log((2*x^4 - 2*x^3 + 7*x^2 + sqrt(5)*(2*x^3 - x^2 + 2*x) - 2*x
 + 2)/(x^4 - x^3 + x^2 - x + 1)) + 1/2*log(x^4 - x^3 + x^2 - x + 1)
```

3.228.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = \left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\log\left(x^2+x\left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+1\right) + \left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\log\left(x^2+x\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+1\right)$$

input
```
integrate((2*x**3-4*x**2+x+2)/(x**4-x**3+x**2-x+1),x)
```

output
```
(1/2 + sqrt(5)/2)*log(x**2 + x*(-1/2 + sqrt(5)/2) + 1) + (1/2 - sqrt(5)/2)
*log(x**2 + x*(-sqrt(5)/2 - 1/2) + 1)
```

3.228.7 Maxima [F]

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = \int \frac{2\,x^3-4\,x^2+x+2}{x^4-x^3+x^2-x+1}\,dx$$

input
```
integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="maxima")
```

output
```
integrate((2*x^3 - 4*x^2 + x + 2)/(x^4 - x^3 + x^2 - x + 1), x)
```

3.228.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = -\frac{1}{2}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right) + \frac{1}{2}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right) + \frac{1}{2}\log\left(x^4-x^3+x^2-x+1\right)$$

input
```
integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="giac")
```

output
```
-1/2*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/2*sqrt(5)*log(x^2 + 1/
2*x*(sqrt(5) - 1) + 1) + 1/2*log(x^4 - x^3 + x^2 - x + 1)
```

3.228.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = \frac{\ln\left(x^2-\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+1\right)}{2}+\frac{\ln\left(\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+x^2+1\right)}{2} - \frac{\sqrt{5}\,\ln\left(x^2-\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+1\right)}{2}+\frac{\sqrt{5}\,\ln\left(\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+x^2+1\right)}{2}$$

input
```
int((x - 4*x^2 + 2*x^3 + 2)/(x^2 - x - x^3 + x^4 + 1),x)
```

output
```
log(x^2 - (5^(1/2)*x)/2 - x/2 + 1)/2 + log((5^(1/2)*x)/2 - x/2 + x^2 + 1)/
2 - (5^(1/2)*log(x^2 - (5^(1/2)*x)/2 - x/2 + 1))/2 + (5^(1/2)*log((5^(1/2)
*x)/2 - x/2 + x^2 + 1))/2
```

3.229 $\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4} dx$

3.229.1 Optimal result

Integrand size = 33, antiderivative size = 14

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx = \frac{1}{3(1+x)^3} + \log(1+x)$$

output
```
1/3/(1+x)^3+ln(1+x)
```

3.229.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx = \frac{1}{3(1+x)^3} + \log(1+x)$$

input
```
Integrate[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

output
```
1/(3*(1 + x)^3) + Log[1 + x]
```

3.229.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2007, 2028, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+3x^2+3x}{x^4+4x^3+6x^2+4x+1} \, dx$$

$$\Big\downarrow \text{2007}$$

$$\int \frac{x^3+3x^2+3x}{(x+1)^4}dx$$

$$\Big\downarrow \text{2028}$$

$$\int \frac{x(x^2+3x+3)}{(x+1)^4}dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{1}{x+1}-\frac{1}{(x+1)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3(x+1)^3}+\log(x+1)$$

input
```
Int[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

output
```
1/(3*(1 + x)^3) + Log[1 + x]
```

3.229.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2007
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Ex
pon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; IntegerQ[p] && Pol
yQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

3.229.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{1}{3(x+1)^3}+\ln(x+1)$	13
norman	$\frac{1}{3(x+1)^3}+\ln(x+1)$	13
risch	$\frac{1}{3x^3+9x^2+9x+3}+\ln(x+1)$	23
parallelrisch	$\frac{3\ln(x+1)x^3+1+9\ln(x+1)x^2+9\ln(x+1)x+3\ln(x+1)}{3x^3+9x^2+9x+3}$	51

input
```
int((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/3/(x+1)^3+ln(x+1)
```

3.229.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(12) = 24.

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.71

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx=\frac{3\left(x^3+3\,x^2+3\,x+1\right)\log\left(x+1\right)+1}{3\left(x^3+3\,x^2+3\,x+1\right)}$$

input
```
integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="fricas")
```

output
```
1/3*(3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 1)/(x^3 + 3*x^2 + 3*x + 1)
```

3.229.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \log{(x+1)} + \frac{1}{3x^3+9x^2+9x+3}$$

input
```
integrate((x**3+3*x**2+3*x)/(x**4+4*x**3+6*x**2+4*x+1),x)
```

output
```
log(x + 1) + 1/(3*x**3 + 9*x**2 + 9*x + 3)
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{1}{3\,(x^3+3\,x^2+3\,x+1)} + \log{(x+1)}$$

input
```
integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")
```

output
```
1/3/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

3.229.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{1}{3\,(x+1)^3} + \log{(|x+1|)}$$

input
```
integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")
```

output
```
1/3/(x + 1)^3 + log(abs(x + 1))
```

3.229.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \ln(x+1) + \frac{1}{3(x+1)^3}$$

input
```
int((3*x + 3*x^2 + x^3)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1),x)
```

output
```
log(x + 1) + 1/(3*(x + 1)^3)
```

3.230 $\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4} dx$

3.230.1 Optimal result

Integrand size = 34, antiderivative size = 28

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4} dx = \frac{8}{3(1+x)^3} - \frac{6}{(1+x)^2} + \frac{6}{1+x} + \log(1+x)$$

output
```
8/3/(1+x)^3-6/(1+x)^2+6/(1+x)+ln(1+x)
```

3.230.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4} dx = \frac{2(4+9x+9x^2)}{3(1+x)^3} + \log(1+x)$$

input
```
Integrate[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

output
```
(2*(4 + 9*x + 9*x^2))/(3*(1 + x)^3) + Log[1 + x]
```

3.230.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2006, 2007, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-3x^2+3x-1}{x^4+4x^3+6x^2+4x+1}\,dx$$

$$\Big\downarrow \text{2006}$$

$$\int \frac{(x-1)^3}{x^4+4x^3+6x^2+4x+1}dx$$

$$\Big\downarrow \text{2007}$$

$$\int \frac{(x-1)^3}{(x+1)^4}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{1}{x+1}-\frac{6}{(x+1)^2}+\frac{12}{(x+1)^3}-\frac{8}{(x+1)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{6}{x+1}-\frac{6}{(x+1)^2}+\frac{8}{3(x+1)^3}+\log(x+1)$$

input
```
Int[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

output
```
8/(3*(1 + x)^3) - 6/(1 + x)^2 + 6/(1 + x) + Log[1 + x]
```

3.230.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2007
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Ex
pon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; IntegerQ[p] && Pol
yQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.230.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

method	result	size
norman	$\frac{6x+6x^2+\frac{8}{3}}{(x+1)^3}+\ln(x+1)$	22
default	$\frac{8}{3(x+1)^3}-\frac{6}{(x+1)^2}+\frac{6}{x+1}+\ln(x+1)$	27
risch	$\frac{6x+6x^2+\frac{8}{3}}{x^3+3x^2+3x+1}+\ln(x+1)$	32
parallelrisch	$\frac{3\ln(x+1)x^3+8+9\ln(x+1)x^2+9\ln(x+1)x+18x^2+3\ln(x+1)+18x}{3x^3+9x^2+9x+3}$	59

input
```
int((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x,method=_RETURNVERBOSE)
```

output
```
(6*x+6*x^2+8/3)/(x+1)^3+ln(x+1)
```

3.230.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{18\,x^2+3\,(x^3+3\,x^2+3\,x+1)\log{(x+1)}+18\,x+8}{3\,(x^3+3\,x^2+3\,x+1)}$$

input
```
integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="fricas")
```

output
```
1/3*(18*x^2 + 3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 8)/(x^3 + 3*x^
2 + 3*x + 1)
```

3.230.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{18x^2+18x+8}{3x^3+9x^2+9x+3}+\log{(x+1)}$$

input
```
integrate((x**3-3*x**2+3*x-1)/(x**4+4*x**3+6*x**2+4*x+1),x)
```

output
```
(18*x**2 + 18*x + 8)/(3*x**3 + 9*x**2 + 9*x + 3) + log(x + 1)
```

3.230.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{2\,(9\,x^2+9\,x+4)}{3\,(x^3+3\,x^2+3\,x+1)}+\log{(x+1)}$$

input
```
integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")
```

output
```
2/3*(9*x^2 + 9*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

3.230.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{2\left(9\,x^2+9\,x+4\right)}{3\left(x+1\right)^3}+\log\left(|x+1|\right)$$

input
```
integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")
```

output
```
2/3*(9*x^2 + 9*x + 4)/(x + 1)^3 + log(abs(x + 1))
```

3.230.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \ln\left(x+1\right)+\frac{6\,x^2+6\,x+\frac{8}{3}}{\left(x+1\right)^3}$$

input
```
int((3*x - 3*x^2 + x^3 - 1)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1),x)
```

output
```
log(x + 1) + (6*x + 6*x^2 + 8/3)/(x + 1)^3
```

3.231 $\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx$

3.231.1 Optimal result

Integrand size = 43, antiderivative size = 59

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{2(1-2x^2)}{(3+2x^2+x^4)^2}-\frac{2x(18+13x^2)}{(3+2x^2+x^4)^2}+\frac{13x}{3+2x^2+x^4}$$

output
```
2*(-2*x^2+1)/(x^4+2*x^2+3)^2-2*x*(13*x^2+18)/(x^4+2*x^2+3)^2+13*x/(x^4+2*x
^2+3)
```

3.231.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{2+3x-4x^2+13x^5}{(3+2x^2+x^4)^2}$$

input
```
Integrate[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*
x^2 + x^4)^3,x]
```

output
```
(2 + 3*x - 4*x^2 + 13*x^5)/(3 + 2*x^2 + x^4)^2
```

3.231.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {2202, 2194, 27, 2191, 24, 2206, 27, 2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-39x^8+26x^6+24x^5+174x^4-18x^2-40x+9}{(x^4+2x^2+3)^3}dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{x(24x^4-40)}{(x^4+2x^2+3)^3}dx+\int \frac{-39x^8+26x^6+174x^4-18x^2+9}{(x^4+2x^2+3)^3}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int -\frac{8(5-3x^4)}{(x^4+2x^2+3)^3}dx^2+\int \frac{-39x^8+26x^6+174x^4-18x^2+9}{(x^4+2x^2+3)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{-39x^8+26x^6+174x^4-18x^2+9}{(x^4+2x^2+3)^3}dx-4\int \frac{5-3x^4}{(x^4+2x^2+3)^3}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\int \frac{-39x^8+26x^6+174x^4-18x^2+9}{(x^4+2x^2+3)^3}dx-4\left(\frac{\int 0dx^2}{16}-\frac{1-2x^2}{2(x^4+2x^2+3)^2}\right)$$

$$\Big\downarrow \text{24}$$

$$\int \frac{-39x^8+26x^6+174x^4-18x^2+9}{(x^4+2x^2+3)^3}dx+\frac{2(1-2x^2)}{(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{96}\int \frac{1248(-3x^4-2x^2+3)}{(x^4+2x^2+3)^2}dx+\frac{2(1-2x^2)}{(x^4+2x^2+3)^2}-\frac{2x(13x^2+18)}{(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{27}$$

$$13\int \frac{-3x^4-2x^2+3}{(x^4+2x^2+3)^2}dx+\frac{2(1-2x^2)}{(x^4+2x^2+3)^2}-\frac{2x(13x^2+18)}{(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2021}$$

$$\frac{13x}{x^4+2x^2+3}-\frac{2(13x^2+18)x}{(x^4+2x^2+3)^2}+\frac{2(1-2x^2)}{(x^4+2x^2+3)^2}$$

input
```
Int[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x^2 +
x^4)^3,x]
```

output
```
(2*(1 - 2*x^2))/(3 + 2*x^2 + x^4)^2 - (2*x*(18 + 13*x^2))/(3 + 2*x^2 + x^4
)^2 + (13*x)/(3 + 2*x^2 + x^4)
```

3.231.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.231. $\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx$

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.231.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{13x^5-4x^2+3x+2}{(x^4+2x^2+3)^2}$	29
norman	$\frac{13x^5-4x^2+3x+2}{(x^4+2x^2+3)^2}$	29
risch	$\frac{13x^5-4x^2+3x+2}{(x^4+2x^2+3)^2}$	29
parallelrisch	$\frac{13x^5-4x^2+3x+2}{(x^4+2x^2+3)^2}$	29
default	$-\frac{-13x^5+4x^2-3x-2}{(x^4+2x^2+3)^2}$	30

input
```
int((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,method
=_RETURNVERBOSE)
```

output
```
(13*x^5-4*x^2+3*x+2)/(x^4+2*x^2+3)^2
```

3.231. $\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx$

3.231.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{13\,x^5-4\,x^2+3\,x+2}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$

input
```
integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
 algorithm="fricas")
```

output
```
(13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

3.231.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = -\frac{-13x^5+4x^2-3x-2}{x^8+4x^6+10x^4+12x^2+9}$$

input
```
integrate((-39*x**8+26*x**6+24*x**5+174*x**4-18*x**2-40*x+9)/(x**4+2*x**2+
3)**3,x)
```

output
```
-(-13*x**5 + 4*x**2 - 3*x - 2)/(x**8 + 4*x**6 + 10*x**4 + 12*x**2 + 9)
```

3.231.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{13\,x^5-4\,x^2+3\,x+2}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$

input
```
integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
 algorithm="maxima")
```

output
```
(13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

3.231.8 Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{13\,x^5-4\,x^2+3\,x+2}{(x^4+2\,x^2+3)^2}$$

input
```
integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
 algorithm="giac")
```

output
```
(13*x^5 - 4*x^2 + 3*x + 2)/(x^4 + 2*x^2 + 3)^2
```

3.231.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{13\,x^5-4\,x^2+3\,x+2}{(x^4+2\,x^2+3)^2}$$

input
```
int((174*x^4 - 18*x^2 - 40*x + 24*x^5 + 26*x^6 - 39*x^8 + 9)/(2*x^2 + x^4
+ 3)^3,x)
```

output
```
(3*x - 4*x^2 + 13*x^5 + 2)/(2*x^2 + x^4 + 3)^2
```

3.232 $\int \frac{-1+4x^5}{(1+x+x^5)^2} \, dx$

3.232.1 Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{-1+4x^5}{(1+x+x^5)^2} \, dx = -\frac{x}{1+x+x^5}$$

output
```
-x/(x^5+x+1)
```

3.232.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2} \, dx = -\frac{x}{1+x+x^5}$$

input
```
Integrate[(-1 + 4*x^5)/(1 + x + x^5)^2,x]
```

output
```
-(x/(1 + x + x^5))
```

3.232.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^5-1}{(x^5+x+1)^2}\,dx$$

$$\Big\downarrow \text{2021}$$

$$-\frac{x}{x^5+x+1}$$

input
```
Int[(-1 + 4*x^5)/(1 + x + x^5)^2,x]
```

output
```
-(x/(1 + x + x^5))
```

3.232.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.232.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{x}{x^5+x+1}$	12
norman	$-\frac{x}{x^5+x+1}$	12
risch	$-\frac{x}{x^5+x+1}$	12
parallelrisch	$-\frac{x}{x^5+x+1}$	12
default	$-\frac{-3x^2+5x-1}{7(x^3-x^2+1)}+\frac{-3x-1}{7x^2+7x+7}$	41

input
```
int((4*x^5-1)/(x^5+x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-x/(x^5+x+1)
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

input
```
integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="fricas")
```

output
```
-x/(x^5 + x + 1)
```

3.232.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

input
```
integrate((4*x**5-1)/(x**5+x+1)**2,x)
```

output
```
-x/(x**5 + x + 1)
```

3.232.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

input
```
integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="maxima")
```

output
```
-x/(x^5 + x + 1)
```

3.232.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

input
```
integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="giac")
```

output
```
-x/(x^5 + x + 1)
```

3.232.9 Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = -\frac{x}{x^5+x+1}$$

input
```
int((4*x^5 - 1)/(x + x^5 + 1)^2,x)
```

output
```
-x/(x + x^5 + 1)
```

3.233 $\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx$

3.233.1 Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx = \frac{x}{16(1-x^2)}+\frac{x(29-5x^2)}{32(1-6x^2+x^4)}+\frac{\operatorname{arctanh}(x)}{4}+\frac{1}{64}\left(\left(3-2\sqrt{2}\right)\operatorname{arctanh}\left(\left(-1+\sqrt{2}\right)x\right)-\left(3+2\sqrt{2}\right)\operatorname{arctanh}\left(\left(1+\sqrt{2}\right)x\right)\right)$$

output
```
1/16*x/(-x^2+1)+1/32*x*(-5*x^2+29)/(x^4-6*x^2+1)+1/4*arctanh(x)+1/64*arcta
nh(x*(2^(1/2)-1))*(3-2*2^(1/2))-1/64*arctanh(x*(1+2^(1/2)))*(3+2*2^(1/2))
```

3.233.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.45

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx = \frac{1}{128}\left(-\frac{4x(31-46x^2+7x^4)}{-1+7x^2-7x^4+x^6}-16\log(1-x)+\left(3+2\sqrt{2}\right)\log\left(-1+\sqrt{2}-x\right)+\left(-3+2\sqrt{2}\right)\log\left(1+\sqrt{2}-x\right)+16\log(1+x)-\left(3+2\sqrt{2}\right)\log\left(-1+\sqrt{2}+x\right)+\left(3-2\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)\right)$$

input
```
Integrate[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2,x]
```

output
```
((-4*x*(31 - 46*x^2 + 7*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 16*Log[1 - x] +
 (3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (-3 + 2*Sqrt[2])*Log[1 + Sqrt[2]
- x] + 16*Log[1 + x] - (3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (3 - 2*Sqrt
[2])*Log[1 + Sqrt[2] + x])/128
```

3.233.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 245 vs. 2(91) = 182.

Time = 0.46 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.69, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{(-x^6+7x^4-7x^2+1)^2}\,dx$$

$$\Big\downarrow \; 2460$$

$$\int \left(\frac{17-7x}{32\left(x^2-2x-1\right)^2}-\frac{1}{4\left(x^2-1\right)}-\frac{3(x-4)}{64\left(x^2-2x-1\right)}+\frac{3(x+4)}{64\left(x^2+2x-1\right)}+\frac{7x+17}{32\left(x^2+2x-1\right)^2}+\frac{1}{32(x-1)^2}+\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{\operatorname{arctanh}(x)}{4}-\frac{12-5x}{64\left(-x^2+2x+1\right)}+\frac{5x+12}{64\left(-x^2-2x+1\right)}+\frac{1}{32(1-x)}-\frac{1}{32(x+1)}-\frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(-x-\sqrt{2}+1\right)+\frac{5\log\left(-x-\sqrt{2}+1\right)}{128\sqrt{2}}-\frac{3}{256}\left(2-3\sqrt{2}\right)\log\left(-x+\sqrt{2}+1\right)-\frac{5\log\left(-x+\sqrt{2}+1\right)}{128\sqrt{2}}+\frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)-\frac{5\log\left(x-\sqrt{2}+1\right)}{128\sqrt{2}}+\frac{3}{256}\left(2-3\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)+\frac{5\log\left(x+\sqrt{2}+1\right)}{128\sqrt{2}}$$

input
```
Int[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2,x]
```

output
```
1/(32*(1 - x)) - 1/(32*(1 + x)) + (12 + 5*x)/(64*(1 - 2*x - x^2)) - (12 -
5*x)/(64*(1 + 2*x - x^2)) + ArcTanh[x]/4 + (5*Log[1 - Sqrt[2] - x])/(128*S
qrt[2]) - (3*(2 + 3*Sqrt[2])*Log[1 - Sqrt[2] - x])/256 - (5*Log[1 + Sqrt[2
] - x])/(128*Sqrt[2]) - (3*(2 - 3*Sqrt[2])*Log[1 + Sqrt[2] - x])/256 - (5*
Log[1 - Sqrt[2] + x])/(128*Sqrt[2]) + (3*(2 + 3*Sqrt[2])*Log[1 - Sqrt[2] +
 x])/256 + (5*Log[1 + Sqrt[2] + x])/(128*Sqrt[2]) + (3*(2 - 3*Sqrt[2])*Log
[1 + Sqrt[2] + x])/256
```

3.233.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

3.233.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

method	result
default	$-\frac{1}{32(x+1)}+\frac{\ln(x+1)}{8}+\frac{-5x-12}{64x^2+128x-64}+\frac{3\ln(x^2+2x-1)}{128}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{32}-\frac{1}{32(x-1)}-\frac{\ln(x-1)}{8}-\frac{5x-}{64(x^2-}$
risch	$\frac{-\frac{7}{32}x^5+\frac{23}{16}x^3-\frac{31}{32}x}{x^6-7x^4+7x^2-1}-\frac{3\ln\left(x-1-\sqrt{2}\right)}{128}+\frac{\ln\left(x-1-\sqrt{2}\right)\sqrt{2}}{64}-\frac{3\ln\left(x-1+\sqrt{2}\right)}{128}-\frac{\ln\left(x-1+\sqrt{2}\right)\sqrt{2}}{64}+\frac{\ln(x+1)}{8}-\frac{\ln(x-1)}{8}$

input
```
int((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/32/(x+1)+1/8*ln(x+1)+1/64*(-5*x-12)/(x^2+2*x-1)+3/128*ln(x^2+2*x-1)-1/3
2*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))-1/32/(x-1)-1/8*ln(x-1)-1/64*(5*x-12
)/(x^2-2*x-1)-3/128*ln(x^2-2*x-1)-1/32*2^(1/2)*arctanh(1/4*(2*x-2)*2^(1/2)
)
```

3.233.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(71) = 142.

Time = 0.34 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.45

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx =$$
$$-\frac{28\,x^5-184\,x^3-2\,\sqrt{2}(x^6-7\,x^4+7\,x^2-1)\log\left(\frac{x^2-2\,\sqrt{2}(x+1)+2\,x+3}{x^2+2\,x-1}\right)-2\,\sqrt{2}(x^6-7\,x^4+7\,x^2-1)\log\left(\right.}{}$$

input
```
integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/128*(28*x^5 - 184*x^3 - 2*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2 -
2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 2*sqrt(2)*(x^6 - 7*x^4 + 7
*x^2 - 1)*log((x^2 - 2*sqrt(2)*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) - 3*(x^
6 - 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x - 1) + 3*(x^6 - 7*x^4 + 7*x^2 - 1)*lo
g(x^2 - 2*x - 1) - 16*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 16*(x^6 - 7*x
^4 + 7*x^2 - 1)*log(x - 1) + 124*x)/(x^6 - 7*x^4 + 7*x^2 - 1)
```

3.233.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(73) = 146.

Time = 0.80 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.99

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx$$

$$= \frac{-7x^5+46x^3-31x}{32x^6-224x^4+224x^2-32} - \frac{\log(x-1)}{8} + \frac{\log(x+1)}{8} + \left(-\frac{3}{128} - \frac{\sqrt{2}}{64}\right)\log\left(x - \frac{38423555}{909328} - \frac{38423555\sqrt{2}}{1363992} + \frac{9549859782656\left(-\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^5}{170499} - \frac{56267374592\left(-\frac{3}{128}-\frac{\sqrt{2}}{64}\right.\cdots}{56833}\right.$$

$$+ \left(-\frac{3}{128} + \frac{\sqrt{2}}{64}\right)\log\left(x - \frac{38423555}{909328} + \frac{9549859782656\left(-\frac{3}{128}+\frac{\sqrt{2}}{64}\right)^5}{170499} - \frac{56267374592\left(-\frac{3}{128}+\frac{\sqrt{2}}{64}\right)^3}{56833} + \frac{38423555\cdots}{1363992}\right.$$

$$+ \left(\frac{3}{128} - \frac{\sqrt{2}}{64}\right)\log\left(x - \frac{38423555\sqrt{2}}{1363992} - \frac{56267374592\left(\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^3}{56833} + \frac{9549859782656\left(\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^5}{170499} + \frac{38423555}{909328}\right)$$

$$+ \left(\frac{\sqrt{2}}{64} + \frac{3}{128}\right)\log\left(x - \frac{56267374592\left(\frac{\sqrt{2}}{64}+\frac{3}{128}\right)^3}{56833} + \frac{9549859782656\left(\frac{\sqrt{2}}{64}+\frac{3}{128}\right)^5}{170499} + \frac{38423555\sqrt{2}}{1363992} + \frac{38423555}{909328}\right)$$

input
```
integrate((x**2+1)/(-x**6+7*x**4-7*x**2+1)**2,x)
```

3.233. $\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx$

output
```
(-7*x**5 + 46*x**3 - 31*x)/(32*x**6 - 224*x**4 + 224*x**2 - 32) - log(x -
1)/8 + log(x + 1)/8 + (-3/128 - sqrt(2)/64)*log(x - 38423555/909328 - 3842
3555*sqrt(2)/1363992 + 9549859782656*(-3/128 - sqrt(2)/64)**5/170499 - 562
67374592*(-3/128 - sqrt(2)/64)**3/56833) + (-3/128 + sqrt(2)/64)*log(x - 3
8423555/909328 + 9549859782656*(-3/128 + sqrt(2)/64)**5/170499 - 562673745
92*(-3/128 + sqrt(2)/64)**3/56833 + 38423555*sqrt(2)/1363992) + (3/128 - s
qrt(2)/64)*log(x - 38423555*sqrt(2)/1363992 - 56267374592*(3/128 - sqrt(2)
/64)**3/56833 + 9549859782656*(3/128 - sqrt(2)/64)**5/170499 + 38423555/90
9328) + (sqrt(2)/64 + 3/128)*log(x - 56267374592*(sqrt(2)/64 + 3/128)**3/5
6833 + 9549859782656*(sqrt(2)/64 + 3/128)**5/170499 + 38423555*sqrt(2)/136
3992 + 38423555/909328)
```

3.233.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.25

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx = \frac{1}{64}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{64}\sqrt{2}\log\left(\frac{x-\sqrt{2}-1}{x+\sqrt{2}-1}\right) - \frac{7\,x^5-46\,x^3+31\,x}{32\,(x^6-7\,x^4+7\,x^2-1)}+\frac{3}{128}\log\left(x^2+2\,x-1\right) - \frac{3}{128}\log\left(x^2-2\,x-1\right)+\frac{1}{8}\log\left(x+1\right)-\frac{1}{8}\log\left(x-1\right)$$

input
```
integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="maxima")
```

output
```
1/64*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/64*sqrt(2)*log((
x - sqrt(2) - 1)/(x + sqrt(2) - 1)) - 1/32*(7*x^5 - 46*x^3 + 31*x)/(x^6 -
7*x^4 + 7*x^2 - 1) + 3/128*log(x^2 + 2*x - 1) - 3/128*log(x^2 - 2*x - 1) +
 1/8*log(x + 1) - 1/8*log(x - 1)
```

3.233.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.47

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx = \frac{1}{64}\sqrt{2}\log\left(\frac{|2x-2\sqrt{2}+2|}{|2x+2\sqrt{2}+2|}\right) + \frac{1}{64}\sqrt{2}\log\left(\frac{|2x-2\sqrt{2}-2|}{|2x+2\sqrt{2}-2|}\right) - \frac{7x^5-46x^3+31x}{32(x^6-7x^4+7x^2-1)} + \frac{3}{128}\log\left(|x^2+2x-1|\right) - \frac{3}{128}\log\left(|x^2-2x-1|\right) + \frac{1}{8}\log\left(|x+1|\right) - \frac{1}{8}\log\left(|x-1|\right)$$

input
```
integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="giac")
```

output
```
1/64*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/64
*sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2)/abs(2*x + 2*sqrt(2) - 2)) - 1/32*(7*
x^5 - 46*x^3 + 31*x)/(x^6 - 7*x^4 + 7*x^2 - 1) + 3/128*log(abs(x^2 + 2*x -
 1)) - 3/128*log(abs(x^2 - 2*x - 1)) + 1/8*log(abs(x + 1)) - 1/8*log(abs(x
 - 1))
```

3.233.9 Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.36

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx = -\frac{\mathrm{atan}(x\,1\mathrm{i})\,1\mathrm{i}}{4} - \frac{\frac{7x^5}{32}-\frac{23x^3}{16}+\frac{31x}{32}}{x^6-7x^4+7x^2-1} + \mathrm{atan}\left(\frac{x\,23313\mathrm{i}}{8192\left(\frac{27309\sqrt{2}}{32768}-\frac{19317}{16384}\right)} - \frac{\sqrt{2}\,x\,65943\mathrm{i}}{32768\left(\frac{27309\sqrt{2}}{32768}-\frac{19317}{16384}\right)}\right)\left(\frac{\sqrt{2}\,1\mathrm{i}}{32}-\frac{3}{64}\mathrm{i}\right) + \mathrm{atan}\left(\frac{x\,23313\mathrm{i}}{8192\left(\frac{27309\sqrt{2}}{32768}+\frac{19317}{16384}\right)} + \frac{\sqrt{2}\,x\,65943\mathrm{i}}{32768\left(\frac{27309\sqrt{2}}{32768}+\frac{19317}{16384}\right)}\right)\left(\frac{\sqrt{2}\,1\mathrm{i}}{32}+\frac{3}{64}\mathrm{i}\right)$$

input
```
int((x^2 + 1)/(7*x^2 - 7*x^4 + x^6 - 1)^2,x)
```

output
```
atan((x*23313i)/(8192*((27309*2^(1/2))/32768 - 19317/16384)) - (2^(1/2)*x*
65943i)/(32768*((27309*2^(1/2))/32768 - 19317/16384)))*((2^(1/2)*1i)/32 -
3i/64) - ((31*x)/32 - (23*x^3)/16 + (7*x^5)/32)/(7*x^2 - 7*x^4 + x^6 - 1)
- (atan(x*1i)*1i)/4 + atan((x*23313i)/(8192*((27309*2^(1/2))/32768 + 19317
/16384)) + (2^(1/2)*x*65943i)/(32768*((27309*2^(1/2))/32768 + 19317/16384)
))*((2^(1/2)*1i)/32 + 3i/64)
```

3.233. $\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx$

3.234 $\int x^m(a+bx+cx^2+dx^3)^p\,(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx$

3.234.1 Optimal result

Integrand size = 56, antiderivative size = 25

$$\int x^m\left(a+bx+cx^2+dx^3\right)^p(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx=x^{1+m}\left(a+bx+cx^2+dx^3\right)^{1+p}$$

output
```
x^(1+m)*(d*x^3+c*x^2+b*x+a)^(p+1)
```

3.234.2 Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x^m\left(a+bx+cx^2+dx^3\right)^p(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx=x^{1+m}(a+x(b+x(c+dx)))^{1+p}$$

input
```
Integrate[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) +
x*(c*(3 + m + 2*p) + d*(4 + m + 3*p)*x))),x]
```

output
```
x^(1 + m)*(a + x*(b + x*(c + d*x)))^(1 + p)
```

3.234.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.018$, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(m+1)+x(b(m+p+2)+x(c(m+2p+3)+dx(m+3p+4)))) \, dx$$

$$\Big\downarrow \text{ 2023}$$

$$x^{m+1}\left(a+bx+cx^2+dx^3\right)^{p+1}$$

input
```
Int[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x*(c*(
3 + m + 2*p) + d*(4 + m + 3*p)*x))),x]
```

output
```
x^(1 + m)*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

3.234.3.1 Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

3.234.4 Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
gosper	$x^{1+m}\left(x^3 d+c\,x^2+bx+a\right)^{1+p}$	26
risch	$\left(x^3 d+c\,x^2+bx+a\right)^p x^m x\left(x^3 d+c\,x^2+bx+a\right)$	38
parallelrisch	$\frac{x^4 x^m \left(x^3 d+c\,x^2+bx+a\right)^p a d+x^3 x^m \left(x^3 d+c\,x^2+bx+a\right)^p a c+x^2 x^m \left(x^3 d+c\,x^2+bx+a\right)^p a b+x\,x^m \left(x^3 d+c\,x^2+bx+a\right)^p a^2}{a}$	109

input
```
int(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+
3*p)*x))),x,method=_RETURNVERBOSE)
```

output
```
x^(1+m)*(d*x^3+c*x^2+b*x+a)^(1+p)
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int x^m (a + bx + cx^2 + dx^3)^p (a(1+m) + x(b(2+m+p) + x(c(3+m+2p)+d(4+m+3p)x)))\,dx = (dx^4 + cx^3 + bx^2 + ax)(dx^3 + cx^2 + bx + a)^p x^m$$

input
```
integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d
*(4+m+3*p)*x))),x, algorithm="fricas")
```

output
```
(d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p*x^m
```

3.234.6 Sympy [F(-1)]

Timed out.

$$\int x^m (a + bx + cx^2 + dx^3)^p (a(1+m) + x(b(2+m+p) + x(c(3+m+2p) + d(4+m+3p)x)))\,dx = \text{Timed out}$$

input
```
integrate(x**m*(d*x**3+c*x**2+b*x+a)**p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*
p)+d*(4+m+3*p)*x))),x)
```

output
```
Timed out
```

3.234.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx$$
$$= \left(dx^4+cx^3+bx^2+ax\right)e^{\left(p\log\left(dx^3+cx^2+bx+a\right)+m\log(x)\right)}$$

input
```
integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d
*(4+m+3*p)*x))),x, algorithm="maxima")
```

output
```
(d*x^4 + c*x^3 + b*x^2 + a*x)*e^(p*log(d*x^3 + c*x^2 + b*x + a) + m*log(x)
)
```

3.234.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(25) = 50.

Time = 0.54 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.96

$$\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(1+m)$$
$$+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx = \left(dx^3+cx^2+bx+a\right)^p dx^4x^m$$
$$+\left(dx^3+cx^2+bx+a\right)^p cx^3x^m$$
$$+\left(dx^3+cx^2+bx+a\right)^p bx^2x^m + \left(dx^3+cx^2+bx+a\right)^p axx^m$$

input
```
integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d
*(4+m+3*p)*x))),x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^p*d*x^4*x^m + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3*
x^m + (d*x^3 + c*x^2 + b*x + a)^p*b*x^2*x^m + (d*x^3 + c*x^2 + b*x + a)^p*
a*x*x^m
```

3.234.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(1+m) +x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx = \left(d\,x^3+c\,x^2+b\,x+a\right)^p\,\left(a\,x\,x^m +b\,x^m\,x^2+c\,x^m\,x^3+d\,x^m\,x^4\right)$$

input
```
int(x^m*(a*(m + 1) + x*(x*(c*(m + 2*p + 3) + d*x*(m + 3*p + 4)) + b*(m + p
 + 2)))*(a + b*x + c*x^2 + d*x^3)^p,x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^p*(a*x*x^m + b*x^m*x^2 + c*x^m*x^3 + d*x^m*x^4)
```

3.235 $\int x^2(a+bx+cx^2+dx^3)^p\,(3a+b(4+p)x+c(5+2$

3.235.1 Optimal result

Integrand size = 51, antiderivative size = 23

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx = x^3\left(a+bx+cx^2+dx^3\right)^{1+p}$$

output
```
x^3*(d*x^3+c*x^2+b*x+a)^(p+1)
```

3.235.2 Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx = x^3(a+x(b+x(c+dx)))^{1+p}$$

input
```
Integrate[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)
*x^2 + d*(6 + 3*p)*x^3),x]
```

output
```
x^3*(a + x*(b + x*(c + d*x)))^(1 + p)
```

3.235.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = $\{2021\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx+cx^2+dx^3\right)^p \left(3a+b(p+4)x+c(2p+5)x^2+d(3p+6)x^3\right) \, dx$$

$$\Big\downarrow \text{2021}$$

$$x^3 \left(a+bx+cx^2+dx^3\right)^{p+1}$$

input
```
Int[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*x^2 +
 d*(6 + 3*p)*x^3),x]
```

output
```
x^3*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

3.235.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.235.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result
gosper	$x^3 \left(x^3 d+c\,x^2+bx+a\right)^{1+p}$
risch	$\left(x^3 d+c\,x^2+bx+a\right)^p x^3 \left(x^3 d+c\,x^2+bx+a\right)$
norman	$a\,x^3 {\mathrm e}^{p \ln\left(x^3 d+c\,x^2+bx+a\right)}+b\,x^4 {\mathrm e}^{p \ln\left(x^3 d+c\,x^2+bx+a\right)}+c\,x^5 {\mathrm e}^{p \ln\left(x^3 d+c\,x^2+bx+a\right)}+d\,x^6 {\mathrm e}^{p \ln\left(x^3 d+c\,x^2+bx+\right.}$
parallelrisch	$\frac{x^6 \left(x^3 d+c\,x^2+bx+a\right)^p c d+x^5 \left(x^3 d+c\,x^2+bx+a\right)^p c^2+x^4 \left(x^3 d+c\,x^2+bx+a\right)^p b c+x^3 \left(x^3 d+c\,x^2+bx+a\right)^p a c}{c}$

input
```
int(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3),
x,method=_RETURNVERBOSE)
```

output
```
x^3*(d*x^3+c*x^2+b*x+a)^(1+p)
```

3.235.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx = \left(dx^6+cx^5+bx^4+ax^3\right)\left(dx^3+cx^2+bx+a\right)^p$$

input
```
integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)
*x^3),x, algorithm="fricas")
```

output
```
(d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

3.235.6 Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx = \text{Timed out}$$

input
```
integrate(x**2*(d*x**3+c*x**2+b*x+a)**p*(3*a+b*(4+p)*x+c*(5+2*p)*x**2+d*(6
+3*p)*x**3),x)
```

output
```
Timed out
```

3.235.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x^2(a+bx+cx^2+dx^3)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx$$
$$=\left(dx^6+cx^5+bx^4+ax^3\right)\left(dx^3+cx^2+bx+a\right)^p$$

input
```
integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)
*x^3),x, algorithm="maxima")
```

output
```
(d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

3.235.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(23) = 46.

Time = 0.53 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.87

$$\int x^2(a+bx+cx^2+dx^3)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx$$
$$=\left(dx^3+cx^2+bx+a\right)^p dx^6+\left(dx^3+cx^2+bx+a\right)^p cx^5$$
$$+\left(dx^3+cx^2+bx+a\right)^p bx^4+\left(dx^3+cx^2+bx+a\right)^p ax^3$$

input
```
integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)
*x^3),x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^p*d*x^6 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^5 + (d
*x^3 + c*x^2 + b*x + a)^p*b*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^3
```

3.235.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x^2(a+bx+cx^2+dx^3)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx$$
$$=\left(d\,x^3+c\,x^2+b\,x+a\right)^p\left(d\,x^6+c\,x^5+b\,x^4+a\,x^3\right)$$

input
```
int(x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*x*(p + 4) + c*x^2*(2*p + 5) +
 d*x^3*(3*p + 6)),x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^p*(a*x^3 + b*x^4 + c*x^5 + d*x^6)
```

3.235. $\int x^2(a+bx+cx^2+dx^3)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx$

3.236 $\int x(a+bx+cx^2+dx^3)^p\left(2a+b(3+p)x+c(4+2p\right.$

3.236.1 Optimal result

Integrand size = 49, antiderivative size = 23

$$\int x(a+bx+cx^2+dx^3)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx = x^2\left(a+bx+cx^2+dx^3\right)^{1+p}$$

output
```
x^2*(d*x^3+c*x^2+b*x+a)^(p+1)
```

3.236.2 Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x(a+bx+cx^2+dx^3)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx = x^2(a+x(b+x(c+dx)))^{1+p}$$

input
```
Integrate[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x
^2 + d*(5 + 3*p)*x^3),x]
```

output
```
x^2*(a + x*(b + x*(c + d*x)))^(1 + p)
```

3.236.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(p+3)x+c(2p+4)x^2+d(3p+5)x^3\right)dx$$

$$\Big\downarrow \text{2021}$$

$$x^2\left(a+bx+cx^2+dx^3\right)^{p+1}$$

input
```
Int[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^2 + d
*(5 + 3*p)*x^3),x]
```

output
```
x^2*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

3.236.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.236.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result
gosper	$x^{2} \left(x^{3} d +c \,x^{2}+b x +a \right)^{1+p}$
risch	$\left(x^{3} d +c \,x^{2}+b x +a \right)^{p} x^{2} \left(x^{3} d +c \,x^{2}+b x +a \right)$
norman	$a \,x^{2} {\mathrm e}^{p \ln \left(x^{3} d +c \,x^{2}+b x +a \right)}+b \,x^{3} {\mathrm e}^{p \ln \left(x^{3} d +c \,x^{2}+b x +a \right)}+c \,x^{4} {\mathrm e}^{p \ln \left(x^{3} d +c \,x^{2}+b x +a \right)}+x^{5} d \,{\mathrm e}^{p \ln \left(x^{3} d +c \,x^{2}+b x +\right.}$
parallelrisch	$\frac{x^{5} \left(x^{3} d +c \,x^{2}+b x +a \right)^{p} a d +x^{4} \left(x^{3} d +c \,x^{2}+b x +a \right)^{p} a c +a b \left(x^{3} d +c \,x^{2}+b x +a \right)^{p} x^{3}+a^{2} \left(x^{3} d +c \,x^{2}+b x +a \right)^{p} x^{2}}{a}$

input
```
int(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3),x,
method=_RETURNVERBOSE)
```

output
```
x^2*(d*x^3+c*x^2+b*x+a)^(1+p)
```

3.236.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx = \left(dx^5+cx^4+bx^3+ax^2\right)\left(dx^3+cx^2+bx+a\right)^p$$

input
```
integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x
^3),x, algorithm="fricas")
```

output
```
(d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

3.236.6 Sympy [F(-1)]

Timed out.

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx = \text{Timed out}$$

input
```
integrate(x*(d*x**3+c*x**2+b*x+a)**p*(2*a+b*(3+p)*x+c*(4+2*p)*x**2+d*(5+3*
p)*x**3),x)
```

output
```
Timed out
```

3.236.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x(a+bx+cx^2+dx^3)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx$$
$$=\left(dx^5+cx^4+bx^3+ax^2\right)\left(dx^3+cx^2+bx+a\right)^p$$

input
```
integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x
^3),x, algorithm="maxima")
```

output
```
(d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

3.236.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(23) = 46.

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.87

$$\int x(a+bx+cx^2+dx^3)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx$$
$$=\left(dx^3+cx^2+bx+a\right)^p dx^5+\left(dx^3+cx^2+bx+a\right)^p cx^4$$
$$+\left(dx^3+cx^2+bx+a\right)^p bx^3+\left(dx^3+cx^2+bx+a\right)^p ax^2$$

input
```
integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x
^3),x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^p*d*x^5 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^4 + (d
*x^3 + c*x^2 + b*x + a)^p*b*x^3 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^2
```

3.236.9 Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int x(a+bx+cx^2+dx^3)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx$$
$$=\left(d\,x^3+c\,x^2+b\,x+a\right)^p\,\left(d\,x^5+c\,x^4+b\,x^3+a\,x^2\right)$$

input
```
int(x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*x*(p + 3) + c*x^2*(2*p + 4) + d
*x^3*(3*p + 5)),x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^p*(a*x^2 + b*x^3 + c*x^4 + d*x^5)
```

3.236. $\int x(a+bx+cx^2+dx^3)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx$

3.237 $\int (a+bx+cx^2+dx^3)^p (a+b(2+p)x+c(3+2p)x$

3.237.1 Optimal result

Integrand size = 46, antiderivative size = 21

$$\int (a+bx+cx^2+dx^3)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right) dx = x\left(a+bx+cx^2+dx^3\right)^{1+p}$$

output
```
x*(d*x^3+c*x^2+b*x+a)^(p+1)
```

3.237.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int (a+bx+cx^2+dx^3)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right) dx = x(a+x(b+x(c+dx)))^{1+p}$$

input
```
Integrate[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 +
 d*(4 + 3*p)*x^3),x]
```

output
```
x*(a + x*(b + x*(c + d*x)))^(1 + p)
```

3.237.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(p+2)x+c(2p+3)x^2+d(3p+4)x^3\right) \, dx$$

$$\Big\downarrow \text{ 2021}$$

$$x\left(a+bx+cx^2+dx^3\right)^{p+1}$$

input
```
Int[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 + d*(4
+ 3*p)*x^3),x]
```

output
```
x*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

3.237.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.237.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result
gosper	$x\left(x^3d+c\,x^2+bx+a\right)^{1+p}$
risch	$\left(x^3d+c\,x^2+bx+a\right)^p x\left(x^3d+c\,x^2+bx+a\right)$
norman	$ax\,e^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+b\,x^2e^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+c\,x^3e^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+d\,x^4e^{p\ln(x^3d+c\,x^2+bx+a}$
parallelrisch	$\frac{x^4\left(x^3d+c\,x^2+bx+a\right)^p d^2+x^3\left(x^3d+c\,x^2+bx+a\right)^p cd+x^2\left(x^3d+c\,x^2+bx+a\right)^p bd+x\left(x^3d+c\,x^2+bx+a\right)^p ad}{d}$

input
```
int((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x,meth
od=_RETURNVERBOSE)
```

output
```
x*(d*x^3+c*x^2+b*x+a)^(1+p)
```

3.237.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int (a+bx+cx^2+dx^3)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right) dx = (dx^4+cx^3+bx^2+ax)(dx^3+cx^2+bx+a)^p$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),
x, algorithm="fricas")
```

output
```
(d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

3.237.6 Sympy [F(-1)]

Timed out.

$$\int (a+bx+cx^2+dx^3)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right) dx = \text{Timed out}$$

input
```
integrate((d*x**3+c*x**2+b*x+a)**p*(a+b*(2+p)*x+c*(3+2*p)*x**2+d*(4+3*p)*x
**3),x)
```

output
```
Timed out
```

3.237.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right) dx$$
$$= \left(dx^4+cx^3+bx^2+ax\right)\left(dx^3+cx^2+bx+a\right)^p$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),
x, algorithm="maxima")
```

output
```
(d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

3.237.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(21) = 42.

Time = 0.40 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.14

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right) dx$$
$$= \left(dx^3+cx^2+bx+a\right)^p dx^4+\left(dx^3+cx^2+bx+a\right)^p cx^3$$
$$+\left(dx^3+cx^2+bx+a\right)^p bx^2+\left(dx^3+cx^2+bx+a\right)^p ax$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),
x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^p*d*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3 + (d
*x^3 + c*x^2 + b*x + a)^p*b*x^2 + (d*x^3 + c*x^2 + b*x + a)^p*a*x
```

3.237.9 Mupad [B] (verification not implemented)

Time = 9.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right) dx$$
$$= \left(d\,x^3+c\,x^2+b\,x+a\right)^p \left(d\,x^4+c\,x^3+b\,x^2+a\,x\right)$$

input
```
int((a + b*x + c*x^2 + d*x^3)^p*(a + b*x*(p + 2) + c*x^2*(2*p + 3) + d*x^3
*(3*p + 4)),x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^p*(a*x + b*x^2 + c*x^3 + d*x^4)
```

3.237. $\int (a+bx+cx^2+dx^3)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right) \, dx$

3.238 $\int \frac{(a+bx+cx^2+dx^3)^p(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx$

3.238.1 Optimal result

Integrand size = 48, antiderivative size = 19

$$\int \frac{(a+bx+cx^2+dx^3)^p(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = (a+bx+cx^2+dx^3)^{1+p}$$

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)
```

3.238.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx+cx^2+dx^3)^p(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = (a+x(b+x(c+dx)))^{1+p}$$

input
```
Integrate[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*
(3 + 3*p)*x^3))/x,x]
```

output
```
(a + x*(b + x*(c + d*x)))^(1 + p)
```

3.238.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {9, 2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b(p+1)x+c(2p+2)x^2+d(3p+3)x^3)\,(a+bx+cx^2+dx^3)^p}{x}\,dx$$

$$\Big\downarrow \; 9$$

$$\int (b(p+1)+2c(p+1)x+3d(p+1)x^2)\,(a+bx+cx^2+dx^3)^p\,dx$$

$$\Big\downarrow \; 2021$$

$$(a+bx+cx^2+dx^3)^{p+1}$$

input
```
Int[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(3 + 3
*p)*x^3))/x,x]
```

output
```
(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

3.238.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.238.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result
gosper	$(x^3 d + c\,x^2 + bx + a)^{1+p}$
risch	$(x^3 d + c\,x^2 + bx + a)^p (x^3 d + c\,x^2 + bx + a)$
norman	$a\,e^{p\ln(x^3 d + c\,x^2 + bx + a)} + bx\,e^{p\ln(x^3 d + c\,x^2 + bx + a)} + c\,x^2 e^{p\ln(x^3 d + c\,x^2 + bx + a)} + x^3 d\,e^{p\ln(x^3 d + c\,x^2 + bx + a)}$
parallelrisch	$\frac{x^3 (x^3 d + c\,x^2 + bx + a)^p d^2 + x^2 (x^3 d + c\,x^2 + bx + a)^p cd + x (x^3 d + c\,x^2 + bx + a)^p bd + (x^3 d + c\,x^2 + bx + a)^p ad}{d}$

input
```
int((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x,meth
od=_RETURNVERBOSE)
```

output
```
(d*x^3+c*x^2+b*x+a)^(1+p)
```

3.238.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{(a + bx + cx^2 + dx^3)^p (b(1 + p)x + c(2 + 2p)x^2 + d(3 + 3p)x^3)}{x}\,dx = (dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,
x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

3.238.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a + bx + cx^2 + dx^3)^p (b(1 + p)x + c(2 + 2p)x^2 + d(3 + 3p)x^3)}{x}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c*x**2+b*x+a)**p*(b*(1+p)*x+c*(2+2*p)*x**2+d*(3+3*p)*x**
3)/x,x)
```

output
```
Timed out
```

3.238.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = (dx^3+cx^2+bx+a)\,(dx^3+cx^2+bx+a)^p$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,
x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

3.238.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(19) = 38.

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.74

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx = \frac{(dx^3+cx^2+bx+a)^{p+1}p}{p+1}+\frac{(dx^3+cx^2+bx+a)^{p+1}}{p+1}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,
x, algorithm="giac")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^(p + 1)*p/(p + 1) + (d*x^3 + c*x^2 + b*x + a)^(p
 + 1)/(p + 1)
```

3.238.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx$$
$$= \left(d\,x^3+c\,x^2+b\,x+a\right)^{p+1}$$

input
```
int(((b*x*(p + 1) + c*x^2*(2*p + 2) + d*x^3*(3*p + 3))*(a + b*x + c*x^2 +
d*x^3)^p)/x,x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^(p + 1)
```

3.239 $\int \frac{(a+bx+cx^2+dx^3)^p(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx$

3.239.1 Optimal result

Integrand size = 49, antiderivative size = 23

$$\int \frac{(a+bx+cx^2+dx^3)^p(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x}$$

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)/x
```

3.239.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2+dx^3)^p(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(a+x(b+x(c+dx)))^{1+p}}{x}$$

input
```
Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(
2 + 3*p)*x^3))/x^2,x]
```

output
```
(a + x*(b + x*(c + d*x)))^(1 + p)/x
```

3.239.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(2p+1)x^2+d(3p+2)x^3)}{x^2}\,dx$$

$$\big\downarrow \text{2023}$$

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x}$$

input
```
Int[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2 + 3*
p)*x^3))/x^2,x]
```

output
```
(a + b*x + c*x^2 + d*x^3)^(1 + p)/x
```

3.239.3.1 Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

3.239.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{(x^3d+c\,x^2+bx+a)^{1+p}}{x}$	24
risch	$\frac{(x^3d+c\,x^2+bx+a)\,(x^3d+c\,x^2+bx+a)^p}{x}$	37
norman	$\frac{a\,\mathrm{e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+bx\,\mathrm{e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+c\,x^2\mathrm{e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+x^3d\,\mathrm{e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}}{x}$	97
parallelrisch	$\frac{x^3(x^3d+c\,x^2+bx+a)^p d^2+x^2(x^3d+c\,x^2+bx+a)^p cd+x(x^3d+c\,x^2+bx+a)^p bd+(x^3d+c\,x^2+bx+a)^p ad}{dx}$	97

input
```
int((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,x,met
hod=_RETURNVERBOSE)
```

output
```
(d*x^3+c*x^2+b*x+a)^(1+p)/x
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2
,x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

3.239.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c*x**2+b*x+a)**p*(-a+b*p*x+c*(1+2*p)*x**2+d*(2+3*p)*x**3
)/x**2,x)
```

output
```
Timed out
```

3.239.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2
,x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

3.239.8 Giac [F]

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \int \frac{(d(3\,p+2)x^3+c(2\,p+1)x^2+bpx-a)(dx^3+cx^2+bx+a)^p}{x^2}\,dx$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2
,x, algorithm="giac")
```

output
```
integrate((d*(3*p + 2)*x^3 + c*(2*p + 1)*x^2 + b*p*x - a)*(d*x^3 + c*x^2 +
 b*x + a)^p/x^2, x)
```

3.239.9 Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx = \frac{(d\,x^3+c\,x^2+b\,x+a)^{p+1}}{x}$$

input
```
int(((a + b*x + c*x^2 + d*x^3)^p*(b*p*x - a + c*x^2*(2*p + 1) + d*x^3*(3*p
 + 2)))/x^2,x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^(p + 1)/x
```

3.239. $\int \frac{(a+bx+cx^2+dx^3)^p(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx$

3.240 $\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx$

3.240.1 Optimal result

Integrand size = 48, antiderivative size = 23

$$\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^2}$$

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)/x^2
```

3.240.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(a+x(b+x(c+dx)))^{1+p}}{x^2}$$

input
```
Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 +
d*(1 + 3*p)*x^3))/x^3,x]
```

output
```
(a + x*(b + x*(c + d*x)))^(1 + p)/x^2
```

3.240.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(p-1)x+2cpx^2+d(3p+1)x^3)}{x^3}\,dx$$

$$\Big\downarrow \; 2023$$

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^2}$$

input
```
Int[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 +
 3*p)*x^3))/x^3,x]
```

output
```
(a + b*x + c*x^2 + d*x^3)^(1 + p)/x^2
```

3.240.3.1 Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

3.240.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{(x^3d+c\,x^2+bx+a)^{1+p}}{x^2}$	24
risch	$\frac{(x^3d+c\,x^2+bx+a)\,(x^3d+c\,x^2+bx+a)^p}{x^2}$	37
norman	$\frac{a\,{\mathrm e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+bx\,{\mathrm e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+c\,x^2{\mathrm e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+x^3d\,{\mathrm e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}}{x^2}$	97
parallelrisch	$\frac{x^3(x^3d+c\,x^2+bx+a)^p cd+x^2(x^3d+c\,x^2+bx+a)^p c^2+x(x^3d+c\,x^2+bx+a)^p bc+(x^3d+c\,x^2+bx+a)^p ac}{x^2c}$	97

input
```
int((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x^3,x,
method=_RETURNVERBOSE)
```

output
```
(d*x^3+c*x^2+b*x+a)^(1+p)/x^2
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^2}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/
x^3,x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

3.240.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c*x**2+b*x+a)**p*(-2*a+b*(-1+p)*x+2*c*p*x**2+d*(1+3*p)*x
**3)/x**3,x)
```

output
```
Timed out
```

3.240.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^2}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/
x^3,x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

3.240.8 Giac [F]

$$\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \int \frac{(d(3p+1)x^3+2cpx^2+b(p-1)x-2a)(dx^3+cx^2+bx+a)^p}{x^3}\,dx$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/
x^3,x, algorithm="giac")
```

output
```
integrate((d*(3*p + 1)*x^3 + 2*c*p*x^2 + b*(p - 1)*x - 2*a)*(d*x^3 + c*x^2
 + b*x + a)^p/x^3, x)
```

3.240.9 Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(d\,x^3+c\,x^2+b\,x+a)^{p+1}}{x^2}$$

input
```
int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 1) - 2*a + 2*c*p*x^2 + d*x^3*(3
*p + 1)))/x^3,x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^(p + 1)/x^2
```

3.240. $\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx$

3.241 $\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx$

3.241.1 Optimal result

Integrand size = 48, antiderivative size = 23

$$\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^3}$$

output
```
(d*x^3+c*x^2+b*x+a)^(p+1)/x^3
```

3.241.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{(a+x(b+x(c+dx)))^{1+p}}{x^3}$$

input
```
Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)
*x^2 + 3*d*p*x^3))/x^4,x]
```

output
```
(a + x*(b + x*(c + d*x)))^(1 + p)/x^3
```

3.241.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2+dx^3)^p\left(-3a+b(p-2)x+c(2p-1)x^2+3dpx^3\right)}{x^4}\,dx$$

$$\Big\downarrow \text{ 2023}$$

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^3}$$

input
```
Int[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*x^2 +
 3*d*p*x^3))/x^4,x]
```

output
```
(a + b*x + c*x^2 + d*x^3)^(1 + p)/x^3
```

3.241.3.1 Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

3.241.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{(x^3d+c\,x^2+bx+a)^{1+p}}{x^3}$	24
risch	$\frac{(x^3d+c\,x^2+bx+a)\,(x^3d+c\,x^2+bx+a)^p}{x^3}$	37
norman	$\frac{a\,\mathrm{e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+bx\,\mathrm{e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+c\,x^2\mathrm{e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}+x^3d\,\mathrm{e}^{p\ln\left(x^3d+c\,x^2+bx+a\right)}}{x^3}$	97
parallelrisch	$\frac{(x^3d+c\,x^2+bx+a)^p ad\,x^3+(x^3d+c\,x^2+bx+a)^p ac\,x^2+(x^3d+c\,x^2+bx+a)^p abx+(x^3d+c\,x^2+bx+a)^p a^2}{x^3a}$	97

input
```
int((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x
,method=_RETURNVERBOSE)
```

output
```
(d*x^3+c*x^2+b*x+a)^(1+p)/x^3
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^3}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)
/x^4,x, algorithm="fricas")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

3.241.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c*x**2+b*x+a)**p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x**2+3*d*p*
x**3)/x**4,x)
```

output
```
Timed out
```

3.241.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^3}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)
/x^4,x, algorithm="maxima")
```

output
```
(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

3.241.8 Giac [F]

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \int \frac{(3\,dpx^3+c(2\,p-1)x^2+b(p-2)x-3\,a)(dx^3+cx^2+bx+a)^p}{x^4}\,dx$$

input
```
integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)
/x^4,x, algorithm="giac")
```

output
```
integrate((3*d*p*x^3 + c*(2*p - 1)*x^2 + b*(p - 2)*x - 3*a)*(d*x^3 + c*x^2
 + b*x + a)^p/x^4, x)
```

3.241.9 Mupad [B] (verification not implemented)

Time = 9.86 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{(d\,x^3+c\,x^2+b\,x+a)^{p+1}}{x^3}$$

input
```
int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 2) - 3*a + 3*d*p*x^3 + c*x^2*(2
*p - 1)))/x^4,x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^(p + 1)/x^3
```

3.241. $\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx$

3.242 $\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

3.242.1 Optimal result

Integrand size = 35, antiderivative size = 97

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{24}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(1+x+x^2\right)-\frac{13}{48}\log\left(2-x+2x^2\right)$$

output
```
5/4*x-3/4*x^2+1/3*x^3+1/4*x^4+1/3*ln(x^2+x+1)-13/48*ln(2*x^2-x+2)+1/72*arc
tan(1/15*(1-4*x)*15^(1/2))*15^(1/2)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/
2)
```

3.242.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{144}\left(180x-108x^2+48x^3+36x^4 - 160\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right) + 48\log\left(1+x+x^2\right)-39\log\left(2-x+2x^2\right)\right)$$

input
```
Integrate[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
(180*x - 108*x^2 + 48*x^3 + 36*x^4 - 160*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]
 - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] - 39*Log[2
 - x + 2*x^2])/144
```

3.242.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(2x^3+3x^2+x+5\right)}{2x^4+x^3+3x^2+x+2}dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(x^3+x^2+\frac{2(x-2)}{3\left(x^2+x+1\right)}+\frac{2-13x}{12\left(2x^2-x+2\right)}-\frac{3x}{2}+\frac{5}{4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{24}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{5x}{4}$$

input
```
Int[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
(5*x)/4 - (3*x^2)/4 + x^3/3 + x^4/4 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]
])/24 - (10*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 -
(13*Log[2 - x + 2*x^2])/48
```

3.242.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.242.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

method	result
default	$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{5x}{4}+\frac{\ln(x^2+x+1)}{3}-\frac{10\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{13\ln(2x^2-x+2)}{48}-\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{72}$
risch	$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{5x}{4}+\frac{\ln(4x^2+4x+4)}{3}-\frac{10\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{13\ln(16x^2-8x+16)}{48}-\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{72}$

input
```
int(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+1/3*x^3-3/4*x^2+5/4*x+1/3*ln(x^2+x+1)-10/9*arctan(1/3*(1+2*x)*3^(1
/2))*3^(1/2)-13/48*ln(2*x^2-x+2)-1/72*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/
2))
```

3.242.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{4}\,x^4+\frac{1}{3}\,x^3-\frac{3}{4}\,x^2-\frac{1}{72}\,\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\,\sqrt{5}\sqrt{3}(4\,x-1)\right) - \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{5}{4}\,x - \frac{13}{48}\,\log\left(2\,x^2-x+2\right)+\frac{1}{3}\,\log\left(x^2+x+1\right)$$

input
```
integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas
")
```

output
```
1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqr
t(3)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/
48*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

3.242.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{5x}{4}-\frac{13\log\left(x^2-\frac{x}{2}+1\right)}{48}+\frac{\log\left(x^2+x+1\right)}{3} - \frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{72}-\frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**4*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

output
```
x**4/4 + x**3/3 - 3*x**2/4 + 5*x/4 - 13*log(x**2 - x/2 + 1)/48 + log(x**2
+ x + 1)/3 - sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/72 - 10*sqrt(3)*
atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

3.242.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{4}\,x^4+\frac{1}{3}\,x^3-\frac{3}{4}\,x^2-\frac{1}{72}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) - \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{5}{4}\,x - \frac{13}{48}\,\log\left(2\,x^2-x+2\right)+\frac{1}{3}\,\log\left(x^2+x+1\right)$$

input
```
integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima
")
```

output
```
1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)
) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 -
 x + 2) + 1/3*log(x^2 + x + 1)
```

3.242.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{4}\,x^4 + \frac{1}{3}\,x^3 - \frac{3}{4}\,x^2 - \frac{1}{72}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) - \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{5}{4}\,x - \frac{13}{48}\,\log\left(2\,x^2-x+2\right) + \frac{1}{3}\,\log\left(x^2+x+1\right)$$

input
```
integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)
) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 -
 x + 2) + 1/3*log(x^2 + x + 1)
```

3.242.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{5\,x}{4} + \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) + \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{13}{48}+\frac{\sqrt{15}\,1\mathrm{i}}{144}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{13}{48}+\frac{\sqrt{15}\,1\mathrm{i}}{144}\right) - \frac{3\,x^2}{4} + \frac{x^3}{3} + \frac{x^4}{4}$$

input
```
int((x^4*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
(5*x)/4 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - log(x + (
3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) + log(x - (15^(1/2)*1i)/4 - 1/
4)*((15^(1/2)*1i)/144 - 13/48) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)
*1i)/144 + 13/48) - (3*x^2)/4 + x^3/3 + x^4/4
```

3.243 $\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

3.243.1 Optimal result

Integrand size = 35, antiderivative size = 90

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{5}{12}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{24}\log\left(2-x+2x^2\right)$$

output
```
-3/2*x+1/2*x^2+1/3*x^3+2/3*ln(x^2+x+1)-1/24*ln(2*x^2-x+2)+5/36*arctan(1/15
*(1-4*x)*15^(1/2))*15^(1/2)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.243.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{72}\left(-108x+36x^2+24x^3+64\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-10\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right)+48\log\left(1+x+x^2\right)-3\log\left(2-x+2x^2\right)\right)$$

input
```
Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
(-108*x + 36*x^2 + 24*x^3 + 64*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 10*Sqrt
[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] - 3*Log[2 - x + 2*x
^2])/72
```

3.243.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (2x^3+3x^2+x+5)}{2x^4+x^3+3x^2+x+2} \, dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(x^2+\frac{2(2x+3)}{3(x^2+x+1)}+\frac{-x-6}{6(2x^2-x+2)}+x-\frac{3}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{12}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{x^3}{3}+\frac{x^2}{2}+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{3x}{2}$$

input
```
Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
(-3*x)/2 + x^2/2 + x^3/3 + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 + (
8*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2
- x + 2*x^2]/24
```

3.243.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.243.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{x^{3}}{3}+\frac{x^{2}}{2}-\frac{3x}{2}+\frac{2\ln\left(x^{2}+x+1\right)}{3}+\frac{8\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{\ln\left(2x^{2}-x+2\right)}{24}-\frac{5\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{36}$	69
risch	$\frac{x^{3}}{3}+\frac{x^{2}}{2}-\frac{3x}{2}-\frac{\ln\left(16x^{2}-8x+16\right)}{24}-\frac{5\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{36}+\frac{2\ln\left(4x^{2}+4x+4\right)}{3}+\frac{8\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}$	73

input
```
int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+1/2*x^2-3/2*x+2/3*ln(x^2+x+1)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1
/2)-1/24*ln(2*x^2-x+2)-5/36*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.82

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{3}\,x^{3}+\frac{1}{2}\,x^{2}-\frac{5}{36}\,\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\,\sqrt{5}\sqrt{3}(4\,x-1)\right)+\frac{8}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{3}{2}\,x-\frac{1}{24}\,\log\left(2\,x^{2}-x+2\right)+\frac{2}{3}\,\log\left(x^{2}+x+1\right)$$

input
```
integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas
")
```

output
```
1/3*x^3 + 1/2*x^2 - 5/36*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x
- 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2
 - x + 2) + 2/3*log(x^2 + x + 1)
```

3.243.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{x^3}{3}+\frac{x^2}{2}-\frac{3x}{2}-\frac{\log\left(x^2-\frac{x}{2}+1\right)}{24}+\frac{2\log\left(x^2+x+1\right)}{3} - \frac{5\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{36}+\frac{8\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

output
```
x**3/3 + x**2/2 - 3*x/2 - log(x**2 - x/2 + 1)/24 + 2*log(x**2 + x + 1)/3 -
 5*sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/36 + 8*sqrt(3)*atan(2*sqrt
(3)*x/3 + sqrt(3)/3)/9
```

3.243.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{3}\,x^3+\frac{1}{2}\,x^2-\frac{5}{36}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) + \frac{8}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{3}{2}\,x - \frac{1}{24}\,\log\left(2\,x^2-x+2\right)+\frac{2}{3}\,\log\left(x^2+x+1\right)$$

input
```
integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima
")
```

output
```
1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sq
rt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/
3*log(x^2 + x + 1)
```

3.243.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{3}\,x^3+\frac{1}{2}\,x^2-\frac{5}{36}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) + \frac{8}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{3}{2}\,x - \frac{1}{24}\,\log\left(2\,x^2-x+2\right)+\frac{2}{3}\,\log\left(x^2+x+1\right)$$

input
```
integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sq
rt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/
3*log(x^2 + x + 1)
```

3.243.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{x^2}{2}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right) + \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{24}+\frac{\sqrt{15}\,5\mathrm{i}}{72}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{24}+\frac{\sqrt{15}\,5\mathrm{i}}{72}\right)-\frac{3\,x}{2}+\frac{x^3}{3}$$

input
```
int((x^3*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) - log(x - (3^(1/2)*1i
)/2 + 1/2)*((3^(1/2)*4i)/9 - 2/3) - (3*x)/2 + log(x - (15^(1/2)*1i)/4 - 1/
4)*((15^(1/2)*5i)/72 - 1/24) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*5
i)/72 + 1/24) + x^2/2 + x^3/3
```

3.244 $\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

3.244.1 Optimal result

Integrand size = 35, antiderivative size = 77

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = x+\frac{x^2}{2}+\frac{1}{6}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \log\left(1+x+x^2\right)+\frac{1}{4}\log\left(2-x+2x^2\right)$$

output
```
x+1/2*x^2-ln(x^2+x+1)+1/4*ln(2*x^2-x+2)+1/18*arctan(1/15*(1-4*x)*15^(1/2))
*15^(1/2)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.244.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{36}\left(8\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right) + 9\left(2x(2+x)-4\log\left(1+x+x^2\right)+\log\left(2-x+2x^2\right)\right)\right)$$

input
```
Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
(8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[1
5]] + 9*(2*x*(2 + x) - 4*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/36
```

3.244.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(2x^3+3x^2+x+5)}{2x^4+x^3+3x^2+x+2}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(-\frac{2(3x+1)}{3(x^2+x+1)}+\frac{3x-2}{3(2x^2-x+2)}+x+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{x^2}{2}-\log\left(x^2+x+1\right)+\frac{1}{4}\log\left(2x^2-x+2\right)+x$$

input
```
Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
x + x^2/2 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)
/Sqrt[3]])/(3*Sqrt[3]) - Log[1 + x + x^2] + Log[2 - x + 2*x^2]/4
```

3.244.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.244.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x^2}{2}+x-\ln\left(x^2+x+1\right)+\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{\ln(2x^2-x+2)}{4}-\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{18}$	62
risch	$\frac{x^2}{2}+x+\frac{\ln(16x^2-8x+16)}{4}-\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{18}-\ln\left(4x^2+4x+4\right)+\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}$	66

input
```
int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+x-ln(x^2+x+1)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/4*ln(2*x^2
-x+2)-1/18*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{2}\,x^2-\frac{1}{18}\,\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\,\sqrt{5}\sqrt{3}(4\,x-1)\right)+\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+x+\frac{1}{4}\,\log\left(2\,x^2-x+2\right)-\log\left(x^2+x+1\right)$$

input
```
integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas
")
```

output
```
1/2*x^2 - 1/18*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 2/
9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log
(x^2 + x + 1)
```

3.244.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{x^2}{2}+x+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{4}-\log\left(x^2+x+1\right) - \frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{18}+\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

output
```
x**2/2 + x + log(x**2 - x/2 + 1)/4 - log(x**2 + x + 1) - sqrt(15)*atan(4*s
qrt(15)*x/15 - sqrt(15)/15)/18 + 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)
/9
```

3.244.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{2}\,x^2-\frac{1}{18}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) + \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+x + \frac{1}{4}\,\log\left(2\,x^2-x+2\right)-\log\left(x^2+x+1\right)$$

input
```
integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima
")
```

output
```
1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arct
an(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)
```

3.244.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{2}\,x^2 - \frac{1}{18}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) + \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + x + \frac{1}{4}\,\log\left(2\,x^2-x+2\right) - \log\left(x^2+x+1\right)$$

input
```
integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arct
an(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)
```

3.244.9 Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = x - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right) + \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{36}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{36}\right) + \frac{x^2}{2}$$

input
```
int((x^2*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
x - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 + 1) + log(x + (3^(1/2)*
1i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1) + log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1
/2)*1i)/36 + 1/4) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/36 - 1/4
) + x^2/2
```

3.245 $\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} \, dx$

3.245.1 Optimal result

Integrand size = 33, antiderivative size = 72

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} \, dx = x-\frac{1}{3}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} +\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{6}\log\left(2-x+2x^2\right)$$

output
```
x+1/3*ln(x^2+x+1)+1/6*ln(2*x^2-x+2)-1/9*arctan(1/15*(1-4*x)*15^(1/2))*15^(
1/2)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.245.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} \, dx = \frac{1}{18}\left(-20\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right) +3\left(6x+2\log\left(1+x+x^2\right)+\log\left(2-x+2x^2\right)\right)\right)$$

input
```
Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
(-20*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt
[15]] + 3*(6*x + 2*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/18
```

3.245.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(2x^3+3x^2+x+5)}{2x^4+x^3+3x^2+x+2}dx$$
$$\Big\downarrow \; 2462$$
$$\int \left(\frac{2(x-2)}{3(x^2+x+1)}+\frac{2(x+1)}{3(2x^2-x+2)}+1\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{1}{3}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(2x^2-x+2\right)+x$$

input
```
Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
x - (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 - (10*ArcTan[(1 + 2*x)/Sqrt[3
]])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 + Log[2 - x + 2*x^2]/6
```

3.245.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.245.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

method	result	size
default	$x+\frac{\ln(x^2+x+1)}{3}-\frac{10\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{\ln(2x^2-x+2)}{6}+\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{9}$	57
risch	$x+\frac{\ln(4x^2+4x+4)}{3}-\frac{10\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{\ln(16x^2-8x+16)}{6}+\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{9}$	61

input
```
int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
x+1/3*ln(x^2+x+1)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/6*ln(2*x^2-x+
2)+1/9*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{9}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x + \frac{1}{6}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

input
```
integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

output
```
1/9*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 10/9*sqrt(3)*
arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 +
 x + 1)
```

3.245.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = x+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{6}+\frac{\log\left(x^2+x+1\right)}{3}+\frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{9}-\frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

output
```
x + log(x**2 - x/2 + 1)/6 + log(x**2 + x + 1)/3 + sqrt(15)*atan(4*sqrt(15)
*x/15 - sqrt(15)/15)/9 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

3.245.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

input
```
integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

output
```
1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

3.245.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x + \frac{1}{6}\log\left(2x^2-x+2\right) + \frac{1}{3}\log\left(x^2+x+1\right)$$

input
```
integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

3.245.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = x + \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) - \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{6}+\frac{\sqrt{15}\,1\mathrm{i}}{18}\right) + \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{6}+\frac{\sqrt{15}\,1\mathrm{i}}{18}\right)$$

input
```
int((x*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
x + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - log(x + (3^(1/2
)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) - log(x - (15^(1/2)*1i)/4 - 1/4)*((1
5^(1/2)*1i)/18 - 1/6) + log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/18 +
 1/6)
```

3.246 $\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4} \, dx$

3.246.1 Optimal result

Integrand size = 32, antiderivative size = 71

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4} \, dx = -\frac{1}{3}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} +\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{6}\log\left(2-x+2x^2\right)$$

output
```
2/3*ln(x^2+x+1)-1/6*ln(2*x^2-x+2)-1/9*arctan(1/15*(1-4*x)*15^(1/2))*15^(1/
2)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.246.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4} \, dx = \frac{1}{18}\left(16\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right) +12\log\left(1+x+x^2\right)-3\log\left(2-x+2x^2\right)\right)$$

input
```
Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
(16*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[
15]] + 12*Log[1 + x + x^2] - 3*Log[2 - x + 2*x^2])/18
```

3.246.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+x+5}{2x^4+x^3+3x^2+x+2}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{3-2x}{3\left(2x^2-x+2\right)}+\frac{2(2x+3)}{3\left(x^2+x+1\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{6}\log\left(2x^2-x+2\right)$$

input
```
Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
-1/3*(Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]]) + (8*ArcTan[(1 + 2*x)/Sqrt[3]]
)/(3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 - x + 2*x^2]/6
```

3.246.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.246.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{2\ln\left(x^{2}+x+1\right)}{3}+\frac{8\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{\ln\left(2x^{2}-x+2\right)}{6}+\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{9}$	56
risch	$\frac{2\ln\left(4x^{2}+4x+4\right)}{3}+\frac{8\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{\ln\left(16x^{2}-8x+16\right)}{6}+\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{9}$	60

input
```
int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
2/3*ln(x^2+x+1)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/6*ln(2*x^2-x+2)+
1/9*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))
```

3.246.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{9}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right) + \frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\log\left(2x^2-x+2\right) + \frac{2}{3}\log\left(x^2+x+1\right)$$

input
```
integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

output
```
1/9*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 8/9*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x +
1)
```

3.246.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = -\frac{\log\left(x^2-\frac{x}{2}+1\right)}{6}+\frac{2\log\left(x^2+x+1\right)}{3}+\frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{9}+\frac{8\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

output
```
-log(x**2 - x/2 + 1)/6 + 2*log(x**2 + x + 1)/3 + sqrt(15)*atan(4*sqrt(15)*
x/15 - sqrt(15)/15)/9 + 8*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

3.246.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

input
```
integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

output
```
1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

3.246.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\log\left(2x^2-x+2\right) + \frac{2}{3}\log\left(x^2+x+1\right)$$

input
```
integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

3.246.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.11

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = -\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,4i}{9}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,4i}{9}\right) - \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1i}{4}\right)\left(\frac{1}{6}+\frac{\sqrt{15}\,1i}{18}\right) + \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1i}{4}\right)\left(-\frac{1}{6}+\frac{\sqrt{15}\,1i}{18}\right)$$

input
```
int((x + 3*x^2 + 2*x^3 + 5)/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) - log(x - (3^(1/2)*1i
)/2 + 1/2)*((3^(1/2)*4i)/9 - 2/3) - log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1
/2)*1i)/18 + 1/6) + log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/18 - 1/6
)
```

3.247 $\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)}\,dx$

3.247.1 Optimal result

Integrand size = 35, antiderivative size = 75

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)}\,dx = \frac{1}{6}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{5\log(x)}{2}-\log\left(1+x+x^2\right)-\frac{1}{4}\log\left(2-x+2x^2\right)$$

output
```
5/2*ln(x)-ln(x^2+x+1)-1/4*ln(2*x^2-x+2)+1/18*arctan(1/15*(1-4*x)*15^(1/2))
*15^(1/2)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.247.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)}\,dx = \frac{1}{36}\left(8\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right)+90\log(x)-36\log\left(1+x+x^2\right)-9\log\left(2-x+2x^2\right)\right)$$

input
```
Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

output
```
(8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[1
5]] + 90*Log[x] - 36*Log[1 + x + x^2] - 9*Log[2 - x + 2*x^2])/36
```

3.247.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+x+5}{x(2x^4+x^3+3x^2+x+2)}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{-6x-1}{6(2x^2-x+2)}-\frac{2(3x+1)}{3(x^2+x+1)}+\frac{5}{2x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(2x^2-x+2\right)+\frac{5\log(x)}{2}$$

input
```
Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

output
```
(Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(
3*Sqrt[3]) + (5*Log[x])/2 - Log[1 + x + x^2] - Log[2 - x + 2*x^2]/4
```

3.247.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.247.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{5\ln(x)}{2}-\ln\left(x^{2}+x+1\right)+\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{\ln(2x^{2}-x+2)}{4}-\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{18}$	60
risch	$-\ln\left(4x^{2}+4x+4\right)+\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{18}-\frac{\ln(16x^{2}-8x+16)}{4}+\frac{5\ln(x)}{2}$	64

input
```
int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
5/2*ln(x)-ln(x^2+x+1)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4*ln(2*x^2
-x+2)-1/18*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))
```

3.247.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int \frac{5+x+3x^2+2x^3}{x\,(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{1}{18}\,\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\,\sqrt{5}\sqrt{3}(4\,x-1)\right) + \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{4}\,\log\left(2\,x^2-x+2\right) - \log\left(x^2+x+1\right) + \frac{5}{2}\,\log\left(x\right)$$

input
```
integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

output
```
-1/18*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 2/9*sqrt(3)
*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)
 + 5/2*log(x)
```

3.247.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

$$\int \frac{5+x+3x^2+2x^3}{x\left(2+x+3x^2+x^3+2x^4\right)}\,dx = \frac{5\log\left(x\right)}{2} - \frac{\log\left(x^2-\frac{x}{2}+1\right)}{4} - \log\left(x^2+x+1\right) - \frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{18} + \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+3*x**2+x+2),x)
```

output
```
5*log(x)/2 - log(x**2 - x/2 + 1)/4 - log(x**2 + x + 1) - sqrt(15)*atan(4*s
qrt(15)*x/15 - sqrt(15)/15)/18 + 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)
/9
```

3.247.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \frac{5+x+3x^2+2x^3}{x\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4\,x-1)\right) + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right) - \frac{1}{4}\log\left(2\,x^2-x+2\right) - \log\left(x^2+x+1\right) + \frac{5}{2}\log\left(x\right)$$

input
```
integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

output
```
-1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) + 5/2*log(x)
```

3.247.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{5+x+3x^2+2x^3}{x\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{4}\log\left(2x^2-x+2\right) - \log\left(x^2+x+1\right) + \frac{5}{2}\log\left(|x|\right)$$

input
```
integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
-1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) + 5/2*log(abs
(x))
```

3.247.9 Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

$$\int \frac{5+x+3x^2+2x^3}{x\left(2+x+3x^2+x^3+2x^4\right)}\,dx = \frac{5\ln(x)}{2} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(1+\frac{\sqrt{3}\,1i}{9}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-1+\frac{\sqrt{3}\,1i}{9}\right) + \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1i}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{15}\,1i}{36}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1i}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{15}\,1i}{36}\right)$$

input
```
int((x + 3*x^2 + 2*x^3 + 5)/(x*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

output
```
(5*log(x))/2 - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 + 1) + log(x
+ (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1) + log(x - (15^(1/2)*1i)/4 - 1
/4)*((15^(1/2)*1i)/36 - 1/4) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1
i)/36 + 1/4)
```

3.248 $\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx$

3.248.1 Optimal result

Integrand size = 35, antiderivative size = 84

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\frac{5}{2x}+\frac{5}{12}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} -\frac{3\log(x)}{4}+\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{24}\log\left(2-x+2x^2\right)$$

output
```
-5/2/x-3/4*ln(x)+1/3*ln(x^2+x+1)+1/24*ln(2*x^2-x+2)+5/36*arctan(1/15*(1-4*
x)*15^(1/2))*15^(1/2)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.248.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\frac{180+80\sqrt{3}x\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+10\sqrt{15}x\arctan\left(\frac{-1+4x}{\sqrt{15}}\right)+54x\log(x)-24x\log\left(1+x+x^2\right)-3x\log(2}{72x}$$

input
```
Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

output
```
-1/72*(180 + 80*Sqrt[3]*x*ArcTan[(1 + 2*x)/Sqrt[3]] + 10*Sqrt[15]*x*ArcTan
[(-1 + 4*x)/Sqrt[15]] + 54*x*Log[x] - 24*x*Log[1 + x + x^2] - 3*x*Log[2 -
x + 2*x^2])/x
```

3.248.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+x+5}{x^2(2x^4+x^3+3x^2+x+2)}dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(\frac{2(x-2)}{3(x^2+x+1)}+\frac{2x-13}{12(2x^2-x+2)}+\frac{5}{2x^2}-\frac{3}{4x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{5}{12}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{5}{2x}-\frac{3\log(x)}{4}$$

input
```
Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

output
```
-5/(2*x) + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 - (10*ArcTan[(1 + 2
*x)/Sqrt[3]])/(3*Sqrt[3]) - (3*Log[x])/4 + Log[1 + x + x^2]/3 + Log[2 - x
+ 2*x^2]/24
```

3.248.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.248.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

method	result
default	$-\frac{5}{2x}-\frac{3\ln(x)}{4}+\frac{\ln(x^2+x+1)}{3}-\frac{10\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{\ln(2x^2-x+2)}{24}-\frac{5\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{36}$
risch	$-\frac{5}{2x}-\frac{5\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{36}+\frac{\ln(16x^2-8x+16)}{24}-\frac{3\ln(x)}{4}+\frac{\ln(25x^2+25x+25)}{3}-\frac{10\sqrt{3}\,\arctan\left(\frac{2\left(5x+\frac{5}{2}\right)\sqrt{3}}{15}\right)}{9}$

input
```
int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
-5/2/x-3/4*ln(x)+1/3*ln(x^2+x+1)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+
1/24*ln(2*x^2-x+2)-5/36*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{10\sqrt{5}\sqrt{3}x\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+80\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-3x\log(2x^2-x+2)-24x}{72x}$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas
")
```

output
```
-1/72*(10*sqrt(5)*sqrt(3)*x*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 80*sq
rt(3)*x*arctan(1/3*sqrt(3)*(2*x + 1)) - 3*x*log(2*x^2 - x + 2) - 24*x*log(
x^2 + x + 1) + 54*x*log(x) + 180)/x
```

3.248.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{3\log(x)}{4} + \frac{\log\left(x^2-\frac{x}{2}+1\right)}{24} + \frac{\log(x^2+x+1)}{3} - \frac{5\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{36} - \frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9} - \frac{5}{2x}$$

input
```
integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+3*x**2+x+2),x)
```

output
```
-3*log(x)/4 + log(x**2 - x/2 + 1)/24 + log(x**2 + x + 1)/3 - 5*sqrt(15)*at
an(4*sqrt(15)*x/15 - sqrt(15)/15)/36 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqr
t(3)/3)/9 - 5/(2*x)
```

3.248.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{5}{2x} + \frac{1}{24}\log\left(2x^2-x+2\right) + \frac{1}{3}\log\left(x^2+x+1\right) - \frac{3}{4}\log(x)$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima
")
```

output
```
-5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x + 1)) - 5/2/x + 1/24*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
 - 3/4*log(x)
```

3.248.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{5}{2x} + \frac{1}{24}\log\left(2x^2-x+2\right) + \frac{1}{3}\log\left(x^2+x+1\right) - \frac{3}{4}\log\left(|x|\right)$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
-5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x + 1)) - 5/2/x + 1/24*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
 - 3/4*log(abs(x))
```

3.248.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{3\ln(x)}{4} + \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) + \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{24}+\frac{\sqrt{15}\,5\mathrm{i}}{72}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{24}+\frac{\sqrt{15}\,5\mathrm{i}}{72}\right) - \frac{5}{2x}$$

input
```
int((x + 3*x^2 + 2*x^3 + 5)/(x^2*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

output
```
log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - (3*log(x))/4 - log(
x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) + log(x - (15^(1/2)*1i)/4
 - 1/4)*((15^(1/2)*5i)/72 + 1/24) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1
/2)*5i)/72 - 1/24) - 5/(2*x)
```

3.249 $\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)}\,dx$

3.249.1 Optimal result

Integrand size = 35, antiderivative size = 91

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)}\,dx=-\frac{5}{4x^2}+\frac{3}{4x}+\frac{1}{24}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{15\log(x)}{8}+\frac{2}{3}\log\left(1+x+x^2\right)+\frac{13}{48}\log\left(2-x+2x^2\right)$$

output
```
-5/4/x^2+3/4/x-15/8*ln(x)+2/3*ln(x^2+x+1)+13/48*ln(2*x^2-x+2)+1/72*arctan(
1/15*(1-4*x)*15^(1/2))*15^(1/2)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.249.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)}\,dx=\frac{1}{144}\left(128\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right)+3\left(-\frac{60}{x^2}+\frac{36}{x}-90\log(x)+32\log\left(1+x+x^2\right)+13\log\left(2-x+2x^2\right)\right)\right)$$

input
```
Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

output
```
(128*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt
[15]] + 3*(-60/x^2 + 36/x - 90*Log[x] + 32*Log[1 + x + x^2] + 13*Log[2 - x
 + 2*x^2]))/144
```

3.249.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+x+5}{x^3\left(2x^4+x^3+3x^2+x+2\right)}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{5}{2x^3}+\frac{2(2x+3)}{3\left(x^2+x+1\right)}+\frac{26x-9}{24\left(2x^2-x+2\right)}-\frac{3}{4x^2}-\frac{15}{8x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{24}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{5}{4x^2}+\frac{2}{3}\log\left(x^2+x+1\right)+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{3}{4x}-\frac{15\log(x)}{8}$$

input
```
Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

output
```
-5/(4*x^2) + 3/(4*x) + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/24 + (8*ArcT
an[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (15*Log[x])/8 + (2*Log[1 + x + x^2])/
3 + (13*Log[2 - x + 2*x^2])/48
```

3.249.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.249.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

method	result	si
default	$-\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\ln(x)}{8}+\frac{2\ln(x^2+x+1)}{3}+\frac{8\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{13\ln(2x^2-x+2)}{48}-\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{72}$	70
risch	$\frac{\frac{3x}{4}-\frac{5}{4}}{x^2}-\frac{\sqrt{15}\,\arctan\left(\frac{(-1+4x)\sqrt{15}}{15}\right)}{72}+\frac{13\ln(16x^2-8x+16)}{48}+\frac{2\ln(4x^2+4x+4)}{3}+\frac{8\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{15\ln(x)}{8}$	7

input
```
int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
-5/4/x^2+3/4/x-15/8*ln(x)+2/3*ln(x^2+x+1)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*
3^(1/2)+13/48*ln(2*x^2-x+2)-1/72*15^(1/2)*arctan(1/15*(-1+4*x)*15^(1/2))
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{2\sqrt{5}\sqrt{3}x^2\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)-128\sqrt{3}x^2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-39x^2\log\left(2x^2-x+2\right)-}{144x^2}$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas
")
```

output
```
-1/144*(2*sqrt(5)*sqrt(3)*x^2*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 128
*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) - 39*x^2*log(2*x^2 - x + 2) - 9
6*x^2*log(x^2 + x + 1) + 270*x^2*log(x) - 108*x + 180)/x^2
```

3.249.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.03

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{15\log(x)}{8} + \frac{13\log\left(x^2-\frac{x}{2}+1\right)}{48} + \frac{2\log(x^2+x+1)}{3} - \frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{72} + \frac{8\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9} + \frac{3x-5}{4x^2}$$

input
```
integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+3*x**2+x+2),x)
```

output
```
-15*log(x)/8 + 13*log(x**2 - x/2 + 1)/48 + 2*log(x**2 + x + 1)/3 - sqrt(15
)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/72 + 8*sqrt(3)*atan(2*sqrt(3)*x/3 +
sqrt(3)/3)/9 + (3*x - 5)/(4*x**2)
```

3.249.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.76

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{3x-5}{4x^2} + \frac{13}{48}\log\left(2x^2-x+2\right) + \frac{2}{3}\log\left(x^2+x+1\right) - \frac{15}{8}\log(x)$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima
")
```

output
```
-1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*log(2*x^2 - x + 2) + 2/3*log(
x^2 + x + 1) - 15/8*log(x)
```

3.249.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\frac{1}{72}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) + \frac{8}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{3\,x-5}{4\,x^2} + \frac{13}{48}\,\log\left(2\,x^2-x+2\right) + \frac{2}{3}\,\log\left(x^2+x+1\right) - \frac{15}{8}\,\log\left(|x|\right)$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
-1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*log(2*x^2 - x + 2) + 2/3*log(
x^2 + x + 1) - 15/8*log(abs(x))
```

3.249.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+3x^2+x^3+2x^4\right)}\,dx = \frac{\frac{3\,x}{4}-\frac{5}{4}}{x^2} - \frac{15\,\ln\left(x\right)}{8} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right) + \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\,\left(\frac{13}{48}+\frac{\sqrt{15}\,1\mathrm{i}}{144}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\,\left(-\frac{13}{48}+\frac{\sqrt{15}\,1\mathrm{i}}{144}\right)$$

input
```
int((x + 3*x^2 + 2*x^3 + 5)/(x^3*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

output
```
((3*x)/4 - 5/4)/x^2 - (15*log(x))/8 - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1
/2)*4i)/9 - 2/3) + log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) +
log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/144 + 13/48) - log(x + (15^(
1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/144 - 13/48)
```

3.250 $\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

3.250.1 Optimal result

Integrand size = 35, antiderivative size = 307

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = -\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{42}\left(7-5i\sqrt{7}\right)x^3+\frac{1}{42}\left(7+5i\sqrt{7}\right)x^3+\frac{11\left(9i+5\sqrt{7}\right)\arctan\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35+i\sqrt{7}\right)}}-\frac{11\left(9i-5\sqrt{7}\right)\arctan\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)+\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)$$

output
```
1/28*x^2*(7-5*I*7^(1/2))+1/42*x^3*(7-5*I*7^(1/2))+1/28*x^2*(7+5*I*7^(1/2))
+1/42*x^3*(7+5*I*7^(1/2))-1/28*x*(35-9*I*7^(1/2))-1/28*x*(35+9*I*7^(1/2))+
3/112*ln(4+4*x^2+x*(1-I*7^(1/2)))*(7-11*I*7^(1/2))+3/112*ln(4+4*x^2+x*(1+I
*7^(1/2)))*(7+11*I*7^(1/2))-11/4*arctan((1+8*x+I*7^(1/2))/(70-2*I*7^(1/2))
^(1/2))*(9*I-5*7^(1/2))/(490-14*I*7^(1/2))^(1/2)+11/4*arctan((1+8*x-I*7^(1
/2))/(70+2*I*7^(1/2))^(1/2))*(9*I+5*7^(1/2))/(490+14*I*7^(1/2))^(1/2)
```

3.250.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.36

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{1}{6}\bigg(x(-15+3x+2x^2)+3\text{RootSum}\bigg[2+\#1+5\#1^2+\#1^3 +2\#1^4\&,\frac{10\log(x-\#1)+\log(x-\#1)\#1+19\log(x-\#1)\#1^2+3\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\bigg]\bigg)$$

input
```
Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

output
```
(x*(-15 + 3*x + 2*x^2) + 3*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (10
*Log[x - #1] + Log[x - #1]*#1 + 19*Log[x - #1]*#1^2 + 3*Log[x - #1]*#1^3)/
(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ])/6
```

3.250.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.74, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(2x^3+3x^2+x+5)}{2x^4+x^3+5x^2+x+2}\,dx$$

$$\Big\downarrow \text{2492}$$

$$\frac{1}{2}\int \left(2x^2+2x+\frac{3(7i-11\sqrt{7})x+2(35i+9\sqrt{7})}{7(4ix^2+(i-\sqrt{7})x+4i)}+\frac{3(7i+11\sqrt{7})x+2(35i-9\sqrt{7})}{7(4ix^2+(i+\sqrt{7})x+4i)}-5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{11(9+5i\sqrt{7})\,\text{arctanh}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2(35-i\sqrt{7})}}\right)}{2\sqrt{14(35-i\sqrt{7})}}+\frac{11(9-5i\sqrt{7})\,\text{arctanh}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2(35+i\sqrt{7})}}\right)}{2\sqrt{14(35+i\sqrt{7})}}+\frac{2x^3}{3}+x^2+\frac{3}{56}(7+11i\sqrt{\right.$$

input
```
Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

output
```
(-5*x + x^2 + (2*x^3)/3 - (11*(9 + (5*I)*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (
8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(2*Sqrt[14*(35 - I*Sqrt[7])]) + (11*(9
- (5*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]]
)/(2*Sqrt[14*(35 + I*Sqrt[7])]) + (3*(7 + (11*I)*Sqrt[7])*Log[4*I + (I - S
qrt[7])*x + (4*I)*x^2])/56 + (3*(7 - (11*I)*Sqrt[7])*Log[4*I + (I + Sqrt[7
])*x + (4*I)*x^2])/56)/2
```

3.250.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

3.250.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.24

method	result	size
default	$\frac{x^3}{3}+\frac{x^2}{2}-\frac{5x}{2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(3\_R^3+19\_R^2+\_R+10\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}\right)}{2}$	74
risch	$\frac{x^3}{3}+\frac{x^2}{2}-\frac{5x}{2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(3\_R^3+19\_R^2+\_R+10\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}\right)}{2}$	74

input
```
int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+1/2*x^2-5/2*x+1/2*sum((3*_R^3+19*_R^2+_R+10)/(8*_R^3+3*_R^2+10*_R+
1)*ln(x-_R),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

3.250.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1202 vs. $2(199) = 398$.

Time = 1.06 (sec) , antiderivative size = 1202, normalized size of antiderivative = 3.92

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas
")
```

output
```
1/3*x^3 + 1/2*x^2 - 1/112*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 5
5/32) - 21)*log(23324*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 5
5/32) + 3/16)^3 - 23765*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) -
 55/32) + 3/16)^2 + 7744*x + 19470*I*sqrt(7) - 33040*sqrt(2101/1568*I*sqrt
(7) - 55/32) + 38950) - 1/112*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7)
 - 55/32) - 21)*log(-23324*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7
) - 55/32) + 3/16)^3 + 49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqr
t(7) - 55/32) + 3/16)^2*(-561*I*sqrt(7) + 952*sqrt(2101/1568*I*sqrt(7) - 5
5/32) - 869) + 1/256*(53312*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(
7) - 55/32) + 3/16)^2 - 11781*I*sqrt(7) + 19992*sqrt(2101/1568*I*sqrt(7) -
 55/32) - 36681)*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 2
1) + 17493*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/1
6)^2 + 7744*x - 15708*I*sqrt(7) + 26656*sqrt(2101/1568*I*sqrt(7) - 55/32)
- 29132) + 1/112*(2*sqrt(7)*sqrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/15
68*I*sqrt(7) - 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/
1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/156
8*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) -
55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 185
9/2) + 28*sqrt(2101/1568*I*sqrt(7) - 55/32) + 28*sqrt(-2101/1568*I*sqrt(7)
 - 55/32) + 21)*log(-49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sq...
```

3.250.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.20

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{x^3}{3}+\frac{x^2}{2}-\frac{5x}{2}$$
$$+\operatorname{RootSum}\left(1372t^4-1029t^3+3136t^2+2688t+512,\left(t\mapsto t\log\left(\frac{5831t^3}{1936}-\frac{23765t^2}{7744}+\frac{2065t}{242}+x+\frac{41}{12}\right.\right.\right.$$

input
```
integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

output
```
x**3/3 + x**2/2 - 5*x/2 + RootSum(1372*_t**4 - 1029*_t**3 + 3136*_t**2 + 2
688*_t + 512, Lambda(_t, _t*log(5831*_t**3/1936 - 23765*_t**2/7744 + 2065*
_t/242 + x + 415/121)))
```

3.250.7 Maxima [F]

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x^3}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

input
```
integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima
")
```

output
```
1/3*x^3 + 1/2*x^2 - 5/2*x + 1/2*integrate((3*x^3 + 19*x^2 + x + 10)/(2*x^4
 + x^3 + 5*x^2 + x + 2), x)
```

3.250.8 Giac [F]

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x^3}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

input
```
integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

output
```
integrate((2*x^3 + 3*x^2 + x + 5)*x^3/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

3.250.9 Mupad [B] (verification not implemented)

Time = 9.86 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.42

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \left(\sum_{k=1}^{4}\ln\left(-29\,x+\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)\left(-\frac{289\,x}{4}+\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)\left(\frac{581\,x}{16}\right.\right.\right.\right.$$
$$\left.\left.+7\right)\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)\right)-\frac{5\,x}{2}+\frac{x^2}{2}+\frac{x^3}{3}$$

input
```
int((x^3*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
symsum(log(root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*
(root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*((581*x)/1
6 - root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*((147*x
)/4 - 49/16) + 1141/64) - (289*x)/4 + 47/4) - 29*x + 7)*root(z^4 - (3*z^3)
/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k), k, 1, 4) - (5*x)/2 + x^2/2 +
 x^3/3
```

3.251 $\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

3.251.1 Optimal result

Integrand size = 35, antiderivative size = 269

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2-\frac{(53i+\sqrt{7})\arctan\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}}+\frac{(53i-\sqrt{7})\arctan\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}}-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)$$

output
```
1/14*x*(7-5*I*7^(1/2))+1/28*x^2*(7-5*I*7^(1/2))+1/14*x*(7+5*I*7^(1/2))+1/2
8*x^2*(7+5*I*7^(1/2))-1/56*ln(4+4*x^2+x*(1+I*7^(1/2)))*(35-9*I*7^(1/2))-1/
56*ln(4+4*x^2+x*(1-I*7^(1/2)))*(35+9*I*7^(1/2))+1/2*arctan((1+8*x+I*7^(1/2
))/(70-2*I*7^(1/2))^(1/2))*(53*I-7^(1/2))/(490-14*I*7^(1/2))^(1/2)-1/2*arc
tan((1+8*x-I*7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(53*I+7^(1/2))/(490+14*I*7^(
1/2))^(1/2)
```

3.251.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.38

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$$
$$= x+\frac{x^2}{2}-\text{RootSum}\Big[2+\#1+5\#1^2+\#1^3$$
$$+2\#1^4\&,\frac{2\log(x-\#1)+3\log(x-\#1)\#1+\log(x-\#1)\#1^2+5\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\Big]$$

input
```
Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

output
```
x + x^2/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (2*Log[x - #1] + 3
*Log[x - #1]*#1 + Log[x - #1]*#1^2 + 5*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1
^2 + 8*#1^3) & ]
```

3.251.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.80, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(2x^3+3x^2+x+5\right)}{2x^4+x^3+5x^2+x+2}\,dx$$
$$\Big\downarrow \text{ 2492}$$
$$\frac{1}{2}\int \left(2x-\frac{2\left(\left(9+5i\sqrt{7}\right)x+2\left(5+i\sqrt{7}\right)\right)}{\sqrt{7}\left(4ix^2+\left(i-\sqrt{7}\right)x+4i\right)}-\frac{2\left(\left(35i-9\sqrt{7}\right)x+2\left(7i-5\sqrt{7}\right)\right)}{7\left(4ix^2+\left(i+\sqrt{7}\right)x+4i\right)}+2\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{(53+i\sqrt{7})\operatorname{arctanh}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2(35-i\sqrt{7})}}\right)}{\sqrt{14\,(35-i\sqrt{7})}}-\frac{(53-i\sqrt{7})\operatorname{arctanh}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2(35+i\sqrt{7})}}\right)}{\sqrt{14\,(35+i\sqrt{7})}}+x^2-\frac{1}{28}\left(35-9i\sqrt{7}\right)\log\left(4ix^2\right.\right.$$

input
```
Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

output
```
(2*x + x^2 + ((53 + I*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35
- I*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] - ((53 - I*Sqrt[7])*ArcTanh[(I +
 Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[14*(35 + I*Sqrt[7])] -
 ((35 - (9*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/28 - ((35 +
 (9*I)*Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/28)/2
```

3.251.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

3.251.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.25

method	result	size
default	$\frac{x^2}{2}+x+\left(\sum_{\_R=\operatorname{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{\left(-5\_R^3-\_R^2-3\_R-2\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}\right)$	67
risch	$\frac{x^2}{2}+x+\left(\sum_{\_R=\operatorname{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{\left(-5\_R^3-\_R^2-3\_R-2\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}\right)$	67

input
```
int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+x+sum((-5*_R^3-_R^2-3*_R-2)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=Ro
otOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

3.251.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1145 vs. 2(171) = 342.

Time = 1.02 (sec) , antiderivative size = 1145, normalized size of antiderivative = 4.26

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas
")
```

output
```
1/2*x^2 - 1/56*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35)*log
(49/4*(135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(9/56*I
*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 10290*(-9/56*I*sq
rt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 - 25725*(-9/56*I*sqrt
(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 3/64*(3920*(-9/56*I*s
qrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4
900*sqrt(-37/392*I*sqrt(7) + 79/56) + 5587)*(-9*I*sqrt(7) + 28*sqrt(37/392
*I*sqrt(7) + 79/56) + 35) + 8384*x + 6615/2*I*sqrt(7) + 10290*sqrt(-37/392
*I*sqrt(7) + 79/56) + 13373/2) + 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqr
t(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/
392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*s
qrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) +
 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2) + 2*sq
rt(37/392*I*sqrt(7) + 79/56) + 2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5)*log(
-49/4*(135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(9/56*I
*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 24304*(-9/56*I*sq
rt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 3/64*(3920*(-9/56*I
*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1575*I*sqrt(7) -
 4900*sqrt(-37/392*I*sqrt(7) + 79/56) + 5587)*(-9*I*sqrt(7) + 28*sqrt(37/3
92*I*sqrt(7) + 79/56) + 35) + 7/64*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(...
```

3.251.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3662 vs. $2(219) = 438$.

Time = 1.59 (sec) , antiderivative size = 3662, normalized size of antiderivative = 13.61

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

output
```
x**2/2 + x + (-5/8 + sqrt(79/448 + sqrt(77)/49))*log(x**2 + x*(-1459*sqrt(
14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 - 15
*sqrt(77)*sqrt(553 + 64*sqrt(77))/2096 - 10391*sqrt(553 + 64*sqrt(77))/268
288 + 1459*sqrt(77)/8384 + 522933/268288 + 45*sqrt(14)*sqrt(553 + 64*sqrt(
77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576) -
510895297*sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(7
7))/71978450944 - 6009493*sqrt(22)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 219
75 + 7648*sqrt(77))/1124663296 - 38714551*sqrt(77)*sqrt(553 + 64*sqrt(77))
/2249326592 - 4417610843*sqrt(553 + 64*sqrt(77))/35989225472 + 153195*sqrt
(22)*sqrt(553 + 64*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7
648*sqrt(77))/2249326592 + 8313499*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(-
333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71978450944 + 2908324
44193/35989225472 + 2303470247*sqrt(77)/2249326592) + (-5/8 - sqrt(79/448
+ sqrt(77)/49))*log(x**2 + x*(-45*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(33
3*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 - 1459*sqrt(14)*
sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 + 10391*s
qrt(553 + 64*sqrt(77))/268288 + 1459*sqrt(77)/8384 + 522933/268288 + 15*sq
rt(77)*sqrt(553 + 64*sqrt(77))/2096) - 510895297*sqrt(14)*sqrt(333*sqrt(55
3 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71978450944 - 6009493*sqrt(22)*s
qrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/1124663296 - 8...
```

3.251.7 Maxima [F]

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x^2}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

input
```
integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima
")
```

output
```
1/2*x^2 + x - integrate((5*x^3 + x^2 + 3*x + 2)/(2*x^4 + x^3 + 5*x^2 + x +
 2), x)
```

3.251.8 Giac [F]

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x^2}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

input
```
integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

output
```
integrate((2*x^3 + 3*x^2 + x + 5)*x^2/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

3.251.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.70

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = x+\frac{x^2}{2} + \left(\sum_{k=1}^{4}\ln\left(-\frac{179\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}{8} - 7\,x - \frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\,x\,459}{8} - \frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^2\,x\,665}{8} - \frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^3\,x\,147}{4} - \frac{35\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^2}{32} + \frac{49\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^3}{16} - 15\right)\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\right)$$

input
```
int((x^2*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
x + x^2/2 + symsum(log((49*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/
343, z, k)^3)/16 - 7*x - (459*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 1
28/343, z, k)*x)/8 - (665*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/3
43, z, k)^2*x)/8 - (147*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343
, z, k)^3*x)/4 - (35*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z
, k)^2)/32 - (179*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k
))/8 - 15)*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k), k, 1
, 4)
```

3.251. $\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

3.252 $\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

3.252.1 Optimal result

Integrand size = 33, antiderivative size = 230

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x - \frac{(19i+7\sqrt{7})\arctan\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}} + \frac{(19i-7\sqrt{7})\arctan\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}} + \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right) + \frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)$$

output
```
1/14*x*(7-5*I*7^(1/2))+1/28*ln(4+4*x^2+x*(1+I*7^(1/2)))*(7-5*I*7^(1/2))+1/
14*x*(7+5*I*7^(1/2))+1/28*ln(4+4*x^2+x*(1-I*7^(1/2)))*(7+5*I*7^(1/2))+arct
an((1+8*x+I*7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(19*I-7*7^(1/2))/(490-14*I*7^
(1/2))^(1/2)-arctan((1+8*x-I*7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(19*I+7*7^(1
/2))/(490+14*I*7^(1/2))^(1/2)
```

3.252.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.41

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$$
$$= x+2\text{RootSum}\Big[2+\#1+5\#1^2+\#1^3$$
$$+2\#1^4\&,\frac{-\log(x-\#1)+2\log(x-\#1)\#1-2\log(x-\#1)\#1^2+\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\Big]$$

input
```
Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

output
```
x + 2*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (-Log[x - #1] + 2*Log[x
- #1]*#1 - 2*Log[x - #1]*#1^2 + Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*
#1^3) & ]
```

3.252.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(2x^3+3x^2+x+5)}{2x^4+x^3+5x^2+x+2}\,dx$$
$$\Big\downarrow \text{2492}$$
$$\frac{1}{2}\int \left(-\frac{4(7i+5\sqrt{7})(1-x)}{7(4ix^2+(i-\sqrt{7})x+4i)}-\frac{4(7i-5\sqrt{7})(1-x)}{7(4ix^2+(i+\sqrt{7})x+4i)}+2\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(19+7i\sqrt{7})\operatorname{arctanh}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2(35-i\sqrt{7})}}\right)}{\sqrt{\frac{7}{2}(35-i\sqrt{7})}}-\frac{(19-7i\sqrt{7})\operatorname{arctanh}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2(35+i\sqrt{7})}}\right)}{\sqrt{\frac{7}{2}(35+i\sqrt{7})}}+\frac{1}{14}(7-5i\sqrt{7})\log\left(4ix^2+\left(-\right.\right.\right.$$

input
```
Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

output
```
(2*x + ((19 + (7*I)*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 -
I*Sqrt[7])]])/Sqrt[(7*(35 - I*Sqrt[7]))/2] - ((19 - (7*I)*Sqrt[7])*ArcTanh
[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[(7*(35 + I*Sqrt[7
]))/2] + ((7 - (5*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/14 +
 ((7 + (5*I)*Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/14)/2
```

3.252.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

3.252.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.27

method	result	size
default	$x+2\left(\sum_{\_R=\operatorname{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(\_R^3-2\_R^2+2\_R-1)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)$	62
risch	$x+2\left(\sum_{\_R=\operatorname{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(\_R^3-2\_R^2+2\_R-1)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)$	62

input
```
int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
x+2*sum((_R^3-2*_R^2+2*_R-1)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=RootOf(2*
_Z^4+_Z^3+5*_Z^2+_Z+2))
```

3.252.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1190 vs. 2(151) = 302.

Time = 1.00 (sec) , antiderivative size = 1190, normalized size of antiderivative = 5.17

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \text{Too large to display}$$

input
```
integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

output
```
-1/28*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)*log(49/4*(55*I
*sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1/14) + 147)*(5/28*I*sqrt(7) - 1/2*s
qrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 3773*(-5/28*I*sqrt(7) - 1/2*sqrt(5
3/98*I*sqrt(7) - 1/14) + 1/4)^3 + 3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I
*sqrt(7) - 1/14) + 1/4)^2 + 11/16*(196*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I
*sqrt(7) - 1/14) + 1/4)^2 + 35*I*sqrt(7) + 98*sqrt(53/98*I*sqrt(7) - 1/14)
 + 15)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 304*x + 115
5/2*I*sqrt(7) + 1617*sqrt(53/98*I*sqrt(7) - 1/14) + 1903/2) + 1/28*(2*sqrt
(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2
- 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(
5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqr
t(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) -
 1/14) - 27/2) + 7*sqrt(53/98*I*sqrt(7) - 1/14) + 7*sqrt(-53/98*I*sqrt(7)
- 1/14) + 7)*log(-49/4*(55*I*sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1/14) +
147)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 2744*(
-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 11/16*(196*(
-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 35*I*sqrt(7)
 + 98*sqrt(53/98*I*sqrt(7) - 1/14) + 15)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7) + 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*s
qrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7...
```

3.252.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.21

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = x + \text{RootSum}\left(343t^4-343t^3+294t^2-336t+128, \left(t \mapsto t\log\left(\frac{3773t^3}{304}-\frac{1029t^2}{304}+\frac{1001t}{152}+x-\frac{121}{19}\right)\right)\right)$$

input
```
integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

output
```
x + RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _
t*log(3773*_t**3/304 - 1029*_t**2/304 + 1001*_t/152 + x - 121/19)))
```

3.252.7 Maxima [F]

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

input
```
integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

output
```
x + 2*integrate((x^3 - 2*x^2 + 2*x - 1)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

3.252.8 Giac [F]

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

input
```
integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

output
```
integrate((2*x^3 + 3*x^2 + x + 5)*x/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

3.252.9 Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.80

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = x+\left(\sum_{k=1}^{4}\ln\left(\frac{115\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)}{8}+15\,x-\frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)\,x\,137}{8}+\frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^2\,x\,133}{8}-\frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^3\,x\,147}{4}-\frac{189\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^2}{16}+\frac{49\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^3}{16}-4\right)\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)\right)$$

input
```
int((x*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
x + symsum(log((115*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k
))/8 + 15*x - (137*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)
*x)/8 + (133*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2*x)/
8 - (147*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3*x)/4 -
(189*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2)/16 + (49*r
oot(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3)/16 - 4)*root(z^4
 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k), k, 1, 4)
```

3.253 $\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx$

3.253.1 Optimal result

Integrand size = 32, antiderivative size = 198

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = \frac{\left(19i+7\sqrt{7}\right)\arctan\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}} - \frac{\left(19i-7\sqrt{7}\right)\arctan\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}} + \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right) + \frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)$$

output
```
1/28*ln(4+4*x^2+x*(1+I*7^(1/2)))*(7-5*I*7^(1/2))+1/28*ln(4+4*x^2+x*(1-I*7^
(1/2)))*(7+5*I*7^(1/2))-arctan((1+8*x+I*7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(
19*I-7*7^(1/2))/(490-14*I*7^(1/2))^(1/2)+arctan((1+8*x-I*7^(1/2))/(70+2*I*
7^(1/2))^(1/2))*(19*I+7*7^(1/2))/(490+14*I*7^(1/2))^(1/2)
```

3.253.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.45

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = \text{RootSum}\Big[2+\#1+5\#1^2+\#1^3 + 2\#1^4\&, \frac{5\log(x-\#1)+\log(x-\#1)\#1+3\log(x-\#1)\#1^2+2\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\Big]$$

input
```
Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

output
```
RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (5*Log[x - #1] + Log[x - #1]*#
1 + 3*Log[x - #1]*#1^2 + 2*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3)
 & ]
```

3.253.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+x+5}{2x^4+x^3+5x^2+x+2}\,dx$$

$$\Big\downarrow \text{2492}$$

$$\frac{1}{2}\int \left(\frac{2i\left(2\left(5i+\sqrt{7}\right)x+5\sqrt{7}+9i\right)}{\sqrt{7}\left(4ix^2+\left(i+\sqrt{7}\right)x+4i\right)}-\frac{2i\left(2\left(5i-\sqrt{7}\right)x-5\sqrt{7}+9i\right)}{\sqrt{7}\left(4ix^2+\left(i-\sqrt{7}\right)x+4i\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{(19+7i\sqrt{7})\,\text{arctanh}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2(35-i\sqrt{7})}}\right)}{\sqrt{\frac{7}{2}(35-i\sqrt{7})}}+\frac{(19-7i\sqrt{7})\,\text{arctanh}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2(35+i\sqrt{7})}}\right)}{\sqrt{\frac{7}{2}(35+i\sqrt{7})}}+\frac{1}{14}(7-5i\sqrt{7})\log\left(4ix^2+\left(\right.\right.\right.$$

input
```
Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

output
```
(-(((19 + (7*I)*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sq
rt[7])]])/Sqrt[(7*(35 - I*Sqrt[7]))/2]) + ((19 - (7*I)*Sqrt[7])*ArcTanh[(I
 + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[(7*(35 + I*Sqrt[7]))
/2] + ((7 - (5*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/14 + ((
7 + (5*I)*Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/14)/2
```

3.253.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

3.253.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.29

method	result	size
default	$\sum_{\_R=\text{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)} \frac{(2\_R^3+3\_R^2+\_R+5)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}$	58
risch	$\sum_{\_R=\text{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)} \frac{(2\_R^3+3\_R^2+\_R+5)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}$	58

input
```
int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
sum((2*_R^3+3*_R^2+_R+5)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=RootOf(2*_Z^4
+_Z^3+5*_Z^2+_Z+2))
```

3.253.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1189 vs. 2(129) = 258.

Time = 1.00 (sec) , antiderivative size = 1189, normalized size of antiderivative = 6.01

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = \text{Too large to display}$$

input
```
integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

output
```
-1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/
14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/
4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqr
t(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/9
8*I*sqrt(7) - 1/14) - 27/2) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 7*sqrt(-53/
98*I*sqrt(7) - 1/14) - 7)*log(49/4*(105*I*sqrt(7) + 294*sqrt(53/98*I*sqrt(
7) - 1/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/
4)^2 + 4900*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 +
 1/16*(4116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 +
 735*I*sqrt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-5*I*sqrt(7) + 1
4*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2
*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(5
3/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt
(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) -
5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((21*sqrt(7)*(5*I*s
qrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7))*(-5*I*sqrt(7)
 + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7)*(5*I*sqrt(7) + 14*s
qrt(53/98*I*sqrt(7) - 1/14) - 7) + 7040*sqrt(7)) + 608*x + 325*I*sqrt(7) +
 910*sqrt(53/98*I*sqrt(7) - 1/14) - 1247) + 1/28*(2*sqrt(7)*sqrt(-21*(5/28
*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*s...
```

3.253.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.23

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx$$
$$=\operatorname{RootSum}\left(343t^4-343t^3+294t^2-336t+128,\left(t\mapsto t\log\left(-\frac{7203t^3}{304}+\frac{2303t^2}{304}-\frac{2177t}{152}+x+\frac{250}{19}\right)\right)\right)$$

input
```
integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

output
```
RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _t*lo
g(-7203*_t**3/304 + 2303*_t**2/304 - 2177*_t/152 + x + 250/19)))
```

3.253.7 Maxima [F]

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx=\int \frac{2\,x^3+3\,x^2+x+5}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

input
```
integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

output
```
integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

3.253.8 Giac [F]

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx=\int \frac{2\,x^3+3\,x^2+x+5}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

input
```
integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

output
```
integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

3.253.9 Mupad [B] (verification not implemented)

Time = 9.83 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.91

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = \sum_{k=1}^{4} \ln\left(-\frac{193\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)}{8}+4\,x-\frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)\,x\,137}{8}+\frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^2\,x\,651}{16}-\frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^3\,x\,147}{4}+\frac{273\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^2}{16}+\frac{49\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^3}{16}+7\right)\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)$$

input
```
int((x + 3*x^2 + 2*x^3 + 5)/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
symsum(log(4*x - (193*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z,
 k))/8 - (137*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)*x)/8
 + (651*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2*x)/16 -
(147*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3*x)/4 + (273
*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2)/16 + (49*root(
z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3)/16 + 7)*root(z^4 - z
^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k), k, 1, 4)
```

3.254 $\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)} \, dx$

3.254.1 Optimal result

Integrand size = 35, antiderivative size = 245

$$\begin{aligned}\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)} \, dx = &-\frac{\left(53+i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}} \\ &+\frac{\left(53-i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}} \\ &+\frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x)+\frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x) \\ &-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right) \\ &-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)\end{aligned}$$

output
```
1/28*ln(x)*(35-9*I*7^(1/2))-1/56*ln(4*I+4*I*x^2+x*(I-7^(1/2)))*(35-9*I*7^(
1/2))+1/28*ln(x)*(35+9*I*7^(1/2))-1/56*ln(4*I+4*I*x^2+x*(I+7^(1/2)))*(35+9
*I*7^(1/2))-1/2*arctanh((I+8*I*x-7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(53+I*7^
(1/2))/(490-14*I*7^(1/2))^(1/2)+1/2*arctanh((I+8*I*x+7^(1/2))/(70+2*I*7^(1
/2))^(1/2))*(53-I*7^(1/2))/(490+14*I*7^(1/2))^(1/2)
```

3.254.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.41

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx = \frac{5\log(x)}{2}-\frac{1}{2}\text{RootSum}\Big[2+\#1+5\#1^2+\#1^3 +2\#1^4\&,\frac{3\log(x-\#1)+19\log(x-\#1)\#1+\log(x-\#1)\#1^2+10\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\Big]$$

input
```
Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

output
```
(5*Log[x])/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (3*Log[x - #1]
+ 19*Log[x - #1]*#1 + Log[x - #1]*#1^2 + 10*Log[x - #1]*#1^3)/(1 + 10*#1 +
 3*#1^2 + 8*#1^3) & ]/2
```

3.254.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.87, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2496, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+x+5}{x(2x^4+x^3+5x^2+x+2)}\,dx$$

$$\Big\downarrow \text{2496}$$

$$\frac{1}{2}\int \left(-\frac{2(35i-9\sqrt{7})x+3(7i+11\sqrt{7})}{7(4ix^2+(i+\sqrt{7})x+4i)}+\frac{5}{x}-\frac{2(35i+9\sqrt{7})x+3(7i-11\sqrt{7})}{7(4ix^2+(i-\sqrt{7})x+4i)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{(53+i\sqrt{7})\,\text{arctanh}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2(35-i\sqrt{7})}}\right)}{\sqrt{14(35-i\sqrt{7})}}+\frac{(53-i\sqrt{7})\,\text{arctanh}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2(35+i\sqrt{7})}}\right)}{\sqrt{14(35+i\sqrt{7})}}-\frac{1}{28}(35-9i\sqrt{7})\log\left(4ix^2+(\right.\right.$$

input
```
Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

output
```
(-(((53 + I*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7
])]])/Sqrt[14*(35 - I*Sqrt[7])]) + ((53 - I*Sqrt[7])*ArcTanh[(I + Sqrt[7]
+ (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[14*(35 + I*Sqrt[7])] + 5*Log[x]
 - ((35 - (9*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/28 - ((35
 + (9*I)*Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/28)/2
```

3.254.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2496
```
Int[(Px_.)*(x_)^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e
_.)*(x_)^4)^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[x^m*Px*(b/d +
 ((d + Sqrt[e*((b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((
d - Sqrt[e*((b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2
- b^2*e, 0]
```

3.254.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.21

method	result
risch	$\frac{5\ln(x)}{2}+\left(\sum_{\_R=\mathrm{RootOf}\left(686\_Z^4+1715\_Z^3+1372\_Z^2+448\_Z+256\right)}\_R\ln\left(2058\_R^3+20825\_R^2+25844\_R\right.\right.$
default	$\frac{5\ln(x)}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(-10\_R^3-\_R^2-19\_R-3\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}\right)}{2}$

input
```
int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
5/2*ln(x)+sum(_R*ln(2058*_R^3+20825*_R^2+25844*_R+8384*x+6816),_R=RootOf(6
86*_Z^4+1715*_Z^3+1372*_Z^2+448*_Z+256))
```

3.254.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1143 vs. 2(148) = 296.

Time = 1.02 (sec) , antiderivative size = 1143, normalized size of antiderivative = 4.67

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx = \text{Too large to display}$$

input
```
integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

output
```
-1/56*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35)*log(49/4*(27
*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56) + 1385)*(9/56*I*sqrt(7) -
1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 2058*(-9/56*I*sqrt(7) - 1/2*
sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 - 5145*(-9/56*I*sqrt(7) - 1/2*sqr
t(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 1/64*(2352*(-9/56*I*sqrt(7) - 1/2*
sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 945*I*sqrt(7) - 2940*sqrt(-37/3
92*I*sqrt(7) + 79/56) - 28507)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) +
79/56) + 35) + 8384*x + 1323/2*I*sqrt(7) + 2058*sqrt(-37/392*I*sqrt(7) + 7
9/56) + 16089/2) + 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqr
t(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7)
+ 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56
) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I
*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2) + 2*sqrt(37/392*I*sq
rt(7) + 79/56) + 2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5)*log(-49/4*(27*I*sq
rt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56) + 1385)*(9/56*I*sqrt(7) - 1/2*s
qrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 15680*(-9/56*I*sqrt(7) - 1/2*sqrt
(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/64*(2352*(-9/56*I*sqrt(7) - 1/2*s
qrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 945*I*sqrt(7) - 2940*sqrt(-37/39
2*I*sqrt(7) + 79/56) - 28507)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 7
9/56) + 35) + 7/64*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7)...
```

3.254.6 Sympy [A] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.24

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx = \frac{5\log(x)}{2} + \operatorname{RootSum}\left(686t^4+1715t^3+1372t^2+448t+256, \left(t \mapsto t\log\left(-\frac{160344611t^4}{532759184} - \frac{16880402t^3}{33297449} + \frac{401052}{21310}\right.\right.\right.$$

input
```
integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+5*x**2+x+2),x)
```

output
```
5*log(x)/2 + RootSum(686*_t**4 + 1715*_t**3 + 1372*_t**2 + 448*_t + 256, L
ambda(_t, _t*log(-160344611*_t**4/532759184 - 16880402*_t**3/33297449 + 40
10520787*_t**2/2131036736 + 1537535671*_t/532759184 + x + 46660495/6659489
8)))
```

3.254.7 Maxima [F]

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx = \int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x}\,dx$$

input
```
integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

output
```
-1/2*integrate((10*x^3 + x^2 + 19*x + 3)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
 + 5/2*log(x)
```

3.254.8 Giac [F]

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx = \int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x}\,dx$$

input
```
integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

output
```
integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x), x)
```

3.254.9 Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.97

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx = \frac{5\,\ln(x)}{2} + \left(\sum_{k=1}^{4}\ln\left(\frac{223\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}{8} - \frac{31\,x}{2} + \frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\,x\,71}{16} - \frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^2\,x\,4463}{64} + \frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^3\,x\,1449}{16} + \frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^4\,x\,3675}{32} + \frac{257\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^2}{32} + \frac{1673\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^3}{64} - \frac{441\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^4}{32} + 10\right)\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\right)$$

input
```
int((x + 3*x^2 + 2*x^3 + 5)/(x*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)
```

output
```
(5*log(x))/2 + symsum(log((223*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 +
128/343, z, k))/8 - (31*x)/2 + (71*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/4
9 + 128/343, z, k)*x)/16 - (4463*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49
+ 128/343, z, k)^2*x)/64 + (1449*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49
+ 128/343, z, k)^3*x)/16 + (3675*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49
+ 128/343, z, k)^4*x)/32 + (257*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 +
 128/343, z, k)^2)/32 + (1673*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 1
28/343, z, k)^3)/64 - (441*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/
343, z, k)^4)/32 + 10)*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343,
 z, k), k, 1, 4)
```

3.254. $\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx$

3.255 $\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx$

3.255.1 Optimal result

Integrand size = 35, antiderivative size = 281

$$\begin{aligned}\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx =& -\frac{35-9i\sqrt{7}}{28x}-\frac{35+9i\sqrt{7}}{28x} \\ &+\frac{11\left(9+5i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}} \\ &-\frac{11\left(9-5i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35+i\sqrt{7}\right)}} \\ &-\frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x)-\frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x) \\ &+\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right) \\ &+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)\end{aligned}$$

output
```
1/28*(-35+9*I*7^(1/2))/x+1/28*(-35-9*I*7^(1/2))/x-3/56*ln(x)*(7-11*I*7^(1/
2))+3/112*ln(4*I+4*I*x^2+x*(I+7^(1/2)))*(7-11*I*7^(1/2))-3/56*ln(x)*(7+11*
I*7^(1/2))+3/112*ln(4*I+4*I*x^2+x*(I-7^(1/2)))*(7+11*I*7^(1/2))+11/4*arcta
nh((I+8*I*x-7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(9+5*I*7^(1/2))/(490-14*I*7^(
1/2))^(1/2)-11/4*arctanh((I+8*I*x+7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(9-5*I*
7^(1/2))/(490+14*I*7^(1/2))^(1/2)
```

3.255.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.39

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx = -\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{1}{4}\text{RootSum}\Big[2+\#1+5\#1^2+\#1^3 +2\#1^4\&, \frac{-35\log(x-\#1)+13\log(x-\#1)\#1-17\log(x-\#1)\#1^2+6\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\Big]$$

input
```
Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

output
```
-5/(2*x) - (3*Log[x])/4 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (-35
*Log[x - #1] + 13*Log[x - #1]*#1 - 17*Log[x - #1]*#1^2 + 6*Log[x - #1]*#1^
3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]/4
```

3.255.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.80, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2496, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+x+5}{x^2(2x^4+x^3+5x^2+x+2)}\,dx$$

$$\Big\downarrow \text{ 2496}$$

$$\frac{1}{2}\int \left(-\frac{6(11-i\sqrt{7})x+7(9+5i\sqrt{7})}{2\sqrt{7}(4ix^2+(i-\sqrt{7})x+4i)}-\frac{3}{2x}-\frac{7(35i-9\sqrt{7})-6(7i+11\sqrt{7})x}{14(4ix^2+(i+\sqrt{7})x+4i)}+\frac{5}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{11(9+5i\sqrt{7})\text{arctanh}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2(35-i\sqrt{7})}}\right)}{2\sqrt{14(35-i\sqrt{7})}}-\frac{11(9-5i\sqrt{7})\text{arctanh}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2(35+i\sqrt{7})}}\right)}{2\sqrt{14(35+i\sqrt{7})}}+\frac{3}{56}(7+11i\sqrt{7})\log(4ix^2+\right.$$

input
```
Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

output
```
(-5/x + (11*(9 + (5*I)*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35
 - I*Sqrt[7])]])/(2*Sqrt[14*(35 - I*Sqrt[7])]) - (11*(9 - (5*I)*Sqrt[7])*A
rcTanh[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 +
 I*Sqrt[7])]) - (3*Log[x])/2 + (3*(7 + (11*I)*Sqrt[7])*Log[4*I + (I - Sqrt
[7])*x + (4*I)*x^2])/56 + (3*(7 - (11*I)*Sqrt[7])*Log[4*I + (I + Sqrt[7])*
x + (4*I)*x^2])/56)/2
```

3.255.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2496
```
Int[(Px_.)*(x_)^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e
_.)*(x_)^4)^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[x^m*Px*(b/d +
 ((d + Sqrt[e*((b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((
d - Sqrt[e*((b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2
- b^2*e, 0]
```

3.255.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.21

method	result
risch	$-\frac{5}{2x}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(686\_Z^4-1029\_Z^3+6272\_Z^2+10752\_Z+4096\right)}\_R\ln\left(-45962\_R^3+98735\_R^2-497168\_R+61952x-38\right.\right.}{2}$
default	$-\frac{5}{2x}-\frac{3\ln(x)}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(6\_R^3-17\_R^2+13\_R-35\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}\right)}{4}$

input
```
int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
-5/2/x+1/2*sum(_R*ln(-45962*_R^3+98735*_R^2-497168*_R+61952*x-384256),_R=R
ootOf(686*_Z^4-1029*_Z^3+6272*_Z^2+10752*_Z+4096))-3/4*ln(x)
```

3.255.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1245 vs. 2(172) = 344.

Time = 1.01 (sec) , antiderivative size = 1245, normalized size of antiderivative = 4.43

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+5x^2+x^3+2x^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas
")
```

output
```
-1/224*(2*x*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*lo
g(91924*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^
3 - 49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/1
6)^2*(-2211*I*sqrt(7) + 3752*sqrt(2101/1568*I*sqrt(7) - 55/32) - 3839) - 1
/256*(210112*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3
/16)^2 - 46431*I*sqrt(7) + 78792*sqrt(2101/1568*I*sqrt(7) - 55/32) - 11748
3)*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - 68943*(33
/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 15488*x
 + 61908*I*sqrt(7) - 105056*sqrt(2101/1568*I*sqrt(7) - 55/32) + 123428) +
2*x*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21)*log(-91924
*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^3 + 987
35*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 1
5488*x - 146487/2*I*sqrt(7) + 124292*sqrt(2101/1568*I*sqrt(7) - 55/32) - 2
85347/2) + (4*sqrt(7)*sqrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*s
qrt(7) - 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I
*sqrt(7) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sq
rt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32)
 + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2)*x
 - x*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - x*(-33*
I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) - 84*x)*log(49/4...
```

3.255.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25507 vs. $2(241) = 482$.

Time = 18.69 (sec) , antiderivative size = 25507, normalized size of antiderivative = 90.77

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx = \text{Too large to display}$$

input
```
integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+5*x**2+x+2),x)
```

output
```
-3*log(x)/4 + (3/16 - sqrt(-55/256 + 11*sqrt(77)/196))*log(x**2 + x*(10896
479943156192*sqrt(77)/(-39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) -
815992034457600 + 6974290892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454
628044800*sqrt(77)) + 1720992726634016*sqrt(7)*sqrt(-245 + 64*sqrt(77))/(-
39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034457600 + 697429
0892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628044800*sqrt(77)) + 39
6034568160*sqrt(14)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-24
5 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77
) + 5983777)/(-39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034
457600 + 6974290892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628044800
*sqrt(77)) + 1300300581888*sqrt(154)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*s
qrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983
777)/(-39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034457600 +
 6974290892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628044800*sqrt(77
)) - 278094051039*sqrt(22)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589*sqrt(11)*s
qrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*
sqrt(77) + 5983777)/(-39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 81
5992034457600 + 6974290892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 22545462
8044800*sqrt(77)) - 29480043023893*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245
+ 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(7...
```

3.255.7 Maxima [F]

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx = \int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x^2}\,dx$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima
")
```

output
```
-5/2/x + 1/4*integrate((6*x^3 - 17*x^2 + 13*x - 35)/(2*x^4 + x^3 + 5*x^2 +
 x + 2), x) - 3/4*log(x)
```

3.255.8 Giac [F]

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+5x^2+x^3+2x^4\right)}\,dx=\int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x^2}\,dx$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

output
```
integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^2), x)
```

3.255.9 Mupad [B] (verification not implemented)

Time = 9.67 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.86

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx = \left(\sum_{k=1}^{4}\ln\left(\frac{1199\,\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)}{32}+25\,x+\frac{\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)\,x\,4169}{32}+\frac{\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)^2\,x\,43993}{256}+\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)^3\,x\,28+\frac{\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)^4\,x\,3675}{32}+\frac{11647\,\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)^2}{128}+\frac{7273\,\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)^3}{128}-\frac{441\,\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)^4}{32}+\frac{21}{4}\right)\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)\right)-\frac{3\,\ln(x)}{4}-\frac{5}{2\,x}$$

input

```
int((x + 3*x^2 + 2*x^3 + 5)/(x^2*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)
```

output
```
symsum(log((1199*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343,
z, k))/32 + 25*x + (4169*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 1
28/343, z, k)*x)/32 + (43993*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49
 + 128/343, z, k)^2*x)/256 + 28*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)
/49 + 128/343, z, k)^3*x + (3675*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z
)/49 + 128/343, z, k)^4*x)/32 + (11647*root(z^4 - (3*z^3)/4 + (16*z^2)/7 +
 (96*z)/49 + 128/343, z, k)^2)/128 + (7273*root(z^4 - (3*z^3)/4 + (16*z^2)
/7 + (96*z)/49 + 128/343, z, k)^3)/128 - (441*root(z^4 - (3*z^3)/4 + (16*z
^2)/7 + (96*z)/49 + 128/343, z, k)^4)/32 + 21/4)*root(z^4 - (3*z^3)/4 + (1
6*z^2)/7 + (96*z)/49 + 128/343, z, k), k, 1, 4) - (3*log(x))/4 - 5/(2*x)
```

3.255. $\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx$

3.256 $\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx$

3.256.1 Optimal result

Integrand size = 35, antiderivative size = 317

$$\begin{aligned}\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+5x^2+x^3+2x^4\right)}\,dx &= -\frac{35-9i\sqrt{7}}{56x^2}-\frac{35+9i\sqrt{7}}{56x^2}\\ &+\frac{3\left(7-11i\sqrt{7}\right)}{56x}+\frac{3\left(7+11i\sqrt{7}\right)}{56x}\\ &+\frac{\left(355-73i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{8\sqrt{14\left(35-i\sqrt{7}\right)}}\\ &-\frac{\left(355+73i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{8\sqrt{14\left(35+i\sqrt{7}\right)}}\\ &-\frac{1}{16}\left(35-9i\sqrt{7}\right)\log(x)-\frac{1}{16}\left(35+9i\sqrt{7}\right)\log(x)\\ &+\frac{1}{32}\left(35-9i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)\\ &+\frac{1}{32}\left(35+9i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)\end{aligned}$$

output
```
1/56*(-35+9*I*7^(1/2))/x^2-1/16*ln(x)*(35-9*I*7^(1/2))+1/32*ln(4*I+4*I*x^2
+x*(I-7^(1/2)))*(35-9*I*7^(1/2))+1/56*(-35-9*I*7^(1/2))/x^2-1/16*ln(x)*(35
+9*I*7^(1/2))+1/32*ln(4*I+4*I*x^2+x*(I+7^(1/2)))*(35+9*I*7^(1/2))+3/56*(7-
11*I*7^(1/2))/x+3/56*(7+11*I*7^(1/2))/x+1/8*arctanh((I+8*I*x-7^(1/2))/(70-
2*I*7^(1/2))^(1/2))*(355-73*I*7^(1/2))/(490-14*I*7^(1/2))^(1/2)-1/8*arctan
h((I+8*I*x+7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(355+73*I*7^(1/2))/(490+14*I*7
^(1/2))^(1/2)
```

3.256.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.37

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx = -\frac{5}{4x^2}+\frac{3}{4x}-\frac{35\log(x)}{8}+\frac{1}{8}\text{RootSum}\Big[2+\#1+5\#1^2+\#1^3 +2\#1^4\&,\frac{61\log(x-\#1)+141\log(x-\#1)\#1+47\log(x-\#1)\#1^2+70\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\Big]$$

input
```
Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

output
```
-5/(4*x^2) + 3/(4*x) - (35*Log[x])/8 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*
#1^4 & , (61*Log[x - #1] + 141*Log[x - #1]*#1 + 47*Log[x - #1]*#1^2 + 70*L
og[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]/8
```

3.256.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.74, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2496, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+x+5}{x^3(2x^4+x^3+5x^2+x+2)}\,dx$$

$$\Big\downarrow \; 2496$$

$$\frac{1}{2}\int\left(\frac{14(35i-9\sqrt{7})x+223\sqrt{7}+427i}{28\left(4ix^2+(i+\sqrt{7})x+4i\right)}-\frac{35}{4x}-\frac{-14(9+5i\sqrt{7})x-61i\sqrt{7}+223}{4\sqrt{7}\left(4ix^2+(i-\sqrt{7})x+4i\right)}-\frac{3}{2x^2}+\frac{5}{x^3}\right)dx$$

$\big\downarrow$ 2009

$$\frac{1}{2}\left(\frac{(355-73i\sqrt{7})\operatorname{arctanh}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2(35-i\sqrt{7})}}\right)}{4\sqrt{14(35-i\sqrt{7})}}-\frac{(355+73i\sqrt{7})\operatorname{arctanh}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2(35+i\sqrt{7})}}\right)}{4\sqrt{14(35+i\sqrt{7})}}-\frac{5}{2x^2}+\frac{1}{16}(35-9i\sqrt{7})\log\right.$$

input
```
Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

output
```
(-5/(2*x^2) + 3/(2*x) + ((355 - (73*I)*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*
I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(4*Sqrt[14*(35 - I*Sqrt[7])]) - ((355 + (
73*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/
(4*Sqrt[14*(35 + I*Sqrt[7])]) - (35*Log[x])/4 + ((35 - (9*I)*Sqrt[7])*Log[
4*I + (I - Sqrt[7])*x + (4*I)*x^2])/16 + ((35 + (9*I)*Sqrt[7])*Log[4*I + (
I + Sqrt[7])*x + (4*I)*x^2])/16)/2
```

3.256.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2496
```
Int[(Px_.)*(x_)^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e
_.)*(x_)^4)^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[x^m*Px*(b/d +
 ((d + Sqrt[e*((b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((
d - Sqrt[e*((b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2
- b^2*e, 0]
```

3.256.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.20

method	result
risch	$\frac{\frac{3x}{4}-\frac{5}{4}}{x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(686\_Z^4-12005\_Z^3+73696\_Z^2-50176\_Z+65536\right)}\_R\ln\left(-2261742\_R^3+41411909\_R^2-249593568\_R\right.\right.}{4}$
default	$-\frac{5}{4x^2}+\frac{3}{4x}-\frac{35\ln(x)}{8}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(70\_R^3+47\_R^2+141\_R+61\right)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)}{8}$

input
```
int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
(3/4*x-5/4)/x^2+1/4*sum(_R*ln(-2261742*_R^3+41411909*_R^2-249593568*_R+154
597376*x+130505728),_R=RootOf(686*_Z^4-12005*_Z^3+73696*_Z^2-50176*_Z+6553
6))-35/8*ln(x)
```

3.256.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1274 vs. 2(196) = 392.

Time = 0.98 (sec) , antiderivative size = 1274, normalized size of antiderivative = 4.02

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+5x^2+x^3+2x^4\right)}\,dx=\text{Too large to display}$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas
")
```

output
```
-1/448*(14*x^2*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 3
5)*log(-49/4*(207711*I*sqrt(7) + 369264*sqrt(-9803/6272*I*sqrt(7) + 2815/8
96) - 957269)*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) +
 35/32)^2 + 9046968*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 281
5/896) + 35/32)^3 - 39580485*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt
(7) + 2815/896) + 35/32)^2 - 21/1024*(13785856*(-9/32*I*sqrt(7) - 1/2*sqrt
(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 16963065*I*sqrt(7) + 301565
60*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 68488563)*(-9*I*sqrt(7) + 16*sq
rt(9803/6272*I*sqrt(7) + 2815/896) - 35) + 9662336*x - 68336919/4*I*sqrt(7
) - 30371964*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 257023549/4) + 14*x^2
*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35)*log(-904696
8*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^3
 + 41411909*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) +
35/32)^2 + 9662336*x + 70198191/4*I*sqrt(7) + 31199196*sqrt(-9803/6272*I*s
qrt(7) + 2815/896) - 240366533/4) + 1960*x^2*log(x) + (4*sqrt(7)*sqrt(-134
4*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 -
1344*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)
^2 - 7/8*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 105)*(-
9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/2*I*sqr
t(7) - 1960*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*x^2 - 7*x^2...
```

3.256.6 Sympy [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.22

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+5x^2+x^3+2x^4\right)}\,dx = -\frac{35\log\left(x\right)}{8} + \operatorname{RootSum}\left(2744t^4-12005t^3+18424t^2-3136t+1024, \left(t\mapsto t\log\left(-\frac{20101387287723t^4}{91907904361586}+\frac{94451521}{45953952}\right.\right.\right. + \frac{3x-5}{4x^2}$$

input
```
integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+5*x**2+x+2),x)
```

output
```
-35*log(x)/8 + RootSum(2744*_t**4 - 12005*_t**3 + 18424*_t**2 - 3136*_t +
1024, Lambda(_t, _t*log(-20101387287723*_t**4/91907904361586 + 94451521449
6*_t**3/45953952180793 + 16572327093911939*_t**2/5882105879141504 - 456447
1749800865*_t/735263234892688 + x + 70084064010625/91907904361586))) + (3*
x - 5)/(4*x**2)
```

3.256.7 Maxima [F]

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx = \int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x^3}\,dx$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima
")
```

output
```
1/4*(3*x - 5)/x^2 + 1/8*integrate((70*x^3 + 47*x^2 + 141*x + 61)/(2*x^4 +
x^3 + 5*x^2 + x + 2), x) - 35/8*log(x)
```

3.256.8 Giac [F]

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx = \int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x^3}\,dx$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

output
```
integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^3), x)
```

3.256.9 Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.78

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx = \left(\sum_{k=1}^{4}\ln\left(-\frac{8939\,\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)}{128}-\frac{69\,x}{8}+\frac{\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)\,x\,14945}{128}-\frac{\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^2\,x\,269991}{1024}-\frac{\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^3\,x\,1393}{8}+\frac{\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^4\,x\,3675}{32}-\frac{35697\,\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^2}{512}-\frac{18487\,\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^3}{256}-\frac{441\,\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^4}{32}+\frac{245}{8}\right)\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)\right)-\frac{35\,\ln(x)}{8}+\frac{\frac{3\,x}{4}-\frac{5}{4}}{x^2}$$

input
```
int((x + 3*x^2 + 2*x^3 + 5)/(x^3*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)
```

output
```
symsum(log((14945*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343,
z, k)*x)/128 - (69*x)/8 - (8939*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)
/7 + 128/343, z, k))/128 - (269991*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8
*z)/7 + 128/343, z, k)^2*x)/1024 - (1393*root(z^4 - (35*z^3)/8 + (47*z^2)/
7 - (8*z)/7 + 128/343, z, k)^3*x)/8 + (3675*root(z^4 - (35*z^3)/8 + (47*z^
2)/7 - (8*z)/7 + 128/343, z, k)^4*x)/32 - (35697*root(z^4 - (35*z^3)/8 + (
47*z^2)/7 - (8*z)/7 + 128/343, z, k)^2)/512 - (18487*root(z^4 - (35*z^3)/8
 + (47*z^2)/7 - (8*z)/7 + 128/343, z, k)^3)/256 - (441*root(z^4 - (35*z^3)
/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z, k)^4)/32 + 245/8)*root(z^4 - (35*z
^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z, k), k, 1, 4) - (35*log(x))/8 +
((3*x)/4 - 5/4)/x^2
```

3.256. $\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx$

3.257 $\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx$

3.257.1 Optimal result

Integrand size = 40, antiderivative size = 19

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \frac{\arctan\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

output
```
arctan(c*x^3/(b*x^2+a))/c
```

3.257.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.58

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \frac{1}{2}\text{RootSum}\Big[a^2+2ab\#1^2+b^2\#1^4 + c^2\#1^6\&, \frac{3a\log(x-\#1)\#1+b\log(x-\#1)\#1^3}{2ab+2b^2\#1^2+3c^2\#1^4}\&\Big]$$

input
```
Integrate[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]
```

output
```
RootSum[a^2 + 2*a*b*#1^2 + b^2*#1^4 + c^2*#1^6 & , (3*a*Log[x - #1]*#1 + b
*Log[x - #1]*#1^3)/(2*a*b + 2*b^2*#1^2 + 3*c^2*#1^4) & ]/2
```

3.257.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2520, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}dx$$

$$\Big\downarrow \text{2520}$$

$$3a^2\int \frac{1}{\frac{a^2c^2x^6}{(bx^2+a)^2}+a^2}d\frac{x^3}{3(bx^2+a)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

input
```
Int[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]
```

output
```
ArcTan[(c*x^3)/(a + b*x^2)]/c
```

3.257.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2520
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Simp[A^2*((m - n + 1)/(m + 1)
)   Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2,
2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0]
&& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

3.257.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.58

method	result	size
parallelrisch	$\frac{i\ln\left(ib\,x^2+c\,x^3+ia\right)-i\ln\left(-ib\,x^2+c\,x^3-ia\right)}{2c}$	49
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c^2\_Z^6+b^2\_Z^4+2a\_Z^2b+a^2\right)}\frac{\left(\_R^4b+3\_R^2a\right)\ln\left(x-\_R\right)}{3\_R^5c^2+2\_R^3b^2+2ab\_R}\right)}{2}$	75
risch	$-\frac{\arctan\left(\frac{c\,x^5b}{a^2}-\frac{c\,x^3}{a}+\frac{b^3x^3}{a^2c}+\frac{b^2x}{ac}\right)}{c}-\frac{\arctan\left(-\frac{c\,x^3}{a}+\frac{cx}{b}-\frac{b^2x}{ac}\right)}{c}+\frac{\arctan\left(\frac{cx}{b}\right)}{c}$	96

input
```
int(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOS
E)
```

output
```
1/2*(I*ln(c*x^3+I*b*x^2+I*a)-I*ln(-I*b*x^2+c*x^3-I*a))/c
```

3.257.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(19) = 38$.

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 4.37

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \frac{\arctan\left(\frac{cx}{b}\right)-\arctan\left(\frac{bc^2x^5+ab^2x+(b^3-ac^2)x^3}{a^2c}\right)+\arctan\left(\frac{bc^2x^3+(b^3-ac^2)x}{abc}\right)}{c}$$

input
```
integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fr
icas")
```

output
```
(arctan(c*x/b) - arctan((b*c^2*x^5 + a*b^2*x + (b^3 - a*c^2)*x^3)/(a^2*c))
 + arctan((b*c^2*x^3 + (b^3 - a*c^2)*x)/(a*b*c)))/c
```

3.257.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \frac{-\frac{i\log\left(-\frac{ia}{c}-\frac{ibx^2}{c}+x^3\right)}{2}+\frac{i\log\left(\frac{ia}{c}+\frac{ibx^2}{c}+x^3\right)}{2}}{c}$$

input
```
integrate(x**2*(b*x**2+3*a)/(c**2*x**6+b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
(-I*log(-I*a/c - I*b*x**2/c + x**3)/2 + I*log(I*a/c + I*b*x**2/c + x**3)/2
)/c
```

3.257.7 Maxima [F]

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \int \frac{(bx^2+3\,a)x^2}{c^2x^6+b^2x^4+2\,abx^2+a^2}\,dx$$

input
```
integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="ma
xima")
```

output
```
integrate((b*x^2 + 3*a)*x^2/(c^2*x^6 + b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

3.257.8 Giac [F]

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx = \int \frac{(bx^2+3\,a)x^2}{c^2x^6+b^2x^4+2\,abx^2+a^2}\,dx$$

input
```
integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="gi
ac")
```

output
```
sage0*x
```

3.257.9 Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 252, normalized size of antiderivative = 13.26

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{27\,a\,c^5\,x^3}{27\,a^2\,c^4-4\,a\,b^3\,c^2}-\frac{27\,b\,c^5\,x^5}{27\,a^2\,c^4-4\,a\,b^3\,c^2}-\frac{31\,b^3\,c^3\,x^3}{27\,a^2\,c^4-4\,a\,b^3\,c^2}+\frac{4\,b^6\,c\,x^3}{27\,a^3\,c^4-4\,a^2\,b^3\,c^2}+\frac{4\,b^5\,c\,x}{27\,a^2\,c^4-4\,a\,b^3\,c^2}+\frac{4\,b^4\,c^3\,x^5}{27\,a^3\,c^4-4\,a^2\,b^3\,c^2}-\right.}{c}$$

input
```
int((x^2*(3*a + b*x^2))/(a^2 + b^2*x^4 + c^2*x^6 + 2*a*b*x^2),x)
```

output
```
(atan((27*a*c^5*x^3)/(27*a^2*c^4 - 4*a*b^3*c^2) - (27*b*c^5*x^5)/(27*a^2*c
^4 - 4*a*b^3*c^2) - (31*b^3*c^3*x^3)/(27*a^2*c^4 - 4*a*b^3*c^2) + (4*b^6*c
*x^3)/(27*a^3*c^4 - 4*a^2*b^3*c^2) + (4*b^5*c*x)/(27*a^2*c^4 - 4*a*b^3*c^2
) + (4*b^4*c^3*x^5)/(27*a^3*c^4 - 4*a^2*b^3*c^2) - (27*a*b^2*c^3*x)/(27*a^
2*c^4 - 4*a*b^3*c^2)) + atan((c*x^3)/a - (c*x)/b + (b^2*x)/(a*c)) + atan((
c*x)/b))/c
```

3.258 $\int \frac{1-3x^4}{(-2+x)(1+x^2)^2} \, dx$

3.258.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{1-3x^4}{(-2+x)(1+x^2)^2} \, dx = -\frac{1-2x}{5(1+x^2)} - \frac{46\arctan(x)}{25} - \frac{47}{25}\log(2-x) - \frac{14}{25}\log(1+x^2)$$

output
```
1/5*(-1+2*x)/(x^2+1)-46/25*arctan(x)-47/25*ln(2-x)-14/25*ln(x^2+1)
```

3.258.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

$$\int \frac{1-3x^4}{(-2+x)(1+x^2)^2} \, dx = \frac{3+2(-2+x)}{5(5+4(-2+x)+(-2+x)^2)} - \frac{46\arctan(x)}{25} - \frac{14}{25}\log\left(5+4(-2+x)+(-2+x)^2\right) - \frac{47}{25}\log(-2+x)$$

input
```
Integrate[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2),x]
```

output
```
(3 + 2*(-2 + x))/(5*(5 + 4*(-2 + x) + (-2 + x)^2)) - (46*ArcTan[x])/25 - (
14*Log[5 + 4*(-2 + x) + (-2 + x)^2])/25 - (47*Log[-2 + x])/25
```

3.258.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2178, 27, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-3x^4}{(x-2)\left(x^2+1\right)^2}dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{1}{2}\int \frac{2\left(-15x^2+2x+9\right)}{5(2-x)\left(x^2+1\right)}dx-\frac{1-2x}{5\left(x^2+1\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{5}\int \frac{-15x^2+2x+9}{(2-x)\left(x^2+1\right)}dx-\frac{1-2x}{5\left(x^2+1\right)}$$

$$\Big\downarrow \text{2160}$$

$$-\frac{1}{5}\int \left(\frac{2(14x+23)}{5\left(x^2+1\right)}+\frac{47}{5(x-2)}\right)dx-\frac{1-2x}{5\left(x^2+1\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\left(-\frac{46\arctan(x)}{5}-\frac{14}{5}\log\left(x^2+1\right)-\frac{47}{5}\log(2-x)\right)-\frac{1-2x}{5\left(x^2+1\right)}$$

input
```
Int[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2),x]
```

output
```
-1/5*(1 - 2*x)/(1 + x^2) + ((-46*ArcTan[x])/5 - (47*Log[2 - x])/5 - (14*Lo
g[1 + x^2])/5)/5
```

3.258.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.258.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result
risch	$\frac{\frac{2x}{5}-\frac{1}{5}}{x^2+1}-\frac{14\ln(x^2+1)}{25}-\frac{46\arctan(x)}{25}-\frac{47\ln(x-2)}{25}$
default	$-\frac{2(-5x+\frac{5}{2})}{25(x^2+1)}-\frac{14\ln(x^2+1)}{25}-\frac{46\arctan(x)}{25}-\frac{47\ln(x-2)}{25}$
parallelrisch	$-\frac{-23i\ln(x-i)x^2+23i\ln(x+i)x^2+47\ln(x-2)x^2+14\ln(x-i)x^2+14\ln(x+i)x^2+5-23i\ln(x-i)+23i\ln(x+i)+47\ln(x-2)+14\ln}{25(x^2+1)}$

input
```
int((-3*x^4+1)/(x-2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
(2/5*x-1/5)/(x^2+1)-14/25*ln(x^2+1)-46/25*arctan(x)-47/25*ln(x-2)
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{1-3x^4}{(-2+x)(1+x^2)^2}\,dx = -\frac{46\,(x^2+1)\arctan(x)+14\,(x^2+1)\log(x^2+1)+47\,(x^2+1)\log(x-2)-10\,x+5}{25\,(x^2+1)}$$

input
```
integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/25*(46*(x^2 + 1)*arctan(x) + 14*(x^2 + 1)*log(x^2 + 1) + 47*(x^2 + 1)*l
og(x - 2) - 10*x + 5)/(x^2 + 1)
```

3.258.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{1-3x^4}{(-2+x)(1+x^2)^2}\,dx = -\frac{1-2x}{5x^2+5} - \frac{47\log(x-2)}{25} - \frac{14\log(x^2+1)}{25} - \frac{46\,\mathrm{atan}(x)}{25}$$

input
```
integrate((-3*x**4+1)/(-2+x)/(x**2+1)**2,x)
```

output
```
-(1 - 2*x)/(5*x**2 + 5) - 47*log(x - 2)/25 - 14*log(x**2 + 1)/25 - 46*atan
(x)/25
```

3.258.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1-3x^4}{(-2+x)(1+x^2)^2}\,dx = \frac{2\,x-1}{5\,(x^2+1)} - \frac{46}{25}\arctan(x) - \frac{14}{25}\log(x^2+1) - \frac{47}{25}\log(x-2)$$

input
```
integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/5*(2*x - 1)/(x^2 + 1) - 46/25*arctan(x) - 14/25*log(x^2 + 1) - 47/25*log
(x - 2)
```

3.258.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1-3x^4}{(-2+x)\left(1+x^2\right)^2}\,dx = \frac{2\,x-1}{5\,(x^2+1)} - \frac{46}{25}\,\arctan\left(x\right) - \frac{14}{25}\,\log\left(x^2+1\right) - \frac{47}{25}\,\log\left(|x-2|\right)$$

input
```
integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="giac")
```

output
```
1/5*(2*x - 1)/(x^2 + 1) - 46/25*arctan(x) - 14/25*log(x^2 + 1) - 47/25*log
(abs(x - 2))
```

3.258.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{1-3x^4}{(-2+x)\left(1+x^2\right)^2}\,dx = \frac{\frac{2\,x}{5}-\frac{1}{5}}{x^2+1} - \frac{47\,\ln\left(x-2\right)}{25} + \ln\left(x-\mathrm{i}\right)\left(-\frac{14}{25}+\frac{23}{25}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\left(-\frac{14}{25}-\frac{23}{25}\mathrm{i}\right)$$

input
```
int(-(3*x^4 - 1)/((x^2 + 1)^2*(x - 2)),x)
```

output
```
((2*x)/5 - 1/5)/(x^2 + 1) - log(x - 1i)*(14/25 - 23i/25) - log(x + 1i)*(14
/25 + 23i/25) - (47*log(x - 2))/25
```

3.259 $\int \frac{-9-9x+2x^2}{-9x+x^3} dx$

3.259.1 Optimal result

Integrand size = 20, antiderivative size = 17

$$\int \frac{-9-9x+2x^2}{-9x+x^3} dx = -\log(3-x)+\log(x)+2\log(3+x)$$

output
```
-ln(3-x)+ln(x)+2*ln(3+x)
```

3.259.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-9-9x+2x^2}{-9x+x^3} dx = -\log(3-x)+\log(x)+2\log(3+x)$$

input
```
Integrate[(-9 - 9*x + 2*x^2)/(-9*x + x^3),x]
```

output
```
-Log[3 - x] + Log[x] + 2*Log[3 + x]
```

3.259.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2026, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2-9x-9}{x^3-9x}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{2x^2-9x-9}{x(x^2-9)}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{1}{x}+\frac{2}{x+3}+\frac{1}{3-x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\log(3-x)+\log(x)+2\log(x+3)$$

input
```
Int[(-9 - 9*x + 2*x^2)/(-9*x + x^3),x]
```

output
```
-Log[3 - x] + Log[x] + 2*Log[3 + x]
```

3.259.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.259.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\ln(x)-\ln(-3+x)+2\ln(3+x)$	16
norman	$\ln(x)-\ln(-3+x)+2\ln(3+x)$	16
risch	$\ln(x)-\ln(-3+x)+2\ln(3+x)$	16
parallelrisch	$\ln(x)-\ln(-3+x)+2\ln(3+x)$	16
meijerg	$\frac{\ln\left(1-\frac{x^2}{9}\right)}{2}+\ln(x)-\ln(3)+\frac{i\pi}{2}+3\,\mathrm{arctanh}\left(\frac{x}{3}\right)$	28

input
```
int((2*x^2-9*x-9)/(x^3-9*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-ln(-3+x)+2*ln(3+x)
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx = 2\,\log(x+3)-\log(x-3)+\log(x)$$

input
```
integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="fricas")
```

output
```
2*log(x + 3) - log(x - 3) + log(x)
```

3.259.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx = \log(x)-\log(x-3)+2\log(x+3)$$

input
```
integrate((2*x**2-9*x-9)/(x**3-9*x),x)
```

output
```
log(x) - log(x - 3) + 2*log(x + 3)
```

3.259.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx = 2\log(x+3) - \log(x-3) + \log(x)$$

input
```
integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="maxima")
```

output
```
2*log(x + 3) - log(x - 3) + log(x)
```

3.259.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx = 2\log(|x+3|) - \log(|x-3|) + \log(|x|)$$

input
```
integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="giac")
```

output
```
2*log(abs(x + 3)) - log(abs(x - 3)) + log(abs(x))
```

3.259.9 Mupad [B] (verification not implemented)

Time = 10.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx = 2\ln(x+3) - 2\,\mathrm{atanh}\left(\frac{1296}{18\,x+162} - 7\right)$$

input
```
int((9*x - 2*x^2 + 9)/(9*x - x^3),x)
```

output
```
2*log(x + 3) - 2*atanh(1296/(18*x + 162) - 7)
```

3.260 $\int \frac{1+2x^2+x^5}{-x+x^3} \, dx$

3.260.1 Optimal result

Integrand size = 20, antiderivative size = 25

$$\int \frac{1+2x^2+x^5}{-x+x^3} \, dx = x + \frac{x^3}{3} + 2\log(1-x) - \log(x) + \log(1+x)$$

output
```
x+1/3*x^3+2*ln(1-x)-ln(x)+ln(1+x)
```

3.260.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1+2x^2+x^5}{-x+x^3} \, dx = x + \frac{x^3}{3} + 2\log(1-x) - \log(x) + \log(1+x)$$

input
```
Integrate[(1 + 2*x^2 + x^5)/(-x + x^3),x]
```

output
```
x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]
```

3.260.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2026, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5+2x^2+1}{x^3-x}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{x^5+2x^2+1}{x\left(x^2-1\right)}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(x^2+\frac{2}{x-1}+\frac{1}{x+1}-\frac{1}{x}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3}{3}+x+2\log(1-x)-\log(x)+\log(x+1)$$

input
```
Int[(1 + 2*x^2 + x^5)/(-x + x^3),x]
```

output
```
x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]
```

3.260.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.260.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{x^{3}}{3}+x-\ln\left(x\right)+\ln\left(x+1\right)+2\ln\left(x-1\right)$	22
norman	$\frac{x^{3}}{3}+x-\ln\left(x\right)+\ln\left(x+1\right)+2\ln\left(x-1\right)$	22
risch	$\frac{x^{3}}{3}+x-\ln\left(x\right)+\ln\left(x+1\right)+2\ln\left(x-1\right)$	22
parallelrisch	$\frac{x^{3}}{3}+x-\ln\left(x\right)+\ln\left(x+1\right)+2\ln\left(x-1\right)$	22
meijerg	$\frac{3\ln\left(-x^{2}+1\right)}{2}-\ln\left(x\right)-\frac{i\pi}{2}+\frac{i\left(-\frac{2ix\left(5x^{2}+15\right)}{15}+2i\,\mathrm{arctanh}\left(x\right)\right)}{2}$	40

input
```
int((x^5+2*x^2+1)/(x^3-x),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+x-ln(x)+ln(x+1)+2*ln(x-1)
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1+2x^2+x^5}{-x+x^3}\,dx=\frac{1}{3}\,x^{3}+x+\log\left(x+1\right)+2\,\log\left(x-1\right)-\log\left(x\right)$$

input
```
integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="fricas")
```

output
```
1/3*x^3 + x + log(x + 1) + 2*log(x - 1) - log(x)
```

3.260.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1+2x^2+x^5}{-x+x^3}\,dx=\frac{x^{3}}{3}+x-\log\left(x\right)+2\log\left(x-1\right)+\log\left(x+1\right)$$

input
```
integrate((x**5+2*x**2+1)/(x**3-x),x)
```

output
```
x**3/3 + x - log(x) + 2*log(x - 1) + log(x + 1)
```

3.260.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1+2x^2+x^5}{-x+x^3}\,dx = \frac{1}{3}\,x^3 + x + \log\left(x+1\right) + 2\,\log\left(x-1\right) - \log\left(x\right)$$

input
```
integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="maxima")
```

output
```
1/3*x^3 + x + log(x + 1) + 2*log(x - 1) - log(x)
```

3.260.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1+2x^2+x^5}{-x+x^3}\,dx = \frac{1}{3}\,x^3 + x + \log\left(|x+1|\right) + 2\,\log\left(|x-1|\right) - \log\left(|x|\right)$$

input
```
integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="giac")
```

output
```
1/3*x^3 + x + log(abs(x + 1)) + 2*log(abs(x - 1)) - log(abs(x))
```

3.260.9 Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{1+2x^2+x^5}{-x+x^3}\,dx = x + 2\,\ln\left(x-1\right) + \frac{x^3}{3} + \mathrm{atan}\left(\frac{48\mathrm{i}}{11\,\left(22\,x-2\right)} + \frac{13}{11}\mathrm{i}\right)\,2\mathrm{i}$$

input
```
int(-(2*x^2 + x^5 + 1)/(x - x^3),x)
```

output
```
x + 2*log(x - 1) + atan(48i/(11*(22*x - 2)) + 13i/11)*2i + x^3/3
```

3.261 $\int \frac{3+2x^2}{(-1+x)^2x} \, dx$

3.261.1 Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{3+2x^2}{(-1+x)^2x} \, dx = \frac{5}{1-x} - \log(1-x) + 3\log(x)$$

output
```
5/(1-x)-ln(1-x)+3*ln(x)
```

3.261.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{3+2x^2}{(-1+x)^2x} \, dx = -\frac{5}{-1+x} - \log(1-x) + 3\log(x)$$

input
```
Integrate[(3 + 2*x^2)/((-1 + x)^2*x),x]
```

output
```
-5/(-1 + x) - Log[1 - x] + 3*Log[x]
```

3.261.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+3}{(x-1)^2x}\,dx$$

$$\Big\downarrow \text{522}$$

$$\int \left(\frac{3}{x}+\frac{5}{(x-1)^2}+\frac{1}{1-x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{1-x}-\log(1-x)+3\log(x)$$

input
```
Int[(3 + 2*x^2)/((-1 + x)^2*x),x]
```

output
```
5/(1 - x) - Log[1 - x] + 3*Log[x]
```

3.261.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.261.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$3\ln(x)-\frac{5}{x-1}-\ln(x-1)$	19
norman	$3\ln(x)-\frac{5}{x-1}-\ln(x-1)$	19
risch	$3\ln(x)-\frac{5}{x-1}-\ln(x-1)$	19
parallelrisch	$\frac{3\ln(x)x-\ln(x-1)x-5-3\ln(x)+\ln(x-1)}{x-1}$	29
meijerg	$\frac{2x}{1-x}-\ln(1-x)+\frac{6x}{-2x+2}+3+3\ln(x)+3i\pi$	39

input
```
int((2*x^2+3)/(x-1)^2/x,x,method=_RETURNVERBOSE)
```

output
```
3*ln(x)-5/(x-1)-ln(x-1)
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{3+2x^2}{(-1+x)^2x}\,dx = -\frac{(x-1)\log(x-1)-3(x-1)\log(x)+5}{x-1}$$

input
```
integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="fricas")
```

output
```
-((x - 1)*log(x - 1) - 3*(x - 1)*log(x) + 5)/(x - 1)
```

3.261.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{3+2x^2}{(-1+x)^2x}\,dx = 3\log(x)-\log(x-1)-\frac{5}{x-1}$$

input
```
integrate((2*x**2+3)/(-1+x)**2/x,x)
```

output
```
3*log(x) - log(x - 1) - 5/(x - 1)
```

3.261.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{3+2x^2}{(-1+x)^2x}\,dx = -\frac{5}{x-1} - \log(x-1) + 3\log(x)$$

input
```
integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="maxima")
```

output
```
-5/(x - 1) - log(x - 1) + 3*log(x)
```

3.261.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{3+2x^2}{(-1+x)^2x}\,dx = -\frac{5}{x-1} + 2\log(|x-1|) + 3\log\left(\left|-\frac{1}{x-1}-1\right|\right)$$

input
```
integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="giac")
```

output
```
-5/(x - 1) + 2*log(abs(x - 1)) + 3*log(abs(-1/(x - 1) - 1))
```

3.261.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{3+2x^2}{(-1+x)^2x}\,dx = 3\ln(x) - \ln(x-1) - \frac{5}{x-1}$$

input
```
int((2*x^2 + 3)/(x*(x - 1)^2),x)
```

output
```
3*log(x) - log(x - 1) - 5/(x - 1)
```

3.262 $\int \frac{-1+2x^2}{(-1+4x)(1+x^2)}\,dx$

3.262.1 Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \frac{-1+2x^2}{(-1+4x)(1+x^2)}\,dx = \frac{3\arctan(x)}{17} - \frac{7}{34}\log(1-4x) + \frac{6}{17}\log\left(1+x^2\right)$$

output
```
3/17*arctan(x)-7/34*ln(1-4*x)+6/17*ln(x^2+1)
```

3.262.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{-1+2x^2}{(-1+4x)(1+x^2)}\,dx = \frac{3\arctan(x)}{17} - \frac{7}{34}\log(-1+4x) + \frac{6}{17}\log\left(17+2(-1+4x)+(-1+4x)^2\right)$$

input
```
Integrate[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)),x]
```

output
```
(3*ArcTan[x])/17 - (7*Log[-1 + 4*x])/34 + (6*Log[17 + 2*(-1 + 4*x) + (-1 +
 4*x)^2])/17
```

3.262.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2-1}{(4x-1)(x^2+1)}\,dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{3(4x+1)}{17(x^2+1)}-\frac{14}{17(4x-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\arctan(x)}{17}+\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(1-4x)$$

input
```
Int[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)),x]
```

output
```
(3*ArcTan[x])/17 - (7*Log[1 - 4*x])/34 + (6*Log[1 + x^2])/17
```

3.262.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.262.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{6\ln(x^2+1)}{17}+\frac{3\arctan(x)}{17}-\frac{7\ln(-1+4x)}{34}$	22
risch	$\frac{6\ln(x^2+1)}{17}+\frac{3\arctan(x)}{17}-\frac{7\ln(-1+4x)}{34}$	22
parallelrisch	$-\frac{7\ln(x-\frac{1}{4})}{34}+\frac{6\ln(x-i)}{17}-\frac{3i\ln(x-i)}{34}+\frac{6\ln(x+i)}{17}+\frac{3i\ln(x+i)}{34}$	38

input
```
int((2*x^2-1)/(-1+4*x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
6/17*ln(x^2+1)+3/17*arctan(x)-7/34*ln(-1+4*x)
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{-1+2x^2}{(-1+4x)(1+x^2)}\,dx = \frac{3}{17}\arctan(x)+\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(4x-1)$$

input
```
integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="fricas")
```

output
```
3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)
```

3.262.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{-1+2x^2}{(-1+4x)(1+x^2)}\,dx = -\frac{7\log\left(x-\frac{1}{4}\right)}{34}+\frac{6\log\left(x^2+1\right)}{17}+\frac{3\,\mathrm{atan}(x)}{17}$$

input
```
integrate((2*x**2-1)/(-1+4*x)/(x**2+1),x)
```

output
```
-7*log(x - 1/4)/34 + 6*log(x**2 + 1)/17 + 3*atan(x)/17
```

3.262.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{-1+2x^2}{(-1+4x)(1+x^2)}\,dx = \frac{3}{17}\arctan(x) + \frac{6}{17}\log\left(x^2+1\right) - \frac{7}{34}\log(4x-1)$$

input
```
integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="maxima")
```

output
```
3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)
```

3.262.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{-1+2x^2}{(-1+4x)(1+x^2)}\,dx = \frac{3}{17}\arctan(x) + \frac{6}{17}\log\left(x^2+1\right) - \frac{7}{34}\log(|4x-1|)$$

input
```
integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="giac")
```

output
```
3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(abs(4*x - 1))
```

3.262.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{-1+2x^2}{(-1+4x)(1+x^2)}\,dx = -\frac{7\ln\left(x-\frac{1}{4}\right)}{34} + \ln(x-\mathrm{i})\left(\frac{6}{17}-\frac{3}{34}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(\frac{6}{17}+\frac{3}{34}\mathrm{i}\right)$$

input
```
int((2*x^2 - 1)/((4*x - 1)*(x^2 + 1)),x)
```

output
```
log(x - 1i)*(6/17 - 3i/34) - (7*log(x - 1/4))/34 + log(x + 1i)*(6/17 + 3i/
34)
```

3.263 $\int \frac{-3+2x-3x^2+x^3}{1+x^2} dx$

3.263.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{-3+2x-3x^2+x^3}{1+x^2} dx = -3x+\frac{x^2}{2}+\frac{1}{2}\log\left(1+x^2\right)$$

output
```
-3*x+1/2*x^2+1/2*ln(x^2+1)
```

3.263.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{-3+2x-3x^2+x^3}{1+x^2} dx = -3x+\frac{x^2}{2}+\frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2),x]
```

output
```
-3*x + x^2/2 + Log[1 + x^2]/2
```

3.263.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-3x^2+2x-3}{x^2+1} \, dx$$

$$\Big\downarrow \; 2341$$

$$\int \left(\frac{x}{x^2+1}+x-3\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2+1\right)-3x$$

input
```
Int[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2),x]
```

output
```
-3*x + x^2/2 + Log[1 + x^2]/2
```

3.263.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.263.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$-3x+\frac{x^2}{2}+\frac{\ln(x^2+1)}{2}$	18
norman	$-3x+\frac{x^2}{2}+\frac{\ln(x^2+1)}{2}$	18
meijerg	$-3x+\frac{x^2}{2}+\frac{\ln(x^2+1)}{2}$	18
risch	$-3x+\frac{x^2}{2}+\frac{\ln(x^2+1)}{2}$	18
parallelrisch	$-3x+\frac{x^2}{2}+\frac{\ln(x^2+1)}{2}$	18

input
```
int((x^3-3*x^2+2*x-3)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-3*x+1/2*x^2+1/2*ln(x^2+1)
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx=\frac{1}{2}\,x^2-3\,x+\frac{1}{2}\,\log\left(x^2+1\right)$$

input
```
integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

3.263.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx=\frac{x^2}{2}-3x+\frac{\log\left(x^2+1\right)}{2}$$

input
```
integrate((x**3-3*x**2+2*x-3)/(x**2+1),x)
```

output
```
x**2/2 - 3*x + log(x**2 + 1)/2
```

3.263.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx = \frac{1}{2}\,x^2 - 3\,x + \frac{1}{2}\,\log\left(x^2+1\right)$$

input
```
integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="maxima")
```

output
```
1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

3.263.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx = \frac{1}{2}\,x^2 - 3\,x + \frac{1}{2}\,\log\left(x^2+1\right)$$

input
```
integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="giac")
```

output
```
1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

3.263.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx = \frac{\ln\left(x^2+1\right)}{2} - 3\,x + \frac{x^2}{2}$$

input
```
int((2*x - 3*x^2 + x^3 - 3)/(x^2 + 1),x)
```

output
```
log(x^2 + 1)/2 - 3*x + x^2/2
```

3.264 $\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2} \, dx$

3.264.1 Optimal result

Integrand size = 26, antiderivative size = 27

$$\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2} \, dx = \frac{x^3}{3} - 3\arctan(3+x) + \frac{1}{2}\log\left(10+6x+x^2\right)$$

output
```
1/3*x^3-3*arctan(3+x)+1/2*ln(x^2+6*x+10)
```

3.264.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2} \, dx = \frac{x^3}{3} - 3\arctan(3+x) + \frac{1}{2}\log\left(10+6x+x^2\right)$$

input
```
Integrate[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2),x]
```

output
```
x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2
```

3.264.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2029, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+6x^3+10x^2+x}{x^2+6x+10}\,dx$$

$$\Big\downarrow \text{ 2029}$$

$$\int \frac{x\left(x^3+6x^2+10x+1\right)}{x^2+6x+10}dx$$

$$\Big\downarrow \text{ 2159}$$

$$\int \left(x^2+\frac{x}{x^2+6x+10}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-3\arctan(x+3)+\frac{x^3}{3}+\frac{1}{2}\log\left(x^2+6x+10\right)$$

input
```
Int[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2),x]
```

output
```
x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2
```

3.264.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2029
```
Int[(Fx_.)*((d_.)*(x_)^(q_.) + (a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*
(x_)^(t_.))^(p_.), x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r)
+ d*x^(q - r))^p*Fx, x] /; FreeQ[{a, b, c, d, r, s, t, q}, x] && IntegerQ[p
] && PosQ[s - r] && PosQ[t - r] && PosQ[q - r] &&  !(EqQ[p, 1] && EqQ[u, 1]
)
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.264.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{x^3}{3}-3\arctan\left(3+x\right)+\frac{\ln\left(x^2+6x+10\right)}{2}$	24
risch	$\frac{x^3}{3}-3\arctan\left(3+x\right)+\frac{\ln\left(x^2+6x+10\right)}{2}$	24
parallelrisch	$\frac{x^3}{3}+\frac{\ln(x+3-i)}{2}+\frac{3i\ln(x+3-i)}{2}+\frac{\ln(x+3+i)}{2}-\frac{3i\ln(x+3+i)}{2}$	41

input
```
int((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3-3*arctan(3+x)+1/2*ln(x^2+6*x+10)
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2}\,dx = \frac{1}{3}\,x^3-3\,\arctan\left(x+3\right)+\frac{1}{2}\,\log\left(x^2+6\,x+10\right)$$

input
```
integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="fricas")
```

output
```
1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

3.264.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x + 10x^2 + 6x^3 + x^4}{10 + 6x + x^2} \, dx = \frac{x^3}{3} + \frac{\log \left(x^2 + 6x + 10\right)}{2} - 3 \operatorname{atan}(x + 3)$$

input
```
integrate((x**4+6*x**3+10*x**2+x)/(x**2+6*x+10),x)
```

output
```
x**3/3 + log(x**2 + 6*x + 10)/2 - 3*atan(x + 3)
```

3.264.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x + 10x^2 + 6x^3 + x^4}{10 + 6x + x^2} \, dx = \frac{1}{3} x^3 - 3 \arctan(x + 3) + \frac{1}{2} \log \left(x^2 + 6x + 10\right)$$

input
```
integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="maxima")
```

output
```
1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

3.264.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x + 10x^2 + 6x^3 + x^4}{10 + 6x + x^2} \, dx = \frac{1}{3} x^3 - 3 \arctan(x + 3) + \frac{1}{2} \log \left(x^2 + 6x + 10\right)$$

input
```
integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="giac")
```

output
```
1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

3.264.9 Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x + 10x^2 + 6x^3 + x^4}{10 + 6x + x^2} \, dx = \frac{\ln(x^2 + 6\,x + 10)}{2} - 3\,\mathrm{atan}(x + 3) + \frac{x^3}{3}$$

input
```
int((x + 10*x^2 + 6*x^3 + x^4)/(6*x + x^2 + 10),x)
```

output
```
log(6*x + x^2 + 10)/2 - 3*atan(x + 3) + x^3/3
```

3.265 $\int \frac{1}{-18+27x-7x^2-3x^3+x^4} \, dx$

3.265.1 Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \frac{1}{-18+27x-7x^2-3x^3+x^4} \, dx = \frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(3+x)$$

output
```
1/8*ln(1-x)-1/5*ln(2-x)+1/12*ln(3-x)-1/120*ln(3+x)
```

3.265.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{-18+27x-7x^2-3x^3+x^4} \, dx = \frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(3+x)$$

input
```
Integrate[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1),x]
```

output
```
Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120
```

3.265.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4-3x^3-7x^2+27x-18}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(-\frac{1}{5(x-2)}+\frac{1}{8(x-1)}-\frac{1}{120(x+3)}+\frac{1}{12(x-3)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(x+3)$$

input
```
Int[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1),x]
```

output
```
Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120
```

3.265.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.265.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\ln(-3+x)}{12}-\frac{\ln(3+x)}{120}+\frac{\ln(x-1)}{8}-\frac{\ln(x-2)}{5}$	26
norman	$\frac{\ln(-3+x)}{12}-\frac{\ln(3+x)}{120}+\frac{\ln(x-1)}{8}-\frac{\ln(x-2)}{5}$	26
risch	$\frac{\ln(-3+x)}{12}-\frac{\ln(3+x)}{120}+\frac{\ln(x-1)}{8}-\frac{\ln(x-2)}{5}$	26
parallelrisch	$\frac{\ln(-3+x)}{12}-\frac{\ln(3+x)}{120}+\frac{\ln(x-1)}{8}-\frac{\ln(x-2)}{5}$	26

input
```
int(1/(x^4-3*x^3-7*x^2+27*x-18),x,method=_RETURNVERBOSE)
```

output
```
1/12*ln(-3+x)-1/120*ln(3+x)+1/8*ln(x-1)-1/5*ln(x-2)
```

3.265.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx = -\frac{1}{120}\log(x+3)+\frac{1}{8}\log(x-1) - \frac{1}{5}\log(x-2)+\frac{1}{12}\log(x-3)$$

input
```
integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="fricas")
```

output
```
-1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x - 3)
```

3.265.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx = \frac{\log(x-3)}{12}-\frac{\log(x-2)}{5}+\frac{\log(x-1)}{8}-\frac{\log(x+3)}{120}$$

input
```
integrate(1/(x**4-3*x**3-7*x**2+27*x-18),x)
```

output
```
log(x - 3)/12 - log(x - 2)/5 + log(x - 1)/8 - log(x + 3)/120
```

3.265.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx = -\frac{1}{120}\log(x+3)+\frac{1}{8}\log(x-1) - \frac{1}{5}\log(x-2)+\frac{1}{12}\log(x-3)$$

input
```
integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="maxima")
```

output
```
-1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x - 3)
```

3.265.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx = -\frac{1}{120}\log(|x+3|)+\frac{1}{8}\log(|x-1|) - \frac{1}{5}\log(|x-2|)+\frac{1}{12}\log(|x-3|)$$

input
```
integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="giac")
```

output
```
-1/120*log(abs(x + 3)) + 1/8*log(abs(x - 1)) - 1/5*log(abs(x - 2)) + 1/12*
log(abs(x - 3))
```

3.265.9 Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx = \frac{\ln(x-1)}{8}-\frac{\ln(x-2)}{5}+\frac{\ln(x-3)}{12}-\frac{\ln(x+3)}{120}$$

input
```
int(-1/(7*x^2 - 27*x + 3*x^3 - x^4 + 18),x)
```

output
```
log(x - 1)/8 - log(x - 2)/5 + log(x - 3)/12 - log(x + 3)/120
```

3.266 $\int \frac{1+x^3}{-2+x} \, dx$

3.266.1 Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{1+x^3}{-2+x} \, dx = 4x + x^2 + \frac{x^3}{3} + 9\log(2-x)$$

output
```
4*x+x^2+1/3*x^3+9*ln(2-x)
```

3.266.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1+x^3}{-2+x} \, dx = -\frac{44}{3} + 4x + x^2 + \frac{x^3}{3} + 9\log(-2+x)$$

input
```
Integrate[(1 + x^3)/(-2 + x),x]
```

output
```
-44/3 + 4*x + x^2 + x^3/3 + 9*Log[-2 + x]
```

3.266.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+1}{x-2} \, dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(x^2+2x+\frac{9}{x-2}+4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3}{3}+x^2+4x+9\log(2-x)$$

input
```
Int[(1 + x^3)/(-2 + x),x]
```

output
```
4*x + x^2 + x^3/3 + 9*Log[2 - x]
```

3.266.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.266.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x^3}{3}+x^2+4x+9\ln\left(x-2\right)$	19
norman	$\frac{x^3}{3}+x^2+4x+9\ln\left(x-2\right)$	19
risch	$\frac{x^3}{3}+x^2+4x+9\ln\left(x-2\right)$	19
parallelrisch	$\frac{x^3}{3}+x^2+4x+9\ln\left(x-2\right)$	19
meijerg	$9\ln\left(1-\frac{x}{2}\right)+\frac{x(x^2+3x+12)}{3}$	21

input
```
int((x^3+1)/(x-2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+x^2+4*x+9*ln(x-2)
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3}{-2+x}\,dx = \frac{1}{3}\,x^3+x^2+4\,x+9\,\log\left(x-2\right)$$

input
```
integrate((x^3+1)/(-2+x),x, algorithm="fricas")
```

output
```
1/3*x^3 + x^2 + 4*x + 9*log(x - 2)
```

3.266.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1+x^3}{-2+x}\,dx = \frac{x^3}{3}+x^2+4x+9\log\left(x-2\right)$$

input
```
integrate((x**3+1)/(-2+x),x)
```

output
```
x**3/3 + x**2 + 4*x + 9*log(x - 2)
```

3.266.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3}{-2+x}\,dx = \frac{1}{3}\,x^3 + x^2 + 4\,x + 9\,\log\left(x-2\right)$$

input
```
integrate((x^3+1)/(-2+x),x, algorithm="maxima")
```

output
```
1/3*x^3 + x^2 + 4*x + 9*log(x - 2)
```

3.266.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1+x^3}{-2+x}\,dx = \frac{1}{3}\,x^3 + x^2 + 4\,x + 9\,\log\left(|x-2|\right)$$

input
```
integrate((x^3+1)/(-2+x),x, algorithm="giac")
```

output
```
1/3*x^3 + x^2 + 4*x + 9*log(abs(x - 2))
```

3.266.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3}{-2+x}\,dx = 4\,x + 9\,\ln\left(x-2\right) + x^2 + \frac{x^3}{3}$$

input
```
int((x^3 + 1)/(x - 2),x)
```

output
```
4*x + 9*log(x - 2) + x^2 + x^3/3
```

3.267 $\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx$

3.267.1 Optimal result

Integrand size = 22, antiderivative size = 15

$$\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx = -4x+\frac{3x^2}{2}+4\arctan(x)$$

output
```
-4*x+3/2*x^2+4*arctan(x)
```

3.267.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx = -4x+\frac{3x^2}{2}+4\arctan(x)$$

input
```
Integrate[(3*x - 4*x^2 + 3*x^3)/(1 + x^2),x]
```

output
```
-4*x + (3*x^2)/2 + 4*ArcTan[x]
```

3.267.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2028, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^3-4x^2+3x}{x^2+1}\,dx$$

$$\Big\downarrow \text{ 2028}$$

$$\int \frac{x(3x^2-4x+3)}{x^2+1}dx$$

$$\Big\downarrow \text{ 2333}$$

$$\int \left(\frac{4}{x^2+1}+3x-4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$4\arctan(x)+\frac{3x^2}{2}-4x$$

input
```
Int[(3*x - 4*x^2 + 3*x^3)/(1 + x^2),x]
```

output
```
-4*x + (3*x^2)/2 + 4*ArcTan[x]
```

3.267.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.267.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-4x+\frac{3x^2}{2}+4\arctan\left(x\right)$	14
meijerg	$-4x+\frac{3x^2}{2}+4\arctan\left(x\right)$	14
risch	$-4x+\frac{3x^2}{2}+4\arctan\left(x\right)$	14
parallelrisch	$-4x+\frac{3x^2}{2}+2i\ln\left(x+i\right)-2i\ln\left(x-i\right)$	26

input
```
int((3*x^3-4*x^2+3*x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-4*x+3/2*x^2+4*arctan(x)
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx=\frac{3}{2}\,x^2-4\,x+4\,\arctan\left(x\right)$$

input
```
integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="fricas")
```

output
```
3/2*x^2 - 4*x + 4*arctan(x)
```

3.267.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx = \frac{3x^2}{2} - 4x + 4\,\mathrm{atan}\,(x)$$

input
```
integrate((3*x**3-4*x**2+3*x)/(x**2+1),x)
```

output
```
3*x**2/2 - 4*x + 4*atan(x)
```

3.267.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx = \frac{3}{2}\,x^2 - 4\,x + 4\,\arctan\left(x\right)$$

input
```
integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="maxima")
```

output
```
3/2*x^2 - 4*x + 4*arctan(x)
```

3.267.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx = \frac{3}{2}\,x^2 - 4\,x + 4\,\arctan\left(x\right)$$

input
```
integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="giac")
```

output
```
3/2*x^2 - 4*x + 4*arctan(x)
```

3.267.9 Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx = 4\,\mathrm{atan}(x) - 4\,x + \frac{3\,x^2}{2}$$

input
```
int((3*x - 4*x^2 + 3*x^3)/(x^2 + 1),x)
```

output
```
4*atan(x) - 4*x + (3*x^2)/2
```

3.268 $\int \frac{5+3x}{1-x-x^2+x^3}\,dx$

3.268.1 Optimal result

Integrand size = 21, antiderivative size = 12

$$\int \frac{5+3x}{1-x-x^2+x^3}\,dx = \frac{4}{1-x} + \operatorname{arctanh}(x)$$

output
```
4/(1-x)+arctanh(x)
```

3.268.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \frac{5+3x}{1-x-x^2+x^3}\,dx = -\frac{4}{-1+x} - \frac{1}{2}\log(-1+x) + \frac{1}{2}\log(1+x)$$

input
```
Integrate[(5 + 3*x)/(1 - x - x^2 + x^3),x]
```

output
```
-4/(-1 + x) - Log[-1 + x]/2 + Log[1 + x]/2
```

3.268.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x+5}{x^3-x^2-x+1} \, dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(\frac{1}{1-x^2}+\frac{4}{(x-1)^2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\text{arctanh}(x)+\frac{4}{1-x}$$

input
```
Int[(5 + 3*x)/(1 - x - x^2 + x^3),x]
```

output
```
4/(1 - x) + ArcTanh[x]
```

3.268.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.268.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

method	result	size
default	$\frac{\ln(x+1)}{2}-\frac{4}{x-1}-\frac{\ln(x-1)}{2}$	21
norman	$\frac{\ln(x+1)}{2}-\frac{4}{x-1}-\frac{\ln(x-1)}{2}$	21
risch	$\frac{\ln(x+1)}{2}-\frac{4}{x-1}-\frac{\ln(x-1)}{2}$	21
parallelrisch	$-\frac{\ln(x-1)x-\ln(x+1)x+8-\ln(x-1)+\ln(x+1)}{2(x-1)}$	33

input
```
int((5+3*x)/(x^3-x^2-x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x+1)-4/(x-1)-1/2*ln(x-1)
```

3.268.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{5+3x}{1-x-x^2+x^3}\,dx = \frac{(x-1)\log(x+1)-(x-1)\log(x-1)-8}{2(x-1)}$$

input
```
integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="fricas")
```

output
```
1/2*((x - 1)*log(x + 1) - (x - 1)*log(x - 1) - 8)/(x - 1)
```

3.268.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{5+3x}{1-x-x^2+x^3}\,dx = -\frac{\log(x-1)}{2}+\frac{\log(x+1)}{2}-\frac{4}{x-1}$$

input
```
integrate((5+3*x)/(x**3-x**2-x+1),x)
```

output
```
-log(x - 1)/2 + log(x + 1)/2 - 4/(x - 1)
```

3.268.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{5+3x}{1-x-x^2+x^3}\,dx = -\frac{4}{x-1}+\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

input
```
integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="maxima")
```

output
```
-4/(x - 1) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

3.268.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{5+3x}{1-x-x^2+x^3}\,dx = -\frac{4}{x-1}+\frac{1}{2}\log(|x+1|)-\frac{1}{2}\log(|x-1|)$$

input
```
integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="giac")
```

output
```
-4/(x - 1) + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

3.268.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{5+3x}{1-x-x^2+x^3}\,dx = \operatorname{atanh}(x)-\frac{4}{x-1}$$

input
```
int(-(3*x + 5)/(x + x^2 - x^3 - 1),x)
```

output
```
atanh(x) - 4/(x - 1)
```

3.269 $\int \frac{-1-x-x^3+x^4}{-x^2+x^3} \, dx$

3.269.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{-1-x-x^3+x^4}{-x^2+x^3} \, dx = -\frac{1}{x}+\frac{x^2}{2}-2\log(1-x)+2\log(x)$$

output
```
-1/x+1/2*x^2-2*ln(1-x)+2*ln(x)
```

3.269.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{-1-x-x^3+x^4}{-x^2+x^3} \, dx = -\frac{1}{x}+\frac{x^2}{2}-2\log(1-x)+2\log(x)$$

input
```
Integrate[(-1 - x - x^3 + x^4)/(-x^2 + x^3),x]
```

output
```
-x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]
```

3.269.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2026, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-x^3-x-1}{x^3-x^2} \, dx$$

$$\Big\downarrow \; 2026$$

$$\int \frac{x^4-x^3-x-1}{(x-1)x^2}dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{1}{x^2}+x-\frac{2}{x-1}+\frac{2}{x}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^2}{2}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

input
```
Int[(-1 - x - x^3 + x^4)/(-x^2 + x^3),x]
```

output
```
-x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]
```

3.269.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.269.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{x^2}{2} - \frac{1}{x} + 2\ln(x) - 2\ln(x-1)$	22
risch	$\frac{x^2}{2} - \frac{1}{x} + 2\ln(x) - 2\ln(x-1)$	22
norman	$\frac{-1+\frac{x^3}{2}}{x} + 2\ln(x) - 2\ln(x-1)$	23
parallelrisch	$\frac{x^3+4\ln(x)x-4\ln(x-1)x-2}{2x}$	23
meijerg	$2\ln(x) + 2i\pi - \frac{1}{x} + \frac{x(6+3x)}{6} - x - 2\ln(1-x)$	34

input
```
int((x^4-x^3-x-1)/(x^3-x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-1/x+2*ln(x)-2*ln(x-1)
```

3.269.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{-1-x-x^3+x^4}{-x^2+x^3}\,dx = \frac{x^3 - 4\,x\log(x-1) + 4\,x\log(x) - 2}{2\,x}$$

input
```
integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="fricas")
```

output
```
1/2*(x^3 - 4*x*log(x - 1) + 4*x*log(x) - 2)/x
```

3.269.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-1-x-x^3+x^4}{-x^2+x^3}\,dx = \frac{x^2}{2} + 2\log(x) - 2\log(x-1) - \frac{1}{x}$$

input
```
integrate((x**4-x**3-x-1)/(x**3-x**2),x)
```

output
```
x**2/2 + 2*log(x) - 2*log(x - 1) - 1/x
```

3.269.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{-1-x-x^3+x^4}{-x^2+x^3}\,dx = \frac{1}{2}\,x^2 - \frac{1}{x} - 2\,\log\left(x-1\right) + 2\,\log\left(x\right)$$

input
```
integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/x - 2*log(x - 1) + 2*log(x)
```

3.269.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{-1-x-x^3+x^4}{-x^2+x^3}\,dx = \frac{1}{2}\,x^2 - \frac{1}{x} - 2\,\log\left(|x-1|\right) + 2\,\log\left(|x|\right)$$

input
```
integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/x - 2*log(abs(x - 1)) + 2*log(abs(x))
```

3.269.9 Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-1-x-x^3+x^4}{-x^2+x^3}\,dx = 4\,\mathrm{atanh}(2\,x-1) - \frac{1}{x} + \frac{x^2}{2}$$

input
```
int((x + x^3 - x^4 + 1)/(x^2 - x^3),x)
```

output
```
4*atanh(2*x - 1) - 1/x + x^2/2
```

3.270 $\int \frac{2+x+x^2+x^3}{2+3x^2+x^4} dx$

3.270.1 Optimal result

Integrand size = 22, antiderivative size = 13

$$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4} dx = \arctan(x) + \frac{1}{2}\log(2+x^2)$$

output
```
arctan(x)+1/2*ln(x^2+2)
```

3.270.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4} dx = \arctan(x) + \frac{1}{2}\log(2+x^2)$$

input
```
Integrate[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4),x]
```

output
```
ArcTan[x] + Log[2 + x^2]/2
```

3.270.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2202, 1387, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+x+2}{x^4+3x^2+2} \, dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{x(x^2+1)}{x^4+3x^2+2}dx+\int \frac{x^2+2}{x^4+3x^2+2}dx$$

$$\Big\downarrow \text{ 1387}$$

$$\int \frac{1}{x^2+1}dx+\int \frac{x}{x^2+2}dx$$

$$\Big\downarrow \text{ 216}$$

$$\int \frac{x}{x^2+2}dx+\arctan(x)$$

$$\Big\downarrow \text{ 240}$$

$$\arctan(x)+\frac{1}{2}\log\left(x^2+2\right)$$

input
```
Int[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4),x]
```

output
```
ArcTan[x] + Log[2 + x^2]/2
```

3.270.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 1387
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*
(x_)^(n_))^(q_.), x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^
p, x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

3.270.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\arctan(x)+\frac{\ln(x^2+2)}{2}$	12
risch	$\arctan(x)+\frac{\ln(x^2+2)}{2}$	12
parallelrisch	$\frac{i\ln(x+i)}{2}-\frac{i\ln(x-i)}{2}+\frac{\ln(x^2+2)}{2}$	26

input
```
int((x^3+x^2+x+2)/(x^4+3*x^2+2),x,method=_RETURNVERBOSE)
```

output
```
arctan(x)+1/2*ln(x^2+2)
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx=\arctan(x)+\frac{1}{2}\log\left(x^2+2\right)$$

input
```
integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="fricas")
```

output
```
arctan(x) + 1/2*log(x^2 + 2)
```

3.270.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx = \frac{\log(x^2+2)}{2} + \operatorname{atan}(x)$$

input
```
integrate((x**3+x**2+x+2)/(x**4+3*x**2+2),x)
```

output
```
log(x**2 + 2)/2 + atan(x)
```

3.270.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx = \arctan(x) + \frac{1}{2}\log\left(x^2+2\right)$$

input
```
integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="maxima")
```

output
```
arctan(x) + 1/2*log(x^2 + 2)
```

3.270.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx = \arctan(x) + \frac{1}{2}\log\left(x^2+2\right)$$

input
```
integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="giac")
```

output
```
arctan(x) + 1/2*log(x^2 + 2)
```

3.270.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx = \frac{\ln(x^2+2)}{2} + \operatorname{atan}(x)$$

input
```
int((x + x^2 + x^3 + 2)/(3*x^2 + x^4 + 2),x)
```

output
```
log(x^2 + 2)/2 + atan(x)
```

3.271 $\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3} \, dx$

3.271.1 Optimal result

Integrand size = 31, antiderivative size = 35

$$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3} \, dx = -\frac{1}{(2+x^2)^2} - \frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{1}{2}\log(2+x^2)$$

output
```
-1/(x^2+2)^2+1/2*ln(x^2+2)-1/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.271.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3} \, dx = -\frac{1}{(2+x^2)^2} - \frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{1}{2}\log(2+x^2)$$

input
```
Integrate[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3,x]
```

output
```
-(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2
```

3.271.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2345, 27, 2019, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5-x^4+4x^3-4x^2+8x-4}{(x^2+2)^3}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{1}{8}\int \frac{8(-x^3+x^2-2x+2)}{(x^2+2)^2}dx-\frac{1}{(x^2+2)^2}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{-x^3+x^2-2x+2}{(x^2+2)^2}dx-\frac{1}{(x^2+2)^2}$$

$$\Big\downarrow \text{2019}$$

$$-\int \frac{1-x}{x^2+2}dx-\frac{1}{(x^2+2)^2}$$

$$\Big\downarrow \text{452}$$

$$-\int \frac{1}{x^2+2}dx+\int \frac{x}{x^2+2}dx-\frac{1}{(x^2+2)^2}$$

$$\Big\downarrow \text{216}$$

$$\int \frac{x}{x^2+2}dx-\frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{(x^2+2)^2}$$

$$\Big\downarrow \text{240}$$

$$-\frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{(x^2+2)^2}+\frac{1}{2}\log\left(x^2+2\right)$$

input
```
Int[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3,x]
```

output
```
-(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2
```

3.271.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.271. $\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3}\,dx$

3.271.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

method	result
default	$-\frac{1}{\left(x^2+2\right)^2}+\frac{\ln\left(x^2+2\right)}{2}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}$
risch	$-\frac{1}{\left(x^2+2\right)^2}+\frac{\ln\left(x^2+2\right)}{2}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}$
meijerg	$-\frac{\sqrt{2}\left(\frac{x\sqrt{2}\left(\frac{3x^2}{2}+5\right)}{4\left(1+\frac{x^2}{2}\right)^2}+\frac{3\arctan\left(\frac{x\sqrt{2}}{2}\right)}{2}\right)}{8}-\frac{x^2\left(\frac{9x^2}{2}+6\right)}{24\left(1+\frac{x^2}{2}\right)^2}+\frac{\ln\left(1+\frac{x^2}{2}\right)}{2}-\frac{\sqrt{2}\left(-\frac{x\sqrt{2}\left(\frac{25x^2}{2}+15\right)}{20\left(1+\frac{x^2}{2}\right)^2}+\frac{3\arctan\left(\frac{x\sqrt{2}}{2}\right)}{2}\right)}{8}+\frac{x^4}{8\left(1+\cdots\right.}$

input
```
int((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/(x^2+2)^2+1/2*ln(x^2+2)-1/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.271.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

$$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3}\,dx$$
$$=-\frac{\sqrt{2}(x^4+4\,x^2+4)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-(x^4+4\,x^2+4)\log\left(x^2+2\right)+2}{2\,(x^4+4\,x^2+4)}$$

input
```
integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*(x^4 + 4*x^2 + 4)*arctan(1/2*sqrt(2)*x) - (x^4 + 4*x^2 + 4)*
log(x^2 + 2) + 2)/(x^4 + 4*x^2 + 4)
```

3.271.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3}\,dx = \frac{\log(x^2+2)}{2} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2} - \frac{1}{x^4+4x^2+4}$$

input
```
integrate((x**5-x**4+4*x**3-4*x**2+8*x-4)/(x**2+2)**3,x)
```

output
```
log(x**2 + 2)/2 - sqrt(2)*atan(sqrt(2)*x/2)/2 - 1/(x**4 + 4*x**2 + 4)
```

3.271.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3}\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{x^4+4x^2+4} + \frac{1}{2}\log(x^2+2)$$

input
```
integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/(x^4 + 4*x^2 + 4) + 1/2*log(x^2 + 2
)
```

3.271.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3}\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{(x^2+2)^2} + \frac{1}{2}\log(x^2+2)$$

input
```
integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/(x^2 + 2)^2 + 1/2*log(x^2 + 2)
```

3.271.9 Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3}\,dx = \frac{\ln(x^2+2)}{2} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2} - \frac{1}{x^4+4\,x^2+4}$$

input
```
int((8*x - 4*x^2 + 4*x^3 - x^4 + x^5 - 4)/(x^2 + 2)^3,x)
```

output
```
log(x^2 + 2)/2 - (2^(1/2)*atan((2^(1/2)*x)/2))/2 - 1/(4*x^2 + x^4 + 4)
```

3.272 $\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx$

3.272.1 Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx = -\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(2+x)$$

output
```
-ln(1-x)+1/2*ln(x)+3/2*ln(2+x)
```

3.272.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx = -\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(2+x)$$

input
```
Integrate[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3),x]
```

output
```
-Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2
```

3.272.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2026, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-3x-1}{x^3+x^2-2x} \, dx$$
$$\Big\downarrow \text{2026}$$
$$\int \frac{x^2-3x-1}{x\left(x^2+x-2\right)}dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{1}{2x}+\frac{3}{2(x+2)}+\frac{1}{1-x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(x+2)$$

input
```
Int[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3),x]
```

output
```
-Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2
```

3.272.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.272.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\ln(x)}{2}+\frac{3\ln(x+2)}{2}-\ln(x-1)$	18
norman	$\frac{\ln(x)}{2}+\frac{3\ln(x+2)}{2}-\ln(x-1)$	18
risch	$\frac{\ln(x)}{2}+\frac{3\ln(x+2)}{2}-\ln(x-1)$	18
parallelrisch	$\frac{\ln(x)}{2}+\frac{3\ln(x+2)}{2}-\ln(x-1)$	18

input
```
int((x^2-3*x-1)/(x^3+x^2-2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x)+3/2*ln(x+2)-ln(x-1)
```

3.272.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx=\frac{3}{2}\log(x+2)-\log(x-1)+\frac{1}{2}\log(x)$$

input
```
integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="fricas")
```

output
```
3/2*log(x + 2) - log(x - 1) + 1/2*log(x)
```

3.272.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx=\frac{\log(x)}{2}-\log(x-1)+\frac{3\log(x+2)}{2}$$

input
```
integrate((x**2-3*x-1)/(x**3+x**2-2*x),x)
```

output
```
log(x)/2 - log(x - 1) + 3*log(x + 2)/2
```

3.272.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx = \frac{3}{2}\log(x+2) - \log(x-1) + \frac{1}{2}\log(x)$$

input
```
integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="maxima")
```

output
```
3/2*log(x + 2) - log(x - 1) + 1/2*log(x)
```

3.272.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx = \frac{3}{2}\log(|x+2|) - \log(|x-1|) + \frac{1}{2}\log(|x|)$$

input
```
integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="giac")
```

output
```
3/2*log(abs(x + 2)) - log(abs(x - 1)) + 1/2*log(abs(x))
```

3.272.9 Mupad [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{-1-3x+x^2}{-2x+x^2+x^3}\,dx = \frac{3\ln(x+2)}{2} - \ln(x-1) + \frac{\ln(x)}{2}$$

input
```
int(-(3*x - x^2 + 1)/(x^2 - 2*x + x^3),x)
```

output
```
(3*log(x + 2))/2 - log(x - 1) + log(x)/2
```

3.273 $\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx$

3.273.1 Optimal result

Integrand size = 33, antiderivative size = 23

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx = \frac{x^2}{2}+\log(x)-\frac{1}{2}\log\left(3-2x+x^2\right)$$

output
```
1/2*x^2+ln(x)-1/2*ln(x^2-2*x+3)
```

3.273.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx = \frac{x^2}{2}+\log(x)-\frac{1}{2}\log\left(3-2x+x^2\right)$$

input
```
Integrate[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3),x]
```

output
```
x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2
```

3.273.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2026, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-2x^3+3x^2-x+3}{x^3-2x^2+3x}dx$$

$$\Big\downarrow \; 2026$$

$$\int \frac{x^4-2x^3+3x^2-x+3}{x\left(x^2-2x+3\right)}dx$$

$$\Big\downarrow \; 2159$$

$$\int \left(\frac{1-x}{x^2-2x+3}+x+\frac{1}{x}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2-2x+3\right)+\log(x)$$

input
```
Int[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3),x]
```

output
```
x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2
```

3.273.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.273.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x^2}{2}+\ln\left(x\right)-\frac{\ln\left(x^2-2x+3\right)}{2}$	20
norman	$\frac{x^2}{2}+\ln\left(x\right)-\frac{\ln\left(x^2-2x+3\right)}{2}$	20
risch	$\frac{x^2}{2}+\ln\left(x\right)-\frac{\ln\left(x^2-2x+3\right)}{2}$	20
parallelrisch	$\frac{x^2}{2}+\ln\left(x\right)-\frac{\ln\left(x^2-2x+3\right)}{2}$	20

input
```
int((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+ln(x)-1/2*ln(x^2-2*x+3)
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx = \frac{1}{2}\,x^2-\frac{1}{2}\,\log\left(x^2-2\,x+3\right)+\log\left(x\right)$$

input
```
integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="fricas")
```

output
```
1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)
```

3.273.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx = \frac{x^2}{2}+\log\left(x\right)-\frac{\log\left(x^2-2x+3\right)}{2}$$

input
```
integrate((x**4-2*x**3+3*x**2-x+3)/(x**3-2*x**2+3*x),x)
```

output
```
x**2/2 + log(x) - log(x**2 - 2*x + 3)/2
```

3.273.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(x^2-2\,x+3\right) + \log\left(x\right)$$

input
```
integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)
```

3.273.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(x^2-2\,x+3\right) + \log\left(|x|\right)$$

input
```
integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(abs(x))
```

3.273.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx = \ln\left(x\right) - \frac{\ln\left(x^2-2\,x+3\right)}{2} + \frac{x^2}{2}$$

input
```
int((3*x^2 - x - 2*x^3 + x^4 + 3)/(3*x - 2*x^2 + x^3),x)
```

output
```
log(x) - log(x^2 - 2*x + 3)/2 + x^2/2
```

3.274 $\int \frac{-1+x+x^3}{(1+x^2)^2} dx$

3.274.1 Optimal result

Integrand size = 14, antiderivative size = 29

$$\int \frac{-1+x+x^3}{(1+x^2)^2} dx = -\frac{x}{2(1+x^2)} - \frac{\arctan(x)}{2} + \frac{1}{2}\log(1+x^2)$$

output
```
-1/2*x/(x^2+1)-1/2*arctan(x)+1/2*ln(x^2+1)
```

3.274.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{-1+x+x^3}{(1+x^2)^2} dx = \frac{1}{2}\left(-\frac{x}{1+x^2} - \arctan(x) + \log(1+x^2)\right)$$

input
```
Integrate[(-1 + x + x^3)/(1 + x^2)^2,x]
```

output
```
(-(x/(1 + x^2)) - ArcTan[x] + Log[1 + x^2])/2
```

3.274.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2345, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x-1}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{1}{2}\int \frac{1-2x}{x^2+1}dx-\frac{x}{2(x^2+1)}$$

$$\Big\downarrow \text{452}$$

$$\frac{1}{2}\left(2\int \frac{x}{x^2+1}dx-\int \frac{1}{x^2+1}dx\right)-\frac{x}{2(x^2+1)}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(2\int \frac{x}{x^2+1}dx-\arctan(x)\right)-\frac{x}{2(x^2+1)}$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{2}\left(\log\left(x^2+1\right)-\arctan(x)\right)-\frac{x}{2(x^2+1)}$$

input
```
Int[(-1 + x + x^3)/(1 + x^2)^2,x]
```

output
```
-1/2*x/(1 + x^2) + (-ArcTan[x] + Log[1 + x^2])/2
```

3.274.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.274.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{x}{2(x^2+1)}-\frac{\arctan(x)}{2}+\frac{\ln(x^2+1)}{2}$	24
risch	$-\frac{x}{2(x^2+1)}-\frac{\arctan(x)}{2}+\frac{\ln(x^2+1)}{2}$	24
meijerg	$\frac{\ln(x^2+1)}{2}-\frac{x}{2x^2+2}-\frac{\arctan(x)}{2}$	26
parallelrisch	$\frac{i\ln(x-i)x^2-i\ln(x+i)x^2+2\ln(x-i)x^2+2\ln(x+i)x^2+i\ln(x-i)-i\ln(x+i)+2\ln(x-i)+2\ln(x+i)-2x}{4x^2+4}$	86

input
```
int((x^3+x-1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(x^2+1)-1/2*arctan(x)+1/2*ln(x^2+1)
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{-1+x+x^3}{(1+x^2)^2}\,dx=-\frac{(x^2+1)\arctan(x)-(x^2+1)\log(x^2+1)+x}{2(x^2+1)}$$

input
```
integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/2*((x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + x)/(x^2 + 1)
```

3.274.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \frac{-1+x+x^3}{(1+x^2)^2} \, dx = -\frac{x}{2x^2+2} + \frac{\log(x^2+1)}{2} - \frac{\operatorname{atan}(x)}{2}$$

input
```
integrate((x**3+x-1)/(x**2+1)**2,x)
```

output
```
-x/(2*x**2 + 2) + log(x**2 + 1)/2 - atan(x)/2
```

3.274.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{-1+x+x^3}{(1+x^2)^2} \, dx = -\frac{x}{2(x^2+1)} - \frac{1}{2}\arctan(x) + \frac{1}{2}\log(x^2+1)$$

input
```
integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/2*x/(x^2 + 1) - 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

3.274.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{-1+x+x^3}{(1+x^2)^2} \, dx = -\frac{x}{2(x^2+1)} - \frac{1}{2}\arctan(x) + \frac{1}{2}\log(x^2+1)$$

input
```
integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/2*x/(x^2 + 1) - 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

3.274.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{-1+x+x^3}{\left(1+x^2\right)^2}\,dx = \frac{\ln\left(x^2+1\right)}{2} - \frac{\operatorname{atan}(x)}{2} - \frac{x}{2\,\left(x^2+1\right)}$$

input
```
int((x + x^3 - 1)/(x^2 + 1)^2,x)
```

output
```
log(x^2 + 1)/2 - atan(x)/2 - x/(2*(x^2 + 1))
```

3.275 $\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx$

3.275.1 Optimal result

Integrand size = 33, antiderivative size = 44

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx = -\frac{3}{1+x}-\frac{2\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-2\log(1+x)+\log\left(1-x+x^2\right)$$

output
```
-3/(1+x)+ln(x)-2*ln(1+x)+ln(x^2-x+1)-2/3*arctan(1/3*(1-2*x)*3^(1/
2))
```

3.275.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx = -\frac{3}{1+x}+\frac{2\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-2\log(1+x)+\log\left(1-x+x^2\right)$$

input
```
Integrate[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]
```

output
```
-3/(1 + x) + (2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*Log[1 + x
] + Log[1 - x + x^2]
```

3.275.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2026, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+8x^3-x^2+2x+1}{(x^2+x)(x^3+1)}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{x^4+8x^3-x^2+2x+1}{x(x+1)(x^3+1)}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{2x}{x^2-x+1}-\frac{2}{x+1}+\frac{3}{(x+1)^2}+\frac{1}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^2-x+1\right)-\frac{3}{x+1}+\log(x)-2\log(x+1)$$

input
```
Int[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]
```

output
```
-3/(1 + x) - (2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*Log[1 + x]
 + Log[1 - x + x^2]
```

3.275.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.275.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

method	result	size
default	$\ln(x)-\frac{3}{x+1}-2\ln(x+1)+\ln(x^2-x+1)+\frac{2\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	42
risch	$-\frac{3}{x+1}+\ln(4x^2-4x+4)+\frac{2\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-2\ln(x+1)+\ln(x)$	44

input
```
int((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-3/(x+1)-2*ln(x+1)+ln(x^2-x+1)+2/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2)
)
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx = \frac{2\sqrt{3}(x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+3(x+1)\log(x^2-x+1)-6(x+1)\log(x+1)+3(x+1)\log(x)}{3(x+1)}$$

input
```
integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(3)*(x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*(x + 1)*log(x^2 -
 x + 1) - 6*(x + 1)*log(x + 1) + 3*(x + 1)*log(x) - 9)/(x + 1)
```

3.275.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx = \log(x) - 2\log(x+1) + \log(x^2-x+1) + \frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3} - \frac{3}{x+1}$$

input
```
integrate((x**4+8*x**3-x**2+2*x+1)/(x**2+x)/(x**3+1),x)
```

output
```
log(x) - 2*log(x + 1) + log(x**2 - x + 1) + 2*sqrt(3)*atan(2*sqrt(3)*x/3 -
 sqrt(3)/3)/3 - 3/(x + 1)
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{x+1} + \log(x^2-x+1) - 2\log(x+1) + \log(x)$$

input
```
integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/(x + 1) + log(x^2 - x + 1) -
 2*log(x + 1) + log(x)
```

3.275.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{x+1} + \log(x^2-x+1) - 2\log(|x+1|) + \log(|x|)$$

input
```
integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/(x + 1) + log(x^2 - x + 1) -
 2*log(abs(x + 1)) + log(abs(x))
```

3.275.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx = \ln(x) - 2\,\ln(x+1) - \frac{3}{x+1} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(1+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)$$

input
```
int((2*x - x^2 + 8*x^3 + x^4 + 1)/((x^3 + 1)*(x + x^2)),x)
```

output
```
log(x) - 2*log(x + 1) - 3/(x + 1) - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/3 - 1) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/3 + 1)
```

3.276 $\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx$

3.276.1 Optimal result

Integrand size = 29, antiderivative size = 46

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = -\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)+\frac{5\arctan\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)$$

output
```
1/2*ln(x^2+2*x+3)+5/2*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)-arctan(1/5*x*5^(1/
2))*5^(1/2)
```

3.276.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = -\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)+\frac{5\arctan\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)$$

input
```
Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

output
```
-(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
 + 2*x + x^2]/2
```

3.276.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2-5x+15}{(x^2+5)(x^2+2x+3)}dx$$
$$\Big\downarrow \text{7276}$$
$$\int \left(\frac{x+6}{x^2+2x+3}-\frac{5}{x^2+5}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)+\frac{5\arctan\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(x^2+2x+3\right)$$

input
```
Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

output
```
-(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
 + 2*x + x^2]/2
```

3.276.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.276.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{\ln(x^2+2x+3)}{2}+\frac{5\arctan\left(\frac{(x+1)\sqrt{2}}{2}\right)\sqrt{2}}{2}-\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}$	39
default	$-\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}+\frac{\ln(x^2+2x+3)}{2}+\frac{5\sqrt{2}\arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}$	41

input
```
int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2+2*x+3)+5/2*arctan(1/2*(x+1)*2^(1/2))*2^(1/2)-arctan(1/5*x*5^(1/
2))*5^(1/2)
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx=\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

input
```
integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="fricas")
```

output
```
5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) +
1/2*log(x^2 + 2*x + 3)
```

3.276.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx=\frac{\log\left(x^2+2x+3\right)}{2}-\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)+\frac{5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}+\frac{\sqrt{2}}{2}\right)}{2}$$

input
```
integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)
```

output
```
log(x**2 + 2*x + 3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*atan(sqrt(2)
*x/2 + sqrt(2)/2)/2
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) + \frac{1}{2}\log\left(x^2+2x+3\right)$$

input
```
integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="maxima")
```

output
```
5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) +
1/2*log(x^2 + 2*x + 3)
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) + \frac{1}{2}\log\left(x^2+2x+3\right)$$

input
```
integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="giac")
```

output
```
5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) +
1/2*log(x^2 + 2*x + 3)
```

3.276.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.91

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)}{2}+\frac{\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)}{2} + \sqrt{5}\,\mathrm{atan}\left(\frac{2000\,\sqrt{5}}{2000\,x+1120}-\frac{224\,\sqrt{5}\,x}{2000\,x+1120}\right) - \frac{\sqrt{2}\,\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)\,5\mathrm{i}}{4}+\frac{\sqrt{2}\,\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)\,5\mathrm{i}}{4}$$

input
```
int((x^2 - 5*x + x^3 + 15)/((x^2 + 5)*(2*x + x^2 + 3)),x)
```

output
```
log(x - 2^(1/2)*1i + 1)/2 + log(x + 2^(1/2)*1i + 1)/2 + 5^(1/2)*atan((2000
*5^(1/2))/(2000*x + 1120) - (224*5^(1/2)*x)/(2000*x + 1120)) - (2^(1/2)*lo
g(x - 2^(1/2)*1i + 1)*5i)/4 + (2^(1/2)*log(x + 2^(1/2)*1i + 1)*5i)/4
```

3.277 $\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx$

3.277.1 Optimal result

Integrand size = 44, antiderivative size = 33

$$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx = -\frac{3}{1+x^2}+\frac{1}{2+x+x^2}+\log(1+x^2)-\log(2+x+x^2)$$

output
```
-3/(x^2+1)+1/(x^2+x+2)+ln(x^2+1)-ln(x^2+x+2)
```

3.277.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx = -\frac{3}{1+x^2}+\frac{1}{2+x+x^2}+\log(1+x^2)-\log(2+x+x^2)$$

input
```
Integrate[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^
2*(2 + x + x^2)^2),x]
```

output
```
-3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^2]
```

3.277.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6+7x^5+15x^4+32x^3+23x^2+25x-3}{(x^2+1)^2(x^2+x+2)^2}\,dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(\frac{-2x-1}{x^2+x+2}+\frac{-2x-1}{(x^2+x+2)^2}+\frac{2x}{x^2+1}+\frac{6x}{(x^2+1)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

input
```
Int[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2*(2 +
 x + x^2)^2),x]
```

output
```
-3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^2]
```

3.277.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.277.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

method	result
default	$-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\ln\left(x^2+1\right)-\ln\left(x^2+x+2\right)$
norman	$\frac{-2x^2-3x-5}{(x^2+x+2)(x^2+1)}-\ln\left(x^2+x+2\right)+\ln\left(x^2+1\right)$
risch	$\frac{-2x^2-3x-5}{(x^2+x+2)(x^2+1)}-\ln\left(x^2+x+2\right)+\ln\left(x^2+1\right)$
parallelrisch	$\frac{\ln(x^2+1)x^4-\ln(x^2+x+2)x^4-5+\ln(x^2+1)x^3-\ln(x^2+x+2)x^3+3\ln(x^2+1)x^2-3\ln(x^2+x+2)x^2+\ln(x^2+1)x-\ln(x^2+x+2)x}{(x^2+1)(x^2+x+2)}$

input
```
int((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,method
=_RETURNVERBOSE)
```

output
```
-3/(x^2+1)+1/(x^2+x+2)+ln(x^2+1)-ln(x^2+x+2)
```

3.277.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(33) = 66.

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.18

$$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2\,(2+x+x^2)^2}\,dx = -\frac{2\,x^2+(x^4+x^3+3\,x^2+x+2)\log\left(x^2+x+2\right)-(x^4+x^3+3\,x^2+x+2)\log\left(x^2+1\right)+3\,x+5}{x^4+x^3+3\,x^2+x+2}$$

input
```
integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
 algorithm="fricas")
```

output
```
-(2*x^2 + (x^4 + x^3 + 3*x^2 + x + 2)*log(x^2 + x + 2) - (x^4 + x^3 + 3*x^
2 + x + 2)*log(x^2 + 1) + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2)
```

3.277.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx = \frac{-2x^2-3x-5}{x^4+x^3+3x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

input
```
integrate((x**6+7*x**5+15*x**4+32*x**3+23*x**2+25*x-3)/(x**2+1)**2/(x**2+x
+2)**2,x)
```

output
```
(-2*x**2 - 3*x - 5)/(x**4 + x**3 + 3*x**2 + x + 2) + log(x**2 + 1) - log(x
**2 + x + 2)
```

3.277.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx = -\frac{2\,x^2+3\,x+5}{x^4+x^3+3\,x^2+x+2}-\log\left(x^2+x+2\right)+\log\left(x^2+1\right)$$

input
```
integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
 algorithm="maxima")
```

output
```
-(2*x^2 + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2) - log(x^2 + x + 2) + log(x^
2 + 1)
```

3.277.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx = -\frac{2\,x^2+3\,x+5}{x^4+x^3+3\,x^2+x+2}-\log\left(x^2+x+2\right)+\log\left(x^2+1\right)$$

input
```
integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
 algorithm="giac")
```

output
```
-(2*x^2 + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2) - log(x^2 + x + 2) + log(x^
2 + 1)
```

3.277.9 Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx$$
$$= -\frac{2x^2+3x+5}{x^4+x^3+3x^2+x+2} + \operatorname{atan}\left(\frac{\frac{x\,224\mathrm{i}}{11}+\frac{224}{11}\mathrm{i}}{44x^2+16x+60}-\frac{3}{11}\mathrm{i}\right)2\mathrm{i}$$

input
```
int((25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6 - 3)/((x^2 + 1)^2*(x +
x^2 + 2)^2),x)
```

output
```
atan(((x*224i)/11 + 224i/11)/(16*x + 44*x^2 + 60) - 3i/11)*2i - (3*x + 2*x
^2 + 5)/(x + 3*x^2 + x^3 + x^4 + 2)
```

3.278 $\int \frac{1}{(1+x^2)(4+x^2)}\,dx$

3.278.1 Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{1}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{6}\arctan\left(\frac{x}{2}\right) + \frac{\arctan(x)}{3}$$

output
```
-1/6*arctan(1/2*x)+1/3*arctan(x)
```

3.278.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)(4+x^2)}\,dx = \frac{1}{6}\arctan\left(\frac{2}{x}\right) + \frac{\arctan(x)}{3}$$

input
```
Integrate[1/((1 + x^2)*(4 + x^2)),x]
```

output
```
ArcTan[2/x]/6 + ArcTan[x]/3
```

3.278.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {303, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)(x^2+4)}dx$$

$$\Big\downarrow \; 303$$

$$\frac{1}{3}\int \frac{1}{x^2+1}dx-\frac{1}{3}\int \frac{1}{x^2+4}dx$$

$$\Big\downarrow \; 216$$

$$\frac{\arctan(x)}{3}-\frac{1}{6}\arctan\left(\frac{x}{2}\right)$$

input
```
Int[1/((1 + x^2)*(4 + x^2)),x]
```

output
```
-1/6*ArcTan[x/2] + ArcTan[x]/3
```

3.278.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

3.278.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{\arctan\left(\frac{x}{2}\right)}{6}+\frac{\arctan\left(x\right)}{3}$	12
risch	$-\frac{\arctan\left(\frac{x}{2}\right)}{6}+\frac{\arctan\left(x\right)}{3}$	12
parallelrisch	$\frac{i\ln\left(x+i\right)}{6}-\frac{i\ln\left(x-i\right)}{6}+\frac{i\ln\left(x-2i\right)}{12}-\frac{i\ln\left(x+2i\right)}{12}$	34

input
```
int(1/(x^2+1)/(x^2+4),x,method=_RETURNVERBOSE)
```

output
```
-1/6*arctan(1/2*x)+1/3*arctan(x)
```

3.278.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{6}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan\left(x\right)$$

input
```
integrate(1/(x^2+1)/(x^2+4),x, algorithm="fricas")
```

output
```
-1/6*arctan(1/2*x) + 1/3*arctan(x)
```

3.278.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{1}{(1+x^2)(4+x^2)}\,dx = -\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{6}+\frac{\operatorname{atan}\left(x\right)}{3}$$

input
```
integrate(1/(x**2+1)/(x**2+4),x)
```

output
```
-atan(x/2)/6 + atan(x)/3
```

3.278.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{6}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

input
```
integrate(1/(x^2+1)/(x^2+4),x, algorithm="maxima")
```

output
```
-1/6*arctan(1/2*x) + 1/3*arctan(x)
```

3.278.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{6}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

input
```
integrate(1/(x^2+1)/(x^2+4),x, algorithm="giac")
```

output
```
-1/6*arctan(1/2*x) + 1/3*arctan(x)
```

3.278.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{(1+x^2)(4+x^2)}\,dx = \frac{\mathrm{atan}(x)}{3} - \frac{\mathrm{atan}\left(\frac{x}{2}\right)}{6}$$

input
```
int(1/((x^2 + 1)*(x^2 + 4)),x)
```

output
```
atan(x)/3 - atan(x/2)/6
```

3.279 $\int \frac{a+bx^3}{1+x^2} \, dx$

3.279.1 Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{a+bx^3}{1+x^2} \, dx = \frac{bx^2}{2} + a\arctan(x) - \frac{1}{2}b\log\left(1+x^2\right)$$

output
```
1/2*b*x^2+a*arctan(x)-1/2*b*ln(x^2+1)
```

3.279.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^3}{1+x^2} \, dx = a\arctan(x) + \frac{1}{2}b\left(x^2 - \log\left(1+x^2\right)\right)$$

input
```
Integrate[(a + b*x^3)/(1 + x^2),x]
```

output
```
a*ArcTan[x] + (b*(x^2 - Log[1 + x^2]))/2
```

3.279.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3}{x^2+1} \, dx$$

$$\Big\downarrow \text{ 2341}$$

$$\int \left(\frac{a-bx}{x^2+1}+bx\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$a \arctan(x)+\frac{bx^2}{2}-\frac{1}{2}b\log\left(x^2+1\right)$$

input
```
Int[(a + b*x^3)/(1 + x^2),x]
```

output
```
(b*x^2)/2 + a*ArcTan[x] - (b*Log[1 + x^2])/2
```

3.279.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.279.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{b\,x^{2}}{2}+a\arctan\left(x\right)-\frac{b\ln\left(x^{2}+1\right)}{2}$	21
meijerg	$\frac{b\left(x^{2}-\ln\left(x^{2}+1\right)\right)}{2}+a\arctan\left(x\right)$	21
risch	$\frac{b\,x^{2}}{2}+a\arctan\left(x\right)-\frac{b\ln\left(x^{2}+1\right)}{2}$	21
parallelrisch	$\frac{b\,x^{2}}{2}-\frac{\ln\left(x-i\right)b}{2}-\frac{i\ln\left(x-i\right)a}{2}-\frac{\ln\left(x+i\right)b}{2}+\frac{i\ln\left(x+i\right)a}{2}$	42

input
```
int((b*x^3+a)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*x^2+a*arctan(x)-1/2*b*ln(x^2+1)
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^3}{1+x^2}\,dx=\frac{1}{2}\,bx^{2}+a\arctan\left(x\right)-\frac{1}{2}\,b\log\left(x^{2}+1\right)$$

input
```
integrate((b*x^3+a)/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

3.279.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{a+bx^3}{1+x^2}\,dx=\frac{bx^{2}}{2}+\left(-\frac{ia}{2}-\frac{b}{2}\right)\log\left(x-i\right)+\left(\frac{ia}{2}-\frac{b}{2}\right)\log\left(x+i\right)$$

input
```
integrate((b*x**3+a)/(x**2+1),x)
```

output
```
b*x**2/2 + (-I*a/2 - b/2)*log(x - I) + (I*a/2 - b/2)*log(x + I)
```

3.279.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^3}{1+x^2}\,dx = \frac{1}{2}\,bx^2 + a\arctan\left(x\right) - \frac{1}{2}\,b\log\left(x^2+1\right)$$

input
```
integrate((b*x^3+a)/(x^2+1),x, algorithm="maxima")
```

output
```
1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

3.279.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^3}{1+x^2}\,dx = \frac{1}{2}\,bx^2 + a\arctan\left(x\right) - \frac{1}{2}\,b\log\left(x^2+1\right)$$

input
```
integrate((b*x^3+a)/(x^2+1),x, algorithm="giac")
```

output
```
1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

3.279.9 Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^3}{1+x^2}\,dx = \frac{b\,x^2}{2} - \frac{b\,\ln\left(x^2+1\right)}{2} + a\,\mathrm{atan}(x)$$

input
```
int((a + b*x^3)/(x^2 + 1),x)
```

output
```
(b*x^2)/2 - (b*log(x^2 + 1))/2 + a*atan(x)
```

3.280 $\int \frac{x+x^2}{(4+x)(-4+x^2)} dx$

3.280.1 Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{x+x^2}{(4+x)(-4+x^2)} dx = -\frac{1}{2}\text{arctanh}\left(\frac{x}{2}\right) + \log(4+x)$$

output
```
-1/2*arctanh(1/2*x)+ln(4+x)
```

3.280.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{x+x^2}{(4+x)(-4+x^2)} dx = \frac{1}{4}\log(2-x) - \frac{1}{4}\log(2+x) + \log(4+x)$$

input
```
Integrate[(x + x^2)/((4 + x)*(-4 + x^2)),x]
```

output
```
Log[2 - x]/4 - Log[2 + x]/4 + Log[4 + x]
```

3.280.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2027, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x}{(x+4)(x^2-4)}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x(x+1)}{(x+4)(x^2-4)}dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{1}{x^2-4}+\frac{1}{x+4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(x+4)-\frac{1}{2}\text{arctanh}\left(\frac{x}{2}\right)$$

input
```
Int[(x + x^2)/((4 + x)*(-4 + x^2)),x]
```

output
```
-1/2*ArcTanh[x/2] + Log[4 + x]
```

3.280.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.280.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{\ln(x+2)}{4}+\ln(x+4)+\frac{\ln(x-2)}{4}$	18
norman	$-\frac{\ln(x+2)}{4}+\ln(x+4)+\frac{\ln(x-2)}{4}$	18
risch	$-\frac{\ln(x+2)}{4}+\ln(x+4)+\frac{\ln(x-2)}{4}$	18
parallelrisch	$-\frac{\ln(x+2)}{4}+\ln(x+4)+\frac{\ln(x-2)}{4}$	18

input
```
int((x^2+x)/(x+4)/(x^2-4),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(x+2)+ln(x+4)+1/4*ln(x-2)
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{x+x^2}{(4+x)(-4+x^2)}\,dx = \log(x+4)-\frac{1}{4}\log(x+2)+\frac{1}{4}\log(x-2)$$

input
```
integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="fricas")
```

output
```
log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)
```

3.280.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{x+x^2}{(4+x)(-4+x^2)}\,dx = \frac{\log(x-2)}{4}-\frac{\log(x+2)}{4}+\log(x+4)$$

input
```
integrate((x**2+x)/(4+x)/(x**2-4),x)
```

output
```
log(x - 2)/4 - log(x + 2)/4 + log(x + 4)
```

3.280.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{x+x^2}{(4+x)(-4+x^2)}\,dx = \log(x+4) - \frac{1}{4}\log(x+2) + \frac{1}{4}\log(x-2)$$

input
```
integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="maxima")
```

output
```
log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)
```

3.280.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{x+x^2}{(4+x)(-4+x^2)}\,dx = \log(|x+4|) - \frac{1}{4}\log(|x+2|) + \frac{1}{4}\log(|x-2|)$$

input
```
integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="giac")
```

output
```
log(abs(x + 4)) - 1/4*log(abs(x + 2)) + 1/4*log(abs(x - 2))
```

3.280.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{x+x^2}{(4+x)(-4+x^2)}\,dx = \ln(x+4) + \frac{\operatorname{atanh}\left(\frac{90}{7(21x+48)} - \frac{8}{7}\right)}{2}$$

input
```
int((x + x^2)/((x^2 - 4)*(x + 4)),x)
```

output
```
log(x + 4) + atanh(90/(7*(21*x + 48)) - 8/7)/2
```

3.281 $\int \frac{4+x^2}{(1+x^2)(2+x^2)} dx$

3.281.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{4+x^2}{(1+x^2)(2+x^2)} dx = 3\arctan(x) - \sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

output
```
3*arctan(x)-arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.281.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{4+x^2}{(1+x^2)(2+x^2)} dx = 3\arctan(x) - \sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

input
```
Integrate[(4 + x^2)/((1 + x^2)*(2 + x^2)),x]
```

output
```
3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]
```

3.281.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+4}{(x^2+1)(x^2+2)} \, dx$$

$$\Big\downarrow \text{397}$$

$$3\int \frac{1}{x^2+1}dx-2\int \frac{1}{x^2+2}dx$$

$$\Big\downarrow \text{216}$$

$$3\arctan(x)-\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

input
```
Int[(4 + x^2)/((1 + x^2)*(2 + x^2)),x]
```

output
```
3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]
```

3.281.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

3.281.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result	size
default	$3 \arctan(x) - \arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	18
risch	$3 \arctan(x) - \arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	18

input
```
int((x^2+4)/(x^2+1)/(x^2+2),x,method=_RETURNVERBOSE)
```

output
```
3*arctan(x)-arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.281.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx = -\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 3\arctan(x)$$

input
```
integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="fricas")
```

output
```
-sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

3.281.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx = 3\,\mathrm{atan}(x) - \sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

input
```
integrate((x**2+4)/(x**2+1)/(x**2+2),x)
```

output
```
3*atan(x) - sqrt(2)*atan(sqrt(2)*x/2)
```

3.281.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx = -\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 3\arctan(x)$$

input
```
integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

output
```
-sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

3.281.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx = -\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 3\arctan(x)$$

input
```
integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

output
```
-sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

3.281.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx = 3\,\mathrm{atan}(x) - \sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

input
```
int((x^2 + 4)/((x^2 + 1)*(x^2 + 2)),x)
```

output
```
3*atan(x) - 2^(1/2)*atan((2^(1/2)*x)/2)
```

3.282 $\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx$

3.282.1 Optimal result

Integrand size = 26, antiderivative size = 37

$$\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx = \frac{5}{2(1-x)}+x+2\arctan(x)+\frac{1}{2}\log(1-x)+\frac{3}{4}\log\left(1+x^2\right)$$

output
```
5/2/(1-x)+x+2*arctan(x)+1/2*ln(1-x)+3/4*ln(x^2+1)
```

3.282.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx = \frac{5}{2-2x}+x+2\arctan(x)+\frac{1}{2}\log(-1+x)+\frac{3}{4}\log\left(1+x^2\right)$$

input
```
Integrate[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)),x]
```

output
```
5/(2 - 2*x) + x + 2*ArcTan[x] + Log[-1 + x]/2 + (3*Log[1 + x^2])/4
```

3.282.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+3x^2-4x+5}{(x-1)^2\left(x^2+1\right)}dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{3x+4}{2\left(x^2+1\right)}+\frac{1}{2(x-1)}+\frac{5}{2(x-1)^2}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\arctan(x)+\frac{3}{4}\log\left(x^2+1\right)+x+\frac{5}{2(1-x)}+\frac{1}{2}\log(1-x)$$

input
```
Int[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)),x]
```

output
```
5/(2*(1 - x)) + x + 2*ArcTan[x] + Log[1 - x]/2 + (3*Log[1 + x^2])/4
```

3.282.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.282.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result
default	$x+\frac{3\ln(x^2+1)}{4}+2\arctan(x)-\frac{5}{2(x-1)}+\frac{\ln(x-1)}{2}$
risch	$x-\frac{5}{2(x-1)}+\frac{\ln(x-1)}{2}+\frac{3\ln(16x^2+16)}{4}+2\arctan(x)$
parallelrisch	$\frac{-4i\ln(x-i)x+4i\ln(x+i)x+2\ln(x-1)x+4i\ln(x-i)+3\ln(x-i)x-4i\ln(x+i)+3\ln(x+i)x+4x^2-14-2\ln(x-1)-3\ln(x-i)-3\ln}{4x-4}$

input
```
int((x^4+3*x^2-4*x+5)/(x-1)^2/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
x+3/4*ln(x^2+1)+2*arctan(x)-5/2/(x-1)+1/2*ln(x-1)
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx = \frac{4\,x^2+8\,(x-1)\arctan(x)+3\,(x-1)\log(x^2+1)+2\,(x-1)\log(x-1)-4\,x-10}{4\,(x-1)}$$

input
```
integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
```

output
```
1/4*(4*x^2 + 8*(x - 1)*arctan(x) + 3*(x - 1)*log(x^2 + 1) + 2*(x - 1)*log(
x - 1) - 4*x - 10)/(x - 1)
```

3.282.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx = x+\frac{\log(x-1)}{2}+\frac{3\log(x^2+1)}{4}+2\,\mathrm{atan}(x)-\frac{5}{2x-2}$$

input
```
integrate((x**4+3*x**2-4*x+5)/(-1+x)**2/(x**2+1),x)
```

output
```
x + log(x - 1)/2 + 3*log(x**2 + 1)/4 + 2*atan(x) - 5/(2*x - 2)
```

3.282.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx = x-\frac{5}{2(x-1)}+2\arctan(x)+\frac{3}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x-1)$$

input
```
integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
```

output
```
x - 5/2/(x - 1) + 2*arctan(x) + 3/4*log(x^2 + 1) + 1/2*log(x - 1)
```

3.282.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(29) = 58.

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx = \frac{1}{2}\pi-2\pi\left\lfloor\frac{\pi+4\arctan(x)}{4\pi}+\frac{1}{2}\right\rfloor+x-\frac{5}{2(x-1)}+2\arctan(x) + \frac{3}{4}\log\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)+2\log(|x-1|)-1$$

input
```
integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="giac")
```

output
```
1/2*pi - 2*pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + x - 5/2/(x - 1) + 2
*arctan(x) + 3/4*log(2/(x - 1) + 2/(x - 1)^2 + 1) + 2*log(abs(x - 1)) - 1
```

3.282.9 Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx = x+\frac{\ln(x-1)}{2}-\frac{5}{2\,(x-1)} + \ln(x-\mathrm{i})\left(\frac{3}{4}-\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{3}{4}+1\mathrm{i}\right)$$

input
```
int((3*x^2 - 4*x + x^4 + 5)/((x^2 + 1)*(x - 1)^2),x)
```

output
```
x + log(x - 1)/2 + log(x - 1i)*(3/4 - 1i) + log(x + 1i)*(3/4 + 1i) - 5/(2*
(x - 1))
```

3.283 $\int \frac{1+x^4}{2+x^2}\,dx$

3.283.1 Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{1+x^4}{2+x^2}\,dx = -2x+\frac{x^3}{3}+\frac{5\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
-2*x+1/3*x^3+5/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.283.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{2+x^2}\,dx = -2x+\frac{x^3}{3}+\frac{5\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Integrate[(1 + x^4)/(2 + x^2),x]
```

output
```
-2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

3.283.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x^2+2}\,dx$$

$$\Big\downarrow \text{ 1468}$$

$$\int \left(x^2+\frac{5}{x^2+2}-2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{5\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{x^3}{3}-2x$$

input
```
Int[(1 + x^4)/(2 + x^2),x]
```

output
```
-2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

3.283.3.1 Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.283.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

method	result	size
default	$-2x+\frac{x^3}{3}+\frac{5\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}$	22
risch	$-2x+\frac{x^3}{3}+\frac{5\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}$	22
meijerg	$\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}+\sqrt{2}\left(-\frac{x\sqrt{2}\left(-\frac{5x^2}{2}+15\right)}{15}+2\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)$	41

input
```
int((x^4+1)/(x^2+2),x,method=_RETURNVERBOSE)
```

output
```
-2*x+1/3*x^3+5/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.283.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1+x^4}{2+x^2}\,dx=\frac{1}{3}\,x^3+\frac{5}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-2\,x$$

input
```
integrate((x^4+1)/(x^2+2),x, algorithm="fricas")
```

output
```
1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

3.283.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{2+x^2}\,dx=\frac{x^3}{3}-2x+\frac{5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

input
```
integrate((x**4+1)/(x**2+2),x)
```

output
```
x**3/3 - 2*x + 5*sqrt(2)*atan(sqrt(2)*x/2)/2
```

3.283.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1+x^4}{2+x^2}\,dx = \frac{1}{3}\,x^3 + \frac{5}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 2\,x$$

input
```
integrate((x^4+1)/(x^2+2),x, algorithm="maxima")
```

output
```
1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

3.283.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1+x^4}{2+x^2}\,dx = \frac{1}{3}\,x^3 + \frac{5}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 2\,x$$

input
```
integrate((x^4+1)/(x^2+2),x, algorithm="giac")
```

output
```
1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

3.283.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1+x^4}{2+x^2}\,dx = \frac{5\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2} - 2\,x + \frac{x^3}{3}$$

input
```
int((x^4 + 1)/(x^2 + 2),x)
```

output
```
(5*2^(1/2)*atan((2^(1/2)*x)/2))/2 - 2*x + x^3/3
```

3.284 $\int \frac{2+2x+x^4}{x^4+x^5}\,dx$

3.284.1 Optimal result

Integrand size = 18, antiderivative size = 12

$$\int \frac{2+2x+x^4}{x^4+x^5}\,dx=-\frac{2}{3x^3}+\log(1+x)$$

output
```
-2/3/x^3+ln(1+x)
```

3.284.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{2+2x+x^4}{x^4+x^5}\,dx=-\frac{2}{3x^3}+\log(1+x)$$

input
```
Integrate[(2 + 2*x + x^4)/(x^4 + x^5),x]
```

output
```
-2/(3*x^3) + Log[1 + x]
```

3.284.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2026, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+2x+2}{x^5+x^4} \, dx$$

$$\Big\downarrow \; 2026$$

$$\int \frac{x^4+2x+2}{x^4(x+1)}dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{2}{x^4}+\frac{1}{x+1}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\log(x+1)-\frac{2}{3x^3}$$

input
```
Int[(2 + 2*x + x^4)/(x^4 + x^5),x]
```

output
```
-2/(3*x^3) + Log[1 + x]
```

3.284.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.284.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{2}{3x^3}+\ln\left(x+1\right)$	11
norman	$-\frac{2}{3x^3}+\ln\left(x+1\right)$	11
meijerg	$-\frac{2}{3x^3}+\ln\left(x+1\right)$	11
risch	$-\frac{2}{3x^3}+\ln\left(x+1\right)$	11
parallelrisch	$\frac{3\ln(x+1)x^3-2}{3x^3}$	17

input
```
int((x^4+2*x+2)/(x^5+x^4),x,method=_RETURNVERBOSE)
```

output
```
-2/3/x^3+ln(x+1)
```

3.284.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{2+2x+x^4}{x^4+x^5}\,dx=\frac{3\,x^3\log\left(x+1\right)-2}{3\,x^3}$$

input
```
integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="fricas")
```

output
```
1/3*(3*x^3*log(x + 1) - 2)/x^3
```

3.284.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{2+2x+x^4}{x^4+x^5}\,dx=\log\left(x+1\right)-\frac{2}{3x^3}$$

input
```
integrate((x**4+2*x+2)/(x**5+x**4),x)
```

output
```
log(x + 1) - 2/(3*x**3)
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{2+2x+x^4}{x^4+x^5}\,dx = -\frac{2}{3\,x^3} + \log\left(x+1\right)$$

input
```
integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="maxima")
```

output
```
-2/3/x^3 + log(x + 1)
```

3.284.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{2+2x+x^4}{x^4+x^5}\,dx = -\frac{2}{3\,x^3} + \log\left(|x+1|\right)$$

input
```
integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="giac")
```

output
```
-2/3/x^3 + log(abs(x + 1))
```

3.284.9 Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{2+2x+x^4}{x^4+x^5}\,dx = \ln\left(x+1\right) - \frac{2}{3\,x^3}$$

input
```
int((2*x + x^4 + 2)/(x^4 + x^5),x)
```

output
```
log(x + 1) - 2/(3*x^3)
```

3.285 $\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx$

3.285.1 Optimal result

Integrand size = 26, antiderivative size = 21

$$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx = 2\log(1-x)-\log(2-x)+\log(1+x)$$

output
```
2*ln(1-x)-ln(2-x)+ln(1+x)
```

3.285.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx = 2\log(1-x)-\log(2-x)+\log(1+x)$$

input
```
Integrate[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3),x]
```

output
```
2*Log[1 - x] - Log[2 - x] + Log[1 + x]
```

3.285.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2-5x-1}{x^3-2x^2-x+2}\,dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{2}{x-1}+\frac{1}{x+1}+\frac{1}{2-x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\log(1-x)-\log(2-x)+\log(x+1)$$

input
```
Int[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3),x]
```

output
```
2*Log[1 - x] - Log[2 - x] + Log[1 + x]
```

3.285.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.285.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\ln(x+1)+2\ln(x-1)-\ln(x-2)$	18
norman	$\ln(x+1)+2\ln(x-1)-\ln(x-2)$	18
risch	$\ln(x+1)+2\ln(x-1)-\ln(x-2)$	18
parallelrisch	$\ln(x+1)+2\ln(x-1)-\ln(x-2)$	18

input
```
int((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x,method=_RETURNVERBOSE)
```

output
```
ln(x+1)+2*ln(x-1)-ln(x-2)
```

3.285.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx = \log(x+1)+2\,\log(x-1)-\log(x-2)$$

input
```
integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="fricas")
```

output
```
log(x + 1) + 2*log(x - 1) - log(x - 2)
```

3.285.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx = -\log(x-2)+2\log(x-1)+\log(x+1)$$

input
```
integrate((2*x**2-5*x-1)/(x**3-2*x**2-x+2),x)
```

output
```
-log(x - 2) + 2*log(x - 1) + log(x + 1)
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx = \log(x+1) + 2\log(x-1) - \log(x-2)$$

input
```
integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="maxima")
```

output
```
log(x + 1) + 2*log(x - 1) - log(x - 2)
```

3.285.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx = \log(|x+1|) + 2\log(|x-1|) - \log(|x-2|)$$

input
```
integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="giac")
```

output
```
log(abs(x + 1)) + 2*log(abs(x - 1)) - log(abs(x - 2))
```

3.285.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx = 2\ln(x-1) - 2\,\mathrm{atanh}\left(\frac{144}{11\,(22\,x-50)} + \frac{13}{11}\right)$$

input
```
int((5*x - 2*x^2 + 1)/(x + 2*x^2 - x^3 - 2),x)
```

output
```
2*log(x - 1) - 2*atanh(144/(11*(22*x - 50)) + 13/11)
```

3.286 $\int \frac{2+x+x^3}{1+2x^2+x^4} \, dx$

3.286.1 Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{2+x+x^3}{1+2x^2+x^4} \, dx = \frac{x}{1+x^2} + \arctan(x) + \frac{1}{2} \log \left(1+x^2\right)$$

output
```
x/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

3.286.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{2+x+x^3}{1+2x^2+x^4} \, dx = \frac{x}{1+x^2} + \arctan(x) + \frac{1}{2} \log \left(1+x^2\right)$$

input
```
Integrate[(2 + x + x^3)/(1 + 2*x^2 + x^4),x]
```

output
```
x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

3.286.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1380, 2345, 27, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x+2}{x^4+2x^2+1} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^3+x+2}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{x}{x^2+1}-\frac{1}{2}\int -\frac{2(x+1)}{x^2+1}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x+1}{x^2+1}dx+\frac{x}{x^2+1}$$

$$\Big\downarrow \text{452}$$

$$\int \frac{1}{x^2+1}dx+\int \frac{x}{x^2+1}dx+\frac{x}{x^2+1}$$

$$\Big\downarrow \text{216}$$

$$\int \frac{x}{x^2+1}dx+\arctan(x)+\frac{x}{x^2+1}$$

$$\Big\downarrow \text{240}$$

$$\arctan(x)+\frac{x}{x^2+1}+\frac{1}{2}\log\left(x^2+1\right)$$

input
```
Int[(2 + x + x^3)/(1 + 2*x^2 + x^4),x]
```

output
```
x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

3.286.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.286.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{x}{x^2+1}+\arctan\left(x\right)+\frac{\ln\left(x^2+1\right)}{2}$	21
risch	$\frac{x}{x^2+1}+\arctan\left(x\right)+\frac{\ln\left(x^2+1\right)}{2}$	21
parallelrisch	$\frac{i\ln\left(x+i\right)x^2-i\ln\left(x-i\right)x^2+\ln\left(x+i\right)x^2+\ln\left(x-i\right)x^2+i\ln\left(x+i\right)-i\ln\left(x-i\right)+\ln\left(x+i\right)+\ln\left(x-i\right)+2x}{2x^2+2}$	80

input
```
int((x^3+x+2)/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
x/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{2+x+x^3}{1+2x^2+x^4}\,dx = \frac{2\,(x^2+1)\arctan{(x)}+(x^2+1)\log{(x^2+1)}+2\,x}{2\,(x^2+1)}$$

input
```
integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="fricas")
```

output
```
1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) + 2*x)/(x^2 + 1)
```

3.286.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{2+x+x^3}{1+2x^2+x^4}\,dx = \frac{x}{x^2+1}+\frac{\log{(x^2+1)}}{2}+\operatorname{atan}{(x)}$$

input
```
integrate((x**3+x+2)/(x**4+2*x**2+1),x)
```

output
```
x/(x**2 + 1) + log(x**2 + 1)/2 + atan(x)
```

3.286.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{2+x+x^3}{1+2x^2+x^4}\,dx = \frac{x}{x^2+1}+\arctan{(x)}+\frac{1}{2}\,\log{\left(x^2+1\right)}$$

input
```
integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="maxima")
```

output
```
x/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

3.286.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{2+x+x^3}{1+2x^2+x^4}\,dx = \frac{x}{x^2+1} + \arctan\left(x\right) + \frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="giac")
```

output
```
x/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

3.286.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{2+x+x^3}{1+2x^2+x^4}\,dx = \frac{\ln\left(x^2+1\right)}{2} + \mathrm{atan}(x) + \frac{x}{x^2+1}$$

input
```
int((x + x^3 + 2)/(2*x^2 + x^4 + 1),x)
```

output
```
log(x^2 + 1)/2 + atan(x) + x/(x^2 + 1)
```

3.287 $\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4} \, dx$

3.287.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4} \, dx = -\frac{1}{2(1+x^2)} + \arctan(x) + \frac{1}{2}\log\left(1+x^2\right)$$

output
```
-1/2/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

3.287.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4} \, dx = -\frac{1}{2(1+x^2)} + \arctan(x) + \frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4),x]
```

output
```
-1/2*1/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

3.287.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1380, 2345, 27, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+2x+1}{x^4+2x^2+1} \, dx$$

$$\Big\downarrow \text{ 1380}$$

$$\int \frac{x^3+x^2+2x+1}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{1}{2}\int -\frac{2(x+1)}{x^2+1}dx-\frac{1}{2(x^2+1)}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x+1}{x^2+1}dx-\frac{1}{2(x^2+1)}$$

$$\Big\downarrow \text{ 452}$$

$$\int \frac{1}{x^2+1}dx+\int \frac{x}{x^2+1}dx-\frac{1}{2(x^2+1)}$$

$$\Big\downarrow \text{ 216}$$

$$\int \frac{x}{x^2+1}dx+\arctan(x)-\frac{1}{2(x^2+1)}$$

$$\Big\downarrow \text{ 240}$$

$$\arctan(x)-\frac{1}{2(x^2+1)}+\frac{1}{2}\log\left(x^2+1\right)$$

input
```
Int[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4),x]
```

output
```
-1/2*1/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

3.287.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.287. $\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4} \, dx$

3.287.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{1}{2(x^2+1)}+\arctan(x)+\frac{\ln(x^2+1)}{2}$	21
risch	$-\frac{1}{2(x^2+1)}+\arctan(x)+\frac{\ln(x^2+1)}{2}$	21
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(x+i)x^2-\ln(x-i)x^2-\ln(x+i)x^2+1+i\ln(x-i)-i\ln(x+i)-\ln(x-i)-\ln(x+i)}{2(x^2+1)}$	84

input
```
int((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

3.287.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx = \frac{2\,(x^2+1)\arctan(x)+(x^2+1)\log(x^2+1)-1}{2\,(x^2+1)}$$

input
```
integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="fricas")
```

output
```
1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 1)/(x^2 + 1)
```

3.287.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx = \frac{\log(x^2+1)}{2}+\operatorname{atan}(x)-\frac{1}{2x^2+2}$$

input
```
integrate((x**3+x**2+2*x+1)/(x**4+2*x**2+1),x)
```

output
```
log(x**2 + 1)/2 + atan(x) - 1/(2*x**2 + 2)
```

3.287.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx = -\frac{1}{2\left(x^2+1\right)} + \arctan\left(x\right) + \frac{1}{2}\,\log\left(x^2+1\right)$$

input
```
integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="maxima")
```

output
```
-1/2/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

3.287.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx = -\frac{1}{2\left(x^2+1\right)} + \arctan\left(x\right) + \frac{1}{2}\,\log\left(x^2+1\right)$$

input
```
integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="giac")
```

output
```
-1/2/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

3.287.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx = \frac{\ln\left(x^2+1\right)}{2} + \mathrm{atan}(x) - \frac{1}{2\,\left(x^2+1\right)}$$

input
```
int((2*x + x^2 + x^3 + 1)/(2*x^2 + x^4 + 1),x)
```

output
```
log(x^2 + 1)/2 + atan(x) - 1/(2*(x^2 + 1))
```

3.288 $\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx$

3.288.1 Optimal result

Integrand size = 20, antiderivative size = 36

$$\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx = 3\arctan(x) - \frac{3\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + 2\log\left(1+x^2\right) - 2\log\left(2+x^2\right)$$

output
```
3*arctan(x)+2*ln(x^2+1)-2*ln(x^2+2)-3/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.288.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx = 3\arctan(x) - \frac{3\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + 2\log\left(1+x^2\right) - 2\log\left(2+x^2\right)$$

input
```
Integrate[(3 + 4*x)/((1 + x^2)*(2 + x^2)),x]
```

output
```
3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*Log[2 + x
^2]
```

3.288.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1343, 303, 216, 353, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x+3}{(x^2+1)(x^2+2)}dx$$

$$\Big\downarrow \text{1343}$$

$$3\int \frac{1}{(x^2+1)(x^2+2)}dx+4\int \frac{x}{(x^2+1)(x^2+2)}dx$$

$$\Big\downarrow \text{303}$$

$$3\left(\int \frac{1}{x^2+1}dx-\int \frac{1}{x^2+2}dx\right)+4\int \frac{x}{(x^2+1)(x^2+2)}dx$$

$$\Big\downarrow \text{216}$$

$$4\int \frac{x}{(x^2+1)(x^2+2)}dx+3\left(\arctan(x)-\frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{353}$$

$$2\int \frac{1}{(x^2+1)(x^2+2)}dx^2+3\left(\arctan(x)-\frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{47}$$

$$2\left(\int \frac{1}{x^2+1}dx^2-\int \frac{1}{x^2+2}dx^2\right)+3\left(\arctan(x)-\frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}\right)$$

$$\Big\downarrow \text{16}$$

$$3\left(\arctan(x)-\frac{\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}\right)+2\left(\log\left(x^2+1\right)-\log\left(x^2+2\right)\right)$$

input
```
Int[(3 + 4*x)/((1 + x^2)*(2 + x^2)),x]
```

output
```
3*(ArcTan[x] - ArcTan[x/Sqrt[2]]/Sqrt[2]) + 2*(Log[1 + x^2] - Log[2 + x^2]
)
```

3.288.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1343
```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Simp[g   Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Simp[h
  Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p,
q}, x]
```

3.288. $\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx$

3.288.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

method	result	size
default	$3\arctan(x)+2\ln(x^2+1)-2\ln(x^2+2)-\frac{3\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}$	34
risch	$3\arctan(x)+2\ln(x^2+1)-2\ln(x^2+2)-\frac{3\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{2}$	34

input
```
int((3+4*x)/(x^2+1)/(x^2+2),x,method=_RETURNVERBOSE)
```

output
```
3*arctan(x)+2*ln(x^2+1)-2*ln(x^2+2)-3/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx = -\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\arctan(x) - 2\log\left(x^2+2\right)+2\log\left(x^2+1\right)$$

input
```
integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="fricas")
```

output
```
-3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(
x^2 + 1)
```

3.288.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx = 2\log\left(x^2+1\right)-2\log\left(x^2+2\right)+3\,\mathrm{atan}(x)-\frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

input
```
integrate((3+4*x)/(x**2+1)/(x**2+2),x)
```

output
```
2*log(x**2 + 1) - 2*log(x**2 + 2) + 3*atan(x) - 3*sqrt(2)*atan(sqrt(2)*x/2
)/2
```

3.288.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx = -\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 3\arctan(x) - 2\log\left(x^2+2\right) + 2\log\left(x^2+1\right)$$

input
```
integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

output
```
-3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(
x^2 + 1)
```

3.288.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx = -\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 3\arctan(x) - 2\log\left(x^2+2\right) + 2\log\left(x^2+1\right)$$

input
```
integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

output
```
-3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(
x^2 + 1)
```

3.288.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

$$\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx = \ln(x-\mathrm{i})\left(2-\frac{3}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(2+\frac{3}{2}\mathrm{i}\right) + \ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(-2+\frac{\sqrt{2}\,3\mathrm{i}}{4}\right) - \ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(2+\frac{\sqrt{2}\,3\mathrm{i}}{4}\right)$$

input
```
int((4*x + 3)/((x^2 + 1)*(x^2 + 2)),x)
```

output
```
log(x - 1i)*(2 - 3i/2) + log(x + 1i)*(2 + 3i/2) + log(x - 2^(1/2)*1i)*((2^
(1/2)*3i)/4 - 2) - log(x + 2^(1/2)*1i)*((2^(1/2)*3i)/4 + 2)
```

3.289 $\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx$

3.289.1 Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx=-\frac{1}{3}\arctan\left(\frac{x}{2}\right)+\frac{2\arctan(x)}{3}+\frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}\log\left(4+x^2\right)$$

output
```
-1/3*arctan(1/2*x)+2/3*arctan(x)+1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

3.289.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx=-\frac{1}{3}\arctan\left(\frac{x}{2}\right)+\frac{2\arctan(x)}{3}+\frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}\log\left(4+x^2\right)$$

input
```
Integrate[(2 + x)/((1 + x^2)*(4 + x^2)),x]
```

output
```
-1/3*ArcTan[x/2] + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/6
```

3.289.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1343, 303, 216, 353, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{(x^2+1)(x^2+4)}dx$$

$$\Big\downarrow \text{1343}$$

$$2\int \frac{1}{(x^2+1)(x^2+4)}dx+\int \frac{x}{(x^2+1)(x^2+4)}dx$$

$$\Big\downarrow \text{303}$$

$$2\left(\frac{1}{3}\int \frac{1}{x^2+1}dx-\frac{1}{3}\int \frac{1}{x^2+4}dx\right)+\int \frac{x}{(x^2+1)(x^2+4)}dx$$

$$\Big\downarrow \text{216}$$

$$\int \frac{x}{(x^2+1)(x^2+4)}dx+2\left(\frac{\arctan(x)}{3}-\frac{1}{6}\arctan\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{(x^2+1)(x^2+4)}dx^2+2\left(\frac{\arctan(x)}{3}-\frac{1}{6}\arctan\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{x^2+1}dx^2-\frac{1}{3}\int \frac{1}{x^2+4}dx^2\right)+2\left(\frac{\arctan(x)}{3}-\frac{1}{6}\arctan\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{16}$$

$$2\left(\frac{\arctan(x)}{3}-\frac{1}{6}\arctan\left(\frac{x}{2}\right)\right)+\frac{1}{2}\left(\frac{1}{3}\log\left(x^2+1\right)-\frac{1}{3}\log\left(x^2+4\right)\right)$$

input
```
Int[(2 + x)/((1 + x^2)*(4 + x^2)),x]
```

output
```
2*(-1/6*ArcTan[x/2] + ArcTan[x]/3) + (Log[1 + x^2]/3 - Log[4 + x^2]/3)/2
```

3.289.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1343
```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Simp[g   Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Simp[h
  Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p,
q}, x]
```

3.289.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{\arctan\left(\frac{x}{2}\right)}{3}+\frac{2\arctan\left(x\right)}{3}+\frac{\ln\left(x^{2}+1\right)}{6}-\frac{\ln\left(x^{2}+4\right)}{6}$	28
risch	$-\frac{\arctan\left(\frac{x}{2}\right)}{3}+\frac{2\arctan\left(x\right)}{3}+\frac{\ln\left(x^{2}+1\right)}{6}-\frac{\ln\left(x^{2}+4\right)}{6}$	28
parallelrisch	$-\frac{\ln\left(x-2i\right)}{6}+\frac{i\ln\left(x-2i\right)}{6}+\frac{\ln\left(x-i\right)}{6}-\frac{i\ln\left(x-i\right)}{3}+\frac{\ln\left(x+i\right)}{6}+\frac{i\ln\left(x+i\right)}{3}-\frac{\ln\left(x+2i\right)}{6}-\frac{i\ln\left(x+2i\right)}{6}$	62

input
```
int((x+2)/(x^2+1)/(x^2+4),x,method=_RETURNVERBOSE)
```

output
```
-1/3*arctan(1/2*x)+2/3*arctan(x)+1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

3.289.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{3}\arctan\left(\frac{1}{2}x\right) + \frac{2}{3}\arctan(x) - \frac{1}{6}\log\left(x^2+4\right) + \frac{1}{6}\log\left(x^2+1\right)$$

input
```
integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="fricas")
```

output
```
-1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

3.289.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx = \frac{\log(x^2+1)}{6} - \frac{\log(x^2+4)}{6} - \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{3} + \frac{2\operatorname{atan}(x)}{3}$$

input
```
integrate((2+x)/(x**2+1)/(x**2+4),x)
```

output
```
log(x**2 + 1)/6 - log(x**2 + 4)/6 - atan(x/2)/3 + 2*atan(x)/3
```

3.289.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{3}\arctan\left(\frac{1}{2}x\right) + \frac{2}{3}\arctan(x) - \frac{1}{6}\log\left(x^2+4\right) + \frac{1}{6}\log\left(x^2+1\right)$$

input
```
integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="maxima")
```

output
```
-1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

3.289.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{3}\arctan\left(\frac{1}{2}x\right)+\frac{2}{3}\arctan(x)-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

input
```
integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="giac")
```

output
```
-1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

3.289.9 Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx = \ln(x-\mathrm{i})\left(\frac{1}{6}-\frac{1}{3}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{1}{6}+\frac{1}{3}\mathrm{i}\right) + \ln(x-2\mathrm{i})\left(-\frac{1}{6}+\frac{1}{6}\mathrm{i}\right)+\ln(x+2\mathrm{i})\left(-\frac{1}{6}-\frac{1}{6}\mathrm{i}\right)$$

input
```
int((x + 2)/((x^2 + 1)*(x^2 + 4)),x)
```

output
```
log(x - 1i)*(1/6 - 1i/3) + log(x + 1i)*(1/6 + 1i/3) - log(x - 2i)*(1/6 - 1
i/6) - log(x + 2i)*(1/6 + 1i/6)
```

3.290 $\int \frac{2-x+x^3}{-7-6x+x^2} dx$

3.290.1 Optimal result

Integrand size = 19, antiderivative size = 29

$$\int \frac{2-x+x^3}{-7-6x+x^2} \, dx = 6x+\frac{x^2}{2}+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(1+x)$$

output
```
6*x+1/2*x^2+169/4*ln(7-x)-1/4*ln(1+x)
```

3.290.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{2-x+x^3}{-7-6x+x^2} \, dx = 6x+\frac{x^2}{2}+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(1+x)$$

input
```
Integrate[(2 - x + x^3)/(-7 - 6*x + x^2),x]
```

output
```
6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4
```

3.290.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-x+2}{x^2-6x-7}\,dx$$

$$\Big\downarrow \; 2188$$

$$\int \left(\frac{2(21x+22)}{x^2-6x-7}+x+6\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^2}{2}+6x+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(x+1)$$

input
```
Int[(2 - x + x^3)/(-7 - 6*x + x^2),x]
```

output
```
6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4
```

3.290.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.290.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{x^2}{2}+6x+\frac{169\ln(x-7)}{4}-\frac{\ln(x+1)}{4}$	22
norman	$\frac{x^2}{2}+6x+\frac{169\ln(x-7)}{4}-\frac{\ln(x+1)}{4}$	22
risch	$\frac{x^2}{2}+6x+\frac{169\ln(x-7)}{4}-\frac{\ln(x+1)}{4}$	22
parallelrisch	$\frac{x^2}{2}+6x+\frac{169\ln(x-7)}{4}-\frac{\ln(x+1)}{4}$	22

input
```
int((x^3-x+2)/(x^2-6*x-7),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+6*x+169/4*ln(x-7)-1/4*ln(x+1)
```

3.290.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{2-x+x^3}{-7-6x+x^2}\,dx = \frac{1}{2}\,x^2+6\,x-\frac{1}{4}\,\log\left(x+1\right)+\frac{169}{4}\,\log\left(x-7\right)$$

input
```
integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="fricas")
```

output
```
1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)
```

3.290.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{2-x+x^3}{-7-6x+x^2}\,dx = \frac{x^2}{2}+6x+\frac{169\log\left(x-7\right)}{4}-\frac{\log\left(x+1\right)}{4}$$

input
```
integrate((x**3-x+2)/(x**2-6*x-7),x)
```

output
```
x**2/2 + 6*x + 169*log(x - 7)/4 - log(x + 1)/4
```

3.290.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{2-x+x^3}{-7-6x+x^2}\,dx = \frac{1}{2}\,x^2+6\,x-\frac{1}{4}\,\log\left(x+1\right)+\frac{169}{4}\,\log\left(x-7\right)$$

input
```
integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="maxima")
```

output
```
1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)
```

3.290.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{2-x+x^3}{-7-6x+x^2}\,dx = \frac{1}{2}\,x^2+6\,x-\frac{1}{4}\,\log\left(|x+1|\right)+\frac{169}{4}\,\log\left(|x-7|\right)$$

input
```
integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="giac")
```

output
```
1/2*x^2 + 6*x - 1/4*log(abs(x + 1)) + 169/4*log(abs(x - 7))
```

3.290.9 Mupad [B] (verification not implemented)

Time = 9.81 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{2-x+x^3}{-7-6x+x^2}\,dx = 6\,x-\frac{\ln\left(x+1\right)}{4}+\frac{169\,\ln\left(x-7\right)}{4}+\frac{x^2}{2}$$

input
```
int(-(x^3 - x + 2)/(6*x - x^2 + 7),x)
```

output
```
6*x - log(x + 1)/4 + (169*log(x - 7))/4 + x^2/2
```

3.291 $\int \frac{-1+x^5}{-1+x^2} \, dx$

3.291.1 Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{-1+x^5}{-1+x^2} \, dx = \frac{x^2}{2} + \frac{x^4}{4} + \log(1+x)$$

output
```
1/2*x^2+1/4*x^4+ln(1+x)
```

3.291.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^5}{-1+x^2} \, dx = \frac{x^2}{2} + \frac{x^4}{4} + \log(1+x)$$

input
```
Integrate[(-1 + x^5)/(-1 + x^2),x]
```

output
```
x^2/2 + x^4/4 + Log[1 + x]
```

3.291.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5-1}{x^2-1} \, dx$$

$$\Big\downarrow \; 2341$$

$$\int \left(x^3-\frac{1-x}{x^2-1}+x\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^4}{4}+\frac{x^2}{2}+\log(x+1)$$

input
```
Int[(-1 + x^5)/(-1 + x^2),x]
```

output
```
x^2/2 + x^4/4 + Log[1 + x]
```

3.291.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.291.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{x^2}{2}+\frac{x^4}{4}+\ln\left(x+1\right)$	16
norman	$\frac{x^2}{2}+\frac{x^4}{4}+\ln\left(x+1\right)$	16
parallelrisch	$\frac{x^2}{2}+\frac{x^4}{4}+\ln\left(x+1\right)$	16
risch	$\frac{x^4}{4}+\frac{x^2}{2}+\frac{1}{4}+\ln\left(x+1\right)$	17
meijerg	$\operatorname{arctanh}\left(x\right)+\frac{x^2\left(3x^2+6\right)}{12}+\frac{\ln\left(-x^2+1\right)}{2}$	26

input
```
int((x^5-1)/(x^2-1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/4*x^4+ln(x+1)
```

3.291.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{-1+x^5}{-1+x^2}\,dx=\frac{1}{4}\,x^4+\frac{1}{2}\,x^2+\log\left(x+1\right)$$

input
```
integrate((x^5-1)/(x^2-1),x, algorithm="fricas")
```

output
```
1/4*x^4 + 1/2*x^2 + log(x + 1)
```

3.291.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{-1+x^5}{-1+x^2}\,dx=\frac{x^4}{4}+\frac{x^2}{2}+\log\left(x+1\right)$$

input
```
integrate((x**5-1)/(x**2-1),x)
```

output
```
x**4/4 + x**2/2 + log(x + 1)
```

3.291.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{-1+x^5}{-1+x^2}\,dx = \frac{1}{4}\,x^4 + \frac{1}{2}\,x^2 + \log\left(x+1\right)$$

input
```
integrate((x^5-1)/(x^2-1),x, algorithm="maxima")
```

output
```
1/4*x^4 + 1/2*x^2 + log(x + 1)
```

3.291.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{-1+x^5}{-1+x^2}\,dx = \frac{1}{4}\,x^4 + \frac{1}{2}\,x^2 + \log\left(|x+1|\right)$$

input
```
integrate((x^5-1)/(x^2-1),x, algorithm="giac")
```

output
```
1/4*x^4 + 1/2*x^2 + log(abs(x + 1))
```

3.291.9 Mupad [B] (verification not implemented)

Time = 9.67 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{-1+x^5}{-1+x^2}\,dx = \ln\left(x+1\right) + \frac{x^2}{2} + \frac{x^4}{4}$$

input
```
int((x^5 - 1)/(x^2 - 1),x)
```

output
```
log(x + 1) + x^2/2 + x^4/4
```

3.292 $\int \frac{5+2x-x^2+x^3}{1+x+x^2} dx$

3.292.1 Optimal result

Integrand size = 22, antiderivative size = 41

$$\int \frac{5+2x-x^2+x^3}{1+x+x^2} dx = -2x + \frac{x^2}{2} + \frac{11 \arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(1+x+x^2\right)$$

output
```
-2*x+1/2*x^2+3/2*ln(x^2+x+1)+11/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.292.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{5+2x-x^2+x^3}{1+x+x^2} dx = -2x + \frac{x^2}{2} + \frac{11 \arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(1+x+x^2\right)$$

input
```
Integrate[(5 + 2*x - x^2 + x^3)/(1 + x + x^2),x]
```

output
```
-2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1 + x + x^2
])/2
```

3.292.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-x^2+2x+5}{x^2+x+1} \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{3x+7}{x^2+x+1}+x-2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{11\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^2}{2}+\frac{3}{2}\log\left(x^2+x+1\right)-2x$$

input
```
Int[(5 + 2*x - x^2 + x^3)/(1 + x + x^2),x]
```

output
```
-2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1 + x + x^2
])/2
```

3.292.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.292.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

method	result	size
default	$-2x+\frac{x^2}{2}+\frac{3\ln\left(x^2+x+1\right)}{2}+\frac{11\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	35
risch	$-2x+\frac{x^2}{2}+\frac{3\ln\left(4x^2+4x+4\right)}{2}+\frac{11\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	39

input
```
int((x^3-x^2+2*x+5)/(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
-2*x+1/2*x^2+3/2*ln(x^2+x+1)+11/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.292.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx=\frac{1}{2}\,x^2+\frac{11}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-2\,x+\frac{3}{2}\,\log\left(x^2+x+1\right)$$

input
```
integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="fricas")
```

output
```
1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
 x + 1)
```

3.292.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx=\frac{x^2}{2}-2x+\frac{3\log\left(x^2+x+1\right)}{2}+\frac{11\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((x**3-x**2+2*x+5)/(x**2+x+1),x)
```

output
```
x**2/2 - 2*x + 3*log(x**2 + x + 1)/2 + 11*sqrt(3)*atan(2*sqrt(3)*x/3 + sqr
t(3)/3)/3
```

3.292.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx = \frac{1}{2}\,x^2+\frac{11}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-2\,x+\frac{3}{2}\,\log\left(x^2+x+1\right)$$

input
```
integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="maxima")
```

output
```
1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
 x + 1)
```

3.292.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx = \frac{1}{2}\,x^2+\frac{11}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-2\,x+\frac{3}{2}\,\log\left(x^2+x+1\right)$$

input
```
integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="giac")
```

output
```
1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
 x + 1)
```

3.292.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx = \frac{3\,\ln\left(x^2+x+1\right)}{2}-2\,x+\frac{11\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{x^2}{2}$$

input
```
int((2*x - x^2 + x^3 + 5)/(x + x^2 + 1),x)
```

output
```
(3*log(x + x^2 + 1))/2 - 2*x + (11*3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/
3))/3 + x^2/2
```

3.293 $\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx$

3.293.1 Optimal result

Integrand size = 24, antiderivative size = 41

$$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx = \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+6\arctan(2-x)+\frac{3}{4}\log\left(5-4x+x^2\right)$$

output
```
3/2*x+1/2*x^2+1/6*x^3-6*arctan(-2+x)+3/4*ln(x^2-4*x+5)
```

3.293.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx = \frac{1}{2}\left(3x+x^2+\frac{x^3}{3}+12\arctan(2-x)+\frac{3}{2}\log\left(5-4x+x^2\right)\right)$$

input
```
Integrate[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2),x]
```

output
```
(3*x + x^2 + x^3/3 + 12*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2])/2)/2
```

3.293.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-2x^3+x-3}{2x^2-8x+10} \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{x^2}{2}-\frac{3(6-x)}{2x^2-8x+10}+x+\frac{3}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$6\arctan(2-x)+\frac{x^3}{6}+\frac{x^2}{2}+\frac{3}{4}\log\left(x^2-4x+5\right)+\frac{3x}{2}$$

input
```
Int[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2),x]
```

output
```
(3*x)/2 + x^2/2 + x^3/6 + 6*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2])/4
```

3.293.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.293.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-6\arctan\left(x-2\right)+\frac{3\ln\left(x^2-4x+5\right)}{4}$	32
risch	$\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-6\arctan\left(x-2\right)+\frac{3\ln\left(x^2-4x+5\right)}{4}$	32
parallelrisch	$\frac{x^3}{6}+\frac{x^2}{2}+\frac{3x}{2}+\frac{3\ln(x-2-i)}{4}+3i\ln\left(x-2-i\right)+\frac{3\ln(x-2+i)}{4}-3i\ln\left(x-2+i\right)$	49

input
```
int((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x,method=_RETURNVERBOSE)
```

output
```
3/2*x+1/2*x^2+1/6*x^3-6*arctan(x-2)+3/4*ln(x^2-4*x+5)
```

3.293.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx = \frac{1}{6}\,x^3+\frac{1}{2}\,x^2+\frac{3}{2}\,x-6\,\arctan\left(x-2\right)+\frac{3}{4}\,\log\left(x^2-4\,x+5\right)$$

input
```
integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="fricas")
```

output
```
1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

3.293.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx = \frac{x^3}{6}+\frac{x^2}{2}+\frac{3x}{2}+\frac{3\log\left(x^2-4x+5\right)}{4}-6\,\mathrm{atan}\left(x-2\right)$$

input
```
integrate((x**4-2*x**3+x-3)/(2*x**2-8*x+10),x)
```

output
```
x**3/6 + x**2/2 + 3*x/2 + 3*log(x**2 - 4*x + 5)/4 - 6*atan(x - 2)
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx = \frac{1}{6}\,x^3+\frac{1}{2}\,x^2+\frac{3}{2}\,x-6\,\arctan\left(x-2\right)+\frac{3}{4}\,\log\left(x^2-4\,x+5\right)$$

input
```
integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="maxima")
```

output
```
1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

3.293.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx = \frac{1}{6}\,x^3+\frac{1}{2}\,x^2+\frac{3}{2}\,x-6\,\arctan\left(x-2\right)+\frac{3}{4}\,\log\left(x^2-4\,x+5\right)$$

input
```
integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="giac")
```

output
```
1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

3.293.9 Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx = \frac{3\,x}{2}-6\,\mathrm{atan}(x-2)+\frac{3\,\ln\left(x^2-4\,x+5\right)}{4}+\frac{x^2}{2}+\frac{x^3}{6}$$

input
```
int((x - 2*x^3 + x^4 - 3)/(2*x^2 - 8*x + 10),x)
```

output
```
(3*x)/2 - 6*atan(x - 2) + (3*log(x^2 - 4*x + 5))/4 + x^2/2 + x^3/6
```

3.294 $\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx$

3.294.1 Optimal result

Integrand size = 29, antiderivative size = 30

$$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx = x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)$$

output
```
x+7/2*ln(1-x)-25*ln(2-x)+61/2*ln(3-x)
```

3.294.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx = x+\frac{61}{2}\log(-3+x)-25\log(-2+x)+\frac{7}{2}\log(-1+x)$$

input
```
Integrate[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

output
```
x + (61*Log[-3 + x])/2 - 25*Log[-2 + x] + (7*Log[-1 + x])/2
```

3.294.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+3x^2+2x+1}{(x-3)(x-2)(x-1)}\,dx$$

$$\Big\downarrow \text{2115}$$

$$\int \left(-\frac{25}{x-2}+\frac{7}{2(x-1)}+\frac{61}{2(x-3)}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)$$

input
```
Int[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

output
```
x + (7*Log[1 - x])/2 - 25*Log[2 - x] + (61*Log[3 - x])/2
```

3.294.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

3.294.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

method	result	size
default	$x+\frac{61 \ln(-3+x)}{2}+\frac{7 \ln(x-1)}{2}-25 \ln(x-2)$	21
norman	$x+\frac{61 \ln(-3+x)}{2}+\frac{7 \ln(x-1)}{2}-25 \ln(x-2)$	21
risch	$x+\frac{61 \ln(-3+x)}{2}+\frac{7 \ln(x-1)}{2}-25 \ln(x-2)$	21
parallelrisch	$x+\frac{61 \ln(-3+x)}{2}+\frac{7 \ln(x-1)}{2}-25 \ln(x-2)$	21

input
```
int((x^3+3*x^2+2*x+1)/(-3+x)/(x-2)/(x-1),x,method=_RETURNVERBOSE)
```

output
```
x+61/2*ln(-3+x)+7/2*ln(x-1)-25*ln(x-2)
```

3.294.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx = x+\frac{7}{2}\log(x-1)-25\log(x-2)+\frac{61}{2}\log(x-3)$$

input
```
integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
```

output
```
x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)
```

3.294.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx = x+\frac{61\log(x-3)}{2}-25\log(x-2)+\frac{7\log(x-1)}{2}$$

input
```
integrate((x**3+3*x**2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x)
```

output
```
x + 61*log(x - 3)/2 - 25*log(x - 2) + 7*log(x - 1)/2
```

3.294.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx = x+\frac{7}{2}\,\log\left(x-1\right)-25\,\log\left(x-2\right)+\frac{61}{2}\,\log\left(x-3\right)$$

input
```
integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
```

output
```
x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)
```

3.294.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx = x+\frac{7}{2}\,\log\left(|x-1|\right)-25\,\log\left(|x-2|\right)+\frac{61}{2}\,\log\left(|x-3|\right)$$

input
```
integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
```

output
```
x + 7/2*log(abs(x - 1)) - 25*log(abs(x - 2)) + 61/2*log(abs(x - 3))
```

3.294.9 Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx = x+\frac{7\,\ln\left(x-1\right)}{2}-25\,\ln\left(x-2\right)+\frac{61\,\ln\left(x-3\right)}{2}$$

input
```
int((2*x + 3*x^2 + x^3 + 1)/((x - 1)*(x - 2)*(x - 3)),x)
```

output
```
x + (7*log(x - 1))/2 - 25*log(x - 2) + (61*log(x - 3))/2
```

3.295 $\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx$

3.295.1 Optimal result

Integrand size = 30, antiderivative size = 35

$$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx=-2x+\frac{x^2}{2}+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(2+x)+20\log(3+x)$$

output
```
-2*x+1/2*x^2+13/3*ln(4-x)-22/3*ln(2+x)+20*ln(3+x)
```

3.295.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx=-2x+\frac{x^2}{2}+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(2+x)+20\log(3+x)$$

input
```
Integrate[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3),x]
```

output
```
-2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 + x]
```

3.295.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-x^3+x^2-7x+2}{x^3+x^2-14x-24}\,dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(x+\frac{13}{3(x-4)}-\frac{22}{3(x+2)}+\frac{20}{x+3}-2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}-2x+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(x+2)+20\log(x+3)$$

input
```
Int[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3),x]
```

output
```
-2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 + x]
```

3.295.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.295.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x^2}{2}-2x-\frac{22\ln(x+2)}{3}+20\ln(3+x)+\frac{13\ln(x-4)}{3}$	28
norman	$\frac{x^2}{2}-2x-\frac{22\ln(x+2)}{3}+20\ln(3+x)+\frac{13\ln(x-4)}{3}$	28
risch	$\frac{x^2}{2}-2x-\frac{22\ln(x+2)}{3}+20\ln(3+x)+\frac{13\ln(x-4)}{3}$	28
parallelrisch	$\frac{x^2}{2}-2x-\frac{22\ln(x+2)}{3}+20\ln(3+x)+\frac{13\ln(x-4)}{3}$	28

input
```
int((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-2*x-22/3*ln(x+2)+20*ln(3+x)+13/3*ln(x-4)
```

3.295.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx=\frac{1}{2}\,x^2-2\,x+20\,\log\left(x+3\right)-\frac{22}{3}\,\log\left(x+2\right)+\frac{13}{3}\,\log\left(x-4\right)$$

input
```
integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="fricas")
```

output
```
1/2*x^2 - 2*x + 20*log(x + 3) - 22/3*log(x + 2) + 13/3*log(x - 4)
```

3.295.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx=\frac{x^2}{2}-2x+\frac{13\log\left(x-4\right)}{3}-\frac{22\log\left(x+2\right)}{3}+20\log\left(x+3\right)$$

input
```
integrate((x**4-x**3+x**2-7*x+2)/(x**3+x**2-14*x-24),x)
```

output
```
x**2/2 - 2*x + 13*log(x - 4)/3 - 22*log(x + 2)/3 + 20*log(x + 3)
```

3.295.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx = \frac{1}{2}\,x^2 - 2\,x + 20\,\log\left(x+3\right) - \frac{22}{3}\,\log\left(x+2\right) + \frac{13}{3}\,\log\left(x-4\right)$$

input
```
integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="maxima")
```

output
```
1/2*x^2 - 2*x + 20*log(x + 3) - 22/3*log(x + 2) + 13/3*log(x - 4)
```

3.295.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx = \frac{1}{2}\,x^2 - 2\,x + 20\,\log\left(|x+3|\right) - \frac{22}{3}\,\log\left(|x+2|\right) + \frac{13}{3}\,\log\left(|x-4|\right)$$

input
```
integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="giac")
```

output
```
1/2*x^2 - 2*x + 20*log(abs(x + 3)) - 22/3*log(abs(x + 2)) + 13/3*log(abs(x
 - 4))
```

3.295.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx = 20\,\ln\left(x+3\right) - \frac{22\,\ln\left(x+2\right)}{3} - 2\,x + \frac{13\,\ln\left(x-4\right)}{3} + \frac{x^2}{2}$$

input
```
int(-(x^2 - 7*x - x^3 + x^4 + 2)/(14*x - x^2 - x^3 + 24),x)
```

output
```
20*log(x + 3) - (22*log(x + 2))/3 - 2*x + (13*log(x - 4))/3 + x^2/2
```

3.296 $\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx$

3.296.1 Optimal result

Integrand size = 19, antiderivative size = 34

$$\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx = \frac{3}{2(1-x)} - \frac{5}{4}\log(1-x) + 2\log(x) - \frac{3}{4}\log(1+x)$$

output
```
3/2/(1-x)-5/4*ln(1-x)+2*ln(x)-3/4*ln(1+x)
```

3.296.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx = -\frac{3}{2(-1+x)} - \frac{5}{4}\log(1-x) + 2\log(x) - \frac{3}{4}\log(1+x)$$

input
```
Integrate[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]
```

output
```
-3/(2*(-1 + x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4
```

3.296.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+2}{(x-1)^2x(x+1)}\,dx$$

$$\Big\downarrow \text{2115}$$

$$\int \left(\frac{2}{x}-\frac{3}{4(x+1)}-\frac{5}{4(x-1)}+\frac{3}{2(x-1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2(1-x)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(x+1)$$

input
```
Int[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]
```

output
```
3/(2*(1 - x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4
```

3.296.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

3.296.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
default	$2\ln\left(x\right)-\frac{3\ln\left(x+1\right)}{4}-\frac{3}{2\left(x-1\right)}-\frac{5\ln\left(x-1\right)}{4}$	25
norman	$2\ln\left(x\right)-\frac{3\ln\left(x+1\right)}{4}-\frac{3}{2\left(x-1\right)}-\frac{5\ln\left(x-1\right)}{4}$	25
risch	$2\ln\left(x\right)-\frac{3\ln\left(x+1\right)}{4}-\frac{3}{2\left(x-1\right)}-\frac{5\ln\left(x-1\right)}{4}$	25
parallelrisch	$\frac{8\ln\left(x\right)x-5\ln\left(x-1\right)x-3\ln\left(x+1\right)x-6-8\ln\left(x\right)+5\ln\left(x-1\right)+3\ln\left(x+1\right)}{4x-4}$	45

input
```
int((x^2+2)/(x-1)^2/x/(x+1),x,method=_RETURNVERBOSE)
```

output
```
2*ln(x)-3/4*ln(x+1)-3/2/(x-1)-5/4*ln(x-1)
```

3.296.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx = -\frac{3\left(x-1\right)\log\left(x+1\right)+5\left(x-1\right)\log\left(x-1\right)-8\left(x-1\right)\log\left(x\right)+6}{4\left(x-1\right)}$$

input
```
integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="fricas")
```

output
```
-1/4*(3*(x - 1)*log(x + 1) + 5*(x - 1)*log(x - 1) - 8*(x - 1)*log(x) + 6)/
(x - 1)
```

3.296.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx = 2\log{(x)} - \frac{5\log{(x-1)}}{4} - \frac{3\log{(x+1)}}{4} - \frac{3}{2x-2}$$

input
```
integrate((x**2+2)/(-1+x)**2/x/(1+x),x)
```

output
```
2*log(x) - 5*log(x - 1)/4 - 3*log(x + 1)/4 - 3/(2*x - 2)
```

3.296.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx = -\frac{3}{2\,(x-1)} - \frac{3}{4}\,\log\left(x+1\right) - \frac{5}{4}\,\log\left(x-1\right) + 2\,\log\left(x\right)$$

input
```
integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="maxima")
```

output
```
-3/2/(x - 1) - 3/4*log(x + 1) - 5/4*log(x - 1) + 2*log(x)
```

3.296.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx = -\frac{3}{2\,(x-1)} + 2\,\log\left(\left|-\frac{1}{x-1} - 1\right|\right) - \frac{3}{4}\,\log\left(\left|-\frac{2}{x-1} - 1\right|\right)$$

input
```
integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="giac")
```

output
```
-3/2/(x - 1) + 2*log(abs(-1/(x - 1) - 1)) - 3/4*log(abs(-2/(x - 1) - 1))
```

3.296.9 Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx = 2\,\ln(x) - \frac{3\,\ln(x+1)}{4} - \frac{5\,\ln(x-1)}{4} - \frac{3}{2\,(x-1)}$$

input
```
int((x^2 + 2)/(x*(x - 1)^2*(x + 1)),x)
```

output
```
2*log(x) - (3*log(x + 1))/4 - (5*log(x - 1))/4 - 3/(2*(x - 1))
```

3.297 $\int \frac{3+x^2+x^3}{(2+x^2)^2} \, dx$

3.297.1 Optimal result

Integrand size = 16, antiderivative size = 42

$$\int \frac{3+x^2+x^3}{(2+x^2)^2} \, dx = \frac{4+x}{4(2+x^2)} + \frac{5\arctan\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}} + \frac{1}{2}\log\left(2+x^2\right)$$

output
```
1/4*(4+x)/(x^2+2)+1/2*ln(x^2+2)+5/8*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.297.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{3+x^2+x^3}{(2+x^2)^2} \, dx = \frac{4+x}{4(2+x^2)} + \frac{5\arctan\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}} + \frac{1}{2}\log\left(2+x^2\right)$$

input
```
Integrate[(3 + x^2 + x^3)/(2 + x^2)^2,x]
```

output
```
(4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2 + x^2]/2
```

3.297.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2345, 25, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+3}{(x^2+2)^2}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{x+4}{4(x^2+2)}-\frac{1}{4}\int -\frac{4x+5}{x^2+2}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int \frac{4x+5}{x^2+2}dx+\frac{x+4}{4(x^2+2)}$$

$$\Big\downarrow \text{452}$$

$$\frac{1}{4}\left(5\int \frac{1}{x^2+2}dx+4\int \frac{x}{x^2+2}dx\right)+\frac{x+4}{4(x^2+2)}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(4\int \frac{x}{x^2+2}dx+\frac{5\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}\right)+\frac{x+4}{4(x^2+2)}$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{4}\left(\frac{5\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+2\log\left(x^2+2\right)\right)+\frac{x+4}{4(x^2+2)}$$

input
```
Int[(3 + x^2 + x^3)/(2 + x^2)^2,x]
```

output
```
(4 + x)/(4*(2 + x^2)) + ((5*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[2 + x^2])/4
```

3.297.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.297.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\frac{x}{4}+1}{x^2+2}+\frac{\ln(x^2+2)}{2}+\frac{5\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{8}$	35
risch	$\frac{\frac{x}{4}+1}{x^2+2}+\frac{\ln(x^2+2)}{2}+\frac{5\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{8}$	35
meijerg	$\frac{3\sqrt{2}\left(\frac{x\sqrt{2}}{x^2+2}+\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{8}-\frac{x^2}{4\left(1+\frac{x^2}{2}\right)}+\frac{\ln\left(1+\frac{x^2}{2}\right)}{2}+\frac{\sqrt{2}\left(-\frac{x\sqrt{2}}{2\left(1+\frac{x^2}{2}\right)}+\arctan\left(\frac{x\sqrt{2}}{2}\right)\right)}{4}$	79

input
```
int((x^3+x^2+3)/(x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*x+1)/(x^2+2)+1/2*ln(x^2+2)+5/8*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.297.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{3+x^2+x^3}{(2+x^2)^2}\,dx = \frac{5\sqrt{2}(x^2+2)\arctan\left(\frac{1}{2}\sqrt{2}x\right)+4\,(x^2+2)\log\left(x^2+2\right)+2\,x+8}{8\,(x^2+2)}$$

input
```
integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="fricas")
```

output
```
1/8*(5*sqrt(2)*(x^2 + 2)*arctan(1/2*sqrt(2)*x) + 4*(x^2 + 2)*log(x^2 + 2)
+ 2*x + 8)/(x^2 + 2)
```

3.297.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{3+x^2+x^3}{(2+x^2)^2}\,dx = \frac{x+4}{4x^2+8}+\frac{\log\left(x^2+2\right)}{2}+\frac{5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

input
```
integrate((x**3+x**2+3)/(x**2+2)**2,x)
```

output
```
(x + 4)/(4*x**2 + 8) + log(x**2 + 2)/2 + 5*sqrt(2)*atan(sqrt(2)*x/2)/8
```

3.297.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{3+x^2+x^3}{(2+x^2)^2}\,dx = \frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{x+4}{4\,(x^2+2)}+\frac{1}{2}\log\left(x^2+2\right)$$

input
```
integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="maxima")
```

output
```
5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*log(x^2 +
2)
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{3+x^2+x^3}{(2+x^2)^2}\,dx = \frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{x+4}{4\,(x^2+2)}+\frac{1}{2}\,\log\left(x^2+2\right)$$

input
```
integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="giac")
```

output
```
5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*log(x^2 +
2)
```

3.297.9 Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{3+x^2+x^3}{(2+x^2)^2}\,dx = \frac{\ln(x^2+2)}{2}+\frac{5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8}+\frac{x}{4\,(x^2+2)}+\frac{1}{x^2+2}$$

input
```
int((x^2 + x^3 + 3)/(x^2 + 2)^2,x)
```

output
```
log(x^2 + 2)/2 + (5*2^(1/2)*atan((2^(1/2)*x)/2))/8 + x/(4*(x^2 + 2)) + 1/(
x^2 + 2)
```

3.298 $\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx$

3.298.1 Optimal result

Integrand size = 36, antiderivative size = 49

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = -\frac{15033\arctan(5-x)}{1025} - \frac{4607\arctan\left(\frac{1}{4}(-1+x)\right)}{4100} + \frac{1003\log(26-10x+x^2)}{1025} + \frac{22\log(17-2x+x^2)}{1025}$$

output
```
15033/1025*arctan(-5+x)-4607/4100*arctan(-1/4+1/4*x)+1003/1025*ln(x^2-10*x
+26)+22/1025*ln(x^2-2*x+17)
```

3.298.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = -\frac{15033\arctan(5-x)}{1025} - \frac{4607\arctan\left(\frac{1}{4}(-1+x)\right)}{4100} + \frac{1003\log(26-10x+x^2)}{1025} + \frac{22\log(17-2x+x^2)}{1025}$$

input
```
Integrate[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2)
),x]
```

output
```
(-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1003*Log[2
6 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025
```

3.298.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3-4x^2+70x-35}{(x^2-10x+26)(x^2-2x+17)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{44x-4651}{1025(x^2-2x+17)}+\frac{2006x+5003}{1025(x^2-10x+26)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{15033\arctan(5-x)}{1025}-\frac{4607\arctan\left(\frac{x-1}{4}\right)}{4100}+\frac{1003\log\left(x^2-10x+26\right)}{1025}+\frac{22\log\left(x^2-2x+17\right)}{1025}$$

input
```
Int[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2)),x]
```

output
```
(-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1003*Log[2
6 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025
```

3.298.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.298.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

method	result
default	$\frac{15033 \arctan(-5+x)}{1025}-\frac{4607 \arctan\left(-\frac{1}{4}+\frac{x}{4}\right)}{4100}+\frac{1003 \ln\left(x^{2}-10x+26\right)}{1025}+\frac{22 \ln\left(x^{2}-2x+17\right)}{1025}$
risch	$\frac{15033 \arctan(-5+x)}{1025}-\frac{4607 \arctan\left(-\frac{1}{4}+\frac{x}{4}\right)}{4100}+\frac{1003 \ln\left(x^{2}-10x+26\right)}{1025}+\frac{22 \ln\left(x^{2}-2x+17\right)}{1025}$
parallelrisch	$\frac{1003 \ln(x-5-i)}{1025}-\frac{15033 i \ln(x-5-i)}{2050}+\frac{1003 \ln(x-5+i)}{1025}+\frac{15033 i \ln(x-5+i)}{2050}+\frac{22 \ln(x-1-4i)}{1025}+\frac{4607 i \ln(x-1-4i)}{8200}-$

input
```
int((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x,method=_RETURNVERBO
SE)
```

output
```
15033/1025*arctan(-5+x)-4607/4100*arctan(-1/4+1/4*x)+1003/1025*ln(x^2-10*x
+26)+22/1025*ln(x^2-2*x+17)
```

3.298.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = \frac{15033}{1025}\arctan(x-5)-\frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right)+\frac{22}{1025}\log\left(x^2-2x+17\right)+\frac{1003}{1025}\log\left(x^2-10x+26\right)$$

input
```
integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="f
ricas")
```

output
```
15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
 - 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

3.298.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = \frac{1003\log(x^2-10x+26)}{1025} + \frac{22\log(x^2-2x+17)}{1025} - \frac{4607\,\mathrm{atan}\left(\frac{x}{4}-\frac{1}{4}\right)}{4100} + \frac{15033\,\mathrm{atan}(x-5)}{1025}$$

input
```
integrate((2*x**3-4*x**2+70*x-35)/(x**2-10*x+26)/(x**2-2*x+17),x)
```

output
```
1003*log(x**2 - 10*x + 26)/1025 + 22*log(x**2 - 2*x + 17)/1025 - 4607*atan
(x/4 - 1/4)/4100 + 15033*atan(x - 5)/1025
```

3.298.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = \frac{15033}{1025}\arctan(x-5) - \frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right) + \frac{22}{1025}\log\left(x^2-2x+17\right) + \frac{1003}{1025}\log\left(x^2-10x+26\right)$$

input
```
integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="m
axima")
```

output
```
15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
 - 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

3.298.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = \frac{15033}{1025}\arctan(x-5) - \frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right) + \frac{22}{1025}\log\left(x^2-2x+17\right) + \frac{1003}{1025}\log\left(x^2-10x+26\right)$$

input
```
integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="g
iac")
```

output
```
15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
 - 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

3.298.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = \ln(x-1-4\mathrm{i})\left(\frac{22}{1025}+\frac{4607}{8200}\mathrm{i}\right) + \ln(x-1+4\mathrm{i})\left(\frac{22}{1025}-\frac{4607}{8200}\mathrm{i}\right) + \ln(x-5-\mathrm{i})\left(\frac{1003}{1025}-\frac{15033}{2050}\mathrm{i}\right) + \ln(x-5+1\mathrm{i})\left(\frac{1003}{1025}+\frac{15033}{2050}\mathrm{i}\right)$$

input
```
int((70*x - 4*x^2 + 2*x^3 - 35)/((x^2 - 2*x + 17)*(x^2 - 10*x + 26)),x)
```

output
```
log(x - (1 + 4i))*(22/1025 + 4607i/8200) + log(x - (1 - 4i))*(22/1025 - 46
07i/8200) + log(x - (5 + 1i))*(1003/1025 - 15033i/2050) + log(x - (5 - 1i)
)*(1003/1025 + 15033i/2050)
```

3.299 $\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx$

3.299.1 Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx = -\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(4+x)$$

output
```
-11/14*ln(3-x)+3/2*ln(5-x)+2/7*ln(4+x)
```

3.299.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx = -\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(4+x)$$

input
```
Integrate[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)),x]
```

output
```
(-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7
```

3.299.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+2}{(x-5)(x-3)(x+4)} \, dx$$
$$\Big\downarrow \text{2115}$$
$$\int \left(-\frac{11}{14(x-3)}+\frac{2}{7(x+4)}+\frac{3}{2(x-5)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(x+4)$$

input
```
Int[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)),x]
```

output
```
(-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7
```

3.299.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

3.299.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{3\ln(-5+x)}{2}-\frac{11\ln(-3+x)}{14}+\frac{2\ln(x+4)}{7}$	20
norman	$\frac{3\ln(-5+x)}{2}-\frac{11\ln(-3+x)}{14}+\frac{2\ln(x+4)}{7}$	20
risch	$\frac{3\ln(-5+x)}{2}-\frac{11\ln(-3+x)}{14}+\frac{2\ln(x+4)}{7}$	20
parallelrisch	$\frac{3\ln(-5+x)}{2}-\frac{11\ln(-3+x)}{14}+\frac{2\ln(x+4)}{7}$	20

input
```
int((x^2+2)/(-5+x)/(-3+x)/(x+4),x,method=_RETURNVERBOSE)
```

output
```
3/2*ln(-5+x)-11/14*ln(-3+x)+2/7*ln(x+4)
```

3.299.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx = \frac{2}{7}\log(x+4)-\frac{11}{14}\log(x-3)+\frac{3}{2}\log(x-5)$$

input
```
integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="fricas")
```

output
```
2/7*log(x + 4) - 11/14*log(x - 3) + 3/2*log(x - 5)
```

3.299.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx = \frac{3\log(x-5)}{2}-\frac{11\log(x-3)}{14}+\frac{2\log(x+4)}{7}$$

input
```
integrate((x**2+2)/(-5+x)/(-3+x)/(4+x),x)
```

output
```
3*log(x - 5)/2 - 11*log(x - 3)/14 + 2*log(x + 4)/7
```

3.299.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx = \frac{2}{7}\log(x+4) - \frac{11}{14}\log(x-3) + \frac{3}{2}\log(x-5)$$

input
```
integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="maxima")
```

output
```
2/7*log(x + 4) - 11/14*log(x - 3) + 3/2*log(x - 5)
```

3.299.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx = \frac{2}{7}\log(|x+4|) - \frac{11}{14}\log(|x-3|) + \frac{3}{2}\log(|x-5|)$$

input
```
integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="giac")
```

output
```
2/7*log(abs(x + 4)) - 11/14*log(abs(x - 3)) + 3/2*log(abs(x - 5))
```

3.299.9 Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx = \frac{2\ln(x+4)}{7} - \frac{11\ln(x-3)}{14} + \frac{3\ln(x-5)}{2}$$

input
```
int((x^2 + 2)/((x - 3)*(x + 4)*(x - 5)),x)
```

output
```
(2*log(x + 4))/7 - (11*log(x - 3))/14 + (3*log(x - 5))/2
```

3.300 $\int \frac{x^4}{(-1+x)(2+x^2)}\,dx$

3.300.1 Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{x^4}{(-1+x)(2+x^2)}\,dx = x+\frac{x^2}{2}-\frac{2}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{1}{3}\log(1-x)-\frac{2}{3}\log\left(2+x^2\right)$$

output
```
x+1/2*x^2+1/3*ln(1-x)-2/3*ln(x^2+2)-2/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.300.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{(-1+x)(2+x^2)}\,dx = \frac{1}{6}\left(-9+6x+3x^2-4\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+2\log(-1+x)-4\log\left(2+x^2\right)\right)$$

input
```
Integrate[x^4/((-1 + x)*(2 + x^2)),x]
```

output
```
(-9 + 6*x + 3*x^2 - 4*Sqrt[2]*ArcTan[x/Sqrt[2]] + 2*Log[-1 + x] - 4*Log[2
+ x^2])/6
```

3.300.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {604, 27, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(x-1)(x^2+2)}dx$$

$$\Big\downarrow \text{604}$$

$$\frac{1}{2}\int \frac{2(-2x^3+3x^2-4x+2)}{(1-x)(x^2+2)}dx+\frac{1}{2}(1-x)^2$$

$$\Big\downarrow \text{27}$$

$$\int \frac{-2x^3+3x^2-4x+2}{(1-x)(x^2+2)}dx+\frac{1}{2}(1-x)^2$$

$$\Big\downarrow \text{2160}$$

$$\int \left(-\frac{4(x+1)}{3(x^2+2)}+\frac{1}{3(x-1)}+2\right)dx+\frac{1}{2}(1-x)^2$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{3}\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)-\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{2}(1-x)^2+2x+\frac{1}{3}\log(1-x)$$

input
```
Int[x^4/((-1 + x)*(2 + x^2)),x]
```

output
```
(1 - x)^2/2 + 2*x - (2*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 + Log[1 - x]/3 - (2*Lo
g[2 + x^2])/3
```

3.300.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 604
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(b*d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*
x^2)^p*ExpandToSum[b*d^m*(m + n + 2*p + 1)*x^m - b*(m + n + 2*p + 1)*(c + d
*x)^m - (c + d*x)^(m - 2)*(a*d^2*(m + n - 1) - b*c^2*(m + n + 2*p + 1) - 2*
b*c*d*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m,
 1] && NeQ[m + n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.300.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{x^2}{2}+x-\frac{2\ln\left(x^2+2\right)}{3}-\frac{2\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}+\frac{\ln(x-1)}{3}$	34
risch	$\frac{x^2}{2}+x-\frac{2\ln\left(x^2+2\right)}{3}-\frac{2\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{3}+\frac{\ln(x-1)}{3}$	34

input
```
int(x^4/(x-1)/(x^2+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+x-2/3*ln(x^2+2)-2/3*arctan(1/2*x*2^(1/2))*2^(1/2)+1/3*ln(x-1)
```

3.300.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^4}{(-1+x)(2+x^2)}\,dx = \frac{1}{2}\,x^2 - \frac{2}{3}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + x - \frac{2}{3}\,\log\left(x^2+2\right) + \frac{1}{3}\,\log\left(x-1\right)$$

input
```
integrate(x^4/(-1+x)/(x^2+2),x, algorithm="fricas")
```

output
```
1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*l
og(x - 1)
```

3.300.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{(-1+x)(2+x^2)}\,dx = \frac{x^2}{2} + x + \frac{\log\left(x-1\right)}{3} - \frac{2\log\left(x^2+2\right)}{3} - \frac{2\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3}$$

input
```
integrate(x**4/(-1+x)/(x**2+2),x)
```

output
```
x**2/2 + x + log(x - 1)/3 - 2*log(x**2 + 2)/3 - 2*sqrt(2)*atan(sqrt(2)*x/2
)/3
```

3.300.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^4}{(-1+x)(2+x^2)}\,dx = \frac{1}{2}\,x^2 - \frac{2}{3}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + x - \frac{2}{3}\,\log\left(x^2+2\right) + \frac{1}{3}\,\log\left(x-1\right)$$

input
```
integrate(x^4/(-1+x)/(x^2+2),x, algorithm="maxima")
```

output
```
1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*l
og(x - 1)
```

3.300.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{(-1+x)(2+x^2)}\,dx = \frac{1}{2}\,x^2 - \frac{2}{3}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + x - \frac{2}{3}\,\log\left(x^2+2\right) + \frac{1}{3}\,\log\left(|x-1|\right)$$

input
```
integrate(x^4/(-1+x)/(x^2+2),x, algorithm="giac")
```

output
```
1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*l
og(abs(x - 1))
```

3.300.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{x^4}{(-1+x)(2+x^2)}\,dx = x + \frac{\ln(x-1)}{3} + \ln\left(x - \sqrt{2}\,1\mathrm{i}\right)\left(-\frac{2}{3} + \frac{\sqrt{2}\,1\mathrm{i}}{3}\right) - \ln\left(x + \sqrt{2}\,1\mathrm{i}\right)\left(\frac{2}{3} + \frac{\sqrt{2}\,1\mathrm{i}}{3}\right) + \frac{x^2}{2}$$

input
```
int(x^4/((x^2 + 2)*(x - 1)),x)
```

output
```
x + log(x - 1)/3 + log(x - 2^(1/2)*1i)*((2^(1/2)*1i)/3 - 2/3) - log(x + 2^
(1/2)*1i)*((2^(1/2)*1i)/3 + 2/3) + x^2/2
```

3.301 $\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3} \, dx$

3.301.1 Optimal result

Integrand size = 24, antiderivative size = 16

$$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3} \, dx = -\frac{3}{1+x} + 2\log(1-x)$$

output
```
-3/(1+x)+2*ln(1-x)
```

3.301.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3} \, dx = -\frac{3}{1+x} + 2\log(-1+x)$$

input
```
Integrate[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3),x]
```

output
```
-3/(1 + x) + 2*Log[-1 + x]
```

3.301.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+7x-1}{x^3+x^2-x-1}\,dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(\frac{3}{(x+1)^2}+\frac{2}{x-1}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$2\log(1-x)-\frac{3}{x+1}$$

input
```
Int[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3),x]
```

output
```
-3/(1 + x) + 2*Log[1 - x]
```

3.301.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.301.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$2\ln\left(x-1\right)-\frac{3}{x+1}$	15
norman	$2\ln\left(x-1\right)-\frac{3}{x+1}$	15
risch	$2\ln\left(x-1\right)-\frac{3}{x+1}$	15
parallelrisch	$\frac{2\ln(x-1)x-3+2\ln(x-1)}{x+1}$	22

input
```
int((2*x^2+7*x-1)/(x^3+x^2-x-1),x,method=_RETURNVERBOSE)
```

output
```
2*ln(x-1)-3/(x+1)
```

3.301.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx=\frac{2\left(x+1\right)\log\left(x-1\right)-3}{x+1}$$

input
```
integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="fricas")
```

output
```
(2*(x + 1)*log(x - 1) - 3)/(x + 1)
```

3.301.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx=2\log\left(x-1\right)-\frac{3}{x+1}$$

input
```
integrate((2*x**2+7*x-1)/(x**3+x**2-x-1),x)
```

output
```
2*log(x - 1) - 3/(x + 1)
```

3.301.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx = -\frac{3}{x+1}+2\log(x-1)$$

input
```
integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="maxima")
```

output
```
-3/(x + 1) + 2*log(x - 1)
```

3.301.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx = -\frac{3}{x+1}+2\log(|x-1|)$$

input
```
integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="giac")
```

output
```
-3/(x + 1) + 2*log(abs(x - 1))
```

3.301.9 Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx = 2\ln(x-1)-\frac{3}{x+1}$$

input
```
int(-(7*x + 2*x^2 - 1)/(x - x^2 - x^3 + 1),x)
```

output
```
2*log(x - 1) - 3/(x + 1)
```

3.302 $\int \frac{1+2x}{-1+3x-3x^2+x^3} \, dx$

3.302.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1+2x}{-1+3x-3x^2+x^3} \, dx = -\frac{3}{2(1-x)^2} + \frac{2}{1-x}$$

output
```
-3/2/(1-x)^2+2/(1-x)
```

3.302.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{1+2x}{-1+3x-3x^2+x^3} \, dx = \frac{1-4x}{2(-1+x)^2}$$

input
```
Integrate[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3),x]
```

output
```
(1 - 4*x)/(2*(-1 + x)^2)
```

3.302.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2007, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+1}{x^3-3x^2+3x-1} \, dx$$
$$\Big\downarrow \ 2007$$
$$\int \frac{2x+1}{(x-1)^3}dx$$
$$\Big\downarrow \ 48$$
$$-\frac{(2x+1)^2}{6(1-x)^2}$$

input
```
Int[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3),x]
```

output
```
-1/6*(1 + 2*x)^2/(1 - x)^2
```

3.302.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 2007
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Ex
pon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; IntegerQ[p] && Pol
yQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

3.302.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

method	result	size
norman	$\frac{-2x+\frac{1}{2}}{(x-1)^2}$	12
default	$-\frac{2}{x-1}-\frac{3}{2(x-1)^2}$	16
risch	$\frac{-2x+\frac{1}{2}}{x^2-2x+1}$	17
gosper	$-\frac{-1+4x}{2(x^2-2x+1)}$	18
parallelrisch	$\frac{1-4x}{2x^2-4x+2}$	18

input
```
int((1+2*x)/(x^3-3*x^2+3*x-1),x,method=_RETURNVERBOSE)
```

output
```
(-2*x+1/2)/(x-1)^2
```

3.302.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx = -\frac{4\,x-1}{2\,(x^2-2\,x+1)}$$

input
```
integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="fricas")
```

output
```
-1/2*(4*x - 1)/(x^2 - 2*x + 1)
```

3.302.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx = \frac{1-4x}{2x^2-4x+2}$$

input
```
integrate((1+2*x)/(x**3-3*x**2+3*x-1),x)
```

output
```
(1 - 4*x)/(2*x**2 - 4*x + 2)
```

3.302.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx = -\frac{4\,x-1}{2\,(x^2-2\,x+1)}$$

input
```
integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="maxima")
```

output
```
-1/2*(4*x - 1)/(x^2 - 2*x + 1)
```

3.302.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

$$\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx = -\frac{4\,x-1}{2\,(x-1)^2}$$

input
```
integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="giac")
```

output
```
-1/2*(4*x - 1)/(x - 1)^2
```

3.302.9 Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

$$\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx = -\frac{4\,x-1}{2\,(x-1)^2}$$

input
```
int((2*x + 1)/(3*x - 3*x^2 + x^3 - 1),x)
```

output
```
-(4*x - 1)/(2*(x - 1)^2)
```

3.303 $\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx$

3.303.1 Optimal result

Integrand size = 24, antiderivative size = 15

$$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx = \frac{1}{1-x} - \frac{2}{(1+x)^2}$$

output
```
1/(1-x)-2/(1+x)^2
```

3.303.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx = -\frac{1}{-1+x} - \frac{2}{(1+x)^2}$$

input
```
Integrate[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3),x]
```

output
```
-(-1 + x)^(-1) - 2/(1 + x)^2
```

3.303.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+7x^2-5x+5}{(x-1)^2(x+1)^3} \, dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{4}{(x+1)^3}+\frac{1}{(x-1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{1-x}-\frac{2}{(x+1)^2}$$

input
```
Int[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3),x]
```

output
```
(1 - x)^(-1) - 2/(1 + x)^2
```

3.303.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.303.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{1}{x-1}-\frac{2}{(x+1)^2}$	16
gosper	$-\frac{x^2+4x-1}{(x-1)(x+1)^2}$	21
norman	$\frac{-x^2-4x+1}{(x-1)(x+1)^2}$	22
risch	$\frac{-x^2-4x+1}{(x-1)(x+1)^2}$	22
parallelrisch	$\frac{-x^2-4x+1}{(x-1)(x+1)^2}$	22

input
```
int((x^3+7*x^2-5*x+5)/(x-1)^2/(x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/(x-1)-2/(x+1)^2
```

3.303.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx = -\frac{x^2+4\,x-1}{x^3+x^2-x-1}$$

input
```
integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="fricas")
```

output
```
-(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)
```

3.303.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx = \frac{-x^2-4x+1}{x^3+x^2-x-1}$$

input
```
integrate((x**3+7*x**2-5*x+5)/(-1+x)**2/(1+x)**3,x)
```

output
```
(-x**2 - 4*x + 1)/(x**3 + x**2 - x - 1)
```

3.303.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx = -\frac{x^2+4\,x-1}{x^3+x^2-x-1}$$

input
```
integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="maxima")
```

output
```
-(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)
```

3.303.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

$$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx = -\frac{1}{x-1} + \frac{\frac{4}{x-1}+1}{2\left(\frac{2}{x-1}+1\right)^2}$$

input
```
integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="giac")
```

output
```
-1/(x - 1) + 1/2*(4/(x - 1) + 1)/(2/(x - 1) + 1)^2
```

3.303.9 Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx = -\frac{1}{x-1} - \frac{2}{(x+1)^2}$$

input
```
int((7*x^2 - 5*x + x^3 + 5)/((x - 1)^2*(x + 1)^3),x)
```

output
```
- 1/(x - 1) - 2/(x + 1)^2
```

3.304 $\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx$

3.304.1 Optimal result

Integrand size = 26, antiderivative size = 31

$$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx=-\frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+x)+\log\left(1+x+x^2\right)$$

output
```
ln(1+x)+ln(x^2+x+1)-2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.304.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx=-\frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+x)+\log\left(1+x+x^2\right)$$

input
```
Integrate[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3),x]
```

output
```
(-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x + x^2]
```

3.304.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+3x+1}{x^3+2x^2+2x+1} \, dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(\frac{2x}{x^2+x+1}+\frac{1}{x+1}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^2+x+1\right)+\log(x+1)$$

input
```
Int[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3),x]
```

output
```
(-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x + x^2]
```

3.304.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.304.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
default	$\ln(x+1)+\ln(x^2+x+1)-\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	29
risch	$\ln(4x^2+4x+4)-\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\ln(x+1)$	33

input
```
int((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x+1)+ln(x^2+x+1)-2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.304.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\log\left(x^2+x+1\right)+\log(x+1)$$

input
```
integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="fricas")
```

output
```
-2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x + 1)
```

3.304.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.10

$$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx = \log(x+1)$$

input
```
integrate((3*x**2+3*x+1)/(x**3+2*x**2+2*x+1),x)
```

output
```
log(x + 1)
```

3.304.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \log\left(x^2+x+1\right) + \log\left(x+1\right)$$

input
```
integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="maxima")
```

output
```
-2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x + 1)
```

3.304.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \log\left(x^2+x+1\right) + \log\left(|x+1|\right)$$

input
```
integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="giac")
```

output
```
-2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(abs(x
+ 1))
```

3.304.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.84

$$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx = \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) + \ln\left(x+1\right) + \frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{3} - \frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{3}$$

input
```
int((3*x + 3*x^2 + 1)/(2*x + 2*x^2 + x^3 + 1),x)
```

output
```
log(x - (3^(1/2)*1i)/2 + 1/2) + log(x + (3^(1/2)*1i)/2 + 1/2) + log(x + 1)
 + (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*1i)/3 - (3^(1/2)*log(x + (3^(1/2
)*1i)/2 + 1/2)*1i)/3
```

3.305 $\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx$

3.305.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx=\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(2+x)$$

output
```
1/10*ln(1-2*x)+1/2*ln(x)-1/10*ln(2+x)
```

3.305.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx=\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(2+x)$$

input
```
Integrate[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
```

output
```
Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

3.305.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2026, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+2x-1}{2x^3+3x^2-2x}dx$$
$$\Big\downarrow \text{ 2026}$$
$$\int \frac{x^2+2x-1}{x(2x^2+3x-2)}dx$$
$$\Big\downarrow \text{ 2159}$$
$$\int \left(-\frac{1}{10(x+2)}+\frac{1}{5(2x-1)}+\frac{1}{2x}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

input
```
Int[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
```

output
```
Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

3.305.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.305.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
parallelrisch	$\frac{\ln(x)}{2}-\frac{\ln(x+2)}{10}+\frac{\ln\left(x-\frac{1}{2}\right)}{10}$	18
default	$\frac{\ln(x)}{2}-\frac{\ln(x+2)}{10}+\frac{\ln(2x-1)}{10}$	20
norman	$\frac{\ln(x)}{2}-\frac{\ln(x+2)}{10}+\frac{\ln(2x-1)}{10}$	20
risch	$\frac{\ln(x)}{2}-\frac{\ln(x+2)}{10}+\frac{\ln(2x-1)}{10}$	20

input
```
int((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x)-1/10*ln(x+2)+1/10*ln(x-1/2)
```

3.305.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx = \frac{1}{10}\,\log\left(2\,x-1\right)-\frac{1}{10}\,\log\left(x+2\right)+\frac{1}{2}\,\log\left(x\right)$$

input
```
integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="fricas")
```

output
```
1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

3.305.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx = \frac{\log\left(x\right)}{2}+\frac{\log\left(x-\frac{1}{2}\right)}{10}-\frac{\log\left(x+2\right)}{10}$$

input
```
integrate((x**2+2*x-1)/(2*x**3+3*x**2-2*x),x)
```

output
```
log(x)/2 + log(x - 1/2)/10 - log(x + 2)/10
```

3.305.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx = \frac{1}{10}\log(2x-1)-\frac{1}{10}\log(x+2)+\frac{1}{2}\log(x)$$

input
```
integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="maxima")
```

output
```
1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

3.305.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx = \frac{1}{10}\log(|2x-1|)-\frac{1}{10}\log(|x+2|)+\frac{1}{2}\log(|x|)$$

input
```
integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="giac")
```

output
```
1/10*log(abs(2*x - 1)) - 1/10*log(abs(x + 2)) + 1/2*log(abs(x))
```

3.305.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx = \frac{\operatorname{atanh}\left(\frac{24}{145\left(\frac{29x}{100}-\frac{11}{50}\right)}+\frac{35}{29}\right)}{5}+\frac{\ln(x)}{2}$$

input
```
int((2*x + x^2 - 1)/(3*x^2 - 2*x + 2*x^3),x)
```

output
```
atanh(24/(145*((29*x)/100 - 11/50)) + 35/29)/5 + log(x)/2
```

3.306 $\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} \, dx$

3.306.1 Optimal result

Integrand size = 29, antiderivative size = 30

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} \, dx = \frac{2}{1-x}+x+\frac{x^2}{2}+\log(1-x)-\log(1+x)$$

output
```
2/(1-x)+x+1/2*x^2+ln(1-x)-ln(1+x)
```

3.306.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} \, dx = -\frac{2}{-1+x}+\frac{1}{2}(1+x)^2+\log(1-x)-\log(1+x)$$

input
```
Integrate[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
```

output
```
-2/(-1 + x) + (1 + x)^2/2 + Log[1 - x] - Log[1 + x]
```

3.306.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1} \, dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(x+\frac{1}{-x-1}+\frac{1}{x-1}+\frac{2}{(x-1)^2}+1\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}+x+\frac{2}{1-x}+\log(1-x)-\log(x+1)$$

input
```
Int[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
```

output
```
2/(1 - x) + x + x^2/2 + Log[1 - x] - Log[1 + x]
```

3.306.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.306.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^2}{2}+x-\ln\left(x+1\right)+\ln\left(x-1\right)-\frac{2}{x-1}$	25
risch	$\frac{x^2}{2}+x-\ln\left(x+1\right)+\ln\left(x-1\right)-\frac{2}{x-1}$	25
norman	$\frac{\frac{1}{2}x^2+\frac{1}{2}x^3-3}{x-1}-\ln\left(x+1\right)+\ln\left(x-1\right)$	30
parallelrisch	$\frac{x^3+2\ln(x-1)x-2\ln(x+1)x+x^2-6-2\ln(x-1)+2\ln(x+1)}{2x-2}$	42

input
```
int((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+x-ln(x+1)+ln(x-1)-2/(x-1)
```

3.306.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx=\frac{x^3+x^2-2\left(x-1\right)\log\left(x+1\right)+2\left(x-1\right)\log\left(x-1\right)-2\,x-4}{2\left(x-1\right)}$$

input
```
integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="fricas")
```

output
```
1/2*(x^3 + x^2 - 2*(x - 1)*log(x + 1) + 2*(x - 1)*log(x - 1) - 2*x - 4)/(x
 - 1)
```

3.306.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx=\frac{x^2}{2}+x+\log\left(x-1\right)-\log\left(x+1\right)-\frac{2}{x-1}$$

input
```
integrate((x**4-2*x**2+4*x+1)/(x**3-x**2-x+1),x)
```

output
```
x**2/2 + x + log(x - 1) - log(x + 1) - 2/(x - 1)
```

3.306.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx = \frac{1}{2}\,x^2+x-\frac{2}{x-1}-\log\left(x+1\right)+\log\left(x-1\right)$$

input
```
integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="maxima")
```

output
```
1/2*x^2 + x - 2/(x - 1) - log(x + 1) + log(x - 1)
```

3.306.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx = \frac{1}{2}\,x^2+x-\frac{2}{x-1}-\log\left(|x+1|\right)+\log\left(|x-1|\right)$$

input
```
integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="giac")
```

output
```
1/2*x^2 + x - 2/(x - 1) - log(abs(x + 1)) + log(abs(x - 1))
```

3.306.9 Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx = x-\frac{2}{x-1}+\frac{x^2}{2}+\mathrm{atan}(x\,1\mathrm{i})\;2\mathrm{i}$$

input
```
int(-(4*x - 2*x^2 + x^4 + 1)/(x + x^2 - x^3 - 1),x)
```

output
```
x + atan(x*1i)*2i - 2/(x - 1) + x^2/2
```

3.307 $\int \frac{4-x+2x^2}{4x+x^3} \, dx$

3.307.1 Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \frac{4-x+2x^2}{4x+x^3} \, dx = -\frac{1}{2} \arctan\left(\frac{x}{2}\right) + \log(x) + \frac{1}{2} \log\left(4+x^2\right)$$

output
```
-1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)
```

3.307.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{4-x+2x^2}{4x+x^3} \, dx = -\frac{1}{2} \arctan\left(\frac{x}{2}\right) + \log(x) + \frac{1}{2} \log\left(4+x^2\right)$$

input
```
Integrate[(4 - x + 2*x^2)/(4*x + x^3),x]
```

output
```
-1/2*ArcTan[x/2] + Log[x] + Log[4 + x^2]/2
```

3.307.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2026, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2-x+4}{x^3+4x}dx$$
$$\Big\downarrow \text{2026}$$
$$\int \frac{2x^2-x+4}{x\left(x^2+4\right)}dx$$
$$\Big\downarrow \text{2333}$$
$$\int \left(\frac{x-1}{x^2+4}+\frac{1}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{2}\arctan\left(\frac{x}{2}\right)+\frac{1}{2}\log\left(x^2+4\right)+\log(x)$$

input
```
Int[(4 - x + 2*x^2)/(4*x + x^3),x]
```

output
```
-1/2*ArcTan[x/2] + Log[x] + Log[4 + x^2]/2
```

3.307.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.307.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln\left(x\right)+\frac{\ln\left(x^2+4\right)}{2}$	18
risch	$-\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln\left(x\right)+\frac{\ln\left(x^2+4\right)}{2}$	18
meijerg	$\frac{\ln\left(1+\frac{x^2}{4}\right)}{2}+\ln\left(x\right)-\ln\left(2\right)-\frac{\arctan\left(\frac{x}{2}\right)}{2}$	24
parallelrisch	$\ln\left(x\right)+\frac{\ln(x-2i)}{2}+\frac{i\ln(x-2i)}{4}+\frac{\ln(x+2i)}{2}-\frac{i\ln(x+2i)}{4}$	34

input
```
int((2*x^2-x+4)/(x^3+4*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)
```

3.307.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx=-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(x\right)$$

input
```
integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="fricas")
```

output
```
-1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

3.307.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx=\log\left(x\right)+\frac{\log\left(x^2+4\right)}{2}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

input
```
integrate((2*x**2-x+4)/(x**3+4*x),x)
```

output
```
log(x) + log(x**2 + 4)/2 - atan(x/2)/2
```

3.307.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx = -\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(x\right)$$

input
```
integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="maxima")
```

output
```
-1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

3.307.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx = -\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(|x|\right)$$

input
```
integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="giac")
```

output
```
-1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(abs(x))
```

3.307.9 Mupad [B] (verification not implemented)

Time = 9.81 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx = \ln\left(x\right)+\ln\left(x-2\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{1}{4}\mathrm{i}\right)+\ln\left(x+2\mathrm{i}\right)\,\left(\frac{1}{2}-\frac{1}{4}\mathrm{i}\right)$$

input
```
int((2*x^2 - x + 4)/(4*x + x^3),x)
```

output
```
log(x - 2i)*(1/2 + 1i/4) + log(x + 2i)*(1/2 - 1i/4) + log(x)
```

3.308 $\int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)}\,dx$

3.308.1 Optimal result

Integrand size = 32, antiderivative size = 103

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx=\frac{1+x}{8\left(1+x^2\right)^2}-\frac{3(1-x)}{8\left(1+x^2\right)}+\frac{3x}{16\left(1+x^2\right)}+\frac{7\arctan(x)}{16}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{8}\log(1-x)-\log(x)+\frac{15}{16}\log\left(1+x^2\right)-\frac{1}{2}\log\left(1+x+x^2\right)$$

output
```
1/8*(1+x)/(x^2+1)^2-3/8*(1-x)/(x^2+1)+3/16*x/(x^2+1)+7/16*arctan(x)+1/8*ln
(1-x)-ln(x)+15/16*ln(x^2+1)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2)
)*3^(1/2)
```

3.308.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx=\frac{1}{48}\left(\frac{6(1+x)}{\left(1+x^2\right)^2}+\frac{9(-2+3x)}{1+x^2}+21\arctan(x)-16\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+20\log(1-x)-48\log(x)+45\log\left(1+x^2\right)-10\log\left(1+x+x^2\right)-14\log\left(1-x^3\right)\right)$$

input
```
Integrate[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

output
```
((6*(1 + x))/(1 + x^2)^2 + (9*(-2 + 3*x))/(1 + x^2) + 21*ArcTan[x] - 16*Sq
rt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 20*Log[1 - x] - 48*Log[x] + 45*Log[1 + x
^2] - 10*Log[1 + x + x^2] - 14*Log[1 - x^3])/48
```

3.308.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+1}{(x-1)x\left(x^2+1\right)^3\left(x^2+x+1\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-x-1}{x^2+x+1}+\frac{15x-1}{8\left(x^2+1\right)}+\frac{3(x+1)}{4\left(x^2+1\right)^2}+\frac{1-x}{2\left(x^2+1\right)^3}+\frac{1}{8(x-1)}-\frac{1}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{7\arctan(x)}{16}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{3(1-x)}{8\left(x^2+1\right)}+\frac{3x}{16\left(x^2+1\right)}+\frac{x+1}{8\left(x^2+1\right)^2}+\frac{15}{16}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{1}{8}\log(1-x)-\log(x)$$

input
```
Int[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

output
```
(1 + x)/(8*(1 + x^2)^2) - (3*(1 - x))/(8*(1 + x^2)) + (3*x)/(16*(1 + x^2))
 + (7*ArcTan[x])/16 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/8 - L
og[x] + (15*Log[1 + x^2])/16 - Log[1 + x + x^2]/2
```

3.308.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.308.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.70

method	result
risch	$\frac{\frac{9}{16}x^{3}-\frac{3}{8}x^{2}+\frac{11}{16}x-\frac{1}{4}}{\left(x^{2}+1\right)^{2}}+\frac{\ln\left(x-1\right)}{8}-\ln\left(x\right)+\frac{15\ln\left(49x^{2}+49\right)}{16}+\frac{7\arctan\left(x\right)}{16}-\frac{\ln\left(x^{2}+x+1\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$
default	$-\ln\left(x\right)+\frac{\frac{9}{2}x^{3}-3x^{2}+\frac{11}{2}x-2}{8\left(x^{2}+1\right)^{2}}+\frac{15\ln\left(x^{2}+1\right)}{16}+\frac{7\arctan\left(x\right)}{16}-\frac{\ln\left(x^{2}+x+1\right)}{2}-\frac{\arctan\left(\frac{\left(1+2x\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln\left(x-1\right)}{8}$

input
```
int((x^3+x^2+1)/(x-1)/x/(x^2+1)^3/(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
(9/16*x^3-3/8*x^2+11/16*x-1/4)/(x^2+1)^2+1/8*ln(x-1)-ln(x)+15/16*ln(49*x^2
+49)+7/16*arctan(x)-1/2*ln(x^2+x+1)-1/3*3^(1/2)*arctan(2/3*(x+1/2)*3^(1/2)
)
```

3.308.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.32

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx$$
$$= \frac{27\,x^{3}-16\,\sqrt{3}(x^{4}+2\,x^{2}+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-18\,x^{2}+21\,\left(x^{4}+2\,x^{2}+1\right)\arctan\left(x\right)-24\,\left(x^{4}+2\right.}{}$$

input
```
integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="fricas")
```

output
```
1/48*(27*x^3 - 16*sqrt(3)*(x^4 + 2*x^2 + 1)*arctan(1/3*sqrt(3)*(2*x + 1))
- 18*x^2 + 21*(x^4 + 2*x^2 + 1)*arctan(x) - 24*(x^4 + 2*x^2 + 1)*log(x^2 +
 x + 1) + 45*(x^4 + 2*x^2 + 1)*log(x^2 + 1) + 6*(x^4 + 2*x^2 + 1)*log(x -
1) - 48*(x^4 + 2*x^2 + 1)*log(x) + 33*x - 12)/(x^4 + 2*x^2 + 1)
```

3.308.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = -\log\left(x\right)+\frac{\log\left(x-1\right)}{8}+\frac{15\log\left(x^2+1\right)}{16} - \frac{\log\left(x^2+x+1\right)}{2}+\frac{7\operatorname{atan}\left(x\right)}{16} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{9x^3-6x^2+11x-4}{16x^4+32x^2+16}$$

input
```
integrate((x**3+x**2+1)/(-1+x)/x/(x**2+1)**3/(x**2+x+1),x)
```

output
```
-log(x) + log(x - 1)/8 + 15*log(x**2 + 1)/16 - log(x**2 + x + 1)/2 + 7*ata
n(x)/16 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3 + (9*x**3 - 6*x**2 + 1
1*x - 4)/(16*x**4 + 32*x**2 + 16)
```

3.308.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{9x^3-6x^2+11x-4}{16\left(x^4+2x^2+1\right)}+\frac{7}{16}\arctan\left(x\right) - \frac{1}{2}\log\left(x^2+x+1\right)+\frac{15}{16}\log\left(x^2+1\right) + \frac{1}{8}\log\left(x-1\right)-\log\left(x\right)$$

input
```
integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x -
4)/(x^4 + 2*x^2 + 1) + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x
^2 + 1) + 1/8*log(x - 1) - log(x)
```

3.308.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.72

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3(1+x+x^2)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{9x^3-6x^2+11x-4}{16\left(x^2+1\right)^2} + \frac{7}{16}\arctan(x) - \frac{1}{2}\log\left(x^2+x+1\right) + \frac{15}{16}\log\left(x^2+1\right) + \frac{1}{8}\log(|x-1|) - \log(|x|)$$

input
```
integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x -
4)/(x^2 + 1)^2 + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2 + 1
) + 1/8*log(abs(x - 1)) - log(abs(x))
```

3.308.9 Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3(1+x+x^2)}\,dx = \frac{\ln(x-1)}{8} - \ln(x) + \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \frac{\frac{9x^3}{16}-\frac{3x^2}{8}+\frac{11x}{16}-\frac{1}{4}}{x^4+2x^2+1} + \ln(x-\mathrm{i})\left(\frac{15}{16}-\frac{7}{32}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(\frac{15}{16}+\frac{7}{32}\mathrm{i}\right)$$

input
```
int((x^2 + x^3 + 1)/(x*(x^2 + 1)^3*(x - 1)*(x + x^2 + 1)),x)
```

output
```
log(x - 1)/8 + log(x - 1i)*(15/16 - 7i/32) + log(x + 1i)*(15/16 + 7i/32) -
 log(x) + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/2) - log(x + (
3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/2) + ((11*x)/16 - (3*x^2)/8 + (9*
x^3)/16 - 1/4)/(2*x^2 + x^4 + 1)
```

3.308. $\int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)}\,dx$

3.309 $\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2}\,dx$

3.309.1 Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2}\,dx = \frac{2-x}{2(1+x^2)} + \frac{3\arctan(x)}{2} - \frac{1}{2}\log\left(1+x^2\right)$$

output
```
1/2*(2-x)/(x^2+1)+3/2*arctan(x)-1/2*ln(x^2+1)
```

3.309.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2}\,dx = \frac{1}{2}\left(\frac{2-x}{1+x^2} + 3\arctan(x) - \log\left(1+x^2\right)\right)$$

input
```
Integrate[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2,x]
```

output
```
((2 - x)/(1 + x^2) + 3*ArcTan[x] - Log[1 + x^2])/2
```

3.309.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2345, 25, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^3+2x^2-3x+1}{(x^2+1)^2} \, dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{2-x}{2(x^2+1)}-\frac{1}{2}\int -\frac{3-2x}{x^2+1}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{3-2x}{x^2+1}dx+\frac{2-x}{2(x^2+1)}$$

$$\Big\downarrow \text{452}$$

$$\frac{1}{2}\left(3\int \frac{1}{x^2+1}dx-2\int \frac{x}{x^2+1}dx\right)+\frac{2-x}{2(x^2+1)}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(3\arctan(x)-2\int \frac{x}{x^2+1}dx\right)+\frac{2-x}{2(x^2+1)}$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{2}\left(3\arctan(x)-\log\left(x^2+1\right)\right)+\frac{2-x}{2(x^2+1)}$$

input
```
Int[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2,x]
```

output
```
(2 - x)/(2*(1 + x^2)) + (3*ArcTan[x] - Log[1 + x^2])/2
```

3.309.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.309.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{1-\frac{x}{2}}{x^2+1}-\frac{\ln(x^2+1)}{2}+\frac{3\arctan(x)}{2}$	27
default	$-\frac{\frac{x}{2}-1}{x^2+1}-\frac{\ln(x^2+1)}{2}+\frac{3\arctan(x)}{2}$	28
meijerg	$-\frac{x^2}{x^2+1}-\frac{\ln(x^2+1)}{2}-\frac{x}{x^2+1}+\frac{3\arctan(x)}{2}+\frac{x}{2x^2+2}$	47
parallelrisch	$-\frac{3i\ln(x-i)x^2-3i\ln(x+i)x^2+2\ln(x-i)x^2+2\ln(x+i)x^2-4+3i\ln(x-i)-3i\ln(x+i)+2\ln(x-i)+2\ln(x+i)+2x}{4(x^2+1)}$	87

input
```
int((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
(1-1/2*x)/(x^2+1)-1/2*ln(x^2+1)+3/2*arctan(x)
```

3.309.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2}\,dx = \frac{3\,(x^2+1)\arctan{(x)}-(x^2+1)\log{(x^2+1)}-x+2}{2\,(x^2+1)}$$

input
```
integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/2*(3*(x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) - x + 2)/(x^2 + 1)
```

3.309.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2}\,dx = -\frac{x-2}{2x^2+2}-\frac{\log{(x^2+1)}}{2}+\frac{3\operatorname{atan}{(x)}}{2}$$

input
```
integrate((-x**3+2*x**2-3*x+1)/(x**2+1)**2,x)
```

output
```
-(x - 2)/(2*x**2 + 2) - log(x**2 + 1)/2 + 3*atan(x)/2
```

3.309.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2}\,dx = -\frac{x-2}{2\,(x^2+1)}+\frac{3}{2}\arctan{(x)}-\frac{1}{2}\log{\left(x^2+1\right)}$$

input
```
integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/2*(x - 2)/(x^2 + 1) + 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

3.309.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2}\,dx = -\frac{x-2}{2\,(x^2+1)} + \frac{3}{2}\arctan(x) - \frac{1}{2}\log(x^2+1)$$

input
```
integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/2*(x - 2)/(x^2 + 1) + 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

3.309.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2}\,dx = \frac{3\,\mathrm{atan}(x)}{2} - \frac{\ln(x^2+1)}{2} - \frac{x}{2\,(x^2+1)} + \frac{1}{x^2+1}$$

input
```
int(-(3*x - 2*x^2 + x^3 - 1)/(x^2 + 1)^2,x)
```

output
```
(3*atan(x))/2 - log(x^2 + 1)/2 - x/(2*(x^2 + 1)) + 1/(x^2 + 1)
```

3.310 $\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2}\,dx$

3.310.1 Optimal result

Integrand size = 26, antiderivative size = 33

$$\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2}\,dx = -\frac{1+2x}{2(1+x^2)} - 2\arctan(x) + \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

output
```
1/2*(-1-2*x)/(x^2+1)-2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

3.310.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2}\,dx = \frac{-1-2x}{2(1+x^2)} - 2\arctan(x) + \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
```

output
```
(-1 - 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

3.310.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2336, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^3+2x^2-3x+1}{x(x^2+1)^2}\,dx$$

$$\Big\downarrow \text{2336}$$

$$-\frac{1}{2}\int -\frac{2(1-2x)}{x(x^2+1)}dx-\frac{2x+1}{2(x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1-2x}{x(x^2+1)}dx-\frac{2x+1}{2(x^2+1)}$$

$$\Big\downarrow \text{523}$$

$$\int \left(\frac{-x-2}{x^2+1}+\frac{1}{x}\right)dx-\frac{2x+1}{2(x^2+1)}$$

$$\Big\downarrow \text{2009}$$

$$-2\arctan(x)-\frac{2x+1}{2(x^2+1)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

input
```
Int[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
```

output
```
-1/2*(1 + 2*x)/(1 + x^2) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

3.310.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.310.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	s
default	$\ln\left(x\right)-\frac{x+\frac{1}{2}}{x^{2}+1}-\frac{\ln\left(x^{2}+1\right)}{2}-2\arctan\left(x\right)$	2
risch	$\frac{-x-\frac{1}{2}}{x^{2}+1}+\ln\left(x\right)-\frac{\ln\left(4x^{2}+4\right)}{2}-2\arctan\left(x\right)$	3
meijerg	$-\frac{2x}{2x^{2}+2}-2\arctan\left(x\right)+\frac{x^{2}}{x^{2}+1}-\frac{x^{2}}{2x^{2}+2}-\frac{\ln\left(x^{2}+1\right)}{2}+\frac{1}{2}+\ln\left(x\right)$	6
parallelrisch	$\frac{2i\ln\left(x-i\right)x^{2}-2i\ln\left(x+i\right)x^{2}+2\ln\left(x\right)x^{2}-\ln\left(x-i\right)x^{2}-\ln\left(x+i\right)x^{2}-1+2i\ln\left(x-i\right)-2i\ln\left(x+i\right)+2\ln\left(x\right)-\ln\left(x-i\right)-\ln\left(x+i\right)-2x}{2x^{2}+2}$	9

input
```
int((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
ln(x)-(x+1/2)/(x^2+1)-1/2*ln(x^2+1)-2*arctan(x)
```

3.310.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx = -\frac{4\left(x^{2}+1\right)\arctan\left(x\right)+\left(x^{2}+1\right)\log\left(x^{2}+1\right)-2\left(x^{2}+1\right)\log\left(x\right)+2\,x+1}{2\left(x^{2}+1\right)}$$

input
```
integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/2*(4*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1)*log(x)
+ 2*x + 1)/(x^2 + 1)
```

3.310.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx = -\frac{2x+1}{2x^2+2} + \log\left(x\right) - \frac{\log\left(x^2+1\right)}{2} - 2\,\mathrm{atan}\left(x\right)$$

input
```
integrate((-x**3+2*x**2-3*x+1)/x/(x**2+1)**2,x)
```

output
```
-(2*x + 1)/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 - 2*atan(x)
```

3.310.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx = -\frac{2\,x+1}{2\left(x^2+1\right)} - 2\,\arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right) + \log\left(x\right)$$

input
```
integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

3.310.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx = -\frac{2\,x+1}{2\left(x^2+1\right)} - 2\,\arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right) + \log\left(|x|\right)$$

input
```
integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

3.310.9 Mupad [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx = \ln(x) - \frac{x+\frac{1}{2}}{x^2+1} + \ln(x-\mathrm{i})\left(-\frac{1}{2}+1\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{2}-\mathrm{i}\right)$$

input
```
int(-(3*x - 2*x^2 + x^3 - 1)/(x*(x^2 + 1)^2),x)
```

output
```
log(x) - log(x + 1i)*(1/2 + 1i) - log(x - 1i)*(1/2 - 1i) - (x + 1/2)/(x^2
+ 1)
```

3.311 $\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} dx$

3.311.1 Optimal result

Integrand size = 26, antiderivative size = 25

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} dx = x+\frac{x^2}{2}-\log(x)+\frac{1}{2}\log\left(1-x^2\right)$$

output
```
x+1/2*x^2-ln(x)+1/2*ln(-x^2+1)
```

3.311.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} dx = x+\frac{x^2}{2}-\log(x)+\frac{1}{2}\log\left(1-x^2\right)$$

input
```
Integrate[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
```

output
```
x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

3.311.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2026, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+x^3-x^2-x+1}{x^3-x}dx$$

$$\Big\downarrow \ 2026$$

$$\int \frac{x^4+x^3-x^2-x+1}{x\left(x^2-1\right)}dx$$

$$\Big\downarrow \ 2333$$

$$\int \left(\frac{x}{x^2-1}+x-\frac{1}{x}+1\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

input
```
Int[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
```

output
```
x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

3.311.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.311.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{x^2}{2}+x-\ln\left(x\right)+\frac{\ln\left(x^2-1\right)}{2}$	20
default	$\frac{x^2}{2}+x-\ln\left(x\right)+\frac{\ln(x+1)}{2}+\frac{\ln(x-1)}{2}$	24
norman	$\frac{x^2}{2}+x-\ln\left(x\right)+\frac{\ln(x+1)}{2}+\frac{\ln(x-1)}{2}$	24
parallelrisch	$\frac{x^2}{2}+x-\ln\left(x\right)+\frac{\ln(x+1)}{2}+\frac{\ln(x-1)}{2}$	24
meijerg	$\frac{\ln(-x^2+1)}{2}-\ln\left(x\right)-\frac{i\pi}{2}+\frac{x^2}{2}-\frac{i(2ix-2i\,\mathrm{arctanh}(x))}{2}+\mathrm{arctanh}\left(x\right)$	40

input
```
int((x^4+x^3-x^2-x+1)/(x^3-x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+x-ln(x)+1/2*ln(x^2-1)
```

3.311.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx=\frac{1}{2}\,x^2+x+\frac{1}{2}\,\log\left(x^2-1\right)-\log\left(x\right)$$

input
```
integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="fricas")
```

output
```
1/2*x^2 + x + 1/2*log(x^2 - 1) - log(x)
```

3.311.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx=\frac{x^2}{2}+x-\log\left(x\right)+\frac{\log\left(x^2-1\right)}{2}$$

input
```
integrate((x**4+x**3-x**2-x+1)/(x**3-x),x)
```

output
```
x**2/2 + x - log(x) + log(x**2 - 1)/2
```

3.311.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx = \frac{1}{2}\,x^2+x+\frac{1}{2}\,\log\left(x+1\right)+\frac{1}{2}\,\log\left(x-1\right)-\log\left(x\right)$$

input
```
integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="maxima")
```

output
```
1/2*x^2 + x + 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

3.311.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx = \frac{1}{2}\,x^2+x+\frac{1}{2}\,\log\left(|x+1|\right)+\frac{1}{2}\,\log\left(|x-1|\right)-\log\left(|x|\right)$$

input
```
integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="giac")
```

output
```
1/2*x^2 + x + 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1)) - log(abs(x))
```

3.311.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx = x+\frac{\ln\left(x^2-1\right)}{2}-\ln\left(x\right)+\frac{x^2}{2}$$

input
```
int(-(x^3 - x^2 - x + x^4 + 1)/(x - x^3),x)
```

output
```
x + log(x^2 - 1)/2 - log(x) + x^2/2
```

3.312 $\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx$

3.312.1 Optimal result

Integrand size = 25, antiderivative size = 36

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = 6\arctan(x) - 5\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{2}\log\left(1+x^2\right) + \log\left(2+x^2\right)$$

output
```
6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.312.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = 6\arctan(x) - 5\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{2}\log\left(1+x^2\right) + \log\left(2+x^2\right)$$

input
```
Integrate[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

output
```
6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

3.312.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-4x^2+2}{(x^2+1)(x^2+2)}\,dx$$

$$\Big\downarrow \text{ 7276}$$

$$\int \left(\frac{6-x}{x^2+1}+\frac{2(x-5)}{x^2+2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$6\arctan(x)-5\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)$$

input
```
Int[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

output
```
6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

3.312.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.312.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$6\arctan\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}+\ln\left(x^{2}+2\right)-5\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	32
risch	$6\arctan\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}+\ln\left(x^{2}+2\right)-5\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	32

input
```
int((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x,method=_RETURNVERBOSE)
```

output
```
6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.312.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx=-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan\left(x\right)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="fricas")
```

output
```
-5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^
2 + 1)
```

3.312.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx=-\frac{\log\left(x^2+1\right)}{2}+\log\left(x^2+2\right)+6\,\mathrm{atan}\left(x\right)-5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

input
```
integrate((x**3-4*x**2+2)/(x**2+1)/(x**2+2),x)
```

output
```
-log(x**2 + 1)/2 + log(x**2 + 2) + 6*atan(x) - 5*sqrt(2)*atan(sqrt(2)*x/2)
```

3.312.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = -5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan(x)+\log(x^2+2)-\frac{1}{2}\log(x^2+1)$$

input
```
integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

output
```
-5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^
2 + 1)
```

3.312.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = -5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan(x)+\log(x^2+2)-\frac{1}{2}\log(x^2+1)$$

input
```
integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

output
```
-5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^
2 + 1)
```

3.312.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

$$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx = \ln(x-\mathrm{i})\left(-\frac{1}{2}-3\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(-\frac{1}{2}+3\mathrm{i}\right) + \ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(1+\frac{\sqrt{2}\,5\mathrm{i}}{2}\right)-\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(-1+\frac{\sqrt{2}\,5\mathrm{i}}{2}\right)$$

input
```
int((x^3 - 4*x^2 + 2)/((x^2 + 1)*(x^2 + 2)),x)
```

output
```
log(x - 2^(1/2)*1i)*((2^(1/2)*5i)/2 + 1) - log(x + 1i)*(1/2 - 3i) - log(x
- 1i)*(1/2 + 3i) - log(x + 2^(1/2)*1i)*((2^(1/2)*5i)/2 - 1)
```

3.313 $\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx$

3.313.1 Optimal result

Integrand size = 23, antiderivative size = 29

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx=-\frac{13x}{24(4+x^2)}+\frac{25}{144}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{9}$$

output
```
-13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)
```

3.313.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx=-\frac{13x}{24(4+x^2)}+\frac{25}{144}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{9}$$

input
```
Integrate[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
```

output
```
(-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

3.313.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+x^2+1}{(x^2+1)(x^2+4)^2}dx$$

$$\Big\downarrow \text{ 7276}$$

$$\int \left(\frac{8}{9(x^2+4)}-\frac{13}{3(x^2+4)^2}+\frac{1}{9(x^2+1)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{25}{144}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{9}-\frac{13x}{24(x^2+4)}$$

input
```
Int[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
```

output
```
(-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

3.313.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.313.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result
default	$-\frac{13x}{24(x^2+4)}+\frac{25\arctan\left(\frac{x}{2}\right)}{144}+\frac{\arctan(x)}{9}$
risch	$-\frac{13x}{24(x^2+4)}+\frac{25\arctan\left(\frac{x}{2}\right)}{144}+\frac{\arctan(x)}{9}$
parallelrisch	$-\frac{25i\ln(x-2i)x^2+16i\ln(x-i)x^2-16i\ln(x+i)x^2-25i\ln(x+2i)x^2+100i\ln(x-2i)+64i\ln(x-i)-64i\ln(x+i)-100i\ln(x+2i)+1}{288(x^2+4)}$

input
```
int((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x,method=_RETURNVERBOSE)
```

output
```
-13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)
```

3.313.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx=\frac{25\,(x^2+4)\arctan\left(\frac{1}{2}\,x\right)+16\,(x^2+4)\arctan(x)-78\,x}{144\,(x^2+4)}$$

input
```
integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="fricas")
```

output
```
1/144*(25*(x^2 + 4)*arctan(1/2*x) + 16*(x^2 + 4)*arctan(x) - 78*x)/(x^2 +
4)
```

3.313.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx=-\frac{13x}{24x^2+96}+\frac{25\,\mathrm{atan}\left(\frac{x}{2}\right)}{144}+\frac{\mathrm{atan}\,(x)}{9}$$

input
```
integrate((x**4+x**2+1)/(x**2+1)/(x**2+4)**2,x)
```

output
```
-13*x/(24*x**2 + 96) + 25*atan(x/2)/144 + atan(x)/9
```

3.313.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx = -\frac{13\,x}{24\,(x^2+4)} + \frac{25}{144}\arctan\left(\frac{1}{2}\,x\right) + \frac{1}{9}\arctan(x)$$

input
```
integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="maxima")
```

output
```
-13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

3.313.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx = -\frac{13\,x}{24\,(x^2+4)} + \frac{25}{144}\arctan\left(\frac{1}{2}\,x\right) + \frac{1}{9}\arctan(x)$$

input
```
integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="giac")
```

output
```
-13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

3.313.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx = \frac{25\,\mathrm{atan}\left(\frac{x}{2}\right)}{144} + \frac{\mathrm{atan}(x)}{9} - \frac{13\,x}{24\,(x^2+4)}$$

input
```
int((x^2 + x^4 + 1)/((x^2 + 1)*(x^2 + 4)^2),x)
```

output
```
(25*atan(x/2))/144 + atan(x)/9 - (13*x)/(24*(x^2 + 4))
```

3.314 $\int \frac{1+x^2+x^3}{2x^2+x^3+x^4} dx$

3.314.1 Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4} dx = -\frac{1}{2x} + \frac{\arctan\left(\frac{1+2x}{\sqrt{7}}\right)}{4\sqrt{7}} - \frac{\log(x)}{4} + \frac{5}{8}\log\left(2+x+x^2\right)$$

output
```
-1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*arctan(1/7*(1+2*x)*7^(1/2))*7^(1/2)
```

3.314.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4} dx = -\frac{1}{2x} + \frac{\arctan\left(\frac{1+2x}{\sqrt{7}}\right)}{4\sqrt{7}} - \frac{\log(x)}{4} + \frac{5}{8}\log\left(2+x+x^2\right)$$

input
```
Integrate[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

output
```
-1/2*1/x + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
 + x^2])/8
```

3.314.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2026, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+1}{x^4+x^3+2x^2}dx$$
$$\Big\downarrow \text{ 2026}$$
$$\int \frac{x^3+x^2+1}{x^2\left(x^2+x+2\right)}dx$$
$$\Big\downarrow \text{ 2159}$$
$$\int \left(\frac{5x+3}{4\left(x^2+x+2\right)}+\frac{1}{2x^2}-\frac{1}{4x}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{\arctan\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}+\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}$$

input
```
Int[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

output
```
-1/2*1/x + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
 + x^2])/8
```

3.314.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.314.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{1}{2x}-\frac{\ln(x)}{4}+\frac{5\ln(x^2+x+2)}{8}+\frac{\arctan\left(\frac{(1+2x)\sqrt{7}}{7}\right)\sqrt{7}}{28}$	36
risch	$-\frac{1}{2x}+\frac{5\ln(4x^2+4x+8)}{8}+\frac{\arctan\left(\frac{(1+2x)\sqrt{7}}{7}\right)\sqrt{7}}{28}-\frac{\ln(x)}{4}$	40

input
```
int((x^3+x^2+1)/(x^4+x^3+2*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*arctan(1/7*(1+2*x)*7^(1/2))*7^(1/2)
```

3.314.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx = \frac{2\sqrt{7}x\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)+35x\log\left(x^2+x+2\right)-14x\log(x)-28}{56x}$$

input
```
integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="fricas")
```

output
```
1/56*(2*sqrt(7)*x*arctan(1/7*sqrt(7)*(2*x + 1)) + 35*x*log(x^2 + x + 2) -
14*x*log(x) - 28)/x
```

3.314.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx = -\frac{\log(x)}{4}+\frac{5\log(x^2+x+2)}{8}+\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x}{7}+\frac{\sqrt{7}}{7}\right)}{28}-\frac{1}{2x}$$

input
```
integrate((x**3+x**2+1)/(x**4+x**3+2*x**2),x)
```

output
```
-log(x)/4 + 5*log(x**2 + x + 2)/8 + sqrt(7)*atan(2*sqrt(7)*x/7 + sqrt(7)/7
)/28 - 1/(2*x)
```

3.314.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx = \frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)-\frac{1}{2x}+\frac{5}{8}\log(x^2+x+2)-\frac{1}{4}\log(x)$$

input
```
integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="maxima")
```

output
```
1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2)
- 1/4*log(x)
```

3.314.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx = \frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)-\frac{1}{2x}+\frac{5}{8}\log(x^2+x+2)-\frac{1}{4}\log(|x|)$$

input
```
integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="giac")
```

output
```
1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2)
- 1/4*log(abs(x))
```

3.314.9 Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx = -\frac{\ln(x)}{4} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)\left(-\frac{5}{8}+\frac{\sqrt{7}\,1\mathrm{i}}{56}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)\left(\frac{5}{8}+\frac{\sqrt{7}\,1\mathrm{i}}{56}\right) - \frac{1}{2\,x}$$

input
```
int((x^2 + x^3 + 1)/(2*x^2 + x^3 + x^4),x)
```

output
```
log(x + (7^(1/2)*1i)/2 + 1/2)*((7^(1/2)*1i)/56 + 5/8) - log(x - (7^(1/2)*1
i)/2 + 1/2)*((7^(1/2)*1i)/56 - 5/8) - log(x)/4 - 1/(2*x)
```

3.315 $\int \frac{1-12x+x^2+x^3}{-12+x+x^2} \, dx$

3.315.1 Optimal result

Integrand size = 20, antiderivative size = 22

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2} \, dx = \frac{x^2}{2} - \frac{2}{7}\operatorname{arctanh}\left(\frac{1}{7}(1+2x)\right)$$

output
```
1/2*x^2-2/7*arctanh(1/7+2/7*x)
```

3.315.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2} \, dx = \frac{x^2}{2} + \frac{1}{7}\log(3-x) - \frac{1}{7}\log(4+x)$$

input
```
Integrate[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
```

output
```
x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

3.315.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2-12x+1}{x^2+x-12}dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{1}{x^2+x-12}+x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(x+4)$$

input
```
Int[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
```

output
```
x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

3.315.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.315.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x^2}{2}+\frac{\ln(-3+x)}{7}-\frac{\ln(x+4)}{7}$	19
norman	$\frac{x^2}{2}+\frac{\ln(-3+x)}{7}-\frac{\ln(x+4)}{7}$	19
risch	$\frac{x^2}{2}+\frac{\ln(-3+x)}{7}-\frac{\ln(x+4)}{7}$	19
parallelrisch	$\frac{x^2}{2}+\frac{\ln(-3+x)}{7}-\frac{\ln(x+4)}{7}$	19

input
```
int((x^3+x^2-12*x+1)/(x^2+x-12),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/7*ln(-3+x)-1/7*ln(x+4)
```

3.315.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{7}\,\log\left(x+4\right) + \frac{1}{7}\,\log\left(x-3\right)$$

input
```
integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="fricas")
```

output
```
1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

3.315.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx = \frac{x^2}{2} + \frac{\log\left(x-3\right)}{7} - \frac{\log\left(x+4\right)}{7}$$

input
```
integrate((x**3+x**2-12*x+1)/(x**2+x-12),x)
```

output
```
x**2/2 + log(x - 3)/7 - log(x + 4)/7
```

3.315.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{7}\,\log\left(x+4\right) + \frac{1}{7}\,\log\left(x-3\right)$$

input
```
integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

3.315.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{7}\,\log\left(|x+4|\right) + \frac{1}{7}\,\log\left(|x-3|\right)$$

input
```
integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/7*log(abs(x + 4)) + 1/7*log(abs(x - 3))
```

3.315.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx = \frac{x^2}{2} - \frac{2\,\mathrm{atanh}\left(\frac{2\,x}{7}+\frac{1}{7}\right)}{7}$$

input
```
int((x^2 - 12*x + x^3 + 1)/(x + x^2 - 12),x)
```

output
```
x^2/2 - (2*atanh((2*x)/7 + 1/7))/7
```

3.316 $\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx$

3.316.1 Optimal result

Integrand size = 25, antiderivative size = 17

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = 2\log(1-x)+\log(x)+3\log(3+x)$$

output
```
2*ln(1-x)+ln(x)+3*ln(3+x)
```

3.316.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = 2\log(1-x)+\log(x)+3\log(3+x)$$

input
```
Integrate[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

output
```
2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

3.316.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2026, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{6x^2+5x-3}{x^3+2x^2-3x}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{6x^2+5x-3}{x(x^2+2x-3)}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{1}{x}+\frac{3}{x+3}+\frac{2}{x-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\log(1-x)+\log(x)+3\log(x+3)$$

input
```
Int[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

output
```
2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

3.316.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.316.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\ln(x)+3\ln(3+x)+2\ln(x-1)$	16
norman	$\ln(x)+3\ln(3+x)+2\ln(x-1)$	16
risch	$\ln(x)+3\ln(3+x)+2\ln(x-1)$	16
parallelrisch	$\ln(x)+3\ln(3+x)+2\ln(x-1)$	16

input
```
int((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+3*ln(3+x)+2*ln(x-1)
```

3.316.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = 3\,\log(x+3)+2\,\log(x-1)+\log(x)$$

input
```
integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="fricas")
```

output
```
3*log(x + 3) + 2*log(x - 1) + log(x)
```

3.316.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = \log(x)+2\log(x-1)+3\log(x+3)$$

input
```
integrate((6*x**2+5*x-3)/(x**3+2*x**2-3*x),x)
```

output
```
log(x) + 2*log(x - 1) + 3*log(x + 3)
```

3.316.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = 3\log(x+3)+2\log(x-1)+\log(x)$$

input
```
integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="maxima")
```

output
```
3*log(x + 3) + 2*log(x - 1) + log(x)
```

3.316.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = 3\log(|x+3|)+2\log(|x-1|)+\log(|x|)$$

input
```
integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="giac")
```

output
```
3*log(abs(x + 3)) + 2*log(abs(x - 1)) + log(abs(x))
```

3.316.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx = 2\ln(x-1)+3\ln(x+3)+\ln(x)$$

input
```
int((5*x + 6*x^2 - 3)/(2*x^2 - 3*x + x^3),x)
```

output
```
2*log(x - 1) + 3*log(x + 3) + log(x)
```

3.317 $\int \frac{-2+3x+5x^2}{2x^2+x^3} \, dx$

3.317.1 Optimal result

Integrand size = 22, antiderivative size = 14

$$\int \frac{-2+3x+5x^2}{2x^2+x^3} \, dx = \frac{1}{x} + 2\log(x) + 3\log(2+x)$$

output
```
1/x+2*ln(x)+3*ln(2+x)
```

3.317.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-2+3x+5x^2}{2x^2+x^3} \, dx = \frac{1}{x} + 2\log(x) + 3\log(2+x)$$

input
```
Integrate[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

output
```
x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

3.317.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2026, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+3x-2}{x^3+2x^2}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{5x^2+3x-2}{x^2(x+2)}dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(-\frac{1}{x^2}+\frac{3}{x+2}+\frac{2}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{x}+2\log(x)+3\log(x+2)$$

input
```
Int[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

output
```
x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

3.317.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.317.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{1}{x}+2\ln\left(x\right)+3\ln\left(x+2\right)$	15
norman	$\frac{1}{x}+2\ln\left(x\right)+3\ln\left(x+2\right)$	15
risch	$\frac{1}{x}+2\ln\left(x\right)+3\ln\left(x+2\right)$	15
parallelrisch	$\frac{2\ln(x)x+3\ln(x+2)x+1}{x}$	19
meijerg	$3\ln\left(1+\frac{x}{2}\right)+2\ln\left(x\right)-2\ln\left(2\right)+\frac{1}{x}$	21

input
```
int((5*x^2+3*x-2)/(x^3+2*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/x+2*ln(x)+3*ln(x+2)
```

3.317.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx=\frac{3\,x\log\left(x+2\right)+2\,x\log\left(x\right)+1}{x}$$

input
```
integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="fricas")
```

output
```
(3*x*log(x + 2) + 2*x*log(x) + 1)/x
```

3.317.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = 2\log(x)+3\log(x+2)+\frac{1}{x}$$

input
```
integrate((5*x**2+3*x-2)/(x**3+2*x**2),x)
```

output
```
2*log(x) + 3*log(x + 2) + 1/x
```

3.317.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = \frac{1}{x}+3\,\log(x+2)+2\,\log(x)$$

input
```
integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="maxima")
```

output
```
1/x + 3*log(x + 2) + 2*log(x)
```

3.317.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = \frac{1}{x}+3\,\log(|x+2|)+2\,\log(|x|)$$

input
```
integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="giac")
```

output
```
1/x + 3*log(abs(x + 2)) + 2*log(abs(x))
```

3.317.9 Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = 3\,\ln(x+2) + 2\,\ln(x) + \frac{1}{x}$$

input
```
int((3*x + 5*x^2 - 2)/(2*x^2 + x^3),x)
```

output
```
3*log(x + 2) + 2*log(x) + 1/x
```

3.318 $\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx$

3.318.1 Optimal result

Integrand size = 24, antiderivative size = 19

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = \log(1-x) - 2\log(2+x) - 3\log(3+x)$$

output
```
ln(1-x)-2*ln(2+x)-3*ln(3+x)
```

3.318.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = -2\left(-\frac{1}{2}\log(1-x) + \log(2+x) + \frac{3}{2}\log(3+x)\right)$$

input
```
Integrate[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
```

output
```
-2*(-1/2*Log[1 - x] + Log[2 + x] + (3*Log[3 + x])/2)
```

3.318.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-4x^2-2x+18}{x^3+4x^2+x-6}\,dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(-\frac{2}{x+2}-\frac{3}{x+3}+\frac{1}{x-1}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\log(1-x)-2\log(x+2)-3\log(x+3)$$

input
```
Int[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
```

output
```
Log[1 - x] - 2*Log[2 + x] - 3*Log[3 + x]
```

3.318.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.318.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$-2\ln(x+2)-3\ln(3+x)+\ln(x-1)$	18
norman	$-2\ln(x+2)-3\ln(3+x)+\ln(x-1)$	18
risch	$-2\ln(x+2)-3\ln(3+x)+\ln(x-1)$	18
parallelrisch	$-2\ln(x+2)-3\ln(3+x)+\ln(x-1)$	18

input
```
int((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x,method=_RETURNVERBOSE)
```

output
```
-2*ln(x+2)-3*ln(3+x)+ln(x-1)
```

3.318.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = -3\,\log(x+3)-2\,\log(x+2)+\log(x-1)$$

input
```
integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="fricas")
```

output
```
-3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

3.318.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = \log(x-1)-2\log(x+2)-3\log(x+3)$$

input
```
integrate((-4*x**2-2*x+18)/(x**3+4*x**2+x-6),x)
```

output
```
log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

3.318.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = -3\,\log(x+3) - 2\,\log(x+2) + \log(x-1)$$

input
```
integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="maxima")
```

output
```
-3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

3.318.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = -3\,\log(|x+3|) - 2\,\log(|x+2|) + \log(|x-1|)$$

input
```
integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="giac")
```

output
```
-3*log(abs(x + 3)) - 2*log(abs(x + 2)) + log(abs(x - 1))
```

3.318.9 Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = \ln(x-1) - 2\,\ln(x+2) - 3\,\ln(x+3)$$

input
```
int(-(2*x + 4*x^2 - 18)/(x + 4*x^2 + x^3 - 6),x)
```

output
```
log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

3.319 $\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} dx$

3.319.1 Optimal result

Integrand size = 24, antiderivative size = 23

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} dx = -\frac{3}{2}\arctan\left(\frac{x}{2}\right)+\arctan(x)+\frac{1}{2}\log\left(4+x^2\right)$$

output
```
-3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)
```

3.319.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} dx = -\frac{3}{2}\arctan\left(\frac{x}{2}\right)+\arctan(x)+\frac{1}{2}\log\left(4+x^2\right)$$

input
```
Integrate[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
```

output
```
(-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

3.319.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2202, 1387, 240, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-2x^2+x+1}{x^4+5x^2+4}dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{1-2x^2}{x^4+5x^2+4}dx+\int \frac{x\left(x^2+1\right)}{x^4+5x^2+4}dx$$

$$\Big\downarrow \text{ 1387}$$

$$\int \frac{x}{x^2+4}dx+\int \frac{1-2x^2}{x^4+5x^2+4}dx$$

$$\Big\downarrow \text{ 240}$$

$$\int \frac{1-2x^2}{x^4+5x^2+4}dx+\frac{1}{2}\log\left(x^2+4\right)$$

$$\Big\downarrow \text{ 1480}$$

$$\int \frac{1}{x^2+1}dx-3\int \frac{1}{x^2+4}dx+\frac{1}{2}\log\left(x^2+4\right)$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{3}{2}\arctan\left(\frac{x}{2}\right)+\arctan(x)+\frac{1}{2}\log\left(x^2+4\right)$$

input
```
Int[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
```

output
```
(-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

3.319.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 1387
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*
(x_)^(n_))^(q_.), x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^
p, x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

3.319.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{3\arctan\left(\frac{x}{2}\right)}{2}+\arctan\left(x\right)+\frac{\ln\left(x^{2}+4\right)}{2}$	18
risch	$-\frac{3\arctan\left(\frac{x}{2}\right)}{2}+\arctan\left(x\right)+\frac{\ln\left(x^{2}+4\right)}{2}$	18
parallelrisch	$\frac{\ln(x-2i)}{2}+\frac{3i\ln(x-2i)}{4}-\frac{i\ln(x-i)}{2}+\frac{i\ln(x+i)}{2}+\frac{\ln(x+2i)}{2}-\frac{3i\ln(x+2i)}{4}$	48

input
```
int((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
-3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)
```

3.319.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx = -\frac{3}{2}\arctan\left(\frac{1}{2}\,x\right)+\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^2+4\right)$$

input
```
integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="fricas")
```

output
```
-3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

3.319.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx = \frac{\log\left(x^2+4\right)}{2}-\frac{3\,\mathrm{atan}\left(\frac{x}{2}\right)}{2}+\mathrm{atan}\left(x\right)$$

input
```
integrate((x**3-2*x**2+x+1)/(x**4+5*x**2+4),x)
```

output
```
log(x**2 + 4)/2 - 3*atan(x/2)/2 + atan(x)
```

3.319.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx = -\frac{3}{2}\arctan\left(\frac{1}{2}\,x\right)+\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^2+4\right)$$

input
```
integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="maxima")
```

output
```
-3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

3.319.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx = -\frac{3}{2}\arctan\left(\frac{1}{2}x\right)+\arctan(x)+\frac{1}{2}\log\left(x^2+4\right)$$

input
```
integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="giac")
```

output
```
-3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

3.319.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx = -\mathrm{atan}\left(\frac{1305}{4\,(144\,x-162)}+\frac{9}{8}\right) + \ln(x-2\mathrm{i})\,\left(\frac{1}{2}+\frac{3}{4}\mathrm{i}\right)+\ln(x+2\mathrm{i})\,\left(\frac{1}{2}-\frac{3}{4}\mathrm{i}\right)$$

input
```
int((x - 2*x^2 + x^3 + 1)/(5*x^2 + x^4 + 4),x)
```

output
```
log(x - 2i)*(1/2 + 3i/4) + log(x + 2i)*(1/2 - 3i/4) - atan(1305/(4*(144*x
- 162)) + 9/8)
```

3.320 $\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5} dx$

3.320.1 Optimal result

Integrand size = 43, antiderivative size = 63

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5} dx = \frac{3988\arctan\left(\frac{1+2x}{\sqrt{19}}\right)}{13685\sqrt{19}} - \frac{3146\log(7-3x)}{80155} - \frac{334}{323}\log(1+2x) + \frac{4822\log(2+5x)}{4879} + \frac{11049\log(5+x+x^2)}{260015}$$

output
```
-3146/80155*ln(7-3*x)-334/323*ln(1+2*x)+4822/4879*ln(2+5*x)+11049/260015*l
n(x^2+x+5)+3988/260015*arctan(1/19*(1+2*x)*19^(1/2))*19^(1/2)
```

3.320.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5} dx = \frac{163508\sqrt{19}\arctan\left(\frac{1+2x}{\sqrt{19}}\right) - 418418\log(7-3x) - 11023670\log(1+2x) + 10536070\log(2+5x) + 45300}{10660615}$$

input
```
Integrate[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 1
3*x^4 + 30*x^5),x]
```

output
```
(163508*Sqrt[19]*ArcTan[(1 + 2*x)/Sqrt[19]] - 418418*Log[7 - 3*x] - 110236
70*Log[1 + 2*x] + 10536070*Log[2 + 5*x] + 453009*Log[5 + x + x^2])/1066061
5
```

3.320.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^3-27x^2+5x-32}{30x^5-13x^4+50x^3-286x^2-299x-70} \, dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{22098x+48935}{260015\left(x^2+x+5\right)}-\frac{668}{323(2x+1)}-\frac{9438}{80155(3x-7)}+\frac{24110}{4879(5x+2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3988\arctan\left(\frac{2x+1}{\sqrt{19}}\right)}{13685\sqrt{19}}+\frac{11049\log\left(x^2+x+5\right)}{260015}-\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(2x+1)+\frac{4822\log(5x+2)}{4879}$$

input
```
Int[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4
+ 30*x^5),x]
```

output
```
(3988*ArcTan[(1 + 2*x)/Sqrt[19]])/(13685*Sqrt[19]) - (3146*Log[7 - 3*x])/8
0155 - (334*Log[1 + 2*x])/323 + (4822*Log[2 + 5*x])/4879 + (11049*Log[5 +
x + x^2])/260015
```

3.320.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.320.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

method	result
default	$\frac{4822 \ln(2+5x)}{4879}-\frac{334 \ln(1+2x)}{323}+\frac{11049 \ln(x^{2}+x+5)}{260015}+\frac{3988 \arctan\left(\frac{(1+2x)\sqrt{19}}{19}\right)\sqrt{19}}{260015}-\frac{3146 \ln(3x-7)}{80155}$
risch	$-\frac{3146 \ln(3x-7)}{80155}-\frac{334 \ln(1+2x)}{323}+\frac{11049 \ln(15904144x^{2}+15904144x+79520720)}{260015}+\frac{3988\sqrt{19}\, \arctan\left(\frac{(3988x+1994)\sqrt{19}}{37886}\right)}{260015}+$

input
```
int((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,method
=_RETURNVERBOSE)
```

output
```
4822/4879*ln(2+5*x)-334/323*ln(1+2*x)+11049/260015*ln(x^2+x+5)+3988/260015
*arctan(1/19*(1+2*x)*19^(1/2))*19^(1/2)-3146/80155*ln(3*x-7)
```

3.320.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = \frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2\,x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right) + \frac{4822}{4879}\log\left(5\,x+2\right)-\frac{3146}{80155}\log\left(3\,x-7\right)-\frac{334}{323}\log\left(2\,x+1\right)$$

input
```
integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
 algorithm="fricas")
```

output
```
3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^
2 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*lo
g(2*x + 1)
```

3.320.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = -\frac{3146\log\left(x-\frac{7}{3}\right)}{80155}+\frac{4822\log\left(x+\frac{2}{5}\right)}{4879}-\frac{334\log\left(x+\frac{1}{2}\right)}{323}+\frac{11049\log\left(x^2+x+5\right)}{260015}+\frac{3988\sqrt{19}\,\mathrm{atan}\left(\frac{2\sqrt{19}x}{19}+\frac{\sqrt{19}}{19}\right)}{260015}$$

input
```
integrate((4*x**3-27*x**2+5*x-32)/(30*x**5-13*x**4+50*x**3-286*x**2-299*x-
70),x)
```

output
```
-3146*log(x - 7/3)/80155 + 4822*log(x + 2/5)/4879 - 334*log(x + 1/2)/323 +
 11049*log(x**2 + x + 5)/260015 + 3988*sqrt(19)*atan(2*sqrt(19)*x/19 + sqr
t(19)/19)/260015
```

3.320.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = \frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right)+\frac{4822}{4879}\log\left(5x+2\right)-\frac{3146}{80155}\log\left(3x-7\right)-\frac{334}{323}\log\left(2x+1\right)$$

input
```
integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
 algorithm="maxima")
```

output
```
3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^
2 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*lo
g(2*x + 1)
```

3.320.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx$$
$$=\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right)$$
$$+\frac{4822}{4879}\log\left(|5x+2|\right)-\frac{3146}{80155}\log\left(|3x-7|\right)-\frac{334}{323}\log\left(|2x+1|\right)$$

input
```
integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
 algorithm="giac")
```

output
```
3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^
2 + x + 5) + 4822/4879*log(abs(5*x + 2)) - 3146/80155*log(abs(3*x - 7)) -
334/323*log(abs(2*x + 1))
```

3.320.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx$$
$$=\frac{4822\ln\left(x+\frac{2}{5}\right)}{4879}-\frac{334\ln\left(x+\frac{1}{2}\right)}{323}-\frac{3146\ln\left(x-\frac{7}{3}\right)}{80155}$$
$$-\ln\left(x+\frac{1}{2}-\frac{\sqrt{19}\,1\mathrm{i}}{2}\right)\left(-\frac{11049}{260015}+\frac{\sqrt{19}\,1994\mathrm{i}}{260015}\right)$$
$$+\ln\left(x+\frac{1}{2}+\frac{\sqrt{19}\,1\mathrm{i}}{2}\right)\left(\frac{11049}{260015}+\frac{\sqrt{19}\,1994\mathrm{i}}{260015}\right)$$

input
```
int(-(5*x - 27*x^2 + 4*x^3 - 32)/(299*x + 286*x^2 - 50*x^3 + 13*x^4 - 30*x
^5 + 70),x)
```

output
```
(4822*log(x + 2/5))/4879 - (334*log(x + 1/2))/323 - (3146*log(x - 7/3))/80
155 - log(x - (19^(1/2)*1i)/2 + 1/2)*((19^(1/2)*1994i)/260015 - 11049/2600
15) + log(x + (19^(1/2)*1i)/2 + 1/2)*((19^(1/2)*1994i)/260015 + 11049/2600
15)
```

3.321 $\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx$

3.321.1 Optimal result

Integrand size = 50, antiderivative size = 69

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \frac{5828}{9075(2-5x)}-\frac{313+502x}{1452\,(1+2x^2)}+\frac{503\arctan\left(\sqrt{2}x\right)}{7986\sqrt{2}} - \frac{59096\log(2-5x)}{99825}+\frac{2843\log\left(1+2x^2\right)}{7986}$$

output
```
5828/9075/(2-5*x)+1/1452*(-313-502*x)/(2*x^2+1)-59096/99825*ln(2-5*x)+2843
/7986*ln(2*x^2+1)+503/15972*arctan(x*2^(1/2))*2^(1/2)
```

3.321.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \frac{-\frac{33(2554+4675x+36458x^2)}{-2+5x-4x^2+10x^3}+12575\sqrt{2}\arctan\left(\sqrt{2}x\right)-236384\log(2-5x)+142150\log\left(1+2x^2\right)}{399300}$$

input
```
Integrate[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*
x^4 - 80*x^5 + 100*x^6),x]
```

output
```
((-33*(2554 + 4675*x + 36458*x^2))/(-2 + 5*x - 4*x^2 + 10*x^3) + 12575*Sqr
t[2]*ArcTan[Sqrt[2]*x] - 236384*Log[2 - 5*x] + 142150*Log[1 + 2*x^2])/3993
00
```

3.321.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{12x^5-7x^3-13x^2+8}{100x^6-80x^5+116x^4-80x^3+41x^2-20x+4}dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(\frac{313x-251}{363\left(2x^2+1\right)^2}+\frac{2(2843x+816)}{3993\left(2x^2+1\right)}-\frac{59096}{19965(5x-2)}+\frac{5828}{1815(5x-2)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{272\sqrt{2}\arctan\left(\sqrt{2}x\right)}{1331}-\frac{251\arctan\left(\sqrt{2}x\right)}{726\sqrt{2}}-\frac{502x+313}{1452\left(2x^2+1\right)}+\frac{2843\log\left(2x^2+1\right)}{7986}+\frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}$$

input
```
Int[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 -
80*x^5 + 100*x^6),x]
```

output
```
5828/(9075*(2 - 5*x)) - (313 + 502*x)/(1452*(1 + 2*x^2)) - (251*ArcTan[Sqr
t[2]*x])/(726*Sqrt[2]) + (272*Sqrt[2]*ArcTan[Sqrt[2]*x])/1331 - (59096*Log
[2 - 5*x])/99825 + (2843*Log[1 + 2*x^2])/7986
```

3.321.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

3.321.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{5828}{9075(5x-2)}-\frac{59096\ln(5x-2)}{99825}+\frac{-\frac{2761x}{4}-\frac{3443}{8}}{3993x^2+\frac{3993}{2}}+\frac{2843\ln(2x^2+1)}{7986}+\frac{503\arctan\left(x\sqrt{2}\right)\sqrt{2}}{15972}$	54
risch	$\frac{-\frac{18229}{60500}x^2-\frac{17}{440}x-\frac{1277}{60500}}{x^3-\frac{2}{5}x^2+\frac{1}{2}x-\frac{1}{5}}-\frac{59096\ln(5x-2)}{99825}+\frac{2843\ln\left(\frac{253009}{2}+253009x^2\right)}{7986}+\frac{503\arctan\left(x\sqrt{2}\right)\sqrt{2}}{15972}$	57

input
```
int((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4),
x,method=_RETURNVERBOSE)
```

output
```
-5828/9075/(5*x-2)-59096/99825*ln(5*x-2)+1/3993*(-2761/4*x-3443/8)/(x^2+1/
2)+2843/7986*ln(2*x^2+1)+503/15972*arctan(x*2^(1/2))*2^(1/2)
```

3.321.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.49

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx$$
$$=\frac{12575\sqrt{2}(10\,x^3-4\,x^2+5\,x-2)\arctan\left(\sqrt{2}x\right)-1203114\,x^2+142150\,(10\,x^3-4\,x^2+5\,x-2)\log\left(2\,x^2\right.}{399300\,(10\,x^3-4\,x^2+5\,x-2)}$$

input
```
integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20
*x+4),x, algorithm="fricas")
```

output
```
1/399300*(12575*sqrt(2)*(10*x^3 - 4*x^2 + 5*x - 2)*arctan(sqrt(2)*x) - 120
3114*x^2 + 142150*(10*x^3 - 4*x^2 + 5*x - 2)*log(2*x^2 + 1) - 236384*(10*x
^3 - 4*x^2 + 5*x - 2)*log(5*x - 2) - 154275*x - 84282)/(10*x^3 - 4*x^2 + 5
*x - 2)
```

3.321.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \frac{-36458x^2-4675x-2554}{121000x^3-48400x^2+60500x-24200} - \frac{59096\log\left(x-\frac{2}{5}\right)}{99825} + \frac{2843\log\left(x^2+\frac{1}{2}\right)}{7986} + \frac{503\sqrt{2}\operatorname{atan}\left(\sqrt{2}x\right)}{15972}$$

input
```
integrate((12*x**5-7*x**3-13*x**2+8)/(100*x**6-80*x**5+116*x**4-80*x**3+41
*x**2-20*x+4),x)
```

output
```
(-36458*x**2 - 4675*x - 2554)/(121000*x**3 - 48400*x**2 + 60500*x - 24200)
 - 59096*log(x - 2/5)/99825 + 2843*log(x**2 + 1/2)/7986 + 503*sqrt(2)*atan
(sqrt(2)*x)/15972
```

3.321.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}x\right) - \frac{36458\,x^2+4675\,x+2554}{12100\,(10\,x^3-4\,x^2+5\,x-2)} + \frac{2843}{7986}\log\left(2\,x^2+1\right) - \frac{59096}{99825}\log\left(5\,x-2\right)$$

input
```
integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20
*x+4),x, algorithm="maxima")
```

output
```
503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/
(10*x^3 - 4*x^2 + 5*x - 2) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(5*
x - 2)
```

3.321.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}x\right)-\frac{36458\,x^2+4675\,x+2554}{12100\,(2\,x^2+1)(5\,x-2)} + \frac{2843}{7986}\,\log\left(2\,x^2+1\right)-\frac{59096}{99825}\,\log\left(|5\,x-2|\right)$$

input
```
integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20
*x+4),x, algorithm="giac")
```

output
```
503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/
((2*x^2 + 1)*(5*x - 2)) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(abs(5
*x - 2))
```

3.321.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = -\frac{59096\,\ln\left(x-\frac{2}{5}\right)}{99825}-\frac{\frac{18229\,x^2}{60500}+\frac{17\,x}{440}+\frac{1277}{60500}}{x^3-\frac{2\,x^2}{5}+\frac{x}{2}-\frac{1}{5}} - \ln\left(x-\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)\left(-\frac{2843}{7986}+\frac{\sqrt{2}\,503\mathrm{i}}{31944}\right)+\ln\left(x+\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)\left(\frac{2843}{7986}+\frac{\sqrt{2}\,503\mathrm{i}}{31944}\right)$$

input
```
int(-(13*x^2 + 7*x^3 - 12*x^5 - 8)/(41*x^2 - 20*x - 80*x^3 + 116*x^4 - 80*
x^5 + 100*x^6 + 4),x)
```

output
```
log(x + (2^(1/2)*1i)/2)*((2^(1/2)*503i)/31944 + 2843/7986) - ((17*x)/440 +
 (18229*x^2)/60500 + 1277/60500)/(x/2 - (2*x^2)/5 + x^3 - 1/5) - log(x - (
2^(1/2)*1i)/2)*((2^(1/2)*503i)/31944 - 2843/7986) - (59096*log(x - 2/5))/9
9825
```

3.321. $\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx$

3.322 $\int \frac{9+x^4}{x^2(9+x^2)} dx$

3.322.1 Optimal result

Integrand size = 16, antiderivative size = 17

$$\int \frac{9+x^4}{x^2(9+x^2)} dx = -\frac{1}{x}+x-\frac{10}{3}\arctan\left(\frac{x}{3}\right)$$

output
```
-1/x+x-10/3*arctan(1/3*x)
```

3.322.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{9+x^4}{x^2(9+x^2)} dx = -\frac{1}{x}+x-\frac{10}{3}\arctan\left(\frac{x}{3}\right)$$

input
```
Integrate[(9 + x^4)/(x^2*(9 + x^2)),x]
```

output
```
-x^(-1) + x - (10*ArcTan[x/3])/3
```

3.322.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1585, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+9}{x^2 (x^2+9)} \, dx$$

$$\Big\downarrow \; 1585$$

$$\int \left(\frac{1}{x^2}-\frac{10}{x^2+9}+1\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{10}{3}\arctan\left(\frac{x}{3}\right)+x-\frac{1}{x}$$

input
```
Int[(9 + x^4)/(x^2*(9 + x^2)),x]
```

output
```
-x^(-1) + x - (10*ArcTan[x/3])/3
```

3.322.3.1 Defintions of rubi rules used

rule 1585
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.322.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{1}{x}+x-\frac{10\arctan\left(\frac{x}{3}\right)}{3}$	14
meijerg	$-\frac{1}{x}+x-\frac{10\arctan\left(\frac{x}{3}\right)}{3}$	14
risch	$-\frac{1}{x}+x-\frac{10\arctan\left(\frac{x}{3}\right)}{3}$	14
parallelrisch	$\frac{5i\ln(x-3i)x-5i\ln(x+3i)x+3x^2-3}{3x}$	31

input
```
int((x^4+9)/x^2/(x^2+9),x,method=_RETURNVERBOSE)
```

output
```
-1/x+x-10/3*arctan(1/3*x)
```

3.322.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{9+x^4}{x^2(9+x^2)}\,dx = \frac{3\,x^2-10\,x\arctan\left(\frac{1}{3}\,x\right)-3}{3\,x}$$

input
```
integrate((x^4+9)/x^2/(x^2+9),x, algorithm="fricas")
```

output
```
1/3*(3*x^2 - 10*x*arctan(1/3*x) - 3)/x
```

3.322.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{9+x^4}{x^2(9+x^2)}\,dx = x-\frac{10\,\mathrm{atan}\left(\frac{x}{3}\right)}{3}-\frac{1}{x}$$

input
```
integrate((x**4+9)/x**2/(x**2+9),x)
```

output
```
x - 10*atan(x/3)/3 - 1/x
```

3.322.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{9+x^4}{x^2\left(9+x^2\right)}\,dx = x - \frac{1}{x} - \frac{10}{3}\,\arctan\left(\frac{1}{3}\,x\right)$$

input
```
integrate((x^4+9)/x^2/(x^2+9),x, algorithm="maxima")
```

output
```
x - 1/x - 10/3*arctan(1/3*x)
```

3.322.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{9+x^4}{x^2\left(9+x^2\right)}\,dx = x - \frac{1}{x} - \frac{10}{3}\,\arctan\left(\frac{1}{3}\,x\right)$$

input
```
integrate((x^4+9)/x^2/(x^2+9),x, algorithm="giac")
```

output
```
x - 1/x - 10/3*arctan(1/3*x)
```

3.322.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{9+x^4}{x^2\left(9+x^2\right)}\,dx = x - \frac{10\,\mathrm{atan}\left(\frac{x}{3}\right)}{3} - \frac{1}{x}$$

input
```
int((x^4 + 9)/(x^2*(x^2 + 9)),x)
```

output
```
x - (10*atan(x/3))/3 - 1/x
```

3.323 $\int \frac{2x+x^4}{1+x^2}\,dx$

3.323.1 Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{2x+x^4}{1+x^2}\,dx=-x+\frac{x^3}{3}+\arctan(x)+\log\left(1+x^2\right)$$

output
```
-x+1/3*x^3+arctan(x)+ln(x^2+1)
```

3.323.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{2x+x^4}{1+x^2}\,dx=-x+\frac{x^3}{3}+\arctan(x)+\log\left(1+x^2\right)$$

input
```
Integrate[(2*x + x^4)/(1 + x^2),x]
```

output
```
-x + x^3/3 + ArcTan[x] + Log[1 + x^2]
```

3.323.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2027, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+2x}{x^2+1}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x(x^3+2)}{x^2+1}dx$$

$$\Big\downarrow \text{ 2333}$$

$$\int \left(x^2+\frac{2x+1}{x^2+1}-1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\arctan(x)+\frac{x^3}{3}+\log\left(x^2+1\right)-x$$

input
```
Int[(2*x + x^4)/(1 + x^2),x]
```

output
```
-x + x^3/3 + ArcTan[x] + Log[1 + x^2]
```

3.323.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.323.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$-x+\frac{x^{3}}{3}+\arctan\left(x\right)+\ln\left(x^{2}+1\right)$	18
risch	$-x+\frac{x^{3}}{3}+\arctan\left(x\right)+\ln\left(x^{2}+1\right)$	18
meijerg	$-\frac{x\left(-5x^{2}+15\right)}{15}+\arctan\left(x\right)+\ln\left(x^{2}+1\right)$	20
parallelrisch	$\frac{x^{3}}{3}-x+\ln\left(x-i\right)-\frac{i\ln\left(x-i\right)}{2}+\ln\left(x+i\right)+\frac{i\ln\left(x+i\right)}{2}$	36

input
```
int((x^4+2*x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-x+1/3*x^3+arctan(x)+ln(x^2+1)
```

3.323.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{2x+x^4}{1+x^2}\,dx=\frac{1}{3}\,x^{3}-x+\arctan\left(x\right)+\log\left(x^{2}+1\right)$$

input
```
integrate((x^4+2*x)/(x^2+1),x, algorithm="fricas")
```

output
```
1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

3.323.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{2x+x^4}{1+x^2}\,dx=\frac{x^{3}}{3}-x+\log\left(x^{2}+1\right)+\operatorname{atan}\left(x\right)$$

input
```
integrate((x**4+2*x)/(x**2+1),x)
```

output
```
x**3/3 - x + log(x**2 + 1) + atan(x)
```

3.323.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{2x + x^4}{1 + x^2} \, dx = \frac{1}{3} x^3 - x + \arctan(x) + \log(x^2 + 1)$$

input
```
integrate((x^4+2*x)/(x^2+1),x, algorithm="maxima")
```

output
```
1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

3.323.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{2x + x^4}{1 + x^2} \, dx = \frac{1}{3} x^3 - x + \arctan(x) + \log(x^2 + 1)$$

input
```
integrate((x^4+2*x)/(x^2+1),x, algorithm="giac")
```

output
```
1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

3.323.9 Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{2x + x^4}{1 + x^2} \, dx = \ln(x^2 + 1) - x + \text{atan}(x) + \frac{x^3}{3}$$

input
```
int((2*x + x^4)/(x^2 + 1),x)
```

output
```
log(x^2 + 1) - x + atan(x) + x^3/3
```

3.324 $\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx$

3.324.1 Optimal result

Integrand size = 20, antiderivative size = 9

$$\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx = \arctan(x)+\log(1-x)$$

output
```
arctan(x)+ln(1-x)
```

3.324.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx = \arctan(x)+\log(1-x)$$

input
```
Integrate[(-x + x^3)/((-1 + x)^2*(1 + x^2)),x]
```

output
```
ArcTan[x] + Log[1 - x]
```

3.324.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2019, 2027, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-x}{(x-1)^2 (x^2+1)} \, dx$$
$$\Big\downarrow \; 2019$$
$$\int \frac{x^2+x}{(x-1)(x^2+1)}dx$$
$$\Big\downarrow \; 2027$$
$$\int \frac{x(x+1)}{(x-1)(x^2+1)}dx$$
$$\Big\downarrow \; 2160$$
$$\int \left(\frac{1}{x^2+1}+\frac{1}{x-1}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\arctan(x)+\log(1-x)$$

input
```
Int[(-x + x^3)/((-1 + x)^2*(1 + x^2)),x]
```

output
```
ArcTan[x] + Log[1 - x]
```

3.324.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.324.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
default	$\arctan(x) + \ln(x-1)$	8
risch	$\arctan(x) + \ln(x-1)$	8
parallelrisch	$\frac{i \ln(x+i)}{2} - \frac{i \ln(x-i)}{2} + \ln(x-1)$	22

input
```
int((x^3-x)/(x-1)^2/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
arctan(x)+ln(x-1)
```

3.324.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx = \arctan(x) + \log(x-1)$$

input
```
integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
```

output
```
arctan(x) + log(x - 1)
```

3.324.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx = \log(x-1) + \operatorname{atan}(x)$$

input
```
integrate((x**3-x)/(-1+x)**2/(x**2+1),x)
```

output
```
log(x - 1) + atan(x)
```

3.324.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx = \arctan(x) + \log(x-1)$$

input
```
integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
```

output
```
arctan(x) + log(x - 1)
```

3.324.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(9) = 18.

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 3.11

$$\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx = \frac{1}{4}\pi - \pi\left\lfloor \frac{\pi+4\arctan(x)}{4\pi} + \frac{1}{2}\right\rfloor + \arctan(x) + \log(|x-1|)$$

input
```
integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="giac")
```

output
```
1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + arctan(x) + log(abs(x
 - 1))
```

3.324.9 Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int \frac{-x+x^3}{(-1+x)^2\,(1+x^2)}\,dx = \ln(x-1) - \operatorname{atan}\left(\frac{5}{4\,x+2} - \frac{1}{2}\right)$$

input
```
int(-(x - x^3)/((x^2 + 1)*(x - 1)^2),x)
```

output
```
log(x - 1) - atan(5/(4*x + 2) - 1/2)
```

3.325 $\int \frac{2+5x+3x^2+2x^3}{1+x+x^2} \, dx$

3.325.1 Optimal result

Integrand size = 24, antiderivative size = 12

$$\int \frac{2+5x+3x^2+2x^3}{1+x+x^2} \, dx = x+x^2+\log\left(1+x+x^2\right)$$

output
```
x+x^2+ln(x^2+x+1)
```

3.325.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{2+5x+3x^2+2x^3}{1+x+x^2} \, dx = x+x^2+\log\left(1+x+x^2\right)$$

input
```
Integrate[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2),x]
```

output
```
x + x^2 + Log[1 + x + x^2]
```

3.325.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+5x+2}{x^2+x+1}\,dx$$
$$\Big\downarrow \text{2188}$$
$$\int \left(\frac{2x+1}{x^2+x+1}+2x+1\right)dx$$
$$\Big\downarrow \text{2009}$$
$$x^2+\log\left(x^2+x+1\right)+x$$

input
```
Int[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2),x]
```

output
```
x + x^2 + Log[1 + x + x^2]
```

3.325.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.325.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$x + x^2 + \ln(x^2 + x + 1)$	13
norman	$x + x^2 + \ln(x^2 + x + 1)$	13
risch	$x + x^2 + \ln(x^2 + x + 1)$	13
parallelrisch	$x + x^2 + \ln(x^2 + x + 1)$	13

input
```
int((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
x+x^2+ln(x^2+x+1)
```

3.325.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{2 + 5x + 3x^2 + 2x^3}{1 + x + x^2}\, dx = x^2 + x + \log\left(x^2 + x + 1\right)$$

input
```
integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="fricas")
```

output
```
x^2 + x + log(x^2 + x + 1)
```

3.325.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{2 + 5x + 3x^2 + 2x^3}{1 + x + x^2}\, dx = x^2 + x + \log\left(x^2 + x + 1\right)$$

input
```
integrate((2*x**3+3*x**2+5*x+2)/(x**2+x+1),x)
```

output
```
x**2 + x + log(x**2 + x + 1)
```

3.325.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{2+5x+3x^2+2x^3}{1+x+x^2}\,dx = x^2+x+\log\left(x^2+x+1\right)$$

input
```
integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="maxima")
```

output
```
x^2 + x + log(x^2 + x + 1)
```

3.325.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{2+5x+3x^2+2x^3}{1+x+x^2}\,dx = x^2+x+\log\left(x^2+x+1\right)$$

input
```
integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="giac")
```

output
```
x^2 + x + log(x^2 + x + 1)
```

3.325.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{2+5x+3x^2+2x^3}{1+x+x^2}\,dx = x+\ln\left(x^2+x+1\right)+x^2$$

input
```
int((5*x + 3*x^2 + 2*x^3 + 2)/(x + x^2 + 1),x)
```

output
```
x + log(x + x^2 + 1) + x^2
```

3.326 $\int \frac{3-4x-5x^2+3x^3}{x^3(-1+x+x^2)}\,dx$

3.326.1 Optimal result

Integrand size = 27, antiderivative size = 65

$$\int \frac{3-4x-5x^2+3x^3}{x^3(-1+x+x^2)}\,dx = \frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)$$

output
```
3/2/x^2-1/x+3*ln(x)-1/10*ln(1+2*x-5^(1/2))*(15-5^(1/2))-1/10*ln(1+2*x+5^(1
/2))*(15+5^(1/2))
```

3.326.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{3-4x-5x^2+3x^3}{x^3(-1+x+x^2)}\,dx = \frac{1}{10}\left(\frac{15}{x^2}-\frac{10}{x}+\left(-15+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x\right)+30\log(x)-\left(15+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)\right)$$

input
```
Integrate[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)),x]
```

output
```
(15/x^2 - 10/x + (-15 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x] + 30*Log[x] - (15
 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10
```

3.326.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^3-5x^2-4x+3}{x^3(x^2+x-1)}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(-\frac{3}{x^3}+\frac{-3x-1}{x^2+x-1}+\frac{1}{x^2}+\frac{3}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

input
```
Int[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)),x]
```

output
```
3/(2*x^2) - x^(-1) + 3*Log[x] - ((15 - Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/10
 - ((15 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10
```

3.326.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.326.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63

method	result	size
default	$-\frac{1}{x}+\frac{3}{2x^{2}}+3\ln\left(x\right)-\frac{3\ln\left(x^{2}+x-1\right)}{2}-\frac{\operatorname{arctanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)\sqrt{5}}{5}$	41
risch	$\frac{-x+\frac{3}{2}}{x^{2}}-\frac{3\ln\left(1+2x-\sqrt{5}\right)}{2}+\frac{\ln\left(1+2x-\sqrt{5}\right)\sqrt{5}}{10}-\frac{3\ln\left(1+2x+\sqrt{5}\right)}{2}-\frac{\ln\left(1+2x+\sqrt{5}\right)\sqrt{5}}{10}+3\ln\left(x\right)$	69

input
```
int((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x,method=_RETURNVERBOSE)
```

output
```
-1/x+3/2/x^2+3*ln(x)-3/2*ln(x^2+x-1)-1/5*arctanh(1/5*(1+2*x)*5^(1/2))*5^(1
/2)
```

3.326.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)}\,dx = \frac{\sqrt{5}x^{2}\log\left(\frac{2\,x^{2}-\sqrt{5}(2\,x+1)+2\,x+3}{x^{2}+x-1}\right)-15\,x^{2}\log\left(x^{2}+x-1\right)+30\,x^{2}\log\left(x\right)-10\,x+15}{10\,x^{2}}$$

input
```
integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="fricas")
```

output
```
1/10*(sqrt(5)*x^2*log((2*x^2 - sqrt(5)*(2*x + 1) + 2*x + 3)/(x^2 + x - 1))
 - 15*x^2*log(x^2 + x - 1) + 30*x^2*log(x) - 10*x + 15)/x^2
```

3.326.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.52

$$\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)}\,dx = 3\log\left(x\right) + \left(-\frac{3}{2}+\frac{\sqrt{5}}{10}\right)\log\left(x-\frac{405}{202}-\frac{35\sqrt{5}}{202}+\frac{110\left(-\frac{3}{2}+\frac{\sqrt{5}}{10}\right)^2}{101}\right) + \left(-\frac{3}{2}-\frac{\sqrt{5}}{10}\right)\log\left(x-\frac{405}{202}+\frac{35\sqrt{5}}{202}+\frac{110\left(-\frac{3}{2}-\frac{\sqrt{5}}{10}\right)^2}{101}\right) + \frac{3-2x}{2x^2}$$

input
```
integrate((3*x**3-5*x**2-4*x+3)/x**3/(x**2+x-1),x)
```

output
```
3*log(x) + (-3/2 + sqrt(5)/10)*log(x - 405/202 - 35*sqrt(5)/202 + 110*(-3/
2 + sqrt(5)/10)**2/101) + (-3/2 - sqrt(5)/10)*log(x - 405/202 + 35*sqrt(5)
/202 + 110*(-3/2 - sqrt(5)/10)**2/101) + (3 - 2*x)/(2*x**2)
```

3.326.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)}\,dx = \frac{1}{10}\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}+1}{2\,x+\sqrt{5}+1}\right) - \frac{2\,x-3}{2\,x^2} - \frac{3}{2}\,\log\left(x^2+x-1\right) + 3\,\log\left(x\right)$$

input
```
integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="maxima")
```

output
```
1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/2*(2*x - 3)/
x^2 - 3/2*log(x^2 + x - 1) + 3*log(x)
```

3.326.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)}\,dx = \frac{1}{10}\sqrt{5}\log\left(\frac{\left|2\,x-\sqrt{5}+1\right|}{\left|2\,x+\sqrt{5}+1\right|}\right)-\frac{2\,x-3}{2\,x^2}-\frac{3}{2}\,\log\left(\left|x^2+x-1\right|\right)+3\,\log\left(\left|x\right|\right)$$

input
```
integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="giac")
```

output
```
1/10*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) - 1/2*(2*x
 - 3)/x^2 - 3/2*log(abs(x^2 + x - 1)) + 3*log(abs(x))
```

3.326.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)}\,dx = 3\,\ln\left(x\right)-\frac{x-\frac{3}{2}}{x^2}+\ln\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\left(\frac{\sqrt{5}}{10}-\frac{3}{2}\right)-\ln\left(x+\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\left(\frac{\sqrt{5}}{10}+\frac{3}{2}\right)$$

input
```
int(-(4*x + 5*x^2 - 3*x^3 - 3)/(x^3*(x + x^2 - 1)),x)
```

output
```
3*log(x) - (x - 3/2)/x^2 + log(x - 5^(1/2)/2 + 1/2)*(5^(1/2)/10 - 3/2) - l
og(x + 5^(1/2)/2 + 1/2)*(5^(1/2)/10 + 3/2)
```

3.327 $\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2} dx$

3.327.1 Optimal result

Integrand size = 26, antiderivative size = 28

$$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2} dx = -\frac{1}{2+2x+x^2} - \arctan(1+x) + \log(2+2x+x^2)$$

output
```
-1/(x^2+2*x+2)-arctan(1+x)+ln(x^2+2*x+2)
```

3.327.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2} dx = -\frac{1}{2+2x+x^2} - \arctan(1+x) + \log(2+2x+x^2)$$

input
```
Integrate[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]
```

output
```
-(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]
```

3.327.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2191, 27, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+5x^2+8x+4}{(x^2+2x+2)^2}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{4}\int \frac{4(2x+1)}{x^2+2x+2}dx-\frac{1}{x^2+2x+2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{2x+1}{x^2+2x+2}dx-\frac{1}{x^2+2x+2}$$

$$\Big\downarrow \text{1142}$$

$$-\int \frac{1}{x^2+2x+2}dx+\int \frac{2(x+1)}{x^2+2x+2}dx-\frac{1}{x^2+2x+2}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{1}{x^2+2x+2}dx+2\int \frac{x+1}{x^2+2x+2}dx-\frac{1}{x^2+2x+2}$$

$$\Big\downarrow \text{1082}$$

$$2\int \frac{x+1}{x^2+2x+2}dx+\int \frac{1}{-(x+1)^2-1}d(x+1)-\frac{1}{x^2+2x+2}$$

$$\Big\downarrow \text{217}$$

$$2\int \frac{x+1}{x^2+2x+2}dx-\arctan(x+1)-\frac{1}{x^2+2x+2}$$

$$\Big\downarrow \text{1103}$$

$$-\arctan(x+1)-\frac{1}{x^2+2x+2}+\log\left(x^2+2x+2\right)$$

input
```
Int[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]
```

output
```
-(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]
```

3.327.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.327. $\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2} dx$

3.327.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result
default	$-\frac{1}{x^2+2x+2}-\arctan(x+1)+\ln(x^2+2x+2)$
risch	$-\frac{1}{x^2+2x+2}-\arctan(x+1)+\ln(x^2+2x+2)$
parallelrisch	$\frac{2i\ln(x+1-i)-i\ln(x+1+i)x^2-2i\ln(x+1+i)+2\ln(x+1-i)x^2+2i\ln(x+1-i)x+2\ln(x+1+i)x^2-2+i\ln(x+1-i)x^2+4\ln(x+1-}{2x^2+4x+4}$

input
```
int((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(x^2+2*x+2)-arctan(x+1)+ln(x^2+2*x+2)
```

3.327.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2}\,dx = -\frac{(x^2+2\,x+2)\arctan(x+1)-(x^2+2\,x+2)\log(x^2+2\,x+2)+1}{x^2+2\,x+2}$$

input
```
integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="fricas")
```

output
```
-((x^2 + 2*x + 2)*arctan(x + 1) - (x^2 + 2*x + 2)*log(x^2 + 2*x + 2) + 1)/
(x^2 + 2*x + 2)
```

3.327.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2}\,dx = \log(x^2+2x+2)-\operatorname{atan}(x+1)-\frac{1}{x^2+2x+2}$$

input
```
integrate((2*x**3+5*x**2+8*x+4)/(x**2+2*x+2)**2,x)
```

output
```
log(x**2 + 2*x + 2) - atan(x + 1) - 1/(x**2 + 2*x + 2)
```

3.327.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2}\,dx = -\frac{1}{x^2+2\,x+2} - \arctan\left(x+1\right) + \log\left(x^2+2\,x+2\right)$$

input
```
integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="maxima")
```

output
```
-1/(x^2 + 2*x + 2) - arctan(x + 1) + log(x^2 + 2*x + 2)
```

3.327.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2}\,dx = -\frac{1}{x^2+2\,x+2} - \arctan\left(x+1\right) + \log\left(x^2+2\,x+2\right)$$

input
```
integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="giac")
```

output
```
-1/(x^2 + 2*x + 2) - arctan(x + 1) + log(x^2 + 2*x + 2)
```

3.327.9 Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2}\,dx = \ln\left(x^2+2\,x+2\right) - \mathrm{atan}(x+1) - \frac{1}{x^2+2\,x+2}$$

input
```
int((8*x + 5*x^2 + 2*x^3 + 4)/(2*x + x^2 + 2)^2,x)
```

output
```
log(2*x + x^2 + 2) - atan(x + 1) - 1/(2*x + x^2 + 2)
```

3.328 $\int \frac{(-1+x)^4x^4}{1+x^2}\,dx$

3.328.1 Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{(-1+x)^4x^4}{1+x^2}\,dx = 4x-\frac{4x^3}{3}+x^5-\frac{2x^6}{3}+\frac{x^7}{7}-4\arctan(x)$$

output
```
4*x-4/3*x^3+x^5-2/3*x^6+1/7*x^7-4*arctan(x)
```

3.328.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(-1+x)^4x^4}{1+x^2}\,dx = 4x-\frac{4x^3}{3}+x^5-\frac{2x^6}{3}+\frac{x^7}{7}-4\arctan(x)$$

input
```
Integrate[((-1 + x)^4*x^4)/(1 + x^2),x]
```

output
```
4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]
```

3.328.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {525, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x-1)^4x^4}{x^2+1}dx$$

$$\Big\downarrow \text{525}$$

$$\int \frac{x^4(-4x^3+5x^2-4x+1)}{x^2+1}dx+\frac{x^7}{7}$$

$$\Big\downarrow \text{2333}$$

$$\int \left(-4x^5+5x^4-4x^2-\frac{4}{x^2+1}+4\right)dx+\frac{x^7}{7}$$

$$\Big\downarrow \text{2009}$$

$$-4\arctan(x)+\frac{x^7}{7}-\frac{2x^6}{3}+x^5-\frac{4x^3}{3}+4x$$

input
```
Int[((-1 + x)^4*x^4)/(1 + x^2),x]
```

output
```
4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]
```

3.328.3.1 Defintions of rubi rules used

rule 525
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_))^(n_))/((a_) + (b_.)*(x_)^2), x_Symbol]
:> Simp[d^n*(x^(m + n - 1)/(b*(m + n - 1))), x] + Simp[1/b   Int[x^m*(Expan
dToSum[b*(c + d*x)^n - b*d^n*x^n - a*d^n*x^(n - 2), x]/(a + b*x^2)), x], x]
 /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 1] && IGtQ[m, -2] && NeQ[m + n - 1, 0
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.328.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result
default	$4x-\frac{4x^3}{3}+x^5-\frac{2x^6}{3}+\frac{x^7}{7}-4\arctan\left(x\right)$
risch	$4x-\frac{4x^3}{3}+x^5-\frac{2x^6}{3}+\frac{x^7}{7}-4\arctan\left(x\right)$
parallelrisch	$\frac{x^7}{7}-\frac{2x^6}{3}+x^5-\frac{4x^3}{3}+4x+2i\ln\left(x-i\right)-2i\ln\left(x+i\right)$
meijerg	$-\frac{x(-45x^6+63x^4-105x^2+315)}{315}-4\arctan\left(x\right)-\frac{x^2(4x^4-6x^2+12)}{6}+\frac{2x(21x^4-35x^2+105)}{35}+\frac{x^2(-3x^2+6)}{3}-$

input
```
int((x-1)^4*x^4/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
4*x-4/3*x^3+x^5-2/3*x^6+1/7*x^7-4*arctan(x)
```

3.328.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{(-1+x)^4x^4}{1+x^2}\,dx=\frac{1}{7}\,x^7-\frac{2}{3}\,x^6+x^5-\frac{4}{3}\,x^3+4\,x-4\,\arctan\left(x\right)$$

input
```
integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="fricas")
```

output
```
1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

3.328.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{(-1+x)^4x^4}{1+x^2}\,dx = \frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\,\mathrm{atan}\,(x)$$

input
```
integrate((-1+x)**4*x**4/(x**2+1),x)
```

output
```
x**7/7 - 2*x**6/3 + x**5 - 4*x**3/3 + 4*x - 4*atan(x)
```

3.328.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{(-1+x)^4x^4}{1+x^2}\,dx = \frac{1}{7}\,x^7 - \frac{2}{3}\,x^6 + x^5 - \frac{4}{3}\,x^3 + 4\,x - 4\,\arctan\,(x)$$

input
```
integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="maxima")
```

output
```
1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

3.328.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{(-1+x)^4x^4}{1+x^2}\,dx = \frac{1}{7}\,x^7 - \frac{2}{3}\,x^6 + x^5 - \frac{4}{3}\,x^3 + 4\,x - 4\,\arctan\,(x)$$

input
```
integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="giac")
```

output
```
1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

3.328.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{(-1+x)^4x^4}{1+x^2}\,dx = 4\,x - 4\,\mathrm{atan}(x) - \frac{4\,x^3}{3} + x^5 - \frac{2\,x^6}{3} + \frac{x^7}{7}$$

input
```
int((x^4*(x - 1)^4)/(x^2 + 1),x)
```

output
```
4*x - 4*atan(x) - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7
```

3.329 $\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx$

3.329.1 Optimal result

Integrand size = 22, antiderivative size = 31

$$\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx=\log(1-x)-\frac{1}{2}\log(3-x)+\frac{3}{2}\log(1+x)-2\log(3+x)$$

output
```
ln(1-x)-1/2*ln(3-x)+3/2*ln(1+x)-2*ln(3+x)
```

3.329.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx=4\left(\frac{1}{4}\log(1-x)-\frac{1}{8}\log(3-x)+\frac{3}{8}\log(1+x)-\frac{1}{2}\log(3+x)\right)$$

input
```
Integrate[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4),x]
```

output
```
4*(Log[1 - x]/4 - Log[3 - x]/8 + (3*Log[1 + x])/8 - Log[3 + x]/2)
```

3.329.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2027, 2193, 27, 1432, 1081, 1450, 220, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2-20x}{x^4-10x^2+9}dx$$
$$\Big\downarrow \text{ 2027}$$
$$\int \frac{x(4x-20)}{x^4-10x^2+9}dx$$
$$\Big\downarrow \text{ 2193}$$
$$\int -\frac{20x}{x^4-10x^2+9}dx+\int \frac{4x^2}{x^4-10x^2+9}dx$$
$$\Big\downarrow \text{ 27}$$
$$4\int \frac{x^2}{x^4-10x^2+9}dx-20\int \frac{x}{x^4-10x^2+9}dx$$
$$\Big\downarrow \text{ 1432}$$
$$4\int \frac{x^2}{x^4-10x^2+9}dx-10\int \frac{1}{x^4-10x^2+9}dx^2$$
$$\Big\downarrow \text{ 1081}$$
$$4\int \frac{x^2}{x^4-10x^2+9}dx-10\int \left(\frac{1}{8(1-x^2)}-\frac{1}{8(9-x^2)}\right)dx^2$$
$$\Big\downarrow \text{ 1450}$$
$$4\left(\frac{9}{8}\int \frac{1}{x^2-9}dx-\frac{1}{8}\int \frac{1}{x^2-1}dx\right)-10\int \left(\frac{1}{8(1-x^2)}-\frac{1}{8(9-x^2)}\right)dx^2$$
$$\Big\downarrow \text{ 220}$$
$$4\left(\frac{\text{arctanh}(x)}{8}-\frac{3}{8}\text{arctanh}\left(\frac{x}{3}\right)\right)-10\int \left(\frac{1}{8(1-x^2)}-\frac{1}{8(9-x^2)}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$4\left(\frac{\text{arctanh}(x)}{8}-\frac{3}{8}\text{arctanh}\left(\frac{x}{3}\right)\right)-10\left(\frac{1}{8}\log\left(9-x^2\right)-\frac{1}{8}\log\left(1-x^2\right)\right)$$

input
```
Int[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4),x]
```

output
```
4*((-3*ArcTanh[x/3])/8 + ArcTanh[x]/8) - 10*(-1/8*Log[1 - x^2] + Log[9 - x
^2]/8)
```

3.329.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.329. $\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx$

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

3.329.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\ln(-3+x)}{2}+\frac{3\ln(x+1)}{2}-2\ln\left(3+x\right)+\ln\left(x-1\right)$	24
norman	$-\frac{\ln(-3+x)}{2}+\frac{3\ln(x+1)}{2}-2\ln\left(3+x\right)+\ln\left(x-1\right)$	24
risch	$-\frac{\ln(-3+x)}{2}+\frac{3\ln(x+1)}{2}-2\ln\left(3+x\right)+\ln\left(x-1\right)$	24
parallelrisch	$-\frac{\ln(-3+x)}{2}+\frac{3\ln(x+1)}{2}-2\ln\left(3+x\right)+\ln\left(x-1\right)$	24

input
```
int((4*x^2-20*x)/(x^4-10*x^2+9),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(-3+x)+3/2*ln(x+1)-2*ln(3+x)+ln(x-1)
```

3.329.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx = -2\,\log\left(x+3\right)+\frac{3}{2}\,\log\left(x+1\right)+\log\left(x-1\right)-\frac{1}{2}\,\log\left(x-3\right)$$

input
```
integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="fricas")
```

output
```
-2*log(x + 3) + 3/2*log(x + 1) + log(x - 1) - 1/2*log(x - 3)
```

3.329.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx = -\frac{\log{(x-3)}}{2} + \log{(x-1)} + \frac{3\log{(x+1)}}{2} - 2\log{(x+3)}$$

input
```
integrate((4*x**2-20*x)/(x**4-10*x**2+9),x)
```

output
```
-log(x - 3)/2 + log(x - 1) + 3*log(x + 1)/2 - 2*log(x + 3)
```

3.329.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx = -2\,\log\left(x+3\right) + \frac{3}{2}\,\log\left(x+1\right) + \log\left(x-1\right) - \frac{1}{2}\,\log\left(x-3\right)$$

input
```
integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="maxima")
```

output
```
-2*log(x + 3) + 3/2*log(x + 1) + log(x - 1) - 1/2*log(x - 3)
```

3.329.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx = -2\,\log\left(|x+3|\right) + \frac{3}{2}\,\log\left(|x+1|\right) + \log\left(|x-1|\right) - \frac{1}{2}\,\log\left(|x-3|\right)$$

input
```
integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="giac")
```

output
```
-2*log(abs(x + 3)) + 3/2*log(abs(x + 1)) + log(abs(x - 1)) - 1/2*log(abs(x
 - 3))
```

3.329.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx = \ln(x-1) + \frac{3\ln(x+1)}{2} - \frac{\ln(x-3)}{2} - 2\ln(x+3)$$

input
```
int(-(20*x - 4*x^2)/(x^4 - 10*x^2 + 9),x)
```

output
```
log(x - 1) + (3*log(x + 1))/2 - log(x - 3)/2 - 2*log(x + 3)
```

3.330 $\int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)}\,dx$

3.330.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)}\,dx = -\frac{1}{x}+\arctan(x)+2\log(1-x)-\log\left(1+x^2\right)$$

output
```
-1/x+arctan(x)+2*ln(1-x)-ln(x^2+1)
```

3.330.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)}\,dx = -\frac{1}{x}+\arctan(x)+2\log(1-x)-\log\left(1+x^2\right)$$

input
```
Integrate[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)),x]
```

output
```
-x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]
```

3.330.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^3+x-1}{(x-1)x^2(x^2+1)}dx$$

$$\Big\downarrow \text{ 2353}$$

$$\int \left(\frac{1-2x}{x^2+1}+\frac{1}{x^2}+\frac{2}{x-1}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\arctan(x)-\log\left(x^2+1\right)-\frac{1}{x}+2\log(1-x)$$

input
```
Int[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)),x]
```

output
```
-x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]
```

3.330.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

3.330.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$-\ln\left(x^2+1\right)+\arctan\left(x\right)-\frac{1}{x}+2\ln\left(x-1\right)$	23
risch	$-\ln\left(x^2+1\right)+\arctan\left(x\right)-\frac{1}{x}+2\ln\left(x-1\right)$	23
parallelrisch	$\frac{-i\ln(x-i)x+i\ln(x+i)x+4\ln(x-1)x-2\ln(x-i)x-2\ln(x+i)x-2}{2x}$	49

input
```
int((4*x^3+x-1)/(x-1)/x^2/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-ln(x^2+1)+arctan(x)-1/x+2*ln(x-1)
```

3.330.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{-1+x+4x^3}{(-1+x)x^2\left(1+x^2\right)}\,dx=\frac{x\arctan\left(x\right)-x\log\left(x^2+1\right)+2\,x\log\left(x-1\right)-1}{x}$$

input
```
integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="fricas")
```

output
```
(x*arctan(x) - x*log(x^2 + 1) + 2*x*log(x - 1) - 1)/x
```

3.330.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{-1+x+4x^3}{(-1+x)x^2\left(1+x^2\right)}\,dx=2\log\left(x-1\right)-\log\left(x^2+1\right)+\operatorname{atan}\left(x\right)-\frac{1}{x}$$

input
```
integrate((4*x**3+x-1)/(-1+x)/x**2/(x**2+1),x)
```

output
```
2*log(x - 1) - log(x**2 + 1) + atan(x) - 1/x
```

3.330.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{-1+x+4x^3}{(-1+x)x^2\left(1+x^2\right)}\,dx = -\frac{1}{x} + \arctan(x) - \log\left(x^2+1\right) + 2\log(x-1)$$

input
```
integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="maxima")
```

output
```
-1/x + arctan(x) - log(x^2 + 1) + 2*log(x - 1)
```

3.330.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{-1+x+4x^3}{(-1+x)x^2\left(1+x^2\right)}\,dx = -\frac{1}{x} + \arctan(x) - \log\left(x^2+1\right) + 2\log(|x-1|)$$

input
```
integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="giac")
```

output
```
-1/x + arctan(x) - log(x^2 + 1) + 2*log(abs(x - 1))
```

3.330.9 Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{-1+x+4x^3}{(-1+x)x^2\left(1+x^2\right)}\,dx = 2\ln(x-1) - \frac{1}{x} + \ln(x-\mathrm{i})\left(-1-\frac{1}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-1+\frac{1}{2}\mathrm{i}\right)$$

input
```
int((x + 4*x^3 - 1)/(x^2*(x^2 + 1)*(x - 1)),x)
```

output
```
2*log(x - 1) - log(x - 1i)*(1 + 1i/2) - log(x + 1i)*(1 - 1i/2) - 1/x
```

3.331 $\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3}\,dx$

3.331.1 Optimal result

Integrand size = 26, antiderivative size = 23

$$\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3}\,dx = -\frac{1}{4(1+x^2)^2}+\frac{2}{1+x^2}+\arctan(x)$$

output
```
-1/4/(x^2+1)^2+2/(x^2+1)+arctan(x)
```

3.331.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3}\,dx = -\frac{1}{4(1+x^2)^2}+\frac{2}{1+x^2}+\arctan(x)$$

input
```
Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]
```

output
```
-1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]
```

3.331.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2345, 27, 2345, 27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-4x^3+2x^2-3x+1}{(x^2+1)^3}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{1}{4}\int -\frac{4(x^2-4x+1)}{(x^2+1)^2}dx-\frac{1}{4(x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^2-4x+1}{(x^2+1)^2}dx-\frac{1}{4(x^2+1)^2}$$

$$\Big\downarrow \text{2345}$$

$$-\frac{1}{2}\int -\frac{2}{x^2+1}dx+\frac{2}{x^2+1}-\frac{1}{4(x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^2+1}dx+\frac{2}{x^2+1}-\frac{1}{4(x^2+1)^2}$$

$$\Big\downarrow \text{216}$$

$$\arctan(x)+\frac{2}{x^2+1}-\frac{1}{4(x^2+1)^2}$$

input
```
Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]
```

output
```
-1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]
```

3.331.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.331.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2x^2+\frac{7}{4}}{(x^2+1)^2}+\arctan(x)$	19
risch	$\frac{2x^2+\frac{7}{4}}{(x^2+1)^2}+\arctan(x)$	19
parallelrisch	$-\frac{2i\ln(x-i)x^4-2i\ln(x+i)x^4-3+4i\ln(x-i)x^2-4i\ln(x+i)x^2+4x^4+2i\ln(x-i)-2i\ln(x+i)}{4(x^2+1)^2}$	77
meijerg	$\frac{x(3x^2+5)}{8(x^2+1)^2}+\arctan(x)-\frac{x(25x^2+15)}{40(x^2+1)^2}-\frac{x^4}{(x^2+1)^2}-\frac{x(-3x^2+3)}{12(x^2+1)^2}-\frac{3x^2(x^2+2)}{4(x^2+1)^2}$	84

input
```
int((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*x^2+7/4)/(x^2+1)^2+arctan(x)
```

3.331.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3}\,dx = \frac{8\,x^2+4\,(x^4+2\,x^2+1)\arctan(x)+7}{4\,(x^4+2\,x^2+1)}$$

input
```
integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="fricas")
```

output
```
1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*arctan(x) + 7)/(x^4 + 2*x^2 + 1)
```

3.331.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3}\,dx = \frac{8x^2+7}{4x^4+8x^2+4}+\operatorname{atan}(x)$$

input
```
integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**2+1)**3,x)
```

output
```
(8*x**2 + 7)/(4*x**4 + 8*x**2 + 4) + atan(x)
```

3.331.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3}\,dx = \frac{8\,x^2+7}{4\,(x^4+2\,x^2+1)}+\arctan(x)$$

input
```
integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="maxima")
```

output
```
1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + arctan(x)
```

3.331.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3}\,dx = \frac{8\,x^2+7}{4\,(x^2+1)^2} + \arctan(x)$$

input
```
integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="giac")
```

output
```
1/4*(8*x^2 + 7)/(x^2 + 1)^2 + arctan(x)
```

3.331.9 Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3}\,dx = \operatorname{atan}(x) + \frac{2\,x^2+\frac{7}{4}}{x^4+2\,x^2+1}$$

input
```
int((2*x^2 - 3*x - 4*x^3 + x^4 + 1)/(x^2 + 1)^3,x)
```

output
```
atan(x) + (2*x^2 + 7/4)/(2*x^2 + x^4 + 1)
```

3.332 $\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx$

3.332.1 Optimal result

Integrand size = 36, antiderivative size = 23

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx=-\frac{1}{4\left(1+x^2\right)^2}+\frac{2}{1+x^2}+\arctan(x)$$

output
```
-1/4/(x^2+1)^2+2/(x^2+1)+arctan(x)
```

3.332.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx=-\frac{1}{4\left(1+x^2\right)^2}+\frac{2}{1+x^2}+\arctan(x)$$

input
```
Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6),x]
```

output
```
-1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]
```

3.332.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2070, 2345, 27, 2345, 27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-4x^3+2x^2-3x+1}{x^6+3x^4+3x^2+1} \, dx$$

$$\Big\downarrow \text{ 2070}$$

$$\int \frac{x^4-4x^3+2x^2-3x+1}{(x^2+1)^3}dx$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{1}{4}\int -\frac{4(x^2-4x+1)}{(x^2+1)^2}dx-\frac{1}{4(x^2+1)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^2-4x+1}{(x^2+1)^2}dx-\frac{1}{4(x^2+1)^2}$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{1}{2}\int -\frac{2}{x^2+1}dx+\frac{2}{x^2+1}-\frac{1}{4(x^2+1)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{x^2+1}dx+\frac{2}{x^2+1}-\frac{1}{4(x^2+1)^2}$$

$$\Big\downarrow \text{ 216}$$

$$\arctan(x)+\frac{2}{x^2+1}-\frac{1}{4(x^2+1)^2}$$

input
```
Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6),x]
```

output
```
-1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]
```

3.332.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2070
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x^2, 0], Expon[Px
, x^2]], b = Rt[Coeff[Px, x^2, Expon[Px, x^2]], Expon[Px, x^2]]}, Int[u*(a
+ b*x^2)^(Expon[Px, x^2]*p), x] /; EqQ[Px, (a + b*x^2)^Expon[Px, x^2]]] /;
IntegerQ[p] && PolyQ[Px, x^2] && GtQ[Expon[Px, x^2], 1] && NeQ[Coeff[Px, x^
2, 0], 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.332.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2x^2+\frac{7}{4}}{(x^2+1)^2}+\arctan(x)$	19
risch	$\frac{2x^2+\frac{7}{4}}{x^4+2x^2+1}+\arctan(x)$	24
parallelrisch	$-\frac{2i\ln(x-i)x^4-2i\ln(x+i)x^4-3+4i\ln(x-i)x^2-4i\ln(x+i)x^2+4x^4+2i\ln(x-i)-2i\ln(x+i)}{4(x^4+2x^2+1)}$	82

input
```
int((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
(2*x^2+7/4)/(x^2+1)^2+arctan(x)
```

3.332.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx = \frac{8\,x^2+4\,(x^4+2\,x^2+1)\arctan(x)+7}{4\,(x^4+2\,x^2+1)}$$

input
```
integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="fricas
")
```

output
```
1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*arctan(x) + 7)/(x^4 + 2*x^2 + 1)
```

3.332.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx = \frac{8x^2+7}{4x^4+8x^2+4}+\operatorname{atan}(x)$$

input
```
integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**6+3*x**4+3*x**2+1),x)
```

output
```
(8*x**2 + 7)/(4*x**4 + 8*x**2 + 4) + atan(x)
```

3.332.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx = \frac{8\,x^2+7}{4\,(x^4+2\,x^2+1)}+\arctan(x)$$

input
```
integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="maxima
")
```

output
```
1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + arctan(x)
```

3.332.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx=\frac{8\,x^2+7}{4\,(x^2+1)^2}+\arctan(x)$$

input
```
integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="giac")
```

output
```
1/4*(8*x^2 + 7)/(x^2 + 1)^2 + arctan(x)
```

3.332.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx=\mathrm{atan}(x)+\frac{2\,x^2+\frac{7}{4}}{x^4+2\,x^2+1}$$

input
```
int((2*x^2 - 3*x - 4*x^3 + x^4 + 1)/(3*x^2 + 3*x^4 + x^6 + 1),x)
```

output
```
atan(x) + (2*x^2 + 7/4)/(2*x^2 + x^4 + 1)
```

3.333 $\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4} dx$

3.333.1 Optimal result

Integrand size = 26, antiderivative size = 13

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4} dx = -\frac{1}{x} + \log \left(1+x+x^2\right)$$

output
```
-1/x+ln(x^2+x+1)
```

3.333.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4} dx = -\frac{1}{x} + \log \left(1+x+x^2\right)$$

input
```
Integrate[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4),x]
```

output
```
-x^(-1) + Log[1 + x + x^2]
```

3.333.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2026, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+2x^2+x+1}{x^4+x^3+x^2}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{2x^3+2x^2+x+1}{x^2(x^2+x+1)}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{2x+1}{x^2+x+1}+\frac{1}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log\left(x^2+x+1\right)-\frac{1}{x}$$

input
```
Int[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4),x]
```

output
```
-x^(-1) + Log[1 + x + x^2]
```

3.333.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.333.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{1}{x}+\ln\left(x^{2}+x+1\right)$	14
norman	$-\frac{1}{x}+\ln\left(x^{2}+x+1\right)$	14
risch	$-\frac{1}{x}+\ln\left(x^{2}+x+1\right)$	14
parallelrisch	$\frac{\ln(x^{2}+x+1)x-1}{x}$	16

input
```
int((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/x+ln(x^2+x+1)
```

3.333.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx=\frac{x\log\left(x^2+x+1\right)-1}{x}$$

input
```
integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="fricas")
```

output
```
(x*log(x^2 + x + 1) - 1)/x
```

3.333.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx=\log\left(x^2+x+1\right)-\frac{1}{x}$$

input
```
integrate((2*x**3+2*x**2+x+1)/(x**4+x**3+x**2),x)
```

output
```
log(x**2 + x + 1) - 1/x
```

3.333.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx = -\frac{1}{x} + \log\left(x^2+x+1\right)$$

input
```
integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="maxima")
```

output
```
-1/x + log(x^2 + x + 1)
```

3.333.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx = -\frac{1}{x} + \log\left(x^2+x+1\right)$$

input
```
integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="giac")
```

output
```
-1/x + log(x^2 + x + 1)
```

3.333.9 Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx = \ln\left(x^2+x+1\right) - \frac{1}{x}$$

input
```
int((x + 2*x^2 + 2*x^3 + 1)/(x^2 + x^3 + x^4),x)
```

output
```
log(x + x^2 + 1) - 1/x
```

3.334 $\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx$

3.334.1 Optimal result

Integrand size = 20, antiderivative size = 206

$$\begin{aligned}\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx &= \frac{2cdx}{b}+\frac{d^2x^2}{2b}+\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}\\ &\quad-\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}\\ &\quad+\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}}+\frac{c^2\log\left(a+bx^3\right)}{3b}\end{aligned}$$

output
```
2*c*d*x/b+1/2*d^2*x^2/b-1/3*a^(1/3)*d*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b
^(1/3)*x)/b^(5/3)+1/6*a^(1/3)*d*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/b^(5/3)+1/3*c^2*ln(b*x^3+a)/b+1/3*a^(1/3)*d*(2*b^(
1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3
)*3^(1/2)
```

3.334.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.94

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx$$
$$= \frac{12b^{2/3}cdx + 3b^{2/3}d^2x^2 + 2\sqrt{3}\sqrt[3]{a}d\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\sqrt[3]{a}d\left(-2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a} \right.}{6b^{5/3}}$$

input
```
Integrate[(x^2*(c + d*x)^2)/(a + b*x^3),x]
```

output
```
(12*b^(2/3)*c*d*x + 3*b^(2/3)*d^2*x^2 + 2*Sqrt[3]*a^(1/3)*d*(2*b^(1/3)*c +
 a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*a^(1/3)*d*(-2*
b^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x] - a^(1/3)*d*(-2*b^(1/3)*c
+ a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)*c^
2*Log[a + b*x^3])/(6*b^(5/3))
```

3.334.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx$$
$$\Big\downarrow \text{2426}$$
$$\int \left(-\frac{2acd+ad^2x-bc^2x^2}{b\left(a+bx^3\right)}+\frac{2cd}{b}+\frac{d^2x}{b}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}}+\frac{\sqrt[3]{a}d\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)}{\sqrt{3}b^{5/3}}-\frac{\sqrt[3]{a}d\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{c^2\log\left(a+bx^3\right)}{3b}+\frac{2cdx}{b}+\frac{d^2x^2}{2b}$$

input
```
Int[(x^2*(c + d*x)^2)/(a + b*x^3),x]
```

output
```
(2*c*d*x)/b + (d^2*x^2)/(2*b) + (a^(1/3)*d*(2*b^(1/3)*c + a^(1/3)*d)*ArcTa
n[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(1/3)
*d*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1
/3)*d*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(6*b^(5/3)) + (c^2*Log[a + b*x^3])/(3*b)
```

3.334.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.334.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.78 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.33

method	result
risch	$\frac{d^2x^2}{2b}+\frac{2cdx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^3b+a\right)}\frac{\left(\_R^2bc^2-\_Ra\,d^2-2acd\right)\ln\left(x-\_R\right)}{\_R^2}}{3b^2}$
default	$\frac{d\left(\frac{1}{2}d\,x^2+2cx\right)}{b}+\frac{-2acd\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-a\,d^2\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2\right.}{}\right.}{b}$

input
```
int(x^2*(d*x+c)^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*d^2*x^2/b+2*c*d*x/b+1/3/b^2*sum((_R^2*b*c^2-_R*a*d^2-2*a*c*d)/_R^2*ln(
x-_R),_R=RootOf(_Z^3*b+a))
```

3.334.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 4545, normalized size of antiderivative = 22.06

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/12*(6*d^2*x^2 + 24*c*d*x - 2*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^
3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c
^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) +
 (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d
^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) +
1) - 2*c^2/b)*b*log(1/4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)
*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*
a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^
(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2
/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*
c^2/b)^2*b^3 + 3*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sq
rt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3
)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(
2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 +
(b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*
b^2*c^2 + 5*b*c^4 + 4*a*c*d^3 + (8*b*c^3*d + a*d^4)*x) + ((2*(1/2)^(2/3)*(
c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3
+ a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*
d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d
^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2...
```

3.334.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.67

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx$$
$$= \mathrm{RootSum}\left(27t^3b^5-27t^2b^4c^2+t\left(18ab^2cd^3+9b^3c^4\right)-a^2d^6+2abc^3d^3-b^2c^6,\left(t\mapsto t\log\left(x+\frac{9t^2b^3-18\right.\right.\right.$$
$$+\frac{2cdx}{b}+\frac{d^2x^2}{2b}$$

input
```
integrate(x**2*(d*x+c)**2/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*b**5 - 27*_t**2*b**4*c**2 + _t*(18*a*b**2*c*d**3 + 9*b**3
*c**4) - a**2*d**6 + 2*a*b*c**3*d**3 - b**2*c**6, Lambda(_t, _t*log(x + (9
*_t**2*b**3 - 18*_t*b**2*c**2 + 4*a*c*d**3 + 5*b*c**4)/(a*d**4 + 8*b*c**3*
d))) + 2*c*d*x/b + d**2*x**2/(2*b)
```

3.334.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.97

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx = -\frac{\sqrt{3}\left(ad^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,acd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{d^2x^2+4\,cdx}{2\,b}$$
$$+\frac{\left(2\,bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,acd\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,acd\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*(a*d^2*(a/b)^(2/3) + 2*a*c*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/2*(d^2*x^2 + 4*c*d*x)/b + 1/6*(
2*b*c^2*(a/b)^(2/3) - a*d^2*(a/b)^(1/3) + 2*a*c*d)*log(x^2 - x*(a/b)^(1/3)
 + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*(b*c^2*(a/b)^(2/3) + a*d^2*(a/b)^(
1/3) - 2*a*c*d)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

3.334.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.01

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx = \frac{c^2\log\left(|bx^3+a|\right)}{3\,b} - \frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}bcd - \left(-ab^2\right)^{\frac{2}{3}}d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3} + \frac{bd^2x^2+4\,bcdx}{2\,b^2} - \frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}bcd + \left(-ab^2\right)^{\frac{2}{3}}d^2\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3} + \frac{\left(ab^4d^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,ab^4cd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^5}$$

input
```
integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*c^2*log(abs(b*x^3 + a))/b - 1/3*sqrt(3)*(2*(-a*b^2)^(1/3)*b*c*d - (-a*
b^2)^(2/3)*d^2)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3
+ 1/2*(b*d^2*x^2 + 4*b*c*d*x)/b^2 - 1/6*(2*(-a*b^2)^(1/3)*b*c*d + (-a*b^2)
^(2/3)*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 + 1/3*(a*b^4*d^2*
(-a/b)^(1/3) + 2*a*b^4*c*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5
)
```

3.334.9 Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.73

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx$$

$$= \left(\sum_{k=1}^{3} \ln\left(\frac{a\left(b\,c^4 + \mathrm{root}(27\,b^5\,z^3 - 27\,b^4\,c^2\,z^2 + 18\,a\,b^2\,c\,d^3\,z + 9\,b^3\,c^4\,z + 2\,a\,b\,c^3\,d^3 - b^2\,c^6 - a^2\,d^6, z, k)^2\,b\right.}{}\right.\right.$$

$$\left.\left. - 27\,b^4\,c^2\,z^2 + 18\,a\,b^2\,c\,d^3\,z + 9\,b^3\,c^4\,z + 2\,a\,b\,c^3\,d^3 - b^2\,c^6 - a^2\,d^6, z, k)\right) + \frac{d^2\,x^2}{2\,b} + \frac{2\,c\,d\,x}{b}$$

input
```
int((x^2*(c + d*x)^2)/(a + b*x^3),x)
```

output
```
symsum(log((a*(b*c^4 + 9*root(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3
*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 - a^2*d^6, z, k)^2*b^3 - 6*root
(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*
d^3 - b^2*c^6 - a^2*d^6, z, k)*b^2*c^2 + 2*a*c*d^3 + a*d^4*x + 2*b*c^3*d*x
 - 6*root(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2
*a*b*c^3*d^3 - b^2*c^6 - a^2*d^6, z, k)*b^2*c*d*x))/b)*root(27*b^5*z^3 - 2
7*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 -
 a^2*d^6, z, k), k, 1, 3) + (d^2*x^2)/(2*b) + (2*c*d*x)/b
```

3.334. $\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx$

3.335 $\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx$

3.335.1 Optimal result

Integrand size = 27, antiderivative size = 45

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \frac{5-7x^2}{8(3+2x^2+x^4)} + \frac{9\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}$$

output
```
1/8*(-7*x^2+5)/(x^4+2*x^2+3)+9/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

3.335.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \frac{5-7x^2}{8(3+2x^2+x^4)} + \frac{9\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}$$

input
```
Integrate[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[
2])
```

3.335.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2028, 2194, 25, 2191, 27, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^5+2x^3-x}{\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \text{ 2028}$$

$$\int \frac{x\left(4x^4+2x^2-1\right)}{\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \text{ 2194}$$

$$\frac{1}{2}\int -\frac{-4x^4-2x^2+1}{\left(x^4+2x^2+3\right)^2}dx^2$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{2}\int \frac{-4x^4-2x^2+1}{\left(x^4+2x^2+3\right)^2}dx^2$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{2}\left(\frac{5-7x^2}{4\left(x^4+2x^2+3\right)}-\frac{1}{8}\int -\frac{18}{x^4+2x^2+3}dx^2\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{9}{4}\int \frac{1}{x^4+2x^2+3}dx^2+\frac{5-7x^2}{4\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(\frac{5-7x^2}{4\left(x^4+2x^2+3\right)}-\frac{9}{2}\int \frac{1}{-x^4-8}d\left(2x^2+2\right)\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(\frac{9\arctan\left(\frac{2x^2+2}{2\sqrt{2}}\right)}{4\sqrt{2}}+\frac{5-7x^2}{4\left(x^4+2x^2+3\right)}\right)$$

input
```
Int[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]
```

output
```
((5 - 7*x^2)/(4*(3 + 2*x^2 + x^4)) + (9*ArcTan[(2 + 2*x^2)/(2*Sqrt[2])])/(
4*Sqrt[2]))/2
```

3.335.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.335.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{-\frac{7x^2}{8}+\frac{5}{8}}{x^4+2x^2+3}+\frac{9\arctan\left(\frac{(x^2+1)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	38
default	$\frac{-\frac{7x^2}{4}+\frac{5}{4}}{2x^4+4x^2+6}+\frac{9\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16}$	41

input
```
int((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(-7/8*x^2+5/8)/(x^4+2*x^2+3)+9/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

3.335.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \frac{9\,\sqrt{2}(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)-14\,x^2+10}{16\,(x^4+2\,x^2+3)}$$

input
```
integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/16*(9*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 14*x^2 +
 10)/(x^4 + 2*x^2 + 3)
```

3.335.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \frac{5-7x^2}{8x^4+16x^2+24} + \frac{9\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

input
```
integrate((4*x**5+2*x**3-x)/(x**4+2*x**2+3)**2,x)
```

output
```
(5 - 7*x**2)/(8*x**4 + 16*x**2 + 24) + 9*sqrt(2)*atan(sqrt(2)*x**2/2 + sqr
t(2)/2)/16
```

3.335.7 Maxima [F]

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \int \frac{4\,x^5+2\,x^3-x}{(x^4+2\,x^2+3)^2}\,dx$$

input
```
integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
-1/8*(7*x^2 - 5)/(x^4 + 2*x^2 + 3) + 9/4*integrate(x/(x^4 + 2*x^2 + 3), x)
```

3.335.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \frac{9}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{7\,x^2-5}{8\,(x^4+2\,x^2+3)}$$

input
```
integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/8*(7*x^2 - 5)/(x^4 + 2*x^2
+ 3)
```

3.335.9 Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \frac{9\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16} - \frac{\frac{7\,x^2}{8}-\frac{5}{8}}{x^4+2\,x^2+3}$$

input
```
int((2*x^3 - x + 4*x^5)/(2*x^2 + x^4 + 3)^2,x)
```

output
```
(9*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 - ((7*x^2)/8 - 5/8)/(2*x^
2 + x^4 + 3)
```

3.336 $\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx$

3.336.1 Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \frac{3+4x^2}{16(1+2x^2+2x^4)^2} + \frac{1+2x^2}{2(1+2x^2+2x^4)} + \arctan(1+2x^2)$$

output
```
1/16*(4*x^2+3)/(2*x^4+2*x^2+1)^2+1/2*(2*x^2+1)/(2*x^4+2*x^2+1)+arctan(2*x^
2+1)
```

3.336.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \frac{11+36x^2+48x^4+32x^6}{16(1+2x^2+2x^4)^2} + \arctan(1+2x^2)$$

input
```
Integrate[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]
```

output
```
(11 + 36*x^2 + 48*x^4 + 32*x^6)/(16*(1 + 2*x^2 + 2*x^4)^2) + ArcTan[1 + 2*
x^2]
```

3.336.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2027, 2194, 2191, 27, 1086, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5+x}{\left(2x^4+2x^2+1\right)^3}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x\left(x^4+1\right)}{\left(2x^4+2x^2+1\right)^3}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{x^4+1}{\left(2x^4+2x^2+1\right)^3}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(\frac{1}{8}\int \frac{16}{\left(2x^4+2x^2+1\right)^2}dx^2+\frac{4x^2+3}{8\left(2x^4+2x^2+1\right)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(2\int \frac{1}{\left(2x^4+2x^2+1\right)^2}dx^2+\frac{4x^2+3}{8\left(2x^4+2x^2+1\right)^2}\right)$$

$$\Big\downarrow \text{1086}$$

$$\frac{1}{2}\left(2\left(\int \frac{1}{2x^4+2x^2+1}dx^2+\frac{2x^2+1}{2\left(2x^4+2x^2+1\right)}\right)+\frac{4x^2+3}{8\left(2x^4+2x^2+1\right)^2}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(2\left(\frac{2x^2+1}{2\left(2x^4+2x^2+1\right)}-\int \frac{1}{-x^4-1}d\left(2x^2+1\right)\right)+\frac{4x^2+3}{8\left(2x^4+2x^2+1\right)^2}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(2\left(\arctan\left(2x^2+1\right)+\frac{2x^2+1}{2\left(2x^4+2x^2+1\right)}\right)+\frac{4x^2+3}{8\left(2x^4+2x^2+1\right)^2}\right)$$

input
```
Int[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]
```

output
```
((3 + 4*x^2)/(8*(1 + 2*x^2 + 2*x^4)^2) + 2*((1 + 2*x^2)/(2*(1 + 2*x^2 + 2*
x^4)) + ArcTan[1 + 2*x^2]))/2
```

3.336.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.336. $\int \frac{x+x^5}{\left(1+2x^2+2x^4\right)^3} \, dx$

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

3.336.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

method	result
default	$\frac{2x^6+3x^4+\frac{9}{4}x^2+\frac{11}{16}}{(2x^4+2x^2+1)^2}+\arctan(2x^2+1)$
risch	$\frac{2x^6+3x^4+\frac{9}{4}x^2+\frac{11}{16}}{(2x^4+2x^2+1)^2}+\arctan(2x^2+1)$
parallelrisch	$-\frac{-8i\ln(x^2+\frac{1}{2}+\frac{i}{2})+8i\ln(x^2+\frac{1}{2}-\frac{i}{2})-5+64i\ln(x^2+\frac{1}{2}-\frac{i}{2})x^4-32i\ln(x^2+\frac{1}{2}+\frac{i}{2})x^8+24x^8+32i\ln(x^2+\frac{1}{2}-\frac{i}{2})x^2+64i\ln(x^2+}{16(2x^4+2x^2+1)}$

input
```
int((x^5+x)/(2*x^4+2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(x^6+3/2*x^4+9/8*x^2+11/32)/(2*x^4+2*x^2+1)^2+arctan(2*x^2+1)
```

3.336.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \frac{32\,x^6+48\,x^4+36\,x^2+16\,(4\,x^8+8\,x^6+8\,x^4+4\,x^2+1)\arctan(2\,x^2+1)+11}{16\,(4\,x^8+8\,x^6+8\,x^4+4\,x^2+1)}$$

input
```
integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="fricas")
```

output
```
1/16*(32*x^6 + 48*x^4 + 36*x^2 + 16*(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)*ar
ctan(2*x^2 + 1) + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)
```

3.336.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \frac{32x^6+48x^4+36x^2+11}{64x^8+128x^6+128x^4+64x^2+16} + \operatorname{atan}\left(2x^2+1\right)$$

input
```
integrate((x**5+x)/(2*x**4+2*x**2+1)**3,x)
```

output
```
(32*x**6 + 48*x**4 + 36*x**2 + 11)/(64*x**8 + 128*x**6 + 128*x**4 + 64*x**
2 + 16) + atan(2*x**2 + 1)
```

3.336.7 Maxima [F]

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \int \frac{x^5+x}{(2\,x^4+2\,x^2+1)^3}\,dx$$

input
```
integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="maxima")
```

output
```
1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1) +
 2*integrate(x/(2*x^4 + 2*x^2 + 1), x)
```

3.336.8 Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \frac{32\,x^6+48\,x^4+36\,x^2+11}{16\,(2\,x^4+2\,x^2+1)^2} + \arctan\left(2\,x^2+1\right)$$

input
```
integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="giac")
```

output
```
1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(2*x^4 + 2*x^2 + 1)^2 + arctan(2*x^2
+ 1)
```

3.336.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \operatorname{atan}\left(2\,x^2+1\right)+\frac{\frac{x^6}{2}+\frac{3\,x^4}{4}+\frac{9\,x^2}{16}+\frac{11}{64}}{x^8+2\,x^6+2\,x^4+x^2+\frac{1}{4}}$$

input
```
int((x + x^5)/(2*x^2 + 2*x^4 + 1)^3,x)
```

output
```
atan(2*x^2 + 1) + ((9*x^2)/16 + (3*x^4)/4 + x^6/2 + 11/64)/(x^2 + 2*x^4 +
2*x^6 + x^8 + 1/4)
```

3.337 $\int \frac{a+bx+cx^2}{d+ex^2+fx^4} \, dx$

3.337.1 Optimal result

Integrand size = 25, antiderivative size = 209

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4} \, dx = \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}} - \frac{b\operatorname{arctanh}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

output
```
-b*arctanh((2*f*x^2+e)/(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)+1/2*arctan(x
*2^(1/2)*f^(1/2)/(e-(-4*d*f+e^2)^(1/2))^(1/2))*(c+(2*a*f-c*e)/(-4*d*f+e^2)
^(1/2))*2^(1/2)/f^(1/2)/(e-(-4*d*f+e^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*
f^(1/2)/(e+(-4*d*f+e^2)^(1/2))^(1/2))*(c+(-2*a*f+c*e)/(-4*d*f+e^2)^(1/2))*
2^(1/2)/f^(1/2)/(e+(-4*d*f+e^2)^(1/2))^(1/2)
```

3.337.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.12

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4} \, dx = \frac{\frac{\sqrt{2}\left(2af+c\left(-e+\sqrt{e^2-4df}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\sqrt{2}\left(-2af+c\left(e+\sqrt{e^2-4df}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}} + b\log\left(-e+\sqrt{e^2-\right.}{2\sqrt{e^2-4df}}$$

input
```
Integrate[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
```

output
```
((Sqrt[2]*(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/
Sqrt[e - Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + (Sqr
t[2]*(-2*a*f + c*(e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[
e + Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) + b*Log[-e
+ Sqrt[e^2 - 4*d*f] - 2*f*x^2] - b*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x^2])/(
2*Sqrt[e^2 - 4*d*f])
```

3.337.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2202, 27, 1432, 1083, 219, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4} \, dx$$

$$\Big\downarrow \; 2202$$

$$\int \frac{cx^2+a}{fx^4+ex^2+d}dx+\int \frac{bx}{fx^4+ex^2+d}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{cx^2+a}{fx^4+ex^2+d}dx+b\int \frac{x}{fx^4+ex^2+d}dx$$

$$\Big\downarrow \; 1432$$

$$\int \frac{cx^2+a}{fx^4+ex^2+d}dx+\frac{1}{2}b\int \frac{1}{fx^4+ex^2+d}dx^2$$

$$\Big\downarrow \; 1083$$

$$\int \frac{cx^2+a}{fx^4+ex^2+d}dx-b\int \frac{1}{-x^4+e^2-4df}d\left(2fx^2+e\right)$$

$$\Big\downarrow \; 219$$

$$\int \frac{cx^2+a}{fx^4+ex^2+d}dx-\frac{b\,\text{arctanh}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

$$\downarrow \text{1480}$$

$$\frac{1}{2}\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\int \frac{1}{fx^2+\frac{1}{2}\left(e-\sqrt{e^2-4df}\right)}dx+$$
$$\frac{1}{2}\left(\frac{ce-2af}{\sqrt{e^2-4df}}+c\right)\int \frac{1}{fx^2+\frac{1}{2}\left(e+\sqrt{e^2-4df}\right)}dx-\frac{b\text{arctanh}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

$$\downarrow \text{218}$$

$$\frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\left(\frac{ce-2af}{\sqrt{e^2-4df}}+c\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{2}\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}}-$$
$$\frac{b\text{arctanh}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

input
```
Int[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
```

output
```
((c - (c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e -
 Sqrt[e^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + ((c
+ (c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e + Sqr
t[e^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) - (b*ArcTa
nh[(e + 2*f*x^2)/Sqrt[e^2 - 4*d*f]])/Sqrt[e^2 - 4*d*f]
```

3.337.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

3.337.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(f\_Z^{4}+e\_Z^{2}+d\right)}\frac{\left(c\_R^{2}+b\_R+a\right)\ln\left(x-\_R\right)}{2\_R^{3}f+\_Re}\right)}{2}$
default	$4f\left(-\frac{\sqrt{-4df+e^{2}}\left(\frac{b\ln\left(-2fx^{2}+\sqrt{-4df+e^{2}}-e\right)}{2}+\frac{\left(-\sqrt{-4df+e^{2}}\,c-2af+ec\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{fx\sqrt{2}}{\sqrt{\left(\sqrt{-4df+e^{2}}-e\right)f}}\right)}{2\sqrt{\left(\sqrt{-4df+e^{2}}-e\right)f}}\right)}{4f\left(4df-e^{2}\right)}-\frac{\sqrt{-4df+e^{2}}\left(-\right.}{}\right.$

input
```
int((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((_R^2*c+_R*b+a)/(2*_R^3*f+_R*e)*ln(x-_R),_R=RootOf(_Z^4*f+_Z^2*e+d
))
```

3.337.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.02 (sec) , antiderivative size = 578003, normalized size of antiderivative = 2765.56

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.337.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/(f*x**4+e*x**2+d),x)
```

output
```
Timed out
```

3.337.7 Maxima [F]

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx = \int \frac{cx^2+bx+a}{fx^4+ex^2+d}\,dx$$

input
```
integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)/(f*x^4 + e*x^2 + d), x)
```

3.337.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1714 vs. $2(171) = 342$.

Time = 1.12 (sec) , antiderivative size = 1714, normalized size of antiderivative = 8.20

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="giac")
```

output
```
-1/2*(e^2*f^2 - 4*d*f^3 - 2*e*f^3 + f^4)*sqrt(e^2 - 4*d*f)*b*log(x^2 + 1/2
*(e - sqrt(e^2 - 4*d*f))/f)/((e^4 - 8*d*e^2*f - 2*e^3*f + 16*d^2*f^2 + 8*d
*e*f^2 + e^2*f^2 - 4*d*f^3)*f^2) + 1/4*((sqrt(2)*sqrt(e*f + sqrt(e^2 - 4*d
*f)*f)*e^4 - 8*sqrt(2)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*e^2*f - 2*sqrt(2)
*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*e^3*f - 2*e^4*f + 16*sqrt(2)*sqrt(e*f + s
qrt(e^2 - 4*d*f)*f)*d^2*f^2 + 8*sqrt(2)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*
e*f^2 + sqrt(2)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*e^2*f^2 + 16*d*e^2*f^2 + 2
*e^3*f^2 - 4*sqrt(2)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*f^3 - 32*d^2*f^3 -
8*d*e*f^3 - sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*e^3
+ 4*sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*e*f + 2*sq
rt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*e^2*f - sqrt(2)*sq
rt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*e*f^2 + 2*(e^2 - 4*d*f)*e^
2*f - 8*(e^2 - 4*d*f)*d*f^2 - 2*(e^2 - 4*d*f)*e*f^2)*a - 2*(2*d*e^2*f^2 -
8*d^2*f^3 - sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*e^
2 + 4*sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d^2*f + 2*
sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*e*f - sqrt(2)*
sqrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*f^2 - 2*(e^2 - 4*d*f)*
d*f^2)*c)*arctan(2*sqrt(1/2)*x/sqrt((e + sqrt(e^2 - 4*d*f))/f))/((d*e^4 -
8*d^2*e^2*f - 2*d*e^3*f + 16*d^3*f^2 + 8*d^2*e*f^2 + d*e^2*f^2 - 4*d^2*f^3
)*abs(f)) + 1/4*((sqrt(2)*sqrt(e*f - sqrt(e^2 - 4*d*f)*f)*e^4 - 8*sqrt(...
```

3.337.9 Mupad [B] (verification not implemented)

Time = 10.02 (sec) , antiderivative size = 3942, normalized size of antiderivative = 18.86

$$\int \frac{a + bx + cx^2}{d + ex^2 + fx^4} \, dx = \text{Too large to display}$$

input
```
int((a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x)
```

output
```
symsum(log(a*b^2*f^2 - a^2*c*f^2 + b^3*f^2*x - c^3*d*f - 8*root(16*d*e^4*f
*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2
*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2
 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f
*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f
+ 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f +
b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^3*e^3*f^2*x + a*c^2*e*f -
 16*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d
*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 +
 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z
^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2
*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d
*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*a*d*
f^3 - 4*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a
*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z
^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3
*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d
*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c
^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*a^
2*f^3*x + 4*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4...
```

3.337. $\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx$

3.338 $\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx$

3.338.1 Optimal result

Integrand size = 22, antiderivative size = 224

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{2de\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
-2*d*e*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)+1/2*arct
an(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e^2+(-b*e^2+2*c*d^2)/(
-4*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan
(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e^2+(b*e^2-2*c*d^2)/(-4*
a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

3.338.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \frac{\frac{\sqrt{2}\left(2cd^2+\left(-b+\sqrt{b^2-4ac}\right)e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(-2cd^2+\left(b+\sqrt{b^2-4ac}\right)e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} + 2de\log\left(-b+\sqrt{b\right.}}{2\sqrt{b^2-4ac}}$$

input
```
Integrate[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```

output
```
((Sqrt[2]*(2*c*d^2 + (-b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
(Sqrt[2]*(-2*c*d^2 + (b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 2
*d*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - 2*d*e*Log[b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2])/(2*Sqrt[b^2 - 4*a*c])
```

3.338.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2202, 27, 1432, 1083, 219, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{d^2+e^2x^2}{cx^4+bx^2+a}dx+\int \frac{2dex}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{d^2+e^2x^2}{cx^4+bx^2+a}dx+2de\int \frac{x}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{ 1432}$$

$$\int \frac{d^2+e^2x^2}{cx^4+bx^2+a}dx+de\int \frac{1}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{ 1083}$$

$$\int \frac{d^2+e^2x^2}{cx^4+bx^2+a}dx-2de\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)$$

$$\Big\downarrow \text{ 219}$$

$$\int \frac{d^2+e^2x^2}{cx^4+bx^2+a}dx-\frac{2de\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{1}{2}\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}+e^2\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+$$
$$\frac{1}{2}\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx-\frac{2de\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

$\big\downarrow$ 218

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}+e^2\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-$$
$$\frac{2de\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```

output
```
((e^2 + (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ ((e^2 - (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]
) - (2*d*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

3.338.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

3.338.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.72 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.24

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln\left(x-\_R\right)}{2c\_R^3+b\_R}\right)}{2}$
default	$4c\left(-\frac{\sqrt{-4ac+b^2}\left(-de\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(e^2\sqrt{-4ac+b^2}+be^2-2cd^2\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4(4ac-b^2)c}-\frac{\sqrt{-4ac+b^2}\cdots}{\cdots}\right.$

input
```
int((e*x+d)^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+
_Z^2*b+a))
```

3.338.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 80.46 (sec) , antiderivative size = 540080, normalized size of antiderivative = 2411.07

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.338.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**2/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

3.338.7 Maxima [F]

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \int \frac{(ex+d)^2}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2/(c*x^4 + b*x^2 + a), x)
```

3.338.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1724 vs. $2(186) = 372$.

Time = 1.19 (sec) , antiderivative size = 1724, normalized size of antiderivative = 7.70

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c)*d*e*log(x^2 + 1/2*(
b - sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b
*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/2*(b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16
*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4 + (b^4*c - 6*a*b^2*c^2 - 2*
b^3*c^2 + 8*a^2*c^3 + 4*a*b*c^3 + b^2*c^3 - 2*a*c^4)*sqrt(b^2 - 4*a*c))*d*
e*log(x^2 + 1/2*(b + sqrt(b^2 - 4*a*c))/c)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3
*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2) + 1/4*((sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*
c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*b
*c^2)*d^2 - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(...
```

3.338.9 Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 3046, normalized size of antiderivative = 13.60

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^2/(a + b*x^2 + c*x^4),x)
```

output
```
symsum(log(3*c^2*d^4*e^2 - a*c*e^6 - 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c
^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 1
6*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^
4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*
e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*
d^8 + a^2*e^8, z, k)^3*b^3*c^2*x + 4*c^2*d^3*e^3*x + 4*root(16*a*b^4*c*z^4
 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2
*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*
z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*
e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^
2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*b^2*c^2*d^2 + b*c*d^2*e^4 - 4*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^
2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2
+ 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z -
 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*
b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*c^3*d^4*x - 16*root(16*
a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z
^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4
*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32
*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*...
```

3.338. $\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx$

3.339 $\int \frac{x^2}{(a+bx)(c+dx)}\,dx$

3.339.1 Optimal result

Integrand size = 18, antiderivative size = 56

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = \frac{x}{bd}+\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}$$

output
```
x/b/d+a^2*ln(b*x+a)/b^2/(-a*d+b*c)-c^2*ln(d*x+c)/d^2/(-a*d+b*c)
```

3.339.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = \frac{x}{bd}+\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}$$

input
```
Integrate[x^2/((a + b*x)*(c + d*x)),x]
```

output
```
x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x])/(d^2*(
b*c - a*d))
```

3.339.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx)(c+dx)} \, dx$$
$$\Big\downarrow \text{93}$$
$$\int \left(\frac{a^2}{b(a+bx)(bc-ad)}+\frac{c^2}{d(c+dx)(ad-bc)}+\frac{1}{bd}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}+\frac{x}{bd}$$

input
```
Int[x^2/((a + b*x)*(c + d*x)),x]
```

output
```
x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x])/(d^2*(
b*c - a*d))
```

3.339.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.339.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{x}{bd}+\frac{c^{2}\ln(dx+c)}{d^{2}(da-bc)}-\frac{a^{2}\ln(bx+a)}{b^{2}(da-bc)}$	57
norman	$\frac{x}{bd}+\frac{c^{2}\ln(dx+c)}{d^{2}(da-bc)}-\frac{a^{2}\ln(bx+a)}{b^{2}(da-bc)}$	57
risch	$\frac{x}{bd}+\frac{c^{2}\ln(-dx-c)}{d^{2}(da-bc)}-\frac{a^{2}\ln(bx+a)}{b^{2}(da-bc)}$	60
parallelrisch	$-\frac{a^{2}\ln(bx+a)d^{2}-c^{2}\ln(dx+c)b^{2}-ab\,d^{2}x+x\,b^{2}cd}{b^{2}d^{2}(da-bc)}$	62

input
```
int(x^2/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
x/b/d+1/d^2*c^2/(a*d-b*c)*ln(d*x+c)-1/b^2*a^2/(a*d-b*c)*ln(b*x+a)
```

3.339.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx=\frac{a^2d^2\log(bx+a)-b^2c^2\log(dx+c)+(b^2cd-abd^2)x}{b^3cd^2-ab^2d^3}$$

input
```
integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
(a^2*d^2*log(b*x + a) - b^2*c^2*log(d*x + c) + (b^2*c*d - a*b*d^2)*x)/(b^3
*c*d^2 - a*b^2*d^3)
```

3.339.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(44) = 88$.

Time = 0.61 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.39

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx=-\frac{a^2\log\left(x+\frac{\frac{a^4d^3}{b(ad-bc)}-\frac{2a^3cd^2}{ad-bc}+\frac{a^2bc^2d}{ad-bc}+a^2cd+abc^2}{a^2d^2+b^2c^2}\right)}{b^2(ad-bc)}+\frac{c^2\log\left(x+\frac{-\frac{a^2bc^2d}{ad-bc}+a^2cd+\frac{2ab^2c^3}{ad-bc}+abc^2-\frac{b^3c^4}{d(ad-bc)}}{a^2d^2+b^2c^2}\right)}{d^2(ad-bc)}+\frac{x}{bd}$$

input
```
integrate(x**2/(b*x+a)/(d*x+c),x)
```

output
```
-a**2*log(x + (a**4*d**3/(b*(a*d - b*c)) - 2*a**3*c*d**2/(a*d - b*c) + a**
2*b*c**2*d/(a*d - b*c) + a**2*c*d + a*b*c**2)/(a**2*d**2 + b**2*c**2))/(b*
*2*(a*d - b*c)) + c**2*log(x + (-a**2*b*c**2*d/(a*d - b*c) + a**2*c*d + 2*
a*b**2*c**3/(a*d - b*c) + a*b*c**2 - b**3*c**4/(d*(a*d - b*c)))/(a**2*d**2
 + b**2*c**2))/(d**2*(a*d - b*c)) + x/(b*d)
```

3.339.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = \frac{a^2\log(bx+a)}{b^3c-ab^2d} - \frac{c^2\log(dx+c)}{bcd^2-ad^3} + \frac{x}{bd}$$

input
```
integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
a^2*log(b*x + a)/(b^3*c - a*b^2*d) - c^2*log(d*x + c)/(b*c*d^2 - a*d^3) +
x/(b*d)
```

3.339.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = \frac{a^2\log(|bx+a|)}{b^3c-ab^2d} - \frac{c^2\log(|dx+c|)}{bcd^2-ad^3} + \frac{x}{bd}$$

input
```
integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
a^2*log(abs(b*x + a))/(b^3*c - a*b^2*d) - c^2*log(abs(d*x + c))/(b*c*d^2 -
 a*d^3) + x/(b*d)
```

3.339.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx = -\frac{a^2\,d^2\,\ln(a+b\,x)-b^2\,c^2\,\ln(c+d\,x)-a\,b\,d^2\,x+b^2\,c\,d\,x}{b^2\,d^2\,(a\,d-b\,c)}$$

input
```
int(x^2/((a + b*x)*(c + d*x)),x)
```

output
```
-(a^2*d^2*log(a + b*x) - b^2*c^2*log(c + d*x) - a*b*d^2*x + b^2*c*d*x)/(b^
2*d^2*(a*d - b*c))
```

3.340 $\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx$

3.340.1 Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx = -\frac{\sqrt{a}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\,(bc^2+ad^2)}+\frac{c^2\log(c+dx)}{d\,(bc^2+ad^2)}+\frac{ad\log\left(a+bx^2\right)}{2b\,(bc^2+ad^2)}$$

output
```
c^2*ln(d*x+c)/d/(a*d^2+b*c^2)+1/2*a*d*ln(b*x^2+a)/b/(a*d^2+b*c^2)-c*arctan
(x*b^(1/2)/a^(1/2))*a^(1/2)/(a*d^2+b*c^2)/b^(1/2)
```

3.340.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx = \frac{-2\sqrt{a}\sqrt{b}cd\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+2bc^2\log(c+dx)+ad^2\log\left(a+bx^2\right)}{2b^2c^2d+2abd^3}$$

input
```
Integrate[x^2/((c + d*x)*(a + b*x^2)),x]
```

output
```
(-2*Sqrt[a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*b*c^2*Log[c + d*x]
 + a*d^2*Log[a + b*x^2])/(2*b^2*c^2*d + 2*a*b*d^3)
```

3.340.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)(c+dx)}\,dx$$

$$\Big\downarrow \text{615}$$

$$\int \left(\frac{c^2}{(c+dx)(ad^2+bc^2)}-\frac{a(c-dx)}{(a+bx^2)(ad^2+bc^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{a}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\,(ad^2+bc^2)}+\frac{ad\log\left(a+bx^2\right)}{2b\,(ad^2+bc^2)}+\frac{c^2\log(c+dx)}{d\,(ad^2+bc^2)}$$

input
```
Int[x^2/((c + d*x)*(a + b*x^2)),x]
```

output
```
-((Sqrt[a]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c^2 + a*d^2))) + (c^
2*Log[c + d*x])/(d*(b*c^2 + a*d^2)) + (a*d*Log[a + b*x^2])/(2*b*(b*c^2 + a
*d^2))
```

3.340.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.340.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

method	result
default	$-\frac{a\left(-\frac{d\ln\left(b\,x^{2}+a\right)}{2b}+\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a\,d^{2}+b\,c^{2}}+\frac{c^{2}\ln(dx+c)}{d(a\,d^{2}+b\,c^{2})}$
risch	$\frac{\ln((-3a^{2}bc\,d^{3}+5a\,b^{2}c^{3}d+\sqrt{-ab}\,a^{2}d^{4}-5\sqrt{-ab}\,ab\,c^{2}d^{2}+2\sqrt{-ab}\,b^{2}c^{4})x-5a^{2}b\,c^{2}d^{2}+3\sqrt{-ab}\,a^{2}c\,d^{3}-5\sqrt{-ab}\,ab\,c^{3}d+a^{3}d^{4}+2a\,b^{2}c^{4})c\sqrt{}}{2(a\,d^{2}+b\,c^{2})b}$

input
```
int(x^2/(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-a/(a*d^2+b*c^2)*(-1/2*d*ln(b*x^2+a)/b+c/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2
)))+c^2*ln(d*x+c)/d/(a*d^2+b*c^2)
```

3.340.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.69

$$\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx = \left[\frac{bcd\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+ad^2\log\left(bx^2+a\right)+2\,bc^2\log\left(dx+c\right)}{2\left(b^2c^2d+abd^3\right)}, -\frac{2\,bcd\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-ad^2\log\left(bx^2+a\right)-2\,bc^2\log\left(dx+c\right)}{2\left(b^2c^2d+abd^3\right)}\right]$$

input
```
integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(b*c*d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + a
*d^2*log(b*x^2 + a) + 2*b*c^2*log(d*x + c))/(b^2*c^2*d + a*b*d^3), -1/2*(2
*b*c*d*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - a*d^2*log(b*x^2 + a) - 2*b*c^2*
log(d*x + c))/(b^2*c^2*d + a*b*d^3)]
```

3.340.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+dx)\,(a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(d*x+c)/(b*x**2+a),x)
```

output
```
Timed out
```

3.340.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(c+dx)\,(a+bx^2)}\,dx = \frac{ad\log\left(bx^2+a\right)}{2\,(b^2c^2+abd^2)} + \frac{c^2\log\left(dx+c\right)}{bc^2d+ad^3} - \frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(bc^2+ad^2)\sqrt{ab}}$$

input
```
integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*a*d*log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) + c^2*log(d*x + c)/(b*c^2*d + a
*d^3) - a*c*arctan(b*x/sqrt(a*b))/((b*c^2 + a*d^2)*sqrt(a*b))
```

3.340.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(c+dx)\,(a+bx^2)}\,dx = \frac{ad\log\left(bx^2+a\right)}{2\,(b^2c^2+abd^2)} + \frac{c^2\log\left(|dx+c|\right)}{bc^2d+ad^3} - \frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(bc^2+ad^2)\sqrt{ab}}$$

input
```
integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*a*d*log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) + c^2*log(abs(d*x + c))/(b*c^2*
d + a*d^3) - a*c*arctan(b*x/sqrt(a*b))/((b*c^2 + a*d^2)*sqrt(a*b))
```

3.340.9 Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.61

$$
\int \frac{x^2}{(c+dx)\,(a+bx^2)}\,dx
$$

$$
= \frac{\ln\left(a\,c+a\,d\,x+\frac{\left(c\,\sqrt{-a\,b^3}+a\,b\,d\right)\,\left(x\,\left(2\,b^2\,c^2-5\,a\,b\,d^2\right)-5\,a\,b\,c\,d+\frac{2\,b^2\,d\,\left(c\,\sqrt{-a\,b^3}+a\,b\,d\right)\,\left(-b\,x\,c^2+4\,a\,c\,d+3\,a\,x\,d^2\right)}{2\,b^3\,c^2+2\,a\,b^2\,d^2}\right)}{2\,b^3\,c^2+2\,a\,b^2\,d^2}\right)\,\left(c\,\sqrt{-a\,b^3}+\cdots\right.}{2\,b^3\,c^2+2\,a\,b^2\,d^2}
$$

$$
-\frac{\ln\left(a\,c+a\,d\,x+\frac{\left(c\,\sqrt{-a\,b^3}-a\,b\,d\right)\,\left(b\,x\,\left(5\,a\,d^2-2\,b\,c^2\right)+5\,a\,b\,c\,d+\frac{d\,\left(c\,\sqrt{-a\,b^3}-a\,b\,d\right)\,\left(-b\,x\,c^2+4\,a\,c\,d+3\,a\,x\,d^2\right)}{b\,c^2+a\,d^2}\right)}{2\,b^2\,\left(b\,c^2+a\,d^2\right)}\right)\,\left(c\,\sqrt{-a\,b^3}-a\cdots\right.}{2\,\left(b^3\,c^2+a\,b^2\,d^2\right)}
$$

$$
+\frac{c^2\,\ln\left(c+d\,x\right)}{b\,c^2\,d+a\,d^3}
$$

input
```
int(x^2/((a + b*x^2)*(c + d*x)),x)
```

output
```
(log(a*c + a*d*x + ((c*(-a*b^3)^(1/2) + a*b*d)*(x*(2*b^2*c^2 - 5*a*b*d^2)
- 5*a*b*c*d + (2*b^2*d*(c*(-a*b^3)^(1/2) + a*b*d)*(4*a*c*d + 3*a*d^2*x - b
*c^2*x))/(2*b^3*c^2 + 2*a*b^2*d^2)))/(2*b^3*c^2 + 2*a*b^2*d^2))*(c*(-a*b^3
)^(1/2) + a*b*d))/(2*b^3*c^2 + 2*a*b^2*d^2) - (log(a*c + a*d*x + ((c*(-a*b
^3)^(1/2) - a*b*d)*(b*x*(5*a*d^2 - 2*b*c^2) + 5*a*b*c*d + (d*(c*(-a*b^3)^(
1/2) - a*b*d)*(4*a*c*d + 3*a*d^2*x - b*c^2*x))/(a*d^2 + b*c^2)))/(2*b^2*(a
*d^2 + b*c^2)))*(c*(-a*b^3)^(1/2) - a*b*d))/(2*(b^3*c^2 + a*b^2*d^2)) + (c
^2*log(c + d*x))/(a*d^3 + b*c^2*d)
```

3.341 $\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx$

3.341.1 Optimal result

Integrand size = 20, antiderivative size = 264

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx = -\frac{\sqrt[3]{a}d\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}\left(b^{2/3}c^2+\sqrt[3]{a}\sqrt[3]{b}cd+a^{2/3}d^2\right)} + \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc^3-ad^3)} - \frac{c^2\log(c+dx)}{bc^3-ad^3} - \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}(bc^3-ad^3)} + \frac{c^2\log(a+bx^3)}{3(bc^3-ad^3)}$$

output
```
1/3*a^(1/3)*d*(b^(1/3)*c+a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/(-a*d^3+
b*c^3)-c^2*ln(d*x+c)/(-a*d^3+b*c^3)-1/6*a^(1/3)*d*(b^(1/3)*c+a^(1/3)*d)*ln
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)/(-a*d^3+b*c^3)+1/3*c^2*ln(
b*x^3+a)/(-a*d^3+b*c^3)-1/3*a^(1/3)*d*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/b^(2/3)/(b^(2/3)*c^2+a^(1/3)*b^(1/3)*c*d+a^(2/3)*d^2)*3^(1/2
)
```

3.341.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx$$
$$= \frac{2\sqrt{3}\sqrt[3]{a}d\left(-\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-6b^{2/3}c^2\log(c+dx}{6b^{2/3}(bc}$$

input
```
Integrate[x^2/((c + d*x)*(a + b*x^3)),x]
```

output
```
(2*Sqrt[3]*a^(1/3)*d*(-(b^(1/3)*c) + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]] + 2*a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1
/3)*x] - 6*b^(2/3)*c^2*Log[c + d*x] - a^(1/3)*b^(1/3)*c*d*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*d^2*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2] + 2*b^(2/3)*c^2*Log[a + b*x^3])/(6*b^(2/3)*(b*c^3 - a*d^
3))
```

3.341.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)(c+dx)}\,dx$$
$$\Big\downarrow \; 7276$$
$$\int \left(\frac{acd-ad^2x+bc^2x^2}{(a+bx^3)(bc^3-ad^3)}-\frac{c^2d}{(c+dx)(bc^3-ad^3)}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{\sqrt[3]{a}d\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}\left(a^{2/3}d^2+\sqrt[3]{a}\sqrt[3]{b}cd+b^{2/3}c^2\right)}-\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}\left(bc^3-ad^3\right)}+\frac{\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)}+\frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)}-\frac{c^2\log(c+dx)}{bc^3-ad^3}$$

input
```
Int[x^2/((c + d*x)*(a + b*x^3)),x]
```

output
```
-((a^(1/3)*d*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b
^(2/3)*(b^(2/3)*c^2 + a^(1/3)*b^(1/3)*c*d + a^(2/3)*d^2))) + (a^(1/3)*d*(b
^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(2/3)*(b*c^3 - a*d^3)
) - (c^2*Log[c + d*x])/(b*c^3 - a*d^3) - (a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(b*c^3 - a*d^
3)) + (c^2*Log[a + b*x^3])/(3*(b*c^3 - a*d^3))
```

3.341.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.341. $\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx$

3.341.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.84 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.67

method	result
risch	$\frac{c^2\ln(-dx-c)}{a\,d^3-b\,c^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a\,b^2d^3-b^3c^3\right)\_Z^3+3b^2c^2\_Z^2-3bc\_Z\right)}\_R\ln\left(\left(\left(-4a\,b^2d^4-2b^3c^3d\right)\_R^3-3\_R^2b^2c^2d+8\_R\right.\right.\right.}{3}$
default	$\frac{c^2\ln(dx+c)}{a\,d^3-b\,c^3}+\frac{-acd\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+a\,d^2\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right.\right.}{6b}\right.}{a\,d^3-b\,c^3}$

input
```
int(x^2/(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
c^2/(a*d^3-b*c^3)*ln(-d*x-c)+1/3*sum(_R*ln(((-4*a*b^2*d^4-2*b^3*c^3*d)*_R^
3-3*_R^2*b^2*c^2*d+8*_R*b*c*d-3*d)*x+(-5*a*b^2*c*d^3-b^3*c^4)*_R^3+(a*b*d^
3-b^2*c^3)*_R^2+5*b*c^2*_R-3*c),_R=RootOf(1+(a*b^2*d^3-b^3*c^3)*_Z^3+3*b^2
*c^2*_Z^2-3*b*c*_Z))
```

3.341.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 5975, normalized size of antiderivative = 22.63

$$\int\frac{x^2}{(c+dx)\,(a+bx^3)}\,dx=\text{Too large to display}$$

input
```
integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.341.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(d*x+c)/(b*x**3+a),x)
```

output
```
Timed out
```

3.341.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx = -\frac{c^2\log(dx+c)}{bc^3-ad^3} - \frac{\sqrt{3}\left(ad^2\left(\frac{a}{b}\right)^{\frac{2}{3}} - acd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}} - abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\left(2\,bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}} - ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}} - acd\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}} - abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{\left(bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}} + ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}} + acd\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}} - abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}$$

input
```
integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-c^2*log(d*x + c)/(b*c^3 - a*d^3) - 1/3*sqrt(3)*(a*d^2*(a/b)^(2/3) - a*c*d
*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c^
3*(a/b)^(2/3) - a*b*d^3*(a/b)^(2/3))*(a/b)^(1/3)) + 1/6*(2*b*c^2*(a/b)^(2/
3) - a*d^2*(a/b)^(1/3) - a*c*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^
2*c^3*(a/b)^(2/3) - a*b*d^3*(a/b)^(2/3)) + 1/3*(b*c^2*(a/b)^(2/3) + a*d^2*
(a/b)^(1/3) + a*c*d)*log(x + (a/b)^(1/3))/(b^2*c^3*(a/b)^(2/3) - a*b*d^3*(
a/b)^(2/3))
```

3.341.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx$$
$$= -\frac{c^2d\log(|dx+c|)}{bc^3d-ad^4} + \frac{c^2\log(|bx^3+a|)}{3(bc^3-ad^3)} + \frac{(-ab^2)^{\frac{1}{3}}\,d\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2-\sqrt{3}(-ab^2)^{\frac{1}{3}}bcd+\sqrt{3}(-ab^2)^{\frac{2}{3}}d^2} + \frac{\left(ab^2c^3d^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2bd^5\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^2c^4d+a^2bcd^4\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3(ab^3c^6-2a^2b^2c^3d^3+a^3bd^6)} + \frac{\left((-ab^2)^{\frac{1}{3}}bcd-(-ab^2)^{\frac{2}{3}}d^2\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(b^3c^3-ab^2d^3)}$$

input
```
integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
-c^2*d*log(abs(d*x + c))/(b*c^3*d - a*d^4) + 1/3*c^2*log(abs(b*x^3 + a))/(
b*c^3 - a*d^3) + (-a*b^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/
(-a/b)^(1/3))/(sqrt(3)*b^2*c^2 - sqrt(3)*(-a*b^2)^(1/3)*b*c*d + sqrt(3)*(-
a*b^2)^(2/3)*d^2) + 1/3*(a*b^2*c^3*d^2*(-a/b)^(1/3) - a^2*b*d^5*(-a/b)^(1/
3) - a*b^2*c^4*d + a^2*b*c*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a
*b^3*c^6 - 2*a^2*b^2*c^3*d^3 + a^3*b*d^6) + 1/6*((-a*b^2)^(1/3)*b*c*d - (-
a*b^2)^(2/3)*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^3*c^3 - a*b^
2*d^3)
```

3.341.9 Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.16

$$\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(-a\,b\,d\left(c+d\,x+\mathrm{root}\left(27\,a\,b^2\,d^3\,z^3-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)^2 b^2\,c^3\,3+\mathrm{root}\left(27\,a\right.\right.\right.\right.$$
$$\left.\left.-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)\right) + \frac{c^2\,\ln(c+d\,x)}{a\,d^3-b\,c^3}$$

input
```
int(x^2/((a + b*x^3)*(c + d*x)),x)
```

output
```
symsum(log(-a*b*d*(c + d*x + 3*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27
*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^2*b^2*c^3 + 9*root(27*a*b^2*d^3*z^3 - 27
*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*b^3*c^4 - 5*root(27*a
*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)*b*c^2
- 3*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1,
 z, k)^2*a*b*d^3 - 8*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z
^2 - 9*b*c*z + 1, z, k)*b*c*d*x + 45*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^
3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*a*b^2*c*d^3 + 36*root(27*a*b^2*d
^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*a*b^2*d^4*
x + 9*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z +
1, z, k)^2*b^2*c^2*d*x + 18*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^
2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*b^3*c^3*d*x))*root(27*a*b^2*d^3*z^3 - 27*
b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k), k, 1, 3) + (c^2*log(c +
 d*x))/(a*d^3 - b*c^3)
```

3.341. $\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx$

3.342 $\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx$

3.342.1 Optimal result

Integrand size = 20, antiderivative size = 417

$$\begin{aligned}\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx &= \frac{\sqrt{a}d^3\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,(bc^4+ad^4)} - \frac{c\left(\sqrt{b}c^2-\sqrt{a}d^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(bc^4+ad^4)} \\ &\quad + \frac{c\left(\sqrt{b}c^2-\sqrt{a}d^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(bc^4+ad^4)} + \frac{c^2d\log(c+dx)}{bc^4+ad^4} \\ &\quad + \frac{c\left(\sqrt{b}c^2+\sqrt{a}d^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(bc^4+ad^4)} \\ &\quad - \frac{c\left(\sqrt{b}c^2+\sqrt{a}d^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(bc^4+ad^4)} \\ &\quad - \frac{c^2d\log\left(a+bx^4\right)}{4\,(bc^4+ad^4)}\end{aligned}$$

output
```
c^2*d*ln(d*x+c)/(a*d^4+b*c^4)-1/4*c^2*d*ln(b*x^4+a)/(a*d^4+b*c^4)+1/2*d^3*
arctan(x^2*b^(1/2)/a^(1/2))*a^(1/2)/(a*d^4+b*c^4)/b^(1/2)+1/4*c*arctan(-1+
b^(1/4)*x*2^(1/2)/a^(1/4))*(-d^2*a^(1/2)+b^(1/2)*c^2)/a^(1/4)/b^(1/4)/(a*d
^4+b*c^4)*2^(1/2)+1/4*c*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(-d^2*a^(1/2)+
b^(1/2)*c^2)/a^(1/4)/b^(1/4)/(a*d^4+b*c^4)*2^(1/2)+1/8*c*ln(-a^(1/4)*b^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(d^2*a^(1/2)+b^(1/2)*c^2)/a^(1/4)/b^(1/4
)/(a*d^4+b*c^4)*2^(1/2)-1/8*c*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(
1/2))*(d^2*a^(1/2)+b^(1/2)*c^2)/a^(1/4)/b^(1/4)/(a*d^4+b*c^4)*2^(1/2)
```

3.342.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx$$
$$= \frac{-2\left(\sqrt{2}b^{3/4}c^3-\sqrt{2}\sqrt{a}\sqrt[4]{b}cd^2+2a^{3/4}d^3\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}b^{3/4}c^3-\sqrt{2}\sqrt{a}\sqrt[4]{b}cd^2-2a^{3/4}d^3\right)\,a}{}$$

input
```
Integrate[x^2/((c + d*x)*(a + b*x^4)),x]
```

output
```
(-2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]*Sqrt[a]*b^(1/4)*c*d^2 + 2*a^(3/4)*d^3)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]
*Sqrt[a]*b^(1/4)*c*d^2 - 2*a^(3/4)*d^3)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(
1/4)] + b^(1/4)*c*(8*a^(1/4)*b^(1/4)*c*d*Log[c + d*x] + Sqrt[2]*(Sqrt[b]*c
^2 + Sqrt[a]*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] -
 Sqrt[2]*Sqrt[b]*c^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
] - Sqrt[2]*Sqrt[a]*d^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2] - 2*a^(1/4)*b^(1/4)*c*d*Log[a + b*x^4]))/(8*a^(1/4)*Sqrt[b]*(b*c^4 +
a*d^4))
```

3.342.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^4)(c+dx)}\,dx$$
$$\Big\downarrow \text{ 7276}$$
$$\int \left(\frac{(c-dx)(bc^2x^2-ad^2)}{(a+bx^4)(ad^4+bc^4)}+\frac{c^2d^2}{(c+dx)(ad^4+bc^4)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a}d^3\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}\left(ad^4+bc^4\right)}-\frac{c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}c^2-\sqrt{a}d^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}+\frac{c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}c^2-\sqrt{a}d^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}-\frac{c^2d\log\left(a+bx^4\right)}{4\left(ad^4+bc^4\right)}+\frac{c^2d\log(c+dx)}{ad^4+bc^4}+\frac{c\left(\sqrt{a}d^2+\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}-\frac{c\left(\sqrt{a}d^2+\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}$$

input
```
Int[x^2/((c + d*x)*(a + b*x^4)),x]
```

output
```
(Sqrt[a]*d^3*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[b]*(b*c^4 + a*d^4)) -
(c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2
*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) + (c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^
4 + a*d^4)) + (c^2*d*Log[c + d*x])/(b*c^4 + a*d^4) + (c*(Sqrt[b]*c^2 + Sqr
t[a]*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[
2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) - (c*(Sqrt[b]*c^2 + Sqrt[a]*d^2)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*b^(1
/4)*(b*c^4 + a*d^4)) - (c^2*d*Log[a + b*x^4])/(4*(b*c^4 + a*d^4))
```

3.342.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.342.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.84 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.54

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a^{2}b^{2}d^{4}+a\,b^{3}c^{4}\right)\_Z^{4}+4a\,b^{2}c^{2}d\,\_Z^{3}+2a\,\_Z^{2}d^{2}b\right)}\_R\ln\left(\left(\left(5a^{2}b^{2}d^{6}-3a\,b^{3}c^{4}d^{2}\right)\_R^{4}+10\,\_R^{3}a\,b^{2}c^{2}d^{3}+(9ab\,d^{4}+\cdots\right.\right.\right.}{4}$
default	$\frac{-\frac{d^{2}c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8}+\frac{a\,d^{3}\arctan\left(x^{2}\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{c^{3}\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}\cdots}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}\cdots}\right)\cdots\right)}{\cdots}}{a\,d^{4}+b\,c^{4}}$

input
```
int(x^2/(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(((5*a^2*b^2*d^6-3*a*b^3*c^4*d^2)*_R^4+10*_R^3*a*b^2*c^2*d^3+
(9*a*b*d^4+b^2*c^4)*_R^2-5*_R*b*c^2*d+4*d^2)*x+(6*a^2*b^2*c*d^5-2*a*b^3*c^
5*d)*_R^4+6*a*b^2*c^3*d^2*_R^3+8*a*b*c*d^3*_R^2-b*c^3*_R+4*c*d),_R=RootOf(
1+(a^2*b^2*d^4+a*b^3*c^4)*_Z^4+4*a*b^2*c^2*d*_Z^3+2*a*_Z^2*d^2*b))+c^2*d*l
n(d*x+c)/(a*d^4+b*c^4)
```

3.342.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 23.56 (sec) , antiderivative size = 259898, normalized size of antiderivative = 623.26

$$\int \frac{x^2}{(c+dx)\left(a+bx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.342.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+dx)\left(a+bx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

3.342.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{(c+dx)\left(a+bx^4\right)}\,dx = \frac{c^2 d\log\left(dx+c\right)}{bc^4+ad^4}$$

$$-\frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}c^2d+\sqrt{a}b^{\frac{3}{2}}c^3+abcd^2\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}c^2d-\sqrt{a}b^{\frac{3}{2}}c^3-abcd^2\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}-\frac{2(}{}}{8(bc}$$

input
```
integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
c^2*d*log(d*x + c)/(b*c^4 + a*d^4) - 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)
*c^2*d + sqrt(a)*b^(3/2)*c^3 + a*b*c*d^2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4
)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4
)*c^2*d - sqrt(a)*b^(3/2)*c^3 - a*b*c*d^2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) - 2*(sqrt(2)*a^(3/4)*b^(7/4)*c^3
 - sqrt(2)*a^(5/4)*b^(5/4)*c*d^2 - 2*a^(3/2)*b*d^3)*arctan(1/2*sqrt(2)*(2*
sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(
sqrt(a)*sqrt(b))*b^(5/4)) - 2*(sqrt(2)*a^(3/4)*b^(7/4)*c^3 - sqrt(2)*a^(5/
4)*b^(5/4)*c*d^2 + 2*a^(3/2)*b*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt
(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))
*b^(5/4)))/(b*c^4 + a*d^4)
```

3.342.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx = \frac{c^2d^2\log(|dx+c|)}{bc^4d+ad^5} - \frac{c^2d\log(|bx^4+a|)}{4(bc^4+ad^4)} - \frac{\left(\sqrt{2}ab^2d-(ab^3)^{\frac{3}{4}}c\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^3c^2+\sqrt{2}\sqrt{ab}ab^2d^2-2(ab^3)^{\frac{1}{4}}ab^2cd\right)} + \frac{\left(\sqrt{2}ab^2d+(ab^3)^{\frac{3}{4}}c\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^3c^2+\sqrt{2}\sqrt{ab}ab^2d^2+2(ab^3)^{\frac{1}{4}}ab^2cd\right)} - \frac{\left((ab^3)^{\frac{1}{4}}abcd^2+(ab^3)^{\frac{3}{4}}c^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^3c^4+\sqrt{2}a^2b^2d^4\right)} + \frac{\left((ab^3)^{\frac{1}{4}}abcd^2+(ab^3)^{\frac{3}{4}}c^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^3c^4+\sqrt{2}a^2b^2d^4\right)}$$

input
```
integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
c^2*d^2*log(abs(d*x + c))/(b*c^4*d + a*d^5) - 1/4*c^2*d*log(abs(b*x^4 + a)
)/(b*c^4 + a*d^4) - 1/2*(sqrt(2)*a*b^2*d - (a*b^3)^(3/4)*c)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^2 + sqrt(2)
*sqrt(a*b)*a*b^2*d^2 - 2*(a*b^3)^(1/4)*a*b^2*c*d) + 1/2*(sqrt(2)*a*b^2*d +
 (a*b^3)^(3/4)*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*a*b^3*c^2 + sqrt(2)*sqrt(a*b)*a*b^2*d^2 + 2*(a*b^3)^(1/4)*a*b
^2*c*d) - 1/4*((a*b^3)^(1/4)*a*b*c*d^2 + (a*b^3)^(3/4)*c^3)*log(x^2 + sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c^4 + sqrt(2)*a^2*b^2*d^4) +
 1/4*((a*b^3)^(1/4)*a*b*c*d^2 + (a*b^3)^(3/4)*c^3)*log(x^2 - sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c^4 + sqrt(2)*a^2*b^2*d^4)
```

3.342.9 Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 823, normalized size of antiderivative = 1.97

$$\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx$$
$$= \left(\sum_{k=1}^{4} \ln\left(a\,b^2\,d\left(c\,d+d^2\,x-\mathrm{root}\left(256\,a^2\,b^2\,d^4\,z^4+256\,a\,b^3\,c^4\,z^4+256\,a\,b^2\,c^2\,d\,z^3+32\,a\,b\,d^2\,z^2+1,z,k\right)\,b\,c^3+\right.\right.\right.$$
$$\left.\left. + 256\,a\,b^3\,c^4\,z^4+256\,a\,b^2\,c^2\,d\,z^3+32\,a\,b\,d^2\,z^2+1,z,k\right)\right)+\frac{c^2\,d\,\ln\left(c+d\,x\right)}{b\,c^4+a\,d^4}$$

input
```
int(x^2/((a + b*x^4)*(c + d*x)),x)
```

output
```
symsum(log(a*b^2*d*(c*d + d^2*x - root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4
*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)*b*c^3 + 4*root(256*
a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2
 + 1, z, k)^2*b^2*c^4*x + 36*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4
+ 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^2*a*b*d^4*x - 128*root(2
56*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*
z^2 + 1, z, k)^4*a*b^3*c^5*d - 5*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*
z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)*b*c^2*d*x + 96*root(
256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2
*z^2 + 1, z, k)^3*a*b^2*c^3*d^2 + 384*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3
*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^4*a^2*b^2*c*d^5
 + 320*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3
+ 32*a*b*d^2*z^2 + 1, z, k)^4*a^2*b^2*d^6*x + 32*root(256*a^2*b^2*d^4*z^4
+ 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^2*a*
b*c*d^3 + 160*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2
*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^3*a*b^2*c^2*d^3*x - 192*root(256*a^2*b^
2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1,
z, k)^4*a*b^3*c^4*d^2*x))*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 2
56*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k), k, 1, 4) + (c^2*d*log(c +
d*x))/(a*d^4 + b*c^4)
```

3.343 $\int \frac{x}{(1-x)(1+x)^2} dx$

3.343.1 Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{x}{(1-x)(1+x)^2} dx = \frac{1}{2(1+x)} + \frac{\operatorname{arctanh}(x)}{2}$$

output
```
1/2/(1+x)+1/2*arctanh(x)
```

3.343.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{x}{(1-x)(1+x)^2} dx = \frac{1}{4}\left(\frac{2}{1+x} - \log(1-x) + \log(1+x)\right)$$

input
```
Integrate[x/((1 - x)*(1 + x)^2),x]
```

output
```
(2/(1 + x) - Log[1 - x] + Log[1 + x])/4
```

3.343.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-x)(x+1)^2} \, dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{1}{2(x^2-1)}-\frac{1}{2(x+1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}(x)}{2}+\frac{1}{2(x+1)}$$

input
```
Int[x/((1 - x)*(1 + x)^2),x]
```

output
```
1/(2*(1 + x)) + ArcTanh[x]/2
```

3.343.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.343.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

method	result	size
default	$\frac{1}{2x+2}+\frac{\ln(x+1)}{4}-\frac{\ln(x-1)}{4}$	21
norman	$\frac{1}{2x+2}+\frac{\ln(x+1)}{4}-\frac{\ln(x-1)}{4}$	21
risch	$\frac{1}{2x+2}+\frac{\ln(x+1)}{4}-\frac{\ln(x-1)}{4}$	21
parallelrisch	$-\frac{\ln(x-1)x-\ln(x+1)x-2+\ln(x-1)-\ln(x+1)}{4(x+1)}$	33

input
```
int(x/(1-x)/(x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(x+1)+1/4*ln(x+1)-1/4*ln(x-1)
```

3.343.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{x}{(1-x)(1+x)^2}\,dx = \frac{(x+1)\log(x+1)-(x+1)\log(x-1)+2}{4(x+1)}$$

input
```
integrate(x/(1-x)/(1+x)^2,x, algorithm="fricas")
```

output
```
1/4*((x + 1)*log(x + 1) - (x + 1)*log(x - 1) + 2)/(x + 1)
```

3.343.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{x}{(1-x)(1+x)^2}\,dx = -\frac{\log(x-1)}{4}+\frac{\log(x+1)}{4}+\frac{1}{2x+2}$$

input
```
integrate(x/(1-x)/(1+x)**2,x)
```

output
```
-log(x - 1)/4 + log(x + 1)/4 + 1/(2*x + 2)
```

3.343.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{x}{(1-x)(1+x)^2}\,dx = \frac{1}{2\,(x+1)} + \frac{1}{4}\,\log\left(x+1\right) - \frac{1}{4}\,\log\left(x-1\right)$$

input
```
integrate(x/(1-x)/(1+x)^2,x, algorithm="maxima")
```

output
```
1/2/(x + 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

3.343.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{x}{(1-x)(1+x)^2}\,dx = \frac{1}{2\,(x+1)} - \frac{1}{4}\,\log\left(\left|-\frac{2}{x+1}+1\right|\right)$$

input
```
integrate(x/(1-x)/(1+x)^2,x, algorithm="giac")
```

output
```
1/2/(x + 1) - 1/4*log(abs(-2/(x + 1) + 1))
```

3.343.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{(1-x)(1+x)^2}\,dx = \frac{\mathrm{atanh}(x)}{2} + \frac{1}{2\,(x+1)}$$

input
```
int(-x/((x - 1)*(x + 1)^2),x)
```

output
```
atanh(x)/2 + 1/(2*(x + 1))
```

3.344 $\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx$

3.344.1 Optimal result

Integrand size = 20, antiderivative size = 19

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx = -\frac{x}{4(1+x^2)} + \frac{\operatorname{arctanh}(x)}{4}$$

output
```
-1/4*x/(x^2+1)+1/4*arctanh(x)
```

3.344.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx = \frac{1}{8}\left(-\frac{2x}{1+x^2} - \log(1-x) + \log(1+x)\right)$$

input
```
Integrate[x^2/((1 - x^2)*(1 + x^2)^2),x]
```

output
```
((-2*x)/(1 + x^2) - Log[1 - x] + Log[1 + x])/8
```

3.344.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {373, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-x^2)(x^2+1)^2}dx$$

$$\Big\downarrow \ 373$$

$$\frac{1}{4}\int \frac{1}{1-x^2}dx-\frac{x}{4(x^2+1)}$$

$$\Big\downarrow \ 219$$

$$\frac{\text{arctanh}(x)}{4}-\frac{x}{4(x^2+1)}$$

input
```
Int[x^2/((1 - x^2)*(1 + x^2)^2),x]
```

output
```
-1/4*x/(1 + x^2) + ArcTanh[x]/4
```

3.344.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

3.344.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\ln(x+1)}{8}-\frac{x}{4(x^2+1)}-\frac{\ln(x-1)}{8}$	24
norman	$\frac{\ln(x+1)}{8}-\frac{x}{4(x^2+1)}-\frac{\ln(x-1)}{8}$	24
risch	$\frac{\ln(x+1)}{8}-\frac{x}{4(x^2+1)}-\frac{\ln(x-1)}{8}$	24
parallelrisch	$-\frac{\ln(x-1)x^2-\ln(x+1)x^2+\ln(x-1)-\ln(x+1)+2x}{8(x^2+1)}$	41

input
```
int(x^2/(-x^2+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(x+1)-1/4*x/(x^2+1)-1/8*ln(x-1)
```

3.344.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(15) = 30.

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx = \frac{(x^2+1)\log(x+1)-(x^2+1)\log(x-1)-2x}{8(x^2+1)}$$

input
```
integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/8*((x^2 + 1)*log(x + 1) - (x^2 + 1)*log(x - 1) - 2*x)/(x^2 + 1)
```

3.344.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx = -\frac{x}{4x^2+4}-\frac{\log(x-1)}{8}+\frac{\log(x+1)}{8}$$

input
```
integrate(x**2/(-x**2+1)/(x**2+1)**2,x)
```

output
```
-x/(4*x**2 + 4) - log(x - 1)/8 + log(x + 1)/8
```

3.344.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx = -\frac{x}{4(x^2+1)} + \frac{1}{8}\log(x+1) - \frac{1}{8}\log(x-1)$$

input
```
integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/4*x/(x^2 + 1) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

3.344.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx = -\frac{1}{4\left(x+\frac{1}{x}\right)} + \frac{1}{16}\log\left(\left|x+\frac{1}{x}+2\right|\right) - \frac{1}{16}\log\left(\left|x+\frac{1}{x}-2\right|\right)$$

input
```
integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/4/(x + 1/x) + 1/16*log(abs(x + 1/x + 2)) - 1/16*log(abs(x + 1/x - 2))
```

3.344.9 Mupad [B] (verification not implemented)

Time = 9.91 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx = \frac{\operatorname{atanh}(x)}{4} - \frac{x}{4(x^2+1)}$$

input
```
int(-x^2/((x^2 - 1)*(x^2 + 1)^2),x)
```

output
```
atanh(x)/4 - x/(4*(x^2 + 1))
```

3.345 $\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx$

3.345.1 Optimal result

Integrand size = 20, antiderivative size = 97

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = -\frac{x}{6(1+x^3)} + \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{12}\log(1-x) - \frac{1}{36}\log(1+x) + \frac{1}{72}\log\left(1-x+x^2\right) + \frac{1}{24}\log\left(1+x+x^2\right)$$

output
```
-1/6*x/(x^3+1)-1/12*ln(1-x)-1/36*ln(1+x)+1/72*ln(x^2-x+1)+1/24*ln(x^2+x+1)
+1/36*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1+2*x)*3^(1/2))
*3^(1/2)
```

3.345.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = \frac{1}{72}\left(-\frac{12x}{1+x^3} - 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 6\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 6\log(1-x) - 2\log(1+x) + \log\left(1-x+x^2\right) + 3\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[x^3/((1 - x^3)*(1 + x^3)^2),x]
```

output
```
((-12*x)/(1 + x^3) - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 6*Sqrt[3]*ArcT
an[(1 + 2*x)/Sqrt[3]] - 6*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] + 3
*Log[1 + x + x^2])/72
```

3.345.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {971, 1020, 750, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(1-x^3)(x^3+1)^2}\,dx$$

$$\Big\downarrow \text{971}$$

$$\frac{1}{6}\int \frac{2x^3+1}{(1-x^3)(x^3+1)}dx-\frac{x}{6(x^3+1)}$$

$$\Big\downarrow \text{1020}$$

$$\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{1-x^3}dx-\frac{1}{2}\int \frac{1}{x^3+1}dx\right)-\frac{x}{6(x^3+1)}$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(-\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{3}\int \frac{1}{x+1}dx\right)+\frac{3}{2}\left(\frac{1}{3}\int \frac{x+2}{x^2+x+1}dx+\frac{1}{3}\int \frac{1}{1-x}dx\right)\right)-\frac{x}{6(x^3+1)}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(-\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{1}{3}\int \frac{x+2}{x^2+x+1}dx-\frac{1}{3}\log(1-x)\right)\right)-\frac{x}{6(x^3+1)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+}{x^2+x}\right.\right.\right.$$
$$\frac{x}{6(x^3+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{x^2+x+1}dx+\frac{1}{2}\int\frac{2x+}{x^2+x}\right.\right.\right.$$
$$\frac{x}{6\left(x^3+1\right)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{3}\left(3\int\frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx-3\right.\right.\right.$$
$$\frac{x}{6\left(x^3+1\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{3}\left(-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx+\sqrt{3}\text{arctar}\right.\right.\right.$$
$$\frac{x}{6\left(x^3+1\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^2\right.\right.\right.\right.$$
$$\frac{x}{6\left(x^3+1\right)}$$

input
```
Int[x^3/((1 - x^3)*(1 + x^3)^2),x]
```

output
```
-1/6*x/(1 + x^3) + ((-1/3*Log[1 + x] + (-(Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3
]]) + Log[1 - x + x^2]/2)/3)/2 + (3*(-1/3*Log[1 - x] + (Sqrt[3]*ArcTan[(1
+ 2*x)/Sqrt[3]] + Log[1 + x + x^2]/2)/3))/2)/6
```

3.345.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

3.345. $\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.345. $\int \frac{x^3}{(1-x^3)(1+x^3)^2} \, dx$

3.345.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{x}{6(x^3+1)}-\frac{\ln(x-1)}{12}-\frac{\ln(x+1)}{36}+\frac{\ln(x^2+x+1)}{24}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}+\frac{\ln(x^2-x+1)}{72}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{36}$
default	$\frac{1}{18x+18}-\frac{\ln(x+1)}{36}+\frac{-2x-2}{36x^2-36x+36}+\frac{\ln(x^2-x+1)}{72}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{36}+\frac{\ln(x^2+x+1)}{24}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}$ -

input
```
int(x^3/(-x^3+1)/(x^3+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*x/(x^3+1)-1/12*ln(x-1)-1/36*ln(x+1)+1/24*ln(x^2+x+1)+1/12*3^(1/2)*arc
tan(2/3*(x+1/2)*3^(1/2))+1/72*ln(x^2-x+1)-1/36*3^(1/2)*arctan(2/3*(x-1/2)*
3^(1/2))
```

3.345.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(1-x^3)\,(1+x^3)^2}\,dx = \frac{6\,\sqrt{3}(x^3+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-2\,\sqrt{3}(x^3+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+3\,(x^3+1)\log\left(x^2+x+1\right)}{72\,(x^3+1)}$$

input
```
integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="fricas")
```

output
```
1/72*(6*sqrt(3)*(x^3 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*(x^3 +
 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*(x^3 + 1)*log(x^2 + x + 1) + (x^3 +
1)*log(x^2 - x + 1) - 2*(x^3 + 1)*log(x + 1) - 6*(x^3 + 1)*log(x - 1) - 12
*x)/(x^3 + 1)
```

3.345.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = -\frac{x}{6x^3+6} - \frac{\log(x-1)}{12} - \frac{\log(x+1)}{36} + \frac{\log(x^2-x+1)}{72} + \frac{\log(x^2+x+1)}{24} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{36} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x**3/(-x**3+1)/(x**3+1)**2,x)
```

output
```
-x/(6*x**3 + 6) - log(x - 1)/12 - log(x + 1)/36 + log(x**2 - x + 1)/72 + l
og(x**2 + x + 1)/24 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/36 + sqrt(3)
*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/12
```

3.345.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{36}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{x}{6(x^3+1)} + \frac{1}{24}\log\left(x^2+x+1\right) + \frac{1}{72}\log\left(x^2-x+1\right) - \frac{1}{36}\log(x+1) - \frac{1}{12}\log(x-1)$$

input
```
integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*log(x^2 + x + 1) + 1/72*log(x^2 - x
 + 1) - 1/36*log(x + 1) - 1/12*log(x - 1)
```

3.345.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{36}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{x}{6(x^3+1)} + \frac{1}{24}\log\left(x^2+x+1\right) + \frac{1}{72}\log\left(x^2-x+1\right) - \frac{1}{36}\log\left(|x+1|\right) - \frac{1}{12}\log\left(|x-1|\right)$$

input
```
integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*log(x^2 + x + 1) + 1/72*log(x^2 - x
 + 1) - 1/36*log(abs(x + 1)) - 1/12*log(abs(x - 1))
```

3.345.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = -\frac{\ln(x-1)}{12} - \frac{\ln(x+1)}{36} - \frac{x}{6\ (x^3+1)} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right) + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{72}+\frac{\sqrt{3}\,1\mathrm{i}}{72}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{72}+\frac{\sqrt{3}\,1\mathrm{i}}{72}\right)$$

input
```
int(-x^3/((x^3 - 1)*(x^3 + 1)^2),x)
```

output
```
log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/24 + 1/24) - log(x + 1)/36 - x
/(6*(x^3 + 1)) - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/24 - 1/24) -
log(x - 1)/12 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/72 + 1/72) - l
og(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/72 - 1/72)
```

3.346 $\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx$

3.346.1 Optimal result

Integrand size = 26, antiderivative size = 15

$$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx = 3\arctan(x) + \frac{1}{2}\log\left(3+x^2\right)$$

output
```
3*arctan(x)+1/2*ln(x^2+3)
```

3.346.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx = 3\arctan(x) + \frac{1}{2}\log\left(3+x^2\right)$$

input
```
Integrate[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

output
```
3*ArcTan[x] + Log[3 + x^2]/2
```

3.346.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+3x^2+x+9}{(x^2+1)(x^2+3)}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{x}{x^2+3}+\frac{3}{x^2+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$3\arctan(x)+\frac{1}{2}\log\left(x^2+3\right)$$

input
```
Int[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

output
```
3*ArcTan[x] + Log[3 + x^2]/2
```

3.346.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.346.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$3\arctan\left(x\right)+\frac{\ln\left(x^{2}+3\right)}{2}$	14
risch	$3\arctan\left(x\right)+\frac{\ln\left(x^{2}+3\right)}{2}$	14
parallelrisch	$\frac{3i\ln\left(x+i\right)}{2}-\frac{3i\ln\left(x-i\right)}{2}+\frac{\ln\left(x^{2}+3\right)}{2}$	26

input
```
int((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x,method=_RETURNVERBOSE)
```

output
```
3*arctan(x)+1/2*ln(x^2+3)
```

3.346.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx=3\,\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^{2}+3\right)$$

input
```
integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="fricas")
```

output
```
3*arctan(x) + 1/2*log(x^2 + 3)
```

3.346.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx=\frac{\log\left(x^{2}+3\right)}{2}+3\operatorname{atan}\left(x\right)$$

input
```
integrate((x**3+3*x**2+x+9)/(x**2+1)/(x**2+3),x)
```

output
```
log(x**2 + 3)/2 + 3*atan(x)
```

3.346.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx = 3 \arctan(x) + \frac{1}{2} \log(x^2+3)$$

input
```
integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="maxima")
```

output
```
3*arctan(x) + 1/2*log(x^2 + 3)
```

3.346.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx = 3 \arctan(x) + \frac{1}{2} \log(x^2+3)$$

input
```
integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="giac")
```

output
```
3*arctan(x) + 1/2*log(x^2 + 3)
```

3.346.9 Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx = \frac{\ln(x^2+3)}{2} + 3\,\mathrm{atan}(x)$$

input
```
int((x + 3*x^2 + x^3 + 9)/((x^2 + 1)*(x^2 + 3)),x)
```

output
```
log(x^2 + 3)/2 + 3*atan(x)
```

3.347 $\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx$

3.347.1 Optimal result

Integrand size = 24, antiderivative size = 13

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx=\arctan(x)+\frac{1}{2}\log\left(3+x^2\right)$$

output
```
arctan(x)+1/2*ln(x^2+3)
```

3.347.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx=\arctan(x)+\frac{1}{2}\log\left(3+x^2\right)$$

input
```
Integrate[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

output
```
ArcTan[x] + Log[3 + x^2]/2
```

3.347.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+x+3}{(x^2+1)(x^2+3)}\,dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{x}{x^2+3}+\frac{1}{x^2+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\arctan(x)+\frac{1}{2}\log\left(x^2+3\right)$$

input
```
Int[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

output
```
ArcTan[x] + Log[3 + x^2]/2
```

3.347.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.347.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\arctan(x)+\frac{\ln(x^2+3)}{2}$	12
risch	$\arctan(x)+\frac{\ln(x^2+3)}{2}$	12
parallelrisch	$\frac{i\ln(x+i)}{2}-\frac{i\ln(x-i)}{2}+\frac{\ln(x^2+3)}{2}$	26

input
```
int((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x,method=_RETURNVERBOSE)
```

output
```
arctan(x)+1/2*ln(x^2+3)
```

3.347.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx = \arctan(x)+\frac{1}{2}\log\left(x^2+3\right)$$

input
```
integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="fricas")
```

output
```
arctan(x) + 1/2*log(x^2 + 3)
```

3.347.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx = \frac{\log(x^2+3)}{2}+\operatorname{atan}(x)$$

input
```
integrate((x**3+x**2+x+3)/(x**2+1)/(x**2+3),x)
```

output
```
log(x**2 + 3)/2 + atan(x)
```

3.347.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx = \arctan(x) + \frac{1}{2}\log\left(x^2+3\right)$$

input
```
integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="maxima")
```

output
```
arctan(x) + 1/2*log(x^2 + 3)
```

3.347.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx = \arctan(x) + \frac{1}{2}\log\left(x^2+3\right)$$

input
```
integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="giac")
```

output
```
arctan(x) + 1/2*log(x^2 + 3)
```

3.347.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx = \frac{\ln\left(x^2+3\right)}{2} + \operatorname{atan}(x)$$

input
```
int((x + x^2 + x^3 + 3)/((x^2 + 1)*(x^2 + 3)),x)
```

output
```
log(x^2 + 3)/2 + atan(x)
```

3.348 $\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx$

3.348.1 Optimal result

Integrand size = 30, antiderivative size = 29

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx = -3\arctan(x)+\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(1+x^2\right)$$

output
```
-3*arctan(x)+3/2*ln(x^2+1)+arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.348.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx = -3\arctan(x)+\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(1+x^2\right)$$

input
```
Integrate[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
```

output
```
-3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

3.348.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^3-x^2+6x-4}{(x^2+1)(x^2+2)}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{3(x-1)}{x^2+1}+\frac{2}{x^2+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-3\arctan(x)+\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(x^2+1\right)$$

input
```
Int[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
```

output
```
-3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

3.348.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.348.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

method	result	size
default	$-3\arctan\left(x\right)+\frac{3\ln\left(x^{2}+1\right)}{2}+\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	25
risch	$-3\arctan\left(x\right)+\frac{3\ln\left(x^{2}+1\right)}{2}+\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$	25

input
```
int((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x,method=_RETURNVERBOSE)
```

output
```
-3*arctan(x)+3/2*ln(x^2+1)+arctan(1/2*x*2^(1/2))*2^(1/2)
```

3.348.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx=\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-3\arctan\left(x\right)+\frac{3}{2}\log\left(x^2+1\right)$$

input
```
integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="fricas")
```

output
```
sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

3.348.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx=\frac{3\log\left(x^2+1\right)}{2}-3\operatorname{atan}\left(x\right)+\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

input
```
integrate((3*x**3-x**2+6*x-4)/(x**2+1)/(x**2+2),x)
```

output
```
3*log(x**2 + 1)/2 - 3*atan(x) + sqrt(2)*atan(sqrt(2)*x/2)
```

3.348.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx = \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3\arctan(x) + \frac{3}{2}\log\left(x^2+1\right)$$

input
```
integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

output
```
sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

3.348.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx = \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3\arctan(x) + \frac{3}{2}\log\left(x^2+1\right)$$

input
```
integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

output
```
sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

3.348.9 Mupad [B] (verification not implemented)

Time = 9.89 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.76

$$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx = -\sqrt{2}\,\mathrm{atan}\left(\frac{24\sqrt{2}}{24\,x-64}+\frac{32\sqrt{2}\,x}{24\,x-64}\right) + \ln(x-\mathrm{i})\left(\frac{3}{2}+\frac{3}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)$$

input
```
int((6*x - x^2 + 3*x^3 - 4)/((x^2 + 1)*(x^2 + 2)),x)
```

output
```
log(x - 1i)*(3/2 + 3i/2) + log(x + 1i)*(3/2 - 3i/2) - 2^(1/2)*atan((24*2^(
1/2))/(24*x - 64) + (32*2^(1/2)*x)/(24*x - 64))
```

3.349 $\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx$

3.349.1 Optimal result

Integrand size = 21, antiderivative size = 14

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = \frac{1}{2-x} + \arctan(2-x)$$

output
```
1/(2-x)-arctan(-2+x)
```

3.349.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{1}{-2+x} + \arctan(2-x)$$

input
```
Integrate[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
```

output
```
-(-2 + x)^(-1) + ArcTan[2 - x]
```

3.349.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1294, 1117, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-4x+4)(x^2-4x+5)}dx$$
$$\Big\downarrow \text{1294}$$
$$\int \frac{1}{(2-x)^2(x^2-4x+5)}dx$$
$$\Big\downarrow \text{1117}$$
$$\frac{1}{2-x}-\int \frac{1}{x^2-4x+5}dx$$
$$\Big\downarrow \text{1083}$$
$$2\int \frac{1}{-(2x-4)^2-4}d(2x-4)+\frac{1}{2-x}$$
$$\Big\downarrow \text{217}$$
$$\frac{1}{2-x}-\arctan\left(\frac{1}{2}(2x-4)\right)$$

input
```
Int[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
```

output
```
(2 - x)^(-1) - ArcTan[(-4 + 2*x)/2]
```

3.349.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1117
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[-2*b*d*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(d^2*(m
+ 1)*(b^2 - 4*a*c))), x] + Simp[b^2*((m + 2*p + 3)/(d^2*(m + 1)*(b^2 - 4*a*
c)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[m, -1] &
& (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) || IntegerQ[(m + 2*p + 3)
/2])
```

rule 1294
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_.), x_Symbol] :> Simp[1/c^p   Int[(b/2 + c*x)^(2*p)*(d + e*x + f*x
^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && EqQ[b^2 - 4*a*c, 0] &&
IntegerQ[p]
```

3.349.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$-\arctan(x-2)-\frac{1}{x-2}$	15
risch	$-\arctan(x-2)-\frac{1}{x-2}$	15
parallelrisch	$\frac{i\ln(x-2-i)x-i\ln(x-2+i)x-2i\ln(x-2-i)+2i\ln(x-2+i)-x}{2x-4}$	50

input
```
int(1/(x^2-4*x+4)/(x^2-4*x+5),x,method=_RETURNVERBOSE)
```

output
```
-arctan(x-2)-1/(x-2)
```

3.349.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{(x-2)\arctan(x-2)+1}{x-2}$$

input
```
integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="fricas")
```

output
```
-((x - 2)*arctan(x - 2) + 1)/(x - 2)
```

3.349.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\operatorname{atan}(x-2) - \frac{1}{x-2}$$

input
```
integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)
```

output
```
-atan(x - 2) - 1/(x - 2)
```

3.349.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{1}{x-2} - \arctan(x-2)$$

input
```
integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="maxima")
```

output
```
-1/(x - 2) - arctan(x - 2)
```

3.349.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\frac{1}{x-2} - \arctan(x-2)$$

input
```
integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="giac")
```

output
```
-1/(x - 2) - arctan(x - 2)
```

3.349.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx = -\mathrm{atan}(x-2) - \frac{1}{x-2}$$

input
```
int(1/((x^2 - 4*x + 4)*(x^2 - 4*x + 5)),x)
```

output
```
- atan(x - 2) - 1/(x - 2)
```

3.350 $\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx$

3.350.1 Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx = -\frac{1}{x}+\log(3-x)$$

output
```
-1/x+ln(3-x)
```

3.350.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx = -\frac{1}{x}+\log(3-x)$$

input
```
Integrate[(-3 + x + x^2)/((-3 + x)*x^2),x]
```

output
```
-x^(-1) + Log[3 - x]
```

3.350.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x-3}{(x-3)x^2}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{1}{x^2}+\frac{1}{x-3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(3-x)-\frac{1}{x}$$

input
```
Int[(-3 + x + x^2)/((-3 + x)*x^2),x]
```

output
```
-x^(-1) + Log[3 - x]
```

3.350.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.350.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$\ln(-3+x)-\frac{1}{x}$	11
norman	$\ln(-3+x)-\frac{1}{x}$	11
risch	$\ln(-3+x)-\frac{1}{x}$	11
meijerg	$\ln\left(1-\frac{x}{3}\right)-\frac{1}{x}$	13
parallelrisch	$\frac{\ln(-3+x)x-1}{x}$	13

input
```
int((x^2+x-3)/(-3+x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
ln(-3+x)-1/x
```

3.350.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx=\frac{x\log(x-3)-1}{x}$$

input
```
integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="fricas")
```

output
```
(x*log(x - 3) - 1)/x
```

3.350.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx=\log(x-3)-\frac{1}{x}$$

input
```
integrate((x**2+x-3)/(-3+x)/x**2,x)
```

output
```
log(x - 3) - 1/x
```

3.350.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx = -\frac{1}{x} + \log(x-3)$$

input
```
integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="maxima")
```

output
```
-1/x + log(x - 3)
```

3.350.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx = -\frac{1}{x} + \log(|x-3|)$$

input
```
integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="giac")
```

output
```
-1/x + log(abs(x - 3))
```

3.350.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx = \ln(x-3) - \frac{1}{x}$$

input
```
int((x + x^2 - 3)/(x^2*(x - 3)),x)
```

output
```
log(x - 3) - 1/x
```

3.351 $\int \frac{1+x+4x^2}{x+4x^3} dx$

3.351.1 Optimal result

Integrand size = 18, antiderivative size = 11

$$\int \frac{1+x+4x^2}{x+4x^3} dx = \frac{1}{2}\arctan(2x) + \log(x)$$

output
```
1/2*arctan(2*x)+ln(x)
```

3.351.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1+x+4x^2}{x+4x^3} dx = \frac{1}{2}\arctan(2x) + \log(x)$$

input
```
Integrate[(1 + x + 4*x^2)/(x + 4*x^3),x]
```

output
```
ArcTan[2*x]/2 + Log[x]
```

3.351.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2026, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+x+1}{4x^3+x} \, dx$$
$$\Big\downarrow \; 2026$$
$$\int \frac{4x^2+x+1}{x(4x^2+1)} dx$$
$$\Big\downarrow \; 2333$$
$$\int \left(\frac{1}{4x^2+1}+\frac{1}{x}\right) dx$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{2}\arctan(2x)+\log(x)$$

input
```
Int[(1 + x + 4*x^2)/(x + 4*x^3),x]
```

output
```
ArcTan[2*x]/2 + Log[x]
```

3.351.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.351.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\arctan(2x)}{2}+\ln\left(x\right)$	10
risch	$\frac{\arctan(2x)}{2}+\ln\left(x\right)$	10
meijerg	$\frac{\arctan(2x)}{2}+\ln\left(x\right)+\ln\left(2\right)$	12
parallelrisch	$\ln\left(x\right)-\frac{i\ln\left(x-\frac{i}{2}\right)}{4}+\frac{i\ln\left(x+\frac{i}{2}\right)}{4}$	20

input
```
int((4*x^2+x+1)/(4*x^3+x),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(2*x)+ln(x)
```

3.351.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx = \frac{1}{2}\arctan\left(2\,x\right)+\log\left(x\right)$$

input
```
integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="fricas")
```

output
```
1/2*arctan(2*x) + log(x)
```

3.351.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx = \log\left(x\right)+\frac{\operatorname{atan}\left(2x\right)}{2}$$

input
```
integrate((4*x**2+x+1)/(4*x**3+x),x)
```

output
```
log(x) + atan(2*x)/2
```

3.351.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx = \frac{1}{2}\arctan(2x) + \log(x)$$

input
```
integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="maxima")
```

output
```
1/2*arctan(2*x) + log(x)
```

3.351.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx = \frac{1}{2}\arctan(2x) + \log(|x|)$$

input
```
integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="giac")
```

output
```
1/2*arctan(2*x) + log(abs(x))
```

3.351.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx = \ln(x) - \frac{\operatorname{atan}\left(\frac{17}{32\left(\frac{x}{16}-\frac{1}{8}\right)}+4\right)}{2}$$

input
```
int((x + 4*x^2 + 1)/(x + 4*x^3),x)
```

output
```
log(x) - atan(17/(32*(x/16 - 1/8)) + 4)/2
```

3.352 $\int \frac{1-x+3x^2}{-x^2+x^3} dx$

3.352.1 Optimal result

Integrand size = 22, antiderivative size = 12

$$\int \frac{1-x+3x^2}{-x^2+x^3} dx = \frac{1}{x} + 3\log(1-x)$$

output
```
1/x+3*ln(1-x)
```

3.352.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1-x+3x^2}{-x^2+x^3} dx = \frac{1}{x} + 3\log(1-x)$$

input
```
Integrate[(1 - x + 3*x^2)/(-x^2 + x^3),x]
```

output
```
x^(-1) + 3*Log[1 - x]
```

3.352.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2026, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2-x+1}{x^3-x^2}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{3x^2-x+1}{(x-1)x^2}dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{3}{x-1}-\frac{1}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{x}+3\log(1-x)$$

input
```
Int[(1 - x + 3*x^2)/(-x^2 + x^3),x]
```

output
```
x^(-1) + 3*Log[1 - x]
```

3.352.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])
```

3.352.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{1}{x}+3\ln\left(x-1\right)$	11
norman	$\frac{1}{x}+3\ln\left(x-1\right)$	11
risch	$\frac{1}{x}+3\ln\left(x-1\right)$	11
meijerg	$\frac{1}{x}+3\ln\left(1-x\right)$	13
parallelrisch	$\frac{3\ln(x-1)x+1}{x}$	14

input
```
int((3*x^2-x+1)/(x^3-x^2),x,method=_RETURNVERBOSE)
```

output
```
1/x+3*ln(x-1)
```

3.352.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx = \frac{3\,x\log\left(x-1\right)+1}{x}$$

input
```
integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="fricas")
```

output
```
(3*x*log(x - 1) + 1)/x
```

3.352.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx = 3\log(x-1)+\frac{1}{x}$$

input
```
integrate((3*x**2-x+1)/(x**3-x**2),x)
```

output
```
3*log(x - 1) + 1/x
```

3.352.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx = \frac{1}{x}+3\log(x-1)$$

input
```
integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="maxima")
```

output
```
1/x + 3*log(x - 1)
```

3.352.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx = \frac{1}{x}+3\log(|x-1|)$$

input
```
integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="giac")
```

output
```
1/x + 3*log(abs(x - 1))
```

3.352.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1 - x + 3x^2}{-x^2 + x^3}\, dx = 3\, \ln{(x - 1)} + \frac{1}{x}$$

input
```
int(-(3*x^2 - x + 1)/(x^2 - x^3),x)
```

output
```
3*log(x - 1) + 1/x
```

3.353 $\int \frac{4+3x+x^2}{x+x^2} dx$

3.353.1 Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{4+3x+x^2}{x+x^2} dx = x+4\log(x)-2\log(1+x)$$

output
```
x+4*ln(x)-2*ln(1+x)
```

3.353.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{4+3x+x^2}{x+x^2} dx = x+4\log(x)-2\log(1+x)$$

input
```
Integrate[(4 + 3*x + x^2)/(x + x^2),x]
```

output
```
x + 4*Log[x] - 2*Log[1 + x]
```

3.353.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2026, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+3x+4}{x^2+x} \, dx$$
$$\Big\downarrow \text{2026}$$
$$\int \frac{x^2+3x+4}{x(x+1)}dx$$
$$\Big\downarrow \text{1195}$$
$$\int \left(-\frac{2}{x+1}+\frac{4}{x}+1\right)dx$$
$$\Big\downarrow \text{2009}$$
$$x+4\log(x)-2\log(x+1)$$

input
```
Int[(4 + 3*x + x^2)/(x + x^2),x]
```

output
```
x + 4*Log[x] - 2*Log[1 + x]
```

3.353.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.353.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$x+4\ln(x)-2\ln(x+1)$	13
norman	$x+4\ln(x)-2\ln(x+1)$	13
meijerg	$x+4\ln(x)-2\ln(x+1)$	13
risch	$x+4\ln(x)-2\ln(x+1)$	13
parallelrisch	$x+4\ln(x)-2\ln(x+1)$	13

input
```
int((x^2+3*x+4)/(x^2+x),x,method=_RETURNVERBOSE)
```

output
```
x+4*ln(x)-2*ln(x+1)
```

3.353.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{4+3x+x^2}{x+x^2}\,dx = x-2\,\log(x+1)+4\,\log(x)$$

input
```
integrate((x^2+3*x+4)/(x^2+x),x, algorithm="fricas")
```

output
```
x - 2*log(x + 1) + 4*log(x)
```

3.353.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{4+3x+x^2}{x+x^2}\,dx = x+4\log(x)-2\log(x+1)$$

input
```
integrate((x**2+3*x+4)/(x**2+x),x)
```

output
```
x + 4*log(x) - 2*log(x + 1)
```

3.353.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{4+3x+x^2}{x+x^2}\,dx = x - 2\,\log\left(x+1\right) + 4\,\log\left(x\right)$$

input
```
integrate((x^2+3*x+4)/(x^2+x),x, algorithm="maxima")
```

output
```
x - 2*log(x + 1) + 4*log(x)
```

3.353.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{4+3x+x^2}{x+x^2}\,dx = x - 2\,\log\left(|x+1|\right) + 4\,\log\left(|x|\right)$$

input
```
integrate((x^2+3*x+4)/(x^2+x),x, algorithm="giac")
```

output
```
x - 2*log(abs(x + 1)) + 4*log(abs(x))
```

3.353.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{4+3x+x^2}{x+x^2}\,dx = x - 2\,\ln\left(x+1\right) + 4\,\ln\left(x\right)$$

input
```
int((3*x + x^2 + 4)/(x + x^2),x)
```

output
```
x - 2*log(x + 1) + 4*log(x)
```

3.354 $\int \frac{4+x+3x^2}{x+x^3} \, dx$

3.354.1 Optimal result

Integrand size = 16, antiderivative size = 17

$$\int \frac{4+x+3x^2}{x+x^3} \, dx = \arctan(x) + 4\log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

output
```
arctan(x)+4*ln(x)-1/2*ln(x^2+1)
```

3.354.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{4+x+3x^2}{x+x^3} \, dx = \arctan(x) + 4\log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[(4 + x + 3*x^2)/(x + x^3),x]
```

output
```
ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

3.354.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2026, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+x+4}{x^3+x}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{3x^2+x+4}{x(x^2+1)}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{1-x}{x^2+1}+\frac{4}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+4\log(x)$$

input
```
Int[(4 + x + 3*x^2)/(x + x^3),x]
```

output
```
ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

3.354.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.354.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\arctan(x)+4\ln(x)-\frac{\ln(x^2+1)}{2}$	16
meijerg	$\arctan(x)+4\ln(x)-\frac{\ln(x^2+1)}{2}$	16
risch	$\arctan(x)+4\ln(x)-\frac{\ln(x^2+1)}{2}$	16
parallelrisch	$4\ln(x)-\frac{\ln(x-i)}{2}-\frac{i\ln(x-i)}{2}-\frac{\ln(x+i)}{2}+\frac{i\ln(x+i)}{2}$	36

input
```
int((3*x^2+x+4)/(x^3+x),x,method=_RETURNVERBOSE)
```

output
```
arctan(x)+4*ln(x)-1/2*ln(x^2+1)
```

3.354.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = \arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + 4\log(x)$$

input
```
integrate((3*x^2+x+4)/(x^3+x),x, algorithm="fricas")
```

output
```
arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)
```

3.354.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = 4\log(x) - \frac{\log(x^2+1)}{2} + \operatorname{atan}(x)$$

input
```
integrate((3*x**2+x+4)/(x**3+x),x)
```

output
```
4*log(x) - log(x**2 + 1)/2 + atan(x)
```

3.354.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = \arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + 4\log(x)$$

input
```
integrate((3*x^2+x+4)/(x^3+x),x, algorithm="maxima")
```

output
```
arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)
```

3.354.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = \arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + 4\log(|x|)$$

input
```
integrate((3*x^2+x+4)/(x^3+x),x, algorithm="giac")
```

output
```
arctan(x) - 1/2*log(x^2 + 1) + 4*log(abs(x))
```

3.354.9 Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = 4\ln(x) + \ln(x-\mathrm{i})\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

input
```
int((x + 3*x^2 + 4)/(x + x^3),x)
```

output
```
4*log(x) - log(x + 1i)*(1/2 - 1i/2) - log(x - 1i)*(1/2 + 1i/2)
```

3.355 $\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)} dx$

3.355.1 Optimal result

Integrand size = 25, antiderivative size = 13

$$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)} dx = -\arctan(x) + 2\log(1+4x)$$

output
```
-arctan(x)+2*ln(1+4*x)
```

3.355.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)} dx = -\arctan(x) + 2\log(1+4x)$$

input
```
Integrate[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]
```

output
```
-ArcTan[x] + 2*Log[1 + 4*x]
```

3.355.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{8x^2-4x+7}{(4x+1)(x^2+1)}dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{1}{-x^2-1}+\frac{8}{4x+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\log(4x+1)-\arctan(x)$$

input
```
Int[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]
```

output
```
-ArcTan[x] + 2*Log[1 + 4*x]
```

3.355.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.355.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-\arctan(x)+2\ln(1+4x)$	14
risch	$-\arctan(x)+2\ln(1+4x)$	14
parallelrisch	$2\ln\left(x+\frac{1}{4}\right)+\frac{i\ln(x-i)}{2}-\frac{i\ln(x+i)}{2}$	24

input
```
int((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-arctan(x)+2*ln(1+4*x)
```

3.355.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{7-4x+8x^2}{(1+4x)\left(1+x^2\right)}\,dx = -\arctan(x)+2\,\log(4\,x+1)$$

input
```
integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="fricas")
```

output
```
-arctan(x) + 2*log(4*x + 1)
```

3.355.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{7-4x+8x^2}{(1+4x)\left(1+x^2\right)}\,dx = 2\log\left(x+\frac{1}{4}\right)-\operatorname{atan}(x)$$

input
```
integrate((8*x**2-4*x+7)/(1+4*x)/(x**2+1),x)
```

output
```
2*log(x + 1/4) - atan(x)
```

3.355.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx = -\arctan(x) + 2\log(4x+1)$$

input
```
integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="maxima")
```

output
```
-arctan(x) + 2*log(4*x + 1)
```

3.355.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx = -\arctan(x) + 2\log(|4x+1|)$$

input
```
integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="giac")
```

output
```
-arctan(x) + 2*log(abs(4*x + 1))
```

3.355.9 Mupad [B] (verification not implemented)

Time = 10.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx = \operatorname{atan}\left(\frac{4x+1}{x-4}\right) + 2\ln\left(x+\frac{1}{4}\right)$$

input
```
int((8*x^2 - 4*x + 7)/((4*x + 1)*(x^2 + 1)),x)
```

output
```
atan((4*x + 1)/(x - 4)) + 2*log(x + 1/4)
```

3.356 $\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx$

3.356.1 Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx=\frac{1}{2(1+x)}+\frac{1}{4}\log(1-x)+\frac{3}{4}\log(1+x)$$

output
```
1/2/(1+x)+1/4*ln(1-x)+3/4*ln(1+x)
```

3.356.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx=\frac{1}{4}\left(\frac{2}{1+x}+\log(-1+x)+3\log(1+x)\right)$$

input
```
Integrate[x^2/((-1 + x)*(1 + 2*x + x^2)),x]
```

output
```
(2/(1 + x) + Log[-1 + x] + 3*Log[1 + x])/4
```

3.356.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1184, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x-1)(x^2+2x+1)}dx$$

$$\Big\downarrow \text{1184}$$

$$\int -\frac{x^2}{(1-x)(x+1)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{x^2}{(1-x)(x+1)^2}dx$$

$$\Big\downarrow \text{99}$$

$$-\int \left(-\frac{3}{4(x+1)}+\frac{1}{2(x+1)^2}-\frac{1}{4(x-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)+\frac{3}{4}\log(x+1)$$

input
```
Int[x^2/((-1 + x)*(1 + 2*x + x^2)),x]
```

output
```
1/(2*(1 + x)) + Log[1 - x]/4 + (3*Log[1 + x])/4
```

3.356.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.356.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{1}{2x+2}+\frac{3\ln(x+1)}{4}+\frac{\ln(x-1)}{4}$	21
norman	$\frac{1}{2x+2}+\frac{3\ln(x+1)}{4}+\frac{\ln(x-1)}{4}$	21
risch	$\frac{1}{2x+2}+\frac{3\ln(x+1)}{4}+\frac{\ln(x-1)}{4}$	21
parallelrisch	$\frac{\ln(x-1)x+3\ln(x+1)x+2+\ln(x-1)+3\ln(x+1)}{4+4x}$	33

input
```
int(x^2/(x-1)/(x^2+2*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2/(x+1)+3/4*ln(x+1)+1/4*ln(x-1)
```

3.356.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx=\frac{3\,(x+1)\log(x+1)+(x+1)\log(x-1)+2}{4\,(x+1)}$$

input
```
integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="fricas")
```

output
```
1/4*(3*(x + 1)*log(x + 1) + (x + 1)*log(x - 1) + 2)/(x + 1)
```

3.356.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx = \frac{\log(x-1)}{4} + \frac{3\log(x+1)}{4} + \frac{1}{2x+2}$$

input
```
integrate(x**2/(-1+x)/(x**2+2*x+1),x)
```

output
```
log(x - 1)/4 + 3*log(x + 1)/4 + 1/(2*x + 2)
```

3.356.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx = \frac{1}{2(x+1)} + \frac{3}{4}\log(x+1) + \frac{1}{4}\log(x-1)$$

input
```
integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="maxima")
```

output
```
1/2/(x + 1) + 3/4*log(x + 1) + 1/4*log(x - 1)
```

3.356.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx = \frac{1}{2(x+1)} + \frac{3}{4}\log(|x+1|) + \frac{1}{4}\log(|x-1|)$$

input
```
integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="giac")
```

output
```
1/2/(x + 1) + 3/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

3.356.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx = \frac{\ln(x-1)}{4} + \frac{3\ln(x+1)}{4} + \frac{1}{2(x+1)}$$

input
```
int(x^2/((x - 1)*(2*x + x^2 + 1)),x)
```

output
```
log(x - 1)/4 + (3*log(x + 1))/4 + 1/(2*(x + 1))
```

3.357 $\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx$

3.357.1 Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = -\frac{9}{32(1-2x)} + \frac{41}{128}\log(1-2x) - \frac{25}{128}\log(3+2x)$$

output
```
-9/32/(1-2*x)+41/128*ln(1-2*x)-25/128*ln(3+2*x)
```

3.357.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = \frac{9}{32(-1+2x)} + \frac{41}{128}\log(1-2x) - \frac{25}{128}\log(3+2x)$$

input
```
Integrate[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
```

output
```
9/(32*(-1 + 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

3.357.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+3x-4}{(2x-1)^2(2x+3)}\,dx$$

$$\Big\downarrow \ 1195$$

$$\int \left(-\frac{25}{64(2x+3)}+\frac{41}{64(2x-1)}-\frac{9}{16(2x-1)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

input
```
Int[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
```

output
```
-9/(32*(1 - 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

3.357.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.357.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{9}{64\left(x-\frac{1}{2}\right)}+\frac{41\ln\left(2x-1\right)}{128}-\frac{25\ln\left(2x+3\right)}{128}$	25
default	$-\frac{25\ln\left(2x+3\right)}{128}+\frac{9}{32\left(2x-1\right)}+\frac{41\ln\left(2x-1\right)}{128}$	27
norman	$\frac{9x}{16\left(2x-1\right)}+\frac{41\ln\left(2x-1\right)}{128}-\frac{25\ln\left(2x+3\right)}{128}$	28
parallelrisch	$\frac{82\ln\left(x-\frac{1}{2}\right)x-50\ln\left(x+\frac{3}{2}\right)x-41\ln\left(x-\frac{1}{2}\right)+25\ln\left(x+\frac{3}{2}\right)+72x}{256x-128}$	40

input
```
int((x^2+3*x-4)/(2*x-1)^2/(2*x+3),x,method=_RETURNVERBOSE)
```

output
```
9/64/(x-1/2)+41/128*ln(2*x-1)-25/128*ln(2*x+3)
```

3.357.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = -\frac{25\,(2\,x-1)\log\left(2\,x+3\right)-41\,(2\,x-1)\log\left(2\,x-1\right)-36}{128\,(2\,x-1)}$$

input
```
integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="fricas")
```

output
```
-1/128*(25*(2*x - 1)*log(2*x + 3) - 41*(2*x - 1)*log(2*x - 1) - 36)/(2*x -
 1)
```

3.357.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = \frac{41\log\left(x-\frac{1}{2}\right)}{128}-\frac{25\log\left(x+\frac{3}{2}\right)}{128}+\frac{9}{64x-32}$$

input
```
integrate((x**2+3*x-4)/(-1+2*x)**2/(3+2*x),x)
```

output
```
41*log(x - 1/2)/128 - 25*log(x + 3/2)/128 + 9/(64*x - 32)
```

3.357.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = \frac{9}{32\,(2\,x-1)} - \frac{25}{128}\,\log\left(2\,x+3\right) + \frac{41}{128}\,\log\left(2\,x-1\right)$$

input
```
integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="maxima")
```

output
```
9/32/(2*x - 1) - 25/128*log(2*x + 3) + 41/128*log(2*x - 1)
```

3.357.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = \frac{9}{32\,(2\,x-1)} - \frac{1}{8}\,\log\left(\frac{|2\,x-1|}{2\,(2\,x-1)^2}\right) - \frac{25}{128}\,\log\left(\left|-\frac{4}{2\,x-1}-1\right|\right)$$

input
```
integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="giac")
```

output
```
9/32/(2*x - 1) - 1/8*log(1/2*abs(2*x - 1)/(2*x - 1)^2) - 25/128*log(abs(-4
/(2*x - 1) - 1))
```

3.357.9 Mupad [B] (verification not implemented)

Time = 9.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx = \frac{41\,\ln\left(x-\frac{1}{2}\right)}{128} - \frac{25\,\ln\left(x+\frac{3}{2}\right)}{128} + \frac{9}{64\,\left(x-\frac{1}{2}\right)}$$

input
```
int((3*x + x^2 - 4)/((2*x - 1)^2*(2*x + 3)),x)
```

output
```
(41*log(x - 1/2))/128 - (25*log(x + 3/2))/128 + 9/(64*(x - 1/2))
```

3.358 $\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx$

3.358.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = -3\arctan(x)+2\log(1-x)+\frac{1}{2}\log\left(1+x^2\right)$$

output
```
-3*arctan(x)+2*ln(1-x)+1/2*ln(x^2+1)
```

3.358.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = -3\arctan(x)+\frac{1}{2}\log\left(2+2(-1+x)+(-1+x)^2\right)+2\log(-1+x)$$

input
```
Integrate[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
```

output
```
-3*ArcTan[x] + Log[2 + 2*(-1 + x) + (-1 + x)^2]/2 + 2*Log[-1 + x]
```

3.358.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2-4x+5}{(x-1)\left(x^2+1\right)}dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{x-3}{x^2+1}+\frac{2}{x-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-3\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)+2\log(1-x)$$

input
```
Int[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
```

output
```
-3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2
```

3.358.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.358.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\ln(x^2+1)}{2} - 3\arctan(x) + 2\ln(x-1)$	20
risch	$2\ln(x-1) + \frac{\ln(9x^2+9)}{2} - 3\arctan(x)$	22
parallelrisch	$2\ln(x-1) + \frac{\ln(x-i)}{2} + \frac{3i\ln(x-i)}{2} + \frac{\ln(x+i)}{2} - \frac{3i\ln(x+i)}{2}$	38

input
```
int((3*x^2-4*x+5)/(x-1)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2+1)-3*arctan(x)+2*ln(x-1)
```

3.358.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = -3\arctan(x) + \frac{1}{2}\log(x^2+1) + 2\log(x-1)$$

input
```
integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="fricas")
```

output
```
-3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

3.358.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = 2\log(x-1) + \frac{\log(x^2+1)}{2} - 3\,\mathrm{atan}(x)$$

input
```
integrate((3*x**2-4*x+5)/(-1+x)/(x**2+1),x)
```

output
```
2*log(x - 1) + log(x**2 + 1)/2 - 3*atan(x)
```

3.358.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)}\,dx = -3\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)+2\log(x-1)$$

input
```
integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="maxima")
```

output
```
-3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

3.358.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)}\,dx = -3\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)+2\log(|x-1|)$$

input
```
integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="giac")
```

output
```
-3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(abs(x - 1))
```

3.358.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)}\,dx = 2\ln(x-1)+\ln(x-\mathrm{i})\left(\frac{1}{2}+\frac{3}{2}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{1}{2}-\frac{3}{2}\mathrm{i}\right)$$

input
```
int((3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)),x)
```

output
```
2*log(x - 1) + log(x - 1i)*(1/2 + 3i/2) + log(x + 1i)*(1/2 - 3i/2)
```

3.359 $\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx$

3.359.1 Optimal result

Integrand size = 21, antiderivative size = 24

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \frac{1}{-1+x} + \arctan(x) + \log(1-x) - \frac{1}{2}\log\left(1+x^2\right)$$

output
```
1/(-1+x)+arctan(x)+ln(1-x)-1/2*ln(x^2+1)
```

3.359.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \frac{1}{-1+x} + \arctan(x) + \log(-1+x) - \frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

output
```
(-1 + x)^(-1) + ArcTan[x] + Log[-1 + x] - Log[1 + x^2]/2
```

3.359.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-2x-1}{(x-1)^2(x^2+1)}dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{1-x}{x^2+1}+\frac{1}{x-1}-\frac{1}{(x-1)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)$$

input
```
Int[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

output
```
(-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

3.359.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.359.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\ln(x^2+1)}{2}+\arctan(x)+\ln(x-1)+\frac{1}{x-1}$	21
risch	$-\frac{\ln(x^2+1)}{2}+\arctan(x)+\ln(x-1)+\frac{1}{x-1}$	21
parallelrisch	$-\frac{i\ln(x-i)x-i\ln(x+i)x-i\ln(x-i)+i\ln(x+i)-2\ln(x-1)x+\ln(x-i)x+\ln(x+i)x-2+2\ln(x-1)-\ln(x-i)-\ln(x+i)}{2(x-1)}$	85

input
```
int((x^2-2*x-1)/(x-1)^2/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(x^2+1)+arctan(x)+ln(x-1)+1/(x-1)
```

3.359.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int\frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx=\frac{2(x-1)\arctan(x)-(x-1)\log(x^2+1)+2(x-1)\log(x-1)+2}{2(x-1)}$$

input
```
integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*(2*(x - 1)*arctan(x) - (x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x - 1) + 2
)/(x - 1)
```

3.359.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int\frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx=\log(x-1)-\frac{\log(x^2+1)}{2}+\operatorname{atan}(x)+\frac{1}{x-1}$$

input
```
integrate((x**2-2*x-1)/(-1+x)**2/(x**2+1),x)
```

output
```
log(x - 1) - log(x**2 + 1)/2 + atan(x) + 1/(x - 1)
```

3.359.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \frac{1}{x-1} + \arctan(x) - \frac{1}{2}\log(x^2+1) + \log(x-1)$$

input
```
integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
```

output
```
1/(x - 1) + arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

3.359.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(22) = 44.

Time = 0.39 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \frac{1}{4}\pi - \pi\left\lfloor\frac{\pi+4\arctan(x)}{4\pi}+\frac{1}{2}\right\rfloor + \frac{1}{x-1} + \arctan(x) - \frac{1}{2}\log\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)$$

input
```
integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="giac")
```

output
```
1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/(x - 1) + arctan(x)
 - 1/2*log(2/(x - 1) + 2/(x - 1)^2 + 1)
```

3.359.9 Mupad [B] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \ln(x-1) + \frac{1}{x-1} + \ln(x-\mathrm{i})\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

input
```
int(-(2*x - x^2 + 1)/((x^2 + 1)*(x - 1)^2),x)
```

output
```
log(x - 1) - log(x - 1i)*(1/2 + 1i/2) - log(x + 1i)*(1/2 - 1i/2) + 1/(x -
1)
```

3.360 $\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx$

3.360.1 Optimal result

Integrand size = 28, antiderivative size = 49

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = -\frac{261}{221}\arctan(1-2x)-\frac{1026}{221}\arctan(3-x)+\frac{56}{221}\log\left(10-6x+x^2\right)+\frac{109}{442}\log\left(1-2x+2x^2\right)$$

output
```
261/221*arctan(-1+2*x)+1026/221*arctan(-3+x)+56/221*ln(x^2-6*x+10)+109/442
*ln(2*x^2-2*x+1)
```

3.360.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = -\frac{261}{221}\arctan(1-2x)-\frac{1026}{221}\arctan(3-x)+\frac{56}{221}\log\left(10-6x+x^2\right)+\frac{109}{442}\log\left(1-2x+2x^2\right)$$

input
```
Integrate[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]
```

output
```
(-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x +
 x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```

3.360.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+5}{(x^2-6x+10)\left(x^2-x+\frac{1}{2}\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{2(56x+345)}{221(x^2-6x+10)}+\frac{2(109x+76)}{221(2x^2-2x+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{261}{221}\arctan(1-2x)-\frac{1026}{221}\arctan(3-x)+\frac{56}{221}\log\left(x^2-6x+10\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)$$

input
```
Int[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]
```

output
```
(-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x +
 x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```

3.360.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.360.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result
default	$\frac{261 \arctan(2x-1)}{221}+\frac{1026 \arctan(-3+x)}{221}+\frac{56 \ln(x^2-6x+10)}{221}+\frac{109 \ln(2x^2-2x+1)}{442}$
risch	$\frac{56 \ln(x^2-6x+10)}{221}+\frac{1026 \arctan(-3+x)}{221}+\frac{109 \ln(4x^2-4x+2)}{442}+\frac{261 \arctan(2x-1)}{221}$
parallelrisch	$\frac{56 \ln(x-3-i)}{221}-\frac{513i \ln(x-3-i)}{221}+\frac{56 \ln(x-3+i)}{221}+\frac{513i \ln(x-3+i)}{221}+\frac{109 \ln(x-\frac{1}{2}-\frac{i}{2})}{442}-\frac{261i \ln(x-\frac{1}{2}-\frac{i}{2})}{442}+\frac{109 \ln}{}$

input
```
int((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x,method=_RETURNVERBOSE)
```

output
```
261/221*arctan(2*x-1)+1026/221*arctan(-3+x)+56/221*ln(x^2-6*x+10)+109/442*
ln(2*x^2-2*x+1)
```

3.360.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = \frac{261}{221}\arctan\left(2x-1\right)+\frac{1026}{221}\arctan\left(x-3\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)+\frac{56}{221}\log\left(x^2-6x+10\right)$$

input
```
integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="fricas")
```

output
```
261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
 + 1) + 56/221*log(x^2 - 6*x + 10)
```

3.360.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = \frac{56\log\left(x^2-6x+10\right)}{221}+\frac{109\log\left(x^2-x+\frac{1}{2}\right)}{442}+\frac{1026\,\mathrm{atan}\left(x-3\right)}{221}+\frac{261\,\mathrm{atan}\left(2x-1\right)}{221}$$

input
```
integrate((x**3+5)/(x**2-6*x+10)/(1/2-x+x**2),x)
```

output
```
56*log(x**2 - 6*x + 10)/221 + 109*log(x**2 - x + 1/2)/442 + 1026*atan(x -
3)/221 + 261*atan(2*x - 1)/221
```

3.360.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = \frac{261}{221}\arctan(2x-1) + \frac{1026}{221}\arctan(x-3) + \frac{109}{442}\log\left(2x^2-2x+1\right) + \frac{56}{221}\log\left(x^2-6x+10\right)$$

input
```
integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="maxima")
```

output
```
261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
 + 1) + 56/221*log(x^2 - 6*x + 10)
```

3.360.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = \frac{261}{221}\arctan(2x-1) + \frac{1026}{221}\arctan(x-3) + \frac{109}{442}\log\left(2x^2-2x+1\right) + \frac{56}{221}\log\left(x^2-6x+10\right)$$

input
```
integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="giac")
```

output
```
261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
 + 1) + 56/221*log(x^2 - 6*x + 10)
```

3.360.9 Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx = \ln(x-3-\mathrm{i})\left(\frac{56}{221}-\frac{513}{221}\mathrm{i}\right) + \ln(x-3+1\mathrm{i})\left(\frac{56}{221}+\frac{513}{221}\mathrm{i}\right) + \ln\left(x-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{109}{442}-\frac{261}{442}\mathrm{i}\right) + \ln\left(x-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{109}{442}+\frac{261}{442}\mathrm{i}\right)$$

input
```
int((x^3 + 5)/((x^2 - x + 1/2)*(x^2 - 6*x + 10)),x)
```

output
```
log(x - (3 + 1i))*(56/221 - 513i/221) + log(x - (3 - 1i))*(56/221 + 513i/2
21) + log(x - (1/2 + 1i/2))*(109/442 - 261i/442) + log(x - (1/2 - 1i/2))*(
109/442 + 261i/442)
```

3.361 $\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx$

3.361.1 Optimal result

Integrand size = 24, antiderivative size = 25

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 4\log(1-x)-14\log(2-x)+11\log(3-x)$$

output
```
4*ln(1-x)-14*ln(2-x)+11*ln(3-x)
```

3.361.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 11\log(-3+x)-14\log(-2+x)+4\log(-1+x)$$

input
```
Integrate[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

output
```
11*Log[-3 + x] - 14*Log[-2 + x] + 4*Log[-1 + x]
```

3.361.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+3x+4}{(x-3)(x-2)(x-1)}dx$$

$$\Big\downarrow \text{ 2115}$$

$$\int \left(-\frac{14}{x-2}+\frac{4}{x-1}+\frac{11}{x-3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$4\log(1-x)-14\log(2-x)+11\log(3-x)$$

input
```
Int[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

output
```
4*Log[1 - x] - 14*Log[2 - x] + 11*Log[3 - x]
```

3.361.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

3.361.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$11\ln(-3+x)+4\ln(x-1)-14\ln(x-2)$	20
norman	$11\ln(-3+x)+4\ln(x-1)-14\ln(x-2)$	20
risch	$11\ln(-3+x)+4\ln(x-1)-14\ln(x-2)$	20
parallelrisch	$11\ln(-3+x)+4\ln(x-1)-14\ln(x-2)$	20

input
```
int((x^2+3*x+4)/(-3+x)/(x-2)/(x-1),x,method=_RETURNVERBOSE)
```

output
```
11*ln(-3+x)+4*ln(x-1)-14*ln(x-2)
```

3.361.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 4\,\log(x-1) - 14\,\log(x-2) + 11\,\log(x-3)$$

input
```
integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
```

output
```
4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

3.361.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 11\log(x-3) - 14\log(x-2) + 4\log(x-1)$$

input
```
integrate((x**2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x)
```

output
```
11*log(x - 3) - 14*log(x - 2) + 4*log(x - 1)
```

3.361.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 4\log(x-1) - 14\log(x-2) + 11\log(x-3)$$

input
```
integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
```

output
```
4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

3.361.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 4\log(|x-1|) - 14\log(|x-2|) + 11\log(|x-3|)$$

input
```
integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
```

output
```
4*log(abs(x - 1)) - 14*log(abs(x - 2)) + 11*log(abs(x - 3))
```

3.361.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 4\ln(x-1) - 14\ln(x-2) + 11\ln(x-3)$$

input
```
int((3*x + x^2 + 4)/((x - 1)*(x - 2)*(x - 3)),x)
```

output
```
4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

3.362 $\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)}\,dx$

3.362.1 Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\,(1+x+x^2)}\,dx = -\frac{79}{273(5+x)}+\frac{451\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2793\sqrt{3}}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}-\frac{481\log\left(1+x+x^2\right)}{5586}$$

output
```
-79/273/(5+x)+200/3211*ln(3-2*x)+2731/24843*ln(5+x)-481/5586*ln(x^2+x+1)+4
51/8379*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.362.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\,(1+x+x^2)}\,dx = \frac{-\frac{819546}{5+x}+152438\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+176400\log(3-2x)+311334\log(5+x)-243867\log\left(1+x+x^2\right)}{2832102}$$

input
```
Integrate[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

output
```
(-819546/(5 + x) + 152438*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 176400*Log[3
 - 2*x] + 311334*Log[5 + x] - 243867*Log[1 + x + x^2])/2832102
```

3.362.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{16x+1}{(x+5)^2(2x-3)\left(x^2+x+1\right)}dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{-481x-15}{2793\left(x^2+x+1\right)}+\frac{2731}{24843(x+5)}+\frac{400}{3211(2x-3)}+\frac{79}{273(x+5)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{451\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}-\frac{481\log\left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(x+5)}{24843}$$

input
```
Int[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

output
```
-79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]) + (200*
Log[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[1 + x + x^2])/5586
```

3.362.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

3.362.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{79}{273(5+x)}+\frac{2731\ln(5+x)}{24843}-\frac{481\ln(x^2+x+1)}{5586}+\frac{451\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{8379}+\frac{200\ln(2x-3)}{3211}$	48
risch	$-\frac{79}{273(5+x)}-\frac{481\ln(4x^2+4x+4)}{5586}+\frac{451\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{8379}+\frac{200\ln(2x-3)}{3211}+\frac{2731\ln(5+x)}{24843}$	52

input
```
int((1+16*x)/(5+x)^2/(2*x-3)/(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
-79/273/(5+x)+2731/24843*ln(5+x)-481/5586*ln(x^2+x+1)+451/8379*arctan(1/3*
(1+2*x)*3^(1/2))*3^(1/2)+200/3211*ln(2*x-3)
```

3.362.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\,(1+x+x^2)}\,dx$$
$$=\frac{152438\sqrt{3}(x+5)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-243867\,(x+5)\log\left(x^2+x+1\right)+176400\,(x+5)\log\left(2x-3\right)}{2832102\,(x+5)}$$

input
```
integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="fricas")
```

output
```
1/2832102*(152438*sqrt(3)*(x + 5)*arctan(1/3*sqrt(3)*(2*x + 1)) - 243867*(
x + 5)*log(x^2 + x + 1) + 176400*(x + 5)*log(2*x - 3) + 311334*(x + 5)*log
(x + 5) - 819546)/(x + 5)
```

3.362.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\,(1+x+x^2)}\,dx=\frac{200\log\left(x-\frac{3}{2}\right)}{3211}+\frac{2731\log\left(x+5\right)}{24843}-\frac{481\log\left(x^2+x+1\right)}{5586}+\frac{451\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{8379}-\frac{79}{273x+1365}$$

input
```
integrate((1+16*x)/(5+x)**2/(-3+2*x)/(x**2+x+1),x)
```

output
```
200*log(x - 3/2)/3211 + 2731*log(x + 5)/24843 - 481*log(x**2 + x + 1)/5586
 + 451*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/8379 - 79/(273*x + 1365)
```

3.362.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\,(1+x+x^2)}\,dx = \frac{451}{8379}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{79}{273\,(x+5)} - \frac{481}{5586}\,\log\left(x^2+x+1\right) + \frac{200}{3211}\,\log\left(2\,x-3\right) + \frac{2731}{24843}\,\log\left(x+5\right)$$

input
```
integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="maxima")
```

output
```
451/8379*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 79/273/(x + 5) - 481/5586
*log(x^2 + x + 1) + 200/3211*log(2*x - 3) + 2731/24843*log(x + 5)
```

3.362.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\,(1+x+x^2)}\,dx = \frac{451}{8379}\,\sqrt{3}\arctan\left(-\sqrt{3}\left(\frac{14}{x+5}-3\right)\right) - \frac{79}{273\,(x+5)} - \frac{481}{5586}\,\log\left(-\frac{9}{x+5}+\frac{21}{(x+5)^2}+1\right) + \frac{200}{3211}\,\log\left(\left|-\frac{13}{x+5}+2\right|\right)$$

input
```
integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="giac")
```

output
```
451/8379*sqrt(3)*arctan(-sqrt(3)*(14/(x + 5) - 3)) - 79/273/(x + 5) - 481/
5586*log(-9/(x + 5) + 21/(x + 5)^2 + 1) + 200/3211*log(abs(-13/(x + 5) + 2
))
```

3.362.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\,(1+x+x^2)}\,dx = \frac{200\,\ln\left(x-\frac{3}{2}\right)}{3211}+\frac{2731\,\ln\left(x+5\right)}{24843}-\frac{79}{273\,\left(x+5\right)} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{481}{5586}+\frac{\sqrt{3}\,451\mathrm{i}}{16758}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{481}{5586}+\frac{\sqrt{3}\,451\mathrm{i}}{16758}\right)$$

input
```
int((16*x + 1)/((2*x - 3)*(x + 5)^2*(x + x^2 + 1)),x)
```

output
```
(200*log(x - 3/2))/3211 + (2731*log(x + 5))/24843 - 79/(273*(x + 5)) - log
(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 + 481/5586) + log(x + (3^
(1/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 - 481/5586)
```

3.363 $\int \frac{-1+x^3}{1+x+x^2} \, dx$

3.363.1 Optimal result

Integrand size = 14, antiderivative size = 11

$$\int \frac{-1+x^3}{1+x+x^2} \, dx = -x+\frac{x^2}{2}$$

output
```
-x+1/2*x^2
```

3.363.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^3}{1+x+x^2} \, dx = -x+\frac{x^2}{2}$$

input
```
Integrate[(-1 + x^3)/(1 + x + x^2),x]
```

output
```
-x + x^2/2
```

3.363.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2019, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-1}{x^2+x+1}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int (x-1)dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{1}{2}(1-x)^2$$

input
```
Int[(-1 + x^3)/(1 + x + x^2),x]
```

output
```
(1 - x)^2/2
```

3.363.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.363.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x(x-2)}{2}$	7
default	$-x+\frac{1}{2}x^2$	10
norman	$-x+\frac{1}{2}x^2$	10
risch	$-x+\frac{1}{2}x^2$	10
parallelrisch	$-x+\frac{1}{2}x^2$	10
parts	$-x+\frac{1}{2}x^2$	10

input
```
int((x^3-1)/(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x-2)
```

3.363.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-1+x^3}{1+x+x^2}\,dx = \frac{1}{2}\,x^2 - x$$

input
```
integrate((x^3-1)/(x^2+x+1),x, algorithm="fricas")
```

output
```
1/2*x^2 - x
```

3.363.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.45

$$\int \frac{-1+x^3}{1+x+x^2}\,dx = \frac{x^2}{2} - x$$

input
```
integrate((x**3-1)/(x**2+x+1),x)
```

output
```
x**2/2 - x
```

3.363.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-1+x^3}{1+x+x^2}\,dx = \frac{1}{2}\,x^2 - x$$

input
```
integrate((x^3-1)/(x^2+x+1),x, algorithm="maxima")
```

output
```
1/2*x^2 - x
```

3.363.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-1+x^3}{1+x+x^2}\,dx = \frac{1}{2}\,x^2 - x$$

input
```
integrate((x^3-1)/(x^2+x+1),x, algorithm="giac")
```

output
```
1/2*x^2 - x
```

3.363.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{-1+x^3}{1+x+x^2}\,dx = \frac{x\,(x-2)}{2}$$

input
```
int((x^3 - 1)/(x + x^2 + 1),x)
```

output
```
(x*(x - 2))/2
```

3.364 $\int \frac{-3+x^3}{-7-6x+x^2} \, dx$

3.364.1 Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{-3+x^3}{-7-6x+x^2} \, dx = 6x + \frac{x^2}{2} + \frac{85}{2}\log(7-x) + \frac{1}{2}\log(1+x)$$

output
```
6*x+1/2*x^2+85/2*ln(7-x)+1/2*ln(1+x)
```

3.364.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{-3+x^3}{-7-6x+x^2} \, dx = 6x + \frac{x^2}{2} + \frac{85}{2}\log(7-x) + \frac{1}{2}\log(1+x)$$

input
```
Integrate[(-3 + x^3)/(-7 - 6*x + x^2),x]
```

output
```
6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2
```

3.364.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-3}{x^2-6x-7}dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{43x+39}{x^2-6x-7}+x+6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}+6x+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(x+1)$$

input
```
Int[(-3 + x^3)/(-7 - 6*x + x^2),x]
```

output
```
6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2
```

3.364.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.364.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{x^2}{2}+6x+\frac{85\ln(x-7)}{2}+\frac{\ln(x+1)}{2}$	22
norman	$\frac{x^2}{2}+6x+\frac{85\ln(x-7)}{2}+\frac{\ln(x+1)}{2}$	22
risch	$\frac{x^2}{2}+6x+\frac{85\ln(x-7)}{2}+\frac{\ln(x+1)}{2}$	22
parallelrisch	$\frac{x^2}{2}+6x+\frac{85\ln(x-7)}{2}+\frac{\ln(x+1)}{2}$	22

input
```
int((x^3-3)/(x^2-6*x-7),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+6*x+85/2*ln(x-7)+1/2*ln(x+1)
```

3.364.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{-3+x^3}{-7-6x+x^2}\,dx = \frac{1}{2}\,x^2+6\,x+\frac{1}{2}\,\log\left(x+1\right)+\frac{85}{2}\,\log\left(x-7\right)$$

input
```
integrate((x^3-3)/(x^2-6*x-7),x, algorithm="fricas")
```

output
```
1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)
```

3.364.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{-3+x^3}{-7-6x+x^2}\,dx = \frac{x^2}{2}+6x+\frac{85\log\left(x-7\right)}{2}+\frac{\log\left(x+1\right)}{2}$$

input
```
integrate((x**3-3)/(x**2-6*x-7),x)
```

output
```
x**2/2 + 6*x + 85*log(x - 7)/2 + log(x + 1)/2
```

3.364.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{-3+x^3}{-7-6x+x^2}\,dx = \frac{1}{2}\,x^2+6\,x+\frac{1}{2}\,\log\left(x+1\right)+\frac{85}{2}\,\log\left(x-7\right)$$

input
```
integrate((x^3-3)/(x^2-6*x-7),x, algorithm="maxima")
```

output
```
1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)
```

3.364.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{-3+x^3}{-7-6x+x^2}\,dx = \frac{1}{2}\,x^2+6\,x+\frac{1}{2}\,\log\left(|x+1|\right)+\frac{85}{2}\,\log\left(|x-7|\right)$$

input
```
integrate((x^3-3)/(x^2-6*x-7),x, algorithm="giac")
```

output
```
1/2*x^2 + 6*x + 1/2*log(abs(x + 1)) + 85/2*log(abs(x - 7))
```

3.364.9 Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{-3+x^3}{-7-6x+x^2}\,dx = 6\,x+\frac{\ln\left(x+1\right)}{2}+\frac{85\,\ln\left(x-7\right)}{2}+\frac{x^2}{2}$$

input
```
int(-(x^3 - 3)/(6*x - x^2 + 7),x)
```

output
```
6*x + log(x + 1)/2 + (85*log(x - 7))/2 + x^2/2
```

3.365 $\int \frac{1+x^3}{(13+4x+x^2)^2}\,dx$

3.365.1 Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{1+x^3}{(13+4x+x^2)^2}\,dx = \frac{67+47x}{18(13+4x+x^2)} - \frac{61}{54}\arctan\left(\frac{2+x}{3}\right) + \frac{1}{2}\log\left(13+4x+x^2\right)$$

output
```
1/18*(67+47*x)/(x^2+4*x+13)-61/54*arctan(2/3+1/3*x)+1/2*ln(x^2+4*x+13)
```

3.365.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1+x^3}{(13+4x+x^2)^2}\,dx = \frac{67+47x}{18(13+4x+x^2)} - \frac{61}{54}\arctan\left(\frac{2+x}{3}\right) + \frac{1}{2}\log\left(13+4x+x^2\right)$$

input
```
Integrate[(1 + x^3)/(13 + 4*x + x^2)^2,x]
```

output
```
(67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + Log[13 + 4
*x + x^2]/2
```

3.365.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2191, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+1}{(x^2+4x+13)^2}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{36}\int -\frac{2(25-18x)}{x^2+4x+13}dx+\frac{47x+67}{18(x^2+4x+13)}$$

$$\Big\downarrow \text{27}$$

$$\frac{47x+67}{18(x^2+4x+13)}-\frac{1}{18}\int \frac{25-18x}{x^2+4x+13}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{18}\left(9\int \frac{2(x+2)}{x^2+4x+13}dx-61\int \frac{1}{x^2+4x+13}dx\right)+\frac{47x+67}{18(x^2+4x+13)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{18}\left(18\int \frac{x+2}{x^2+4x+13}dx-61\int \frac{1}{x^2+4x+13}dx\right)+\frac{47x+67}{18(x^2+4x+13)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{18}\left(18\int \frac{x+2}{x^2+4x+13}dx+122\int \frac{1}{-(2x+4)^2-36}d(2x+4)\right)+\frac{47x+67}{18(x^2+4x+13)}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{18}\left(18\int \frac{x+2}{x^2+4x+13}dx-\frac{61}{3}\arctan\left(\frac{1}{6}(2x+4)\right)\right)+\frac{47x+67}{18(x^2+4x+13)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{18}\left(9\log\left(x^2+4x+13\right)-\frac{61}{3}\arctan\left(\frac{1}{6}(2x+4)\right)\right)+\frac{47x+67}{18(x^2+4x+13)}$$

input
```
Int[(1 + x^3)/(13 + 4*x + x^2)^2,x]
```

output
```
(67 + 47*x)/(18*(13 + 4*x + x^2)) + ((-61*ArcTan[(4 + 2*x)/6])/3 + 9*Log[1
3 + 4*x + x^2])/18
```

3.365.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.365. $\int \frac{1+x^3}{(13+4x+x^2)^2} \, dx$

3.365.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result
default	$\frac{\frac{47x}{18}+\frac{67}{18}}{x^2+4x+13}+\frac{\ln(x^2+4x+13)}{2}-\frac{61\arctan\left(\frac{2}{3}+\frac{x}{3}\right)}{54}$
risch	$\frac{\frac{47x}{18}+\frac{67}{18}}{x^2+4x+13}+\frac{\ln(x^2+4x+13)}{2}-\frac{61\arctan\left(\frac{2}{3}+\frac{x}{3}\right)}{54}$
parallelrisch	$\frac{-793i\ln(x+2+3i)x^2+10309i\ln(x+2-3i)+3172i\ln(x+2-3i)x+702\ln(x+2-3i)x^2-3172i\ln(x+2+3i)x+702\ln(x+2+3i)x^2+}{1404x^2}$

input
```
int((x^3+1)/(x^2+4*x+13)^2,x,method=_RETURNVERBOSE)
```

output
```
(47/18*x+67/18)/(x^2+4*x+13)+1/2*ln(x^2+4*x+13)-61/54*arctan(2/3+1/3*x)
```

3.365.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int \frac{1+x^3}{(13+4x+x^2)^2}\,dx = -\frac{61\,(x^2+4\,x+13)\arctan\left(\frac{1}{3}\,x+\frac{2}{3}\right)-27\,(x^2+4\,x+13)\log\left(x^2+4\,x+13\right)-141\,x-201}{54\,(x^2+4\,x+13)}$$

input
```
integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="fricas")
```

output
```
-1/54*(61*(x^2 + 4*x + 13)*arctan(1/3*x + 2/3) - 27*(x^2 + 4*x + 13)*log(x
^2 + 4*x + 13) - 141*x - 201)/(x^2 + 4*x + 13)
```

3.365.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3}{(13+4x+x^2)^2}\,dx = \frac{47x+67}{18x^2+72x+234}+\frac{\log\left(x^2+4x+13\right)}{2}-\frac{61\,\mathrm{atan}\left(\frac{x}{3}+\frac{2}{3}\right)}{54}$$

input
```
integrate((x**3+1)/(x**2+4*x+13)**2,x)
```

output
```
(47*x + 67)/(18*x**2 + 72*x + 234) + log(x**2 + 4*x + 13)/2 - 61*atan(x/3
+ 2/3)/54
```

3.365.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3}{(13+4x+x^2)^2}\,dx = \frac{47\,x+67}{18\,(x^2+4\,x+13)} - \frac{61}{54}\arctan\left(\frac{1}{3}\,x+\frac{2}{3}\right) + \frac{1}{2}\log\left(x^2+4\,x+13\right)$$

input
```
integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="maxima")
```

output
```
1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*arctan(1/3*x + 2/3) + 1/2*log(x^
2 + 4*x + 13)
```

3.365.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3}{(13+4x+x^2)^2}\,dx = \frac{47\,x+67}{18\,(x^2+4\,x+13)} - \frac{61}{54}\arctan\left(\frac{1}{3}\,x+\frac{2}{3}\right) + \frac{1}{2}\log\left(x^2+4\,x+13\right)$$

input
```
integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="giac")
```

output
```
1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*arctan(1/3*x + 2/3) + 1/2*log(x^
2 + 4*x + 13)
```

3.365.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int \frac{1+x^3}{(13+4x+x^2)^2}\,dx = \frac{\ln\left(x^2+4\,x+13\right)}{2} - \frac{61\,\mathrm{atan}\left(\frac{x}{3}+\frac{2}{3}\right)}{54} + \frac{47\,x}{18\,\left(x^2+4\,x+13\right)} + \frac{67}{18\,\left(x^2+4\,x+13\right)}$$

input
```
int((x^3 + 1)/(4*x + x^2 + 13)^2,x)
```

output
```
log(4*x + x^2 + 13)/2 - (61*atan(x/3 + 2/3))/54 + (47*x)/(18*(4*x + x^2 +
13)) + 67/(18*(4*x + x^2 + 13))
```

3.365. $\int \frac{1+x^3}{(13+4x+x^2)^2} \, dx$

3.366 $\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2} dx$

3.366.1 Optimal result

Integrand size = 43, antiderivative size = 32

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2} dx = \frac{1}{4+x^2}+\frac{1}{2}\arctan\left(\frac{x}{2}\right)+2\arctan(x)-2\log(x)+\log\left(4+x^2\right)$$

output
```
1/(x^2+4)+1/2*arctan(1/2*x)+2*arctan(x)-2*ln(x)+ln(x^2+4)
```

3.366.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2} dx = \frac{1}{4+x^2}+\frac{1}{2}\arctan\left(\frac{x}{2}\right)+2\arctan(x)-2\log(x)+\log\left(4+x^2\right)$$

input
```
Integrate[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4
+ x^2)^2),x]
```

output
```
(4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 + x^2]
```

3.366.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^5-10x^4+21x^3-42x^2+36x-32}{x(x^2+1)(x^2+4)^2}dx$$
$$\Big\downarrow \text{7276}$$
$$\int \left(-\frac{2x}{(x^2+4)^2}+\frac{2}{x^2+1}+\frac{2x+1}{x^2+4}-\frac{2}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\arctan\left(\frac{x}{2}\right)+2\arctan(x)+\frac{1}{x^2+4}+\log\left(x^2+4\right)-2\log(x)$$

input
```
Int[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4 + x^2)
^2),x]
```

output
```
(4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 + x^2]
```

3.366.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.366.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result
default	$\frac{1}{x^2+4}+\frac{\arctan\left(\frac{x}{2}\right)}{2}+2\arctan\left(x\right)-2\ln\left(x\right)+\ln\left(x^2+4\right)$
risch	$\frac{1}{x^2+4}+\frac{\arctan\left(\frac{x}{2}\right)}{2}+2\arctan\left(x\right)-2\ln\left(x\right)+\ln\left(x^2+4\right)$
parallelrisch	$-\frac{4i\ln(x-2i)-i\ln(x+2i)x^2+16i\ln(x-i)+4i\ln(x-i)x^2+8\ln(x)x^2-4\ln(x-2i)x^2-4\ln(x+2i)x^2-4i\ln(x+2i)-4i\ln(x+i)x^2-}{4(x^2+4)}$

input
```
int((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x,method=_RET
URNVERBOSE)
```

output
```
1/(x^2+4)+1/2*arctan(1/2*x)+2*arctan(x)-2*ln(x)+ln(x^2+4)
```

3.366.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2}\,dx$$
$$=\frac{(x^2+4)\arctan\left(\frac{1}{2}x\right)+4(x^2+4)\arctan(x)+2(x^2+4)\log(x^2+4)-4(x^2+4)\log(x)+2}{2(x^2+4)}$$

input
```
integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algo
rithm="fricas")
```

output
```
1/2*((x^2 + 4)*arctan(1/2*x) + 4*(x^2 + 4)*arctan(x) + 2*(x^2 + 4)*log(x^2
 + 4) - 4*(x^2 + 4)*log(x) + 2)/(x^2 + 4)
```

3.366.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2}\,dx$$
$$= -2\log(x)+\log\left(x^2+4\right)+\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}+2\operatorname{atan}(x)+\frac{1}{x^2+4}$$

input
```
integrate((3*x**5-10*x**4+21*x**3-42*x**2+36*x-32)/x/(x**2+1)/(x**2+4)**2,
x)
```

output
```
-2*log(x) + log(x**2 + 4) + atan(x/2)/2 + 2*atan(x) + 1/(x**2 + 4)
```

3.366.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2}\,dx$$
$$= \frac{1}{x^2+4}+\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+2\arctan(x)+\log\left(x^2+4\right)-2\log(x)$$

input
```
integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algo
rithm="maxima")
```

output
```
1/(x^2 + 4) + 1/2*arctan(1/2*x) + 2*arctan(x) + log(x^2 + 4) - 2*log(x)
```

3.366.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2}\,dx$$
$$= \frac{1}{x^2+4}+\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+2\arctan(x)+\log\left(x^2+4\right)-2\log(|x|)$$

input
```
integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algo
rithm="giac")
```

output
```
1/(x^2 + 4) + 1/2*arctan(1/2*x) + 2*arctan(x) + log(x^2 + 4) - 2*log(abs(x
))
```

3.366.9 Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2}\,dx = \frac{1}{x^2+4} - 2\ln(x) - 2\,\mathrm{atan}\left(\frac{328000}{7\,(36288\,x-19584)}+\frac{34}{63}\right) + \ln(x-2\mathrm{i})\,\left(1-\frac{1}{4}\mathrm{i}\right) + \ln(x+2\mathrm{i})\,\left(1+\frac{1}{4}\mathrm{i}\right)$$

input
```
int((36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5 - 32)/(x*(x^2 + 1)*(x^2 + 4)^
2),x)
```

output
```
log(x - 2i)*(1 - 1i/4) + log(x + 2i)*(1 + 1i/4) - 2*atan(328000/(7*(36288*
x - 19584)) + 34/63) - 2*log(x) + 1/(x^2 + 4)
```

3.367 $\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx$

3.367.1 Optimal result

Integrand size = 26, antiderivative size = 148

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx = \frac{x^2}{2}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}7^{3/4}}+\frac{\arctan\left(1+\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}7^{3/4}}-\frac{\operatorname{arctanh}(x^2)}{2}-\frac{\log\left(\sqrt{7}-\sqrt{2}\sqrt[4]{7}x+x^2\right)}{4\sqrt{2}7^{3/4}}+\frac{\log\left(\sqrt{7}+\sqrt{2}\sqrt[4]{7}x+x^2\right)}{4\sqrt{2}7^{3/4}}$$

output
```
1/2*x^2-1/2*arctanh(x^2)+1/28*arctan(-1+1/7*x*2^(1/2)*7^(3/4))*7^(1/4)*2^(
1/2)+1/28*arctan(1+1/7*x*2^(1/2)*7^(3/4))*7^(1/4)*2^(1/2)-1/56*ln(x^2-7^(1
/4)*x*2^(1/2)+7^(1/2))*7^(1/4)*2^(1/2)+1/56*ln(x^2+7^(1/4)*x*2^(1/2)+7^(1/
2))*7^(1/4)*2^(1/2)
```

3.367.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx = \frac{1}{56}\left(28x^2-2\sqrt{2}\sqrt[4]{7}\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)+2\sqrt{2}\sqrt[4]{7}\arctan\left(1+\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)+14\log(1-x)+14\log(1+x)-14\log\left(1+x^2\right)-\sqrt{2}\sqrt[4]{7}\log\left(7-\sqrt{2}7^{3/4}x+\sqrt{7}x^2\right)+\sqrt{2}\sqrt[4]{7}\log\left(7+\sqrt{2}7^{3/4}x+\sqrt{7}x^2\right)\right)$$

input
```
Integrate[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8),x]
```

output
```
(28*x^2 - 2*Sqrt[2]*7^(1/4)*ArcTan[1 - (Sqrt[2]*x)/7^(1/4)] + 2*Sqrt[2]*7^
(1/4)*ArcTan[1 + (Sqrt[2]*x)/7^(1/4)] + 14*Log[1 - x] + 14*Log[1 + x] - 14
*Log[1 + x^2] - Sqrt[2]*7^(1/4)*Log[7 - Sqrt[2]*7^(3/4)*x + Sqrt[7]*x^2] +
 Sqrt[2]*7^(1/4)*Log[7 + Sqrt[2]*7^(3/4)*x + Sqrt[7]*x^2])/56
```

3.367.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2322, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9+7x^5+x^4-1}{x^8+6x^4-7}dx$$

$$\Big\downarrow \text{2322}$$

$$\int \left(\frac{x^4-1}{x^8+6x^4-7}+\frac{x(x^8+7x^4)}{x^8+6x^4-7}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}7^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{7}}+1\right)}{2\sqrt{2}7^{3/4}}-\frac{\text{arctanh}(x^2)}{2}+\frac{x^2}{2}-\frac{\log\left(x^2-\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}7^{3/4}}+\frac{\log\left(x^2+\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}7^{3/4}}$$

input
```
Int[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8),x]
```

output
```
x^2/2 - ArcTan[1 - (Sqrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) + ArcTan[1 + (
Sqrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) - ArcTanh[x^2]/2 - Log[Sqrt[7] - S
qrt[2]*7^(1/4)*x + x^2]/(4*Sqrt[2]*7^(3/4)) + Log[Sqrt[7] + Sqrt[2]*7^(1/4
)*x + x^2]/(4*Sqrt[2]*7^(3/4))
```

3.367.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2322
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

3.367.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.30

method	result
risch	$\frac{x^2}{2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(343\_Z^4+1\right)}\_R\ln\left(x+7\_R\right)\right)}{4}+\frac{\ln\left(x^2-1\right)}{4}-\frac{\ln\left(x^2+1\right)}{4}$
default	$\frac{x^2}{2}+\frac{\ln(x+1)}{4}+\frac{7^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+7^{\frac{1}{4}}x\sqrt{2}+\sqrt{7}}{x^2-7^{\frac{1}{4}}x\sqrt{2}+\sqrt{7}}\right)+2\arctan\left(1+\frac{x\sqrt{2}\,7^{\frac{3}{4}}}{7}\right)+2\arctan\left(-1+\frac{x\sqrt{2}\,7^{\frac{3}{4}}}{7}\right)\right)}{56}-\frac{\ln\left(x^2+1\right)}{4}+\frac{\ln(x-1)}{4}$

input
```
int((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/4*sum(_R*ln(x+7*_R),_R=RootOf(343*_Z^4+1))+1/4*ln(x^2-1)-1/4*ln(
x^2+1)
```

3.367.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.72

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx = \left(\frac{1}{2744}i+\frac{1}{2744}\right)\cdot 343^{\frac{3}{4}}\sqrt{2}\log\left((i+1)\cdot 343^{\frac{3}{4}}\sqrt{2}+98\,x\right) - \left(\frac{1}{2744}i-\frac{1}{2744}\right)\cdot 343^{\frac{3}{4}}\sqrt{2}\log\left(-(i-1)\cdot 343^{\frac{3}{4}}\sqrt{2}+98\,x\right) + \left(\frac{1}{2744}i-\frac{1}{2744}\right)\cdot 343^{\frac{3}{4}}\sqrt{2}\log\left((i-1)\cdot 343^{\frac{3}{4}}\sqrt{2}+98\,x\right) - \left(\frac{1}{2744}i+\frac{1}{2744}\right)\cdot 343^{\frac{3}{4}}\sqrt{2}\log\left(-(i+1)\cdot 343^{\frac{3}{4}}\sqrt{2}+98\,x\right) + \frac{1}{2}\,x^2-\frac{1}{4}\,\log\left(x^2+1\right)+\frac{1}{4}\,\log\left(x^2-1\right)$$

input
```
integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="fricas")
```

output
```
(1/2744*I + 1/2744)*343^(3/4)*sqrt(2)*log((I + 1)*343^(3/4)*sqrt(2) + 98*x
) - (1/2744*I - 1/2744)*343^(3/4)*sqrt(2)*log(-(I - 1)*343^(3/4)*sqrt(2) +
 98*x) + (1/2744*I - 1/2744)*343^(3/4)*sqrt(2)*log((I - 1)*343^(3/4)*sqrt(
2) + 98*x) - (1/2744*I + 1/2744)*343^(3/4)*sqrt(2)*log(-(I + 1)*343^(3/4)*
sqrt(2) + 98*x) + 1/2*x^2 - 1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)
```

3.367.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx = \frac{x^2}{2}+\frac{\log\left(x^2-1\right)}{4}-\frac{\log\left(x^2+1\right)}{4} - \frac{\sqrt{2}\cdot\sqrt[4]{7}\log\left(x^2-\sqrt{2}\cdot\sqrt[4]{7}x+\sqrt{7}\right)}{56} + \frac{\sqrt{2}\cdot\sqrt[4]{7}\log\left(x^2+\sqrt{2}\cdot\sqrt[4]{7}x+\sqrt{7}\right)}{56} + \frac{\sqrt{2}\cdot\sqrt[4]{7}\operatorname{atan}\left(\frac{\sqrt{2}\cdot 7^{\frac{3}{4}}x}{7}-1\right)}{28} + \frac{\sqrt{2}\cdot\sqrt[4]{7}\operatorname{atan}\left(\frac{\sqrt{2}\cdot 7^{\frac{3}{4}}x}{7}+1\right)}{28}$$

input
```
integrate((x**9+7*x**5+x**4-1)/(x**8+6*x**4-7),x)
```

output
```
x**2/2 + log(x**2 - 1)/4 - log(x**2 + 1)/4 - sqrt(2)*7**(1/4)*log(x**2 - s
qrt(2)*7**(1/4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*log(x**2 + sqrt(2)*7**(
1/4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 - 1)/28
+ sqrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 + 1)/28
```

3.367.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx = {} & \frac{1}{2}\,x^2+\frac{1}{28}\cdot 7^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2\,x+7^{\frac{1}{4}}\sqrt{2}\right)\right)\\ & +\frac{1}{28}\cdot 7^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2\,x-7^{\frac{1}{4}}\sqrt{2}\right)\right)\\ & +\frac{1}{56}\cdot 7^{\frac{1}{4}}\sqrt{2}\log\left(x^2+7^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right)\\ & -\frac{1}{56}\cdot 7^{\frac{1}{4}}\sqrt{2}\log\left(x^2-7^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right)\\ & -\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4}\log\left(x+1\right)+\frac{1}{4}\log\left(x-1\right)\end{aligned}$$

input
```
integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="maxima")
```

output
```
1/2*x^2 + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x + 7^(1/4)*
sqrt(2))) + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x - 7^(1/4
)*sqrt(2))) + 1/56*7^(1/4)*sqrt(2)*log(x^2 + 7^(1/4)*sqrt(2)*x + sqrt(7))
- 1/56*7^(1/4)*sqrt(2)*log(x^2 - 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/4*log(x^
2 + 1) + 1/4*log(x + 1) + 1/4*log(x - 1)
```

3.367.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx = \frac{1}{2}\,x^2+\frac{1}{28}\cdot 28^{\frac{1}{4}}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2\,x+7^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{28}\cdot 28^{\frac{1}{4}}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2\,x-7^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{56}\cdot 28^{\frac{1}{4}}\log\left(x^2+7^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right)-\frac{1}{56}\cdot 28^{\frac{1}{4}}\log\left(x^2-7^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right)-\frac{1}{4}\,\log\left(x^2+1\right)+\frac{1}{4}\,\log\left(|x+1|\right)+\frac{1}{4}\,\log\left(|x-1|\right)$$

input
```
integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="giac")
```

output
```
1/2*x^2 + 1/28*28^(1/4)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x + 7^(1/4)*sqrt(2)
)) + 1/28*28^(1/4)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x - 7^(1/4)*sqrt(2))) +
1/56*28^(1/4)*log(x^2 + 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/56*28^(1/4)*log(x
^2 - 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/4*log(x^2 + 1) + 1/4*log(abs(x + 1))
 + 1/4*log(abs(x - 1))
```

3.367.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx = \frac{\operatorname{atan}(x^2\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{x^2}{2}+\sqrt{2}\,7^{1/4}\operatorname{atan}\left(\frac{\sqrt{2}\,7^{1/4}\,x\left(\frac{89653248}{2401}+\frac{89653248}{2401}\mathrm{i}\right)}{-\frac{1048576}{49}+\frac{\sqrt{7}\,179306496\mathrm{i}}{2401}}+\frac{\sqrt{2}\,7^{3/4}\,x\left(-\frac{524288}{343}+\frac{524288}{343}\mathrm{i}\right)}{-\frac{1048576}{49}+\frac{\sqrt{7}\,179306496\mathrm{i}}{2401}}\right)\left(\frac{1}{28}+\frac{1}{28}\mathrm{i}\right)+\sqrt{2}\,7^{1/4}\operatorname{atan}\left(\frac{\sqrt{2}\,7^{1/4}\,x\left(\frac{89653248}{2401}-\frac{89653248}{2401}\mathrm{i}\right)}{\frac{1048576}{49}+\frac{\sqrt{7}\,179306496\mathrm{i}}{2401}}+\frac{\sqrt{2}\,7^{3/4}\,x}{\frac{1048576}{49}}\right.$$

input
```
int((x^4 + 7*x^5 + x^9 - 1)/(6*x^4 + x^8 - 7),x)
```

output
```
(atan(x^2*1i)*1i)/2 + x^2/2 + 2^(1/2)*7^(1/4)*atan((2^(1/2)*7^(1/4)*x*(896
53248/2401 + 89653248i/2401))/((7^(1/2)*179306496i)/2401 - 1048576/49) - (
2^(1/2)*7^(3/4)*x*(524288/343 - 524288i/343))/((7^(1/2)*179306496i)/2401 -
 1048576/49))*(1/28 + 1i/28) - 2^(1/2)*7^(1/4)*atan((2^(1/2)*7^(1/4)*x*(89
653248/2401 - 89653248i/2401))/((7^(1/2)*179306496i)/2401 + 1048576/49) -
(2^(1/2)*7^(3/4)*x*(524288/343 + 524288i/343))/((7^(1/2)*179306496i)/2401
+ 1048576/49))*(1/28 - 1i/28)
```

3.367. $\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8} \, dx$

3.368 $\int \frac{1+x^3+x^6}{x+x^5}\,dx$

3.368.1 Optimal result

Integrand size = 16, antiderivative size = 112

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx=\frac{x^2}{2}-\frac{\arctan(x^2)}{2}-\frac{\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}x\right)}{2\sqrt{2}}+\log(x)+\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}-\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}-\frac{1}{4}\log\left(1+x^4\right)$$

output
```
1/2*x^2-1/2*arctan(x^2)+ln(x)-1/4*ln(x^4+1)+1/4*arctan(-1+x*2^(1/2))*2^(1/
2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/8*ln(
1+x^2+x*2^(1/2))*2^(1/2)
```

3.368.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx=\frac{1}{8}\left(4x^2-2\left(-2+\sqrt{2}\right)\arctan\left(1-\sqrt{2}x\right)+2\left(2+\sqrt{2}\right)\arctan\left(1+\sqrt{2}x\right)+8\log(x)+\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right)-\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)-2\log\left(1+x^4\right)\right)$$

input
```
Integrate[(1 + x^3 + x^6)/(x + x^5),x]
```

output
```
(4*x^2 - 2*(-2 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(2 + Sqrt[2])*ArcTan[1
 + Sqrt[2]*x] + 8*Log[x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[
1 + Sqrt[2]*x + x^2] - 2*Log[1 + x^4])/8
```

3.368.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2026, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6+x^3+1}{x^5+x}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{x^6+x^3+1}{x(x^4+1)}dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(\frac{x^6+1}{(x^4+1)x}+\frac{x^2}{x^4+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan(x^2)}{2}-\frac{\arctan(1-\sqrt{2}x)}{2\sqrt{2}}+\frac{\arctan(\sqrt{2}x+1)}{2\sqrt{2}}-\frac{1}{4}\log(x^4+1)+\frac{x^2}{2}+\frac{\log(x^2-\sqrt{2}x+1)}{4\sqrt{2}}-\frac{\log(x^2+\sqrt{2}x+1)}{4\sqrt{2}}+\log(x)$$

input
```
Int[(1 + x^3 + x^6)/(x + x^5),x]
```

output
```
x^2/2 - ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqr
t[2]*x]/(2*Sqrt[2]) + Log[x] + Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[
1 + Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + x^4]/4
```

3.368.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

3.368.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.48

method	result
risch	$\frac{x^2}{2}+\ln(x)+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+4\_Z^3+8\_Z^2+4\_Z+1\right)}\_R\ln\left(-\_R^3-5\_R^2-10\_R+3x-5\right)\right)}{4}$
default	$\frac{x^2}{2}+\ln(x)-\frac{\arctan(x^2)}{2}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{8}-\frac{\ln(x^4+1)}{4}$
meijerg	$\frac{x^2}{2}-\frac{\arctan(x^2)}{2}+\frac{x^3\sqrt{2}\,\ln\left(1-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}+\frac{x^3}{}$

input
```
int((x^6+x^3+1)/(x^5+x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+ln(x)+1/4*sum(_R*ln(-_R^3-5*_R^2-10*_R+3*x-5),_R=RootOf(_Z^4+4*_Z^
3+8*_Z^2+4*_Z+1))
```

3.368.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 515, normalized size of antiderivative = 4.60

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx = \text{Too large to display}$$

input
```
integrate((x^6+x^3+1)/(x^5+x),x, algorithm="fricas")
```

output
```
1/2*x^2 - 1/4*(2*sqrt(1/4*I) + I + 1)*log((2*sqrt(1/4*I) + I + 1)^3 - 5*(2
*sqrt(1/4*I) + I + 1)^2 + 3*x + 20*sqrt(1/4*I) + 10*I + 5) - 1/4*(2*sqrt(-
1/4*I) - I + 1)*log(-(2*sqrt(1/4*I) + I + 1)^3 - (2*sqrt(1/4*I) + I + 2)*(
2*sqrt(-1/4*I) - I + 1)^2 + 4*(2*sqrt(1/4*I) + I + 1)^2 - ((2*sqrt(1/4*I)
+ I + 1)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(-1/4*I) - I + 1) + 3*x - 16*
sqrt(1/4*I) - 8*I - 9) + 1/4*(sqrt(1/4*I) + sqrt(-1/4*I) - 2*sqrt(-3/16*(2
*sqrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*I) + I - 3)*(2*sqrt(-1/4*I) - I
+ 1) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sqrt(1/4*I) + 1/2*I - 1/2) - 1)*l
og(1/2*(2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1)^2 + 1/2*(2*sqrt(1/
4*I) + I + 1)^2 + 1/2*((2*sqrt(1/4*I) + I + 1)^2 - 8*sqrt(1/4*I) - 4*I - 6
)*(2*sqrt(-1/4*I) - I + 1) + 2*sqrt(-3/16*(2*sqrt(1/4*I) + I + 1)^2 - 1/8*
(2*sqrt(1/4*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) - 3/16*(2*sqrt(-1/4*I) -
I + 1)^2 + sqrt(1/4*I) + 1/2*I - 1/2)*((2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/
4*I) - I + 1) + 2*sqrt(1/4*I) + I - 1) + 3*x - 2*sqrt(1/4*I) - I + 2) + 1/
4*(sqrt(1/4*I) + sqrt(-1/4*I) + 2*sqrt(-3/16*(2*sqrt(1/4*I) + I + 1)^2 - 1
/8*(2*sqrt(1/4*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) - 3/16*(2*sqrt(-1/4*I)
 - I + 1)^2 + sqrt(1/4*I) + 1/2*I - 1/2) - 1)*log(1/2*(2*sqrt(1/4*I) + I +
 2)*(2*sqrt(-1/4*I) - I + 1)^2 + 1/2*(2*sqrt(1/4*I) + I + 1)^2 + 1/2*((2*s
qrt(1/4*I) + I + 1)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(-1/4*I) - I + 1)
- 2*sqrt(-3/16*(2*sqrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*I) + I - 3)*...
```

3.368.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx = \frac{x^2}{2} + \log(x) + \operatorname{RootSum}\left(256t^4 + 256t^3 + 128t^2 + 16t + 1, \left(t \mapsto t\log\left(\frac{1792t^4}{73} + \frac{704t^3}{219} - \frac{3152t^2}{219} - \frac{2584t}{219} + x - \frac{344}{219}\right.\right.\right.$$

input
```
integrate((x**6+x**3+1)/(x**5+x),x)
```

output
```
x**2/2 + log(x) + RootSum(256*_t**4 + 256*_t**3 + 128*_t**2 + 16*_t + 1, L
ambda(_t, _t*log(1792*_t**4/73 + 704*_t**3/219 - 3152*_t**2/219 - 2584*_t/
219 + x - 344/219)))
```

3.368.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$
\begin{aligned}
\int \frac{1+x^3+x^6}{x+x^5}\,dx = {} & \frac{1}{4}\sqrt{2}\left(\sqrt{2}+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) \\
& -\frac{1}{4}\sqrt{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) \\
& -\frac{1}{8}\sqrt{2}\left(\sqrt{2}+1\right)\log\left(x^2+\sqrt{2}x+1\right) \\
& -\frac{1}{8}\sqrt{2}\left(\sqrt{2}-1\right)\log\left(x^2-\sqrt{2}x+1\right)+\frac{1}{2}\,x^2+\log\left(x\right)
\end{aligned}
$$

input
```
integrate((x^6+x^3+1)/(x^5+x),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*(sqrt(2) + 1)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/4*sqrt(2
)*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*(sqrt(2)
 + 1)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*(sqrt(2) - 1)*log(x^2 - sqrt(
2)*x + 1) + 1/2*x^2 + log(x)
```

3.368.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82

$$
\begin{aligned}
\int \frac{1+x^3+x^6}{x+x^5}\,dx = {} & \frac{1}{2}\,x^2+\frac{1}{4}\left(\sqrt{2}+2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) \\
& +\frac{1}{4}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) \\
& +\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{1}{4}\log\left(x^4+1\right)+\log\left(|x|\right)
\end{aligned}
$$

input
```
integrate((x^6+x^3+1)/(x^5+x),x, algorithm="giac")
```

output
```
1/2*x^2 + 1/4*(sqrt(2) + 2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(sqr
t(2) - 2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt
(2)*x + 1) + 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*log(x^4 + 1) + log
(abs(x))
```

3.368.9 Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.52

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx = \ln(x) + \left(\sum_{k=1}^{4} \ln\left(\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\right)\left(8\,\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\right)+x+\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\right)\right)\right) + \frac{x^2}{2}$$

input
```
int((x^3 + x^6 + 1)/(x + x^5),x)
```

output
```
log(x) + symsum(log(root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)*(8*root(z
^4 + z^3 + z^2/2 + z/16 + 1/256, z, k) + x + 96*root(z^4 + z^3 + z^2/2 + z
/16 + 1/256, z, k)*x + 240*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^2*
x + 320*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^3*x - 16*root(z^4 + z
^3 + z^2/2 + z/16 + 1/256, z, k)^2 + 8))*root(z^4 + z^3 + z^2/2 + z/16 + 1
/256, z, k), k, 1, 4) + x^2/2
```

3.369 $\int \frac{1+x^2}{-x+x^2} dx$

3.369.1 Optimal result

Integrand size = 15, antiderivative size = 14

$$\int \frac{1+x^2}{-x+x^2} dx = x + 2\log(1-x) - \log(x)$$

output
```
x+2*ln(1-x)-ln(x)
```

3.369.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{-x+x^2} dx = x + 2\log(1-x) - \log(x)$$

input
```
Integrate[(1 + x^2)/(-x + x^2),x]
```

output
```
x + 2*Log[1 - x] - Log[x]
```

3.369.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2026, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^2-x}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{x^2+1}{(x-1)x}dx$$

$$\Big\downarrow \text{522}$$

$$\int \left(-\frac{1}{x}+\frac{2}{x-1}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x+2\log(1-x)-\log(x)$$

input
```
Int[(1 + x^2)/(-x + x^2),x]
```

output
```
x + 2*Log[1 - x] - Log[x]
```

3.369.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.369.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$x-\ln(x)+2\ln(x-1)$	13
norman	$x-\ln(x)+2\ln(x-1)$	13
risch	$x-\ln(x)+2\ln(x-1)$	13
parallelrisch	$x-\ln(x)+2\ln(x-1)$	13
meijerg	$2\ln(1-x)-\ln(x)-i\pi+x$	19

input
```
int((x^2+1)/(x^2-x),x,method=_RETURNVERBOSE)
```

output
```
x-ln(x)+2*ln(x-1)
```

3.369.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{-x+x^2}\,dx = x+2\,\log(x-1)-\log(x)$$

input
```
integrate((x^2+1)/(x^2-x),x, algorithm="fricas")
```

output
```
x + 2*log(x - 1) - log(x)
```

3.369.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1+x^2}{-x+x^2}\,dx = x-\log(x)+2\log(x-1)$$

input
```
integrate((x**2+1)/(x**2-x),x)
```

output
```
x - log(x) + 2*log(x - 1)
```

3.369.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{-x+x^2} \, dx = x + 2 \log(x-1) - \log(x)$$

input
```
integrate((x^2+1)/(x^2-x),x, algorithm="maxima")
```

output
```
x + 2*log(x - 1) - log(x)
```

3.369.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{-x+x^2} \, dx = x + 2 \log(|x-1|) - \log(|x|)$$

input
```
integrate((x^2+1)/(x^2-x),x, algorithm="giac")
```

output
```
x + 2*log(abs(x - 1)) - log(abs(x))
```

3.369.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{-x+x^2} \, dx = x + 2 \ln(x-1) - \ln(x)$$

input
```
int(-(x^2 + 1)/(x - x^2),x)
```

output
```
x + 2*log(x - 1) - log(x)
```

3.370 $\int \frac{1+x^3}{-x+x^3} \, dx$

3.370.1 Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{1+x^3}{-x+x^3} \, dx = x+\log(1-x)-\log(x)$$

output
```
x+ln(1-x)-ln(x)
```

3.370.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+x^3}{-x+x^3} \, dx = x+\log(1-x)-\log(x)$$

input
```
Integrate[(1 + x^3)/(-x + x^3),x]
```

output
```
x + Log[1 - x] - Log[x]
```

3.370.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2026, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+1}{x^3-x} \, dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{x^3+1}{x\left(x^2-1\right)}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(-\frac{1}{x}+\frac{1}{x-1}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x+\log(1-x)-\log(x)$$

input
```
Int[(1 + x^3)/(-x + x^3),x]
```

output
```
x + Log[1 - x] - Log[x]
```

3.370.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.370.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$x - \ln(x) + \ln(x-1)$	11
norman	$x - \ln(x) + \ln(x-1)$	11
risch	$x - \ln(x) + \ln(x-1)$	11
parallelrisch	$x - \ln(x) + \ln(x-1)$	11
meijerg	$\frac{\ln(-x^2+1)}{2} - \ln(x) - \frac{i\pi}{2} - \frac{i(2ix-2i\,\mathrm{arctanh}(x))}{2}$	33

input
```
int((x^3+1)/(x^3-x),x,method=_RETURNVERBOSE)
```

output
```
x-ln(x)+ln(x-1)
```

3.370.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^3}{-x+x^3}\,dx = x + \log(x-1) - \log(x)$$

input
```
integrate((x^3+1)/(x^3-x),x, algorithm="fricas")
```

output
```
x + log(x - 1) - log(x)
```

3.370.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1+x^3}{-x+x^3}\,dx = x - \log(x) + \log(x-1)$$

input
```
integrate((x**3+1)/(x**3-x),x)
```

output
```
x - log(x) + log(x - 1)
```

3.370.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^3}{-x+x^3}\,dx = x + \log(x-1) - \log(x)$$

input
```
integrate((x^3+1)/(x^3-x),x, algorithm="maxima")
```

output
```
x + log(x - 1) - log(x)
```

3.370.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+x^3}{-x+x^3}\,dx = x + \log(|x-1|) - \log(|x|)$$

input
```
integrate((x^3+1)/(x^3-x),x, algorithm="giac")
```

output
```
x + log(abs(x - 1)) - log(abs(x))
```

3.370.9 Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^3}{-x+x^3}\,dx = x - 2\,\mathrm{atanh}(2\,x-1)$$

input
```
int(-(x^3 + 1)/(x - x^3),x)
```

output
```
x - 2*atanh(2*x - 1)
```

3.371 $\int \frac{1+x^3}{-x^2+x^3} dx$

3.371.1 Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1+x^3}{-x^2+x^3} dx = \frac{1}{x} + x + 2\log(1-x) - \log(x)$$

output
```
1/x+x+2*ln(1-x)-ln(x)
```

3.371.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1+x^3}{-x^2+x^3} dx = \frac{1}{x} + x + 2\log(1-x) - \log(x)$$

input
```
Integrate[(1 + x^3)/(-x^2 + x^3),x]
```

output
```
x^(-1) + x + 2*Log[1 - x] - Log[x]
```

3.371.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2026, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+1}{x^3-x^2}dx$$
$$\Big\downarrow \text{ 2026}$$
$$\int \frac{x^3+1}{(x-1)x^2}dx$$
$$\Big\downarrow \text{ 2123}$$
$$\int \left(-\frac{1}{x^2}-\frac{1}{x}+\frac{2}{x-1}+1\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$x+\frac{1}{x}+2\log(1-x)-\log(x)$$

input
```
Int[(1 + x^3)/(-x^2 + x^3),x]
```

output
```
x^(-1) + x + 2*Log[1 - x] - Log[x]
```

3.371.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.371.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$x+\frac{1}{x}-\ln\left(x\right)+2\ln\left(x-1\right)$	16
risch	$x+\frac{1}{x}-\ln\left(x\right)+2\ln\left(x-1\right)$	16
norman	$\frac{x^2+1}{x}-\ln\left(x\right)+2\ln\left(x-1\right)$	21
meijerg	$2\ln\left(1-x\right)-\ln\left(x\right)-i\pi+\frac{1}{x}+x$	22
parallelrisch	$-\frac{\ln(x)x-2\ln(x-1)x-x^2-1}{x}$	24

input
```
int((x^3+1)/(x^3-x^2),x,method=_RETURNVERBOSE)
```

output
```
x+1/x-ln(x)+2*ln(x-1)
```

3.371.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{1+x^3}{-x^2+x^3}\,dx=\frac{x^2+2\,x\log\left(x-1\right)-x\log\left(x\right)+1}{x}$$

input
```
integrate((x^3+1)/(x^3-x^2),x, algorithm="fricas")
```

output
```
(x^2 + 2*x*log(x - 1) - x*log(x) + 1)/x
```

3.371.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3}{-x^2+x^3}\,dx=x-\log\left(x\right)+2\log\left(x-1\right)+\frac{1}{x}$$

input
```
integrate((x**3+1)/(x**3-x**2),x)
```

output
```
x - log(x) + 2*log(x - 1) + 1/x
```

3.371.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^3}{-x^2+x^3}\,dx = x + \frac{1}{x} + 2\,\log\left(x-1\right) - \log\left(x\right)$$

input
```
integrate((x^3+1)/(x^3-x^2),x, algorithm="maxima")
```

output
```
x + 1/x + 2*log(x - 1) - log(x)
```

3.371.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1+x^3}{-x^2+x^3}\,dx = x + \frac{1}{x} + 2\,\log\left(|x-1|\right) - \log\left(|x|\right)$$

input
```
integrate((x^3+1)/(x^3-x^2),x, algorithm="giac")
```

output
```
x + 1/x + 2*log(abs(x - 1)) - log(abs(x))
```

3.371.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^3}{-x^2+x^3}\,dx = x + 2\,\ln\left(x-1\right) - \ln\left(x\right) + \frac{1}{x}$$

input
```
int(-(x^3 + 1)/(x^2 - x^3),x)
```

output
```
x + 2*log(x - 1) - log(x) + 1/x
```

3.372 $\int \frac{-1+x^5}{-x+x^3}\,dx$

3.372.1 Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{-1+x^5}{-x+x^3}\,dx = x+\frac{x^3}{3}+\log(x)-\log(1+x)$$

output
```
x+1/3*x^3+ln(x)-ln(1+x)
```

3.372.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^5}{-x+x^3}\,dx = x+\frac{x^3}{3}+\log(x)-\log(1+x)$$

input
```
Integrate[(-1 + x^5)/(-x + x^3),x]
```

output
```
x + x^3/3 + Log[x] - Log[1 + x]
```

3.372.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2026, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5-1}{x^3-x}\,dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{x^5-1}{x\left(x^2-1\right)}dx$$

$$\Big\downarrow \text{ 2333}$$

$$\int \left(x^2+\frac{1}{-x-1}+\frac{1}{x}+1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^3}{3}+x+\log(x)-\log(x+1)$$

input
```
Int[(-1 + x^5)/(-x + x^3),x]
```

output
```
x + x^3/3 + Log[x] - Log[1 + x]
```

3.372.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.372.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$x+\frac{x^{3}}{3}+\ln \left(x \right)-\ln \left(x +1\right)$	16
norman	$x+\frac{x^{3}}{3}+\ln \left(x \right)-\ln \left(x +1\right)$	16
risch	$x+\frac{x^{3}}{3}+\ln \left(x \right)-\ln \left(x +1\right)$	16
parallelrisch	$x+\frac{x^{3}}{3}+\ln \left(x \right)-\ln \left(x +1\right)$	16
meijerg	$-\frac{\ln \left(-x^{2}+1\right)}{2}+\ln \left(x \right)+\frac{i \pi}{2}+\frac{i \left(-\frac{2 i x \left(5 x^{2}+15\right)}{15}+2 i \,\operatorname{arctanh}\left(x \right)\right)}{2}$	38

input
```
int((x^5-1)/(x^3-x),x,method=_RETURNVERBOSE)
```

output
```
x+1/3*x^3+ln(x)-ln(x+1)
```

3.372.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^5}{-x+x^3}\,dx=\frac{1}{3}\,x^{3}+x-\log\left(x+1\right)+\log\left(x\right)$$

input
```
integrate((x^5-1)/(x^3-x),x, algorithm="fricas")
```

output
```
1/3*x^3 + x - log(x + 1) + log(x)
```

3.372.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{-1+x^5}{-x+x^3}\,dx=\frac{x^{3}}{3}+x+\log\left(x\right)-\log\left(x+1\right)$$

input
```
integrate((x**5-1)/(x**3-x),x)
```

output
```
x**3/3 + x + log(x) - log(x + 1)
```

3.372.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^5}{-x+x^3}\,dx = \frac{1}{3}\,x^3 + x - \log\left(x+1\right) + \log\left(x\right)$$

input
```
integrate((x^5-1)/(x^3-x),x, algorithm="maxima")
```

output
```
1/3*x^3 + x - log(x + 1) + log(x)
```

3.372.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^5}{-x+x^3}\,dx = \frac{1}{3}\,x^3 + x - \log\left(|x+1|\right) + \log\left(|x|\right)$$

input
```
integrate((x^5-1)/(x^3-x),x, algorithm="giac")
```

output
```
1/3*x^3 + x - log(abs(x + 1)) + log(abs(x))
```

3.372.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^5}{-x+x^3}\,dx = x - 2\,\mathrm{atanh}(2\,x+1) + \frac{x^3}{3}$$

input
```
int(-(x^5 - 1)/(x - x^3),x)
```

output
```
x - 2*atanh(2*x + 1) + x^3/3
```

3.373 $\int \frac{1+x^4}{x^3+x^5} \, dx$

3.373.1 Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{1+x^4}{x^3+x^5} \, dx = -\frac{1}{2x^2} - \log(x) + \log\left(1+x^2\right)$$

output
```
-1/2/x^2-ln(x)+ln(x^2+1)
```

3.373.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{x^3+x^5} \, dx = -\frac{1}{2x^2} - \log(x) + \log\left(1+x^2\right)$$

input
```
Integrate[(1 + x^4)/(x^3 + x^5),x]
```

output
```
-1/2*1/x^2 - Log[x] + Log[1 + x^2]
```

3.373.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2026, 1579, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x^5+x^3} \, dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{x^4+1}{x^3 \left(x^2+1\right)}dx$$

$$\Big\downarrow \text{1579}$$

$$\frac{1}{2} \int \frac{x^4+1}{x^4 \left(x^2+1\right)}dx^2$$

$$\Big\downarrow \text{522}$$

$$\frac{1}{2} \int \left(-\frac{1}{x^2}+\frac{1}{x^4}+\frac{2}{x^2+1}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(-\frac{1}{x^2}-\log \left(x^2\right)+2 \log \left(x^2+1\right)\right)$$

input
```
Int[(1 + x^4)/(x^3 + x^5),x]
```

output
```
(-x^(-2) - Log[x^2] + 2*Log[1 + x^2])/2
```

3.373.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.373.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{1}{2x^2}-\ln\left(x\right)+\ln\left(x^2+1\right)$	17
norman	$-\frac{1}{2x^2}-\ln\left(x\right)+\ln\left(x^2+1\right)$	17
meijerg	$-\frac{1}{2x^2}-\ln\left(x\right)+\ln\left(x^2+1\right)$	17
risch	$-\frac{1}{2x^2}-\ln\left(x\right)+\ln\left(x^2+1\right)$	17
parallelrisch	$-\frac{2\ln(x)x^2-2\ln(x^2+1)x^2+1}{2x^2}$	26

input
```
int((x^4+1)/(x^5+x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2-ln(x)+ln(x^2+1)
```

3.373.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{1+x^4}{x^3+x^5}\,dx=\frac{2\,x^2\log\left(x^2+1\right)-2\,x^2\log\left(x\right)-1}{2\,x^2}$$

input
```
integrate((x^4+1)/(x^5+x^3),x, algorithm="fricas")
```

output
```
1/2*(2*x^2*log(x^2 + 1) - 2*x^2*log(x) - 1)/x^2
```

3.373.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1+x^4}{x^3+x^5}\,dx = -\log(x) + \log\left(x^2+1\right) - \frac{1}{2x^2}$$

input
```
integrate((x**4+1)/(x**5+x**3),x)
```

output
```
-log(x) + log(x**2 + 1) - 1/(2*x**2)
```

3.373.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1+x^4}{x^3+x^5}\,dx = -\frac{1}{2\,x^2} + \log\left(x^2+1\right) - \log(x)$$

input
```
integrate((x^4+1)/(x^5+x^3),x, algorithm="maxima")
```

output
```
-1/2/x^2 + log(x^2 + 1) - log(x)
```

3.373.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1+x^4}{x^3+x^5}\,dx = \frac{x^2-1}{2\,x^2} + \log\left(x^2+1\right) - \frac{1}{2}\log\left(x^2\right)$$

input
```
integrate((x^4+1)/(x^5+x^3),x, algorithm="giac")
```

output
```
1/2*(x^2 - 1)/x^2 + log(x^2 + 1) - 1/2*log(x^2)
```

3.373.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1+x^4}{x^3+x^5}\,dx = \ln\left(x^2+1\right) - \ln\left(x\right) - \frac{1}{2\,x^2}$$

input
```
int((x^4 + 1)/(x^3 + x^5),x)
```

output
```
log(x^2 + 1) - log(x) - 1/(2*x^2)
```

3.374 $\int \frac{1+x^2}{x+2x^2+x^3} dx$

3.374.1 Optimal result

Integrand size = 18, antiderivative size = 10

$$\int \frac{1+x^2}{x+2x^2+x^3} \, dx = \frac{2}{1+x} + \log(x)$$

output
```
2/(1+x)+ln(x)
```

3.374.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{x+2x^2+x^3} \, dx = \frac{2}{1+x} + \log(x)$$

input
```
Integrate[(1 + x^2)/(x + 2*x^2 + x^3),x]
```

output
```
2/(1 + x) + Log[x]
```

3.374.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2026, 1332, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^3+2x^2+x}dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{x^2+1}{x(x^2+2x+1)}dx$$

$$\Big\downarrow \text{ 1332}$$

$$\int \frac{x^2+1}{x(x+1)^2}dx$$

$$\Big\downarrow \text{ 522}$$

$$\int \left(\frac{1}{x}-\frac{2}{(x+1)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{x+1}+\log(x)$$

input
```
Int[(1 + x^2)/(x + 2*x^2 + x^3),x]
```

output
```
2/(1 + x) + Log[x]
```

3.374.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 1332
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d
_.) + (f_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/c^p   Int[(g + h*x)^m*(b/2 +
 c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, q}, x]
&& EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.374.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{2}{x+1}+\ln\left(x\right)$	11
norman	$\frac{2}{x+1}+\ln\left(x\right)$	11
risch	$\frac{2}{x+1}+\ln\left(x\right)$	11
parallelrisch	$\frac{\ln(x)x+2+\ln(x)}{x+1}$	15

input
```
int((x^2+1)/(x^3+2*x^2+x),x,method=_RETURNVERBOSE)
```

output
```
2/(x+1)+ln(x)
```

3.374.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx=\frac{(x+1)\log\left(x\right)+2}{x+1}$$

input
```
integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="fricas")
```

output
```
((x + 1)*log(x) + 2)/(x + 1)
```

3.374.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx = \log(x) + \frac{2}{x+1}$$

input
```
integrate((x**2+1)/(x**3+2*x**2+x),x)
```

output
```
log(x) + 2/(x + 1)
```

3.374.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx = \frac{2}{x+1} + \log(x)$$

input
```
integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="maxima")
```

output
```
2/(x + 1) + log(x)
```

3.374.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx = \frac{2}{x+1} + \log(|x|)$$

input
```
integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="giac")
```

output
```
2/(x + 1) + log(abs(x))
```

3.374.9 Mupad [B] (verification not implemented)

Time = 9.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx=\ln(x)+\frac{2}{x+1}$$

input
```
int((x^2 + 1)/(x + 2*x^2 + x^3),x)
```

output
```
log(x) + 2/(x + 1)
```

3.375 $\int \frac{1+x^5}{-10x-3x^2+x^3} \, dx$

3.375.1 Optimal result

Integrand size = 20, antiderivative size = 42

$$\int \frac{1+x^5}{-10x-3x^2+x^3} \, dx = 19x + \frac{3x^2}{2} + \frac{x^3}{3} + \frac{3126}{35} \log(5-x) - \frac{\log(x)}{10} - \frac{31}{14} \log(2+x)$$

output
```
19*x+3/2*x^2+1/3*x^3+3126/35*ln(5-x)-1/10*ln(x)-31/14*ln(2+x)
```

3.375.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1+x^5}{-10x-3x^2+x^3} \, dx = 19x + \frac{3x^2}{2} + \frac{x^3}{3} + \frac{3126}{35} \log(5-x) - \frac{\log(x)}{10} - \frac{31}{14} \log(2+x)$$

input
```
Integrate[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

output
```
19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 +
x])/14
```

3.375.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2026, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5+1}{x^3-3x^2-10x}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{x^5+1}{x\left(x^2-3x-10\right)}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(x^2+3x+\frac{3126}{35(x-5)}-\frac{31}{14(x+2)}-\frac{1}{10x}+19\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

input
```
Int[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

output
```
19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 +
x])/14
```

3.375.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.375.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{\ln(x)}{10}+\frac{3126\ln(-5+x)}{35}-\frac{31\ln(x+2)}{14}$	31
norman	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{\ln(x)}{10}+\frac{3126\ln(-5+x)}{35}-\frac{31\ln(x+2)}{14}$	31
risch	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{\ln(x)}{10}+\frac{3126\ln(-5+x)}{35}-\frac{31\ln(x+2)}{14}$	31
parallelrisch	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{\ln(x)}{10}+\frac{3126\ln(-5+x)}{35}-\frac{31\ln(x+2)}{14}$	31

input
```
int((x^5+1)/(x^3-3*x^2-10*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+3/2*x^2+19*x-1/10*ln(x)+3126/35*ln(-5+x)-31/14*ln(x+2)
```

3.375.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx=\frac{1}{3}\,x^3+\frac{3}{2}\,x^2+19\,x-\frac{31}{14}\,\log\left(x+2\right)+\frac{3126}{35}\,\log\left(x-5\right)-\frac{1}{10}\,\log\left(x\right)$$

input
```
integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="fricas")
```

output
```
1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*lo
g(x)
```

3.375.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \frac{x^3}{3} + \frac{3x^2}{2} + 19x - \frac{\log(x)}{10} + \frac{3126\log(x-5)}{35} - \frac{31\log(x+2)}{14}$$

input
```
integrate((x**5+1)/(x**3-3*x**2-10*x),x)
```

output
```
x**3/3 + 3*x**2/2 + 19*x - log(x)/10 + 3126*log(x - 5)/35 - 31*log(x + 2)/
14
```

3.375.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \frac{1}{3}\,x^3 + \frac{3}{2}\,x^2 + 19\,x - \frac{31}{14}\,\log(x+2) + \frac{3126}{35}\,\log(x-5) - \frac{1}{10}\,\log(x)$$

input
```
integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="maxima")
```

output
```
1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*lo
g(x)
```

3.375.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \frac{1}{3}\,x^3 + \frac{3}{2}\,x^2 + 19\,x - \frac{31}{14}\,\log(|x+2|) + \frac{3126}{35}\,\log(|x-5|) - \frac{1}{10}\,\log(|x|)$$

input
```
integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="giac")
```

output
```
1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(abs(x + 2)) + 3126/35*log(abs(x - 5))
 - 1/10*log(abs(x))
```

3.375.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = 19\,x - \frac{31\,\ln(x+2)}{14} + \frac{3126\,\ln(x-5)}{35} - \frac{\ln(x)}{10} + \frac{3\,x^2}{2} + \frac{x^3}{3}$$

input
```
int(-(x^5 + 1)/(10*x + 3*x^2 - x^3),x)
```

output
```
19*x - (31*log(x + 2))/14 + (3126*log(x - 5))/35 - log(x)/10 + (3*x^2)/2 +
 x^3/3
```

3.376 $\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx$

3.376.1 Optimal result

Integrand size = 29, antiderivative size = 46

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx=-\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)+\frac{5\arctan\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)$$

output
```
1/2*ln(x^2+2*x+3)+5/2*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)-arctan(1/5*x*5^(1/
2))*5^(1/2)
```

3.376.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx=-\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)+\frac{5\arctan\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)$$

input
```
Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

output
```
-(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
 + 2*x + x^2]/2
```

3.376.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2-5x+15}{(x^2+5)(x^2+2x+3)}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{x+6}{x^2+2x+3}-\frac{5}{x^2+5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)+\frac{5\arctan\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(x^2+2x+3\right)$$

input
```
Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

output
```
-(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
 + 2*x + x^2]/2
```

3.376.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.376.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{\ln(x^2+2x+3)}{2}+\frac{5\arctan\left(\frac{(x+1)\sqrt{2}}{2}\right)\sqrt{2}}{2}-\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}$	39
default	$-\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}+\frac{\ln(x^2+2x+3)}{2}+\frac{5\sqrt{2}\arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}$	41

input
```
int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2+2*x+3)+5/2*arctan(1/2*(x+1)*2^(1/2))*2^(1/2)-arctan(1/5*x*5^(1/
2))*5^(1/2)
```

3.376.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) + \frac{1}{2}\log\left(x^2+2x+3\right)$$

input
```
integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="fricas")
```

output
```
5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) +
1/2*log(x^2 + 2*x + 3)
```

3.376.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{\log(x^2+2x+3)}{2} - \sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}x}{5}\right) + \frac{5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}+\frac{\sqrt{2}}{2}\right)}{2}$$

input
```
integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)
```

output
```
log(x**2 + 2*x + 3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*atan(sqrt(2)
*x/2 + sqrt(2)/2)/2
```

3.376.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) + \frac{1}{2}\log\left(x^2+2x+3\right)$$

input
```
integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="maxima")
```

output
```
5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) +
1/2*log(x^2 + 2*x + 3)
```

3.376.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) + \frac{1}{2}\log\left(x^2+2x+3\right)$$

input
```
integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="giac")
```

output
```
5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) +
1/2*log(x^2 + 2*x + 3)
```

3.376.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.91

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)}{2}+\frac{\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)}{2} + \sqrt{5}\,\mathrm{atan}\left(\frac{2000\sqrt{5}}{2000\,x+1120}-\frac{224\sqrt{5}\,x}{2000\,x+1120}\right) - \frac{\sqrt{2}\,\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)\,5\mathrm{i}}{4}+\frac{\sqrt{2}\,\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)\,5\mathrm{i}}{4}$$

input
```
int((x^2 - 5*x + x^3 + 15)/((x^2 + 5)*(2*x + x^2 + 3)),x)
```

output
```
log(x - 2^(1/2)*1i + 1)/2 + log(x + 2^(1/2)*1i + 1)/2 + 5^(1/2)*atan((2000
*5^(1/2))/(2000*x + 1120) - (224*5^(1/2)*x)/(2000*x + 1120)) - (2^(1/2)*lo
g(x - 2^(1/2)*1i + 1)*5i)/4 + (2^(1/2)*log(x + 2^(1/2)*1i + 1)*5i)/4
```

3.377 $\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx$

3.377.1 Optimal result

Integrand size = 22, antiderivative size = 19

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = -\frac{1}{8}\log(3+x)+\frac{1}{8}\log(1+3x)$$

output
```
-1/8*ln(3+x)+1/8*ln(1+3*x)
```

3.377.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = -\frac{1}{8}\log(3+x)+\frac{1}{8}\log(1+3x)$$

input
```
Integrate[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]
```

output
```
-1/8*Log[3 + x] + Log[1 + 3*x]/8
```

3.377.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {7239, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)\left(\frac{10x}{x^2+1}+3\right)}dx$$

$$\Big\downarrow \text{ 7239}$$

$$\int \frac{1}{3x^2+10x+3}dx$$

$$\Big\downarrow \text{ 1081}$$

$$3\int \left(\frac{1}{8(3x+1)}-\frac{1}{24(x+3)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$3\left(\frac{1}{24}\log(3x+1)-\frac{1}{24}\log(x+3)\right)$$

input
```
Int[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]
```

output
```
3*(-1/24*Log[3 + x] + Log[1 + 3*x]/24)
```

3.377.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

3.377.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$-\frac{\ln(3+x)}{8}+\frac{\ln(\frac{1}{3}+x)}{8}$	14
default	$-\frac{\ln(3+x)}{8}+\frac{\ln(1+3x)}{8}$	16
norman	$-\frac{\ln(3+x)}{8}+\frac{\ln(1+3x)}{8}$	16
risch	$-\frac{\ln(3+x)}{8}+\frac{\ln(1+3x)}{8}$	16

input
```
int(1/(x^2+1)/(3+10*x/(x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(3+x)+1/8*ln(1/3+x)
```

3.377.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = \frac{1}{8}\log(3x+1) - \frac{1}{8}\log(x+3)$$

input
```
integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="fricas")
```

output
```
1/8*log(3*x + 1) - 1/8*log(x + 3)
```

3.377.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = \frac{\log\left(x+\frac{1}{3}\right)}{8} - \frac{\log(x+3)}{8}$$

input
```
integrate(1/(x**2+1)/(3+10*x/(x**2+1)),x)
```

output
```
log(x + 1/3)/8 - log(x + 3)/8
```

3.377.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = \frac{1}{8}\,\log\left(3\,x+1\right) - \frac{1}{8}\,\log\left(x+3\right)$$

input
```
integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="maxima")
```

output
```
1/8*log(3*x + 1) - 1/8*log(x + 3)
```

3.377.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = \frac{1}{8}\,\log\left(|3\,x+1|\right) - \frac{1}{8}\,\log\left(|x+3|\right)$$

input
```
integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="giac")
```

output
```
1/8*log(abs(3*x + 1)) - 1/8*log(abs(x + 3))
```

3.377.9 Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = -\frac{\operatorname{atanh}\left(\frac{3\,x}{4}+\frac{5}{4}\right)}{4}$$

input
```
int(1/((x^2 + 1)*((10*x)/(x^2 + 1) + 3)),x)
```

output
```
-atanh((3*x)/4 + 5/4)/4
```

3.378 $\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx$

3.378.1 Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx = \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}$$

output
```
139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

3.378.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx = \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}$$

input
```
Integrate[x^3/(13 + 2/x + 15*x),x]
```

output
```
(139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x
]/4375
```

3.378.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1722, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{15x+\frac{2}{x}+13}\,dx$$

$$\Big\downarrow \ 1722$$

$$\int \frac{x^4}{15x^2+13x+2}dx$$

$$\Big\downarrow \ 1141$$

$$15\int \left(\frac{x^2}{225}-\frac{13x}{3375}-\frac{16}{2835(3x+2)}+\frac{1}{13125(5x+1)}+\frac{139}{50625}\right)dx$$

$$\Big\downarrow \ 2009$$

$$15\left(\frac{x^3}{675}-\frac{13x^2}{6750}+\frac{139x}{50625}-\frac{16\log(3x+2)}{8505}+\frac{\log(5x+1)}{65625}\right)$$

input
```
Int[x^3/(13 + 2/x + 15*x),x]
```

output
```
15*((139*x)/50625 - (13*x^2)/6750 + x^3/675 - (16*Log[2 + 3*x])/8505 + Log
[1 + 5*x]/65625)
```

3.378.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1722
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.378.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}+\frac{\ln\left(x+\frac{1}{5}\right)}{4375}-\frac{16\ln\left(x+\frac{2}{3}\right)}{567}$	27
default	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(3x+2)}{567}+\frac{\ln(1+5x)}{4375}$	31
norman	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(3x+2)}{567}+\frac{\ln(1+5x)}{4375}$	31
risch	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(3x+2)}{567}+\frac{\ln(1+5x)}{4375}$	31

input
```
int(x^3/(13+2/x+15*x),x,method=_RETURNVERBOSE)
```

output
```
1/45*x^3-13/450*x^2+139/3375*x+1/4375*ln(x+1/5)-16/567*ln(x+2/3)
```

3.378.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx=\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^3/(13+2/x+15*x),x, algorithm="fricas")
```

output
```
1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x
+ 2)
```

3.378.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx = \frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\log\left(x+\frac{1}{5}\right)}{4375} - \frac{16\log\left(x+\frac{2}{3}\right)}{567}$$

input
```
integrate(x**3/(13+2/x+15*x),x)
```

output
```
x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/5
67
```

3.378.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx = \frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\,\log(5\,x+1) - \frac{16}{567}\,\log(3\,x+2)$$

input
```
integrate(x^3/(13+2/x+15*x),x, algorithm="maxima")
```

output
```
1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x
+ 2)
```

3.378.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx = \frac{1}{45}\,x^3 - \frac{13}{450}\,x^2 + \frac{139}{3375}\,x + \frac{1}{4375}\,\log(|5\,x+1|) - \frac{16}{567}\,\log(|3\,x+2|)$$

input
```
integrate(x^3/(13+2/x+15*x),x, algorithm="giac")
```

output
```
1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log
(abs(3*x + 2))
```

3.378.9 Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx = \frac{139\,x}{3375} - \frac{16\,\ln\left(x+\frac{2}{3}\right)}{567} + \frac{\ln\left(x+\frac{1}{5}\right)}{4375} - \frac{13\,x^2}{450} + \frac{x^3}{45}$$

input
```
int(x^3/(15*x + 2/x + 13),x)
```

output
```
(139*x)/3375 - (16*log(x + 2/3))/567 + log(x + 1/5)/4375 - (13*x^2)/450 +
x^3/45
```

3.379 $\int \frac{x^2}{13+\frac{2}{x}+15x} \, dx$

3.379.1 Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{x^2}{13+\frac{2}{x}+15x} \, dx = -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)$$

output
```
-13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

3.379.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{13+\frac{2}{x}+15x} \, dx = -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)$$

input
```
Integrate[x^2/(13 + 2/x + 15*x),x]
```

output
```
(-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

3.379.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1722, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{15x+\frac{2}{x}+13}\,dx$$

$$\Big\downarrow \text{1722}$$

$$\int \frac{x^3}{15x^2+13x+2}dx$$

$$\Big\downarrow \text{1141}$$

$$15\int \left(\frac{x}{225}+\frac{8}{945(3x+2)}-\frac{1}{2625(5x+1)}-\frac{13}{3375}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15\left(\frac{x^2}{450}-\frac{13x}{3375}+\frac{8\log(3x+2)}{2835}-\frac{\log(5x+1)}{13125}\right)$$

input
```
Int[x^2/(13 + 2/x + 15*x),x]
```

output
```
15*((-13*x)/3375 + x^2/450 + (8*Log[2 + 3*x])/2835 - Log[1 + 5*x]/13125)
```

3.379.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1722
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.379.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{x^2}{30}-\frac{13x}{225}-\frac{\ln\left(x+\frac{1}{5}\right)}{875}+\frac{8\ln\left(x+\frac{2}{3}\right)}{189}$	22
default	$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8\ln(3x+2)}{189}-\frac{\ln(1+5x)}{875}$	26
norman	$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8\ln(3x+2)}{189}-\frac{\ln(1+5x)}{875}$	26
risch	$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8\ln(3x+2)}{189}-\frac{\ln(1+5x)}{875}$	26

input
```
int(x^2/(13+2/x+15*x),x,method=_RETURNVERBOSE)
```

output
```
1/30*x^2-13/225*x-1/875*ln(x+1/5)+8/189*ln(x+2/3)
```

3.379.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx=\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^2/(13+2/x+15*x),x, algorithm="fricas")
```

output
```
1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

3.379.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx = \frac{x^2}{30} - \frac{13x}{225} - \frac{\log\left(x+\frac{1}{5}\right)}{875} + \frac{8\log\left(x+\frac{2}{3}\right)}{189}$$

input
```
integrate(x**2/(13+2/x+15*x),x)
```

output
```
x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

3.379.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx = \frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\,\log\left(5\,x+1\right) + \frac{8}{189}\,\log\left(3\,x+2\right)$$

input
```
integrate(x^2/(13+2/x+15*x),x, algorithm="maxima")
```

output
```
1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

3.379.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx = \frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\,\log\left(|5\,x+1|\right) + \frac{8}{189}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(x^2/(13+2/x+15*x),x, algorithm="giac")
```

output
```
1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))
```

3.379.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx = \frac{8\,\ln\left(x+\frac{2}{3}\right)}{189} - \frac{13\,x}{225} - \frac{\ln\left(x+\frac{1}{5}\right)}{875} + \frac{x^2}{30}$$

input
```
int(x^2/(15*x + 2/x + 13),x)
```

output
```
(8*log(x + 2/3))/189 - (13*x)/225 - log(x + 1/5)/875 + x^2/30
```

3.380 $\int \frac{x}{13+\frac{2}{x}+15x}\,dx$

3.380.1 Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \frac{x}{13+\frac{2}{x}+15x}\,dx=\frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)$$

output
```
1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

3.380.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{13+\frac{2}{x}+15x}\,dx=\frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)$$

input
```
Integrate[x/(13 + 2/x + 15*x),x]
```

output
```
x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

3.380.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1722, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{15x+\frac{2}{x}+13}dx$$
$$\Big\downarrow \text{1722}$$
$$\int \frac{x^2}{15x^2+13x+2}dx$$
$$\Big\downarrow \text{1141}$$
$$15\int \left(\frac{1}{525(5x+1)}+\frac{1}{225}-\frac{4}{315(3x+2)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$15\left(\frac{x}{225}-\frac{4}{945}\log(3x+2)+\frac{\log(5x+1)}{2625}\right)$$

input
```
Int[x/(13 + 2/x + 15*x),x]
```

output
```
15*(x/225 - (4*Log[2 + 3*x])/945 + Log[1 + 5*x]/2625)
```

3.380.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1722
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.380.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{x}{15}+\frac{\ln\left(x+\frac{1}{5}\right)}{175}-\frac{4\ln\left(x+\frac{2}{3}\right)}{63}$	17
default	$\frac{x}{15}-\frac{4\ln\left(3x+2\right)}{63}+\frac{\ln\left(1+5x\right)}{175}$	21
norman	$\frac{x}{15}-\frac{4\ln\left(3x+2\right)}{63}+\frac{\ln\left(1+5x\right)}{175}$	21
risch	$\frac{x}{15}-\frac{4\ln\left(3x+2\right)}{63}+\frac{\ln\left(1+5x\right)}{175}$	21

input
```
int(x/(13+2/x+15*x),x,method=_RETURNVERBOSE)
```

output
```
1/15*x+1/175*ln(x+1/5)-4/63*ln(x+2/3)
```

3.380.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x}{13+\frac{2}{x}+15x}\,dx=\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

input
```
integrate(x/(13+2/x+15*x),x, algorithm="fricas")
```

output
```
1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

3.380.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x}{13+\frac{2}{x}+15x}\,dx = \frac{x}{15} + \frac{\log\left(x+\frac{1}{5}\right)}{175} - \frac{4\log\left(x+\frac{2}{3}\right)}{63}$$

input
```
integrate(x/(13+2/x+15*x),x)
```

output
```
x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

3.380.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x}{13+\frac{2}{x}+15x}\,dx = \frac{1}{15}\,x + \frac{1}{175}\,\log\left(5\,x+1\right) - \frac{4}{63}\,\log\left(3\,x+2\right)$$

input
```
integrate(x/(13+2/x+15*x),x, algorithm="maxima")
```

output
```
1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

3.380.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x}{13+\frac{2}{x}+15x}\,dx = \frac{1}{15}\,x + \frac{1}{175}\,\log\left(|5\,x+1|\right) - \frac{4}{63}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(x/(13+2/x+15*x),x, algorithm="giac")
```

output
```
1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))
```

3.380.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{x}{13+\frac{2}{x}+15x}\,dx = \frac{x}{15} - \frac{4\ln\left(x+\frac{2}{3}\right)}{63} + \frac{\ln\left(x+\frac{1}{5}\right)}{175}$$

input
```
int(x/(15*x + 2/x + 13),x)
```

output
```
x/15 - (4*log(x + 2/3))/63 + log(x + 1/5)/175
```

3.381 $\int \frac{1}{13+\frac{2}{x}+15x}\,dx$

3.381.1 Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{1}{13+\frac{2}{x}+15x}\,dx=\frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)$$

output
```
2/21*ln(2+3*x)-1/35*ln(1+5*x)
```

3.381.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{13+\frac{2}{x}+15x}\,dx=\frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)$$

input
```
Integrate[(13 + 2/x + 15*x)^(-1),x]
```

output
```
(2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

3.381.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1688, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{15x+\frac{2}{x}+13}dx$$

$$\Big\downarrow \text{1688}$$

$$\int \frac{x}{15x^2+13x+2}dx$$

$$\Big\downarrow \text{1141}$$

$$15\int \left(\frac{2}{105(3x+2)}-\frac{1}{105(5x+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15\left(\frac{2}{315}\log(3x+2)-\frac{1}{525}\log(5x+1)\right)$$

input
```
Int[(13 + 2/x + 15*x)^(-1),x]
```

output
```
15*((2*Log[2 + 3*x])/315 - Log[1 + 5*x]/525)
```

3.381.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1688
```
Int[((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_), x_Symbol] :> Int[(b
+ a*x^n + c*x^(2*n))^p/x^(n*p), x] /; FreeQ[{a, b, c, n}, x] && EqQ[mn, -n]
 && IntegerQ[p] && PosQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.381.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$-\frac{\ln\left(x+\frac{1}{5}\right)}{35}+\frac{2\ln\left(x+\frac{2}{3}\right)}{21}$	14
default	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18
norman	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18
risch	$\frac{2\ln(3x+2)}{21}-\frac{\ln(1+5x)}{35}$	18

input
```
int(1/(13+2/x+15*x),x,method=_RETURNVERBOSE)
```

output
```
-1/35*ln(x+1/5)+2/21*ln(x+2/3)
```

3.381.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{13+\frac{2}{x}+15x}\,dx = -\frac{1}{35}\,\log\left(5\,x+1\right)+\frac{2}{21}\,\log\left(3\,x+2\right)$$

input
```
integrate(1/(13+2/x+15*x),x, algorithm="fricas")
```

output
```
-1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

3.381.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{13+\frac{2}{x}+15x}\,dx = -\frac{\log\left(x+\frac{1}{5}\right)}{35} + \frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

input
```
integrate(1/(13+2/x+15*x),x)
```

output
```
-log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

3.381.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{13+\frac{2}{x}+15x}\,dx = -\frac{1}{35}\,\log\left(5\,x+1\right) + \frac{2}{21}\,\log\left(3\,x+2\right)$$

input
```
integrate(1/(13+2/x+15*x),x, algorithm="maxima")
```

output
```
-1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

3.381.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{13+\frac{2}{x}+15x}\,dx = -\frac{1}{35}\,\log\left(|5\,x+1|\right) + \frac{2}{21}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(1/(13+2/x+15*x),x, algorithm="giac")
```

output
```
-1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))
```

3.381.9 Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{13+\frac{2}{x}+15x}\,dx = \frac{2\,\ln\left(x+\frac{2}{3}\right)}{21}-\frac{\ln\left(x+\frac{1}{5}\right)}{35}$$

input
```
int(1/(15*x + 2/x + 13),x)
```

output
```
(2*log(x + 2/3))/21 - log(x + 1/5)/35
```

3.382 $\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx$

3.382.1 Optimal result

Integrand size = 16, antiderivative size = 21

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)$$

output
```
-1/7*ln(2+3*x)+1/7*ln(1+5*x)
```

3.382.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)$$

input
```
Integrate[1/(x*(13 + 2/x + 15*x)),x]
```

output
```
-1/7*Log[2 + 3*x] + Log[1 + 5*x]/7
```

3.382.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1722, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(15x+\frac{2}{x}+13\right)}dx$$
$$\Big\downarrow \; 1722$$
$$\int \frac{1}{15x^2+13x+2}dx$$
$$\Big\downarrow \; 1081$$
$$15\int \left(\frac{1}{21(5x+1)}-\frac{1}{35(3x+2)}\right)dx$$
$$\Big\downarrow \; 2009$$
$$15\left(\frac{1}{105}\log(5x+1)-\frac{1}{105}\log(3x+2)\right)$$

input
```
Int[1/(x*(13 + 2/x + 15*x)),x]
```

output
```
15*(-1/105*Log[2 + 3*x] + Log[1 + 5*x]/105)
```

3.382.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1722
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.382.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln\left(x+\frac{1}{5}\right)}{7}-\frac{\ln\left(x+\frac{2}{3}\right)}{7}$	14
default	$\frac{\ln(1+5x)}{7}-\frac{\ln(3x+2)}{7}$	18
norman	$\frac{\ln(1+5x)}{7}-\frac{\ln(3x+2)}{7}$	18
risch	$\frac{\ln(1+5x)}{7}-\frac{\ln(3x+2)}{7}$	18

input
```
int(1/x/(13+2/x+15*x),x,method=_RETURNVERBOSE)
```

output
```
1/7*ln(x+1/5)-1/7*ln(x+2/3)
```

3.382.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx=\frac{1}{7}\,\log\left(5\,x+1\right)-\frac{1}{7}\,\log\left(3\,x+2\right)$$

input
```
integrate(1/x/(13+2/x+15*x),x, algorithm="fricas")
```

output
```
1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

3.382.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx=\frac{\log\left(x+\frac{1}{5}\right)}{7}-\frac{\log\left(x+\frac{2}{3}\right)}{7}$$

input
```
integrate(1/x/(13+2/x+15*x),x)
```

output
```
log(x + 1/5)/7 - log(x + 2/3)/7
```

3.382.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx = \frac{1}{7}\log\left(5\,x+1\right) - \frac{1}{7}\log\left(3\,x+2\right)$$

input
```
integrate(1/x/(13+2/x+15*x),x, algorithm="maxima")
```

output
```
1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

3.382.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx = \frac{1}{7}\log\left(|5\,x+1|\right) - \frac{1}{7}\log\left(|3\,x+2|\right)$$

input
```
integrate(1/x/(13+2/x+15*x),x, algorithm="giac")
```

output
```
1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))
```

3.382.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{30\,x}{7}+\frac{13}{7}\right)}{7}$$

input
```
int(1/(x*(15*x + 2/x + 13)),x)
```

output
```
-(2*atanh((30*x)/7 + 13/7))/7
```

3.383 $\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx$

3.383.1 Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx=\frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)$$

output
```
1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

3.383.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx=\frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)$$

input
```
Integrate[1/(x^2*(13 + 2/x + 15*x)),x]
```

output
```
Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

3.383.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1722, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(15x + \frac{2}{x} + 13\right)} dx$$

$$\Big\downarrow \text{ 1722}$$

$$\int \frac{1}{x \left(15x^2 + 13x + 2\right)} dx$$

$$\Big\downarrow \text{ 1141}$$

$$15 \int \left(\frac{3}{70(3x+2)} - \frac{5}{21(5x+1)} + \frac{1}{30x}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$15 \left(\frac{\log(x)}{30} + \frac{1}{70} \log(3x+2) - \frac{1}{21} \log(5x+1)\right)$$

input
```
Int[1/(x^2*(13 + 2/x + 15*x)),x]
```

output
```
15*(Log[x]/30 + Log[2 + 3*x]/70 - Log[1 + 5*x]/21)
```

3.383.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1722
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.383.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln(x)}{2}-\frac{5\ln\left(x+\frac{1}{5}\right)}{7}+\frac{3\ln\left(x+\frac{2}{3}\right)}{14}$	18
default	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22
norman	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22
risch	$\frac{\ln(x)}{2}+\frac{3\ln(3x+2)}{14}-\frac{5\ln(1+5x)}{7}$	22

input
```
int(1/x^2/(13+2/x+15*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x)-5/7*ln(x+1/5)+3/14*ln(x+2/3)
```

3.383.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{5}{7}\,\log\left(5\,x+1\right)+\frac{3}{14}\,\log\left(3\,x+2\right)+\frac{1}{2}\,\log\left(x\right)$$

input
```
integrate(1/x^2/(13+2/x+15*x),x, algorithm="fricas")
```

output
```
-5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

3.383.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx=\frac{\log\left(x\right)}{2}-\frac{5\log\left(x+\frac{1}{5}\right)}{7}+\frac{3\log\left(x+\frac{2}{3}\right)}{14}$$

input
```
integrate(1/x**2/(13+2/x+15*x),x)
```

output
```
log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

3.383.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{5}{7}\,\log\left(5\,x+1\right)+\frac{3}{14}\,\log\left(3\,x+2\right)+\frac{1}{2}\,\log\left(x\right)$$

input
```
integrate(1/x^2/(13+2/x+15*x),x, algorithm="maxima")
```

output
```
-5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

3.383.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{5}{7}\,\log\left(|5\,x+1|\right)+\frac{3}{14}\,\log\left(|3\,x+2|\right)+\frac{1}{2}\,\log\left(|x|\right)$$

input
```
integrate(1/x^2/(13+2/x+15*x),x, algorithm="giac")
```

output
```
-5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

3.383.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx=\frac{3\ln\left(x+\frac{2}{3}\right)}{14}-\frac{5\ln\left(x+\frac{1}{5}\right)}{7}+\frac{\ln\left(x\right)}{2}$$

input
```
int(1/(x^2*(15*x + 2/x + 13)),x)
```

output
```
(3*log(x + 2/3))/14 - (5*log(x + 1/5))/7 + log(x)/2
```

3.384 $\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx$

3.384.1 Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

output
```
-1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

3.384.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

input
```
Integrate[1/(x^3*(13 + 2/x + 15*x)),x]
```

output
```
-1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

3.384.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1722, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(15x+\frac{2}{x}+13\right)} \, dx$$

$$\Big\downarrow \ 1722$$

$$\int \frac{1}{x^2 \left(15x^2+13x+2\right)}dx$$

$$\Big\downarrow \ 1141$$

$$15 \int \left(-\frac{9}{140(3x+2)}+\frac{25}{21(5x+1)}-\frac{13}{60x}+\frac{1}{30x^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$15 \left(-\frac{1}{30x}-\frac{13\log(x)}{60}-\frac{3}{140}\log(3x+2)+\frac{5}{21}\log(5x+1)\right)$$

input
```
Int[1/(x^3*(13 + 2/x + 15*x)),x]
```

output
```
15*(-1/30*1/x - (13*Log[x])/60 - (3*Log[2 + 3*x])/140 + (5*Log[1 + 5*x])/2
1)
```

3.384.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1722
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.384.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
norman	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
risch	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(3x+2)}{28}+\frac{25\ln(1+5x)}{7}$	27
parallelrisch	$-\frac{91\ln(x)x-100\ln\left(x+\frac{1}{5}\right)x+9\ln\left(x+\frac{2}{3}\right)x+14}{28x}$	27

input
```
int(1/x^3/(13+2/x+15*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x-13/4*ln(x)-9/28*ln(3*x+2)+25/7*ln(1+5*x)
```

3.384.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx=\frac{100\,x\log\left(5\,x+1\right)-9\,x\log\left(3\,x+2\right)-91\,x\log\left(x\right)-14}{28\,x}$$

input
```
integrate(1/x^3/(13+2/x+15*x),x, algorithm="fricas")
```

output
```
1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

3.384.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^3 \left(13+\frac{2}{x}+15x\right)} \, dx = -\frac{13\log{(x)}}{4} + \frac{25\log{\left(x+\frac{1}{5}\right)}}{7} - \frac{9\log{\left(x+\frac{2}{3}\right)}}{28} - \frac{1}{2x}$$

input
```
integrate(1/x**3/(13+2/x+15*x),x)
```

output
```
-13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

3.384.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3 \left(13+\frac{2}{x}+15x\right)} \, dx = -\frac{1}{2\,x} + \frac{25}{7}\log\left(5\,x+1\right) - \frac{9}{28}\log\left(3\,x+2\right) - \frac{13}{4}\log\left(x\right)$$

input
```
integrate(1/x^3/(13+2/x+15*x),x, algorithm="maxima")
```

output
```
-1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

3.384.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3 \left(13+\frac{2}{x}+15x\right)} \, dx = -\frac{1}{2\,x} + \frac{25}{7}\log\left(|5\,x+1|\right) - \frac{9}{28}\log\left(|3\,x+2|\right) - \frac{13}{4}\log\left(|x|\right)$$

input
```
integrate(1/x^3/(13+2/x+15*x),x, algorithm="giac")
```

output
```
-1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x)
)
```

3.384.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx = \frac{25\ln\left(x+\frac{1}{5}\right)}{7} - \frac{9\ln\left(x+\frac{2}{3}\right)}{28} - \frac{13\ln(x)}{4} - \frac{1}{2\,x}$$

input
```
int(1/(x^3*(15*x + 2/x + 13)),x)
```

output
```
(25*log(x + 1/5))/7 - (9*log(x + 2/3))/28 - (13*log(x))/4 - 1/(2*x)
```

3.385 $\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx$

3.385.1 Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

output
```
-1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

3.385.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

input
```
Integrate[1/(x^4*(13 + 2/x + 15*x)),x]
```

output
```
-1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
 + 5*x])/7
```

3.385.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1722, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(15 x+\frac{2}{x}+13\right)} \, dx$$
$$\Big\downarrow \text{1722}$$
$$\int \frac{1}{x^3 \left(15 x^2+13 x+2\right)}dx$$
$$\Big\downarrow \text{1141}$$
$$15 \int \left(\frac{27}{280(3 x+2)}-\frac{125}{21(5 x+1)}+\frac{139}{120 x}-\frac{13}{60 x^2}+\frac{1}{30 x^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$15 \left(-\frac{1}{60 x^2}+\frac{13}{60 x}+\frac{139 \log (x)}{120}+\frac{9}{280} \log (3 x+2)-\frac{25}{21} \log (5 x+1)\right)$$

input
```
Int[1/(x^4*(13 + 2/x + 15*x)),x]
```

output
```
15*(-1/60*1/x^2 + 13/(60*x) + (139*Log[x])/120 + (9*Log[2 + 3*x])/280 - (2
5*Log[1 + 5*x])/21)
```

3.385.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1722
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.385.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\frac{13x}{4}-\frac{1}{4}}{x^2}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	31
default	$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	32
norman	$\frac{-\frac{1}{4}x+\frac{13}{4}x^2}{x^3}+\frac{139\ln(x)}{8}+\frac{27\ln(3x+2)}{56}-\frac{125\ln(1+5x)}{7}$	35
parallelrisch	$\frac{973\ln(x)x^2-1000\ln(x+\frac{1}{5})x^2+27\ln(x+\frac{2}{3})x^2-14+182x}{56x^2}$	36

input
```
int(1/x^4/(13+2/x+15*x),x,method=_RETURNVERBOSE)
```

output
```
(13/4*x-1/4)/x^2+139/8*ln(x)+27/56*ln(3*x+2)-125/7*ln(1+5*x)
```

3.385.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{1000\,x^2\log\left(5\,x+1\right)-27\,x^2\log\left(3\,x+2\right)-973\,x^2\log\left(x\right)-182\,x+14}{56\,x^2}$$

input
```
integrate(1/x^4/(13+2/x+15*x),x, algorithm="fricas")
```

output
```
-1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*
x + 14)/x^2
```

3.385.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4 \left(13+\frac{2}{x}+15x\right)}\,dx = \frac{139\log(x)}{8} - \frac{125\log\left(x+\frac{1}{5}\right)}{7} + \frac{27\log\left(x+\frac{2}{3}\right)}{56} + \frac{13x-1}{4x^2}$$

input
```
integrate(1/x**4/(13+2/x+15*x),x)
```

output
```
139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**
2)
```

3.385.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4 \left(13+\frac{2}{x}+15x\right)}\,dx = \frac{13\,x-1}{4\,x^2} - \frac{125}{7}\log(5\,x+1) + \frac{27}{56}\log(3\,x+2) + \frac{139}{8}\log(x)$$

input
```
integrate(1/x^4/(13+2/x+15*x),x, algorithm="maxima")
```

output
```
1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x
)
```

3.385.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^4 \left(13+\frac{2}{x}+15x\right)}\,dx = \frac{13\,x-1}{4\,x^2} - \frac{125}{7}\log(|5\,x+1|) + \frac{27}{56}\log(|3\,x+2|) + \frac{139}{8}\log(|x|)$$

input
```
integrate(1/x^4/(13+2/x+15*x),x, algorithm="giac")
```

output
```
1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 1
39/8*log(abs(x))
```

3.385.9 Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^4 \left(13+\frac{2}{x}+15x\right)}\,dx=\frac{27\ln\left(x+\frac{2}{3}\right)}{56}-\frac{125\ln\left(x+\frac{1}{5}\right)}{7}+\frac{139\ln\left(x\right)}{8}+\frac{\frac{13\,x}{4}-\frac{1}{4}}{x^2}$$

input
```
int(1/(x^4*(15*x + 2/x + 13)),x)
```

output
```
(27*log(x + 2/3))/56 - (125*log(x + 1/5))/7 + (139*log(x))/8 + ((13*x)/4 -
 1/4)/x^2
```

3.386 $\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx$

3.386.1 Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

output
```
-1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

3.386.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx=-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

input
```
Integrate[1/(x^5*(13 + 2/x + 15*x)),x]
```

output
```
-1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])
/112 + (625*Log[1 + 5*x])/7
```

3.386.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1722, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(15 x+\frac{2}{x}+13\right)} \, dx$$

$$\Big\downarrow \text{1722}$$

$$\int \frac{1}{x^4 \left(15 x^2+13 x+2\right)}dx$$

$$\Big\downarrow \text{1141}$$

$$15 \int \left(-\frac{81}{560 (3 x+2)}+\frac{625}{21 (5 x+1)}-\frac{1417}{240 x}+\frac{139}{120 x^2}-\frac{13}{60 x^3}+\frac{1}{30 x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$15 \left(-\frac{1}{90 x^3}+\frac{13}{120 x^2}-\frac{139}{120 x}-\frac{1417 \log (x)}{240}-\frac{27}{560} \log (3 x+2)+\frac{125}{21} \log (5 x+1)\right)$$

input
```
Int[1/(x^5*(13 + 2/x + 15*x)),x]
```

output
```
15*(-1/90*1/x^3 + 13/(120*x^2) - 139/(120*x) - (1417*Log[x])/240 - (27*Log
[2 + 3*x])/560 + (125*Log[1 + 5*x])/21)
```

3.386.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1722
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.386.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{-\frac{139}{8}x^2+\frac{13}{8}x-\frac{1}{6}}{x^3}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	36
default	$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	37
norman	$\frac{-\frac{1}{6}x+\frac{13}{8}x^2-\frac{139}{8}x^3}{x^4}-\frac{1417\ln(x)}{16}-\frac{81\ln(3x+2)}{112}+\frac{625\ln(1+5x)}{7}$	40
parallelrisch	$-\frac{29757\ln(x)x^3-30000\ln\left(x+\frac{1}{5}\right)x^3+243\ln\left(x+\frac{2}{3}\right)x^3+56+5838x^2-546x}{336x^3}$	41

input
```
int(1/x^5/(13+2/x+15*x),x,method=_RETURNVERBOSE)
```

output
```
(-139/8*x^2+13/8*x-1/6)/x^3-1417/16*ln(x)-81/112*ln(3*x+2)+625/7*ln(1+5*x)
```

3.386.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx = \frac{30000\,x^3\log\left(5\,x+1\right)-243\,x^3\log\left(3\,x+2\right)-29757\,x^3\log\left(x\right)-5838\,x^2+546\,x-56}{336\,x^3}$$

input
```
integrate(1/x^5/(13+2/x+15*x),x, algorithm="fricas")
```

output
```
1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) -
5838*x^2 + 546*x - 56)/x^3
```

3.386.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{1417\log(x)}{16}+\frac{625\log\left(x+\frac{1}{5}\right)}{7}-\frac{81\log\left(x+\frac{2}{3}\right)}{112}+\frac{-417x^2+39x-4}{24x^3}$$

input
```
integrate(1/x**5/(13+2/x+15*x),x)
```

output
```
-1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 + (-417*x**2 +
39*x - 4)/(24*x**3)
```

3.386.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\log(5\,x+1)-\frac{81}{112}\log(3\,x+2)-\frac{1417}{16}\log(x)$$

input
```
integrate(1/x^5/(13+2/x+15*x),x, algorithm="maxima")
```

output
```
-1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2)
- 1417/16*log(x)
```

3.386.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx = -\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\log(|5\,x+1|)-\frac{81}{112}\log(|3\,x+2|)-\frac{1417}{16}\log(|x|)$$

input
```
integrate(1/x^5/(13+2/x+15*x),x, algorithm="giac")
```

output
```
-1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(
3*x + 2)) - 1417/16*log(abs(x))
```

3.386.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^5 \left(13+\frac{2}{x}+15x\right)} \, dx = \frac{625 \ln \left(x+\frac{1}{5}\right)}{7} - \frac{81 \ln \left(x+\frac{2}{3}\right)}{112} - \frac{1417 \ln (x)}{16} - \frac{\frac{139\,x^2}{8} - \frac{13\,x}{8} + \frac{1}{6}}{x^3}$$

input
```
int(1/(x^5*(15*x + 2/x + 13)),x)
```

output
```
(625*log(x + 1/5))/7 - (81*log(x + 2/3))/112 - (1417*log(x))/16 - ((139*x^
2)/8 - (13*x)/8 + 1/6)/x^3
```

3.387 $\int \frac{x^2}{2-(1+x^2)^4} \, dx$

3.387.1 Optimal result

Integrand size = 17, antiderivative size = 157

$$\int \frac{x^2}{2-(1+x^2)^4} \, dx = \frac{i\sqrt{1-i\sqrt[4]{2}}\arctan\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} - \frac{i\sqrt{1+i\sqrt[4]{2}}\arctan\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} - \frac{\sqrt{1+\sqrt[4]{2}}\arctan\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} + \frac{\sqrt{-1+\sqrt[4]{2}}\text{arctanh}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}$$

output
```
1/8*I*arctan(x/(1-I*2^(1/4))^(1/2))*(1-I*2^(1/4))^(1/2)*2^(1/4)-1/8*I*arct
an(x/(1+I*2^(1/4))^(1/2))*(1+I*2^(1/4))^(1/2)*2^(1/4)+1/8*arctanh(x/(-1+2^
(1/4))^(1/2))*(-1+2^(1/4))^(1/2)*2^(1/4)-1/8*arctan(x/(1+2^(1/4))^(1/2))*(
1+2^(1/4))^(1/2)*2^(1/4)
```

3.387.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{2-(1+x^2)^4} \, dx = -\frac{1}{8}\text{RootSum}\left[-1+4\#1^2+6\#1^4+4\#1^6 + \#1^8\&, \frac{\log(x-\#1)\#1}{1+3\#1^2+3\#1^4+\#1^6}\&\right]$$

input
```
Integrate[x^2/(2 - (1 + x^2)^4),x]
```

output
```
-1/8*RootSum[-1 + 4*#1^2 + 6*#1^4 + 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(1
+ 3*#1^2 + 3*#1^4 + #1^6) & ]
```

3.387.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7291, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{2-(x^2+1)^4}\,dx$$

$$\Big\downarrow \text{7291}$$

$$\int \left(\frac{-\sqrt[4]{2}-\sqrt{2}}{8\left(x^2+\sqrt[4]{2}+1\right)}+\frac{-\sqrt[4]{2}-i\sqrt{2}}{8\left(\sqrt[4]{2}-i\left(x^2+1\right)\right)}+\frac{-\sqrt[4]{2}+i\sqrt{2}}{8\left(\sqrt[4]{2}+i\left(x^2+1\right)\right)}+\frac{\sqrt{2}-\sqrt[4]{2}}{8\left(-x^2+\sqrt[4]{2}-1\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\sqrt{1-i\sqrt[4]{2}}\arctan\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1+i\sqrt[4]{2}}\arctan\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt{1+\sqrt[4]{2}}\arctan\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{\sqrt[4]{2}-1}\text{arctanh}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\ 2^{3/4}}$$

input
```
Int[x^2/(2 - (1 + x^2)^4),x]
```

output
```
((I/4)*Sqrt[1 - I*2^(1/4)]*ArcTan[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4) - ((I/4)
*Sqrt[1 + I*2^(1/4)]*ArcTan[x/Sqrt[1 + I*2^(1/4)]])/2^(3/4) - (Sqrt[1 + 2^
(1/4)]*ArcTan[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4)) + (Sqrt[-1 + 2^(1/4)]*ArcT
anh[x/Sqrt[-1 + 2^(1/4)]])/(4*2^(3/4))
```

3.387.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7291
```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

3.387.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.34

method	result	size
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2-1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7+3\_R^5+3\_R^3+\_R}\right)}{8}$	54
risch	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2-1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7+3\_R^5+3\_R^3+\_R}\right)}{8}$	54

input
```
int(x^2/(2-(x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/8*sum(_R^2/(_R^7+3*_R^5+3*_R^3+_R)*ln(x-_R),_R=RootOf(_Z^8+4*_Z^6+6*_Z^
4+4*_Z^2-1))
```

3.387.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. 2(97) = 194.

Time = 1.02 (sec) , antiderivative size = 1506, normalized size of antiderivative = 9.59

$$\int \frac{x^2}{2-(1+x^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(2-(x^2+1)^4),x, algorithm="fricas")
```

output
```
-1/16*sqrt(2)*sqrt(1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2
^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*l
og(1/4*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sq
rt(2)*(2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*
(2^(3/4) - sqrt(2)) + 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) +
sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(
2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqr
t(2)) - 4*sqrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*s
qrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/
4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2)*sqr
t(1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))
*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4*((sqrt(2)
*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sqrt(2)*(2^(3/4) +
 sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(3/4) - sqrt(
2)) + 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4
) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(2^(3/4) + sqrt(2
)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) - 4*sqrt(2
)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) + sqrt(-3
/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) -
3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) - 1/16*sqrt(2)*sqrt(1/2*sqrt(2)...
```

3.387.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.26

$$\int \frac{x^2}{2-(1+x^2)^4}\,dx =$$
$$-\operatorname{RootSum}\left(1073741824t^8 - 65536t^4 + 1024t^2 - 1, \left(t \mapsto t\log\left(-\frac{67108864t^7}{3} - \frac{262144t^5}{3} - \frac{40t}{3} + x\right)\right)\right)$$

input
```
integrate(x**2/(2-(x**2+1)**4),x)
```

output
```
-RootSum(1073741824*_t**8 - 65536*_t**4 + 1024*_t**2 - 1, Lambda(_t, _t*lo
g(-67108864*_t**7/3 - 262144*_t**5/3 - 40*_t/3 + x)))
```

3.387.7 Maxima [F]

$$\int \frac{x^2}{2-(1+x^2)^4}\,dx=\int -\frac{x^2}{(x^2+1)^4-2}\,dx$$

input
```
integrate(x^2/(2-(x^2+1)^4),x, algorithm="maxima")
```

output
```
-integrate(x^2/((x^2 + 1)^4 - 2), x)
```

3.387.8 Giac [F]

$$\int \frac{x^2}{2-(1+x^2)^4}\,dx=\int -\frac{x^2}{(x^2+1)^4-2}\,dx$$

input
```
integrate(x^2/(2-(x^2+1)^4),x, algorithm="giac")
```

output
```
integrate(-x^2/((x^2 + 1)^4 - 2), x)
```

3.387.9 Mupad [B] (verification not implemented)

Time = 9.91 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{2-(1+x^2)^4}\,dx=\sum_{k=1}^{8}\ln\left(-\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(56\,x\right.\right.$$
$$-\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\right.\left(\right.$$
$$\left.-1\right)\,\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)$$

input
```
int(-x^2/((x^2 + 1)^4 - 2),x)
```

output
```
symsum(log(- root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)*(56*
x - root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)*(root(z^8 - z
^4/16384 + z^2/1048576 - 1/1073741824, z, k)*(4096*x - root(z^8 - z^4/1638
4 + z^2/1048576 - 1/1073741824, z, k)^2*(262144*x + 67108864*root(z^8 - z^
4/16384 + z^2/1048576 - 1/1073741824, z, k)^2*x)) + 256)) - 1)*root(z^8 -
z^4/16384 + z^2/1048576 - 1/1073741824, z, k), k, 1, 8)
```

3.387. $\int \frac{x^2}{2-(1+x^2)^4} \, dx$

3.388 $\int \frac{x^2}{2-(1-x^2)^4}\,dx$

3.388.1 Optimal result

Integrand size = 19, antiderivative size = 157

$$\int \frac{x^2}{2-(1-x^2)^4}\,dx = -\frac{\sqrt{-1+\sqrt[4]{2}}\arctan\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\,2^{3/4}} - \frac{i\sqrt{1-i\sqrt[4]{2}}\text{arctanh}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\,2^{3/4}} + \frac{i\sqrt{1+i\sqrt[4]{2}}\text{arctanh}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\,2^{3/4}} + \frac{\sqrt{1+\sqrt[4]{2}}\text{arctanh}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\,2^{3/4}}$$

output
```
-1/8*I*arctanh(x/(1-I*2^(1/4))^(1/2))*(1-I*2^(1/4))^(1/2)*2^(1/4)+1/8*I*ar
ctanh(x/(1+I*2^(1/4))^(1/2))*(1+I*2^(1/4))^(1/2)*2^(1/4)-1/8*arctan(x/(-1+
2^(1/4))^(1/2))*(-1+2^(1/4))^(1/2)*2^(1/4)+1/8*arctanh(x/(1+2^(1/4))^(1/2)
)*(1+2^(1/4))^(1/2)*2^(1/4)
```

3.388.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{2-(1-x^2)^4}\,dx = -\frac{1}{8}\text{RootSum}\left[-1-4\#1^2+6\#1^4-4\#1^6+\#1^8\&,\frac{\log(x-\#1)\#1}{-1+3\#1^2-3\#1^4+\#1^6}\&\right]$$

input
```
Integrate[x^2/(2 - (1 - x^2)^4),x]
```

output
```
-1/8*RootSum[-1 - 4*#1^2 + 6*#1^4 - 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(-1
 + 3*#1^2 - 3*#1^4 + #1^6) & ]
```

3.388.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {7291, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{2-\left(1-x^2\right)^4}\,dx$$

$$\Big\downarrow \text{7291}$$

$$\int \left(\frac{\sqrt[4]{2}-\sqrt{2}}{8\left(x^2+\sqrt[4]{2}-1\right)}+\frac{\sqrt[4]{2}+i\sqrt{2}}{8\left(\sqrt[4]{2}-i\left(1-x^2\right)\right)}+\frac{\sqrt[4]{2}-i\sqrt{2}}{8\left(\sqrt[4]{2}+i\left(1-x^2\right)\right)}+\frac{\sqrt[4]{2}+\sqrt{2}}{8\left(-x^2+\sqrt[4]{2}+1\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\sqrt[4]{2}-1}\arctan\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1-i\sqrt[4]{2}}\text{arctanh}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{i\sqrt{1+i\sqrt[4]{2}}\text{arctanh}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{1+\sqrt[4]{2}}\text{arctanh}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}$$

input
```
Int[x^2/(2 - (1 - x^2)^4),x]
```

output
```
-1/4*(Sqrt[-1 + 2^(1/4)]*ArcTan[x/Sqrt[-1 + 2^(1/4)]])/2^(3/4) - ((I/4)*Sq
rt[1 - I*2^(1/4)]*ArcTanh[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4) + ((I/4)*Sqrt[1
+ I*2^(1/4)]*ArcTanh[x/Sqrt[1 + I*2^(1/4)]])/2^(3/4) + (Sqrt[1 + 2^(1/4)]*
ArcTanh[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4))
```

3.388.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7291
```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

3.388.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.36

method	result	size
default	$-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}\right)}{8}$	56
risch	$-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}\right)}{8}$	56

input
```
int(x^2/(2-(-x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/8*sum(_R^2/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8-4*_Z^6+6*_Z^
4-4*_Z^2-1))
```

3.388.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1546 vs. 2(97) = 194.

Time = 0.99 (sec) , antiderivative size = 1546, normalized size of antiderivative = 9.85

$$\int \frac{x^2}{2-(1-x^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(2-(-x^2+1)^4),x, algorithm="fricas")
```

output
```
-1/16*sqrt(2)*sqrt(-1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(
2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*
log(1/4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2))^2 - s
qrt(2)*(2^(3/4) - sqrt(2))^2 - (sqrt(2)*(2^(3/4) - sqrt(2))^2 - 4*sqrt(2))
*(2^(3/4) + sqrt(2)) + 4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4)
 + sqrt(2)) + sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-3/16*(2^(3/4)
 + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4
) - sqrt(2))^2 + 1) - 4*sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-1/2
*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(
3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2)*s
qrt(-1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(
2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4*((sqrt
(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2))^2 - sqrt(2)*(2^(3/4
) - sqrt(2))^2 - (sqrt(2)*(2^(3/4) - sqrt(2))^2 - 4*sqrt(2))*(2^(3/4) + sq
rt(2)) + 4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2)) +
sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-3/16*(2^(3/4) + sqrt(2))^2
+ 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2
 + 1) - 4*sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-1/2*sqrt(2) + sqr
t(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)
) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) - 1/16*sqrt(2)*sqrt(-1/2*sq...
```

3.388.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.26

$$\int \frac{x^2}{2-(1-x^2)^4}\,dx =$$
$$-\operatorname{RootSum}\left(1073741824t^8 - 65536t^4 - 1024t^2 - 1, \left(t \mapsto t\log\left(-\frac{67108864t^7}{3} + \frac{262144t^5}{3} + \frac{40t}{3} + x\right)\right)\right)$$

input
```
integrate(x**2/(2-(-x**2+1)**4),x)
```

output
```
-RootSum(1073741824*_t**8 - 65536*_t**4 - 1024*_t**2 - 1, Lambda(_t, _t*lo
g(-67108864*_t**7/3 + 262144*_t**5/3 + 40*_t/3 + x)))
```

3.388.7 Maxima [F]

$$\int \frac{x^2}{2-(1-x^2)^4}\,dx = \int -\frac{x^2}{(x^2-1)^4-2}\,dx$$

input
```
integrate(x^2/(2-(-x^2+1)^4),x, algorithm="maxima")
```

output
```
-integrate(x^2/((x^2 - 1)^4 - 2), x)
```

3.388.8 Giac [F]

$$\int \frac{x^2}{2-(1-x^2)^4}\,dx = \int -\frac{x^2}{(x^2-1)^4-2}\,dx$$

input
```
integrate(x^2/(2-(-x^2+1)^4),x, algorithm="giac")
```

output
```
integrate(-x^2/((x^2 - 1)^4 - 2), x)
```

3.388.9 Mupad [B] (verification not implemented)

Time = 9.94 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{2-(1-x^2)^4}\,dx = \sum_{k=1}^{8} \ln\left(-\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(56\,x\right.\right.$$
$$+\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\right)\left(\right.$$
$$\left.\left.-1\right)\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)$$

input
```
int(-x^2/((x^2 - 1)^4 - 2),x)
```

output
```
symsum(log(- root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)*(56*
x + root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)*(root(z^8 - z
^4/16384 - z^2/1048576 - 1/1073741824, z, k)*(4096*x + root(z^8 - z^4/1638
4 - z^2/1048576 - 1/1073741824, z, k)^2*(262144*x - 67108864*root(z^8 - z^
4/16384 - z^2/1048576 - 1/1073741824, z, k)^2*x)) + 256)) - 1)*root(z^8 -
z^4/16384 - z^2/1048576 - 1/1073741824, z, k), k, 1, 8)
```

3.388. $\int \frac{x^2}{2-(1-x^2)^4}\,dx$

3.389 $\int \frac{x^2}{2+(1+x^2)^4}\,dx$

3.389.1 Optimal result

Integrand size = 15, antiderivative size = 188

$$\int \frac{x^2}{2+(1+x^2)^4}\,dx = \frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\arctan\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} - \frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\arctan\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} - \frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\arctan\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} + \frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{(1+i)+2^{3/4}}}\arctan\left(\sqrt{\frac{1+i}{(1+i)+2^{3/4}}}x\right)$$

output
```
1/8*(-1)^(1/4)*arctan(x/(1-(-2)^(1/4))^(1/2))*(1-(-2)^(1/4))^(1/2)*2^(1/4)
-1/8*(-1)^(3/4)*2^(1/4)*arctan(x/(1+I*(-2)^(1/4))^(1/2))*(1+I*(-2)^(1/4))^
(1/2)-1/8*(-1)^(1/4)*arctan(x/(1+(-2)^(1/4))^(1/2))*(1+(-2)^(1/4))^(1/2)*2
^(1/4)+1/8*I*arctan(x*((1+I)/(1+I+2^(3/4)))^(1/2))*((-2)^(1/4)+2^(1/2))*((
1+I)/(1+I+2^(3/4)))^(1/2)
```

3.389.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.32

$$\int \frac{x^2}{2+(1+x^2)^4}\,dx = \frac{1}{8}\text{RootSum}\left[3+4\#1^2+6\#1^4+4\#1^6 +\#1^8\&,\frac{\log(x-\#1)\#1}{1+3\#1^2+3\#1^4+\#1^6}\&\right]$$

input
```
Integrate[x^2/(2 + (1 + x^2)^4),x]
```

output
```
RootSum[3 + 4*#1^2 + 6*#1^4 + 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(1 + 3*#1
^2 + 3*#1^4 + #1^6) & ]/8
```

3.389.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7291, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^2+1)^4+2}\,dx$$

$$\Big\downarrow \text{7291}$$

$$\int \left(\frac{-\sqrt[4]{-2}-i\sqrt{2}}{8\left(x^2+\sqrt[4]{-2}+1\right)}+\frac{\sqrt{2}-\sqrt[4]{-2}}{8\left(\sqrt[4]{-2}-i\left(x^2+1\right)\right)}+\frac{-\sqrt[4]{-2}-\sqrt{2}}{8\left(\sqrt[4]{-2}+i\left(x^2+1\right)\right)}+\frac{-\sqrt[4]{-2}+i\sqrt{2}}{8\left(-x^2+\sqrt[4]{-2}-1\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\arctan\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\arctan\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\arctan\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}\arctan\left(\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}x\right)$$

input
```
Int[x^2/(2 + (1 + x^2)^4),x]
```

output
```
((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTan[x/Sqrt[1 - (-2)^(1/4)]])/(4*2^(3/4
)) - ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTan[x/Sqrt[1 + I*(-2)^(1/4)]])/
(4*2^(3/4)) - ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*ArcTan[x/Sqrt[1 + (-2)^(1/4
)]])/(4*2^(3/4)) + (I/8)*((-2)^(1/4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^
(3/4))]*ArcTan[Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*x]
```

3.389.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7291
```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

3.389.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.29

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+4\_Z^{6}+6\_Z^{4}+4\_Z^{2}+3\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{\_R^{7}+3\_R^{5}+3\_R^{3}+\_R}\right)}{8}$	54
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+4\_Z^{6}+6\_Z^{4}+4\_Z^{2}+3\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{\_R^{7}+3\_R^{5}+3\_R^{3}+\_R}\right)}{8}$	54

input
```
int(x^2/(2+(x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum(_R^2/(_R^7+3*_R^5+3*_R^3+_R)*ln(x-_R),_R=RootOf(_Z^8+4*_Z^6+6*_Z^4
+4*_Z^2+3))
```

3.389.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2271 vs. 2(118) = 236.

Time = 0.98 (sec) , antiderivative size = 2271, normalized size of antiderivative = 12.08

$$\int \frac{x^2}{2+(1+x^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(2+(x^2+1)^4),x, algorithm="fricas")
```

output
```
-1/16*sqrt(2)*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2
)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*
sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqr
t(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log(
(16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt
(2) + 128*sqrt(1/8192*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-
1/8192*I*sqrt(2))) - sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)
))^2 - 16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 -
sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/2
56*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(
-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1)*((sqrt(
2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - sqrt(2))*(-I*sqrt(2) + 128*
sqrt(1/8192*I*sqrt(2))) - sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))
)) + sqrt(2))*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2
)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*
sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqr
t(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*
x) + 1/16*sqrt(2)*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sq
rt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8
*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/819...
```

3.389.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.21

$$\int \frac{x^2}{2+(1+x^2)^4}\,dx$$
$$= \mathrm{RootSum}\left(1073741824t^8+65536t^4+1024t^2+3,\left(t\mapsto t\log\left(67108864t^7-262144t^5+4096t^3+40t+x\right)\right)\right)$$

input
```
integrate(x**2/(2+(x**2+1)**4),x)
```

output
```
RootSum(1073741824*_t**8 + 65536*_t**4 + 1024*_t**2 + 3, Lambda(_t, _t*log
(67108864*_t**7 - 262144*_t**5 + 4096*_t**3 + 40*_t + x)))
```

3.389.7 Maxima [F]

$$\int \frac{x^2}{2+\left(1+x^2\right)^4}\,dx=\int \frac{x^2}{\left(x^2+1\right)^4+2}\,dx$$

input
```
integrate(x^2/(2+(x^2+1)^4),x, algorithm="maxima")
```

output
```
integrate(x^2/((x^2 + 1)^4 + 2), x)
```

3.389.8 Giac [F]

$$\int \frac{x^2}{2+\left(1+x^2\right)^4}\,dx=\int \frac{x^2}{\left(x^2+1\right)^4+2}\,dx$$

input
```
integrate(x^2/(2+(x^2+1)^4),x, algorithm="giac")
```

output
```
integrate(x^2/((x^2 + 1)^4 + 2), x)
```

3.389.9 Mupad [B] (verification not implemented)

Time = 10.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{2+\left(1+x^2\right)^4}\,dx=\sum_{k=1}^{8}\ln\left(\mathrm{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(40\,x\right.\right.$$
$$+\mathrm{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(\mathrm{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\right.\left(\right.$$
$$\left.\left.-3\right)\mathrm{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)$$

input
```
int(x^2/((x^2 + 1)^4 + 2),x)
```

output
```
symsum(log(root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k)*(40*x
+ root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k)*(root(z^8 + z^4
/16384 + z^2/1048576 + 3/1073741824, z, k)*(4096*x - root(z^8 + z^4/16384
+ z^2/1048576 + 3/1073741824, z, k)^2*(786432*x - 67108864*root(z^8 + z^4/
16384 + z^2/1048576 + 3/1073741824, z, k)^2*x)) - 768)) - 3)*root(z^8 + z^
4/16384 + z^2/1048576 + 3/1073741824, z, k), k, 1, 8)
```

3.389. $\int \frac{x^2}{2+(1+x^2)^4} \, dx$

3.390 $\int \frac{x^2}{2+(1-x^2)^4} \, dx$

3.390.1 Optimal result

Integrand size = 17, antiderivative size = 188

$$\int \frac{x^2}{2+(1-x^2)^4} \, dx = -\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\operatorname{arctanh}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} + \frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\operatorname{arctanh}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} + \frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}} - \frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{(1+i)+2^{3/4}}}\operatorname{arctanh}\left(\sqrt{\frac{1+i}{(1+i)+2^{3/4}}}x\right)$$

output
```
-1/8*(-1)^(1/4)*arctanh(x/(1-(-2)^(1/4))^(1/2))*(1-(-2)^(1/4))^(1/2)*2^(1/
4)+1/8*(-1)^(3/4)*2^(1/4)*arctanh(x/(1+I*(-2)^(1/4))^(1/2))*(1+I*(-2)^(1/4
))^(1/2)+1/8*(-1)^(1/4)*arctanh(x/(1+(-2)^(1/4))^(1/2))*(1+(-2)^(1/4))^(1/
2)*2^(1/4)-1/8*I*arctanh(x*((1+I)/(1+I+2^(3/4)))^(1/2))*((-2)^(1/4)+2^(1/2
))*((1+I)/(1+I+2^(3/4)))^(1/2)
```

3.390.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.32

$$\int \frac{x^2}{2+(1-x^2)^4}\,dx = \frac{1}{8}\text{RootSum}\left[3-4\#1^2+6\#1^4-4\#1^6+\#1^8\&,\frac{\log(x-\#1)\#1}{-1+3\#1^2-3\#1^4+\#1^6}\&\right]$$

input
```
Integrate[x^2/(2 + (1 - x^2)^4),x]
```

output
```
RootSum[3 - 4*#1^2 + 6*#1^4 - 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(-1 + 3*#
1^2 - 3*#1^4 + #1^6) & ]/8
```

3.390.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7291, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-x^2)^4+2}\,dx$$

$$\Big\downarrow \; 7291$$

$$\int \left(\frac{\sqrt[4]{-2}-i\sqrt{2}}{8\left(x^2+\sqrt[4]{-2}-1\right)}+\frac{\sqrt[4]{-2}-\sqrt{2}}{8\left(\sqrt[4]{-2}-i\left(1-x^2\right)\right)}+\frac{\sqrt[4]{-2}+\sqrt{2}}{8\left(\sqrt[4]{-2}+i\left(1-x^2\right)\right)}+\frac{\sqrt[4]{-2}+i\sqrt{2}}{8\left(-x^2+\sqrt[4]{-2}+1\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\text{arctanh}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\text{arctanh}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\text{arctanh}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}\text{arctanh}\left(\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}x\right)$$

input
```
Int[x^2/(2 + (1 - x^2)^4),x]
```

output
```
-1/4*((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTanh[x/Sqrt[1 - (-2)^(1/4)]])/2^(
3/4) + ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTanh[x/Sqrt[1 + I*(-2)^(1/4)]
])/(4*2^(3/4)) + ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*ArcTanh[x/Sqrt[1 + (-2)^
(1/4)]])/(4*2^(3/4)) - (I/8)*((-2)^(1/4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I)
+ 2^(3/4))]*ArcTanh[Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*x]
```

3.390.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7291
```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

3.390.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.30

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3\right)} \frac{\_R^2 \ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}}{8}$	56
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3\right)} \frac{\_R^2 \ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}}{8}$	56

input
```
int(x^2/(2+(-x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum(_R^2/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8-4*_Z^6+6*_Z^4
-4*_Z^2+3))
```

3.390.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2259 vs. 2(118) = 236.

Time = 0.99 (sec) , antiderivative size = 2259, normalized size of antiderivative = 12.02

$$\int \frac{x^2}{2+(1-x^2)^4} \, dx = \text{Too large to display}$$

input
```
integrate(x^2/(2+(-x^2+1)^4),x, algorithm="fricas")
```

output

```
-1/16*sqrt(2)*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(
2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*
sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqr
t(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log(
(16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(
2) + 128*sqrt(-1/8192*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(1
/8192*I*sqrt(2))) + sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)
))^2 + 16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - s
qrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/2
56*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1
/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1)*((sqrt(2
)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2))*(-I*sqrt(2) + 128*sq
rt(-1/8192*I*sqrt(2))) + sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))))
 - sqrt(2))*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)
))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sq
rt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(
2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x)
 + 1/16*sqrt(2)*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqr
t(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(
I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*...
```

3.390.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.21

$$\int \frac{x^2}{2+(1-x^2)^4}\,dx$$
$$= \text{RootSum}\left(1073741824t^8+65536t^4-1024t^2+3,\left(t\mapsto t\log\left(67108864t^7+262144t^5+4096t^3-40t+x\right)\right)\right)$$

input

```
integrate(x**2/(2+(-x**2+1)**4),x)
```

output

```
RootSum(1073741824*_t**8 + 65536*_t**4 - 1024*_t**2 + 3, Lambda(_t, _t*log
(67108864*_t**7 + 262144*_t**5 + 4096*_t**3 - 40*_t + x)))
```

3.390.7 Maxima [F]

$$\int \frac{x^2}{2+(1-x^2)^4}\,dx = \int \frac{x^2}{(x^2-1)^4+2}\,dx$$

input
```
integrate(x^2/(2+(-x^2+1)^4),x, algorithm="maxima")
```

output
```
integrate(x^2/((x^2 - 1)^4 + 2), x)
```

3.390.8 Giac [F]

$$\int \frac{x^2}{2+(1-x^2)^4}\,dx = \int \frac{x^2}{(x^2-1)^4+2}\,dx$$

input
```
integrate(x^2/(2+(-x^2+1)^4),x, algorithm="giac")
```

output
```
integrate(x^2/((x^2 - 1)^4 + 2), x)
```

3.390.9 Mupad [B] (verification not implemented)

Time = 10.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{2+(1-x^2)^4}\,dx = \sum_{k=1}^{8} \ln\left(\operatorname{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(40\,x\right.\right.$$
$$-\operatorname{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(\operatorname{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\right.\Big($$
$$\left.\left.-3\right)\right)\operatorname{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)$$

input
```
int(x^2/((x^2 - 1)^4 + 2),x)
```

output
```
symsum(log(root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k)*(40*x
- root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k)*(root(z^8 + z^4
/16384 - z^2/1048576 + 3/1073741824, z, k)*(4096*x + root(z^8 + z^4/16384
- z^2/1048576 + 3/1073741824, z, k)^2*(786432*x + 67108864*root(z^8 + z^4/
16384 - z^2/1048576 + 3/1073741824, z, k)^2*x)) - 768)) - 3)*root(z^8 + z^
4/16384 - z^2/1048576 + 3/1073741824, z, k), k, 1, 8)
```

3.390. $\int \frac{x^2}{2+(1-x^2)^4} \, dx$

3.391 $\int \frac{1-x^2}{a+b(1-x^2)^4}\,dx$

3.391.1 Optimal result

Integrand size = 23, antiderivative size = 663

$$\begin{aligned}
&\int \frac{1-x^2}{a+b(1-x^2)^4}\,dx \\
&= -\frac{\arctan\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\,b^{3/8}} \\
&+ \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\,b^{3/8}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,b^{3/8}} \\
&+ \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt[8]{b}x+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\,b^{3/8}} \\
&- \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt[8]{b}x+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\,b^{3/8}}
\end{aligned}$$

output
```
-1/4*arctan(b^(1/8)*x/((-a)^(1/4)-b^(1/4))^(1/2))/b^(3/8)/(-a)^(1/2)/((-a)
^(1/4)-b^(1/4))^(1/2)+1/4*arctanh(b^(1/8)*x/((-a)^(1/4)+b^(1/4))^(1/2))/b^
(3/8)/(-a)^(1/2)/((-a)^(1/4)+b^(1/4))^(1/2)-1/8*arctan((-b^(1/8)*x*2^(1/2)
+(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))/(-b^(1/4)+((-a)^(1/2)+b^(1/2)
)^(1/2))^(1/2))*(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2
)/(-a)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/2)+1/8*arctan((b^(1/8)*x*2^(1/2)+(b^(
1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))/(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/
2))^(1/2))*(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a
)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/2)+1/16*ln(b^(1/4)*x^2+((-a)^(1/2)+b^(1/2)
)^(1/2)-b^(1/8)*x*2^(1/2)*(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))*(b^(
1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/((-a)^(1
/2)+b^(1/2))^(1/2)-1/16*ln(b^(1/4)*x^2+((-a)^(1/2)+b^(1/2))^(1/2)+b^(1/8)*
x*2^(1/2)*(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))*(b^(1/4)+((-a)^(1/2)
+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/
2)
```

3.391.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.10

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx = -\frac{\text{RootSum}\left[a+b-4b\#1^2+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{\log(x-\#1)}{\#1-2\#1^3+\#1^5}\&\right]}{8b}$$

input
```
Integrate[(1 - x^2)/(a + b*(1 - x^2)^4),x]
```

output
```
-1/8*RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[x - #
1]/(#1 - 2*#1^3 + #1^5) & ]/b
```

3.391.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7291, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{a+b(1-x^2)^4} \, dx$$

$$\Big\downarrow \text{7291}$$

$$\int \left(-\frac{\sqrt{b}(1-x^2)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}-b(1-x^2)^2\right)}-\frac{\sqrt{b}(1-x^2)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}+b(1-x^2)^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(-\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}$$

input
```
Int[(1 - x^2)/(a + b*(1 - x^2)^4),x]
```

output
```
-1/4*ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(Sqrt[-a]*Sqrt[(-a)^(1
/4) - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan
[(Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[
Sqrt[-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[
b]]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt
[Sqrt[-a] + Sqrt[b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] +
Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)
) + ArcTanh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^
(1/4) + b^(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[
Sqrt[Sqrt[-a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)
]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b
^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sq
rt[b]] + Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1
/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

3.391.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7291
```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

3.391.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6\_Z^4b-4\_Z^2b+a+b\right)}\frac{\left(-\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}}{8b}$	69
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6\_Z^4b-4\_Z^2b+a+b\right)}\frac{\left(-\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}}{8b}$	69

input
```
int((-x^2+1)/(a+b*(-x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8*b-4*_Z
^6*b+6*_Z^4*b-4*_Z^2*b+a+b))
```

3.391.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 322185, normalized size of antiderivative = 485.95

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4} \, dx = \text{Too large to display}$$

input
```
integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="fricas")
```

output
```
Too large to include
```

3.391.6 Sympy [A] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.20

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4} \, dx = -\operatorname{RootSum}\left(t^8\cdot\left(16777216a^5b^3+16777216a^4b^4\right)+1048576t^6a^3b^3+24576t^4a^2b^2+256t^2ab+1,\left(t\mapsto t\,\mathrm{l}\right.\right.$$

input
```
integrate((-x**2+1)/(a+b*(-x**2+1)**4),x)
```

output
```
-RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 1048576*_t**6*a
**3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, Lambda(_t, _t*log(-6
291456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**4 + 65536*_t**5*a**3*b**2 -
 327680*_t**5*a**2*b**3 - 512*_t**3*a**2*b - 5632*_t**3*a*b**2 - 32*_t*b +
 x)))
```

3.391.7 Maxima [F]

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx=\int -\frac{x^2-1}{\left(x^2-1\right)^4b+a}\,dx$$

input
```
integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

3.391.8 Giac [F]

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx=\int -\frac{x^2-1}{\left(x^2-1\right)^4b+a}\,dx$$

input
```
integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

3.391.9 Mupad [B] (verification not implemented)

Time = 9.83 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.49

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx$$
$$=\sum_{k=1}^{8}\ln\Big(a\,b^5\,\Big(\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+$$
$$+1\Big)\,\Big(\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k$$
$$+\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\big)^2\,a\,b$$
$$-\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\big)^5\,a^3$$
$$+1\Big)\Big)\,\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4$$
$$+256\,a\,b\,z^2+1,z,k\big)$$

input
```
int(-(x^2 - 1)/(a + b*(x^2 - 1)^4),x)
```

output
```
symsum(log(a*b^5*(64*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 10
48576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b + 1)*
(4096*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z
^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^4*a^2*b^2 + 128*root(16777
216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b
^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b - 32768*root(16777216*a^5*b^3*z^8 +
16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z
^2 + 1, z, k)^5*a^3*b^2*x + 1))*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b
^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k),
 k, 1, 8)
```

3.391. $\int \frac{1-x^2}{a+b(1-x^2)^4} \, dx$

3.392 $\int \frac{1-x^2}{a+b(-1+x^2)^4}\,dx$

3.392.1 Optimal result

Integrand size = 21, antiderivative size = 663

$$
\begin{aligned}
\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx
&= -\frac{\arctan\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}}
-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\,b^{3/8}} \\
&\quad+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\,b^{3/8}}
+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,b^{3/8}} \\
&\quad+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt[8]{b}x+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\,b^{3/8}} \\
&\quad-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt[8]{b}x+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\,b^{3/8}}
\end{aligned}
$$

output
```
-1/4*arctan(b^(1/8)*x/((-a)^(1/4)-b^(1/4))^(1/2))/b^(3/8)/(-a)^(1/2)/((-a)
^(1/4)-b^(1/4))^(1/2)+1/4*arctanh(b^(1/8)*x/((-a)^(1/4)+b^(1/4))^(1/2))/b^
(3/8)/(-a)^(1/2)/((-a)^(1/4)+b^(1/4))^(1/2)-1/8*arctan((-b^(1/8)*x*2^(1/2)
+(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))/(-b^(1/4)+((-a)^(1/2)+b^(1/2)
)^(1/2))^(1/2))*(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2
)/(-a)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/2)+1/8*arctan((b^(1/8)*x*2^(1/2)+(b^(
1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))/(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/
2))^(1/2))*(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a
)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/2)+1/16*ln(b^(1/4)*x^2+((-a)^(1/2)+b^(1/2)
)^(1/2)-b^(1/8)*x*2^(1/2)*(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))*(b^(
1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/((-a)^(1
/2)+b^(1/2))^(1/2)-1/16*ln(b^(1/4)*x^2+((-a)^(1/2)+b^(1/2))^(1/2)+b^(1/8)*
x*2^(1/2)*(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))*(b^(1/4)+((-a)^(1/2)
+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/
2)
```

3.392.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.10

$$\int \frac{1-x^2}{a+b(-1+x^2)^4}\,dx = -\frac{\text{RootSum}\left[a+b-4b\#1^2+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{\log(x-\#1)}{\#1-2\#1^3+\#1^5}\&\right]}{8b}$$

input
```
Integrate[(1 - x^2)/(a + b*(-1 + x^2)^4),x]
```

output
```
-1/8*RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[x - #
1]/(#1 - 2*#1^3 + #1^5) & ]/b
```

3.392.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {7291, 25, 25, 7291, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{a+b\left(x^2-1\right)^4}dx$$

$$\Big\downarrow \text{7291}$$

$$\int -\frac{x^2-1}{a+b\left(x^2-1\right)^4}dx$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{1-x^2}{b\left(1-x^2\right)^4+a}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}dx$$

$$\Big\downarrow \text{7291}$$

$$\int \left(-\frac{\sqrt{b}\left(1-x^2\right)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}-b\left(1-x^2\right)^2\right)}-\frac{\sqrt{b}\left(1-x^2\right)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}+b\left(1-x^2\right)^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(-\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}$$

input
```
Int[(1 - x^2)/(a + b*(-1 + x^2)^4),x]
```

output
```
-1/4*ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(Sqrt[-a]*Sqrt[(-a)^(1
/4) - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan
[(Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[
Sqrt[-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[
b]]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt
[Sqrt[-a] + Sqrt[b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] +
Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)
) + ArcTanh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^
(1/4) + b^(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[
Sqrt[Sqrt[-a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)
]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b
^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sq
rt[b]] + Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1
/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

3.392. $\int \frac{1-x^2}{a+b(-1+x^2)^4}\,dx$

3.392.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7291
```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

3.392.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6\_Z^4b-4\_Z^2b+a+b\right)}\frac{\left(-\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}}{8b}$	69
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6\_Z^4b-4\_Z^2b+a+b\right)}\frac{\left(-\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}}{8b}$	69

input
```
int((-x^2+1)/(a+b*(x^2-1)^4),x,method=_RETURNVERBOSE)
```

output
```
1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8*b-4*_Z
^6*b+6*_Z^4*b-4*_Z^2*b+a+b))
```

3.392.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 322185, normalized size of antiderivative = 485.95

$$\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="fricas")
```

output
```
Too large to include
```

3.392.6 Sympy [A] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.20

$$\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx = -\operatorname{RootSum}\left(t^8\cdot\left(16777216a^5b^3+16777216a^4b^4\right)+1048576t^6a^3b^3+24576t^4a^2b^2+256t^2ab+1,\left(t\mapsto t\,\mathrm{l}\right.\right.$$

input
```
integrate((-x**2+1)/(a+b*(x**2-1)**4),x)
```

output
```
-RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 1048576*_t**6*a
**3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, Lambda(_t, _t*log(-6
291456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**4 + 65536*_t**5*a**3*b**2 -
 327680*_t**5*a**2*b**3 - 512*_t**3*a**2*b - 5632*_t**3*a*b**2 - 32*_t*b +
 x)))
```

3.392.7 Maxima [F]

$$\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx = \int -\frac{x^2-1}{\left(x^2-1\right)^4b+a}\,dx$$

input
```
integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

3.392.8 Giac [F]

$$\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx = \int -\frac{x^2-1}{\left(x^2-1\right)^4 b+a}\,dx$$

input
```
integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

3.392.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.49

$$\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx$$
$$=\sum_{k=1}^{8}\ln\Big(a\,b^5\,\Big(\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+$$
$$+1\Big)\,\Big(\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k$$
$$+\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\big)^2\,a\,b$$
$$-\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\big)^5\,a^3$$
$$+1\Big)\Big)\,\mathrm{root}\big(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4$$
$$+256\,a\,b\,z^2+1,z,k\big)$$

input
```
int(-(x^2 - 1)/(a + b*(x^2 - 1)^4),x)
```

output
```
symsum(log(a*b^5*(64*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 10
48576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b + 1)*
(4096*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z
^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^4*a^2*b^2 + 128*root(16777
216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b
^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b - 32768*root(16777216*a^5*b^3*z^8 +
16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z
^2 + 1, z, k)^5*a^3*b^2*x + 1))*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b
^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k),
 k, 1, 8)
```

3.393 $\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx$

3.393.1 Optimal result

Integrand size = 25, antiderivative size = 168

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}x}{\sqrt[6]{b}}\right)}{3\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}b^{5/6}}+\frac{\arctan\left(\frac{\sqrt{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}}x}{\sqrt[6]{b}}\right)}{3\sqrt{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}}b^{5/6}}+\frac{\arctan\left(\frac{\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}x}{\sqrt[6]{b}}\right)}{3\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}b^{5/6}}$$

output
```
1/3*arctan(x*(a^(1/3)+b^(1/3))^(1/2)/b^(1/6))/b^(5/6)/(a^(1/3)+b^(1/3))^(1
/2)+1/3*arctan(x*(-(-1)^(1/3)*a^(1/3)+b^(1/3))^(1/2)/b^(1/6))/b^(5/6)/(-(-
1)^(1/3)*a^(1/3)+b^(1/3))^(1/2)+1/3*arctan(x*((-1)^(2/3)*a^(1/3)+b^(1/3))^
(1/2)/b^(1/6))/b^(5/6)/((-1)^(2/3)*a^(1/3)+b^(1/3))^(1/2)
```

3.393.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.57

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \frac{1}{6}\text{RootSum}\left[b+3b\#1^2+3b\#1^4+a\#1^6 +b\#1^6\&, \frac{\log(x-\#1)+2\log(x-\#1)\#1^2+\log(x-\#1)\#1^4}{b\#1+2b\#1^3+a\#1^5+b\#1^5}\&\right]$$

input
```
Integrate[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]
```

output
```
RootSum[b + 3*b*#1^2 + 3*b*#1^4 + a*#1^6 + b*#1^6 & , (Log[x - #1] + 2*Log
[x - #1]*#1^2 + Log[x - #1]*#1^4)/(b*#1 + 2*b*#1^3 + a*#1^5 + b*#1^5) & ]/
6
```

3.393.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)^2}{ax^6+b(x^2+1)^3}\,dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(\frac{x^4}{ax^6\left(\frac{b}{a}+1\right)+3bx^4+3bx^2+b}+\frac{2x^2}{ax^6\left(\frac{b}{a}+1\right)+3bx^4+3bx^2+b}+\frac{1}{ax^6\left(\frac{b}{a}+1\right)+3bx^4+3bx^2+b}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\int \frac{1}{a\left(\frac{b}{a}+1\right)x^6+3bx^4+3bx^2+b}dx+2\int \frac{x^2}{a\left(\frac{b}{a}+1\right)x^6+3bx^4+3bx^2+b}dx+\int \frac{x^4}{a\left(\frac{b}{a}+1\right)x^6+3bx^4+3bx^2+b}dx$$

input
```
Int[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]
```

output
```
$Aborted
```

3.393.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.393.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.82 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^6+3\_Z^4b+3\_Z^2b+b\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^5a+\_R^5b+2\_R^3b+b\_R}\right)}{6}$	67
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^6+3\_Z^4b+3\_Z^2b+b\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^5a+\_R^5b+2\_R^3b+b\_R}\right)}{6}$	67

input
```
int((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum((_R^4+2*_R^2+1)/(_R^5*a+_R^5*b+2*_R^3*b+_R*b)*ln(x-_R),_R=RootOf((
a+b)*_Z^6+3*_Z^4*b+3*_Z^2*b+b))
```

3.393.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 5653, normalized size of antiderivative = 33.65

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.393.6 Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.25

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx$$
$$= \text{RootSum}\left(t^6\cdot\left(46656ab^5+46656b^6\right)+3888t^4b^4+108t^2b^2+1,(t\mapsto t\log(6tb+x))\right)$$

input
```
integrate((x**2+1)**2/(a*x**6+b*(x**2+1)**3),x)
```

output
```
RootSum(_t**6*(46656*a*b**5 + 46656*b**6) + 3888*_t**4*b**4 + 108*_t**2*b*
*2 + 1, Lambda(_t, _t*log(6*_t*b + x)))
```

3.393.7 Maxima [F]

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \int \frac{(x^2+1)^2}{ax^6+(x^2+1)^3b}\,dx$$

input
```
integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b), x)
```

3.393.8 Giac [F]

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \int \frac{(x^2+1)^2}{ax^6+(x^2+1)^3b}\,dx$$

input
```
integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.393.9 Mupad [B] (verification not implemented)

Time = 10.48 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.00

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3} dx$$
$$= \sum_{k=1}^{6} \ln\Big(-a^3(a+b)\Big(-\mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^2\,b^2\,60 - \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^4\,b^4\,864 - \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^4\,a\,b^3\,864 + \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^3\,b^3\,x\,504 + \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^5\,b^5\,x\,7776 + \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)\,a\,x\,2 + \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)\,b\,x\,8 + \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^2\,a\,b\,12 - \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^3\,a\,b^2\,x\,144 + \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^5\,a\,b^4\,x\,7776 - 1\Big)\,3\Big)\,\mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)$$

input
```
int((x^2 + 1)^2/(b*(x^2 + 1)^3 + a*x^6),x)
```

output
```
symsum(log(-3*a^3*(a + b)*(504*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888
*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^3*b^3*x - 864*root(46656*a*b^5*z^6 + 466
56*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^4*b^4 - 864*root(46656*
a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^4*a*b^3
- 60*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1
, z, k)^2*b^2 + 7776*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 +
 108*b^2*z^2 + 1, z, k)^5*b^5*x + 2*root(46656*a*b^5*z^6 + 46656*b^6*z^6 +
 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)*a*x + 8*root(46656*a*b^5*z^6 + 4665
6*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)*b*x + 12*root(46656*a*b^
5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^2*a*b - 144*
root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z,
k)^3*a*b^2*x + 7776*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 +
108*b^2*z^2 + 1, z, k)^5*a*b^4*x - 1))*root(46656*a*b^5*z^6 + 46656*b^6*z^
6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k), k, 1, 6)
```

3.394 $\int \frac{(d+ex)^3}{a+cx^4}\,dx$

3.394.1 Optimal result

Integrand size = 17, antiderivative size = 320

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx = \frac{3d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} - \frac{d\left(\sqrt{c}d^2+3\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{d\left(\sqrt{c}d^2+3\sqrt{a}e^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} - \frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{e^3\log\left(a+cx^4\right)}{4c}$$

output
```
1/4*e^3*ln(c*x^4+a)/c+3/2*d^2*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(1/2)/c^(1/2
)-1/8*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*e^2*a^(1/2)
+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/8*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*c^(1/2))*(-3*e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1
/4*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e^2*a^(1/2)+d^2*c^(1/2))/a^(3
/4)/c^(3/4)*2^(1/2)+1/4*d*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e^2*a^(1/
2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)
```

3.394.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^3}{a+cx^4} \, dx$$
$$= \frac{-2\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{2}\sqrt{c}d^2+6\sqrt[4]{a}\sqrt[4]{c}de+3\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{2}\sqrt{c}d^2-6\sqrt[4]{a}\sqrt[4]{c}de+\right.}{}$$

input
```
Integrate[(d + e*x)^3/(a + c*x^4),x]
```

output
```
(-2*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 6*a^(1/4)*c^(1/4)*d*e + 3*Sqr
t[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*c^(1
/4)*d*(Sqrt[2]*Sqrt[c]*d^2 - 6*a^(1/4)*c^(1/4)*d*e + 3*Sqrt[2]*Sqrt[a]*e^2
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[
c]*d^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c
]*x^2] + Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 2*a*e^3*Log[a + c*x^4])/(8
*a*c)
```

3.394.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{a+cx^4} \, dx$$
$$\Big\downarrow \text{2415}$$
$$\int \left(\frac{d^3+3de^2x^2}{a+cx^4}+\frac{x(3d^2e+e^3x^2)}{a+cx^4}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(3\sqrt{a}e^2+\sqrt{c}d^2)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(3\sqrt{a}e^2+\sqrt{c}d^2)}{2\sqrt{2}a^{3/4}c^{3/4}}-\frac{d(\sqrt{c}d^2-3\sqrt{a}e^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{d(\sqrt{c}d^2-3\sqrt{a}e^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{3d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}+\frac{e^3\log(a+cx^4)}{4c}$$

input
```
Int[(d + e*x)^3/(a + c*x^4),x]
```

output
```
(3*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*(Sqrt[c]*
d^2 + 3*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a
^(3/4)*c^(3/4)) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - (d*(Sqrt[c]*d^2 - 3*Sqrt[a
]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
a^(3/4)*c^(3/4)) + (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + (e^3*Log[a
 + c*x^4])/(4*c)
```

3.394.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.394.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.77 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.17

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(\_R^3 e^3+3\_R^2 d e^2+3\_R d^2 e+d^3\right)\ln\left(x-\_R\right)}{\_R^3}}{4c}$
default	$\frac{d^3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{3d^2 e\arctan\left(x^2\sqrt{\frac{c}{a}}\right)}{2\sqrt{ac}}+\frac{3d e^2\sqrt{2}\left(\ln\right.}{}$

input
```
int((e*x+d)^3/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/_R^3*ln(x-_R),_R=RootOf(_
Z^4*c+a))
```

3.394.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.63 (sec) , antiderivative size = 141845, normalized size of antiderivative = 443.27

$$\int\frac{(d+ex)^3}{a+cx^4}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^3/(c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.394.6 Sympy [A] (verification not implemented)

Time = 28.21 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.20

$$\int\frac{(d+ex)^3}{a+cx^4}\,dx$$
$$=\mathrm{RootSum}\left(256t^4a^3c^4-256t^3a^3c^3e^3+t^2\cdot\left(96a^3c^2e^6+480a^2c^3d^4e^2\right)+t\left(-16a^3ce^9+192a^2c^2d^4e^5-48ac^3\right.\right.$$

input
```
integrate((e*x+d)**3/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*c**4 - 256*_t**3*a**3*c**3*e**3 + _t**2*(96*a**3*c*
*2*e**6 + 480*a**2*c**3*d**4*e**2) + _t*(-16*a**3*c*e**9 + 192*a**2*c**2*d
**4*e**5 - 48*a*c**3*d**8*e) + a**3*e**12 + 3*a**2*c*d**4*e**8 + 3*a*c**2*
d**8*e**4 + c**3*d**12, Lambda(_t, _t*log(x + (1728*_t**3*a**4*c**3*e**6 +
 960*_t**3*a**3*c**4*d**4*e**2 - 1296*_t**2*a**4*c**2*e**9 - 2016*_t**2*a*
*3*c**3*d**4*e**5 + 48*_t**2*a**2*c**4*d**8*e + 324*_t*a**4*c*e**12 + 4716
*_t*a**3*c**2*d**4*e**8 + 1452*_t*a**2*c**3*d**8*e**4 + 4*_t*a*c**4*d**12
- 27*a**4*e**15 + 1119*a**3*c*d**4*e**11 - 609*a**2*c**2*d**8*e**7 - 91*a*
c**3*d**12*e**3)/(729*a**3*c*d**3*e**12 - 1053*a**2*c**2*d**7*e**8 - 117*a
*c**3*d**11*e**4 + c**4*d**15))))
```

3.394.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx = \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3+cd^3-3\sqrt{a}\sqrt{c}de^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3-cd^3+3\sqrt{a}\sqrt{c}de^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{5}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{5}{4}}d^3+3\sqrt{2}a^{\frac{3}{4}}c^{\frac{3}{4}}de^2-6\sqrt{a}cd^2e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{5}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{5}{4}}d^3+3\sqrt{2}a^{\frac{3}{4}}c^{\frac{3}{4}}de^2+6\sqrt{a}cd^2e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{5}{4}}}$$

input
```
integrate((e*x+d)^3/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*e^3 + c*d^3 - 3*sqrt(a)*sqrt(c)*d*e^2
)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4))
 + 1/8*sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*e^3 - c*d^3 + 3*sqrt(a)*sqrt(c)*d*
e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/
4)) + 1/4*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + 3*sqrt(2)*a^(3/4)*c^(3/4)*d*e^2 -
 6*sqrt(a)*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1
/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) + 1/4*
(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + 3*sqrt(2)*a^(3/4)*c^(3/4)*d*e^2 + 6*sqrt(a)
*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(
sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4))
```

3.394.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx$$
$$= \frac{e^3\log\left(|cx^4+a|\right)}{4\,c} + \frac{\sqrt{2}\left(3\sqrt{2}\sqrt{ac}c^2d^2e+(ac^3)^{\frac{1}{4}}c^2d^3+3\,(ac^3)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}\left(3\sqrt{2}\sqrt{ac}c^2d^2e+(ac^3)^{\frac{1}{4}}c^2d^3+3\,(ac^3)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d^3-3\,(ac^3)^{\frac{3}{4}}de^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d^3-3\,(ac^3)^{\frac{3}{4}}de^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$

input
```
integrate((e*x+d)^3/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*e^3*log(abs(c*x^4 + a))/c + 1/4*sqrt(2)*(3*sqrt(2)*sqrt(a*c)*c^2*d^2*e
 + (a*c^3)^(1/4)*c^2*d^3 + 3*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x
+ sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*(3*sqrt(2)*sqrt(
a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + 3*(a*c^3)^(3/4)*d*e^2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((
a*c^3)^(1/4)*c^2*d^3 - 3*(a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1
/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - 3*(a*c^3)^
(3/4)*d*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

3.394.9 Mupad [B] (verification not implemented)

Time = 9.67 (sec) , antiderivative size = 894, normalized size of antiderivative = 2.79

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx = \sum_{k=1}^{4} \ln\Big(-c\,d^2\,\Big(-3\,c\,d^5\,e^2 + 5\,a\,d\,e^6 + 3\,a\,e^7\,x$$
$$+\mathrm{root}\big(256\,a^3\,c^4\,z^4 - 256\,a^3\,c^3\,e^3\,z^3 + 480\,a^2\,c^3\,d^4\,e^2\,z^2 + 96\,a^3\,c^2\,e^6\,z^2 + 192\,a^2\,c^2\,d^4\,e^5\,z - 48\,a\,c^3\,d^8\,e\,z -$$
$$+\mathrm{root}\big(256\,a^3\,c^4\,z^4 - 256\,a^3\,c^3\,e^3\,z^3 + 480\,a^2\,c^3\,d^4\,e^2\,z^2 + 96\,a^3\,c^2\,e^6\,z^2 + 192\,a^2\,c^2\,d^4\,e^5\,z - 48\,a\,c^3\,d^8\,e\,z - 16\,a^3$$
$$- 5\,c\,d^4\,e^3\,x$$
$$-\mathrm{root}\big(256\,a^3\,c^4\,z^4 - 256\,a^3\,c^3\,e^3\,z^3 + 480\,a^2\,c^3\,d^4\,e^2\,z^2 + 96\,a^3\,c^2\,e^6\,z^2 + 192\,a^2\,c^2\,d^4\,e^5\,z - 48\,a\,c^3\,d^8\,e\,z -$$
$$+\mathrm{root}\big(256\,a^3\,c^4\,z^4 - 256\,a^3\,c^3\,e^3\,z^3 + 480\,a^2\,c^3\,d^4\,e^2\,z^2 + 96\,a^3\,c^2\,e^6\,z^2 + 192\,a^2\,c^2\,d^4\,e^5\,z - 48\,a\,c^3\,d^8\,e\,z - 16\,a^3$$
$$-\mathrm{root}\big(256\,a^3\,c^4\,z^4 - 256\,a^3\,c^3\,e^3\,z^3 + 480\,a^2\,c^3\,d^4\,e^2\,z^2 + 96\,a^3\,c^2\,e^6\,z^2 + 192\,a^2\,c^2\,d^4\,e^5\,z - 48\,a\,c^3\,d^8\,e\,z - 16\,a^3$$
$$- 256\,a^3\,c^3\,e^3\,z^3 + 480\,a^2\,c^3\,d^4\,e^2\,z^2 + 96\,a^3\,c^2\,e^6\,z^2 + 192\,a^2\,c^2\,d^4\,e^5\,z - 48\,a\,c^3\,d^8\,e\,z$$
$$- 16\,a^3\,c\,e^9\,z + 3\,a^2\,c\,d^4\,e^8 + 3\,a\,c^2\,d^8\,e^4 + c^3\,d^{12} + a^3\,e^{12}, z, k\big)$$

input
```
int((d + e*x)^3/(a + c*x^4),x)
```

output
```
symsum(log(-2*c*d^2*(5*a*d*e^6 - 3*c*d^5*e^2 + 3*a*e^7*x + 8*root(256*a^3*
c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z
^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d
^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)^2*a*c^2*d + 2*root(2
56*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^
2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*
a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)*c^2*d^4*x - 5
*c*d^4*e^3*x - 24*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3
*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e
*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e
^12, z, k)^2*a*c^2*e*x + 32*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 4
80*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a
*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^
12 + a^3*e^12, z, k)*a*c*d*e^3 - 6*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*
z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z
 - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 +
 c^3*d^12 + a^3*e^12, z, k)*a*c*e^4*x))*root(256*a^3*c^4*z^4 - 256*a^3*c^3
*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*
e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*
e^4 + c^3*d^12 + a^3*e^12, z, k), k, 1, 4)
```

3.395 $\int \frac{(d+ex)^2}{a+cx^4}\,dx$

3.395.1 Optimal result

Integrand size = 17, antiderivative size = 291

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = \frac{de\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} - \frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}$$

output
```
d*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(1/2)/c^(1/2)-1/8*ln(-a^(1/4)*c^(1/4)*x*
2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^
(1/2)+1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+
d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/
4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*arctan(1+c^(1/4)
*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)
```

3.395.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = \frac{-2\left(\sqrt{2}\sqrt{c}d^2+4\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt{c}d^2-4\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2\right)\text{arctan}}{8a^{3/}}$$

input
```
Integrate[(d + e*x)^2/(a + c*x^4),x]
```

output
```
(-2*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*Sqrt[c]*d^2 - 4*a^(1/4)
*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
] - Sqrt[2]*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2]))/(8*a^(3/4)*c^(3/4))
```

3.395.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx$$
$$\Big\downarrow \text{2415}$$
$$\int \left(\frac{d^2+e^2x^2}{a+cx^4}+\frac{2dex}{a+cx^4}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}-\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{de\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

input
```
Int[(d + e*x)^2/(a + c*x^4),x]
```

output
```
(d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - ((Sqrt[c]*d^2 + Sq
rt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(
3/4)) + ((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4
)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) +
 ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

3.395.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.395.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.81 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.15

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)} \frac{\left(\_R^2 e^2+2\_R d e+d^2\right)\ln\left(x-\_R\right)}{\_R^3}}{4c}$
default	$\frac{d^2\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{ed\arctan\left(x^2\sqrt{\frac{c}{a}}\right)}{\sqrt{ac}}+\frac{e^2\sqrt{2}\left(\ln\left(\frac{x^2}{x^2}\right.\right.}{}$

input
```
int((e*x+d)^2/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*sum((_R^2*e^2+2*_R*d*e+d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

3.395.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 86139, normalized size of antiderivative = 296.01

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.395.6 Sympy [A] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx$$
$$=\mathrm{RootSum}\left(256t^4a^3c^3+192t^2a^2c^2d^2e^2+t\left(32a^2cde^5-32ac^2d^5e\right)+a^2e^8+2acd^4e^4+c^2d^8,\left(t\mapsto t\log\left(x\right.\right.\right.$$

input
```
integrate((e*x+d)**2/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*c**3 + 192*_t**2*a**2*c**2*d**2*e**2 + _t*(32*a**2*
c*d*e**5 - 32*a*c**2*d**5*e) + a**2*e**8 + 2*a*c*d**4*e**4 + c**2*d**8, La
mbda(_t, _t*log(x + (64*_t**3*a**4*c**2*e**6 + 448*_t**3*a**3*c**3*d**4*e*
*2 - 160*_t**2*a**3*c**2*d**3*e**5 + 32*_t**2*a**2*c**3*d**7*e + 60*_t*a**
3*c*d**2*e**8 + 256*_t*a**2*c**2*d**6*e**4 + 4*_t*a*c**3*d**10 + 6*a**3*d*
e**11 - 24*a**2*c*d**5*e**7 - 30*a*c**2*d**9*e**3)/(a**3*e**12 - 33*a**2*c
*d**4*e**8 - 33*a*c**2*d**8*e**4 + c**3*d**12))))
```

3.395.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = \frac{\sqrt{2}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2-4\,\sqrt{a}\sqrt{c}de\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{3}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2+4\,\sqrt{a}\sqrt{c}de\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{3}{4}}}$$

input
```
integrate((e*x+d)^2/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*
c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - 1/8*sqrt(2)*(sqrt(c)*d^2 - sqrt(a
)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(
3/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 4
*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^
(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 1/
4*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 4*sqrt(a)*s
qrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqr
t(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4))
```

3.395.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}c^2de+(ac^3)^{\frac{1}{4}}c^2d^2+(ac^3)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}c^2de+(ac^3)^{\frac{1}{4}}c^2d^2+(ac^3)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d^2-(ac^3)^{\frac{3}{4}}e^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d^2-(ac^3)^{\frac{3}{4}}e^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$

input
```
integrate((e*x+d)^2/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*e + (a*c^3)^(1/4)*c^2*d^2 + (a*c^3)
^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a
*c^3) + 1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*e + (a*c^3)^(1/4)*c^2*d^2 +
 (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(
1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*lo
g(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^
(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(
a/c))/(a*c^3)
```

3.395.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.91

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = \sum_{k=1}^{4} \ln\Big(3\,c^2\,d^4\,e^2 - a\,c\,e^6 + 4\,c^2\,d^3\,e^3\,x - \mathrm{root}\big(256\,a^3\,c^3\,z^4 + 192\,a^2\,c^2\,d^2\,e^2\,z^2$$
$$+ 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8 + a^2\,e^8, z, k\big)\ c^3\,d^4\,x\,4$$
$$-\mathrm{root}\big(256\,a^3\,c^3\,z^4 + 192\,a^2\,c^2\,d^2\,e^2\,z^2 + 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8 + a^2\,e^8, z, k\big)^2\,a\,c$$
$$+\mathrm{root}\big(256\,a^3\,c^3\,z^4 + 192\,a^2\,c^2\,d^2\,e^2\,z^2 + 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8$$
$$+ a^2\,e^8, z, k\big)\ a\,c^2\,e^4\,x\,4 - \mathrm{root}\big(256\,a^3\,c^3\,z^4$$
$$+192\,a^2\,c^2\,d^2\,e^2\,z^2 + 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8 + a^2\,e^8, z, k\big)\ a\,c^2\,d\,e^3\,16$$
$$+\mathrm{root}\big(256\,a^3\,c^3\,z^4 + 192\,a^2\,c^2\,d^2\,e^2\,z^2 + 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8 + a^2\,e^8, z, k\big)^2\,a\,c$$
$$+ 192\,a^2\,c^2\,d^2\,e^2\,z^2 + 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8 + a^2\,e^8, z, k\big)$$

input
```
int((d + e*x)^2/(a + c*x^4),x)
```

output
```
symsum(log(3*c^2*d^4*e^2 - a*c*e^6 + 4*c^2*d^3*e^3*x - 4*root(256*a^3*c^3*
z^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*
c*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*c^3*d^4*x - 16*root(256*a^3*c^3*z^4 +
 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4
*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*a*c^3*d^2 + 4*root(256*a^3*c^3*z^4 + 192
*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4
 + c^2*d^8 + a^2*e^8, z, k)*a*c^2*e^4*x - 16*root(256*a^3*c^3*z^4 + 192*a^
2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 +
c^2*d^8 + a^2*e^8, z, k)*a*c^2*d*e^3 + 32*root(256*a^3*c^3*z^4 + 192*a^2*c
^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2
*d^8 + a^2*e^8, z, k)^2*a*c^3*d*e*x)*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^
2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8
+ a^2*e^8, z, k), k, 1, 4)
```

3.396 $\int \frac{d+ex}{a+cx^4} dx$

3.396.1 Optimal result

Integrand size = 15, antiderivative size = 219

$$\int \frac{d+ex}{a+cx^4} dx = \frac{e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} - \frac{d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

output
```
1/4*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)+1/4*d*a
rctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)-1/8*d*ln(-a^(1/
4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/8*d*ln
(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/
2*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(1/2)/c^(1/2)
```

3.396.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.84

$$\int \frac{d+ex}{a+cx^4} dx = \frac{-2\left(\sqrt{2}\sqrt[4]{c}d+2\sqrt[4]{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt[4]{c}d-2\sqrt[4]{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+\sqrt{2}\sqrt[4]{c}d(-\log}{8a^{3/4}\sqrt{c}}$$

input
```
Integrate[(d + e*x)/(a + c*x^4),x]
```

output
```
(-2*(Sqrt[2]*c^(1/4)*d + 2*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)] + 2*(Sqrt[2]*c^(1/4)*d - 2*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)] + Sqrt[2]*c^(1/4)*d*(-Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(8*a
^(3/4)*Sqrt[c])
```

3.396.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{a+cx^4} \, dx$$

$$\Big\downarrow \; 2415$$

$$\int \left(\frac{d}{a+cx^4}+\frac{ex}{a+cx^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{d \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d \arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{d \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{e \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}$$

input
```
Int[(d + e*x)/(a + c*x^4),x]
```

output
```
(e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*ArcTan[1 - (Sqr
t[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*ArcTan[1 + (Sqr
t[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - (d*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)
*c^(1/4))
```

3.396.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.396.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.15

method	result	size
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+a\right)}\frac{\left(\_Re+d\right)\ln\left(x-\_R\right)}{\_R^3}}{4c}$	32
default	$\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{e\arctan\left(x^2\sqrt{\frac{c}{a}}\right)}{2\sqrt{ac}}$	124

input
```
int((e*x+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*sum((_R*e+d)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

3.396.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 41851, normalized size of antiderivative = 191.10

$$\int \frac{d+ex}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.396.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.57

$$\int \frac{d+ex}{a+cx^4}\,dx$$
$$=\text{RootSum}\left(256t^4a^3c^2+32t^2a^2ce^2-16tacd^2e+ae^4+cd^4,\left(t\mapsto t\log\left(x+\frac{-128t^3a^3ce^2-16t^2a^2cd^2e-}{4ade^4-}\right.\right.\right.$$

input
```
integrate((e*x+d)/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*c**2 + 32*_t**2*a**2*c*e**2 - 16*_t*a*c*d**2*e + a*
e**4 + c*d**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*c*e**2 - 16*_t**2*a*
*2*c*d**2*e - 8*_t*a**2*e**4 - 4*_t*a*c*d**4 + 5*a*d**2*e**3)/(4*a*d*e**4
- c*d**5))))
```

3.396.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.95

$$\int \frac{d+ex}{a+cx^4}\,dx = \frac{\sqrt{2}d\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}d\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}}$$
$$+\frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d-2\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{1}{4}}}$$
$$+\frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d+2\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{1}{4}}}$$

input
```
integrate((e*x+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*d*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3
/4)*c^(1/4)) - 1/8*sqrt(2)*d*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x +
 sqrt(a))/(a^(3/4)*c^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(1/4)*d - 2*sqrt(a)*e
)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*
sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*c
^(1/4)*d + 2*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*
c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4))
```

3.396.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.97

$$\int \frac{d+ex}{a+cx^4}\,dx = \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac} - \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ac}ce-(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^2} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ac}ce-(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^2}$$

input
```
integrate((e*x+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(
a/c))/(a*c) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*c)*c*e - (a*c^3)^(1/4)*c*d)*arct
an(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2) - 1/4*sqrt
(2)*(sqrt(2)*sqrt(a*c)*c*e - (a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2)
```

3.396.9 Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.73

$$\int \frac{d+ex}{a+cx^4}\,dx = \begin{cases} -\frac{2\,d+3\,e\,x}{6\,c\,x^3} & \text{if} \\ \frac{\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/4}\,x}{a^{1/4}}-1\right)\left(2\,a^{1/4}\,e+\sqrt{2}\,c^{1/4}\,d\right)}{4\,a^{3/4}\,\sqrt{c}} - \frac{\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/4}\,x}{a^{1/4}}+1\right)\left(4\,a^{1/4}\,e-2\,\sqrt{2}\,c^{1/4}\,d\right)}{8\,a^{3/4}\,\sqrt{c}} + \frac{\sqrt{2}\,d\,\ln\left(\frac{\sqrt{a}+\sqrt{c}\,x^2+\sqrt{2}\,a^{1/4}\,c^{1/4}\,x}{\sqrt{a}+\sqrt{c}\,x^2-\sqrt{2}\,a^{1/4}\,c^{1/4}\,x}\right)}{8\,a^{3/4}\,c^{1/4}} & \text{if} \end{cases}$$

input
```
int((d + e*x)/(a + c*x^4),x)
```

output
```
piecewise(a == 0, -(2*d + 3*e*x)/(6*c*x^3), a ~= 0, (atan((2^(1/2)*c^(1/4)
*x)/a^(1/4) - 1)*(2*a^(1/4)*e + 2^(1/2)*c^(1/4)*d))/(4*a^(3/4)*c^(1/2)) -
(atan((2^(1/2)*c^(1/4)*x)/a^(1/4) + 1)*(4*a^(1/4)*e - 2*2^(1/2)*c^(1/4)*d)
)/(8*a^(3/4)*c^(1/2)) + (2^(1/2)*d*log((a^(1/2) + c^(1/2)*x^2 + 2^(1/2)*a^
(1/4)*c^(1/4)*x)/(a^(1/2) + c^(1/2)*x^2 - 2^(1/2)*a^(1/4)*c^(1/4)*x)))/(8*
a^(3/4)*c^(1/4)))
```

3.396. $\int \frac{d+ex}{a+cx^4}\,dx$

3.397 $\int \frac{1}{a+cx^4} dx$

3.397.1 Optimal result

Integrand size = 9, antiderivative size = 185

$$\int \frac{1}{a+cx^4} dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

output
```
1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)+1/4*arcta
n(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)-1/8*ln(-a^(1/4)*c^(
1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/8*ln(a^(1/4)
*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)
```

3.397.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.72

$$\int \frac{1}{a+cx^4} dx = \frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

input
```
Integrate[(a + c*x^4)^(-1),x]
```

output
```
(-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
c^(1/4))
```

3.397.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+cx^4}dx$$

$$\Big\downarrow \text{755}$$

$$\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

input
```
Int[(a + c*x^4)^(-1),x]
```

output
```
(-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + Ar
cTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a
]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]
*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a])
```

3.397. $\int \frac{1}{a+cx^4}\,dx$

3.397.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.397.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.15

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}}{4c}$	27
default	$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}$	102

input
```
int(1/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

3.397.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.61

$$\int \frac{1}{a+cx^4}\,dx = \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)+\frac{1}{4}i\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(i\,a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right) - \frac{1}{4}i\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-i\,a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right) - \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)$$

input
```
integrate(1/(c*x^4+a),x, algorithm="fricas")
```

output
```
1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/(a^3*c))^(1/4) + x) + 1/4*I*(-1/(a^3*c))^
(1/4)*log(I*a*(-1/(a^3*c))^(1/4) + x) - 1/4*I*(-1/(a^3*c))^(1/4)*log(-I*a*
(-1/(a^3*c))^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4)
 + x)
```

3.397.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.11

$$\int \frac{1}{a+cx^4}\,dx = \text{RootSum}\left(256t^4a^3c+1,\left(t\mapsto t\log\left(4ta+x\right)\right)\right)$$

input
```
integrate(1/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

3.397.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.91

$$\int \frac{1}{a+cx^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}}$$

input
```
integrate(1/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqr
t(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/4*sqrt(2)*arctan(1
/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/8*sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^
(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 1/8*sqrt(2)*log(sqrt(c)*x^2
 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4))
```

3.397.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+cx^4}\,dx = \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac} - \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac}$$

input
```
integrate(1/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2
+ sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*log
(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c)
```

3.397.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.18

$$\int \frac{1}{a+cx^4}\,dx = -\frac{\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)+\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{2\,(-a)^{3/4}\,c^{1/4}}$$

input
```
int(1/(a + c*x^4),x)
```

output
```
-(atan((c^(1/4)*x)/(-a)^(1/4)) + atanh((c^(1/4)*x)/(-a)^(1/4)))/(2*(-a)^(3
/4)*c^(1/4))
```

3.398 $\int \frac{1}{(d+ex)(a+cx^4)}\,dx$

3.398.1 Optimal result

Integrand size = 17, antiderivative size = 416

$$\begin{aligned}\int \frac{1}{(d+ex)(a+cx^4)}\,dx = &-\frac{\sqrt{c}d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(cd^4+ae^4)}\\ &-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)}\\ &+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)}+\frac{e^3\log(d+ex)}{cd^4+ae^4}\\ &-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)}\\ &+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)}\\ &-\frac{e^3\log\left(a+cx^4\right)}{4\,(cd^4+ae^4)}\end{aligned}$$

output
```
e^3*ln(e*x+d)/(a*e^4+c*d^4)-1/4*e^3*ln(c*x^4+a)/(a*e^4+c*d^4)-1/2*d^2*e*ar
ctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)/a^(1/2)-1/8*c^(1/4)*d*ln(-
a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/
a^(3/4)/(a*e^4+c*d^4)*2^(1/2)+1/8*c^(1/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)*2^(1/
2)+1/4*c^(1/4)*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(
1/2))/a^(3/4)/(a*e^4+c*d^4)*2^(1/2)+1/4*c^(1/4)*d*arctan(1+c^(1/4)*x*2^(1/
2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)*2^(1/2)
```

3.398.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)(a+cx^4)}\,dx$$

$$= \frac{-2\sqrt[4]{c}d\left(\sqrt{2}\sqrt{c}d^2-2\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{c}d\left(\sqrt{2}\sqrt{c}d^2+2\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e\right.}{}$$

input
```
Integrate[1/((d + e*x)*(a + c*x^4)),x]
```

output
```
(-2*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 - 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[
a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^(1/4)*d*(Sqrt[2]*Sqr
t[c]*d^2 + 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)] + 8*a^(3/4)*e^3*Log[d + e*x] - Sqrt[2]*c^(3/4)*d^3*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*Sqrt[a]*c^
(1/4)*d*e^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[
2]*c^(3/4)*d^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sq
rt[2]*Sqrt[a]*c^(1/4)*d*e^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2] - 2*a^(3/4)*e^3*Log[a + c*x^4])/(8*a^(3/4)*(c*d^4 + a*e^4))
```

3.398.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)(d+ex)}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{e^4}{(d+ex)(ae^4+cd^4)}+\frac{c(d^3-d^2ex+de^2x^2-e^3x^3)}{(a+cx^4)(ae^4+cd^4)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt[4]{c}d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}(ae^4+cd^4)}+\frac{\sqrt[4]{c}d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}(ae^4+cd^4)}-\frac{\sqrt[4]{c}d(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^4+cd^4)}+\frac{\sqrt[4]{c}d(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^4+cd^4)}-\frac{\sqrt{c}d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(ae^4+cd^4)}-\frac{e^3\log(a+cx^4)}{4(ae^4+cd^4)}+\frac{e^3\log(d+ex)}{ae^4+cd^4}$$

input
```
Int[1/((d + e*x)*(a + c*x^4)),x]
```

output
```
-1/2*(Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4
)) - (c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 +
 Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(c*d^4 + a*e^4)) + (e^3*Log[d + e*x])/(c*d^4 + a*e^4) - (c^(1/4)*d*(Sqrt[c
]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 - Sqrt[a]
*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a
^(3/4)*(c*d^4 + a*e^4)) - (e^3*Log[a + c*x^4])/(4*(c*d^4 + a*e^4))
```

3.398.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.398.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.89 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.51

method	result
risch	$\frac{e^3 \ln(ex+d)}{e^4 a+d^4 c}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a^4 e^4+a^3 c\, d^4\right)\_Z^4+4 a^3 e^3 \_Z^3+6 \_Z^2 a^2 e^2+4 \_Z a e\right)} \_R \ln\left(\left(\left(5 e^6 a^3-3 a^2 d^4 e^2 c\right)\_R^3+\left(15 a^2 e^5-3\right.\right.\right.}{}$
default	$\frac{c\left(\frac{d^3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{d^2 e \arctan\left(x^2\sqrt{\frac{c}{a}}\right)}{2\sqrt{ac}}+\frac{d\,e^2\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{}}\right.\right.}{}\right.}{e^4 a+d^4 c}$

input
```
int(1/(e*x+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
e^3*ln(e*x+d)/(a*e^4+c*d^4)+1/4*sum(_R*ln(((5*a^3*e^6-3*a^2*c*d^4*e^2)*_R^
3+(15*a^2*e^5-3*a*c*d^4*e)*_R^2+(15*a*e^4-c*d^4)*_R+5*e^3)*x+(6*a^3*d*e^5-
2*a^2*c*d^5*e)*_R^3+(13*a^2*d*e^4-a*c*d^5)*_R^2+8*a*d*e^3*_R+d*e^2),_R=Roo
tOf(1+(a^4*e^4+a^3*c*d^4)*_Z^4+4*a^3*e^3*_Z^3+6*_Z^2*a^2*e^2+4*_Z*a*e))
```

3.398.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 97.11 (sec) , antiderivative size = 352864, normalized size of antiderivative = 848.23

$$\int \frac{1}{(d+ex)\,(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

3.398.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\,(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x**4+a),x)
```

output
```
Timed out
```

3.398.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d+ex)\,(a+cx^4)}\,dx = \frac{e^3\log(ex+d)}{cd^4+ae^4} - \frac{c\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3-cd^3+\sqrt{a}\sqrt{c}de^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3+cd^3-\sqrt{a}\sqrt{c}de^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}} - \cdots\right)}{8\,(\cdots}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
e^3*log(e*x + d)/(c*d^4 + a*e^4) - 1/8*c*(sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)
*e^3 - c*d^3 + sqrt(a)*sqrt(c)*d*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^
(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*e^
3 + c*d^3 - sqrt(a)*sqrt(c)*d*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + sqrt(
2)*a^(3/4)*c^(3/4)*d*e^2 + 2*sqrt(a)*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c
)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a
)*sqrt(c))*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + sqrt(2)*a^(3/4)*c^(
3/4)*d*e^2 - 2*sqrt(a)*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*
a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(
5/4)))/(c*d^4 + a*e^4)
```

3.398.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex)(a+cx^4)}\,dx = \frac{e^4\log(|ex+d|)}{cd^4e+ae^5} - \frac{e^3\log(|cx^4+a|)}{4(cd^4+ae^4)} + \frac{(ac^3)^{\frac{1}{4}}\,cd\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^2d^2+\sqrt{2}\sqrt{ac}ace^2-2\,(ac^3)^{\frac{1}{4}}\,acde\right)} + \frac{(ac^3)^{\frac{1}{4}}\,cd\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^2d^2+\sqrt{2}\sqrt{ac}ace^2+2\,(ac^3)^{\frac{1}{4}}\,acde\right)} + \frac{\left((ac^3)^{\frac{1}{4}}\,c^2d^3-(ac^3)^{\frac{3}{4}}\,de^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^4+\sqrt{2}a^2c^2e^4\right)} - \frac{\left((ac^3)^{\frac{1}{4}}\,c^2d^3-(ac^3)^{\frac{3}{4}}\,de^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^4+\sqrt{2}a^2c^2e^4\right)}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
e^4*log(abs(e*x + d))/(c*d^4*e + a*e^5) - 1/4*e^3*log(abs(c*x^4 + a))/(c*d
^4 + a*e^4) + 1/2*(a*c^3)^(1/4)*c*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^2*d^2 + sqrt(2)*sqrt(a*c)*a*c*e^2 - 2*(
a*c^3)^(1/4)*a*c*d*e) + 1/2*(a*c^3)^(1/4)*c*d*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^2*d^2 + sqrt(2)*sqrt(a*c)*a*c
*e^2 + 2*(a*c^3)^(1/4)*a*c*d*e) + 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/
4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4
+ sqrt(2)*a^2*c^2*e^4) - 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)
*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + sqrt(2)
*a^2*c^2*e^4)
```

3.398.9 Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 874, normalized size of antiderivative = 2.10

$$\int \frac{1}{(d+ex)(a+cx^4)} dx$$
$$= \left(\sum_{k=1}^{4} \ln \left(\mathrm{root}\left(256\,a^3\,c\,d^4\,z^4+256\,a^4\,e^4\,z^4+256\,a^3\,e^3\,z^3+96\,a^2\,e^2\,z^2+16\,a\,e\,z+1,z,k\right)\,c^4\,e\,\left(d\,e^2+5\,e^3\,x+\mathrm{ro}\right.\right.\right.$$
$$\left.\left. + 256\,a^4\,e^4\,z^4 + 256\,a^3\,e^3\,z^3 + 96\,a^2\,e^2\,z^2 + 16\,a\,e\,z + 1, z, k\right)\right) + \frac{e^3\,\ln\left(d+e\,x\right)}{c\,d^4+a\,e^4}$$

input
```
int(1/((a + c*x^4)*(d + e*x)),x)
```

output
```
symsum(log(root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96
*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)*c^4*e*(d*e^2 + 5*e^3*x + 240*root(256*a
^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e
*z + 1, z, k)^2*a^2*e^5*x + 320*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 +
 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^3*e^6*x + 32*r
oot(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2
 + 16*a*e*z + 1, z, k)*a*d*e^3 + 60*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z
^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)*a*e^4*x - 4*ro
ot(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2
+ 16*a*e*z + 1, z, k)*c*d^4*x - 16*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^
4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^2*a*c*d^5 + 208
*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z
^2 + 16*a*e*z + 1, z, k)^2*a^2*d*e^4 + 384*root(256*a^3*c*d^4*z^4 + 256*a^
4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^3*d
*e^5 - 128*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96
*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^2*c*d^5*e - 192*root(256*a^3*c*d^4*
z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z
, k)^3*a^2*c*d^4*e^2*x - 48*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256
*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^2*a*c*d^4*e*x))*root(2
56*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 +...
```

3.398. $\int \frac{1}{(d+ex)(a+cx^4)}\,dx$

3.399 $\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx$

3.399.1 Optimal result

Integrand size = 17, antiderivative size = 552

$$\begin{aligned}
\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx
&= -\frac{e^3}{(cd^4+ae^4)(d+ex)} - \frac{\sqrt{c}de(cd^4-ae^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\left(cd^4+ae^4\right)^2} \\
&\quad - \frac{\sqrt[4]{c}\left(\sqrt{c}d^2(cd^4-3ae^4)+\sqrt{a}e^2(3cd^4-ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2} \\
&\quad + \frac{\sqrt[4]{c}\left(\sqrt{c}d^2(cd^4-3ae^4)+\sqrt{a}e^2(3cd^4-ae^4)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2} \\
&\quad + \frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2} \\
&\quad - \frac{\sqrt[4]{c}\left(\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2} \\
&\quad + \frac{\sqrt[4]{c}\left(\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2} \\
&\quad - \frac{cd^3e^3\log\left(a+cx^4\right)}{\left(cd^4+ae^4\right)^2}
\end{aligned}$$

output
```
-e^3/(a*e^4+c*d^4)/(e*x+d)+4*c*d^3*e^3*ln(e*x+d)/(a*e^4+c*d^4)^2-c*d^3*e^3
*ln(c*x^4+a)/(a*e^4+c*d^4)^2-d*e*(-a*e^4+c*d^4)*arctan(x^2*c^(1/2)/a^(1/2)
)*c^(1/2)/(a*e^4+c*d^4)^2/a^(1/2)-1/8*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2
)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4)
*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/8*c^(1/4)*ln(a^(1/4)*c^(1/4)*x
*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4
+c*d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/4*c^(1/4)*arctan(-1+c^(
1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4)
*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/4*c^(1/4)*arctan(1+c^(1/4)*x*2
^(1/2)/a^(1/4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4)*c^(1/2)
)/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)
```

3.399.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 524, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx$$

$$= \frac{-\frac{8e^3(cd^4+ae^4)}{d+ex} + \frac{2\sqrt[4]{c}(-\sqrt{c}d^2+\sqrt{a}e^2)\left(\sqrt{2}cd^4-4\sqrt[4]{a}c^{3/4}d^3e+4\sqrt{2}\sqrt{a}\sqrt{c}d^2e^2-4a^{3/4}\sqrt[4]{c}de^3+\sqrt{2}ae^4\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{2\sqrt[4]{c}(\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[1/((d + e*x)^2*(a + c*x^4)),x]
```

output
```
((-8*e^3*(c*d^4 + a*e^4))/(d + e*x) + (2*c^(1/4)*(-(Sqrt[c]*d^2) + Sqrt[a]
*e^2)*(Sqrt[2]*c*d^4 - 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]
*d^2*e^2 - 4*a^(3/4)*c^(1/4)*d*e^3 + Sqrt[2]*a*e^4)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/a^(3/4) + (2*c^(1/4)*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[
2]*c*d^4 + 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*d^2*e^2 + 4
*a^(3/4)*c^(1/4)*d*e^3 + Sqrt[2]*a*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/a^(3/4) + 32*c*d^3*e^3*Log[d + e*x] - (Sqrt[2]*c^(1/4)*(c^(3/2)*d^6
 - 3*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4) + (Sqrt[2]*c^(1/4)*(c^(3
/2)*d^6 - 3*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4) - 8*c*d^3*e^3*Log
[a + c*x^4])/(8*(c*d^4 + a*e^4)^2)
```

3.399.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)(d+ex)^2}\,dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{e^4}{(d+ex)^2(ae^4+cd^4)}+\frac{4cd^3e^4}{(d+ex)(ae^4+cd^4)^2}+\frac{c\left(-2dex(cd^4-ae^4)+e^2x^2(3cd^4-ae^4)+d^2(cd^4-3ae^4)\right.}{(a+cx^4)(ae^4+cd^4)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e^2(3cd^4-ae^4)+\sqrt{c}d^2(cd^4-3ae^4)\right)}{2\sqrt{2}a^{3/4}(ae^4+cd^4)^2}+\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e^2(3cd^4-ae^4)+\sqrt{c}d^2(cd^4-3ae^4)\right)}{2\sqrt{2}a^{3/4}(ae^4+cd^4)^2}-\frac{\sqrt[4]{c}\left(\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^4+cd^4)^2}+\frac{\sqrt[4]{c}\left(\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^4+cd^4)^2}-\frac{\sqrt{c}de\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)(cd^4-ae^4)}{\sqrt{a}(ae^4+cd^4)^2}-\frac{e^3}{(d+ex)(ae^4+cd^4)}-\frac{cd^3e^3\log\left(a+cx^4\right)}{(ae^4+cd^4)^2}+\frac{4cd^3e^3\log(d+ex)}{(ae^4+cd^4)^2}$$

input
```
Int[1/((d + e*x)^2*(a + c*x^4)),x]
```

output
```
-(e^3/((c*d^4 + a*e^4)*(d + e*x))) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)*ArcTan[(
Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^2) - (c^(1/4)*(Sqrt[c]*d^2
*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[
c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (4*c*d^3*
e^3*Log[d + e*x])/(c*d^4 + a*e^4)^2 - (c^(1/4)*(Sqrt[c]*d^2*(c*d^4 - 3*a*e
^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[
c]*d^2*(c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)
^2) - (c*d^3*e^3*Log[a + c*x^4])/(c*d^4 + a*e^4)^2
```

3.399.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.399.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.64

method	result
default	$-\frac{c\left(\frac{\left(3a\,d^2e^4-c\,d^6\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{\left(-2ad\,e^5+2c\,d^5e\right)\arctan\left(x^2\sqrt{\frac{c}{a}}\right.}{2\sqrt{ac}}\right.}{\left(e^4a+d^4c\right)^2}$
risch	$-\frac{e^3}{\left(e^4a+d^4c\right)\left(ex+d\right)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^5e^8+2d^4c\,a^4e^4+c^2d^8a^3\right)\_Z^4+16a^3c\,d^3e^3\_Z^3+20a^2c\,d^2e^2\_Z^2+8acde\_Z+c\right)}\_R\ln\left(\left(\left((5a^5e^1\right.\right.\right.\right.}{}$

input
```
int(1/(e*x+d)^2/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-c/(a*e^4+c*d^4)^2*(1/8*(3*a*d^2*e^4-c*d^6)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2
+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)
))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/
2*(-2*a*d*e^5+2*c*d^5*e)/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*(a*e^6-3*
c*d^4*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2
))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x
+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+d^3*e^3*ln(c*x^4+a))-e^3/(a*e^4+c*d
^4)/(e*x+d)+4*c*d^3*e^3*ln(e*x+d)/(a*e^4+c*d^4)^2
```

3.399.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\,(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.399.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\,(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**2/(c*x**4+a),x)
```

output
```
Timed out
```

3.399.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.02

$$\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx = \frac{4\,cd^3e^3\log(ex+d)}{c^2d^8+2\,acd^4e^4+a^2e^8} - \frac{e^3}{cd^5+ade^4+(cd^4e+ae^5)x}$$

$$- \frac{c\left(\frac{\sqrt{2}\left(4\sqrt{2}a^{\frac{3}{4}}c^{\frac{5}{4}}d^3e^3-c^2d^6+3\sqrt{a}c^{\frac{3}{2}}d^4e^2+3\,acd^2e^4-a^{\frac{3}{2}}\sqrt{c}e^6\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}} + \frac{\sqrt{2}\left(4\sqrt{2}a^{\frac{3}{4}}c^{\frac{5}{4}}d^3e^3+c^2d^6-3\sqrt{a}c^{\frac{3}{2}}d^4e^2-\cdots\right.}{}\right.}{}$$

input

```
integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="maxima")
```

output

```
4*c*d^3*e^3*log(e*x + d)/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) - e^3/(c*d^5
+ a*d*e^4 + (c*d^4*e + a*e^5)*x) - 1/8*c*(sqrt(2)*(4*sqrt(2)*a^(3/4)*c^(5/
4)*d^3*e^3 - c^2*d^6 + 3*sqrt(a)*c^(3/2)*d^4*e^2 + 3*a*c*d^2*e^4 - a^(3/2)
*sqrt(c)*e^6)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3
/4)*c^(5/4)) + sqrt(2)*(4*sqrt(2)*a^(3/4)*c^(5/4)*d^3*e^3 + c^2*d^6 - 3*sq
rt(a)*c^(3/2)*d^4*e^2 - 3*a*c*d^2*e^4 + a^(3/2)*sqrt(c)*e^6)*log(sqrt(c)*x
^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a
^(1/4)*c^(9/4)*d^6 + 3*sqrt(2)*a^(3/4)*c^(7/4)*d^4*e^2 - 3*sqrt(2)*a^(5/4)
*c^(5/4)*d^2*e^4 - sqrt(2)*a^(7/4)*c^(3/4)*e^6 + 4*sqrt(a)*c^2*d^5*e - 4*a
^(3/2)*c*d*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))
/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(sqrt(
2)*a^(1/4)*c^(9/4)*d^6 + 3*sqrt(2)*a^(3/4)*c^(7/4)*d^4*e^2 - 3*sqrt(2)*a^(
5/4)*c^(5/4)*d^2*e^4 - sqrt(2)*a^(7/4)*c^(3/4)*e^6 - 4*sqrt(a)*c^2*d^5*e +
 4*a^(3/2)*c*d*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1
/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(c^2*
d^8 + 2*a*c*d^4*e^4 + a^2*e^8)
```

3.399.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.21

$$\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx = \frac{4\,cd^3e^4\log(|ex+d|)}{c^2d^8e+2\,acd^4e^5+a^2e^9} - \frac{cd^3e^3\log(|cx^4+a|)}{c^2d^8+2\,acd^4e^4+a^2e^8}$$
$$+ \frac{\left((ac^3)^{\frac{1}{4}}\,c^2d^2-(ac^3)^{\frac{3}{4}}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^4+\sqrt{2}a^2c^2e^4+4\sqrt{2}\sqrt{ac}ac^2d^2e^2-4\,(ac^3)^{\frac{1}{4}}\,ac^2d^3e-4\,(ac^3)^{\frac{3}{4}}\,ade^3\right)}$$
$$+ \frac{\left((ac^3)^{\frac{1}{4}}\,c^2d^2-(ac^3)^{\frac{3}{4}}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^4+\sqrt{2}a^2c^2e^4+4\sqrt{2}\sqrt{ac}ac^2d^2e^2+4\,(ac^3)^{\frac{1}{4}}\,ac^2d^3e+4\,(ac^3)^{\frac{3}{4}}\,ade^3\right)}$$
$$+ \frac{\left(\sqrt{2}(ac^3)^{\frac{1}{4}}\,c^3d^6-3\sqrt{2}(ac^3)^{\frac{1}{4}}\,ac^2d^2e^4-3\sqrt{2}(ac^3)^{\frac{3}{4}}\,cd^4e^2+\sqrt{2}(ac^3)^{\frac{3}{4}}\,ae^6\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\left(ac^4d^8+2\,a^2c^3d^4e^4+a^3c^2e^8\right)}$$
$$- \frac{\left(\sqrt{2}(ac^3)^{\frac{1}{4}}\,c^3d^6-3\sqrt{2}(ac^3)^{\frac{1}{4}}\,ac^2d^2e^4-3\sqrt{2}(ac^3)^{\frac{3}{4}}\,cd^4e^2+\sqrt{2}(ac^3)^{\frac{3}{4}}\,ae^6\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\left(ac^4d^8+2\,a^2c^3d^4e^4+a^3c^2e^8\right)}$$
$$- \frac{cd^4e^3+ae^7}{(cd^4+ae^4)^2(ex+d)}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="giac")
```

output
```
4*c*d^3*e^4*log(abs(e*x + d))/(c^2*d^8*e + 2*a*c*d^4*e^5 + a^2*e^9) - c*d^
3*e^3*log(abs(c*x^4 + a))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) + 1/2*((a*c^
3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + sqrt(2)*a^2*c^2*e^4 + 4*sqrt(
2)*sqrt(a*c)*a*c^2*d^2*e^2 - 4*(a*c^3)^(1/4)*a*c^2*d^3*e - 4*(a*c^3)^(3/4)
*a*d*e^3) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + sqrt(2)
*a^2*c^2*e^4 + 4*sqrt(2)*sqrt(a*c)*a*c^2*d^2*e^2 + 4*(a*c^3)^(1/4)*a*c^2*d
^3*e + 4*(a*c^3)^(3/4)*a*d*e^3) + 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^6 - 3*s
qrt(2)*(a*c^3)^(1/4)*a*c^2*d^2*e^4 - 3*sqrt(2)*(a*c^3)^(3/4)*c*d^4*e^2 + s
qrt(2)*(a*c^3)^(3/4)*a*e^6)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a*c^4*d^8 + 2*a^2*c^3*d^4*e^4 + a^3*c^2*e^8) - 1/8*(sqrt(2)*(a*c^3)^(1/4)*
c^3*d^6 - 3*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^2*e^4 - 3*sqrt(2)*(a*c^3)^(3/4)*
c*d^4*e^2 + sqrt(2)*(a*c^3)^(3/4)*a*e^6)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) +
 sqrt(a/c))/(a*c^4*d^8 + 2*a^2*c^3*d^4*e^4 + a^3*c^2*e^8) - (c*d^4*e^3 + a
*e^7)/((c*d^4 + a*e^4)^2*(e*x + d))
```

3.399.9 Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 2436, normalized size of antiderivative = 4.41

$$\int \frac{1}{(d+ex)^2\,(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)*(d + e*x)^2),x)
```

output
```
symsum(log((c^5*d*e^6 + c^5*e^7*x + 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^
3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e
^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^4*c^4*e^13 + 256*root(512*a^4*c*d^4*e
^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 +
320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a^2*c^5*d^3*e^8 + 496*ro
ot(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^
3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^2*c^
6*d^8*e^5 + 528*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5
*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z +
 c, z, k)^3*a^3*c^5*d^4*e^9 - 128*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2
*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^
2 + 32*a*c*d*e*z + c, z, k)^4*a^2*c^7*d^13*e^2 + 128*root(512*a^4*c*d^4*e^
4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 3
20*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^3*c^6*d^9*e^6 + 640*roo
t(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3
*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^4*c^5
*d^5*e^10 + 32*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*
e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z +
c, z, k)*a*c^5*d^2*e^7 - 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z
^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 +...
```

3.400 $\int \frac{1}{(d+ex)^3(a+cx^4)}\,dx$

3.400.1 Optimal result

Integrand size = 17, antiderivative size = 680

$$\begin{aligned}\int \frac{1}{(d+ex)^3(a+cx^4)}\,dx &= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2}\\
&-\frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)}-\frac{\sqrt{c}e(3c^2d^8-12acd^4e^4+a^2e^8)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(cd^4+ae^4)^3}\\
&-\frac{c^{3/4}d\left(c^2d^8-12acd^4e^4+3a^2e^8+2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^3}\\
&+\frac{c^{3/4}d\left(c^2d^8-12acd^4e^4+3a^2e^8+2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^3}\\
&+\frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^3}\\
&-\frac{c^{3/4}d\left(c^2d^8-12acd^4e^4+3a^2e^8-2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^3}\\
&+\frac{c^{3/4}d\left(c^2d^8-12acd^4e^4+3a^2e^8-2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^3}\\
&-\frac{cd^2e^3(5cd^4-3ae^4)\log(a+cx^4)}{2(cd^4+ae^4)^3}\end{aligned}$$

output
```
-1/2*e^3/(a*e^4+c*d^4)/(e*x+d)^2-4*c*d^3*e^3/(a*e^4+c*d^4)^2/(e*x+d)+2*c*d
^2*e^3*(-3*a*e^4+5*c*d^4)*ln(e*x+d)/(a*e^4+c*d^4)^3-1/2*c*d^2*e^3*(-3*a*e^
4+5*c*d^4)*ln(c*x^4+a)/(a*e^4+c*d^4)^3-1/2*e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2
*d^8)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^3/a^(1/2)-1/8*c^(3
/4)*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c^2*d^8-12*a*c*d
^4*e^4+3*a^2*e^8-2*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*
e^4+c*d^4)^3*2^(1/2)+1/8*c^(3/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^
2*c^(1/2))*(c^2*d^8-12*a*c*d^4*e^4+3*a^2*e^8-2*d^2*e^2*(-5*a*e^4+3*c*d^4)*
a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(3/4)*d*arctan(-1+c
^(1/4)*x*2^(1/2)/a^(1/4))*(c^2*d^8-12*a*c*d^4*e^4+3*a^2*e^8+2*d^2*e^2*(-5*
a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(3/4
)*d*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(c^2*d^8-12*a*c*d^4*e^4+3*a^2*e^8+
2*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1
/2)
```

3.400.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 738, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex)^3(a+cx^4)}\,dx$$
$$= \frac{-4a^{3/4}e^3(cd^4+ae^4)^2 - 32a^{3/4}cd^3e^3(cd^4+ae^4)(d+ex) - 2\sqrt{c}\left(\sqrt{2}c^{9/4}d^9 - 6\sqrt[4]{a}c^2d^8e + 6\sqrt{2}\sqrt{a}c^{7/4}d^7e^2 \cdots\right.}{}$$

input
```
Integrate[1/((d + e*x)^3*(a + c*x^4)),x]
```

output
```
(-4*a^(3/4)*e^3*(c*d^4 + a*e^4)^2 - 32*a^(3/4)*c*d^3*e^3*(c*d^4 + a*e^4)*(
d + e*x) - 2*Sqrt[c]*(Sqrt[2]*c^(9/4)*d^9 - 6*a^(1/4)*c^2*d^8*e + 6*Sqrt[2
]*Sqrt[a]*c^(7/4)*d^7*e^2 - 12*Sqrt[2]*a*c^(5/4)*d^5*e^4 + 24*a^(5/4)*c*d^
4*e^5 - 10*Sqrt[2]*a^(3/2)*c^(3/4)*d^3*e^6 + 3*Sqrt[2]*a^2*c^(1/4)*d*e^8 -
 2*a^(9/4)*e^9)*(d + e*x)^2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sq
rt[c]*(Sqrt[2]*c^(9/4)*d^9 + 6*a^(1/4)*c^2*d^8*e + 6*Sqrt[2]*Sqrt[a]*c^(7/
4)*d^7*e^2 - 12*Sqrt[2]*a*c^(5/4)*d^5*e^4 - 24*a^(5/4)*c*d^4*e^5 - 10*Sqrt
[2]*a^(3/2)*c^(3/4)*d^3*e^6 + 3*Sqrt[2]*a^2*c^(1/4)*d*e^8 + 2*a^(9/4)*e^9)
*(d + e*x)^2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 16*a^(3/4)*c*d^2*e^
3*(5*c*d^4 - 3*a*e^4)*(d + e*x)^2*Log[d + e*x] - Sqrt[2]*c^(3/4)*d*(c^2*d^
8 - 6*Sqrt[a]*c^(3/2)*d^6*e^2 - 12*a*c*d^4*e^4 + 10*a^(3/2)*Sqrt[c]*d^2*e^
6 + 3*a^2*e^8)*(d + e*x)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2] + Sqrt[2]*c^(3/4)*d*(c^2*d^8 - 6*Sqrt[a]*c^(3/2)*d^6*e^2 - 12*a*c*
d^4*e^4 + 10*a^(3/2)*Sqrt[c]*d^2*e^6 + 3*a^2*e^8)*(d + e*x)^2*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 4*a^(3/4)*c*d^2*e^3*(-5*c*d^4
 + 3*a*e^4)*(d + e*x)^2*Log[a + c*x^4])/(8*a^(3/4)*(c*d^4 + a*e^4)^3*(d +
e*x)^2)
```

3.400.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)(d+ex)^3}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{c\left(-ex\left(a^2e^8-12acd^4e^4+3c^2d^8\right)+d\left(3a^2e^8-12acd^4e^4+c^2d^8\right)+2cd^3e^2x^2\left(3cd^4-5ae^4\right)-2cd^2e^3x^3\left(5cd^4-\right.\right.}{(a+cx^4)\left(ae^4+cd^4\right)^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c}e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)\left(a^2e^8-12acd^4e^4+3c^2d^8\right)}{2\sqrt{a}\left(ae^4+cd^4\right)^3}-\frac{c^{3/4}d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(3a^2e^8-12acd^4e^4+2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}+\frac{c^{3/4}d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(3a^2e^8-12acd^4e^4+2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}-\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4-2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}+\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4-2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}-\frac{e^3}{2(d+ex)^2\left(ae^4+cd^4\right)}-\frac{4cd^3e^3}{(d+ex)\left(ae^4+cd^4\right)^2}-\frac{cd^2e^3\left(5cd^4-3ae^4\right)\log\left(a+cx^4\right)}{2\left(ae^4+cd^4\right)^3}+\frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(ae^4+cd^4\right)^3}$$

input
```
Int[1/((d + e*x)^3*(a + c*x^4)),x]
```

output
```
-1/2*e^3/((c*d^4 + a*e^4)*(d + e*x)^2) - (4*c*d^3*e^3)/((c*d^4 + a*e^4)^2*
(d + e*x)) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqr
t[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*(c^2*d^8 -
12*a*c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4)
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e
^4)^3) + (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*Sqrt
[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (2*c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*L
og[d + e*x])/(c*d^4 + a*e^4)^3 - (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*
a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^
3) + (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*
d^2*e^2*(3*c*d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c*d^2*e^3*(5*c*d^4 - 3
*a*e^4)*Log[a + c*x^4])/(2*(c*d^4 + a*e^4)^3)
```

3.400. $\int \frac{1}{(d+ex)^3(a+cx^4)}\,dx$

3.400.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.400.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.66

method	result
default	$\frac{c\left(\frac{\left(3a^2 d e^8-12ac d^5 e^4+c^2 d^9\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{\left(-a^2 e^9+12ac d^4 e^5-3c^2 \cdots\right.}{2\sqrt{a \cdots}} \cdots\right.}{\cdots}$
risch	$\frac{-\frac{4d^3 c e^4 x}{a^2 e^8+2ac d^4 e^4+c^2 d^8}-\frac{\left(e^4 a+9d^4 c\right)e^3}{2\left(a^2 e^8+2ac d^4 e^4+c^2 d^8\right)}}{(ex+d)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^6 e^{12}+3a^5 c d^4 e^8+3a^4 c^2 d^8 e^4+a^3 c^3 d^{12}\right)\_Z^4+\left(-24a^4 c d^2 e^7+40a^3 c^2 \cdots\right.\right.}\cdots\right.}{\cdots}$

input
```
int(1/(e*x+d)^3/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
c/(a*e^4+c*d^4)^3*(1/8*(3*a^2*d*e^8-12*a*c*d^5*e^4+c^2*d^9)*(a/c)^(1/4)/a*
2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(
1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c
)^(1/4)*x-1))+1/2*(-a^2*e^9+12*a*c*d^4*e^5-3*c^2*d^8*e)/(a*c)^(1/2)*arctan
(x^2*(c/a)^(1/2))+1/8*(-10*a*c*d^3*e^6+6*c^2*d^7*e^2)/c/(a/c)^(1/4)*2^(1/2
)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(
a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4
)*x-1))+1/4*(6*a*c*d^2*e^7-10*c^2*d^6*e^3)/c*ln(c*x^4+a))-1/2*e^3/(a*e^4+c
*d^4)/(e*x+d)^2-4*c*d^3*e^3/(a*e^4+c*d^4)^2/(e*x+d)-2*c*d^2*e^3*(3*a*e^4-5
*c*d^4)/(a*e^4+c*d^4)^3*ln(e*x+d)
```

3.400.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.400.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**3/(c*x**4+a),x)
```

output
```
Timed out
```

3.400.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 817, normalized size of antiderivative = 1.20

$$\int \frac{1}{(d+ex)^3(a+cx^4)}\,dx = -\frac{c\left(\frac{\sqrt{2}\left(10\sqrt{2}a^{\frac{3}{4}}c^{\frac{9}{4}}d^6e^3-6\sqrt{2}a^{\frac{7}{4}}c^{\frac{5}{4}}d^2e^7-c^3d^9+6\sqrt{a}c^{\frac{5}{2}}d^7e^2+12\,ac^2d^5e^4-10\,a^{\frac{3}{2}}c^{\frac{3}{2}}d^3e^6-3\,a^2cde^8\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}+\frac{\sqrt{2}(1\cdots}{\cdots}\right.}{\cdots} + \frac{2\left(5\,c^2d^6e^3-3\,acd^2e^7\right)\log\left(ex+d\right)}{c^3d^{12}+3\,ac^2d^8e^4+3\,a^2cd^4e^8+a^3e^{12}} - \frac{8\,cd^3e^4x+9\,cd^4e^3+ae^7}{2\left(c^2d^{10}+2\,acd^6e^4+a^2d^2e^8+\left(c^2d^8e^2+2\,acd^4e^6+a^2e^{10}\right)x^2+2\left(c^2d^9e+2\,acd^5e^5+a^2de^9\right)x\right)}$$

input
```
integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="maxima")
```

output
```
-1/8*c*(sqrt(2)*(10*sqrt(2)*a^(3/4)*c^(9/4)*d^6*e^3 - 6*sqrt(2)*a^(7/4)*c^
(5/4)*d^2*e^7 - c^3*d^9 + 6*sqrt(a)*c^(5/2)*d^7*e^2 + 12*a*c^2*d^5*e^4 - 1
0*a^(3/2)*c^(3/2)*d^3*e^6 - 3*a^2*c*d*e^8)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/
4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(10*sqrt(2)*a^(3/4)*c^
(9/4)*d^6*e^3 - 6*sqrt(2)*a^(7/4)*c^(5/4)*d^2*e^7 + c^3*d^9 - 6*sqrt(a)*c^
(5/2)*d^7*e^2 - 12*a*c^2*d^5*e^4 + 10*a^(3/2)*c^(3/2)*d^3*e^6 + 3*a^2*c*d*
e^8)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/
4)) - 2*(sqrt(2)*a^(1/4)*c^(13/4)*d^9 + 6*sqrt(2)*a^(3/4)*c^(11/4)*d^7*e^2
 - 12*sqrt(2)*a^(5/4)*c^(9/4)*d^5*e^4 - 10*sqrt(2)*a^(7/4)*c^(7/4)*d^3*e^6
 + 3*sqrt(2)*a^(9/4)*c^(5/4)*d*e^8 + 6*sqrt(a)*c^3*d^8*e - 24*a^(3/2)*c^2*
d^4*e^5 + 2*a^(5/2)*c*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/
4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4))
 - 2*(sqrt(2)*a^(1/4)*c^(13/4)*d^9 + 6*sqrt(2)*a^(3/4)*c^(11/4)*d^7*e^2 -
12*sqrt(2)*a^(5/4)*c^(9/4)*d^5*e^4 - 10*sqrt(2)*a^(7/4)*c^(7/4)*d^3*e^6 +
3*sqrt(2)*a^(9/4)*c^(5/4)*d*e^8 - 6*sqrt(a)*c^3*d^8*e + 24*a^(3/2)*c^2*d^4
*e^5 - 2*a^(5/2)*c*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*
c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(
c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + 2*(5*c^2*d^6*e^
3 - 3*a*c*d^2*e^7)*log(e*x + d)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*
e^8 + a^3*e^12) - 1/2*(8*c*d^3*e^4*x + 9*c*d^4*e^3 + a*e^7)/(c^2*d^10 +...
```

3.400.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 941, normalized size of antiderivative = 1.38

$$\int \frac{1}{(d+ex)^3(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*(2*a*c^2*e^3 + sqrt(2)*(a*c^3)^(1/4)*c^2*d^3 - 3*sqrt(2)*(a*c^3)^(3/4)
*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3
*d^6 + 9*a^2*c^2*d^2*e^4 - 3*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^5*e - 3*sqrt(2)
*(a*c^3)^(1/4)*a^2*c*d*e^5 + 9*sqrt(a*c)*a*c^2*d^4*e^2 + sqrt(a*c)*a^2*c*e
^6 - 8*sqrt(2)*(a*c^3)^(3/4)*a*d^3*e^3) - 1/4*(2*a*c^2*e^3 - sqrt(2)*(a*c^
3)^(1/4)*c^2*d^3 + 3*sqrt(2)*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x
- sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3*d^6 + 9*a^2*c^2*d^2*e^4 + 3*sqr
t(2)*(a*c^3)^(1/4)*a*c^2*d^5*e + 3*sqrt(2)*(a*c^3)^(1/4)*a^2*c*d*e^5 - 9*s
qrt(a*c)*a*c^2*d^4*e^2 + sqrt(a*c)*a^2*c*e^6 + 8*sqrt(2)*(a*c^3)^(3/4)*a*d
^3*e^3) + 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^9 - 12*sqrt(2)*(a*c^3)^(1/4)*a*
c^2*d^5*e^4 + 3*sqrt(2)*(a*c^3)^(1/4)*a^2*c*d*e^8 - 6*sqrt(2)*(a*c^3)^(3/4
)*c*d^7*e^2 + 10*sqrt(2)*(a*c^3)^(3/4)*a*d^3*e^6)*log(x^2 + sqrt(2)*x*(a/c
)^(1/4) + sqrt(a/c))/(a*c^4*d^12 + 3*a^2*c^3*d^8*e^4 + 3*a^3*c^2*d^4*e^8 +
 a^4*c*e^12) - 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^9 - 12*sqrt(2)*(a*c^3)^(1/
4)*a*c^2*d^5*e^4 + 3*sqrt(2)*(a*c^3)^(1/4)*a^2*c*d*e^8 - 6*sqrt(2)*(a*c^3)
^(3/4)*c*d^7*e^2 + 10*sqrt(2)*(a*c^3)^(3/4)*a*d^3*e^6)*log(x^2 - sqrt(2)*x
*(a/c)^(1/4) + sqrt(a/c))/(a*c^4*d^12 + 3*a^2*c^3*d^8*e^4 + 3*a^3*c^2*d^4*
e^8 + a^4*c*e^12) - 1/2*(5*c^2*d^6*e^3 - 3*a*c*d^2*e^7)*log(abs(c*x^4 + a)
)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + 2*(5*c^2*d^6
*e^4 - 3*a*c*d^2*e^8)*log(abs(e*x + d))/(c^3*d^12*e + 3*a*c^2*d^8*e^5 +...
```

3.400.9 Mupad [B] (verification not implemented)

Time = 9.89 (sec) , antiderivative size = 1955, normalized size of antiderivative = 2.88

$$\int \frac{1}{(d+ex)^3 (a+cx^4)} dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)*(d + e*x)^3),x)
```

output
```
symsum(log((c^7*d^5*e^6 + a*c^6*d*e^10)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^1
2*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) + root(768*a^5*c*d^4*e^8*z^4
 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 153
6*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 +
 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((208*a*c^7*d^7*e^7 - 48
*a^2*c^6*d^3*e^11)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e
^12 + 6*a^2*c^2*d^8*e^8) + root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^
4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 +
 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 4
8*a*c^2*d^2*e*z + c^2, z, k)*((144*a*c^8*d^13*e^4 + 16*a^4*c^5*d*e^16 + 26
08*a^2*c^7*d^9*e^8 - 592*a^3*c^6*d^5*e^12)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*
d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) - root(768*a^5*c*d^4*e^8*
z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 -
1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^
2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((896*a^4*c^6*d^7*e^1
3 - 1120*a^3*c^7*d^11*e^9 - 1024*a^2*c^8*d^15*e^5 + 976*a^5*c^5*d^3*e^17 +
 16*a*c^9*d^19*e)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^
12 + 6*a^2*c^2*d^8*e^8) - root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4
*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 +
2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 +...
```

3.400. $\int \frac{1}{(d+ex)^3(a+cx^4)}\,dx$

3.401 $\int \frac{(d+ex)^3}{(a+cx^4)^2} dx$

3.401.1 Optimal result

Integrand size = 17, antiderivative size = 349

$$\begin{aligned}\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx = & -\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac\,(a+cx^4)}+\frac{3d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} \\ & -\frac{3d(\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} \\ & +\frac{3d(\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} \\ & -\frac{3d(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}} \\ & +\frac{3d(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}\end{aligned}$$

output
```
1/4*(-a*e^3+c*x*(3*d*e^2*x^2+3*d^2*e*x+d^3))/a/c/(c*x^4+a)+3/4*d^2*e*arcta
n(x^2*c^(1/2)/a^(1/2))/a^(3/2)/c^(1/2)-3/32*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2
)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+
3/32*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2
*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+3/16*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/
4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+3/16*d*arctan(1+c^(1
/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)
```

3.401.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx$$
$$= \frac{-\frac{8a(ae^3-cdx(d^2+3dex+3e^2x^2))}{a+cx^4} - 6\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{2}\sqrt{c}d^2+4\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+6\sqrt[4]{a}\sqrt[4]{c}}{}$$

input
```
Integrate[(d + e*x)^3/(a + c*x^4)^2,x]
```

output
```
((-8*a*(a*e^3 - c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(a + c*x^4) - 6*a^(1/4
)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]
*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*a^(1/4)*c^(1/4)*d*(Sqrt[
2]*Sqrt[c]*d^2 - 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*c^(1/4)*(-(a^(1/4)*Sqrt[c]*d^3) +
a^(3/4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*
Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(32*a^2*c)
```

3.401.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2393, 27, 2006, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx$$
$$\Big\downarrow \; 2393$$
$$-\frac{\int -\frac{3(d^3+2exd^2+e^2x^2d)}{cx^4+a}dx}{4a} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac(a+cx^4)}$$
$$\Big\downarrow \; 27$$
$$\frac{3\int \frac{d^3+2exd^2+e^2x^2d}{cx^4+a}dx}{4a} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac(a+cx^4)}$$

$$\Big\downarrow \text{2006}$$

$$\frac{3\int \frac{\left(d^{3/2}+ex\sqrt{d}\right)^2}{cx^4+a}dx}{4a}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}$$

$$\Big\downarrow \text{2415}$$

$$\frac{3\int \left(\frac{2exd^2}{cx^4+a}+\frac{d^3+e^2x^2d}{cx^4+a}\right)dx}{4a}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}-\frac{d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\right.}{4a}$$

$$\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}$$

input

```
Int[(d + e*x)^3/(a + c*x^4)^2,x]
```

output

```
-1/4*(a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(a*c*(a + c*x^4)) + (3*
((d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - (d*(Sqrt[c]*d^2
 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4
)*c^(3/4)) + (d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - (d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c
^(3/4)) + (d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))))/(4*a)
```

3.401. $\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx$

3.401.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.401.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.85 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.29

method	result
risch	$\frac{\frac{3d\,e^2x^3}{4a}+\frac{3e\,x^2d^2}{4a}+\frac{d^3x}{4a}-\frac{e^3}{4c}}{c\,x^4+a}+\frac{3d\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(e^2\_R^2+2ed\_R+d^2\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{16ac}$
default	$d^3\left(\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+3d^2e\left(\frac{x^2}{4a(c\,x^4}\right.$

input
```
int((e*x+d)^3/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(3/4*d*e^2/a*x^3+3/4*e/a*x^2*d^2+1/4*d^3/a*x-1/4*e^3/c)/(c*x^4+a)+3/16*d/a
/c*sum((_R^2*e^2+2*_R*d*e+d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

3.401.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.49 (sec) , antiderivative size = 91191, normalized size of antiderivative = 261.29

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.401.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**3/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.401.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx$$
$$= \frac{3\,d\left(\frac{\sqrt{2}(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{2\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2-4\sqrt{a}\cdots\right.}{a^{\frac{3}{4}}\sqrt{\cdots}}\right.}{32\,a}$$
$$+ \frac{3\,cde^2x^3+3\,cd^2ex^2+cd^3x-ae^3}{4\,(ac^2x^4+a^2c)}$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
3/32*d*(sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1
/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d^2 - sqrt(a
)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(
3/4)) + 2*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 4*s
qrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1
/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(s
qrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 4*sqrt(a)*sqrt(
c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sq
rt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a + 1/4*(3*c*d*e^
2*x^3 + 3*c*d^2*e*x^2 + c*d^3*x - a*e^3)/(a*c^2*x^4 + a^2*c)
```

3.401.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx = \frac{3\,cde^2x^3+3\,cd^2ex^2+cd^3x-ae^3}{4\,(cx^4+a)ac} + \frac{3\,\sqrt{2}\left(2\,\sqrt{2}\sqrt{ac}c^2d^2e+(ac^3)^{\frac{1}{4}}\,c^2d^3+(ac^3)^{\frac{3}{4}}\,de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{3\,\sqrt{2}\left(2\,\sqrt{2}\sqrt{ac}c^2d^2e+(ac^3)^{\frac{1}{4}}\,c^2d^3+(ac^3)^{\frac{3}{4}}\,de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{3\,\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^2d^3-(ac^3)^{\frac{3}{4}}\,de^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3} - \frac{3\,\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^2d^3-(ac^3)^{\frac{3}{4}}\,de^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(3*c*d*e^2*x^3 + 3*c*d^2*e*x^2 + c*d^3*x - a*e^3)/((c*x^4 + a)*a*c) +
3/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + (a*c
^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4
))/(a^2*c^3) + 3/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)
*c^2*d^3 + (a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1
/4))/(a/c)^(1/4))/(a^2*c^3) + 3/32*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3
)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 3/
32*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 - sqrt(2)
*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)
```

3.401.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.92

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx$$
$$= \left(\sum_{k=1}^{4} \ln\left(\frac{c\,d^2\left(27\,c\,d^5\,e^2 - 9\,a\,d\,e^6 + 36\,c\,d^4\,e^3\,x - \mathrm{root}(65536\,a^7\,c^3\,z^4 + 27648\,a^4\,c^2\,d^4\,e^2\,z^2 + 3456\,a^3\,c\,d^4\right.}{}\right.\right.$$
$$+ 27648\,a^4\,c^2\,d^4\,e^2\,z^2 + 3456\,a^3\,c\,d^4\,e^5\,z - 3456\,a^2\,c^2\,d^8\,e\,z + 162\,a\,c\,d^8\,e^4 + 81\,a^2\,d^4\,e^8$$
$$\left.\left. + 81\,c^2\,d^{12}, z, k)\right)\right) + \frac{\frac{d^3\,x}{4\,a} - \frac{e^3}{4\,c} + \frac{3\,d^2\,e\,x^2}{4\,a} + \frac{3\,d\,e^2\,x^3}{4\,a}}{c\,x^4 + a}$$

input
```
int((d + e*x)^3/(a + c*x^4)^2,x)
```

output
```
symsum(log((3*c*d^2*(27*c*d^5*e^2 - 9*a*d*e^6 + 36*c*d^4*e^3*x - 256*root(
65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 345
6*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)^
2*a^3*c^2*d - 48*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456
*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8
 + 81*c^2*d^12, z, k)*a*c^2*d^4*x + 48*root(65536*a^7*c^3*z^4 + 27648*a^4*
c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^
8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)*a^2*c*e^4*x + 512*root(65536*a
^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c
^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)^2*a^3*c
^2*e*x - 192*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3
*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 8
1*c^2*d^12, z, k)*a^2*c*d*e^3))/(64*a^3))*root(65536*a^7*c^3*z^4 + 27648*a
^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c
*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k), k, 1, 4) + ((d^3*x)/(4*a)
- e^3/(4*c) + (3*d^2*e*x^2)/(4*a) + (3*d*e^2*x^3)/(4*a))/(a + c*x^4)
```

3.402 $\int \frac{(d+ex)^2}{(a+cx^4)^2} dx$

3.402.1 Optimal result

Integrand size = 17, antiderivative size = 322

$$\int \frac{(d+ex)^2}{(a+cx^4)^2} dx = \frac{x(d+ex)^2}{4a(a+cx^4)} + \frac{de\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} - \frac{(3\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} - \frac{(3\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}c^{3/4}}$$

output
```
1/4*x*(e*x+d)^2/a/(c*x^4+a)+1/2*d*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(3/2)/c^
(1/2)-1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2
)+3*d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+1/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)
+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)
+1/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+3*d^2*c^(1/2))/a^(
7/4)/c^(3/4)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)
+3*d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)
```

3.402.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx$$
$$= \frac{\frac{8ax(d+ex)^2}{a+cx^4} - \frac{2\sqrt[4]{a}\left(3\sqrt{2}\sqrt{c}d^2+8\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{2\sqrt[4]{a}\left(3\sqrt{2}\sqrt{c}d^2-8\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1+\right.}{c^{3/4}}}{32a}$$

input
```
Integrate[(d + e*x)^2/(a + c*x^4)^2,x]
```

output
```
((8*a*x*(d + e*x)^2)/(a + c*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 + 8*a
^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/c^(3/4) + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 - 8*a^(1/4)*c^(1/4)*d
*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4)
 + (Sqrt[2]*(-3*a^(1/4)*Sqrt[c]*d^2 + a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[c]*d^2
 - a^(3/4)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^
(3/4))/(32*a^2)
```

3.402.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx$$
$$\Big\downarrow \ 2394$$
$$\frac{x(d+ex)^2}{4a(a+cx^4)} - \frac{\int -\frac{3d^2+4exd+e^2x^2}{cx^4+a}dx}{4a}$$
$$\Big\downarrow \ 25$$
$$\frac{\int \frac{3d^2+4exd+e^2x^2}{cx^4+a}dx}{4a} + \frac{x(d+ex)^2}{4a(a+cx^4)}$$

$$\Big\downarrow \; 2415$$

$$\frac{\int \left(\frac{4dex}{cx^4+a}+\frac{3d^2+e^2x^2}{cx^4+a}\right)dx}{4a}+\frac{x(d+ex)^2}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e^2+3\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e^2+3\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}-\frac{\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{(3\sqrt{c}}{}}{4a}$$

$$\frac{x(d+ex)^2}{4a\left(a+cx^4\right)}$$

input
```
Int[(d + e*x)^2/(a + c*x^4)^2,x]
```

output
```
(x*(d + e*x)^2)/(4*a*(a + c*x^4)) + ((2*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])
/(Sqrt[a]*Sqrt[c]) - ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 + Sqrt[a]
*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4))
 - ((3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
+ Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 - Sqrt[a]*e^
2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3
/4)*c^(3/4)))/(4*a)
```

3.402.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.402.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.82 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.28

method	result
risch	$\frac{\frac{e^2x^3}{4a}+\frac{ed\,x^2}{2a}+\frac{d^2x}{4a}}{c\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(e^2\_R^2+4ed\_R+3d^2\right)\ln\left(x-\_R\right)}{\_R^3}}{16ac}$
default	$d^2\left(\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+2ed\Bigg(\frac{x^2}{4a(c\,x^4 \cdots$

input
```
int((e*x+d)^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*e^2/a*x^3+1/2*e*d/a*x^2+1/4/a*d^2*x)/(c*x^4+a)+1/16/a/c*sum((_R^2*e^2
+4*_R*d*e+3*d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

3.402.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.74 (sec) , antiderivative size = 90963, normalized size of antiderivative = 282.49

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.402.6 Sympy [A] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx$$
$$= \operatorname{RootSum}\left(65536t^4a^7c^3 + 11264t^2a^4c^2d^2e^2 + t\left(256a^3cde^5 - 2304a^2c^2d^5e\right) + a^2e^8 + 82acd^4e^4 + 81c^2d^8, \left(\right.\right.$$
$$+ \frac{d^2x + 2dex^2 + e^2x^3}{4a^2 + 4acx^4}$$

input
```
integrate((e*x+d)**2/(c*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*c**3 + 11264*_t**2*a**4*c**2*d**2*e**2 + _t*(256*
a**3*c*d*e**5 - 2304*a**2*c**2*d**5*e) + a**2*e**8 + 82*a*c*d**4*e**4 + 81
*c**2*d**8, Lambda(_t, _t*log(x + (4096*_t**3*a**7*c**2*e**6 + 356352*_t**
3*a**6*c**3*d**4*e**2 - 23552*_t**2*a**5*c**2*d**3*e**5 + 27648*_t**2*a**4
*c**3*d**7*e + 912*_t*a**4*c*d**2*e**8 + 43584*_t*a**3*c**2*d**6*e**4 + 38
88*_t*a**2*c**3*d**10 + 12*a**3*d*e**11 - 1088*a**2*c*d**5*e**7 - 7020*a*c
**2*d**9*e**3)/(a**3*e**12 - 649*a**2*c*d**4*e**8 - 5841*a*c**2*d**8*e**4
+ 729*c**3*d**12))))) + (d**2*x + 2*d*e*x**2 + e**2*x**3)/(4*a**2 + 4*a*c*x
**4)
```

3.402.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx = \frac{e^2x^3 + 2\,dex^2 + d^2x}{4\,(acx^4 + a^2)}$$
$$+ \frac{\frac{\sqrt{2}(3\sqrt{c}d^2 - \sqrt{a}e^2)\log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(3\sqrt{c}d^2 - \sqrt{a}e^2)\log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{2\left(3\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2 + \sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2 - 8\sqrt{c}\right.}{a^{\frac{3}{4}}\sqrt{}}}{32\,a}$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(e^2*x^3 + 2*d*e*x^2 + d^2*x)/(a*c*x^4 + a^2) + 1/32*(sqrt(2)*(3*sqrt(
c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a
))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x
^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(3*sqrt(2)
*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 8*sqrt(a)*sqrt(c)*d*e
)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*
sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*c
^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 8*sqrt(a)*sqrt(c)*d*e)*arctan(1
/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/
(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a
```

3.402.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx$$
$$= \frac{e^2x^3 + 2\,dex^2 + d^2x}{4\,(cx^4+a)a} + \frac{\sqrt{2}\left(4\,\sqrt{2}\sqrt{ac}c^2de + 3\,(ac^3)^{\frac{1}{4}}\,c^2d^2 + (ac^3)^{\frac{3}{4}}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{\sqrt{2}\left(4\,\sqrt{2}\sqrt{ac}c^2de + 3\,(ac^3)^{\frac{1}{4}}\,c^2d^2 + (ac^3)^{\frac{3}{4}}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2 - (ac^3)^{\frac{3}{4}}\,e^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3} - \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2 - (ac^3)^{\frac{3}{4}}\,e^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(e^2*x^3 + 2*d*e*x^2 + d^2*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(4*sqrt(2
)*sqrt(a*c)*c^2*d*e + 3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sqrt
(2)*(4*sqrt(2)*sqrt(a*c)*c^2*d*e + 3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)
*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3
) + 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 + s
qrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4
)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c)
)/(a^2*c^3)
```

3.402.9 Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx = \frac{\frac{d^2\,x}{4\,a}+\frac{e^2\,x^3}{4\,a}+\frac{d\,e\,x^2}{2\,a}}{c\,x^4+a}+\left(\sum_{k=1}^{4}\ln\left(\frac{39\,c^2\,d^4\,e^2-a\,c\,e^6}{64\,a^3}\right.\right.$$
$$-\mathrm{root}\left(65536\,a^7\,c^3\,z^4+11264\,a^4\,c^2\,d^2\,e^2\,z^2-2304\,a^2\,c^2\,d^5\,e\,z+256\,a^3\,c\,d\,e^5\,z+82\,a\,c\,d^4\,e^4+81\,c^2\,d^8+a^2\,e^8,z\right.$$
$$\left.+\frac{5\,c^2\,d^3\,e^3\,x}{8\,a^3}\right)\mathrm{root}\left(65536\,a^7\,c^3\,z^4+11264\,a^4\,c^2\,d^2\,e^2\,z^2-2304\,a^2\,c^2\,d^5\,e\,z\right.$$
$$\left.\left.+256\,a^3\,c\,d\,e^5\,z+82\,a\,c\,d^4\,e^4+81\,c^2\,d^8+a^2\,e^8,z,k\right)\right)$$

input
```
int((d + e*x)^2/(a + c*x^4)^2,x)
```

output
```
((d^2*x)/(4*a) + (e^2*x^3)/(4*a) + (d*e*x^2)/(2*a))/(a + c*x^4) + symsum(l
og((39*c^2*d^4*e^2 - a*c*e^6)/(64*a^3) - root(65536*a^7*c^3*z^4 + 11264*a^
4*c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*
e^4 + 81*c^2*d^8 + a^2*e^8, z, k)*(root(65536*a^7*c^3*z^4 + 11264*a^4*c^2*
d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4 +
81*c^2*d^8 + a^2*e^8, z, k)*(12*c^3*d^2 - 16*c^3*d*e*x) + (x*(18*a*c^3*d^4
 - 2*a^2*c^2*e^4))/(8*a^3) + (2*c^2*d*e^3)/a) + (5*c^2*d^3*e^3*x)/(8*a^3))
*root(65536*a^7*c^3*z^4 + 11264*a^4*c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z
 + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4 + 81*c^2*d^8 + a^2*e^8, z, k), k, 1,
 4)
```

3.403 $\int \frac{d+ex}{(a+cx^4)^2} dx$

3.403.1 Optimal result

Integrand size = 15, antiderivative size = 241

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx = \frac{x(d+ex)}{4a\,(a+cx^4)} + \frac{e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3d\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

output
```
1/4*x*(e*x+d)/a/(c*x^4+a)+3/16*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7
/4)/c^(1/4)*2^(1/2)+3/16*d*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(
1/4)*2^(1/2)-3/32*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(
7/4)/c^(1/4)*2^(1/2)+3/32*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/
2))/a^(7/4)/c^(1/4)*2^(1/2)+1/4*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(3/2)/c^(1
/2)
```

3.403.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.93

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx$$
$$= \frac{\frac{8a^{3/4}x(d+ex)}{a+cx^4} - \frac{2\left(3\sqrt{2}\sqrt[4]{c}d+4\sqrt[4]{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}} + \frac{2\left(3\sqrt{2}\sqrt[4]{c}d-4\sqrt[4]{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}} - \frac{3\sqrt{2}d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{\sqrt[4]{c}}}{32a^{7/4}}$$

input
```
Integrate[(d + e*x)/(a + c*x^4)^2,x]
```

output
```
((8*a^(3/4)*x*(d + e*x))/(a + c*x^4) - (2*(3*Sqrt[2]*c^(1/4)*d + 4*a^(1/4)
*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] + (2*(3*Sqrt[2]*c^(1/
4)*d - 4*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] - (3*
Sqrt[2]*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4)
+ (3*Sqrt[2]*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(
1/4))/(32*a^(7/4))
```

3.403.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx$$
$$\Big\downarrow \text{2394}$$
$$\frac{x(d+ex)}{4a(a+cx^4)} - \frac{\int -\frac{3d+2ex}{cx^4+a}dx}{4a}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{3d+2ex}{cx^4+a}dx}{4a} + \frac{x(d+ex)}{4a(a+cx^4)}$$
$$\Big\downarrow \text{2415}$$

$$\frac{\int \left(\frac{3d}{cx^4+a}+\frac{2ex}{cx^4+a}\right)dx}{4a}+\frac{x(d+ex)}{4a\left(a+cx^4\right)}$$

$\big\downarrow$ 2009

$$\frac{-\frac{3d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{3d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{3d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{3d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{e\arcta}{}}{4a}$$
$$\frac{x(d+ex)}{4a\left(a+cx^4\right)}$$

input
```
Int[(d + e*x)/(a + c*x^4)^2,x]
```

output
```
(x*(d + e*x))/(4*a*(a + c*x^4)) + ((e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt
[a]*Sqrt[c]) - (3*d*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^
(3/4)*c^(1/4)) + (3*d*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*
a^(3/4)*c^(1/4)) - (3*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + (3*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4)))/(4*a)
```

3.403.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.403.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.87 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.27

method	result
risch	$\frac{\frac{e\,x^2}{4a}+\frac{dx}{4a}}{c\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(2e\_R+3d\right)\ln\left(x-\_R\right)}{\_R^3}}{16ac}$
default	$d\left(\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+e\left(\frac{x^2}{4a(c\,x^4+a)}\right.$

input
```
int((e*x+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*e/a*x^2+1/4*d/a*x)/(c*x^4+a)+1/16/a/c*sum((2*_R*e+3*d)/_R^3*ln(x-_R),
_R=RootOf(_Z^4*c+a))
```

3.403.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 43065, normalized size of antiderivative = 178.69

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.403.6 Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.64

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx$$
$$=\mathrm{RootSum}\left(65536t^4a^7c^2+2048t^2a^4ce^2-1152ta^2cd^2e+16ae^4+81cd^4,\left(t\mapsto t\log\left(x+\frac{-32768t^3a^6ce^2-\right.\right.\right.$$
$$+\frac{dx+ex^2}{4a^2+4acx^4}$$

input
```
integrate((e*x+d)/(c*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*c**2 + 2048*_t**2*a**4*c*e**2 - 1152*_t*a**2*c*d*
*2*e + 16*a*e**4 + 81*c*d**4, Lambda(_t, _t*log(x + (-32768*_t**3*a**6*c*e
**2 - 4608*_t**2*a**4*c*d**2*e - 512*_t*a**3*e**4 - 1296*_t*a**2*c*d**4 +
360*a*d**2*e**3)/(192*a*d*e**4 - 243*c*d**5)))) + (d*x + e*x**2)/(4*a**2 +
 4*a*c*x**4)
```

3.403.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.99

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx=\frac{ex^2+dx}{4\,(acx^4+a^2)}$$
$$+\frac{\frac{3\sqrt{2}d\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{3\sqrt{2}d\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d-4\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{1}{4}}}+\frac{2}{}}{32\,a}$$

input
```
integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(e*x^2 + d*x)/(a*c*x^4 + a^2) + 1/32*(3*sqrt(2)*d*log(sqrt(c)*x^2 + sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 3*sqrt(2)*d*log(sqr
t(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) + 2*(3*s
qrt(2)*a^(1/4)*c^(1/4)*d - 4*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x +
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt
(c))*c^(1/4)) + 2*(3*sqrt(2)*a^(1/4)*c^(1/4)*d + 4*sqrt(a)*e)*arctan(1/2*s
qrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(
3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4)))/a
```

3.403.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.99

$$\int \frac{d+ex}{\left(a+cx^4\right)^2}\,dx = \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c} - \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c} + \frac{ex^2+dx}{4\,(cx^4+a)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}ce+3\,(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^2} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}ce+3\,(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^2}$$

input
```
integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
3/32*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/
(a^2*c) - 3/32*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + s
qrt(a/c))/(a^2*c) + 1/4*(e*x^2 + d*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(2*sq
rt(2)*sqrt(a*c)*c*e + 3*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)
*c*e + 3*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))
/(a/c)^(1/4))/(a^2*c^2)
```

3.403.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.17

$$\int \frac{d+ex}{\left(a+cx^4\right)^2}\,dx = \left(\sum_{k=1}^{4}\ln\left(\frac{c^2\left(3\,d\,e^2+2\,e^3\,x-\mathrm{root}(65536\,a^7\,c^2\,z^4+2048\,a^4\,c\,e^2\,z^2-1152\,a^2\,c\,d^2\,e\,z+81\,c\,d^4+16\,a\,e^4,z,\right.}{\quad}\right.\right.$$

$$\left.\left.+2048\,a^4\,c\,e^2\,z^2-1152\,a^2\,c\,d^2\,e\,z+81\,c\,d^4+16\,a\,e^4,z,k)\right)\right) + \frac{\frac{e\,x^2}{4\,a}+\frac{d\,x}{4\,a}}{c\,x^4+a}$$

input
```
int((d + e*x)/(a + c*x^4)^2,x)
```

output
```
symsum(log((c^2*(3*d*e^2 + 2*e^3*x - 192*root(65536*a^7*c^2*z^4 + 2048*a^4
*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)^2*a^3*c*d + 1
28*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c
*d^4 + 16*a*e^4, z, k)^2*a^3*c*e*x - 36*root(65536*a^7*c^2*z^4 + 2048*a^4*
c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)*a*c*d^2*x))/(1
6*a^3))*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z +
 81*c*d^4 + 16*a*e^4, z, k), k, 1, 4) + ((e*x^2)/(4*a) + (d*x)/(4*a))/(a +
 c*x^4)
```

3.403. $\int \frac{d+ex}{(a+cx^4)^2}\,dx$

3.404 $\int \frac{1}{(a+cx^4)^2} dx$

3.404.1 Optimal result

Integrand size = 9, antiderivative size = 202

$$\int \frac{1}{(a+cx^4)^2} dx = \frac{x}{4a(a+cx^4)} - \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

output
```
1/4*x/a/(c*x^4+a)+3/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4
)*2^(1/2)+3/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)*2^(1/2)
-3/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*2
^(1/2)+3/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1
/4)*2^(1/2)
```

3.404.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+cx^4)^2} dx = \frac{\frac{8a^{3/4}x}{a+cx^4} - \frac{6\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} - \frac{3\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\right.}{\sqrt[4]{c}}}{32a^{7/4}}$$

input
```
Integrate[(a + c*x^4)^(-2),x]
```

output
```
((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/
4) - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^
(1/4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/c^(1/4))/(32*a^(7/4))
```

3.404.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {749, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+cx^4\right)^2}dx$$

$$\Big\downarrow \text{749}$$

$$\frac{3\int \frac{1}{cx^4+a}dx}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{755}$$

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \ 217$$

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \ 1479$$

$$\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \ 27$$

3.404. $\int \frac{1}{\left(a+cx^4\right)^2}\,dx$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$\big\downarrow$ 1103

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

input
```
Int[(a + c*x^4)^(-2),x]
```

output
```
x/(4*a*(a + c*x^4)) + (3*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt
[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a
^(1/4)*c^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a])))/(4*a
)
```

3.404.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.404. $\int \frac{1}{(a+cx^4)^2} dx$

3.404.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.76 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.23

method	result	size
risch	$\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(c\,\_Z^4+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16ac}$	46
default	$\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}$	118

input
```
int(1/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x/a/(c*x^4+a)+3/16/a/c*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

3.404.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+cx^4)^2}\,dx$$
$$= \frac{3\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)-3\left(-i\,acx^4-i\,a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(i\,a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)-3\left(i\,acx^4\right.}{16\left(acx^4+a\right.}$$

input
```
integrate(1/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*(3*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x)
 - 3*(-I*a*c*x^4 - I*a^2)*(-1/(a^7*c))^(1/4)*log(I*a^2*(-1/(a^7*c))^(1/4)
+ x) - 3*(I*a*c*x^4 + I*a^2)*(-1/(a^7*c))^(1/4)*log(-I*a^2*(-1/(a^7*c))^(1
/4) + x) - 3*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4
) + x) + 4*x)/(a*c*x^4 + a^2)
```

3.404.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.19

$$\int \frac{1}{\left(a+cx^4\right)^2}\,dx=\frac{x}{4a^2+4acx^4}+\operatorname{RootSum}\left(65536t^4a^7c+81,\left(t\mapsto t\log\left(\frac{16ta^2}{3}+x\right)\right)\right)$$

input
```
integrate(1/(c*x**4+a)**2,x)
```

output
```
x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda(_t, _t*l
og(16*_t*a**2/3 + x)))
```

3.404.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+cx^4\right)^2}\,dx=\frac{x}{4\left(acx^4+a^2\right)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac14}c^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac14}c^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac14}c^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}c^{\frac14}}-\frac{\sqrt{2}\log\left(\sqrt{c}x^2-\right.}{a^{\frac34}}\right.}{32\,a}$$

input
```
integrate(1/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*x/(a*c*x^4 + a^2) + 3/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x +
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt
(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4
))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sq
rt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(
2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)
))/a
```

3.404.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4\,(cx^4+a)a} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c} - \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c}$$

input
```
integrate(1/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt(2)*(a*c^3)^(1/4)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/32
*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c
) - 3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c)
)/(a^2*c)
```

3.404.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.29

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4\,a\,\left(c\,x^4+a\right)} + \frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{7/4}\,c^{1/4}} + \frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{7/4}\,c^{1/4}}$$

input
```
int(1/(a + c*x^4)^2,x)
```

output
```
x/(4*a*(a + c*x^4)) + (3*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/
4)) + (3*atanh((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4))
```

3.405 $\int \frac{1}{(d+ex)(a+cx^4)^2}\,dx$

3.405.1 Optimal result

Integrand size = 17, antiderivative size = 855

$$\int \frac{1}{(d+ex)(a+cx^4)^2}\,dx = \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} - \frac{\sqrt{c}d^2e^5\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(cd^4+ae^4)^2} - \frac{\sqrt{c}d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}(cd^4+ae^4)} - \frac{\sqrt[4]{c}de^4(\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^2} - \frac{\sqrt[4]{c}d(3\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^4+ae^4)} + \frac{\sqrt[4]{c}de^4(\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^2} + \frac{\sqrt[4]{c}d(3\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^4+ae^4)} + \frac{e^7\log(d+ex)}{(cd^4+ae^4)^2} - \frac{\sqrt[4]{c}de^4(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^2} - \frac{\sqrt[4]{c}d(3\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(cd^4+ae^4)} + \frac{\sqrt[4]{c}de^4(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^2} + \frac{\sqrt[4]{c}d(3\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(cd^4+ae^4)} - \frac{e^7\log(a+cx^4)}{4(cd^4+ae^4)^2}$$

output
```
1/4*(a*e^3+c*x*(d*e^2*x^2-d^2*e*x+d^3))/a/(a*e^4+c*d^4)/(c*x^4+a)+e^7*ln(e
*x+d)/(a*e^4+c*d^4)^2-1/4*e^7*ln(c*x^4+a)/(a*e^4+c*d^4)^2-1/4*d^2*e*arctan
(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4)-1/2*d^2*e^5*arctan(x^2
*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^2/a^(1/2)-1/8*c^(1/4)*d*e^4*ln(-a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^
(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/8*c^(1/4)*d*e^4*ln(a^(1/4)*c^(1/4)*x*2^(1/
2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2
*2^(1/2)+1/4*c^(1/4)*d*e^4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/
2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/4*c^(1/4)*d*e^4*arctan(1
+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4
)^2*2^(1/2)-1/32*c^(1/4)*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/
2))*(-e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)*2^(1/2)+1/32*c^(1/4
)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+3*d^2*
c^(1/2))/a^(7/4)/(a*e^4+c*d^4)*2^(1/2)+1/16*c^(1/4)*d*arctan(-1+c^(1/4)*x*
2^(1/2)/a^(1/4))*(e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)*2^(1/2)
+1/16*c^(1/4)*d*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+3*d^2*c^(
1/2))/a^(7/4)/(a*e^4+c*d^4)*2^(1/2)
```

3.405.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 558, normalized size of antiderivative = 0.65

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx$$

$$= \frac{\frac{8(cd^4+ae^4)\left(ae^3+cdx(d^2-dex+e^2x^2)\right)}{a(a+cx^4)} - \frac{2\sqrt[4]{c}d\left(3\sqrt{2}c^{3/2}d^6-4\sqrt[4]{a}c^{5/4}d^5e+\sqrt{2}\sqrt{a}cd^4e^2+7\sqrt{2}a\sqrt{c}d^2e^4-12a^{5/4}\sqrt[4]{c}de^5+5\sqrt{2}a^{3/2}e^6\right)\text{arcta}}{a^{7/4}}}{}$$

input
```
Integrate[1/((d + e*x)*(a + c*x^4)^2),x]
```

output
```
((8*(c*d^4 + a*e^4)*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(a + c*x^4
)) - (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 - 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt[
2]*Sqrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 - 12*a^(5/4)*c^(1/4)*d*
e^5 + 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7
/4) + (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 + 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt
[2]*Sqrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 + 12*a^(5/4)*c^(1/4)*d
*e^5 + 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(
7/4) + 32*e^7*Log[d + e*x] + (Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^7 + Sqrt[a]*c*
d^5*e^2 - 7*a*Sqrt[c]*d^3*e^4 + 5*a^(3/2)*d*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^7 -
 Sqrt[a]*c*d^5*e^2 + 7*a*Sqrt[c]*d^3*e^4 - 5*a^(3/2)*d*e^6)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 8*e^7*Log[a + c*x^4])/
(32*(c*d^4 + a*e^4)^2)
```

3.405.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 855, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^2(d+ex)}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{e^8}{(d+ex)(ae^4+cd^4)^2}-\frac{ce^4(-d^3+d^2ex-de^2x^2+e^3x^3)}{(a+cx^4)(ae^4+cd^4)^2}+\frac{c(d^3-d^2ex+de^2x^2-e^3x^3)}{(a+cx^4)^2(ae^4+cd^4)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(d+ex)e^7}{(cd^4+ae^4)^2}-\frac{\log\left(cx^4+a\right)e^7}{4\left(cd^4+ae^4\right)^2}-\frac{\sqrt{c}d^2\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{2\sqrt{a}\left(cd^4+ae^4\right)^2}-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^2}-\frac{\sqrt{c}d^2\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{4a^{3/2}\left(cd^4+ae^4\right)}+\frac{ae^3+cx\left(d^3-exd^2+e^2x^2d\right)}{4a\left(cd^4+ae^4\right)\left(cx^4+a\right)}-\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)}+\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)}-\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)}+\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)}$$

input
```
Int[1/((d + e*x)*(a + c*x^4)^2),x]
```

3.405. $\int \frac{1}{(d+ex)(a+cx^4)^2}\,dx$

output
```
(a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(4*a*(c*d^4 + a*e^4)*(a + c*x^4)
) - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*
e^4)^2) - (Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4
+ a*e^4)) - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]
*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(
3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*S
qrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^
2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*
e^4)^2) + (c^(1/4)*d*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (e^7*Log[d + e*x])
/(c*d^4 + a*e^4)^2 - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 +
a*e^4)^2) - (c^(1/4)*d*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) +
(c^(1/4)*d*e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*
d*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) - (e^7*Log[a + c*x^4])/
(4*(c*d^4 + a*e^4)^2)
```

3.405.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.405.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.50

method	result
default	$c\left(\frac{\frac{d e^{2}\left(e^{4} a+d^{4} c\right) x^{3}}{4 a}-\frac{d^{2} e\left(e^{4} a+d^{4} c\right) x^{2}}{4 a}+\frac{d^{3}\left(e^{4} a+d^{4} c\right) x}{4 a}+\frac{e^{3}\left(e^{4} a+d^{4} c\right)}{4 c}}{c x^{4}+a}+\frac{\frac{\left(7 a d^{3} e^{4}+3 c d^{7}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2 \arctan\left(\right.\right.}{8 a}}{}\right.$
risch	$\frac{\frac{c d e^{2} x^{3}}{4 a\left(e^{4} a+d^{4} c\right)}-\frac{d^{2} c e x^{2}}{4 a\left(e^{4} a+d^{4} c\right)}+\frac{d^{3} c x}{4 a\left(e^{4} a+d^{4} c\right)}+\frac{e^{3}}{4 e^{4} a+4 d^{4} c}}{c x^{4}+a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a^{9} e^{8}+2 a^{8} c d^{4} e^{4}+a^{7} c^{2} d^{8}\right) \_Z^{4}+16 a^{7} e^{7} \_Z^{3}+\left(96 a^{5} e^{6}+20 a^{4}\right.\right.}\right.}{}$

input
```
int(1/(e*x+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
c/(a*e^4+c*d^4)^2*((1/4*d*e^2*(a*e^4+c*d^4)/a*x^3-1/4*d^2*e*(a*e^4+c*d^4)/
a*x^2+1/4*d^3*(a*e^4+c*d^4)/a*x+1/4*e^3*(a*e^4+c*d^4)/c)/(c*x^4+a)+1/4/a*(
1/8*(7*a*d^3*e^4+3*c*d^7)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(
1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2
)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/2*(-6*a*d^2*e^5-2*
c*d^6*e)/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*(5*a*d*e^6+c*d^5*e^2)/c/(
a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^
(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2
^(1/2)/(a/c)^(1/4)*x-1))-a*e^7/c*ln(c*x^4+a))+e^7*ln(e*x+d)/(a*e^4+c*d^4)
^2
```

3.405.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+cx^{4}\right)^{2}}\, dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.405.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.405.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 601, normalized size of antiderivative = 0.70

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx = \frac{e^7\log(ex+d)}{c^2d^8+2\,acd^4e^4+a^2e^8} - \frac{c\left(\frac{\sqrt{2}\left(4\sqrt{2}a^{\frac{7}{4}}c^{\frac{1}{4}}e^7-3\,c^2d^7+\sqrt{a}c^{\frac{3}{2}}d^5e^2-7\,acd^3e^4+5\,a^{\frac{3}{2}}\sqrt{c}de^6\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}} + \frac{\sqrt{2}\left(4\sqrt{2}a^{\frac{7}{4}}c^{\frac{1}{4}}e^7+3\,c^2d^7-\sqrt{a}c^{\frac{3}{2}}d^5e^2+7\cdots\right.}{\cdots}\cdots\right.}{\cdots} + \frac{cde^2x^3-cd^2ex^2+cd^3x+ae^3}{4\left(a^2cd^4+a^3e^4+\left(ac^2d^4+a^2ce^4\right)x^4\right)}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
e^7*log(e*x + d)/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) - 1/32*c*(sqrt(2)*(4*
sqrt(2)*a^(7/4)*c^(1/4)*e^7 - 3*c^2*d^7 + sqrt(a)*c^(3/2)*d^5*e^2 - 7*a*c*
d^3*e^4 + 5*a^(3/2)*sqrt(c)*d*e^6)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(4*sqrt(2)*a^(7/4)*c^(1/4)*e^7
 + 3*c^2*d^7 - sqrt(a)*c^(3/2)*d^5*e^2 + 7*a*c*d^3*e^4 - 5*a^(3/2)*sqrt(c)
*d*e^6)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^
(5/4)) - 2*(3*sqrt(2)*a^(1/4)*c^(9/4)*d^7 + sqrt(2)*a^(3/4)*c^(7/4)*d^5*e^
2 + 7*sqrt(2)*a^(5/4)*c^(5/4)*d^3*e^4 + 5*sqrt(2)*a^(7/4)*c^(3/4)*d*e^6 +
4*sqrt(a)*c^2*d^6*e + 12*a^(3/2)*c*d^2*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*
x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*
sqrt(c))*c^(5/4)) - 2*(3*sqrt(2)*a^(1/4)*c^(9/4)*d^7 + sqrt(2)*a^(3/4)*c^(
7/4)*d^5*e^2 + 7*sqrt(2)*a^(5/4)*c^(5/4)*d^3*e^4 + 5*sqrt(2)*a^(7/4)*c^(3/
4)*d*e^6 - 4*sqrt(a)*c^2*d^6*e - 12*a^(3/2)*c*d^2*e^5)*arctan(1/2*sqrt(2)*
(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sq
rt(sqrt(a)*sqrt(c))*c^(5/4)))/(a*c^2*d^8 + 2*a^2*c*d^4*e^4 + a^3*e^8) + 1/
4*(c*d*e^2*x^3 - c*d^2*e*x^2 + c*d^3*x + a*e^3)/(a^2*c*d^4 + a^3*e^4 + (a*
c^2*d^4 + a^2*c*e^4)*x^4)
```

3.405.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 795, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx = \frac{e^8\log\left(|ex+d|\right)}{c^2d^8e+2\,acd^4e^5+a^2e^9} - \frac{e^7\log\left(|cx^4+a|\right)}{4\left(c^2d^8+2\,acd^4e^4+a^2e^8\right)}$$
$$+ \frac{\left(4\,\sqrt{2}\sqrt{ac}c^2d^2e+3\left(ac^3\right)^{\frac14}c^2d^3+5\left(ac^3\right)^{\frac34}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{8\left(\sqrt{2}a^2c^3d^4+\sqrt{2}a^3c^2e^4+4\,\sqrt{2}\sqrt{ac}a^2c^2d^2e^2-4\left(ac^3\right)^{\frac14}a^2c^2d^3e-4\left(ac^3\right)^{\frac34}a^2de^3\right)}$$
$$+ \frac{\left(4\,\sqrt{2}\sqrt{ac}c^2d^2e+3\left(ac^3\right)^{\frac14}c^2d^3+5\left(ac^3\right)^{\frac34}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{8\left(\sqrt{2}a^2c^3d^4+\sqrt{2}a^3c^2e^4+4\,\sqrt{2}\sqrt{ac}a^2c^2d^2e^2+4\left(ac^3\right)^{\frac14}a^2c^2d^3e+4\left(ac^3\right)^{\frac34}a^2de^3\right)}$$
$$+ \frac{\left(3\,\sqrt{2}(ac^3)^{\frac14}c^3d^7+7\,\sqrt{2}(ac^3)^{\frac14}ac^2d^3e^4-\sqrt{2}(ac^3)^{\frac34}cd^5e^2-5\,\sqrt{2}(ac^3)^{\frac34}ade^6\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{}\right.}{32\left(a^2c^4d^8+2\,a^3c^3d^4e^4+a^4c^2e^8\right)}$$
$$- \frac{\left(3\,\sqrt{2}(ac^3)^{\frac14}c^3d^7+7\,\sqrt{2}(ac^3)^{\frac14}ac^2d^3e^4-\sqrt{2}(ac^3)^{\frac34}cd^5e^2-5\,\sqrt{2}(ac^3)^{\frac34}ade^6\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{}\right.}{32\left(a^2c^4d^8+2\,a^3c^3d^4e^4+a^4c^2e^8\right)}$$
$$+ \frac{acd^4e^3+a^2e^7+\left(c^2d^5e^2+acde^6\right)x^3-\left(c^2d^6e+acd^2e^5\right)x^2+\left(c^2d^7+acd^3e^4\right)x}{4\left(cd^4+ae^4\right)^2\left(cx^4+a\right)a}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
e^8*log(abs(e*x + d))/(c^2*d^8*e + 2*a*c*d^4*e^5 + a^2*e^9) - 1/4*e^7*log(
abs(c*x^4 + a))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) + 1/8*(4*sqrt(2)*sqrt(
a*c)*c^2*d^2*e + 3*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^4 +
 sqrt(2)*a^3*c^2*e^4 + 4*sqrt(2)*sqrt(a*c)*a^2*c^2*d^2*e^2 - 4*(a*c^3)^(1/
4)*a^2*c^2*d^3*e - 4*(a*c^3)^(3/4)*a^2*d*e^3) + 1/8*(4*sqrt(2)*sqrt(a*c)*c
^2*d^2*e + 3*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^4 + sqrt(
2)*a^3*c^2*e^4 + 4*sqrt(2)*sqrt(a*c)*a^2*c^2*d^2*e^2 + 4*(a*c^3)^(1/4)*a^2
*c^2*d^3*e + 4*(a*c^3)^(3/4)*a^2*d*e^3) + 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*c^
3*d^7 + 7*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^3*e^4 - sqrt(2)*(a*c^3)^(3/4)*c*d^
5*e^2 - 5*sqrt(2)*(a*c^3)^(3/4)*a*d*e^6)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) +
 sqrt(a/c))/(a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^2*e^8) - 1/32*(3*sqrt
(2)*(a*c^3)^(1/4)*c^3*d^7 + 7*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^3*e^4 - sqrt(2
)*(a*c^3)^(3/4)*c*d^5*e^2 - 5*sqrt(2)*(a*c^3)^(3/4)*a*d*e^6)*log(x^2 - sqr
t(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^2
*e^8) + 1/4*(a*c*d^4*e^3 + a^2*e^7 + (c^2*d^5*e^2 + a*c*d*e^6)*x^3 - (c^2*
d^6*e + a*c*d^2*e^5)*x^2 + (c^2*d^7 + a*c*d^3*e^4)*x)/((c*d^4 + a*e^4)^2*(
c*x^4 + a)*a)
```

3.405.9 Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 1591, normalized size of antiderivative = 1.86

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)^2*(d + e*x)),x)
```

output
```
e^3/(4*(a^2*e^4 + c^2*d^4*x^4 + a*c*d^4 + a*c*e^4*x^4)) + symsum(log((81*c
^5*d^5*e^6 + 64*a*c^4*d*e^10)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^
4)) + root(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^
8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1
152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(root(131
072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*
a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*
e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(root(131072*a^8*c*d^4*
e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 +
5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3
*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(root(131072*a^8*c*d^4*e^4*z^4 + 6553
6*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4
*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*
d^4 + 256*a*e^4, z, k)*((98304*a^9*c^4*d*e^14 - 32768*a^6*c^7*d^13*e^2 + 3
2768*a^7*c^6*d^9*e^6 + 163840*a^8*c^5*d^5*e^10)/(256*(a^6*e^8 + a^4*c^2*d^
8 + 2*a^5*c*d^4*e^4)) + (x*(81920*a^9*c^4*e^15 - 49152*a^6*c^7*d^12*e^3 -
16384*a^7*c^6*d^8*e^7 + 114688*a^8*c^5*d^4*e^11))/(256*(a^6*e^8 + a^4*c^2*
d^8 + 2*a^5*c*d^4*e^4))) + (52224*a^7*c^4*d*e^13 - 3072*a^4*c^7*d^13*e + 1
3312*a^5*c^6*d^9*e^5 + 68608*a^6*c^5*d^5*e^9)/(256*(a^6*e^8 + a^4*c^2*d^8
+ 2*a^5*c*d^4*e^4)) + (x*(61440*a^7*c^4*e^14 - 8192*a^4*c^7*d^12*e^2 - ...
```

3.405. $\int \frac{1}{(d+ex)(a+cx^4)^2}\,dx$

3.406 $\int \frac{1}{(d+ex)^2 (a+cx^4)^2} \, dx$

3.406.1 Optimal result

Integrand size = 17, antiderivative size = 1141

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^2}\,dx$$
$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)\,x+e^2(3cd^4-ae^4)\,x^2))}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$
$$- \frac{\sqrt{c}de^5(3cd^4-ae^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\left(cd^4+ae^4\right)^3} - \frac{\sqrt{c}de(cd^4-ae^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(cd^4+ae^4\right)^2}$$
$$- \frac{\sqrt[4]{c}\left(3\sqrt{c}d^2(cd^4-3ae^4)+\sqrt{a}e^2(3cd^4-ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}$$
$$- \frac{\sqrt[4]{c}e^4\left(\sqrt{c}d^2(5cd^4-3ae^4)+\sqrt{a}e^2(7cd^4-ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}$$
$$+ \frac{\sqrt[4]{c}\left(3\sqrt{c}d^2(cd^4-3ae^4)+\sqrt{a}e^2(3cd^4-ae^4)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}$$
$$+ \frac{\sqrt[4]{c}e^4\left(\sqrt{c}d^2(5cd^4-3ae^4)+\sqrt{a}e^2(7cd^4-ae^4)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}$$
$$+ \frac{8cd^3e^7\log(d+ex)}{\left(cd^4+ae^4\right)^3}$$
$$- \frac{\sqrt[4]{c}\left(3\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}$$
$$- \frac{\sqrt[4]{c}e^4\left(\sqrt{c}d^2(5cd^4-3ae^4)-\sqrt{a}e^2(7cd^4-ae^4)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}$$
$$+ \frac{\sqrt[4]{c}\left(3\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}$$
$$+ \frac{\sqrt[4]{c}e^4\left(\sqrt{c}d^2(5cd^4-3ae^4)-\sqrt{a}e^2(7cd^4-ae^4)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}$$
$$- \frac{2cd^3e^7\log\left(a+cx^4\right)}{\left(cd^4+ae^4\right)^3}$$

output
```
-e^7/(a*e^4+c*d^4)^2/(e*x+d)+1/4*c*(4*a*d^3*e^3+x*(d^2*(-3*a*e^4+c*d^4)-2*
d*e*(-a*e^4+c*d^4)*x+e^2*(-a*e^4+3*c*d^4)*x^2))/a/(a*e^4+c*d^4)^2/(c*x^4+a
)+8*c*d^3*e^7*ln(e*x+d)/(a*e^4+c*d^4)^3-2*c*d^3*e^7*ln(c*x^4+a)/(a*e^4+c*d
^4)^3-1/2*d*e*(-a*e^4+c*d^4)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(
a*e^4+c*d^4)^2-d*e^5*(-a*e^4+3*c*d^4)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/
(a*e^4+c*d^4)^3/a^(1/2)-1/32*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*c^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+c*d^4)*c^(1/2
))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/32*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/
2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+c*d
^4)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/16*c^(1/4)*arctan(-1+c^(1/4
)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+c*d^4)*
c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/16*c^(1/4)*arctan(1+c^(1/4)*x*2
^(1/2)/a^(1/4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+c*d^4)*c^(1/
2))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)-1/8*c^(1/4)*e^4*ln(-a^(1/4)*c^(1/4)*x*
2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+
5*c*d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/8*c^(1/4)*e^4*ln(a^(1/
4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d
^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(1/4)
*e^4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^
2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(1/...
```

3.406.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 807, normalized size of antiderivative = 0.71

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^2}\,dx$$

$$= \frac{-\frac{32e^7(cd^4+ae^4)}{d+ex} + \frac{8c(cd^4+ae^4)(cd^4x(d^2-2dex+3e^2x^2)+ae^3(4d^3-3d^2ex+2de^2x^2-e^3x^3))}{a(a+cx^4)} + \frac{2\sqrt[4]{c}\left(-3\sqrt{2}c^{5/2}d^{10}+8\sqrt[4]{a}c^{9/4}d^9e-3\sqrt{2}\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[1/((d + e*x)^2*(a + c*x^4)^2),x]
```

output
```
((-32*e^7*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(c*d^4*x*(d^2
- 2*d*e*x + 3*e^2*x^2) + a*e^3*(4*d^3 - 3*d^2*e*x + 2*d*e^2*x^2 - e^3*x^3)
))/(a*(a + c*x^4)) + (2*c^(1/4)*(-3*Sqrt[2]*c^(5/2)*d^10 + 8*a^(1/4)*c^(9/
4)*d^9*e - 3*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 - 14*Sqrt[2]*a*c^(3/2)*d^6*e^4 +
48*a^(5/4)*c^(5/4)*d^5*e^5 - 30*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 21*Sqrt[2]*a^2
*Sqrt[c]*d^2*e^8 - 24*a^(9/4)*c^(1/4)*d*e^9 + 5*Sqrt[2]*a^(5/2)*e^10)*ArcT
an[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*c^(1/4)*(3*Sqrt[2]*c^(5/
2)*d^10 + 8*a^(1/4)*c^(9/4)*d^9*e + 3*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 + 14*Sqr
t[2]*a*c^(3/2)*d^6*e^4 + 48*a^(5/4)*c^(5/4)*d^5*e^5 + 30*Sqrt[2]*a^(3/2)*c
*d^4*e^6 - 21*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 - 24*a^(9/4)*c^(1/4)*d*e^9 - 5*S
qrt[2]*a^(5/2)*e^10)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + 25
6*c*d^3*e^7*Log[d + e*x] - (Sqrt[2]*c^(1/4)*(3*c^(5/2)*d^10 - 3*Sqrt[a]*c^
2*d^8*e^2 + 14*a*c^(3/2)*d^6*e^4 - 30*a^(3/2)*c*d^4*e^6 - 21*a^2*Sqrt[c]*d
^2*e^8 + 5*a^(5/2)*e^10)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]
*x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(5/2)*d^10 - 3*Sqrt[a]*c^2*d^8*e^2
+ 14*a*c^(3/2)*d^6*e^4 - 30*a^(3/2)*c*d^4*e^6 - 21*a^2*Sqrt[c]*d^2*e^8 + 5
*a^(5/2)*e^10)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(
7/4) - 64*c*d^3*e^7*Log[a + c*x^4])/(32*(c*d^4 + a*e^4)^3)
```

3.406.3 Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 1141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^2 (d+ex)^2} dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(\frac{e^8}{(d+ex)^2 (ae^4+cd^4)^2}+\frac{8cd^3e^8}{(d+ex)(ae^4+cd^4)^3}+\frac{ce^4(-2dex(3cd^4-ae^4)+e^2x^2(7cd^4-ae^4)+d^2(5cd^4-3}{(a+cx^4)(ae^4+cd^4)^3}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{8cd^3\log(d+ex)e^7}{(cd^4+ae^4)^3}-\frac{2cd^3\log(cx^4+a)e^7}{(cd^4+ae^4)^3}-\frac{e^7}{(cd^4+ae^4)^2(d+ex)}-\frac{\sqrt{c}d(3cd^4-ae^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{\sqrt{a}(cd^4+ae^4)^3}-\frac{\sqrt[4]{c}\left(\sqrt{c}(5cd^4-3ae^4)d^2+\sqrt{a}e^2(7cd^4-ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^3}+\frac{\sqrt[4]{c}\left(\sqrt{c}(5cd^4-3ae^4)d^2+\sqrt{a}e^2(7cd^4-ae^4)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^3}-\frac{\sqrt[4]{c}\left(\sqrt{c}d^2(5cd^4-3ae^4)-\sqrt{a}e^2(7cd^4-ae^4)\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^3}+\frac{\sqrt[4]{c}\left(\sqrt{c}d^2(5cd^4-3ae^4)-\sqrt{a}e^2(7cd^4-ae^4)\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^3}-\frac{\sqrt{c}d(cd^4-ae^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{2a^{3/2}(cd^4+ae^4)^2}+\frac{c\left(4ad^3e^3+x\left((cd^4-3ae^4)d^2-2e(cd^4-ae^4)xd+e^2(3cd^4-ae^4)x^2\right)\right)}{4a(cd^4+ae^4)^2(cx^4+a)}-\frac{\sqrt[4]{c}\left(3\sqrt{c}(cd^4-3ae^4)d^2+\sqrt{a}e^2(3cd^4-ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^4+ae^4)^2}+\frac{\sqrt[4]{c}\left(3\sqrt{c}(cd^4-3ae^4)d^2+\sqrt{a}e^2(3cd^4-ae^4)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}(cd^4+ae^4)^2}-\frac{\sqrt[4]{c}\left(3\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}(cd^4+ae^4)^2}+\frac{\sqrt[4]{c}\left(3\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}(cd^4+ae^4)^2}$$

input
```
Int[1/((d + e*x)^2*(a + c*x^4)^2),x]
```

3.406. $\int \frac{1}{(d+ex)^2(a+cx^4)^2}\,dx$

output
```
-(e^7/((c*d^4 + a*e^4)^2*(d + e*x))) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3
*a*e^4) - 2*d*e*(c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d
^4 + a*e^4)^2*(a + c*x^4)) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*ArcTan[(Sqrt
[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^3) - (Sqrt[c]*d*e*(c*d^4 - a*e
^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^2) - (c^(1/4
)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan
[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) -
 (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*
e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 +
 a*e^4)^3) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*
d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[
a]*e^2*(7*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (8*c*d^3*e^7*Log[d + e*x])/(c*d^4 + a*e^4
)^3 - (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a
*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]
*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4
) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*(3*Sqrt[
c]*d^2*(c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] +...
```

3.406.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.406.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 534, normalized size of antiderivative = 0.47

method	result
default	$-\frac{c\left(\frac{\frac{e^2\left(a^2e^8-2acd^4e^4-3c^2d^8\right)x^3}{4a}-\frac{ed\left(a^2e^8-c^2d^8\right)x^2}{2a}+\frac{d^2\left(3a^2e^8+2acd^4e^4-c^2d^8\right)x}{4a}-d^3e^3\left(e^4a+d^4c\right)}{cx^4+a}+\frac{\left(21a^2d^2e^8-14acd^6e^4-3c^2d^{10}\right)\left(\right.}{}\right.}{}$
risch	$\frac{-\frac{e^3c\left(5e^4a-3d^4c\right)x^4}{4a\left(e^4a+d^4c\right)^2}+\frac{cde^2x^3}{4a\left(e^4a+d^4c\right)}-\frac{d^2cex^2}{4a\left(e^4a+d^4c\right)}+\frac{d^3cx}{4a\left(e^4a+d^4c\right)}-\frac{e^3\left(e^4a-d^4c\right)}{\left(e^4a+d^4c\right)^2}}{(ex+d)(cx^4+a)}+\frac{8d^3e^7c\ln(ex+d)}{a^3e^{12}+3a^2cd^4e^8+3ac^2d^8e^4+c^3d^{12}}+\left(\_R=\right.$

input
```
int(1/(e*x+d)^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-c/(a*e^4+c*d^4)^3*((1/4*e^2*(a^2*e^8-2*a*c*d^4*e^4-3*c^2*d^8)/a*x^3-1/2*e
*d*(a^2*e^8-c^2*d^8)/a*x^2+1/4*d^2*(3*a^2*e^8+2*a*c*d^4*e^4-c^2*d^8)/a*x-d
^3*e^3*(a*e^4+c*d^4))/(c*x^4+a)+1/4/a*(1/8*(21*a^2*d^2*e^8-14*a*c*d^6*e^4-
3*c^2*d^10)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/
2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*
x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/2*(-12*a^2*d*e^9+24*a*c*d^5*e^5+
4*c^2*d^9*e)/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*(5*a^2*e^10-30*a*c*d^
4*e^6-3*c^2*d^8*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+
(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/
c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+8*a*d^3*e^7*ln(c*x^4+a)))
-e^7/(a*e^4+c*d^4)^2/(e*x+d)+8*c*d^3*e^7*ln(e*x+d)/(a*e^4+c*d^4)^3
```

3.406.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.406.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**2/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.406.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 961, normalized size of antiderivative = 0.84

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
8*c*d^3*e^7*log(e*x + d)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a
^3*e^12) - 1/32*c*(sqrt(2)*(32*sqrt(2)*a^(7/4)*c^(5/4)*d^3*e^7 - 3*c^3*d^1
0 + 3*sqrt(a)*c^(5/2)*d^8*e^2 - 14*a*c^2*d^6*e^4 + 30*a^(3/2)*c^(3/2)*d^4*
e^6 + 21*a^2*c*d^2*e^8 - 5*a^(5/2)*sqrt(c)*e^10)*log(sqrt(c)*x^2 + sqrt(2)
*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(32*sqrt(2)*a^(7
/4)*c^(5/4)*d^3*e^7 + 3*c^3*d^10 - 3*sqrt(a)*c^(5/2)*d^8*e^2 + 14*a*c^2*d^
6*e^4 - 30*a^(3/2)*c^(3/2)*d^4*e^6 - 21*a^2*c*d^2*e^8 + 5*a^(5/2)*sqrt(c)*
e^10)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5
/4)) - 2*(3*sqrt(2)*a^(1/4)*c^(13/4)*d^10 + 3*sqrt(2)*a^(3/4)*c^(11/4)*d^8
*e^2 + 14*sqrt(2)*a^(5/4)*c^(9/4)*d^6*e^4 + 30*sqrt(2)*a^(7/4)*c^(7/4)*d^4
*e^6 - 21*sqrt(2)*a^(9/4)*c^(5/4)*d^2*e^8 - 5*sqrt(2)*a^(11/4)*c^(3/4)*e^1
0 + 8*sqrt(a)*c^3*d^9*e + 48*a^(3/2)*c^2*d^5*e^5 - 24*a^(5/2)*c*d*e^9)*arc
tan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(
c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(3*sqrt(2)*a^(1/4)*c^(13/
4)*d^10 + 3*sqrt(2)*a^(3/4)*c^(11/4)*d^8*e^2 + 14*sqrt(2)*a^(5/4)*c^(9/4)*
d^6*e^4 + 30*sqrt(2)*a^(7/4)*c^(7/4)*d^4*e^6 - 21*sqrt(2)*a^(9/4)*c^(5/4)*
d^2*e^8 - 5*sqrt(2)*a^(11/4)*c^(3/4)*e^10 - 8*sqrt(a)*c^3*d^9*e - 48*a^(3/
2)*c^2*d^5*e^5 + 24*a^(5/2)*c*d*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqr
t(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c)
)*c^(5/4)))/(a*c^3*d^12 + 3*a^2*c^2*d^8*e^4 + 3*a^3*c*d^4*e^8 + a^4*e^1...
```

3.406.8 Giac [A] (verification not implemented)

Time = 6.61 (sec) , antiderivative size = 1145, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
8*c*d^3*e^8*log(abs(e*x + d))/(c^3*d^12*e + 3*a*c^2*d^8*e^5 + 3*a^2*c*d^4*
e^9 + a^3*e^13) - 2*c*d^3*e^7*log(abs(c*x^4 + a))/(c^3*d^12 + 3*a*c^2*d^8*
e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + 1/8*(3*sqrt(2)*a*c^2*d*e^3 + 5*sqrt(2)
*sqrt(a*c)*c^2*d^3*e + 3*(a*c^3)^(1/4)*c^2*d^4 - 5*(a*c^3)^(1/4)*a*c*e^4 +
 6*(a*c^3)^(3/4)*d^2*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^6 + 9*sqrt(2)*a^3*c^2*d^2*e^4 + 9*sqrt(2)*s
qrt(a*c)*a^2*c^2*d^4*e^2 + sqrt(2)*sqrt(a*c)*a^3*c*e^6 - 6*(a*c^3)^(1/4)*a
^2*c^2*d^5*e - 6*(a*c^3)^(1/4)*a^3*c*d*e^5 - 16*(a*c^3)^(3/4)*a^2*d^3*e^3)
 - 1/8*(3*sqrt(2)*a*c^2*d*e^3 - 5*sqrt(2)*sqrt(a*c)*c^2*d^3*e - 3*(a*c^3)^
(1/4)*c^2*d^4 + 5*(a*c^3)^(1/4)*a*c*e^4 - 6*(a*c^3)^(3/4)*d^2*e^2)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^6
+ 9*sqrt(2)*a^3*c^2*d^2*e^4 + 9*sqrt(2)*sqrt(a*c)*a^2*c^2*d^4*e^2 + sqrt(2
)*sqrt(a*c)*a^3*c*e^6 + 6*(a*c^3)^(1/4)*a^2*c^2*d^5*e + 6*(a*c^3)^(1/4)*a^
3*c*d*e^5 + 16*(a*c^3)^(3/4)*a^2*d^3*e^3) + 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*
c^4*d^10 + 14*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^6*e^4 - 21*sqrt(2)*(a*c^3)^(1/
4)*a^2*c^2*d^2*e^8 - 3*sqrt(2)*(a*c^3)^(3/4)*c^2*d^8*e^2 - 30*sqrt(2)*(a*c
^3)^(3/4)*a*c*d^4*e^6 + 5*sqrt(2)*(a*c^3)^(3/4)*a^2*e^10)*log(x^2 + sqrt(2
)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^5*d^12 + 3*a^3*c^4*d^8*e^4 + 3*a^4*c^3
*d^4*e^8 + a^5*c^2*e^12) - 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*c^4*d^10 + 14*sqr
t(2)*(a*c^3)^(1/4)*a*c^3*d^6*e^4 - 21*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^2...
```

3.406.9 Mupad [B] (verification not implemented)

Time = 10.59 (sec) , antiderivative size = 2246, normalized size of antiderivative = 1.97

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)^2*(d + e*x)^2),x)
```

output

```
symsum(log(root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65
536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 18
1248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z
+ 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((120*a*c^8*d^14*e
^3 + 2664*a^2*c^7*d^10*e^7 - 10904*a^3*c^6*d^6*e^11 + 19320*a^4*c^5*d^2*e^
15)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4
+ 6*a^6*c^2*d^8*e^8)) + root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8
*e^4*z^4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3
*e^7*z^3 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2
*c^2*d^5*e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((409
6*a^3*c^8*d^15*e^4 + 54272*a^4*c^7*d^11*e^8 - 2048*a^5*c^6*d^7*e^12 + 1443
84*a^6*c^5*d^3*e^16)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*
a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + root(196608*a^9*c*d^4*e^8*z^4 + 1
96608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 +
 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e
^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2
*d^4, z, k)*(root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 +
65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 +
181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*
z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((98304*a^11*...
```

3.406. $\int \frac{1}{(d+ex)^2(a+cx^4)^2}\,dx$

3.407 $\int \frac{1}{(d+ex)^3 (a+cx^4)^2} \, dx$

3.407.1 Optimal result

Integrand size = 17, antiderivative size = 1384

$$
\begin{aligned}
\int \frac{1}{(d+ex)^3\left(a+cx^4\right)^2}\,dx &= -\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2}-\frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)} \\
&+\frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d(c^2d^8-12acd^4e^4+3a^2e^8)-e(3c^2d^8-12acd^4e^4+a^2e^8)\,x+2cd^3e^2(3cd^4-}{4a\left(cd^4+ae^4\right)^3\left(a+cx^4\right)} \\
&-\frac{\sqrt{c}e^5(21c^2d^8-26acd^4e^4+a^2e^8)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^4} \\
&-\frac{\sqrt{c}e(3c^2d^8-12acd^4e^4+a^2e^8)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^4+ae^4\right)^3} \\
&-\frac{c^{3/4}d\left(3c^2d^8-36acd^4e^4+9a^2e^8+2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3} \\
&-\frac{c^{3/4}de^4\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)+3(5c^2d^8-10acd^4e^4+a^2e^8)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4} \\
&+\frac{c^{3/4}d\left(3c^2d^8-36acd^4e^4+9a^2e^8+2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3} \\
&+\frac{c^{3/4}de^4\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)+3(5c^2d^8-10acd^4e^4+a^2e^8)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4} \\
&+\frac{12cd^2e^7(3cd^4-ae^4)\log(d+ex)}{\left(cd^4+ae^4\right)^4} \\
&-\frac{c^{3/4}d\left(3c^2d^8-36acd^4e^4+9a^2e^8-2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3} \\
&+\frac{c^{3/4}de^4\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)-3(5c^2d^8-10acd^4e^4+a^2e^8)\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4} \\
&+\frac{c^{3/4}d\left(3c^2d^8-36acd^4e^4+9a^2e^8-2\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3} \\
&-\frac{c^{3/4}de^4\left(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)-3(5c^2d^8-10acd^4e^4+a^2e^8)\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4} \\
&-\frac{3cd^2e^7(3cd^4-ae^4)\log\left(a+cx^4\right)}{\left(cd^4+ae^4\right)^4}
\end{aligned}
$$

output
```
-1/2*e^7/(a*e^4+c*d^4)^2/(e*x+d)^2-8*c*d^3*e^7/(a*e^4+c*d^4)^3/(e*x+d)+1/4
*c*(2*a*d^2*e^3*(-3*a*e^4+5*c*d^4)+x*(d*(3*a^2*e^8-12*a*c*d^4*e^4+c^2*d^8)
-e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*x+2*c*d^3*e^2*(-5*a*e^4+3*c*d^4)*x^2
))/a/(a*e^4+c*d^4)^3/(c*x^4+a)+12*c*d^2*e^7*(-a*e^4+3*c*d^4)*ln(e*x+d)/(a*
e^4+c*d^4)^4-3*c*d^2*e^7*(-a*e^4+3*c*d^4)*ln(c*x^4+a)/(a*e^4+c*d^4)^4-1/4*
e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a
^(3/2)/(a*e^4+c*d^4)^3-1/2*e^5*(a^2*e^8-26*a*c*d^4*e^4+21*c^2*d^8)*arctan(
x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^4/a^(1/2)-1/32*c^(3/4)*d*ln(-a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c^2*d^8-36*a*c*d^4*e^4+9*a
^2*e^8-2*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)
^3*2^(1/2)+1/32*c^(3/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2)
)*(3*c^2*d^8-36*a*c*d^4*e^4+9*a^2*e^8-2*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)
*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c^(3/4)*d*arctan(-1+c^(1/4)
*x*2^(1/2)/a^(1/4))*(3*c^2*d^8-36*a*c*d^4*e^4+9*a^2*e^8+2*d^2*e^2*(-5*a*e^
4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c^(3/4)*d
*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*c^2*d^8-36*a*c*d^4*e^4+9*a^2*e^8+2
*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/
2)+1/8*c^(3/4)*d*e^4*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-
3*a^2*e^8+30*a*c*d^4*e^4-15*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2)*c
^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)-1/8*c^(3/4)*d*e^4*ln(a^(1/4)*c^...
```

3.407.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 996, normalized size of antiderivative = 0.72

$$\int \frac{1}{(d+ex)^3 \left(a+cx^4\right)^2} \, dx$$

$$= \frac{-\frac{16e^7\left(cd^4+ae^4\right)^2}{(d+ex)^2} - \frac{256cd^3e^7\left(cd^4+ae^4\right)}{d+ex} + \frac{8c\left(cd^4+ae^4\right)\left(-a^2e^7\left(6d^2-3dex+e^2x^2\right)+c^2d^7x\left(d^2-3dex+6e^2x^2\right)+2acd^3e^3\left(5d^3-6d^2ex+6de^2x\ldots\right.\right.}{a\left(a+cx^4\right)}}{\ldots}$$

input
```
Integrate[1/((d + e*x)^3*(a + c*x^4)^2),x]
```

output
```
((-16*e^7*(c*d^4 + a*e^4)^2)/(d + e*x)^2 - (256*c*d^3*e^7*(c*d^4 + a*e^4))
/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(-(a^2*e^7*(6*d^2 - 3*d*e*x + e^2*x^2))
+ c^2*d^7*x*(d^2 - 3*d*e*x + 6*e^2*x^2) + 2*a*c*d^3*e^3*(5*d^3 - 6*d^2*e*x
 + 6*d*e^2*x^2 - 5*e^3*x^3)))/(a*(a + c*x^4)) - (6*Sqrt[c]*(Sqrt[2]*c^(13/
4)*d^13 - 4*a^(1/4)*c^3*d^12*e + 2*Sqrt[2]*Sqrt[a]*c^(11/4)*d^11*e^2 + 9*S
qrt[2]*a*c^(9/4)*d^9*e^4 - 44*a^(5/4)*c^2*d^8*e^5 + 36*Sqrt[2]*a^(3/2)*c^(
7/4)*d^7*e^6 - 49*Sqrt[2]*a^2*c^(5/4)*d^5*e^8 + 84*a^(9/4)*c*d^4*e^9 - 30*
Sqrt[2]*a^(5/2)*c^(3/4)*d^3*e^10 + 7*Sqrt[2]*a^3*c^(1/4)*d*e^12 - 4*a^(13/
4)*e^13)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (6*Sqrt[c]*(Sq
rt[2]*c^(13/4)*d^13 + 4*a^(1/4)*c^3*d^12*e + 2*Sqrt[2]*Sqrt[a]*c^(11/4)*d^
11*e^2 + 9*Sqrt[2]*a*c^(9/4)*d^9*e^4 + 44*a^(5/4)*c^2*d^8*e^5 + 36*Sqrt[2]
*a^(3/2)*c^(7/4)*d^7*e^6 - 49*Sqrt[2]*a^2*c^(5/4)*d^5*e^8 - 84*a^(9/4)*c*d
^4*e^9 - 30*Sqrt[2]*a^(5/2)*c^(3/4)*d^3*e^10 + 7*Sqrt[2]*a^3*c^(1/4)*d*e^1
2 + 4*a^(13/4)*e^13)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + 38
4*c*d^2*e^7*(3*c*d^4 - a*e^4)*Log[d + e*x] - (3*Sqrt[2]*c^(3/4)*(c^3*d^13
- 2*Sqrt[a]*c^(5/2)*d^11*e^2 + 9*a*c^2*d^9*e^4 - 36*a^(3/2)*c^(3/2)*d^7*e^
6 - 49*a^2*c*d^5*e^8 + 30*a^(5/2)*Sqrt[c]*d^3*e^10 + 7*a^3*d*e^12)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) + (3*Sqrt[2]*c^(3
/4)*(c^3*d^13 - 2*Sqrt[a]*c^(5/2)*d^11*e^2 + 9*a*c^2*d^9*e^4 - 36*a^(3/2)*
c^(3/2)*d^7*e^6 - 49*a^2*c*d^5*e^8 + 30*a^(5/2)*Sqrt[c]*d^3*e^10 + 7*a^...
```

3.407.3 Rubi [A] (verified)

Time = 2.36 (sec) , antiderivative size = 1384, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^2 (d+ex)^3} \, dx$$

$$\big\downarrow \text{7293}$$

$$\int \left(\frac{ce^4\left(-ex\left(a^2e^8-26acd^4e^4+21c^2d^8\right)+3d\left(a^2e^8-10acd^4e^4+5c^2d^8\right)+4cd^3e^2x^2\left(7cd^4-5ae^4\right)-12cd^2e^3x^3(3\right.}{(a+cx^4)\left(ae^4+cd^4\right)^4}\right.$$

$$\big\downarrow \text{2009}$$

$$\frac{12cd^2\left(3cd^4-ae^4\right)\log(d+ex)e^7}{\left(cd^4+ae^4\right)^4}-\frac{3cd^2\left(3cd^4-ae^4\right)\log\left(cx^4+a\right)e^7}{\left(cd^4+ae^4\right)^4}-\frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}-$$
$$\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2}-\frac{\sqrt{c}\left(21c^2d^8-26ace^4d^4+a^2e^8\right)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{2\sqrt{a}\left(cd^4+ae^4\right)^4}-$$
$$\frac{c^{3/4}d\left(4\sqrt{a}\sqrt{c}d^2\left(7cd^4-5ae^4\right)e^2+3\left(5c^2d^8-10ace^4d^4+a^2e^8\right)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4}+$$
$$\frac{c^{3/4}d\left(4\sqrt{a}\sqrt{c}d^2\left(7cd^4-5ae^4\right)e^2+3\left(5c^2d^8-10ace^4d^4+a^2e^8\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4}+$$
$$\frac{c^{3/4}d\left(4\sqrt{a}\sqrt{c}d^2e^2\left(7cd^4-5ae^4\right)-3\left(5c^2d^8-10ace^4d^4+a^2e^8\right)\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4}-$$
$$\frac{c^{3/4}d\left(4\sqrt{a}\sqrt{c}d^2e^2\left(7cd^4-5ae^4\right)-3\left(5c^2d^8-10ace^4d^4+a^2e^8\right)\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^4}-$$
$$\frac{\sqrt{c}\left(3c^2d^8-12ace^4d^4+a^2e^8\right)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{4a^{3/2}\left(cd^4+ae^4\right)^3}+$$
$$\frac{c\left(2ad^2\left(5cd^4-3ae^4\right)e^3+x\left(2ce^2\left(3cd^4-5ae^4\right)x^2d^3+\left(c^2d^8-12ace^4d^4+3a^2e^8\right)d-e\left(3c^2d^8-12ace^4d^4+a^2e^8\right)\right.\right.}{4a\left(cd^4+ae^4\right)^3\left(cx^4+a\right)}$$
$$\frac{c^{3/4}d\left(3c^2d^8-36ace^4d^4+2\sqrt{a}\sqrt{c}e^2\left(3cd^4-5ae^4\right)d^2+9a^2e^8\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3}+$$
$$\frac{c^{3/4}d\left(3c^2d^8-36ace^4d^4+2\sqrt{a}\sqrt{c}e^2\left(3cd^4-5ae^4\right)d^2+9a^2e^8\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3}-$$
$$\frac{c^{3/4}d\left(3c^2d^8-36ace^4d^4-2\sqrt{a}\sqrt{c}e^2\left(3cd^4-5ae^4\right)d^2+9a^2e^8\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3}+$$
$$\frac{c^{3/4}d\left(3c^2d^8-36ace^4d^4-2\sqrt{a}\sqrt{c}e^2\left(3cd^4-5ae^4\right)d^2+9a^2e^8\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^3}$$

input
```
Int[1/((d + e*x)^3*(a + c*x^4)^2),x]
```

3.407. $\int \frac{1}{(d+ex)^3(a+cx^4)^2}\,dx$

output
```
-1/2*e^7/((c*d^4 + a*e^4)^2*(d + e*x)^2) - (8*c*d^3*e^7)/((c*d^4 + a*e^4)^
3*(d + e*x)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e^4) + x*(d*(c^2*d^8 - 12*a*
c*d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 2*c*
d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^4)) - (
Sqrt[c]*e^5*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/S
qrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^
4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a*e^4)
^3) - (c^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*Sqrt[
c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(
8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d
^2*e^2*(7*c*d^4 - 5*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Arc
Tan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4
) + (c^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]
*d^2*e^2*(3*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*
Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2
*e^2*(7*c*d^4 - 5*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTa
n[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4)
+ (12*c*d^2*e^7*(3*c*d^4 - a*e^4)*Log[d + e*x])/(c*d^4 + a*e^4)^4 - (c^(3/
4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(
3*c*d^4 - 5*a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x...
```

3.407.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.407. $\int \frac{1}{(d+ex)^3(a+cx^4)^2}\,dx$

3.407.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 680, normalized size of antiderivative = 0.49

method	result
default	$c\left(\frac{-\frac{c\,d^3e^2\left(5a^2e^8+2ac\,d^4e^4-3c^2d^8\right)x^3}{2a}-\frac{e\left(a^3e^{12}-11a^2c\,d^4e^8-9a\,c^2d^8e^4+3c^3d^{12}\right)x^2}{4a}+\frac{d\left(3a^3e^{12}-9a^2c\,d^4e^8-11a\,c^2d^8e^4+c^3d^{12}\right)x}{4a}-\frac{d^2e^3\left(3c\right.}{}}{c\,x^4+a}\right.$
risch	Expression too large to display

input
```
int(1/(e*x+d)^3/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
c/(a*e^4+c*d^4)^4*((-1/2*c*d^3*e^2*(5*a^2*e^8+2*a*c*d^4*e^4-3*c^2*d^8)/a*x
^3-1/4*e*(a^3*e^12-11*a^2*c*d^4*e^8-9*a*c^2*d^8*e^4+3*c^3*d^12)/a*x^2+1/4*
d*(3*a^3*e^12-9*a^2*c*d^4*e^8-11*a*c^2*d^8*e^4+c^3*d^12)/a*x-1/2*d^2*e^3*(
3*a^2*e^8-2*a*c*d^4*e^4-5*c^2*d^8))/(c*x^4+a)+3/4/a*(1/8*(7*a^3*d*e^12-49*
a^2*c*d^5*e^8+9*a*c^2*d^9*e^4+c^3*d^13)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/
c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2
*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/2*(-
2*a^3*e^13+42*a^2*c*d^4*e^9-22*a*c^2*d^8*e^5-2*c^3*d^12*e)/(a*c)^(1/2)*arc
tan(x^2*(c/a)^(1/2))+1/8*(-30*a^2*c*d^3*e^10+36*a*c^2*d^7*e^6+2*c^3*d^11*e
^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2
+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*a
rctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/4*(16*a^2*c*d^2*e^11-48*a*c^2*d^6*e^7)/c
*ln(c*x^4+a)))-1/2*e^7/(a*e^4+c*d^4)^2/(e*x+d)^2-8*c*d^3*e^7/(a*e^4+c*d^4)
^3/(e*x+d)-12*e^7*c*d^2*(a*e^4-3*c*d^4)/(a*e^4+c*d^4)^4*ln(e*x+d)
```

3.407.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\left(a+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.407.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3 (a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**3/(c*x**4+a)**2,x)
```

output
```
Timed out
```

3.407.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1394, normalized size of antiderivative = 1.01

$$\int \frac{1}{(d+ex)^3 (a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-3/32*c*(sqrt(2)*(48*sqrt(2)*a^(7/4)*c^(9/4)*d^6*e^7 - 16*sqrt(2)*a^(11/4)
*c^(5/4)*d^2*e^11 - c^4*d^13 + 2*sqrt(a)*c^(7/2)*d^11*e^2 - 9*a*c^3*d^9*e^
4 + 36*a^(3/2)*c^(5/2)*d^7*e^6 + 49*a^2*c^2*d^5*e^8 - 30*a^(5/2)*c^(3/2)*d
^3*e^10 - 7*a^3*c*d*e^12)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sq
rt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(48*sqrt(2)*a^(7/4)*c^(9/4)*d^6*e^7 - 1
6*sqrt(2)*a^(11/4)*c^(5/4)*d^2*e^11 + c^4*d^13 - 2*sqrt(a)*c^(7/2)*d^11*e^
2 + 9*a*c^3*d^9*e^4 - 36*a^(3/2)*c^(5/2)*d^7*e^6 - 49*a^2*c^2*d^5*e^8 + 30
*a^(5/2)*c^(3/2)*d^3*e^10 + 7*a^3*c*d*e^12)*log(sqrt(c)*x^2 - sqrt(2)*a^(1
/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(17/4)*d
^13 + 2*sqrt(2)*a^(3/4)*c^(15/4)*d^11*e^2 + 9*sqrt(2)*a^(5/4)*c^(13/4)*d^9
*e^4 + 36*sqrt(2)*a^(7/4)*c^(11/4)*d^7*e^6 - 49*sqrt(2)*a^(9/4)*c^(9/4)*d^
5*e^8 - 30*sqrt(2)*a^(11/4)*c^(7/4)*d^3*e^10 + 7*sqrt(2)*a^(13/4)*c^(5/4)*
d*e^12 + 4*sqrt(a)*c^4*d^12*e + 44*a^(3/2)*c^3*d^8*e^5 - 84*a^(5/2)*c^2*d^
4*e^9 + 4*a^(7/2)*c*e^13)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4
)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4))
- 2*(sqrt(2)*a^(1/4)*c^(17/4)*d^13 + 2*sqrt(2)*a^(3/4)*c^(15/4)*d^11*e^2 +
 9*sqrt(2)*a^(5/4)*c^(13/4)*d^9*e^4 + 36*sqrt(2)*a^(7/4)*c^(11/4)*d^7*e^6
- 49*sqrt(2)*a^(9/4)*c^(9/4)*d^5*e^8 - 30*sqrt(2)*a^(11/4)*c^(7/4)*d^3*e^1
0 + 7*sqrt(2)*a^(13/4)*c^(5/4)*d*e^12 - 4*sqrt(a)*c^4*d^12*e - 44*a^(3/2)*
c^3*d^8*e^5 + 84*a^(5/2)*c^2*d^4*e^9 - 4*a^(7/2)*c*e^13)*arctan(1/2*sqr...
```

3.407.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 1557, normalized size of antiderivative = 1.12

$$\int \frac{1}{(d+ex)^3 (a+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
3/8*(4*sqrt(2)*a*c^3*d^2*e^3 + 2*sqrt(2)*sqrt(a*c)*c^3*d^4*e + 2*sqrt(2)*s
qrt(a*c)*a*c^2*e^5 + (a*c^3)^(1/4)*c^3*d^5 - 9*(a*c^3)^(1/4)*a*c^2*d*e^4 +
 2*(a*c^3)^(3/4)*c*d^3*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))
/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^8 + 34*sqrt(2)*a^3*c^3*d^4*e^4 + sqrt(2)*
a^4*c^2*e^8 + 16*sqrt(2)*sqrt(a*c)*a^2*c^3*d^6*e^2 + 16*sqrt(2)*sqrt(a*c)*
a^3*c^2*d^2*e^6 - 8*(a*c^3)^(1/4)*a^2*c^3*d^7*e - 40*(a*c^3)^(1/4)*a^3*c^2
*d^3*e^5 - 40*(a*c^3)^(3/4)*a^2*c*d^5*e^3 - 8*(a*c^3)^(3/4)*a^3*d*e^7) - 3
/8*(4*sqrt(2)*a*c^3*d^2*e^3 - 2*sqrt(2)*sqrt(a*c)*c^3*d^4*e - 2*sqrt(2)*sq
rt(a*c)*a*c^2*e^5 - (a*c^3)^(1/4)*c^3*d^5 + 9*(a*c^3)^(1/4)*a*c^2*d*e^4 -
2*(a*c^3)^(3/4)*c*d^3*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^8 + 34*sqrt(2)*a^3*c^3*d^4*e^4 + sqrt(2)*a
^4*c^2*e^8 + 16*sqrt(2)*sqrt(a*c)*a^2*c^3*d^6*e^2 + 16*sqrt(2)*sqrt(a*c)*a
^3*c^2*d^2*e^6 + 8*(a*c^3)^(1/4)*a^2*c^3*d^7*e + 40*(a*c^3)^(1/4)*a^3*c^2*
d^3*e^5 + 40*(a*c^3)^(3/4)*a^2*c*d^5*e^3 + 8*(a*c^3)^(3/4)*a^3*d*e^7) + 3/
32*(sqrt(2)*(a*c^3)^(1/4)*c^4*d^13 + 9*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^9*e^4
 - 49*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^5*e^8 + 7*sqrt(2)*(a*c^3)^(1/4)*a^3*
c*d*e^12 - 2*sqrt(2)*(a*c^3)^(3/4)*c^2*d^11*e^2 - 36*sqrt(2)*(a*c^3)^(3/4)
*a*c*d^7*e^6 + 30*sqrt(2)*(a*c^3)^(3/4)*a^2*d^3*e^10)*log(x^2 + sqrt(2)*x*
(a/c)^(1/4) + sqrt(a/c))/(a^2*c^5*d^16 + 4*a^3*c^4*d^12*e^4 + 6*a^4*c^3*d^
8*e^8 + 4*a^5*c^2*d^4*e^12 + a^6*c*e^16) - 3/32*(sqrt(2)*(a*c^3)^(1/4)*...
```

3.407.9 Mupad [B] (verification not implemented)

Time = 10.85 (sec) , antiderivative size = 3256, normalized size of antiderivative = 2.35

$$\int \frac{1}{(d+ex)^3 (a+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)^2*(d + e*x)^3),x)
```

output
```
symsum(log(root(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*e^8*z^4 +
262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11*e^16*z^4
 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712*a^5*c^
2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 + 58752*a
^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4, z,
k)*((108*a*c^10*d^19*e^3 + 3888*a^2*c^9*d^15*e^7 - 99576*a^3*c^8*d^11*e^11
 + 591408*a^4*c^7*d^7*e^15 - 79380*a^5*c^6*d^3*e^19)/(256*(a^10*e^24 + a^4
*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 +
20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16)) + root(262144*a^10*c*d^4*e^12
*z^4 + 393216*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^
7*c^4*d^16*z^4 + 65536*a^11*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296
*a^7*c^2*d^6*e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*
z^2 + 18432*a^6*c*e^10*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*
z + 1296*a*c^2*e^4 + 81*c^3*d^4, z, k)*((6912*a^8*c^5*d*e^24 + 4608*a^3*c^
10*d^21*e^4 + 154368*a^4*c^9*d^17*e^8 - 331776*a^5*c^8*d^13*e^12 + 5976576
*a^6*c^7*d^9*e^16 - 612864*a^7*c^6*d^5*e^20)/(256*(a^10*e^24 + a^4*c^6*d^2
4 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c
^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16)) + root(262144*a^10*c*d^4*e^12*z^4 + 3
93216*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^
16*z^4 + 65536*a^11*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*...
```

3.407. $\int \frac{1}{(d+ex)^3(a+cx^4)^2}\,dx$

3.408 $\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx$

3.408.1 Optimal result

Integrand size = 17, antiderivative size = 394

$$\begin{aligned}\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx &= \frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2(a+cx^4)} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2} \\ &\quad + \frac{9d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{3d(7\sqrt{c}d^2+5\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} \\ &\quad + \frac{3d(7\sqrt{c}d^2+5\sqrt{a}e^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} \\ &\quad - \frac{3d(7\sqrt{c}d^2-5\sqrt{a}e^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}} \\ &\quad + \frac{3d(7\sqrt{c}d^2-5\sqrt{a}e^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}\end{aligned}$$

output
```
1/32*x*(15*d*e^2*x^2+18*d^2*e*x+7*d^3)/a^2/(c*x^4+a)+1/8*(-a*e^3+c*x*(3*d*
e^2*x^2+3*d^2*e*x+d^3))/a/c/(c*x^4+a)^2+9/16*d^2*e*arctan(x^2*c^(1/2)/a^(1
/2))/a^(5/2)/c^(1/2)-3/256*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(
1/2))*(-5*e^2*a^(1/2)+7*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^(1/2)+3/256*d*ln(a
^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-5*e^2*a^(1/2)+7*d^2*c^(1/2
))/a^(11/4)/c^(3/4)*2^(1/2)+3/128*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(
5*e^2*a^(1/2)+7*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^(1/2)+3/128*d*arctan(1+c^(
1/4)*x*2^(1/2)/a^(1/4))*(5*e^2*a^(1/2)+7*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^(
1/2)
```

3.408.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx$$
$$= \frac{\frac{8adx(7d^2+18dex+15e^2x^2)}{a+cx^4} - \frac{32a^2(ae^3-cdx(d^2+3dex+3e^2x^2))}{c(a+cx^4)^2} - \frac{6\sqrt[4]{a}d\left(7\sqrt{2}\sqrt{c}d^2+24\sqrt[4]{a}\sqrt[4]{c}de+5\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}} +}{}$$

input
```
Integrate[(d + e*x)^3/(a + c*x^4)^3,x]
```

output
```
((8*a*d*x*(7*d^2 + 18*d*e*x + 15*e^2*x^2))/(a + c*x^4) - (32*a^2*(a*e^3 -
c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(c*(a + c*x^4)^2) - (6*a^(1/4)*d*(7*Sq
rt[2]*Sqrt[c]*d^2 + 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan
[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (6*a^(1/4)*d*(7*Sqrt[2]*Sqrt[
c]*d^2 - 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[
2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (3*Sqrt[2]*(-7*a^(1/4)*Sqrt[c]*d^3 + 5*a
^(3/4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3
/4) + (3*Sqrt[2]*(7*a^(1/4)*Sqrt[c]*d^3 - 5*a^(3/4)*d*e^2)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(256*a^3)
```

3.408.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2393, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx$$
$$\Big\downarrow \text{ 2393}$$
$$-\frac{\int -\frac{7d^3+18exd^2+15e^2x^2d}{(cx^4+a)^2}dx}{8a} - \frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2}$$
$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{7d^3+18exd^2+15e^2x^2d}{(cx^4+a)^2}dx}{8a} - \frac{ae^3 - cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2}$$

$$\Big\downarrow \; 2394$$

$$\frac{\frac{x(7d^3+18d^2ex+15de^2x^2)}{4a(a+cx^4)} - \frac{\int -\frac{3(7d^3+12exd^2+5e^2x^2d)}{cx^4+a}dx}{4a}}{8a} - \frac{ae^3 - cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{7d^3+12exd^2+5e^2x^2d}{cx^4+a}dx}{4a} + \frac{x(7d^3+18d^2ex+15de^2x^2)}{4a(a+cx^4)}}{8a} - \frac{ae^3 - cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2}$$

$$\Big\downarrow \; 2415$$

$$\frac{\frac{3\int \left(\frac{12exd^2}{cx^4+a}+\frac{7d^3+5e^2x^2d}{cx^4+a}\right)dx}{4a} + \frac{x(7d^3+18d^2ex+15de^2x^2)}{4a(a+cx^4)}}{8a} - \frac{ae^3 - cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{3\left(-\frac{d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}e^2+7\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}e^2+7\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}} - \frac{d\left(7\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{d\left(7\sqrt{c}d^2-5\right.}{}}{4a}}{8a}$$

$$\frac{ae^3 - cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2}$$

input
```
Int[(d + e*x)^3/(a + c*x^4)^3,x]
```

output
```
-1/8*(a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(a*c*(a + c*x^4)^2) + (
(x*(7*d^3 + 18*d^2*e*x + 15*d*e^2*x^2))/(4*a*(a + c*x^4)) + (3*((6*d^2*e*A
rcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - (d*(7*Sqrt[c]*d^2 + 5*Sq
rt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(
3/4)) + (d*(7*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - (d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4
)*c^(3/4)) + (d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))))/(4*a))/(8*a)
```

3.408. $\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx$

3.408.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.408.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.81 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.36

method	result
risch	$\frac{\frac{15cd\,e^2x^7}{32a^2}+\frac{9c\,d^2e\,x^6}{16a^2}+\frac{7d^3c\,x^5}{32a^2}+\frac{27d\,e^2x^3}{32a}+\frac{15e\,x^2d^2}{16a}+\frac{11d^3x}{32a}-\frac{e^3}{8c}}{(c\,x^4+a)^2}+\frac{3d\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+a)}\frac{\left(5e^2\_R^2+12ed\_R+7d^2\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{128a^2c}$
default	$d^3\left(\frac{x}{8a(c\,x^4+a)^2}+\frac{\frac{7x}{32a(c\,x^4+a)}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}\right)+3d^2e$

input
```
int((e*x+d)^3/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(15/32*c*d*e^2/a^2*x^7+9/16*c*d^2*e/a^2*x^6+7/32*d^3*c/a^2*x^5+27/32*d*e^2
/a*x^3+15/16*e/a*x^2*d^2+11/32*d^3/a*x-1/8*e^3/c)/(c*x^4+a)^2+3/128/a^2*d/
c*sum((5*_R^2*e^2+12*_R*d*e+7*d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

3.408.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.23 (sec) , antiderivative size = 95566, normalized size of antiderivative = 242.55

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.408.6 Sympy [A] (verification not implemented)

Time = 131.88 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx$$
$$= \mathrm{RootSum}\left(268435456t^4a^{11}c^3 + 63111168t^2a^6c^2d^4e^2 + t\left(4147200a^4cd^4e^5 - 8128512a^3c^2d^8e\right) + 50625a^2d^4\right.$$
$$+ \frac{-4a^2e^3 + 11acd^3x + 30acd^2ex^2 + 27acde^2x^3 + 7c^2d^3x^5 + 18c^2d^2ex^6 + 15c^2de^2x^7}{32a^4c + 64a^3c^2x^4 + 32a^2c^3x^8}$$

input
```
integrate((e*x+d)**3/(c*x**4+a)**3,x)
```

output
```
RootSum(268435456*_t**4*a**11*c**3 + 63111168*_t**2*a**6*c**2*d**4*e**2 +
_t*(4147200*a**4*c*d**4*e**5 - 8128512*a**3*c**2*d**8*e) + 50625*a**2*d**4
*e**8 + 245106*a*c*d**8*e**4 + 194481*c**2*d**12, Lambda(_t, _t*log(x + (2
62144000*_t**3*a**10*c**2*e**6 + 3714056192*_t**3*a**9*c**3*d**4*e**2 - 53
9688960*_t**2*a**7*c**2*d**4*e**5 + 202309632*_t**2*a**6*c**3*d**8*e + 773
28000*_t*a**5*c*d**4*e**8 + 660699648*_t*a**4*c**2*d**8*e**4 + 19361664*_t
*a**3*c**3*d**12 + 3037500*a**3*d**4*e**11 - 26360640*a**2*c*d**8*e**7 - 6
0566940*a*c**2*d**12*e**3)/(421875*a**3*d**3*e**12 - 29598075*a**2*c*d**7*
e**8 - 58012227*a*c**2*d**11*e**4 + 3176523*c**3*d**15))) + (-4*a**2*e**3
 + 11*a*c*d**3*x + 30*a*c*d**2*e*x**2 + 27*a*c*d*e**2*x**3 + 7*c**2*d**3*x
**5 + 18*c**2*d**2*e*x**6 + 15*c**2*d*e**2*x**7)/(32*a**4*c + 64*a**3*c**2
*x**4 + 32*a**2*c**3*x**8)
```

3.408.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx$$
$$= \frac{15\,c^2de^2x^7 + 18\,c^2d^2ex^6 + 7\,c^2d^3x^5 + 27\,acde^2x^3 + 30\,acd^2ex^2 + 11\,acd^3x - 4\,a^2e^3}{32\,(a^2c^3x^8 + 2\,a^3c^2x^4 + a^4c)}$$
$$+ \frac{3\,d\left(\frac{\sqrt{2}(7\,\sqrt{c}d^2 - 5\,\sqrt{a}e^2)\log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(7\,\sqrt{c}d^2 - 5\,\sqrt{a}e^2)\log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{2\left(7\,\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2 + 5\,\sqrt{2}a^{\frac{3}{4}}\right.}{}\right.}{256\,a}$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(15*c^2*d*e^2*x^7 + 18*c^2*d^2*e*x^6 + 7*c^2*d^3*x^5 + 27*a*c*d*e^2*x
^3 + 30*a*c*d^2*e*x^2 + 11*a*c*d^3*x - 4*a^2*e^3)/(a^2*c^3*x^8 + 2*a^3*c^2
*x^4 + a^4*c) + 3/256*d*(sqrt(2)*(7*sqrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(
c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*
(7*sqrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*
x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt
(2)*a^(3/4)*c^(1/4)*e^2 - 24*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sq
rt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sq
rt(a)*sqrt(c))*c^(3/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt(2)*a^(
3/4)*c^(1/4)*e^2 + 24*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x
 - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*s
qrt(c))*c^(3/4)))/a^2
```

3.408.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx = \frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{ac}c^2d^2e+7\left(ac^3\right)^{\frac{1}{4}}c^2d^3+5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3} + \frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{ac}c^2d^2e+7\left(ac^3\right)^{\frac{1}{4}}c^2d^3+5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3} + \frac{3\sqrt{2}\left(7\left(ac^3\right)^{\frac{1}{4}}c^2d^3-5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3} - \frac{3\sqrt{2}\left(7\left(ac^3\right)^{\frac{1}{4}}c^2d^3-5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3} + \frac{15\,c^2de^2x^7+18\,c^2d^2ex^6+7\,c^2d^3x^5+27\,acde^2x^3+30\,acd^2ex^2+11\,acd^3x-4\,a^2e^3}{32\left(cx^4+a\right)^2a^2c}$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*c^3)^(1/4)*c^2*d^3 +
5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c
)^(1/4))/(a^3*c^3) + 3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*
c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c
^2*d^3 - 5*(a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c
))/(a^3*c^3) - 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^3 - 5*(a*c^3)^(3/4)*d*
e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) + 1/32*(15*c^2
*d*e^2*x^7 + 18*c^2*d^2*e*x^6 + 7*c^2*d^3*x^5 + 27*a*c*d*e^2*x^3 + 30*a*c*
d^2*e*x^2 + 11*a*c*d^3*x - 4*a^2*e^3)/((c*x^4 + a)^2*a^2*c)
```

3.408.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.83

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx = \frac{\frac{11\,d^3\,x}{32\,a}-\frac{e^3}{8\,c}+\frac{7\,c\,d^3\,x^5}{32\,a^2}+\frac{15\,d^2\,e\,x^2}{16\,a}+\frac{27\,d\,e^2\,x^3}{32\,a}+\frac{9\,c\,d^2\,e\,x^6}{16\,a^2}+\frac{15\,c\,d\,e^2\,x^7}{32\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}$$
$$+\left(\sum_{k=1}^{4}\ln\left(\frac{c\,d^2\left(6867\,c\,d^5\,e^2-1125\,a\,d\,e^6+7992\,c\,d^4\,e^3\,x-\mathrm{root}(268435456\,a^{11}\,c^3\,z^4+63111168\,a^6\,c^2\,d^{\ldots}}{\ldots}\right.\right.$$
$$+63111168\,a^6\,c^2\,d^4\,e^2\,z^2-8128512\,a^3\,c^2\,d^8\,e\,z+4147200\,a^4\,c\,d^4\,e^5\,z+245106\,a\,c\,d^8\,e^4$$
$$\left.+50625\,a^2\,d^4\,e^8+194481\,c^2\,d^{12},z,k)\right)$$

input
```
int((d + e*x)^3/(a + c*x^4)^3,x)
```

output
```
((11*d^3*x)/(32*a) - e^3/(8*c) + (7*c*d^3*x^5)/(32*a^2) + (15*d^2*e*x^2)/(
16*a) + (27*d*e^2*x^3)/(32*a) + (9*c*d^2*e*x^6)/(16*a^2) + (15*c*d*e^2*x^7
)/(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + symsum(log((3*c*d^2*(6867*c*d^5*
e^2 - 1125*a*d*e^6 + 7992*c*d^4*e^3*x - 114688*root(268435456*a^11*c^3*z^4
 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*
d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k
)^2*a^5*c^2*d + 9600*root(268435456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^
2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8
*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)*a^3*c*e^4*x - 18816*root
(268435456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d
^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8
+ 194481*c^2*d^12, z, k)*a^2*c^2*d^4*x + 196608*root(268435456*a^11*c^3*z^
4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c
*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^12, z,
k)^2*a^5*c^2*e*x - 46080*root(268435456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^
4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c
*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)*a^3*c*d*e^3))/(32768
*a^6))*root(268435456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 812851
2*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a
^2*d^4*e^8 + 194481*c^2*d^12, z, k), k, 1, 4)
```

3.408. $\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx$

3.409 $\int \frac{(d+ex)^2}{(a+cx^4)^3} dx$

3.409.1 Optimal result

Integrand size = 17, antiderivative size = 360

$$\begin{aligned}\int \frac{(d+ex)^2}{(a+cx^4)^3} dx = & \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{3de\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} \\ & - \frac{(21\sqrt{c}d^2+5\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} \\ & + \frac{(21\sqrt{c}d^2+5\sqrt{a}e^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} \\ & - \frac{(21\sqrt{c}d^2-5\sqrt{a}e^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}} \\ & + \frac{(21\sqrt{c}d^2-5\sqrt{a}e^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}c^{3/4}}\end{aligned}$$

output
```
1/8*x*(e*x+d)^2/a/(c*x^4+a)^2+1/32*x*(5*e^2*x^2+12*d*e*x+7*d^2)/a^2/(c*x^4
+a)+3/8*d*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/c^(1/2)-1/256*ln(-a^(1/4)*
c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(
11/4)/c^(3/4)*2^(1/2)+1/256*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/
2))*(-5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^(1/2)+1/128*arctan(
-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/4)/c^(3
/4)*2^(1/2)+1/128*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(5*e^2*a^(1/2)+21*d^
2*c^(1/2))/a^(11/4)/c^(3/4)*2^(1/2)
```

3.409.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx$$

$$= \frac{\frac{32a^2x(d+ex)^2}{(a+cx^4)^2}+\frac{8ax(7d^2+12dex+5e^2x^2)}{a+cx^4}-\frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{c}d^2+48\sqrt[4]{a}\sqrt[4]{c}de+5\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}}+\frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{c}d^2-\right.}{}}{}$$

input
```
Integrate[(d + e*x)^2/(a + c*x^4)^3,x]
```

output
```
((32*a^2*x*(d + e*x)^2)/(a + c*x^4)^2 + (8*a*x*(7*d^2 + 12*d*e*x + 5*e^2*x
^2))/(a + c*x^4) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*d^2 + 48*a^(1/4)*c^(1/4)
*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(
3/4) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*d^2 - 48*a^(1/4)*c^(1/4)*d*e + 5*Sqr
t[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (Sqrt
[2]*(-21*a^(1/4)*Sqrt[c]*d^2 + 5*a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[c]*d^2 - 5
*a^(3/4)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3
/4))/(256*a^3)
```

3.409.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2394, 25, 2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \; 2394$$

$$\frac{x(d+ex)^2}{8a(a+cx^4)^2}-\frac{\int -\frac{7d^2+12exd+5e^2x^2}{(cx^4+a)^2}dx}{8a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{7d^2+12exd+5e^2x^2}{(cx^4+a)^2}dx}{8a}+\frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 2394

$$\frac{\frac{x(7d^2+12dex+5e^2x^2)}{4a(a+cx^4)}-\frac{\int -\frac{21d^2+24exd+5e^2x^2}{cx^4+a}dx}{4a}}{8a}+\frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{21d^2+24exd+5e^2x^2}{cx^4+a}dx}{4a}+\frac{x(7d^2+12dex+5e^2x^2)}{4a(a+cx^4)}}{8a}+\frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 2415

$$\frac{\frac{\int \left(\frac{24dex}{cx^4+a}+\frac{21d^2+5e^2x^2}{cx^4+a}\right)dx}{4a}+\frac{x(7d^2+12dex+5e^2x^2)}{4a(a+cx^4)}}{8a}+\frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 2009

$$\frac{\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}e^2+21\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}e^2+21\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}-\frac{\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{\left(21\sqrt{c}d^2-5\sqrt{a}e^{\cdots}\right.}{}}{4a}}{8a}$$

$$\frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}$$

input
```
Int[(d + e*x)^2/(a + c*x^4)^3,x]
```

output
```
(x*(d + e*x)^2)/(8*a*(a + c*x^4)^2) + ((x*(7*d^2 + 12*d*e*x + 5*e^2*x^2))/
(4*a*(a + c*x^4)) + ((12*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[
c]) - ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*Arc
Tan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((21*S
qrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)
*c^(3/4)))/(4*a))/(8*a)
```

3.409. $\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx$

3.409.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.409.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.84 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\frac{5c\,e^2x^7}{32a^2}+\frac{3dce\,x^6}{8a^2}+\frac{7c\,d^2x^5}{32a^2}+\frac{9e^2x^3}{32a}+\frac{5ed\,x^2}{8a}+\frac{11d^2x}{32a}}{(c\,x^4+a)^2}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+a\right)}\frac{\left(5e^2\_R^2+24ed\_R+21d^2\right)\ln\left(x-\_R\right)}{\_R^3}}{128a^2c}$
default	$d^2\left(\frac{x}{8a(c\,x^4+a)^2}+\frac{\frac{7x}{32a\left(c\,x^4+a\right)}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}\right)+2ed$

input
```
int((e*x+d)^2/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(5/32*c*e^2/a^2*x^7+3/8*d*c*e/a^2*x^6+7/32*c*d^2/a^2*x^5+9/32*e^2/a*x^3+5/
8*e*d/a*x^2+11/32/a*d^2*x)/(c*x^4+a)^2+1/128/a^2/c*sum((5*_R^2*e^2+24*_R*d
*e+21*d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

3.409.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.15 (sec) , antiderivative size = 91420, normalized size of antiderivative = 253.94

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.409.6 Sympy [A] (verification not implemented)

Time = 14.65 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx$$
$$=\text{RootSum}\Big(268435456t^4a^{11}c^3+25755648t^2a^6c^2d^2e^2+t\big(307200a^4cde^5-5419008a^3c^2d^5e\big)+625a^2e^8+1$$
$$+\frac{11ad^2x+20adex^2+9ae^2x^3+7cd^2x^5+12cdex^6+5ce^2x^7}{32a^4+64a^3cx^4+32a^2c^2x^8}$$

input
```
integrate((e*x+d)**2/(c*x**4+a)**3,x)
```

output
```
RootSum(268435456*_t**4*a**11*c**3 + 25755648*_t**2*a**6*c**2*d**2*e**2 +
_t*(307200*a**4*c*d*e**5 - 5419008*a**3*c**2*d**5*e) + 625*a**2*e**8 + 111
906*a*c*d**4*e**4 + 194481*c**2*d**8, Lambda(_t, _t*log(x + (262144000*_t*
*3*a**10*c**2*e**6 + 46110081024*_t**3*a**9*c**3*d**4*e**2 - 1645608960*_t
**2*a**7*c**2*d**3*e**5 + 3641573376*_t**2*a**6*c**3*d**7*e + 32688000*_t*
a**5*c*d**2*e**8 + 3128219136*_t*a**4*c**2*d**6*e**4 + 522764928*_t*a**3*c
**3*d**10 + 225000*a**3*d*e**11 - 43338240*a**2*c*d**5*e**7 - 523431720*a*
c**2*d**9*e**3)/(15625*a**3*e**12 - 21357225*a**2*c*d**4*e**8 - 376741449*
a*c**2*d**8*e**4 + 85766121*c**3*d**12)))) + (11*a*d**2*x + 20*a*d*e*x**2
+ 9*a*e**2*x**3 + 7*c*d**2*x**5 + 12*c*d*e*x**6 + 5*c*e**2*x**7)/(32*a**4
+ 64*a**3*c*x**4 + 32*a**2*c**2*x**8)
```

3.409.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx = \frac{5\,ce^2x^7+12\,cdex^6+7\,cd^2x^5+9\,ae^2x^3+20\,adex^2+11\,ad^2x}{32\,(a^2c^2x^8+2\,a^3cx^4+a^4)}$$

$$+\frac{\frac{\sqrt{2}(21\,\sqrt{c}d^2-5\,\sqrt{a}e^2)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac14}c^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}c^{\frac34}}-\frac{\sqrt{2}(21\,\sqrt{c}d^2-5\,\sqrt{a}e^2)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac14}c^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}c^{\frac34}}+\frac{2\left(21\,\sqrt{2}a^{\frac14}c^{\frac34}d^2+5\,\sqrt{2}a^{\frac34}c^{\frac14}\right.}{}}{256\,a^2}$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(5*c*e^2*x^7 + 12*c*d*e*x^6 + 7*c*d^2*x^5 + 9*a*e^2*x^3 + 20*a*d*e*x^
2 + 11*a*d^2*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 1/256*(sqrt(2)*(21*sqr
t(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sq
rt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(21*sqrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sq
rt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(21
*sqrt(2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 48*sqrt(a)*
sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sq
rt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(21*sqrt(
2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 48*sqrt(a)*sqrt(c
)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqr
t(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a^2
```

3.409.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx = \frac{5\,ce^2x^7+12\,cdex^6+7\,cd^2x^5+9\,ae^2x^3+20\,adex^2+11\,ad^2x}{32\,(cx^4+a)^2a^2} + \frac{\sqrt{2}\left(24\,\sqrt{2}\sqrt{ac}c^2de+21\,(ac^3)^{\frac{1}{4}}\,c^2d^2+5\,(ac^3)^{\frac{3}{4}}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3} + \frac{\sqrt{2}\left(24\,\sqrt{2}\sqrt{ac}c^2de+21\,(ac^3)^{\frac{1}{4}}\,c^2d^2+5\,(ac^3)^{\frac{3}{4}}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3} + \frac{\sqrt{2}\left(21\,(ac^3)^{\frac{1}{4}}\,c^2d^2-5\,(ac^3)^{\frac{3}{4}}\,e^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3} - \frac{\sqrt{2}\left(21\,(ac^3)^{\frac{1}{4}}\,c^2d^2-5\,(ac^3)^{\frac{3}{4}}\,e^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3}$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
1/32*(5*c*e^2*x^7 + 12*c*d*e*x^6 + 7*c*d^2*x^5 + 9*a*e^2*x^3 + 20*a*d*e*x^
2 + 11*a*d^2*x)/((c*x^4 + a)^2*a^2) + 1/128*sqrt(2)*(24*sqrt(2)*sqrt(a*c)*
c^2*d*e + 21*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(
2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 1/128*sqrt(2)*(24*
sqrt(2)*sqrt(a*c)*c^2*d*e + 21*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^(3/4)*e^2
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) +
1/256*sqrt(2)*(21*(a*c^3)^(1/4)*c^2*d^2 - 5*(a*c^3)^(3/4)*e^2)*log(x^2 + s
qrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) - 1/256*sqrt(2)*(21*(a*c^3)^(1
/4)*c^2*d^2 - 5*(a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(
a/c))/(a^3*c^3)
```

3.409.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx = \frac{\frac{11\,d^2\,x}{32\,a}+\frac{9\,e^2\,x^3}{32\,a}+\frac{7\,c\,d^2\,x^5}{32\,a^2}+\frac{5\,c\,e^2\,x^7}{32\,a^2}+\frac{5\,d\,e\,x^2}{8\,a}+\frac{3\,c\,d\,e\,x^6}{8\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8} + \left(\sum_{k=1}^{4}\ln\left(-\frac{c\left(125\,a\,e^6-9891\,c\,d^4\,e^2+\mathrm{root}(268435456\,a^{11}\,c^3\,z^4+25755648\,a^6\,c^2\,d^2\,e^2\,z^2-5419008\,a^3\right.}{}\right.\right.$$

$$+\,25755648\,a^6\,c^2\,d^2\,e^2\,z^2-5419008\,a^3\,c^2\,d^5\,e\,z+307200\,a^4\,c\,d\,e^5\,z+111906\,a\,c\,d^4\,e^4$$

$$\left.+\,194481\,c^2\,d^8+625\,a^2\,e^8,z,k)\right)$$

input

```
int((d + e*x)^2/(a + c*x^4)^3,x)
```

output

```
((11*d^2*x)/(32*a) + (9*e^2*x^3)/(32*a) + (7*c*d^2*x^5)/(32*a^2) + (5*c*e^
2*x^7)/(32*a^2) + (5*d*e*x^2)/(8*a) + (3*c*d*e*x^6)/(8*a^2))/(a^2 + c^2*x^
8 + 2*a*c*x^4) + symsum(log(-(c*(125*a*e^6 - 9891*c*d^4*e^2 + 344064*root(
268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^
5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a
^2*e^8, z, k)^2*a^5*c^2*d^2 - 8784*c*d^3*e^3*x - 3200*root(268435456*a^11*
c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*
a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a
^3*c*e^4*x + 56448*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*
z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4
+ 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a^2*c^2*d^4*x + 30720*root(268435456
*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 3
07200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z
, k)*a^3*c*d*e^3 - 393216*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d
^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d
^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)^2*a^5*c^2*d*e*x))/(32768*a^6)
)*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3
*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8
+ 625*a^2*e^8, z, k), k, 1, 4)
```

3.410 $\int \frac{d+ex}{\left(a+cx^4\right)^3}\,dx$

3.410.1 Optimal result

Integrand size = 15, antiderivative size = 266

$$\begin{aligned}\int \frac{d+ex}{\left(a+cx^4\right)^3}\,dx &= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2}+\frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)}+\frac{3e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}\\ &\quad-\frac{21d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{21d\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}\\ &\quad-\frac{21d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}\\ &\quad+\frac{21d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}\end{aligned}$$

output
```
1/8*x*(e*x+d)/a/(c*x^4+a)^2+1/32*x*(6*e*x+7*d)/a^2/(c*x^4+a)+21/128*d*arct
an(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/c^(1/4)*2^(1/2)+21/128*d*arctan(
1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/c^(1/4)*2^(1/2)-21/256*d*ln(-a^(1/4)
*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)+21/256*d*
ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)
+3/16*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/c^(1/2)
```

3.410.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.94

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx$$

$$= \frac{\frac{32a^{7/4}x(d+ex)}{(a+cx^4)^2} + \frac{8a^{3/4}x(7d+6ex)}{a+cx^4} - \frac{6\left(7\sqrt{2}\sqrt[4]{c}d+8\sqrt[4]{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}} + \frac{6\left(7\sqrt{2}\sqrt[4]{c}d-8\sqrt[4]{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}} - \cdots}{256a^{11/4}}$$

input
```
Integrate[(d + e*x)/(a + c*x^4)^3,x]
```

output
```
((32*a^(7/4)*x*(d + e*x))/(a + c*x^4)^2 + (8*a^(3/4)*x*(7*d + 6*e*x))/(a +
 c*x^4) - (6*(7*Sqrt[2]*c^(1/4)*d + 8*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/Sqrt[c] + (6*(7*Sqrt[2]*c^(1/4)*d - 8*a^(1/4)*e)*ArcTan[1
+ (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] - (21*Sqrt[2]*d*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*d*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))
```

3.410.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{x(d+ex)}{8a(a+cx^4)^2} - \frac{\int -\frac{7d+6ex}{(cx^4+a)^2}dx}{8a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{7d+6ex}{(cx^4+a)^2}dx}{8a} + \frac{x(d+ex)}{8a(a+cx^4)^2}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(7d+6ex)}{4a(a+cx^4)}-\frac{\int -\frac{3(7d+4ex)}{cx^4+a}dx}{4a}}{8a}+\frac{x(d+ex)}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{7d+4ex}{cx^4+a}dx}{4a}+\frac{x(7d+6ex)}{4a(a+cx^4)}}{8a}+\frac{x(d+ex)}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\frac{3\int \left(\frac{7d}{cx^4+a}+\frac{4ex}{cx^4+a}\right)dx}{4a}+\frac{x(7d+6ex)}{4a(a+cx^4)}}{8a}+\frac{x(d+ex)}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3\left(-\frac{7d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{7d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{7d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{7d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}\right)}{4a}+\dots}{8a}$$

$$\frac{x(d+ex)}{8a\left(a+cx^4\right)^2}$$

input
```
Int[(d + e*x)/(a + c*x^4)^3,x]
```

output
```
(x*(d + e*x))/(8*a*(a + c*x^4)^2) + ((x*(7*d + 6*e*x))/(4*a*(a + c*x^4)) +
 (3*((2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - (7*d*ArcTan[1
 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + (7*d*ArcTan
[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - (7*d*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(
1/4)) + (7*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sq
rt[2]*a^(3/4)*c^(1/4))))/(4*a))/(8*a)
```

3.410. $\int \frac{d+ex}{(a+cx^4)^3}\,dx$

3.410.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

3.410.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.84 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\frac{3ec\,x^6}{16a^2}+\frac{7cd\,x^5}{32a^2}+\frac{5e\,x^2}{16a}+\frac{11dx}{32a}}{(c\,x^4+a)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+a)}\frac{(4e\_R+7d)\ln(x-\_R)}{\_R^3}\right)}{128a^2c}$
default	$d\left(\frac{x}{8a(c\,x^4+a)^2}+\frac{\frac{7x}{32a(c\,x^4+a)}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}\right)+e\Bigg(\frac{}{8a}$

input
```
int((e*x+d)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/16*e*c/a^2*x^6+7/32/a^2*c*d*x^5+5/16*e/a*x^2+11/32*d/a*x)/(c*x^4+a)^2+3
/128/a^2/c*sum((4*_R*e+7*d)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

3.410.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 43180, normalized size of antiderivative = 162.33

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.410.6 Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.72

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx$$
$$= \text{RootSum}\left(268435456t^4a^{11}c^2 + 4718592t^2a^6ce^2 - 2709504ta^3cd^2e + 20736ae^4 + 194481cd^4, \left(t \mapsto t\log\left(\right.\right.\right.$$
$$+ \frac{11adx + 10aex^2 + 7cdx^5 + 6cex^6}{32a^4 + 64a^3cx^4 + 32a^2c^2x^8}$$

input
```
integrate((e*x+d)/(c*x**4+a)**3,x)
```

output
```
RootSum(268435456*_t**4*a**11*c**2 + 4718592*_t**2*a**6*c*e**2 - 2709504*_
t*a**3*c*d**2*e + 20736*a*e**4 + 194481*c*d**4, Lambda(_t, _t*log(x + (-67
108864*_t**3*a**9*c*e**2 - 9633792*_t**2*a**6*c*d**2*e - 589824*_t*a**4*e*
*4 - 2765952*_t*a**3*c*d**4 + 423360*a*d**2*e**3)/(193536*a*d*e**4 - 45378
9*c*d**5)))) + (11*a*d*x + 10*a*e*x**2 + 7*c*d*x**5 + 6*c*e*x**6)/(32*a**4
 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8)
```

3.410.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.01

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx = \frac{6\,cex^6+7\,cdx^5+10\,aex^2+11\,adx}{32\,(a^2c^2x^8+2\,a^3cx^4+a^4)} + \frac{3\left(\frac{7\sqrt{2}d\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{7\sqrt{2}d\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} + \frac{2\left(7\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d-8\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{1}{4}}}\right.}{256\,a^2}$$

input
```
integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(6*c*e*x^6 + 7*c*d*x^5 + 10*a*e*x^2 + 11*a*d*x)/(a^2*c^2*x^8 + 2*a^3*
c*x^4 + a^4) + 3/256*(7*sqrt(2)*d*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4
)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 7*sqrt(2)*d*log(sqrt(c)*x^2 - sqrt(2)*a
^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(1/
4)*d - 8*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1
/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4)) + 2*(7
*sqrt(2)*a^(1/4)*c^(1/4)*d + 8*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x
- sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sq
rt(c))*c^(1/4)))/a^2
```

3.410.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.96

$$\int \frac{d+ex}{\left(a+cx^4\right)^3}\,dx = \frac{21\,\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c} - \frac{21\,\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c} + \frac{3\,\sqrt{2}\left(4\,\sqrt{2}\sqrt{ac}ce+7\,(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^2} + \frac{3\,\sqrt{2}\left(4\,\sqrt{2}\sqrt{ac}ce+7\,(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^2} + \frac{6\,cex^6+7\,cdx^5+10\,aex^2+11\,adx}{32\,(cx^4+a)^2a^2}$$

input
```
integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
21/256*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c)
)/(a^3*c) - 21/256*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4)
 + sqrt(a/c))/(a^3*c) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c*e + 7*(a*c^3)
^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a
^3*c^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c*e + 7*(a*c^3)^(1/4)*c*d)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^2) + 1/32
*(6*c*e*x^6 + 7*c*d*x^5 + 10*a*e*x^2 + 11*a*d*x)/((c*x^4 + a)^2*a^2)
```

3.410.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.18

$$\int \frac{d+ex}{\left(a+cx^4\right)^3}\,dx = \frac{\frac{5\,e\,x^2}{16\,a}+\frac{11\,d\,x}{32\,a}+\frac{7\,c\,d\,x^5}{32\,a^2}+\frac{3\,c\,e\,x^6}{16\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8} + \left(\sum_{k=1}^{4}\ln\left(\frac{c^2\left(63\,d\,e^2+36\,e^3\,x-\mathrm{root}(268435456\,a^{11}\,c^2\,z^4+4718592\,a^6\,c\,e^2\,z^2-2709504\,a^3\,c\,d^2\,e\,z+19\right.}{}\right.\right.$$

$$\left.+4718592\,a^6\,c\,e^2\,z^2-2709504\,a^3\,c\,d^2\,e\,z+194481\,c\,d^4+20736\,a\,e^4,z,k)\right)$$

input
```
int((d + e*x)/(a + c*x^4)^3,x)
```

output
```
((5*e*x^2)/(16*a) + (11*d*x)/(32*a) + (7*c*d*x^5)/(32*a^2) + (3*c*e*x^6)/(
16*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + symsum(log((3*c^2*(63*d*e^2 + 36*e^
3*x - 7168*root(268435456*a^11*c^2*z^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a
^3*c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4, z, k)^2*a^5*c*d - 1176*root(268
435456*a^11*c^2*z^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 1944
81*c*d^4 + 20736*a*e^4, z, k)*a^2*c*d^2*x + 4096*root(268435456*a^11*c^2*z
^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*d^4 + 20736*
a*e^4, z, k)^2*a^5*c*e*x))/(2048*a^6))*root(268435456*a^11*c^2*z^4 + 47185
92*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4, z,
k), k, 1, 4)
```

3.410. $\int \frac{d+ex}{(a+cx^4)^3} \, dx$

3.411 $\int \frac{1}{(a+cx^4)^3}\,dx$

3.411.1 Optimal result

Integrand size = 9, antiderivative size = 219

$$\int \frac{1}{(a+cx^4)^3}\,dx = \frac{x}{8a(a+cx^4)^2}+\frac{7x}{32a^2(a+cx^4)}-\frac{21\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{21\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}-\frac{21\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{21\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}$$

output
```
1/8*x/a/(c*x^4+a)^2+7/32*x/a^2/(c*x^4+a)+21/128*arctan(-1+c^(1/4)*x*2^(1/2
)/a^(1/4))/a^(11/4)/c^(1/4)*2^(1/2)+21/128*arctan(1+c^(1/4)*x*2^(1/2)/a^(1
/4))/a^(11/4)/c^(1/4)*2^(1/2)-21/256*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)+21/256*ln(a^(1/4)*c^(1/4)*x*2^(1/2)
+a^(1/2)+x^2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)
```

3.411.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+cx^4)^3}\,dx$$
$$= \frac{\frac{32a^{7/4}x}{(a+cx^4)^2}+\frac{56a^{3/4}x}{a+cx^4}-\frac{42\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}+\frac{42\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}-\frac{21\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{\sqrt[4]{c}}+\frac{21\sqrt{2}\log}{}}{256a^{11/4}}$$

input
```
Integrate[(a + c*x^4)^(-3),x]
```

output
```
((32*a^(7/4)*x)/(a + c*x^4)^2 + (56*a^(3/4)*x)/(a + c*x^4) - (42*Sqrt[2]*A
rcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) + (42*Sqrt[2]*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) - (21*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))
```

3.411.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.111$, Rules used = {749, 749, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^3}\,dx$$
$$\Big\downarrow \text{749}$$
$$\frac{7\int \frac{1}{(cx^4+a)^2}dx}{8a}+\frac{x}{8a(a+cx^4)^2}$$
$$\Big\downarrow \text{749}$$
$$\frac{7\left(\frac{3\int \frac{1}{cx^4+a}dx}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a(a+cx^4)^2}$$
$$\Big\downarrow \text{755}$$

$$\frac{7\left(\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$\downarrow$ 1476

$$\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$\downarrow$ 1082

$$\frac{7\left(\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$\downarrow$ 217

3.411. $\int \frac{1}{\left(a+cx^4\right)^3}\,dx$

$$\frac{7\left(\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 1479$$

$$\frac{7\left(\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 25$$

3.411. $\int \frac{1}{(a+cx^4)^3}\,dx$

$$\frac{7\left(\frac{3\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{7\left(\frac{3\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1103$$

3.411. $\int \frac{1}{(a+cx^4)^3}\, dx$

$$\frac{7\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

input

```
Int[(a + c*x^4)^(-3),x]
```

output

```
x/(8*a*(a + c*x^4)^2) + (7*(x/(4*a*(a + c*x^4)) + (3*((-(ArcTan[1 - (Sqrt[
2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) +
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*
c^(1/4)))/(2*Sqrt[a])))/(4*a)))/(8*a)
```

3.411.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.411. $\int \frac{1}{(a+cx^4)^3} dx$

3.411.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.26

method	result	size
risch	$\frac{\frac{7 c \,x^{5}}{32 a^{2}}+\frac{11 x}{32 a}}{\left(c \,x^{4}+a \right)^{2}}+\frac{21 \left(\munderset{\textit{\_R} =\operatorname{RootOf}\left(c \,\textit{\_Z}^{4}+a \right)}{\sum}\frac{\ln \left(x -\textit{\_R} \right)}{\textit{\_R}^{3}}\right)}{128 a^{2} c}$	57
default	$\frac{x}{8 a \left(c \,x^{4}+a \right)^{2}}+\frac{\frac{7 x}{32 a \left(c \,x^{4}+a \right)}+\frac{21 \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256 a^{2}}}{a}$	139

input
```
int(1/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(7/32*c/a^2*x^5+11/32*x/a)/(c*x^4+a)^2+21/128/a^2/c*sum(1/_R^3*ln(x-_R),_R
=RootOf(_Z^4*c+a))
```

3.411.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.16

$$\int \frac{1}{(a+cx^4)^3}\,dx$$
$$= \frac{28\,cx^5 + 21\left(a^2c^2x^8 + 2\,a^3cx^4 + a^4\right)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}\log\left(a^3\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}} + x\right) - 21\left(-i\,a^2c^2x^8 - 2i\,a^3cx^4 - i\,a^4\right)\left(-\frac{}{a}\right.}{}$$

input
```
integrate(1/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/128*(28*c*x^5 + 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c))^(1/4)
*log(a^3*(-1/(a^11*c))^(1/4) + x) - 21*(-I*a^2*c^2*x^8 - 2*I*a^3*c*x^4 - I
*a^4)*(-1/(a^11*c))^(1/4)*log(I*a^3*(-1/(a^11*c))^(1/4) + x) - 21*(I*a^2*c
^2*x^8 + 2*I*a^3*c*x^4 + I*a^4)*(-1/(a^11*c))^(1/4)*log(-I*a^3*(-1/(a^11*c
))^(1/4) + x) - 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c))^(1/4)*l
og(-a^3*(-1/(a^11*c))^(1/4) + x) + 44*a*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^
4)
```

3.411.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.29

$$\int \frac{1}{(a+cx^4)^3}\,dx = \frac{11ax+7cx^5}{32a^4+64a^3cx^4+32a^2c^2x^8} + \operatorname{RootSum}\left(268435456t^4a^{11}c+194481,\left(t\mapsto t\log\left(\frac{128ta^3}{21}+x\right)\right)\right)$$

input
```
integrate(1/(c*x**4+a)**3,x)
```

output
```
(11*a*x + 7*c*x**5)/(32*a**4 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8) + RootS
um(268435456*_t**4*a**11*c + 194481, Lambda(_t, _t*log(128*_t*a**3/21 + x)
))
```

3.411.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+cx^4)^3}\,dx = \frac{7\,cx^5+11\,ax}{32\,(a^2c^2x^8+2\,a^3cx^4+a^4)} + \frac{21\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2 \cdots\right.}{a \cdots}\right.}{256\,a^2}$$

input
```
integrate(1/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(7*c*x^5 + 11*a*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 21/256*(2*sqrt
(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a
)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)
*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*
x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)
*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/a^2
```

3.411.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+cx^4\right)^3}\,dx = \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c} + \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c} + \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c} - \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c} + \frac{7\,cx^5+11\,ax}{32\,\left(cx^4+a\right)^2a^2}$$

input
```
integrate(1/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(a^3*c) + 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c) + 21/256*sqrt(2)*(a*c^3)^(
1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) - 21/256*sqrt(2)
*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) + 1/32
*(7*c*x^5 + 11*a*x)/((c*x^4 + a)^2*a^2)
```

3.411.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.37

$$\int \frac{1}{\left(a+cx^4\right)^3}\,dx = \frac{\frac{11\,x}{32\,a}+\frac{7\,c\,x^5}{32\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8} - \frac{21\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{11/4}\,c^{1/4}} - \frac{21\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{11/4}\,c^{1/4}}$$

input
```
int(1/(a + c*x^4)^3,x)
```

output
```
((11*x)/(32*a) + (7*c*x^5)/(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (21*ata
n((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4)) - (21*atanh((c^(1/4)*x
)/(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4))
```

3.412 $\int \frac{1}{(d+ex)(a+cx^4)^3}\,dx$

3.412.1 Optimal result

Integrand size = 17, antiderivative size = 1352

$$\int \frac{1}{(d+ex)(a+cx^4)^3}\,dx = \frac{cx(7d^3-6d^2ex+5de^2x^2)}{32a^2(cd^4+ae^4)(a+cx^4)} + \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{8a(cd^4+ae^4)(a+cx^4)^2}
+ \frac{e^4(ae^3+cx(d^3-d^2ex+de^2x^2))}{4a(cd^4+ae^4)^2(a+cx^4)} - \frac{\sqrt{c}d^2e^9\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(cd^4+ae^4)^3}
- \frac{\sqrt{c}d^2e^5\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}(cd^4+ae^4)^2} - \frac{3\sqrt{c}d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}(cd^4+ae^4)}
- \frac{\sqrt[4]{c}de^8(\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^3}
- \frac{\sqrt[4]{c}de^4(3\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^4+ae^4)^2}
- \frac{\sqrt[4]{c}d(21\sqrt{c}d^2+5\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}(cd^4+ae^4)}
+ \frac{\sqrt[4]{c}de^8(\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^3}
+ \frac{\sqrt[4]{c}de^4(3\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(cd^4+ae^4)^2}
+ \frac{\sqrt[4]{c}d(21\sqrt{c}d^2+5\sqrt{a}e^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}(cd^4+ae^4)}
+ \frac{e^{11}\log(d+ex)}{(cd^4+ae^4)^3}
- \frac{\sqrt[4]{c}de^8(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^3}
- \frac{\sqrt[4]{c}de^4(3\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(cd^4+ae^4)^2}
- \frac{\sqrt[4]{c}d(21\sqrt{c}d^2-5\sqrt{a}e^2)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}(cd^4+ae^4)}
+ \frac{\sqrt[4]{c}de^8(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^3}
+ \frac{\sqrt[4]{c}de^4(3\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}(cd^4+ae^4)^2}
+ \frac{\sqrt[4]{c}d(21\sqrt{c}d^2-5\sqrt{a}e^2)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}(cd^4+ae^4)}$$

output
```
1/32*c*x*(5*d*e^2*x^2-6*d^2*e*x+7*d^3)/a^2/(a*e^4+c*d^4)/(c*x^4+a)+1/8*(a*
e^3+c*x*(d*e^2*x^2-d^2*e*x+d^3))/a/(a*e^4+c*d^4)/(c*x^4+a)^2+1/4*e^4*(a*e^
3+c*x*(d*e^2*x^2-d^2*e*x+d^3))/a/(a*e^4+c*d^4)^2/(c*x^4+a)+e^11*ln(e*x+d)/
(a*e^4+c*d^4)^3-1/4*e^11*ln(c*x^4+a)/(a*e^4+c*d^4)^3-1/4*d^2*e^5*arctan(x^
2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4)^2-3/16*d^2*e*arctan(x^2*c
^(1/2)/a^(1/2))*c^(1/2)/a^(5/2)/(a*e^4+c*d^4)-1/2*d^2*e^9*arctan(x^2*c^(1/
2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^3/a^(1/2)-1/8*c^(1/4)*d*e^8*ln(-a^(1/4)*
c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/
(a*e^4+c*d^4)^3*2^(1/2)+1/8*c^(1/4)*d*e^8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(
1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/
2)+1/4*c^(1/4)*d*e^8*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2
*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(1/4)*d*e^8*arctan(1+c^(1/
4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^
(1/2)-1/32*c^(1/4)*d*e^4*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2)
)*(-e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/32*c^(1/4
)*d*e^4*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+3*
d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/16*c^(1/4)*d*e^4*arctan(-1+
c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^
4)^2*2^(1/2)+1/16*c^(1/4)*d*e^4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a
^(1/2)+3*d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)-1/256*c^(1/4)*d*l...
```

3.412.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 835, normalized size of antiderivative = 0.62

$$\int \frac{1}{(d+ex)(a+cx^4)^3}\,dx$$

$$= \frac{\frac{32(cd^4+ae^4)^2(ae^3+cdx(d^2-dex+e^2x^2))}{a(a+cx^4)^2} + \frac{8(cd^4+ae^4)(8a^2e^7+c^2d^5x(7d^2-6dex+5e^2x^2)+acde^4x(15d^2-14dex+13e^2x^2))}{a^2(a+cx^4)} - \frac{2\sqrt[4]{c}d\left(21\sqrt{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Integrate[1/((d + e*x)*(a + c*x^4)^3),x]
```

output
```
((32*(c*d^4 + a*e^4)^2*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(a + c*
x^4)^2) + (8*(c*d^4 + a*e^4)*(8*a^2*e^7 + c^2*d^5*x*(7*d^2 - 6*d*e*x + 5*e
^2*x^2) + a*c*d*e^4*x*(15*d^2 - 14*d*e*x + 13*e^2*x^2)))/(a^2*(a + c*x^4))
 - (2*c^(1/4)*d*(21*Sqrt[2]*c^(5/2)*d^10 - 24*a^(1/4)*c^(9/4)*d^9*e + 5*Sq
rt[2]*Sqrt[a]*c^2*d^8*e^2 + 66*Sqrt[2]*a*c^(3/2)*d^6*e^4 - 80*a^(5/4)*c^(5
/4)*d^5*e^5 + 18*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 77*Sqrt[2]*a^2*Sqrt[c]*d^2*e^
8 - 120*a^(9/4)*c^(1/4)*d*e^9 + 45*Sqrt[2]*a^(5/2)*e^10)*ArcTan[1 - (Sqrt[
2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + (2*c^(1/4)*d*(21*Sqrt[2]*c^(5/2)*d^10 +
 24*a^(1/4)*c^(9/4)*d^9*e + 5*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 + 66*Sqrt[2]*a*c
^(3/2)*d^6*e^4 + 80*a^(5/4)*c^(5/4)*d^5*e^5 + 18*Sqrt[2]*a^(3/2)*c*d^4*e^6
 + 77*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 + 120*a^(9/4)*c^(1/4)*d*e^9 + 45*Sqrt[2]
*a^(5/2)*e^10)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + 256*e^1
1*Log[d + e*x] + (Sqrt[2]*c^(1/4)*(-21*c^(5/2)*d^11 + 5*Sqrt[a]*c^2*d^9*e^
2 - 66*a*c^(3/2)*d^7*e^4 + 18*a^(3/2)*c*d^5*e^6 - 77*a^2*Sqrt[c]*d^3*e^8 +
 45*a^(5/2)*d*e^10)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
)/a^(11/4) + (Sqrt[2]*c^(1/4)*(21*c^(5/2)*d^11 - 5*Sqrt[a]*c^2*d^9*e^2 + 6
6*a*c^(3/2)*d^7*e^4 - 18*a^(3/2)*c*d^5*e^6 + 77*a^2*Sqrt[c]*d^3*e^8 - 45*a
^(5/2)*d*e^10)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(
11/4) - 64*e^11*Log[a + c*x^4])/(256*(c*d^4 + a*e^4)^3)
```

3.412.3 Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 1352, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^3(d+ex)}\,dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{e^{12}}{(d+ex)(ae^4+cd^4)^3}-\frac{ce^4(-d^3+d^2ex-de^2x^2+e^3x^3)}{(a+cx^4)^2(ae^4+cd^4)^2}+\frac{c(d^3-d^2ex+de^2x^2-e^3x^3)}{(a+cx^4)^3(ae^4+cd^4)}-\frac{ce^8(-d^3+d^2ex}{(a+cx^4)(}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\log(d+ex)e^{11}}{(cd^4+ae^4)^3}-\frac{\log\left(cx^4+a\right)e^{11}}{4\left(cd^4+ae^4\right)^3}-\frac{\sqrt{c}d^2\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^9}{2\sqrt{a}\left(cd^4+ae^4\right)^3}-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^8}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^8}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^8}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^8}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}-\frac{\sqrt{c}d^2\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{4a^{3/2}\left(cd^4+ae^4\right)^2}+\frac{\left(ae^3+cx\left(d^3-exd^2+e^2x^2d\right)\right)e^4}{4a\left(cd^4+ae^4\right)^2\left(cx^4+a\right)}-\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}-\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}d\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}-\frac{3\sqrt{c}d^2\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{16a^{5/2}\left(cd^4+ae^4\right)}+\frac{ae^3+cx\left(d^3-exd^2+e^2x^2d\right)}{8a\left(cd^4+ae^4\right)\left(cx^4+a\right)^2}-\frac{\sqrt[4]{c}d\left(21\sqrt{c}d^2+5\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\left(cd^4+ae^4\right)}+\frac{\sqrt[4]{c}d\left(21\sqrt{c}d^2+5\sqrt{a}e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}\left(cd^4+ae^4\right)}-\frac{\sqrt[4]{c}d\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{128\sqrt{2}a^{11/4}\left(cd^4+ae^4\right)}+\frac{\sqrt[4]{c}d\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{128\sqrt{2}a^{11/4}\left(cd^4+ae^4\right)}+\frac{cx\left(7d^3-6exd^2+5e^2x^2d\right)}{32a^2\left(cd^4+ae^4\right)\left(cx^4+a\right)}$$

input
```
Int[1/((d + e*x)*(a + c*x^4)^3),x]
```

3.412. $\int \frac{1}{(d+ex)(a+cx^4)^3}\,dx$

output
```
(c*x*(7*d^3 - 6*d^2*e*x + 5*d*e^2*x^2))/(32*a^2*(c*d^4 + a*e^4)*(a + c*x^4
)) + (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(8*a*(c*d^4 + a*e^4)*(a + c
*x^4)^2) + (e^4*(a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2)))/(4*a*(c*d^4 + a
*e^4)^2*(a + c*x^4)) - (Sqrt[c]*d^2*e^9*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*
Sqrt[a]*(c*d^4 + a*e^4)^3) - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]
])/(4*a^(3/2)*(c*d^4 + a*e^4)^2) - (3*Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/S
qrt[a]])/(16*a^(5/2)*(c*d^4 + a*e^4)) - (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt
[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^
4 + a*e^4)^3) - (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (S
qrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/
4)*d*(21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 +
 Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(c*d^4 + a*e^4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (
c^(1/4)*d*(21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (e^11*Log[d + e*x])/(c*d
^4 + a*e^4)^3 - (c^(1/4)*d*e^8*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4
)^3) - (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[...
```

3.412.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.412.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 677, normalized size of antiderivative = 0.50

method	result
default	$c\left(\frac{\frac{cd\,e^{2}\left(13a^{2}e^{8}+18ac\,d^{4}e^{4}+5c^{2}d^{8}\right)x^{7}}{32a^{2}}-\frac{c\,d^{2}e\left(7a^{2}e^{8}+10ac\,d^{4}e^{4}+3c^{2}d^{8}\right)x^{6}}{16a^{2}}+\frac{d^{3}c\left(15a^{2}e^{8}+22ac\,d^{4}e^{4}+7c^{2}d^{8}\right)x^{5}}{32a^{2}}+\left(\frac{1}{4}a\,e^{11}+\frac{1}{4}d^{4}e^{7}c\right)x^{4}+\frac{d\,e^{2}\left(\right.}{}}{\left(c\,x^{4}+a\right)^{2}}\right.$
risch	Expression too large to display

input
```
int(1/(e*x+d)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
c/(a*e^4+c*d^4)^3*((1/32*c*d*e^2*(13*a^2*e^8+18*a*c*d^4*e^4+5*c^2*d^8)/a^2
*x^7-1/16*c*d^2*e*(7*a^2*e^8+10*a*c*d^4*e^4+3*c^2*d^8)/a^2*x^6+1/32*d^3*c*
(15*a^2*e^8+22*a*c*d^4*e^4+7*c^2*d^8)/a^2*x^5+(1/4*a*e^11+1/4*d^4*e^7*c)*x
^4+1/32*d*e^2*(17*a^2*e^8+26*a*c*d^4*e^4+9*c^2*d^8)/a*x^3-1/16*d^2*e*(9*a^
2*e^8+14*a*c*d^4*e^4+5*c^2*d^8)/a*x^2+1/32*d^3*(19*a^2*e^8+30*a*c*d^4*e^4+
11*c^2*d^8)/a*x+1/8*e^3*(3*a^2*e^8+4*a*c*d^4*e^4+c^2*d^8)/c)/(c*x^4+a)^2+1
/32/a^2*(1/8*(77*a^2*d^3*e^8+66*a*c*d^7*e^4+21*c^2*d^11)*(a/c)^(1/4)/a*2^(
1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2
)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(
1/4)*x-1))+1/2*(-60*a^2*d^2*e^9-40*a*c*d^6*e^5-12*c^2*d^10*e)/(a*c)^(1/2)*
arctan(x^2*(c/a)^(1/2))+1/8*(45*a^2*d*e^10+18*a*c*d^5*e^6+5*c^2*d^9*e^2)/c
/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c
)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan
(2^(1/2)/(a/c)^(1/4)*x-1))-8*a^2*e^11/c*ln(c*x^4+a)))+e^11*ln(e*x+d)/(a*e^
4+c*d^4)^3
```

3.412.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+cx^{4}\right)^{3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.412.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x**4+a)**3,x)
```

output
```
Timed out
```

3.412.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1015, normalized size of antiderivative = 0.75

$$\int \frac{1}{(d+ex)(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
e^11*log(e*x + d)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12
) - 1/256*c*(sqrt(2)*(32*sqrt(2)*a^(11/4)*c^(1/4)*e^11 - 21*c^3*d^11 + 5*s
qrt(a)*c^(5/2)*d^9*e^2 - 66*a*c^2*d^7*e^4 + 18*a^(3/2)*c^(3/2)*d^5*e^6 - 7
7*a^2*c*d^3*e^8 + 45*a^(5/2)*sqrt(c)*d*e^10)*log(sqrt(c)*x^2 + sqrt(2)*a^(
1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(32*sqrt(2)*a^(11/4)
*c^(1/4)*e^11 + 21*c^3*d^11 - 5*sqrt(a)*c^(5/2)*d^9*e^2 + 66*a*c^2*d^7*e^4
 - 18*a^(3/2)*c^(3/2)*d^5*e^6 + 77*a^2*c*d^3*e^8 - 45*a^(5/2)*sqrt(c)*d*e^
10)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4
)) - 2*(21*sqrt(2)*a^(1/4)*c^(13/4)*d^11 + 5*sqrt(2)*a^(3/4)*c^(11/4)*d^9*
e^2 + 66*sqrt(2)*a^(5/4)*c^(9/4)*d^7*e^4 + 18*sqrt(2)*a^(7/4)*c^(7/4)*d^5*
e^6 + 77*sqrt(2)*a^(9/4)*c^(5/4)*d^3*e^8 + 45*sqrt(2)*a^(11/4)*c^(3/4)*d*e
^10 + 24*sqrt(a)*c^3*d^10*e + 80*a^(3/2)*c^2*d^6*e^5 + 120*a^(5/2)*c*d^2*e
^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a
)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(21*sqrt(2)*a^(1/4
)*c^(13/4)*d^11 + 5*sqrt(2)*a^(3/4)*c^(11/4)*d^9*e^2 + 66*sqrt(2)*a^(5/4)*
c^(9/4)*d^7*e^4 + 18*sqrt(2)*a^(7/4)*c^(7/4)*d^5*e^6 + 77*sqrt(2)*a^(9/4)*
c^(5/4)*d^3*e^8 + 45*sqrt(2)*a^(11/4)*c^(3/4)*d*e^10 - 24*sqrt(a)*c^3*d^10
*e - 80*a^(3/2)*c^2*d^6*e^5 - 120*a^(5/2)*c*d^2*e^9)*arctan(1/2*sqrt(2)*(2
*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt
(sqrt(a)*sqrt(c))*c^(5/4)))/(a^2*c^3*d^12 + 3*a^3*c^2*d^8*e^4 + 3*a^4*c...
```

3.412.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 1311, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)\,(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
e^12*log(abs(e*x + d))/(c^3*d^12*e + 3*a*c^2*d^8*e^5 + 3*a^2*c*d^4*e^9 + a
^3*e^13) - 1/4*e^11*log(abs(c*x^4 + a))/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^
2*c*d^4*e^8 + a^3*e^12) - 1/64*(75*sqrt(2)*a*c^2*d^2*e^3 - 51*sqrt(2)*sqrt
(a*c)*c^2*d^4*e - 21*(a*c^3)^(1/4)*c^2*d^5 - 45*(a*c^3)^(1/4)*a*c*d*e^4 -
122*(a*c^3)^(3/4)*d^3*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^6 + 9*sqrt(2)*a^4*c^2*d^2*e^4 + 9*sqrt(2)*
sqrt(a*c)*a^3*c^2*d^4*e^2 + sqrt(2)*sqrt(a*c)*a^4*c*e^6 - 6*(a*c^3)^(1/4)*
a^3*c^2*d^5*e - 6*(a*c^3)^(1/4)*a^4*c*d*e^5 - 16*(a*c^3)^(3/4)*a^3*d^3*e^3
) + 1/64*(75*sqrt(2)*a*c^2*d^2*e^3 + 51*sqrt(2)*sqrt(a*c)*c^2*d^4*e + 21*(
a*c^3)^(1/4)*c^2*d^5 + 45*(a*c^3)^(1/4)*a*c*d*e^4 + 122*(a*c^3)^(3/4)*d^3*
e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*
a^3*c^3*d^6 + 9*sqrt(2)*a^4*c^2*d^2*e^4 + 9*sqrt(2)*sqrt(a*c)*a^3*c^2*d^4*
e^2 + sqrt(2)*sqrt(a*c)*a^4*c*e^6 + 6*(a*c^3)^(1/4)*a^3*c^2*d^5*e + 6*(a*c
^3)^(1/4)*a^4*c*d*e^5 + 16*(a*c^3)^(3/4)*a^3*d^3*e^3) + 1/256*(21*sqrt(2)*
(a*c^3)^(1/4)*c^4*d^11 + 66*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^7*e^4 + 77*sqrt(
2)*(a*c^3)^(1/4)*a^2*c^2*d^3*e^8 - 5*sqrt(2)*(a*c^3)^(3/4)*c^2*d^9*e^2 - 1
8*sqrt(2)*(a*c^3)^(3/4)*a*c*d^5*e^6 - 45*sqrt(2)*(a*c^3)^(3/4)*a^2*d*e^10)
*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^5*d^12 + 3*a^4*c^4*d^
8*e^4 + 3*a^5*c^3*d^4*e^8 + a^6*c^2*e^12) - 1/256*(21*sqrt(2)*(a*c^3)^(1/4
)*c^4*d^11 + 66*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^7*e^4 + 77*sqrt(2)*(a*c^3...
```

3.412.9 Mupad [B] (verification not implemented)

Time = 10.55 (sec) , antiderivative size = 2720, normalized size of antiderivative = 2.01

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)^3*(d + e*x)),x)
```

output
```
symsum(log((194481*c^7*d^13*e^6 + 871362*a*c^6*d^9*e^10 + 425984*a^3*c^4*d
*e^18 + 1148881*a^2*c^5*d^5*e^14)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a
^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + root(80530636
8*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3
*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a
^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 +
 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z +
676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(8053063
68*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^
3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*
a^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2
+ 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z +
 676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(805306
368*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c
^3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152
*a^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2
 + 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z
+ 676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(80530
6368*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*
c^3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 4305...
```

3.412. $\int \frac{1}{(d+ex)(a+cx^4)^3}\,dx$

3.413 $\int \frac{1}{(d+ex)^2(a+cx^4)^3} dx$

3.413.1 Optimal result

Integrand size = 17, antiderivative size = 1830

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3} dx = \text{Too large to display}$$

output
```
-e^11/(a*e^4+c*d^4)^3/(e*x+d)+1/32*c*x*(7*d^2*(-3*a*e^4+c*d^4)-12*d*e*(-a*
e^4+c*d^4)*x+5*e^2*(-a*e^4+3*c*d^4)*x^2)/a^2/(a*e^4+c*d^4)^2/(c*x^4+a)+1/8
*c*(4*a*d^3*e^3+x*(d^2*(-3*a*e^4+c*d^4)-2*d*e*(-a*e^4+c*d^4)*x+e^2*(-a*e^4
+3*c*d^4)*x^2))/a/(a*e^4+c*d^4)^2/(c*x^4+a)^2+1/4*c*e^4*(8*a*d^3*e^3+x*(d^
2*(-3*a*e^4+5*c*d^4)-2*d*e*(-a*e^4+3*c*d^4)*x+e^2*(-a*e^4+7*c*d^4)*x^2))/a
/(a*e^4+c*d^4)^3/(c*x^4+a)+12*c*d^3*e^11*ln(e*x+d)/(a*e^4+c*d^4)^4-3*c*d^3
*e^11*ln(c*x^4+a)/(a*e^4+c*d^4)^4-1/2*d*e^5*(-a*e^4+3*c*d^4)*arctan(x^2*c^
(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4)^3-3/8*d*e*(-a*e^4+c*d^4)*arct
an(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(5/2)/(a*e^4+c*d^4)^2-d*e^9*(-a*e^4+5*c*
d^4)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^4/a^(1/2)-1/8*c^(1/
4)*e^8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(9*c^(3/2)*d^6+a
^(3/2)*e^6-11*c*d^4*e^2*a^(1/2)-3*a*d^2*e^4*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)
^4*2^(1/2)+1/8*c^(1/4)*e^8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2
))*(9*c^(3/2)*d^6+a^(3/2)*e^6-11*c*d^4*e^2*a^(1/2)-3*a*d^2*e^4*c^(1/2))/a^
(3/4)/(a*e^4+c*d^4)^4*2^(1/2)-1/256*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+
a^(1/2)+x^2*c^(1/2))*(-5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^2*(-3*a*e^4+c*d
^4)*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/256*c^(1/4)*ln(a^(1/4)*c^(
1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^
2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/128*c^(1/4)
*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+2...
```

3.413.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 1115, normalized size of antiderivative = 0.61

$$\int \frac{1}{(d+ex)^2\,(a+cx^4)^3}\,dx$$

$$= \frac{-\frac{256e^{11}(cd^4+ae^4)}{d+ex}+\frac{8c(cd^4+ae^4)\left(c^2d^8x(7d^2-12dex+15e^2x^2)+2acd^4e^4x(13d^2-24dex+33e^2x^2)+a^2e^7(64d^3-45d^2ex+28de^2x^2-13e^3x^3)\right)}{a^2(a+cx^4)}}{}$$

input
```
Integrate[1/((d + e*x)^2*(a + c*x^4)^3),x]
```

output
```
((-256*e^11*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(c^2*d^8*x*(
7*d^2 - 12*d*e*x + 15*e^2*x^2) + 2*a*c*d^4*e^4*x*(13*d^2 - 24*d*e*x + 33*e
^2*x^2) + a^2*e^7*(64*d^3 - 45*d^2*e*x + 28*d*e^2*x^2 - 13*e^3*x^3)))/(a^2
*(a + c*x^4)) + (32*c*(c*d^4 + a*e^4)^2*(c*d^4*x*(d^2 - 2*d*e*x + 3*e^2*x^
2) + a*e^3*(4*d^3 - 3*d^2*e*x + 2*d*e^2*x^2 - e^3*x^3)))/(a*(a + c*x^4)^2)
 - (6*c^(1/4)*(7*Sqrt[2]*c^(7/2)*d^14 - 16*a^(1/4)*c^(13/4)*d^13*e + 5*Sqr
t[2]*Sqrt[a]*c^3*d^12*e^2 + 33*Sqrt[2]*a*c^(5/2)*d^10*e^4 - 80*a^(5/4)*c^(
9/4)*d^9*e^5 + 27*Sqrt[2]*a^(3/2)*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^(3/2)*d^6
*e^8 - 240*a^(9/4)*c^(5/4)*d^5*e^9 + 135*Sqrt[2]*a^(5/2)*c*d^4*e^10 - 77*S
qrt[2]*a^3*Sqrt[c]*d^2*e^12 + 80*a^(13/4)*c^(1/4)*d*e^13 - 15*Sqrt[2]*a^(7
/2)*e^14)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*c^(1/4)*(
7*Sqrt[2]*c^(7/2)*d^14 + 16*a^(1/4)*c^(13/4)*d^13*e + 5*Sqrt[2]*Sqrt[a]*c^
3*d^12*e^2 + 33*Sqrt[2]*a*c^(5/2)*d^10*e^4 + 80*a^(5/4)*c^(9/4)*d^9*e^5 +
27*Sqrt[2]*a^(3/2)*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^(3/2)*d^6*e^8 + 240*a^(9
/4)*c^(5/4)*d^5*e^9 + 135*Sqrt[2]*a^(5/2)*c*d^4*e^10 - 77*Sqrt[2]*a^3*Sqrt
[c]*d^2*e^12 - 80*a^(13/4)*c^(1/4)*d*e^13 - 15*Sqrt[2]*a^(7/2)*e^14)*ArcTa
n[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + 3072*c*d^3*e^11*Log[d + e*x
] - (3*Sqrt[2]*c^(1/4)*(7*c^(7/2)*d^14 - 5*Sqrt[a]*c^3*d^12*e^2 + 33*a*c^(
5/2)*d^10*e^4 - 27*a^(3/2)*c^2*d^8*e^6 + 77*a^2*c^(3/2)*d^6*e^8 - 135*a^(5
/2)*c*d^4*e^10 - 77*a^3*Sqrt[c]*d^2*e^12 + 15*a^(7/2)*e^14)*Log[Sqrt[a]...
```

3.413.3 Rubi [A] (verified)

Time = 3.13 (sec) , antiderivative size = 1830, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^3 (d+ex)^2} dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(\frac{e^{12}}{(d+ex)^2 (ae^4+cd^4)^3} + \frac{12cd^3e^{12}}{(d+ex)(ae^4+cd^4)^4} + \frac{ce^4(-2dex(3cd^4-ae^4)+e^2x^2(7cd^4-ae^4)+d^2(5cd^4-3}{(a+cx^4)^2 (ae^4+cd^4)^3}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{12cd^3\log(d+ex)e^{11}}{(cd^4+ae^4)^4}-\frac{3cd^3\log\left(cx^4+a\right)e^{11}}{(cd^4+ae^4)^4}-\frac{e^{11}}{(cd^4+ae^4)^3\,(d+ex)}-$$
$$\frac{\sqrt{c}d\left(5cd^4-ae^4\right)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^9}{\sqrt{a}\,(cd^4+ae^4)^4}-$$
$$\frac{\sqrt[4]{c}\left(3\sqrt{c}\left(3cd^4-ae^4\right)d^2+\sqrt{a}e^2\left(11cd^4-ae^4\right)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^8}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^4}+$$
$$\frac{\sqrt[4]{c}\left(3\sqrt{c}\left(3cd^4-ae^4\right)d^2+\sqrt{a}e^2\left(11cd^4-ae^4\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^8}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^4}-$$
$$\frac{\sqrt[4]{c}\left(9c^{3/2}d^6-11\sqrt{a}ce^2d^4-3a\sqrt{c}e^4d^2+a^{3/2}e^6\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^8}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)^4}+$$
$$\frac{\sqrt[4]{c}\left(9c^{3/2}d^6-11\sqrt{a}ce^2d^4-3a\sqrt{c}e^4d^2+a^{3/2}e^6\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^8}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)^4}-$$
$$\frac{\sqrt{c}d\left(3cd^4-ae^4\right)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{2a^{3/2}\,(cd^4+ae^4)^3}+$$
$$\frac{c\left(8ad^3e^3+x\left(\left(5cd^4-3ae^4\right)d^2-2e\left(3cd^4-ae^4\right)xd+e^2\left(7cd^4-ae^4\right)x^2\right)\right)e^4}{4a\,(cd^4+ae^4)^3\,(cx^4+a)}-$$
$$\frac{\sqrt[4]{c}\left(3\sqrt{c}\left(5cd^4-3ae^4\right)d^2+\sqrt{a}e^2\left(7cd^4-ae^4\right)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{8\sqrt{2}a^{7/4}\,(cd^4+ae^4)^3}+$$
$$\frac{\sqrt[4]{c}\left(3\sqrt{c}\left(5cd^4-3ae^4\right)d^2+\sqrt{a}e^2\left(7cd^4-ae^4\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{8\sqrt{2}a^{7/4}\,(cd^4+ae^4)^3}-$$
$$\frac{\sqrt[4]{c}\left(3\sqrt{c}d^2\left(5cd^4-3ae^4\right)-\sqrt{a}e^2\left(7cd^4-ae^4\right)\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{16\sqrt{2}a^{7/4}\,(cd^4+ae^4)^3}+$$
$$\frac{\sqrt[4]{c}\left(3\sqrt{c}d^2\left(5cd^4-3ae^4\right)-\sqrt{a}e^2\left(7cd^4-ae^4\right)\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{16\sqrt{2}a^{7/4}\,(cd^4+ae^4)^3}-$$
$$\frac{3\sqrt{c}d\left(cd^4-ae^4\right)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{8a^{5/2}\,(cd^4+ae^4)^2}+$$
$$\frac{c\left(4ad^3e^3+x\left(\left(cd^4-3ae^4\right)d^2-2e\left(cd^4-ae^4\right)xd+e^2\left(3cd^4-ae^4\right)x^2\right)\right)}{8a\,(cd^4+ae^4)^2\,(cx^4+a)^2}-$$
$$\frac{\sqrt[4]{c}\left(21\sqrt{c}\left(cd^4-3ae^4\right)d^2+5\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\,(cd^4+ae^4)^2}+$$
$$\frac{\sqrt[4]{c}\left(21\sqrt{c}\left(cd^4-3ae^4\right)d^2+5\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}\,(cd^4+ae^4)^2}-$$
$$\frac{\sqrt[4]{c}\left(21\sqrt{c}d^2\left(cd^4-3ae^4\right)-5\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{128\sqrt{2}a^{11/4}\,(cd^4+ae^4)^2}+$$
$$\frac{\sqrt[4]{c}\left(21\sqrt{c}d^2\left(cd^4-3ae^4\right)-5\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{128\sqrt{2}a^{11/4}\,(cd^4+ae^4)^2}+$$
$$\frac{cx\left(7\left(cd^4-3ae^4\right)d^2-12e\left(cd^4-ae^4\right)xd+5e^2\left(3cd^4-ae^4\right)x^2\right)}{32a^2\,(cd^4+ae^4)^2\,(cx^4+a)}$$

3.413. $\int \frac{1}{(d+ex)^2(a+cx^4)^3}\,dx$

input
```
Int[1/((d + e*x)^2*(a + c*x^4)^3),x]
```

output
```
-(e^11/((c*d^4 + a*e^4)^3*(d + e*x))) + (c*x*(7*d^2*(c*d^4 - 3*a*e^4) - 12
*d*e*(c*d^4 - a*e^4)*x + 5*e^2*(3*c*d^4 - a*e^4)*x^2))/(32*a^2*(c*d^4 + a*
e^4)^2*(a + c*x^4)) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3*a*e^4) - 2*d*e*(
c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(8*a*(c*d^4 + a*e^4)^2*(a
+ c*x^4)^2) + (c*e^4*(8*a*d^3*e^3 + x*(d^2*(5*c*d^4 - 3*a*e^4) - 2*d*e*(3*
c*d^4 - a*e^4)*x + e^2*(7*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a
+ c*x^4)) - (Sqrt[c]*d*e^9*(5*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(Sqrt[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*ArcTan[(S
qrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^3) - (3*Sqrt[c]*d*e*(c*d^
4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(8*a^(5/2)*(c*d^4 + a*e^4)^2) -
(c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^
4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 +
 a*e^4)^2) - (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2
*(7*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^
(7/4)*(c*d^4 + a*e^4)^3) - (c^(1/4)*e^8*(3*Sqrt[c]*d^2*(3*c*d^4 - a*e^4) +
 Sqrt[a]*e^2*(11*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 -
3*a*e^4) + 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)
/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(3*Sqrt[
c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*ArcTan[1 + ...
```

3.413.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.413.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 829, normalized size of antiderivative = 0.45

method	result
default	$-\frac{c\left(\frac{c e^{2}\left(13a^{3}e^{12}-53a^{2}c\,d^{4}e^{8}-81a\,c^{2}d^{8}e^{4}-15c^{3}d^{12}\right)x^{7}}{32a^{2}}-\frac{dce\left(7a^{3}e^{12}-5a^{2}c\,d^{4}e^{8}-15a\,c^{2}d^{8}e^{4}-3c^{3}d^{12}\right)x^{6}}{8a^{2}}+\frac{c\,d^{2}\left(45a^{3}e^{12}+19a^{2}c\,d^{4}e^{8}-33a\,c\right.}{32a^{2}}\right.}{}$
risch	Expression too large to display

input
```
int(1/(e*x+d)^2/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-c/(a*e^4+c*d^4)^4*((1/32*c*e^2*(13*a^3*e^12-53*a^2*c*d^4*e^8-81*a*c^2*d^8
*e^4-15*c^3*d^12)/a^2*x^7-1/8*d*c*e*(7*a^3*e^12-5*a^2*c*d^4*e^8-15*a*c^2*d
^8*e^4-3*c^3*d^12)/a^2*x^6+1/32*c*d^2*(45*a^3*e^12+19*a^2*c*d^4*e^8-33*a*c
^2*d^8*e^4-7*c^3*d^12)/a^2*x^5+(-2*a*c*d^3*e^11-2*c^2*d^7*e^7)*x^4+1/32*e^
2*(17*a^3*e^12-57*a^2*c*d^4*e^8-101*a*c^2*d^8*e^4-27*c^3*d^12)/a*x^3-1/8*d
*e*(9*a^3*e^12-3*a^2*c*d^4*e^8-17*a*c^2*d^8*e^4-5*c^3*d^12)/a*x^2+1/32*d^2
*(57*a^3*e^12+39*a^2*c*d^4*e^8-29*a*c^2*d^8*e^4-11*c^3*d^12)/a*x-5/2*a^2*d
^3*e^11-3*a*d^7*e^7*c-1/2*d^11*e^3*c^2)/(c*x^4+a)^2+3/32/a^2*(1/8*(77*a^3*
d^2*e^12-77*a^2*c*d^6*e^8-33*a*c^2*d^10*e^4-7*c^3*d^14)*(a/c)^(1/4)/a*2^(1
/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)
+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1
/4)*x-1))+1/2*(-40*a^3*d*e^13+120*a^2*c*d^5*e^9+40*a*c^2*d^9*e^5+8*c^3*d^1
3*e)/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*(15*a^3*e^14-135*a^2*c*d^4*e^
10-27*a*c^2*d^8*e^6-5*c^3*d^12*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(
1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arc
tan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+32*a^2*d^3
*e^11*ln(c*x^4+a)))-e^11/(a*e^4+c*d^4)^3/(e*x+d)+12*c*d^3*e^11*ln(e*x+d)/(
a*e^4+c*d^4)^4
```

3.413.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.413.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**2/(c*x**4+a)**3,x)
```

output
```
Timed out
```

3.413.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 1564, normalized size of antiderivative = 0.85

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
12*c*d^3*e^11*log(e*x + d)/(c^4*d^16 + 4*a*c^3*d^12*e^4 + 6*a^2*c^2*d^8*e^
8 + 4*a^3*c*d^4*e^12 + a^4*e^16) - 3/256*c*(sqrt(2)*(128*sqrt(2)*a^(11/4)*
c^(5/4)*d^3*e^11 - 7*c^4*d^14 + 5*sqrt(a)*c^(7/2)*d^12*e^2 - 33*a*c^3*d^10
*e^4 + 27*a^(3/2)*c^(5/2)*d^8*e^6 - 77*a^2*c^2*d^6*e^8 + 135*a^(5/2)*c^(3/
2)*d^4*e^10 + 77*a^3*c*d^2*e^12 - 15*a^(7/2)*sqrt(c)*e^14)*log(sqrt(c)*x^2
 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(128*s
qrt(2)*a^(11/4)*c^(5/4)*d^3*e^11 + 7*c^4*d^14 - 5*sqrt(a)*c^(7/2)*d^12*e^2
 + 33*a*c^3*d^10*e^4 - 27*a^(3/2)*c^(5/2)*d^8*e^6 + 77*a^2*c^2*d^6*e^8 - 1
35*a^(5/2)*c^(3/2)*d^4*e^10 - 77*a^3*c*d^2*e^12 + 15*a^(7/2)*sqrt(c)*e^14)
*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4))
- 2*(7*sqrt(2)*a^(1/4)*c^(17/4)*d^14 + 5*sqrt(2)*a^(3/4)*c^(15/4)*d^12*e^2
 + 33*sqrt(2)*a^(5/4)*c^(13/4)*d^10*e^4 + 27*sqrt(2)*a^(7/4)*c^(11/4)*d^8*
e^6 + 77*sqrt(2)*a^(9/4)*c^(9/4)*d^6*e^8 + 135*sqrt(2)*a^(11/4)*c^(7/4)*d^
4*e^10 - 77*sqrt(2)*a^(13/4)*c^(5/4)*d^2*e^12 - 15*sqrt(2)*a^(15/4)*c^(3/4
)*e^14 + 16*sqrt(a)*c^4*d^13*e + 80*a^(3/2)*c^3*d^9*e^5 + 240*a^(5/2)*c^2*
d^5*e^9 - 80*a^(7/2)*c*d*e^13)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a
^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5
/4)) - 2*(7*sqrt(2)*a^(1/4)*c^(17/4)*d^14 + 5*sqrt(2)*a^(3/4)*c^(15/4)*d^1
2*e^2 + 33*sqrt(2)*a^(5/4)*c^(13/4)*d^10*e^4 + 27*sqrt(2)*a^(7/4)*c^(11/4)
*d^8*e^6 + 77*sqrt(2)*a^(9/4)*c^(9/4)*d^6*e^8 + 135*sqrt(2)*a^(11/4)*c^...
```

3.413.8 Giac [A] (verification not implemented)

Time = 46.91 (sec) , antiderivative size = 1809, normalized size of antiderivative = 0.99

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
12*c*d^3*e^12*log(abs(e*x + d))/(c^4*d^16*e + 4*a*c^3*d^12*e^5 + 6*a^2*c^2
*d^8*e^9 + 4*a^3*c*d^4*e^13 + a^4*e^17) - 3*c*d^3*e^11*log(abs(c*x^4 + a))
/(c^4*d^16 + 4*a*c^3*d^12*e^4 + 6*a^2*c^2*d^8*e^8 + 4*a^3*c*d^4*e^12 + a^4
*e^16) - 3/64*(32*sqrt(2)*a*c^3*d^3*e^3 - 20*sqrt(2)*sqrt(a*c)*c^3*d^5*e -
 20*sqrt(2)*sqrt(a*c)*a*c^2*d*e^5 - 7*(a*c^3)^(1/4)*c^3*d^6 - 3*(a*c^3)^(1
/4)*a*c^2*d^2*e^4 - 53*(a*c^3)^(3/4)*c*d^4*e^2 + 15*(a*c^3)^(3/4)*a*e^6)*a
rctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^
4*d^8 + 34*sqrt(2)*a^4*c^3*d^4*e^4 + sqrt(2)*a^5*c^2*e^8 + 16*sqrt(2)*sqrt
(a*c)*a^3*c^3*d^6*e^2 + 16*sqrt(2)*sqrt(a*c)*a^4*c^2*d^2*e^6 - 8*(a*c^3)^(
1/4)*a^3*c^3*d^7*e - 40*(a*c^3)^(1/4)*a^4*c^2*d^3*e^5 - 40*(a*c^3)^(3/4)*a
^3*c*d^5*e^3 - 8*(a*c^3)^(3/4)*a^4*d*e^7) + 3/64*(32*sqrt(2)*a*c^3*d^3*e^3
 + 20*sqrt(2)*sqrt(a*c)*c^3*d^5*e + 20*sqrt(2)*sqrt(a*c)*a*c^2*d*e^5 + 7*(
a*c^3)^(1/4)*c^3*d^6 + 3*(a*c^3)^(1/4)*a*c^2*d^2*e^4 + 53*(a*c^3)^(3/4)*c*
d^4*e^2 - 15*(a*c^3)^(3/4)*a*e^6)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^
(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^4*d^8 + 34*sqrt(2)*a^4*c^3*d^4*e^4 + sq
rt(2)*a^5*c^2*e^8 + 16*sqrt(2)*sqrt(a*c)*a^3*c^3*d^6*e^2 + 16*sqrt(2)*sqrt
(a*c)*a^4*c^2*d^2*e^6 + 8*(a*c^3)^(1/4)*a^3*c^3*d^7*e + 40*(a*c^3)^(1/4)*a
^4*c^2*d^3*e^5 + 40*(a*c^3)^(3/4)*a^3*c*d^5*e^3 + 8*(a*c^3)^(3/4)*a^4*d*e^
7) + 3/256*(7*sqrt(2)*(a*c^3)^(1/4)*c^5*d^14 + 33*sqrt(2)*(a*c^3)^(1/4)*a*
c^4*d^10*e^4 + 77*sqrt(2)*(a*c^3)^(1/4)*a^2*c^3*d^6*e^8 - 77*sqrt(2)*(a...
```

3.413.9 Mupad [B] (verification not implemented)

Time = 11.18 (sec) , antiderivative size = 3572, normalized size of antiderivative = 1.95

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3} \, dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)^3*(d + e*x)^2),x)
```

output
```
symsum(log((194481*c^9*d^17*e^6 + 1527012*a*c^8*d^13*e^10 + 4100625*a^4*c^
5*d*e^22 + 1926342*a^2*c^7*d^9*e^14 - 3102300*a^3*c^6*d^5*e^18)/(1048576*(
a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^1
0*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)) + root(161
0612736*a^13*c^2*d^8*e^8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 10737418
24*a^14*c*d^4*e^12*z^4 + 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16
*z^4 + 3221225472*a^11*c*d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39
518208*a^6*c^3*d^10*e^2*z^2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4
*c^2*d^5*e^5*z + 5419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 11380
50*a*c^2*d^4*e^4 + 4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*(root(1610612
736*a^13*c^2*d^8*e^8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a
^14*c*d^4*e^12*z^4 + 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4
 + 3221225472*a^11*c*d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 395182
08*a^6*c^3*d^10*e^2*z^2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2
*d^5*e^5*z + 5419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a
*c^2*d^4*e^4 + 4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*(root(1610612736*
a^13*c^2*d^8*e^8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*
c*d^4*e^12*z^4 + 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 3
221225472*a^11*c*d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a
^6*c^3*d^10*e^2*z^2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*...
```

3.413. $\int \frac{1}{(d+ex)^2(a+cx^4)^3}\,dx$

3.414 $\int \frac{1}{(d+ex)^3(a+cx^4)^3} dx$

3.414.1 Optimal result

Integrand size = 17, antiderivative size = 2204

$$\int \frac{1}{(d+ex)^3(a+cx^4)^3} dx = \text{Too large to display}$$

output
```
-1/2*e^11/(a*e^4+c*d^4)^3/(e*x+d)^2-12*c*d^3*e^11/(a*e^4+c*d^4)^4/(e*x+d)+
1/32*c*x*(7*d*(3*a^2*e^8-12*a*c*d^4*e^4+c^2*d^8)-6*e*(a^2*e^8-12*a*c*d^4*e
^4+3*c^2*d^8)*x+10*c*d^3*e^2*(-5*a*e^4+3*c*d^4)*x^2)/a^2/(a*e^4+c*d^4)^3/(
c*x^4+a)+1/8*c*(2*a*d^2*e^3*(-3*a*e^4+5*c*d^4)+x*(d*(3*a^2*e^8-12*a*c*d^4*
e^4+c^2*d^8)-e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*x+2*c*d^3*e^2*(-5*a*e^4+
3*c*d^4)*x^2))/a/(a*e^4+c*d^4)^3/(c*x^4+a)^2+1/4*c*e^4*(12*a*d^2*e^3*(-a*e
^4+3*c*d^4)+x*(3*d*(a^2*e^8-10*a*c*d^4*e^4+5*c^2*d^8)-e*(a^2*e^8-26*a*c*d^
4*e^4+21*c^2*d^8)*x+4*c*d^3*e^2*(-5*a*e^4+7*c*d^4)*x^2))/a/(a*e^4+c*d^4)^4
/(c*x^4+a)+6*c*d^2*e^11*(-3*a*e^4+13*c*d^4)*ln(e*x+d)/(a*e^4+c*d^4)^5-3/2*
c*d^2*e^11*(-3*a*e^4+13*c*d^4)*ln(c*x^4+a)/(a*e^4+c*d^4)^5-1/4*e^5*(a^2*e^
8-26*a*c*d^4*e^4+21*c^2*d^8)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(
a*e^4+c*d^4)^4-3/16*e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*arctan(x^2*c^(1/2
)/a^(1/2))*c^(1/2)/a^(5/2)/(a*e^4+c*d^4)^3-1/2*e^9*(a^2*e^8-40*a*c*d^4*e^4
+55*c^2*d^8)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^5/a^(1/2)+1
/256*c^(3/4)*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-63*a^2
*e^8+252*a*c*d^4*e^4-21*c^2*d^8+10*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1
/2))/a^(11/4)/(a*e^4+c*d^4)^3*2^(1/2)-1/256*c^(3/4)*d*ln(a^(1/4)*c^(1/4)*x
*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-63*a^2*e^8+252*a*c*d^4*e^4-21*c^2*d^8+10*d
^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^3*2^(1/2
)+1/128*c^(3/4)*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(63*a^2*e^8-252*...
```

3.414.2 Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 1338, normalized size of antiderivative = 0.61

$$\int \frac{1}{(d+ex)^3 (a+cx^4)^3} \, dx$$

$$= \frac{-\frac{128e^{11}(cd^4+ae^4)^2}{(d+ex)^2} - \frac{3072cd^3e^{11}(cd^4+ae^4)}{d+ex} + \frac{8c(cd^4+ae^4)(a^3e^{11}(-96d^2+45dex-14e^2x^2)+c^3d^{11}x(7d^2-18dex+30e^2x^2)+ac^2d^7e^4x(43d}{a^2(a+cx^4)}}{}$$

input
```
Integrate[1/((d + e*x)^3*(a + c*x^4)^3),x]
```

output
```
((-128*e^11*(c*d^4 + a*e^4)^2)/(d + e*x)^2 - (3072*c*d^3*e^11*(c*d^4 + a*e
^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(a^3*e^11*(-96*d^2 + 45*d*e*x - 14*e
^2*x^2) + c^3*d^11*x*(7*d^2 - 18*d*e*x + 30*e^2*x^2) + a*c^2*d^7*e^4*x*(43
*d^2 - 114*d*e*x + 204*e^2*x^2) + a^2*c*d^3*e^7*(288*d^3 - 303*d^2*e*x + 2
74*d*e^2*x^2 - 210*e^3*x^3)))/(a^2*(a + c*x^4)) + (32*c*(c*d^4 + a*e^4)^2*
(-(a^2*e^7*(6*d^2 - 3*d*e*x + e^2*x^2)) + c^2*d^7*x*(d^2 - 3*d*e*x + 6*e^2
*x^2) + 2*a*c*d^3*e^3*(5*d^3 - 6*d^2*e*x + 6*d*e^2*x^2 - 5*e^3*x^3)))/(a*(
a + c*x^4)^2) - (6*Sqrt[c]*(7*Sqrt[2]*c^(17/4)*d^17 - 24*a^(1/4)*c^4*d^16*
e + 10*Sqrt[2]*Sqrt[a]*c^(15/4)*d^15*e^2 + 50*Sqrt[2]*a*c^(13/4)*d^13*e^4
- 176*a^(5/4)*c^3*d^12*e^5 + 78*Sqrt[2]*a^(3/2)*c^(11/4)*d^11*e^6 + 220*Sq
rt[2]*a^2*c^(9/4)*d^9*e^8 - 960*a^(9/4)*c^2*d^8*e^9 + 702*Sqrt[2]*a^(5/2)*
c^(7/4)*d^7*e^10 - 770*Sqrt[2]*a^3*c^(5/4)*d^5*e^12 + 1200*a^(13/4)*c*d^4*
e^13 - 390*Sqrt[2]*a^(7/2)*c^(3/4)*d^3*e^14 + 77*Sqrt[2]*a^4*c^(1/4)*d*e^1
6 - 40*a^(17/4)*e^17)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) +
(6*Sqrt[c]*(7*Sqrt[2]*c^(17/4)*d^17 + 24*a^(1/4)*c^4*d^16*e + 10*Sqrt[2]*S
qrt[a]*c^(15/4)*d^15*e^2 + 50*Sqrt[2]*a*c^(13/4)*d^13*e^4 + 176*a^(5/4)*c^
3*d^12*e^5 + 78*Sqrt[2]*a^(3/2)*c^(11/4)*d^11*e^6 + 220*Sqrt[2]*a^2*c^(9/4
)*d^9*e^8 + 960*a^(9/4)*c^2*d^8*e^9 + 702*Sqrt[2]*a^(5/2)*c^(7/4)*d^7*e^10
 - 770*Sqrt[2]*a^3*c^(5/4)*d^5*e^12 - 1200*a^(13/4)*c*d^4*e^13 - 390*Sqrt[
2]*a^(7/2)*c^(3/4)*d^3*e^14 + 77*Sqrt[2]*a^4*c^(1/4)*d*e^16 + 40*a^(17/...
```

3.414.3 Rubi [A] (verified)

Time = 3.51 (sec) , antiderivative size = 2204, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^3 (d+ex)^3} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{ce^8\left(-ex\left(a^2e^8-40acd^4e^4+55c^2d^8\right)+3d\left(a^2e^8-16acd^4e^4+15c^2d^8\right)+6cd^3e^2x^2\left(11cd^4-5ae^4\right)-6cd^2e^3x^3\right.}{(a+cx^4)\left(ae^4+cd^4\right)^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{6cd^2(13cd^4-3ae^4)\log(d+ex)e^{11}}{(cd^4+ae^4)^5}-\frac{3cd^2(13cd^4-3ae^4)\log(cx^4+a)e^{11}}{2(cd^4+ae^4)^5}-\frac{12cd^3e^{11}}{(cd^4+ae^4)^4(d+ex)}-$$

$$\frac{e^{11}}{2(cd^4+ae^4)^3(d+ex)^2}-\frac{\sqrt{c}(55c^2d^8-40ace^4d^4+a^2e^8)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^9}{2\sqrt{a}(cd^4+ae^4)^5}-$$

$$\frac{3c^{3/4}d(15c^2d^8-16ace^4d^4+2\sqrt{a}\sqrt{c}e^2(11cd^4-5ae^4)d^2+a^2e^8)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^8}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^5}+$$

$$\frac{3c^{3/4}d(15c^2d^8-16ace^4d^4+2\sqrt{a}\sqrt{c}e^2(11cd^4-5ae^4)d^2+a^2e^8)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^8}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^5}-$$

$$\frac{3c^{3/4}d(15c^2d^8-16ace^4d^4-2\sqrt{a}\sqrt{c}e^2(11cd^4-5ae^4)d^2+a^2e^8)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^8}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^5}+$$

$$\frac{3c^{3/4}d(15c^2d^8-16ace^4d^4-2\sqrt{a}\sqrt{c}e^2(11cd^4-5ae^4)d^2+a^2e^8)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^8}{4\sqrt{2}a^{3/4}(cd^4+ae^4)^5}-$$

$$\frac{\sqrt{c}(21c^2d^8-26ace^4d^4+a^2e^8)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{4a^{3/2}(cd^4+ae^4)^4}+$$

$$\frac{c(12ad^2(3cd^4-ae^4)e^3+x(4ce^2(7cd^4-5ae^4)x^2d^3+3(5c^2d^8-10ace^4d^4+a^2e^8)d-e(21c^2d^8-26ace^4d^4+a^2e}{4a(cd^4+ae^4)^4(cx^4+a)}$$

$$\frac{c^{3/4}d(4\sqrt{a}\sqrt{c}d^2(7cd^4-5ae^4)e^2+9(5c^2d^8-10ace^4d^4+a^2e^8))\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{8\sqrt{2}a^{7/4}(cd^4+ae^4)^4}+$$

$$\frac{c^{3/4}d(4\sqrt{a}\sqrt{c}d^2(7cd^4-5ae^4)e^2+9(5c^2d^8-10ace^4d^4+a^2e^8))\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{8\sqrt{2}a^{7/4}(cd^4+ae^4)^4}+$$

$$\frac{c^{3/4}d(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)-9(5c^2d^8-10ace^4d^4+a^2e^8))\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{16\sqrt{2}a^{7/4}(cd^4+ae^4)^4}-$$

$$\frac{c^{3/4}d(4\sqrt{a}\sqrt{c}d^2e^2(7cd^4-5ae^4)-9(5c^2d^8-10ace^4d^4+a^2e^8))\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{16\sqrt{2}a^{7/4}(cd^4+ae^4)^4}-$$

$$\frac{3\sqrt{c}(3c^2d^8-12ace^4d^4+a^2e^8)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{16a^{5/2}(cd^4+ae^4)^3}+$$

$$\frac{c(2ad^2(5cd^4-3ae^4)e^3+x(2ce^2(3cd^4-5ae^4)x^2d^3+(c^2d^8-12ace^4d^4+3a^2e^8)d-e(3c^2d^8-12ace^4d^4+a^2e^8)}{8a(cd^4+ae^4)^3(cx^4+a)^2}$$

$$\frac{c^{3/4}d(10\sqrt{a}\sqrt{c}d^2(3cd^4-5ae^4)e^2+21(c^2d^8-12ace^4d^4+3a^2e^8))\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}(cd^4+ae^4)^3}+$$

$$\frac{c^{3/4}d(10\sqrt{a}\sqrt{c}d^2(3cd^4-5ae^4)e^2+21(c^2d^8-12ace^4d^4+3a^2e^8))\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}(cd^4+ae^4)^3}+$$

$$\frac{c^{3/4}d(10\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)-21(c^2d^8-12ace^4d^4+3a^2e^8))\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{128\sqrt{2}a^{11/4}(cd^4+ae^4)^3}-$$

$$\frac{c^{3/4}d(10\sqrt{a}\sqrt{c}d^2e^2(3cd^4-5ae^4)-21(c^2d^8-12ace^4d^4+3a^2e^8))\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{128\sqrt{2}a^{11/4}(cd^4+ae^4)^3}+$$

$$\frac{cx(10ce^2(3cd^4-5ae^4)x^2d^3+7(c^2d^8-12ace^4d^4+3a^2e^8)d-6e(3c^2d^8-12ace^4d^4+a^2e^8)x)}{32a^2(cd^4+ae^4)^3(cx^4+a)}$$

3.414. $\int \frac{1}{(d+ex)^3(a+cx^4)^3}\,dx$

input
```
Int[1/((d + e*x)^3*(a + c*x^4)^3),x]
```

output
```
-1/2*e^11/((c*d^4 + a*e^4)^3*(d + e*x)^2) - (12*c*d^3*e^11)/((c*d^4 + a*e^
4)^4*(d + e*x)) + (c*x*(7*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - 6*e*(
3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 10*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)
*x^2))/(32*a^2*(c*d^4 + a*e^4)^3*(a + c*x^4)) + (c*(2*a*d^2*e^3*(5*c*d^4 -
 3*a*e^4) + x*(d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 1
2*a*c*d^4*e^4 + a^2*e^8)*x + 2*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(8*a*(
c*d^4 + a*e^4)^3*(a + c*x^4)^2) + (c*e^4*(12*a*d^2*e^3*(3*c*d^4 - a*e^4) +
 x*(3*d*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8) - e*(21*c^2*d^8 - 26*a*c*d^
4*e^4 + a^2*e^8)*x + 4*c*d^3*e^2*(7*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 +
a*e^4)^4*(a + c*x^4)) - (Sqrt[c]*e^9*(55*c^2*d^8 - 40*a*c*d^4*e^4 + a^2*e^
8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^5) - (Sqrt[c]
*e^5*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(4*a^(3/2)*(c*d^4 + a*e^4)^4) - (3*Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4
 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*(c*d^4 + a*e^4)^3)
- (3*c^(3/4)*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 + 2*Sqrt[a]*Sqrt
[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]
*d^2*e^2*(7*c*d^4 - 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*A
rcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)
^4) - (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) + 21*(...
```

3.414.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.414.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 1008, normalized size of antiderivative = 0.46

method	result	size
default	Expression too large to display	1008
risch	Expression too large to display	2238

input
```
int(1/(e*x+d)^3/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
c/(a*e^4+c*d^4)^5*((-3/16*c^2*d^3*e^2*(35*a^3*e^12+a^2*c*d^4*e^8-39*a*c^2*
d^8*e^4-5*c^3*d^12)/a^2*x^7-1/16*e*c*(7*a^4*e^16-130*a^3*c*d^4*e^12-80*a^2
*c^2*d^8*e^8+66*a*c^3*d^12*e^4+9*c^4*d^16)/a^2*x^6+1/32*c*d*(45*a^4*e^16-2
58*a^3*c*d^4*e^12-260*a^2*c^2*d^8*e^8+50*a*c^3*d^12*e^4+7*c^4*d^16)/a^2*x^
5+(-3*a^2*c*d^2*e^15+6*a*c^2*d^6*e^11+9*c^3*d^10*e^7)*x^4-1/16*c*d^3*e^2*(
125*a^3*e^12+31*a^2*c*d^4*e^8-121*a*c^2*d^8*e^4-27*c^3*d^12)/a*x^3-3/16*e*
(3*a^4*e^16-50*a^3*c*d^4*e^12-40*a^2*c^2*d^8*e^8+18*a*c^3*d^12*e^4+5*c^4*d
^16)/a*x^2+1/32*d*(57*a^4*e^16-282*a^3*c*d^4*e^12-340*a^2*c^2*d^8*e^8+10*a
*c^3*d^12*e^4+11*c^4*d^16)/a*x-15/4*a^3*d^2*e^15+23/4*a^2*d^6*e^11*c+43/4*
a*d^10*e^7*c^2+5/4*d^14*e^3*c^3)/(c*x^4+a)^2+3/32/a^2*(1/8*(77*a^4*d*e^16-
770*a^3*c*d^5*e^12+220*a^2*c^2*d^9*e^8+50*a*c^3*d^13*e^4+7*c^4*d^17)*(a/c)
^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1
/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(
1/2)/(a/c)^(1/4)*x-1))+1/2*(-20*a^4*e^17+600*a^3*c*d^4*e^13-480*a^2*c^2*d^
8*e^9-88*a*c^3*d^12*e^5-12*c^4*d^16*e)/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))
+1/8*(-390*a^3*c*d^3*e^14+702*a^2*c^2*d^7*e^10+78*a*c^3*d^11*e^6+10*c^4*d^
15*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/
(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)
+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/4*(192*a^3*c*d^2*e^15-832*a^2*c^2*d^
6*e^11)/c*ln(c*x^4+a))-1/2*e^11/(a*e^4+c*d^4)^3/(e*x+d)^2-12*c*d^3*e^1...
```

3.414.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3 (a+cx^4)^3} dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.414.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**3/(c*x**4+a)**3,x)
```

output
```
Timed out
```

3.414.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 2198, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)^3(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-3/256*c*(sqrt(2)*(832*sqrt(2)*a^(11/4)*c^(9/4)*d^6*e^11 - 192*sqrt(2)*a^(
15/4)*c^(5/4)*d^2*e^15 - 7*c^5*d^17 + 10*sqrt(a)*c^(9/2)*d^15*e^2 - 50*a*c
^4*d^13*e^4 + 78*a^(3/2)*c^(7/2)*d^11*e^6 - 220*a^2*c^3*d^9*e^8 + 702*a^(5
/2)*c^(5/2)*d^7*e^10 + 770*a^3*c^2*d^5*e^12 - 390*a^(7/2)*c^(3/2)*d^3*e^14
 - 77*a^4*c*d*e^16)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))
/(a^(3/4)*c^(5/4)) + sqrt(2)*(832*sqrt(2)*a^(11/4)*c^(9/4)*d^6*e^11 - 192*
sqrt(2)*a^(15/4)*c^(5/4)*d^2*e^15 + 7*c^5*d^17 - 10*sqrt(a)*c^(9/2)*d^15*e
^2 + 50*a*c^4*d^13*e^4 - 78*a^(3/2)*c^(7/2)*d^11*e^6 + 220*a^2*c^3*d^9*e^8
 - 702*a^(5/2)*c^(5/2)*d^7*e^10 - 770*a^3*c^2*d^5*e^12 + 390*a^(7/2)*c^(3/
2)*d^3*e^14 + 77*a^4*c*d*e^16)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x
 + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(7*sqrt(2)*a^(1/4)*c^(21/4)*d^17 + 10*sq
rt(2)*a^(3/4)*c^(19/4)*d^15*e^2 + 50*sqrt(2)*a^(5/4)*c^(17/4)*d^13*e^4 + 7
8*sqrt(2)*a^(7/4)*c^(15/4)*d^11*e^6 + 220*sqrt(2)*a^(9/4)*c^(13/4)*d^9*e^8
 + 702*sqrt(2)*a^(11/4)*c^(11/4)*d^7*e^10 - 770*sqrt(2)*a^(13/4)*c^(9/4)*d
^5*e^12 - 390*sqrt(2)*a^(15/4)*c^(7/4)*d^3*e^14 + 77*sqrt(2)*a^(17/4)*c^(5
/4)*d*e^16 + 24*sqrt(a)*c^5*d^16*e + 176*a^(3/2)*c^4*d^12*e^5 + 960*a^(5/2
)*c^3*d^8*e^9 - 1200*a^(7/2)*c^2*d^4*e^13 + 40*a^(9/2)*c*e^17)*arctan(1/2*
sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^
(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(7*sqrt(2)*a^(1/4)*c^(21/4)*d^17
+ 10*sqrt(2)*a^(3/4)*c^(19/4)*d^15*e^2 + 50*sqrt(2)*a^(5/4)*c^(17/4)*d^...
```

3.414.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 2200, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)^3 (a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
-3/64*(23*sqrt(2)*a*c^4*d^6*e - 115*sqrt(2)*a^2*c^3*d^2*e^5 + 30*sqrt(2)*s
qrt(a*c)*a*c^3*d^4*e^3 + 20*sqrt(2)*sqrt(a*c)*a^2*c^2*e^7 - 65*(a*c^3)^(1/
4)*a*c^3*d^5*e^2 + 123*(a*c^3)^(1/4)*a^2*c^2*d*e^6 - 7*(a*c^3)^(3/4)*c^2*d
^7 + 65*(a*c^3)^(3/4)*a*c*d^3*e^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)
^(1/4))/(a/c)^(1/4))/(25*sqrt(2)*a^4*c^4*d^8*e^2 + 90*sqrt(2)*a^5*c^3*d^4*
e^6 + sqrt(2)*a^6*c^2*e^10 - sqrt(2)*sqrt(a*c)*a^3*c^4*d^10 - 90*sqrt(2)*s
qrt(a*c)*a^4*c^3*d^6*e^4 - 25*sqrt(2)*sqrt(a*c)*a^5*c^2*d^2*e^8 - 80*(a*c^
3)^(1/4)*a^4*c^3*d^7*e^3 - 80*(a*c^3)^(1/4)*a^5*c^2*d^3*e^7 - 10*(a*c^3)^(
3/4)*a^3*c^2*d^9*e - 148*(a*c^3)^(3/4)*a^4*c*d^5*e^5 - 10*(a*c^3)^(3/4)*a^
5*d*e^9) + 3/64*(23*sqrt(2)*a*c^4*d^6*e - 115*sqrt(2)*a^2*c^3*d^2*e^5 - 30
*sqrt(2)*sqrt(a*c)*a*c^3*d^4*e^3 - 20*sqrt(2)*sqrt(a*c)*a^2*c^2*e^7 + 65*(
a*c^3)^(1/4)*a*c^3*d^5*e^2 - 123*(a*c^3)^(1/4)*a^2*c^2*d*e^6 + 7*(a*c^3)^(
3/4)*c^2*d^7 - 65*(a*c^3)^(3/4)*a*c*d^3*e^4)*arctan(1/2*sqrt(2)*(2*x - sqr
t(2)*(a/c)^(1/4))/(a/c)^(1/4))/(25*sqrt(2)*a^4*c^4*d^8*e^2 + 90*sqrt(2)*a^
5*c^3*d^4*e^6 + sqrt(2)*a^6*c^2*e^10 - sqrt(2)*sqrt(a*c)*a^3*c^4*d^10 - 90
*sqrt(2)*sqrt(a*c)*a^4*c^3*d^6*e^4 - 25*sqrt(2)*sqrt(a*c)*a^5*c^2*d^2*e^8
+ 80*(a*c^3)^(1/4)*a^4*c^3*d^7*e^3 + 80*(a*c^3)^(1/4)*a^5*c^2*d^3*e^7 + 10
*(a*c^3)^(3/4)*a^3*c^2*d^9*e + 148*(a*c^3)^(3/4)*a^4*c*d^5*e^5 + 10*(a*c^3
)^(3/4)*a^5*d*e^9) + 3/128*(7*(a*c^3)^(1/4)*c^5*d^17 + 50*(a*c^3)^(1/4)*a*
c^4*d^13*e^4 + 220*(a*c^3)^(1/4)*a^2*c^3*d^9*e^8 - 770*(a*c^3)^(1/4)*a^...
```

3.414.9 Mupad [B] (verification not implemented)

Time = 12.67 (sec) , antiderivative size = 6280, normalized size of antiderivative = 2.85

$$\int \frac{1}{(d+ex)^3 (a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)^3*(d + e*x)^3),x)
```

output
```
symsum(log(root(2684354560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a^18*c^3*d^
12*e^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^20*c*d^4*e
^36*z^5 + 56371445760*a^17*c^4*d^16*e^24*z^5 + 56371445760*a^15*c^6*d^24*e
^16*z^5 + 12079595520*a^19*c^2*d^8*e^32*z^5 + 12079595520*a^13*c^8*d^32*e^
8*z^5 + 67645734912*a^16*c^5*d^20*e^20*z^5 + 268435456*a^11*c^10*d^40*z^5
+ 268435456*a^21*e^40*z^5 + 45339770880*a^9*c^6*d^20*e^14*z^3 - 7914848256
0*a^13*c^2*d^4*e^30*z^3 + 791941349376*a^12*c^3*d^8*e^26*z^3 + 1239810048*
a^7*c^8*d^28*e^6*z^3 - 1555444924416*a^11*c^4*d^12*e^22*z^3 + 83755008*a^6
*c^9*d^32*e^2*z^3 + 81566760960*a^10*c^5*d^16*e^18*z^3 + 12177506304*a^8*c
^7*d^24*e^10*z^3 + 117964800*a^14*c*e^34*z^3 - 2785204224*a^6*c^6*d^18*e^1
3*z^2 + 8128512*a^3*c^9*d^30*e*z^2 + 2700933120*a^10*c^2*d^2*e^29*z^2 - 54
3361222656*a^8*c^4*d^10*e^21*z^2 + 1048135680*a^5*c^7*d^22*e^9*z^2 + 11849
9328*a^4*c^8*d^26*e^5*z^2 - 55938263040*a^7*c^5*d^14*e^17*z^2 + 1239904972
80*a^9*c^3*d^6*e^25*z^2 + 24139215*a^2*c^7*d^20*e^8*z + 2819286*a*c^8*d^24
*e^4*z + 10462847841*a^6*c^3*d^4*e^24*z - 5777473473*a^4*c^5*d^12*e^16*z -
 43509753450*a^5*c^4*d^8*e^20*z - 548810316*a^3*c^6*d^16*e^12*z + 12960000
*a^7*c^2*e^28*z + 194481*c^9*d^28*z - 977636142*a^2*c^4*d^6*e^19 + 2332800
00*a^3*c^3*d^2*e^23 - 140556060*a*c^5*d^10*e^15 - 15169518*c^6*d^14*e^11,
z, k)*(root(2684354560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a^18*c^3*d^12*e
^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^20*c*d^4*e^...
```

3.414. $\int \frac{1}{(d+ex)^3(a+cx^4)^3}\,dx$

3.415 $\int \frac{-1+x}{1-x+x^2} dx$

3.415.1 Optimal result

Integrand size = 14, antiderivative size = 32

$$\int \frac{-1+x}{1-x+x^2} dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(1-x+x^2\right)$$

output
```
1/2*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.415.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{-1+x}{1-x+x^2} dx = -\frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(1-x+x^2\right)$$

input
```
Integrate[(-1 + x)/(1 - x + x^2),x]
```

output
```
-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

3.415.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-1}{x^2-x+1}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2-x+1}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx$$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\log\left(x^2-x+1\right)-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(-1 + x)/(1 - x + x^2),x]
```

output
```
-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

3.415.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.415.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\ln(x^2-x+1)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	29
risch	$\frac{\ln\left(4x^2-4x+4\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	31

input
```
int((x-1)/(x^2-x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

3.415.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x}{1-x+x^2}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^2-x+1\right)$$

input
```
integrate((-1+x)/(x^2-x+1),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

3.415.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{-1+x}{1-x+x^2}\,dx = \frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((-1+x)/(x**2-x+1),x)
```

output
```
log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

3.415.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x}{1-x+x^2}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^2-x+1\right)$$

input
```
integrate((-1+x)/(x^2-x+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

3.415.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x}{1-x+x^2} \, dx = -\frac{1}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2x-1) \right) + \frac{1}{2} \log \left(x^2 - x + 1 \right)$$

input
```
integrate((-1+x)/(x^2-x+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

3.415.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{-1+x}{1-x+x^2} \, dx = \frac{\ln(x^2-x+1)}{2} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3} \right)}{3}$$

input
```
int((x - 1)/(x^2 - x + 1),x)
```

output
```
log(x^2 - x + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

3.416 $\int \frac{-1+x^2}{1+x^3}\,dx$

3.416.1 Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{-1+x^2}{1+x^3}\,dx=\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)$$

output
```
1/2*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.416.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{-1+x^2}{1+x^3}\,dx=-\frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)$$

input
```
Integrate[(-1 + x^2)/(1 + x^3),x]
```

output
```
-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

3.416.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2411, 25, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-1}{x^3+1} \, dx$$

$$\Big\downarrow \text{ 2411}$$

$$\int -\frac{1-x}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1-x}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2} \int -\frac{1-2x}{x^2-x+1}dx-\frac{1}{2} \int \frac{1}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{2} \int \frac{1}{x^2-x+1}dx-\frac{1}{2} \int \frac{1-2x}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 1083}$$

$$\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2} \int \frac{1-2x}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{1}{2} \int \frac{1-2x}{x^2-x+1}dx-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2} \log\left(x^2-x+1\right)-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(-1 + x^2)/(1 + x^3),x]
```

output
```
-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

3.416.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2411
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Simp[q^2/a
   Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x]] /; EqQ[A - B*(a/b)^(1/3) + C*
(a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

3.416. $\int \frac{-1+x^2}{1+x^3}\,dx$

3.416.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\ln(x^2-x+1)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	29
risch	$\frac{\ln(4x^2-4x+4)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	31
meijerg	$\frac{\ln(x^3+1)}{3}-\frac{x\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{1}{3}}}-\frac{x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{1}{3}}}$	82

input
```
int((x^2-1)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

3.416.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^2}{1+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

input
```
integrate((x^2-1)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

3.416.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{-1+x^2}{1+x^3}\,dx = \frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((x**2-1)/(x**3+1),x)
```

output
```
log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

3.416.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^2}{1+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

input
```
integrate((x^2-1)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

3.416.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^2}{1+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

input
```
integrate((x^2-1)/(x^3+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

3.416.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{-1+x^2}{1+x^3}\,dx = \frac{\ln\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
int((x^2 - 1)/(x^3 + 1),x)
```

output
```
log(x^2 - x + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

3.417 $\int \frac{-4+3x}{4-2x+x^2} \, dx$

3.417.1 Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{-4+3x}{4-2x+x^2} \, dx = \frac{\arctan\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)$$

output
```
3/2*ln(x^2-2*x+4)+1/3*arctan(1/3*(1-x)*3^(1/2))*3^(1/2)
```

3.417.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{-4+3x}{4-2x+x^2} \, dx = -\frac{\arctan\left(\frac{-1+x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)$$

input
```
Integrate[(-4 + 3*x)/(4 - 2*x + x^2),x]
```

output
```
-(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

3.417.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x-4}{x^2-2x+4}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{3}{2}\int -\frac{2(1-x)}{x^2-2x+4}dx-\int \frac{1}{x^2-2x+4}dx$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{1}{x^2-2x+4}dx-3\int \frac{1-x}{x^2-2x+4}dx$$

$$\Big\downarrow \text{1083}$$

$$2\int \frac{1}{-(2x-2)^2-12}d(2x-2)-3\int \frac{1-x}{x^2-2x+4}dx$$

$$\Big\downarrow \text{217}$$

$$-3\int \frac{1-x}{x^2-2x+4}dx-\frac{\arctan\left(\frac{2x-2}{2\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3}{2}\log\left(x^2-2x+4\right)-\frac{\arctan\left(\frac{2x-2}{2\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(-4 + 3*x)/(4 - 2*x + x^2),x]
```

output
```
-(ArcTan[(-2 + 2*x)/(2*Sqrt[3])]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

3.417.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.417.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{3\ln(x^2-2x+4)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(x-1)\sqrt{3}}{3}\right)}{3}$	27
default	$\frac{3\ln(x^2-2x+4)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-2)\sqrt{3}}{6}\right)}{3}$	29

input
```
int((-4+3*x)/(x^2-2*x+4),x,method=_RETURNVERBOSE)
```

output
```
3/2*ln(x^2-2*x+4)-1/3*3^(1/2)*arctan(1/3*(x-1)*3^(1/2))
```

3.417.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-4+3x}{4-2x+x^2}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

input
```
integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

3.417.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{-4+3x}{4-2x+x^2}\,dx = \frac{3\log\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((-4+3*x)/(x**2-2*x+4),x)
```

output
```
3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3
```

3.417.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-4+3x}{4-2x+x^2}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

input
```
integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

3.417.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-4+3x}{4-2x+x^2}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2\,x+4\right)$$

input
```
integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

3.417.9 Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{-4+3x}{4-2x+x^2}\,dx = \frac{3\,\ln\left(x^2-2\,x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
int((3*x - 4)/(x^2 - 2*x + 4),x)
```

output
```
(3*log(x^2 - 2*x + 4))/2 - (3^(1/2)*atan((3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

3.418 $\int \frac{-8+2x+3x^2}{8+x^3}\,dx$

3.418.1 Optimal result

Integrand size = 18, antiderivative size = 32

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx = \frac{\arctan\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)$$

output
```
3/2*ln(x^2-2*x+4)+1/3*arctan(1/3*(1-x)*3^(1/2))*3^(1/2)
```

3.418.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx = -\frac{\arctan\left(\frac{-1+x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)$$

input
```
Integrate[(-8 + 2*x + 3*x^2)/(8 + x^3),x]
```

output
```
-(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

3.418.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2411, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2x-8}{x^3+8} \, dx$$

$$\Big\downarrow \text{ 2411}$$

$$\frac{1}{2} \int -\frac{2(4-3x)}{x^2-2x+4}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{4-3x}{x^2-2x+4}dx$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{3}{2} \int -\frac{2(1-x)}{x^2-2x+4}dx-\int \frac{1}{x^2-2x+4}dx$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{1}{x^2-2x+4}dx-3 \int \frac{1-x}{x^2-2x+4}dx$$

$$\Big\downarrow \text{ 1083}$$

$$2 \int \frac{1}{-(2x-2)^2-12}d(2x-2)-3 \int \frac{1-x}{x^2-2x+4}dx$$

$$\Big\downarrow \text{ 217}$$

$$-3 \int \frac{1-x}{x^2-2x+4}dx-\frac{\arctan\left(\frac{2x-2}{2\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{3}{2} \log \left(x^2-2x+4\right)-\frac{\arctan\left(\frac{2x-2}{2\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(-8 + 2*x + 3*x^2)/(8 + x^3),x]
```

output
```
-(ArcTan[(-2 + 2*x)/(2*Sqrt[3])]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

3.418.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2411
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Simp[q^2/a
   Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x]] /; EqQ[A - B*(a/b)^(1/3) + C*
(a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

3.418. $\int \frac{-8+2x+3x^2}{8+x^3} \, dx$

3.418.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result
risch	$\frac{3\ln(x^2-2x+4)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(x-1)\sqrt{3}}{3}\right)}{3}$
default	$\frac{3\ln(x^2-2x+4)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-2)\sqrt{3}}{6}\right)}{3}$
meijerg	$-\frac{2x\ln\left(1+\frac{(x^3)^{\frac{1}{3}}}{2}\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x\ln\left(1-\frac{(x^3)^{\frac{1}{3}}}{2}+\frac{(x^3)^{\frac{2}{3}}}{4}\right)}{3(x^3)^{\frac{1}{3}}}-\frac{2x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{4-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{1}{3}}}+\ln\left(1+\frac{x^3}{8}\right)-\frac{x^2\ln\left(1+\frac{(x^3)^{\frac{1}{3}}}{2}\right)}{3(x^3)^{\frac{2}{3}}}+$

input
```
int((3*x^2+2*x-8)/(x^3+8),x,method=_RETURNVERBOSE)
```

output
```
3/2*ln(x^2-2*x+4)-1/3*3^(1/2)*arctan(1/3*(x-1)*3^(1/2))
```

3.418.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx=-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(x-1)\right)+\frac{3}{2}\,\log\left(x^2-2\,x+4\right)$$

input
```
integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

3.418.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx=\frac{3\log\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((3*x**2+2*x-8)/(x**3+8),x)
```

output
```
3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3
```

3.418.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

input
```
integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

3.418.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

input
```
integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

3.418.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx = \frac{3\,\ln\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
int((2*x + 3*x^2 - 8)/(x^3 + 8),x)
```

output
```
(3*log(x^2 - 2*x + 4))/2 - (3^(1/2)*atan((3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

3.419 $\int \frac{2+x}{-1+2x+x^2} dx$

3.419.1 Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{2+x}{-1+2x+x^2} dx = \frac{1}{4}\left(2+\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)$$

output
```
1/4*ln(1+x+2^(1/2))*(2-2^(1/2))+1/4*ln(1+x-2^(1/2))*(2+2^(1/2))
```

3.419.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{2+x}{-1+2x+x^2} dx = \frac{1}{4}\left(\left(2+\sqrt{2}\right)\log\left(-1+\sqrt{2}-x\right)-\left(-2+\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)\right)$$

input
```
Integrate[(2 + x)/(-1 + 2*x + x^2),x]
```

output
```
((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x]
)/4
```

3.419.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{x^2+2x-1} \, dx$$
$$\Big\downarrow \text{1141}$$
$$\int \left(\frac{2-\sqrt{2}}{4 \left(x+\sqrt{2}+1\right)}+\frac{2+\sqrt{2}}{4 \left(x-\sqrt{2}+1\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{4} \left(2+\sqrt{2}\right) \log \left(x-\sqrt{2}+1\right)+\frac{1}{4} \left(2-\sqrt{2}\right) \log \left(x+\sqrt{2}+1\right)$$

input
```
Int[(2 + x)/(-1 + 2*x + x^2),x]
```

output
```
((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] +
x])/4
```

3.419.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.419.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\ln(x^2+2x-1)}{2} - \frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}$	29
risch	$\frac{\ln\left(1+x-\sqrt{2}\right)}{2} + \frac{\ln\left(1+x-\sqrt{2}\right)\sqrt{2}}{4} + \frac{\ln\left(1+x+\sqrt{2}\right)}{2} - \frac{\ln\left(1+x+\sqrt{2}\right)\sqrt{2}}{4}$	48

input
```
int((x+2)/(x^2+2*x-1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))
```

3.419.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{-1+2x+x^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}(x+1)+2x+3}{x^2+2x-1}\right) + \frac{1}{2}\log\left(x^2+2x-1\right)$$

input
```
integrate((2+x)/(x^2+2*x-1),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2
*log(x^2 + 2*x - 1)
```

3.419.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{2+x}{-1+2x+x^2}\,dx = \left(\frac{1}{2}-\frac{\sqrt{2}}{4}\right)\log\left(x+1+\sqrt{2}\right) + \left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x-\sqrt{2}+1\right)$$

input
```
integrate((2+x)/(x**2+2*x-1),x)
```

output
```
(1/2 - sqrt(2)/4)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x - sqrt(2)
 + 1)
```

3.419.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{2+x}{-1+2x+x^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right) + \frac{1}{2}\log\left(x^2+2x-1\right)$$

input
```
integrate((2+x)/(x^2+2*x-1),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/2*log(x^2 + 2*x -
 1)
```

3.419.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{2+x}{-1+2x+x^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{|2x-2\sqrt{2}+2|}{|2x+2\sqrt{2}+2|}\right) + \frac{1}{2}\log\left(|x^2+2x-1|\right)$$

input
```
integrate((2+x)/(x^2+2*x-1),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*l
og(abs(x^2 + 2*x - 1))
```

3.419.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \frac{2+x}{-1+2x+x^2}\,dx = \ln\left(x-\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right) - \ln\left(x+\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}-\frac{1}{2}\right)$$

input
```
int((x + 2)/(2*x + x^2 - 1),x)
```

output
```
log(x - 2^(1/2) + 1)*(2^(1/2)/4 + 1/2) - log(x + 2^(1/2) + 1)*(2^(1/2)/4 -
 1/2)
```

3.420 $\int \frac{-4+x^2}{2-5x+x^3} dx$

3.420.1 Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{-4+x^2}{2-5x+x^3} dx = \frac{1}{4}\left(2+\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)$$

output
```
1/4*ln(1+x+2^(1/2))*(2-2^(1/2))+1/4*ln(1+x-2^(1/2))*(2+2^(1/2))
```

3.420.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{-4+x^2}{2-5x+x^3} dx = \frac{1}{4}\left(\left(2+\sqrt{2}\right)\log\left(-1+\sqrt{2}-x\right)-\left(-2+\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)\right)$$

input
```
Integrate[(-4 + x^2)/(2 - 5*x + x^3),x]
```

output
```
((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x]
)/4
```

3.420.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2457, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-4}{x^3-5x+2}dx$$

$$\Big\downarrow \text{ 2457}$$

$$\int \frac{x+2}{x^2+2x-1}dx$$

$$\Big\downarrow \text{ 1141}$$

$$\int \left(\frac{2-\sqrt{2}}{4\left(x+\sqrt{2}+1\right)}+\frac{2+\sqrt{2}}{4\left(x-\sqrt{2}+1\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

input
```
Int[(-4 + x^2)/(2 - 5*x + x^3),x]
```

output
```
((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] +
x])/4
```

3.420.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2457
```
Int[(u_.)*(Px_)*(Qx_)^(q_), x_Symbol] :> Module[{Rx = PolyGCD[Px, Qx, x]},
Int[u*Rx^(q + 1)*PolynomialQuotient[Px, Rx, x]*PolynomialQuotient[Qx, Rx, x
]^q, x] /; NeQ[Rx, 1]] /; ILtQ[q, 0] && PolyQ[Px, x] && PolyQ[Qx, x]
```

3.420.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\ln(x^2+2x-1)}{2}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}$	29
risch	$\frac{\ln\left(1+x-\sqrt{2}\right)}{2}+\frac{\ln\left(1+x-\sqrt{2}\right)\sqrt{2}}{4}+\frac{\ln\left(1+x+\sqrt{2}\right)}{2}-\frac{\ln\left(1+x+\sqrt{2}\right)\sqrt{2}}{4}$	48

input
```
int((x^2-4)/(x^3-5*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))
```

3.420.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}(x+1)+2\,x+3}{x^2+2\,x-1}\right)+\frac{1}{2}\log\left(x^2+2\,x-1\right)$$

input
```
integrate((x^2-4)/(x^3-5*x+2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2
*log(x^2 + 2*x - 1)
```

3.420.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx = \left(\frac{1}{2}-\frac{\sqrt{2}}{4}\right)\log\left(x+1+\sqrt{2}\right)+\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x-\sqrt{2}+1\right)$$

input
```
integrate((x**2-4)/(x**3-5*x+2),x)
```

output
```
(1/2 - sqrt(2)/4)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x - sqrt(2)
 + 1)
```

3.420.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{2}\log\left(x^2+2x-1\right)$$

input
```
integrate((x^2-4)/(x^3-5*x+2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/2*log(x^2 + 2*x -
 1)
```

3.420.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{|2x-2\sqrt{2}+2|}{|2x+2\sqrt{2}+2|}\right)+\frac{1}{2}\log\left(|x^2+2x-1|\right)$$

input
```
integrate((x^2-4)/(x^3-5*x+2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*l
og(abs(x^2 + 2*x - 1))
```

3.420.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx = \ln\left(x-\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)-\ln\left(x+\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}-\frac{1}{2}\right)$$

input
```
int((x^2 - 4)/(x^3 - 5*x + 2),x)
```

output
```
log(x - 2^(1/2) + 1)*(2^(1/2)/4 + 1/2) - log(x + 2^(1/2) + 1)*(2^(1/2)/4 -
 1/2)
```

3.421 $\int \frac{2}{-1+4x^2} \, dx$

3.421.1 Optimal result

Integrand size = 11, antiderivative size = 6

$$\int \frac{2}{-1+4x^2} \, dx = -\operatorname{arctanh}(2x)$$

output
```
-arctanh(2*x)
```

3.421.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(6) = 12.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 3.83

$$\int \frac{2}{-1+4x^2} \, dx = 2\left(\frac{1}{4}\log(1-2x)-\frac{1}{4}\log(1+2x)\right)$$

input
```
Integrate[2/(-1 + 4*x^2),x]
```

output
```
2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)
```

3.421.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {27, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2}{4x^2-1}dx$$
$$\Big\downarrow \text{ 27}$$
$$2\int \frac{1}{4x^2-1}dx$$
$$\Big\downarrow \text{ 220}$$
$$-\text{arctanh}(2x)$$

input
```
Int[2/(-1 + 4*x^2),x]
```

output
```
-ArcTanh[2*x]
```

3.421.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

3.421.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
meijerg	$-\operatorname{arctanh}\left(2x\right)$	7
parallelrisch	$\frac{\ln\left(x-\frac{1}{2}\right)}{2}-\frac{\ln\left(x+\frac{1}{2}\right)}{2}$	14
default	$-\frac{\ln(1+2x)}{2}+\frac{\ln(2x-1)}{2}$	18
norman	$-\frac{\ln(1+2x)}{2}+\frac{\ln(2x-1)}{2}$	18
risch	$-\frac{\ln(1+2x)}{2}+\frac{\ln(2x-1)}{2}$	18

input
```
int(2/(4*x^2-1),x,method=_RETURNVERBOSE)
```

output
```
-arctanh(2*x)
```

3.421.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{2}{-1+4x^2}\,dx = -\frac{1}{2}\,\log\left(2\,x+1\right)+\frac{1}{2}\,\log\left(2\,x-1\right)$$

input
```
integrate(2/(4*x^2-1),x, algorithm="fricas")
```

output
```
-1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

3.421.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(5) = 10.

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{2}{-1+4x^2}\,dx = \frac{\log\left(x-\frac{1}{2}\right)}{2}-\frac{\log\left(x+\frac{1}{2}\right)}{2}$$

input
```
integrate(2/(4*x**2-1),x)
```

output
```
log(x - 1/2)/2 - log(x + 1/2)/2
```

3.421.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{2}{-1+4x^2}\,dx = -\frac{1}{2}\,\log\left(2\,x+1\right)+\frac{1}{2}\,\log\left(2\,x-1\right)$$

input
```
integrate(2/(4*x^2-1),x, algorithm="maxima")
```

output
```
-1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

3.421.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(6) = 12.

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{2}{-1+4x^2}\,dx = -\frac{1}{2}\,\log\left(\left|x+\frac{1}{2}\right|\right)+\frac{1}{2}\,\log\left(\left|x-\frac{1}{2}\right|\right)$$

input
```
integrate(2/(4*x^2-1),x, algorithm="giac")
```

output
```
-1/2*log(abs(x + 1/2)) + 1/2*log(abs(x - 1/2))
```

3.421.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{2}{-1+4x^2}\,dx = -\operatorname{atanh}(2\,x)$$

input
```
int(2/(4*x^2 - 1),x)
```

output
```
-atanh(2*x)
```

3.422 $\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right) dx$

3.422.1 Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right) dx = \frac{1}{2}\log(1-2x)-\frac{1}{2}\log(1+2x)$$

output
```
1/2*ln(1-2*x)-1/2*ln(1+2*x)
```

3.422.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right) dx = 2\left(\frac{1}{4}\log(1-2x)-\frac{1}{4}\log(1+2x)\right)$$

input
```
Integrate[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1),x]
```

output
```
2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)
```

3.422.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{2x-1}-\frac{1}{2x+1}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(2x+1)$$

input
```
Int[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1),x]
```

output
```
Log[1 - 2*x]/2 - Log[1 + 2*x]/2
```

3.422.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.422.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{\ln\left(x-\frac{1}{2}\right)}{2}-\frac{\ln\left(x+\frac{1}{2}\right)}{2}$	14
default	$-\frac{\ln(1+2x)}{2}+\frac{\ln(2x-1)}{2}$	18
norman	$-\frac{\ln(1+2x)}{2}+\frac{\ln(2x-1)}{2}$	18
meijerg	$\frac{\ln(1-2x)}{2}-\frac{\ln(1+2x)}{2}$	18
risch	$-\frac{\ln(1+2x)}{2}+\frac{\ln(2x-1)}{2}$	18

input
```
int(1/(2*x-1)-1/(1+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x-1/2)-1/2*ln(x+1/2)
```

3.422.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=-\frac{1}{2}\,\log\left(2\,x+1\right)+\frac{1}{2}\,\log\left(2\,x-1\right)$$

input
```
integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="fricas")
```

output
```
-1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

3.422.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=\frac{\log\left(x-\frac{1}{2}\right)}{2}-\frac{\log\left(x+\frac{1}{2}\right)}{2}$$

input
```
integrate(1/(-1+2*x)-1/(1+2*x),x)
```

output
```
log(x - 1/2)/2 - log(x + 1/2)/2
```

3.422.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=-\frac{1}{2}\,\log\left(2\,x+1\right)+\frac{1}{2}\,\log\left(2\,x-1\right)$$

input
```
integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="maxima")
```

output
```
-1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

3.422.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=-\frac{1}{2}\,\log\left(|2\,x+1|\right)+\frac{1}{2}\,\log\left(|2\,x-1|\right)$$

input
```
integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="giac")
```

output
```
-1/2*log(abs(2*x + 1)) + 1/2*log(abs(2*x - 1))
```

3.422.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.29

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=-\mathrm{atanh}(2\,x)$$

input
```
int(1/(2*x - 1) - 1/(2*x + 1),x)
```

output
```
-atanh(2*x)
```

3.423 $\int \frac{x}{(1-x^2)^5} dx$

3.423.1 Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{x}{(1-x^2)^5} dx = \frac{1}{8(1-x^2)^4}$$

output
```
1/8/(-x^2+1)^4
```

3.423.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{(1-x^2)^5} dx = \frac{1}{8(-1+x^2)^4}$$

input
```
Integrate[x/(1 - x^2)^5,x]
```

output
```
1/(8*(-1 + x^2)^4)
```

3.423.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-x^2)^5}\,dx$$

$$\Big\downarrow \; 241$$

$$\frac{1}{8(1-x^2)^4}$$

input
```
Int[x/(1 - x^2)^5,x]
```

output
```
1/(8*(1 - x^2)^4)
```

3.423.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

3.423.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{1}{8(x^2-1)^4}$	10
default	$\frac{1}{8(x^2-1)^4}$	10
norman	$\frac{1}{8(x^2-1)^4}$	10
risch	$\frac{1}{8(x^2-1)^4}$	10
parallelrisch	$\frac{1}{8(x^2-1)^4}$	10
derivativedivides	$\frac{1}{8(-x^2+1)^4}$	12
meijerg	$\frac{x^2(-x^6+4x^4-6x^2+4)}{8(-x^2+1)^4}$	32

input
```
int(x/(-x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/8/(x^2-1)^4
```

3.423.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(9) = 18.

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \frac{x}{(1-x^2)^5}\,dx = \frac{1}{8\,(x^8-4\,x^6+6\,x^4-4\,x^2+1)}$$

input
```
integrate(x/(-x^2+1)^5,x, algorithm="fricas")
```

output
```
1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

3.423.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(8) = 16.

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{x}{(1-x^2)^5}\,dx = \frac{1}{8x^8-32x^6+48x^4-32x^2+8}$$

input
```
integrate(x/(-x**2+1)**5,x)
```

output
```
1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)
```

3.423.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x}{(1-x^2)^5}\,dx = \frac{1}{8(x^2-1)^4}$$

input
```
integrate(x/(-x^2+1)^5,x, algorithm="maxima")
```

output
```
1/8/(x^2 - 1)^4
```

3.423.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x}{(1-x^2)^5}\,dx = \frac{1}{8(x^2-1)^4}$$

input
```
integrate(x/(-x^2+1)^5,x, algorithm="giac")
```

output
```
1/8/(x^2 - 1)^4
```

3.423.9 Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x}{(1-x^2)^5} \, dx = \frac{1}{8 (x^2-1)^4}$$

input
```
int(-x/(x^2 - 1)^5,x)
```

output
```
1/(8*(x^2 - 1)^4)
```

3.424 $\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\right.$

3.424.1 Optimal result

Integrand size = 73, antiderivative size = 13

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2} -\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right) dx = \frac{1}{8\left(1-x^2\right)^4}$$

output
```
1/8/(-x^2+1)^4
```

3.424.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2} -\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right) dx = \frac{1}{8\left(-1+x^2\right)^4}$$

input
```
Integrate[-1/32*1/(-1 + x)^5 + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/
(256*(-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3)
 - 5/(256*(1 + x)^2),x]
```

output
```
1/(8*(-1 + x^2)^4)
```

3.424.
$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right) dx$

3.424.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 81 vs. 2(13) = 26.

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 6.23, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.014$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\frac{5}{256(x+1)^2}-\frac{5}{128(x+1)^3}-\frac{3}{64(x+1)^4}-\frac{1}{32(x+1)^5}+\frac{5}{256(x-1)^2}-\frac{5}{128(x-1)^3}+\frac{3}{64(x-1)^4}-\frac{1}{32(x$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{256(x+1)}+\frac{5}{256(x+1)^2}+\frac{1}{64(x+1)^3}+\frac{1}{128(x+1)^4}+\frac{5}{256(1-x)}+\frac{5}{256(1-x)^2}+\frac{1}{64(1-x)^3}+\frac{1}{128(1-x)^4}$$

input
```
Int[-1/32*1/(-1 + x)^5 + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(256*(
-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) - 5/(
256*(1 + x)^2),x]
```

output
```
1/(128*(1 - x)^4) + 1/(64*(1 - x)^3) + 5/(256*(1 - x)^2) + 5/(256*(1 - x))
 + 1/(128*(1 + x)^4) + 1/(64*(1 + x)^3) + 5/(256*(1 + x)^2) + 5/(256*(1 +
x))
```

3.424.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.424.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

3.424. $\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right) dx$

method	result
gosper	$\frac{1}{8(x+1)^4(x-1)^4}$
norman	$\frac{1}{8(x+1)^4(x-1)^4}$
risch	$\frac{1}{8(x+1)^4(x-1)^4}$
parallelrisch	$\frac{1}{8(x+1)^4(x-1)^4}$
default	$\frac{1}{128(x-1)^4}-\frac{1}{64(x-1)^3}+\frac{5}{256(x-1)^2}-\frac{5}{256(x-1)}+\frac{1}{128(x+1)^4}+\frac{1}{64(x+1)^3}+\frac{5}{256(x+1)^2}+\frac{5}{256(x+1)}$
meijerg	$\frac{x(-x^3+4x^2-6x+4)}{128(1-x)^4}+\frac{x(x^2-3x+3)}{64(1-x)^3}+\frac{5x(2-x)}{256(1-x)^2}+\frac{5x}{256(1-x)}-\frac{x(x^3+4x^2+6x+4)}{128(x+1)^4}-\frac{x(x^2+3x+3)}{64(x+1)^3}-\frac{5x(x+2)}{256(x+1)^2}$

input
```
int(-1/32/(x-1)^5+3/64/(x-1)^4-5/128/(x-1)^3+5/256/(x-1)^2-1/32/(x+1)^5-3/
64/(x+1)^4-5/128/(x+1)^3-5/256/(x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/(x+1)^4/(x-1)^4
```

3.424.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(9) = 18$.

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right)dx=\frac{1}{8\left(x^8-4\,x^6+6\,x^4-4\,x^2+1\right)}$$

input
```
integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/
(1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="fricas")
```

output
```
1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

3.424.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right) dx = \frac{1}{8x^8-32x^6+48x^4-32x^2+8}$$

input
```
integrate(-1/32/(-1+x)**5+3/64/(-1+x)**4-5/128/(-1+x)**3+5/256/(-1+x)**2-1
/32/(1+x)**5-3/64/(1+x)**4-5/128/(1+x)**3-5/256/(1+x)**2,x)
```

output
```
1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)
```

3.424.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(9) = 18$.

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.38

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right) dx = \frac{5}{256\,(x+1)}-\frac{5}{256\,(x-1)}+\frac{5}{256\,(x+1)^2}+\frac{5}{256\,(x-1)^2}+\frac{1}{64\,(x+1)^3}-\frac{1}{64\,(x-1)^3}+\frac{1}{128\,(x+1)^4}+\frac{1}{128\,(x-1)^4}$$

input
```
integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/
(1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="maxima")
```

output
```
5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2 + 1/64/(
x + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x - 1)^4
```

3.424.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(9) = 18.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.38

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right)dx=\frac{5}{256\,(x+1)}-\frac{5}{256\,(x-1)}+\frac{5}{256\,(x+1)^2}+\frac{5}{256\,(x-1)^2}+\frac{1}{64\,(x+1)^3}-\frac{1}{64\,(x-1)^3}+\frac{1}{128\,(x+1)^4}+\frac{1}{128\,(x-1)^4}$$

input
```
integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/
(1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="giac")
```

output
```
5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2 + 1/64/(
x + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x - 1)^4
```

3.424.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right)dx=\frac{1}{8\,(x^2-1)^4}$$

input
```
int(5/(256*(x - 1)^2) - 5/(256*(x + 1)^2) - 5/(128*(x - 1)^3) - 5/(128*(x
+ 1)^3) + 3/(64*(x - 1)^4) - 3/(64*(x + 1)^4) - 1/(32*(x - 1)^5) - 1/(32*(
x + 1)^5),x)
```

output
```
1/(8*(x^2 - 1)^4)
```

3.425 $\int \frac{1+x^6}{-1+x^6}\,dx$

3.425.1 Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{1+x^6}{-1+x^6}\,dx = x+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2\text{arctanh}(x)}{3}+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)$$

output
```
x-2/3*arctanh(x)+1/6*ln(x^2-x+1)-1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1-2*x)*3^
(1/2))*3^(1/2)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.425.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{1+x^6}{-1+x^6}\,dx = \frac{1}{6}\left(6x-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+2\log(1-x)-2\log(1+x)+\log\left(1-x+x^2\right)-\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[(1 + x^6)/(-1 + x^6),x]
```

output
```
(6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/S
qrt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2
])/6
```

3.425.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {913, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6+1}{x^6-1}dx$$

$$\Big\downarrow \text{913}$$

$$2\int \frac{1}{x^6-1}dx+x$$

$$\Big\downarrow \text{754}$$

$$2\left(-\frac{1}{3}\int \frac{1}{1-x^2}dx-\frac{1}{3}\int \frac{2-x}{2\left(x^2-x+1\right)}dx-\frac{1}{3}\int \frac{x+2}{2\left(x^2+x+1\right)}dx\right)+x$$

$$\Big\downarrow \text{27}$$

$$2\left(-\frac{1}{3}\int \frac{1}{1-x^2}dx-\frac{1}{6}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{6}\int \frac{x+2}{x^2+x+1}dx\right)+x$$

$$\Big\downarrow \text{219}$$

$$2\left(-\frac{1}{6}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{6}\int \frac{x+2}{x^2+x+1}dx-\frac{\text{arctanh}(x)}{3}\right)+x$$

$$\Big\downarrow \text{1142}$$

$$2\left(\frac{1}{6}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)+\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)-\frac{\text{arctanh}(x)}{3}\right.$$
$$x$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)-\frac{\text{arctanh}(x)}{3}\right.$$
$$x$$

$$\Big\downarrow \text{1083}$$

$$2\left(\frac{1}{6}\left(3\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{6}\left(3\int \frac{1}{-(2x+1)^2-3}d(2x+1)-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}\right.\right.$$
$$x$$

$$\downarrow 217$$

$$2\left(\frac{1}{6}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right)-\frac{\text{arctan}}{3}\right)_x$$

$$\downarrow 1103$$

$$2\left(\frac{1}{6}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2+x+1\right)\right)-\frac{\text{arctanh(}}{3}\right)_x$$

input
```
Int[(1 + x^6)/(-1 + x^6),x]
```

output
```
x + 2*(-1/3*ArcTanh[x] + (-(Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]]) + Log[1 -
x + x^2]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]) - Log[1 + x + x^2]/2
)/6)
```

3.425.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.425.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

method	result
default	$x-\frac{\ln(x^2+x+1)}{6}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(x-1)}{3}-\frac{\ln(x+1)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$
risch	$x-\frac{\ln(4x^2+4x+4)}{6}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(4x^2-4x+4)}{6}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{\ln(x+1)}{3}+\frac{\ln(x-1)}{3}$
meijerg	$\frac{x\left(\ln\left(1-(x^6)^{\frac{1}{6}}\right)-\ln\left(1+(x^6)^{\frac{1}{6}}\right)+\frac{\ln\left(1-(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{2-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)}{2+(x^6)}\right.\right.}{6(x^6)^{\frac{1}{6}}}$

input
```
int((x^6+1)/(x^6-1),x,method=_RETURNVERBOSE)
```

output
```
x-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/3*ln(x-1)-1/3*
ln(x+1)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

3.425.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1+x^6}{-1+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x - \frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

input
```
integrate((x^6+1)/(x^6-1),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x
 + 1) + 1/3*log(x - 1)
```

3.425.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23

$$\int \frac{1+x^6}{-1+x^6}\,dx = x + \frac{\log(x-1)}{3} - \frac{\log(x+1)}{3} + \frac{\log(x^2-x+1)}{6} - \frac{\log(x^2+x+1)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((x**6+1)/(x**6-1),x)
```

output
```
x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/
6 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*atan(2*sqrt(3)*x/3
 + sqrt(3)/3)/3
```

3.425.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1+x^6}{-1+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log(x+1) + \frac{1}{3}\log(x-1)$$

input
```
integrate((x^6+1)/(x^6-1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x
 + 1) + 1/3*log(x - 1)
```

3.425.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{1+x^6}{-1+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log(|x+1|) + \frac{1}{3}\log(|x-1|)$$

input
```
integrate((x^6+1)/(x^6-1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(a
bs(x + 1)) + 1/3*log(abs(x - 1))
```

3.425.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.36

$$\int \frac{1+x^6}{-1+x^6}\,dx = x + \frac{\mathrm{atan}(x\,1\mathrm{i})\;2\mathrm{i}}{3} - \mathrm{atan}\left(\frac{x\,32\mathrm{i}}{-32+\sqrt{3}\,32\mathrm{i}} - \frac{32\,\sqrt{3}\,x}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3} - \frac{1}{3}\mathrm{i}\right) - \mathrm{atan}\left(\frac{x\,32\mathrm{i}}{32+\sqrt{3}\,32\mathrm{i}} + \frac{32\,\sqrt{3}\,x}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3} + \frac{1}{3}\mathrm{i}\right)$$

input
```
int((x^6 + 1)/(x^6 - 1),x)
```

output
```
x + (atan(x*1i)*2i)/3 - atan((x*32i)/(3^(1/2)*32i - 32) - (32*3^(1/2)*x)/(
3^(1/2)*32i - 32))*(3^(1/2)/3 - 1i/3) - atan((x*32i)/(3^(1/2)*32i + 32) +
(32*3^(1/2)*x)/(3^(1/2)*32i + 32))*(3^(1/2)/3 + 1i/3)
```

3.426 $\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx$

3.426.1 Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = x+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2\text{arctanh}(x)}{3}+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)$$

output
```
x-2/3*arctanh(x)+1/6*ln(x^2-x+1)-1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1-2*x)*3^
(1/2))*3^(1/2)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.426.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = \frac{1}{6}\left(6x-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+2\log(1-x)-2\log(1+x)+\log\left(1-x+x^2\right)-\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[(x^(-3) + x^3)/(-x^(-3) + x^3),x]
```

output
```
(6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/S
qrt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2
])/6
```

3.426.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {2027, 10, 25, 913, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+\frac{1}{x^3}}{x^3-\frac{1}{x^3}}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^3\left(x^3+\frac{1}{x^3}\right)}{x^6-1}dx$$

$$\Big\downarrow \text{10}$$

$$\int -\frac{x^6+1}{1-x^6}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{x^6+1}{1-x^6}dx$$

$$\Big\downarrow \text{913}$$

$$x-2\int \frac{1}{1-x^6}dx$$

$$\Big\downarrow \text{754}$$

$$x-2\left(\frac{1}{3}\int \frac{1}{1-x^2}dx+\frac{1}{3}\int \frac{2-x}{2\left(x^2-x+1\right)}dx+\frac{1}{3}\int \frac{x+2}{2\left(x^2+x+1\right)}dx\right)$$

$$\Big\downarrow \text{27}$$

$$x-2\left(\frac{1}{3}\int \frac{1}{1-x^2}dx+\frac{1}{6}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{6}\int \frac{x+2}{x^2+x+1}dx\right)$$

$$\Big\downarrow \text{219}$$

$$x-2\left(\frac{1}{6}\int\frac{2-x}{x^2-x+1}dx+\frac{1}{6}\int\frac{x+2}{x^2+x+1}dx+\frac{\text{arctanh}(x)}{3}\right)$$

$\big\downarrow$ 1142

$$x-2\left(\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int-\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{x^2+x+1}dx+\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx\right)+\frac{\text{arctanh}(x)}{3}\right)$$

$\big\downarrow$ 25

$$x-2\left(\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{x^2-x+1}dx+\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{x^2+x+1}dx+\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx\right)+\frac{\text{arctanh}(x)}{3}\right)$$

$\big\downarrow$ 1083

$$x-2\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx-3\int\frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx-3\int\frac{1}{-(2x+1)^2-3}d(2x+$$

$\big\downarrow$ 217

$$x-2\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx+\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx+\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right)+\frac{\text{arctanh}(x}{3}$$

$\big\downarrow$ 1103

$$x-2\left(\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^2+x+1\right)\right)+\frac{\text{arctanh}(x}{3}$$

input
```
Int[(x^(-3) + x^3)/(-x^(-3) + x^3),x]
```

output
```
x - 2*(ArcTanh[x]/3 + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - Log[1 - x + x^
2]/2)/6 + (Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[1 + x + x^2]/2)/6)
```

3.426.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

3.426. $\int\frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx$

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.426. $\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx$

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.426.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

method	result	size
risch	$x+\frac{\ln(x-1)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}-\frac{\ln(x^2+x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}-\frac{\ln(x+1)}{3}$	63
default	$x-\frac{\ln(x^2+x+1)}{6}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(x-1)}{3}-\frac{\ln(x+1)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	67

input
```
int((1/x^3+x^3)/(-1/x^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
x+1/3*ln(x-1)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))-1/6*
ln(x^2+x+1)-1/3*3^(1/2)*arctan(2/3*(x+1/2)*3^(1/2))-1/3*ln(x+1)
```

3.426.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

input
```
integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x
 + 1) + 1/3*log(x - 1)
```

3.426.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = x+\frac{\log{(x-1)}}{3}-\frac{\log{(x+1)}}{3}+\frac{\log{(x^2-x+1)}}{6}-\frac{\log{(x^2+x+1)}}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((1/x**3+x**3)/(-1/x**3+x**3),x)
```

output
```
x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/
6 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*atan(2*sqrt(3)*x/3
 + sqrt(3)/3)/3
```

3.426.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x - \frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

input
```
integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x
 + 1) + 1/3*log(x - 1)
```

3.426.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(|x+1|\right)+\frac{1}{3}\log\left(|x-1|\right)$$

input
```
integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(a
bs(x + 1)) + 1/3*log(abs(x - 1))
```

3.426.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.36

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = x+\frac{\mathrm{atan}(x\,1\mathrm{i})\;2\mathrm{i}}{3}-\mathrm{atan}\left(\frac{x\,32\mathrm{i}}{-32+\sqrt{3}\,32\mathrm{i}}-\frac{32\,\sqrt{3}\,x}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}-\frac{1}{3}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x\,32\mathrm{i}}{32+\sqrt{3}\,32\mathrm{i}}+\frac{32\,\sqrt{3}\,x}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}+\frac{1}{3}\mathrm{i}\right)$$

input
```
int(-(1/x^3 + x^3)/(1/x^3 - x^3),x)
```

output
```
x + (atan(x*1i)*2i)/3 - atan((x*32i)/(3^(1/2)*32i - 32) - (32*3^(1/2)*x)/(
3^(1/2)*32i - 32))*(3^(1/2)/3 - 1i/3) - atan((x*32i)/(3^(1/2)*32i + 32) +
(32*3^(1/2)*x)/(3^(1/2)*32i + 32))*(3^(1/2)/3 + 1i/3)
```

3.427 $\int \frac{-x+x^3}{6+2x} \, dx$

3.427.1 Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{-x+x^3}{6+2x} \, dx = 4x - \frac{3x^2}{4} + \frac{x^3}{6} - 12\log(3+x)$$

output
```
4*x-3/4*x^2+1/6*x^3-12*ln(3+x)
```

3.427.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{-x+x^3}{6+2x} \, dx = \frac{1}{2}\left(\frac{93}{2} + 8x - \frac{3x^2}{2} + \frac{x^3}{3} - 24\log(3+x)\right)$$

input
```
Integrate[(-x + x^3)/(6 + 2*x),x]
```

output
```
(93/2 + 8*x - (3*x^2)/2 + x^3/3 - 24*Log[3 + x])/2
```

3.427.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2027, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-x}{2x+6}\,dx$$

$$\Big\downarrow \ 2027$$

$$\int \frac{x(x^2-1)}{2x+6}dx$$

$$\Big\downarrow \ 522$$

$$\int \left(\frac{x^2}{2}-\frac{3x}{2}-\frac{12}{x+3}+4\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{x^3}{6}-\frac{3x^2}{4}+4x-12\log(x+3)$$

input
```
Int[(-x + x^3)/(6 + 2*x),x]
```

output
```
4*x - (3*x^2)/4 + x^3/6 - 12*Log[3 + x]
```

3.427.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.427.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\ln\left(3+x\right)$	21
risch	$4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\ln\left(3+x\right)$	21
parallelrisch	$4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\ln\left(3+x\right)$	21
norman	$4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\ln\left(6+2x\right)$	23
meijerg	$\frac{3x\left(\frac{4}{9}x^2-2x+12\right)}{8}-12\ln\left(1+\frac{x}{3}\right)-\frac{x}{2}$	26

input
```
int((x^3-x)/(6+2*x),x,method=_RETURNVERBOSE)
```

output
```
4*x-3/4*x^2+1/6*x^3-12*ln(3+x)
```

3.427.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-x+x^3}{6+2x}\,dx = \frac{1}{6}\,x^3 - \frac{3}{4}\,x^2 + 4\,x - 12\,\log\left(x+3\right)$$

input
```
integrate((x^3-x)/(6+2*x),x, algorithm="fricas")
```

output
```
1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)
```

3.427.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-x+x^3}{6+2x}\,dx = \frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12\log\left(x+3\right)$$

input
```
integrate((x**3-x)/(6+2*x),x)
```

output
```
x**3/6 - 3*x**2/4 + 4*x - 12*log(x + 3)
```

3.427.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-x+x^3}{6+2x}\,dx = \frac{1}{6}\,x^3 - \frac{3}{4}\,x^2 + 4\,x - 12\,\log\left(x+3\right)$$

input
```
integrate((x^3-x)/(6+2*x),x, algorithm="maxima")
```

output
```
1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)
```

3.427.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{-x+x^3}{6+2x}\,dx = \frac{1}{6}\,x^3 - \frac{3}{4}\,x^2 + 4\,x - 12\,\log\left(|x+3|\right)$$

input
```
integrate((x^3-x)/(6+2*x),x, algorithm="giac")
```

output
```
1/6*x^3 - 3/4*x^2 + 4*x - 12*log(abs(x + 3))
```

3.427.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-x+x^3}{6+2x}\,dx = 4\,x - 12\,\ln\left(x+3\right) - \frac{3\,x^2}{4} + \frac{x^3}{6}$$

input
```
int(-(x - x^3)/(2*x + 6),x)
```

output
```
4*x - 12*log(x + 3) - (3*x^2)/4 + x^3/6
```

3.428 $\int \frac{x+x^3}{-1+x} \, dx$

3.428.1 Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{x+x^3}{-1+x} \, dx = 2x + \frac{x^2}{2} + \frac{x^3}{3} + 2\log(1-x)$$

output
```
2*x+1/2*x^2+1/3*x^3+2*ln(1-x)
```

3.428.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{x+x^3}{-1+x} \, dx = \frac{1}{6}\left(-17 + 12x + 3x^2 + 2x^3 + 12\log(-1+x)\right)$$

input
```
Integrate[(x + x^3)/(-1 + x),x]
```

output
```
(-17 + 12*x + 3*x^2 + 2*x^3 + 12*Log[-1 + x])/6
```

3.428.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2027, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x}{x-1} \, dx$$
$$\Big\downarrow \ 2027$$
$$\int \frac{x(x^2+1)}{x-1} dx$$
$$\Big\downarrow \ 522$$
$$\int \left(x^2+x+\frac{2}{x-1}+2\right) dx$$
$$\Big\downarrow \ 2009$$
$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\log(1-x)$$

input
```
Int[(x + x^3)/(-1 + x),x]
```

output
```
2*x + x^2/2 + x^3/3 + 2*Log[1 - x]
```

3.428.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

3.428.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln\left(x-1\right)$	21
norman	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln\left(x-1\right)$	21
risch	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln\left(x-1\right)$	21
parallelrisch	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln\left(x-1\right)$	21
meijerg	$\frac{x\left(4x^2+6x+12\right)}{12}+2\ln\left(1-x\right)+x$	24

input
```
int((x^3+x)/(x-1),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+1/2*x^2+2*x+2*ln(x-1)
```

3.428.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x+x^3}{-1+x}\,dx=\frac{1}{3}\,x^3+\frac{1}{2}\,x^2+2\,x+2\,\log\left(x-1\right)$$

input
```
integrate((x^3+x)/(-1+x),x, algorithm="fricas")
```

output
```
1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

3.428.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{x+x^3}{-1+x}\,dx=\frac{x^3}{3}+\frac{x^2}{2}+2x+2\log\left(x-1\right)$$

input
```
integrate((x**3+x)/(-1+x),x)
```

output
```
x**3/3 + x**2/2 + 2*x + 2*log(x - 1)
```

3.428.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x+x^3}{-1+x}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + 2\,x + 2\,\log\left(x-1\right)$$

input
```
integrate((x^3+x)/(-1+x),x, algorithm="maxima")
```

output
```
1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

3.428.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{x+x^3}{-1+x}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + 2\,x + 2\,\log\left(|x-1|\right)$$

input
```
integrate((x^3+x)/(-1+x),x, algorithm="giac")
```

output
```
1/3*x^3 + 1/2*x^2 + 2*x + 2*log(abs(x - 1))
```

3.428.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x+x^3}{-1+x}\,dx = 2\,x + 2\,\ln\left(x-1\right) + \frac{x^2}{2} + \frac{x^3}{3}$$

input
```
int((x + x^3)/(x - 1),x)
```

output
```
2*x + 2*log(x - 1) + x^2/2 + x^3/3
```

3.429 $\int (ac+(bc+d)x)\,dx$

3.429.1 Optimal result

Integrand size = 11, antiderivative size = 17

$$\int (ac+(bc+d)x)\,dx = acx+\frac{1}{2}(bc+d)x^2$$

output
```
a*c*x+1/2*(b*c+d)*x^2
```

3.429.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int (ac+(bc+d)x)\,dx = acx+\frac{1}{2}bcx^2+\frac{dx^2}{2}$$

input
```
Integrate[a*c + (b*c + d)*x,x]
```

output
```
a*c*x + (b*c*x^2)/2 + (d*x^2)/2
```

3.429.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ac + x(bc + d))\,dx$$

$$\Big\downarrow \; 17$$

$$\frac{(ac + x(bc + d))^2}{2(bc + d)}$$

input
```
Int[a*c + (b*c + d)*x,x]
```

output
```
(a*c + (b*c + d)*x)^2/(2*(b*c + d))
```

3.429.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

3.429.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x(bcx+2ac+dx)}{2}$	16
parallelrisch	$acx + \frac{(bc+d)x^2}{2}$	16
norman	$\left(\frac{bc}{2} + \frac{d}{2}\right)x^2 + acx$	18
default	$\frac{1}{2}bc\,x^2 + acx + \frac{1}{2}d\,x^2$	19
risch	$\frac{1}{2}bc\,x^2 + acx + \frac{1}{2}d\,x^2$	19
parts	$\frac{1}{2}bc\,x^2 + acx + \frac{1}{2}d\,x^2$	19

input
```
int(a*c+(b*c+d)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b*c*x+2*a*c+d*x)
```

3.429.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int (ac+(bc+d)x)\,dx = \frac{1}{2}x^2cb + \frac{1}{2}x^2d + xca$$

input
```
integrate(a*c+(b*c+d)*x,x, algorithm="fricas")
```

output
```
1/2*x^2*c*b + 1/2*x^2*d + x*c*a
```

3.429.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int (ac+(bc+d)x)\,dx = acx + x^2\left(\frac{bc}{2}+\frac{d}{2}\right)$$

input
```
integrate(a*c+(b*c+d)*x,x)
```

output
```
a*c*x + x**2*(b*c/2 + d/2)
```

3.429.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int (ac+(bc+d)x)\,dx = acx + \frac{1}{2}(bc+d)x^2$$

input
```
integrate(a*c+(b*c+d)*x,x, algorithm="maxima")
```

output
```
a*c*x + 1/2*(b*c + d)*x^2
```

3.429.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int (ac+(bc+d)x)\,dx = acx + \frac{1}{2}(bc+d)x^2$$

input
```
integrate(a*c+(b*c+d)*x,x, algorithm="giac")
```

output
```
a*c*x + 1/2*(b*c + d)*x^2
```

3.429.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (ac+(bc+d)x)\,dx = \left(\frac{d}{2}+\frac{b\,c}{2}\right)\,x^2 + a\,c\,x$$

input
```
int(a*c + x*(d + b*c),x)
```

output
```
x^2*(d/2 + (b*c)/2) + a*c*x
```

3.430 $\int (dx + c(a + bx))\, dx$

3.430.1 Optimal result

Integrand size = 11, antiderivative size = 24

$$\int (dx + c(a + bx))\, dx = \frac{dx^2}{2} + \frac{c(a + bx)^2}{2b}$$

output
```
1/2*d*x^2+1/2*c*(b*x+a)^2/b
```

3.430.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int (dx + c(a + bx))\, dx = acx + \frac{1}{2}bcx^2 + \frac{dx^2}{2}$$

input
```
Integrate[d*x + c*(a + b*x),x]
```

output
```
a*c*x + (b*c*x^2)/2 + (d*x^2)/2
```

3.430.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c(a+bx)+dx)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(a+bx)^2}{2b}+\frac{dx^2}{2}$$

input
```
Int[d*x + c*(a + b*x),x]
```

output
```
(d*x^2)/2 + (c*(a + b*x)^2)/(2*b)
```

3.430.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.430.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{x(bcx+2ac+dx)}{2}$	16
norman	$\left(\frac{bc}{2}+\frac{d}{2}\right)x^2+acx$	18
default	$\frac{1}{2}bc\,x^2+acx+\frac{1}{2}d\,x^2$	19
risch	$\frac{1}{2}bc\,x^2+acx+\frac{1}{2}d\,x^2$	19
parallelrisch	$\frac{1}{2}bc\,x^2+acx+\frac{1}{2}d\,x^2$	19
parts	$\frac{1}{2}bc\,x^2+acx+\frac{1}{2}d\,x^2$	19

input
```
int(d*x+c*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b*c*x+2*a*c+d*x)
```

3.430.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int (dx+c(a+bx))\,dx=\frac{1}{2}x^2cb+\frac{1}{2}x^2d+xca$$

input
```
integrate(d*x+c*(b*x+a),x, algorithm="fricas")
```

output
```
1/2*x^2*c*b + 1/2*x^2*d + x*c*a
```

3.430.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int (dx+c(a+bx))\,dx=acx+x^2\left(\frac{bc}{2}+\frac{d}{2}\right)$$

input
```
integrate(d*x+c*(b*x+a),x)
```

output
```
a*c*x + x**2*(b*c/2 + d/2)
```

3.430.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int (dx+c(a+bx))\,dx=\frac{1}{2}\,dx^2+\frac{1}{2}\,(bx^2+2\,ax)c$$

input
```
integrate(d*x+c*(b*x+a),x, algorithm="maxima")
```

output
```
1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c
```

3.430.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int (dx + c(a + bx))\, dx = \frac{1}{2}\, dx^2 + \frac{1}{2}\left(bx^2 + 2\, ax\right)c$$

input
```
integrate(d*x+c*(b*x+a),x, algorithm="giac")
```

output
```
1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c
```

3.430.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int (dx + c(a + bx))\, dx = \left(\frac{d}{2} + \frac{b\,c}{2}\right) x^2 + a\,c\,x$$

input
```
int(d*x + c*(a + b*x),x)
```

output
```
x^2*(d/2 + (b*c)/2) + a*c*x
```

3.431 $\int \frac{4+4x}{x^2(1+x^2)}\,dx$

3.431.1 Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{4+4x}{x^2(1+x^2)}\,dx = -\frac{4}{x} - 4\arctan(x) + 4\log(x) - 2\log\left(1+x^2\right)$$

output
```
-4/x-4*arctan(x)+4*ln(x)-2*ln(x^2+1)
```

3.431.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{4+4x}{x^2(1+x^2)}\,dx = 4\left(-\frac{1}{x} - \arctan(x) + \log(x) - \frac{1}{2}\log\left(1+x^2\right)\right)$$

input
```
Integrate[(4 + 4*x)/(x^2*(1 + x^2)),x]
```

output
```
4*(-x^(-1) - ArcTan[x] + Log[x] - Log[1 + x^2]/2)
```

3.431.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x+4}{x^2(x^2+1)}dx$$
$$\Big\downarrow \text{523}$$
$$\int \left(-\frac{4(x+1)}{x^2+1}+\frac{4}{x^2}+\frac{4}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-4\arctan(x)-2\log\left(x^2+1\right)-\frac{4}{x}+4\log(x)$$

input
```
Int[(4 + 4*x)/(x^2*(1 + x^2)),x]
```

output
```
-4/x - 4*ArcTan[x] + 4*Log[x] - 2*Log[1 + x^2]
```

3.431.3.1 Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.431.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
default	$-\frac{4}{x}-4\arctan\left(x\right)+4\ln\left(x\right)-2\ln\left(x^{2}+1\right)$	23
meijerg	$-\frac{4}{x}-4\arctan\left(x\right)+4\ln\left(x\right)-2\ln\left(x^{2}+1\right)$	23
risch	$-\frac{4}{x}-4\arctan\left(x\right)+4\ln\left(x\right)-2\ln\left(x^{2}+1\right)$	23
parallelrisch	$\frac{2i\ln(x-i)x-2i\ln(x+i)x+4\ln(x)x-2\ln(x-i)x-2\ln(x+i)x-4}{x}$	46

input
```
int((4+4*x)/x^2/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-4/x-4*arctan(x)+4*ln(x)-2*ln(x^2+1)
```

3.431.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{4+4x}{x^2\left(1+x^2\right)}\,dx=-\frac{2\left(2\,x\arctan\left(x\right)+x\log\left(x^2+1\right)-2\,x\log\left(x\right)+2\right)}{x}$$

input
```
integrate((4+4*x)/x^2/(x^2+1),x, algorithm="fricas")
```

output
```
-2*(2*x*arctan(x) + x*log(x^2 + 1) - 2*x*log(x) + 2)/x
```

3.431.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{4+4x}{x^2\left(1+x^2\right)}\,dx=4\log\left(x\right)-2\log\left(x^2+1\right)-4\,\mathrm{atan}\left(x\right)-\frac{4}{x}$$

input
```
integrate((4+4*x)/x**2/(x**2+1),x)
```

output
```
4*log(x) - 2*log(x**2 + 1) - 4*atan(x) - 4/x
```

3.431.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{4+4x}{x^2(1+x^2)}\,dx = -\frac{4}{x} - 4\arctan(x) - 2\log(x^2+1) + 4\log(x)$$

input
```
integrate((4+4*x)/x^2/(x^2+1),x, algorithm="maxima")
```

output
```
-4/x - 4*arctan(x) - 2*log(x^2 + 1) + 4*log(x)
```

3.431.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{4+4x}{x^2(1+x^2)}\,dx = -\frac{4}{x} - 4\arctan(x) - 2\log(x^2+1) + 4\log(|x|)$$

input
```
integrate((4+4*x)/x^2/(x^2+1),x, algorithm="giac")
```

output
```
-4/x - 4*arctan(x) - 2*log(x^2 + 1) + 4*log(abs(x))
```

3.431.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{4+4x}{x^2(1+x^2)}\,dx = 4\ln(x) - \frac{4}{x} + \ln(x-\mathrm{i})(-2+2\mathrm{i}) + \ln(x+1\mathrm{i})(-2-2\mathrm{i})$$

input
```
int((4*x + 4)/(x^2*(x^2 + 1)),x)
```

output
```
4*log(x) - log(x + 1i)*(2 + 2i) - log(x - 1i)*(2 - 2i) - 4/x
```

3.432 $\int \frac{24+8x}{x(-4+x^2)} \, dx$

3.432.1 Optimal result

Integrand size = 16, antiderivative size = 17

$$\int \frac{24+8x}{x(-4+x^2)} \, dx = 5\log(2-x) - 6\log(x) + \log(2+x)$$

output
```
5*ln(2-x)-6*ln(x)+ln(2+x)
```

3.432.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{24+8x}{x(-4+x^2)} \, dx = 8\left(\frac{5}{8}\log(2-x) - \frac{3\log(x)}{4} + \frac{1}{8}\log(2+x)\right)$$

input
```
Integrate[(24 + 8*x)/(x*(-4 + x^2)),x]
```

output
```
8*((5*Log[2 - x])/8 - (3*Log[x])/4 + Log[2 + x]/8)
```

3.432.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{8x+24}{x(x^2-4)}dx$$

$$\Big\downarrow \text{523}$$

$$\int \left(-\frac{6}{x}+\frac{1}{x+2}+\frac{5}{x-2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$5\log(2-x)-6\log(x)+\log(x+2)$$

input
```
Int[(24 + 8*x)/(x*(-4 + x^2)),x]
```

output
```
5*Log[2 - x] - 6*Log[x] + Log[2 + x]
```

3.432.3.1 Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.432.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$-6\ln(x)+\ln(x+2)+5\ln(x-2)$	16
norman	$-6\ln(x)+\ln(x+2)+5\ln(x-2)$	16
risch	$-6\ln(x)+\ln(x+2)+5\ln(x-2)$	16
parallelrisch	$-6\ln(x)+\ln(x+2)+5\ln(x-2)$	16
meijerg	$3\ln\left(1-\frac{x^2}{4}\right)-6\ln(x)+6\ln(2)-3i\pi-4\operatorname{arctanh}\left(\frac{x}{2}\right)$	30

input
```
int((24+8*x)/x/(x^2-4),x,method=_RETURNVERBOSE)
```

output
```
-6*ln(x)+ln(x+2)+5*ln(x-2)
```

3.432.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{24+8x}{x(-4+x^2)}\,dx = \log(x+2)+5\,\log(x-2)-6\,\log(x)$$

input
```
integrate((24+8*x)/x/(x^2-4),x, algorithm="fricas")
```

output
```
log(x + 2) + 5*log(x - 2) - 6*log(x)
```

3.432.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{24+8x}{x(-4+x^2)}\,dx = -6\log(x)+5\log(x-2)+\log(x+2)$$

input
```
integrate((24+8*x)/x/(x**2-4),x)
```

output
```
-6*log(x) + 5*log(x - 2) + log(x + 2)
```

3.432.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{24+8x}{x\,(-4+x^2)}\,dx = \log(x+2) + 5\,\log(x-2) - 6\,\log(x)$$

input
```
integrate((24+8*x)/x/(x^2-4),x, algorithm="maxima")
```

output
```
log(x + 2) + 5*log(x - 2) - 6*log(x)
```

3.432.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{24+8x}{x\,(-4+x^2)}\,dx = \log(|x+2|) + 5\,\log(|x-2|) - 6\,\log(|x|)$$

input
```
integrate((24+8*x)/x/(x^2-4),x, algorithm="giac")
```

output
```
log(abs(x + 2)) + 5*log(abs(x - 2)) - 6*log(abs(x))
```

3.432.9 Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{24+8x}{x\,(-4+x^2)}\,dx = 5\,\ln(x-2) + \ln(x+2) - 6\,\ln(x)$$

input
```
int((8*x + 24)/(x*(x^2 - 4)),x)
```

output
```
5*log(x - 2) + log(x + 2) - 6*log(x)
```

3.433 $\int \frac{-1+x^2}{-2x+x^3} dx$

3.433.1 Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{-1+x^2}{-2x+x^3}\,dx = \frac{\log(x)}{2} + \frac{1}{4}\log\left(2-x^2\right)$$

output
```
1/2*ln(x)+1/4*ln(-x^2+2)
```

3.433.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2}{-2x+x^3}\,dx = \frac{\log(x)}{2} + \frac{1}{4}\log\left(2-x^2\right)$$

input
```
Integrate[(-1 + x^2)/(-2*x + x^3),x]
```

output
```
Log[x]/2 + Log[2 - x^2]/4
```

3.433.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2026, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-1}{x^3-2x}dx$$
$$\Big\downarrow \text{ 2026}$$
$$\int \frac{x^2-1}{x(x^2-2)}dx$$
$$\Big\downarrow \text{ 354}$$
$$\frac{1}{2}\int \frac{1-x^2}{x^2(2-x^2)}dx^2$$
$$\Big\downarrow \text{ 86}$$
$$\frac{1}{2}\int \left(\frac{1}{2x^2}+\frac{1}{2(x^2-2)}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(\frac{\log(x^2)}{2}+\frac{1}{2}\log(2-x^2)\right)$$

input
```
Int[(-1 + x^2)/(-2*x + x^3),x]
```

output
```
(Log[x^2]/2 + Log[2 - x^2]/2)/2
```

3.433.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.433.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\ln(x)}{2}+\frac{\ln(x^2-2)}{4}$	14
norman	$\frac{\ln(x)}{2}+\frac{\ln(x^2-2)}{4}$	14
risch	$\frac{\ln(x)}{2}+\frac{\ln(x^2-2)}{4}$	14
parallelrisch	$\frac{\ln(x)}{2}+\frac{\ln(x^2-2)}{4}$	14
meijerg	$\frac{\ln\left(1-\frac{x^2}{2}\right)}{4}+\frac{\ln(x)}{2}-\frac{\ln(2)}{4}+\frac{i\pi}{4}$	24

input
```
int((x^2-1)/(x^3-2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x)+1/4*ln(x^2-2)
```

3.433.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{-1+x^2}{-2x+x^3}\,dx = \frac{1}{4}\,\log\left(x^2-2\right)+\frac{1}{2}\,\log\left(x\right)$$

input
```
integrate((x^2-1)/(x^3-2*x),x, algorithm="fricas")
```

output
```
1/4*log(x^2 - 2) + 1/2*log(x)
```

3.433.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{-1+x^2}{-2x+x^3}\,dx = \frac{\log\left(x\right)}{2}+\frac{\log\left(x^2-2\right)}{4}$$

input
```
integrate((x**2-1)/(x**3-2*x),x)
```

output
```
log(x)/2 + log(x**2 - 2)/4
```

3.433.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{-1+x^2}{-2x+x^3}\,dx = \frac{1}{4}\,\log\left(x^2-2\right)+\frac{1}{2}\,\log\left(x\right)$$

input
```
integrate((x^2-1)/(x^3-2*x),x, algorithm="maxima")
```

output
```
1/4*log(x^2 - 2) + 1/2*log(x)
```

3.433.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{-1+x^2}{-2x+x^3}\,dx = \frac{1}{4}\log\left(x^2\right) + \frac{1}{4}\log\left(\left|x^2-2\right|\right)$$

input
```
integrate((x^2-1)/(x^3-2*x),x, algorithm="giac")
```

output
```
1/4*log(x^2) + 1/4*log(abs(x^2 - 2))
```

3.433.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{-1+x^2}{-2x+x^3}\,dx = \frac{\ln\left(x^2-2\right)}{4} + \frac{\ln\left(x\right)}{2}$$

input
```
int(-(x^2 - 1)/(2*x - x^3),x)
```

output
```
log(x^2 - 2)/4 + log(x)/2
```

3.434 $\int \frac{1+x^2}{3x+x^3} dx$

3.434.1 Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{1+x^2}{3x+x^3} dx = \frac{1}{3}\log\left(3x+x^3\right)$$

output
```
1/3*ln(x^3+3*x)
```

3.434.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1+x^2}{3x+x^3} dx = \frac{\log(x)}{3} + \frac{1}{3}\log\left(3+x^2\right)$$

input
```
Integrate[(1 + x^2)/(3*x + x^3),x]
```

output
```
Log[x]/3 + Log[3 + x^2]/3
```

3.434.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^3+3x}\,dx$$

$$\Big\downarrow \text{2020}$$

$$\frac{1}{3}\log\left(x^3+3x\right)$$

input
```
Int[(1 + x^2)/(3*x + x^3),x]
```

output
```
Log[3*x + x^3]/3
```

3.434.3.1 Defintions of rubi rules used

rule 2020
```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

3.434.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\ln\left(x\left(x^2+3\right)\right)}{3}$	11
risch	$\frac{\ln\left(x^3+3x\right)}{3}$	11
norman	$\frac{\ln\left(x\right)}{3}+\frac{\ln\left(x^2+3\right)}{3}$	14
parallelrisch	$\frac{\ln\left(x\right)}{3}+\frac{\ln\left(x^2+3\right)}{3}$	14
meijerg	$\frac{\ln\left(1+\frac{x^2}{3}\right)}{3}+\frac{\ln\left(x\right)}{3}-\frac{\ln\left(3\right)}{6}$	20

input
```
int((x^2+1)/(x^3+3*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(x*(x^2+3))
```

3.434.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^2}{3x+x^3}\,dx = \frac{1}{3}\,\log\left(x^3+3\,x\right)$$

input
```
integrate((x^2+1)/(x^3+3*x),x, algorithm="fricas")
```

output
```
1/3*log(x^3 + 3*x)
```

3.434.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1+x^2}{3x+x^3}\,dx = \frac{\log\left(x^3+3x\right)}{3}$$

input
```
integrate((x**2+1)/(x**3+3*x),x)
```

output
```
log(x**3 + 3*x)/3
```

3.434.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^2}{3x+x^3}\,dx = \frac{1}{3}\,\log\left(x^3+3\,x\right)$$

input
```
integrate((x^2+1)/(x^3+3*x),x, algorithm="maxima")
```

output
```
1/3*log(x^3 + 3*x)
```

3.434.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1+x^2}{3x+x^3}\,dx = \frac{1}{3}\log\left(3\left|\frac{1}{3}\,x^3+x\right|\right)$$

input
```
integrate((x^2+1)/(x^3+3*x),x, algorithm="giac")
```

output
```
1/3*log(3*abs(1/3*x^3 + x))
```

3.434.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^2}{3x+x^3}\,dx = \frac{\ln\left(x^3+3\,x\right)}{3}$$

input
```
int((x^2 + 1)/(3*x + x^3),x)
```

output
```
log(3*x + x^3)/3
```

3.435 $\int \frac{a+3bx^2}{ax+bx^3} dx$

3.435.1 Optimal result

Integrand size = 20, antiderivative size = 10

$$\int \frac{a+3bx^2}{ax+bx^3} dx = \log \left(ax+bx^3\right)$$

output
```
ln(b*x^3+a*x)
```

3.435.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{a+3bx^2}{ax+bx^3} dx = \log(x)+\log \left(a+bx^2\right)$$

input
```
Integrate[(a + 3*b*x^2)/(a*x + b*x^3),x]
```

output
```
Log[x] + Log[a + b*x^2]
```

3.435.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx$$
$$\Big\downarrow \; 2020$$
$$\log\left(ax+bx^3\right)$$

input
```
Int[(a + 3*b*x^2)/(a*x + b*x^3),x]
```

output
```
Log[a*x + b*x^3]
```

3.435.3.1 Defintions of rubi rules used

rule 2020
```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

3.435.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\ln\left(b\,x^3+ax\right)$	11
default	$\ln\left(x(b\,x^2+a)\right)$	11
risch	$\ln\left(b\,x^3+ax\right)$	11
norman	$\ln\left(x\right)+\ln\left(b\,x^2+a\right)$	12
parallelrisch	$\ln\left(x\right)+\ln\left(b\,x^2+a\right)$	12

input
```
int((3*b*x^2+a)/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
ln(b*x^3+a*x)
```

3.435.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx = \log\left(bx^3+ax\right)$$

input
```
integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
log(b*x^3 + a*x)
```

3.435.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx = \log\left(ax+bx^3\right)$$

input
```
integrate((3*b*x**2+a)/(b*x**3+a*x),x)
```

output
```
log(a*x + b*x**3)
```

3.435.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx = \log\left(bx^3+ax\right)$$

input
```
integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
log(b*x^3 + a*x)
```

3.435.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx = \log\left(\left|bx^3+ax\right|\right)$$

input
```
integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="giac")
```

output
```
log(abs(b*x^3 + a*x))
```

3.435.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx = \ln\left(b\,x^3+a\,x\right)$$

input
```
int((a + 3*b*x^2)/(a*x + b*x^3),x)
```

output
```
log(a*x + b*x^3)
```

3.436 $\int \frac{-2+4x}{-x+x^3} \, dx$

3.436.1 Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{-2+4x}{-x+x^3} \, dx = \log(1-x) + 2\log(x) - 3\log(1+x)$$

output
```
ln(1-x)+2*ln(x)-3*ln(1+x)
```

3.436.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-2+4x}{-x+x^3} \, dx = \log(1-x) + 2\log(x) - 3\log(1+x)$$

input
```
Integrate[(-2 + 4*x)/(-x + x^3),x]
```

output
```
Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

3.436.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2026, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x-2}{x^3-x}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{4x-2}{x\left(x^2-1\right)}dx$$

$$\Big\downarrow \text{523}$$

$$\int \left(\frac{2}{x}-\frac{3}{x+1}+\frac{1}{x-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(1-x)+2\log(x)-3\log(x+1)$$

input
```
Int[(-2 + 4*x)/(-x + x^3),x]
```

output
```
Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

3.436.3.1 Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.436.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$2\ln(x) - 3\ln(x+1) + \ln(x-1)$	16
norman	$2\ln(x) - 3\ln(x+1) + \ln(x-1)$	16
risch	$2\ln(x) - 3\ln(x+1) + \ln(x-1)$	16
parallelrisch	$2\ln(x) - 3\ln(x+1) + \ln(x-1)$	16
meijerg	$-\ln(-x^2+1) + 2\ln(x) + i\pi - 4\,\mathrm{arctanh}(x)$	24

input
```
int((-2+4*x)/(x^3-x),x,method=_RETURNVERBOSE)
```

output
```
2*ln(x)-3*ln(x+1)+ln(x-1)
```

3.436.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-2+4x}{-x+x^3}\,dx = -3\,\log(x+1) + \log(x-1) + 2\,\log(x)$$

input
```
integrate((-2+4*x)/(x^3-x),x, algorithm="fricas")
```

output
```
-3*log(x + 1) + log(x - 1) + 2*log(x)
```

3.436.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-2+4x}{-x+x^3}\,dx = 2\log(x) + \log(x-1) - 3\log(x+1)$$

input
```
integrate((-2+4*x)/(x**3-x),x)
```

output
```
2*log(x) + log(x - 1) - 3*log(x + 1)
```

3.436.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-2+4x}{-x+x^3}\,dx = -3\,\log\left(x+1\right)+\log\left(x-1\right)+2\,\log\left(x\right)$$

input
```
integrate((-2+4*x)/(x^3-x),x, algorithm="maxima")
```

output
```
-3*log(x + 1) + log(x - 1) + 2*log(x)
```

3.436.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{-2+4x}{-x+x^3}\,dx = -3\,\log\left(|x+1|\right)+\log\left(|x-1|\right)+2\,\log\left(|x|\right)$$

input
```
integrate((-2+4*x)/(x^3-x),x, algorithm="giac")
```

output
```
-3*log(abs(x + 1)) + log(abs(x - 1)) + 2*log(abs(x))
```

3.436.9 Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-2+4x}{-x+x^3}\,dx = \ln\left(x-1\right)-3\,\ln\left(x+1\right)+2\,\ln\left(x\right)$$

input
```
int(-(4*x - 2)/(x - x^3),x)
```

output
```
log(x - 1) - 3*log(x + 1) + 2*log(x)
```

3.437 $\int \frac{4+x}{4x+x^3} dx$

3.437.1 Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{4+x}{4x+x^3} dx = \frac{1}{2}\arctan\left(\frac{x}{2}\right) + \log(x) - \frac{1}{2}\log\left(4+x^2\right)$$

output
```
1/2*arctan(1/2*x)+ln(x)-1/2*ln(x^2+4)
```

3.437.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{4+x}{4x+x^3} dx = \frac{1}{2}\arctan\left(\frac{x}{2}\right) + \log(x) - \frac{1}{2}\log\left(4+x^2\right)$$

input
```
Integrate[(4 + x)/(4*x + x^3),x]
```

output
```
ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2
```

3.437.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2026, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+4}{x^3+4x}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{x+4}{x(x^2+4)}dx$$

$$\Big\downarrow \text{523}$$

$$\int \left(\frac{1-x}{x^2+4}+\frac{1}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\arctan\left(\frac{x}{2}\right)-\frac{1}{2}\log\left(x^2+4\right)+\log(x)$$

input
```
Int[(4 + x)/(4*x + x^3),x]
```

output
```
ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2
```

3.437.3.1 Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])
```

3.437.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln\left(x\right)-\frac{\ln\left(x^{2}+4\right)}{2}$	18
risch	$\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln\left(x\right)-\frac{\ln\left(x^{2}+4\right)}{2}$	18
meijerg	$-\frac{\ln\left(1+\frac{x^{2}}{4}\right)}{2}+\ln\left(x\right)-\ln\left(2\right)+\frac{\arctan\left(\frac{x}{2}\right)}{2}$	24
parallelrisch	$\ln\left(x\right)-\frac{\ln\left(x-2i\right)}{2}-\frac{i\ln\left(x-2i\right)}{4}-\frac{\ln\left(x+2i\right)}{2}+\frac{i\ln\left(x+2i\right)}{4}$	34

input
```
int((x+4)/(x^3+4*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(1/2*x)+ln(x)-1/2*ln(x^2+4)
```

3.437.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4+x}{4x+x^3}\,dx = \frac{1}{2}\arctan\left(\frac{1}{2}x\right)-\frac{1}{2}\log\left(x^2+4\right)+\log\left(x\right)$$

input
```
integrate((4+x)/(x^3+4*x),x, algorithm="fricas")
```

output
```
1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)
```

3.437.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4+x}{4x+x^3}\,dx = \log\left(x\right)-\frac{\log\left(x^2+4\right)}{2}+\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

input
```
integrate((4+x)/(x**3+4*x),x)
```

output
```
log(x) - log(x**2 + 4)/2 + atan(x/2)/2
```

3.437.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4+x}{4x+x^3}\,dx = \frac{1}{2}\arctan\left(\frac{1}{2}x\right) - \frac{1}{2}\log\left(x^2+4\right) + \log(x)$$

input
```
integrate((4+x)/(x^3+4*x),x, algorithm="maxima")
```

output
```
1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)
```

3.437.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{4+x}{4x+x^3}\,dx = \frac{1}{2}\arctan\left(\frac{1}{2}x\right) - \frac{1}{2}\log\left(x^2+4\right) + \log(|x|)$$

input
```
integrate((4+x)/(x^3+4*x),x, algorithm="giac")
```

output
```
1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(abs(x))
```

3.437.9 Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{4+x}{4x+x^3}\,dx = \ln(x) + \ln(x-2\mathrm{i})\left(-\frac{1}{2}-\frac{1}{4}\mathrm{i}\right) + \ln(x+2\mathrm{i})\left(-\frac{1}{2}+\frac{1}{4}\mathrm{i}\right)$$

input
```
int((x + 4)/(4*x + x^3),x)
```

output
```
log(x) - log(x + 2i)*(1/2 - 1i/4) - log(x - 2i)*(1/2 + 1i/4)
```

3.438 $\int \frac{-x+2x^3}{1-x^2+x^4} \, dx$

3.438.1 Optimal result

Integrand size = 22, antiderivative size = 15

$$\int \frac{-x+2x^3}{1-x^2+x^4} \, dx = \frac{1}{2}\log\left(1-x^2+x^4\right)$$

output
```
1/2*ln(x^4-x^2+1)
```

3.438.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{-x+2x^3}{1-x^2+x^4} \, dx = \frac{1}{2}\log\left(1-x^2+x^4\right)$$

input
```
Integrate[(-x + 2*x^3)/(1 - x^2 + x^4),x]
```

output
```
Log[1 - x^2 + x^4]/2
```

3.438.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3-x}{x^4-x^2+1}\,dx$$

$$\Big\downarrow \; 2020$$

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

input
```
Int[(-x + 2*x^3)/(1 - x^2 + x^4),x]
```

output
```
Log[1 - x^2 + x^4]/2
```

3.438.3.1 Defintions of rubi rules used

rule 2020
```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

3.438.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln\left(x^4-x^2+1\right)}{2}$	14
norman	$\frac{\ln\left(x^4-x^2+1\right)}{2}$	14
risch	$\frac{\ln\left(x^4-x^2+1\right)}{2}$	14
parallelrisch	$\frac{\ln\left(x^4-x^2+1\right)}{2}$	14

input
```
int((2*x^3-x)/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^4-x^2+1)
```

3.438.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{1}{2}\log\left(x^4-x^2+1\right)$$

input
```
integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/2*log(x^4 - x^2 + 1)
```

3.438.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{\log\left(x^4-x^2+1\right)}{2}$$

input
```
integrate((2*x**3-x)/(x**4-x**2+1),x)
```

output
```
log(x**4 - x**2 + 1)/2
```

3.438.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{1}{2}\log\left(x^4-x^2+1\right)$$

input
```
integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
1/2*log(x^4 - x^2 + 1)
```

3.438.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{1}{2}\log\left(x^4-x^2+1\right)$$

input
```
integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/2*log(x^4 - x^2 + 1)
```

3.438.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{\ln\left(x^4-x^2+1\right)}{2}$$

input
```
int(-(x - 2*x^3)/(x^4 - x^2 + 1),x)
```

output
```
log(x^4 - x^2 + 1)/2
```

3.439 $\int \frac{-3+x}{2x+3x^2+x^3} dx$

3.439.1 Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{-3+x}{2x+3x^2+x^3} dx = -\frac{3\log(x)}{2} + 4\log(1+x) - \frac{5}{2}\log(2+x)$$

output
```
-3/2*ln(x)+4*ln(1+x)-5/2*ln(2+x)
```

3.439.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{-3+x}{2x+3x^2+x^3} dx = -\frac{3\log(x)}{2} + 4\log(1+x) - \frac{5}{2}\log(2+x)$$

input
```
Integrate[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

output
```
(-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

3.439.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1979, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-3}{x^3+3x^2+2x}dx$$
$$\Big\downarrow \text{1979}$$
$$\int \frac{x-3}{x\left(x^2+3x+2\right)}dx$$
$$\Big\downarrow \text{1200}$$
$$\int \left(\frac{4}{x+1}-\frac{5}{2(x+2)}-\frac{3}{2x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

input
```
Int[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

output
```
(-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

3.439.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1979
```
Int[((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) +
(B_.)*(x_)^(r_.)), x_Symbol] :> Int[x^(p*q)*(A + B*x^(n - q))*(a + b*x^(n -
 q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b, c, A, B, n, q}, x] && EqQ[r, n
 - q] && EqQ[j, 2*n - q] && IntegerQ[p] && PosQ[n - q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.439.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{3\ln(x)}{2}+4\ln(x+1)-\frac{5\ln(x+2)}{2}$	18
norman	$-\frac{3\ln(x)}{2}+4\ln(x+1)-\frac{5\ln(x+2)}{2}$	18
risch	$-\frac{3\ln(x)}{2}+4\ln(x+1)-\frac{5\ln(x+2)}{2}$	18
parallelrisch	$-\frac{3\ln(x)}{2}+4\ln(x+1)-\frac{5\ln(x+2)}{2}$	18

input
```
int((-3+x)/(x^3+3*x^2+2*x),x,method=_RETURNVERBOSE)
```

output
```
-3/2*ln(x)+4*ln(x+1)-5/2*ln(x+2)
```

3.439.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{5}{2}\log(x+2)+4\log(x+1)-\frac{3}{2}\log(x)$$

input
```
integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="fricas")
```

output
```
-5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

3.439.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{3\log{(x)}}{2}+4\log{(x+1)}-\frac{5\log{(x+2)}}{2}$$

input
```
integrate((-3+x)/(x**3+3*x**2+2*x),x)
```

output
```
-3*log(x)/2 + 4*log(x + 1) - 5*log(x + 2)/2
```

3.439.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{5}{2}\log{(x+2)}+4\log{(x+1)}-\frac{3}{2}\log{(x)}$$

input
```
integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="maxima")
```

output
```
-5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

3.439.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = -\frac{5}{2}\log{(|x+2|)}+4\log{(|x+1|)}-\frac{3}{2}\log{(|x|)}$$

input
```
integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="giac")
```

output
```
-5/2*log(abs(x + 2)) + 4*log(abs(x + 1)) - 3/2*log(abs(x))
```

3.439.9 Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx = 4\,\ln(x+1) - \frac{5\,\ln(x+2)}{2} - \frac{3\,\ln(x)}{2}$$

input
```
int((x - 3)/(2*x + 3*x^2 + x^3),x)
```

output
```
4*log(x + 1) - (5*log(x + 2))/2 - (3*log(x))/2
```

3.440 $\int \frac{2+4x}{x^2+2x^3+x^4} dx$

3.440.1 Optimal result

Integrand size = 20, antiderivative size = 10

$$\int \frac{2+4x}{x^2+2x^3+x^4} dx = -\frac{2}{x(1+x)}$$

output
```
-2/x/(1+x)
```

3.440.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{2+4x}{x^2+2x^3+x^4} dx = -\frac{2}{x+x^2}$$

input
```
Integrate[(2 + 4*x)/(x^2 + 2*x^3 + x^4),x]
```

output
```
-2/(x + x^2)
```

3.440.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 1184, 27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x+2}{x^4+2x^3+x^2}dx$$
$$\Big\downarrow \text{ 1979}$$
$$\int \frac{4x+2}{x^2\left(x^2+2x+1\right)}dx$$
$$\Big\downarrow \text{ 1184}$$
$$\int \frac{2(2x+1)}{x^2(x+1)^2}dx$$
$$\Big\downarrow \text{ 27}$$
$$2\int \frac{2x+1}{x^2(x+1)^2}dx$$
$$\Big\downarrow \text{ 83}$$
$$-\frac{2}{x(x+1)}$$

input
```
Int[(2 + 4*x)/(x^2 + 2*x^3 + x^4),x]
```

output
```
-2/(x*(1 + x))
```

3.440.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 1184
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(f + g*x
)^n*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && E
qQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1979
```
Int[((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) +
(B_.)*(x_)^(r_.)), x_Symbol] :> Int[x^(p*q)*(A + B*x^(n - q))*(a + b*x^(n -
 q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b, c, A, B, n, q}, x] && EqQ[r, n
 - q] && EqQ[j, 2*n - q] && IntegerQ[p] && PosQ[n - q]
```

3.440.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
gosper	$-\frac{2}{x(x+1)}$	11
norman	$-\frac{2}{x(x+1)}$	11
risch	$-\frac{2}{x(x+1)}$	11
parallelrisch	$-\frac{2}{x(x+1)}$	11
default	$-\frac{2}{x}+\frac{2}{x+1}$	14

input
```
int((4*x+2)/(x^4+2*x^3+x^2),x,method=_RETURNVERBOSE)
```

output
```
-2/x/(x+1)
```

3.440.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{2+4x}{x^2+2x^3+x^4}\,dx = -\frac{2}{x^2+x}$$

input
```
integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="fricas")
```

output
```
-2/(x^2 + x)
```

3.440.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{2+4x}{x^2+2x^3+x^4}\,dx = -\frac{2}{x^2+x}$$

input
```
integrate((2+4*x)/(x**4+2*x**3+x**2),x)
```

output
```
-2/(x**2 + x)
```

3.440.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{2+4x}{x^2+2x^3+x^4}\,dx = -\frac{2}{x^2+x}$$

input
```
integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="maxima")
```

output
```
-2/(x^2 + x)
```

3.440.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{2+4x}{x^2+2x^3+x^4}\,dx = -\frac{2}{x^2+x}$$

input
```
integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="giac")
```

output
```
-2/(x^2 + x)
```

3.440.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{2+4x}{x^2+2x^3+x^4}\,dx = -\frac{2}{x\,(x+1)}$$

input
```
int((4*x + 2)/(x^2 + 2*x^3 + x^4),x)
```

output
```
-2/(x*(x + 1))
```

3.441 $\int \frac{1+x}{-6x+x^2+x^3} \, dx$

3.441.1 Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{1+x}{-6x+x^2+x^3} \, dx = \frac{3}{10} \log(2-x) - \frac{\log(x)}{6} - \frac{2}{15} \log(3+x)$$

output
```
3/10*ln(2-x)-1/6*ln(x)-2/15*ln(3+x)
```

3.441.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{-6x+x^2+x^3} \, dx = \frac{3}{10} \log(2-x) - \frac{\log(x)}{6} - \frac{2}{15} \log(3+x)$$

input
```
Integrate[(1 + x)/(-6*x + x^2 + x^3),x]
```

output
```
(3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15
```

3.441.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1979, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{x^3+x^2-6x} \, dx$$

$$\Big\downarrow \text{1979}$$

$$\int \frac{x+1}{x\left(x^2+x-6\right)}dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(-\frac{1}{6x}-\frac{2}{15(x+3)}+\frac{3}{10(x-2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(x+3)$$

input
```
Int[(1 + x)/(-6*x + x^2 + x^3),x]
```

output
```
(3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15
```

3.441.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1979
```
Int[((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) +
(B_.)*(x_)^(r_.)), x_Symbol] :> Int[x^(p*q)*(A + B*x^(n - q))*(a + b*x^(n -
 q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b, c, A, B, n, q}, x] && EqQ[r, n
 - q] && EqQ[j, 2*n - q] && IntegerQ[p] && PosQ[n - q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.441.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{\ln(x)}{6}-\frac{2\ln(3+x)}{15}+\frac{3\ln(x-2)}{10}$	18
norman	$-\frac{\ln(x)}{6}-\frac{2\ln(3+x)}{15}+\frac{3\ln(x-2)}{10}$	18
risch	$-\frac{\ln(x)}{6}-\frac{2\ln(3+x)}{15}+\frac{3\ln(x-2)}{10}$	18
parallelrisch	$-\frac{\ln(x)}{6}-\frac{2\ln(3+x)}{15}+\frac{3\ln(x-2)}{10}$	18

input
```
int((x+1)/(x^3+x^2-6*x),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(x)-2/15*ln(3+x)+3/10*ln(x-2)
```

3.441.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1+x}{-6x+x^2+x^3}\,dx = -\frac{2}{15}\log(x+3)+\frac{3}{10}\log(x-2)-\frac{1}{6}\log(x)$$

input
```
integrate((1+x)/(x^3+x^2-6*x),x, algorithm="fricas")
```

output
```
-2/15*log(x + 3) + 3/10*log(x - 2) - 1/6*log(x)
```

3.441.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1+x}{-6x+x^2+x^3}\,dx = -\frac{\log(x)}{6} + \frac{3\log(x-2)}{10} - \frac{2\log(x+3)}{15}$$

input
```
integrate((1+x)/(x**3+x**2-6*x),x)
```

output
```
-log(x)/6 + 3*log(x - 2)/10 - 2*log(x + 3)/15
```

3.441.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1+x}{-6x+x^2+x^3}\,dx = -\frac{2}{15}\log(x+3) + \frac{3}{10}\log(x-2) - \frac{1}{6}\log(x)$$

input
```
integrate((1+x)/(x^3+x^2-6*x),x, algorithm="maxima")
```

output
```
-2/15*log(x + 3) + 3/10*log(x - 2) - 1/6*log(x)
```

3.441.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1+x}{-6x+x^2+x^3}\,dx = -\frac{2}{15}\log(|x+3|) + \frac{3}{10}\log(|x-2|) - \frac{1}{6}\log(|x|)$$

input
```
integrate((1+x)/(x^3+x^2-6*x),x, algorithm="giac")
```

output
```
-2/15*log(abs(x + 3)) + 3/10*log(abs(x - 2)) - 1/6*log(abs(x))
```

3.441.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1+x}{-6x+x^2+x^3}\,dx = \frac{3\,\ln(x-2)}{10} - \frac{2\,\ln(x+3)}{15} - \frac{\ln(x)}{6}$$

input
```
int((x + 1)/(x^2 - 6*x + x^3),x)
```

output
```
(3*log(x - 2))/10 - (2*log(x + 3))/15 - log(x)/6
```

3.442 $\int \frac{4x^2+x^3}{x+x^3} dx$

3.442.1 Optimal result

Integrand size = 17, antiderivative size = 14

$$\int \frac{4x^2+x^3}{x+x^3} dx = x - \arctan(x) + 2\log\left(1+x^2\right)$$

output
```
x-arctan(x)+2*ln(x^2+1)
```

3.442.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{4x^2+x^3}{x+x^3} dx = x - \arctan(x) + 2\log\left(1+x^2\right)$$

input
```
Integrate[(4*x^2 + x^3)/(x + x^3),x]
```

output
```
x - ArcTan[x] + 2*Log[1 + x^2]
```

3.442.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2026, 9, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+4x^2}{x^3+x}\,dx$$

$$\Big\downarrow \ 2026$$

$$\int \frac{x^3+4x^2}{x\left(x^2+1\right)}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x(x+4)}{x^2+1}dx$$

$$\Big\downarrow \ 523$$

$$\int \left(1-\frac{1-4x}{x^2+1}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\arctan(x)+2\log\left(x^2+1\right)+x$$

input
```
Int[(4*x^2 + x^3)/(x + x^3),x]
```

output
```
x - ArcTan[x] + 2*Log[1 + x^2]
```

3.442.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.442.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$x-\arctan \left(x \right)+2 \ln \left(x^{2}+1\right)$	15
meijerg	$x-\arctan \left(x \right)+2 \ln \left(x^{2}+1\right)$	15
risch	$x-\arctan \left(x \right)+2 \ln \left(x^{2}+1\right)$	15
parallelrisch	$x+2 \ln \left(x -i\right)+\frac{i \ln \left(x -i\right)}{2}+2 \ln \left(x +i\right)-\frac{i \ln \left(x +i\right)}{2}$	33

input
```
int((x^3+4*x^2)/(x^3+x),x,method=_RETURNVERBOSE)
```

output
```
x-arctan(x)+2*ln(x^2+1)
```

3.442.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{4 x^2+x^3}{x+x^3}\, dx=x - \arctan \left (x\right) + 2 \, \log \left (x^{2} + 1\right)$$

input
```
integrate((x^3+4*x^2)/(x^3+x),x, algorithm="fricas")
```

output
```
x - arctan(x) + 2*log(x^2 + 1)
```

3.442.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{4x^2+x^3}{x+x^3}\,dx = x + 2\log\left(x^2+1\right) - \operatorname{atan}\left(x\right)$$

input
```
integrate((x**3+4*x**2)/(x**3+x),x)
```

output
```
x + 2*log(x**2 + 1) - atan(x)
```

3.442.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{4x^2+x^3}{x+x^3}\,dx = x - \arctan\left(x\right) + 2\,\log\left(x^2+1\right)$$

input
```
integrate((x^3+4*x^2)/(x^3+x),x, algorithm="maxima")
```

output
```
x - arctan(x) + 2*log(x^2 + 1)
```

3.442.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{4x^2+x^3}{x+x^3}\,dx = x - \arctan\left(x\right) + 2\,\log\left(x^2+1\right)$$

input
```
integrate((x^3+4*x^2)/(x^3+x),x, algorithm="giac")
```

output
```
x - arctan(x) + 2*log(x^2 + 1)
```

3.442.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{4x^2+x^3}{x+x^3}\,dx = x+2\,\ln\left(x^2+1\right)-\operatorname{atan}(x)$$

input
```
int((4*x^2 + x^3)/(x + x^3),x)
```

output
```
x + 2*log(x^2 + 1) - atan(x)
```

3.443 $\int \frac{x+2x^3}{\left(x^2+x^4\right)^3}\,dx$

3.443.1 Optimal result

Integrand size = 17, antiderivative size = 13

$$\int \frac{x+2x^3}{\left(x^2+x^4\right)^3}\,dx=-\frac{1}{4\left(x^2+x^4\right)^2}$$

output
```
-1/4/(x^4+x^2)^2
```

3.443.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{x+2x^3}{\left(x^2+x^4\right)^3}\,dx=-\frac{1}{4x^4\left(1+x^2\right)^2}$$

input
```
Integrate[(x + 2*x^3)/(x^2 + x^4)^3,x]
```

output
```
-1/4*1/(x^4*(1 + x^2)^2)
```

3.443.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+x}{(x^4+x^2)^3} \, dx$$

$$\Big\downarrow \text{2021}$$

$$-\frac{1}{4(x^4+x^2)^2}$$

input
```
Int[(x + 2*x^3)/(x^2 + x^4)^3,x]
```

output
```
-1/4*1/(x^2 + x^4)^2
```

3.443.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.443.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{1}{4x^4(x^2+1)^2}$	13
norman	$-\frac{1}{4x^4(x^2+1)^2}$	13
risch	$-\frac{1}{4x^4(x^2+1)^2}$	13
parallelrisch	$-\frac{1}{4x^4(x^2+1)^2}$	13
default	$-\frac{1}{4x^4}+\frac{1}{2x^2}-\frac{1}{4(x^2+1)^2}-\frac{1}{2(x^2+1)}$	30
meijerg	$\frac{x^2(5x^2+6)}{2(x^2+1)^2}-\frac{3}{4}+\frac{1}{2x^2}-\frac{x^2(7x^2+8)}{4(x^2+1)^2}-\frac{1}{4x^4}$	51

input
```
int((2*x^3+x)/(x^4+x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4/(x^2+1)^2
```

3.443.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{x+2x^3}{(x^2+x^4)^3}\,dx = -\frac{1}{4\,(x^8+2\,x^6+x^4)}$$

input
```
integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="fricas")
```

output
```
-1/4/(x^8 + 2*x^6 + x^4)
```

3.443.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{x+2x^3}{(x^2+x^4)^3}\,dx = -\frac{1}{4x^8+8x^6+4x^4}$$

input
```
integrate((2*x**3+x)/(x**4+x**2)**3,x)
```

output
```
-1/(4*x**8 + 8*x**6 + 4*x**4)
```

3.443.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x+2x^3}{(x^2+x^4)^3}\,dx = -\frac{1}{4\,(x^4+x^2)^2}$$

input
```
integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="maxima")
```

output
```
-1/4/(x^4 + x^2)^2
```

3.443.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x+2x^3}{(x^2+x^4)^3}\,dx = -\frac{1}{4\,(x^4+x^2)^2}$$

input
```
integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="giac")
```

output
```
-1/4/(x^4 + x^2)^2
```

3.443.9 Mupad [B] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{x+2x^3}{(x^2+x^4)^3}\,dx = -\frac{1}{4\,x^8+8\,x^6+4\,x^4}$$

input
```
int((x + 2*x^3)/(x^2 + x^4)^3,x)
```

output
```
-1/(4*x^4 + 8*x^6 + 4*x^8)
```

3.444 $\int \frac{ax^2+bx^3}{cx^2+dx^3}\,dx$

3.444.1 Optimal result

Integrand size = 25, antiderivative size = 26

$$\int \frac{ax^2+bx^3}{cx^2+dx^3}\,dx=\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}$$

output
```
b*x/d-(-a*d+b*c)*ln(d*x+c)/d^2
```

3.444.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{ax^2+bx^3}{cx^2+dx^3}\,dx=\frac{bx}{d}+\frac{(-bc+ad)\log(c+dx)}{d^2}$$

input
```
Integrate[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]
```

output
```
(b*x)/d + ((-(b*c) + a*d)*Log[c + d*x])/d^2
```

3.444.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2026, 9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^2+bx^3}{cx^2+dx^3} \, dx$$
$$\Big\downarrow \text{ 2026}$$
$$\int \frac{ax^2+bx^3}{x^2(c+dx)}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{a+bx}{c+dx}dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(\frac{ad-bc}{d(c+dx)}+\frac{b}{d}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}$$

input
```
Int[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]
```

output
```
(b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2
```

3.444.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.444.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{xb}{d}+\frac{(da-bc)\ln(dx+c)}{d^2}$	26
norman	$\frac{xb}{d}+\frac{(da-bc)\ln(dx+c)}{d^2}$	26
parallelrisch	$\frac{\ln(dx+c)ad-\ln(dx+c)bc+bdx}{d^2}$	29
risch	$\frac{xb}{d}+\frac{\ln(dx+c)a}{d}-\frac{\ln(dx+c)bc}{d^2}$	32

input
```
int((b*x^3+a*x^2)/(d*x^3+c*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*x*b+(a*d-b*c)/d^2*ln(d*x+c)
```

3.444.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{ax^2+bx^3}{cx^2+dx^3}\,dx=\frac{bdx-(bc-ad)\log(dx+c)}{d^2}$$

input
```
integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="fricas")
```

output
```
(b*d*x - (b*c - a*d)*log(d*x + c))/d^2
```

3.444.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{ax^2 + bx^3}{cx^2 + dx^3}\, dx = \frac{bx}{d} + \frac{(ad - bc)\log{(c + dx)}}{d^2}$$

input
```
integrate((b*x**3+a*x**2)/(d*x**3+c*x**2),x)
```

output
```
b*x/d + (a*d - b*c)*log(c + d*x)/d**2
```

3.444.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{ax^2 + bx^3}{cx^2 + dx^3}\, dx = \frac{bx}{d} - \frac{(bc - ad)\log{(dx + c)}}{d^2}$$

input
```
integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="maxima")
```

output
```
b*x/d - (b*c - a*d)*log(d*x + c)/d^2
```

3.444.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{ax^2 + bx^3}{cx^2 + dx^3}\, dx = \frac{bx}{d} - \frac{(bc - ad)\log{(|dx + c|)}}{d^2}$$

input
```
integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="giac")
```

output
```
b*x/d - (b*c - a*d)*log(abs(d*x + c))/d^2
```

3.444.9 Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{ax^2 + bx^3}{cx^2 + dx^3} \, dx = \frac{\ln(c + d\, x) \, (a\, d - b\, c)}{d^2} + \frac{b\, x}{d}$$

input
```
int((a*x^2 + b*x^3)/(c*x^2 + d*x^3),x)
```

output
```
(log(c + d*x)*(a*d - b*c))/d^2 + (b*x)/d
```

3.445 $\int \frac{x+x^2}{-2x-x^2+x^3}\,dx$

3.445.1 Optimal result

Integrand size = 20, antiderivative size = 6

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx = \log(2-x)$$

output
```
ln(2-x)
```

3.445.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx = \log(-2+x)$$

input
```
Integrate[(x + x^2)/(-2*x - x^2 + x^3),x]
```

output
```
Log[-2 + x]
```

3.445.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x}{x^3-x^2-2x} \, dx$$

$$\Big\downarrow \; 2019$$

$$\int \frac{1}{x-2}dx$$

$$\Big\downarrow \; 16$$

$$\log(2-x)$$

input
```
Int[(x + x^2)/(-2*x - x^2 + x^3),x]
```

output
```
Log[2 - x]
```

3.445.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.445.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

method	result	size
default	$\ln(x-2)$	5
norman	$\ln(x-2)$	5
risch	$\ln(x-2)$	5
parallelrisch	$\ln(x-2)$	5

input
```
int((x^2+x)/(x^3-x^2-2*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x-2)
```

3.445.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx=\log(x-2)$$

input
```
integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="fricas")
```

output
```
log(x - 2)
```

3.445.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx=\log(x-2)$$

input
```
integrate((x**2+x)/(x**3-x**2-2*x),x)
```

output
```
log(x - 2)
```

3.445.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx = \log(x-2)$$

input
```
integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="maxima")
```

output
```
log(x - 2)
```

3.445.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx = \log(|x-2|)$$

input
```
integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="giac")
```

output
```
log(abs(x - 2))
```

3.445.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx = \ln(x-2)$$

input
```
int(-(x + x^2)/(2*x + x^2 - x^3),x)
```

output
```
log(x - 2)
```

3.446 $\int \frac{1-5x^2}{x^3(1+x^2)}\,dx$

3.446.1 Optimal result

Integrand size = 18, antiderivative size = 20

$$\int \frac{1-5x^2}{x^3\left(1+x^2\right)}\,dx=-\frac{1}{2x^2}-6\log(x)+3\log\left(1+x^2\right)$$

output
```
-1/2/x^2-6*ln(x)+3*ln(x^2+1)
```

3.446.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1-5x^2}{x^3\left(1+x^2\right)}\,dx=-\frac{1}{2x^2}-6\log(x)+3\log\left(1+x^2\right)$$

input
```
Integrate[(1 - 5*x^2)/(x^3*(1 + x^2)),x]
```

output
```
-1/2*1/x^2 - 6*Log[x] + 3*Log[1 + x^2]
```

3.446.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-5x^2}{x^3(x^2+1)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1-5x^2}{x^4(x^2+1)}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(-\frac{6}{x^2}+\frac{1}{x^4}+\frac{6}{x^2+1}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{1}{x^2}-6\log\left(x^2\right)+6\log\left(x^2+1\right)\right)$$

input
```
Int[(1 - 5*x^2)/(x^3*(1 + x^2)),x]
```

output
```
(-x^(-2) - 6*Log[x^2] + 6*Log[1 + x^2])/2
```

3.446.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.446.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{1}{2x^2}-6\ln(x)+3\ln(x^2+1)$	19
norman	$-\frac{1}{2x^2}-6\ln(x)+3\ln(x^2+1)$	19
meijerg	$-\frac{1}{2x^2}-6\ln(x)+3\ln(x^2+1)$	19
risch	$-\frac{1}{2x^2}-6\ln(x)+3\ln(x^2+1)$	19
parallelrisch	$-\frac{12\ln(x)x^2-6\ln(x^2+1)x^2+1}{2x^2}$	26

input
```
int((-5*x^2+1)/x^3/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2-6*ln(x)+3*ln(x^2+1)
```

3.446.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{1-5x^2}{x^3(1+x^2)}\,dx = \frac{6\,x^2\log(x^2+1)-12\,x^2\log(x)-1}{2\,x^2}$$

input
```
integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*(6*x^2*log(x^2 + 1) - 12*x^2*log(x) - 1)/x^2
```

3.446.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1-5x^2}{x^3(1+x^2)}\,dx = -6\log(x) + 3\log(x^2+1) - \frac{1}{2x^2}$$

input
```
integrate((-5*x**2+1)/x**3/(x**2+1),x)
```

output
```
-6*log(x) + 3*log(x**2 + 1) - 1/(2*x**2)
```

3.446.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1-5x^2}{x^3(1+x^2)}\,dx = -\frac{1}{2\,x^2} + 3\,\log(x^2+1) - 3\,\log(x^2)$$

input
```
integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="maxima")
```

output
```
-1/2/x^2 + 3*log(x^2 + 1) - 3*log(x^2)
```

3.446.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1-5x^2}{x^3(1+x^2)}\,dx = \frac{6\,x^2-1}{2\,x^2} + 3\,\log(x^2+1) - 3\,\log(x^2)$$

input
```
integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="giac")
```

output
```
1/2*(6*x^2 - 1)/x^2 + 3*log(x^2 + 1) - 3*log(x^2)
```

3.446.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1-5x^2}{x^3\left(1+x^2\right)}\,dx=3\,\ln\left(x^2+1\right)-6\,\ln\left(x\right)-\frac{1}{2\,x^2}$$

input
```
int(-(5*x^2 - 1)/(x^3*(x^2 + 1)),x)
```

output
```
3*log(x^2 + 1) - 6*log(x) - 1/(2*x^2)
```

3.447 $\int \frac{2x}{(-1+x)(5+x^2)}\,dx$

3.447.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{2x}{(-1+x)\,(5+x^2)}\,dx = \frac{1}{3}\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right) + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(5+x^2\right)$$

output
```
1/3*ln(1-x)-1/6*ln(x^2+5)+1/3*arctan(1/5*x*5^(1/2))*5^(1/2)
```

3.447.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{2x}{(-1+x)\,(5+x^2)}\,dx = 2\left(\frac{1}{6}\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right) + \frac{1}{6}\log(1-x) - \frac{1}{12}\log\left(5+x^2\right)\right)$$

input
```
Integrate[(2*x)/((-1 + x)*(5 + x^2)),x]
```

output
```
2*((Sqrt[5]*ArcTan[x/Sqrt[5]])/6 + Log[1 - x]/6 - Log[5 + x^2]/12)
```

3.447.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {27, 25, 587, 16, 25, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x}{(x-1)(x^2+5)}dx$$

$$\Big\downarrow \text{27}$$

$$2\int -\frac{x}{(1-x)(x^2+5)}dx$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{x}{(1-x)(x^2+5)}dx$$

$$\Big\downarrow \text{587}$$

$$-2\left(\frac{1}{6}\int -\frac{5-x}{x^2+5}dx+\frac{1}{6}\int \frac{1}{1-x}dx\right)$$

$$\Big\downarrow \text{16}$$

$$-2\left(\frac{1}{6}\int -\frac{5-x}{x^2+5}dx-\frac{1}{6}\log(1-x)\right)$$

$$\Big\downarrow \text{25}$$

$$-2\left(-\frac{1}{6}\int \frac{5-x}{x^2+5}dx-\frac{1}{6}\log(1-x)\right)$$

$$\Big\downarrow \text{452}$$

$$-2\left(\frac{1}{6}\left(\int \frac{x}{x^2+5}dx-5\int \frac{1}{x^2+5}dx\right)-\frac{1}{6}\log(1-x)\right)$$

$$\Big\downarrow \text{216}$$

$$-2\left(\frac{1}{6}\left(\int \frac{x}{x^2+5}dx-\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)\right)-\frac{1}{6}\log(1-x)\right)$$

$$\Big\downarrow \text{240}$$

$$-2\left(\frac{1}{6}\left(\frac{1}{2}\log\left(x^2+5\right)-\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)\right)-\frac{1}{6}\log(1-x)\right)$$

input
```
Int[(2*x)/((-1 + x)*(5 + x^2)),x]
```

output
```
-2*(-1/6*Log[1 - x] + (-(Sqrt[5]*ArcTan[x/Sqrt[5]]) + Log[5 + x^2]/2)/6)
```

3.447.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 587
```
Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(-
c)*(d/(b*c^2 + a*d^2))   Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2)
 Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
^2 + a*d^2, 0]
```

3.447.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{\ln(x^2+5)}{6}+\frac{\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}}{3}+\frac{\ln(x-1)}{3}$	28
risch	$-\frac{\ln(x^2+5)}{6}+\frac{\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}}{3}+\frac{\ln(x-1)}{3}$	28

input
```
int(2*x/(x-1)/(x^2+5),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(x^2+5)+1/3*arctan(1/5*x*5^(1/2))*5^(1/2)+1/3*ln(x-1)
```

3.447.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{2x}{(-1+x)(5+x^2)}\,dx=\frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log\left(x-1\right)$$

input
```
integrate(2*x/(-1+x)/(x^2+5),x, algorithm="fricas")
```

output
```
1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x - 1)
```

3.447.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{2x}{(-1+x)(5+x^2)}\,dx=\frac{\log\left(x-1\right)}{3}-\frac{\log\left(x^2+5\right)}{6}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}x}{5}\right)}{3}$$

input
```
integrate(2*x/(-1+x)/(x**2+5),x)
```

output
```
log(x - 1)/3 - log(x**2 + 5)/6 + sqrt(5)*atan(sqrt(5)*x/5)/3
```

3.447.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{2x}{(-1+x)(5+x^2)}\,dx = \frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) - \frac{1}{6}\log\left(x^2+5\right) + \frac{1}{3}\log\left(x-1\right)$$

input
```
integrate(2*x/(-1+x)/(x^2+5),x, algorithm="maxima")
```

output
```
1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x - 1)
```

3.447.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{2x}{(-1+x)(5+x^2)}\,dx = \frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) - \frac{1}{6}\log\left(x^2+5\right) + \frac{1}{3}\log\left(|x-1|\right)$$

input
```
integrate(2*x/(-1+x)/(x^2+5),x, algorithm="giac")
```

output
```
1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(abs(x - 1))
```

3.447.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{2x}{(-1+x)(5+x^2)}\,dx = \frac{\ln(x-1)}{3} - \ln\left(x-\sqrt{5}\,1i\right)\left(\frac{1}{6}+\frac{\sqrt{5}\,1i}{6}\right) + \ln\left(x+\sqrt{5}\,1i\right)\left(-\frac{1}{6}+\frac{\sqrt{5}\,1i}{6}\right)$$

input
```
int((2*x)/((x^2 + 5)*(x - 1)),x)
```

output
```
log(x - 1)/3 - log(x - 5^(1/2)*1i)*((5^(1/2)*1i)/6 + 1/6) + log(x + 5^(1/2
)*1i)*((5^(1/2)*1i)/6 - 1/6)
```

3.448 $\int \frac{2+x^2}{2+x} dx$

3.448.1 Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{2+x^2}{2+x} dx = -2x + \frac{x^2}{2} + 6\log(2+x)$$

output
```
-2*x+1/2*x^2+6*ln(2+x)
```

3.448.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{2+x^2}{2+x} dx = -6 - 2x + \frac{x^2}{2} + 6\log(2+x)$$

input
```
Integrate[(2 + x^2)/(2 + x),x]
```

output
```
-6 - 2*x + x^2/2 + 6*Log[2 + x]
```

3.448.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+2}{x+2}\,dx$$

$$\Big\downarrow \; 476$$

$$\int \left(x+\frac{6}{x+2}-2\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^2}{2}-2x+6\log(x+2)$$

input
```
Int[(2 + x^2)/(2 + x),x]
```

output
```
-2*x + x^2/2 + 6*Log[2 + x]
```

3.448.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.448.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$-2x+\frac{x^2}{2}+6\ln(x+2)$	16
norman	$-2x+\frac{x^2}{2}+6\ln(x+2)$	16
risch	$-2x+\frac{x^2}{2}+6\ln(x+2)$	16
parallelrisch	$-2x+\frac{x^2}{2}+6\ln(x+2)$	16
meijerg	$6\ln\left(1+\frac{x}{2}\right)-\frac{x\left(-\frac{3x}{2}+6\right)}{3}$	18

input
```
int((x^2+2)/(x+2),x,method=_RETURNVERBOSE)
```

output
```
-2*x+1/2*x^2+6*ln(x+2)
```

3.448.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{2+x^2}{2+x}\,dx = \frac{1}{2}\,x^2 - 2\,x + 6\,\log(x+2)$$

input
```
integrate((x^2+2)/(2+x),x, algorithm="fricas")
```

output
```
1/2*x^2 - 2*x + 6*log(x + 2)
```

3.448.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{2+x^2}{2+x}\,dx = \frac{x^2}{2} - 2x + 6\log(x+2)$$

input
```
integrate((x**2+2)/(2+x),x)
```

output
```
x**2/2 - 2*x + 6*log(x + 2)
```

3.448.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{2+x^2}{2+x}\,dx = \frac{1}{2}\,x^2 - 2\,x + 6\,\log\left(x+2\right)$$

input
```
integrate((x^2+2)/(2+x),x, algorithm="maxima")
```

output
```
1/2*x^2 - 2*x + 6*log(x + 2)
```

3.448.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{2+x^2}{2+x}\,dx = \frac{1}{2}\,x^2 - 2\,x + 6\,\log\left(|x+2|\right)$$

input
```
integrate((x^2+2)/(2+x),x, algorithm="giac")
```

output
```
1/2*x^2 - 2*x + 6*log(abs(x + 2))
```

3.448.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{2+x^2}{2+x}\,dx = 6\,\ln\left(x+2\right) - 2\,x + \frac{x^2}{2}$$

input
```
int((x^2 + 2)/(x + 2),x)
```

output
```
6*log(x + 2) - 2*x + x^2/2
```

3.449 $\int \frac{1}{(-3+x)(4+x^2)}\,dx$

3.449.1 Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{1}{(-3+x)(4+x^2)}\,dx = -\frac{3}{26}\arctan\left(\frac{x}{2}\right)+\frac{1}{13}\log(3-x)-\frac{1}{26}\log\left(4+x^2\right)$$

output
```
-3/26*arctan(1/2*x)+1/13*ln(3-x)-1/26*ln(x^2+4)
```

3.449.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{1}{(-3+x)(4+x^2)}\,dx = -\frac{3}{26}\arctan\left(\frac{x}{2}\right)-\frac{1}{26}\log\left(13+6(-3+x)+(-3+x)^2\right) + \frac{1}{13}\log(-3+x)$$

input
```
Integrate[1/((-3 + x)*(4 + x^2)),x]
```

output
```
(-3*ArcTan[x/2])/26 - Log[13 + 6*(-3 + x) + (-3 + x)^2]/26 + Log[-3 + x]/1
3
```

3.449.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {479, 25, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x-3)(x^2+4)}dx$$

$$\Big\downarrow \text{479}$$

$$\frac{1}{13}\int -\frac{x+3}{x^2+4}dx+\frac{1}{13}\log(3-x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{13}\log(3-x)-\frac{1}{13}\int \frac{x+3}{x^2+4}dx$$

$$\Big\downarrow \text{452}$$

$$\frac{1}{13}\left(-3\int \frac{1}{x^2+4}dx-\int \frac{x}{x^2+4}dx\right)+\frac{1}{13}\log(3-x)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{13}\left(-\int \frac{x}{x^2+4}dx-\frac{3}{2}\arctan\left(\frac{x}{2}\right)\right)+\frac{1}{13}\log(3-x)$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{13}\left(-\frac{3}{2}\arctan\left(\frac{x}{2}\right)-\frac{1}{2}\log\left(x^2+4\right)\right)+\frac{1}{13}\log(3-x)$$

input
```
Int[1/((-3 + x)*(4 + x^2)),x]
```

output
```
Log[3 - x]/13 + ((-3*ArcTan[x/2])/2 - Log[4 + x^2]/2)/13
```

3.449.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

3.449.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{\ln\left(x^{2}+4\right)}{26}-\frac{3\arctan\left(\frac{x}{2}\right)}{26}+\frac{\ln\left(-3+x\right)}{13}$	22
risch	$\frac{\ln\left(-3+x\right)}{13}-\frac{\ln\left(9x^{2}+36\right)}{26}-\frac{3\arctan\left(\frac{x}{2}\right)}{26}$	24
parallelrisch	$\frac{\ln\left(-3+x\right)}{13}-\frac{\ln\left(x-2i\right)}{26}+\frac{3i\ln\left(x-2i\right)}{52}-\frac{\ln\left(x+2i\right)}{26}-\frac{3i\ln\left(x+2i\right)}{52}$	38

input
```
int(1/(-3+x)/(x^2+4),x,method=_RETURNVERBOSE)
```

output
```
-1/26*ln(x^2+4)-3/26*arctan(1/2*x)+1/13*ln(-3+x)
```

3.449.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{(-3+x)\,(4+x^2)}\,dx = -\frac{3}{26}\,\arctan\left(\frac{1}{2}\,x\right) - \frac{1}{26}\,\log\left(x^2+4\right) + \frac{1}{13}\,\log\left(x-3\right)$$

input
```
integrate(1/(-3+x)/(x^2+4),x, algorithm="fricas")
```

output
```
-3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)
```

3.449.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-3+x)\,(4+x^2)}\,dx = \frac{\log\left(x-3\right)}{13} - \frac{\log\left(x^2+4\right)}{26} - \frac{3\,\mathrm{atan}\left(\frac{x}{2}\right)}{26}$$

input
```
integrate(1/(-3+x)/(x**2+4),x)
```

output
```
log(x - 3)/13 - log(x**2 + 4)/26 - 3*atan(x/2)/26
```

3.449.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{(-3+x)\,(4+x^2)}\,dx = -\frac{3}{26}\,\arctan\left(\frac{1}{2}\,x\right) - \frac{1}{26}\,\log\left(x^2+4\right) + \frac{1}{13}\,\log\left(x-3\right)$$

input
```
integrate(1/(-3+x)/(x^2+4),x, algorithm="maxima")
```

output
```
-3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)
```

3.449.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1}{(-3+x)(4+x^2)}\,dx = -\frac{3}{26}\arctan\left(\frac{1}{2}x\right) - \frac{1}{26}\log\left(x^2+4\right) + \frac{1}{13}\log\left(|x-3|\right)$$

input
```
integrate(1/(-3+x)/(x^2+4),x, algorithm="giac")
```

output
```
-3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(abs(x - 3))
```

3.449.9 Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-3+x)(4+x^2)}\,dx = \frac{\ln(x-3)}{13} + \ln(x-2\mathrm{i})\left(-\frac{1}{26}+\frac{3}{52}\mathrm{i}\right) + \ln(x+2\mathrm{i})\left(-\frac{1}{26}-\frac{3}{52}\mathrm{i}\right)$$

input
```
int(1/((x^2 + 4)*(x - 3)),x)
```

output
```
log(x - 3)/13 - log(x - 2i)*(1/26 - 3i/52) - log(x + 2i)*(1/26 + 3i/52)
```

3.450 $\int \frac{-2+3x^6}{x(5+2x^6)}\,dx$

3.450.1 Optimal result

Integrand size = 20, antiderivative size = 19

$$\int \frac{-2+3x^6}{x(5+2x^6)}\,dx = -\frac{2\log(x)}{5}+\frac{19}{60}\log\left(5+2x^6\right)$$

output
```
-2/5*ln(x)+19/60*ln(2*x^6+5)
```

3.450.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{-2+3x^6}{x(5+2x^6)}\,dx = -\frac{2\log(x)}{5}+\frac{19}{60}\log\left(5+2x^6\right)$$

input
```
Integrate[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]
```

output
```
(-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60
```

3.450.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {948, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^6-2}{x(2x^6+5)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{6}\int -\frac{2-3x^6}{x^6(2x^6+5)}dx^6$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{6}\int \frac{2-3x^6}{x^6(2x^6+5)}dx^6$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{6}\int \left(\frac{2}{5x^6}-\frac{19}{5(2x^6+5)}\right)dx^6$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(\frac{19}{10}\log\left(2x^6+5\right)-\frac{2\log\left(x^6\right)}{5}\right)$$

input
```
Int[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]
```

output
```
((-2*Log[x^6])/5 + (19*Log[5 + 2*x^6])/10)/6
```

3.450.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.450.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$-\frac{2\ln(x)}{5}+\frac{19\ln\left(x^{6}+\frac{5}{2}\right)}{60}$	14
default	$-\frac{2\ln(x)}{5}+\frac{19\ln(2x^{6}+5)}{60}$	16
norman	$-\frac{2\ln(x)}{5}+\frac{19\ln(2x^{6}+5)}{60}$	16
risch	$-\frac{2\ln(x)}{5}+\frac{19\ln(2x^{6}+5)}{60}$	16
meijerg	$\frac{19\ln\left(1+\frac{2x^{6}}{5}\right)}{60}-\frac{2\ln(x)}{5}-\frac{\ln(2)}{15}+\frac{\ln(5)}{15}$	24

input
```
int((3*x^6-2)/x/(2*x^6+5),x,method=_RETURNVERBOSE)
```

output
```
-2/5*ln(x)+19/60*ln(x^6+5/2)
```

3.450.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{-2+3x^6}{x(5+2x^6)}\,dx = \frac{19}{60}\,\log\left(2\,x^6+5\right) - \frac{2}{5}\,\log\left(x\right)$$

input
```
integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="fricas")
```

output
```
19/60*log(2*x^6 + 5) - 2/5*log(x)
```

3.450.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{-2+3x^6}{x(5+2x^6)}\,dx = -\frac{2\log\left(x\right)}{5} + \frac{19\log\left(2x^6+5\right)}{60}$$

input
```
integrate((3*x**6-2)/x/(2*x**6+5),x)
```

output
```
-2*log(x)/5 + 19*log(2*x**6 + 5)/60
```

3.450.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{-2+3x^6}{x(5+2x^6)}\,dx = \frac{19}{60}\,\log\left(2\,x^6+5\right) - \frac{1}{15}\,\log\left(x^6\right)$$

input
```
integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="maxima")
```

output
```
19/60*log(2*x^6 + 5) - 1/15*log(x^6)
```

3.450.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{-2+3x^6}{x\left(5+2x^6\right)}\,dx = \frac{19}{60}\,\log\left(2\,x^6+5\right)-\frac{1}{15}\,\log\left(x^6\right)$$

input
```
integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="giac")
```

output
```
19/60*log(2*x^6 + 5) - 1/15*log(x^6)
```

3.450.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{-2+3x^6}{x\left(5+2x^6\right)}\,dx = \frac{19\,\ln\left(x^6+\frac{5}{2}\right)}{60}-\frac{2\,\ln\left(x\right)}{5}$$

input
```
int((3*x^6 - 2)/(x*(2*x^6 + 5)),x)
```

output
```
(19*log(x^6 + 5/2))/60 - (2*log(x))/5
```

3.451 $\int \frac{3+2x}{(-2+x)(5+x)} \, dx$

3.451.1 Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{3+2x}{(-2+x)(5+x)} \, dx = \log(2-x) + \log(5+x)$$

output
```
ln(2-x)+ln(5+x)
```

3.451.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{(-2+x)(5+x)} \, dx = \log(-2+x) + \log(5+x)$$

input
```
Integrate[(3 + 2*x)/((-2 + x)*(5 + x)),x]
```

output
```
Log[-2 + x] + Log[5 + x]
```

3.451.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+3}{(x-2)(x+5)} \, dx$$

$$\Big\downarrow \; 86$$

$$\int \left(\frac{1}{x+5}+\frac{1}{x-2}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\log(2-x)+\log(x+5)$$

input
```
Int[(3 + 2*x)/((-2 + x)*(5 + x)),x]
```

output
```
Log[2 - x] + Log[5 + x]
```

3.451.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.451.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
default	$\ln\left(\left(x-2\right)\left(5+x\right)\right)$	9
norman	$\ln\left(5+x\right)+\ln\left(x-2\right)$	10
risch	$\ln\left(x^{2}+3x-10\right)$	10
parallelrisch	$\ln\left(5+x\right)+\ln\left(x-2\right)$	10

input
```
int((2*x+3)/(x-2)/(5+x),x,method=_RETURNVERBOSE)
```

output
```
ln((x-2)*(5+x))
```

3.451.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log\left(x^{2}+3\,x-10\right)$$

input
```
integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="fricas")
```

output
```
log(x^2 + 3*x - 10)
```

3.451.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log\left(x^{2}+3x-10\right)$$

input
```
integrate((3+2*x)/(-2+x)/(5+x),x)
```

output
```
log(x**2 + 3*x - 10)
```

3.451.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log(x+5) + \log(x-2)$$

input
```
integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="maxima")
```

output
```
log(x + 5) + log(x - 2)
```

3.451.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \log(|x+5|) + \log(|x-2|)$$

input
```
integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="giac")
```

output
```
log(abs(x + 5)) + log(abs(x - 2))
```

3.451.9 Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx = \ln\left(x^2+3\,x-10\right)$$

input
```
int((2*x + 3)/((x - 2)*(x + 5)),x)
```

output
```
log(3*x + x^2 - 10)
```

3.452 $\int \frac{x^4}{4+5x^2+x^4} dx$

3.452.1 Optimal result

Integrand size = 16, antiderivative size = 18

$$\int \frac{x^4}{4+5x^2+x^4} dx = x - \frac{8}{3}\arctan\left(\frac{x}{2}\right) + \frac{\arctan(x)}{3}$$

output
```
x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

3.452.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{4+5x^2+x^4} dx = x + \frac{8}{3}\arctan\left(\frac{2}{x}\right) + \frac{\arctan(x)}{3}$$

input
```
Integrate[x^4/(4 + 5*x^2 + x^4),x]
```

output
```
x + (8*ArcTan[2/x])/3 + ArcTan[x]/3
```

3.452.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1442, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^4+5x^2+4}dx$$
$$\Big\downarrow \text{1442}$$
$$x-\int \frac{5x^2+4}{x^4+5x^2+4}dx$$
$$\Big\downarrow \text{1480}$$
$$\frac{1}{3}\int \frac{1}{x^2+1}dx-\frac{16}{3}\int \frac{1}{x^2+4}dx+x$$
$$\Big\downarrow \text{216}$$
$$-\frac{8}{3}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{3}+x$$

input
```
Int[x^4/(4 + 5*x^2 + x^4),x]
```

output
```
x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

3.452.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.452.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

method	result	size
default	$x-\frac{8\arctan\left(\frac{x}{2}\right)}{3}+\frac{\arctan\left(x\right)}{3}$	13
risch	$x-\frac{8\arctan\left(\frac{x}{2}\right)}{3}+\frac{\arctan\left(x\right)}{3}$	13
parallelrisch	$x+\frac{i\ln\left(x+i\right)}{6}-\frac{i\ln\left(x-i\right)}{6}+\frac{4i\ln\left(x-2i\right)}{3}-\frac{4i\ln\left(x+2i\right)}{3}$	35

input
```
int(x^4/(x^4+5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

3.452.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x-\frac{8}{3}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan\left(x\right)$$

input
```
integrate(x^4/(x^4+5*x^2+4),x, algorithm="fricas")
```

output
```
x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

3.452.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3} + \frac{\mathrm{atan}\,(x)}{3}$$

input
```
integrate(x**4/(x**4+5*x**2+4),x)
```

output
```
x - 8*atan(x/2)/3 + atan(x)/3
```

3.452.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8}{3}\arctan\left(\frac{1}{2}\,x\right) + \frac{1}{3}\arctan\,(x)$$

input
```
integrate(x^4/(x^4+5*x^2+4),x, algorithm="maxima")
```

output
```
x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

3.452.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8}{3}\arctan\left(\frac{1}{2}\,x\right) + \frac{1}{3}\arctan\,(x)$$

input
```
integrate(x^4/(x^4+5*x^2+4),x, algorithm="giac")
```

output
```
x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

3.452.9 Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3} + \frac{\mathrm{atan}(x)}{3}$$

input
```
int(x^4/(5*x^2 + x^4 + 4),x)
```

output
```
x - (8*atan(x/2))/3 + atan(x)/3
```

3.453 $\int \frac{1}{(1+x)(2+x)^2(3+x)^3} dx$

3.453.1 Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3} dx = \frac{1}{2+x} + \frac{1}{4(3+x)^2} + \frac{5}{4(3+x)} + \frac{1}{8}\log(1+x) + 2\log(2+x) - \frac{17}{8}\log(3+x)$$

output
```
1/(2+x)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(1+x)+2*ln(2+x)-17/8*ln(3+x)
```

3.453.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3} dx = \frac{1}{8}\left(\frac{8}{2+x} + \frac{2}{(3+x)^2} + \frac{10}{3+x} + \log(-1-x) + 16\log(2+x) - 17\log(3+x)\right)$$

input
```
Integrate[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

output
```
(8/(2 + x) + 2/(3 + x)^2 + 10/(3 + x) + Log[-1 - x] + 16*Log[2 + x] - 17*L
og[3 + x])/8
```

3.453.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)(x+2)^2(x+3)^3}\,dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(\frac{2}{x+2}-\frac{17}{8(x+3)}-\frac{1}{(x+2)^2}-\frac{5}{4(x+3)^2}-\frac{1}{2(x+3)^3}+\frac{1}{8(x+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

input
```
Int[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

output
```
(2 + x)^(-1) + 1/(4*(3 + x)^2) + 5/(4*(3 + x)) + Log[1 + x]/8 + 2*Log[2 +
x] - (17*Log[3 + x])/8
```

3.453.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.453.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result
default	$\frac{1}{x+2}+\frac{1}{4(3+x)^{2}}+\frac{5}{4(3+x)}+\frac{\ln(x+1)}{8}+2\ln\left(x+2\right)-\frac{17\ln(3+x)}{8}$
norman	$\frac{\frac{9}{4}x^{2}+\frac{25}{2}x+17}{(x+2)(3+x)^{2}}-\frac{17\ln(3+x)}{8}+\frac{\ln(x+1)}{8}+2\ln\left(x+2\right)$
risch	$\frac{\frac{9}{4}x^{2}+\frac{25}{2}x+17}{(x+2)(3+x)^{2}}-\frac{17\ln(3+x)}{8}+\frac{\ln(x+1)}{8}+2\ln\left(x+2\right)$
parallelrisch	$\frac{\ln(x+1)x^{3}+16\ln(x+2)x^{3}-17\ln(3+x)x^{3}+136+8\ln(x+1)x^{2}+128\ln(x+2)x^{2}-136\ln(3+x)x^{2}+21\ln(x+1)x+336\ln(x+2)x-3}{8(x+2)(3+x)^{2}}$

input
```
int(1/(x+1)/(x+2)^2/(3+x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(x+2)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(x+1)+2*ln(x+2)-17/8*ln(3+x)
```

3.453.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(38) = 76.

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80

$$\int\frac{1}{(1+x)(2+x)^{2}(3+x)^{3}}\,dx$$
$$=\frac{18\,x^{2}-17\,(x^{3}+8\,x^{2}+21\,x+18)\log\left(x+3\right)+16\,(x^{3}+8\,x^{2}+21\,x+18)\log\left(x+2\right)+(x^{3}+8\,x^{2}+21\,x}{8\,(x^{3}+8\,x^{2}+21\,x+18)}$$

input
```
integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="fricas")
```

output
```
1/8*(18*x^2 - 17*(x^3 + 8*x^2 + 21*x + 18)*log(x + 3) + 16*(x^3 + 8*x^2 +
21*x + 18)*log(x + 2) + (x^3 + 8*x^2 + 21*x + 18)*log(x + 1) + 100*x + 136
)/(x^3 + 8*x^2 + 21*x + 18)
```

3.453.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{9x^2+50x+68}{4x^3+32x^2+84x+72} + \frac{\log(x+1)}{8} + 2\log(x+2) - \frac{17\log(x+3)}{8}$$

input
```
integrate(1/(1+x)/(2+x)**2/(3+x)**3,x)
```

output
```
(9*x**2 + 50*x + 68)/(4*x**3 + 32*x**2 + 84*x + 72) + log(x + 1)/8 + 2*log
(x + 2) - 17*log(x + 3)/8
```

3.453.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{9\,x^2+50\,x+68}{4\,(x^3+8\,x^2+21\,x+18)} - \frac{17}{8}\,\log(x+3) + 2\,\log(x+2) + \frac{1}{8}\,\log(x+1)$$

input
```
integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="maxima")
```

output
```
1/4*(9*x^2 + 50*x + 68)/(x^3 + 8*x^2 + 21*x + 18) - 17/8*log(x + 3) + 2*lo
g(x + 2) + 1/8*log(x + 1)
```

3.453.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{1}{x+2} - \frac{\frac{7}{x+2}+6}{4\left(\frac{1}{x+2}+1\right)^2} + \frac{1}{8}\,\log\left(\left|-\frac{1}{x+2}+1\right|\right) - \frac{17}{8}\,\log\left(\left|-\frac{1}{x+2}-1\right|\right)$$

input
```
integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="giac")
```

output
```
1/(x + 2) - 1/4*(7/(x + 2) + 6)/(1/(x + 2) + 1)^2 + 1/8*log(abs(-1/(x + 2)
 + 1)) - 17/8*log(abs(-1/(x + 2) - 1))
```

3.453.9 Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx = \frac{\ln(x+1)}{8} + 2\ln(x+2) - \frac{17\ln(x+3)}{8} + \frac{\frac{9x^2}{4} + \frac{25x}{2} + 17}{x^3 + 8x^2 + 21x + 18}$$

input
```
int(1/((x + 1)*(x + 2)^2*(x + 3)^3),x)
```

output
```
log(x + 1)/8 + 2*log(x + 2) - (17*log(x + 3))/8 + ((25*x)/2 + (9*x^2)/4 +
17)/(21*x + 8*x^2 + x^3 + 18)
```

3.454 $\int \frac{x}{-1+x^2} \, dx$

3.454.1 Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{x}{-1+x^2} \, dx = \frac{1}{2} \log \left(1-x^2\right)$$

output
```
1/2*ln(-x^2+1)
```

3.454.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{-1+x^2} \, dx = \frac{1}{2} \log \left(-1+x^2\right)$$

input
```
Integrate[x/(-1 + x^2),x]
```

output
```
Log[-1 + x^2]/2
```

3.454.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^2-1} \, dx$$

$$\Big\downarrow \text{ 240}$$

$$\frac{1}{2} \log \left(1-x^2\right)$$

input
```
Int[x/(-1 + x^2),x]
```

output
```
Log[1 - x^2]/2
```

3.454.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

3.454.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\ln\left(x^{2}-1\right)}{2}$	9
risch	$\frac{\ln\left(x^{2}-1\right)}{2}$	9
meijerg	$\frac{\ln\left(-x^{2}+1\right)}{2}$	11
default	$\frac{\ln\left(x -1\right)}{2}+\frac{\ln\left(x +1\right)}{2}$	14
norman	$\frac{\ln\left(x -1\right)}{2}+\frac{\ln\left(x +1\right)}{2}$	14
parallelrisch	$\frac{\ln\left(x -1\right)}{2}+\frac{\ln\left(x +1\right)}{2}$	14

input
```
int(x/(x^2-1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2-1)
```

3.454.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{-1+x^2}\,dx = \frac{1}{2}\log\left(x^2-1\right)$$

input
```
integrate(x/(x^2-1),x, algorithm="fricas")
```

output
```
1/2*log(x^2 - 1)
```

3.454.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{x}{-1+x^2}\,dx = \frac{\log\left(x^2-1\right)}{2}$$

input
```
integrate(x/(x**2-1),x)
```

output
```
log(x**2 - 1)/2
```

3.454.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{-1+x^2}\,dx = \frac{1}{2}\log\left(x^2-1\right)$$

input
```
integrate(x/(x^2-1),x, algorithm="maxima")
```

output
```
1/2*log(x^2 - 1)
```

3.454.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{x}{-1+x^2}\,dx = \frac{1}{2}\log\left(|x^2-1|\right)$$

input
```
integrate(x/(x^2-1),x, algorithm="giac")
```

output
```
1/2*log(abs(x^2 - 1))
```

3.454.9 Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{-1+x^2}\,dx = \frac{\ln\left(x^2-1\right)}{2}$$

input
```
int(x/(x^2 - 1),x)
```

output
```
log(x^2 - 1)/2
```

3.455 $\int \frac{1}{(-1+x^2)^2} dx$

3.455.1 Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \frac{1}{(-1+x^2)^2} dx = \frac{x}{2(1-x^2)} + \frac{\operatorname{arctanh}(x)}{2}$$

output
```
1/2*x/(-x^2+1)+1/2*arctanh(x)
```

3.455.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{1}{(-1+x^2)^2} dx = \frac{1}{4}\left(-\frac{2x}{-1+x^2} - \log(1-x) + \log(1+x)\right)$$

input
```
Integrate[(-1 + x^2)^(-2),x]
```

output
```
((-2*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

3.455.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {215, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-1)^2}dx$$

$$\Big\downarrow \ 215$$

$$\frac{x}{2(1-x^2)}-\frac{1}{2}\int \frac{1}{x^2-1}dx$$

$$\Big\downarrow \ 220$$

$$\frac{\text{arctanh}(x)}{2}+\frac{x}{2(1-x^2)}$$

input
```
Int[(-1 + x^2)^(-2),x]
```

output
```
x/(2*(1 - x^2)) + ArcTanh[x]/2
```

3.455.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

3.455.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

method	result	size
meijerg	$-\frac{i\left(\frac{2ix}{-2x^2+2}+i\operatorname{arctanh}(x)\right)}{2}$	23
norman	$-\frac{x}{2(x^2-1)}-\frac{\ln(x-1)}{4}+\frac{\ln(x+1)}{4}$	24
risch	$-\frac{x}{2(x^2-1)}-\frac{\ln(x-1)}{4}+\frac{\ln(x+1)}{4}$	24
default	$-\frac{1}{4(x+1)}+\frac{\ln(x+1)}{4}-\frac{1}{4(x-1)}-\frac{\ln(x-1)}{4}$	28
parallelrisch	$-\frac{\ln(x-1)x^2-\ln(x+1)x^2-\ln(x-1)+\ln(x+1)+2x}{4(x^2-1)}$	41

input
```
int(1/(x^2-1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(2*I*x/(-2*x^2+2)+I*arctanh(x))
```

3.455.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(15) = 30.

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{1}{(-1+x^2)^2}\,dx = \frac{(x^2-1)\log(x+1)-(x^2-1)\log(x-1)-2x}{4(x^2-1)}$$

input
```
integrate(1/(x^2-1)^2,x, algorithm="fricas")
```

output
```
1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

3.455.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{(-1+x^2)^2}\,dx = -\frac{x}{2x^2-2} - \frac{\log{(x-1)}}{4} + \frac{\log{(x+1)}}{4}$$

input
```
integrate(1/(x**2-1)**2,x)
```

output
```
-x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

3.455.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{(-1+x^2)^2}\,dx = -\frac{x}{2\,(x^2-1)} + \frac{1}{4}\,\log{(x+1)} - \frac{1}{4}\,\log{(x-1)}$$

input
```
integrate(1/(x^2-1)^2,x, algorithm="maxima")
```

output
```
-1/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

3.455.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{(-1+x^2)^2}\,dx = -\frac{x}{2\,(x^2-1)} + \frac{1}{4}\,\log{(|x+1|)} - \frac{1}{4}\,\log{(|x-1|)}$$

input
```
integrate(1/(x^2-1)^2,x, algorithm="giac")
```

output
```
-1/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

3.455.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-1+x^2)^2}\,dx = \frac{\operatorname{atanh}(x)}{2} - \frac{x}{2\,(x^2-1)}$$

input
```
int(1/(x^2 - 1)^2,x)
```

output
```
atanh(x)/2 - x/(2*(x^2 - 1))
```

3.456 $\int \frac{x^2}{(1+x^2)^2} dx$

3.456.1 Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{x^2}{(1+x^2)^2} \, dx = -\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}$$

output
```
-1/2*x/(x^2+1)+1/2*arctan(x)
```

3.456.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+x^2)^2} \, dx = -\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}$$

input
```
Integrate[x^2/(1 + x^2)^2,x]
```

output
```
-1/2*x/(1 + x^2) + ArcTan[x]/2
```

3.456.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^2+1)^2}\,dx$$

$$\Big\downarrow \ 252$$

$$\frac{1}{2}\int \frac{1}{x^2+1}dx - \frac{x}{2(x^2+1)}$$

$$\Big\downarrow \ 216$$

$$\frac{\arctan(x)}{2} - \frac{x}{2(x^2+1)}$$

input
```
Int[x^2/(1 + x^2)^2,x]
```

output
```
-1/2*x/(1 + x^2) + ArcTan[x]/2
```

3.456.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

3.456.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
meijerg	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
risch	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(x+i)x^2+i\ln(x-i)-i\ln(x+i)+2x}{4(x^2+1)}$	52

input
```
int(x^2/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(x^2+1)+1/2*arctan(x)
```

3.456.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(1+x^2)^2}\,dx=\frac{(x^2+1)\arctan(x)-x}{2(x^2+1)}$$

input
```
integrate(x^2/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

3.456.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{(1+x^2)^2}\,dx=-\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(x**2/(x**2+1)**2,x)
```

output
```
-x/(2*x**2 + 2) + atan(x)/2
```

3.456.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(1+x^2)^2}\,dx = -\frac{x}{2\,(x^2+1)} + \frac{1}{2}\arctan\left(x\right)$$

input
```
integrate(x^2/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

3.456.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(1+x^2)^2}\,dx = -\frac{x}{2\,(x^2+1)} + \frac{1}{2}\arctan\left(x\right)$$

input
```
integrate(x^2/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

3.456.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(1+x^2)^2}\,dx = \frac{\mathrm{atan}(x)}{2} - \frac{x}{2\,\left(x^2+1\right)}$$

input
```
int(x^2/(x^2 + 1)^2,x)
```

output
```
atan(x)/2 - x/(2*(x^2 + 1))
```

3.457 $\int \frac{1}{2+3x} \, dx$

3.457.1 Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \frac{1}{2+3x} \, dx = \frac{1}{3}\log(2+3x)$$

output
```
1/3*ln(2+3*x)
```

3.457.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+3x} \, dx = \frac{1}{3}\log(2+3x)$$

input
```
Integrate[(2 + 3*x)^(-1),x]
```

output
```
Log[2 + 3*x]/3
```

3.457.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x+2} \, dx$$

$$\Big\downarrow \, 16$$

$$\frac{1}{3} \log(3x+2)$$

input
```
Int[(2 + 3*x)^(-1),x]
```

output
```
Log[2 + 3*x]/3
```

3.457.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

3.457.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
parallelrisch	$\frac{\ln\left(x+\frac{2}{3}\right)}{3}$	7
default	$\frac{\ln(3x+2)}{3}$	9
norman	$\frac{\ln(3x+2)}{3}$	9
meijerg	$\frac{\ln\left(1+\frac{3x}{2}\right)}{3}$	9
risch	$\frac{\ln(3x+2)}{3}$	9

input
```
int(1/(3*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(x+2/3)
```

3.457.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{2+3x}\,dx = \frac{1}{3}\,\log\left(3\,x+2\right)$$

input
```
integrate(1/(2+3*x),x, algorithm="fricas")
```

output
```
1/3*log(3*x + 2)
```

3.457.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{2+3x}\,dx = \frac{\log\left(3x+2\right)}{3}$$

input
```
integrate(1/(2+3*x),x)
```

output
```
log(3*x + 2)/3
```

3.457.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{2+3x}\,dx = \frac{1}{3}\,\log\left(3\,x+2\right)$$

input
```
integrate(1/(2+3*x),x, algorithm="maxima")
```

output
```
1/3*log(3*x + 2)
```

3.457.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{2+3x}\,dx = \frac{1}{3}\,\log\left(|3\,x+2|\right)$$

input
```
integrate(1/(2+3*x),x, algorithm="giac")
```

output
```
1/3*log(abs(3*x + 2))
```

3.457.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{2+3x}\,dx = \frac{\ln\left(x+\frac{2}{3}\right)}{3}$$

input
```
int(1/(3*x + 2),x)
```

output
```
log(x + 2/3)/3
```

3.458 $\int \frac{1}{a^2+x^2} dx$

3.458.1 Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{1}{a^2 + x^2} dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

output
```
arctan(x/a)/a
```

3.458.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + x^2} dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

input
```
Integrate[(a^2 + x^2)^(-1),x]
```

output
```
ArcTan[x/a]/a
```

3.458.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2+x^2}\,dx$$
$$\Big\downarrow \ 216$$
$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

input
```
Int[(a^2 + x^2)^(-1),x]
```

output
```
ArcTan[x/a]/a
```

3.458.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.458.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\arctan\left(\frac{x}{a}\right)}{a}$	11
risch	$\frac{\arctan\left(\frac{x}{a}\right)}{a}$	11
parallelrisch	$-\frac{i\ln(-ia+x)-i\ln(ia+x)}{2a}$	27

input
```
int(1/(a^2+x^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(x/a)/a
```

3.458.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

input
```
integrate(1/(a^2+x^2),x, algorithm="fricas")
```

output
```
arctan(x/a)/a
```

3.458.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{-\frac{i\log(-ia+x)}{2} + \frac{i\log(ia+x)}{2}}{a}$$

input
```
integrate(1/(a**2+x**2),x)
```

output
```
(-I*log(-I*a + x)/2 + I*log(I*a + x)/2)/a
```

3.458.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

input
```
integrate(1/(a^2+x^2),x, algorithm="maxima")
```

output
```
arctan(x/a)/a
```

3.458.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

input
```
integrate(1/(a^2+x^2),x, algorithm="giac")
```

output
```
arctan(x/a)/a
```

3.458.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{\mathrm{atan}\left(\frac{x}{a}\right)}{a}$$

input
```
int(1/(a^2 + x^2),x)
```

output
```
atan(x/a)/a
```

3.459 $\int \frac{1}{a+bx^2} dx$

3.459.1 Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

3.459.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

3.459.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^2}\,dx$$

$$\Big\downarrow \; 218$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[(a + b*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

3.459.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.459.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$-\frac{\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}$	41

input
```
int(1/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.459.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{1}{a+bx^2}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

input
```
integrate(1/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt
(a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

3.459.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(22) = 44.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{1}{a+bx^2}\,dx = -\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

input
```
integrate(1/(b*x**2+a),x)
```

output
```
-sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-
1/(a*b)) + x)/2
```

3.459.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(b*x^2+a),x, algorithm="maxima")
```

output
```
arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

3.459.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(b*x^2+a),x, algorithm="giac")
```

output
```
arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

3.459.9 Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{a+bx^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int(1/(a + b*x^2),x)
```

output
```
atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

3.460 $\int \frac{1}{2-x+x^2} dx$

3.460.1 Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \frac{1}{2-x+x^2} dx = -\frac{2\arctan\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

output
```
-2/7*arctan(1/7*(1-2*x)*7^(1/2))*7^(1/2)
```

3.460.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{2-x+x^2} dx = \frac{2\arctan\left(\frac{-1+2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

input
```
Integrate[(2 - x + x^2)^(-1),x]
```

output
```
(2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]
```

3.460.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2-x+2}dx$$
$$\Big\downarrow \text{1083}$$
$$-2\int \frac{1}{-(2x-1)^2-7}d(2x-1)$$
$$\Big\downarrow \text{217}$$
$$\frac{2\arctan\left(\frac{2x-1}{\sqrt{7}}\right)}{\sqrt{7}}$$

input
```
Int[(2 - x + x^2)^(-1),x]
```

output
```
(2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]
```

3.460.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.460.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{2\sqrt{7}\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$	17
risch	$\frac{2\sqrt{7}\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$	17

input
```
int(1/(x^2-x+2),x,method=_RETURNVERBOSE)
```

output
```
2/7*7^(1/2)*arctan(1/7*(2*x-1)*7^(1/2))
```

3.460.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

input
```
integrate(1/(x^2-x+2),x, algorithm="fricas")
```

output
```
2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

3.460.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x}{7}-\frac{\sqrt{7}}{7}\right)}{7}$$

input
```
integrate(1/(x**2-x+2),x)
```

output
```
2*sqrt(7)*atan(2*sqrt(7)*x/7 - sqrt(7)/7)/7
```

3.460.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

input
```
integrate(1/(x^2-x+2),x, algorithm="maxima")
```

output
```
2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

3.460.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

input
```
integrate(1/(x^2-x+2),x, algorithm="giac")
```

output
```
2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

3.460.9 Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{2-x+x^2}\,dx = \frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{\sqrt{7}\,(2x-1)}{7}\right)}{7}$$

input
```
int(1/(x^2 - x + 2),x)
```

output
```
(2*7^(1/2)*atan((7^(1/2)*(2*x - 1))/7))/7
```

3.461 $\int x^2(4-x^2)^2 \, dx$

3.461.1 Optimal result

Integrand size = 13, antiderivative size = 22

$$\int x^2(4-x^2)^2 \, dx = \frac{16x^3}{3} - \frac{8x^5}{5} + \frac{x^7}{7}$$

output
```
16/3*x^3-8/5*x^5+1/7*x^7
```

3.461.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2(4-x^2)^2 \, dx = \frac{16x^3}{3} - \frac{8x^5}{5} + \frac{x^7}{7}$$

input
```
Integrate[x^2*(4 - x^2)^2,x]
```

output
```
(16*x^3)/3 - (8*x^5)/5 + x^7/7
```

3.461.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(4-x^2\right)^2 \, dx$$

$$\Big\downarrow \; 244$$

$$\int \left(x^6-8 x^4+16 x^2\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^7}{7}-\frac{8 x^5}{5}+\frac{16 x^3}{3}$$

input
```
Int[x^2*(4 - x^2)^2,x]
```

output
```
(16*x^3)/3 - (8*x^5)/5 + x^7/7
```

3.461.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.461.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{16}{3}x^3-\frac{8}{5}x^5+\frac{1}{7}x^7$	17
norman	$\frac{16}{3}x^3-\frac{8}{5}x^5+\frac{1}{7}x^7$	17
risch	$\frac{16}{3}x^3-\frac{8}{5}x^5+\frac{1}{7}x^7$	17
parallelrisch	$\frac{16}{3}x^3-\frac{8}{5}x^5+\frac{1}{7}x^7$	17
gosper	$\frac{x^3(15x^4-168x^2+560)}{105}$	18

input
```
int(x^2*(-x^2+4)^2,x,method=_RETURNVERBOSE)
```

output
```
16/3*x^3-8/5*x^5+1/7*x^7
```

3.461.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x^2(4-x^2)^2\,dx = \frac{1}{7}x^7 - \frac{8}{5}x^5 + \frac{16}{3}x^3$$

input
```
integrate(x^2*(-x^2+4)^2,x, algorithm="fricas")
```

output
```
1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

3.461.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int x^2(4-x^2)^2\,dx = \frac{x^7}{7} - \frac{8x^5}{5} + \frac{16x^3}{3}$$

input
```
integrate(x**2*(-x**2+4)**2,x)
```

output
```
x**7/7 - 8*x**5/5 + 16*x**3/3
```

3.461.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x^2 (4 - x^2)^2 \, dx = \frac{1}{7} x^7 - \frac{8}{5} x^5 + \frac{16}{3} x^3$$

input
```
integrate(x^2*(-x^2+4)^2,x, algorithm="maxima")
```

output
```
1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

3.461.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x^2 (4 - x^2)^2 \, dx = \frac{1}{7} x^7 - \frac{8}{5} x^5 + \frac{16}{3} x^3$$

input
```
integrate(x^2*(-x^2+4)^2,x, algorithm="giac")
```

output
```
1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

3.461.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int x^2 (4 - x^2)^2 \, dx = \frac{x^3 (15 x^4 - 168 x^2 + 560)}{105}$$

input
```
int(x^2*(x^2 - 4)^2,x)
```

output
```
(x^3*(15*x^4 - 168*x^2 + 560))/105
```

3.462 $\int x(1-x^3)^2\,dx$

3.462.1 Optimal result

Integrand size = 11, antiderivative size = 22

$$\int x(1-x^3)^2\,dx=\frac{x^2}{2}-\frac{2x^5}{5}+\frac{x^8}{8}$$

output
```
1/2*x^2-2/5*x^5+1/8*x^8
```

3.462.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(1-x^3)^2\,dx=\frac{x^2}{2}-\frac{2x^5}{5}+\frac{x^8}{8}$$

input
```
Integrate[x*(1 - x^3)^2,x]
```

output
```
x^2/2 - (2*x^5)/5 + x^8/8
```

3.462.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(1-x^3\right)^2 dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(x^7-2x^4+x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^8}{8}-\frac{2x^5}{5}+\frac{x^2}{2}$$

input
```
Int[x*(1 - x^3)^2,x]
```

output
```
x^2/2 - (2*x^5)/5 + x^8/8
```

3.462.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.462.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{1}{2}x^2-\frac{2}{5}x^5+\frac{1}{8}x^8$	17
norman	$\frac{1}{2}x^2-\frac{2}{5}x^5+\frac{1}{8}x^8$	17
risch	$\frac{1}{2}x^2-\frac{2}{5}x^5+\frac{1}{8}x^8$	17
parallelrisch	$\frac{1}{2}x^2-\frac{2}{5}x^5+\frac{1}{8}x^8$	17
gosper	$\frac{x^2(5x^6-16x^3+20)}{40}$	18

input
```
int(x*(-x^3+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-2/5*x^5+1/8*x^8
```

3.462.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x(1-x^3)^2\,dx=\frac{1}{8}x^8-\frac{2}{5}x^5+\frac{1}{2}x^2$$

input
```
integrate(x*(-x^3+1)^2,x, algorithm="fricas")
```

output
```
1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

3.462.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int x(1-x^3)^2\,dx=\frac{x^8}{8}-\frac{2x^5}{5}+\frac{x^2}{2}$$

input
```
integrate(x*(-x**3+1)**2,x)
```

output
```
x**8/8 - 2*x**5/5 + x**2/2
```

3.462.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x(1-x^3)^2\,dx = \frac{1}{8}\,x^8 - \frac{2}{5}\,x^5 + \frac{1}{2}\,x^2$$

input
```
integrate(x*(-x^3+1)^2,x, algorithm="maxima")
```

output
```
1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

3.462.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x(1-x^3)^2\,dx = \frac{1}{8}\,x^8 - \frac{2}{5}\,x^5 + \frac{1}{2}\,x^2$$

input
```
integrate(x*(-x^3+1)^2,x, algorithm="giac")
```

output
```
1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

3.462.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int x(1-x^3)^2\,dx = \frac{x^2\,(5\,x^6 - 16\,x^3 + 20)}{40}$$

input
```
int(x*(x^3 - 1)^2,x)
```

output
```
(x^2*(5*x^6 - 16*x^3 + 20))/40
```

3.463 $\int \frac{-4+5x^2+x^3}{x^2} \, dx$

3.463.1 Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{-4+5x^2+x^3}{x^2} \, dx = \frac{4}{x}+5x+\frac{x^2}{2}$$

output
```
4/x+5*x+1/2*x^2
```

3.463.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-4+5x^2+x^3}{x^2} \, dx = \frac{4}{x}+5x+\frac{x^2}{2}$$

input
```
Integrate[(-4 + 5*x^2 + x^3)/x^2,x]
```

output
```
4/x + 5*x + x^2/2
```

3.463.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+5x^2-4}{x^2}\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(-\frac{4}{x^2}+x+5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}+5x+\frac{4}{x}$$

input
```
Int[(-4 + 5*x^2 + x^3)/x^2,x]
```

output
```
4/x + 5*x + x^2/2
```

3.463.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

3.463.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{4}{x}+5x+\frac{x^2}{2}$	15
risch	$\frac{4}{x}+5x+\frac{x^2}{2}$	15
gosper	$\frac{x^3+10x^2+8}{2x}$	16
parallelrisch	$\frac{x^3+10x^2+8}{2x}$	16
norman	$\frac{\frac{1}{2}x^3+5x^2+4}{x}$	17

input
```
int((x^3+5*x^2-4)/x^2,x,method=_RETURNVERBOSE)
```

output
```
4/x+5*x+1/2*x^2
```

3.463.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx=\frac{x^3+10\,x^2+8}{2\,x}$$

input
```
integrate((x^3+5*x^2-4)/x^2,x, algorithm="fricas")
```

output
```
1/2*(x^3 + 10*x^2 + 8)/x
```

3.463.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx=\frac{x^2}{2}+5x+\frac{4}{x}$$

input
```
integrate((x**3+5*x**2-4)/x**2,x)
```

output
```
x**2/2 + 5*x + 4/x
```

3.463.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx = \frac{1}{2}\,x^2 + 5\,x + \frac{4}{x}$$

input
```
integrate((x^3+5*x^2-4)/x^2,x, algorithm="maxima")
```

output
```
1/2*x^2 + 5*x + 4/x
```

3.463.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx = \frac{1}{2}\,x^2 + 5\,x + \frac{4}{x}$$

input
```
integrate((x^3+5*x^2-4)/x^2,x, algorithm="giac")
```

output
```
1/2*x^2 + 5*x + 4/x
```

3.463.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx = \frac{x^3+10\,x^2+8}{2\,x}$$

input
```
int((5*x^2 + x^3 - 4)/x^2,x)
```

output
```
(10*x^2 + x^3 + 8)/(2*x)
```

3.464 $\int \frac{-1+x}{3-4x+3x^2} \, dx$

3.464.1 Optimal result

Integrand size = 16, antiderivative size = 37

$$\int \frac{-1+x}{3-4x+3x^2} \, dx = \frac{\arctan\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}} + \frac{1}{6}\log\left(3-4x+3x^2\right)$$

output
```
1/6*ln(3*x^2-4*x+3)+1/15*arctan(1/5*(2-3*x)*5^(1/2))*5^(1/2)
```

3.464.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{-1+x}{3-4x+3x^2} \, dx = -\frac{\arctan\left(\frac{-2+3x}{\sqrt{5}}\right)}{3\sqrt{5}} + \frac{1}{6}\log\left(3-4x+3x^2\right)$$

input
```
Integrate[(-1 + x)/(3 - 4*x + 3*x^2),x]
```

output
```
-1/3*ArcTan[(-2 + 3*x)/Sqrt[5]]/Sqrt[5] + Log[3 - 4*x + 3*x^2]/6
```

3.464.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-1}{3x^2-4x+3}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{6}\int -\frac{2(2-3x)}{3x^2-4x+3}dx-\frac{1}{3}\int \frac{1}{3x^2-4x+3}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}\int \frac{1}{3x^2-4x+3}dx-\frac{1}{3}\int \frac{2-3x}{3x^2-4x+3}dx$$

$$\Big\downarrow \text{1083}$$

$$\frac{2}{3}\int \frac{1}{-(6x-4)^2-20}d(6x-4)-\frac{1}{3}\int \frac{2-3x}{3x^2-4x+3}dx$$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{3}\int \frac{2-3x}{3x^2-4x+3}dx-\frac{\arctan\left(\frac{6x-4}{2\sqrt{5}}\right)}{3\sqrt{5}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{6}\log\left(3x^2-4x+3\right)-\frac{\arctan\left(\frac{6x-4}{2\sqrt{5}}\right)}{3\sqrt{5}}$$

input
```
Int[(-1 + x)/(3 - 4*x + 3*x^2),x]
```

output
```
-1/3*ArcTan[(-4 + 6*x)/(2*Sqrt[5])]/Sqrt[5] + Log[3 - 4*x + 3*x^2]/6
```

3.464.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.464.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln(3x^2-4x+3)}{6}-\frac{\sqrt{5}\,\arctan\left(\frac{(6x-4)\sqrt{5}}{10}\right)}{15}$	31
risch	$\frac{\ln(9x^2-12x+9)}{6}-\frac{\sqrt{5}\,\arctan\left(\frac{(3x-2)\sqrt{5}}{5}\right)}{15}$	31

input
```
int((x-1)/(3*x^2-4*x+3),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(3*x^2-4*x+3)-1/15*5^(1/2)*arctan(1/10*(6*x-4)*5^(1/2))
```

3.464.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{-1+x}{3-4x+3x^2}\,dx = -\frac{1}{15}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}(3x-2)\right)+\frac{1}{6}\log\left(3x^2-4x+3\right)$$

input
```
integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="fricas")
```

output
```
-1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

3.464.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{-1+x}{3-4x+3x^2}\,dx = \frac{\log\left(x^2-\frac{4x}{3}+1\right)}{6}-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{3\sqrt{5}x}{5}-\frac{2\sqrt{5}}{5}\right)}{15}$$

input
```
integrate((-1+x)/(3*x**2-4*x+3),x)
```

output
```
log(x**2 - 4*x/3 + 1)/6 - sqrt(5)*atan(3*sqrt(5)*x/5 - 2*sqrt(5)/5)/15
```

3.464.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{-1+x}{3-4x+3x^2}\,dx = -\frac{1}{15}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}(3x-2)\right)+\frac{1}{6}\log\left(3x^2-4x+3\right)$$

input
```
integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="maxima")
```

output
```
-1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

3.464.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{-1+x}{3-4x+3x^2}\,dx = -\frac{1}{15}\,\sqrt{5}\arctan\left(\frac{1}{5}\,\sqrt{5}(3\,x-2)\right)+\frac{1}{6}\,\log\left(3\,x^2-4\,x+3\right)$$

input
```
integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="giac")
```

output
```
-1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

3.464.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{-1+x}{3-4x+3x^2}\,dx = \frac{\ln\left(x^2-\frac{4\,x}{3}+1\right)}{6}-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{3\,\sqrt{5}\,x}{5}-\frac{2\,\sqrt{5}}{5}\right)}{15}$$

input
```
int((x - 1)/(3*x^2 - 4*x + 3),x)
```

output
```
log(x^2 - (4*x)/3 + 1)/6 - (5^(1/2)*atan((3*5^(1/2)*x)/5 - (2*5^(1/2))/5))
/15
```

3.465 $\int (2+x^3)^2 \, dx$

3.465.1 Optimal result

Integrand size = 7, antiderivative size = 14

$$\int (2+x^3)^2 \, dx = 4x + x^4 + \frac{x^7}{7}$$

output
```
4*x+x^4+1/7*x^7
```

3.465.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (2+x^3)^2 \, dx = 4x + x^4 + \frac{x^7}{7}$$

input
```
Integrate[(2 + x^3)^2,x]
```

output
```
4*x + x^4 + x^7/7
```

3.465.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^3+2)^2 \, dx$$

$$\Big\downarrow \; 747$$

$$\int (x^6+4x^3+4) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^7}{7}+x^4+4x$$

input
```
Int[(2 + x^3)^2,x]
```

output
```
4*x + x^4 + x^7/7
```

3.465.3.1 Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.465.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$4x + x^4 + \frac{1}{7}x^7$	13
default	$4x + x^4 + \frac{1}{7}x^7$	13
norman	$4x + x^4 + \frac{1}{7}x^7$	13
risch	$4x + x^4 + \frac{1}{7}x^7$	13
parallelrisch	$4x + x^4 + \frac{1}{7}x^7$	13

input
```
int((x^3+2)^2,x,method=_RETURNVERBOSE)
```

output
```
4*x+x^4+1/7*x^7
```

3.465.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(2+x^3\right)^2 \, dx = \frac{1}{7}\,x^7 + x^4 + 4\,x$$

input
```
integrate((x^3+2)^2,x, algorithm="fricas")
```

output
```
1/7*x^7 + x^4 + 4*x
```

3.465.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(2+x^3\right)^2 \, dx = \frac{x^7}{7} + x^4 + 4x$$

input
```
integrate((x**3+2)**2,x)
```

output
```
x**7/7 + x**4 + 4*x
```

3.465.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(2+x^3\right)^2 \, dx = \frac{1}{7}\,x^7 + x^4 + 4\,x$$

input
```
integrate((x^3+2)^2,x, algorithm="maxima")
```

output
```
1/7*x^7 + x^4 + 4*x
```

3.465.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(2+x^3\right)^2 \, dx = \frac{1}{7}\,x^7 + x^4 + 4\,x$$

input
```
integrate((x^3+2)^2,x, algorithm="giac")
```

output
```
1/7*x^7 + x^4 + 4*x
```

3.465.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \left(2+x^3\right)^2 \, dx = \frac{x\,\left(x^6+7\,x^3+28\right)}{7}$$

input
```
int((x^3 + 2)^2,x)
```

output
```
(x*(7*x^3 + x^6 + 28))/7
```

3.466 $\int \frac{-4+x^2}{2+x} \, dx$

3.466.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{-4+x^2}{2+x} \, dx = -2x + \frac{x^2}{2}$$

output
```
-2*x+1/2*x^2
```

3.466.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-4+x^2}{2+x} \, dx = -2x + \frac{x^2}{2}$$

input
```
Integrate[(-4 + x^2)/(2 + x),x]
```

output
```
-2*x + x^2/2
```

3.466.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {456, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-4}{x+2}dx$$

$$\Big\downarrow \text{456}$$

$$\int (x-2)dx$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{2}(2-x)^2$$

input
```
Int[(-4 + x^2)/(2 + x),x]
```

output
```
(2 - x)^2/2
```

3.466.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

3.466.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x(x-4)}{2}$	7
meijerg	$-\frac{x\left(-\frac{3x}{2}+6\right)}{3}$	9
default	$-2x+\frac{1}{2}x^{2}$	10
norman	$-2x+\frac{1}{2}x^{2}$	10
risch	$-2x+\frac{1}{2}x^{2}$	10
parallelrisch	$-2x+\frac{1}{2}x^{2}$	10
parts	$-2x+\frac{1}{2}x^{2}$	10

input
```
int((x^2-4)/(x+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x-4)
```

3.466.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-4+x^2}{2+x}\,dx=\frac{1}{2}\,x^2-2\,x$$

input
```
integrate((x^2-4)/(2+x),x, algorithm="fricas")
```

output
```
1/2*x^2 - 2*x
```

3.466.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{-4+x^2}{2+x}\,dx = \frac{x^2}{2} - 2x$$

input
```
integrate((x**2-4)/(2+x),x)
```

output
```
x**2/2 - 2*x
```

3.466.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-4+x^2}{2+x}\,dx = \frac{1}{2}\,x^2 - 2\,x$$

input
```
integrate((x^2-4)/(2+x),x, algorithm="maxima")
```

output
```
1/2*x^2 - 2*x
```

3.466.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-4+x^2}{2+x}\,dx = \frac{1}{2}\,x^2 - 2\,x$$

input
```
integrate((x^2-4)/(2+x),x, algorithm="giac")
```

output
```
1/2*x^2 - 2*x
```

3.466.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{-4+x^2}{2+x}\,dx = \frac{x\,(x-4)}{2}$$

input
```
int((x^2 - 4)/(x + 2),x)
```

output
```
(x*(x - 4))/2
```

3.467 $\int \frac{1}{(2+x)(1+x^2)}\,dx$

3.467.1 Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{2\arctan(x)}{5} + \frac{1}{5}\log(2+x) - \frac{1}{10}\log\left(1+x^2\right)$$

output
```
2/5*arctan(x)+1/5*ln(2+x)-1/10*ln(x^2+1)
```

3.467.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{2\arctan(x)}{5} + \frac{1}{5}\log(2+x) - \frac{1}{10}\log\left(1+x^2\right)$$

input
```
Integrate[1/((2 + x)*(1 + x^2)),x]
```

output
```
(2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10
```

3.467.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {479, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+2)(x^2+1)}dx$$

$$\Big\downarrow \text{479}$$

$$\frac{1}{5}\int \frac{2-x}{x^2+1}dx+\frac{1}{5}\log(x+2)$$

$$\Big\downarrow \text{452}$$

$$\frac{1}{5}\left(2\int \frac{1}{x^2+1}dx-\int \frac{x}{x^2+1}dx\right)+\frac{1}{5}\log(x+2)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{5}\left(2\arctan(x)-\int \frac{x}{x^2+1}dx\right)+\frac{1}{5}\log(x+2)$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{5}\left(2\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)\right)+\frac{1}{5}\log(x+2)$$

input
```
Int[1/((2 + x)*(1 + x^2)),x]
```

output
```
Log[2 + x]/5 + (2*ArcTan[x] - Log[1 + x^2]/2)/5
```

3.467.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

3.467.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{2\arctan(x)}{5}+\frac{\ln(x+2)}{5}-\frac{\ln(x^2+1)}{10}$	20
risch	$\frac{2\arctan(x)}{5}+\frac{\ln(x+2)}{5}-\frac{\ln(x^2+1)}{10}$	20
parallelrisch	$\frac{\ln(x+2)}{5}-\frac{\ln(x-i)}{10}-\frac{i\ln(x-i)}{5}-\frac{\ln(x+i)}{10}+\frac{i\ln(x+i)}{5}$	38

input
```
int(1/(x+2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
2/5*arctan(x)+1/5*ln(x+2)-1/10*ln(x^2+1)
```

3.467.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{2}{5}\arctan(x) - \frac{1}{10}\log\left(x^2+1\right) + \frac{1}{5}\log(x+2)$$

input
```
integrate(1/(2+x)/(x^2+1),x, algorithm="fricas")
```

output
```
2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)
```

3.467.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{\log(x+2)}{5} - \frac{\log(x^2+1)}{10} + \frac{2\,\mathrm{atan}(x)}{5}$$

input
```
integrate(1/(2+x)/(x**2+1),x)
```

output
```
log(x + 2)/5 - log(x**2 + 1)/10 + 2*atan(x)/5
```

3.467.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{2}{5}\arctan(x) - \frac{1}{10}\log\left(x^2+1\right) + \frac{1}{5}\log(x+2)$$

input
```
integrate(1/(2+x)/(x^2+1),x, algorithm="maxima")
```

output
```
2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)
```

3.467.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{2}{5}\arctan(x) - \frac{1}{10}\log\left(x^2+1\right) + \frac{1}{5}\log(|x+2|)$$

input
```
integrate(1/(2+x)/(x^2+1),x, algorithm="giac")
```

output
```
2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(abs(x + 2))
```

3.467.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+x)(1+x^2)}\,dx = \frac{\ln(x+2)}{5} + \ln(x-\mathrm{i})\left(-\frac{1}{10}-\frac{1}{5}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{10}+\frac{1}{5}\mathrm{i}\right)$$

input
```
int(1/((x^2 + 1)*(x + 2)),x)
```

output
```
log(x + 2)/5 - log(x - 1i)*(1/10 + 1i/5) - log(x + 1i)*(1/10 - 1i/5)
```

3.468 $\int \frac{1}{(1+x)(1+x^2)} dx$

3.468.1 Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{1}{(1+x)(1+x^2)} dx = \frac{\arctan(x)}{2} + \frac{1}{2}\log(1+x) - \frac{1}{4}\log\left(1+x^2\right)$$

output
```
1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

3.468.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)(1+x^2)} dx = \frac{\arctan(x)}{2} + \frac{1}{2}\log(1+x) - \frac{1}{4}\log\left(1+x^2\right)$$

input
```
Integrate[1/((1 + x)*(1 + x^2)),x]
```

output
```
ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

3.468.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {479, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)(x^2+1)}dx$$

$$\Big\downarrow \text{479}$$

$$\frac{1}{2}\int \frac{1-x}{x^2+1}dx+\frac{1}{2}\log(x+1)$$

$$\Big\downarrow \text{452}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2+1}dx-\int \frac{x}{x^2+1}dx\right)+\frac{1}{2}\log(x+1)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\arctan(x)-\int \frac{x}{x^2+1}dx\right)+\frac{1}{2}\log(x+1)$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{2}\left(\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)\right)+\frac{1}{2}\log(x+1)$$

input
```
Int[1/((1 + x)*(1 + x^2)),x]
```

output
```
Log[1 + x]/2 + (ArcTan[x] - Log[1 + x^2]/2)/2
```

3.468.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

3.468.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{2}-\frac{\ln(x^2+1)}{4}$	20
risch	$\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{2}-\frac{\ln(x^2+1)}{4}$	20
parallelrisch	$\frac{\ln(x+1)}{2}-\frac{\ln(x-i)}{4}-\frac{i\ln(x-i)}{4}-\frac{\ln(x+i)}{4}+\frac{i\ln(x+i)}{4}$	38

input
```
int(1/(x+1)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x)+1/2*ln(x+1)-1/4*ln(x^2+1)
```

3.468.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

input
```
integrate(1/(1+x)/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

3.468.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1+x)(1+x^2)}\,dx = \frac{\log(x+1)}{2} - \frac{\log(x^2+1)}{4} + \frac{\mathrm{atan}(x)}{2}$$

input
```
integrate(1/(1+x)/(x**2+1),x)
```

output
```
log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

3.468.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{4}\log(x^2+1) + \frac{1}{2}\log(x+1)$$

input
```
integrate(1/(1+x)/(x^2+1),x, algorithm="maxima")
```

output
```
1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

3.468.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan(x) - \frac{1}{4}\log(x^2+1) + \frac{1}{2}\log(|x+1|)$$

input
```
integrate(1/(1+x)/(x^2+1),x, algorithm="giac")
```

output
```
1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))
```

3.468.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)\left(1+x^2\right)}\,dx = \frac{\ln\left(x+1\right)}{2} + \ln\left(x-\mathrm{i}\right)\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

input
```
int(1/((x^2 + 1)*(x + 1)),x)
```

output
```
log(x + 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)
```

3.469 $\int \frac{x}{(1+x)(1+x^2)} dx$

3.469.1 Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{x}{(1+x)(1+x^2)} dx = \frac{\arctan(x)}{2} - \frac{1}{2}\log(1+x) + \frac{1}{4}\log(1+x^2)$$

output
```
1/2*arctan(x)-1/2*ln(1+x)+1/4*ln(x^2+1)
```

3.469.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)(1+x^2)} dx = \frac{\arctan(x)}{2} - \frac{1}{2}\log(1+x) + \frac{1}{4}\log(1+x^2)$$

input
```
Integrate[x/((1 + x)*(1 + x^2)),x]
```

output
```
ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4
```

3.469.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {587, 16, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)(x^2+1)}dx$$

$$\Big\downarrow \text{587}$$

$$\frac{1}{2}\int \frac{x+1}{x^2+1}dx-\frac{1}{2}\int \frac{1}{x+1}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\int \frac{x+1}{x^2+1}dx-\frac{1}{2}\log(x+1)$$

$$\Big\downarrow \text{452}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2+1}dx+\int \frac{x}{x^2+1}dx\right)-\frac{1}{2}\log(x+1)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\int \frac{x}{x^2+1}dx+\arctan(x)\right)-\frac{1}{2}\log(x+1)$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{2}\left(\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)\right)-\frac{1}{2}\log(x+1)$$

input
```
Int[x/((1 + x)*(1 + x^2)),x]
```

output
```
-1/2*Log[1 + x] + (ArcTan[x] + Log[1 + x^2]/2)/2
```

3.469.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 587
```
Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(-
c)*(d/(b*c^2 + a*d^2))   Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2)
 Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
^2 + a*d^2, 0]
```

3.469.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\arctan(x)}{2} - \frac{\ln(x+1)}{2} + \frac{\ln(x^2+1)}{4}$	20
risch	$\frac{\arctan(x)}{2} - \frac{\ln(x+1)}{2} + \frac{\ln(x^2+1)}{4}$	20
parallelrisch	$-\frac{\ln(x+1)}{2} + \frac{\ln(x-i)}{4} - \frac{i\ln(x-i)}{4} + \frac{\ln(x+i)}{4} + \frac{i\ln(x+i)}{4}$	38

input
```
int(x/(x+1)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x)-1/2*ln(x+1)+1/4*ln(x^2+1)
```

3.469.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{x}{(1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan(x) + \frac{1}{4}\log(x^2+1) - \frac{1}{2}\log(x+1)$$

input
```
integrate(x/(1+x)/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)
```

3.469.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{x}{(1+x)(1+x^2)}\,dx = -\frac{\log(x+1)}{2} + \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(x/(1+x)/(x**2+1),x)
```

output
```
-log(x + 1)/2 + log(x**2 + 1)/4 + atan(x)/2
```

3.469.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{x}{(1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan(x) + \frac{1}{4}\log(x^2+1) - \frac{1}{2}\log(x+1)$$

input
```
integrate(x/(1+x)/(x^2+1),x, algorithm="maxima")
```

output
```
1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)
```

3.469.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{x}{(1+x)(1+x^2)}\,dx = \frac{1}{2}\arctan(x) + \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log\left(|x+1|\right)$$

input
```
integrate(x/(1+x)/(x^2+1),x, algorithm="giac")
```

output
```
1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(abs(x + 1))
```

3.469.9 Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x)(1+x^2)}\,dx = -\frac{\ln(x+1)}{2} + \ln(x-\mathrm{i})\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

input
```
int(x/((x^2 + 1)*(x + 1)),x)
```

output
```
log(x - 1i)*(1/4 - 1i/4) - log(x + 1)/2 + log(x + 1i)*(1/4 + 1i/4)
```

3.470 $\int \frac{2x+x^2}{(1+x)^2} dx$

3.470.1 Optimal result

Integrand size = 13, antiderivative size = 9

$$\int \frac{2x+x^2}{(1+x)^2} dx = \frac{x^2}{1+x}$$

output
```
x^2/(1+x)
```

3.470.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{2x+x^2}{(1+x)^2} dx = x+\frac{1}{1+x}$$

input
```
Integrate[(2*x + x^2)/(1 + x)^2,x]
```

output
```
x + (1 + x)^(-1)
```

3.470.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1107, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+2x}{(x+1)^2}\,dx$$
$$\Big\downarrow \text{1107}$$
$$\int \left(1-\frac{1}{(x+1)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$x+\frac{1}{x+1}$$

input
```
Int[(2*x + x^2)/(1 + x)^2,x]
```

output
```
x + (1 + x)^(-1)
```

3.470.3.1 Defintions of rubi rules used

rule 1107
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[2*c*d - b*e, 0] && IGtQ[p, 0] && !(EqQ[
m, 3] && NeQ[p, 1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.470.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
default	$x+\frac{1}{x+1}$	8
risch	$x+\frac{1}{x+1}$	8
gosper	$\frac{x^{2}}{x+1}$	10
norman	$\frac{x^{2}}{x+1}$	10
parallelrisch	$\frac{x^{2}}{x+1}$	10
meijerg	$\frac{x(6+3x)}{3x+3}-\frac{2x}{x+1}$	23

input
```
int((x^2+2*x)/(x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x+1/(x+1)
```

3.470.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{2x+x^2}{(1+x)^2}\,dx=\frac{x^2+x+1}{x+1}$$

input
```
integrate((x^2+2*x)/(1+x)^2,x, algorithm="fricas")
```

output
```
(x^2 + x + 1)/(x + 1)
```

3.470.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int \frac{2x+x^2}{(1+x)^2}\,dx=x+\frac{1}{x+1}$$

input
```
integrate((x**2+2*x)/(1+x)**2,x)
```

output
```
x + 1/(x + 1)
```

3.470.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{2x+x^2}{(1+x)^2}\,dx = x + \frac{1}{x+1}$$

input
```
integrate((x^2+2*x)/(1+x)^2,x, algorithm="maxima")
```

output
```
x + 1/(x + 1)
```

3.470.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{2x+x^2}{(1+x)^2}\,dx = x + \frac{1}{x+1} + 1$$

input
```
integrate((x^2+2*x)/(1+x)^2,x, algorithm="giac")
```

output
```
x + 1/(x + 1) + 1
```

3.470.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{2x+x^2}{(1+x)^2}\,dx = x + \frac{1}{x+1}$$

input
```
int((2*x + x^2)/(x + 1)^2,x)
```

output
```
x + 1/(x + 1)
```

3.471 $\int \frac{-10+x^2}{4+9x^2+2x^4} \, dx$

3.471.1 Optimal result

Integrand size = 20, antiderivative size = 22

$$\int \frac{-10+x^2}{4+9x^2+2x^4} \, dx = \arctan\left(\frac{x}{2}\right) - \frac{3\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

output
```
arctan(1/2*x)-3/2*arctan(x*2^(1/2))*2^(1/2)
```

3.471.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{-10+x^2}{4+9x^2+2x^4} \, dx = \arctan\left(\frac{x}{2}\right) - \frac{3\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

input
```
Integrate[(-10 + x^2)/(4 + 9*x^2 + 2*x^4),x]
```

output
```
ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]
```

3.471.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-10}{2x^4+9x^2+4}dx$$

$$\Big\downarrow \text{ 1480}$$

$$4\int \frac{1}{2x^2+8}dx-3\int \frac{1}{2x^2+1}dx$$

$$\Big\downarrow \text{ 216}$$

$$\arctan\left(\frac{x}{2}\right)-\frac{3\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

input
```
Int[(-10 + x^2)/(4 + 9*x^2 + 2*x^4),x]
```

output
```
ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]
```

3.471.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.471.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
default	$\arctan\left(\frac{x}{2}\right)-\frac{3\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}$	17
risch	$\arctan\left(\frac{x}{2}\right)-\frac{3\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}$	17

input
```
int((x^2-10)/(2*x^4+9*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
arctan(1/2*x)-3/2*arctan(x*2^(1/2))*2^(1/2)
```

3.471.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx=-\frac{3}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)+\arctan\left(\frac{1}{2}\,x\right)$$

input
```
integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="fricas")
```

output
```
-3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

3.471.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx=\operatorname{atan}\left(\frac{x}{2}\right)-\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}x\right)}{2}$$

input
```
integrate((x**2-10)/(2*x**4+9*x**2+4),x)
```

output
```
atan(x/2) - 3*sqrt(2)*atan(sqrt(2)*x)/2
```

3.471.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx = -\frac{3}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)+\arctan\left(\frac{1}{2}\,x\right)$$

input
```
integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="maxima")
```

output
```
-3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

3.471.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx = -\frac{3}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)+\arctan\left(\frac{1}{2}\,x\right)$$

input
```
integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="giac")
```

output
```
-3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

3.471.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx = \operatorname{atan}\left(\frac{x}{2}\right)-\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x\right)}{2}$$

input
```
int((x^2 - 10)/(9*x^2 + 2*x^4 + 4),x)
```

output
```
atan(x/2) - (3*2^(1/2)*atan(2^(1/2)*x))/2
```

3.472 $\int \frac{31+5x}{11-4x+3x^2} dx$

3.472.1 Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{31+5x}{11-4x+3x^2} dx = -\frac{103 \arctan\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}} + \frac{5}{6}\log\left(11-4x+3x^2\right)$$

output
```
5/6*ln(3*x^2-4*x+11)-103/87*arctan(1/29*(2-3*x)*29^(1/2))*29^(1/2)
```

3.472.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{31+5x}{11-4x+3x^2} dx = \frac{103 \arctan\left(\frac{-2+3x}{\sqrt{29}}\right)}{3\sqrt{29}} + \frac{5}{6}\log\left(11-4x+3x^2\right)$$

input
```
Integrate[(31 + 5*x)/(11 - 4*x + 3*x^2),x]
```

output
```
(103*ArcTan[(-2 + 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x + 3*x^2])
/6
```

3.472.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x+31}{3x^2-4x+11}dx$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{103}{3}\int \frac{1}{3x^2-4x+11}dx+\frac{5}{6}\int -\frac{2(2-3x)}{3x^2-4x+11}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{103}{3}\int \frac{1}{3x^2-4x+11}dx-\frac{5}{3}\int \frac{2-3x}{3x^2-4x+11}dx$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{5}{3}\int \frac{2-3x}{3x^2-4x+11}dx-\frac{206}{3}\int \frac{1}{-(6x-4)^2-116}d(6x-4)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{103\arctan\left(\frac{6x-4}{2\sqrt{29}}\right)}{3\sqrt{29}}-\frac{5}{3}\int \frac{2-3x}{3x^2-4x+11}dx$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{103\arctan\left(\frac{6x-4}{2\sqrt{29}}\right)}{3\sqrt{29}}+\frac{5}{6}\log\left(3x^2-4x+11\right)$$

input
```
Int[(31 + 5*x)/(11 - 4*x + 3*x^2),x]
```

output
```
(103*ArcTan[(-4 + 6*x)/(2*Sqrt[29])])/(3*Sqrt[29]) + (5*Log[11 - 4*x + 3*x
^2])/6
```

3.472.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.472.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{5\ln\left(3x^{2}-4x+11\right)}{6}+\frac{103\sqrt{29}\,\arctan\left(\frac{(6x-4)\sqrt{29}}{58}\right)}{87}$	31
risch	$\frac{5\ln\left(9x^{2}-12x+33\right)}{6}+\frac{103\sqrt{29}\,\arctan\left(\frac{(3x-2)\sqrt{29}}{29}\right)}{87}$	31

input
```
int((31+5*x)/(3*x^2-4*x+11),x,method=_RETURNVERBOSE)
```

output
```
5/6*ln(3*x^2-4*x+11)+103/87*29^(1/2)*arctan(1/58*(6*x-4)*29^(1/2))
```

3.472.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{31+5x}{11-4x+3x^2}\,dx = \frac{103}{87}\sqrt{29}\arctan\left(\frac{1}{29}\sqrt{29}(3\,x-2)\right)+\frac{5}{6}\,\log\left(3\,x^2-4\,x+11\right)$$

input
```
integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="fricas")
```

output
```
103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11
)
```

3.472.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{31+5x}{11-4x+3x^2}\,dx = \frac{5\log\left(x^2-\frac{4x}{3}+\frac{11}{3}\right)}{6}+\frac{103\sqrt{29}\,\mathrm{atan}\left(\frac{3\sqrt{29}x}{29}-\frac{2\sqrt{29}}{29}\right)}{87}$$

input
```
integrate((31+5*x)/(3*x**2-4*x+11),x)
```

output
```
5*log(x**2 - 4*x/3 + 11/3)/6 + 103*sqrt(29)*atan(3*sqrt(29)*x/29 - 2*sqrt(
29)/29)/87
```

3.472.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{31+5x}{11-4x+3x^2}\,dx = \frac{103}{87}\sqrt{29}\arctan\left(\frac{1}{29}\sqrt{29}(3\,x-2)\right)+\frac{5}{6}\,\log\left(3\,x^2-4\,x+11\right)$$

input
```
integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="maxima")
```

output
```
103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11
)
```

3.472.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{31+5x}{11-4x+3x^2}\,dx = \frac{103}{87}\sqrt{29}\arctan\left(\frac{1}{29}\sqrt{29}(3\,x-2)\right)+\frac{5}{6}\log\left(3\,x^2-4\,x+11\right)$$

input
```
integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="giac")
```

output
```
103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11
)
```

3.472.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{31+5x}{11-4x+3x^2}\,dx = \frac{5\,\ln\left(x^2-\frac{4\,x}{3}+\frac{11}{3}\right)}{6}+\frac{103\,\sqrt{29}\,\mathrm{atan}\left(\frac{3\,\sqrt{29}\,x}{29}-\frac{2\,\sqrt{29}}{29}\right)}{87}$$

input
```
int((5*x + 31)/(3*x^2 - 4*x + 11),x)
```

output
```
(5*log(x^2 - (4*x)/3 + 11/3))/6 + (103*29^(1/2)*atan((3*29^(1/2)*x)/29 - (
2*29^(1/2))/29))/87
```

3.473 $\int \frac{-2+x^2+x^3}{x^4} dx$

3.473.1 Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \frac{-2+x^2+x^3}{x^4} dx = \frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

output
```
2/3/x^3-1/x+ln(x)
```

3.473.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{-2+x^2+x^3}{x^4} dx = \frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

input
```
Integrate[(-2 + x^2 + x^3)/x^4,x]
```

output
```
2/(3*x^3) - x^(-1) + Log[x]
```

3.473.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2-2}{x^4} \, dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(-\frac{2}{x^4}+\frac{1}{x^2}+\frac{1}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3x^3}-\frac{1}{x}+\log(x)$$

input
```
Int[(-2 + x^2 + x^3)/x^4,x]
```

output
```
2/(3*x^3) - x^(-1) + Log[x]
```

3.473.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

3.473.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{2}{3x^3}-\frac{1}{x}+\ln\left(x\right)$	14
norman	$\frac{\frac{2}{3}-x^2}{x^3}+\ln\left(x\right)$	15
risch	$\frac{\frac{2}{3}-x^2}{x^3}+\ln\left(x\right)$	15
parallelrisch	$\frac{3\ln(x)x^3+2-3x^2}{3x^3}$	20

input
```
int((x^3+x^2-2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2/3/x^3-1/x+ln(x)
```

3.473.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{-2+x^2+x^3}{x^4}\,dx = \frac{3\,x^3\log\left(x\right)-3\,x^2+2}{3\,x^3}$$

input
```
integrate((x^3+x^2-2)/x^4,x, algorithm="fricas")
```

output
```
1/3*(3*x^3*log(x) - 3*x^2 + 2)/x^3
```

3.473.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{-2+x^2+x^3}{x^4}\,dx = \log\left(x\right)+\frac{2-3x^2}{3x^3}$$

input
```
integrate((x**3+x**2-2)/x**4,x)
```

output
```
log(x) + (2 - 3*x**2)/(3*x**3)
```

3.473.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{-2+x^2+x^3}{x^4}\,dx = -\frac{3\,x^2-2}{3\,x^3}+\log\left(x\right)$$

input
```
integrate((x^3+x^2-2)/x^4,x, algorithm="maxima")
```

output
```
-1/3*(3*x^2 - 2)/x^3 + log(x)
```

3.473.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{-2+x^2+x^3}{x^4}\,dx = -\frac{3\,x^2-2}{3\,x^3}+\log\left(|x|\right)$$

input
```
integrate((x^3+x^2-2)/x^4,x, algorithm="giac")
```

output
```
-1/3*(3*x^2 - 2)/x^3 + log(abs(x))
```

3.473.9 Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-2+x^2+x^3}{x^4}\,dx = \ln\left(x\right)-\frac{x^2-\frac{2}{3}}{x^3}$$

input
```
int((x^2 + x^3 - 2)/x^4,x)
```

output
```
log(x) - (x^2 - 2/3)/x^3
```

3.474 $\int \frac{1+x+x^3}{x^2} dx$

3.474.1 Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \frac{1+x+x^3}{x^2} dx = -\frac{1}{x} + \frac{x^2}{2} + \log(x)$$

output
```
-1/x+1/2*x^2+ln(x)
```

3.474.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^3}{x^2} dx = -\frac{1}{x} + \frac{x^2}{2} + \log(x)$$

input
```
Integrate[(1 + x + x^3)/x^2,x]
```

output
```
-x^(-1) + x^2/2 + Log[x]
```

3.474.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x+1}{x^2} \, dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{1}{x^2}+x+\frac{1}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}-\frac{1}{x}+\log(x)$$

input
```
Int[(1 + x + x^3)/x^2,x]
```

output
```
-x^(-1) + x^2/2 + Log[x]
```

3.474.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

3.474.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{x}+\frac{x^{2}}{2}+\ln \left(x \right)$	14
risch	$-\frac{1}{x}+\frac{x^{2}}{2}+\ln \left(x \right)$	14
norman	$\frac{-1+\frac{x^{3}}{2}}{x}+\ln \left(x \right)$	15
parallelrisch	$\frac{x^{3}+2 \ln \left(x \right) x -2}{2 x}$	16

input
```
int((x^3+x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x+1/2*x^2+ln(x)
```

3.474.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^3}{x^2}\,dx=\frac{x^{3}+2\,x\log\left(x\right)-2}{2\,x}$$

input
```
integrate((x^3+x+1)/x^2,x, algorithm="fricas")
```

output
```
1/2*(x^3 + 2*x*log(x) - 2)/x
```

3.474.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1+x+x^3}{x^2}\,dx=\frac{x^{2}}{2}+\log\left(x\right)-\frac{1}{x}$$

input
```
integrate((x**3+x+1)/x**2,x)
```

output
```
x**2/2 + log(x) - 1/x
```

3.474.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1+x+x^3}{x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{x} + \log\left(x\right)$$

input
```
integrate((x^3+x+1)/x^2,x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/x + log(x)
```

3.474.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1+x+x^3}{x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{x} + \log\left(|x|\right)$$

input
```
integrate((x^3+x+1)/x^2,x, algorithm="giac")
```

output
```
1/2*x^2 - 1/x + log(abs(x))
```

3.474.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1+x+x^3}{x^2}\,dx = \ln\left(x\right) - \frac{1}{x} + \frac{x^2}{2}$$

input
```
int((x + x^3 + 1)/x^2,x)
```

output
```
log(x) - 1/x + x^2/2
```

3.475 $\int \frac{-2+x^2}{x(2+x^2)} \, dx$

3.475.1 Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{-2+x^2}{x(2+x^2)} \, dx = -\log(x) + \log\left(2+x^2\right)$$

output
```
-ln(x)+ln(x^2+2)
```

3.475.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-2+x^2}{x(2+x^2)} \, dx = -\log(x) + \log\left(2+x^2\right)$$

input
```
Integrate[(-2 + x^2)/(x*(2 + x^2)),x]
```

output
```
-Log[x] + Log[2 + x^2]
```

3.475.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {354, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-2}{x(x^2+2)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int -\frac{2-x^2}{x^2(x^2+2)}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{2-x^2}{x^2(x^2+2)}dx^2$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{2}\int \left(\frac{1}{x^2}-\frac{2}{x^2+2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(2\log\left(x^2+2\right)-\log\left(x^2\right)\right)$$

input
```
Int[(-2 + x^2)/(x*(2 + x^2)),x]
```

output
```
(-Log[x^2] + 2*Log[2 + x^2])/2
```

3.475.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.475.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$-\ln(x)+\ln(x^2+2)$	12
norman	$-\ln(x)+\ln(x^2+2)$	12
risch	$-\ln(x)+\ln(x^2+2)$	12
parallelrisch	$-\ln(x)+\ln(x^2+2)$	12
meijerg	$\ln\left(1+\frac{x^2}{2}\right)-\ln(x)+\frac{\ln(2)}{2}$	18

input
```
int((x^2-2)/x/(x^2+2),x,method=_RETURNVERBOSE)
```

output
```
-ln(x)+ln(x^2+2)
```

3.475.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-2+x^2}{x\,(2+x^2)}\,dx = \log\left(x^2+2\right) - \log\left(x\right)$$

input
```
integrate((x^2-2)/x/(x^2+2),x, algorithm="fricas")
```

output
```
log(x^2 + 2) - log(x)
```

3.475.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{-2+x^2}{x\,(2+x^2)}\,dx = -\log\left(x\right) + \log\left(x^2+2\right)$$

input
```
integrate((x**2-2)/x/(x**2+2),x)
```

output
```
-log(x) + log(x**2 + 2)
```

3.475.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{-2+x^2}{x\,(2+x^2)}\,dx = \log\left(x^2+2\right) - \frac{1}{2}\,\log\left(x^2\right)$$

input
```
integrate((x^2-2)/x/(x^2+2),x, algorithm="maxima")
```

output
```
log(x^2 + 2) - 1/2*log(x^2)
```

3.475.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{-2+x^2}{x\left(2+x^2\right)}\,dx=\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2\right)$$

input
```
integrate((x^2-2)/x/(x^2+2),x, algorithm="giac")
```

output
```
log(x^2 + 2) - 1/2*log(x^2)
```

3.475.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-2+x^2}{x\left(2+x^2\right)}\,dx=\ln\left(x^2+2\right)-\ln\left(x\right)$$

input
```
int((x^2 - 2)/(x*(x^2 + 2)),x)
```

output
```
log(x^2 + 2) - log(x)
```

3.476 $\int (-3+x)\left(-7+4x^2\right)\,dx$

3.476.1 Optimal result

Integrand size = 11, antiderivative size = 22

$$\int (-3+x)\left(-7+4x^2\right)\,dx=21x-4x^3+\frac{1}{16}\left(7-4x^2\right)^2$$

output
```
21*x-4*x^3+1/16*(-4*x^2+7)^2
```

3.476.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int (-3+x)\left(-7+4x^2\right)\,dx=21x-\frac{7x^2}{2}-4x^3+x^4$$

input
```
Integrate[(-3 + x)*(-7 + 4*x^2),x]
```

output
```
21*x - (7*x^2)/2 - 4*x^3 + x^4
```

3.476.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {455, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x-3)\left(4x^2-7\right)\,dx$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{16}\left(7-4x^2\right)^2-3\int \left(4x^2-7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\left(7-4x^2\right)^2-3\left(\frac{4x^3}{3}-7x\right)$$

input
```
Int[(-3 + x)*(-7 + 4*x^2),x]
```

output
```
(7 - 4*x^2)^2/16 - 3*(-7*x + (4*x^3)/3)
```

3.476.3.1 Defintions of rubi rules used

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.476.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

method	result	size
gosper	$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$	18
default	$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$	18
norman	$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$	18
risch	$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$	18
parallelrisch	$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$	18

input
```
int((-3+x)*(4*x^2-7),x,method=_RETURNVERBOSE)
```

output
```
x^4-4*x^3-7/2*x^2+21*x
```

3.476.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int (-3+x)\left(-7+4x^2\right)\,dx = x^4 - 4\,x^3 - \frac{7}{2}\,x^2 + 21\,x$$

input
```
integrate((-3+x)*(4*x^2-7),x, algorithm="fricas")
```

output
```
x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

3.476.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int (-3+x)\left(-7+4x^2\right)\,dx = x^4 - 4x^3 - \frac{7x^2}{2} + 21x$$

input
```
integrate((-3+x)*(4*x**2-7),x)
```

output
```
x**4 - 4*x**3 - 7*x**2/2 + 21*x
```

3.476.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int (-3+x)\left(-7+4x^2\right)\,dx = x^4-4\,x^3-\frac{7}{2}\,x^2+21\,x$$

input
```
integrate((-3+x)*(4*x^2-7),x, algorithm="maxima")
```

output
```
x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

3.476.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int (-3+x)\left(-7+4x^2\right)\,dx = x^4-4\,x^3-\frac{7}{2}\,x^2+21\,x$$

input
```
integrate((-3+x)*(4*x^2-7),x, algorithm="giac")
```

output
```
x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

3.476.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int (-3+x)\left(-7+4x^2\right)\,dx = x^4-4\,x^3-\frac{7\,x^2}{2}+21\,x$$

input
```
int((4*x^2 - 7)*(x - 3),x)
```

output
```
21*x - (7*x^2)/2 - 4*x^3 + x^4
```

3.477 $\int (-2+7x)^3 \, dx$

3.477.1 Optimal result

Integrand size = 7, antiderivative size = 11

$$\int (-2+7x)^3 \, dx = \frac{1}{28}(2-7x)^4$$

output
```
1/28*(2-7*x)^4
```

3.477.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (-2+7x)^3 \, dx = \frac{1}{28}(-2+7x)^4$$

input
```
Integrate[(-2 + 7*x)^3,x]
```

output
```
(-2 + 7*x)^4/28
```

3.477.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (7x-2)^3 \, dx$$

$$\Big\downarrow \; 17$$

$$\frac{1}{28}(2-7x)^4$$

input
```
Int[(-2 + 7*x)^3,x]
```

output
```
(2 - 7*x)^4/28
```

3.477.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

3.477.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{(-2+7x)^4}{28}$	10
gosper	$\frac{343}{4}x^4-98x^3+42x^2-8x$	20
norman	$\frac{343}{4}x^4-98x^3+42x^2-8x$	20
parallelrisch	$\frac{343}{4}x^4-98x^3+42x^2-8x$	20
risch	$\frac{343}{4}x^4-98x^3+42x^2-8x+\frac{4}{7}$	21

input
```
int((-2+7*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/28*(-2+7*x)^4
```

3.477.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-2+7x)^3\,dx = \frac{343}{4}\,x^4 - 98\,x^3 + 42\,x^2 - 8\,x$$

input
```
integrate((-2+7*x)^3,x, algorithm="fricas")
```

output
```
343/4*x^4 - 98*x^3 + 42*x^2 - 8*x
```

3.477.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-2+7x)^3\,dx = \frac{343x^4}{4} - 98x^3 + 42x^2 - 8x$$

input
```
integrate((-2+7*x)**3,x)
```

output
```
343*x**4/4 - 98*x**3 + 42*x**2 - 8*x
```

3.477.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-2+7x)^3\,dx = \frac{343}{4}\,x^4 - 98\,x^3 + 42\,x^2 - 8\,x$$

input
```
integrate((-2+7*x)^3,x, algorithm="maxima")
```

output
```
343/4*x^4 - 98*x^3 + 42*x^2 - 8*x
```

3.477.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2+7x)^3 \, dx = \frac{1}{28} (7x-2)^4$$

input
```
integrate((-2+7*x)^3,x, algorithm="giac")
```

output
```
1/28*(7*x - 2)^4
```

3.477.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-2+7x)^3 \, dx = \frac{(7x-2)^4}{28}$$

input
```
int((7*x - 2)^3,x)
```

output
```
(7*x - 2)^4/28
```

3.478 $\int \frac{-7+4x^2}{3+2x} \, dx$

3.478.1 Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{-7+4x^2}{3+2x} \, dx = -3x + x^2 + \log(3+2x)$$

output
```
-3*x+x^2+ln(3+2*x)
```

3.478.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{-7+4x^2}{3+2x} \, dx = -\frac{27}{4} - 3x + x^2 + \log(3+2x)$$

input
```
Integrate[(-7 + 4*x^2)/(3 + 2*x),x]
```

output
```
-27/4 - 3*x + x^2 + Log[3 + 2*x]
```

3.478.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2-7}{2x+3}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(2x+\frac{2}{2x+3}-3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x^2-3x+\log(2x+3)$$

input
```
Int[(-7 + 4*x^2)/(3 + 2*x),x]
```

output
```
-3*x + x^2 + Log[3 + 2*x]
```

3.478.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.478.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$x^2 - 3x + \ln\left(x + \frac{3}{2}\right)$	12
default	$-3x + x^2 + \ln(2x + 3)$	14
norman	$-3x + x^2 + \ln(2x + 3)$	14
risch	$-3x + x^2 + \ln(2x + 3)$	14
meijerg	$\ln\left(1 + \frac{2x}{3}\right) - \frac{x(-2x+6)}{2}$	16

input
```
int((4*x^2-7)/(2*x+3),x,method=_RETURNVERBOSE)
```

output
```
x^2-3*x+ln(x+3/2)
```

3.478.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-7 + 4x^2}{3 + 2x}\, dx = x^2 - 3\, x + \log(2\, x + 3)$$

input
```
integrate((4*x^2-7)/(3+2*x),x, algorithm="fricas")
```

output
```
x^2 - 3*x + log(2*x + 3)
```

3.478.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{-7 + 4x^2}{3 + 2x}\, dx = x^2 - 3x + \log(2x + 3)$$

input
```
integrate((4*x**2-7)/(3+2*x),x)
```

output
```
x**2 - 3*x + log(2*x + 3)
```

3.478.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-7+4x^2}{3+2x}\,dx = x^2 - 3\,x + \log\left(2\,x+3\right)$$

input
```
integrate((4*x^2-7)/(3+2*x),x, algorithm="maxima")
```

output
```
x^2 - 3*x + log(2*x + 3)
```

3.478.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{-7+4x^2}{3+2x}\,dx = x^2 - 3\,x + \log\left(|2\,x+3|\right)$$

input
```
integrate((4*x^2-7)/(3+2*x),x, algorithm="giac")
```

output
```
x^2 - 3*x + log(abs(2*x + 3))
```

3.478.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{-7+4x^2}{3+2x}\,dx = \ln\left(x+\frac{3}{2}\right) - 3\,x + x^2$$

input
```
int((4*x^2 - 7)/(2*x + 3),x)
```

output
```
log(x + 3/2) - 3*x + x^2
```

3.479 $\int \frac{1+x}{(-1+x)x^2} dx$

3.479.1 Optimal result

Integrand size = 12, antiderivative size = 16

$$\int \frac{1+x}{(-1+x)x^2} dx = \frac{1}{x} + 2\log(1-x) - 2\log(x)$$

output
```
1/x+2*ln(1-x)-2*ln(x)
```

3.479.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{(-1+x)x^2} dx = \frac{1}{x} + 2\log(1-x) - 2\log(x)$$

input
```
Integrate[(1 + x)/((-1 + x)*x^2),x]
```

output
```
x^(-1) + 2*Log[1 - x] - 2*Log[x]
```

3.479.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{(x-1)x^2}\,dx$$
$$\Big\downarrow \; 86$$
$$\int \left(-\frac{1}{x^2}-\frac{2}{x}+\frac{2}{x-1}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{x}+2\log(1-x)-2\log(x)$$

input
```
Int[(1 + x)/((-1 + x)*x^2),x]
```

output
```
x^(-1) + 2*Log[1 - x] - 2*Log[x]
```

3.479.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.479.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{1}{x}-2\ln\left(x\right)+2\ln\left(x-1\right)$	15
norman	$\frac{1}{x}-2\ln\left(x\right)+2\ln\left(x-1\right)$	15
risch	$\frac{1}{x}-2\ln\left(x\right)+2\ln\left(x-1\right)$	15
parallelrisch	$-\frac{2\ln(x)x-2\ln(x-1)x-1}{x}$	20
meijerg	$2\ln\left(1-x\right)-2\ln\left(x\right)-2i\pi+\frac{1}{x}$	21

input
```
int((x+1)/(x-1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x-2*ln(x)+2*ln(x-1)
```

3.479.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int\frac{1+x}{(-1+x)x^2}\,dx=\frac{2\,x\log\left(x-1\right)-2\,x\log\left(x\right)+1}{x}$$

input
```
integrate((1+x)/(-1+x)/x^2,x, algorithm="fricas")
```

output
```
(2*x*log(x - 1) - 2*x*log(x) + 1)/x
```

3.479.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int\frac{1+x}{(-1+x)x^2}\,dx=-2\log\left(x\right)+2\log\left(x-1\right)+\frac{1}{x}$$

input
```
integrate((1+x)/(-1+x)/x**2,x)
```

output
```
-2*log(x) + 2*log(x - 1) + 1/x
```

3.479.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x}{(-1+x)x^2}\,dx = \frac{1}{x} + 2\,\log\left(x-1\right) - 2\,\log\left(x\right)$$

input
```
integrate((1+x)/(-1+x)/x^2,x, algorithm="maxima")
```

output
```
1/x + 2*log(x - 1) - 2*log(x)
```

3.479.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{(-1+x)x^2}\,dx = \frac{1}{x} + 2\,\log\left(|x-1|\right) - 2\,\log\left(|x|\right)$$

input
```
integrate((1+x)/(-1+x)/x^2,x, algorithm="giac")
```

output
```
1/x + 2*log(abs(x - 1)) - 2*log(abs(x))
```

3.479.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1+x}{(-1+x)x^2}\,dx = \frac{1}{x} - 4\,\mathrm{atanh}(2\,x-1)$$

input
```
int((x + 1)/(x^2*(x - 1)),x)
```

output
```
1/x - 4*atanh(2*x - 1)
```

3.480 $\int \frac{1}{4x^2+4x^3+x^4}\,dx$

3.480.1 Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx=\frac{1+x}{2\left(1-(1+x)^2\right)}+\frac{1}{2}\text{arctanh}(1+x)$$

output
```
1/2*(1+x)/(1-(1+x)^2)+1/2*arctanh(1+x)
```

3.480.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx=\frac{1}{4}\left(-\frac{2(1+x)}{x(2+x)}-\log(x)+\log(2+x)\right)$$

input
```
Integrate[(4*x^2 + 4*x^3 + x^4)^(-1),x]
```

output
```
((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4
```

3.480.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1949, 1098, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4+4x^3+4x^2}dx$$
$$\Big\downarrow \text{1949}$$
$$\int \frac{1}{x^2(x^2+4x+4)}dx$$
$$\Big\downarrow \text{1098}$$
$$\int \frac{1}{x^2(x+2)^2}dx$$
$$\Big\downarrow \text{54}$$
$$\int \left(\frac{1}{4x^2}+\frac{1}{4(x+2)}+\frac{1}{4(x+2)^2}-\frac{1}{4x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{4x}-\frac{1}{4(x+2)}-\frac{\log(x)}{4}+\frac{1}{4}\log(x+2)$$

input
```
Int[(4*x^2 + 4*x^3 + x^4)^(-1),x]
```

output
```
-1/4*1/x - 1/(4*(2 + x)) - Log[x]/4 + Log[2 + x]/4
```

3.480.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1949
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b,
c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.480.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{1}{4x}-\frac{\ln(x)}{4}-\frac{1}{4(x+2)}+\frac{\ln(x+2)}{4}$	24
norman	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(x+2)}-\frac{\ln(x)}{4}+\frac{\ln(x+2)}{4}$	26
risch	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(x+2)}-\frac{\ln(x)}{4}+\frac{\ln(x+2)}{4}$	26
parallelrisch	$-\frac{\ln(x)x^2-\ln(x+2)x^2+2+2\ln(x)x-2\ln(x+2)x+2x}{4x(x+2)}$	43

input
```
int(1/(x^4+4*x^3+4*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x-1/4*ln(x)-1/4/(x+2)+1/4*ln(x+2)
```

3.480.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx=\frac{(x^2+2\,x)\log(x+2)-(x^2+2\,x)\log(x)-2\,x-2}{4\,(x^2+2\,x)}$$

input
```
integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="fricas")
```

output
```
1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)
```

3.480.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx = \frac{-x-1}{2x^2+4x} - \frac{\log(x)}{4} + \frac{\log(x+2)}{4}$$

input
```
integrate(1/(x**4+4*x**3+4*x**2),x)
```

output
```
(-x - 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

3.480.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx = -\frac{x+1}{2\,(x^2+2\,x)} + \frac{1}{4}\,\log(x+2) - \frac{1}{4}\,\log(x)$$

input
```
integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="maxima")
```

output
```
-1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)
```

3.480.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx = -\frac{x+1}{2\,(x^2+2\,x)} + \frac{1}{4}\,\log(|x+2|) - \frac{1}{4}\,\log(|x|)$$

input
```
integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="giac")
```

output
```
-1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))
```

3.480.9 Mupad [B] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx=\frac{\operatorname{atanh}(x+1)}{2}-\frac{\frac{x}{2}+\frac{1}{2}}{x^2+2\,x}$$

input
```
int(1/(4*x^2 + 4*x^3 + x^4),x)
```

output
```
atanh(x + 1)/2 - (x/2 + 1/2)/(2*x + x^2)
```

3.481 $\int \frac{1+x^2}{1+x} \, dx$

3.481.1 Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1+x^2}{1+x} \, dx = -x + \frac{x^2}{2} + 2\log(1+x)$$

output
```
-x+1/2*x^2+2*ln(1+x)
```

3.481.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1+x^2}{1+x} \, dx = \frac{1}{2}\left(-3-2x+x^2+4\log(1+x)\right)$$

input
```
Integrate[(1 + x^2)/(1 + x),x]
```

output
```
(-3 - 2*x + x^2 + 4*Log[1 + x])/2
```

3.481.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x+1} \, dx$$
$$\Big\downarrow \text{476}$$
$$\int \left(x+\frac{2}{x+1}-1\right) dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{x^2}{2}-x+2\log(x+1)$$

input
```
Int[(1 + x^2)/(1 + x),x]
```

output
```
-x + x^2/2 + 2*Log[1 + x]
```

3.481.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.481.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$-x+\frac{x^2}{2}+2\ln(x+1)$	16
norman	$-x+\frac{x^2}{2}+2\ln(x+1)$	16
meijerg	$-\frac{x(6-3x)}{6}+2\ln(x+1)$	16
risch	$-x+\frac{x^2}{2}+2\ln(x+1)$	16
parallelrisch	$-x+\frac{x^2}{2}+2\ln(x+1)$	16

input
```
int((x^2+1)/(x+1),x,method=_RETURNVERBOSE)
```

output
```
-x+1/2*x^2+2*ln(x+1)
```

3.481.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{1+x}\,dx = \frac{1}{2}\,x^2 - x + 2\,\log(x+1)$$

input
```
integrate((x^2+1)/(1+x),x, algorithm="fricas")
```

output
```
1/2*x^2 - x + 2*log(x + 1)
```

3.481.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{1+x^2}{1+x}\,dx = \frac{x^2}{2} - x + 2\log(x+1)$$

input
```
integrate((x**2+1)/(1+x),x)
```

output
```
x**2/2 - x + 2*log(x + 1)
```

3.481.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{1+x}\,dx = \frac{1}{2}\,x^2 - x + 2\,\log\left(x+1\right)$$

input
```
integrate((x^2+1)/(1+x),x, algorithm="maxima")
```

output
```
1/2*x^2 - x + 2*log(x + 1)
```

3.481.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1+x^2}{1+x}\,dx = \frac{1}{2}\,x^2 - x + 2\,\log\left(|x+1|\right)$$

input
```
integrate((x^2+1)/(1+x),x, algorithm="giac")
```

output
```
1/2*x^2 - x + 2*log(abs(x + 1))
```

3.481.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^2}{1+x}\,dx = 2\,\ln\left(x+1\right) - x + \frac{x^2}{2}$$

input
```
int((x^2 + 1)/(x + 1),x)
```

output
```
2*log(x + 1) - x + x^2/2
```

3.482 $\int \frac{-1+3x-3x^2+x^3}{x^2} \, dx$

3.482.1 Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \frac{-1+3x-3x^2+x^3}{x^2} \, dx = \frac{1}{x} - 3x + \frac{x^2}{2} + 3\log(x)$$

output
```
1/x-3*x+1/2*x^2+3*ln(x)
```

3.482.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-1+3x-3x^2+x^3}{x^2} \, dx = \frac{1}{x} - 3x + \frac{x^2}{2} + 3\log(x)$$

input
```
Integrate[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]
```

output
```
x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

3.482.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2006, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-3x^2+3x-1}{x^2} \, dx$$

$$\Big\downarrow \text{ 2006}$$

$$\int \frac{(x-1)^3}{x^2}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(-\frac{1}{x^2}+x+\frac{3}{x}-3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^2}{2}-3x+\frac{1}{x}+3\log(x)$$

input
```
Int[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]
```

output
```
x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

3.482.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.482.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{1}{x}-3x+\frac{x^{2}}{2}+3\ln\left(x\right)$	17
risch	$\frac{1}{x}-3x+\frac{x^{2}}{2}+3\ln\left(x\right)$	17
parallelrisch	$\frac{x^{3}+6\ln\left(x\right)x-6x^{2}+2}{2x}$	21
norman	$\frac{1-3x^{2}+\frac{1}{2}x^{3}}{x}+3\ln\left(x\right)$	22

input
```
int((x^3-3*x^2+3*x-1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x-3*x+1/2*x^2+3*ln(x)
```

3.482.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx=\frac{x^{3}-6\,x^{2}+6\,x\log\left(x\right)+2}{2\,x}$$

input
```
integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="fricas")
```

output
```
1/2*(x^3 - 6*x^2 + 6*x*log(x) + 2)/x
```

3.482.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx = \frac{x^2}{2} - 3x + 3\log{(x)} + \frac{1}{x}$$

input
```
integrate((x**3-3*x**2+3*x-1)/x**2,x)
```

output
```
x**2/2 - 3*x + 3*log(x) + 1/x
```

3.482.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx = \frac{1}{2}\,x^2 - 3\,x + \frac{1}{x} + 3\,\log{(x)}$$

input
```
integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="maxima")
```

output
```
1/2*x^2 - 3*x + 1/x + 3*log(x)
```

3.482.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx = \frac{1}{2}\,x^2 - 3\,x + \frac{1}{x} + 3\,\log{(|x|)}$$

input
```
integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="giac")
```

output
```
1/2*x^2 - 3*x + 1/x + 3*log(abs(x))
```

3.482.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx = 3\ \ln{(x)} - 3\,x + \frac{1}{x} + \frac{x^2}{2}$$

input
```
int((3*x - 3*x^2 + x^3 - 1)/x^2,x)
```

output
```
3*log(x) - 3*x + 1/x + x^2/2
```

3.483 $\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx$

3.483.1 Optimal result

Integrand size = 29, antiderivative size = 18

$$\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx=-7x+\frac{3x^2}{2}+\frac{x^3}{3}$$

output
```
-7*x+3/2*x^2+1/3*x^3
```

3.483.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx=-7x+\frac{3x^2}{2}+\frac{x^3}{3}$$

input
```
Integrate[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x),x]
```

output
```
-7*x + (3*x^2)/2 + x^3/3
```

3.483.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x+\frac{1}{2}\left(3-\sqrt{37}\right)\right)\left(x+\frac{1}{2}\left(3+\sqrt{37}\right)\right)\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(x^2+3x-7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3}{3}+\frac{3x^2}{2}-7x$$

input
```
Int[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x),x]
```

output
```
-7*x + (3*x^2)/2 + x^3/3
```

3.483.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.483.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
norman	$-7x+\frac{3}{2}x^{2}+\frac{1}{3}x^{3}$	15
risch	$-7x+\frac{3}{2}x^{2}+\frac{1}{3}x^{3}$	15
parallelrisch	$-7x+\frac{3}{2}x^{2}+\frac{1}{3}x^{3}$	15
default	$\frac{x^{3}}{3}+\frac{3x^{2}}{2}+\frac{\left(3-\sqrt{37}\right)\left(3+\sqrt{37}\right)x}{4}$	27
gosper	$-\frac{x(2x^{2}+9x-42)\left(-2x-3+\sqrt{37}\right)\left(2x+3+\sqrt{37}\right)}{24(x^{2}+3x-7)}$	40

input
```
int((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-7*x+3/2*x^2+1/3*x^3
```

3.483.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int\left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx=\frac{1}{3}\,x^{3}+\frac{3}{2}\,x^{2}-7\,x$$

input
```
integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="fricas")
```

output
```
1/3*x^3 + 3/2*x^2 - 7*x
```

3.483.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int\left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx=\frac{x^{3}}{3}+\frac{3x^{2}}{2}-7x$$

input
```
integrate((x+3/2-1/2*37**(1/2))*(x+3/2+1/2*37**(1/2)),x)
```

output
```
x**3/3 + 3*x**2/2 - 7*x
```

3.483.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx=\frac{1}{3}\,x^3+\frac{3}{2}\,x^2-7\,x$$

input
```
integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="maxima")
```

output
```
1/3*x^3 + 3/2*x^2 - 7*x
```

3.483.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx=\frac{1}{3}\,x^3+\frac{3}{2}\,x^2-7\,x$$

input
```
integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="giac")
```

output
```
1/3*x^3 + 3/2*x^2 - 7*x
```

3.483.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx=\frac{x\,(2\,x^2+9\,x-42)}{6}$$

input
```
int((x - 37^(1/2)/2 + 3/2)*(x + 37^(1/2)/2 + 3/2),x)
```

output
```
(x*(9*x + 2*x^2 - 42))/6
```

3.484 $\int \frac{4+3x^2+2x^3}{(1+x)^4} \, dx$

3.484.1 Optimal result

Integrand size = 18, antiderivative size = 23

$$\int \frac{4+3x^2+2x^3}{(1+x)^4} \, dx = -\frac{5}{3(1+x)^3}+\frac{3}{1+x}+2\log(1+x)$$

output
```
-5/3/(1+x)^3+3/(1+x)+2*ln(1+x)
```

3.484.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{4+3x^2+2x^3}{(1+x)^4} \, dx = -\frac{5}{3(1+x)^3}+\frac{3}{1+x}+2\log(1+x)$$

input
```
Integrate[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]
```

output
```
-5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]
```

3.484.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+4}{(x+1)^4}\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{2}{x+1}-\frac{3}{(x+1)^2}+\frac{5}{(x+1)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{x+1}-\frac{5}{3(x+1)^3}+2\log(x+1)$$

input
```
Int[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]
```

output
```
-5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]
```

3.484.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.484.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{5}{3(x+1)^{3}}+\frac{3}{x+1}+2\ln\left(x+1\right)$	22
norman	$\frac{3x^{2}+6x+\frac{4}{3}}{(x+1)^{3}}+2\ln\left(x+1\right)$	24
risch	$\frac{3x^{2}+6x+\frac{4}{3}}{(x+1)^{3}}+2\ln\left(x+1\right)$	24
parallelrisch	$\frac{6\ln(x+1)x^{3}+4+18\ln(x+1)x^{2}+18\ln(x+1)x+9x^{2}+6\ln(x+1)+18x}{3(x+1)^{3}}$	49
meijerg	$\frac{4x(x^{2}+3x+3)}{3(x+1)^{3}}-\frac{x(22x^{2}+30x+12)}{6(x+1)^{3}}+2\ln\left(x+1\right)+\frac{x^{3}}{(x+1)^{3}}$	51

input
```
int((2*x^3+3*x^2+4)/(x+1)^4,x,method=_RETURNVERBOSE)
```

output
```
-5/3/(x+1)^3+3/(x+1)+2*ln(x+1)
```

3.484.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(21) = 42.

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \frac{4+3x^2+2x^3}{(1+x)^4}\,dx=\frac{9\,x^{2}+6\,\left(x^{3}+3\,x^{2}+3\,x+1\right)\log\left(x+1\right)+18\,x+4}{3\,\left(x^{3}+3\,x^{2}+3\,x+1\right)}$$

input
```
integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="fricas")
```

output
```
1/3*(9*x^2 + 6*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 4)/(x^3 + 3*x^2
 + 3*x + 1)
```

3.484.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{4+3x^2+2x^3}{(1+x)^4} dx = \frac{9x^2+18x+4}{3x^3+9x^2+9x+3} + 2\log(x+1)$$

input
```
integrate((2*x**3+3*x**2+4)/(1+x)**4,x)
```

output
```
(9*x**2 + 18*x + 4)/(3*x**3 + 9*x**2 + 9*x + 3) + 2*log(x + 1)
```

3.484.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{4+3x^2+2x^3}{(1+x)^4} dx = \frac{9x^2+18x+4}{3(x^3+3x^2+3x+1)} + 2\log(x+1)$$

input
```
integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="maxima")
```

output
```
1/3*(9*x^2 + 18*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + 2*log(x + 1)
```

3.484.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{4+3x^2+2x^3}{(1+x)^4} dx = \frac{9x^2+18x+4}{3(x+1)^3} + 2\log(|x+1|)$$

input
```
integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="giac")
```

output
```
1/3*(9*x^2 + 18*x + 4)/(x + 1)^3 + 2*log(abs(x + 1))
```

3.484.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{4+3x^2+2x^3}{(1+x)^4}\,dx = 2\,\ln\left(x+1\right)+\frac{3\,x^2+6\,x+\frac{4}{3}}{{\left(x+1\right)}^3}$$

input
```
int((3*x^2 + 2*x^3 + 4)/(x + 1)^4,x)
```

output
```
2*log(x + 1) + (6*x + 3*x^2 + 4/3)/(x + 1)^3
```

3.485 $\int \frac{x}{(1+x)^2(1+x^2)} \, dx$

3.485.1 Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{x}{(1+x)^2 (1+x^2)} \, dx = \frac{1}{2(1+x)} + \frac{\arctan(x)}{2}$$

output
```
1/2/(1+x)+1/2*arctan(x)
```

3.485.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{(1+x)^2 (1+x^2)} \, dx = \frac{1}{2}\left(\frac{1}{1+x} + \arctan(x)\right)$$

input
```
Integrate[x/((1 + x)^2*(1 + x^2)),x]
```

output
```
((1 + x)^(-1) + ArcTan[x])/2
```

3.485.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {594, 25, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)^2 (x^2+1)} \, dx$$
$$\Big\downarrow \ 594$$
$$\frac{1}{2(x+1)} - \frac{1}{2} \int -\frac{1}{x^2+1} dx$$
$$\Big\downarrow \ 25$$
$$\frac{1}{2} \int \frac{1}{x^2+1} dx + \frac{1}{2(x+1)}$$
$$\Big\downarrow \ 216$$
$$\frac{\arctan(x)}{2} + \frac{1}{2(x+1)}$$

input
```
Int[x/((1 + x)^2*(1 + x^2)),x]
```

output
```
1/(2*(1 + x)) + ArcTan[x]/2
```

3.485.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

3.485.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{1}{2x+2}+\frac{\arctan(x)}{2}$	13
risch	$\frac{1}{2x+2}+\frac{\arctan(x)}{2}$	13
parallelrisch	$-\frac{i\ln(x-i)x-i\ln(x+i)x+i\ln(x-i)-i\ln(x+i)-2}{4(x+1)}$	44

input
```
int(x/(x+1)^2/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2/(x+1)+1/2*arctan(x)
```

3.485.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{(1+x)^2(1+x^2)}\,dx=\frac{(x+1)\arctan(x)+1}{2(x+1)}$$

input
```
integrate(x/(1+x)^2/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*((x + 1)*arctan(x) + 1)/(x + 1)
```

3.485.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x}{(1+x)^2(1+x^2)}\,dx = \frac{\operatorname{atan}(x)}{2} + \frac{1}{2x+2}$$

input
```
integrate(x/(1+x)**2/(x**2+1),x)
```

output
```
atan(x)/2 + 1/(2*x + 2)
```

3.485.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{(1+x)^2(1+x^2)}\,dx = \frac{1}{2(x+1)} + \frac{1}{2}\arctan(x)$$

input
```
integrate(x/(1+x)^2/(x^2+1),x, algorithm="maxima")
```

output
```
1/2/(x + 1) + 1/2*arctan(x)
```

3.485.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{x}{(1+x)^2(1+x^2)}\,dx = -\frac{1}{8}\pi - \frac{1}{2}\pi\left\lfloor -\frac{\pi-4\arctan(x)}{4\pi} + \frac{1}{2}\right\rfloor + \frac{1}{2(x+1)} + \frac{1}{2}\arctan(x)$$

input
```
integrate(x/(1+x)^2/(x^2+1),x, algorithm="giac")
```

output
```
-1/8*pi - 1/2*pi*floor(-1/4*(pi - 4*arctan(x))/pi + 1/2) + 1/2/(x + 1) + 1
/2*arctan(x)
```

3.485.9 Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{(1+x)^2 (1+x^2)} \, dx = \frac{\operatorname{atan}(x)}{2} + \frac{1}{2 (x+1)}$$

input
```
int(x/((x^2 + 1)*(x + 1)^2),x)
```

output
```
atan(x)/2 + 1/(2*(x + 1))
```

3.486 $\int \frac{7-2x+3x^2-x^3+x^4}{2+x} dx$

3.486.1 Optimal result

Integrand size = 24, antiderivative size = 29

$$\int \frac{7-2x+3x^2-x^3+x^4}{2+x} dx = -20x + \frac{9x^2}{2} - x^3 + \frac{x^4}{4} + 47\log(2+x)$$

output
```
-20*x+9/2*x^2-x^3+1/4*x^4+47*ln(2+x)
```

3.486.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{7-2x+3x^2-x^3+x^4}{2+x} dx = -70 - 20x + \frac{9x^2}{2} - x^3 + \frac{x^4}{4} + 47\log(2+x)$$

input
```
Integrate[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x),x]
```

output
```
-70 - 20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]
```

3.486.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-x^3+3x^2-2x+7}{x+2}\,dx$$

$$\Big\downarrow \ 2389$$

$$\int \left(x^3-3x^2+9x+\frac{47}{x+2}-20\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{x^4}{4}-x^3+\frac{9x^2}{2}-20x+47\log(x+2)$$

input
```
Int[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x),x]
```

output
```
-20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]
```

3.486.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.486.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$-20x+\frac{9x^2}{2}-x^3+\frac{x^4}{4}+47\ln\left(x+2\right)$	26
norman	$-20x+\frac{9x^2}{2}-x^3+\frac{x^4}{4}+47\ln\left(x+2\right)$	26
risch	$-20x+\frac{9x^2}{2}-x^3+\frac{x^4}{4}+47\ln\left(x+2\right)$	26
parallelrisch	$-20x+\frac{9x^2}{2}-x^3+\frac{x^4}{4}+47\ln\left(x+2\right)$	26
meijerg	$47\ln\left(1+\frac{x}{2}\right)-\frac{2x\left(-\frac{15}{8}x^3+5x^2-15x+60\right)}{15}-\frac{x\left(x^2-3x+12\right)}{3}-x\left(-\frac{3x}{2}+6\right)-2x$	50

input
```
int((x^4-x^3+3*x^2-2*x+7)/(x+2),x,method=_RETURNVERBOSE)
```

output
```
-20*x+9/2*x^2-x^3+1/4*x^4+47*ln(x+2)
```

3.486.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{7-2x+3x^2-x^3+x^4}{2+x}\,dx=\frac{1}{4}\,x^4-x^3+\frac{9}{2}\,x^2-20\,x+47\,\log\left(x+2\right)$$

input
```
integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="fricas")
```

output
```
1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)
```

3.486.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{7-2x+3x^2-x^3+x^4}{2+x}\,dx=\frac{x^4}{4}-x^3+\frac{9x^2}{2}-20x+47\log\left(x+2\right)$$

input
```
integrate((x**4-x**3+3*x**2-2*x+7)/(2+x),x)
```

output
```
x**4/4 - x**3 + 9*x**2/2 - 20*x + 47*log(x + 2)
```

3.486.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{7-2x+3x^2-x^3+x^4}{2+x}\,dx = \frac{1}{4}\,x^4 - x^3 + \frac{9}{2}\,x^2 - 20\,x + 47\,\log\left(x+2\right)$$

input
```
integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="maxima")
```

output
```
1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)
```

3.486.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{7-2x+3x^2-x^3+x^4}{2+x}\,dx = \frac{1}{4}\,x^4 - x^3 + \frac{9}{2}\,x^2 - 20\,x + 47\,\log\left(|x+2|\right)$$

input
```
integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="giac")
```

output
```
1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(abs(x + 2))
```

3.486.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{7-2x+3x^2-x^3+x^4}{2+x}\,dx = 47\,\ln\left(x+2\right) - 20\,x + \frac{9\,x^2}{2} - x^3 + \frac{x^4}{4}$$

input
```
int((3*x^2 - 2*x - x^3 + x^4 + 7)/(x + 2),x)
```

output
```
47*log(x + 2) - 20*x + (9*x^2)/2 - x^3 + x^4/4
```

3.487 $\int \frac{-1+x^3}{-1+x}\,dx$

3.487.1 Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{-1+x^3}{-1+x}\,dx = x+\frac{x^2}{2}+\frac{x^3}{3}$$

output
```
x+1/2*x^2+1/3*x^3
```

3.487.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^3}{-1+x}\,dx = x+\frac{x^2}{2}+\frac{x^3}{3}$$

input
```
Integrate[(-1 + x^3)/(-1 + x),x]
```

output
```
x + x^2/2 + x^3/3
```

3.487.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2019, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-1}{x-1}\,dx$$

$$\Big\downarrow \text{2019}$$

$$\int \left(x^2+x+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

input
```
Int[(-1 + x^3)/(-1 + x),x]
```

output
```
x + x^2/2 + x^3/3
```

3.487.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

3.487.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$x+\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
norman	$x+\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
risch	$x+\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
parallelrisch	$x+\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
parts	$x+\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
gosper	$\frac{x(2x^{2}+3x+6)}{6}$	14
meijerg	$\frac{x(4x^{2}+6x+12)}{12}$	14

input
```
int((x^3-1)/(x-1),x,method=_RETURNVERBOSE)
```

output
```
x+1/2*x^2+1/3*x^3
```

3.487.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+x^3}{-1+x}\,dx=\frac{1}{3}\,x^{3}+\frac{1}{2}\,x^{2}+x$$

input
```
integrate((x^3-1)/(-1+x),x, algorithm="fricas")
```

output
```
1/3*x^3 + 1/2*x^2 + x
```

3.487.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{-1+x^3}{-1+x}\,dx = \frac{x^3}{3} + \frac{x^2}{2} + x$$

input
```
integrate((x**3-1)/(-1+x),x)
```

output
```
x**3/3 + x**2/2 + x
```

3.487.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+x^3}{-1+x}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + x$$

input
```
integrate((x^3-1)/(-1+x),x, algorithm="maxima")
```

output
```
1/3*x^3 + 1/2*x^2 + x
```

3.487.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+x^3}{-1+x}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + x$$

input
```
integrate((x^3-1)/(-1+x),x, algorithm="giac")
```

output
```
1/3*x^3 + 1/2*x^2 + x
```

3.487.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{-1+x^3}{-1+x}\,dx = \frac{x\,(2\,x^2+3\,x+6)}{6}$$

input
```
int((x^3 - 1)/(x - 1),x)
```

output
```
(x*(3*x + 2*x^2 + 6))/6
```

3.488 $\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx$

3.488.1 Optimal result

Integrand size = 18, antiderivative size = 17

$$\int \frac{2+2x}{(-1+x)^3\,(1+x^2)}\,dx = -\frac{1}{(1-x)^2}+\frac{1}{-1+x}+\arctan(x)$$

output
```
-1/(1-x)^2+1/(-1+x)+arctan(x)
```

3.488.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{2+2x}{(-1+x)^3\,(1+x^2)}\,dx = \frac{-2+x+(-1+x)^2\arctan(x)}{(-1+x)^2}$$

input
```
Integrate[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]
```

output
```
(-2 + x + (-1 + x)^2*ArcTan[x])/(-1 + x)^2
```

3.488.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+2}{(x-1)^3 (x^2+1)} \, dx$$

$$\Big\downarrow \ 657$$

$$\int \left(\frac{1}{x^2+1}-\frac{1}{(x-1)^2}+\frac{2}{(x-1)^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\arctan(x)+\frac{1}{x-1}-\frac{1}{(1-x)^2}$$

input
```
Int[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]
```

output
```
-(1 - x)^(-2) + (-1 + x)^(-1) + ArcTan[x]
```

3.488.3.1 Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.488.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{x-2}{(x-1)^{2}}+\arctan\left(x\right)$	13
default	$\arctan\left(x\right)-\frac{1}{(x-1)^{2}}+\frac{1}{x-1}$	16
parallelrisch	$-\frac{i\ln(x-i)x^{2}-i\ln(x+i)x^{2}-2i\ln(x-i)x+2i\ln(x+i)x+3+i\ln(x-i)-i\ln(x+i)-x^{2}}{2(x-1)^{2}}$	71

input
```
int((2*x+2)/(x-1)^3/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
(x-2)/(x-1)^2+arctan(x)
```

3.488.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \frac{2+2x}{(-1+x)^3\,(1+x^2)}\,dx=\frac{(x^2-2\,x+1)\arctan\left(x\right)+x-2}{x^2-2\,x+1}$$

input
```
integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="fricas")
```

output
```
((x^2 - 2*x + 1)*arctan(x) + x - 2)/(x^2 - 2*x + 1)
```

3.488.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{2+2x}{(-1+x)^3\,(1+x^2)}\,dx=\frac{x-2}{x^2-2x+1}+\operatorname{atan}\left(x\right)$$

input
```
integrate((2+2*x)/(-1+x)**3/(x**2+1),x)
```

output
```
(x - 2)/(x**2 - 2*x + 1) + atan(x)
```

3.488.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx = \frac{x-2}{x^2-2\,x+1} + \arctan(x)$$

input
```
integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="maxima")
```

output
```
(x - 2)/(x^2 - 2*x + 1) + arctan(x)
```

3.488.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx = \frac{x-2}{(x-1)^2} + \arctan(x)$$

input
```
integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="giac")
```

output
```
(x - 2)/(x - 1)^2 + arctan(x)
```

3.488.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx = \operatorname{atan}(x) + \frac{x-2}{x^2-2\,x+1}$$

input
```
int((2*x + 2)/((x^2 + 1)*(x - 1)^3),x)
```

output
```
atan(x) + (x - 2)/(x^2 - 2*x + 1)
```

3.489 $\int \frac{1}{bx+c(d+ex)^2} dx$

3.489.1 Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{1}{bx+c(d+ex)^2} dx = -\frac{2\text{arctanh}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

output
```
-2*arctanh((b+2*c*e*(e*x+d))/b^(1/2)/(4*c*d*e+b)^(1/2))/b^(1/2)/(4*c*d*e+b
)^(1/2)
```

3.489.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{1}{bx+c(d+ex)^2} dx = -\frac{2\text{arctanh}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

input
```
Integrate[(b*x + c*(d + e*x)^2)^(-1),x]
```

output
```
(-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(Sqrt[b]*S
qrt[b + 4*c*d*e])
```

3.489.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2080, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{bx+c(d+ex)^2}dx$$

$$\Big\downarrow \text{2080}$$

$$\int \frac{1}{x(b+2cde)+cd^2+ce^2x^2}dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{b(b+4cde)-(2cxe^2+2cde+b)^2}d(2cxe^2+2cde+b)$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{b+2cde+2ce^2x}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

input
```
Int[(b*x + c*(d + e*x)^2)^(-1),x]
```

output
```
(-2*ArcTanh[(b + 2*c*d*e + 2*c*e^2*x)/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(Sqrt[
b]*Sqrt[b + 4*c*d*e])
```

3.489.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2080
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] && !QuadraticMatchQ[u, x]
```

3.489.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{2c\,e^{2}x+2dce+b}{\sqrt{4bcde+b^{2}}}\right)}{\sqrt{4bcde+b^{2}}}$	43
risch	$\frac{\ln\left(-2c\,e^{2}x-2dce+\sqrt{b(4dce+b)}-b\right)}{\sqrt{b(4dce+b)}}-\frac{\ln\left(2c\,e^{2}x+2dce+\sqrt{b(4dce+b)}+b\right)}{\sqrt{b(4dce+b)}}$	81

input
```
int(1/(b*x+c*(e*x+d)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/(4*b*c*d*e+b^2)^(1/2)*arctanh((2*c*e^2*x+2*c*d*e+b)/(4*b*c*d*e+b^2)^(1/
2))
```

3.489.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 4.04

$$\int \frac{1}{bx+c(d+ex)^2}\,dx = \left[\frac{\log\left(\frac{2\,c^2e^4x^2+2\,c^2d^2e^2+4\,bcde+b^2+2\,(2\,c^2de^3+bce^2)x-\sqrt{4\,bcde+b^2}\,(2\,ce^2x+2\,cde+b)}{ce^2x^2+cd^2+(2\,cde+b)x}\right)}{\sqrt{4\,bcde+b^2}}, \frac{2\,\sqrt{-4\,bcde-b^2}\,\arctan\left(\frac{\sqrt{-4\,bcde-b^2}}{4\,bc}\right.}{4\,bcde+b^2}\right.$$

input
```
integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="fricas")
```

output
```
[log((2*c^2*e^4*x^2 + 2*c^2*d^2*e^2 + 4*b*c*d*e + b^2 + 2*(2*c^2*d*e^3 + b
*c*e^2)*x - sqrt(4*b*c*d*e + b^2)*(2*c*e^2*x + 2*c*d*e + b))/(c*e^2*x^2 +
c*d^2 + (2*c*d*e + b)*x))/sqrt(4*b*c*d*e + b^2), 2*sqrt(-4*b*c*d*e - b^2)*
arctan(sqrt(-4*b*c*d*e - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e + b^2))
/(4*b*c*d*e + b^2)]
```

3.489.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(49) = 98$.

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.21

$$\int \frac{1}{bx + c(d + ex)^2} dx = \sqrt{\frac{1}{b(b + 4cde)}} \log\left(x + \frac{-b^2\sqrt{\frac{1}{b(b+4cde)}} - 4bcde\sqrt{\frac{1}{b(b+4cde)}} + b + 2cde}{2ce^2}\right) - \sqrt{\frac{1}{b(b + 4cde)}} \log\left(x + \frac{b^2\sqrt{\frac{1}{b(b+4cde)}} + 4bcde\sqrt{\frac{1}{b(b+4cde)}} + b + 2cde}{2ce^2}\right)$$

input
```
integrate(1/(b*x+c*(e*x+d)**2),x)
```

output
```
sqrt(1/(b*(b + 4*c*d*e)))*log(x + (-b**2*sqrt(1/(b*(b + 4*c*d*e))) - 4*b*c
*d*e*sqrt(1/(b*(b + 4*c*d*e))) + b + 2*c*d*e)/(2*c*e**2)) - sqrt(1/(b*(b +
 4*c*d*e)))*log(x + (b**2*sqrt(1/(b*(b + 4*c*d*e))) + 4*b*c*d*e*sqrt(1/(b*
(b + 4*c*d*e))) + b + 2*c*d*e)/(2*c*e**2))
```

3.489.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{bx + c(d + ex)^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c*d*e+b>0)', see `assume?` for
 more deta
```

3.489.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{1}{bx + c(d + ex)^2}\,dx = \frac{2\,\arctan\left(\frac{2\,ce^2x+2\,cde+b}{\sqrt{-4\,bcde-b^2}}\right)}{\sqrt{-4\,bcde - b^2}}$$

input
```
integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="giac")
```

output
```
2*arctan((2*c*e^2*x + 2*c*d*e + b)/sqrt(-4*b*c*d*e - b^2))/sqrt(-4*b*c*d*e
 - b^2)
```

3.489.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{1}{bx + c(d + ex)^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{2\,c\,x\,e^2+2\,c\,d\,e+b}{\sqrt{b}\,\sqrt{b+4\,c\,d\,e}}\right)}{\sqrt{b}\,\sqrt{b + 4\,c\,d\,e}}$$

input
```
int(1/(c*(d + e*x)^2 + b*x),x)
```

output
```
-(2*atanh((b + 2*c*d*e + 2*c*e^2*x)/(b^(1/2)*(b + 4*c*d*e)^(1/2))))/(b^(1/
2)*(b + 4*c*d*e)^(1/2))
```

3.490 $\int \frac{1}{a+bx+c(d+ex)^2} dx$

3.490.1 Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{a+bx+c(d+ex)^2} dx = -\frac{2\text{arctanh}\left(\frac{b+2ce(d+ex)}{\sqrt{b^2+4bcde-4ace^2}}\right)}{\sqrt{b^2+4bcde-4ace^2}}$$

output
```
-2*arctanh((b+2*c*e*(e*x+d))/(-4*a*c*e^2+4*b*c*d*e+b^2)^(1/2))/(-4*a*c*e^2
+4*b*c*d*e+b^2)^(1/2)
```

3.490.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{1}{a+bx+c(d+ex)^2} dx = \frac{2\arctan\left(\frac{b+2ce(d+ex)}{\sqrt{-b^2-4bcde+4ace^2}}\right)}{\sqrt{-b^2-4bcde+4ace^2}}$$

input
```
Integrate[(a + b*x + c*(d + e*x)^2)^(-1),x]
```

output
```
(2*ArcTan[(b + 2*c*e*(d + e*x))/Sqrt[-b^2 - 4*b*c*d*e + 4*a*c*e^2]])/Sqrt[
-b^2 - 4*b*c*d*e + 4*a*c*e^2]
```

3.490.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2080, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx+c(d+ex)^2}dx$$

$$\Big\downarrow \text{2080}$$

$$\int \frac{1}{a+x(b+2cde)+cd^2+ce^2x^2}dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{b^2+4cdeb-4ace^2-(2cxe^2+2cde+b)^2}d(2cxe^2+2cde+b)$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{b+2cde+2ce^2x}{\sqrt{-4ace^2+b^2+4bcde}}\right)}{\sqrt{-4ace^2+b^2+4bcde}}$$

input
```
Int[(a + b*x + c*(d + e*x)^2)^(-1),x]
```

output
```
(-2*ArcTanh[(b + 2*c*d*e + 2*c*e^2*x)/Sqrt[b^2 + 4*b*c*d*e - 4*a*c*e^2]])/
Sqrt[b^2 + 4*b*c*d*e - 4*a*c*e^2]
```

3.490.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2080
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

3.490.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{2\arctan\left(\frac{2c\,e^2x+2dce+b}{\sqrt{4e^2ac-4bcde-b^2}}\right)}{\sqrt{4e^2ac-4bcde-b^2}}$	61
risch	$-\frac{\ln\left(2c\,e^2x+2dce+\sqrt{-4e^2ac+4bcde+b^2}+b\right)}{\sqrt{-4e^2ac+4bcde+b^2}}+\frac{\ln\left(-2c\,e^2x-2dce+\sqrt{-4e^2ac+4bcde+b^2}-b\right)}{\sqrt{-4e^2ac+4bcde+b^2}}$	113

input
```
int(1/(a+b*x+c*(e*x+d)^2),x,method=_RETURNVERBOSE)
```

output
```
2/(4*a*c*e^2-4*b*c*d*e-b^2)^(1/2)*arctan((2*c*e^2*x+2*c*d*e+b)/(4*a*c*e^2-
4*b*c*d*e-b^2)^(1/2))
```

3.490.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.21

$$\int \frac{1}{a+bx+c(d+ex)^2}\,dx = \left[\frac{\log\left(\frac{2\,c^2e^4x^2+4\,bcde+2\,(c^2d^2-ac)e^2+b^2+2\,(2\,c^2de^3+bce^2)x-\sqrt{4\,bcde-4\,ace^2+b^2}\,(2\,ce^2x+2\,cde+b)}{ce^2x^2+cd^2+(2\,cde+b)x+a}\right)}{\sqrt{4\,bcde-4\,ace^2+b^2}}, -\frac{2\sqrt{-4\,bcde+4\,ace^2-b^2}\arctan\left(-\frac{\sqrt{-4\,bcde+4\,ace^2-b^2}\,(2\,ce^2x+2\,cde+b)}{4\,bcde-4\,ace^2+b^2}\right)}{4\,bcde-4\,ace^2+b^2}\right]$$

input
```
integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="fricas")
```

output
```
[log((2*c^2*e^4*x^2 + 4*b*c*d*e + 2*(c^2*d^2 - a*c)*e^2 + b^2 + 2*(2*c^2*d
*e^3 + b*c*e^2)*x - sqrt(4*b*c*d*e - 4*a*c*e^2 + b^2)*(2*c*e^2*x + 2*c*d*e
 + b))/(c*e^2*x^2 + c*d^2 + (2*c*d*e + b)*x + a))/sqrt(4*b*c*d*e - 4*a*c*e
^2 + b^2), -2*sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2)*arctan(-sqrt(-4*b*c*d*e +
 4*a*c*e^2 - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e - 4*a*c*e^2 + b^2))
/(4*b*c*d*e - 4*a*c*e^2 + b^2)]
```

3.490.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(63) = 126$.

Time = 0.18 (sec) , antiderivative size = 294, normalized size of antiderivative = 5.16

$$\int \frac{1}{a+bx+c(d+ex)^2}\,dx =$$

$$-\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}\log\left(x+\frac{-4ace^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+b^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+4bcde\sqrt{-\frac{1}{4ace^2-b^2-4}}}{2ce^2}\right.$$

$$+\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}\log\left(x+\frac{4ace^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}-b^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}-4bcde\sqrt{-\frac{1}{4ace^2-b^2-4bcd}}}{2ce^2}\right.$$

input
```
integrate(1/(a+b*x+c*(e*x+d)**2),x)
```

output
```
-sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*log(x + (-4*a*c*e**2*sqrt(-1/(4*
a*c*e**2 - b**2 - 4*b*c*d*e)) + b**2*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*
e)) + 4*b*c*d*e*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2*c*d*e)/(2
*c*e**2)) + sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*log(x + (4*a*c*e**2*s
qrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) - b**2*sqrt(-1/(4*a*c*e**2 - b**2
- 4*b*c*d*e)) - 4*b*c*d*e*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2
*c*d*e)/(2*c*e**2))
```

3.490.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a + bx + c(d + ex)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c*e^2>0)', see `assume?` for
 more deta
```

3.490.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{1}{a + bx + c(d + ex)^2} \, dx = \frac{2 \arctan\left(\frac{2 ce^2x+2 cde+b}{\sqrt{-4 bcde+4 ace^2-b^2}}\right)}{\sqrt{-4 bcde + 4 ace^2 - b^2}}$$

input
```
integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="giac")
```

output
```
2*arctan((2*c*e^2*x + 2*c*d*e + b)/sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2))/sqr
t(-4*b*c*d*e + 4*a*c*e^2 - b^2)
```

3.490.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.44

$$\int \frac{1}{a + bx + c(d + ex)^2} \, dx = \frac{2 \operatorname{atan}\left(\frac{b+2 c d e}{\sqrt{-b^2-4 c d b e+4 a c e^2}} + \frac{2 c e^2 x}{\sqrt{-b^2-4 c d b e+4 a c e^2}}\right)}{\sqrt{-b^2 - 4 c d b e + 4 a c e^2}}$$

input
```
int(1/(a + c*(d + e*x)^2 + b*x),x)
```

output
```
(2*atan((b + 2*c*d*e)/(4*a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2) + (2*c*e^2*x)/(4
*a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2)))/(4*a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2)
```

3.491 $\int \frac{x^2}{1+(-1+x^2)^2} \, dx$

3.491.1 Optimal result

Integrand size = 15, antiderivative size = 188

$$\begin{aligned}\int \frac{x^2}{1+(-1+x^2)^2} \, dx = & -\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)} \arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right) \\ & +\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)} \arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right) \\ & +\frac{\log\left(\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} \\ & -\frac{\log\left(\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}\end{aligned}$$

output
```
-1/4*arctan((-2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))
^(1/2)+1/4*arctan((2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(
1/2))^(1/2)+1/4*ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)-
1/4*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)
```

3.491.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.21

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = -\frac{\arctan\left(\frac{x}{\sqrt{-1-i}}\right)}{(-1-i)^{3/2}} - \frac{\arctan\left(\frac{x}{\sqrt{-1+i}}\right)}{(-1+i)^{3/2}}$$

input
```
Integrate[x^2/(1 + (-1 + x^2)^2),x]
```

output
```
-(ArcTan[x/Sqrt[-1 - I]]/(-1 - I)^(3/2)) - ArcTan[x/Sqrt[-1 + I]]/(-1 + I)
^(3/2)
```

3.491.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2086, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^2-1)^2+1}\,dx$$

$$\Big\downarrow \text{2086}$$

$$\int \frac{x^2}{x^4-2x^2+2}dx$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2}\int \frac{x^2+\sqrt{2}}{x^4-2x^2+2}dx - \frac{1}{2}\int \frac{\sqrt{2}-x^2}{x^4-2x^2+2}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx + \frac{1}{2}\int \frac{1}{x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx\right) - \frac{1}{2}\int \frac{\sqrt{2}-x^2}{x^4-2x^2+2}dx$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(-\int \frac{1}{2\left(1-\sqrt{2}\right)-\left(2x-\sqrt{2\left(1+\sqrt{2}\right)}\right)^2}d\left(2x-\sqrt{2\left(1+\sqrt{2}\right)}\right)-\int \frac{1}{2\left(1-\sqrt{2}\right)-\left(2x+\sqrt{2\left(1+\sqrt{2}\right)}\right)^2}\right.$$

$$\frac{1}{2}\int \frac{\sqrt{2}-x^2}{x^4-2x^2+2}dx$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\arctan\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{2}\int \frac{\sqrt{2}-x^2}{x^4-2x^2+2}dx$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx}{2\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\int -\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx}{2\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\arctan\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx}{2\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\int \frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx}{2\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\arctan\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

3.491. $\int \frac{x^2}{1+\left(-1+x^2\right)^2}\,dx$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\arctan\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}\right)$$

input
```
Int[x^2/(1 + (-1 + x^2)^2),x]
```

output
```
(ArcTan[(-Sqrt[2*(1 + Sqrt[2])] + 2*x)/Sqrt[2*(-1 + Sqrt[2])]]/Sqrt[2*(-1
+ Sqrt[2])] + ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*x)/Sqrt[2*(-1 + Sqrt[2])]]
/Sqrt[2*(-1 + Sqrt[2])])/2 + (Log[Sqrt[2] - Sqrt[2*(1 + Sqrt[2])]*x + x^2]
/(2*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/
(2*Sqrt[2*(1 + Sqrt[2])]))/2
```

3.491.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.491. $\int \frac{x^2}{1+(-1+x^2)^2}\,dx$

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 2086
```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

3.491.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.19

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-2\_Z^2+2\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^3-\_R}\right)}{4}$
default	$-\frac{\sqrt{2+2\sqrt{2}}\left(\sqrt{2}-1\right)\left(\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{2+2\sqrt{2}}\right)}{2}-\frac{\sqrt{2+2\sqrt{2}}\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}\right)}{4}+\frac{\sqrt{2+2\sqrt{2}}\left(\sqrt{2}-1\right)\left(\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{2+2\sqrt{2}}\right)}{2}+\ldots\right.}{4}$

input
```
int(x^2/(1+(x^2-1)^2),x,method=_RETURNVERBOSE)
```

3.491. $\int \frac{x^2}{1+(-1+x^2)^2}\,dx$

output
```
1/4*sum(_R^2/(_R^3-_R)*ln(x-_R),_R=RootOf(_Z^4-2*_Z^2+2))
```

3.491.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.28

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = \frac{1}{4}\sqrt{i-1}\log\left(x+i\sqrt{i-1}\right)-\frac{1}{4}\sqrt{i-1}\log\left(x-i\sqrt{i-1}\right) - \frac{1}{4}\sqrt{-i-1}\log\left(x+i\sqrt{-i-1}\right)+\frac{1}{4}\sqrt{-i-1}\log\left(x-i\sqrt{-i-1}\right)$$

input
```
integrate(x^2/(1+(x^2-1)^2),x, algorithm="fricas")
```

output
```
1/4*sqrt(I - 1)*log(x + I*sqrt(I - 1)) - 1/4*sqrt(I - 1)*log(x - I*sqrt(I
- 1)) - 1/4*sqrt(-I - 1)*log(x + I*sqrt(-I - 1)) + 1/4*sqrt(-I - 1)*log(x
- I*sqrt(-I - 1))
```

3.491.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.13

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = \operatorname{RootSum}\left(128t^4+16t^2+1,\left(t\mapsto t\log\left(64t^3+4t+x\right)\right)\right)$$

input
```
integrate(x**2/(1+(x**2-1)**2),x)
```

output
```
RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*log(64*_t**3 + 4*_t + x)))
```

3.491.7 Maxima [F]

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = \int \frac{x^2}{(x^2-1)^2+1}\,dx$$

input
```
integrate(x^2/(1+(x^2-1)^2),x, algorithm="maxima")
```

output
```
integrate(x^2/((x^2 - 1)^2 + 1), x)
```

3.491.8 Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.78

$$\begin{aligned}\int \frac{x^2}{1+(-1+x^2)^2}\,dx &= \frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x+2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\sqrt{-\sqrt{2}+2}}\right)\\ &\quad+\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\sqrt{-\sqrt{2}+2}}\right)\\ &\quad-\frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(x^2+2^{\frac{1}{4}}x\sqrt{\sqrt{2}+2}+\sqrt{2}\right)\\ &\quad+\frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(x^2-2^{\frac{1}{4}}x\sqrt{\sqrt{2}+2}+\sqrt{2}\right)\end{aligned}$$

input
```
integrate(x^2/(1+(x^2-1)^2),x, algorithm="giac")
```

output
```
1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x + 2^(1/4)*sqrt(sqrt(2) + 2
))/sqrt(-sqrt(2) + 2)) + 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x -
 2^(1/4)*sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2*sqrt(2) - 2)*
log(x^2 + 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2)) + 1/8*sqrt(2*sqrt(2) - 2)
*log(x^2 - 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2))
```

3.491.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.54

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = \operatorname{atanh}\left(32\,x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)+\operatorname{atanh}\left(32\,x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)$$

input
```
int(x^2/((x^2 - 1)^2 + 1),x)
```

output
```
atanh(32*x*((- 2^(1/2)/32 - 1/32)^(1/2) + (2^(1/2)/32 - 1/32)^(1/2))^3)*(2
*(- 2^(1/2)/32 - 1/32)^(1/2) + 2*(2^(1/2)/32 - 1/32)^(1/2)) + atanh(32*x*(
(- 2^(1/2)/32 - 1/32)^(1/2) - (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*(- 2^(1/2)/
32 - 1/32)^(1/2) - 2*(2^(1/2)/32 - 1/32)^(1/2))
```

3.492 $\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4} \, dx$

3.492.1 Optimal result

Integrand size = 50, antiderivative size = 60

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4} \, dx = \frac{2}{(3+x+x^4)^3} - \frac{3x}{(3+x+x^4)^3} + \frac{5x^2}{(3+x+x^4)^3} + \frac{x^4}{(3+x+x^4)^3} - \frac{5x^6}{(3+x+x^4)^3}$$

output
```
2/(x^4+x+3)^3-3*x/(x^4+x+3)^3+5*x^2/(x^4+x+3)^3+x^4/(x^4+x+3)^3-5*x^6/(x^4
+x+3)^3
```

3.492.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.45

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4} \, dx = \frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}$$

input
```
Integrate[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^
7 - 30*x^9)/(3 + x + x^4)^4),x]
```

output
```
(2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

3.492.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {25, 2527, 27, 2527, 27, 2527, 27, 2527, 27, 2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{-30x^9+8x^7+15x^6+140x^5-34x^4+12x^3+5x^2-36x+15}{(x^4+x+3)^4}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{-30x^9+8x^7+15x^6+140x^5-34x^4+12x^3+5x^2-36x+15}{(x^4+x+3)^4}dx$$

$$\Big\downarrow \ 2527$$

$$\frac{1}{6}\int -\frac{6\left(8x^7+50x^5-34x^4+12x^3+5x^2-36x+15\right)}{(x^4+x+3)^4}dx-\frac{5x^6}{(x^4+x+3)^3}$$

$$\Big\downarrow \ 27$$

$$-\int \frac{8x^7+50x^5-34x^4+12x^3+5x^2-36x+15}{(x^4+x+3)^4}dx-\frac{5x^6}{(x^4+x+3)^3}$$

$$\Big\downarrow \ 2527$$

$$\frac{1}{8}\int -\frac{8\left(50x^5-33x^4+24x^3+5x^2-36x+15\right)}{(x^4+x+3)^4}dx+\frac{x^4}{(x^4+x+3)^3}-\frac{5x^6}{(x^4+x+3)^3}$$

$$\Big\downarrow \ 27$$

$$-\int \frac{50x^5-33x^4+24x^3+5x^2-36x+15}{(x^4+x+3)^4}dx+\frac{x^4}{(x^4+x+3)^3}-\frac{5x^6}{(x^4+x+3)^3}$$

$$\Big\downarrow \ 2527$$

$$\frac{1}{10}\int -\frac{30\left(-11x^4+8x^3-2x+5\right)}{(x^4+x+3)^4}dx+\frac{x^4}{(x^4+x+3)^3}-\frac{5x^6}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}$$

$$\Big\downarrow \ 27$$

$$-3\int \frac{-11x^4+8x^3-2x+5}{(x^4+x+3)^4}dx+\frac{x^4}{(x^4+x+3)^3}-\frac{5x^6}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}$$

$$\Big\downarrow \ 2527$$

$$-3\left(\frac{x}{(x^4+x+3)^3}-\frac{1}{11}\int -\frac{22\left(4x^3+1\right)}{(x^4+x+3)^4}dx\right)+\frac{x^4}{(x^4+x+3)^3}-\frac{5x^6}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}$$

$$\Big\downarrow \text{27}$$

$$-3\left(2\int \frac{4x^3+1}{(x^4+x+3)^4}dx+\frac{x}{(x^4+x+3)^3}\right)+\frac{x^4}{(x^4+x+3)^3}-\frac{5x^6}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}$$

$$\Big\downarrow \text{2021}$$

$$\frac{x^4}{(x^4+x+3)^3}-3\left(\frac{x}{(x^4+x+3)^3}-\frac{2}{3\,(x^4+x+3)^3}\right)-\frac{5x^6}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}$$

input
```
Int[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30
*x^9)/(3 + x + x^4)^4),x]
```

output
```
(5*x^2)/(3 + x + x^4)^3 + x^4/(3 + x + x^4)^3 - (5*x^6)/(3 + x + x^4)^3 -
3*(-2/(3*(3 + x + x^4)^3) + x/(3 + x + x^4)^3)
```

3.492.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

rule 2527
```
Int[(Pm_)*(Qn_)^(p_.), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x
]}, Simp[Coeff[Pm, x, m]*x^(m - n + 1)*(Qn^(p + 1)/((m + n*p + 1)*Coeff[Qn,
 x, n])), x] + Simp[1/((m + n*p + 1)*Coeff[Qn, x, n])   Int[ExpandToSum[(m
+ n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn +
 (p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 <
0] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]
```

3.492. $\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx$

3.492.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{-5x^{6}+x^{4}+5x^{2}-3x+2}{\left(x^{4}+x+3\right)^{3}}$	28
norman	$\frac{-5x^{6}+x^{4}+5x^{2}-3x+2}{\left(x^{4}+x+3\right)^{3}}$	28
risch	$\frac{-5x^{6}+x^{4}+5x^{2}-3x+2}{\left(x^{4}+x+3\right)^{3}}$	28
parallelrisch	$\frac{-5x^{6}+x^{4}+5x^{2}-3x+2}{\left(x^{4}+x+3\right)^{3}}$	28
gosper	$-\frac{5x^{6}-x^{4}-5x^{2}+3x-2}{\left(x^{4}+x+3\right)^{3}}$	31

input
```
int((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+3)^4,
x,method=_RETURNVERBOSE)
```

output
```
(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

3.492.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx = -\frac{5\,x^{6}-x^{4}-5\,x^{2}+3\,x-2}{x^{12}+3\,x^{9}+9\,x^{8}+3\,x^{6}+18\,x^{5}+27\,x^{4}+x^{3}+9\,x^{2}+27\,x+27}$$

input
```
integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x
+3)^4,x, algorithm="fricas")
```

output
```
-(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 +
27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

3.492.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx = \frac{-5x^6+x^4+5x^2-3x+2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

input
```
integrate((30*x**9-8*x**7-15*x**6-140*x**5+34*x**4-12*x**3-5*x**2+36*x-15)
/(x**4+x+3)**4,x)
```

output
```
(-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18
*x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

3.492.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx = -\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

input
```
integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x
+3)^4,x, algorithm="maxima")
```

output
```
-(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 +
27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

3.492.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.50

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx = -\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{(x^4+x+3)^3}$$

input
```
integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x
+3)^4,x, algorithm="giac")
```

output
```
-(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^4 + x + 3)^3
```

3.492.9 Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.45

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx$$
$$=\frac{-5\,x^6+x^4+5\,x^2-3\,x+2}{{\left(x^4+x+3\right)}^3}$$

input
```
int(-(5*x^2 - 36*x + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*x^9 +
 15)/(x + x^4 + 3)^4,x)
```

output
```
(5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

3.493 $\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x}\right.$

3.493.1 Optimal result

Integrand size = 61, antiderivative size = 27

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right) dx = \frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}$$

output
```
(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

3.493.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right) dx = \frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}$$

input
```
Integrate[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320
*x - 75*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]
```

output
```
(2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

3.493.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{30x}{(x^4+x+3)^2}+\frac{-8x^3-75x^2-320x+42}{(x^4+x+3)^3}+\frac{3(19x^3+120x^2+228x-47)}{(x^4+x+3)^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{621}{4}\int \frac{1}{(x^4+x+3)^4}dx+684\int \frac{x}{(x^4+x+3)^4}dx+44\int \frac{1}{(x^4+x+3)^3}dx-320\int \frac{x}{(x^4+x+3)^3}dx+30\int \frac{x}{(x^4+x+3)^2}dx+360\int \frac{x^2}{(x^4+x+3)^4}dx-75\int \frac{x^2}{(x^4+x+3)^3}dx+\frac{1}{(x^4+x+3)^2}-\frac{19}{4(x^4+x+3)^3}$$

input
```
Int[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*x - 7
5*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]
```

output
```
$Aborted
```

3.493.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.493.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result
norman	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$
parallelrisch	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$
gosper	$-\frac{5x^6-x^4-5x^2+3x-2}{(x^4+x+3)^3}$
default	$\frac{\frac{377432}{195075}x^7-\frac{1404328}{195075}x^6+\frac{234517}{195075}x^5+\frac{660506}{195075}x^4-\frac{208792}{195075}x^3-\frac{13339729}{390150}x^2+\frac{89881}{13005}x+\frac{121303}{21675}}{(x^4+x+3)^2}+\frac{\left(\sum_{\_R=\text{RootOf}(\_Z^4+\_Z+3)}\frac{(377432}{}\right.}{}$
risch	$\frac{\frac{377432}{195075}x^7-\frac{1404328}{195075}x^6+\frac{234517}{195075}x^5+\frac{660506}{195075}x^4-\frac{208792}{195075}x^3-\frac{13339729}{390150}x^2+\frac{89881}{13005}x+\frac{121303}{21675}}{(x^4+x+3)^2}+\frac{\left(\sum_{\_R=\text{RootOf}(\_Z^4+\_Z+3)}\frac{(377432}{}\right.}{}$

input
```
int(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/(x^4+
x+3)^3+30*x/(x^4+x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

3.493.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(30) = 60.

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int\left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)dx$$
$$=-\frac{5x^6-x^4-5x^2+3x-2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

input
```
integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)
/(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="fricas")
```

output
```
-(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 +
27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

3.493.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(26) = 52.

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)dx$$
$$=\frac{-5x^6+x^4+5x^2-3x+2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

input
```
integrate(3*(19*x**3+120*x**2+228*x-47)/(x**4+x+3)**4+(-8*x**3-75*x**2-320
*x+42)/(x**4+x+3)**3+30*x/(x**4+x+3)**2,x)
```

output
```
(-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18
*x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

3.493.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(30) = 60.

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)dx$$
$$=-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

input
```
integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)
/(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="maxima")
```

output
```
-(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 +
27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

3.493.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 7.30

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)dx$$
$$=\frac{1}{195075}x\left(\frac{377432\,x^2-2808656\,x+703551}{x^4+x+3}-\frac{255032\,x^2-1829456\,x+680601}{x^4+x+3}-\frac{7650\,(16\,x^2-128\,x+}{x^4+x+3}\right.$$
$$-\frac{2\,(16\,x^3-64\,x^2+x+12)}{51\,(x^4+x+3)}$$
$$+\frac{754864\,x^7-2808656\,x^6+469034\,x^5+1321012\,x^4-417584\,x^3-13339729\,x^2+2696430\,x+2183454}{390150\,(x^4+x+3)^2}$$
$$-\frac{510064\,x^{11}-1829456\,x^{10}+453734\,x^9+1402676\,x^8-472048\,x^7-13501313\,x^6+4720744\,x^5+37475}{390150\,(x^4+x+3)^3}$$

input
```
integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)
/(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="giac")
```

output
```
1/195075*x*((377432*x^2 - 2808656*x + 703551)/(x^4 + x + 3) - (255032*x^2
- 1829456*x + 680601)/(x^4 + x + 3) - 7650*(16*x^2 - 128*x + 3)/(x^4 + x +
 3)) - 2/51*(16*x^3 - 64*x^2 + x + 12)/(x^4 + x + 3) + 1/390150*(754864*x^
7 - 2808656*x^6 + 469034*x^5 + 1321012*x^4 - 417584*x^3 - 13339729*x^2 + 2
696430*x + 2183454)/(x^4 + x + 3)^2 - 1/390150*(510064*x^11 - 1829456*x^10
 + 453734*x^9 + 1402676*x^8 - 472048*x^7 - 13501313*x^6 + 4720744*x^5 + 37
47556*x^4 - 10935781*x^3 - 30736107*x^2 + 10203894*x + 4117662)/(x^4 + x +
 3)^3
```

3.493.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)dx$$
$$=\frac{-5\,x^6+x^4+5\,x^2-3\,x+2}{(x^4+x+3)^3}$$

input
```
int((684*x + 360*x^2 + 57*x^3 - 141)/(x + x^4 + 3)^4 - (320*x + 75*x^2 + 8
*x^3 - 42)/(x + x^4 + 3)^3 + (30*x)/(x + x^4 + 3)^2,x)
```

output
```
(5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

3.493. $\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)\,dx$

3.494 $\int \left(\frac{-3+10x+4x^3-30x^5}{\left(3+x+x^4\right)^3}-\frac{3\left(1+4x^3\right)\left(2-3x+5x^2+x^4-5x^6\right)}{\left(3+x+x^4\right)^4}\right)\,dx$

3.494.1 Optimal result

Integrand size = 60, antiderivative size = 27

$$\int \left(\frac{-3+10x+4x^3-30x^5}{\left(3+x+x^4\right)^3}-\frac{3(1+4x^3)\left(2-3x+5x^2+x^4-5x^6\right)}{\left(3+x+x^4\right)^4}\right)\,dx = \frac{2-3x+5x^2+x^4-5x^6}{\left(3+x+x^4\right)^3}$$

output
```
(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

3.494.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(\frac{-3+10x+4x^3-30x^5}{\left(3+x+x^4\right)^3}-\frac{3(1+4x^3)\left(2-3x+5x^2+x^4-5x^6\right)}{\left(3+x+x^4\right)^4}\right)\,dx = \frac{2-3x+5x^2+x^4-5x^6}{\left(3+x+x^4\right)^3}$$

input
```
Integrate[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2
 - 3*x + 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4,x]
```

output
```
(2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

3.494.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{-30x^5+4x^3+10x-3}{(x^4+x+3)^3} - \frac{3(4x^3+1)(-5x^6+x^4+5x^2-3x+2)}{(x^4+x+3)^4} \right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{144}{11}\int \frac{1}{(x^4+x+3)^4}dx + \frac{828}{11}\int \frac{x}{(x^4+x+3)^4}dx - 4\int \frac{1}{(x^4+x+3)^3}dx - 20\int \frac{x}{(x^4+x+3)^3}dx + 18\int \frac{x^2}{(x^4+x+3)^4}dx + \frac{3x^4}{2(x^4+x+3)^3} - \frac{63x}{22(x^4+x+3)^3} - \frac{1}{2(x^4+x+3)^2} + \frac{7}{2(x^4+x+3)^3} - \frac{10x^6}{(x^4+x+3)^3} - \frac{5x^3}{(x^4+x+3)^3} + \frac{5x^2}{(x^4+x+3)^2} - \frac{12x^2}{(x^4+x+3)^3}$$

input
```
Int[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2 - 3*x
 + 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4,x]
```

output
```
$Aborted
```

3.494.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.494.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result
norman	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$
parallelrisch	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$
gosper	$-\frac{5x^6-x^4-5x^2+3x-2}{(x^4+x+3)^3}$
risch	$-\frac{5x^6-x^4-5x^2+3x-2}{(x^4+x+3)^3}$
default	$-\frac{-\frac{34568}{195075}x^7+\frac{73672}{195075}x^6+\frac{15392}{195075}x^5-\frac{60494}{195075}x^4-\frac{68792}{195075}x^3-\frac{583927}{195075}x^2+\frac{3356}{13005}x-\frac{2069}{43350}}{(x^4+x+3)^2} + \frac{-\frac{34568}{195075}x^{11}+\frac{73672}{195075}x^{10}+\frac{15392}{195075}x^9}{}$

input
```
int((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-3*x+2
)/(x^4+x+3)^4,x,method=_RETURNVERBOSE)
```

output
```
(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

3.494.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(30) = 60.

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right)\,dx$$
$$=-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

input
```
integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2
-3*x+2)/(x^4+x+3)^4,x, algorithm="fricas")
```

output
```
-(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 +
27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

3.494.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(26) = 52.

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right)\,dx$$
$$=\frac{-5x^6+x^4+5x^2-3x+2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

input
```
integrate((-30*x**5+4*x**3+10*x-3)/(x**4+x+3)**3-3*(4*x**3+1)*(-5*x**6+x**
4+5*x**2-3*x+2)/(x**4+x+3)**4,x)
```

output
```
(-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18
*x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

3.494.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3} - \frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right) dx = -\frac{5x^6-x^4-5x^2+3x-2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

input
```
integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2
-3*x+2)/(x^4+x+3)^4,x, algorithm="maxima")
```

output
```
-(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 +
27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

3.494.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(30) = 60$.

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.11

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3} - \frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right) dx = \frac{69136x^7-147344x^6-30784x^5+120988x^4+137584x^3+1167854x^2-100680x+18621}{390150(x^4+x+3)^2} - \frac{69136x^{11}-147344x^{10}-30784x^9+190124x^8+197648x^7+2645788x^6-72044x^5+129019x^4+15}{390150(x^4+x+3)^3}$$

input
```
integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2
-3*x+2)/(x^4+x+3)^4,x, algorithm="giac")
```

output
```
1/390150*(69136*x^7 - 147344*x^6 - 30784*x^5 + 120988*x^4 + 137584*x^3 + 1
167854*x^2 - 100680*x + 18621)/(x^4 + x + 3)^2 - 1/390150*(69136*x^11 - 14
7344*x^10 - 30784*x^9 + 190124*x^8 + 197648*x^7 + 2645788*x^6 - 72044*x^5
+ 129019*x^4 + 1580606*x^3 + 1452132*x^2 + 887031*x - 724437)/(x^4 + x + 3
)^3
```

3.494.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)\,(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right)\,dx$$
$$=\frac{-5\,x^6+x^4+5\,x^2-3\,x+2}{\left(x^4+x+3\right)^3}$$

input
```
int((10*x + 4*x^3 - 30*x^5 - 3)/(x + x^4 + 3)^3 - (3*(4*x^3 + 1)*(5*x^2 -
3*x + x^4 - 5*x^6 + 2))/(x + x^4 + 3)^4,x)
```

output
```
(5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```